New developments on **inclusive** V_{cb}

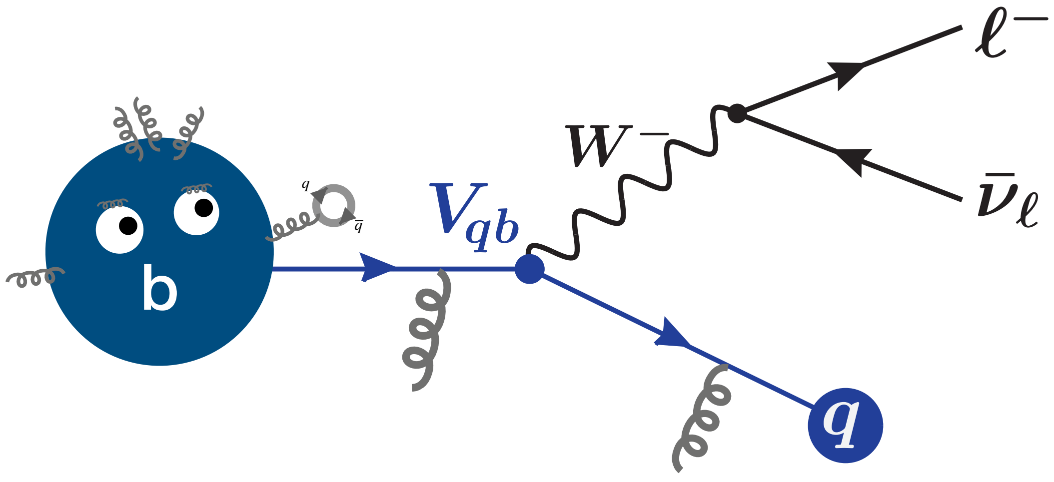
florian.bernlochner@uni-bonn.de

Many thanks to feedback from

…

Puzzles…

It may look cute, but that might be deceiving…



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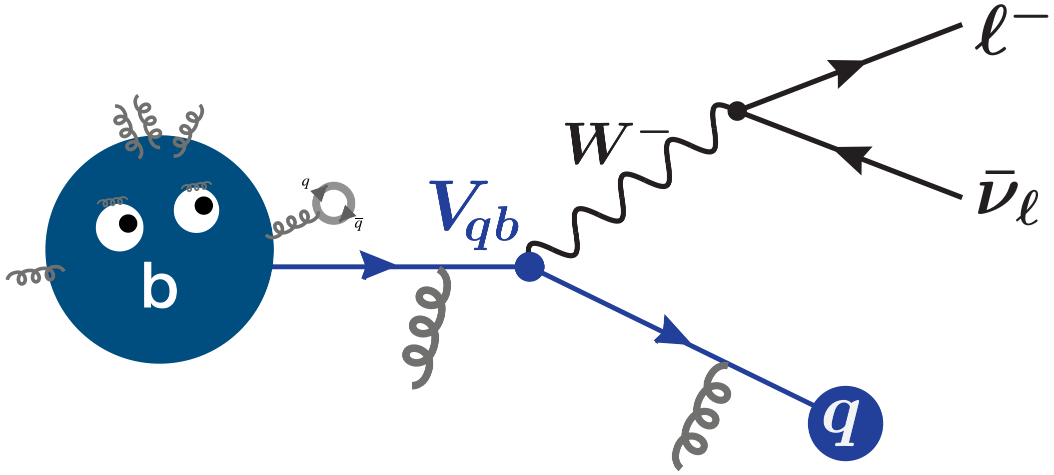
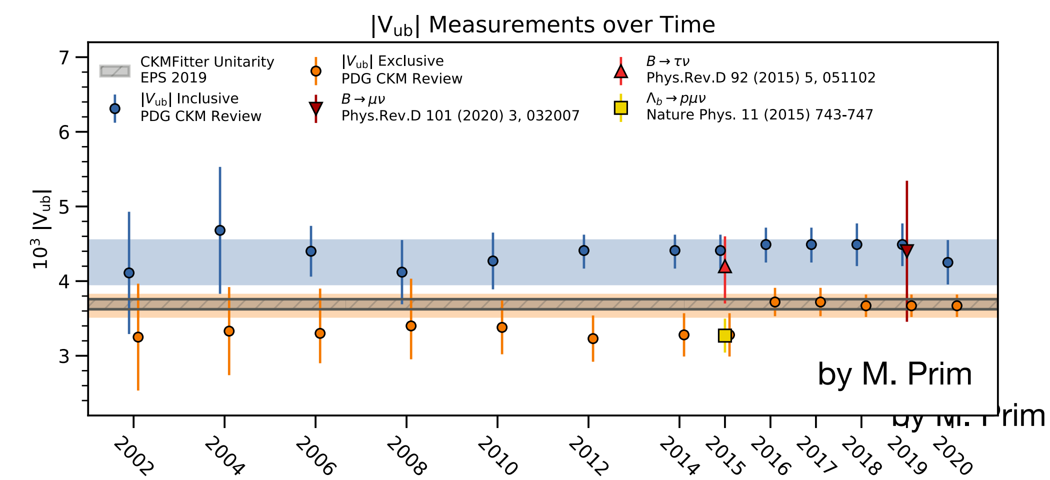
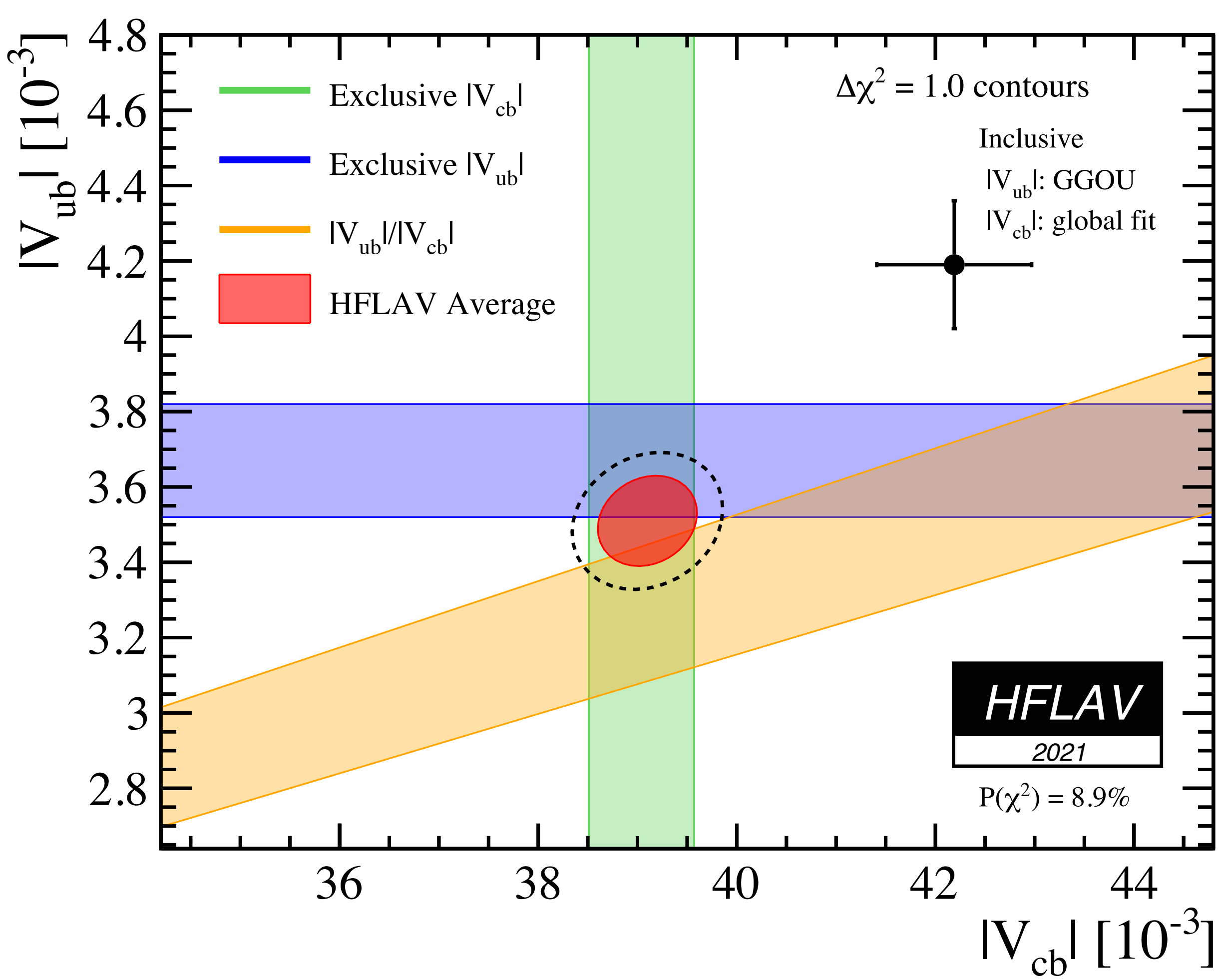


… Long-standing discrepancy since about a decade





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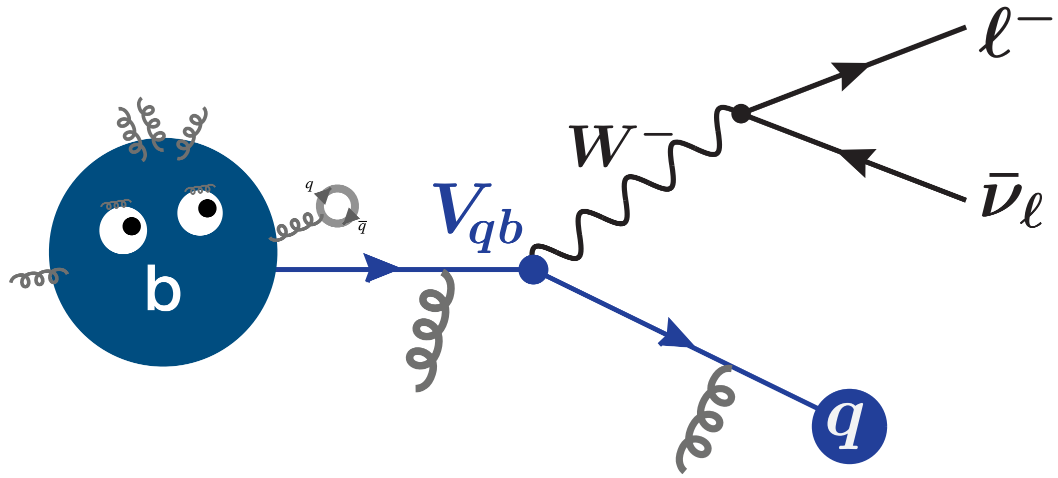

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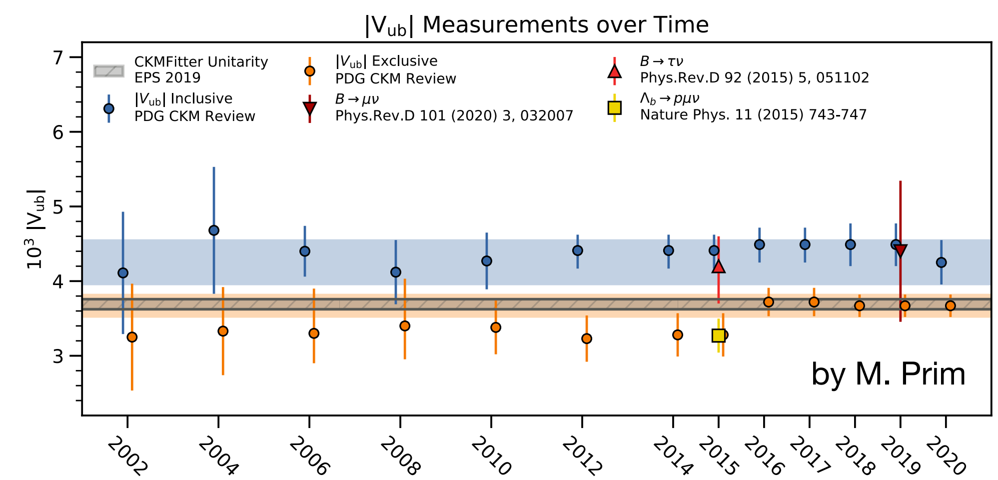

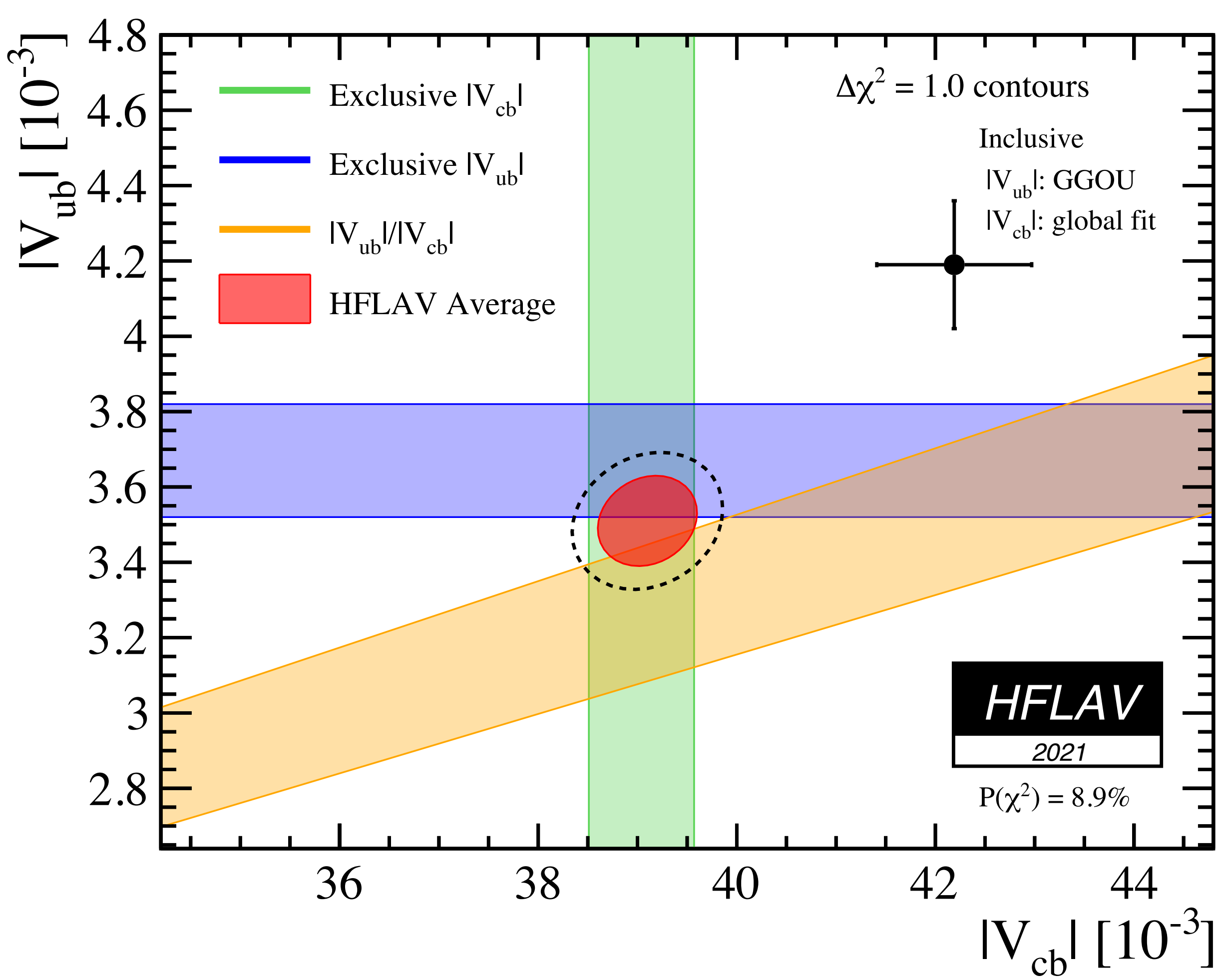

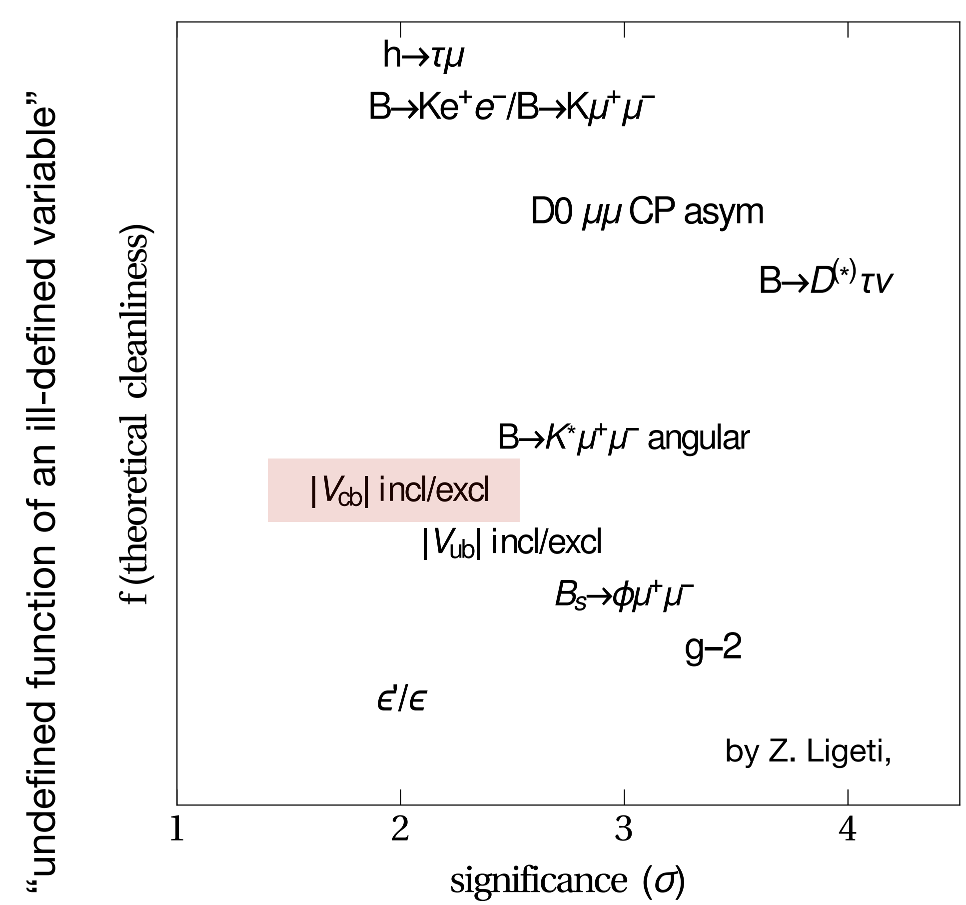

How to inclusive V_{cb}

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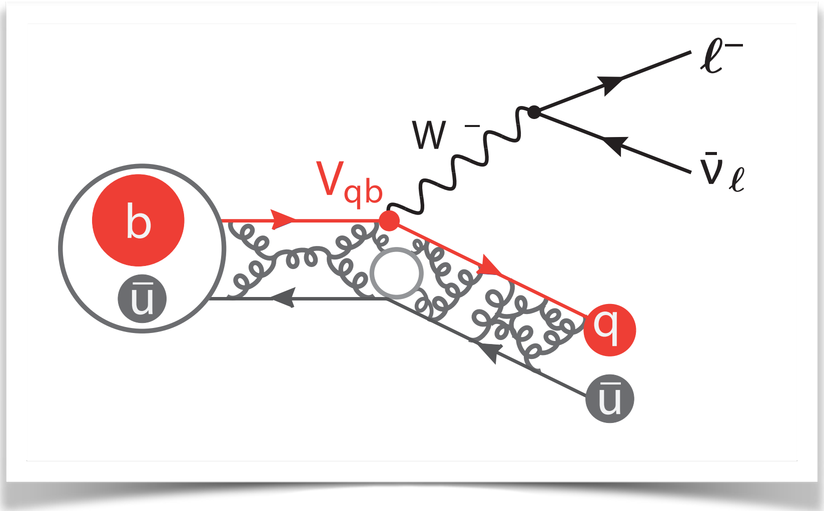


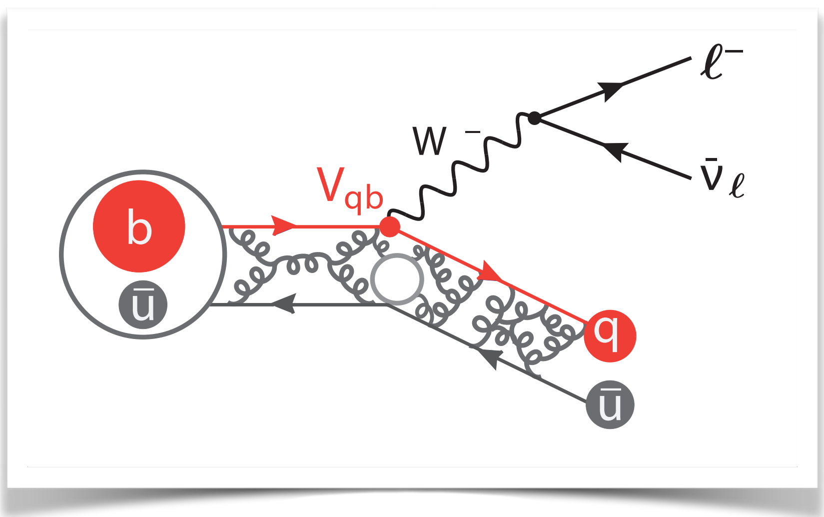
Inclusive $|V_{cb}|$

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

How to inclusive V_{cb}



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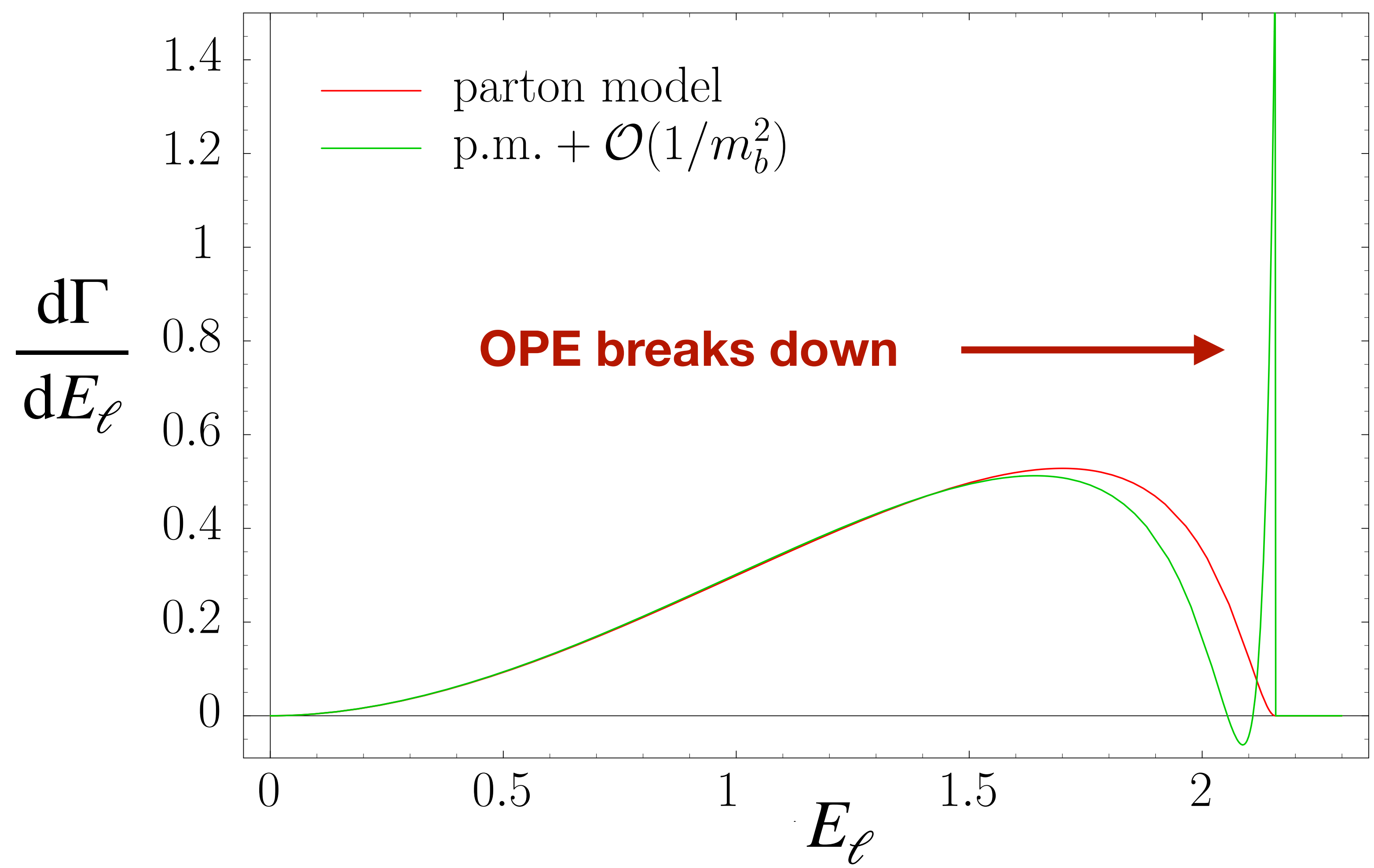
Other complication: OPE does not allow point-by-point predictions



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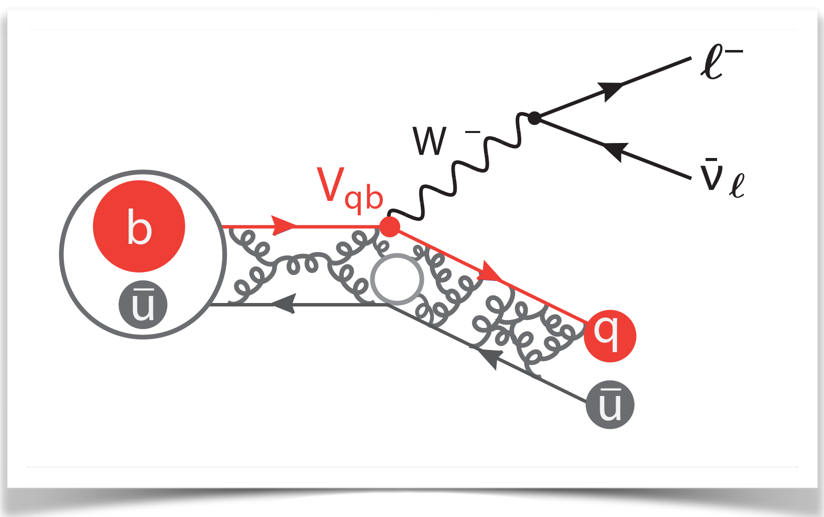

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Other complication: OPE does not allow point-by-point predictions

But converges if integrated over large parts of phase space

$v = p_B / m_B$

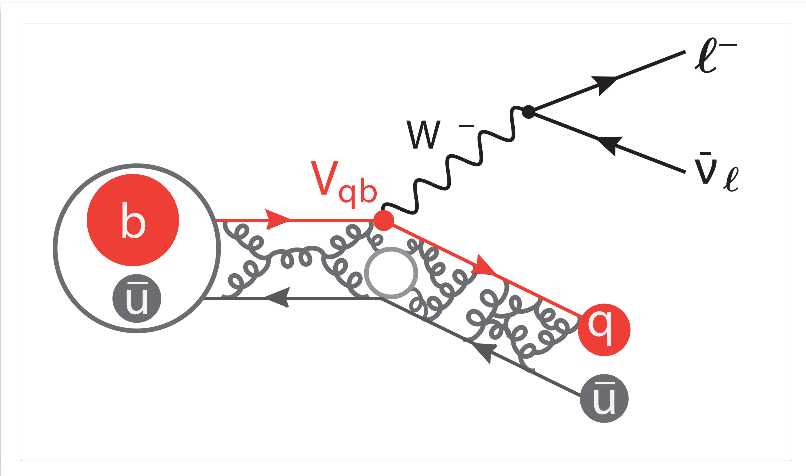
$$\int w^n(v, p_\ell, p_\nu) \frac{d\Gamma}{d\Phi} \, d\Phi$$

weight function

Example weight functions

$w =$	$\left(p_\ell + p_\nu\right)^2$	$= q^2$	four-momentum transfer squared
$w =$	$\left(m_B v - q\right)^2$	$= M_X^2$	invariant mass squared
$w =$	$\left(v \cdot p_\ell\right)$	$= E_\ell^B$	Lepton Energy

How to inclusive V_{cb}



Inclusive $|V_{cb}|$

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Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Established approach: Use **spectral moments** (hadronic mass moments, lepton energy moments etc.) to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

$d\Gamma$ are calculated perturbatively

Available at $\mathcal{O}(\alpha_s^3)$
Fael, Schönwald, Steinhauser
Phys. Rev. D 104, 016003 (2021)

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ encapsulate non-perturbative dynamics

HQE parameters must be extracted from data

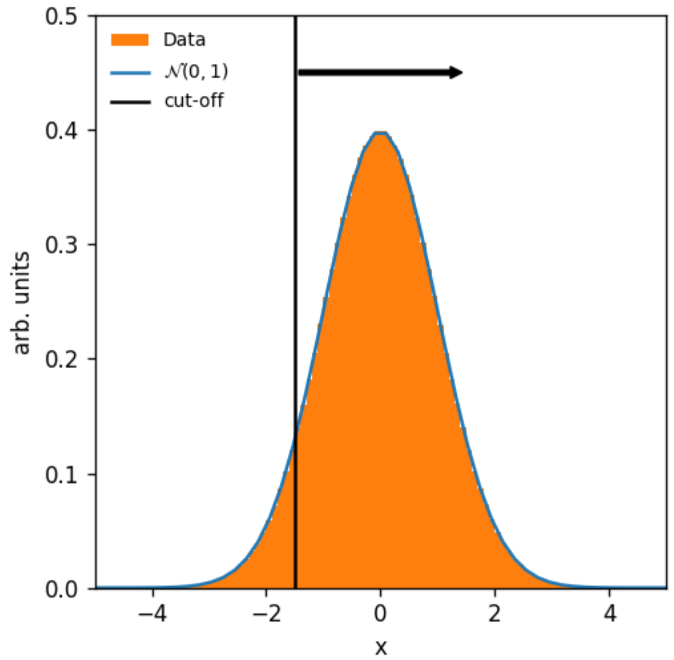
requires the spectral moments of $B \to X_c \ell \nu$

Challenge: Proliferation of HQE parameters at higher order

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Let’s take a moment or two…

Illustrations by Markus Prim



$$\mu_n = \int_{-\infty}^{-\infty} (x-c)^n f(x)dx$$

Raw moment: $c = 0$

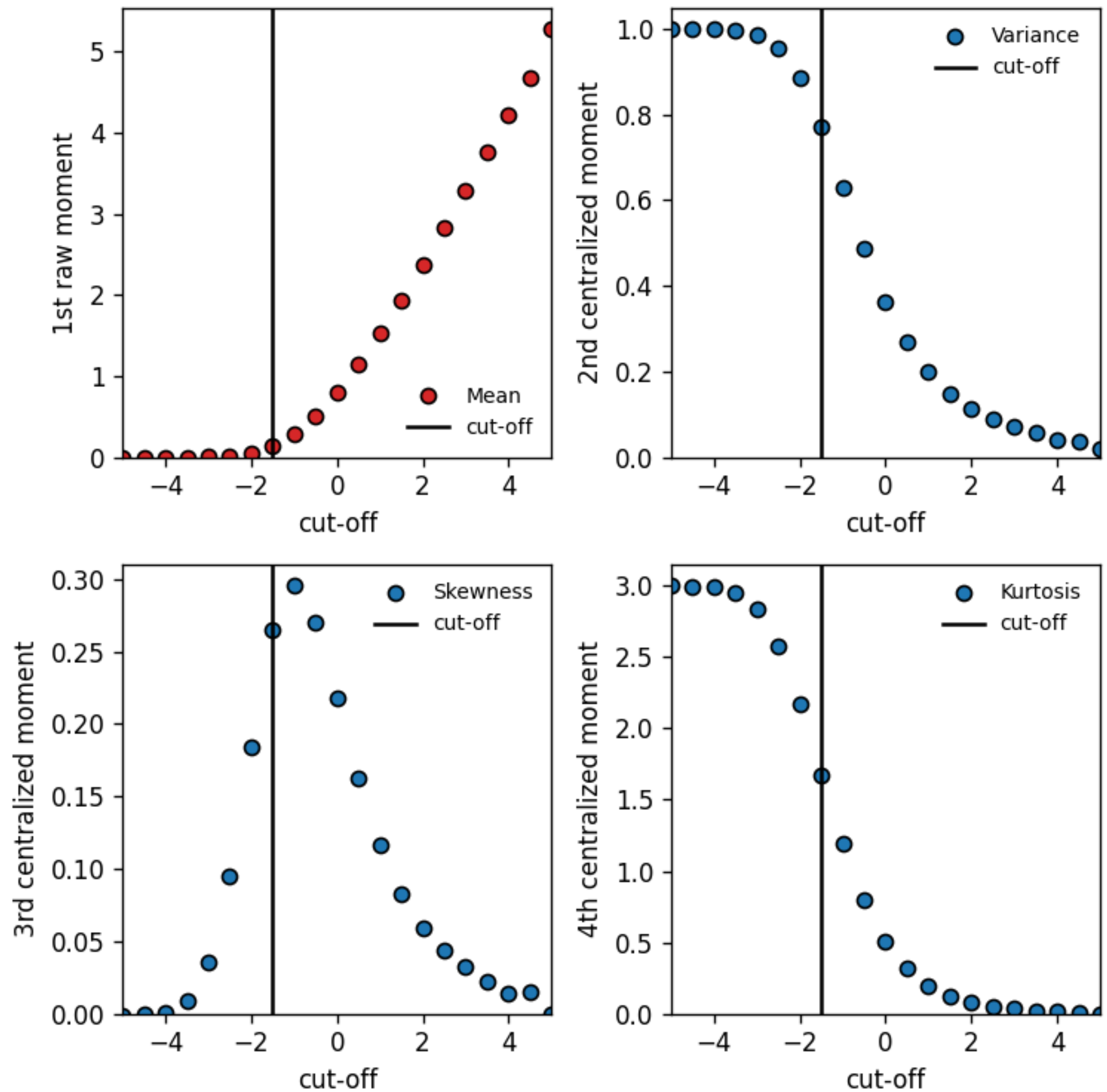
Central moment: $c = \text{Mean}$

First raw moment: Mean
Measures the location

Second central moment: Variance
Measures the spread

Third central moment: Skewness
Measures asymmetry

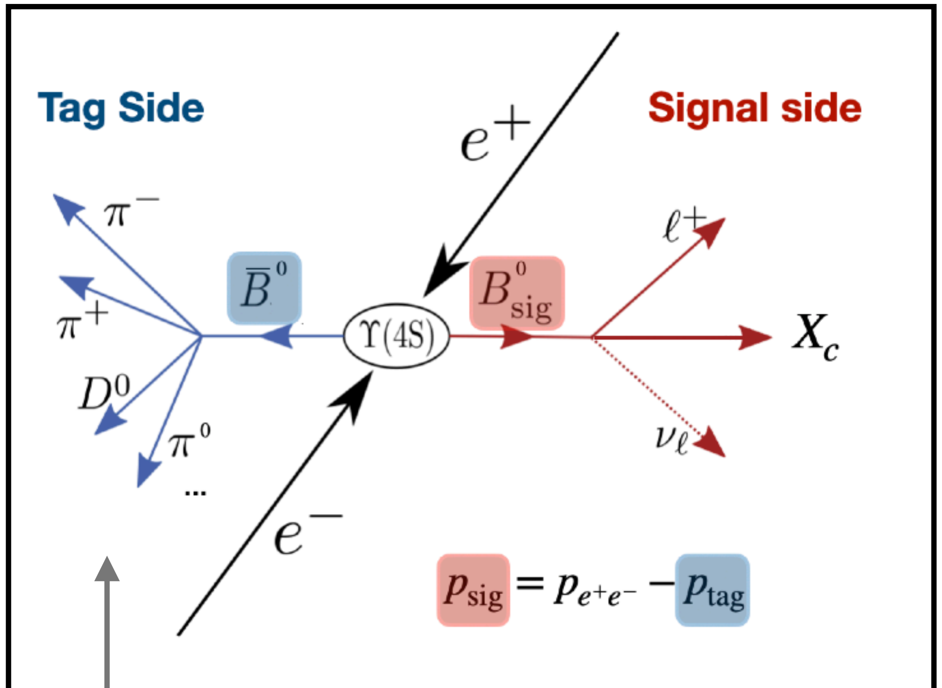
Fourth central moment: Kurtosis
Measures “tailedness”



Moments are measured with progressive cuts in the distribution
→ **highly correlated measurements**

How to measure spectral moments

Key-technique: hadronic tagging



Hadronic Tagging
with **Belle II** algorithm (FEI)

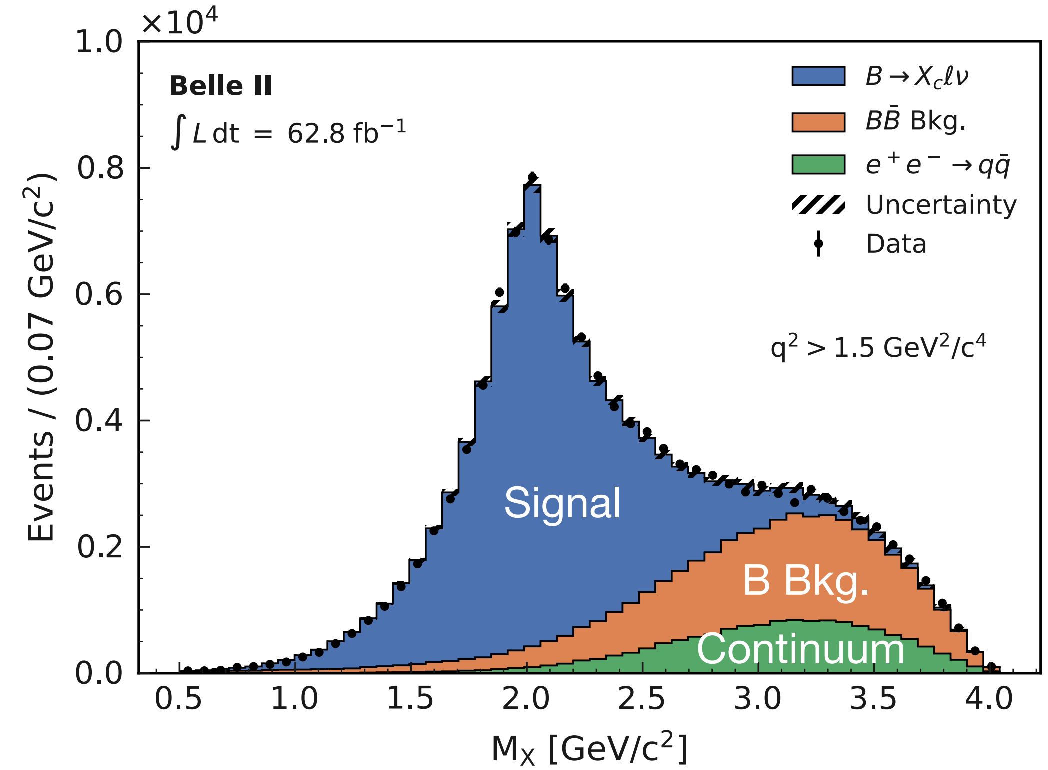
[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

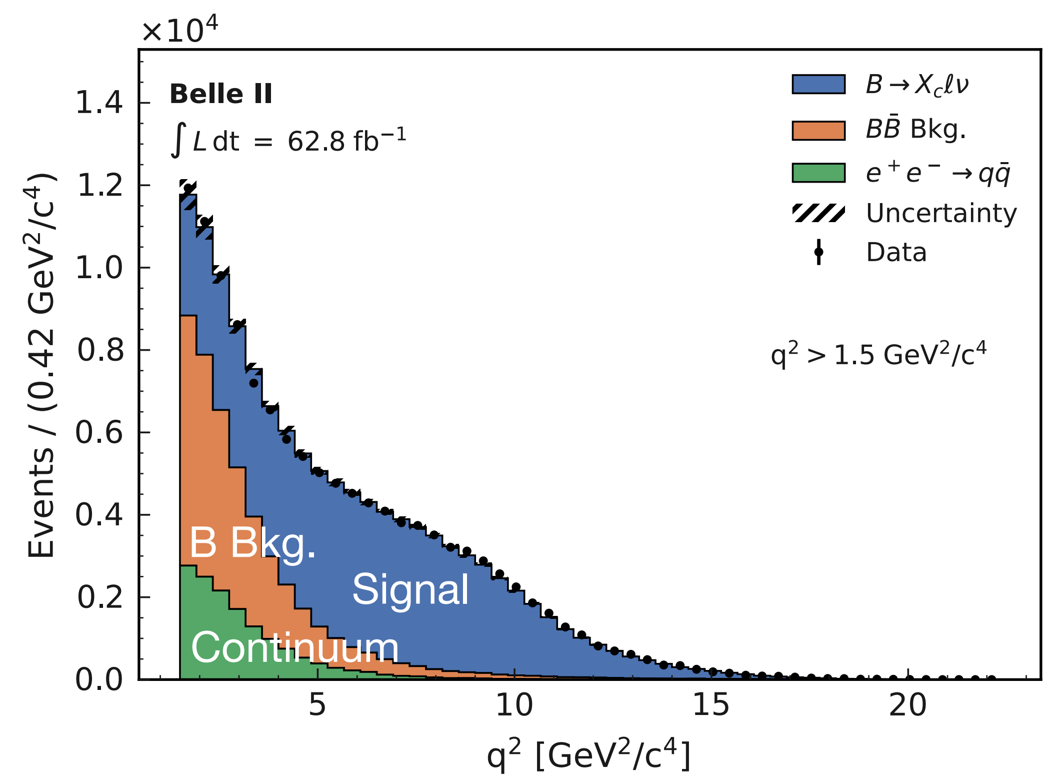
Can identify X_c constituents

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$

[PRD 107, 072002 (2023), arXiv:2205.06372]





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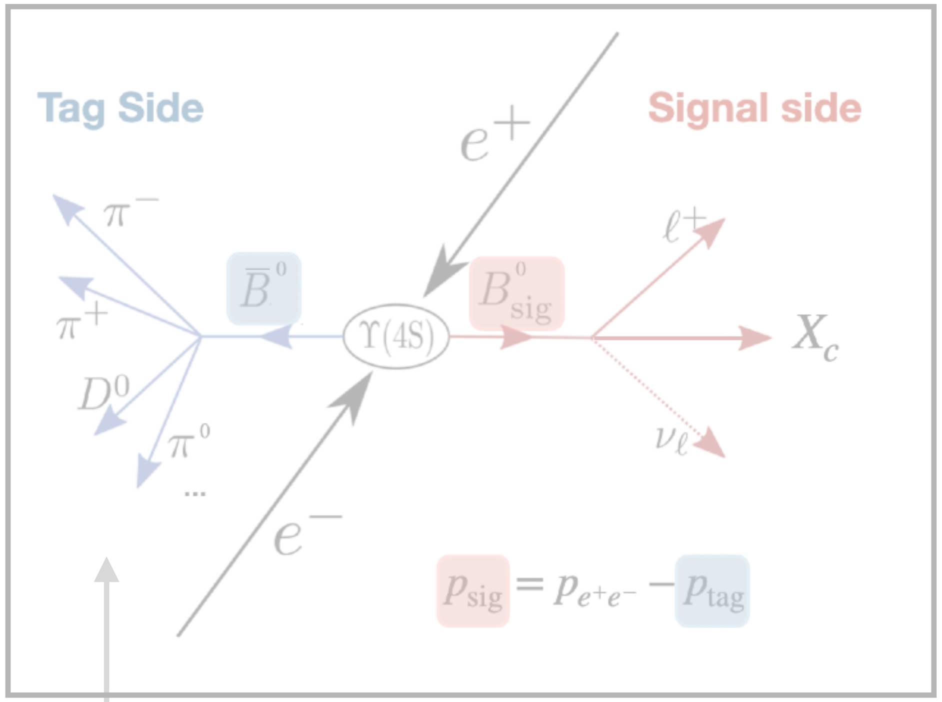

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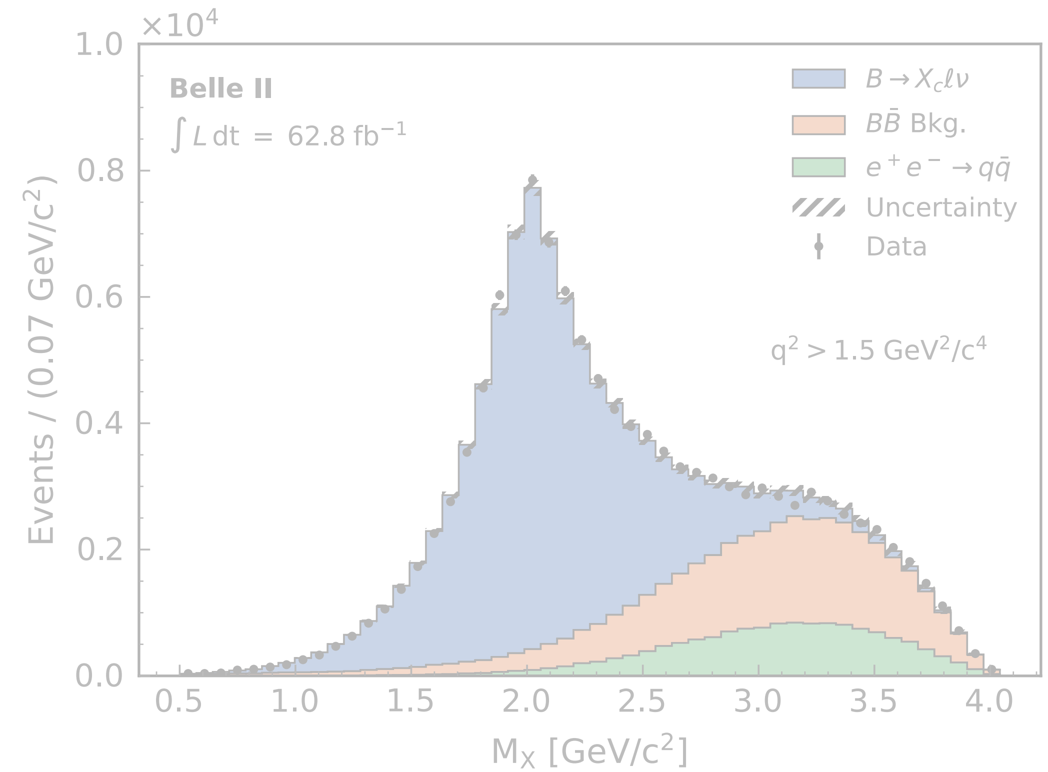

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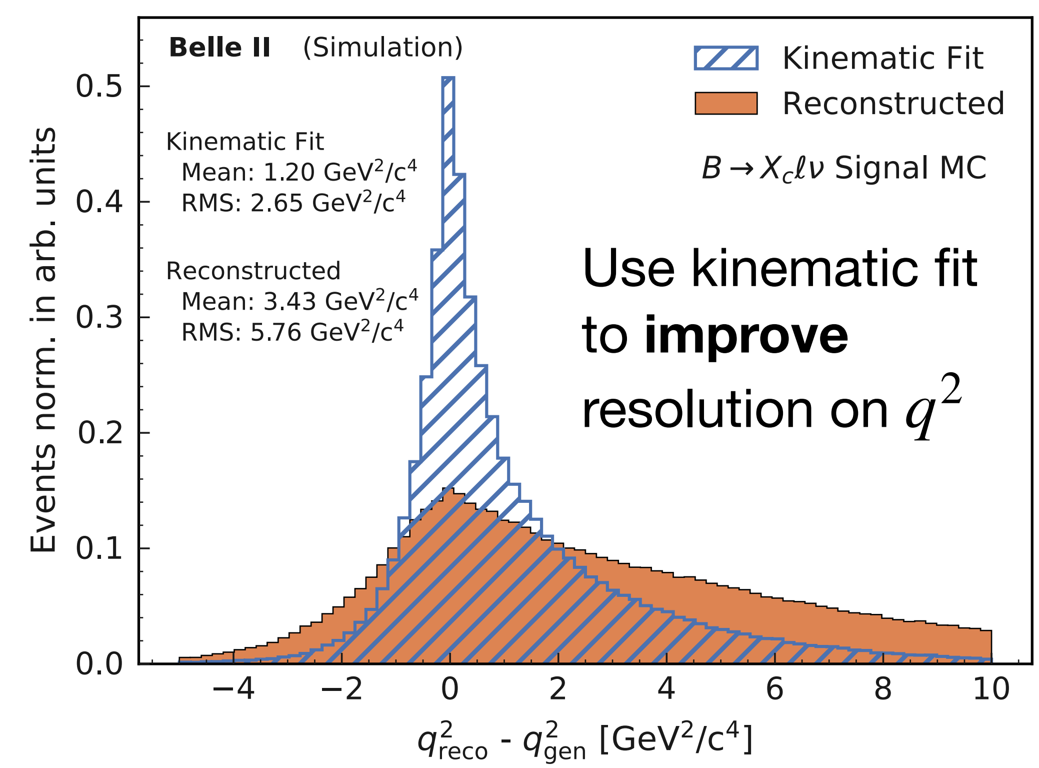
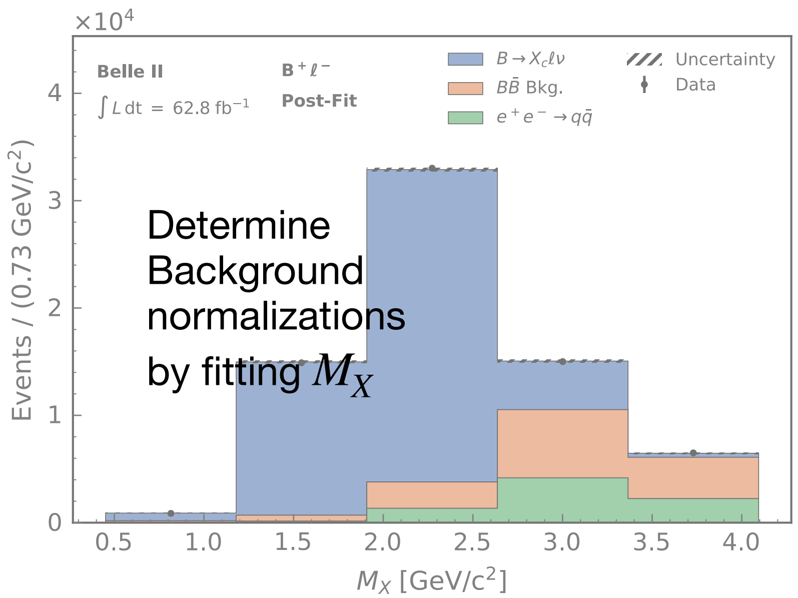
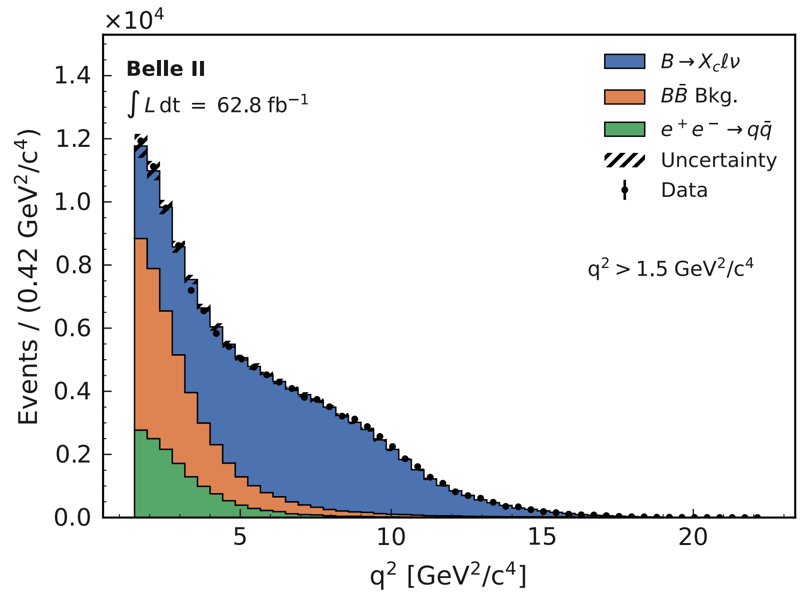


Use kinematic fit to **improve** resolution on q^2

Measurement in a nutshell



→



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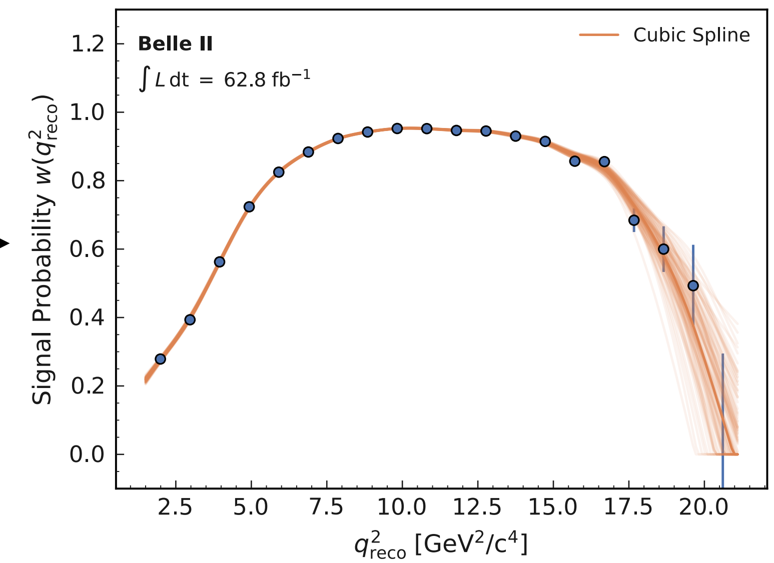


Step #1: Subtract Background

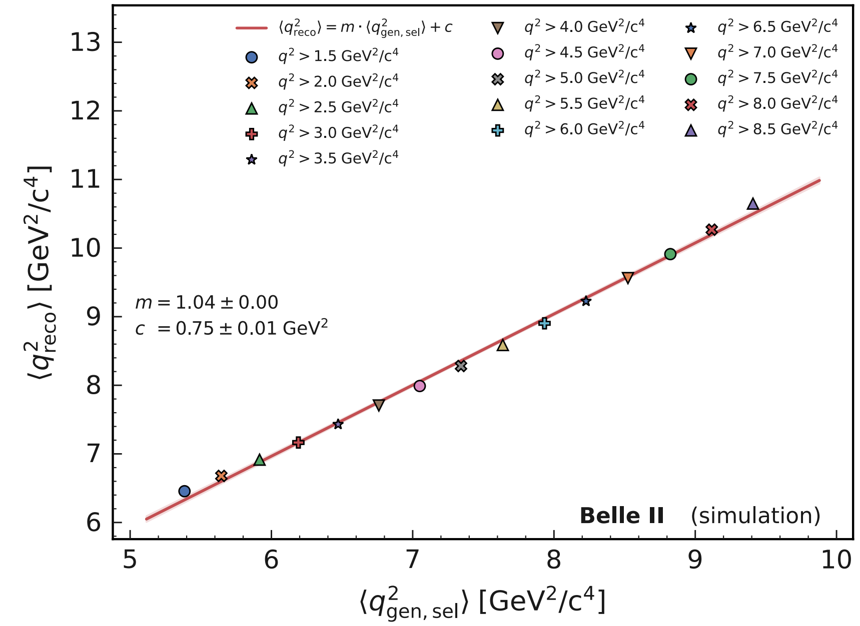
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,i}}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,j}})} \times \mathcal{C}_{\mathrm{calib}} \times \mathcal{C}_{\mathrm{gen}} \, ,$$

Measurement in a nutshell

Exploit linear dependence between rec. & true moments

$$q^{2m}_{\mathrm{cal}\,i} = \left(q^{2m}_{\mathrm{reco}\,i} - c\right)/m$$



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Step #2: Calibrate moment

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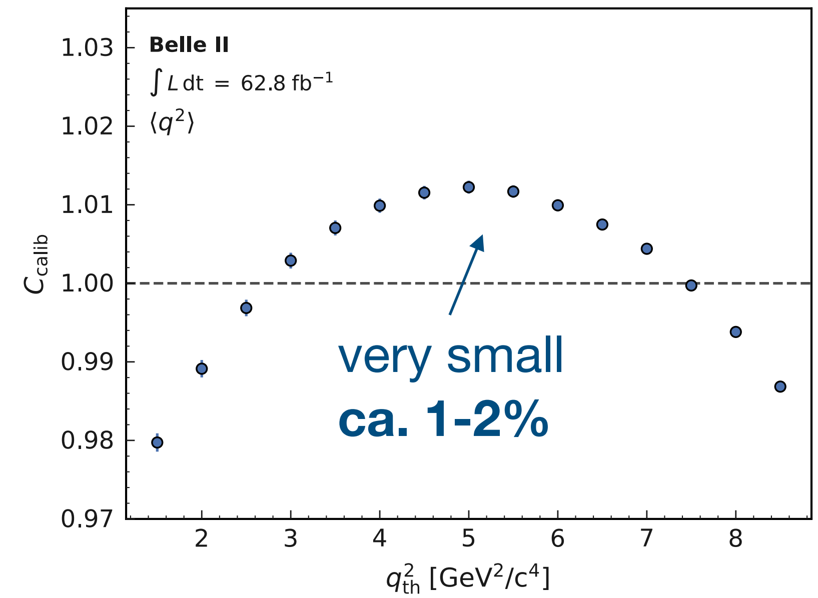
Measurement in a nutshell

Very small deviation from linear behavior between reconstruct and true q^2



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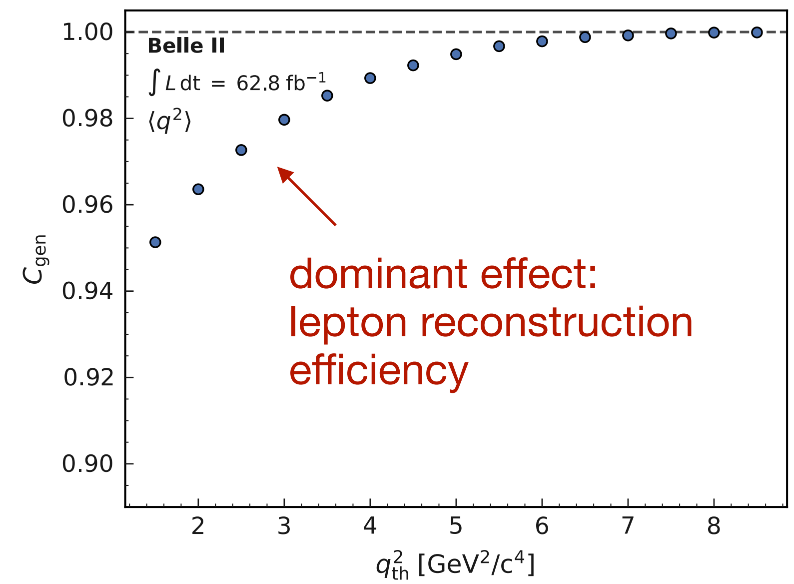
Step #3: If you fail, try again

Measurement in a nutshell

Account for efficiency & acceptance effects



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Step #4: Correct for selection effects

Measurement in a nutshell

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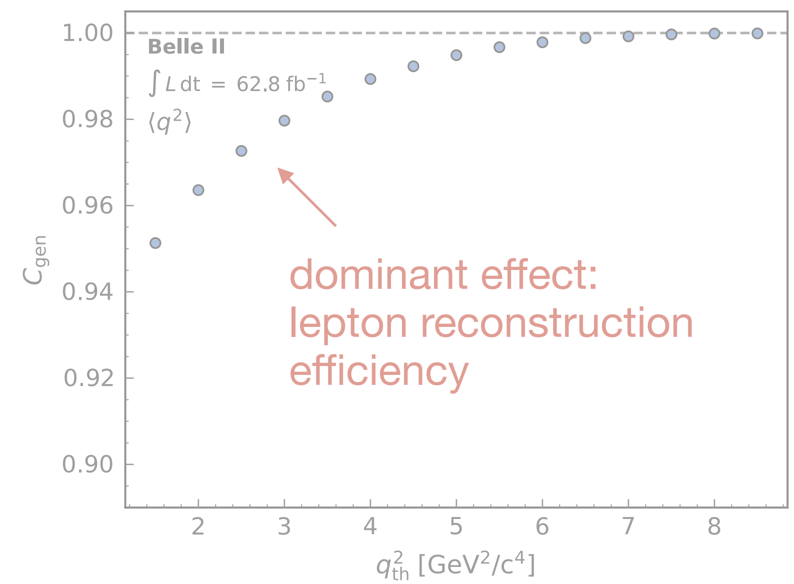

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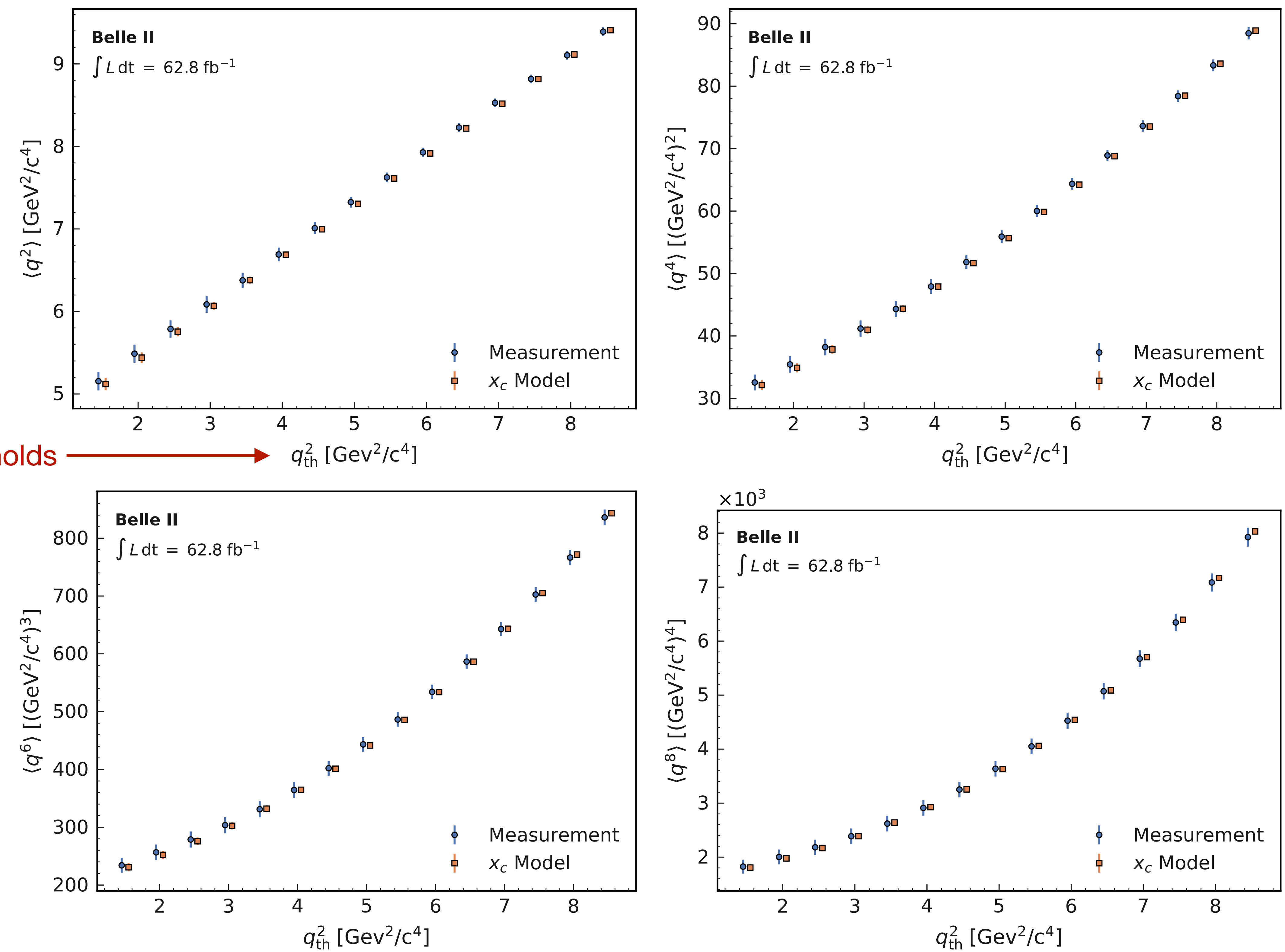
Step #3: If you fail, try again

Step #4: Correct for selection effects

→ Repeat this for many different thresholds cuts $q^2_{\rm th}$

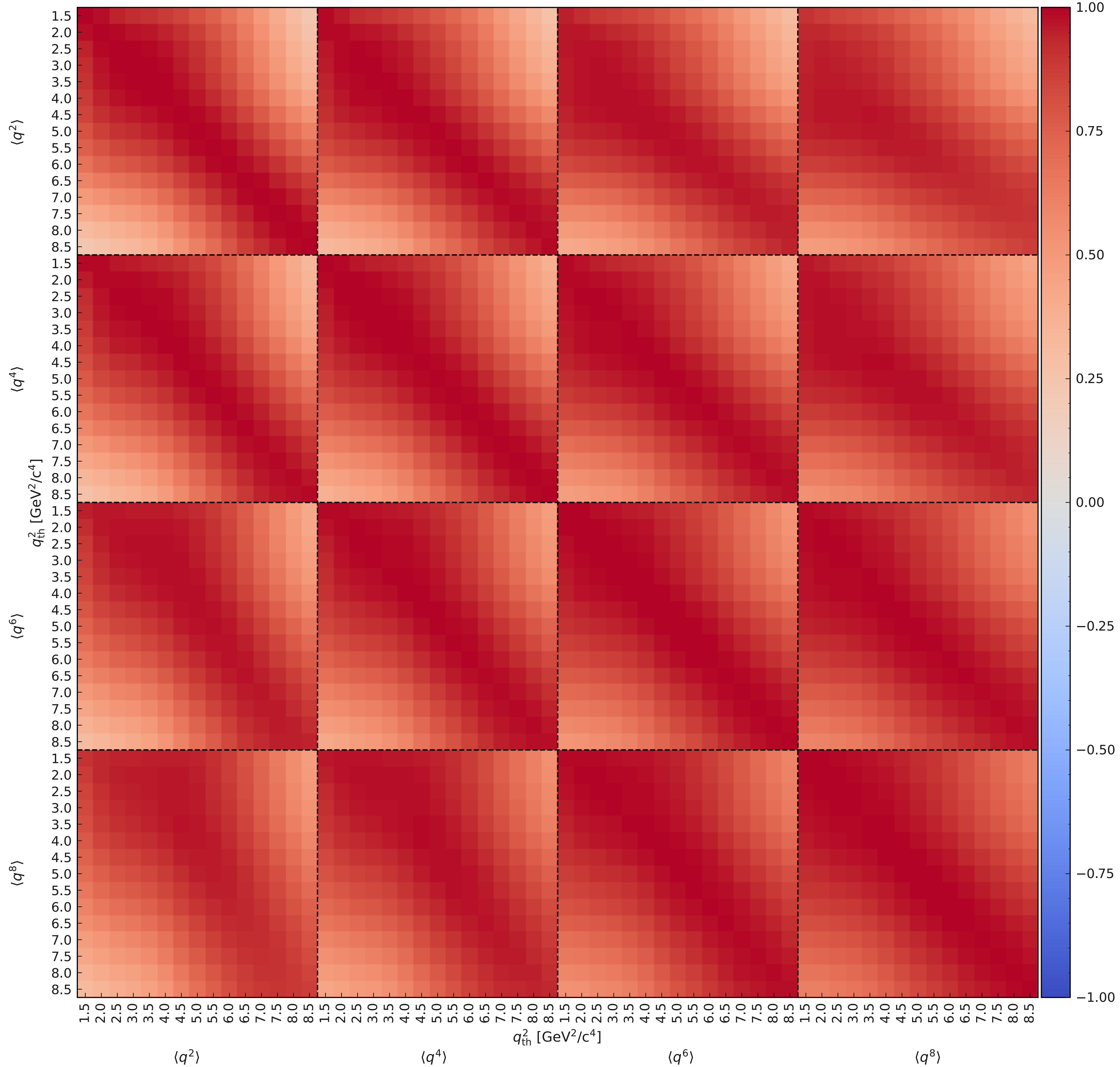
Example: Belle II q^2 spectral moments

q^2 thresholds →



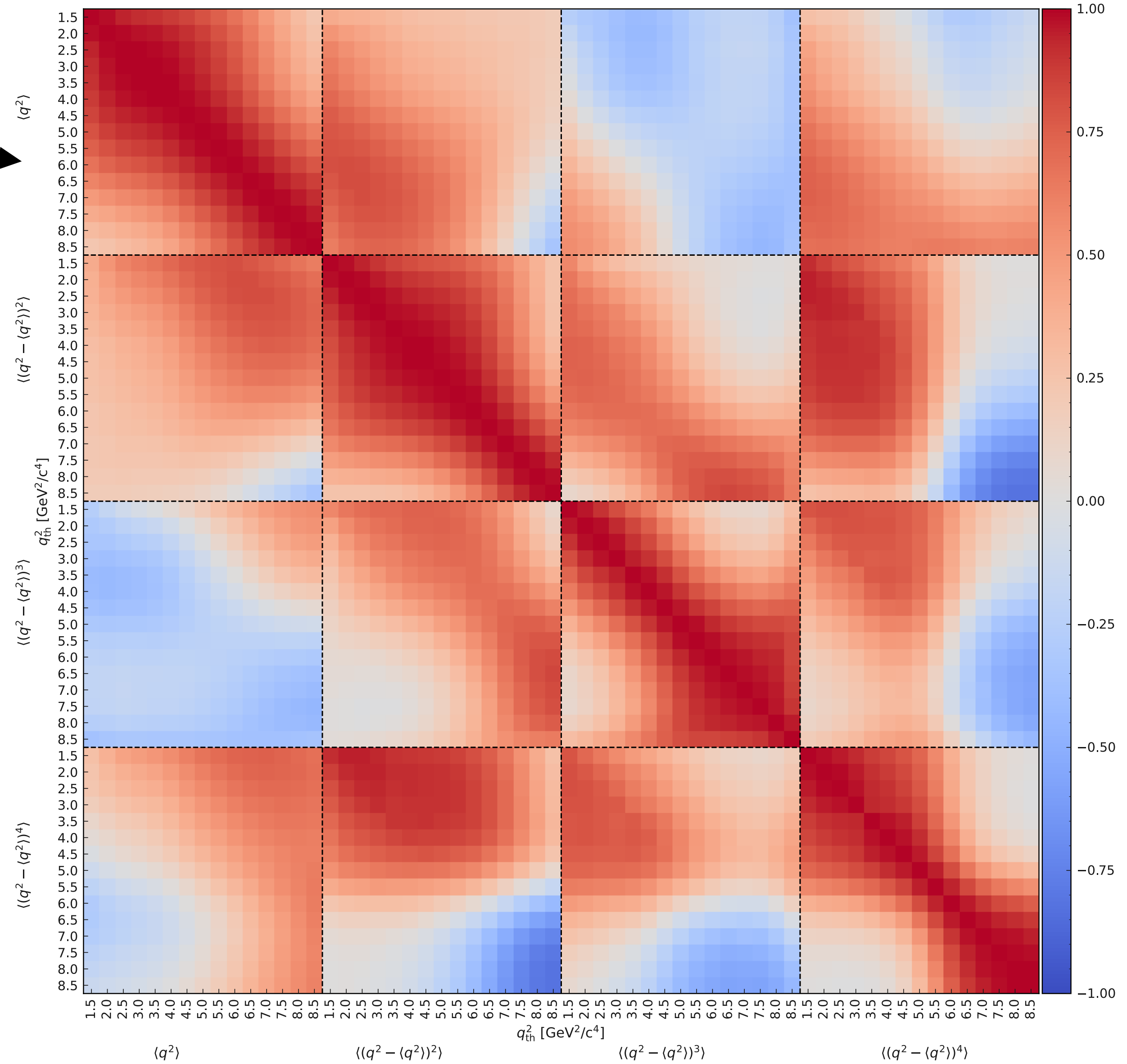
Statistical plus **systematic** correlations

strong correlations!

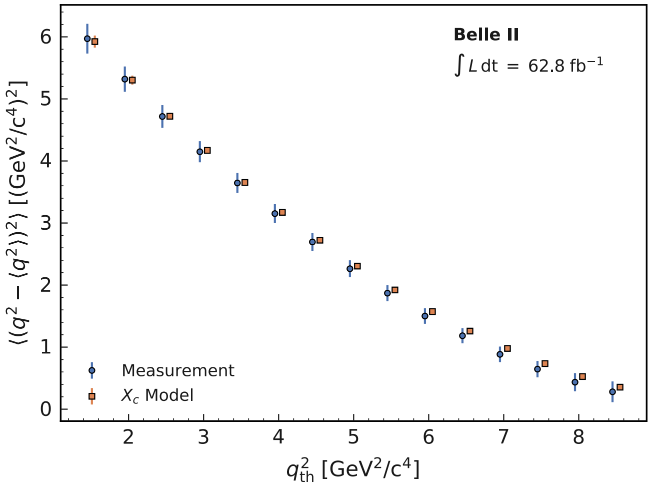


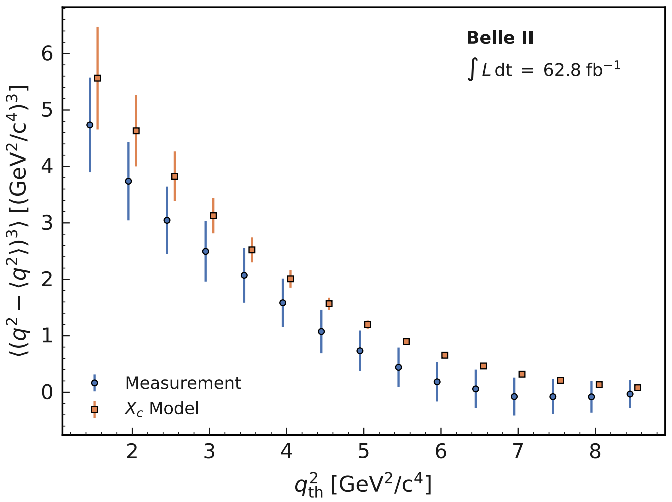
From moments to *central moments*

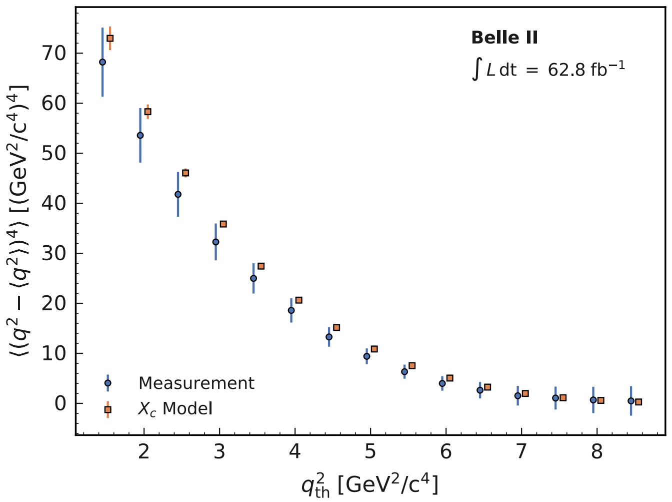
Central moments are **less** strongly correlated



$$\begin{pmatrix} \langle q^2 \rangle \\ \langle q^4 \rangle \\ \langle q^6 \rangle \\ \langle q^8 \rangle \end{pmatrix} \rightarrow \begin{pmatrix} \langle q^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^3 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^4 \rangle \end{pmatrix}$$



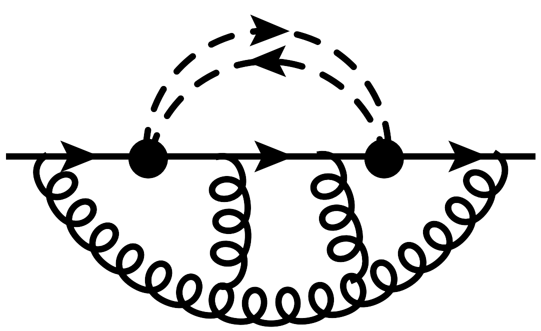




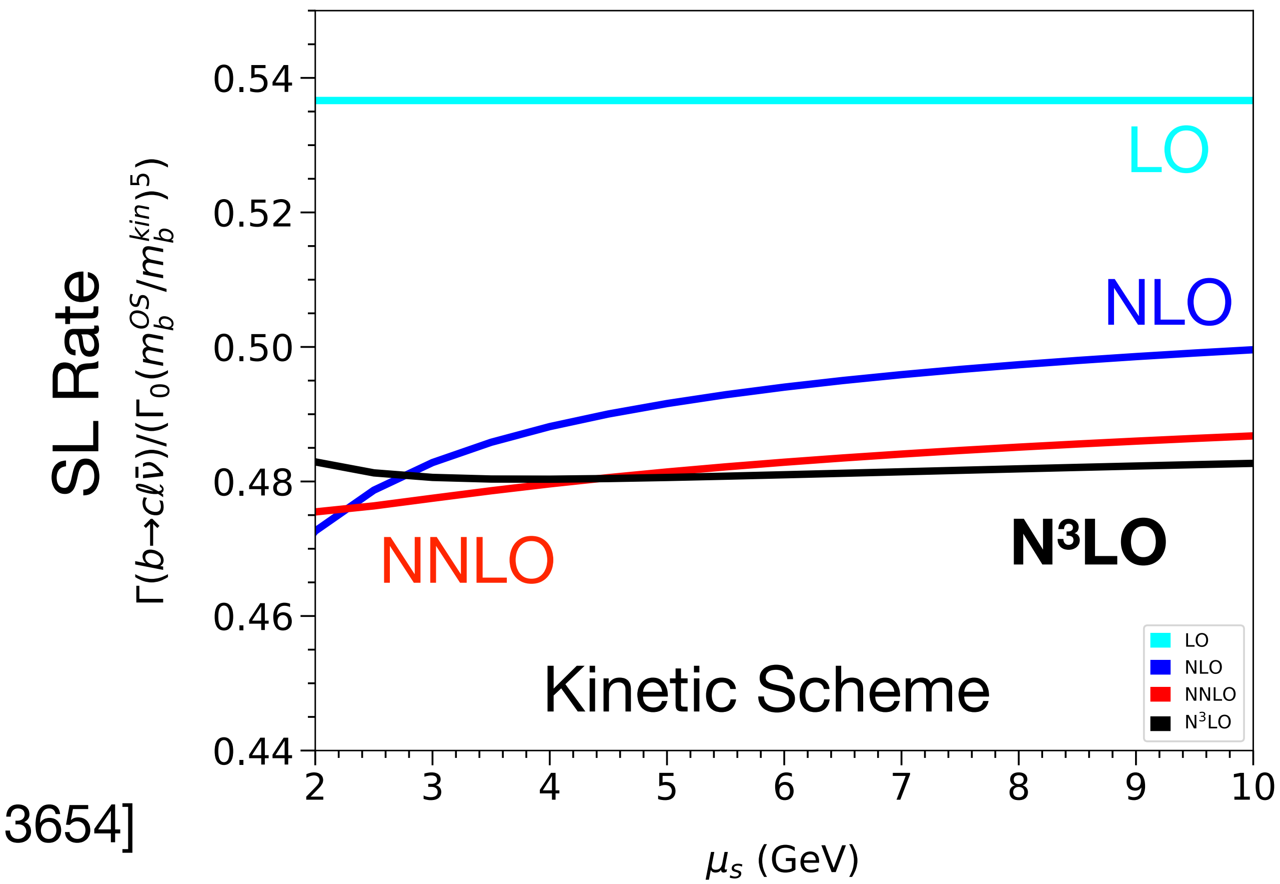
What’s **new?**

State-of-the-art: $|V_{cb}|$ with $E_\ell : M_X^2$

Fantastic progress on the theory side:
semileptonic rate @ **N³LO**!



M. Fael, K. Schönwald, M. Steinhauser
[Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]



Renormalization scale

Updated inclusive fit to $\langle E_\ell \rangle$, $\langle M_X \rangle$ moments:

$$|V_{cb}| = 42.16(30)_{th}(32)_{exp}(25)_\Gamma \; 10^{-3}$$

$$\Delta |V_{cb}| / |V_{cb}| = 1.2\,\%\,!$$

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\|V_{cb}\|$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51
1	0.307	-0.141	0.047	0.612	-0.196	-0.064	-0.420
	1	0.018	-0.010	-0.162	0.048	0.028	0.061
		1	0.735	-0.054	0.067	0.172	0.429
			1	-0.157	-0.149	0.091	0.299
				1	0.001	0.013	-0.225
					1	-0.033	-0.005
						1	0.684
							1

M. Bordone, B. Capdevila, P. Gambino
[Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

See also [Phys.Lett.B 829 (2022) 137068, 2202.01434] for very recent 1S fit finding $|V_{cb}| = (42.5 \pm 1.1) \times 10^{-3}$

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472]
(M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting **reparametrization invariance**, but **not true for every observable**

Spectral moments :

$v = p_B/m_B$

$$\langle M^n[w]\rangle = \int w^n(v, p_\ell, p_\nu)\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi}\,\mathrm{d}\Phi$$

$w = (m_B v - q)^2 \Rightarrow \langle M_X^n\rangle$ Moments	not RPI (depends on v)
$w = v\cdot p_\ell \Rightarrow \langle E_\ell^n\rangle$ Moments	not RPI (depends on v)
$w = q^2 \Rightarrow \langle (q^2)^n\rangle$ Moments	RPI! (does not depend on v)

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

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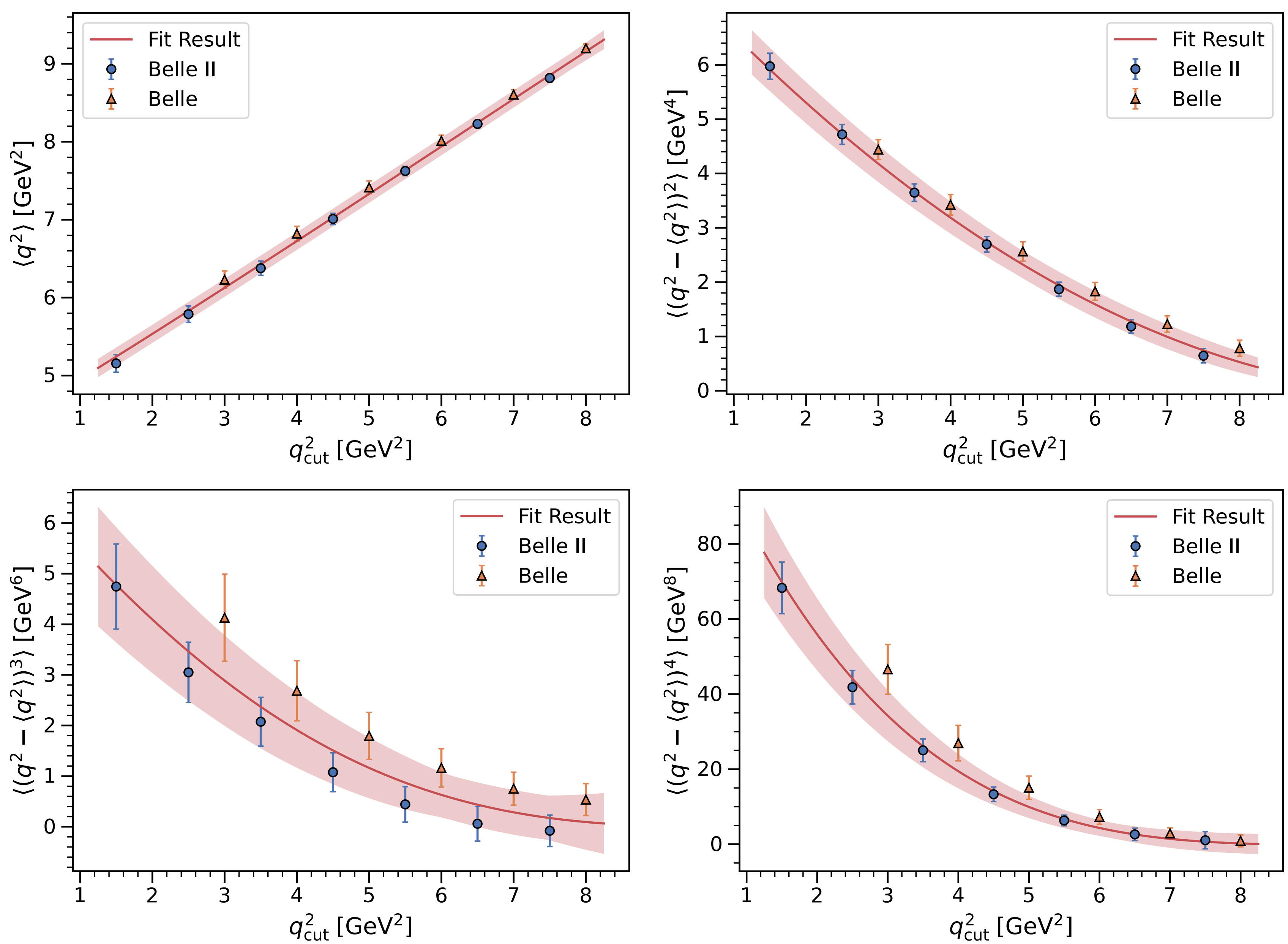
Measurements of q^2 **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment
[PRD 107, 072002 (2023), arXiv:2205.06372]

$|V_{cb}|$ from q^2

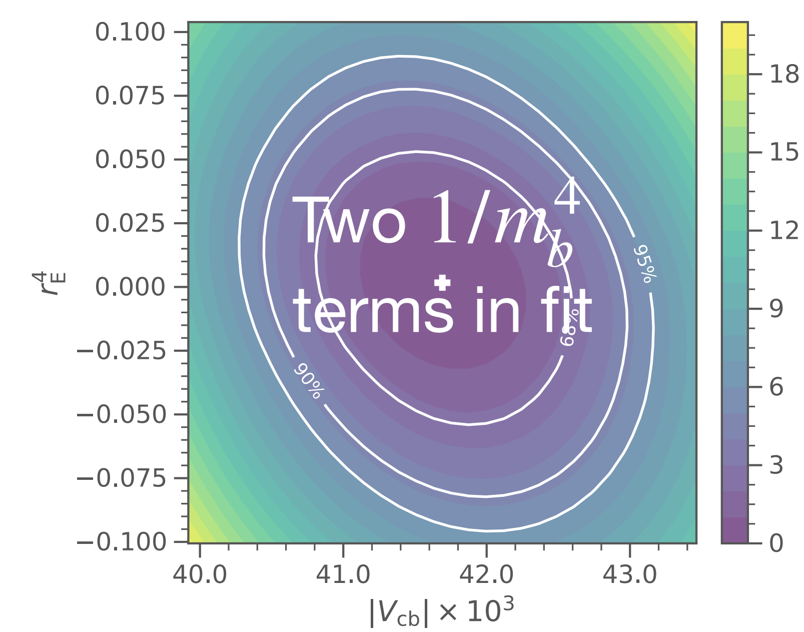
F. Bernlochner, M. Fael, K. Olschewsky, E. Persson, R. Van Tonder, K. Vos, M. Welsch [arXiv:2205.10274]

Extraction of $|V_{cb}|$ from q^2 moments:



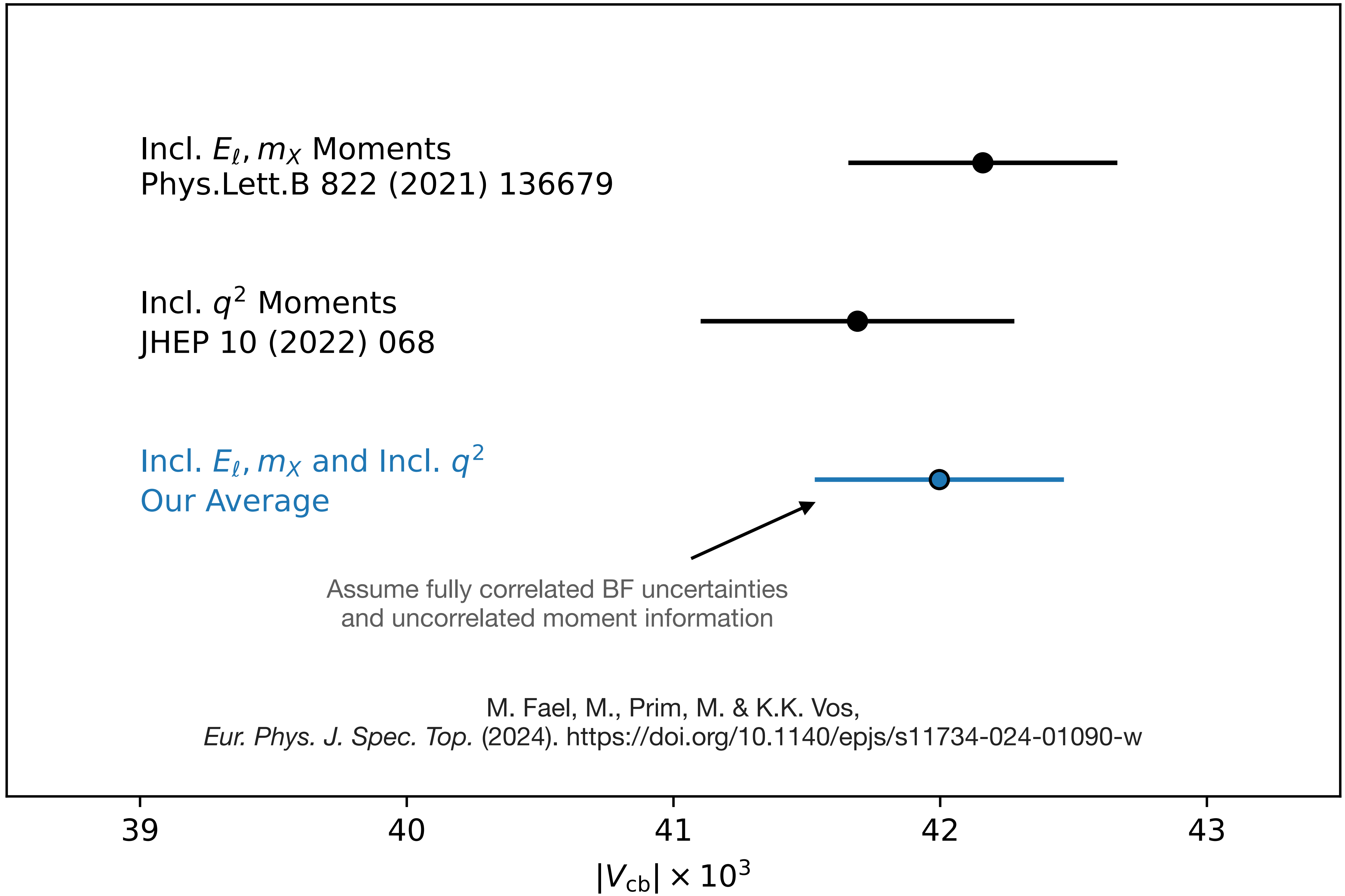
Included corrections on the mom. predictions

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			



$$\rightarrow \quad |V_{cb}| = (41.69 \pm 0.59|_{\rm fit} \pm 0.23|_{\rm h.o.}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

$|V_{cb}|$ from q^2 **versus** $E_\ell : M_X^2$



Moment party: $q^2 : E_\ell^B : M_X^2$

Placeholder

https://arxiv.org/abs/2310.20324

The q^2 moments in inclusive semileptonic B decays

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Interesting **future** directions

LHCb might enter the scene

Placeholder

https://arxiv.org/abs/2312.05147

Inclusive semileptonic B_s^0 meson decays at the LHC via a sum-of-exclusive modes technique: possibilities and prospects

M. De Cian[a], N. Feliks[b,†], M. Rotondo[c] and K. Keri Vos[d,e]

LQCD might enter the scene

Placeholder

https://arxiv.org/abs/2311.09892

On the study of inclusive semileptonic decays of B_s-meson from lattice QCD

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QED will enter the scene

Placeholder

https://arxiv.org/abs/2309.02849

QED effects in inclusive semi-leptonic B decays

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Ulrich Haisch[c] and Andrea Piccione[e]

Discussion items

Experimental and Theory Errors

Placeholder

Isospin and Lifetimes

Placeholder