

KEK IPNS Physics Seminar

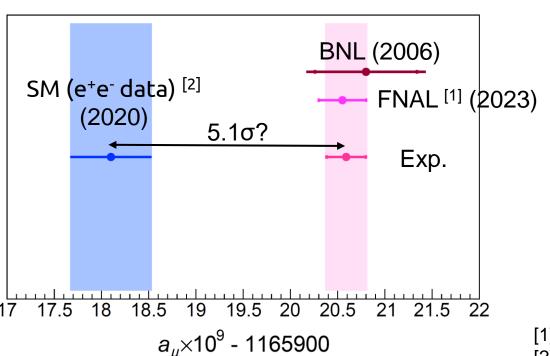
Measurement of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section with the Belle II detector

Yuki Sue, Nagoya University on behalf of Belle II collaboration

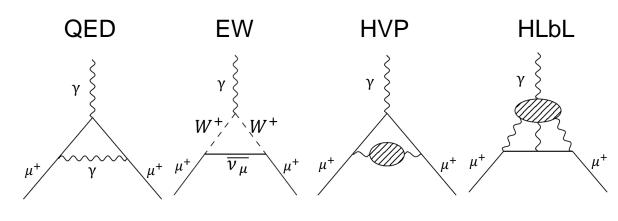
2024.4.10

arXiv:2404.04915

- \Box 5 σ significance through new direct measurements from Fermilab
- Non-negligible uncertainty in theoretical predictions

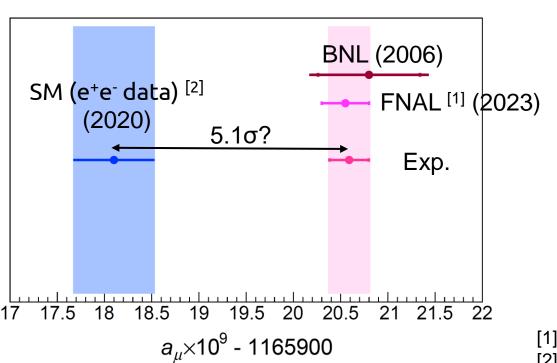


$$a_{\mu}^{\text{SM}} = \frac{g_{\mu}-2}{2} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{HVP}} + a_{\mu}^{\text{HLbL}}$$



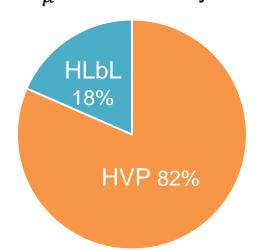
- [1] DPRL 131 161802 (2023)
- [2] Phys. Rept. 887, 1 (2020)

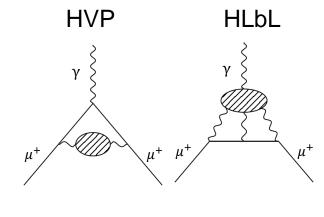
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- Non-negligible uncertainty in theoretical predictions
 - ☐ Major uncertainty is derived from Hadronic Vacuum Polarization (HVP) term



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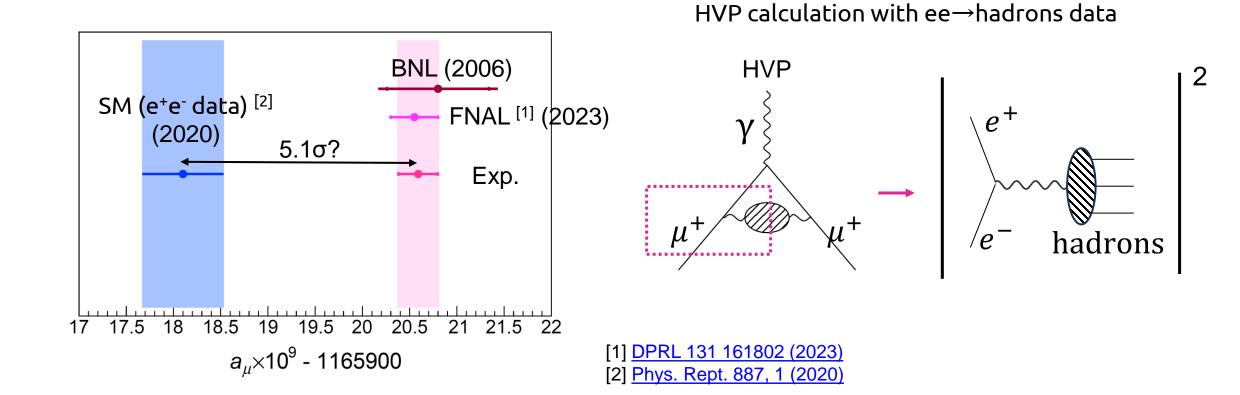
$$a_{\mu}^{\text{SM}} \text{ uncertainty}^{[2]}$$



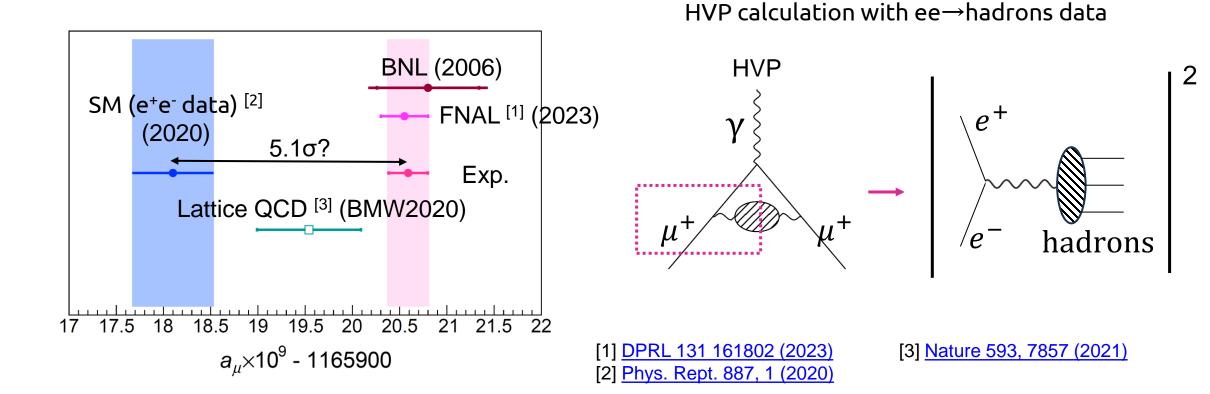


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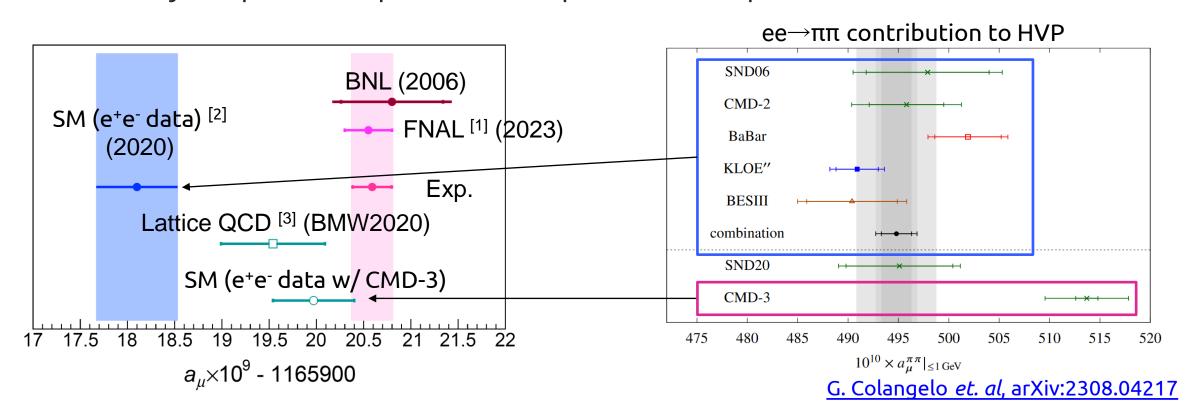
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- \Box 5 σ significance through new direct measurements from Fermilab
- Non-negligible uncertainty in theoretical predictions
 - ☐ Major uncertainty is derived from Hadronic Vacuum Polarization (HVP) term
 - ☐ HVP predictions are different depending on methods: e⁺e⁻ data vs Lattice QCD
 - ☐ Differences among e⁺e⁻ experiments are also non-negligible
- □ Validation by independent experiments is important in HVP prediction



Cross section measurements of exclusive channels

 $\kappa_{\rm S} \kappa_{\rm L}^{\pi^0 \gamma}$ E>1.8 GeV

K+K- $\pi^{+}\pi^{-}\pi^{+}\pi^{-}$

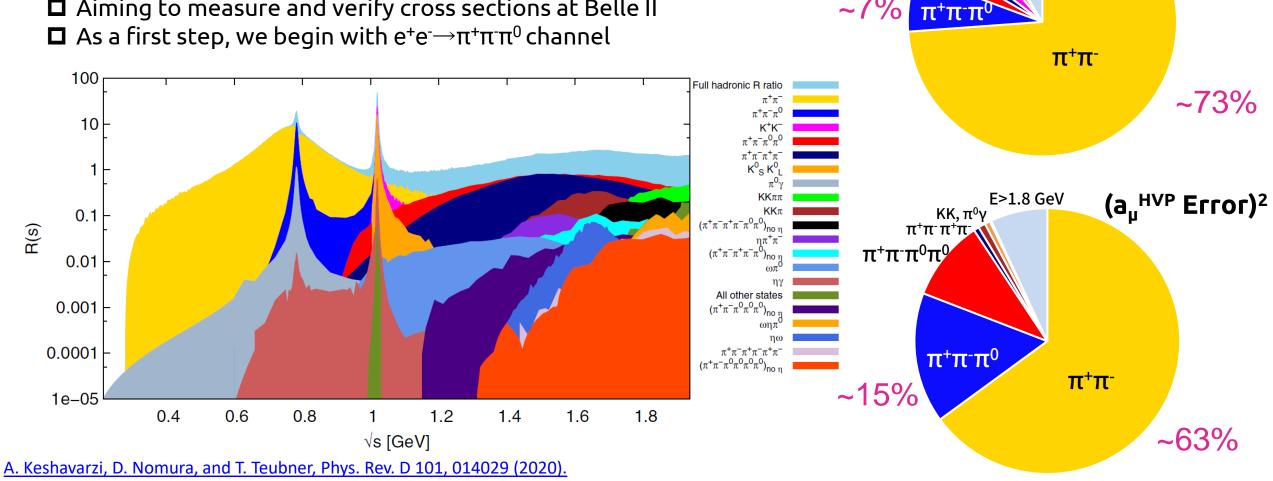
 $\pi^{+}\pi^{-}\pi^{0}\pi^{0}$

 a_{μ}^{HVP}

Leading order HVP contribution

$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{3\pi}\right)^2 \int_{m_{\pi}^2}^{\infty} \frac{\widehat{K}(s)}{s^2} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} ds$$

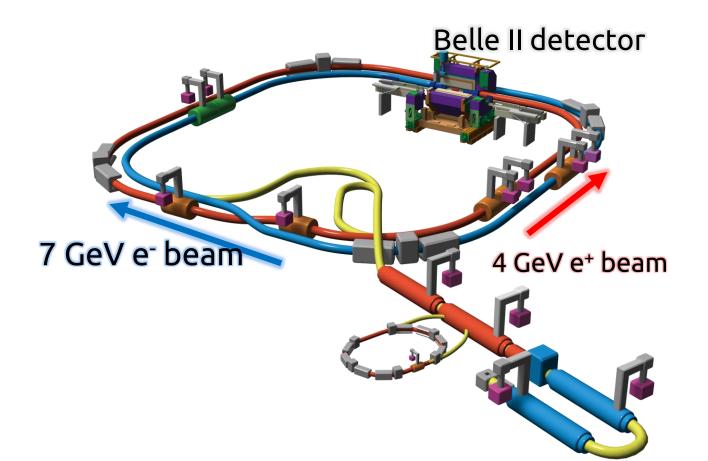
☐ Aiming to measure and verify cross sections at Belle II

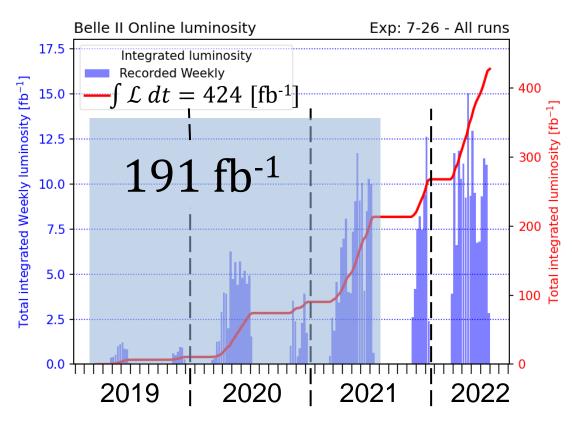


SuperKEKB/Belle II experiment

- ☐ Asymmetric e⁺e⁻ collider at KEK
 - $\sqrt{s} = M(Y(4S)) = 10.58 \text{ GeV}$
 - World record instantaneous luminosity: 4.7×10^{34} /cm²/s
 - ~90% data taking efficiency: 1-2 fb⁻¹/day

- Used dataset in this analysis
 - 2019 2021 Summer dataset
 - Integrated luminosity: 191 fb⁻¹
 - A half of the collected data, 424 fb⁻¹

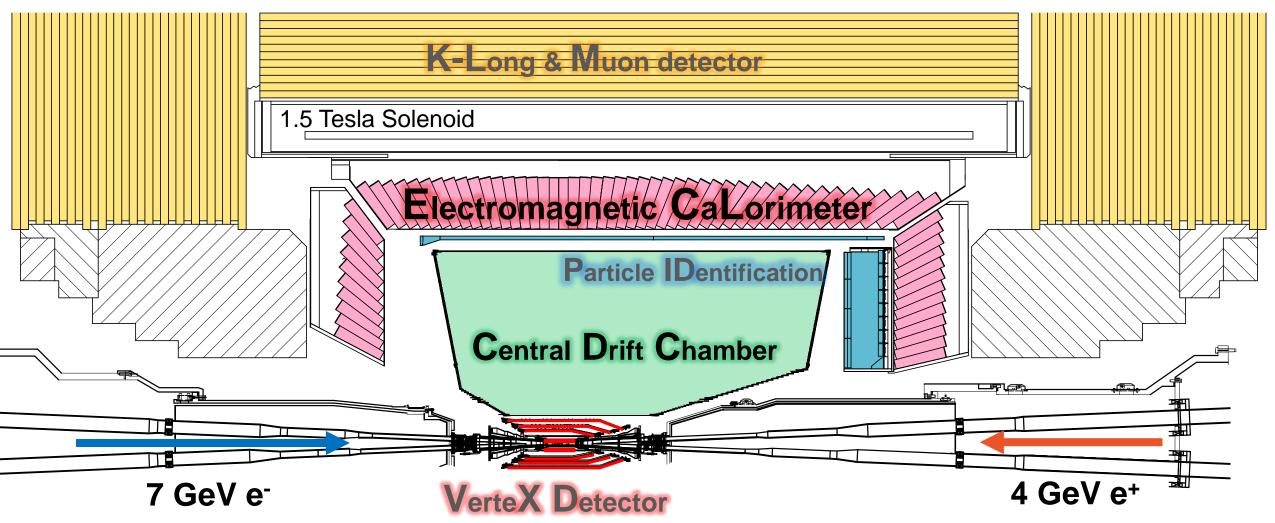




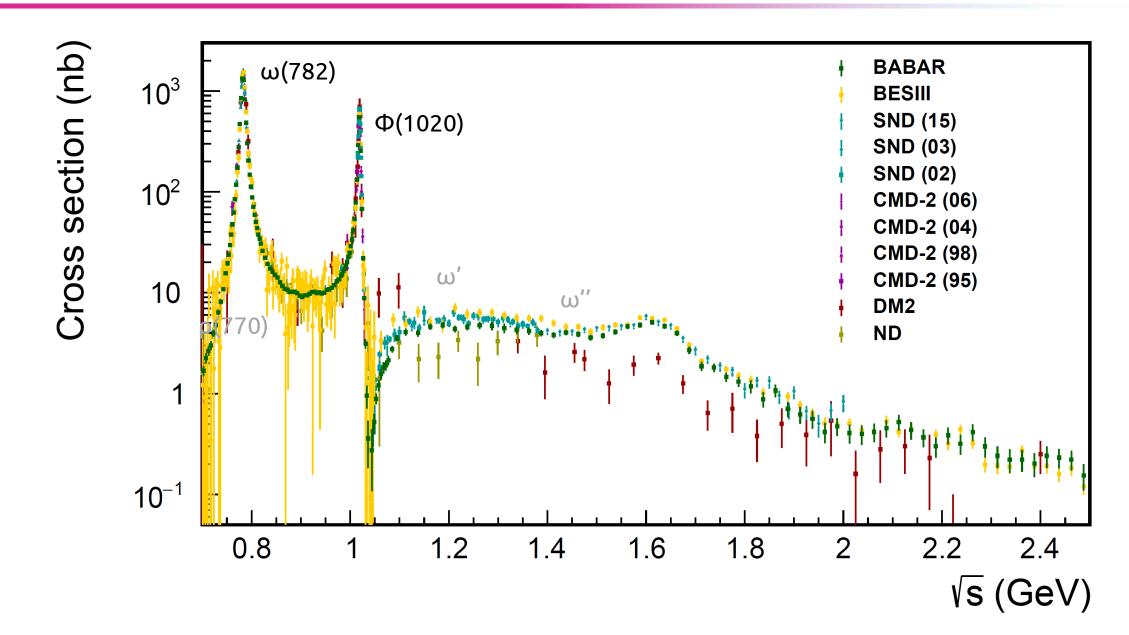
Belle II detector

Trigger & DAQ

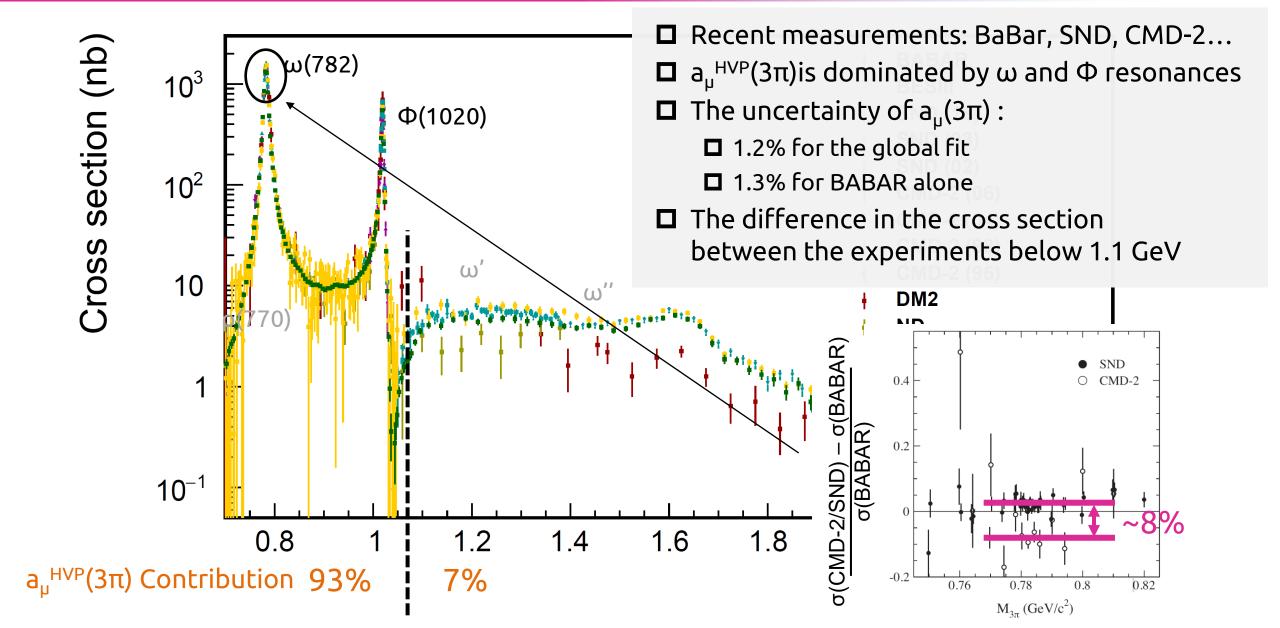
New calorimeter-based trigger enables light-hadron cross section measurements



Previous measurements for $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

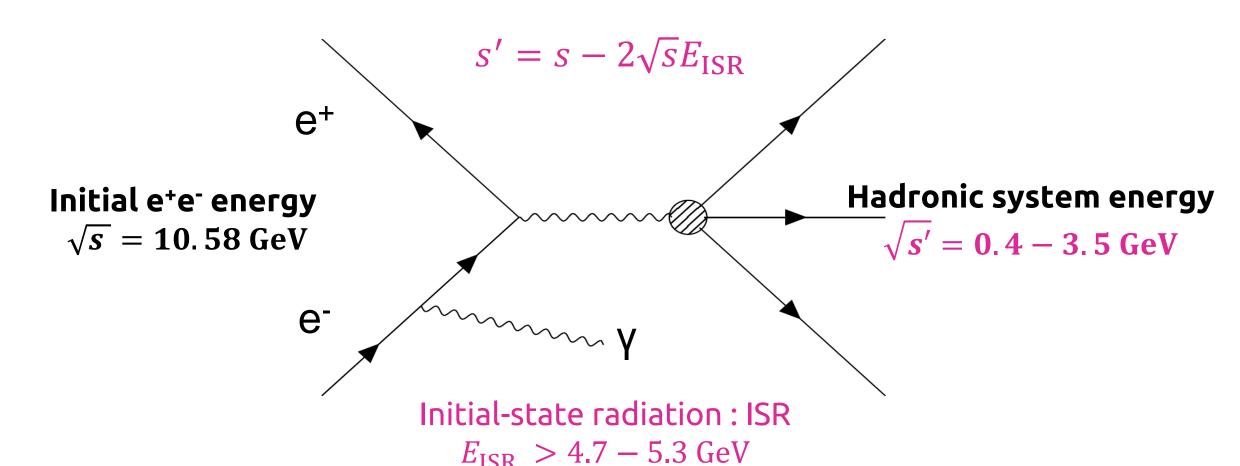


Previous measurements for $e^+e^- \rightarrow \pi^+\pi^-\pi^0$



Radiative return method

- Measure the cross section in the energy range 0.4-3.5 GeV at fixed e⁺e⁻ energy collision
- ☐ Use a process associated with energetic ISR emission
 - Only less than 10% of ISR photons are emitted into detector acceptance



Analysis overview

Cross section
$$\sigma_{3\pi}(M(3\pi)) = \frac{N_{\rm signal}}{\varepsilon(M(3\pi)) \cdot L_{\rm eff}(M(3\pi))}$$

$$3\pi \, {\rm mass}$$
 Signal spectrum
$$N_{\rm signal}$$

$$\varepsilon(M(3\pi)) \cdot L_{\rm eff}(M(3\pi))$$
 Efficiency Integrated luminosity

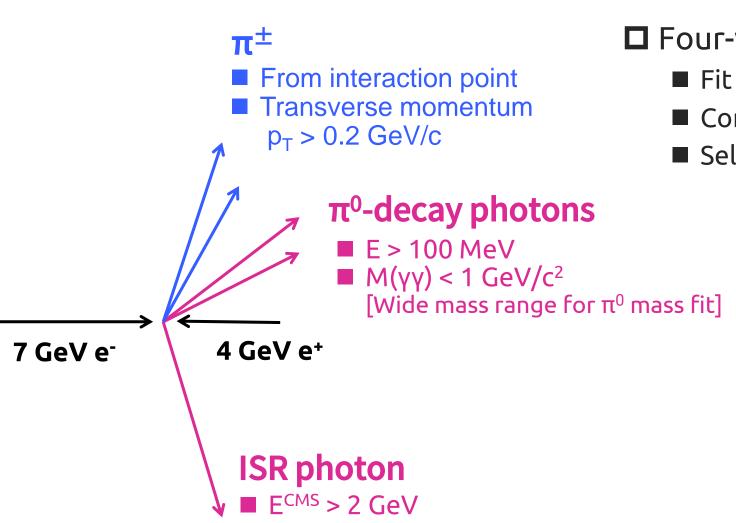
- \square Target: $\delta a_{\mu}^{3\pi}/a_{\mu}^{3\pi} \sim 2\%$ with 191 fb⁻¹ data
- Key items
 - Event selection to extract $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$ process
 - Background suppression and estimation
 - Unfolding to mitigate detector resolution
 - Efficiency corrections between data and simulation
- Blind analysis
 - ☐ Study of analytical methods using MC before examining data

Analysis outline

- Event selection
- Background estimation
- ☐ Signal extraction
- Unfolding
- Efficiency estimation
- \square Cross section and a_{μ} calculation

$e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$ selection

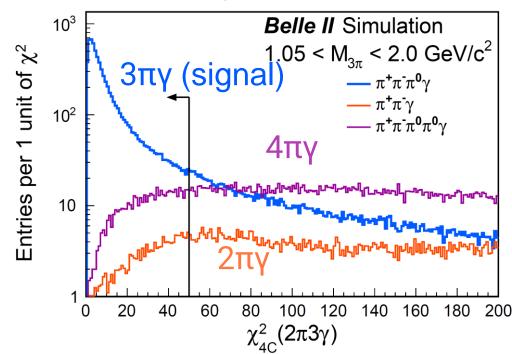
Reconstruct Two tracks + three photons : $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR} \rightarrow \pi^+\pi^-\gamma\gamma\gamma\gamma_{ISR}$



In barrel ECL for trigger

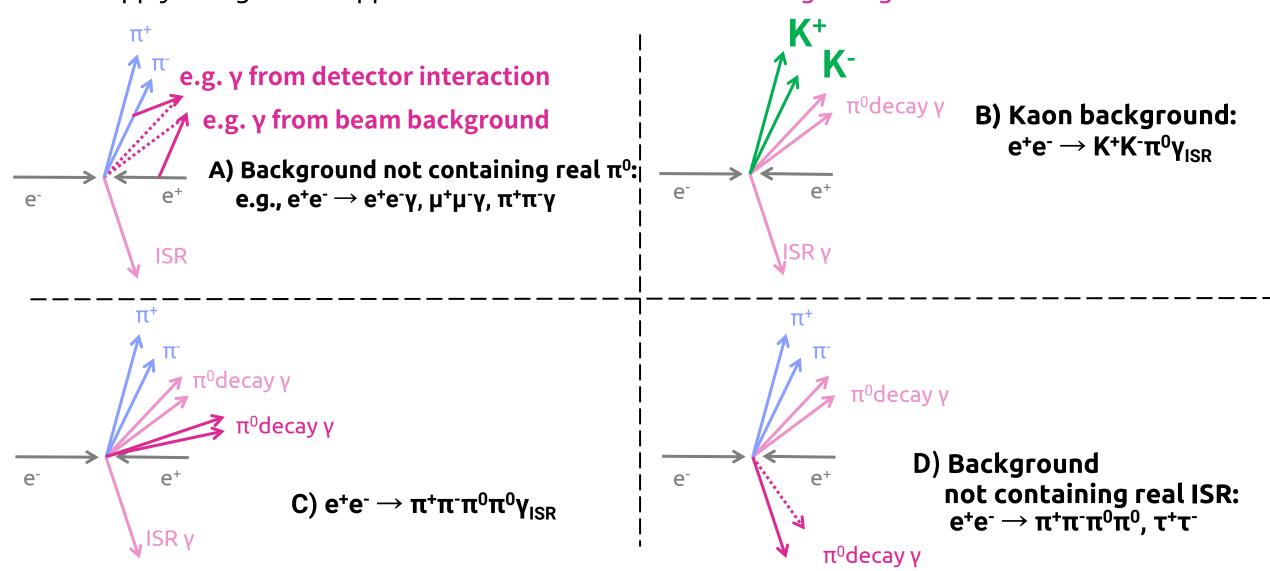
- ☐ Four-vector kinematic fit (4C-KFit)
 - Fit to positions and momenta
 - Constrain to initial e⁺e⁻ four-momentum
 - Select small χ^2 to extract signal-like event

4C-Kfit χ^2 distribution (MC)



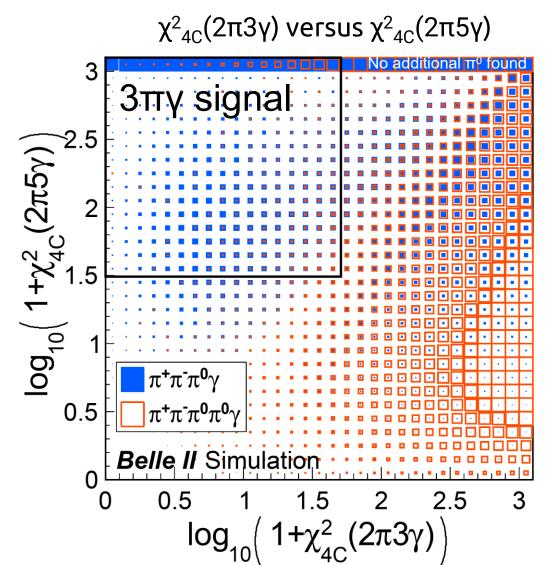
$e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma_{ISR}$ selection: Background suppression

Apply background suppression criteria to reduce remaining backgrounds



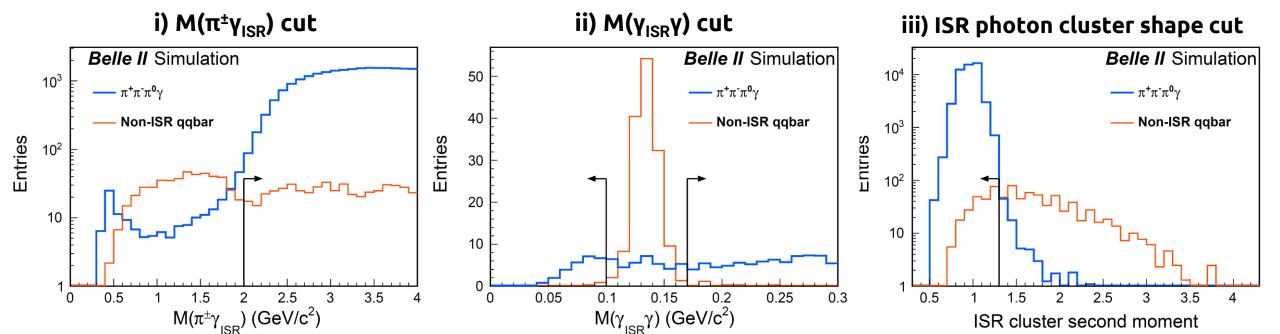
Background suppression (1)

- A) Background not containing real π^0 : $e^+e^- \rightarrow e^+e^- \gamma$, $\pi^+\pi^- \gamma$, $\mu^+\mu^- \gamma$
 - Pion/Electron ID > 0.1
 - $M^2_{recoil}(\pi^+\pi^-) > 4 \text{ GeV}^2/c^4$
- B) Charged kaon : $e^+e^- \rightarrow K^+K^-\pi^0\gamma$
 - Pion/Kaon ID $L(\pi/K) > 0.1$
- C) $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\gamma$
 - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ (with additional π^0)
 - 4C kinematic fit under $\pi^+\pi^-\pi^0\pi^0\gamma$ (2π5γ) hypothesis, and $\chi^2_{4C}(2\pi5\gamma) > 30$



Background suppression (2)

- D) Background not containing real ISR : Non-ISR qqbar (dominated by $\pi^+\pi^-\pi^0\pi^0$) and $\tau^+\tau^$
 - i. $M(\pi^{\pm}\gamma_{ISR}) > 2 \text{ GeV/c}^2$ to reduce high momentum $\rho^{\pm} \to \pi^{+}\pi^{0}$
 - ii. $M(\gamma_{ISR}\gamma)$ cut to reduce ISR candidate from π^0 -decay photon
 - iii. Cluster shape cut to reduce ISR-like photon in which two photons from of π^0 are merged



After applying all selection criteria

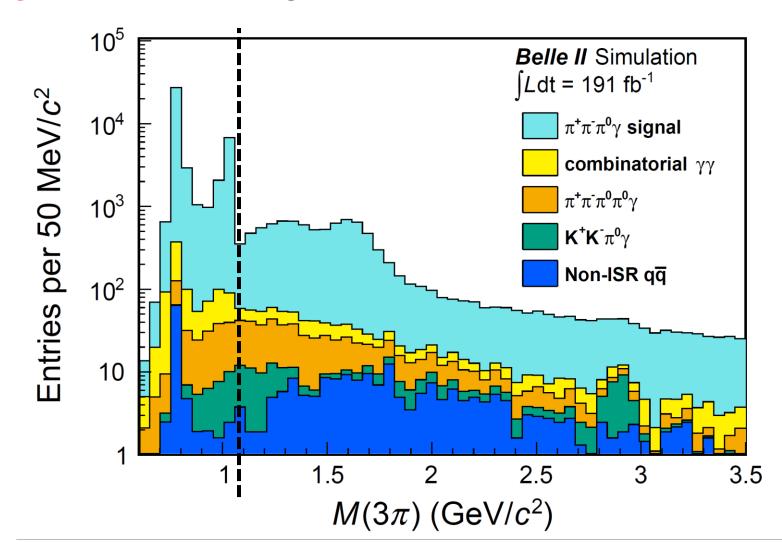
$M(3\pi) < 1.05 \text{ GeV/c}^2$

 $M(3\pi) > 1.05 \text{ GeV/c}^2$

Combinatorial γγ background is dominant bkg.

 \square $\pi^+\pi^-\pi^0\pi^0\gamma$ background is dominant bkg.

☐ Signal purity is 98%



Analysis outline

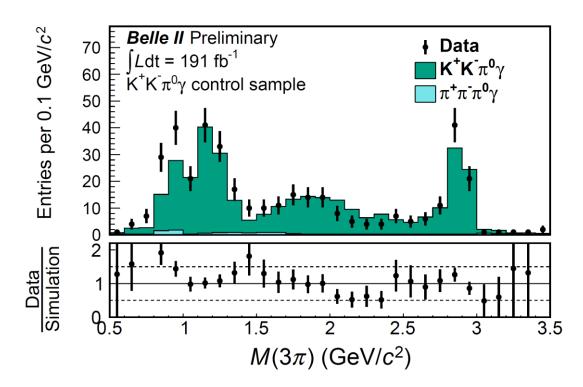
- Event selection
- Background estimation
- Signal extraction
- Unfolding
- Efficiency estimation
- \square Cross section and a_{μ} calculation

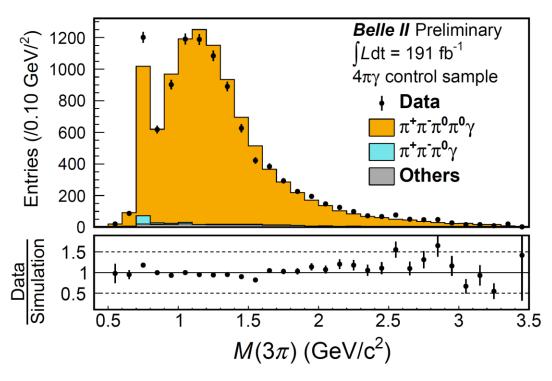
Background estimation

Estimate by determining a mass-dependent data-MC scale factor using a control sample.

$$N_{\text{Signal}}^{\text{data}} = N_{\text{Signal}}^{\text{MC}} \cdot \frac{N_{\text{Control}}^{\text{data}}}{N_{\text{Control}}^{\text{MC}}}$$

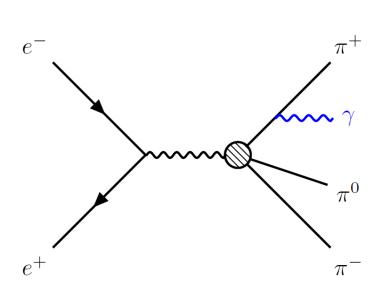
- $e^+e^-\rightarrow K^+K^-\pi^0\gamma$: Invert π/K -ID $L(\pi/K) > 0.1 \Rightarrow L(\pi/K) < 0.1$
- $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- Non-ISR qqbar : $0.10 < M(\gamma_{ISR}\gamma) < 0.17 \text{ GeV} / \text{large cluster second moment}$





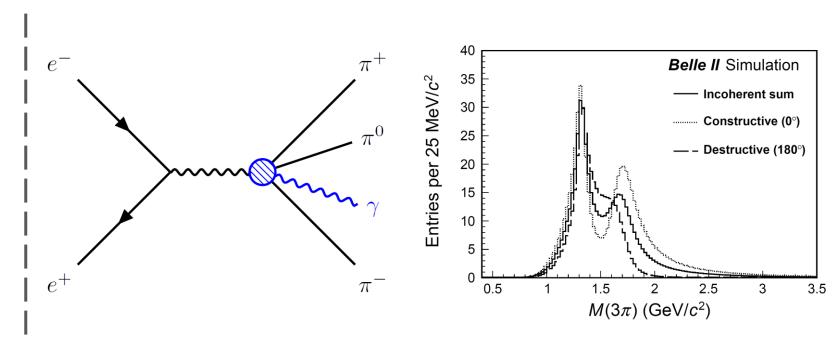
Final-state radiation background

- Difficult to reject FSR background or extract control sample
- Estimate FSR background using pQCD prediction based on the BABAR previous analysis [PRD112003]



FSR emission from final-state pions

 $\sim 0.001 \text{fb} \rightarrow < 1 \text{ event occur}$

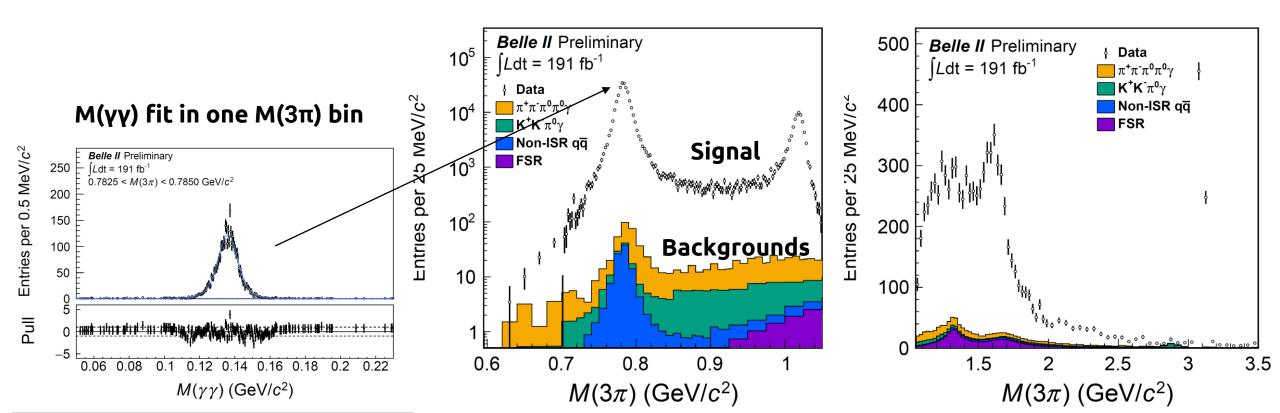


FSR emission from the quark legs

 \blacksquare e⁺e⁻ \rightarrow M $\gamma_{FSR} \rightarrow \pi^{+}\pi^{-}\pi^{0}\gamma_{FSR}$; M= η , $a_1(1260)$, $a_2(1320)$, $a_1(1640)$, $a_2(1700)$, $a_1(1930)$, $a_2(2030)$

Signal extraction

- Fit $M(\gamma\gamma)$ in each $M(3\pi)$ bin to remove the combinatorial background in $\gamma\gamma$
 - Signal: Gaussian + Novosibirsk function
 - Background: linear function
- Fit each bin of M(3π) with fixed signal-shape parameters
- Signals were observed up to 0.62 GeV as the lower limit.

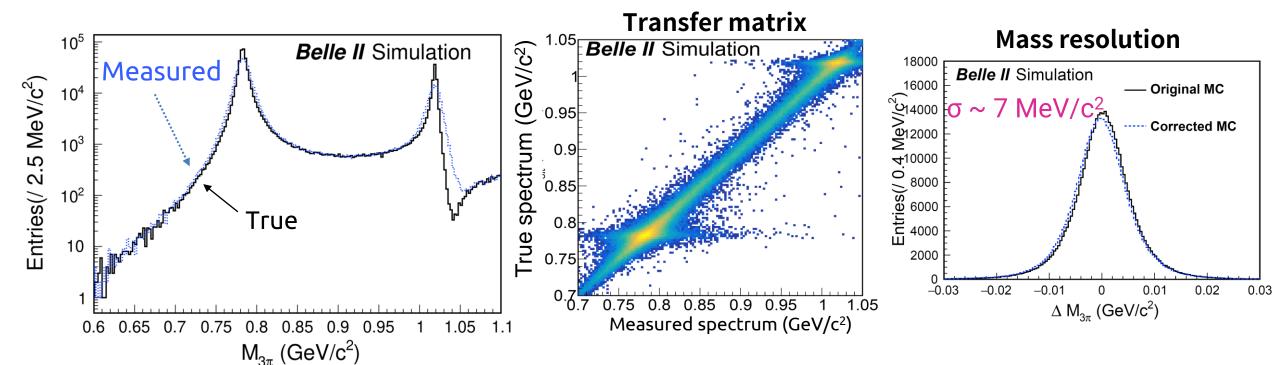


Analysis outline

- Event selection
- Background estimation
- Signal extraction
- Unfolding
- Efficiency estimation
- \square Cross section and a_{μ} calculation

Unfolding

- ☐ The signal spectrum is unfolded to mitigate the effect of detector resolution
 - \blacksquare typically with a mass resolution around 7-10 MeV/c²
- **The data-MC difference of mass bias and resolution is determined** by a Gaussian convolution fit to the ω, Φ, and J/ψ resonances
 - Mass bias of 0.5-1.5 MeV/ c^2 , and resolution of about 1 MeV/ c^2 is corrected



Analysis outline

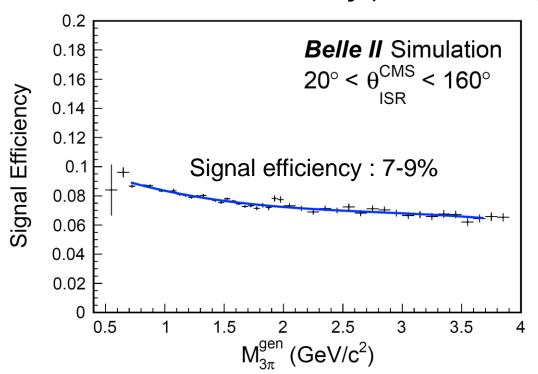
- Event selection
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- ☐ Signal extraction
- Unfolding
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Signal efficiency and data-MC corrections

Efficiency
$$\varepsilon = \varepsilon_{\text{MC}} \prod_{i} (1 + \delta_{i})$$
 Data-MC correction $\delta_{i} \sim O(1)\%$

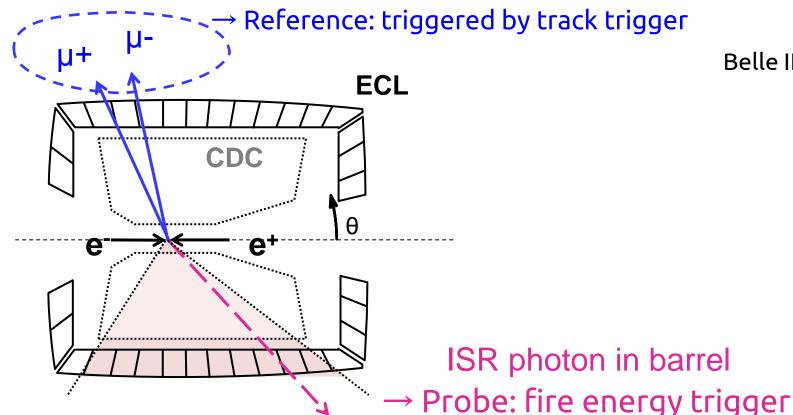
- 1st order signal efficiency is estimated using MC of the x10 larger statistics
- Possible differences between data and MC are checked in data-driven way
 - Trigger efficiency
 - ISR photon efficiency
 - Tracking efficiency
 - $-\pi^0$ efficiency
 - Selection efficiency
 - Higher-order ISR effects

MC detection efficiency (no correction)

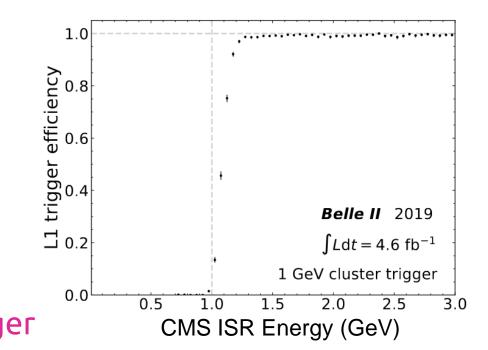


Trigger efficiency

- ☐ ISR events are triggered by the calorimeter
- ☐ The efficiency can be measured by using the events triggered independently by the tracker
 - ☐ Efficiency for energetic ISR in barrel region: 99.9%
- ☐ The uncertainty related is small, 0.1%
- This also benefits other final-state measurements

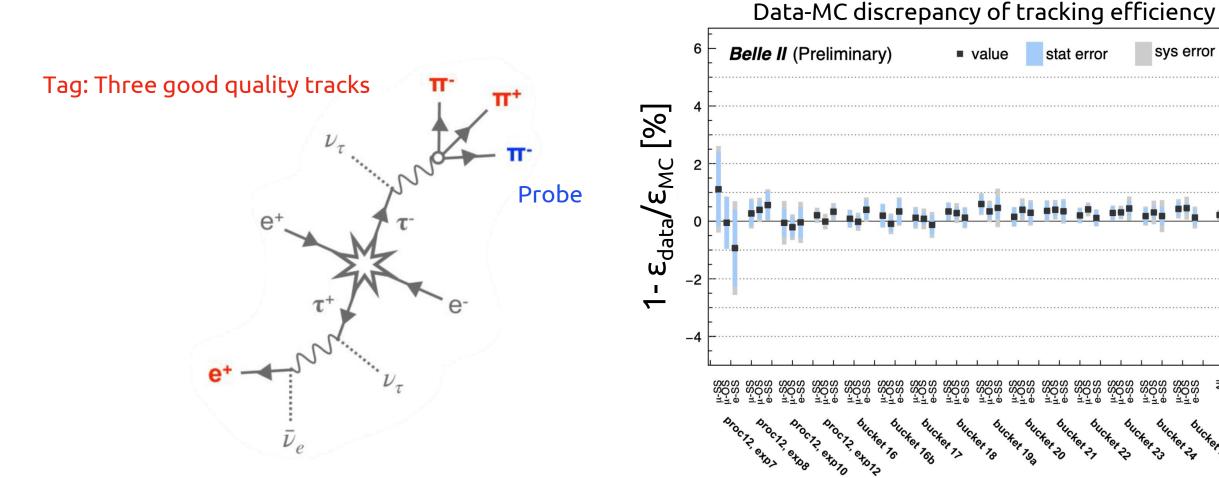


Belle II trigger efficiency measured by $\mu\mu\gamma$ (data)



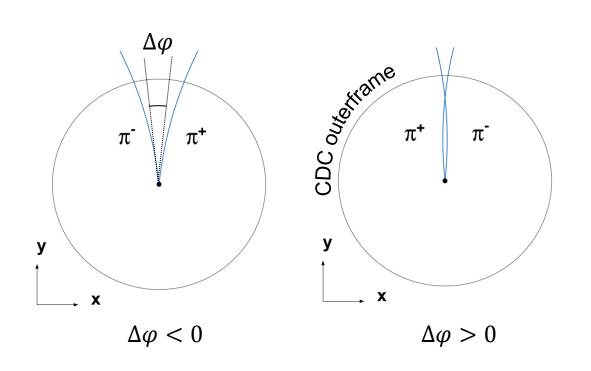
Tracking efficiency

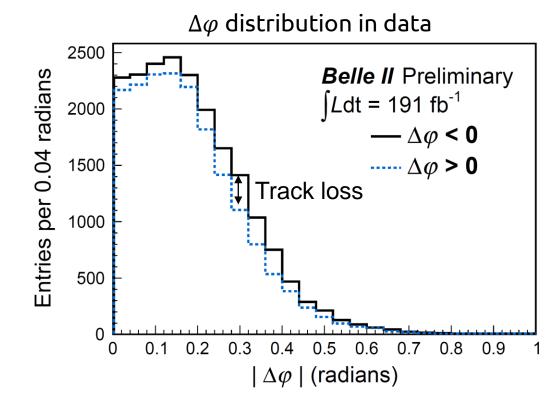
- \Box Tracking efficiency for pions is studied with the e⁺e⁻ \rightarrow τ ⁺ τ ⁻ process.
- □ Data-MC differences are confirmed to be small with 0.3% uncertainty per track.



Tracking efficiency: Track loss

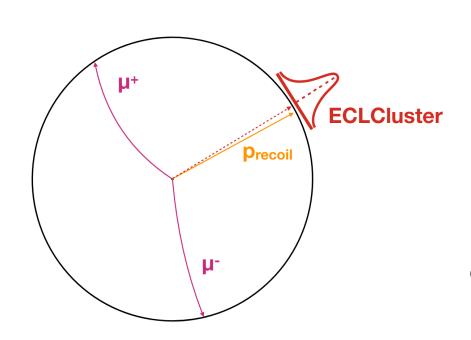
- \square Track loss due to shared hits on the drift chamber is confirmed using the e⁺e⁻ \rightarrow π ⁺ π ⁻ π ⁰ γ
- \square Define $\Delta \varphi \coloneqq \varphi(\pi^+) \varphi(\pi^-)$
- The Inefficiency due to track loss is given by $f = \frac{N(\Delta \varphi < 0) N(\Delta \varphi > 0)}{2N(\Delta \varphi < 0)}$
 - The track loss is 5.0% in data and 4.0% in MC
- \Box In total, the correction factor of tracking is $(-1.4\pm0.8)\%$.
 - Dependency on no. of CDC hits and duplicated tracks are also studied.

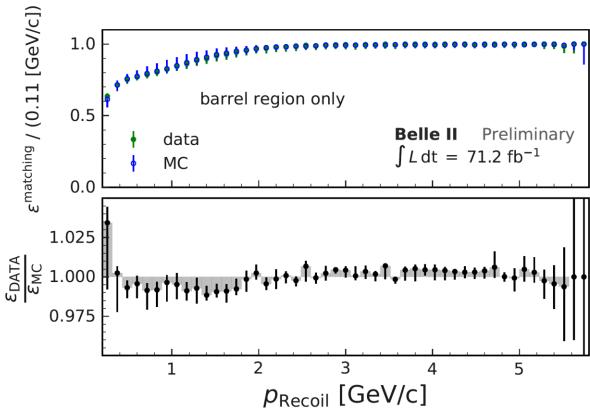




ISR photon detection efficiency

- \square Photon detection efficiency is measured using $e^+e^- \rightarrow \mu^+\mu^-\gamma$ events
 - □ Taking a match between a ECL cluster and the missing momentum of dimuon system
- Efficiency is in good agreement with 0.7% systematic uncertainty

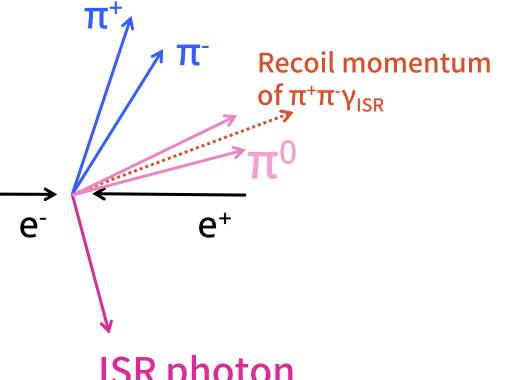


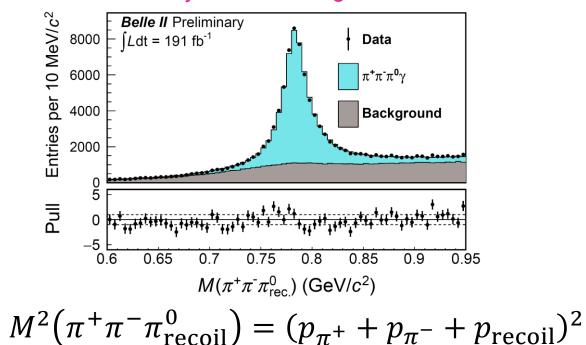


π⁰ efficiency correction

- \square Accurate evaluation of π^0 efficiency in e⁺e⁻ experiment is a challenging task.
 - \blacksquare Exclusive processes that include a π^0 are limited.
- \square Evaluate efficiency using the e⁺e⁻ $\rightarrow \omega \gamma \rightarrow \pi^{+}\pi^{-}\pi^{0}\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction}: \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction}: \gamma_{\text{ISR}}\pi^+\pi^-)} \longrightarrow \text{Count } \omega \rightarrow \pi^+\pi^-\pi^0 \text{ decay without using } \pi^0 \text{ information.}$$



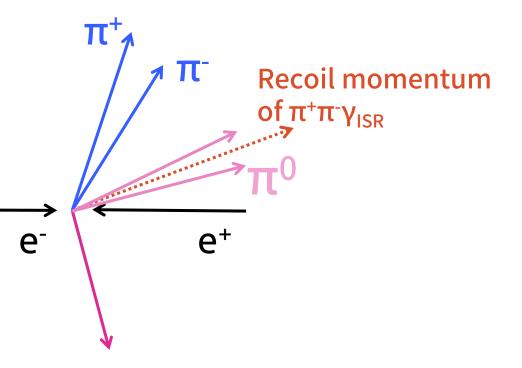


 \blacksquare π^0 momentum p_{recoil} is determined by kinematic fit to $\pi^+\pi^-\gamma$ with hypothesis that recoil mass equals π^0 mass

π⁰ efficiency correction

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 - \blacksquare Exclusive processes that include a π^0 are limited.
- \blacksquare Evaluate efficiency using the e⁺e⁻ $\rightarrow \omega \gamma \rightarrow \pi^{+}\pi^{-}\pi^{0}\gamma$ events.

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}} \pi^+ \pi^- \pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}} \pi^+ \pi^-)} \longrightarrow \text{Count by reconstructing } \pi^0 \text{ and fitting M(}\gamma\gamma)$$



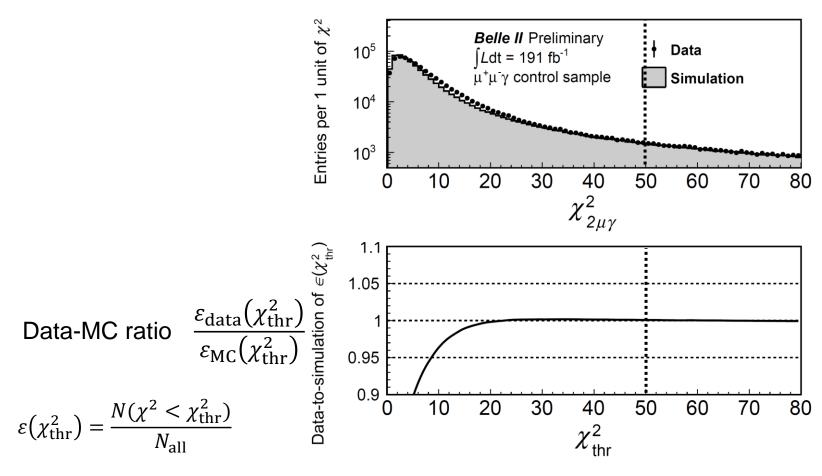
- $\square \varepsilon_{\pi^0}$ are independently evaluated by the data and MC \Box Data/MC ratio = 0.986 ± 0.006_{stat}
- \Box The systematic uncertainty related to π^0 is 1.0%
 - ■The uncertainty is evaluated by variations of the $M(\gamma\gamma)$ signal pdf, background pdfs, and selections

Background suppression efficiency

- Estimated by the ratio of signal yield before/after the criteria
- \Box It is evaluated using ω and Φ , J/ ψ resonances of good S/N
- \blacksquare In M(3 π) < 1.05 GeV/c², efficiency is (89.5 \pm 0.2)% for data
 - \square Correction factor is $(-1.90\pm0.20)\%$
- \square M(3 π) > 1.05 GeV/c² : the number of J/ ψ was obtained by M(3 π) fitting
 - \square Correction factor is $(-1.78 \pm 1.85)\%$
 - ☐ Error is due to statistical errors in the sample

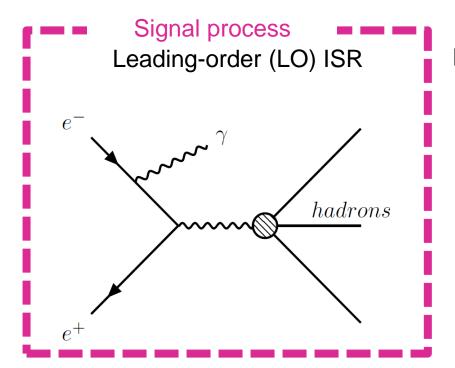
χ2 selection efficiency

- □ ISR and tracks χ^2 -cut efficiency is confirmed using e⁺e⁻ \rightarrow μ ⁺ μ ⁻ γ sample
- ☐ Confirm effects from differences in position, momentum, and energy of ISR and tracks
 - ☐ Agreement confirmed within ±0.6% uncertainty
- Dependence on multi-ISR photon calculations is discussed on the next page

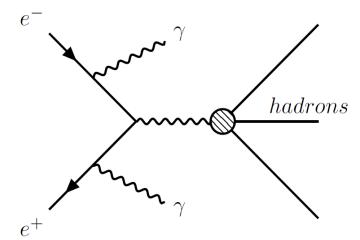


Higher-order ISR effects

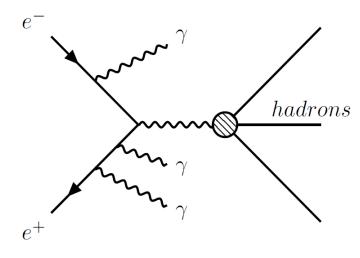
- ☐ Although a one-ISR photon emission process is set as the signal, in reality there are processes with multiple photon emissions.
- ☐ Two effects need to be considered from the existence of multiple photons:
 - A) Effective integrated luminosity $L_{\rm eff}$ (radiative correction): 0.5% unc.
 - B) χ 2 selection efficiency due to ISR photon calculations in generator: 1.2% unc.



Next-to-Leading-order (NLO) ISR



NNLO ISR



Efficiency correction: Summary

	Efficiency correction (%)	
Source	M < 1.05 GeV/c ²	M > 1.05 GeV/c ²
Trigger	-0.1±0.1	-0.1±0.1
ISR photon detection	0.2±0.7	+0.2±0.7
Tracking	-1.4±0.8	-1.7±0.8
π^0 reconstruction	-1.4±1.0	-1.4±1.0
Background suppression	-1.9±0.2	-1.8±1.9
χ^2 distribution	0.0±0.6	0.3±0.3
MC generator	0.0±1.2	0.0±1.2
Total correction	-4.6±2.0	-4.6±2.0

Analysis outline

- Event selection
- Background estimation
- Signal extraction
- Unfolding
- Efficiency estimation
- ☐ Cross section and a_u calculation

Systematic uncertainty for $e^+e^-\rightarrow \pi^+\pi^-\pi^0$ cross section

- Luminosity is measured with Bhabha events and confirmed with $e^+e^- \rightarrow \gamma \gamma$ and $\mu^+\mu^-$ processes
- Major systematic uncertainty comes from MC generator, and π^0 efficiency
 - In M(3 π) > 1.05 GeV, the uncertainty of selection efficiency is dominant

Source -	Systematic uncertainty (%)	
	\sqrt{s} < 1.05 GeV ²	√s > 1.05 GeV
Trigger	0.1	0.2
ISR photon detection	0.7	0.7
Tracking	0.8	8.0
π^0 reconstruction	1.0	1.0
χ^2 criteria efficiency	0.6	0.3
Background suppression	0.2	1.9
MC generator	1.2	1.2
Radiative correction	0.5	0.5
Integrated luminosity	0.6	0.6
Total systematics	2.2	2.8

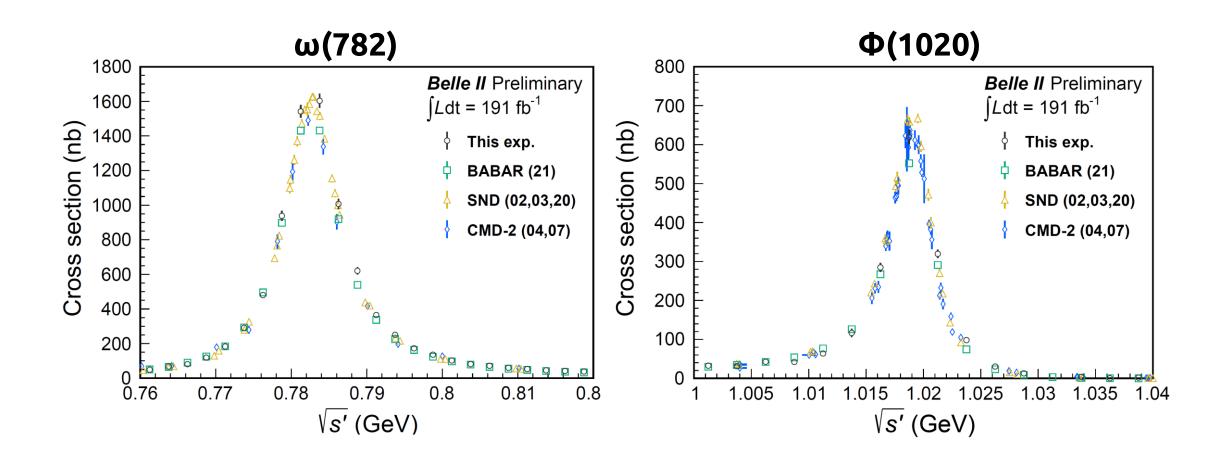
Cross section calculation

Cross section $\sigma_{ee \to 3\pi}(M_i(3\pi)) = \frac{N_{\rm unfolded}, i}{\varepsilon(M_i(3\pi)) \cdot L_{\rm eff}(M_i(3\pi)) \cdot r_{\rm rad}}$ Radiative correction Effective luminosity

Corrected Efficiency

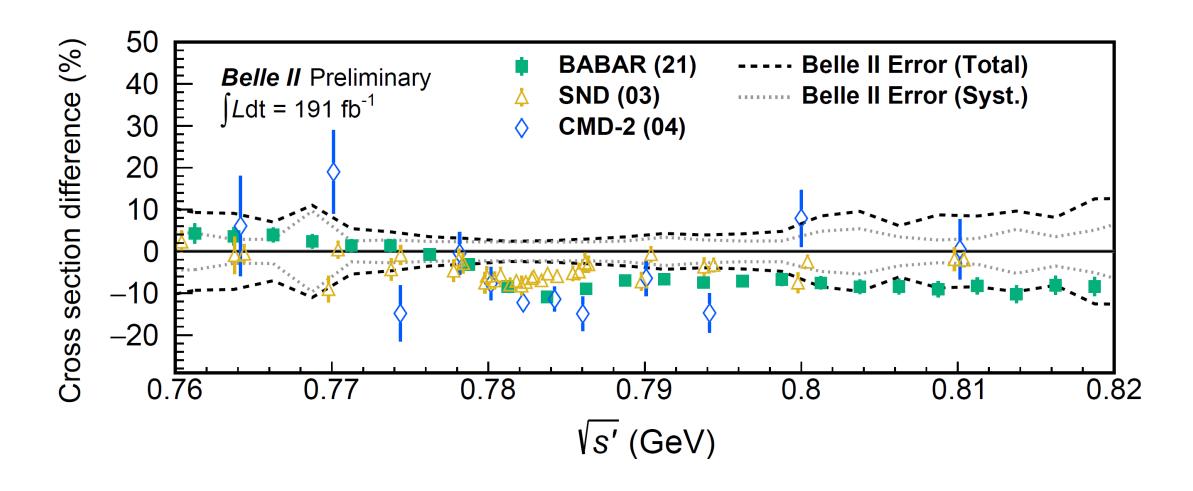
Unfolded signal spectrum

Result: cross section below 1.05 GeV



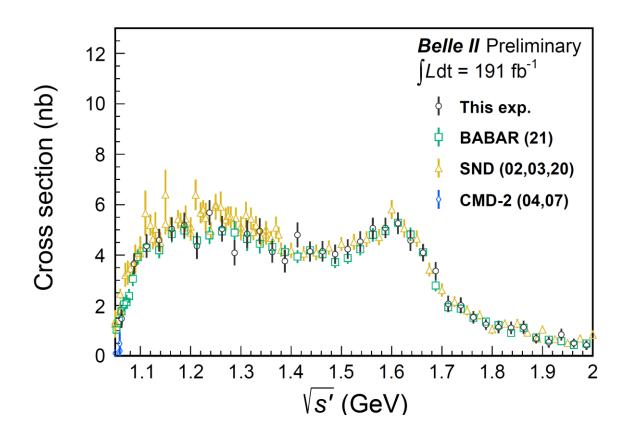
Result: cross section below 1.05 GeV

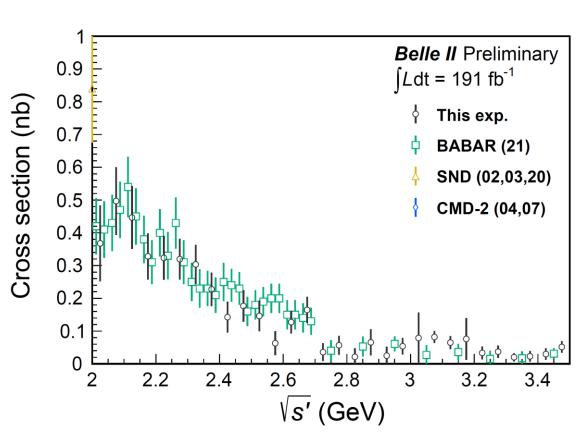
 \blacksquare Cross section at ω resonance is 5-10% higher than SND, BABAR, and CMD-2



Result: cross section above 1.05 GeV

☐ Good agreement with BABAR result





Results: 3π contribution to a_{μ} HVP

$$a_{\mu}^{\text{LO,HVP,3}\pi}(0.62 - 1.8 \text{ GeV}) = (48.91 \pm 0.25_{\text{stat}} \pm 1.07_{\text{syst}}) \times 10^{-10}$$

	a _μ (3π)×10 ¹⁰	Difference×10 ¹⁰	
BABAR alone [PRD104 11 (2021)]	45.86 ± 0.14 ± 0.58	-3.2±1.3 (6.9%)	
Global fit [JHEP08 208 (2023)]	45.91 ± 0.37 ± 0.38	-3.0±1.2 (6.5%)	

- \square 6.5% higher than the global fit result with 2.5 σ significance
- **□** This difference $3x10^{-10}$ corresponds 10% of $\Delta a_{\mu} = a_{\mu}(Exp) a_{\mu}(SM) = 25x10^{-10}$

Systematic uncertainty for a_{μ}

Source	Systematic uncertainty (%)
Efficiency corrections	1.63
Monte Carlo generator	1.20
Integrated luminosity	0.64
Simulated sample size	0.15
Background subtraction	0.02
Unfolding	0.12
Radiative corrections	0.50
Vacuum polarization corrections	0.04
Total	2.19

Next: $e^+e^- \rightarrow \pi^+\pi^-$ at Belle II

- \square Target precision: 0.5% of $a_{\mu}(2\pi)$
- ☐ Trying to follow BABAR methods as a baseline
- Systematics uncertainty dominant analysis
 - BABAR: 232 /fb [Phys. Rev. D 86 (2012), 032013]
 - We can use large dataset to control systematic uncertainties
- \Box Design of data-driven efficiency corrections for tracking, trigger and $\pi/\mu/K$ ID is ongoing

Summary

- ☐ Cross-section measurements are ongoing at the SuperKEKB/Belle II experiment
 - Good trigger efficiency thanks to the upgrade is confirmed
 - Further channel analysis can be expected in the future
- \square We measured the e⁺e- $\rightarrow \pi^+\pi^-\pi^0$ cross section with systematic uncertainty of 2.2%
 - The second largest contribution to HVP term
 - The largest uncertainty arises from NLO/NNLO calculation in MC generator
- \Box Our results are about 2.5 σ greater than BABAR and global fit

$$\blacksquare \ a_{\mu}^{\text{LO,HVP}}(3\pi) = (48.91 \pm 0.25_{\text{stat}} \pm 1.07_{\text{syst}}) \times 10^{-10}$$

