Measurements of Michel parameters and tests of lepton universality in τ decays at Belle and Belle II

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17th International Workshop on Tau Lepton Physics (TAU2023)

Louisville, Kentucky

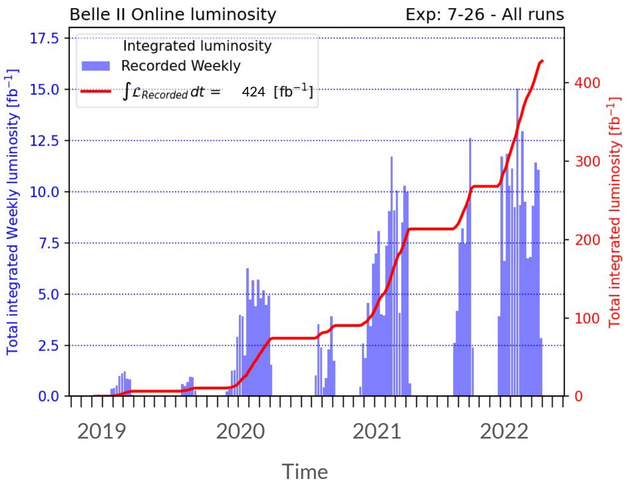
4th December 2023

τ physics at B factories

- Belle/Belle II are general purpose detectors: B and D physics, quarkonium, τ-physics, dark sector, …
- colliding electrons and positrons at $m_{\Upsilon(4S)}$=10.58 GeV/c^2

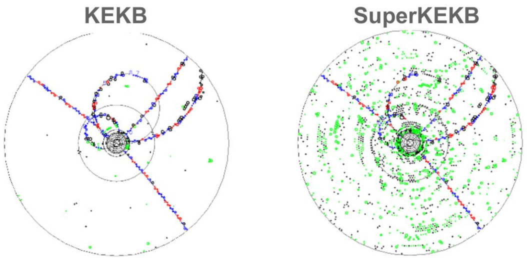
$$\sigma\left(e^+e^- \rightarrow \Upsilon(4S)\right) = 1.05\text{nb}$$
$$\sigma\left(e^+e^- \rightarrow \tau^+\tau^-\right) = 0.919\text{nb}$$

  - Belle: 988 $fb^{-1} \rightarrow$ **~908 million** τ pairs
  - Belle II: 424 $fb^{-1} \rightarrow$ **~390 million** τ pairs
- clean collision environment ($e^+ \leftrightarrows e^-$)
- large solid angle coverage (> 90%)
  - well known missing mass and energy
- good track reconstruction, particle identification



KEKB → SuperKEKB accelerator

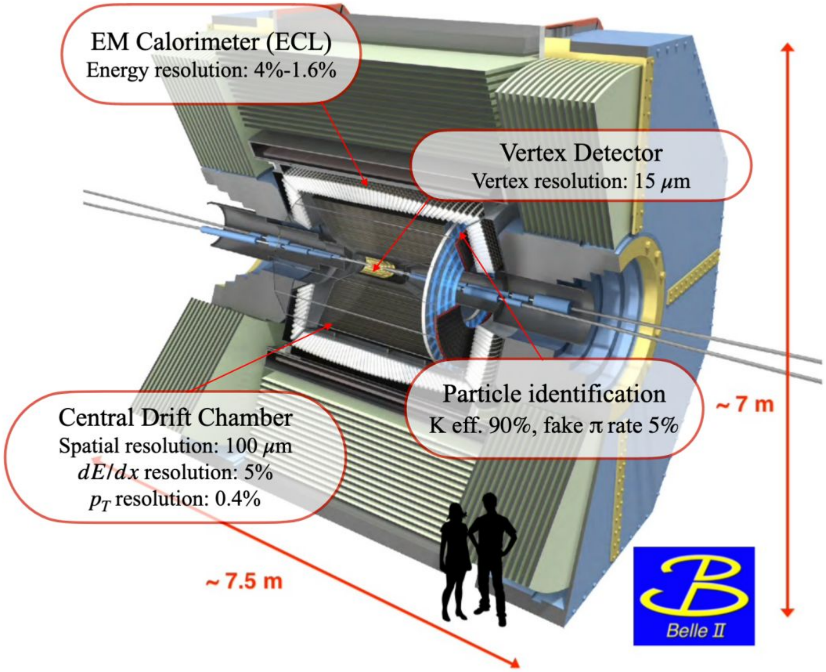
- 2x beam currents, 50 nm vertical beam spot size (“nano beam”)
- design peak luminosity $2.1\times10^{34} \rightarrow$ **6.0×10^{35} cm^{-2} s^{-1}**
- SuperKEKB currently holds world record (4.7×10^{34} cm^{-2}s^{-1})



challenge: increased beam backgrounds and trigger rates

Belle → Belle II detector

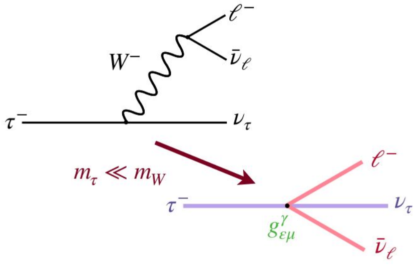
- new 2-layer Pixel Detector with first layer at 1.4cm
- 4-layer Silicon Vertex Detector with larger acceptance
- Central Drift Chamber with larger outer radius
- improved particle ID (K/π separation)
- improved trigger, and faster electronics in general



» arXiv:1011.0352 (Technical Design Report)

Michel parameters in leptonic decays

- Michel parameters (MP) of a lepton decay are bilinear combinations of coupling constants arising in the most general expression for the decay matrix element



$$= \frac{4G_F}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\overline{u}_i(\ell)\Gamma^N v_n(\nu_\ell)\right]\left[\overline{u}_m(\nu_\tau)\Gamma_N u_j(\tau)\right]$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = i\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)/2\sqrt{2}$$

Scalar Vector Tensor

- MP describe the Lorentz structure of the charged current in the theory of weak interaction and can be used to test the SM (only nonzero term in SM is $g_{LL}^V = 1$)
- deviations can be caused by anomalous coupling with the W-boson, new gauge or charged Higgs bosons, presence of massive neutrinos [1]

» [1] JHEP 2022 (2022) 117

Measurement of Michel parameters at Belle

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \frac{m_\tau}{4\pi^3} W_{\ell\tau}^4 G_F^2 \sqrt{x^2 - x_0^2} \left(F_{IS}(x) \pm F_{AS}(x) P_\tau \cos\theta + F_{T_1}(x) P_\tau \sin\theta \zeta_1 + F_{T_2}(x) P_\tau \sin\theta \zeta_2 + (\pm F_{IP}(x) + F_{AP}(x) P_\tau \cos\theta)\zeta_3 \right)$$

$$W_{\ell\tau} = \max E_\ell = \frac{m_\tau^2 + m_\ell^2}{2m_\tau},\ x = \frac{E_\ell}{\max E_\ell},\ x_0 = \frac{m_\ell}{\max E_\ell},\ P_\tau = |\boldsymbol{P}_\tau|$$

parametric functions	MPs	
$F_{IS}(x)$:	ρ, η	polarisation insensitive
$F_{AS}(x)$:	$\xi, \xi\delta$	
$F_{IP}(x)$:	$\xi', \xi, \xi\delta$	polarisation sensitive

	$\mu^- \to e^- \nu_\mu \overline{\nu}_e$	$\tau^- \to e^- \nu_\tau \overline{\nu}_e$	$\tau^- \to \mu^- \nu_\tau \overline{\nu}_\mu$	SM
ρ	0.74979 ± 0.00026	0.747 ± 0.010	0.763 ± 0.020	0.75
η	0.057 ± 0.034	–	0.094 ± 0.073	0
ξ	$1.0009^{+0.0016}_{-0.0007}$	0.994 ± 0.040	1.030 ± 0.059	1
$\xi\delta$	$0.7511^{+0.0012}_{-0.0006}$	0.734 ± 0.028	0.778 ± 0.037	0.75
ξ'	1.00 ± 0.04	–	0.22 ± 1.03	1
ξ''	0.65 ± 0.36	–	–	1

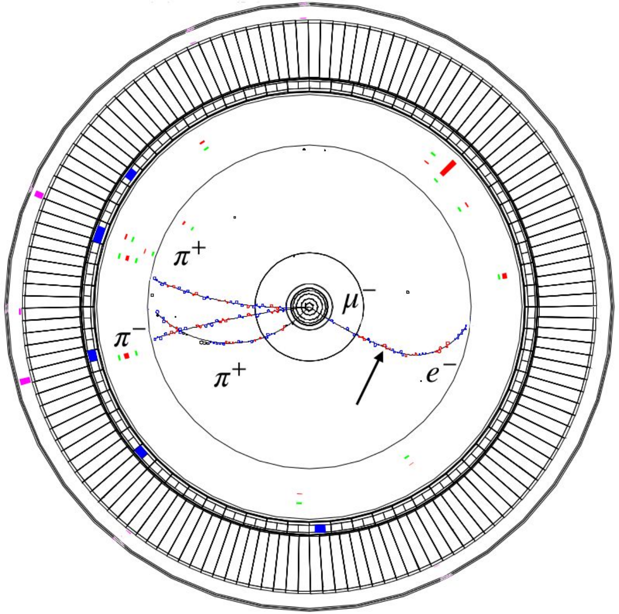
Michel parameters and their most accurate determinations [4]

- ongoing analysis for ϱ, η, ξ and $\xi\delta$ in τ decays [1]
  - ➢ statistical uncertainty at order 10^{-3}
    systematics around 10^{-2}
- measurement of $\bar{\eta}$ and $\xi\kappa$ in radiative leptonic τ-decays [2]
- new measurement of ξ' in $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ [3]

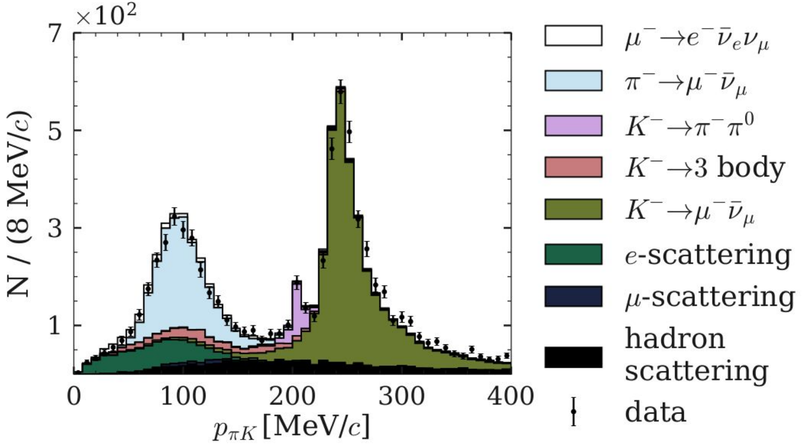
» [1] Nucl.Part.Phys.Proc. 287-288 (2017)
» [2] PTEP 2018 (2018) 2, 023C01
» [3] Phys.Rev.Lett. 131 (2023) 021801
» [4] JHEP 2022 (2022) 117

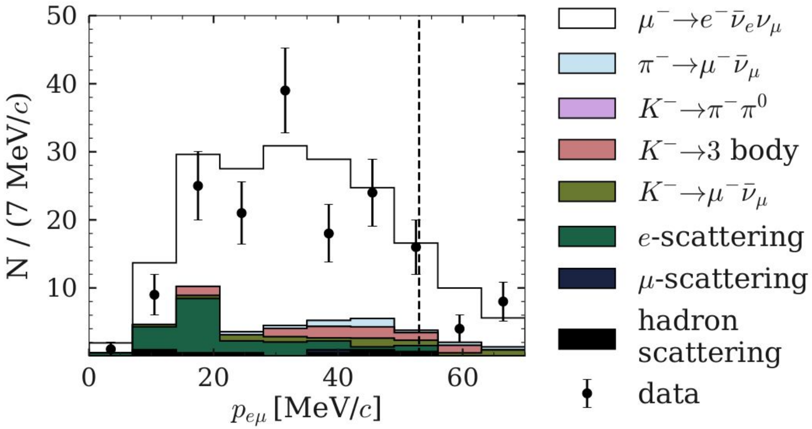
First measurement of the Michel parameter ξ′ in the $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ decay at Belle

- method uses muon decay-in-flight
- searching for kinks inside the tracking detector
- the information about muon spin can be inferred from the daughter electron direction in the muon rest frame due to P-violation in the decay
- using the full Belle data sample of 988 fb^{-1}



Kink candidates

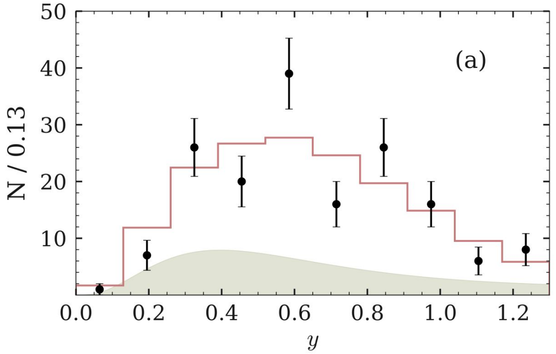




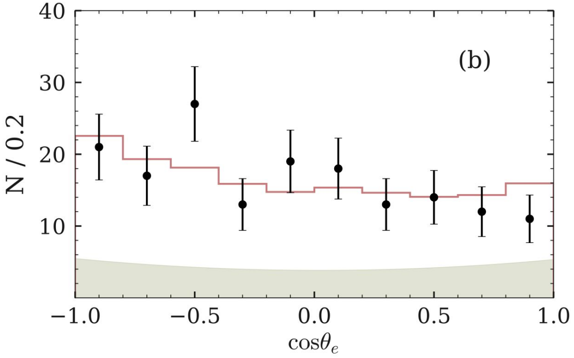
- daughter particle momentum in the rest frame of the mother particle with pion and kaon mass hypotheses
- main background sources are two body decays of π and K

- BDT is used to suppress background by 50 times with the signal efficiency $\varepsilon_{sig} \approx 80\,\%$

Result



$e^{\pm}$ energy in $\mu^{\pm}$ rest frame divided by $m_{\mu}/2$



angle of electron emission direction in the muon rest frame

- 2D unbinned likelihood fit using y and $\cos\theta_e$
- histogram is signal, filled area is the background function
- the measured value is **$\xi' = 0.22 \pm 0.94$ (stat) ± 0.42 (syst)** [1, 2]
  - total uncertainty is 1.03
- Belle II (with 50 ab^{-1}) can improve the statistical uncertainty up to $\sigma_{\xi'} \approx 7 \times 10^{-3}$ [3]
  - enlarged drift chamber, special kink reconstruction algorithm, higher integrated luminosity
  - GNN-based tracking algorithm for displaced tracks [4]
  - systematics can be controlled at the same level with various data samples with kinks

» [1] Phys.Rev.Lett. 131 (2023) 021801
» [2] Phys.Rev.D 108 (2023) 012003
» [3] JHEP 10 (2022) 035
» [4] CHEP 2023

Lepton universality tests at Belle II

- in the SM the electroweak gauge bosons have the same coupling to all generations of leptons
- precise test of μ-e universality by measuring

$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{\frac{\mathcal{B}\left(\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu(\gamma)\right)}{\mathcal{B}\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)\right)} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}} \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x \qquad [1]$$

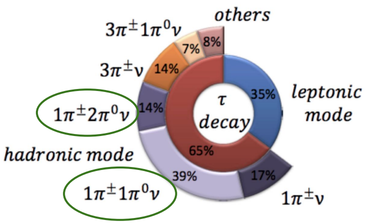
- ratio of leptonic branching fractions

$$R_\mu \equiv \frac{\mathcal{B}\left(\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu(\gamma)\right)}{\mathcal{B}\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)\right)} \stackrel{\text{SM}}{=} 0.9726$$

is sensitive to new physics if it violates lepton flavour [2] or lepton universality in weak charged-currents [3]

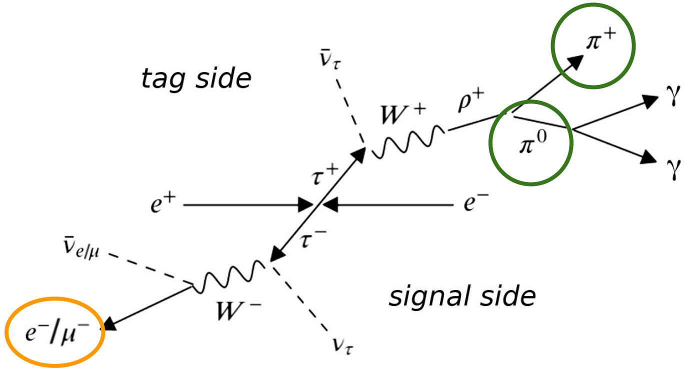
» [1] Phys.Rev.Lett. 61 (1988) 1815
» [2] arXiv:1607.06832
» [3] arXiv:2105.06734

Event topology



using 1-prong decays with one charged hadron and at least one π^0 on the tag side

- large BF (~35%), low backgrounds, high trigger efficiency
- Signal side selection
  - ➢ 1 track: particle ID requirement (μ or e)
- Tag side selection
  - ➢ 1 track: $E_{cluster} / p < 0.8$
  - ➢ $N(\pi^0)_{tag} > 0$



Dataset

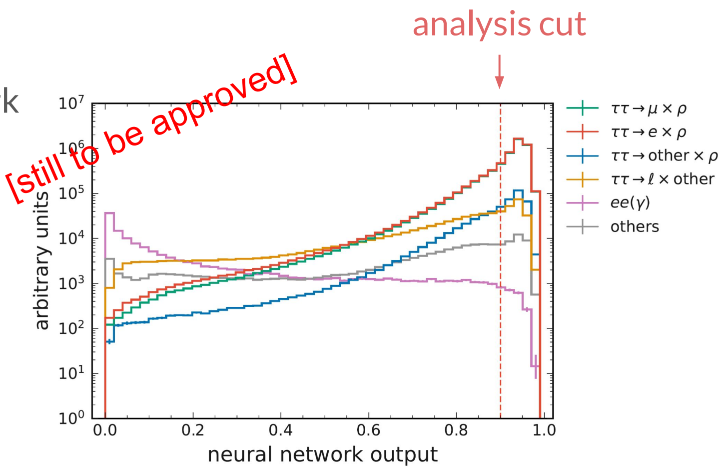
- on-resonance sample corresponding to 362 fb^{-1} (2019 - 2022)

Event selection

- Background suppression with rectangular cuts + neural network
- exactly same selection is used for e and μ sample

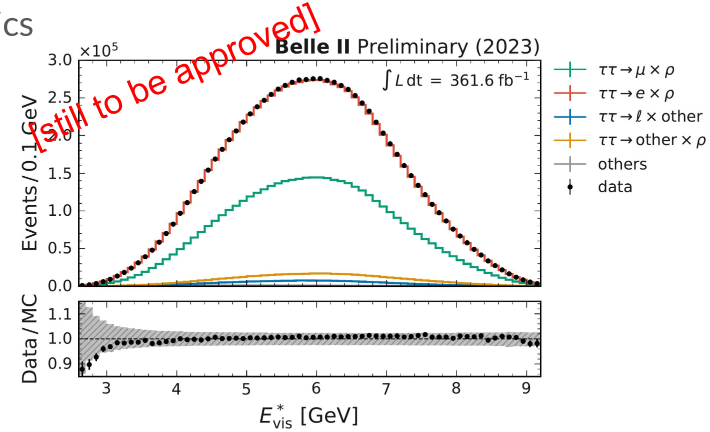
  - thrust value
  - thrust axis θ
  - total visible energy (CMS)
  - missing momentum: p_T, θ (CMS)
  - tag side: p, θ, M (CMS)

$$V_{thrust} \overset{max}{=} \frac{\sum_i |\vec{p}_i^{\,\mathrm{CM}} \cdot \hat{n}_{thrust}|}{\sum_i |\vec{p}_i^{\,\mathrm{CM}}|}$$



- restrict analysis to region least sensitive to particle ID systematics
  - $0.82 < \theta_{lepton} < 2.13$ (barrel of muon detector)
  - $1.5\ \mathrm{GeV} < p_{lepton} < 5.0\ \mathrm{GeV}$

- 94% purity @ 9.6% signal efficiency for combined e+μ sample

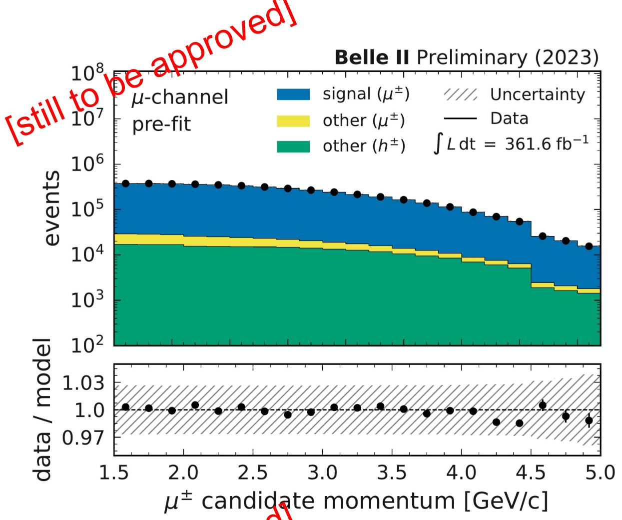


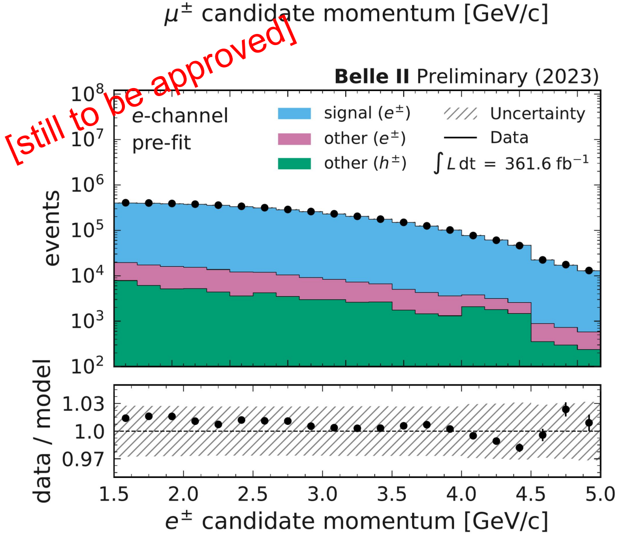
  - $e^+e^- \to \tau^+\tau^-$ ($\pi^\pm$ faking $\mu^\pm/e^\pm$): ~3.3%
  - $e^+e^- \to \tau^+\tau^-$ (wrong tag): ~2.3%
  - $e^+e^- \to e^+e^-\tau^+\tau^-$: 0.2%

main backgrounds

R_μ extraction

- measure R_μ with a *binned maximum likelihood* fit using the *pyhf* library [1]
- 21 bins defined over lepton momentum from 1.5 to 5 GeV
- systematics are included with (constrained) nuisance parameters that modify the templates
- 3 templates for μ and e channel
  - signal decays
  - background with correct lepton on the signal side
  - background with misidentified particle on the signal side





» [1] JOSS 6 (2021) 2823

Systematics

Particle identification (leading) **(0.32%)**

- correction factors and uncertainties derived from calibration channels
  - eff.: $J/\psi \to \ell^+\ell^-$, $e^+e^- \to e^+e^-\ell^+\ell^-$ and $e^+e^- \to \ell^+\ell^-(\gamma)$
  - fakes: $K_S^0 \to \pi^+\pi^-$ and $\tau^\pm \to \pi^\pm\pi^\mp\pi^\pm\nu_\tau$

Trigger (sub-leading) **(0.10%)**

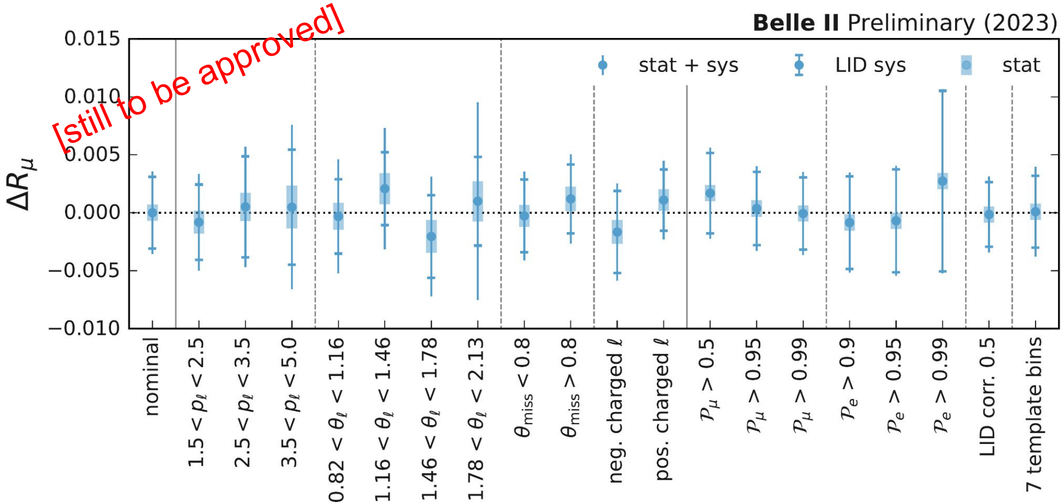
- used triggers are based on EM calorimeter information, targeting low multiplicity events
  - most important: E_{ECL}>1 GeV trigger
- correction factor for MC obtained directly from data
  - ε=99.8% for $\tau^-\to e^-\bar{\nu}_e\nu_\tau$ and ε=96.6% for $\tau^-\to\mu^-\bar{\nu}_\mu\nu_\tau$

Source	Uncertainty [%]
Charged-particle identification:	
Electron identification	0.22
Muon misidentification	0.19
Electron misidentification	0.12
Muon identification	0.05
Trigger	0.10
Imperfections of the simulation:	
Modelling of FSR	0.08
Normalisation of individual processes	0.07
Modelling of the momentum distribution	0.06
Tag side modelling	0.05
π^0 efficiency	0.02
Modelling of ISR	0.01
Photon efficiency	< 0.01
Photon energy	< 0.01
Size of the samples	
Simulated samples	0.06
Luminosity	0.01
Charged-particle reconstruction:	
Particle decay-in-flight	0.02
Tracking efficiency	0.01
Detector misalignment	< 0.01
Momentum correction	< 0.01
Total	0.37

relative systematic uncertainties of R_μ

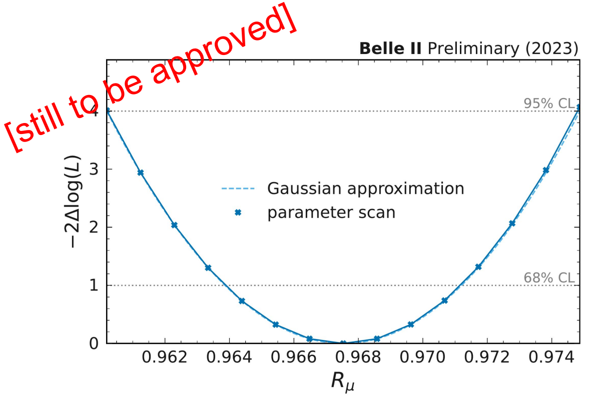
Stability of the result

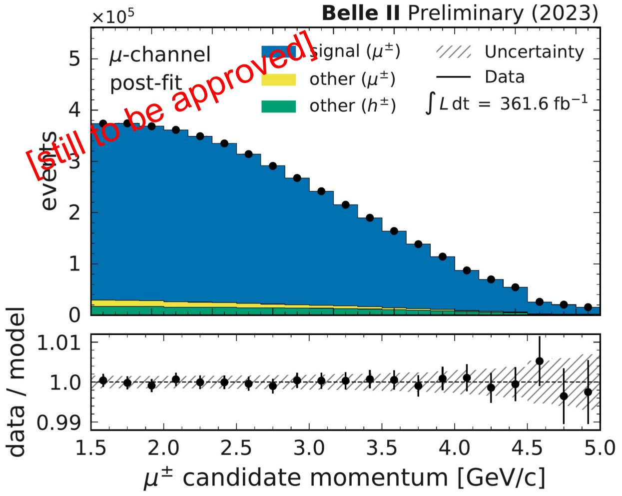
- checked for consistency of the result before unblinding
- sub-regions for different kinematic variables (momentum, polar angle, missing momentum, charge), data-taking periods as well as different requirements for particle-identification
- good agreement between the measured values

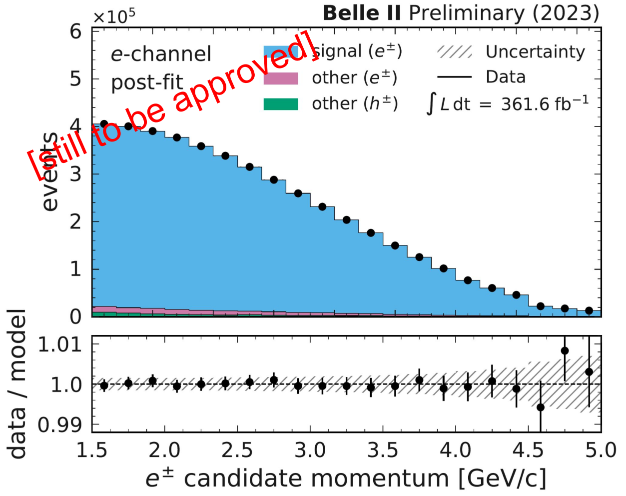


Result

- we measure R_μ = **0.9675 ± 0.0037**
  - statistical uncertainty 0.0007
  - systematic uncertainty 0.0036
- consistent with SM expectation at the level of 1.4 sigma

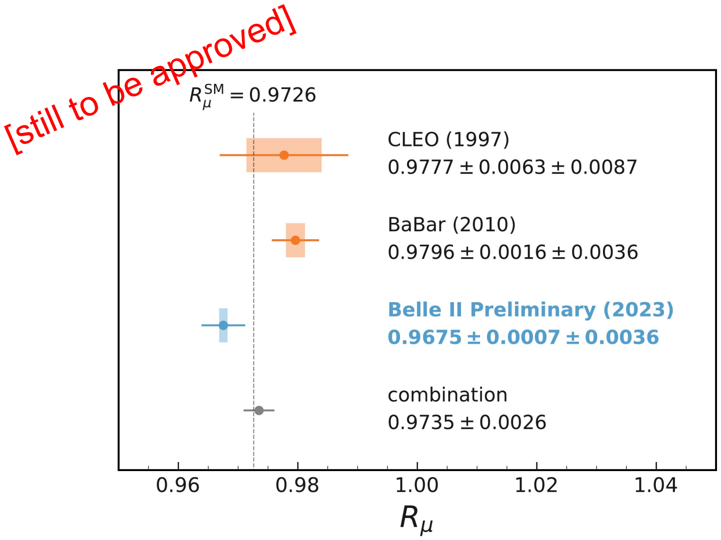


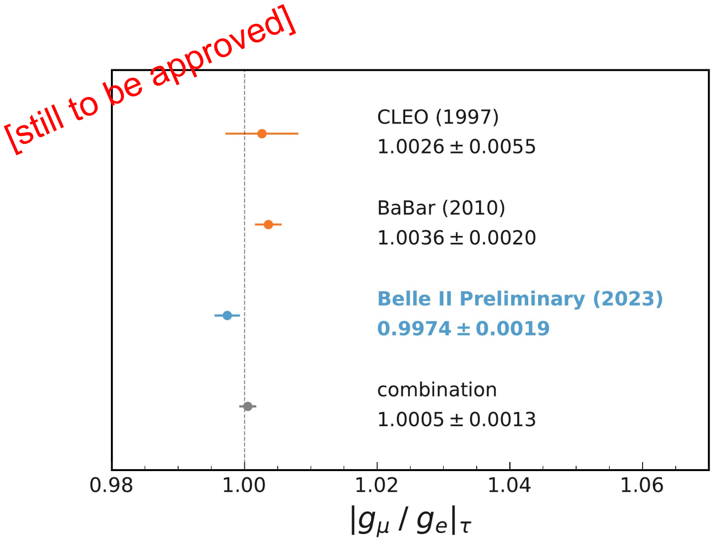




Result

$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{R_\mu \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}}$$



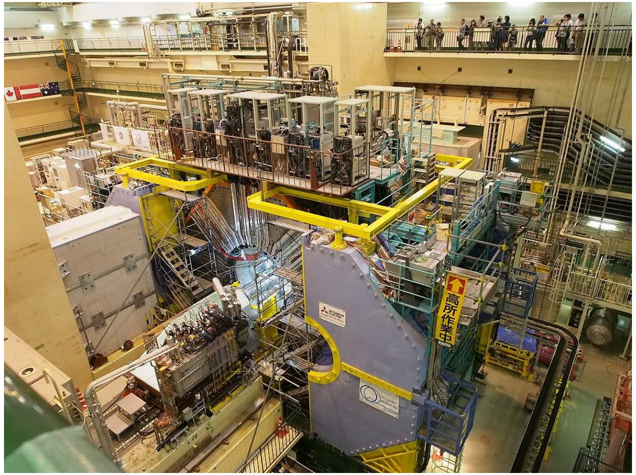


- most precise test of μ-e universality in τ decays from a single measurement
  - determination of g_μ/g_e from global fit to tau BFs: 1.0019 ± 0.0014 [1]
- combination of CLEO, BaBar and Belle II (assuming indep. systematics) yields g_μ/g_e = 1.0005 ± 0.0013

» [1] HFLAV, Phys.Rev.D 107 (2023) 052008

Summary

- Belle data (988 fb^{-1}) is still being actively analysed
  - first measurement of the Michel parameter ξ' using a novel method
- Belle II recorded 424 fb^{-1} so far, resuming data taking soon
- new results for test of μ-e universality
  - world's best determination from single measurement
  - similar systematics as BaBar measurement
  - still room for improvement in systematics
- more (τ) physics to come from Belle/Belle II !



Thank you!

backup slides ⤵

Radiative and five-body leptonic τ decays

- Radiative and five-body leptonic τ-decays provide information about Michel parameters that describe daughter lepton polarization in $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$
- Their understanding is also crucial for LFV studies as they are main background

Radiative leptonic τ-decay

$$\frac{d\Gamma(\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma)}{dE_\ell \, d\Omega_\ell \, dE_\gamma \, d\Omega_\gamma} = (A_0 + \bar{\eta} A_1) + \left(\vec{B}_0 + \xi\kappa \vec{B}_1\right) \cdot \vec{S}_\tau$$

$$\xi\kappa = -1/4(\xi + \xi') + 2/3\xi\delta$$
$$\bar{\eta} = 4/3\rho - 1/4\xi'' - 3/4$$

Belle collaboration measured $\xi\kappa(e) = -0.4 \pm 1.2$, $\xi\kappa(\mu) = 0.8 \pm 0.6$, and $\bar{\eta}(\mu) = -1.3 \pm 1.7$ ($\mathscr{L} = 711\ \text{fb}^{-1}$) PTEP 2018 (2018) 2, 023C01

Five-body leptonic τ-decay

Belle estimations for $\mathscr{L} = 700\ \text{fb}^{-1}$

Mode	SM Br	Measured	Expected N	Systematics
$\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau$	$4.21(1) \times 10^{-5}$	$(1.8 \pm 1.5) \times 10^{-5}$	$1300\ (r_s = 47\%)$	$(6-12)\ \%$
$\tau^- \to \mu^- e^+ e^- \bar{\nu}_e \nu_\tau$	$1.984(4) \times 10^{-5}$	$< 3.2 \times 10^{-5}\ (90\%)$	$430\ (r_s = 50\%)$	$(8-13)\ \%$
$\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$	$1.247(1) \times 10^{-7}$	NM	$8\ (r_s = 37\%)$	$(36-72)\ \%$
$\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau$	$1.183(1) \times 10^{-7}$	NM	$4\ (r_s = 16\%)$	$(36-72)\ \%$

JHEP 04 (2016) 185 J.Phys.Conf.Ser. 912 (2017) 1

Belle II can repeat with better precision!

τ lepton polarization at Belle II

- The beams at Belle II are not polarized, so average τ lepton polarization is zero. Nevertheless, spins of τ leptons are correlated in $e^+e^- \to \tau^+\tau^-$:

$$\frac{d\sigma(e^+e^-(w^-) \to \tau_{\text{sig}}(\vec{s}_{\text{sig}})\tau_{\text{tag}}(\vec{s}_{\text{tag}}))}{d\Omega_\tau} = \frac{\alpha^2\beta}{64E^2}\left[A_0 + D_{ij}(\vec{s}_{\text{sig}})_i(\vec{s}_{\text{tag}})_j\right]$$

$$A_0 = 1 + \cos^2\theta_\tau + \frac{\sin^2\theta_\tau}{\gamma^2} \qquad D_{ij} = \begin{pmatrix} \left(1+\frac{1}{\gamma^2}\right)\sin^2\theta_\tau & 0 & \frac{1}{\gamma}\sin 2\theta_\tau \\ 0 & -\beta^2\sin^2\theta_\tau & 0 \\ \frac{1}{\gamma}\sin 2\theta_\tau & 0 & 1+\cos^2\theta_\tau - \frac{\sin^2\theta_\tau}{\gamma^2} \end{pmatrix}$$

- One can use tagging τ lepton as a spin analyzer with the decay mode $\tau^+ \to \pi^+\pi^0\bar{\nu}_\tau$. This mode has the largest branching fraction (around $25\ \%$), and it is also well-studied

Leptonic differential decay width parametric functions definition

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$F_{AS}(x) = \frac{1}{3}\xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 3 - \frac{x_0^2}{2}\right)\right]$$

$$F_{IP}(x) = \frac{1}{54}\sqrt{x^2 - x_0^2}\left[-9\xi'\left(2x - 3 + \frac{x_0^2}{2}\right) + 4\xi\left(\delta - \frac{3}{4}\right)\left(4x - 3 - \frac{x_0^2}{2}\right)\right]$$

$$F_{AP}(x) = \frac{1}{6}\left[\xi''\left(2x^2 - x - x_0^2\right) + 4\left(\rho - \frac{3}{4}\right)\left(4x^2 - 3x - x_0^2\right) + 2\eta'' x_0(1-x)\right]$$

$$F_{T_1}(x) = -\frac{1}{12}\left[2\left(\xi'' + 12\left(\rho - \frac{3}{4}\right)\right)(1-x)x_0 + 3\eta(x^2 - x_0^2) + \eta''(3x^2 - 4x + x_0^2)\right]$$

$$F_{T_2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2}\left(3\frac{\alpha'}{A}(1-x) + \frac{\beta'}{A}(2 - x_0^2)\right)$$

MP parameters through coupling constants

$$\rho = \frac{3}{4} - \frac{3}{4}\left[\left(|g^V_{RL}|^2 + |g^V_{LR}|^2\right) + 2\left(|g^T_{LR}|^2 + |g^T_{RL}|^2\right) + \Re\left\{g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR}\right\}\right]$$

$$\eta = \frac{1}{2}\Re\left\{g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right) + g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) + \left(g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}\right)\right\}$$

$$\xi = 4\Re\left\{g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL}\right\} + \left(|g^V_{LL}|^2 - |g^V_{RR}|^2\right) + 3\left(|g^V_{LR}|^2 - |g^V_{RL}|^2\right)$$
$$+5\left(|g^T_{LR}|^2 - |g^T_{RL}|^2\right) + \frac{1}{4}\left(|g^S_{LL}|^2 - |g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2\right)$$

$$\xi\delta = \frac{3}{16}\left(|g^S_{LL}|^2 - |g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2\right) + \frac{3}{4}\left(|g^V_{LL}|^2 - |g^V_{RR}|^2 - |g^T_{LR}|^2\right.$$
$$\left. + |g^T_{RL}|^2 + \Re\left\{g^S_{LR}g^{T*}_{LR} - g^S_{RL}g^{T*}_{RL}\right\}\right)$$

MP parameters through coupling constants (2)

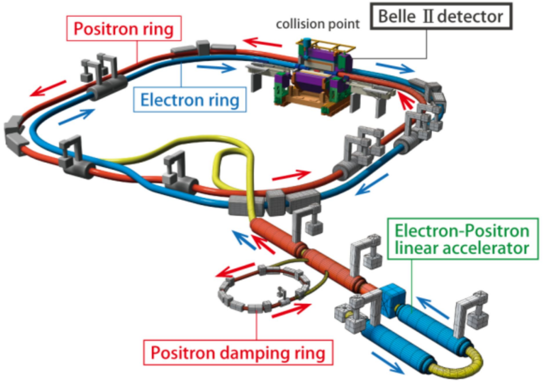
$$\xi' = -\left[3\left(|g^T_{RL}|^2 - |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 - |g^V_{LR}|^2 - |g^V_{LL}|^2\right) + \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 - |g^S_{LR}|^2 - |g^S_{LL}|^2\right)\right]$$

$$\xi'' = 1 - \frac{1}{2}\left(|g^S_{RL}|^2 + |g^S_{LR}|^2\right) + 2\left(|g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^T_{RL}|^2 + |g^T_{LR}|^2\right) + 4\mathfrak{R}\left\{g^S_{RL}g^{T*}_{RL} + g^S_{LR}g^{T*}_{LR}\right\}$$

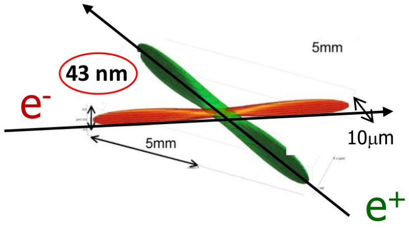
$$\eta'' = \frac{1}{2}\mathfrak{R}\left\{3g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right) + 3g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) - \left(g^V_{RR}g^{S*}_{LL} + g^V_{LL}g^{S*}_{RR}\right)\right\}$$

$$\frac{\alpha'}{A} = \frac{1}{2}\mathfrak{I}\left\{g^V_{LR}\left(g^{S*}_{RL} + 6g^{T*}_{RL}\right) - g^V_{RL}\left(g^{S*}_{LR} + 6g^{T*}_{LR}\right)\right\}$$

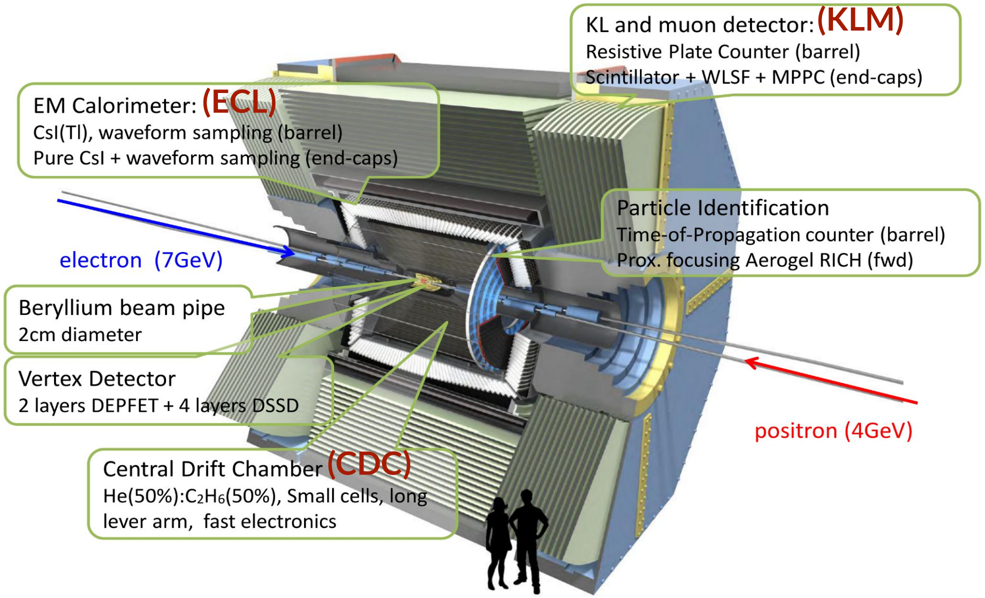
$$\frac{\beta'}{A} = \frac{1}{4}\mathfrak{I}\left\{g^V_{RR}g^{S*}_{LL} - g^V_{LL}g^{S*}_{RR}\right\}$$



SuperKEKB accelerator @Tsukuba, Japan



Belle II detector



» arXiv:1011.0352 (Technical Design Report)