Partial-wave analysis of $\tau^{\mp} \to \pi^{\mp}\pi^{\mp}\pi^{\pm}\nu_{\tau}$

Monday, December 4th, 2023

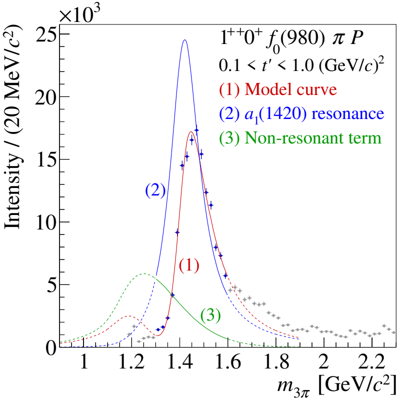
$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$: Motivation

Searches for new particles and interactions not part of the Standard Model: need to know various hadron form-factors

- Disagreement in measurements of $a_1(1260)$ parameters
- COMPASS observed* narrow peak $a_1(1420)$
  - Isospin partner of $f_1(1420)$?
  - K^*K rescattering?
- $\tau \to 3\pi\nu$ provides X-check for COMPASS 3π partial wave analysis (PWA) in different experimental conditions

Improve current model in event generators

$a_1(1420)$ observation at COMPASS



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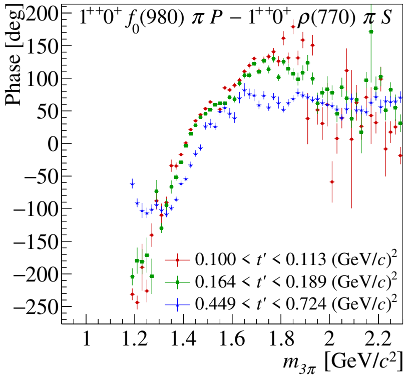

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Only amplitude analysis in $\tau \to 3\pi\nu_\tau$: CLEO-II

Wave	Branching
$1^+[\rho(770)\pi]_S$	68.11%
$1^+[\sigma\pi]_P$	16.18%
$1^+[f_0(1370)\pi]_P$	4.29%
$1^+[\rho(1450)\pi]_D$	0.43%
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Different channel ($\pi^\pm\pi^0\pi^0$)

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Same $a_1(1260)$ shape in all states

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No $1^+[f_0(980)\pi]_P$ wave discovered

BELLE@KEKB

KEKB is asymmetric e^+e^- collider

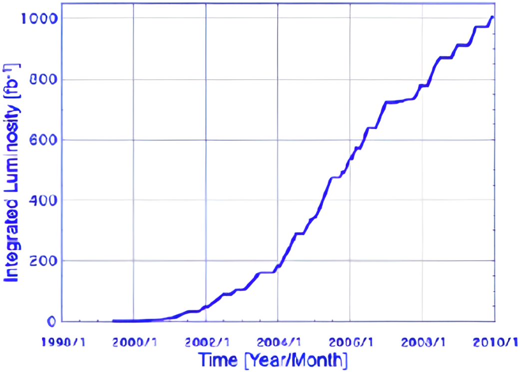
$E_{e^+} = 3.5$ GeV, $E_{e^-} = 8$ GeV

Total luminosity: 980 fb^{-1}

- $\Upsilon(4S)$: 711 fb^{-1}
- 0.9×10^9 tauon pairs produced

Tauon:

- $m = 1.777$ GeV
- $c\tau = 86$ µm
- $\mathcal{B}(\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu_\tau) = 9\%$



Integrated luminosity of Belle

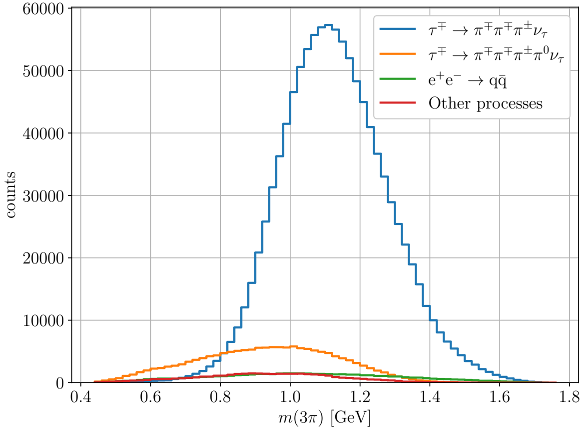
Event-selection criteria

Selection criteria summary:

	Previous*	Current
Efficiency	22 %	32 %
Purity	89 %	82 %
# of events	9×10^6	55×10^6

Major background components:

$\tau \to 3\pi^\mp\pi^0\nu_\tau$	12 %
$e^+e^- \to q\bar{q}$	4 %
$\tau \to K^\mp 2\pi^\mp\nu_\tau$	1 %
$\tau \to 3\pi^\mp N\pi^0\nu_\tau, N \geq 2$	0.8 %

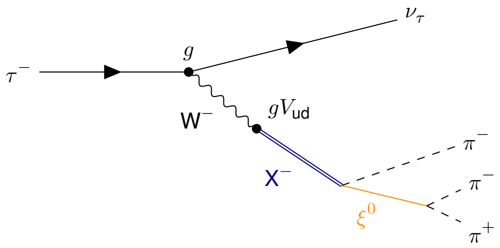


Simulated $m_{3\pi}$ spectrum

Decay rate

* Eur.Phys.J.C 81 (2021) 12, 1073

7D phase space



Decay rate

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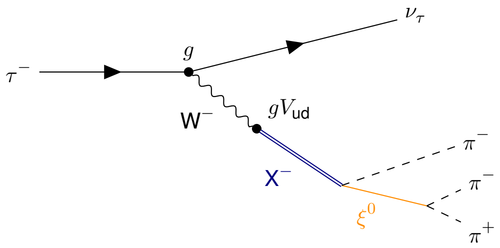

Average intensity over tauon azimuthal angle

Decay rate

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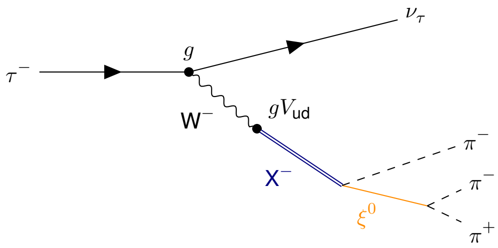
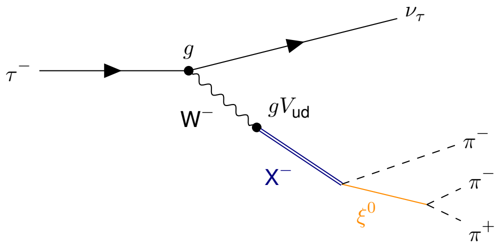


Average intensity over tauon azimuthal angle

Decompose hadron current* J^μ_{had} into partial waves

Decay rate

7D phase space



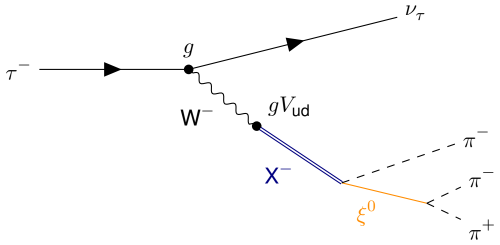
Average intensity over tauon azimuthal angle

Decompose hadron current* J^μ_{had} into partial waves

$$J^\mu_{\text{had}} = \sum_w \mathcal{C}_w j^\mu_w$$

Decay rate

7D phase space



Average intensity over tauon azimuthal angle

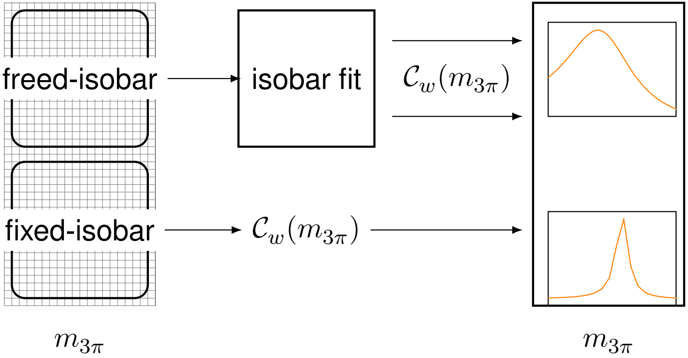
Decompose hadron current* J^μ_{had} into partial waves

$$J^P[\xi^0\pi]_L,$$ 16 partial waves

Fitting strategy

Partial-wave decompositon in 5D

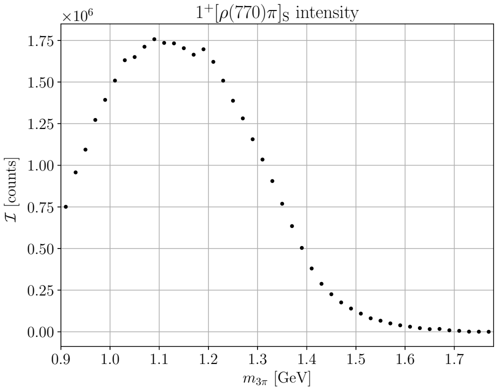
Resonance-model fit in 1D



Decomposition results

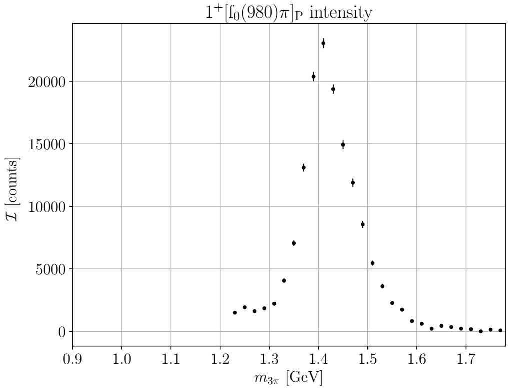
Partial wave	$\mathbb{B}$ [%]
$1^+[\rho(770)\pi]_S$	97.37
$1^+[\sigma\pi]_P$	11.57
$1^+[\rho(770)\pi]_D$	2.48
$1^+[\rho(1450)\pi]_S$	1.77
$1^+[f_0(980)\pi]_P$	0.41
$1^+[f_2(1270)\pi]_P$	0.11
$1^+[f_0(1370)\pi]_P$	0.06
$1^+[f_2(1270)\pi]_F$	0.02
$1^+[\rho(1450)\pi]_D$	0.01
$1^+[f_0(1500)\pi]_P$	0.00
$0^-[\rho(770)\pi]_P$	0.36
$0^-[\sigma\pi]_S$	0.26
$0^-[f_2(1270)\pi]_D$	0.02
$0^-[f_0(980)\pi]_S$	0.01

$1^+[\rho(770)\pi]_S$ intensity

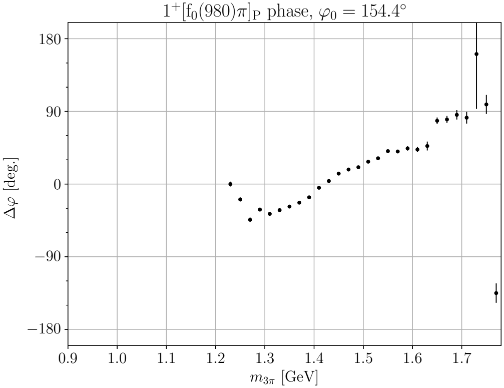


$1^{+}[f_0(980)\pi]_P$-wave

Intensity



Phase



Freed isobar PWA

Conventional PWA:

- Isobar's shape $\Delta(s)$ is fixed
- One complex coefficient $\mathcal{C}_w$ per wave

Breit-Wigner (BW) parametrization:

$$\Delta_\xi(s) = \text{BW}_\xi(s) = \frac{m_\xi^2}{m_\xi^2 - s - i\sqrt{s}\Gamma(s)}$$

$$\Gamma_\xi(s) = \Gamma_\xi \left(\frac{q_s}{q_m}\right)^{2L_\xi+1} \frac{m_\xi}{\sqrt{s}}$$

Freed isobar PWA

Freed isobar PWA:

$$\Delta(s) = \sum_{w\,\text{freed}} \mathcal{C}_{w\,\text{freed}} \Theta_w(s)$$

$$\Theta_w(s) = \begin{cases} 1 & \text{if } s \text{ in the } m_{2\pi} \text{ bin } w \\ 0 & \text{otherwise} \end{cases}$$

1^{--} $m(2\pi)$ binning:

$$\begin{cases} 2\ \text{MeV} & [770, 792]\ \text{MeV} \\ 20\ \text{MeV} & [640, 920]\ \text{MeV} \\ 40\ \text{MeV} & \text{otherwise} \end{cases}$$

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$$\Theta_w(s) = \begin{cases} 1 & \text{if } s \text{ in the } m_{2\pi} \text{ bin } w \\ 0 & \text{otherwise} \end{cases}$$

0^{++} $m(2\pi)$ binning:

$$\begin{cases} 10\ \text{MeV} & [920, 1080]\ \text{MeV} \\ 40\ \text{MeV} & \text{otherwise} \end{cases}$$

Freed isobar PWA

Freed isobar PWA:

$$\Delta(s) = \sum_{w\,\text{freed}} \mathcal{C}_{w\,\text{freed}} \Theta_w(s)$$

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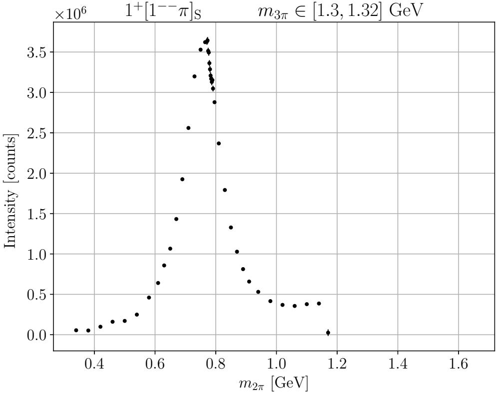
Mathematical ambiguities (zero modes)

`10.1103/PhysRevD.97.114008`

$1^+[1^{--}\pi]_S$ wave

Intensity



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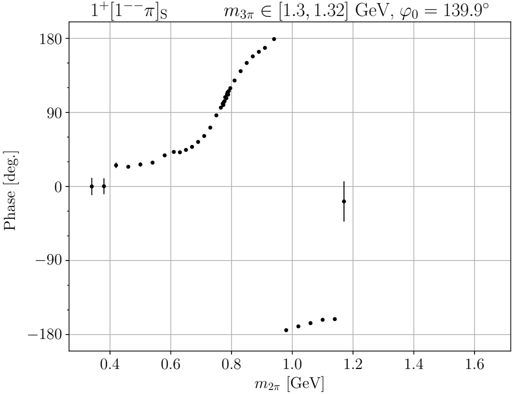
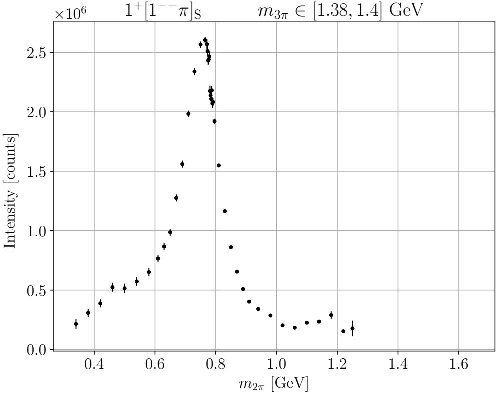


$1^+[1^{--}\pi]_S$ wave

Intensity



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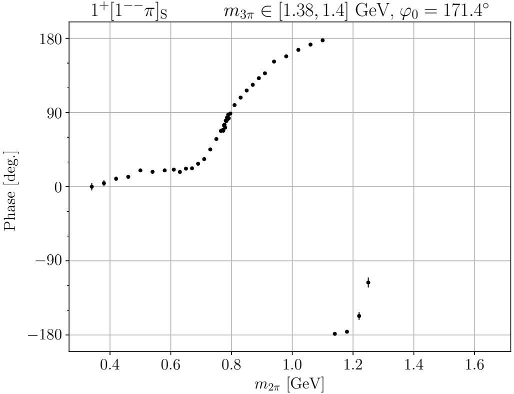
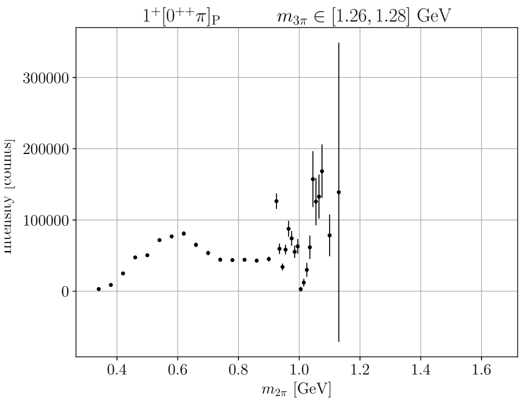


$1^+[0^{++}\pi]_P$ wave

Intensity



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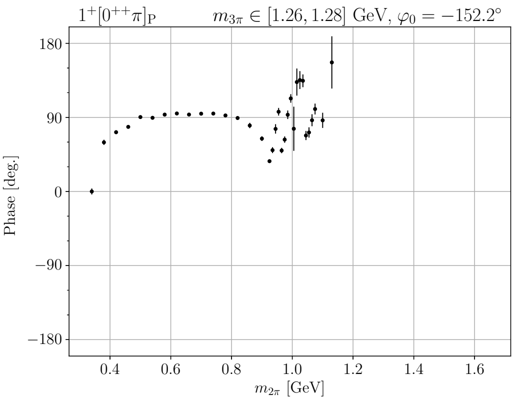

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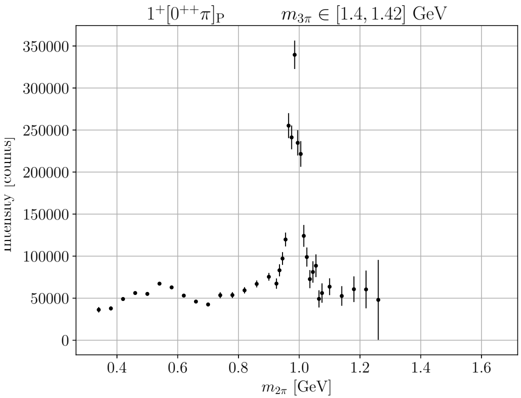
Intensity



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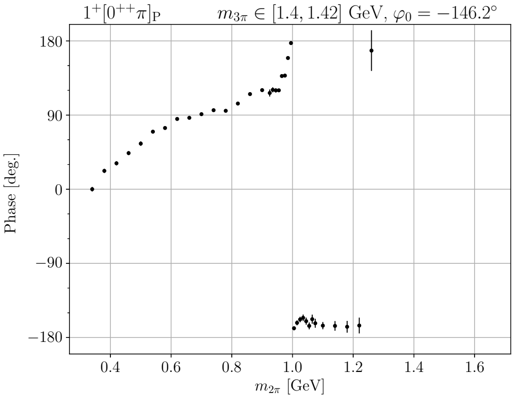

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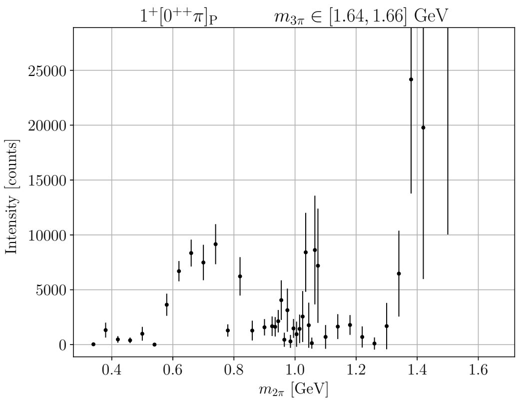

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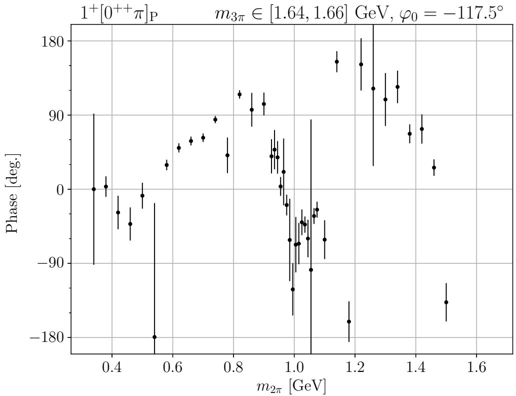
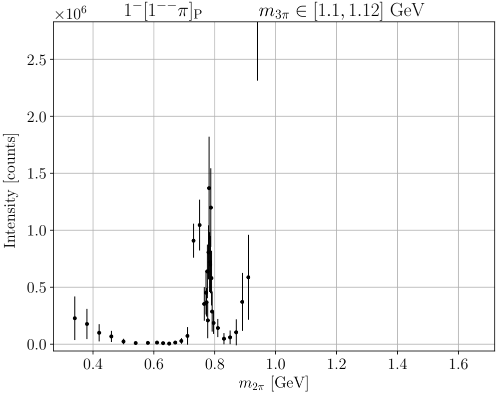


$1^{-}[1^{--}\pi]_{\mathrm{P}}$ wave

Intensity



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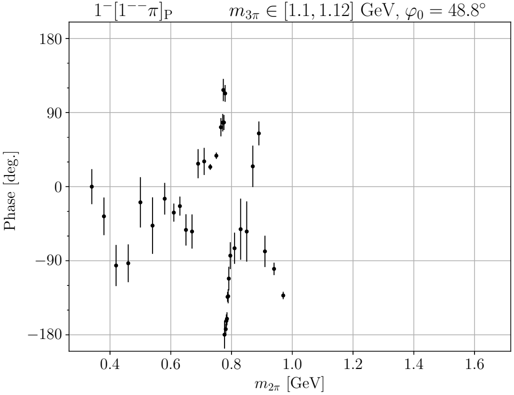
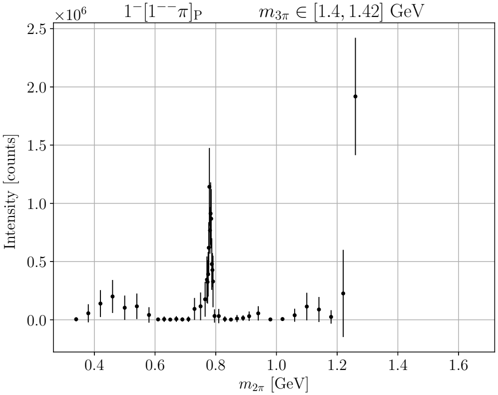


$1^-[1^{--}\pi]_P$ wave

Intensity



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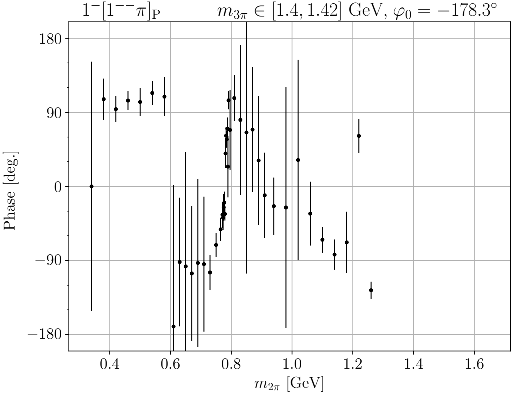

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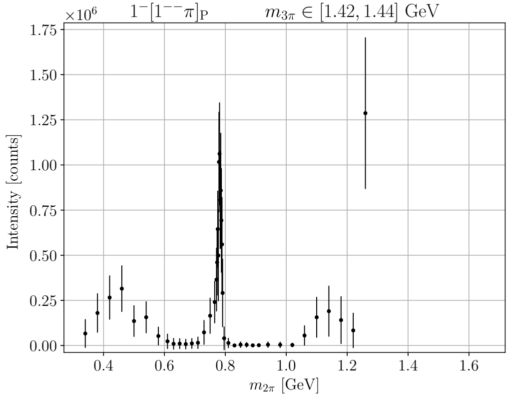

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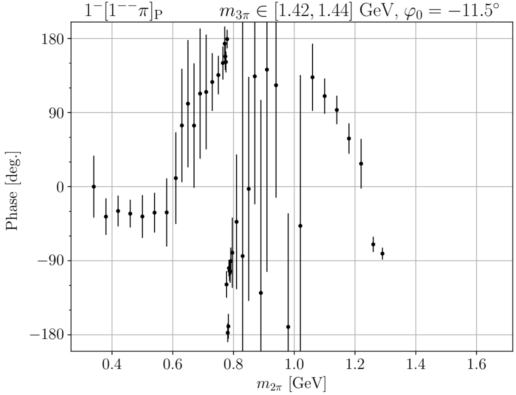

$1^{-}[1^{--}\pi]_{\mathrm{P}}$ wave

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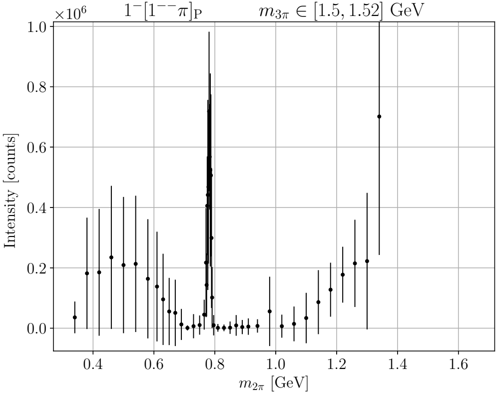

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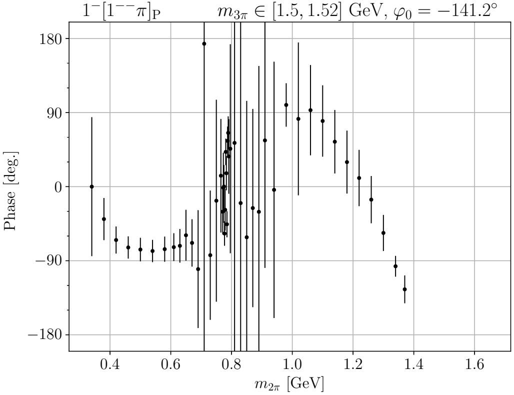

$1^{-}[1^{--}\pi]_{\mathrm{P}}$ wave

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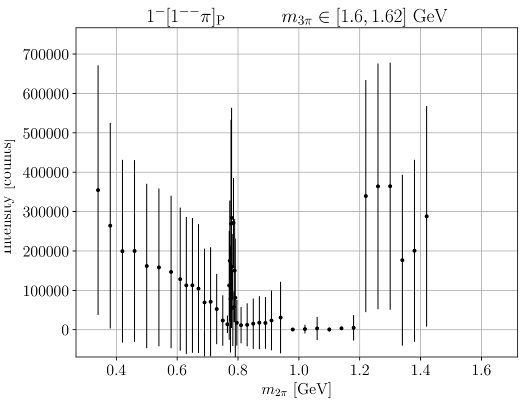

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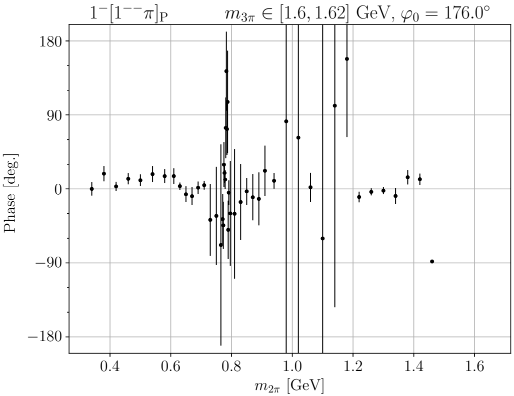
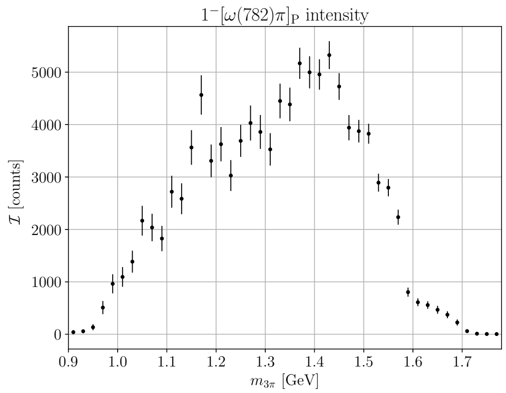
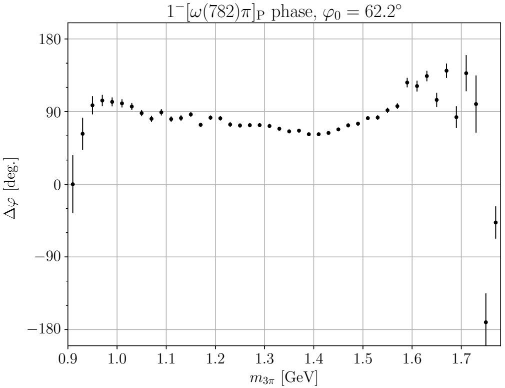


$1^{-}[\omega(782)\pi]_{\mathrm{P}}$-wave

Intensity



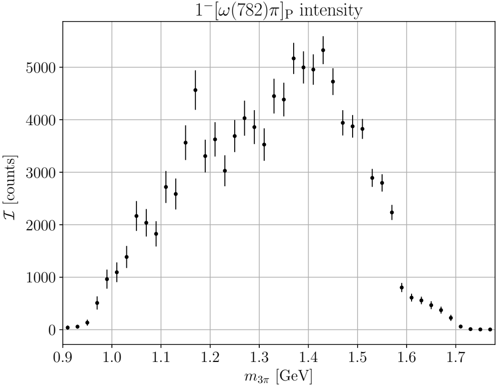
Phase

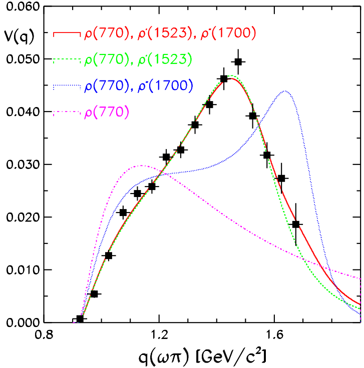


$1^-[\omega(782)\pi]_P$-wave

Intensity

CLEO-II spectrum of $m_{\omega\pi^-}$ for $\tau^- \to \omega\pi^-\nu_\tau$





Summary

Current status:

- Selection criteria
- Background description with neural network
- Partial-wave decomposition
  - Fixed-isobar
  - Freed-isobar
- Resonance-model and isobar fits
- Systematic uncertainties:
  - Model
  - Background
  - Acceptance
  - Resolution

Results:

- $\mathrm{a}_1(1420)$ discovered
- ρ–ω interference observed

Remaining systematic studies:

- Trigger
- PID

Future projects:

- Prepare `TAUOLA-m` for public use
- Measure τEDM and τMDM
- Test for second-class currents

Thank you for your attention!

Backup

Event-selection criteria

Event-class selection

- Standard Belle selection for tauon pairs
- Topology: 3–1
- Boosted Decision Tree

Signal hemisphere

- Tracks identification:
  - Veto signal-side particles being electrons or muons
  - Veto the like-sign signal-side particles being kaons
- Veto pions coming from K_0: $\left|m_{2\pi} - m_{\mathrm{K_S}}\right| < 12$ MeV
- Veto π^0s in signal hemisphere: $\sum E_\gamma < 480$ MeV

List of selections

Good tracks selection:

- $|\Delta r| < 0.5$ cm
- $|\Delta z| < 2.5$ cm
- $p_\perp > 0.1$ GeV

Good photons selection:

- $E_\gamma > 0.04$ GeV
- $w > 0.5$ cm
- $N_{\text{hits}} > 2$
- $E_{\text{seed}}/E_{\text{cluster}} < 0.95$

Preselection: four good tracks with sum charge zero

Skimming:

- `tau_skimB` or `HadronBJ`
- BDT response $b_{\tau\tau} > 0$

Topology: $3 + 1$

Loose π^0-veto in signal hemisphere: $\sum E_\gamma < 0.48$ GeV

Signal tracks PID:

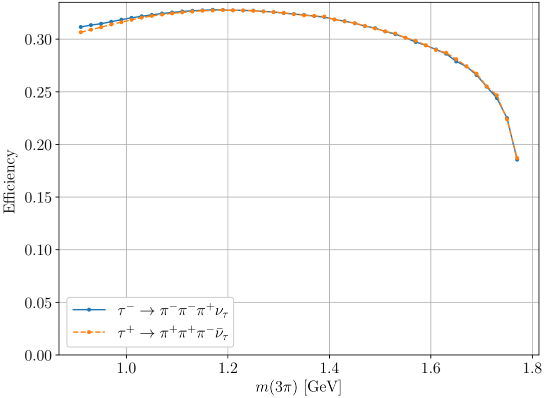
- three tracks: e-veto, μ-veto
- two tracks same charge: K-veto

K_S-veto in signal hemisphere: $\left|m_{2\pi} - m_{\text{K}_\text{S}}\right| < 0.012$ GeV

Efficiency and purity

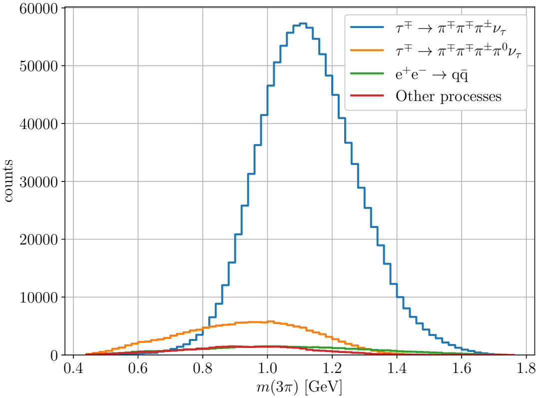
Sequential efficiencies and purities:

Criterion	purity [%]	efficiency [%]
baseline	22.1	44.4
trig	23.1	43.8
skim	24.5	43.1
BDT	50.6	39.9
LID	54.0	37.8
HID	57.4	36.7
PHS	57.7	36.7
ISR	58.2	35.9
KS_veto	60.7	34.3
pi0_veto	81.6	32.3

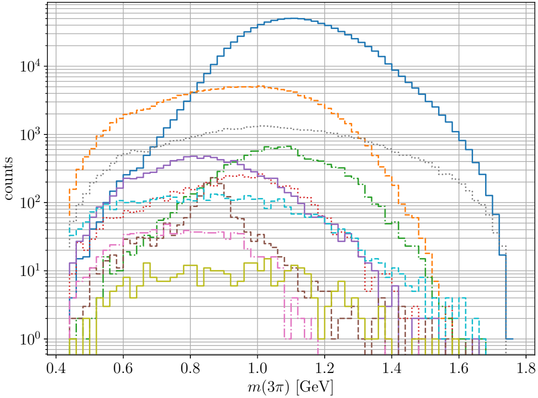


$m(3\pi)$ efficiency

Efficiency and purity



$m(3\pi)$ spectrum in MC

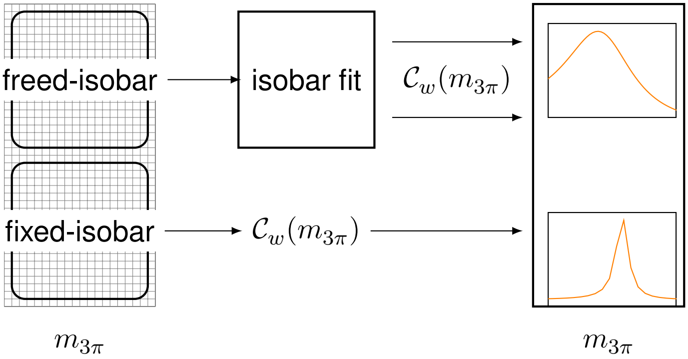


$m(3\pi)$ spectrum in MC, $\log\mathrm{OY}$

Fitting strategies

Partial-wave decompositon in 5D

Resonance-model fit in 1D



Fitting strategies

Conventional PWA:

- Isobars shape is fixed
- Fit in bins of $m_{3\pi}$ (no a$_1$ shape assumptions)
- $2 \times N_{\text{PW}} - 1$ free parameters: partial-waves complex coefficients $\mathcal{C}_w$ (one phase is fixed)

Intensity:

$$\mathcal{I} = \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v}$$

$$\mathcal{I}_{w\,v} = \overline{\mathcal{L}_{\mu\nu}} j_w^{\mu} \left(j_v^{\nu}\right)^*$$

Extended log likelihood function:

$$\ln \mathcal{L} = \sum_{\text{Data}} \ln \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{I}_{w\,v} - \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \mathcal{N}_{w\,v}$$

Integral matrix:

$$\mathcal{N}_{wv} = \frac{\sum_{\text{Acc MC}} \mathcal{I}_{wv} / \left|\mathcal{M}_{\text{MC Generator}}\right|^2}{\sum_{\text{Acc MC}} 1 / \left|\mathcal{M}_{\text{MC Generator}}\right|^2}$$

$\left|\mathcal{M}_{\text{MC Generator}}\right|^2$ TAUOLA intensity (CLEO current for $\tau \to 3\pi\nu_\tau$)

Partial waves

Partial wave	$m(\xi)$ [GeV]	$\Gamma(\xi)$ [GeV]	Threshold [GeV]
$1^+[\sigma\pi]_P$	Broad $[\pi\pi]_S$-wave component*		
$1^+[f_0(980)\pi]_P$	0.990	0.07	1.22
$1^+[f_0(1500)\pi]_P$	1.504	0.109	1.64
$1^+[\rho(770)\pi]_S$	0.769	0.1509	—
$1^+[\rho(770)\pi]_D$			—
$1^+[\rho(1450)\pi]_S$	1.465	0.40	1.18
$1^+[\rho(1450)\pi]_D$			1.34
$1^+[f_2(1270)\pi]_P$	1.2755	0.1867	1.4
$1^+[f_2(1270)\pi]_F$			1.44
$0^-[\sigma\pi]_S$			—
$0^-[f_0(980)\pi]_S$			1.14
$0^-[\rho(770)\pi]_P$			—
$0^-[f_2(1270)\pi]_D$			1.0

Partial wave	Threshold [GeV]
$1^-[\omega(782)\pi]_P$	—
$1^-[\rho(770)\pi]_P$	0.96
$1^-[f_2(1270)\pi]_D$	—

G-parity violating waves

$$G = (-1)^I C = (-1)^{J+L+I}$$

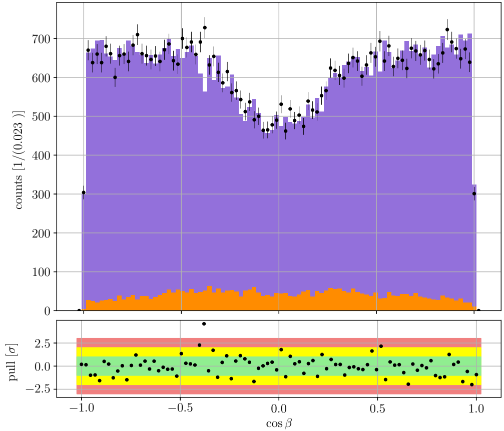
Comparison of simulation with data

Slice of $m_{3\pi} \in [1.50, 1.52]$ GeV

Fit to $20\% L_{\text{int}}$ data and simulate $\tau^{\mp} \to \pi^{\mp}\pi^{\mp}\pi^{\pm}\nu_{\tau}$

Plot legend:

- Black dots: data
- Blue hist: Fit prediction with `TAUOLA-m`
- Orange hist: background



$\cos\beta$

Comparison of simulation with data

Slice of $m_{3\pi} \in [1.50, 1.52]$ GeV

Fit to $20\% L_{\text{int}}$ data and simulate $\tau^{\mp} \to \pi^{\mp}\pi^{\mp}\pi^{\pm}\nu_{\tau}$

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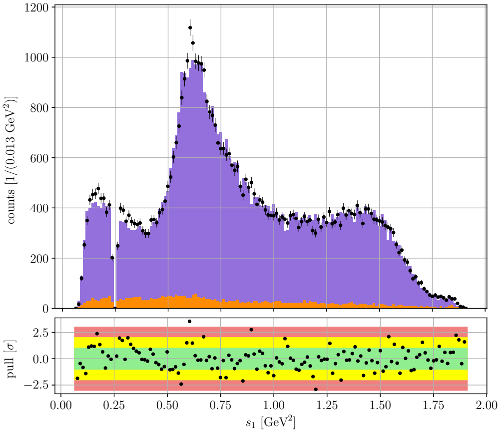
Data overshoots simulation at the $\rho(770)$ peak position



s_1 [GeV^2]

G-parity violating waves

Partial wave	$\mathbb{B}$ [%]
$1^-[\mathrm{f}_2(1270)\pi]_\mathrm{D}$	0.46
$1^-[\omega(782)\pi]_\mathrm{P}$	0.30
$1^-[\rho(770)\pi]_\mathrm{P}$	0.12

Blatt-Weisskopf centrifugal-barrier factor F

F takes into the finite size of a meson

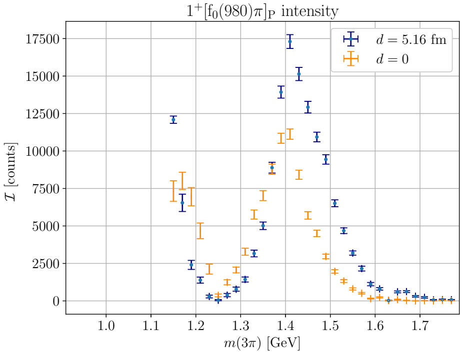
Each partial wave is multiplied by two Fs corresponding to either X^- or ξ^0

F_S for ξ^0 depends on the break-up momentum in the rest frame of ξ^0

F_L for X^- depends on the break-up momentum in the rest frame of X^-

There are two alternative parametrizations for F, I use the relativistic one, for example for a P wave

$$F_P(x) = \sqrt{\frac{1-x_0}{1-x}},\ x = (pd)^2$$



Blatt-Weisskopf centrifugal-barrier factor F

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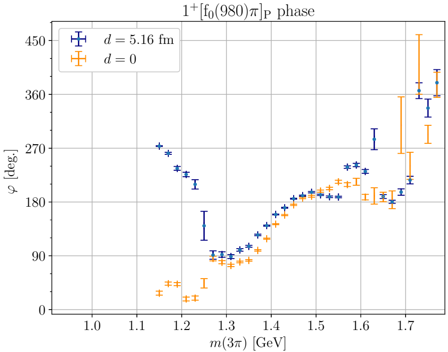

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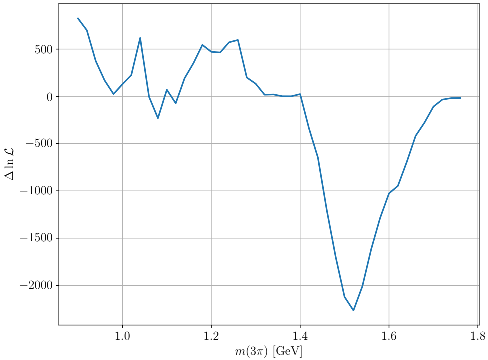
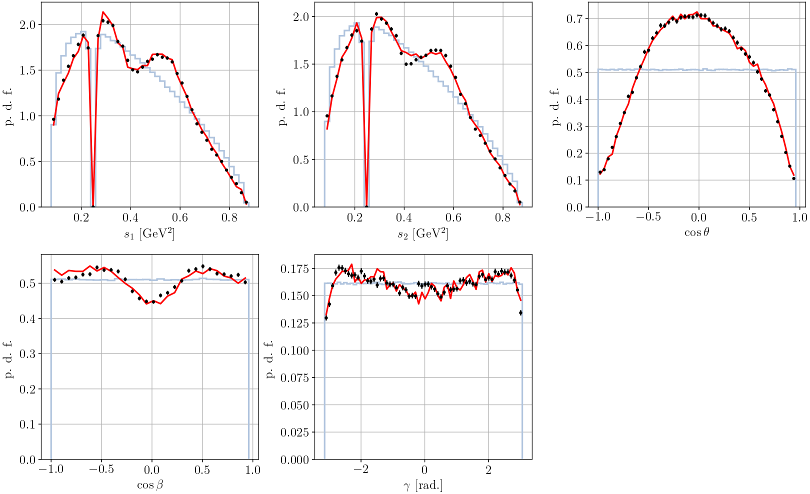


Difference between $\ln \mathcal{L}$ of the fit.

1D projections



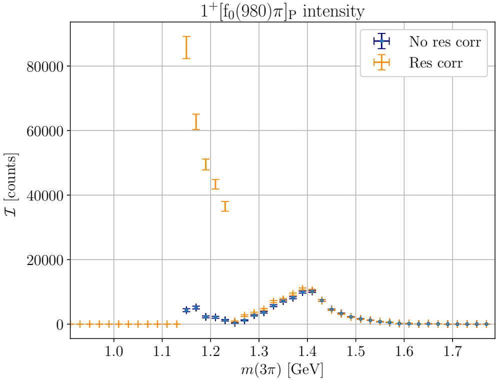
Detector resolution

$$\mathcal{I}_{w\,v}(\Phi) \to \int \mathcal{I}_{w\,v}(\Phi')\varepsilon(\Phi,\Phi')\mathrm{d}\Phi',$$

Φ — reconstructed phase space variables, Φ' — generated phase space variables

Requires MC sampling for each event

Unknown $\varepsilon(\Phi,\Phi')$



Intensity of the $1^+[f_0(980)\pi]_P$ wave.

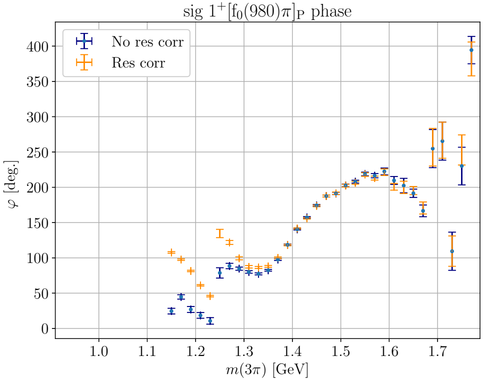
Detector resolution

$$\mathcal{I}_{w\,v}(\Phi) \to \int \mathcal{I}_{w\,v}(\Phi')\varepsilon(\Phi, \Phi')\mathrm{d}\Phi',$$

Φ — reconstructed phase space variables, Φ' — generated phase space variables

Requires MC sampling for each event

Unknown $\varepsilon(\Phi, \Phi')$



Phase of the $1^+[f_0(980)\pi]_P$ wave.

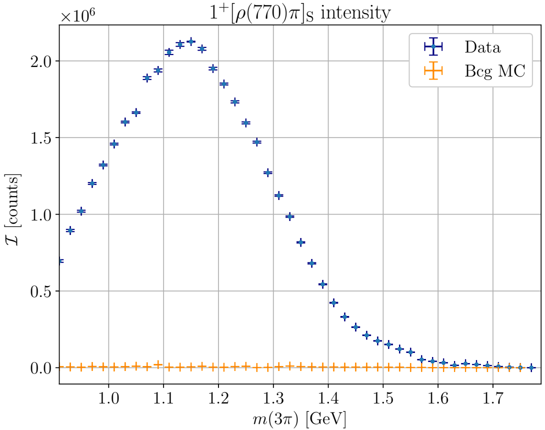
Systematic effects: overview

Four major sources:

- Model:
  - Isobar parametrization
  - Model selection
- Background
  - Model in simulations
  - Neural network parametrization
- Acceptance
  - Stat. uncertainty of $\mathcal{N}_{wv}$
  - Momentum correction
- Detector resolution

Background

- Neural network* trained on simulated data
- Neural network shape fixed in PWA
- Test background leakage on simulated data
- Background still leaks to signal



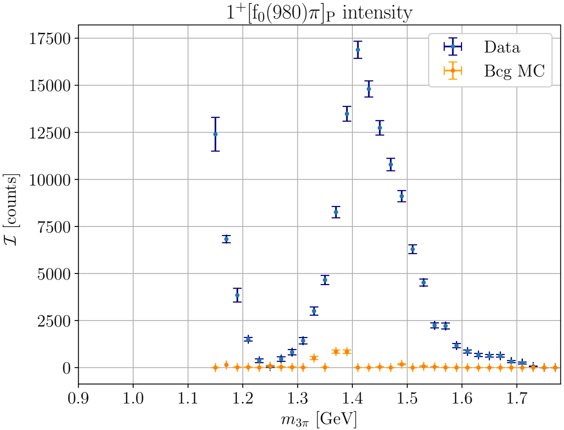
Background

Neural network* trained on simulated data

Neural network shape fixed in PWA

Test background leakage on simulated data

Background still leaks to signal



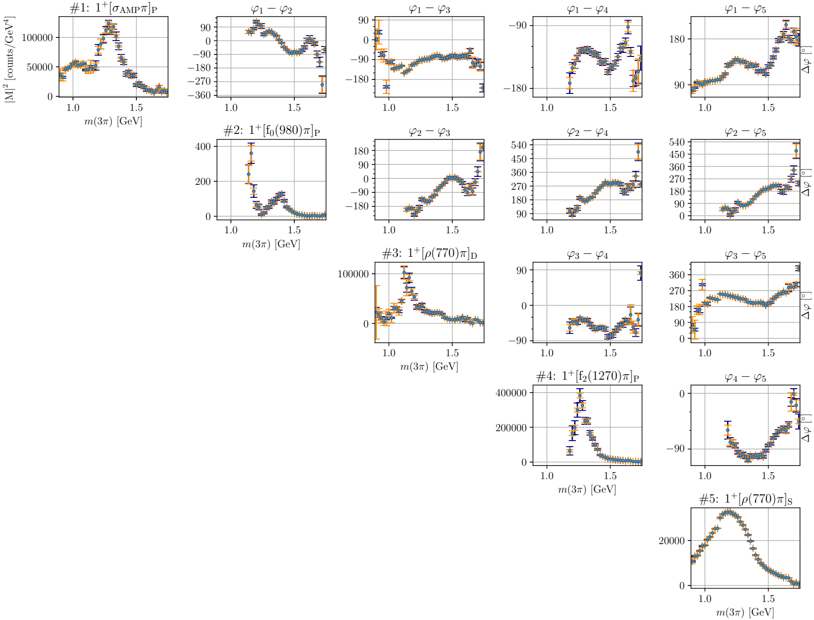
Neural network uncertainty:

Plots' legend:

- Stat. uncertainty
- Syst. uncertainty

Propagate uncertainties by varying neural network parameters before PWA

25–120 networks/bin



Integrals uncertainty:

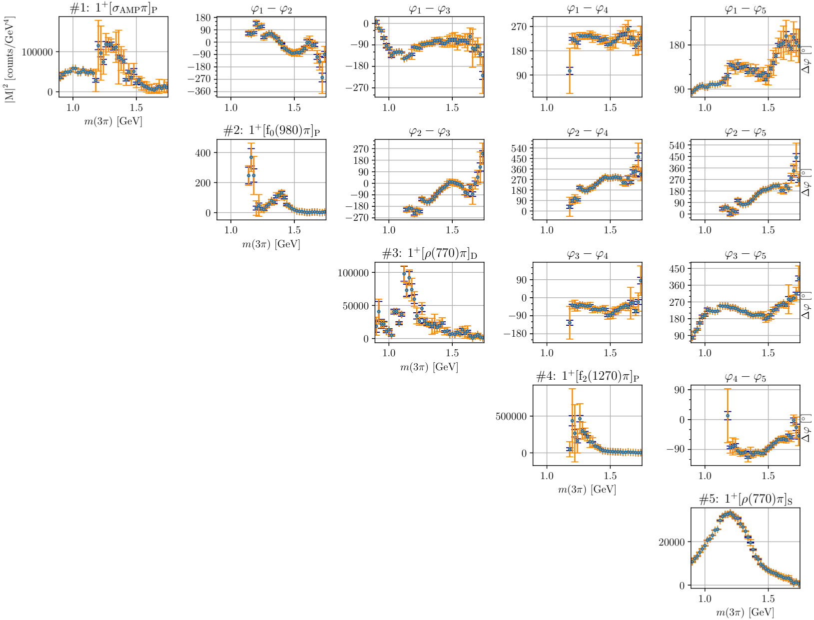
Plots' legend:

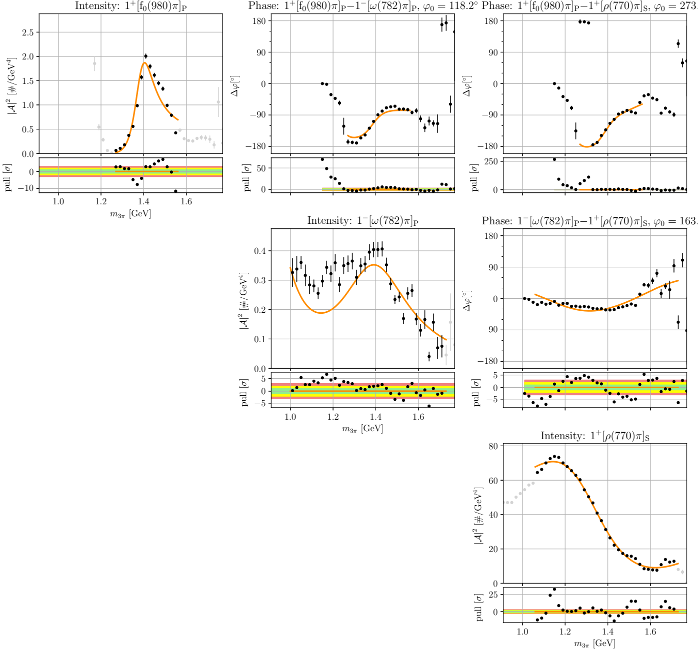
- Stat. uncertainty
- Syst. uncertainty

Propagate statistical uncertainties of integrals by varying integrals within their statistical uncertainties before PWA

Correlations not taken into account

$100\ N_{wv}$/bin




Intensity: $1^+[f_0(980)\pi]_P$
Phase: $1^+[f_0(980)\pi]_P-1^-[\omega(782)\pi]_P$, $\varphi_0 = 118.2°$
Phase: $1^+[f_0(980)\pi]_P-1^+[\rho(770)\pi]_S$, $\varphi_0 = 273.$
Intensity: $1^-[\omega(782)\pi]_P$
Phase: $1^-[\omega(782)\pi]_P-1^+[\rho(770)\pi]_S$, $\varphi_0 = 163.$
Intensity: $1^+[\rho(770)\pi]_S$
$|\mathcal{A}|^2$ [#/GeV^4]
$\Delta\varphi$ [°]
pull [σ]
$m_{3\pi}$ [GeV]