Hadronic Vacuum Polarization measurement at Belle II

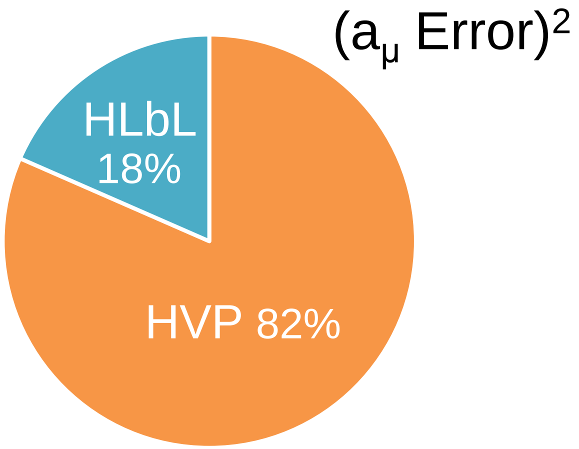
Yuki Sue
Nagoya University
On behalf of the Belle II collaboration

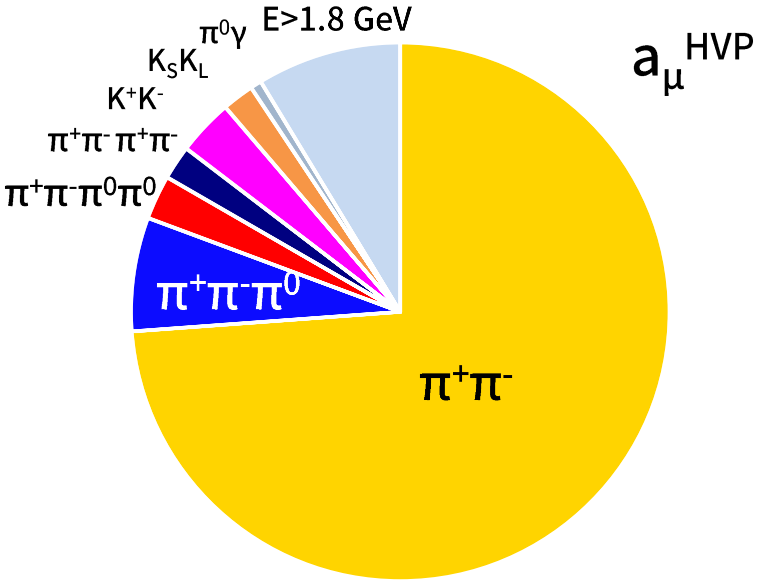
The Muon g-2 Theory Initiative Workshop, September 4th-8th , 2023

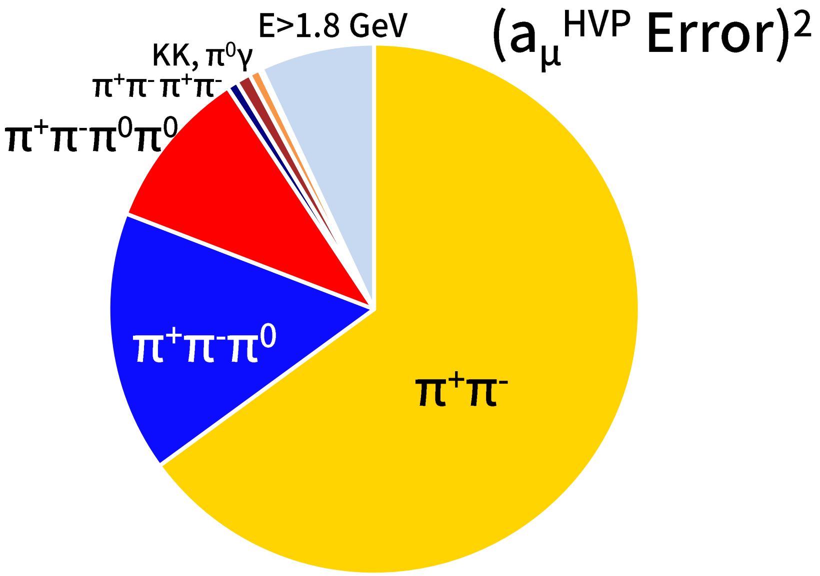
Muon g-2 and Hadronic Vacuum Polarization (HVP)

- HVP contributes to the largest uncertainty in the prediction of muon g-2.
- Two approaches for estimating the HVP contribution of SM predictions
  - Dispersion relations (w/ inputs from ee → hadrons data)
  - Lattice QCD
- Belle II can provide the cross section for $e^+e^- \to$ hadrons to improve the theoretical prediction.

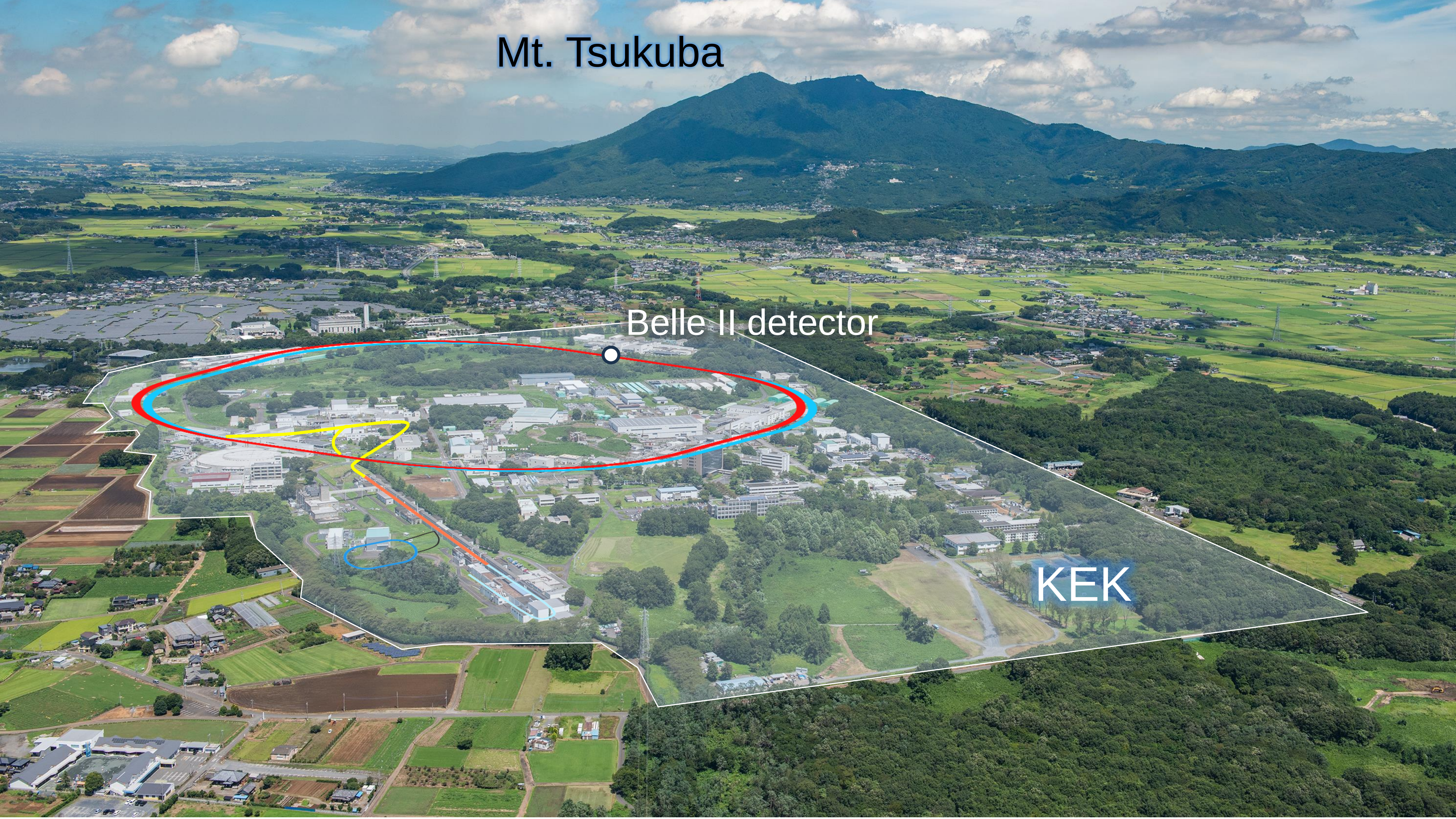
$$a_\mu^{\mathrm{SM}} = \frac{g-2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}}$$





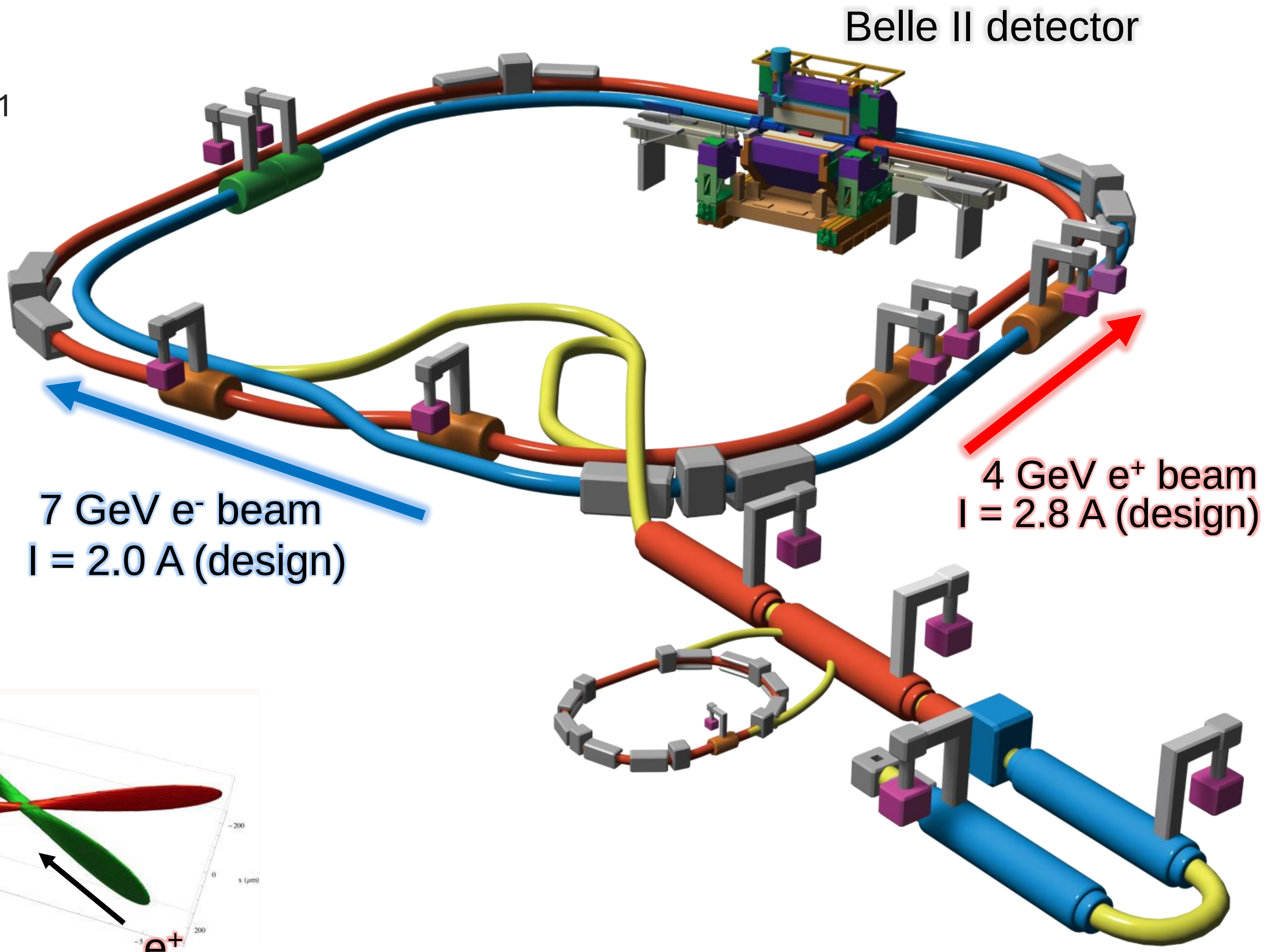


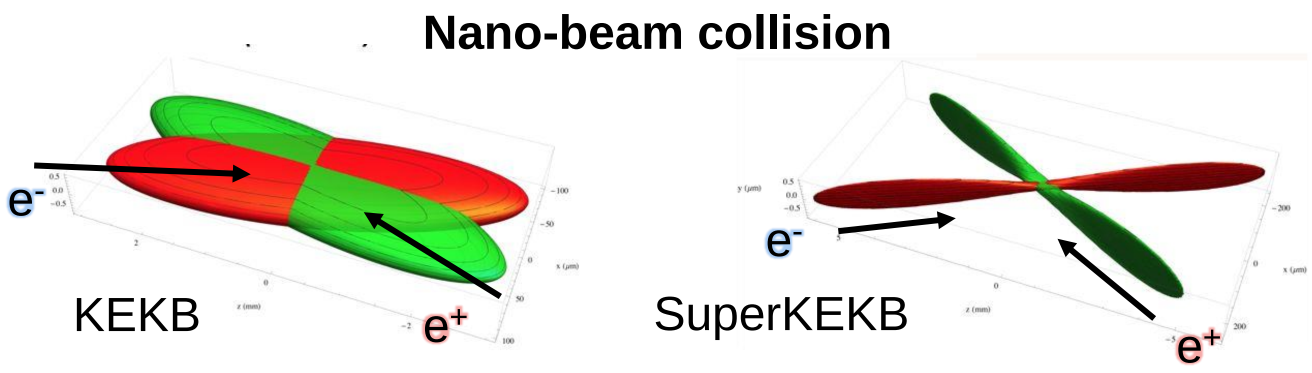
T. Aoyama *et al.*, Phys. Rept. 887 (2020).


Mt. Tsukuba
Belle II detector
KEK

SuperKEKB collider

- Asymmetric e^+e^- collider
  - $\sqrt{s}$ = M(Υ(4S)) = 10.58 GeV
  - Design luminosity : 6×10^{35} cm^{-2}s^{-1}
- Improvements from KEKB
  - Nano beam scheme
  - Higher beam currents



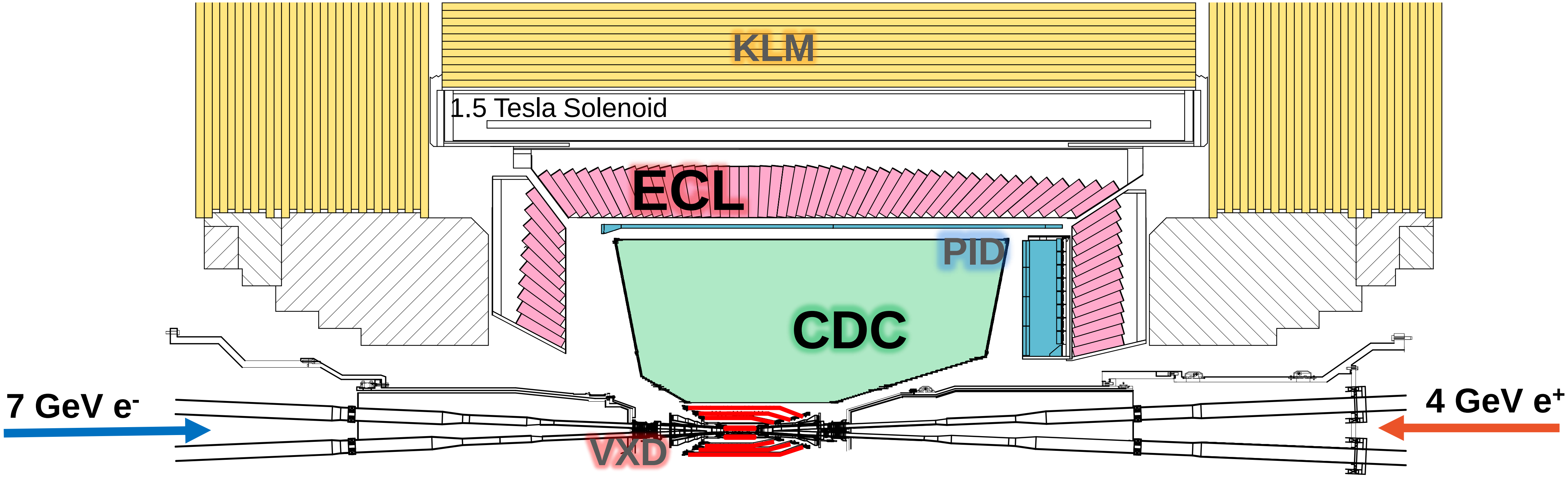


Belle II detector

Particle Identification
- Aerogel RICH in the forward endcap
- Time-of-Propagation counter in the barrel
- **K/π ID : K efficiency 90% at 1.8% π fake**

Electromagnetic Calorimeter (ECL)
- CsI(Tl) crystals + Waveform sampling
- **Electron ID eff. 90% at <0.1% fake**
- **Energy resolution 1.6-4%**
- **94% of solid angle coverage**

K-long and Muon Detector (KLM)
- Alternating iron and detector plates
- Scintillator / Resistive Plate Chamber
- **Muon ID efficiency 90% at 2% fake**



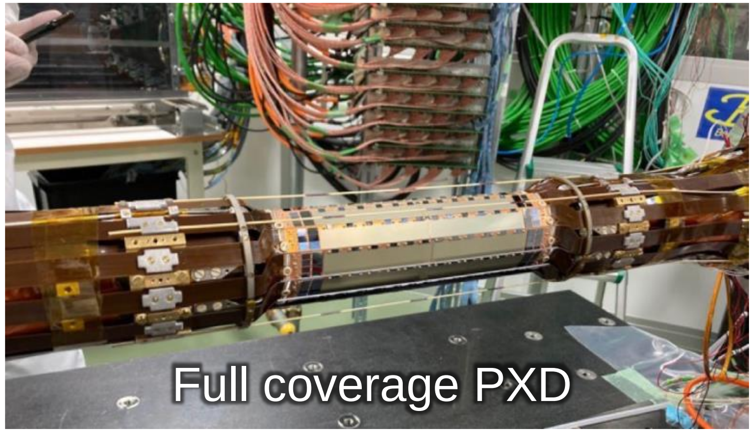
Vertex Detector (VXD)
- Inner 2 layers : Pixel
- Outer 4 layer : Double side strip
- σ(Track impact parameter) ~ 15 μm

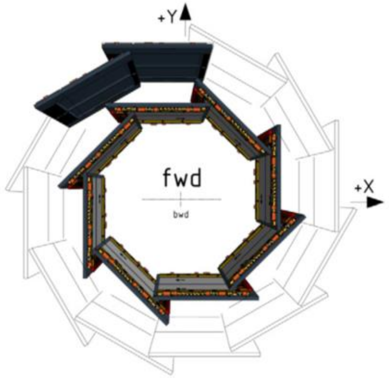
Central Drift Chamber (CDC)
- **91% of solid angle coverage**
- **p_T resolution ~ 0.4%/p_T**
- dE/dx resolution 5% (low-p PID)

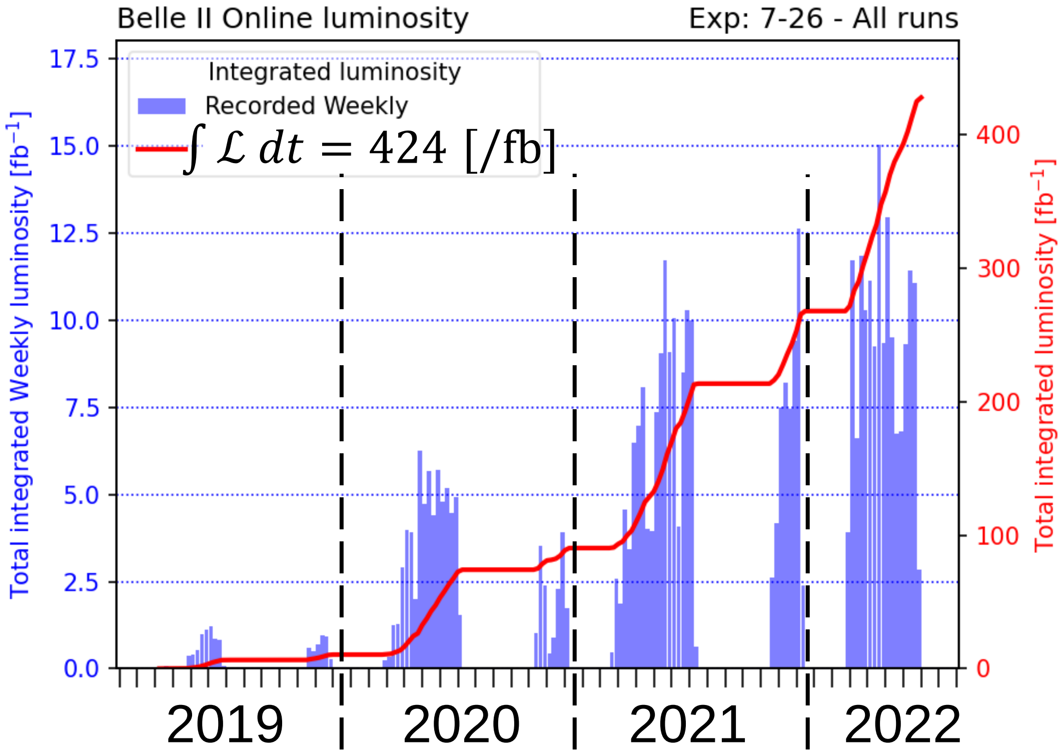
Trigger and DAQ
- L1 Trigger rate 30 kHz (design)
- **New trigger line for low-multiplicity events**
- Constant improvements of trigger algorithm

Operation status

- World record instantaneous luminosity : 4.7 $\times 10^{34}$ /cm^2/s
  - ~90% data taking efficiency : 1-2 fb^{-1}/day
- Recorded data : 424 /fb
  - 363 fb^{-1} at $\sqrt{s}$ = 10.58 GeV
- Long Shutdown 1 is finishing and new run will start at the end of 2023.
  - SuperKEKB upgrade for higher luminosity etc.
  - Full coverage of pixel detector
  - PMT replacement of the barrel PID detector for lifetime and robustness
  - Data-acquisition system upgrade

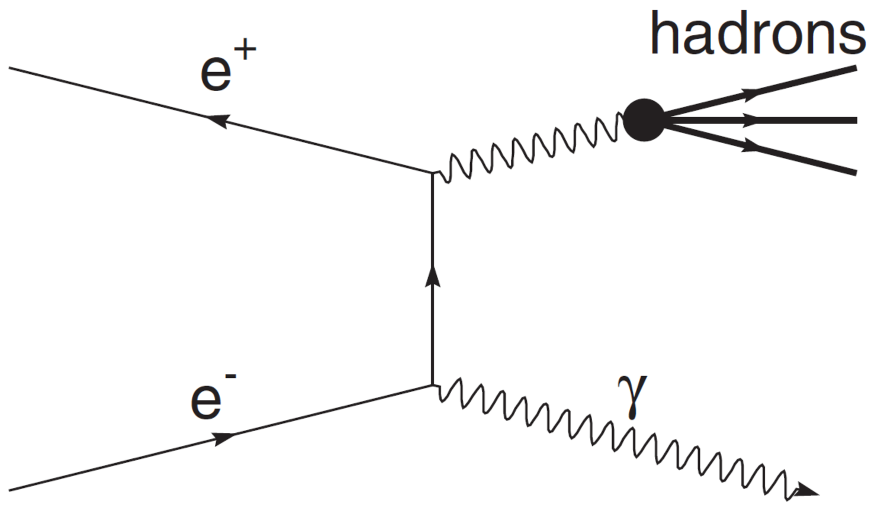




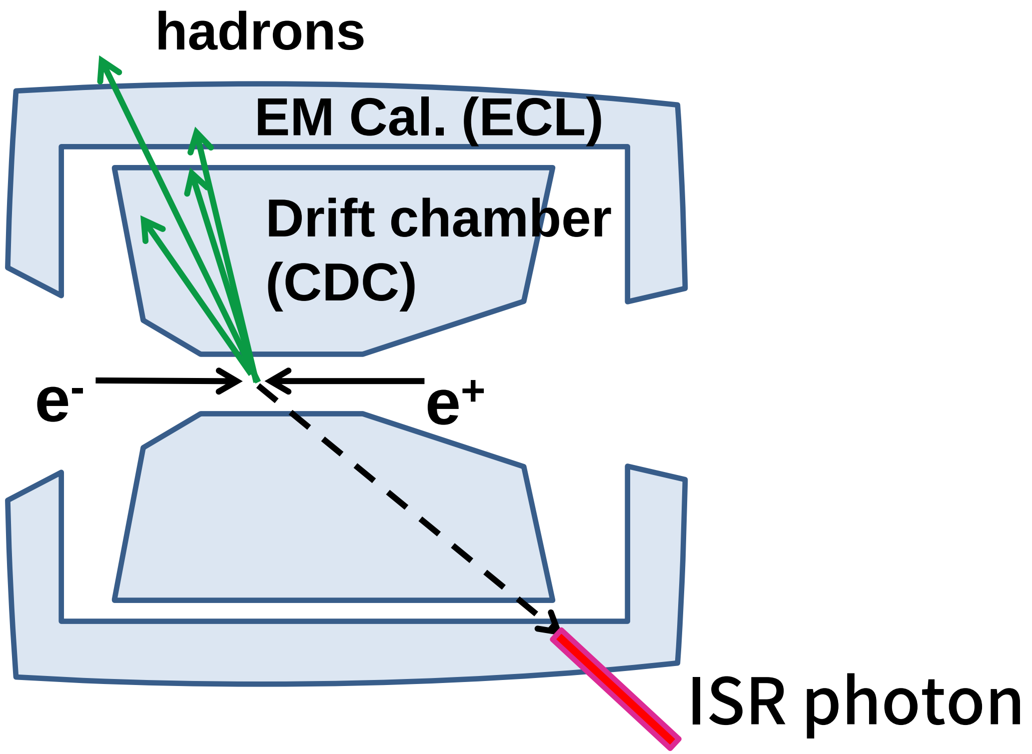


Radiative return method for HVP measurements

- Radiative return is also used in BaBar, KLOE, BESIII
  - Other method is direct scan, e.g., Novosibirsk experiments
- Scan the energy of hadronic system at fixed energy using ISR
- Access to the entire hadronic mass range with single dataset
- Around 7% of ISR photons are produced within the detector acceptance.



Energetic ISR photon



HVP measurements at Belle II

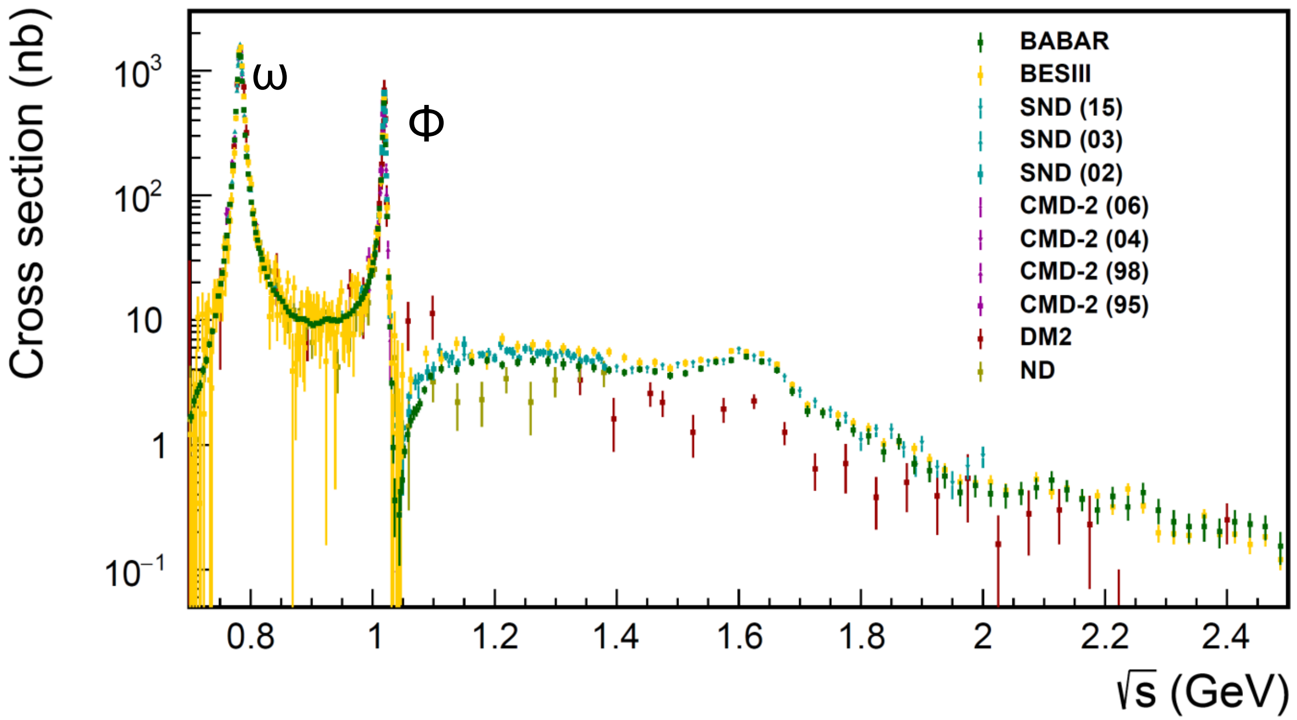
- New low-multiplicity trigger lines enable this physics at Belle II
  - Almost 100% efficiency for energetic ISR
  - Two independent triggers : Tracker and Calorimeter
- Two channels are mainly under study
  1. $e^+e^- \to \pi^+\pi^-$
     - The largest contribution to a_μ^{HVP} ~ 73 %
     - Target 0.5% precision using 363 fb^{-1} data
     - Try to following BaBar methods as a base line
  2. $e^+e^- \to \pi^+\pi^-\pi$
     - The 2nd largest contribution to a_μ^{HVP} ~ 7 %
- Today we report the status of $e^+e^- \to \pi^+\pi^-\pi^0$ analysis
  to demonstrate the capability of Belle II for the ISR processes

A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D101, 014029 (2020).

Previous measurements of $e^+e^- \to \pi^+\pi^-\pi^0$

- Recent measurements
  - Preliminary result from BES III [arXiv:1912.11208]
  - BABAR has updated its results with full data [Phys. Rev. D 104, 112003 (2021)]
- As for the $e^+e^- \to \pi^+\pi^-\pi^0$ contribution $a_\mu(3\pi)$,
  the uncertainty of $a_\mu(3\pi)$ is 2-3% for combination and 1.3% for BABAR alone
  - The difference in the cross section between the experiments below 1.1 GeV produces the error.

Previous measurements of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section



Eur. Phys. J. C 80, 241 (2020)
Phys. Rev. D 101, 014029 (2020)
J. High Energy Phys. 08 137 (2019)

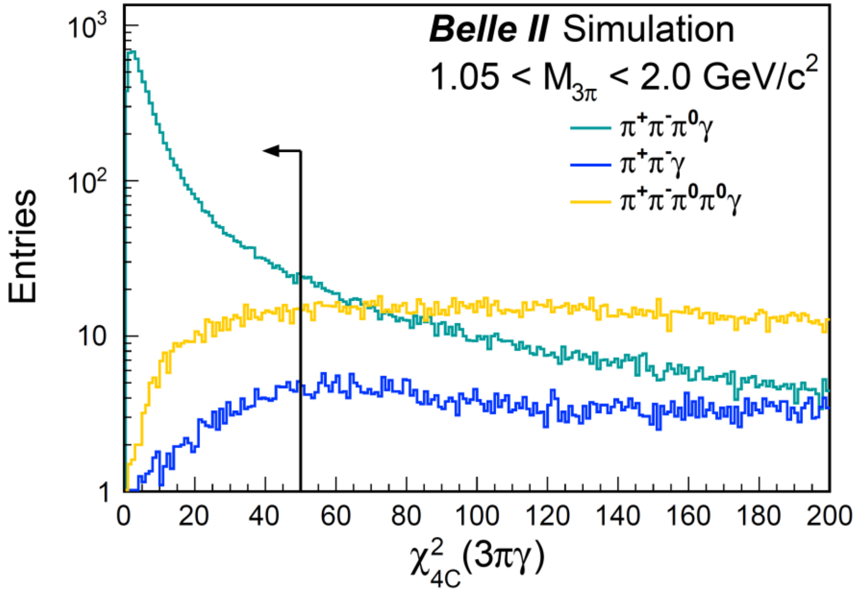
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ Analysis overview

- Target precision : $\delta a_\mu(3\pi) \sim 2\%$
- $\pi^+\pi^-\pi^0$ Mass range : 0.6-3.5 GeV
- Dataset : 2019-2021 Summer 190 fb^{-1}
- Key items
  - Trigger
  - Background reduction and estimation
  - Efficiency corrections
  - Unfolding
- Blind analysis
  - Study of analytical methods using MC and validation using 10% data
  - Final confirmation under way using full data without correction factors

$e^+e^- \to \pi^+\pi^-\pi^0$: Event selection

- Two tracks + three photons : $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR} \to \pi^+\pi^-\gamma\gamma\gamma_{ISR}$
  - Tracks : dr < 0.5 cm and |dz| < 2cm and p_T > 0.2 GeV/c
  - Photons : E > 100 MeV + at least one photon must be energetic ISR (E^{CMS}> 2 GeV in barrel ECL)
- π^0 reconstruction
  - Invariant mass of two photons within 0.123-0.147 GeV/c^2
- Select events using four-momentum kinematic fit (4C-Kfit) χ^2
  - $\chi^2_{4C}(3\pi\gamma)$ < 50 is used for the cross section measurement
- Cuts to reduce remaining backgrounds
  - A) Background not containing real π^0 : $e^+e^- \to e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
  - B) Charged kaon : $e^+e^- \to K^+K^-\pi^0\gamma$
  - C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$
  - D) Background not containing real ISR : Non-ISR qqbar and $\tau^+\tau^-$

4C-Kfit χ2 distribution
in M(3π) > 1.05 GeV

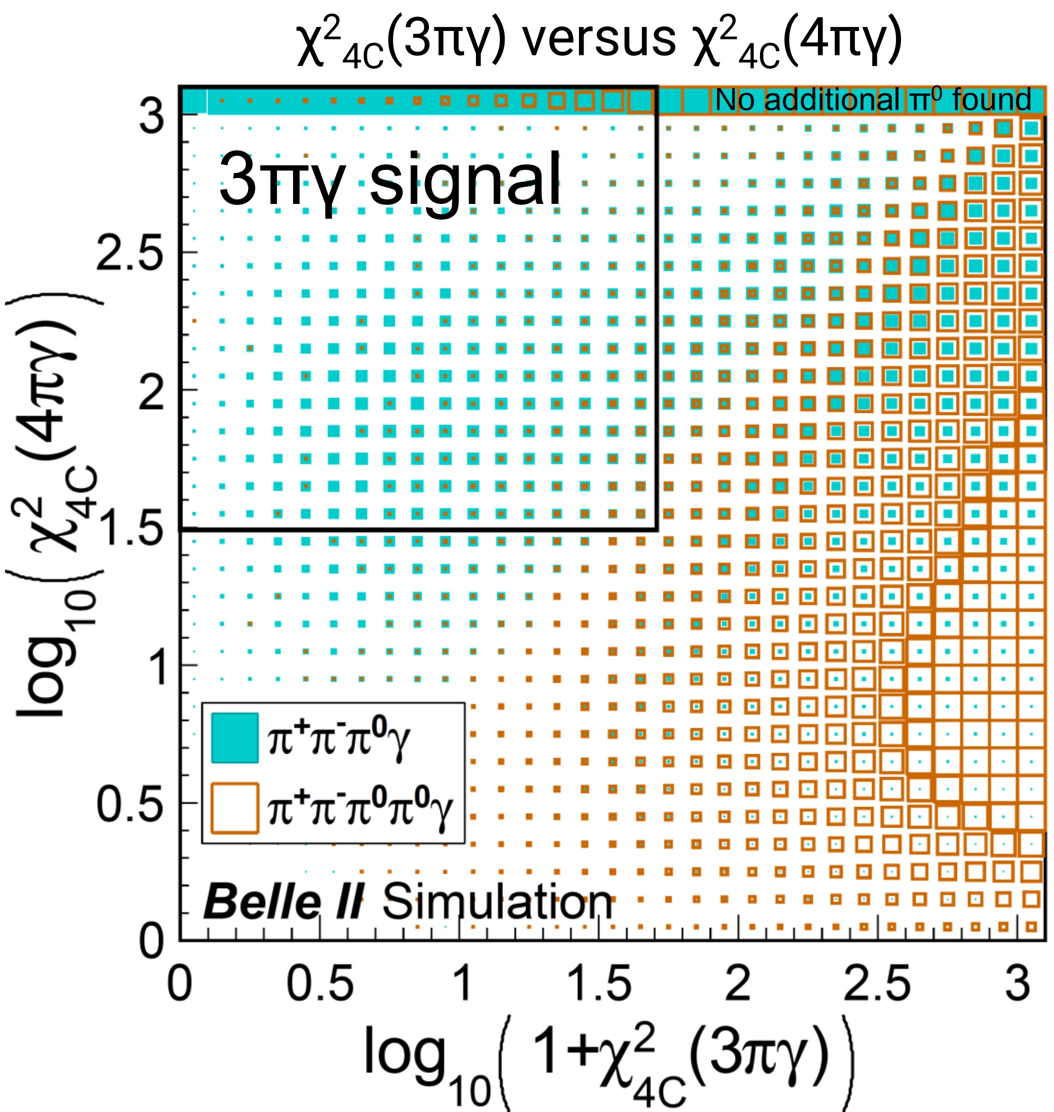


Background reduction cuts (1)

A) Background not containing real π^0 : $e^+e^- \to e^+e^-\gamma, \pi^+\pi^-\gamma, \mu^+\mu^-\gamma$
  - Pion/Electron ID : $L(\pi/e) > 0.1$
  - $M^2_{recoil}(\pi^+\pi^-) > 4\ \mathrm{GeV}^2/c^4$

B) Charged kaon : $e^+e^- \to K^+K^-\pi^0\gamma$
  - Pion/Kaon ID : $L(\pi/K) > 0.1$

C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$
  - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ (with additional π^0)
  - 4C kinematic fit under $\pi^+\pi^-\pi^0\pi^0\gamma$ hypothesis and $\chi^2_{4C}(4\pi\gamma) > 30$



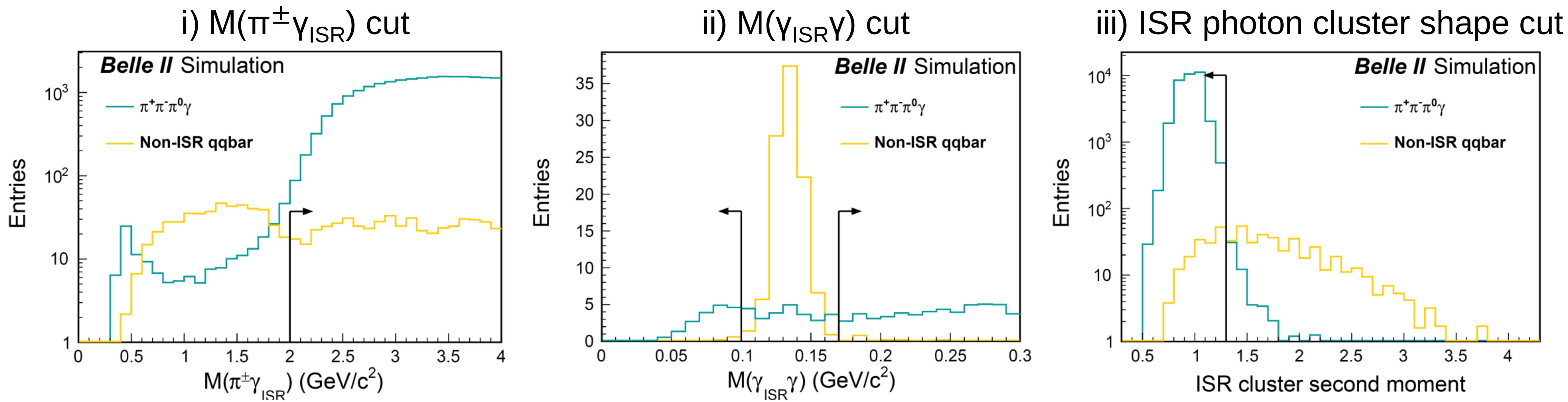
Background reduction cuts (2)

D) Background not containing real ISR : Non-ISR qqbar (dominated by $\pi^+\pi^-\pi^0\pi^0$) and $\tau^+\tau^-$

i. $M(\pi^\pm\gamma_{ISR})$ > 2 GeV/c^2 to reduce high momentum $\rho^\pm \to \pi^+\pi^0$

ii. $M(\gamma_{ISR}\gamma)$ cut to reduce ISR as which a daughter photon of π^0 is reconstructed

iii. Cluster shape cut to reduce ISR as which both photon clusters of π^0 decay are merged.



In total,

$M(3\pi)$ < 1.05 GeV/c^2 : the background fraction is reduced from 8.9% to 2.2% with 9% signal loss.

$M(3\pi)$ > 1.05 GeV/c^2 : the background events are reduced 78% with 11% signal loss.

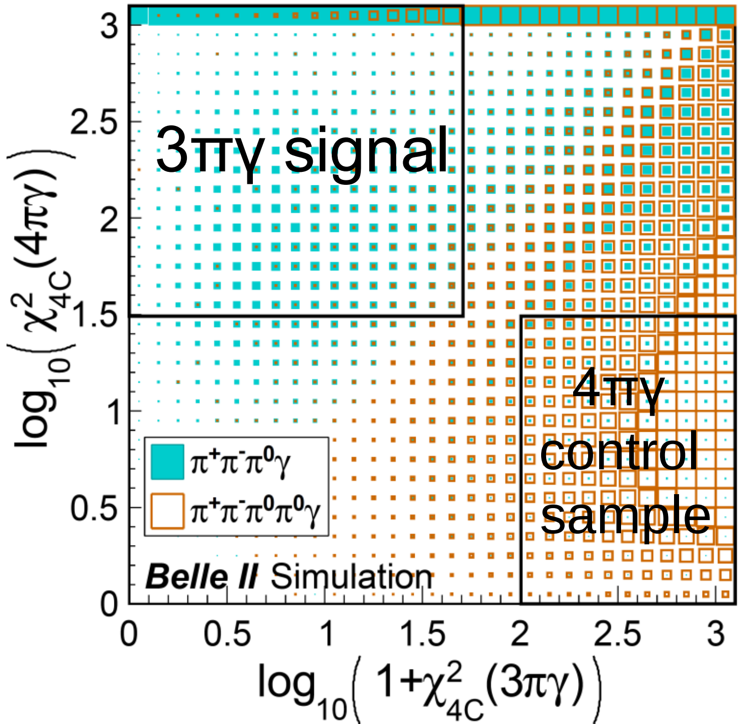
Background estimation

Estimate by determining a mass-dependent data-MC scale factor using a control sample (CS)

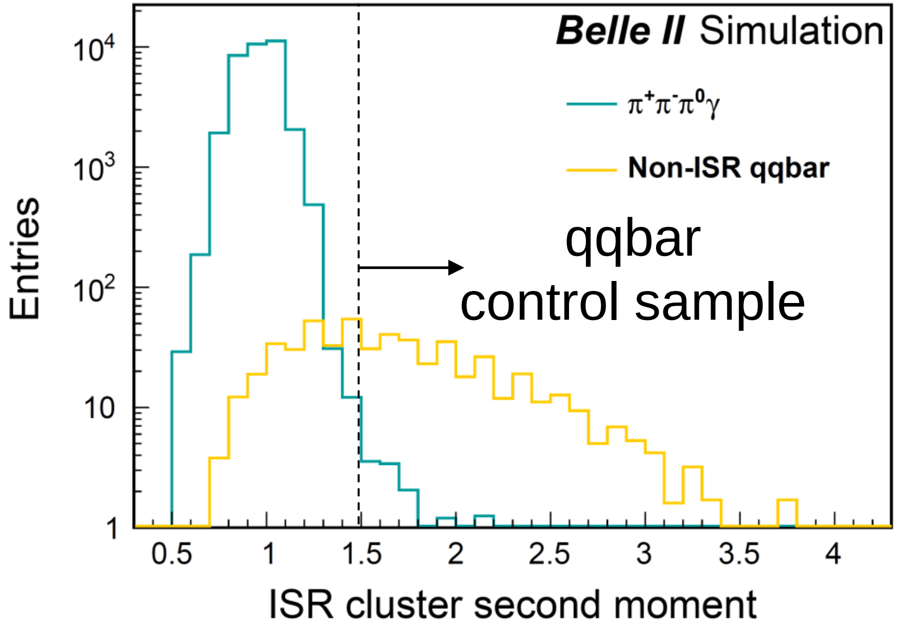
$$N^{\mathrm{data}}_{\mathrm{Signal}} = N^{\mathrm{MC}}_{\mathrm{Signal}} \cdot \frac{N^{\mathrm{data}}_{\mathrm{Control}}}{N^{\mathrm{MC}}_{\mathrm{Control}}}$$

- $e^+e^- \to K^+K^-\pi^0\gamma$: Invert π/K ID : L(π/K) > 0.1 ⇒ L(π/K) < 0.1
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- Non-ISR qqbar : $0.10 < M(\gamma_{ISR}\gamma) < 0.17$ GeV / large cluster second moment

$\chi^2_{4C}(3\pi\gamma)$ versus $\chi^2_{4C}(4\pi\gamma)$



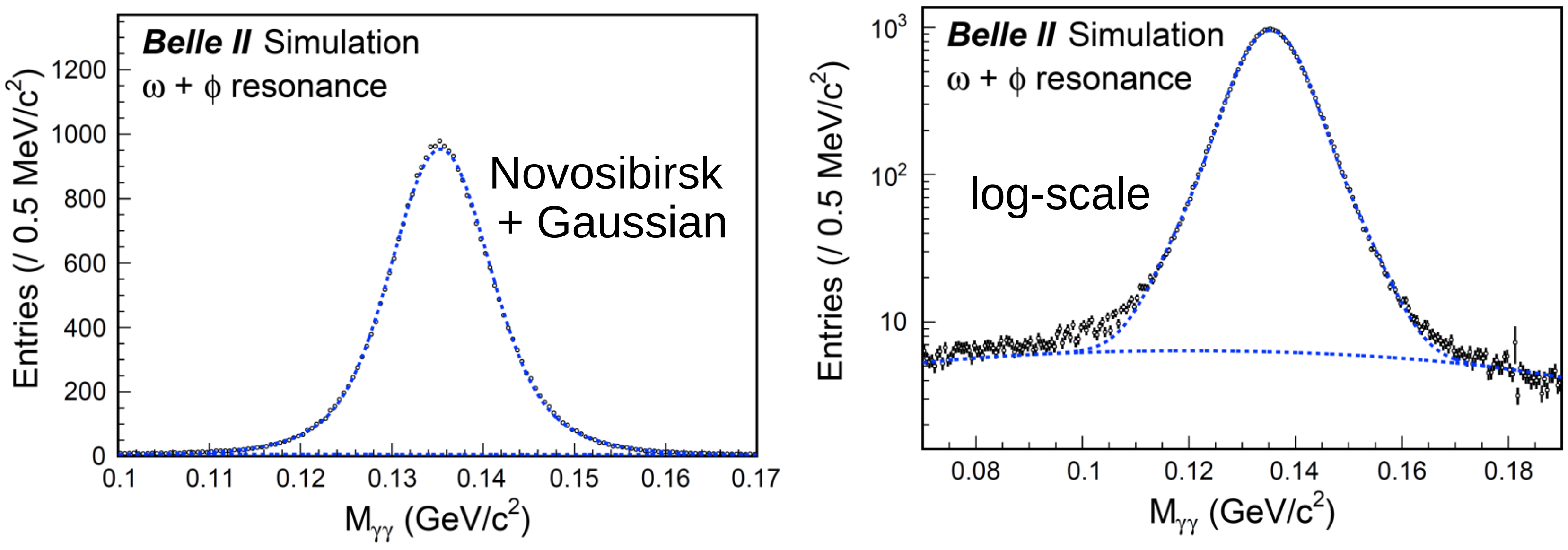
ISR cluster shape



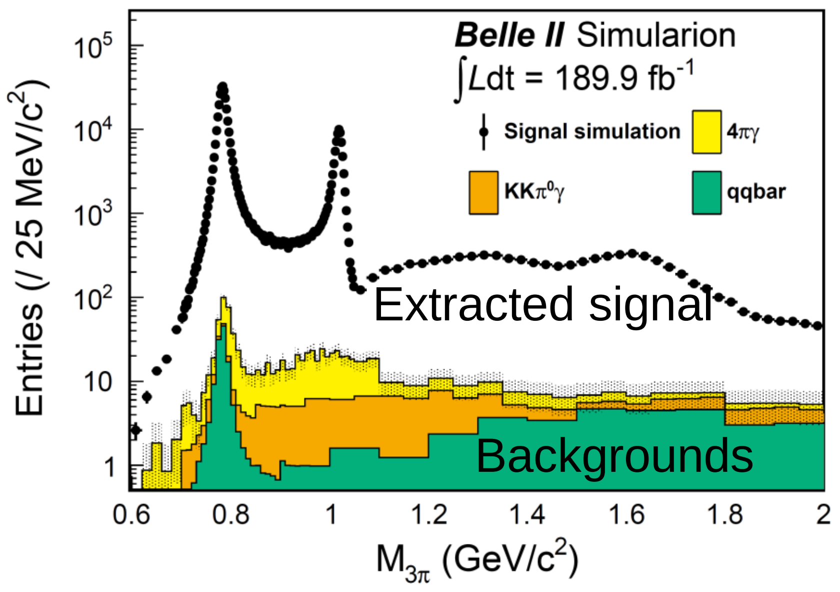
Signals after event selection

- Signal extraction by M(γγ) fit to each M(3π) bin
  - Fit and integral over 0.123-0.147 GeV/c^2
- Estimated background is subtracted from the spectrum

γγ invariant mass fitting



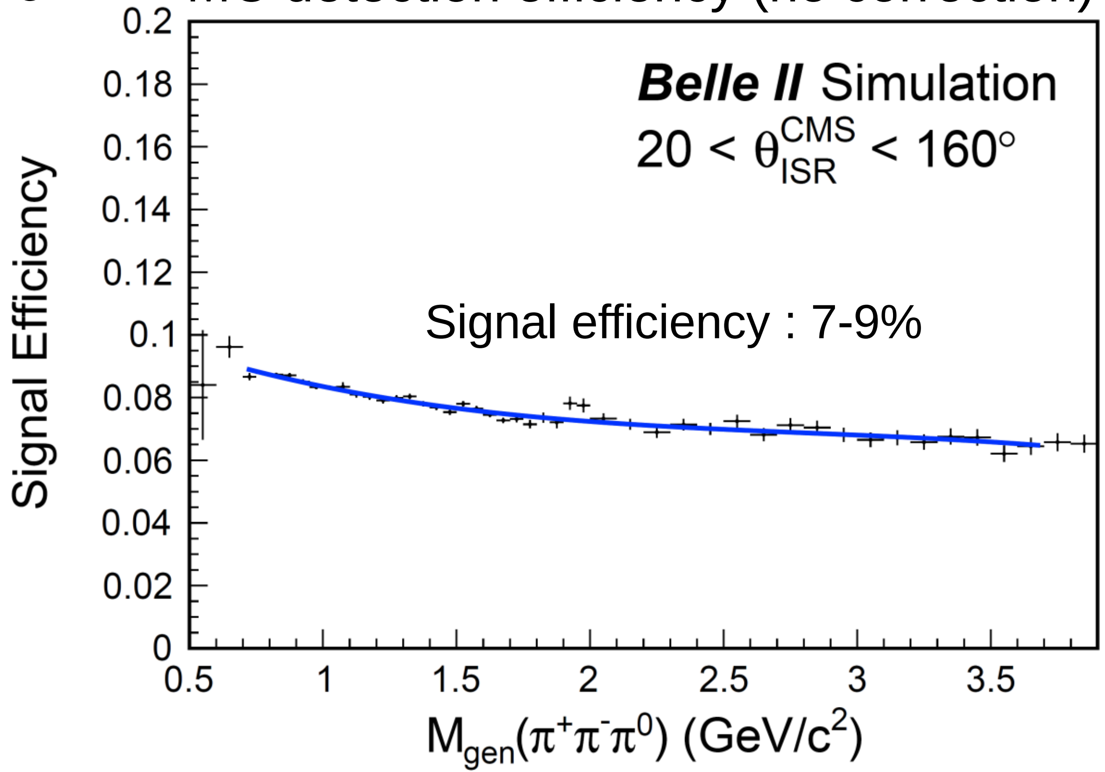
Extracted spectrum and background



Detection efficiency and Data-MC correction

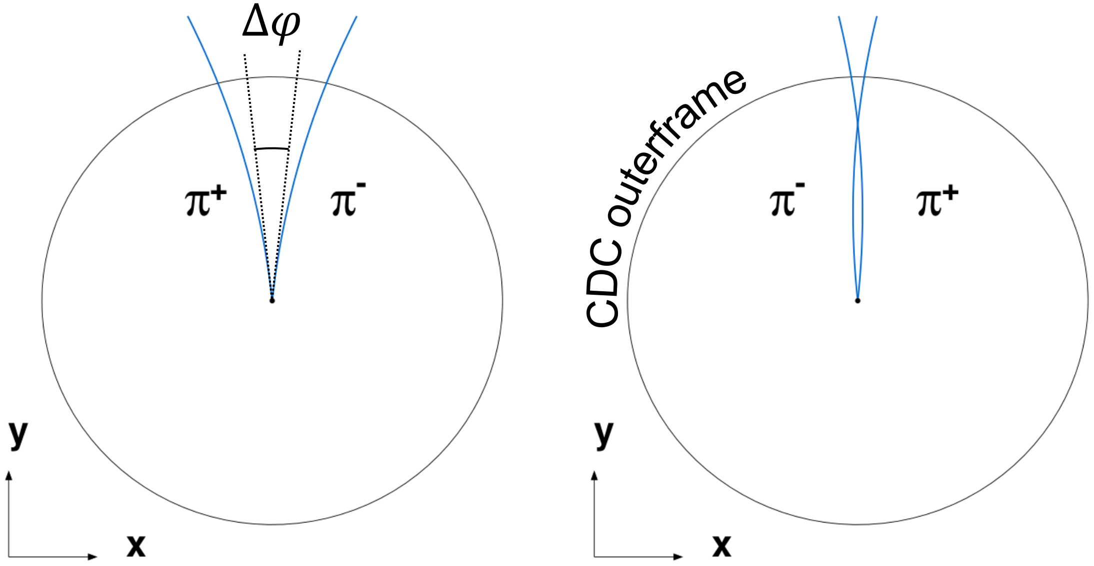
- Detection efficiency is estimated using MC of the x20 larger statistics.
- Possible differences between data and MC must be corrected.
- Estimate data-MC corrections dedicated to this analysis.
  - Trigger efficiency
  - High energy photon detection efficiency
  - Tracking efficiency
  - π^0 efficiency
  - χ^2 distribution
  - Background reduction cut efficiency

MC detection efficiency (no correction)

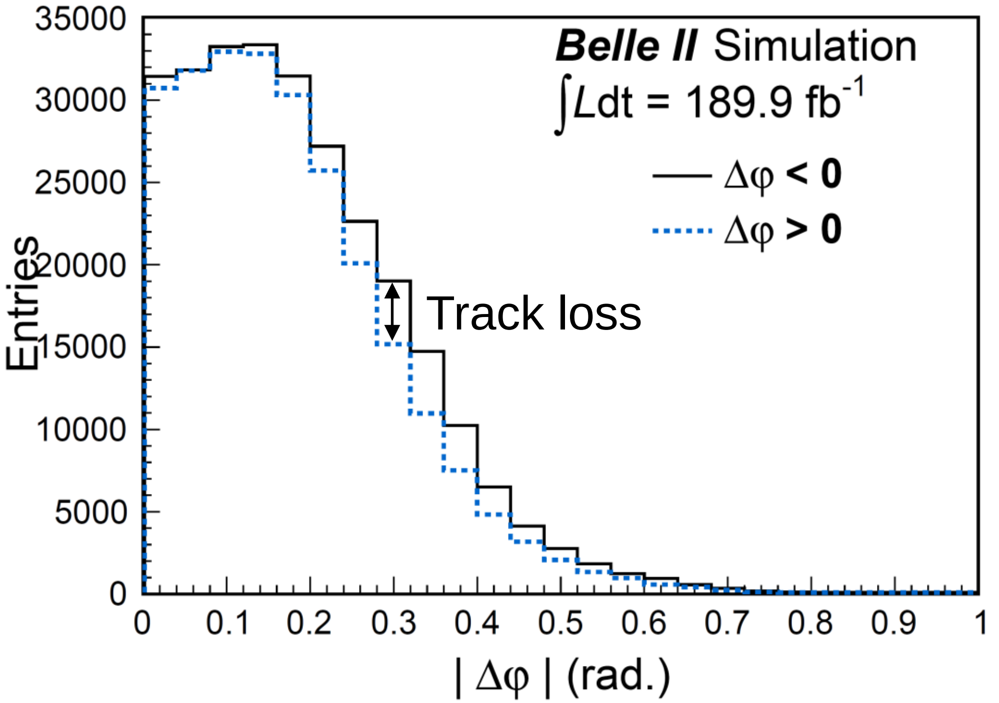


Tracking efficiency

- Tracking efficiency is confirmed by tag-and-probe method using τ pairs.
- Track loss due to crossing on the drift chamber is confirmed.
  - Evaluate using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ process at the ω resonance.
- Define $\Delta\varphi \coloneqq \varphi(\pi^+) - \varphi(\pi^-)$
- The expected inefficiency due to track loss is $f = \frac{N(\Delta\varphi<0) - N(\Delta\varphi>0)}{2N(\Delta\varphi<0)}$
  - The track loss in MC is 4%.
- In total, the systematic uncertainty of tracking is 0.8%.



$\Delta\varphi < 0$ $\Delta\varphi > 0$



π^0 efficiency correction

π^0 detection efficiency is 50-60%.

Evaluate efficiency using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ events of ω resonance.

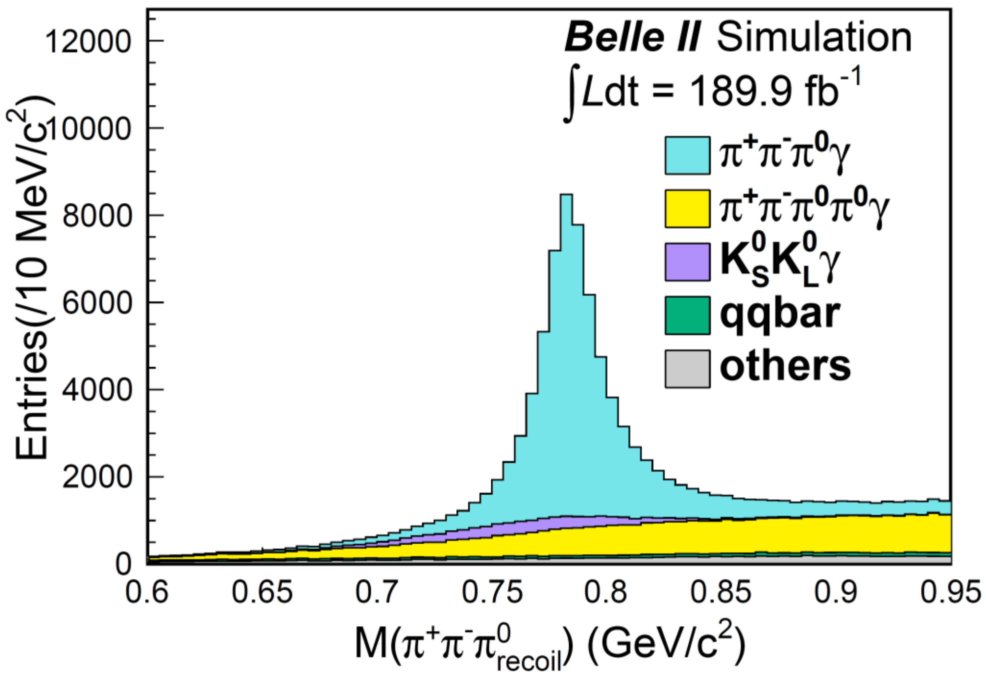
$$\pi^0\text{efficiency} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

Partial reconstruction $\pi^+\pi^-\gamma$: ISR + Two tracks

- Kinematic fit to $\pi^+\pi^-\gamma$ with hypothesis that recoil mass equals π^0 mass. (1-constraint)
- Fit on $M(\pi^+\pi^-\pi^0_{\text{recoil}})$ distribution around ω resonance to estimate the number of $3\pi\gamma$.
  - Count the number of events in ω region.

$$M^2(\pi^+\pi^-\pi^0_{\text{recoil}}) = (p_{\pi^+} + p_{\pi^-} + p_{\text{recoil}})^2$$

$M(\pi^+\pi^-\pi^0_{\text{recoil}})$ distribution



π^0 efficiency correction

π^0 detection efficiency is 50-60%.

Evaluate efficiency using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ events of ω resonance.

$$\pi^0\text{efficiency} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

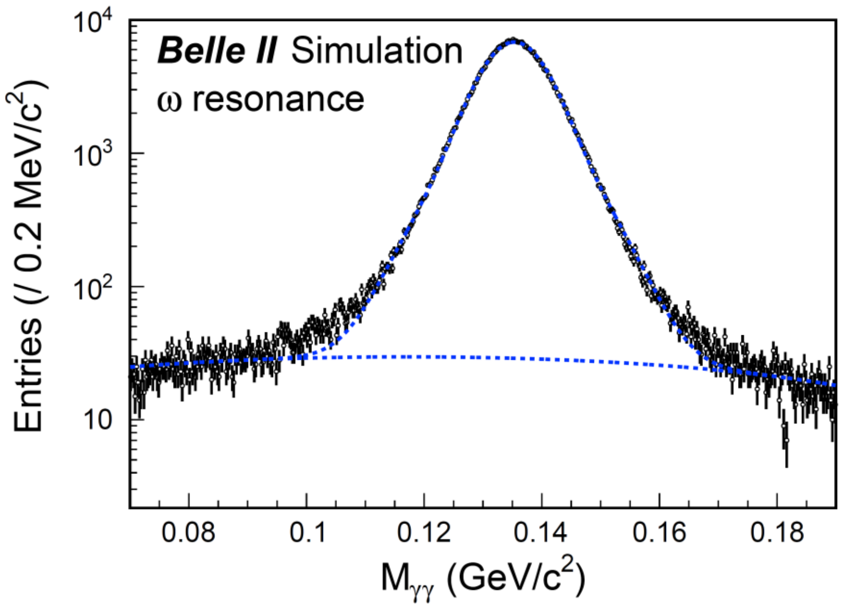
Full reconstruction : Partial reconstruction + π^0 selection + $\chi^2_{4\text{C},3\pi\gamma} < 50$

- Fit M(γγ) with signal extraction parameters at ω region events
  - Signal : Novosibirsk function + Gaussian (Fixed parameters)
  - Background : Quadratic function (Floated parameters)

The systematic uncertainty related to π^0 is 1.0%.

- The uncertainty is evaluated by variations of the M(γγ) signal pdf, background pdfs, and selections.

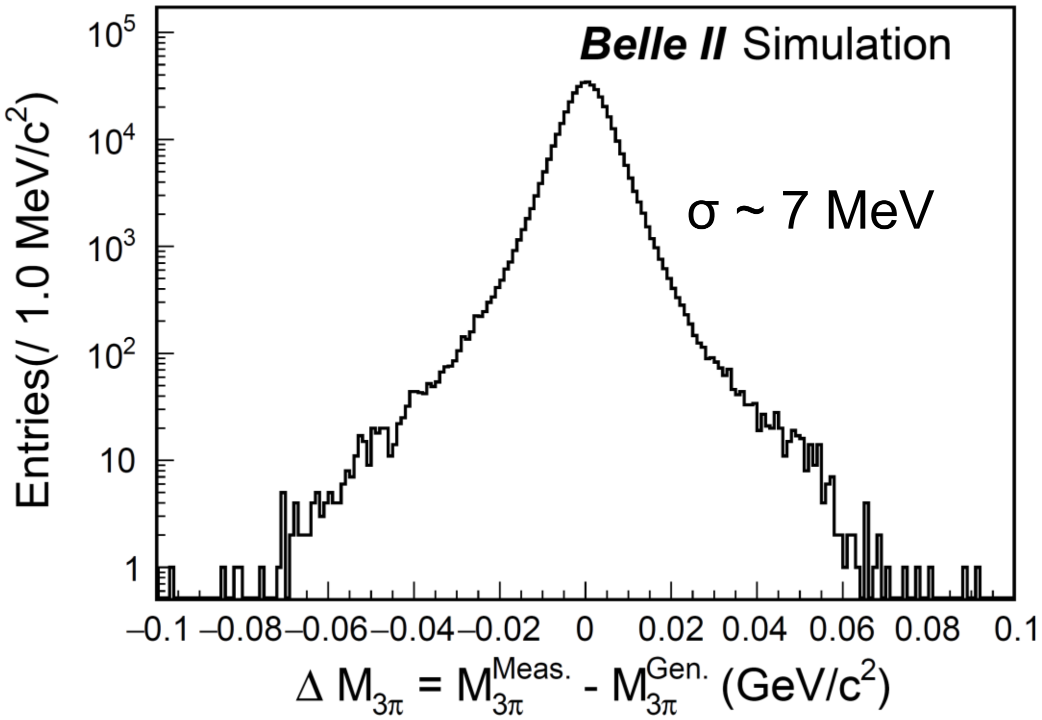
M(γγ) fit (Simulation)



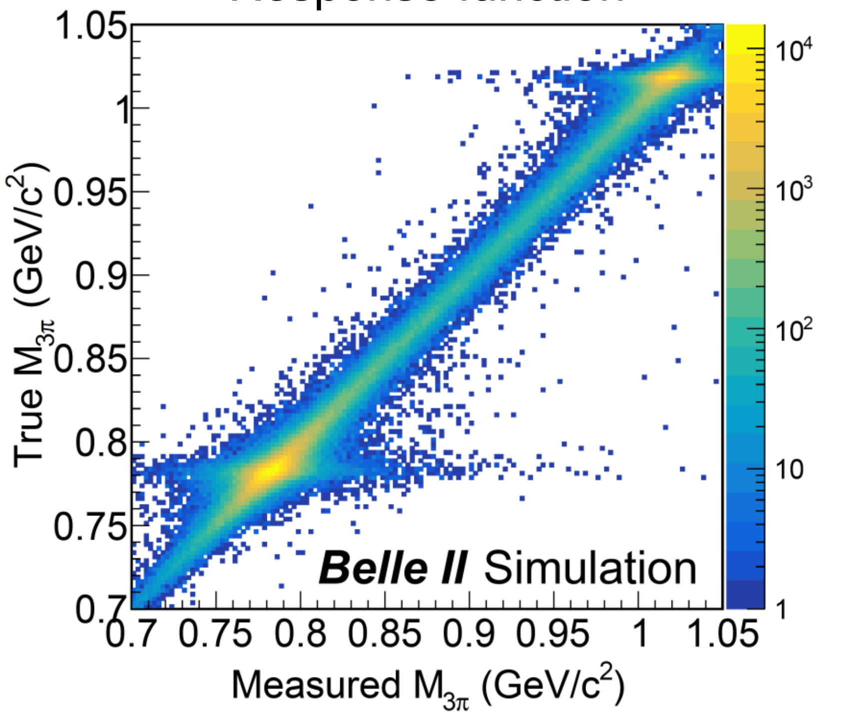
Unfolding

- The background-subtracted spectrum is unfolded to mitigate the effect of detector response and final-state radiation.
- The data-MC resolution difference is determined by a Gaussian convolution fit to the ω, Φ, and J/ψ resonances.
  - The agreement is good typically with a mass resolution around 7-10 MeV.

3π mass resolution



Response function



Systematic uncertainty and prospects

- Major systematic uncertainty comes from π^0 and tracking.
  - In M(3π) > 1.05 GeV, the uncertainty of selection efficiency is dominant.
- For $a_\mu(3\pi)$, the total uncertainty is expected to be 2% including stat. uncertainty of 0.5%.
- The results will be released within a few months.

Systematic uncertainties for $e^+e^-\to\pi^+\pi^-\pi^0$ cross section (Preliminary)

Source	Systematic uncertainty (%)	
	$M < 1.05$ GeVc^2	$M > 1.05$ GeVc^2
Trigger	0.2	0.2
ISR photon detection	0.7	0.7
Tracking	0.8	0.8
π^0 reconstruction	1.0	1.0
χ^2 distribution	0.3	0.3
Selection	0.2	1.9
Integrated luminosity	0.7	0.7
Radiative correction	0.5	0.5
Total systematics	1.8	2.61

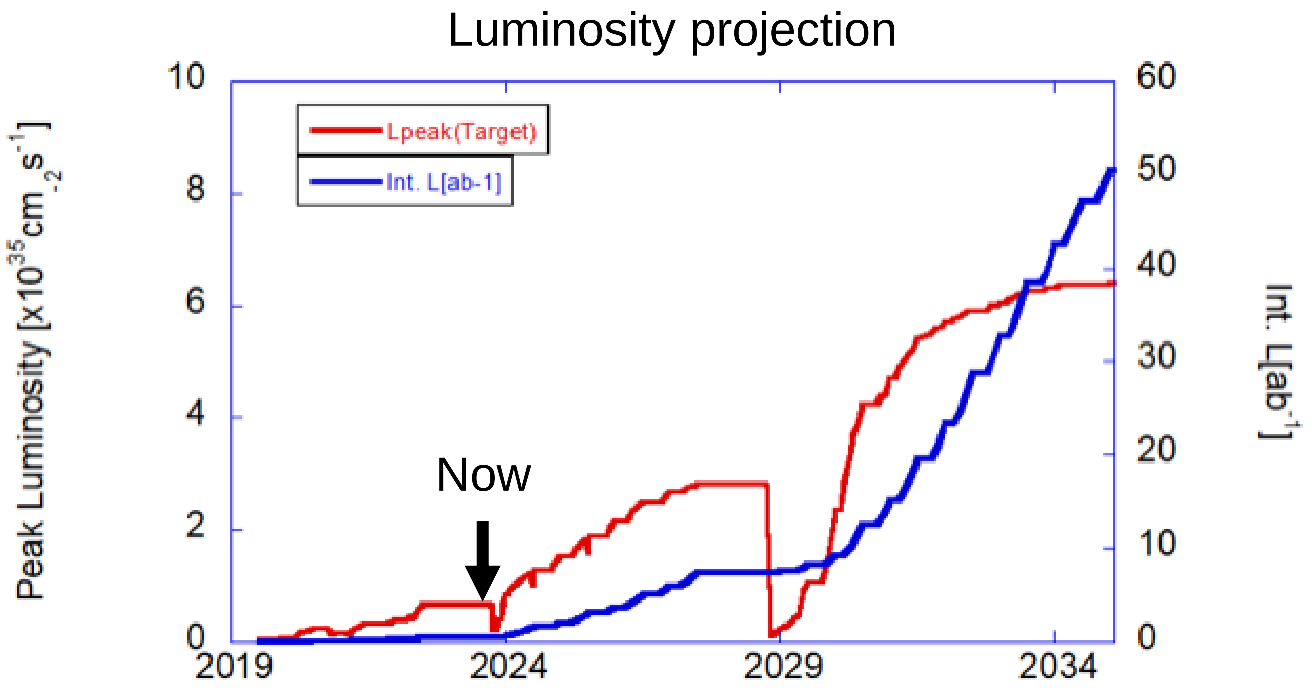
Conclusion

- Belle II has collected 424 fb^{-1} data, and further data taking will be going on for over 10 years
  - Long shutdown 1 is finishing and new run will start from the end of 2023
- Measurements related to muon g-2 are active and in progress at Belle II
- The analysis of $e^+e^-\to\pi^+\pi^-$ targets 0.5% precision
- The analysis of $e^+e^-\to\pi^+\pi^-\pi^0$ is at the final stage
  - We aim at ~2% precision using 190 fb^{-1} data
  - Blind analysis is introduced
  - Major systematic uncertainty comes from π^0 and tracking
  - The results will be released within a few months

Backup

Operation Plans

- Long Shutdown 1 is finishing and new run will start end of 2023.
  - SuperKEKB upgrade for higer luminosity etc.
  - Full coverage of pixel detector
  - PMT replacement of the barrel PID detector for lifetime and robustness
  - Data-acquisition system upgrade
- Another long shutdown around 2027 is under discussion.



$e^+e^- \to \pi^+\pi^-$: Status at Belle II

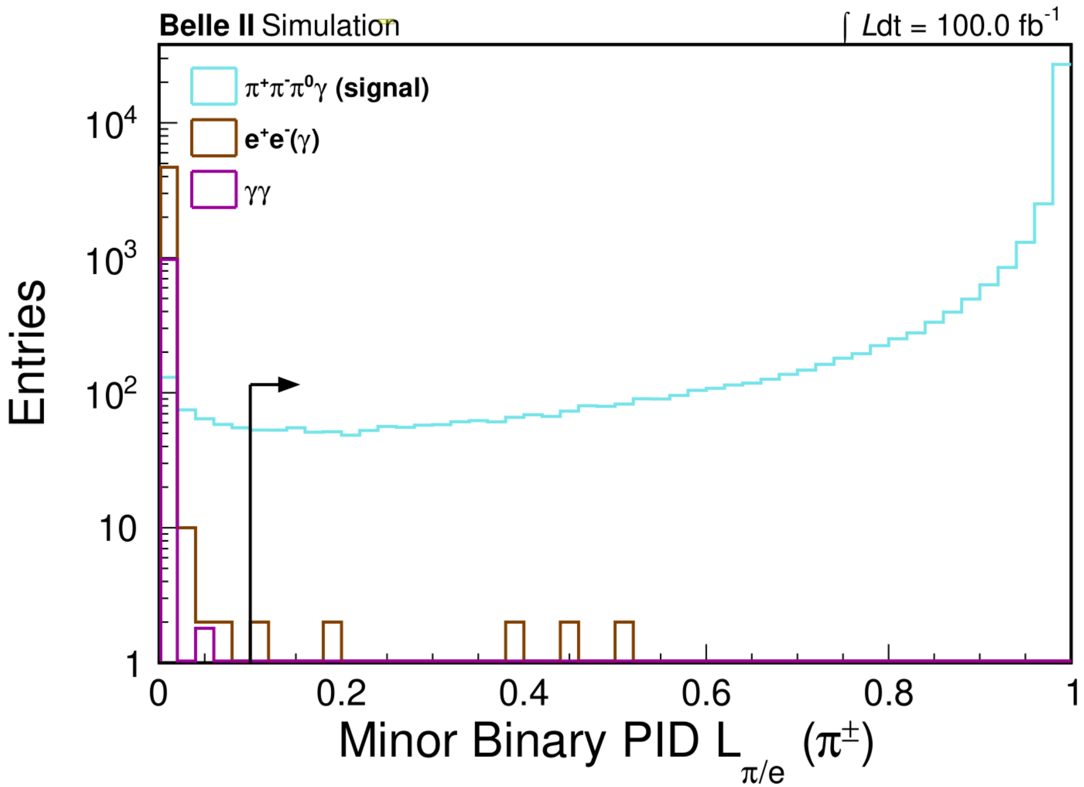
- Target precision : 0.5% of $a_\mu(2\pi)$
- Trying to follow BaBar methods as a base line.
- Systematics uncertainty dominant analysis
  - BaBar : 232 /fb [Phys. Rev. D 86 (2012), 032013]
  - We can use large statistics to control systematic uncertainties.
- Implementation of kinematic fitting tools
  - Useful for reducing background and correction for tracking efficiency.
  - Implementation of basic fitter has been completed.
- Sanity check on signal generator and background MC using < 2 fb^{-1} data .
- Design of data-driven efficiency corrections for tracking, trigger and π/μ/K ID is ongoing.

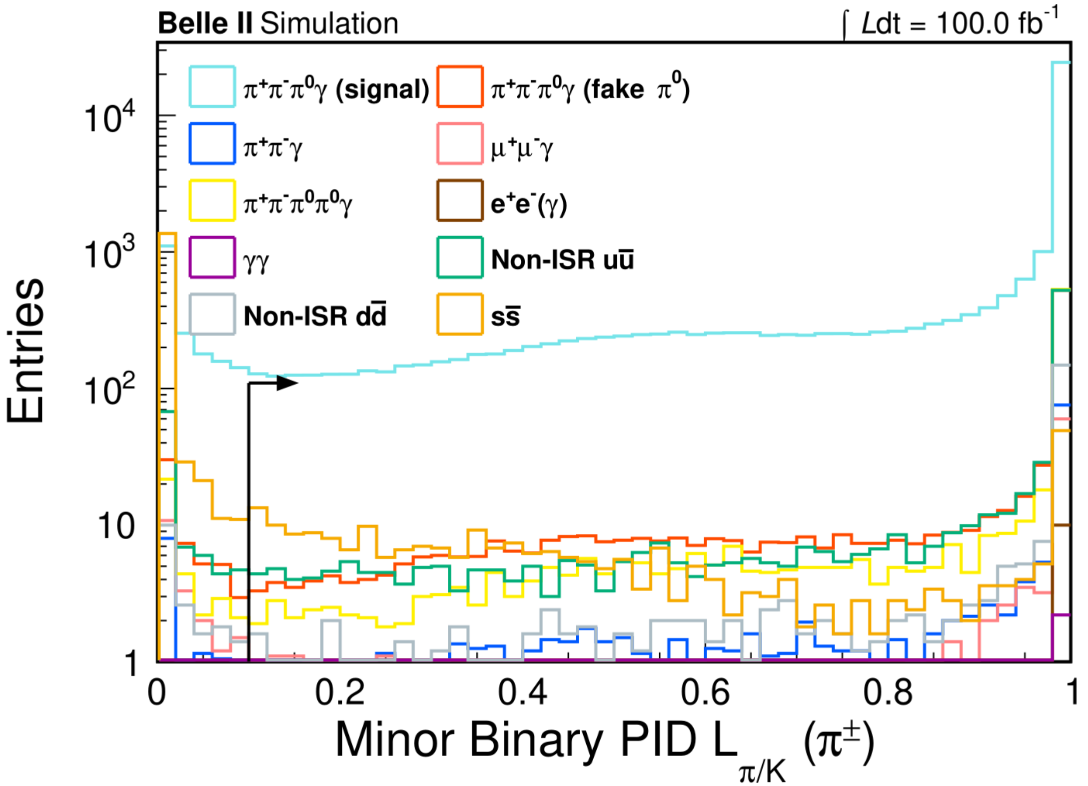
ee→ππ uncertainty (10^{-3}) at BaBar

Sources	0.3–0.4	0.4–0.5	0.5–0.6	0.6–0.9	0.9–1.2
Trigger/filter	5.3	2.7	1.9	1.0	0.7
Tracking	3.8	2.1	2.1	1.1	1.7
π-ID	10.1	2.5	6.2	2.4	4.2
Background	3.5	4.3	5.2	1.0	3.0
Acceptance	1.6	1.6	1.0	1.0	1.6
Kinematic fit (χ^2)	0.9	0.9	0.3	0.3	0.9
Correl. $\mu\mu$ ID loss	3.0	2.0	3.0	1.3	2.0
$\pi\pi/\mu\mu$ non-cancel.	2.7	1.4	1.6	1.1	1.3
Unfolding	1.0	2.7	2.7	1.0	1.3
ISR luminosity	3.4	3.4	3.4	3.4	3.4
Sum (cross section)	13.8	8.1	10.2	5.0	6.5

Event selection : Background reduction cuts

- Electron- and kaon-related backgrounds
  - $L(\pi/e) > 0.1$ for both tracks (no SVD and noTOP)
  - $L(\pi/K) > 0.1$ for both tracks

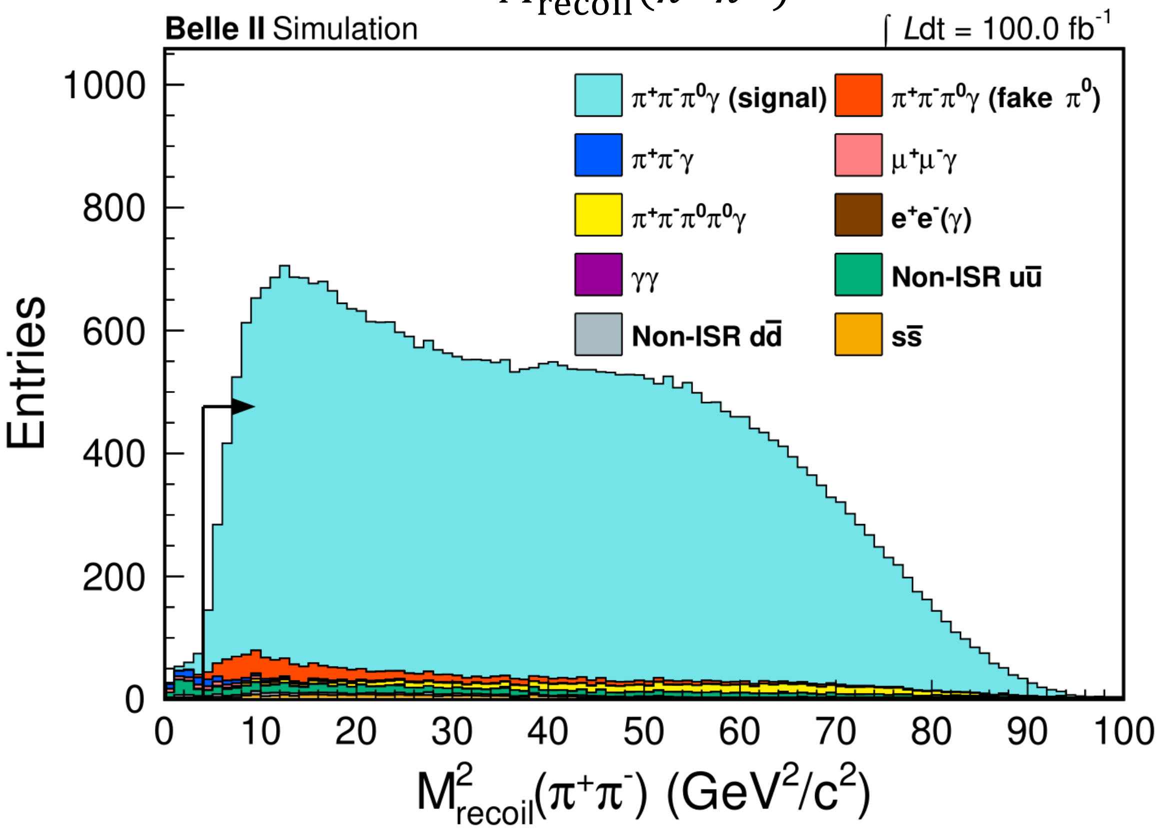




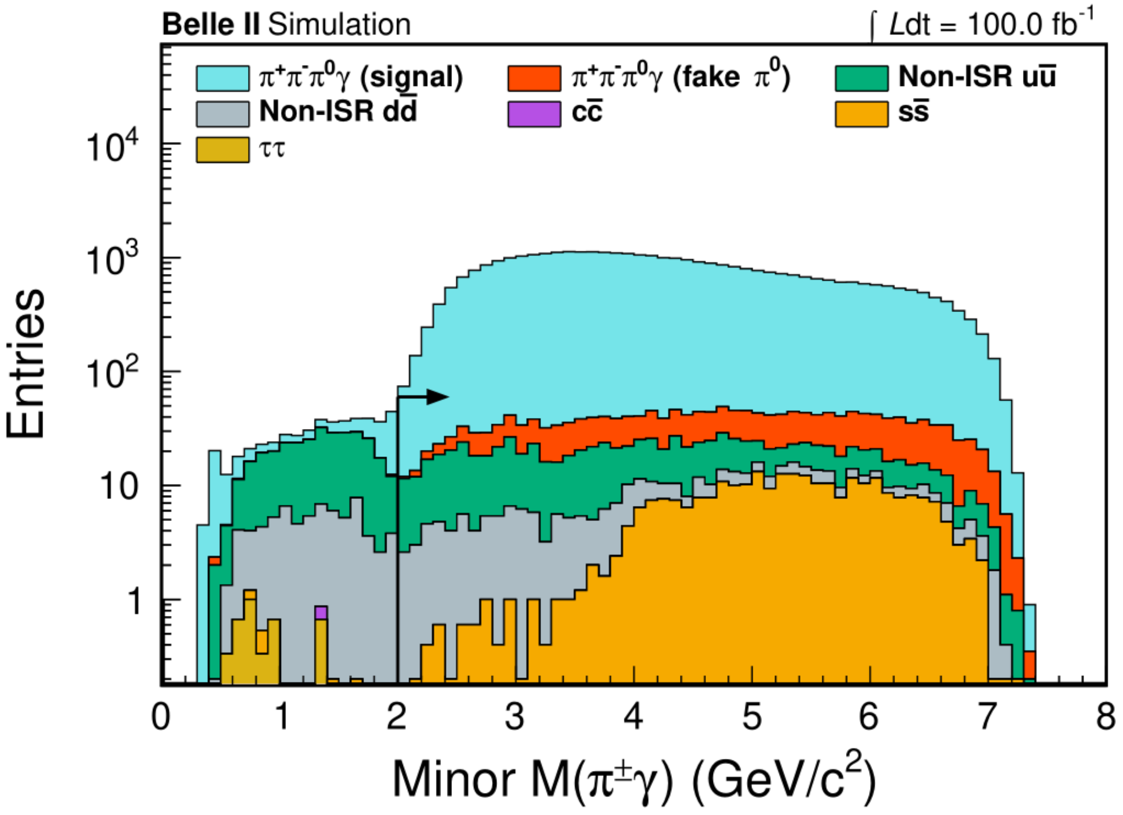
Event selection : Background reduction cuts

- $M^2_{\mathrm{recoil}}(\pi^+\pi^-) > 4\ \mathrm{GeV}^2$ for $\pi^+\pi^-\gamma$ and $\mu^+\mu^-\gamma$ rejection
- Non-ISR qqbar and ττ reduction (1)
  - $M(\pi^\pm\gamma_{\mathrm{ISR}})\ >\ 2$ GeV to reduce $\rho^\pm \to \pi^+\pi^0(\to\gamma\gamma)$

$M^2_{\mathrm{recoil}}(\pi^+\pi^-)$

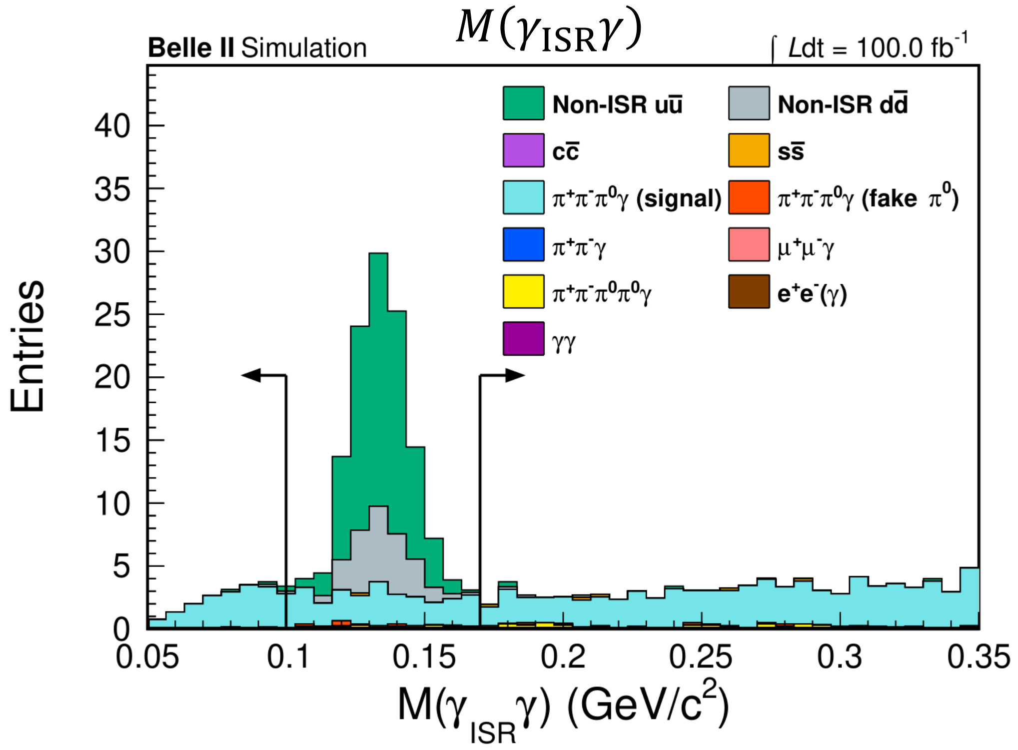


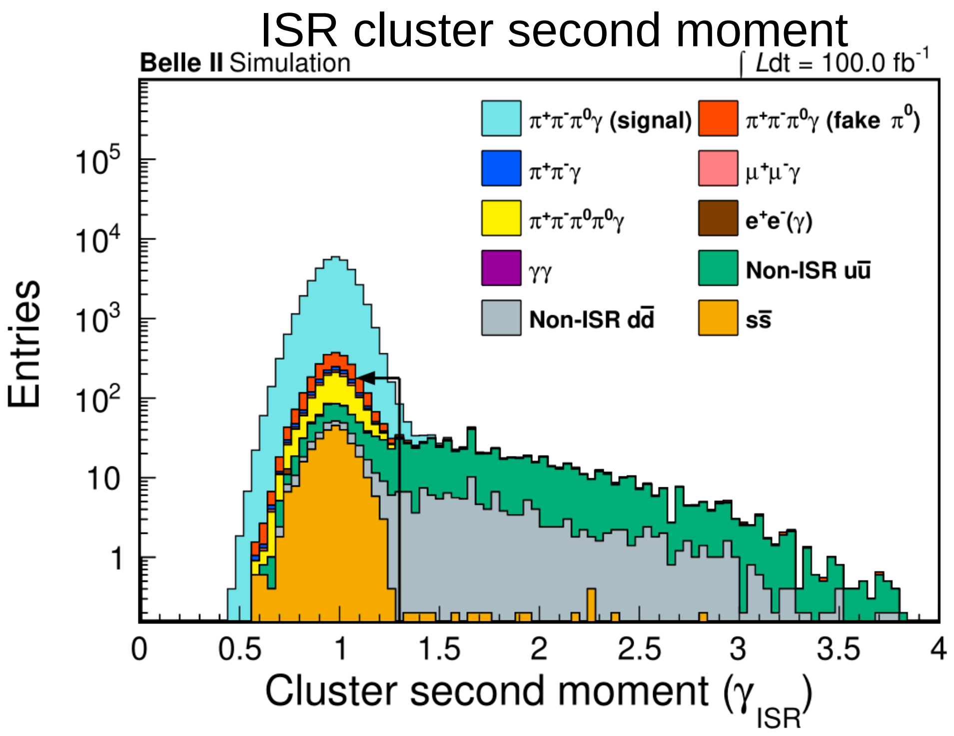
$M(\pi^\pm\gamma_{\mathrm{ISR}})$



Event selection : Background reduction cuts

- Non-ISR qqbar and ττ reduction (2)
  - One daughter photon of π^0 is reconstructed as ISR
    - Reconstruct π^0 from ISR and additional photon
    - Reject the invariant mass $0.110 < M(\gamma_{ISR}\gamma) < 0.170$ GeV
  - Both photon clusters of π^0 decay are merged and reconstructed as ISR
    - Cluster second moment of ISR < 1.3

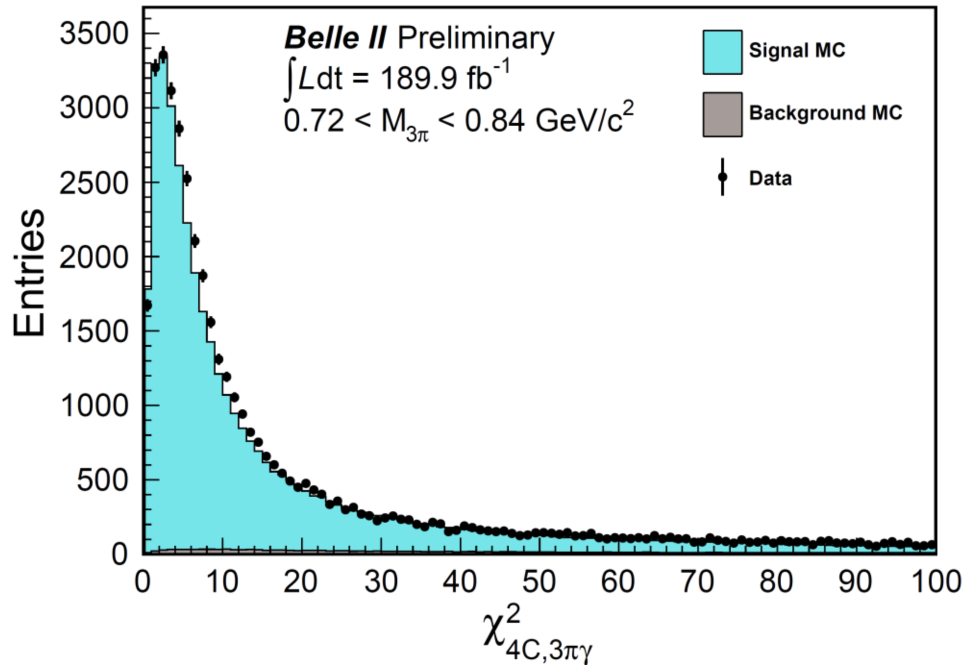




$e^+e^- \to \pi^+\pi^-\pi^0$: Event selection

- Two tracks + three photons : $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR} \to \pi^+\pi^-\gamma\gamma\gamma_{ISR}$
  - Tracks : originate from the interaction point + $p_T > 0.2$ GeV
  - Photons : E > 100 MeV + at least one photon must be energetic ISR (E^{CMS}> 2 GeV in barrel ECL)
- π^0 reconstruction
  - Invariant mass of two photons within 0.123-0.147 GeV/c^2
- Select events using four-momentum kinematic fit (4C-Kfit) χ^2
  - $\chi^2_{4C}(3\pi\gamma) < 50$ is used for the cross section measurement.
- Reduce backgrounds by additional cuts (next pages)

4C-Kfit χ2 at the ω resonance



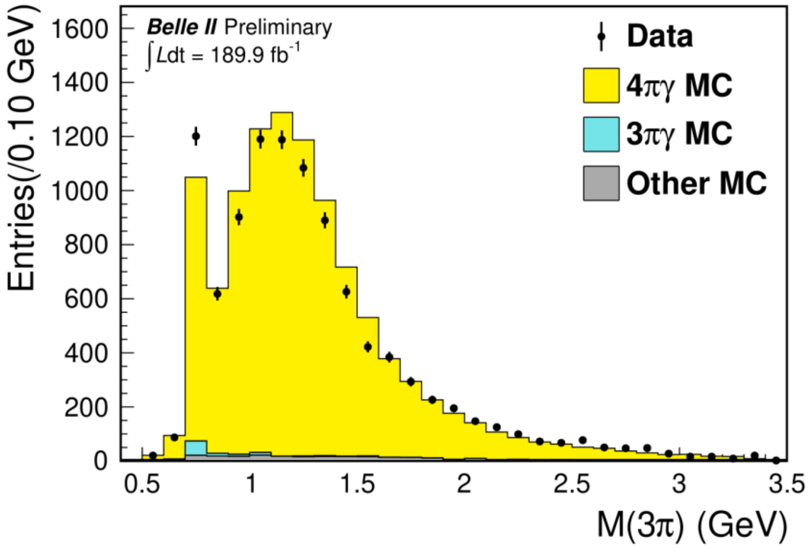
$e^+e^- \to \pi^+\pi^-\pi^0$: Background estimation

Estimate by determining a mass-dependent data-MC scale factor using a control sample (CS)

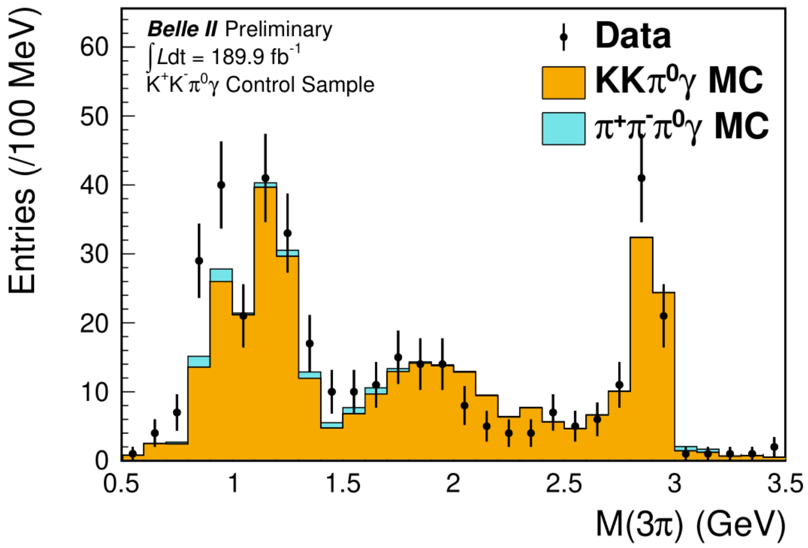
$$N_{SS}^{\mathrm{data}} = N_{SS}^{\mathrm{MC}} \cdot \frac{N_{CS}^{\mathrm{data}}}{N_{CS}^{\mathrm{MC}}}$$

- $\pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- $K^+K^-\pi^0\gamma$: Invert binary π/K ID : L(π/K) > 0.1 ⇒ L(π/K) < 0.1
- Non-ISR qqbar (dominated by $\pi^+\pi^-\pi^0\pi^0$) : $0.10 < M(\gamma_{ISR}\gamma) < 0.17$ GeV / large cluster second moment

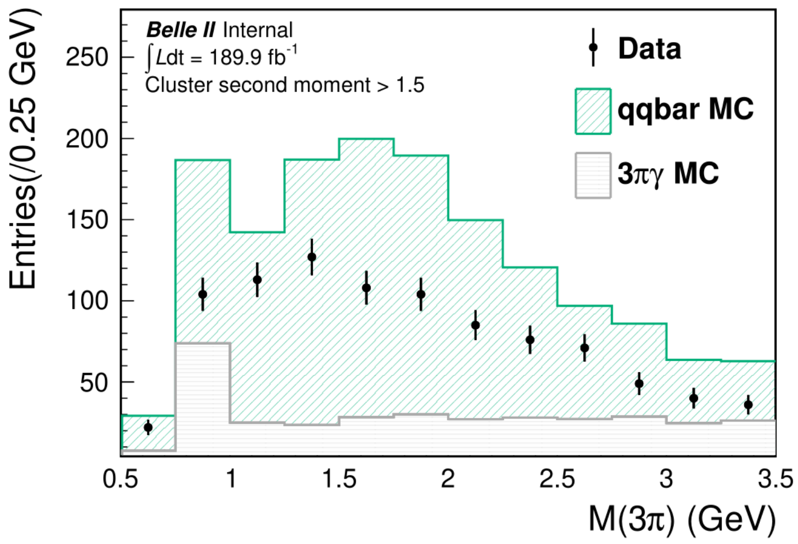
4πγ control sample



KKπ⁰γ control sample

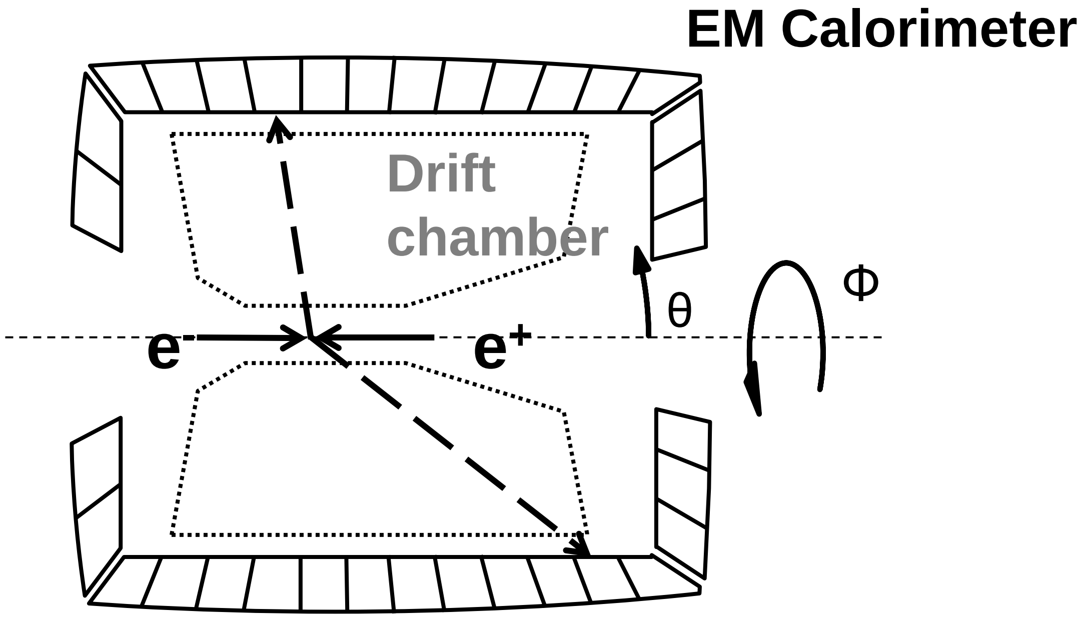


Non-ISR qqbar control sample



Trigger challenge at Belle II

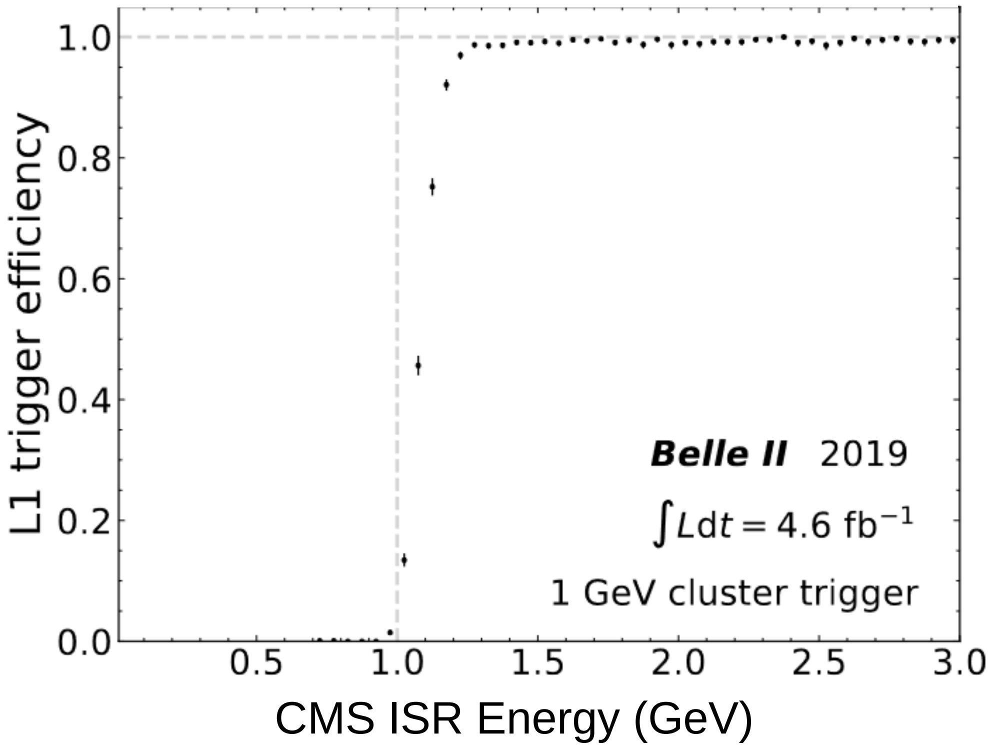
- Light hadron cross section measurement at BELLE was suffered from the trigger efficiency.
  - The measurement for $\sigma(e^+e^-\to\pi^+\pi^-\pi^0)$ was attempted, but could not be published. [J. Crnkovic, PhD thesis, Illinois U. (2013)]
- Bhabha veto has been upgraded to avoid the inefficiency and uncertainty.
  - BELLE bhabha veto was based on only θ angle.
  - Belle II 3D bhabha veto uses θ and Φ angle.
- The trigger efficiency of EM Calorimeter triggers for energetic ISR can be measured by making the orthogonal tracking trigger a reference.



Trigger challenge at Belle II

- Light hadron cross section measurement at BELLE was suffered from the trigger efficiency.
  - The measurement for $\sigma(e^+e^-\to\pi^+\pi^-\pi^0)$ was attempted, but could not be published.
    [J. Crnkovic, PhD thesis, Illinois U. (2013)]
- Bhabha veto has been upgraded to avoid the inefficiency and uncertainty.
  - BELLE bhabha veto was based on only θ angle.
  - Belle II 3D bhabha veto uses θ and Φ angle.
- The trigger efficiency of EM Calorimeter triggers for energetic ISR can be measured by making the orthogonal tracking trigger a reference.
  - Efficiency for energetic ISR > 99%
  - Event loss due to 3D bhabha veto is suppressed in μμγ.
- The high trigger efficiency for energetic ISR is beneficial for most light hadron cross section measurements in the radiative return method.

Belle II trigger efficiency for μμγ (data)

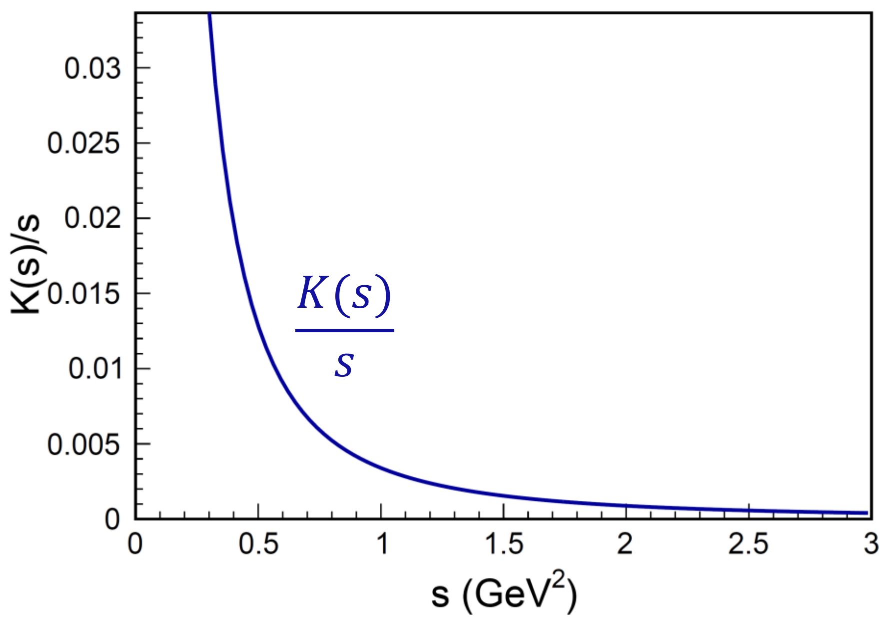


Data-driven method and R-ratio measurement

Leading order HVP contribution using dispersion relation

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)}{s}R(s)ds$$

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



Kernel function

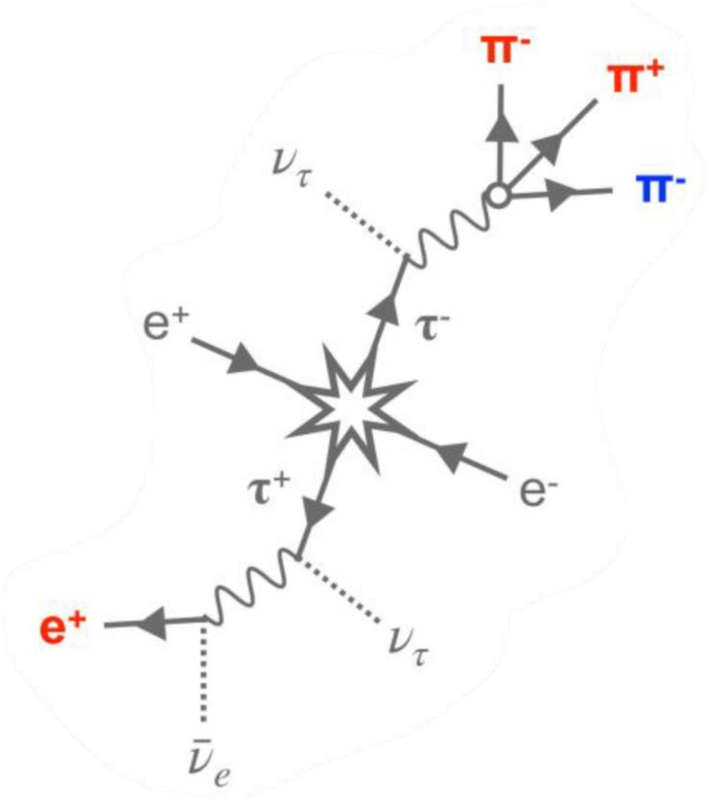
$$K(s) = \frac{x^2}{2}(2-x) + \frac{(1+x^2)(1+x)^2}{x^2}\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2\ln(x)$$

$$x = \frac{1-\beta_\mu}{1+\beta_\mu}, \qquad \beta_\mu = \sqrt{1-\frac{4m_\mu^2}{s}}$$

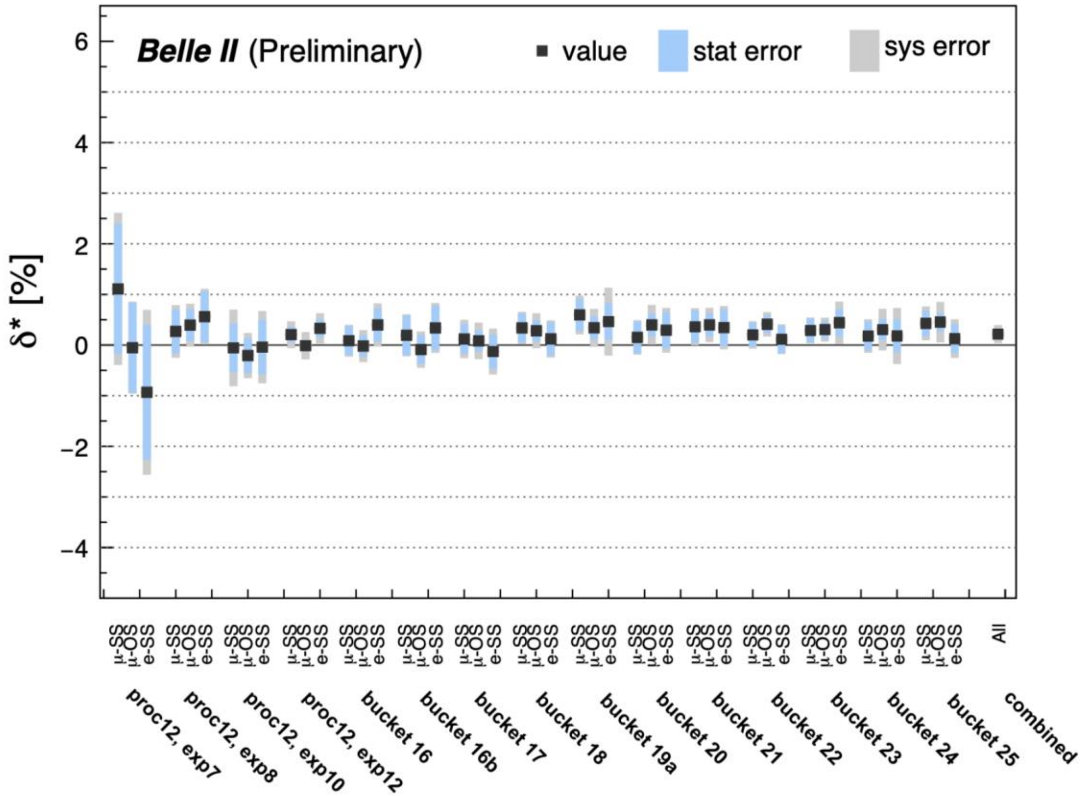
A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D101, 014029 (2020).

Performance : Tracking Efficiency

- Tracking efficiency is measured by tag-and-probe method on ee→ττ→1×3 prong.
  - 3 good quality tracks for tag
  - Look for 4th track for probe
- Uncertainty for tracking efficiency is 0.30% per track.

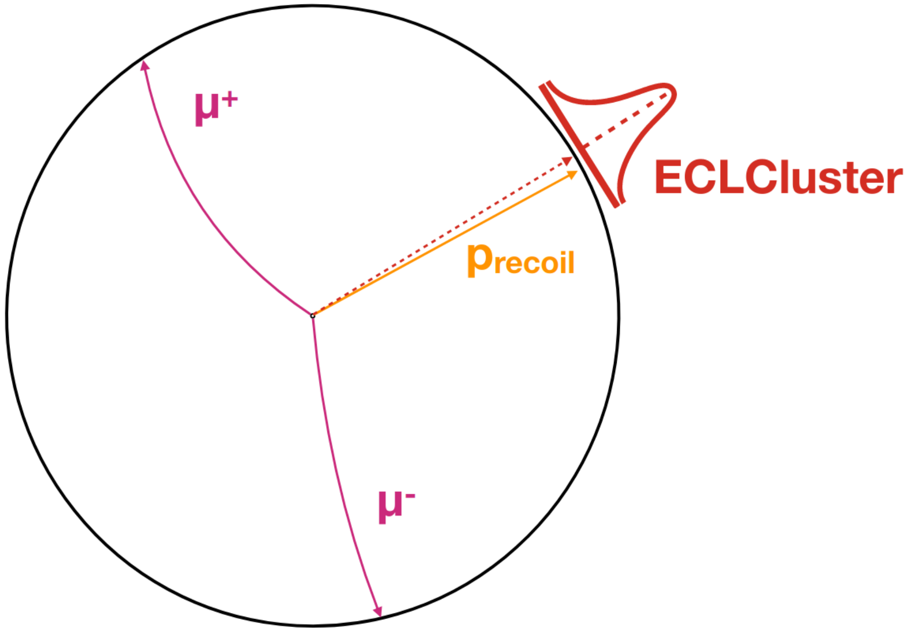


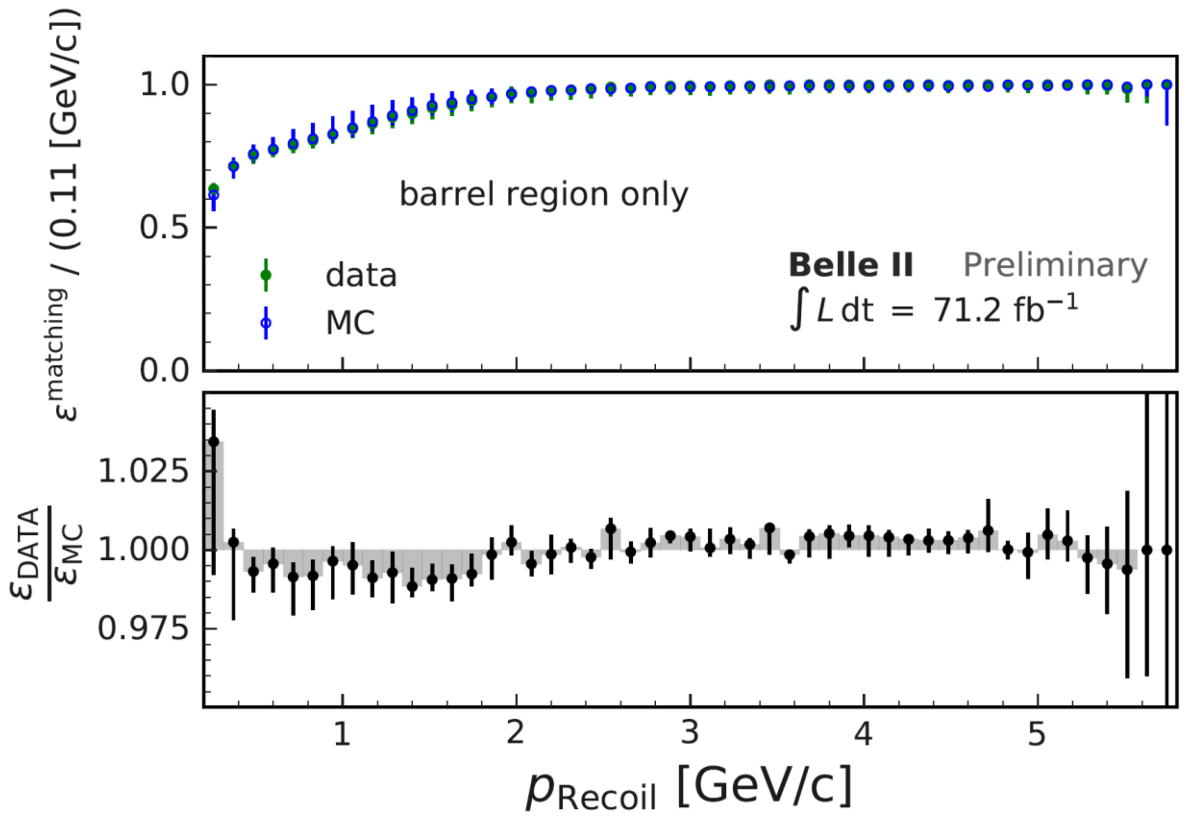
Data/MC discrepancy of tracking efficiency



Performance : Photon Detection Efficiency

- Photon detection efficiency is measured using ee→μμγ events.
  - Detection efficiency is estimated by taking match between a ECL cluster and the missing momentum of dimuon system.
- Data/MC agreement is good. Uncertainty for photon detection efficiency is 0.30%.

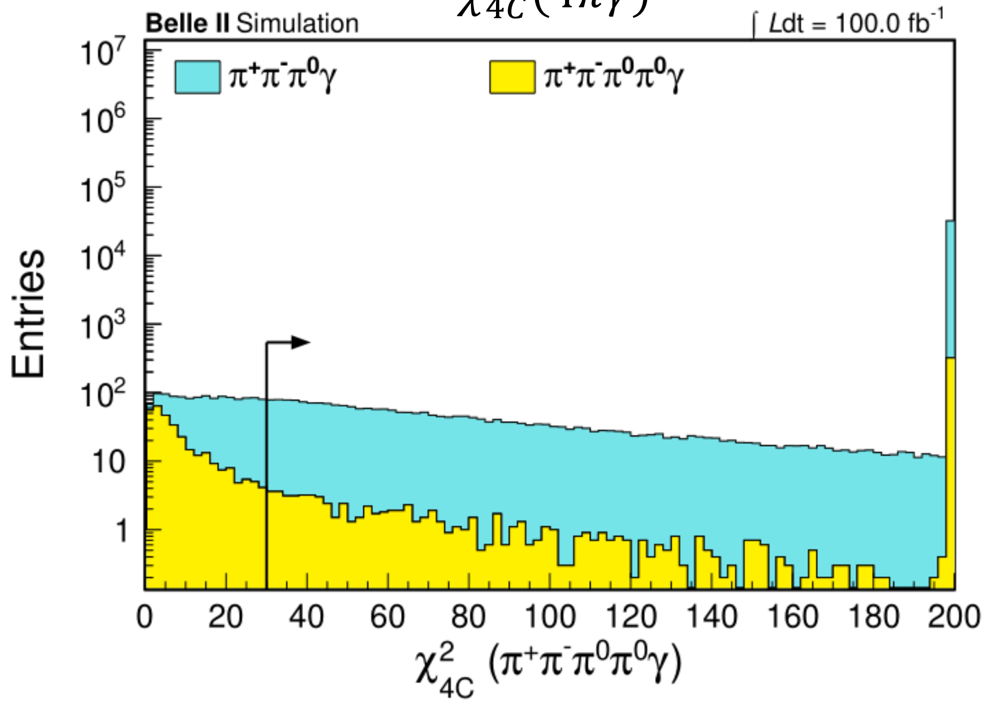


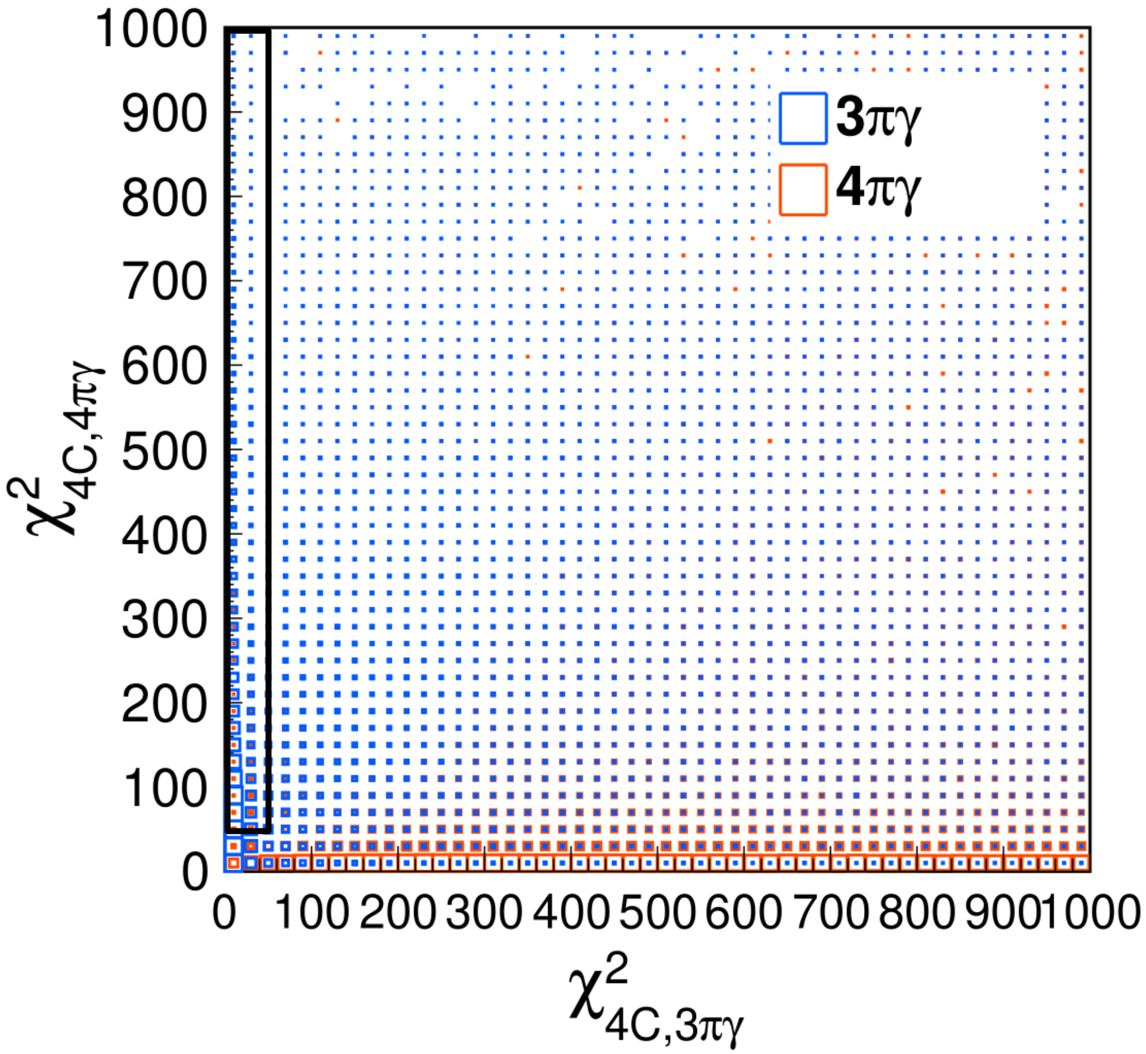


Event selection : Background reduction cuts

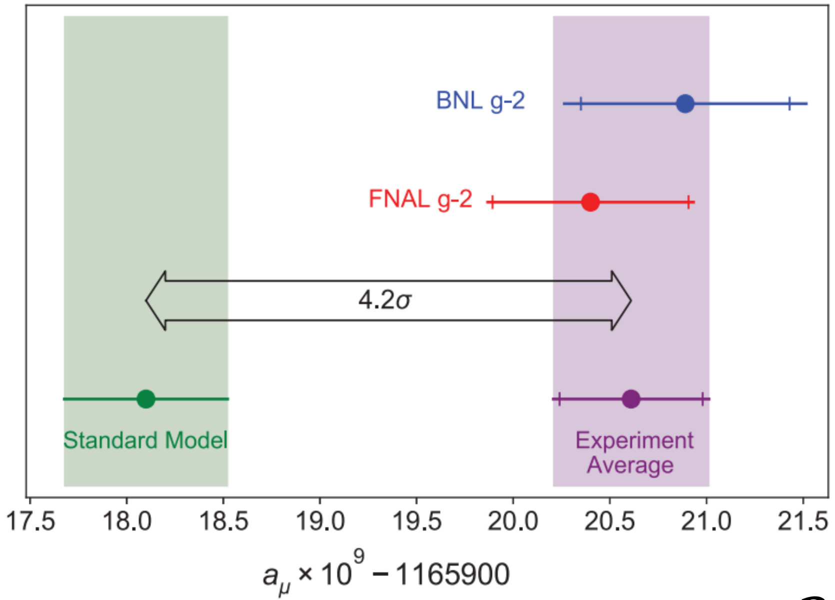
- $\pi^+\pi^-\pi^0\pi^0\gamma$ rejection
  - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ from additional π^0
  - Apply 4C-Kfit under $\pi^+\pi^-\pi^0\pi^0\gamma$ hypothesis
  - $\chi^2_{4C,4\pi\gamma} > 30$

$\chi^2_{4C}(4\pi\gamma)$



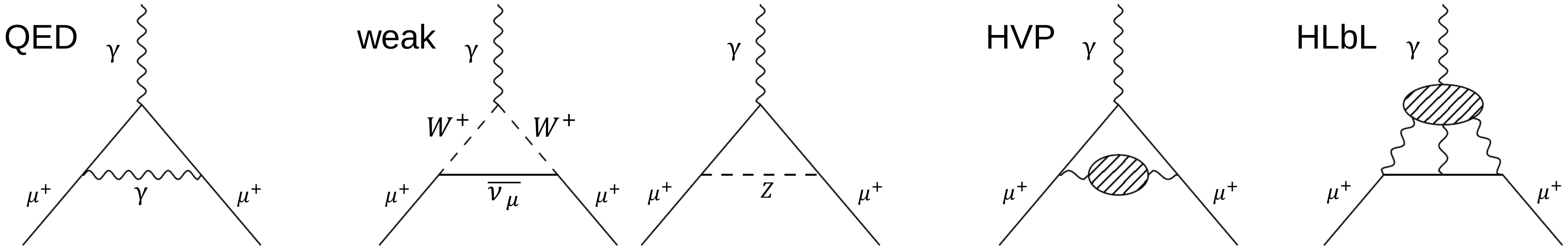


Introduction for muon g-2



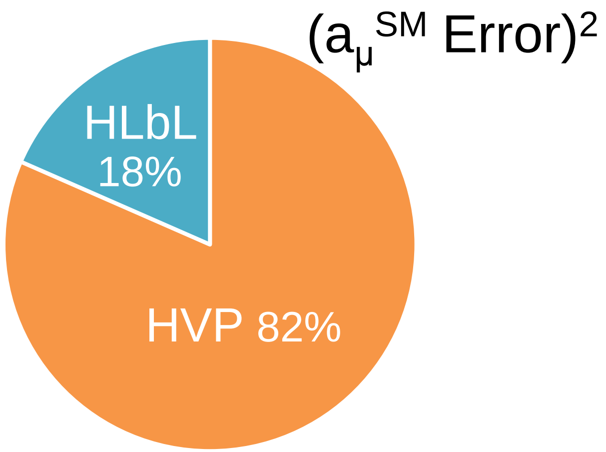
Contribution	Value $a_\mu \times 10^{11}$	Error $\delta a_\mu \times 10^{11}$
QED	116 584718.931	0.104
HVP LO (Leading-Order)	6931	40
HVP HO (Higher-Order)	-85.9	1.2
HLbL (Light-by-Light)	92	19
EW (Electroweak)	153.6	1
SM total (Dispersive)	**116591810**	**43**
Experiment (BNL+FNAL)	**116592061**	**41**
Experiment – SM	**251**	**59**

$$a_\mu^{\mathrm{SM}} = \frac{g-2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$



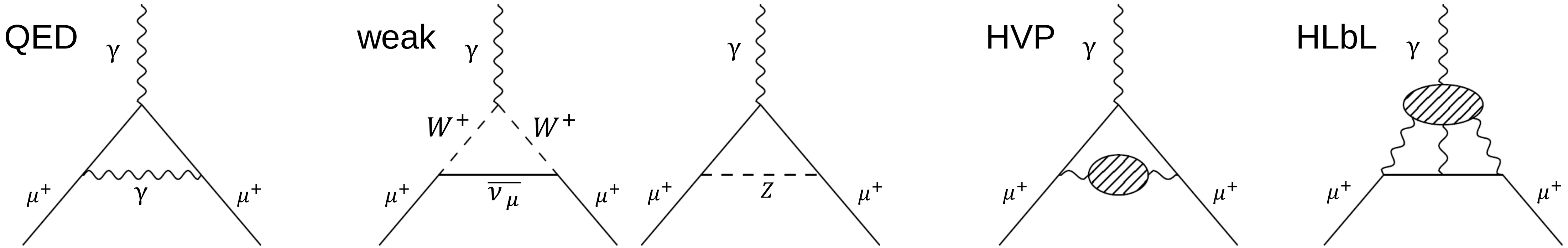
B. Abi *et al*,, PRL126, 141801 (2021)
T. Aoyama *et al.*, Phys. Rept. 887 (2020).

Introduction for muon g-2



Contribution	Value $a_\mu \times 10^{11}$	Error $\delta a_\mu \times 10^{11}$
QED	116 584718.931	0.104
HVP LO (Leading-Order)	6931	40
HVP HO (Higher-Order)	-85.9	1.2
HLbL (Light-by-Light)	92	19
EW (Electroweak)	153.6	1
SM total (Dispersive)	116591810	43
Experiment (BNL+FNAL)	116592061	41
Experiment – SM	251	59

$$a_\mu^{\mathrm{SM}} = \frac{g-2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$



B. Abi *et al,*, PRL126, 141801 (2021)
T. Aoyama *et al.*, Phys. Rept. 887 (2020).

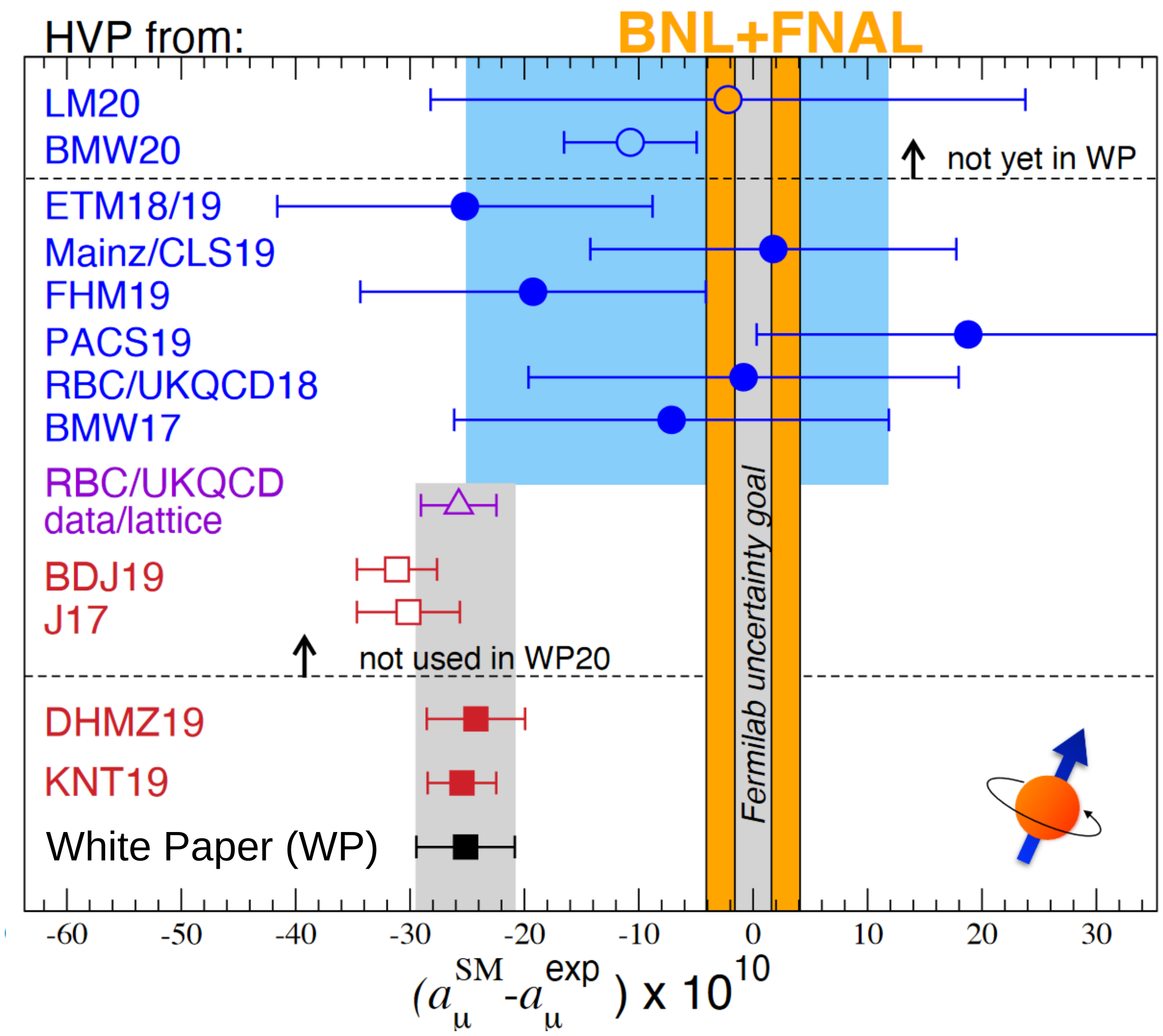
HVP contribution

G. Colangelo *et al.*, arXiv:2203.15810 [hep-ex] (2022)

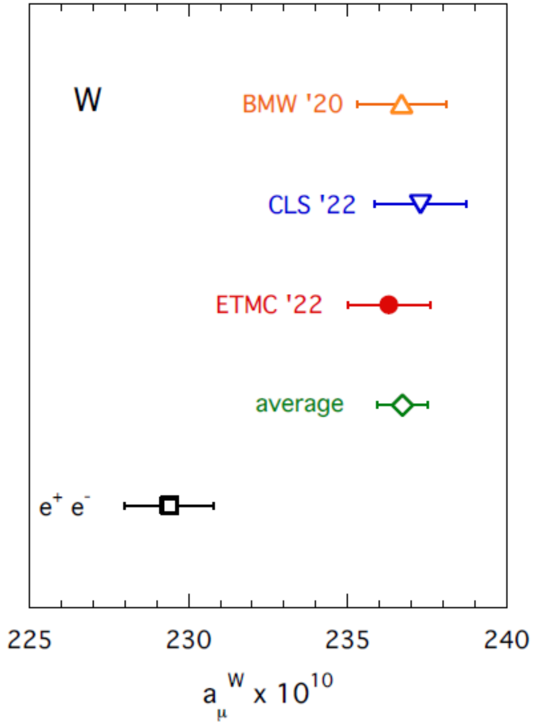
Lattice QCD

Data driven (Dispersive)

MUonE experiment (In future)

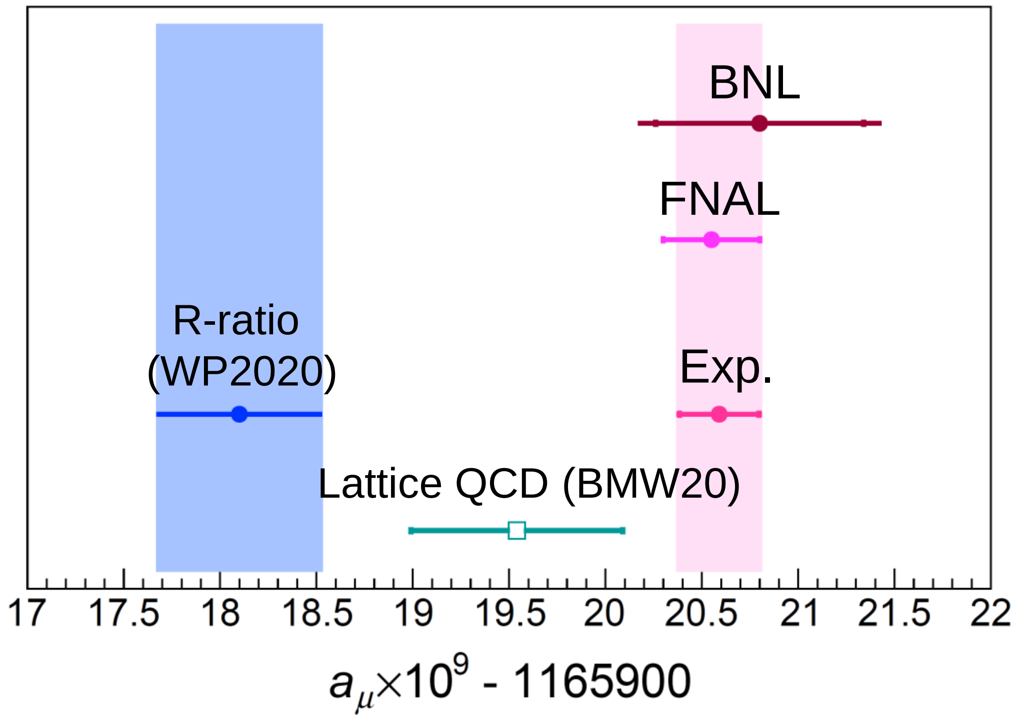


C. Alexandrou *et al.*, arXiv:2206.15084 [hep-lat] (2022)



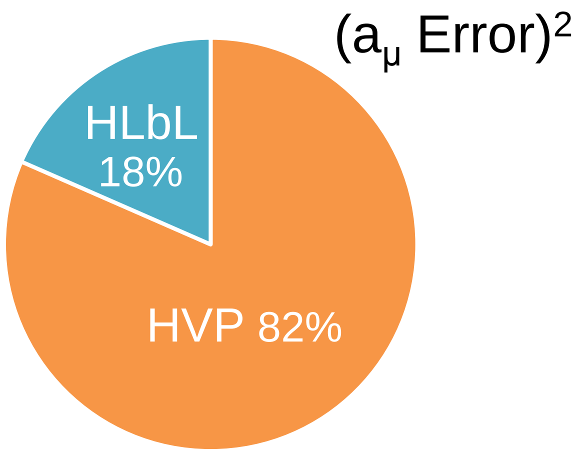
Muon g-2 and Hadronic Vacuum Polarization (HVP)

- HVP contributes to the largest uncertainty in the prediction of muon g-2.
- Two approaches for estimating the HVP contribution of SM predictions
  - Dispersion relations (w/ inputs from ee → hadrons data)
  - Lattice QCD
- Belle II can provide the cross section for $e^+e^- \rightarrow$ hadrons to improve the theoretical prediction.



D. P. Aguillard et al., arXiv:2308.06230 (2023)

$$a_\mu^{\mathrm{SM}} = \frac{g-2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{Had}}$$



HVP measurement

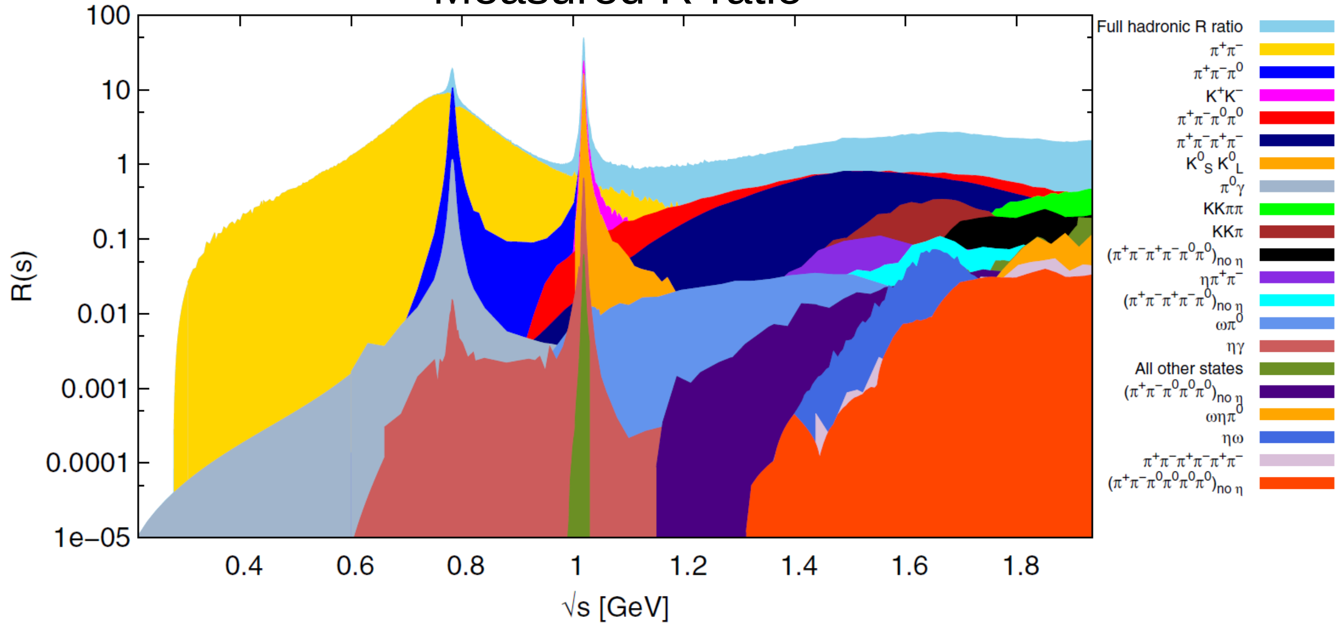
Leading order HVP contribution using dispersion relation :

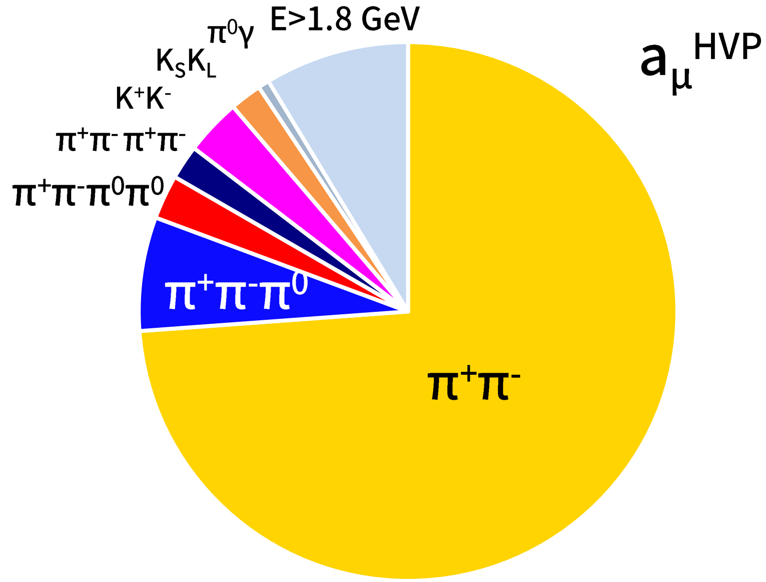
$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R(s)ds$$

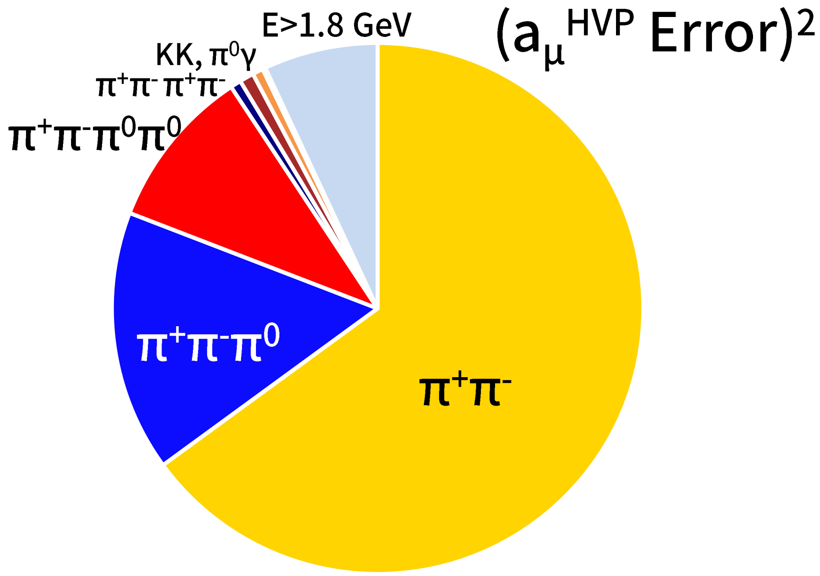
$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$K(s)$: QED kernel function

Measured R-ratio







A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D101, 014029 (2020).

R-ratio measurement

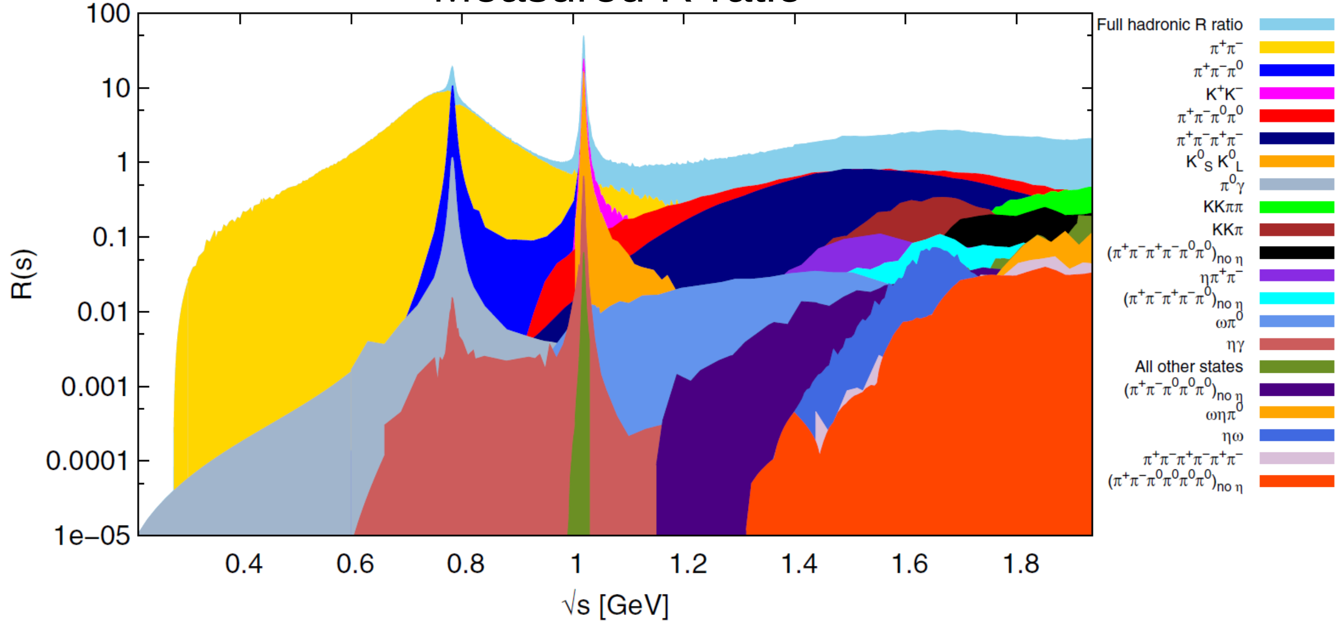
Leading order HVP contribution using dispersion relation :

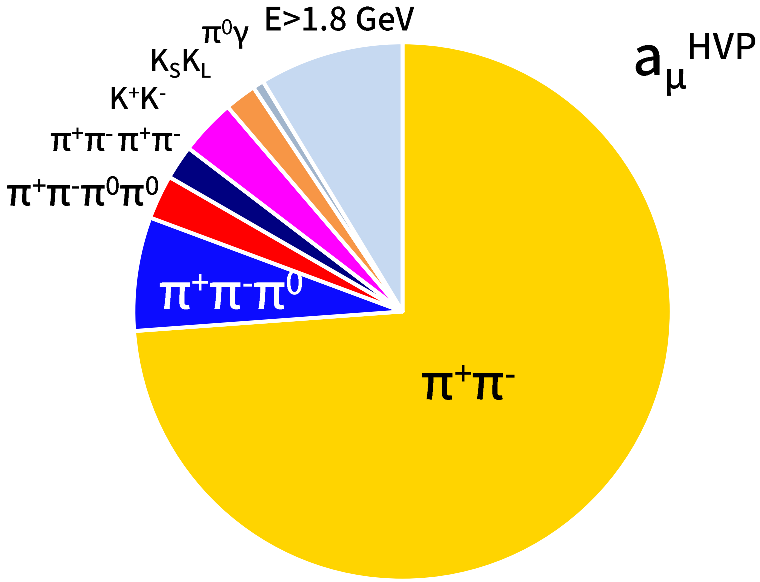
$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R(s)ds$$

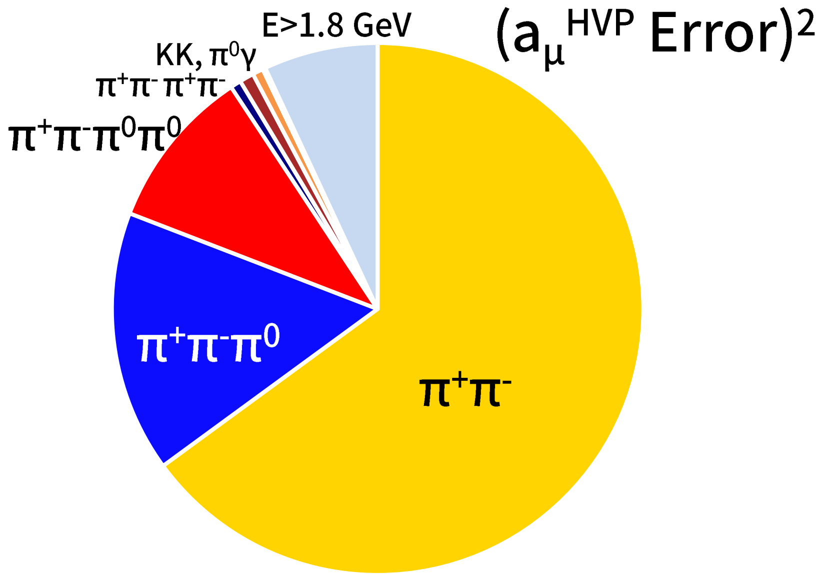
$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$K(s)$: QED kernel function

Measured R-ratio







A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D101, 014029 (2020).