

*Tau Analysis 101**

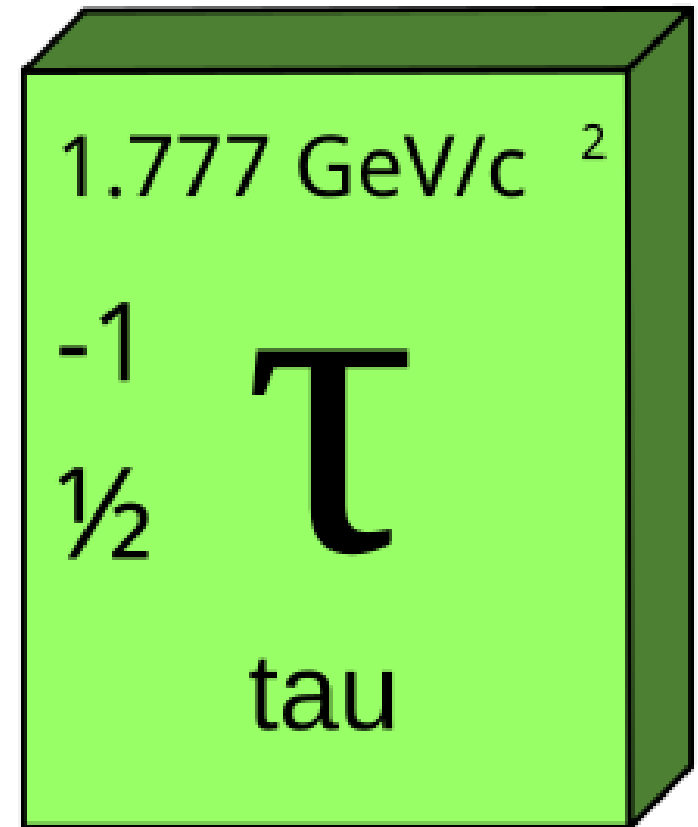
“We use (the name) τ because it appears to be the third charged lepton to be found and *τριτον* means third in Greek.”

– Martin Perl, Proceedings of the XII Recontre de Moriond (1977)

Soeren Prell (Iowa State University)

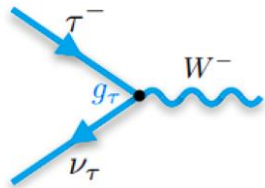
Belle II Physics Week

October 14-18, 2024 @ KEK



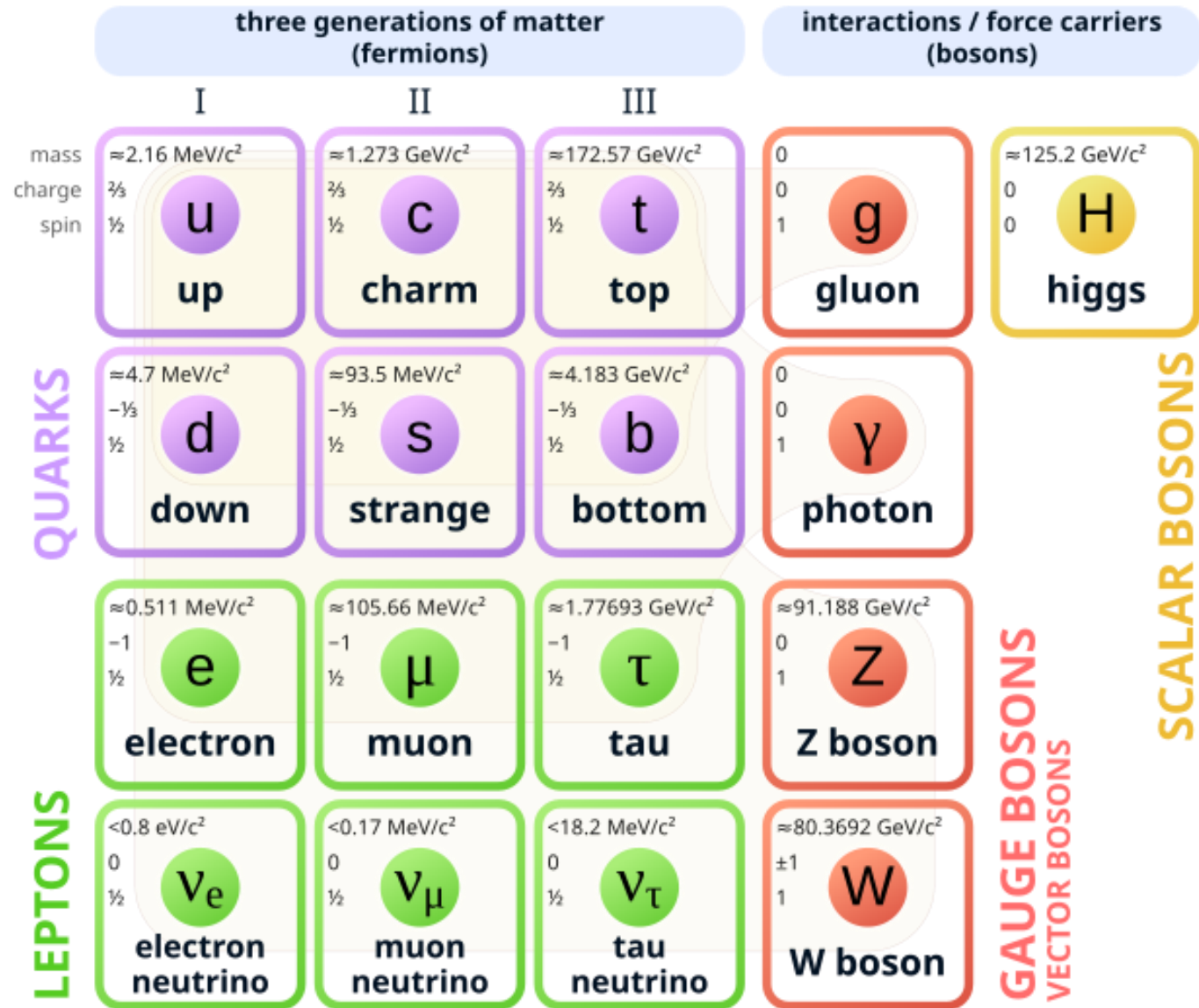
The τ in the Standard Model

- τ is a lepton and a member of a left-handed doublet
 - τ does not interact strongly
- τ lepton number L_τ is conserved
 - τ decays always have a ν_τ in the final state
 - τ only decays via charged weak current



- The τ is heavy
 - Only lepton that decays to hadrons (but not to c , b , and t quarks)

Standard Model of Elementary Particles

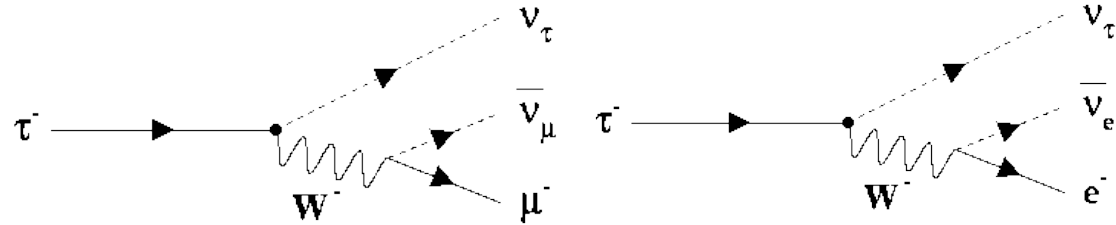


Brief history of heavy fermions

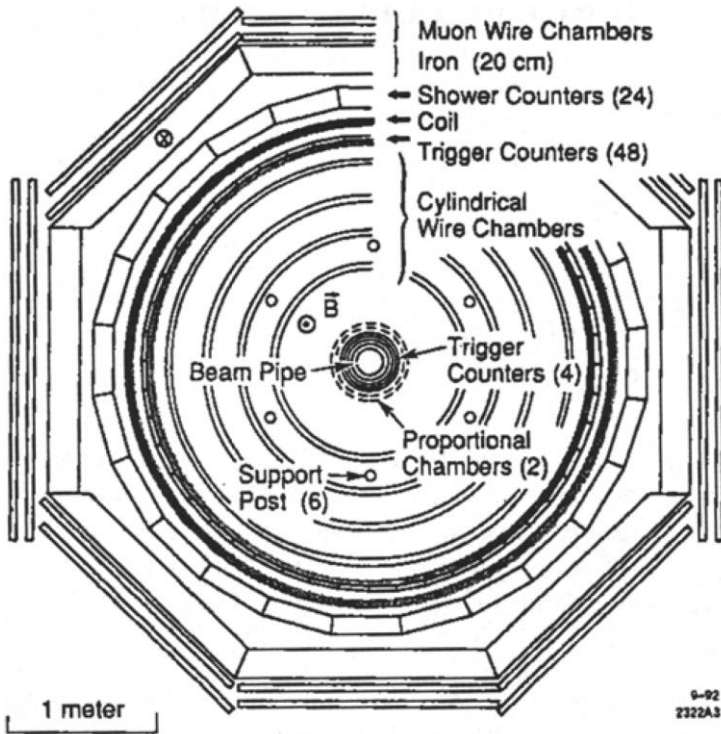
- *1972 Kobayashi & Maskawa predict 3rd generation of quarks to explain CP violation in kaon decays*
- *1974 J/ψ discovered independently at SLAC (Richter et al.) and BNL (Ting et al.) – first strong evidence for the charm quark*
- *1975 τ lepton discovered at SLAC – first evidence for 3rd generation fermions (Perl et al.)*
- *1977 $Y(1S)$ discovered at Fermilab (Lederman et al.) – first evidence for the bottom quark and 3rd generation quarks*
- *1995 top quark discovered at Fermilab (D0 & CDF)*

The τ discovery (1975)

- If a sequential 3rd charged lepton exists, it will decay to the first two generations

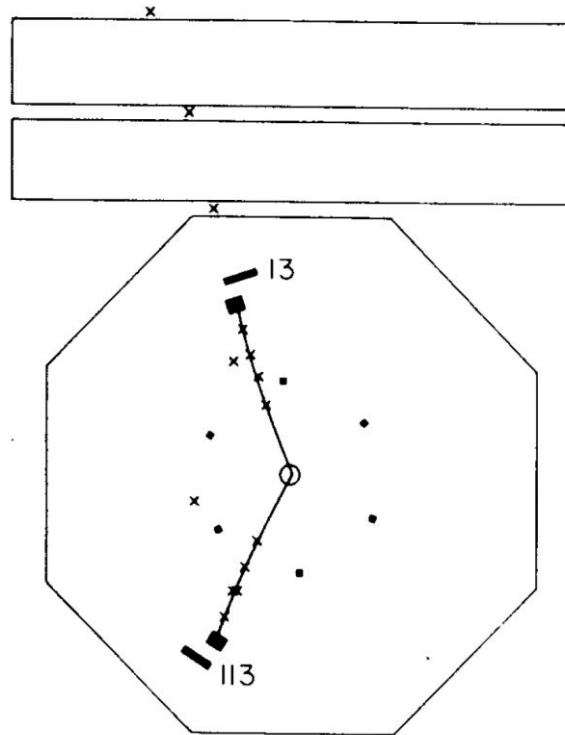


- Looking for $e^+e^- \rightarrow \tau^+\tau^- \rightarrow e^\pm\mu^\mp E_{miss}$

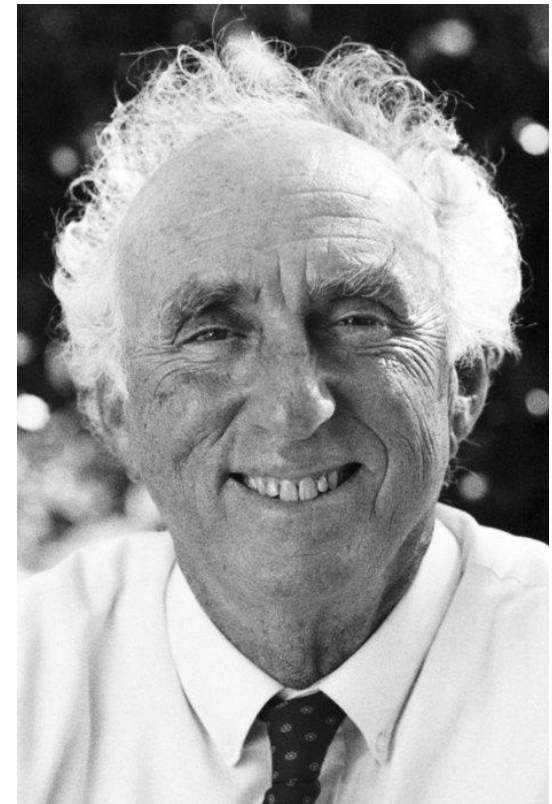


SLAC-LBL detector

Early $e^+e^- \rightarrow e^\pm\mu^\mp E_{miss}$ event



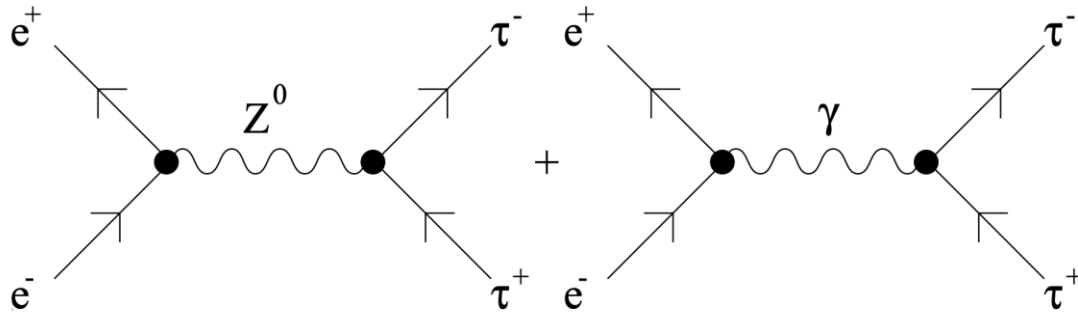
G.J. Feldman at Lepton Photon 1975



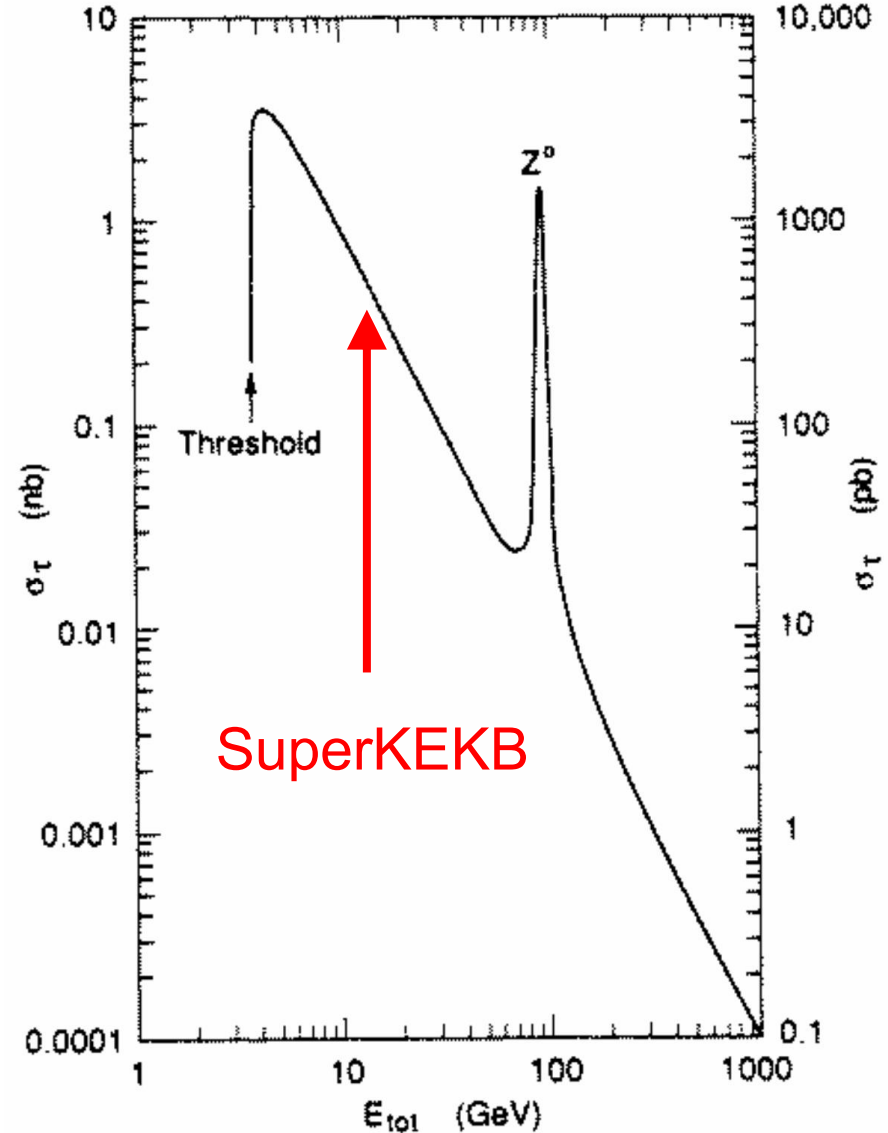
Martin Perl



τ pair production in e^+e^- collisions

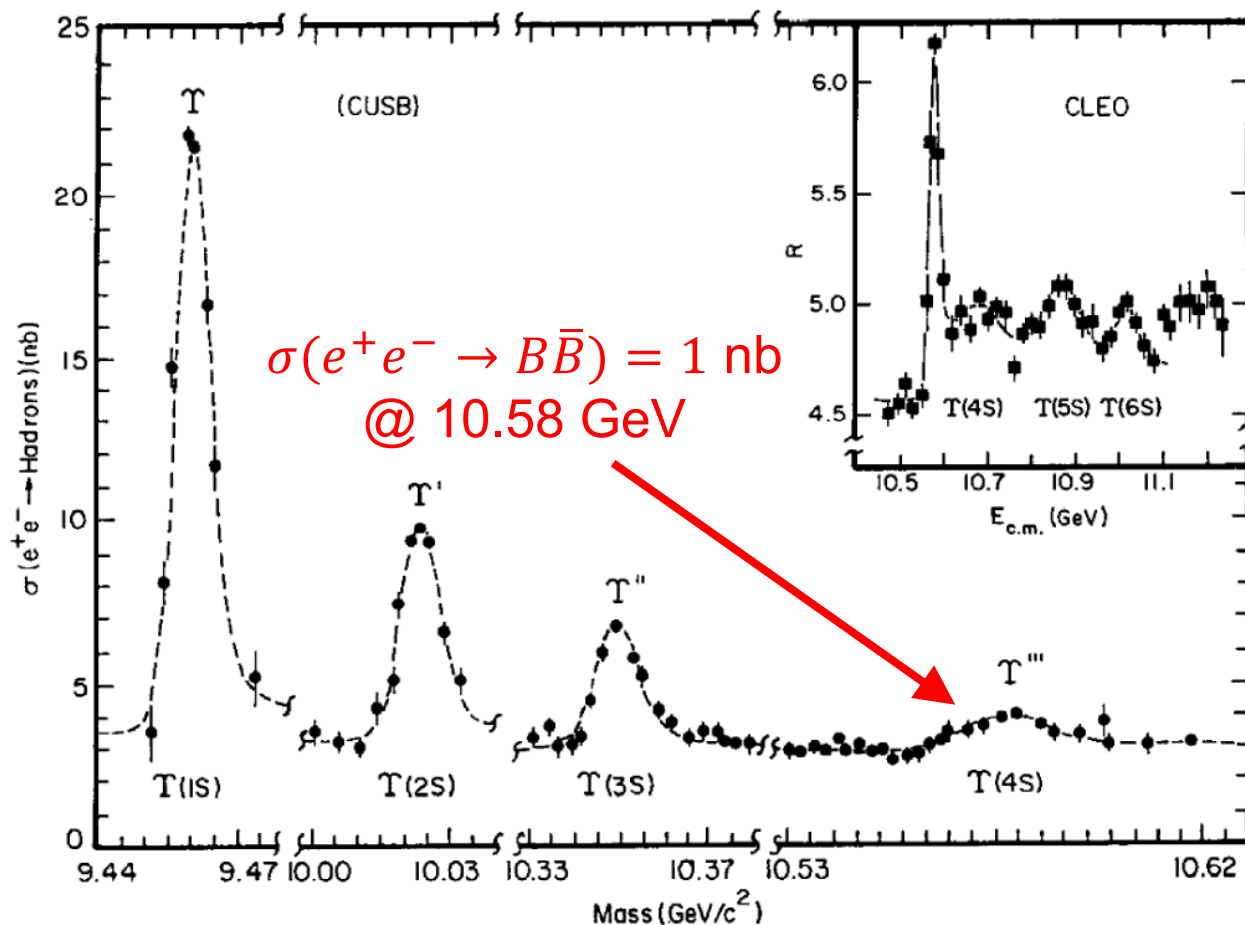


- *1st order diagrams for τ pair production*
- *$ee \rightarrow \tau\tau$ cross-section can be precisely calculated*
 - *Was already calculated before the τ was discovered (assuming that the τ is a point-like fermion of a certain mass)*



Cross-section rises sharply at the $2 \times m(\tau)$ threshold

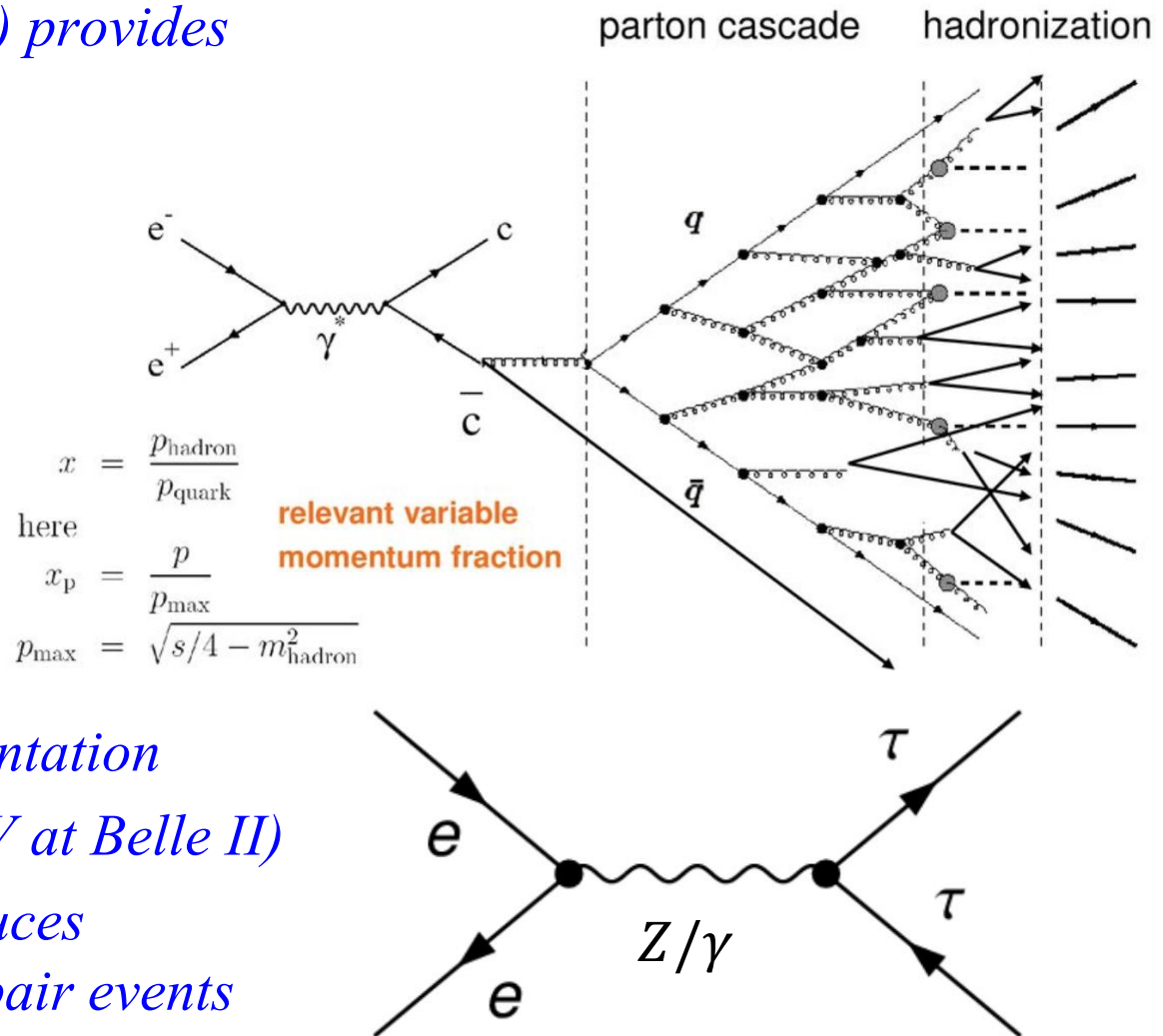
Belle II is τ factory !



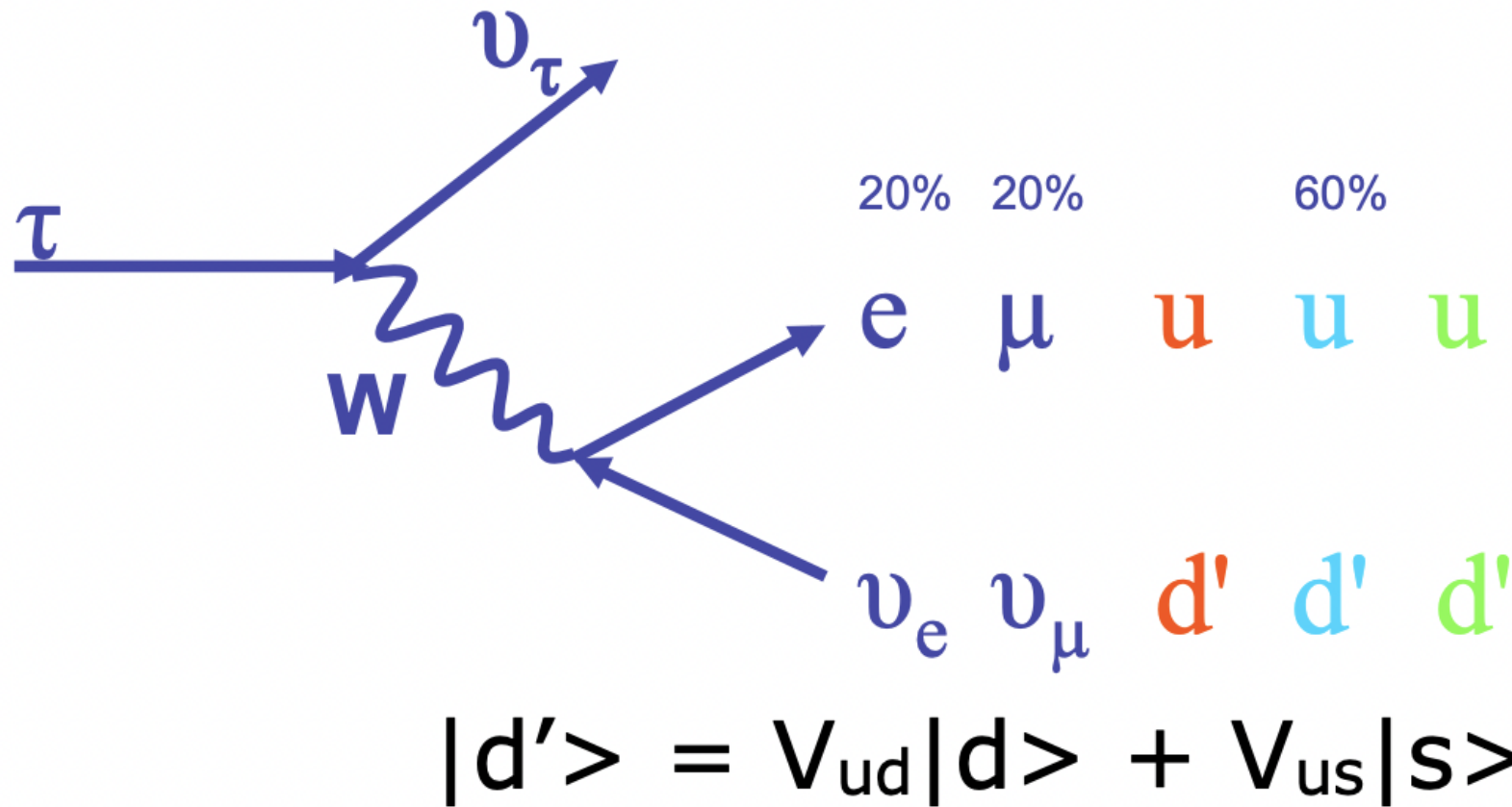
- *We call Belle II a B factory because of the large $e^+e^- \rightarrow B\bar{B}$ cross-section at the $\Upsilon(4S)$*
- *The cross-section $\sigma(e^+e^- \rightarrow \tau^+\tau^-) = 0.919 \pm 0.003 \text{ nb}$ at 10.58 GeV*
 - *We produce 920,000 $\tau^+\tau^-$ events per 1 fb^{-1}*

τ pair production is “clean”

- $B\bar{B}$ production is clean at the $\Upsilon(4S)$
 - Only $e^+e^- \rightarrow B\bar{B}$ is allowed (no additional particles)
 - Not enough energy for $e^+e^- \rightarrow B^*\bar{B}$
 - Reconstruction on B (tag) provides momentum of the other B
- Charm (and light) hadron production is not clean
 - Additional particles from fragmentation
 - Two charm hadrons can be of different types
- τ pair production is clean
 - No particles from fragmentation
 - $E_\tau^* = E_{beam}^*$ (= 5.29 GeV at Belle II)
 - Reconstructing tag τ reduces background from non- τ -pair events



τ decay (simplified)



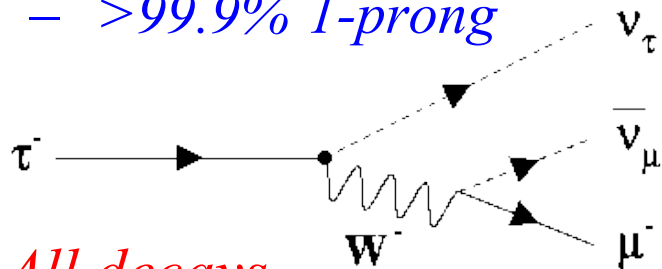
- *Leptonic branching fraction is $\sim 20\%$*
- *(Semi) hadronic final states are mostly non-strange*
 - $|V_{us}|^2 = \sin^2 \theta_C = 5\%$ of hadr. decays have net strangeness

τ branching fractions

- 35% leptonic
 - ~ 50:50 electrons and muons

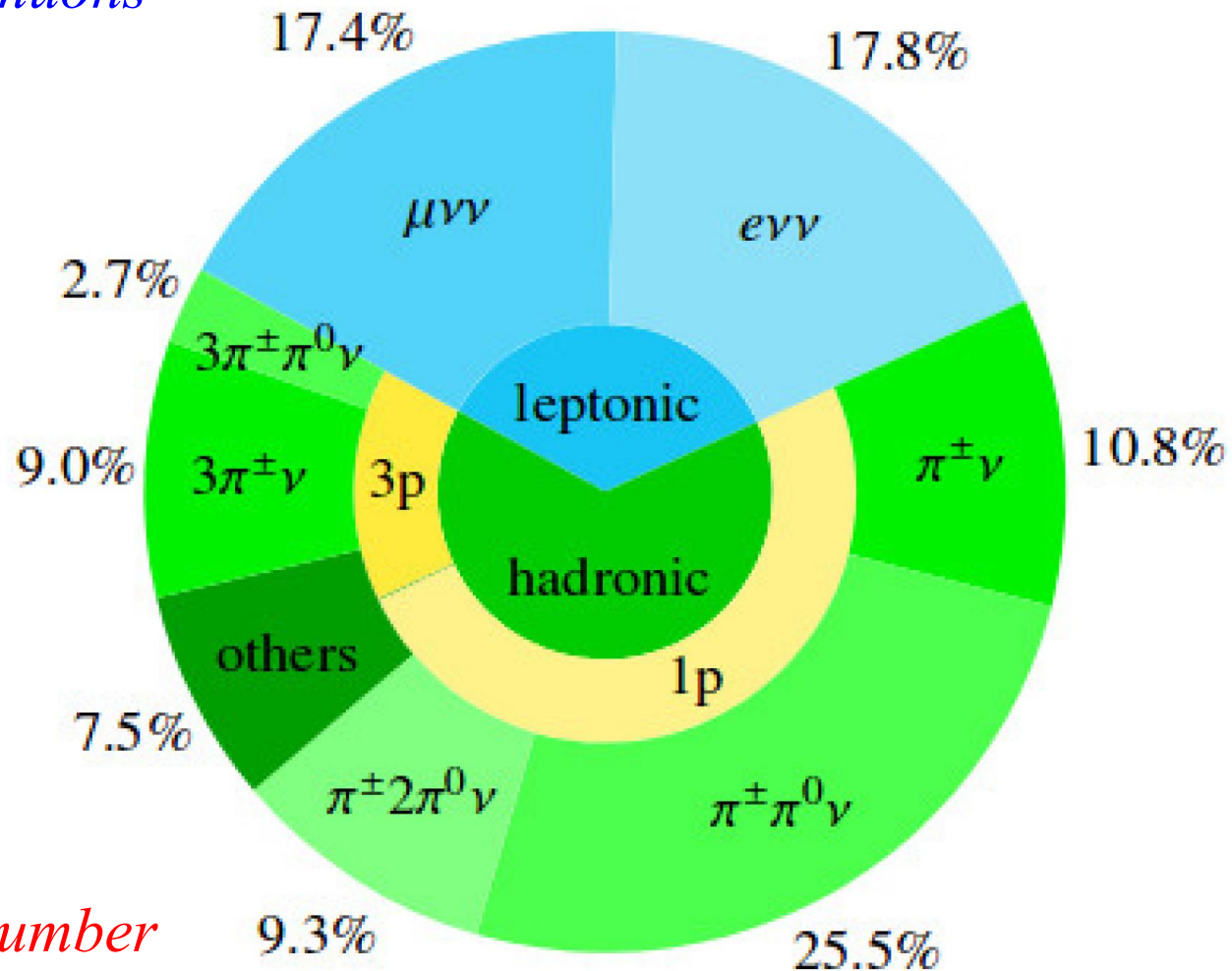
- 65% hadronic

- Leptonic decays
 - >99.9% 1-prong



- All decays
 - 85 % 1-prong
 - 15 % 3-prong
 - 0.1 % 5-prong
 - $< 3 \times 10^{-7}$ 7-prong

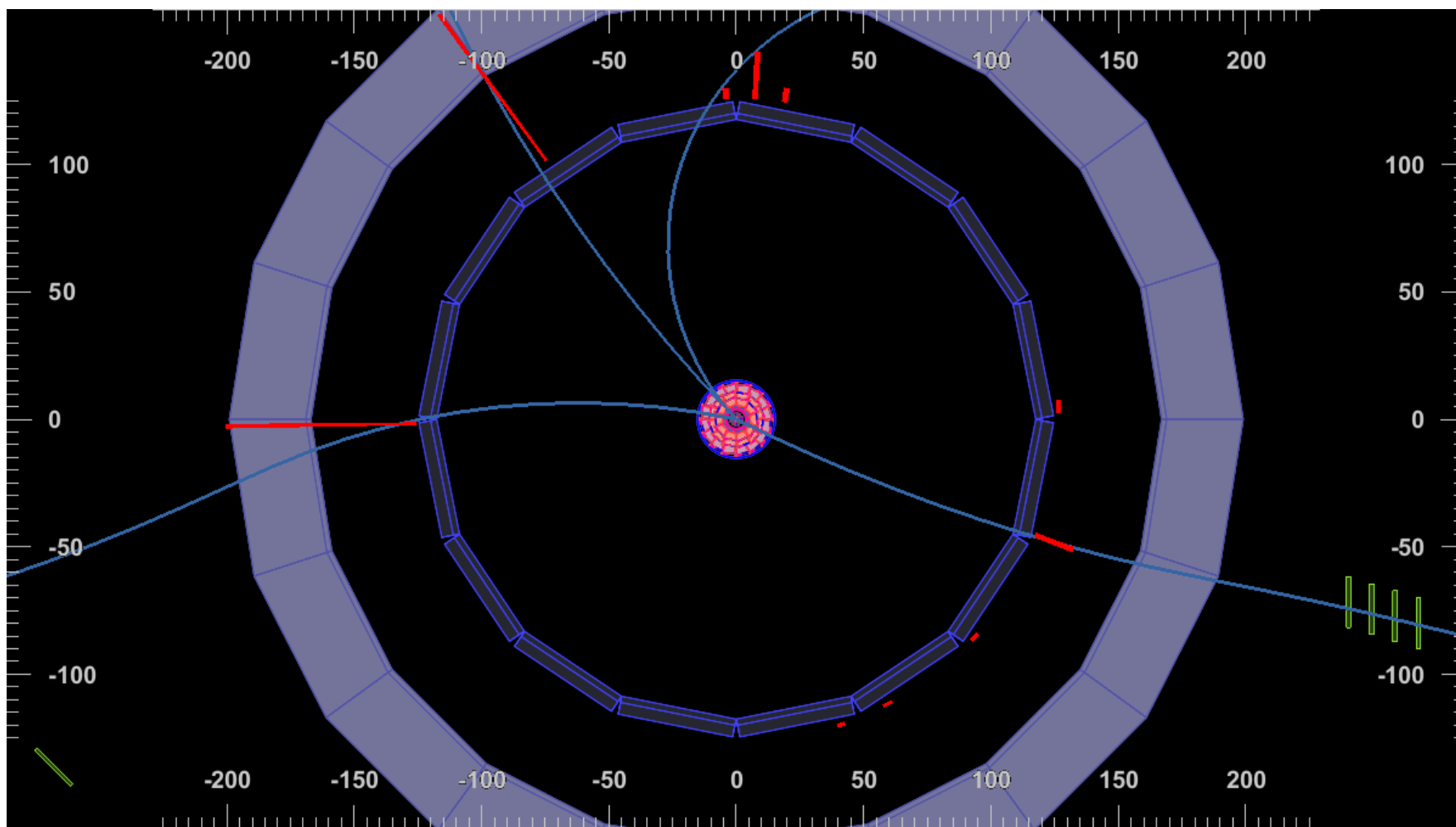
- All τ decays have an odd number of charged particles (prongs*) in the final state



*prong (noun): projecting pointed parts at the end of a fork

Tau events are really clean !

τ pair events have either 2 tracks (73%), 4 tracks (26%), or 6 tracks (2%)



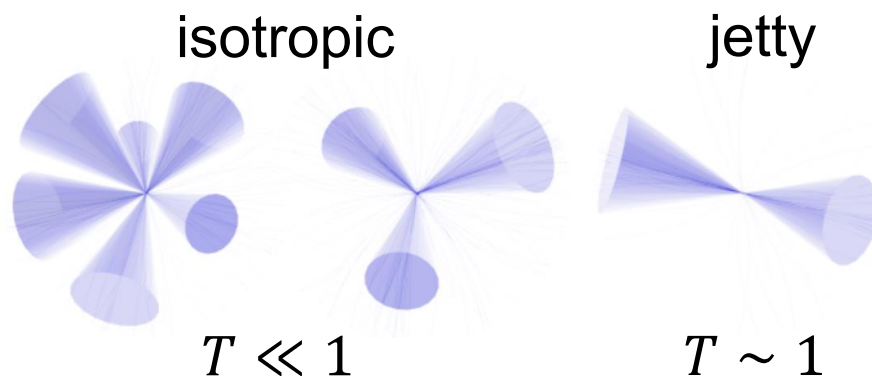
A typical (3x1) event.

Candidate for a $e^+e^- \rightarrow (\tau^+ \rightarrow 3\pi\bar{\nu}_\tau)(\tau^- \rightarrow \mu^-\nu_\tau\bar{\nu}_\mu)$ event

τ pair kinematics

$$e^+e^- \rightarrow \tau^+\tau^-$$

- *Energy conservation (“jetty” τ pairs or boosted τ ’s)*
 - $E_\tau^* = E_{beam}^* = 5.29 \text{ GeV} \rightarrow p_\tau^* = 5.0 \text{ GeV}$ ($m_\tau = 1.777 \text{ GeV}$)
- *Momentum conservation (back-to-back taus)*
 - $\vec{p}^*(\tau^-) = -\vec{p}^*(\tau^+)$ (* indicates center-of-mass system)
- *Unfortunately, we don’t know the direction of the τ ’s*
 - Each τ decays to one or more neutrinos, taking away momentum
- *Approximate the directions of the τ ’s with the event thrust axis \hat{n}_T*
 - The thrust axis maximizes the thrust magnitude T



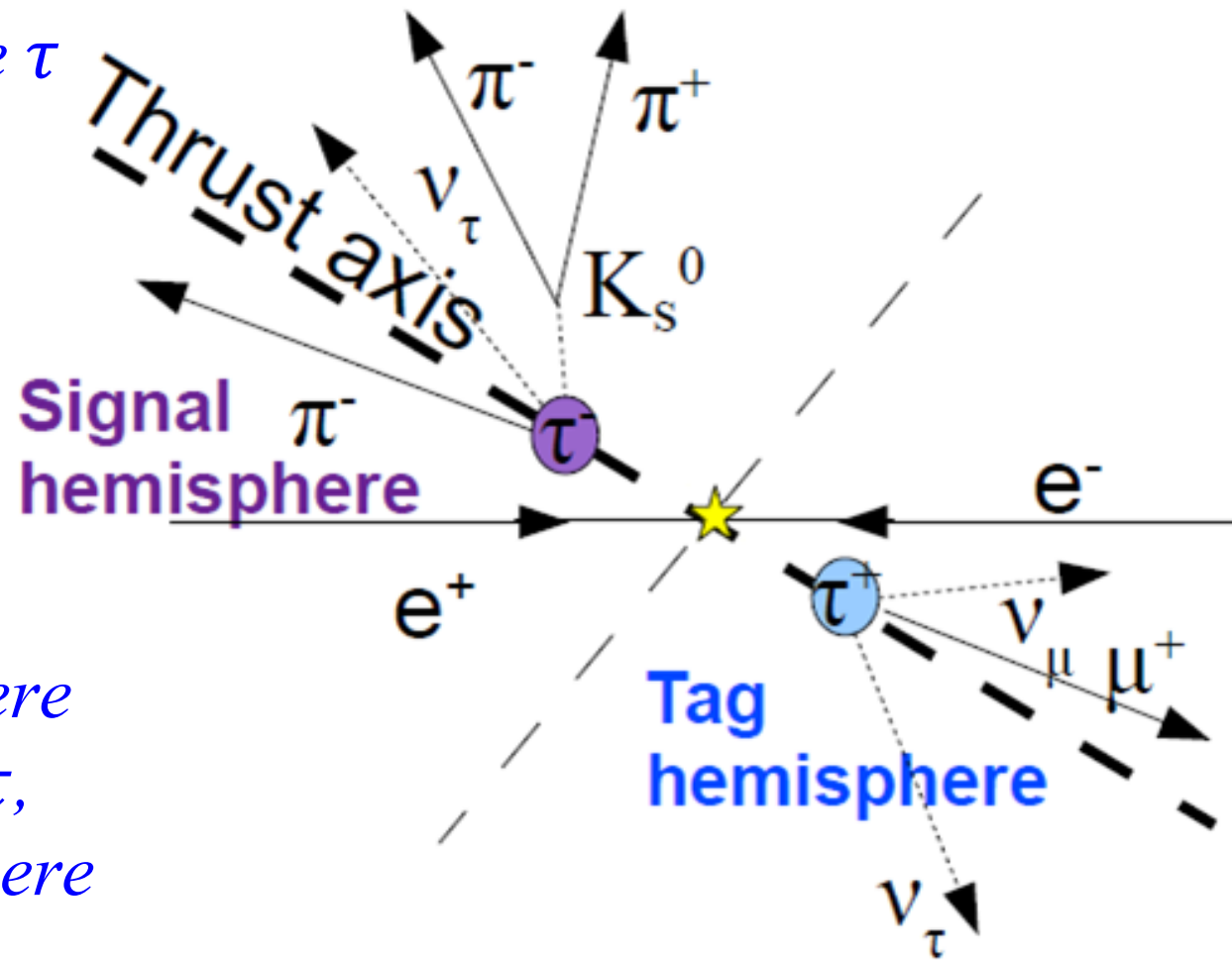
$$T = \frac{\sum |\vec{p}_i^* \cdot \hat{n}_T|}{\sum |\vec{p}_i^*|}$$

i runs over all tracks and neutral particles in the event

Signal τ and tag τ

Use the thrust axis to split event into two hemispheres

– Each hemisphere contains decay products of only one τ



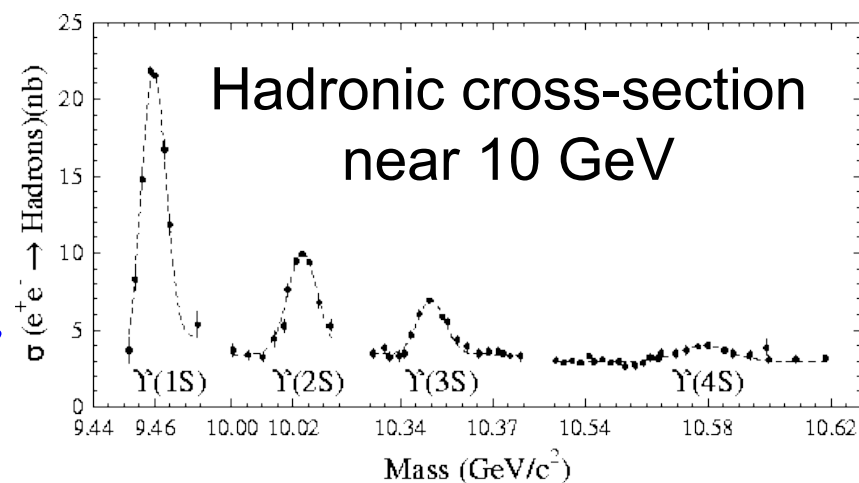
– The signal hemisphere contains the signal τ , and the tag hemisphere contains the tag τ

Backgrounds

- *The actual background in any analysis strongly depends on the final state under study ...*
- *The “usual” backgrounds ...*
 - $B\bar{B}$: many tracks (~ 10 on average), isotropic topology
 - $q\bar{q}$ continuum: many tracks, jetty-ish, few leptons

... can be effectively suppressed requiring a large thrust value, and either an e or μ in the tag hemisphere (lepton tag)
- *The “unusual” backgrounds (low-multiplicity backgrounds)*
 - $e^+e^-(\gamma)$ or Bhabha events
 - $\mu^+\mu^-(\gamma)$ or mu pair events
 - $e^+e^- X$, where X can be a lepton pair, a hadronic resonance or a multi-hadron final state

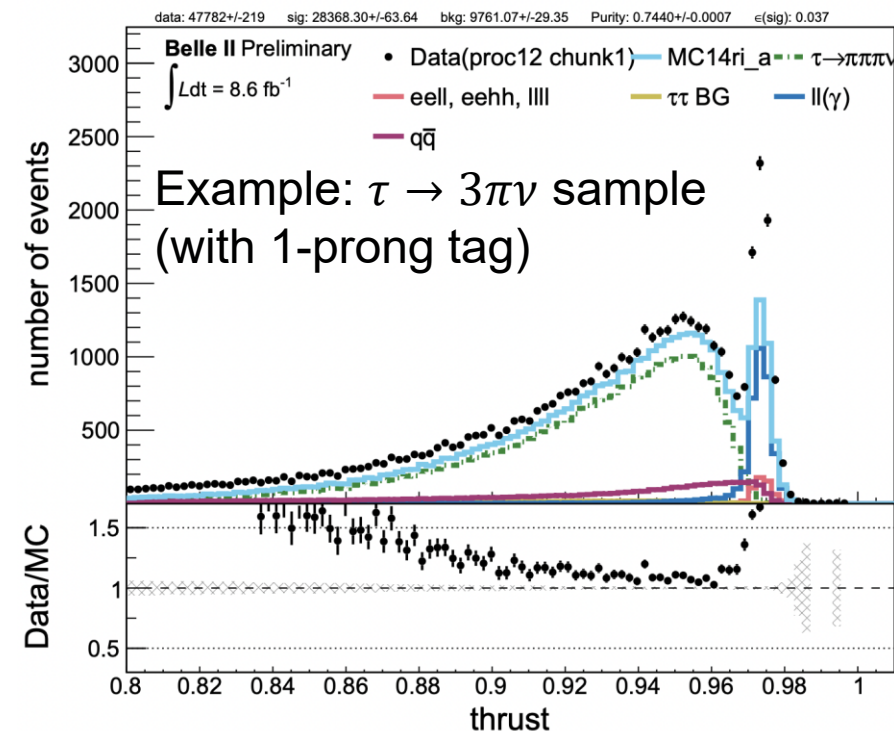
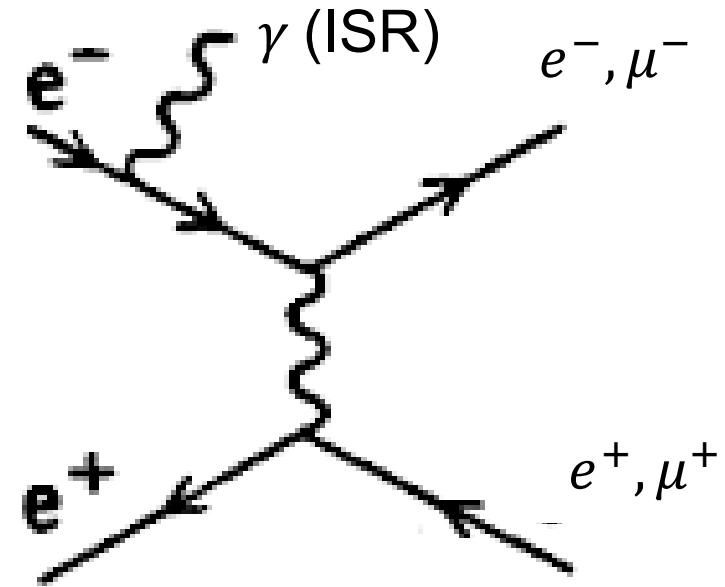
(with or without initial state radiation (ISR)
or final state radiation (FSR))



“unusual” backgrounds don’t show up here

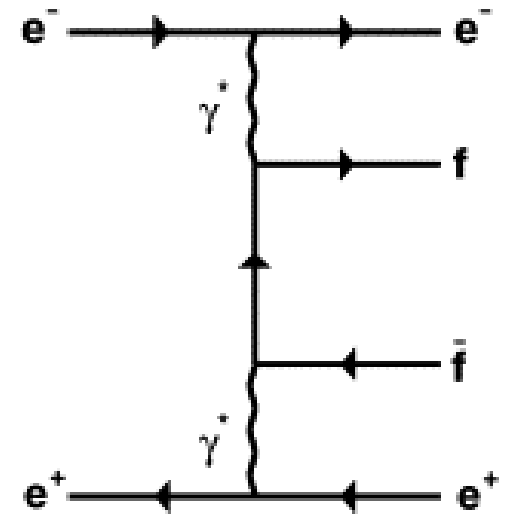
Backgrounds from $ee(\gamma)$ and $\mu\mu(\gamma)$

- γ can come from ISR or FSR, or from interaction with detector material (bremsstrahlung)
- Relatively easy to identify, but huge cross-section ($\gg \sigma(ee \rightarrow \tau\tau)$)
- Even an issue for (3x1) tau events
 - γ can convert in detector material to e^+e^- or (if virtual) turn into a vector meson
- γ is mostly soft, and the leptons have nearly beam energy and remain very collinear
 - $\ell\ell(\gamma)$ events have large thrust value
- Cut on thrust is effective against $ee(\gamma)$ and $\mu\mu(\gamma)$ backgrounds

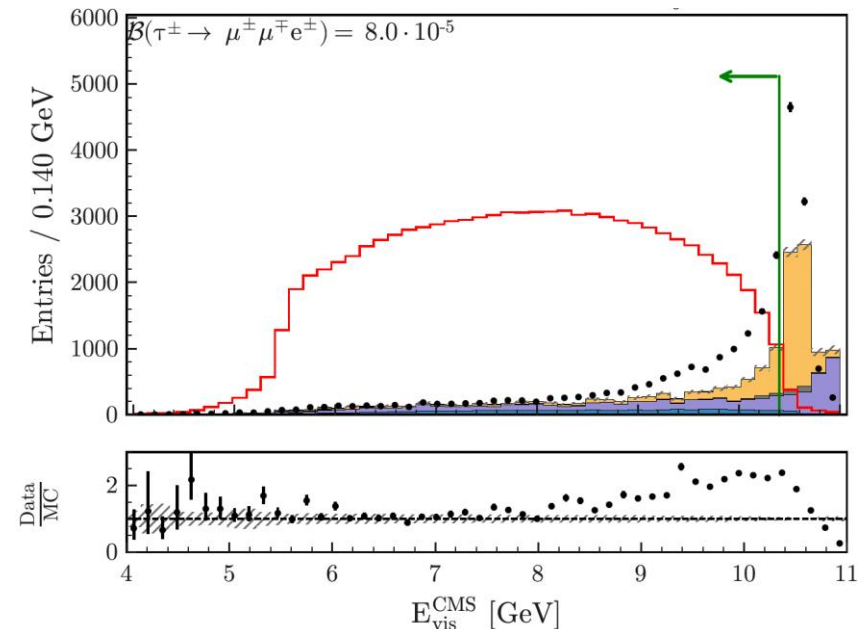


Four-fermion backgrounds (2-photon events)

- Produced fermions $f\bar{f}$ can be leptons or quarks
 - quarks can form hadronic resonances
 - $f\bar{f}$ system can produce 2,3,4, or more hadrons
- The γ^* are often emitted collinear with the beams and the beam electrons disappear in the beam pipe carrying a lot of energy; but not always
- Possible scenarios
 - Beam electrons go down the beam pipe \rightarrow small mass of the $f\bar{f}$ system
 - Beam electrons are scattered into the detector \rightarrow if $f\bar{f}$ system produces 2 tracks, event can mimic (3x1) τ event
 - \rightarrow Contrary to τ events, there are no ν 's and the 4 tracks carry the full CM energy

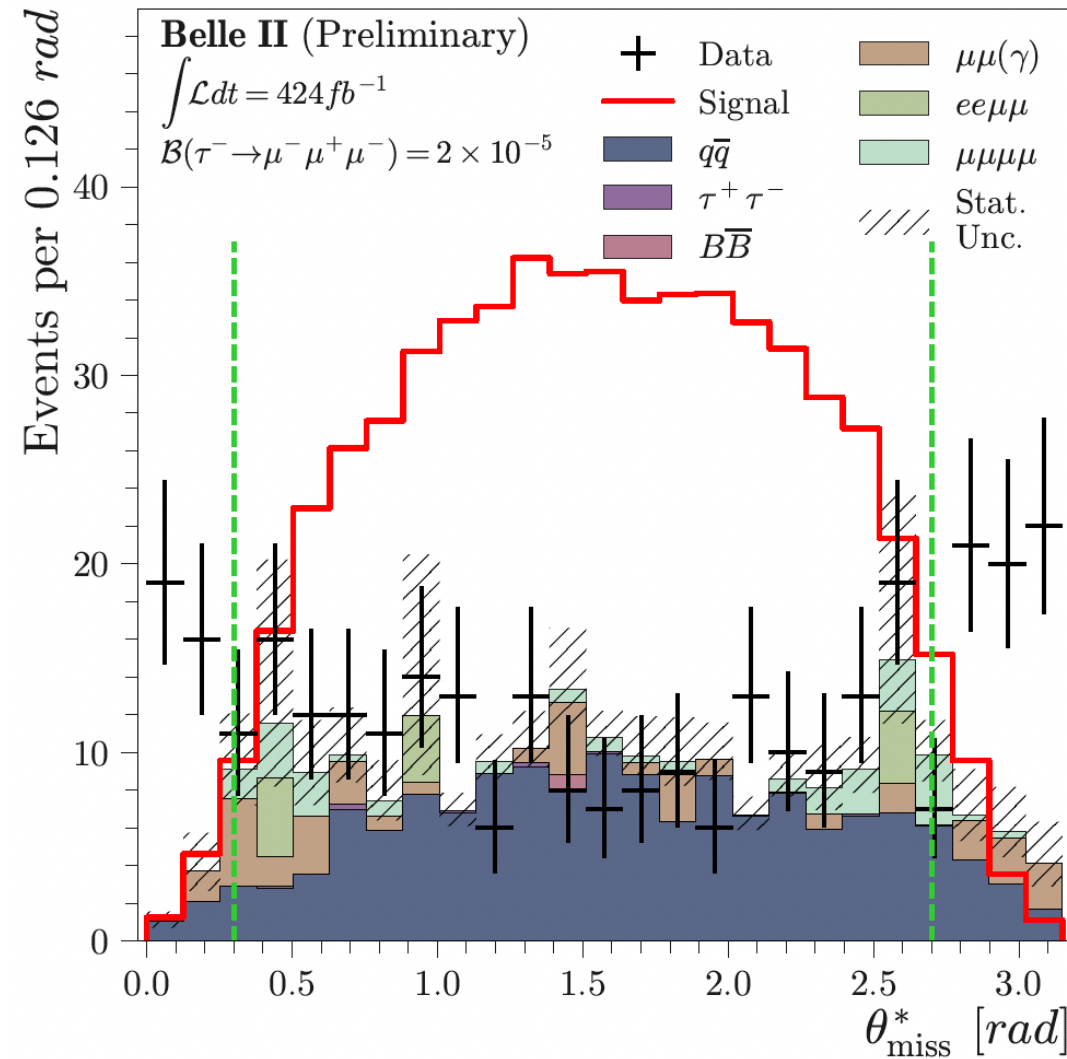


Example: visible energy of $\tau^- \rightarrow \mu^+ \mu^- e^-$ candidates in LFV search



Missing energy/momentum in $\tau^+\tau^-$ events

- τ pairs have at least 2 ν_τ in the SM
 - Hadr. τ decays have 1 neutrino (ν_τ)
 - Leptonic τ decays also have an $\bar{\nu}_\ell$
- Large missing energy in $\tau^+\tau^-$ events
- Missing energy also arises if particles are not detected (e.g., when they go down the beam pipe)
- In reconstructed $\tau^+\tau^-$ events, the missing momentum vector is aligned with visible energy and the thrust axis
- Missing momentum vector points into fiducial detector volume in $\tau^+\tau^-$ events



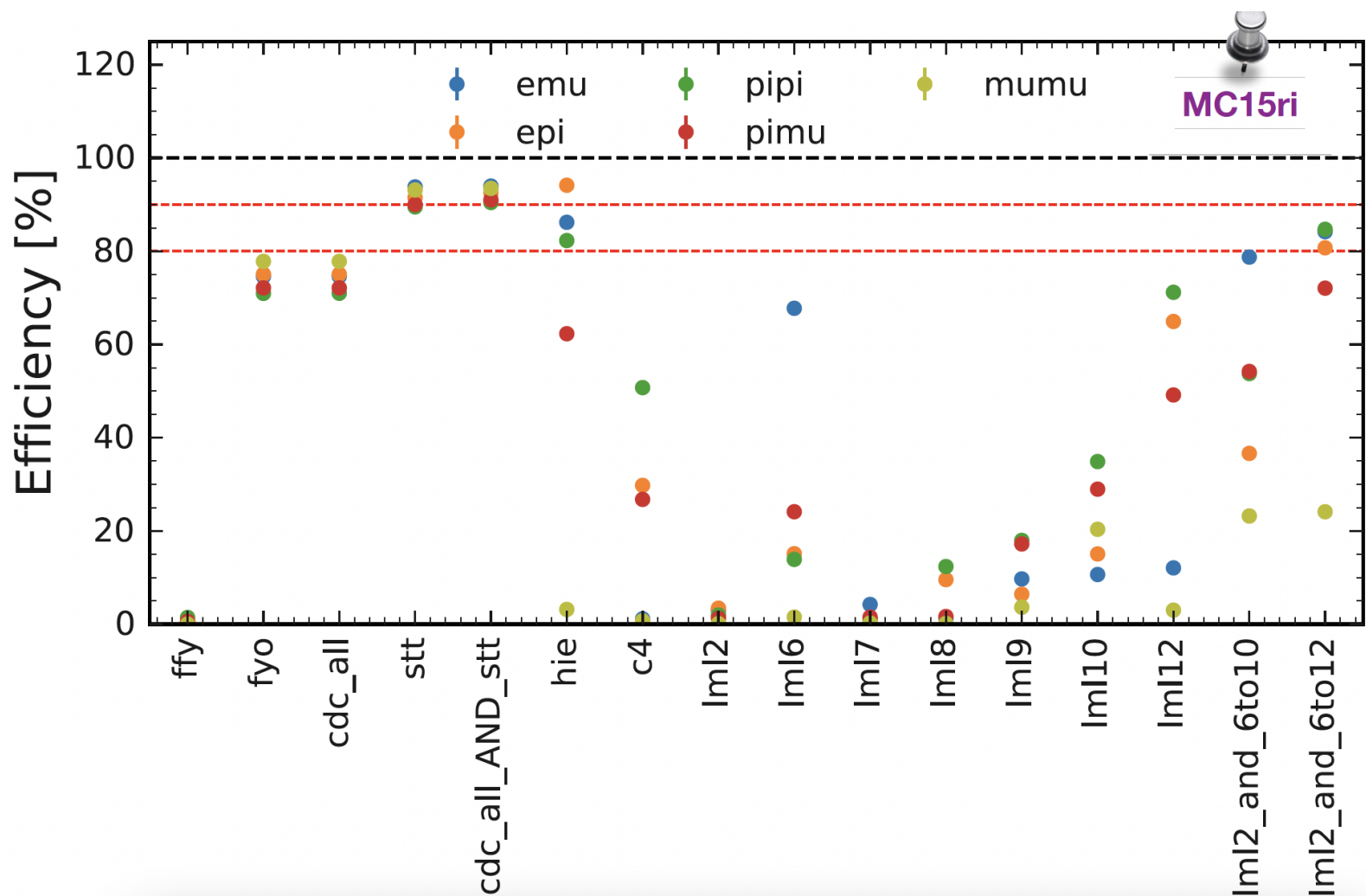
Many low-multiplicity backgrounds are not modeled very well.

Typical τ analysis cuts

- *Object reconstruction*
 - Usual criteria for tracks and neutrals (incl. particle ID)
 - Resonance masses
- *Event variables*
 - Track multiplicity (and neutral multiplicity)
 - Thrust magnitude
 - Visible energy, missing momentum magnitude, missing mass (squared)
 - Missing momentum direction
- *Tag variables*
 - 3-prong tag (e.g., if signal tau decay is one-prong)
 - To reject $\ell\ell(\gamma)$ backgrounds
 - 1-prong tag (leptonic or hadronic)
 - For larger efficiency and to reject $q\bar{q}$ background
 - Inclusive tag (combined many ROE variables in a BDT)

Trigger efficiency uncertainty is not negligible!

- Trigger efficiency is 100% for $B\bar{B}$ events, but not for $\tau^+\tau^-$ events
- ϵ_{trig} and its uncertainty need to be determined
- Worst ϵ_{trig} for (1x1) topologies



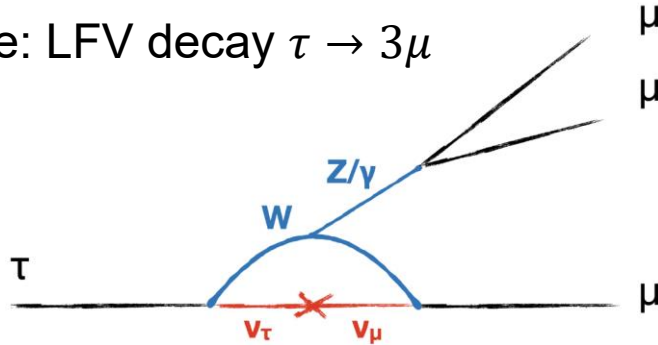
1. Lepton flavor violation (LFV) searches

2. (Precision) tests of the SM

Lepton number/ flavor conservation

- *Lepton flavor is almost conserved in SM*
- *Loop diagrams with ν mixing can give charged lepton flavor violation (cLFV)*

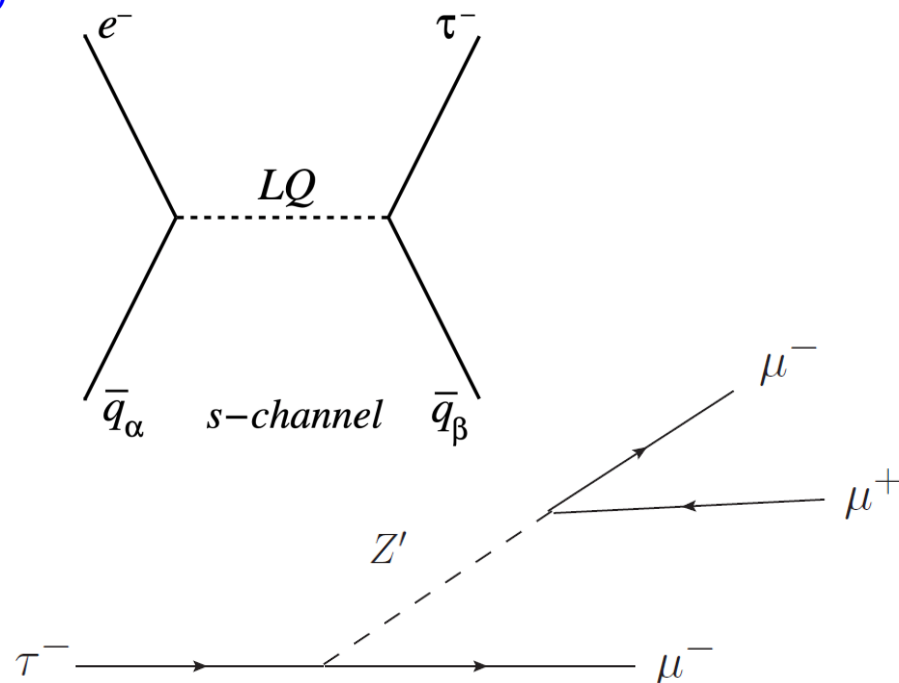
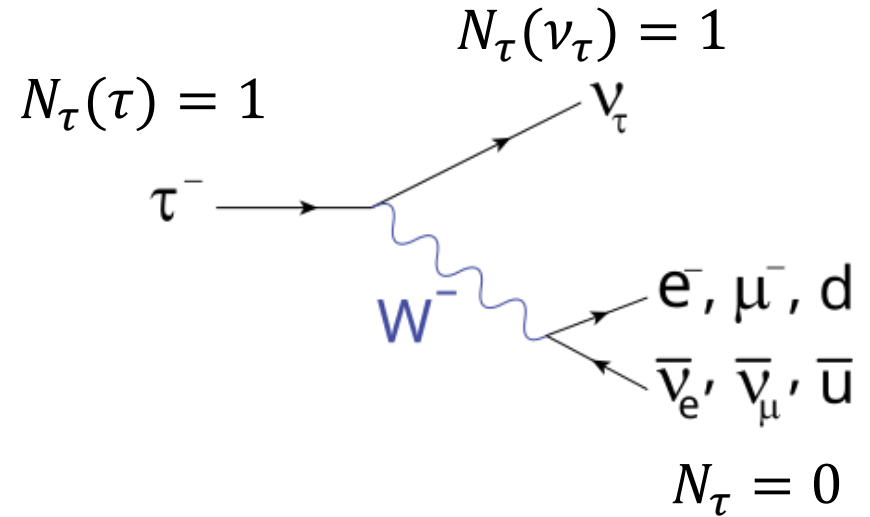
Example: LFV decay $\tau \rightarrow 3\mu$



– *SM cLFV BFs are of order $10^{-(50 \pm 2)}$*

- *Many beyond SM models predict cLFV:*
 - *E.g., Leptoquarks (LQ), Z'*

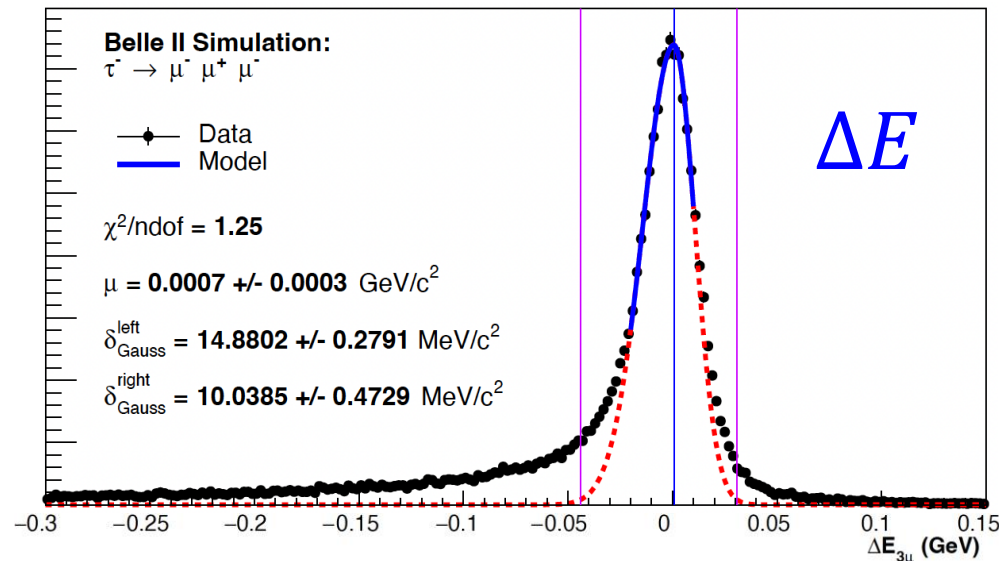
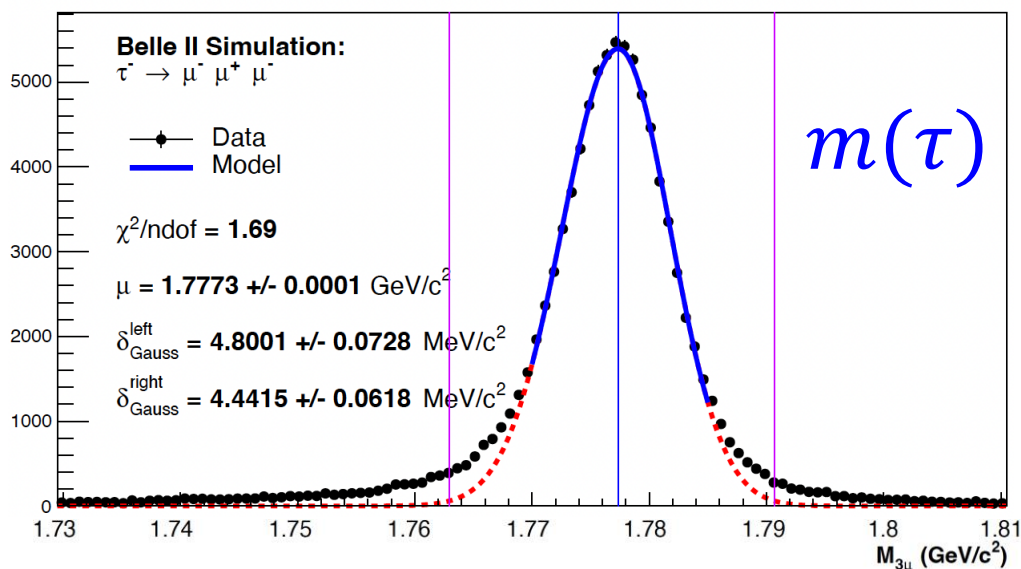
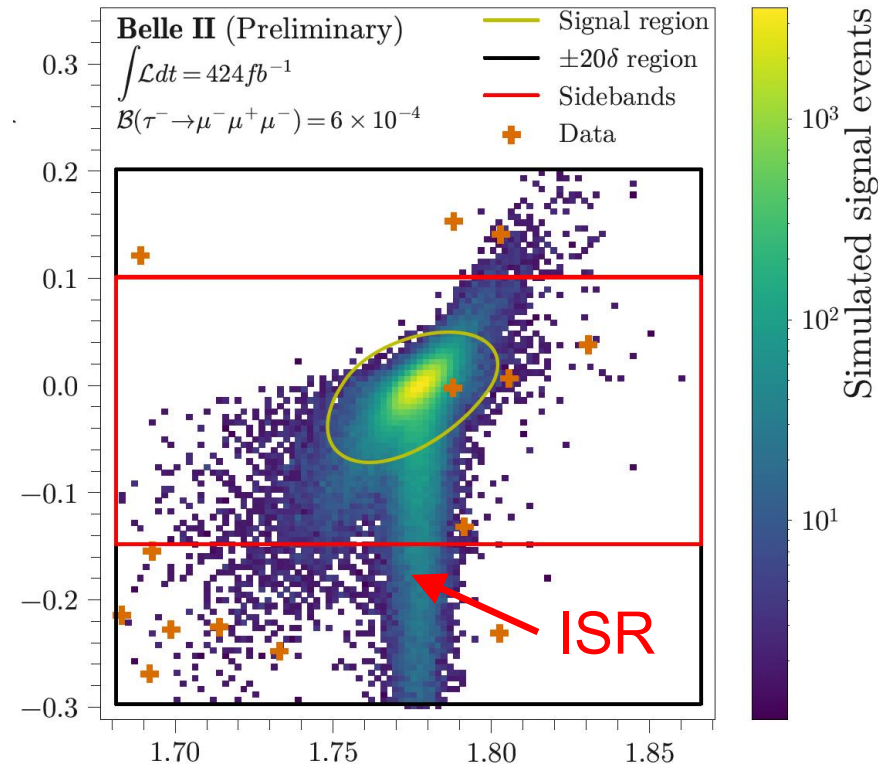
Any observation of cLFV will make you famous !



Fully-reconstructed τ 's

- In (most) LFV τ searches, final state can be fully reconstructed (no neutrinos)
- Important kinematic variables
 - mass of τ candidate $m(\tau)$
 - difference between τ energy and beam energy (in center of mass)
 $\Delta E = E^*(\tau) - \sqrt{s}/2$
 - ΔE tail towards lower values due to ISR
- Signal yield usually estimated in $m(\tau)$ - ΔE signal region

$m(\tau)$ - ΔE signal region



Tests of the SM with τ measurements

- *Tau properties*

- *Lifetime*
- *Mass*
- *Electric and magnetic dipole moment (also of μ)*

- *Couplings*

- *Lepton flavor universality*
- *V_{us}*
- *Michel parameters*
- *Second class currents*
- *α_S*
- *CP violation*

- *Hadronic system*

- *Spectral functions*
- *Partial-wave analyses*

Almost all measurements
are systematically limited:
400M τ pair events !!!

τ lifetime

- The τ decays weakly. τ lifetime is the ratio of the leptonic BF and width

$$\tau_\tau = \frac{1}{\Gamma_{tot}} = \frac{B(\tau \rightarrow l\nu_l\nu_\tau)}{\Gamma(\tau \rightarrow l\nu_l\nu_\tau)}$$

- and the leptonic width can be calculated in the SM

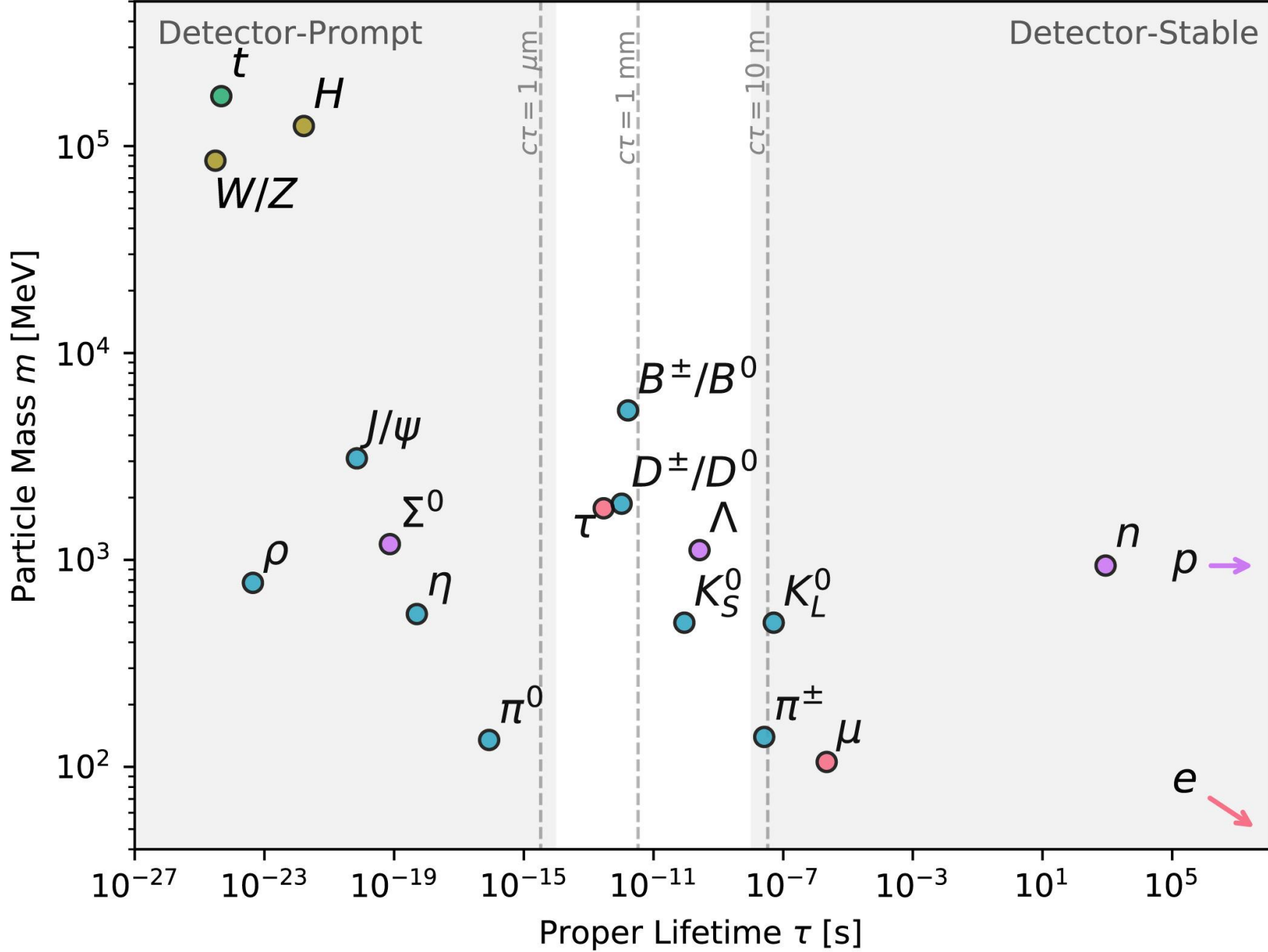
$$\Gamma(\tau^- \rightarrow l^- \nu_\tau \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f\left(\frac{m_l^2}{m_\tau^2}\right) r_{EW}$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$r_{EW} = \frac{\alpha}{2\pi} \left[\frac{25}{4} - \pi^2 + O\left(\frac{m_l^2}{m_\tau^2}\right) \right]$$

$$\tau(\tau) \sim 290 \text{ fs}$$

Not quite stable, not quite prompt



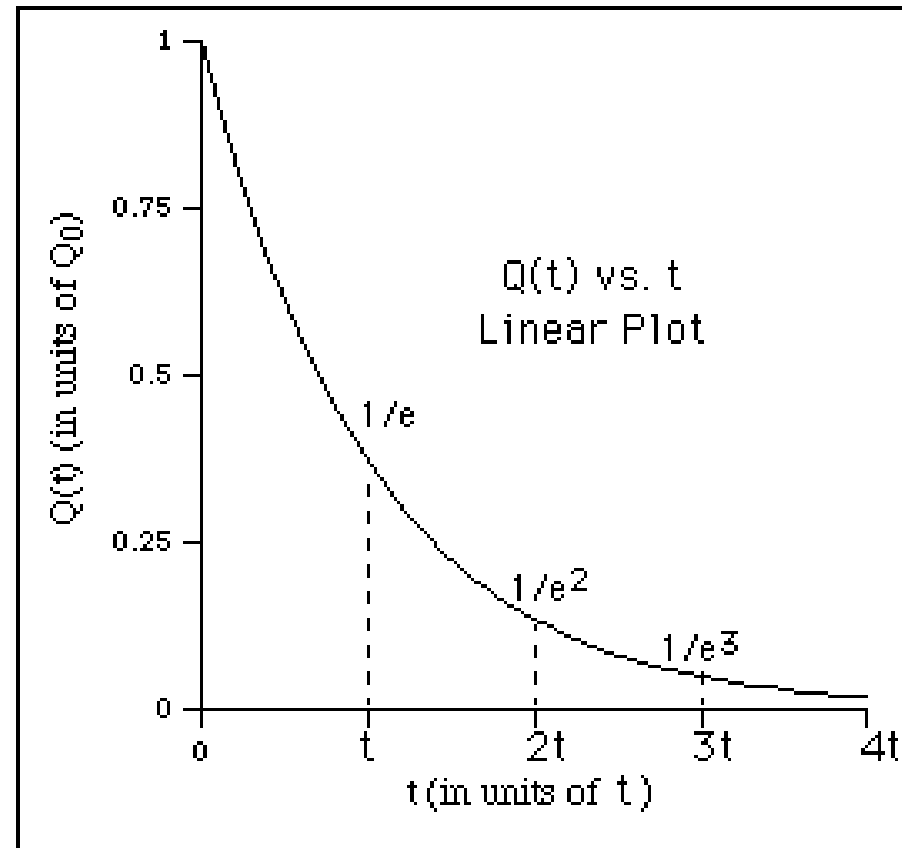
Lifetime measurement

- *Decay time*
 - *distribution is exponential with decay constant $\lambda = 1/\tau$ and average of τ*

$$Q(t) = \frac{N(t)}{N_0} e^{-\lambda t}$$

$$\langle t \rangle = \tau \quad N_0 \text{ is number of particles at } t = 0$$

- *Decay times are too short to measure*
 - *Typical timing resolution in Belle II is of order 1 ns (or 3,000 τ lifetimes)*
- *Determine decay time from decay distance ℓ (taking into account time dilation)*



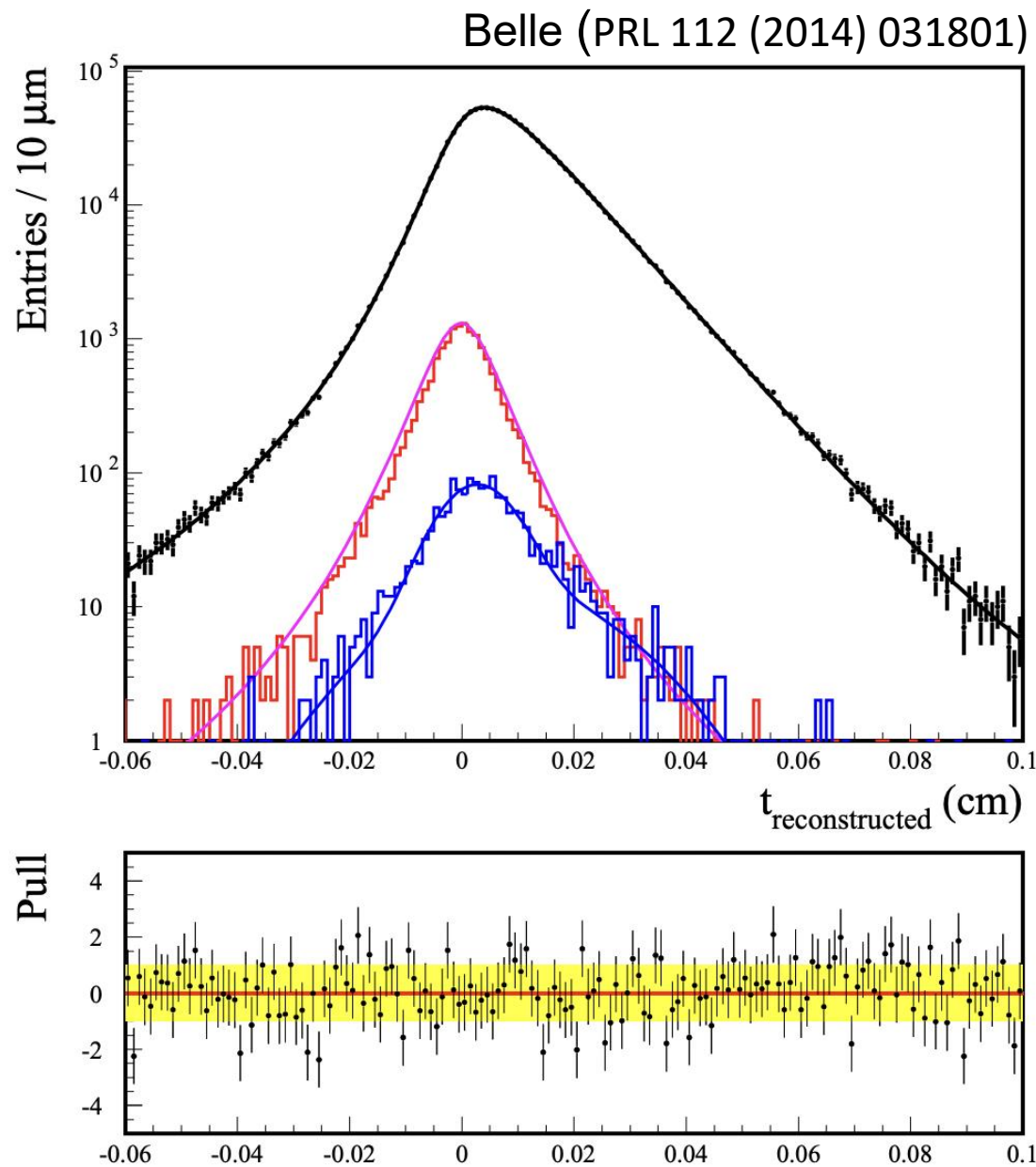
$$t = \frac{\ell}{\beta\gamma c} = \frac{m\ell}{pc}$$

τ lifetime measurement

- *Belle determined τ lifetime from fit to decay time distribution in (3x3) τ pair events*
 - *average t is 245 μm (can be measured with good vertex detector)*
- *Negative decay times result from finite detector resolution*

Best measurement from Belle
(largest syst. error from SVD alignment)

$$\tau(\tau) = 290.17 \pm 0.53 \pm 0.33 \text{ fs}$$



m_τ measurement at $e^+e^- \rightarrow \tau^+\tau^-$ threshold

- $e^+e^- \rightarrow \tau^+\tau^-$ cross-section as function of center-of-mass energy at $\tau^+\tau^-$ threshold depends strongly on m_τ

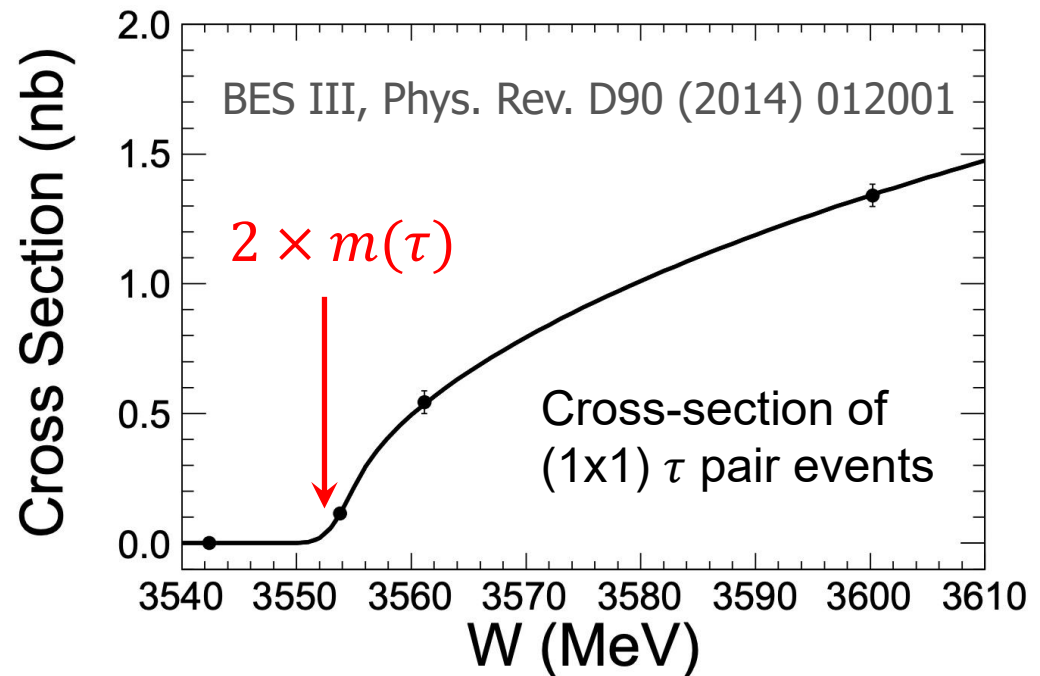
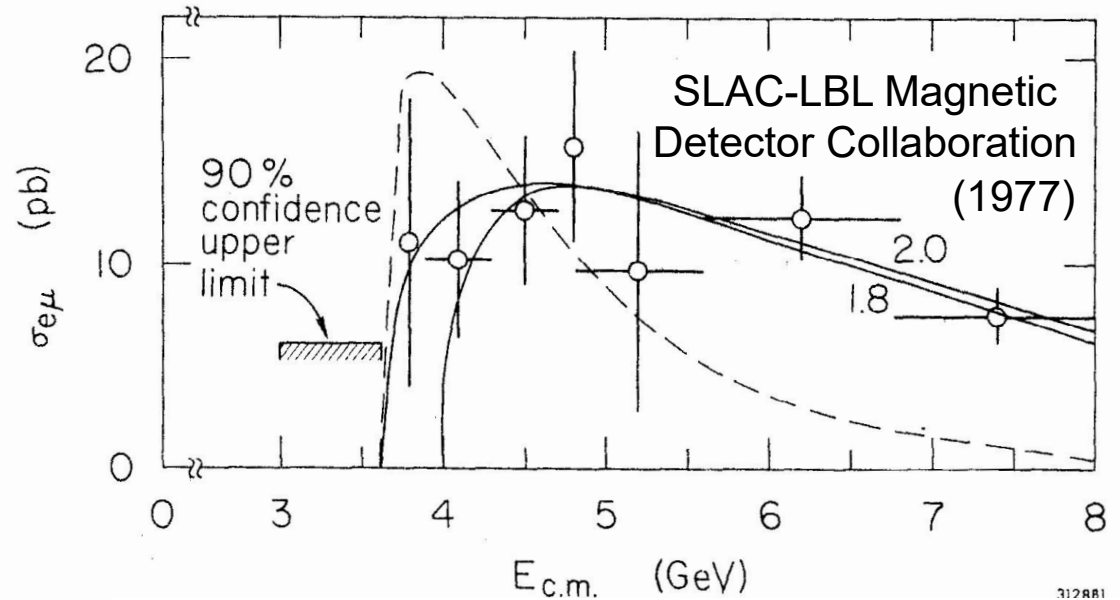
- First m_τ measurements came from the cross-section of $e^+e^- \rightarrow e\mu X$ events

$$m_\tau = (1900 \pm 100) \text{ MeV}$$

- This is still one of the most precise techniques

$$m_\tau = (1776.91 \pm 0.12^{+0.10}_{-0.13}) \text{ MeV}$$

- ... but SuperKEKB operates far away from $\tau^+\tau^-$ threshold

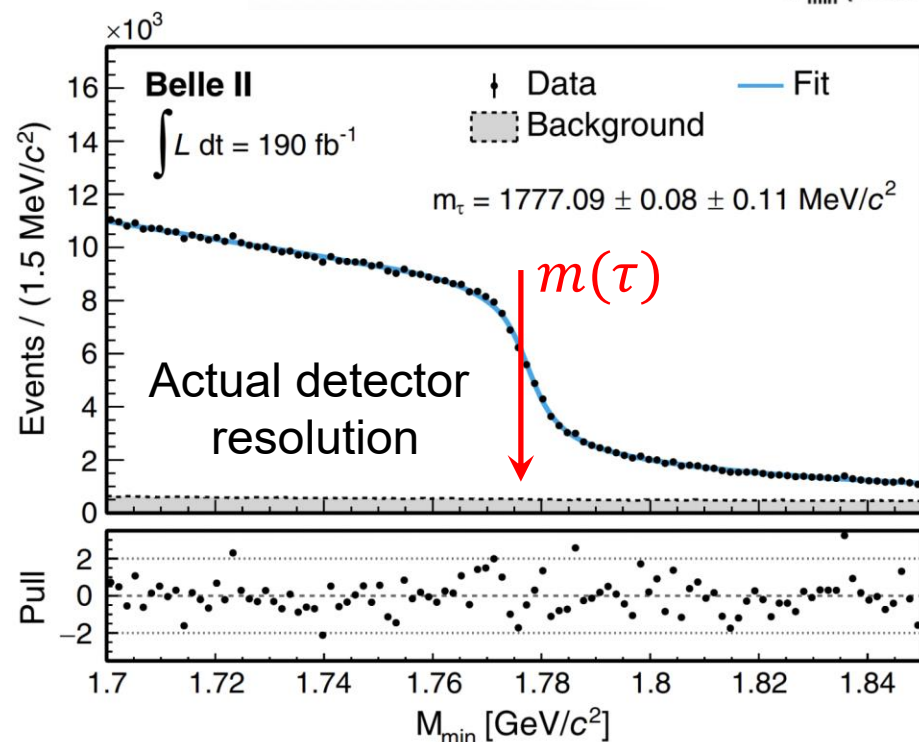
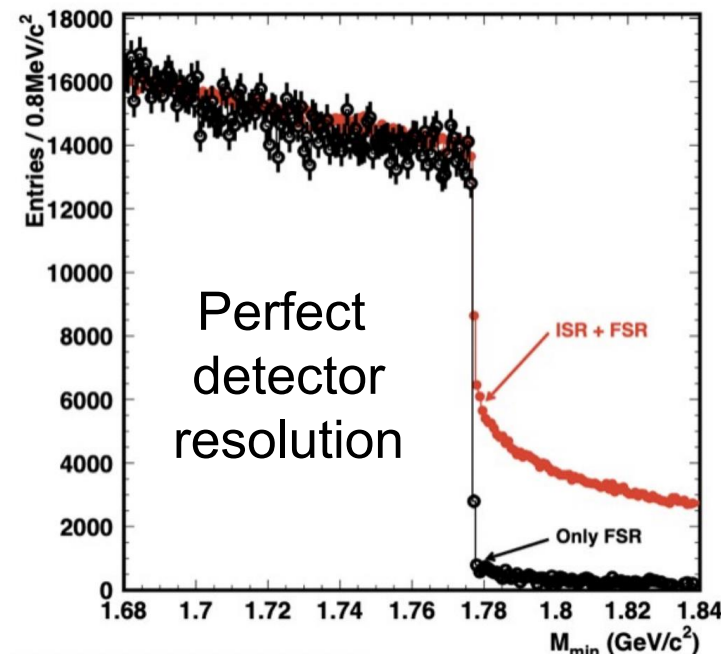


m_τ mass measurement at Belle II

- τ mass measurement with $\tau \rightarrow 3\pi\nu$
- M_{min} approximates m_τ assuming the neutrino direction is the same as the three-pion momentum direction
 - If it's not, $M_{min} < m_\tau$

$$M_{min} = \sqrt{M_{3\pi}^2 + 2(\sqrt{s}/2 - E_{3\pi}^*)(E_{3\pi}^* - p_{3\pi}^*)} \leq m_\tau$$

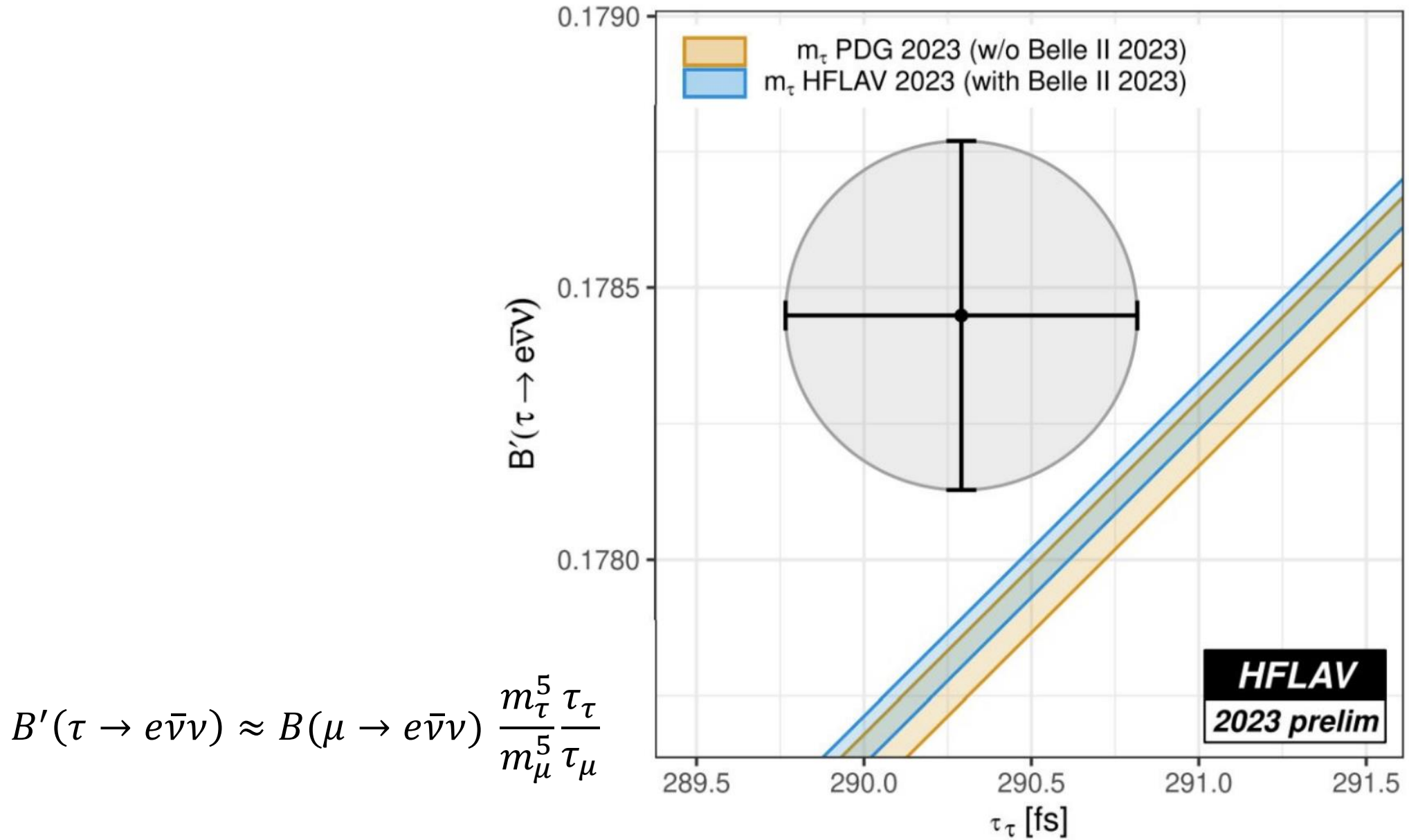
- Sharp drop of M_{min} distribution at m_τ
 - Smeared by detector resolution
- Most precise τ mass measurement
 - Largest systematics from knowledge of beam energy and momentum scale



Belle II, PRD 108 (2023) 032006

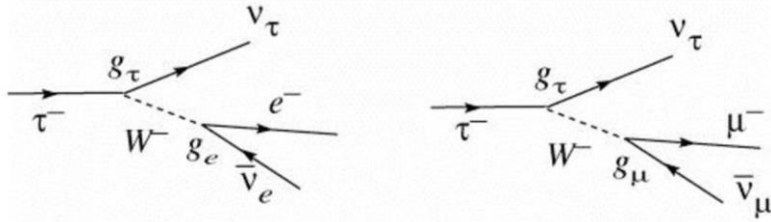
SM test

Test of SM with τ mass and lifetime

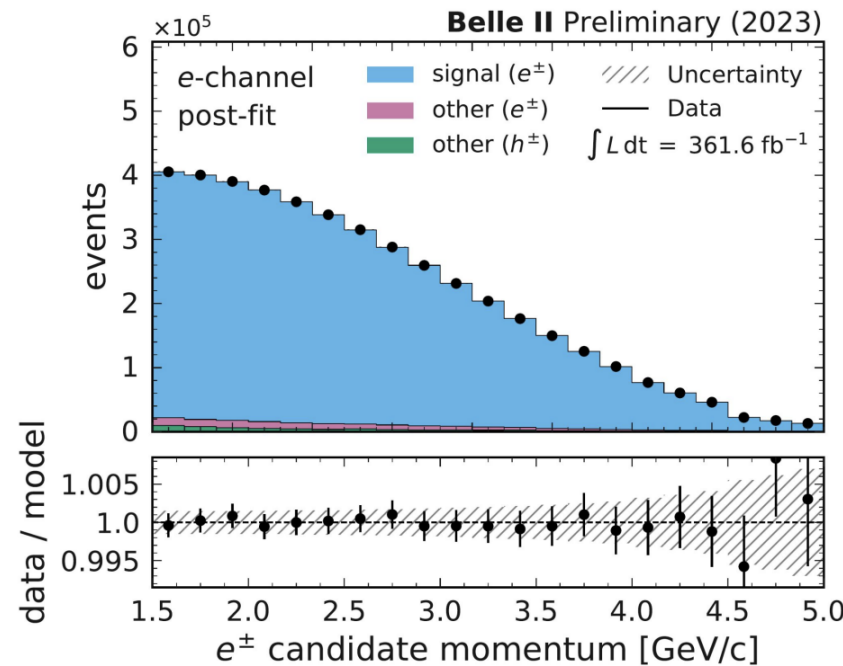
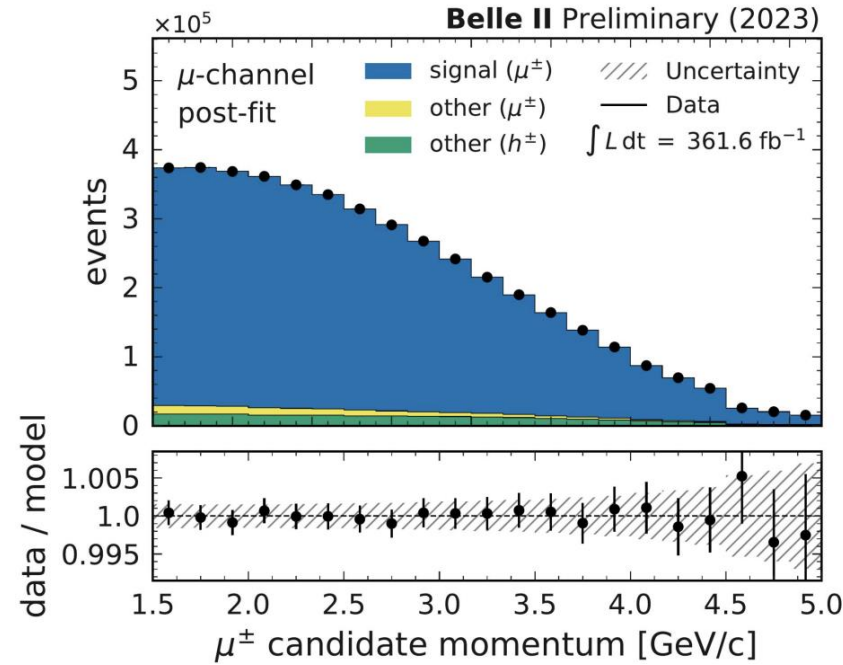
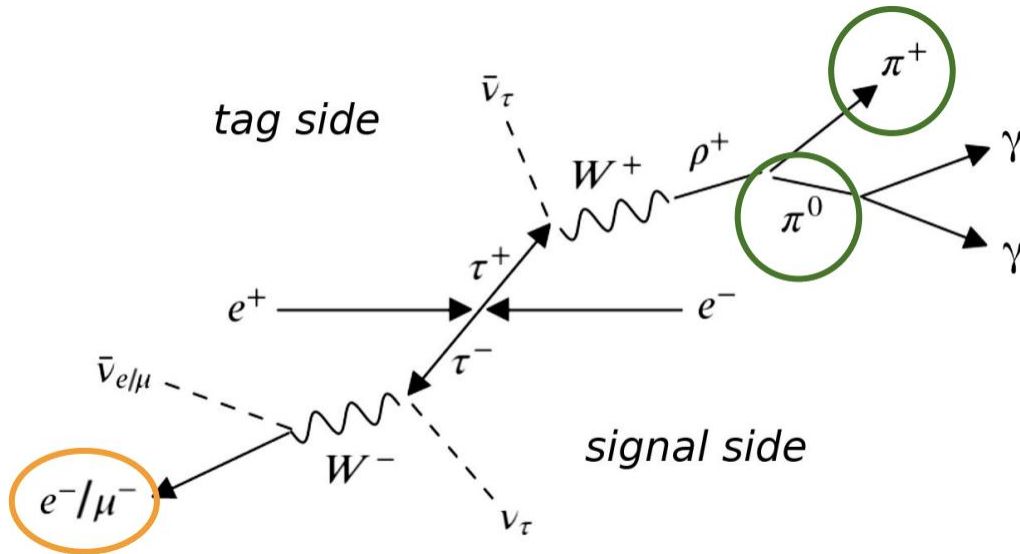


Light-lepton universality

- In the SM, $g_e = g_\mu$



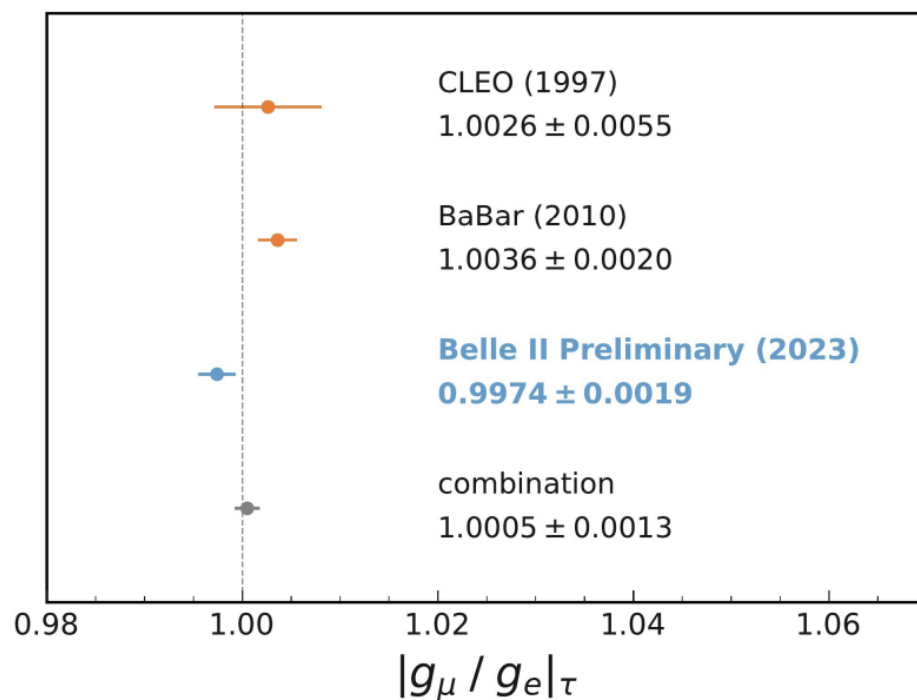
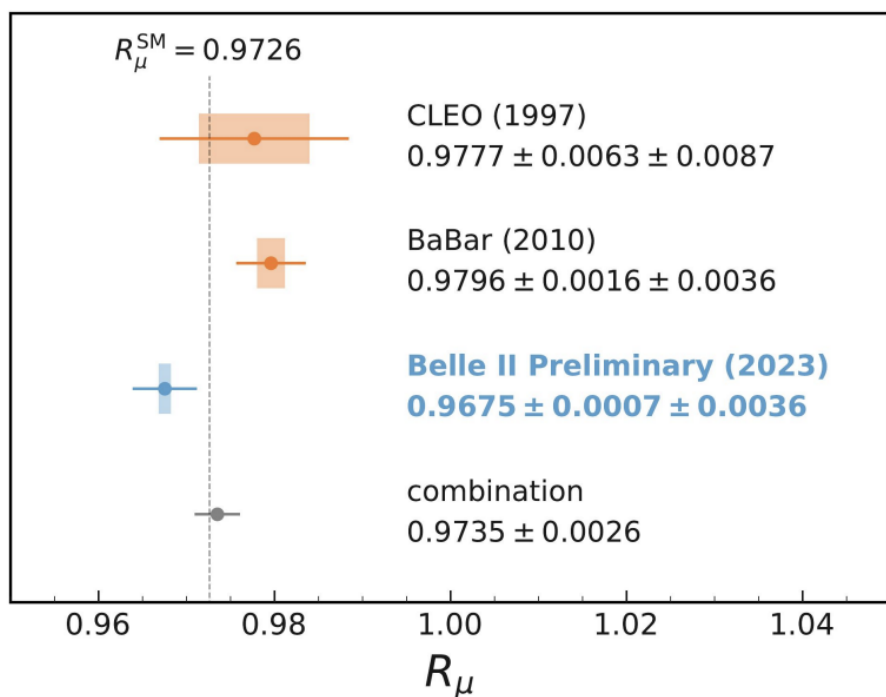
$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu(\gamma)) f(m_e^2/m_\tau^2)}{\mathcal{B}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)) f(m_\mu^2/m_\tau^2)}}$$



Light-lepton universality

$$R_\mu = \frac{\mathcal{B}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu (\gamma))}{\mathcal{B}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e (\gamma))}$$

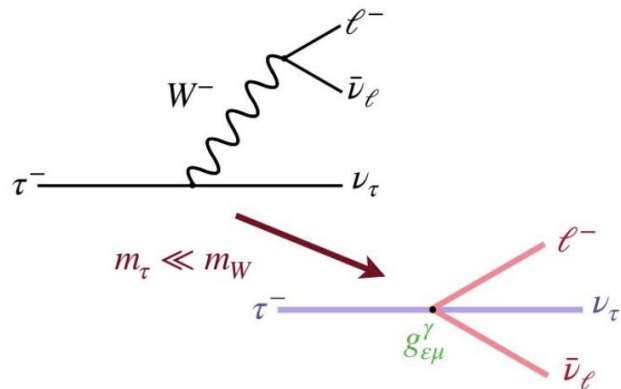
$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{R_\mu \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}}$$



Most precise measurement of R_μ
(largest syst. error from lepton ID)

Michel parameters

- Generalized matrix element



$$= \frac{4G_F}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N [\bar{u}_i(\ell)\Gamma^N v_n(\nu_\ell)] [\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau)]$$

$$\Gamma^S = 1, \Gamma^V = \gamma^\mu, \Gamma^T = i(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) / 2\sqrt{2}$$

Scalar Vector Tensor

- Test Lorentz structure of weak current (in SM $g_{LL}^V = 1$, all other $g_{ij}^N = 0$)

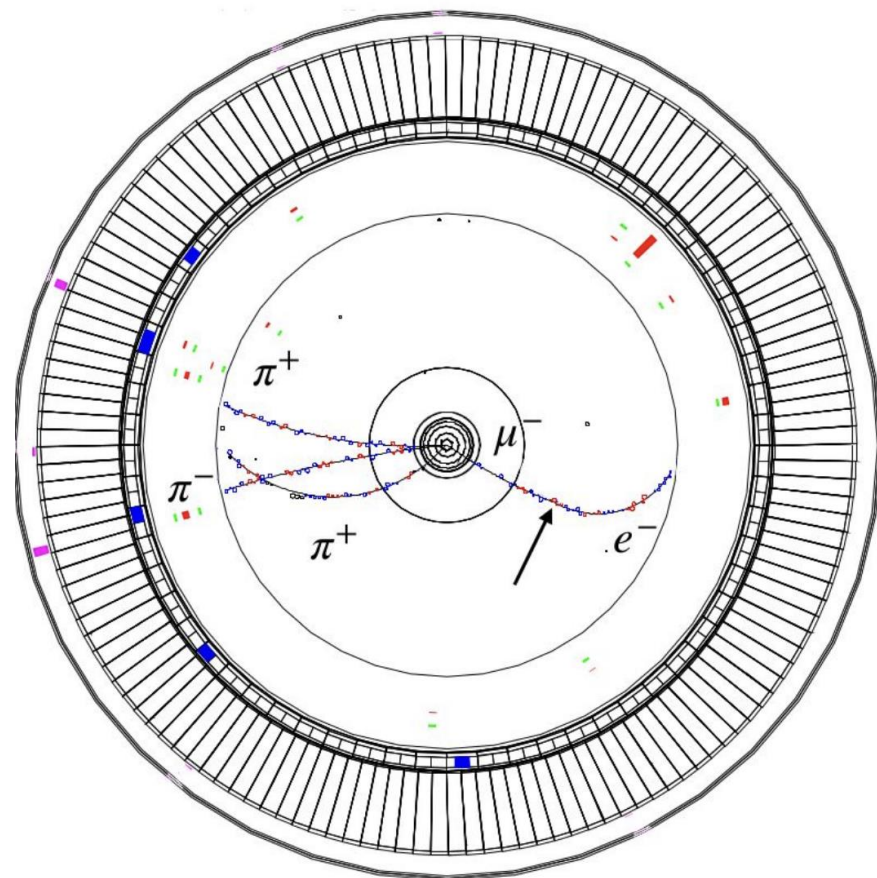
$$\frac{d\Gamma}{dx} = \frac{G_{\tau\ell}^2 m_\tau^5}{192 \pi^3} \left\{ f_0(x) + \rho f_1(x) + \eta \frac{m_\ell}{m_\tau} f_2(x) - P_\tau [\xi g_1(x) + \xi \Delta g_2(x)] \right\}$$

- Michel parameters ρ, η, ξ , and $\xi\delta$ are related to g_{ij}^N in SM

Michel parameters (cont'ed)

- All measurements consistent with SM predictions
- Most precise measurements are from CLEO and LEP experiments
- More Michel parameters can be measured if polarization of outgoing lepton is known
 - $\bar{\eta}$ and $\xi\kappa$ in radiative decays
 - ξ' with decay in flight muons with $\tau \rightarrow \mu\bar{\nu}\nu$ (Belle; PRL 131 (2023) 021801)

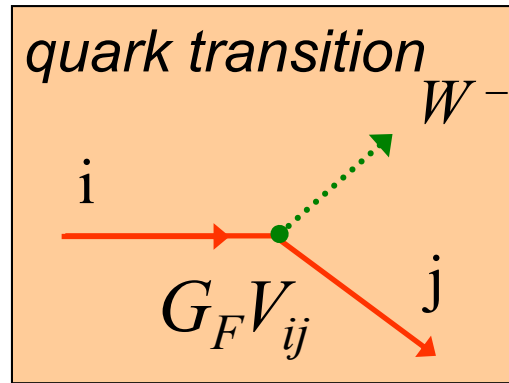
	$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$	$\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e$	$\tau^- \rightarrow \mu^- \nu_\tau \bar{\nu}_\mu$	SM
ρ	0.74979 ± 0.00026	0.747 ± 0.010	0.763 ± 0.020	0.75
η	0.057 ± 0.034	—	0.094 ± 0.073	0
ξ	$1.0009^{+0.0016}_{-0.0007}$	0.994 ± 0.040	1.030 ± 0.059	1
$\xi\delta$	$0.7511^{+0.0012}_{-0.0006}$	0.734 ± 0.028	0.778 ± 0.037	0.75
ξ'	1.00 ± 0.04	—	0.22 ± 1.03	1
ξ''	0.65 ± 0.36	—	—	1



$$\xi' = 0.22 \pm 0.94 \pm 0.42$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix

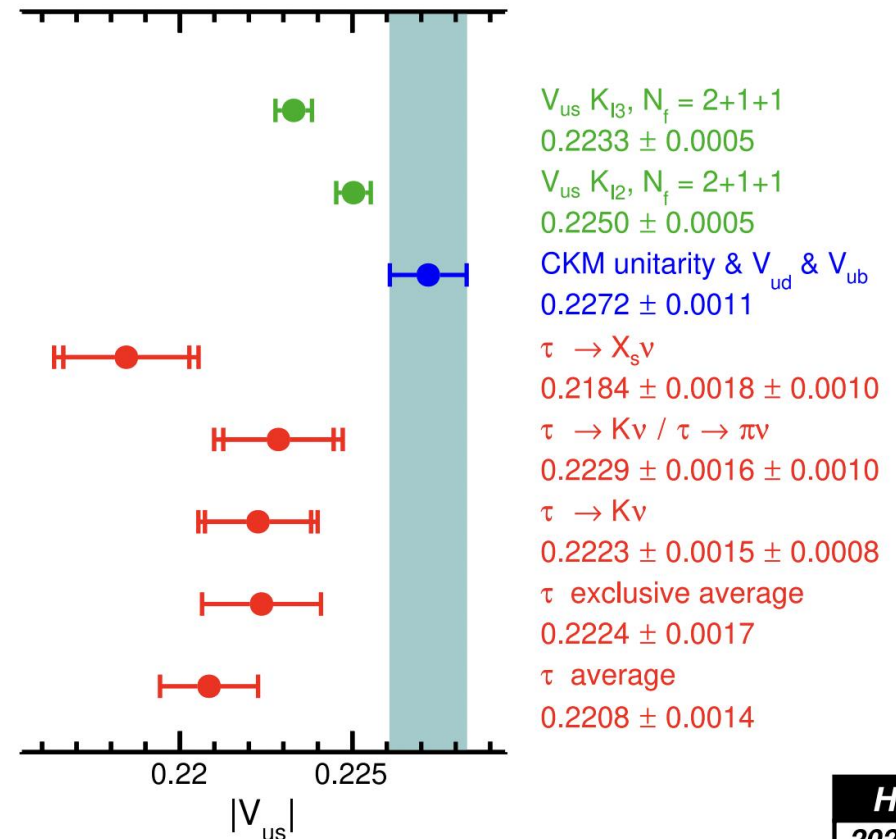
- *Unitary matrix that gives strength of weak quark transitions*
 - *Most relevant for Belle II are $|V_{ub}|$, $|V_{cb}|$, and ϕ_2/α*



$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

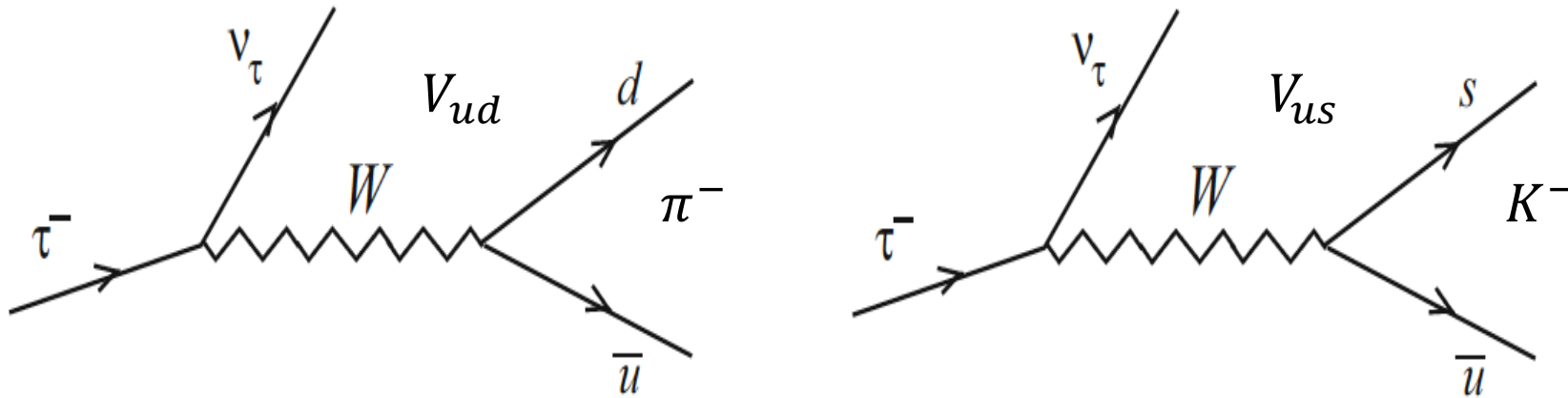
- *Belle II can also measure $|V_{us}|$ with τ decays*
 - *Current measurements with kaon decays and τ decays differ from CKM unitarity*

$$|V_{us}|^2 = 1 - |V_{ud}|^2 - |V_{ub}|^2$$



HFLAV
2023 prelim

$|V_{us}|$ from exclusive τ decays

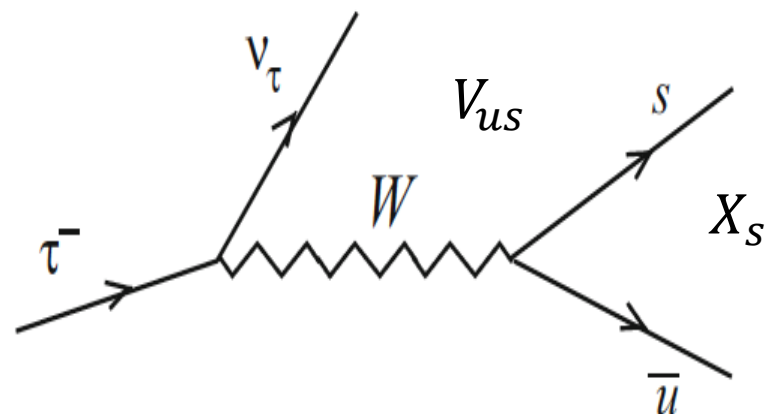
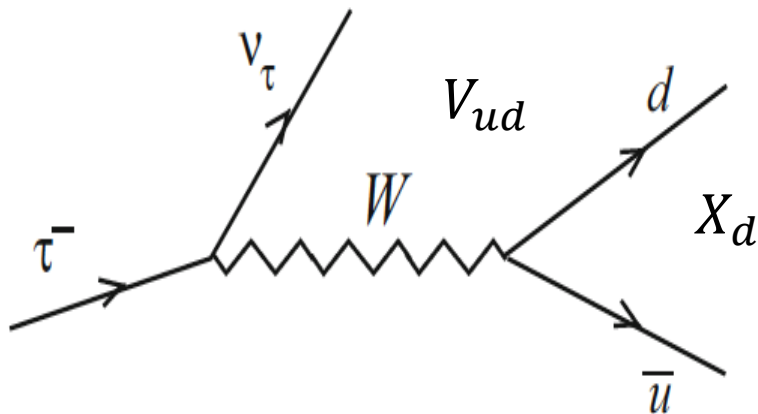


$$\frac{B(\tau \rightarrow K^- \nu_\tau)}{B(\tau \rightarrow \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(1 - m_K^2/m_\tau^2)^2}{(1 - m_\pi^2/m_\tau^2)^2} \delta_{LD}$$

Radiative corrections

Dominant systematic error from hadron ID

$|V_{us}|$ from inclusive τ decays

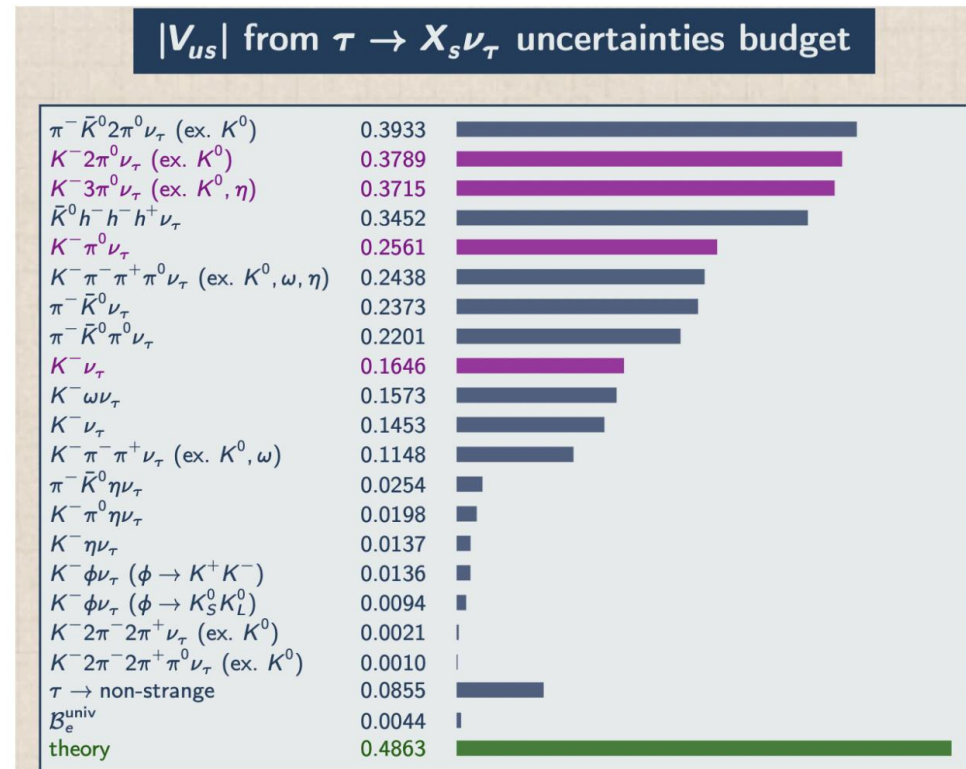


- Determine $|V_{us}|$ from fraction of hadronic τ decays with strangeness
 - Inclusive BF as sum of exclusive BFs
- Measurements can be extended to higher moments of hadronic mass distributions

Spectral Moments:

$$R_{\tau}^{kl} = \int_0^1 dz (1-z)^k z^l \frac{dR_{\tau}}{dz}, \quad z = \frac{q^2}{m_{\tau}^2}$$

- Many spectral function measurements are still from the LEP era



Courtesy: A. Lusiani [Tau2023 slides]

Conclusions

- *τ pair events at the $\Upsilon(4S)$ are clean and provide many constraints on kinematic variables*
 - *τ pair events are quite different from B and charm decays*
- *τ properties and decays provide a wide variety of SM tests and opportunities to search for new physics*
- *Belle II will soon have the largest pile of τ 's in the world*
 - *New physics may be hiding in it ...*

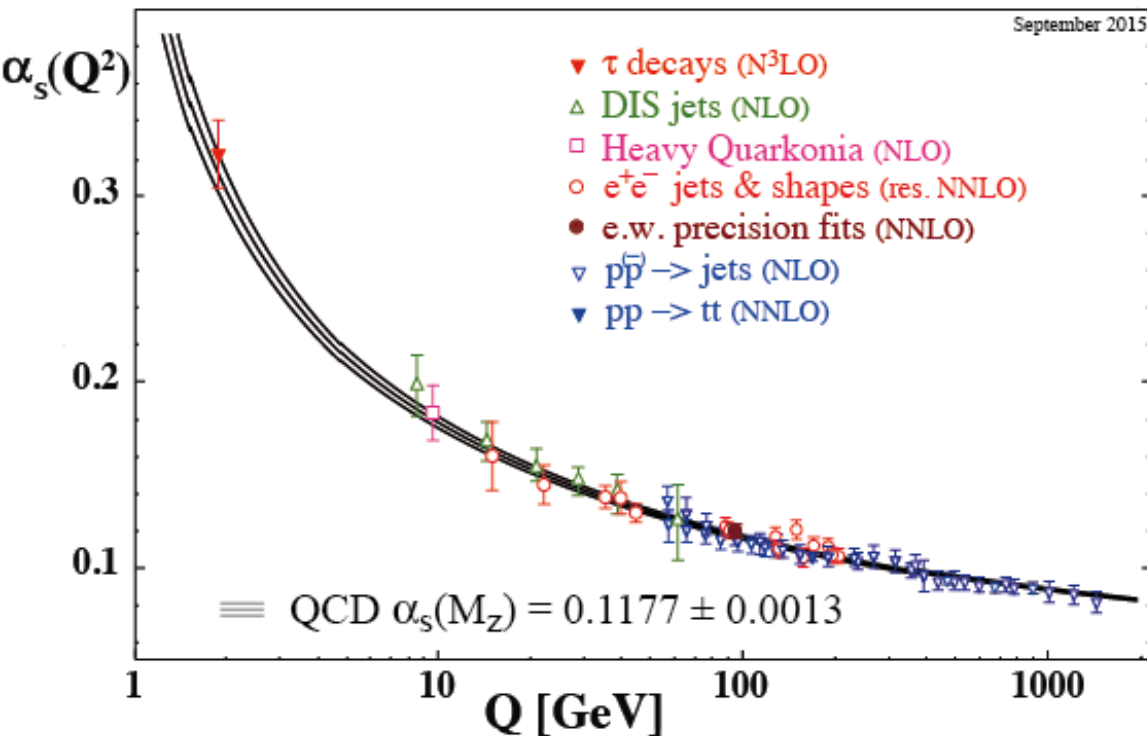


References

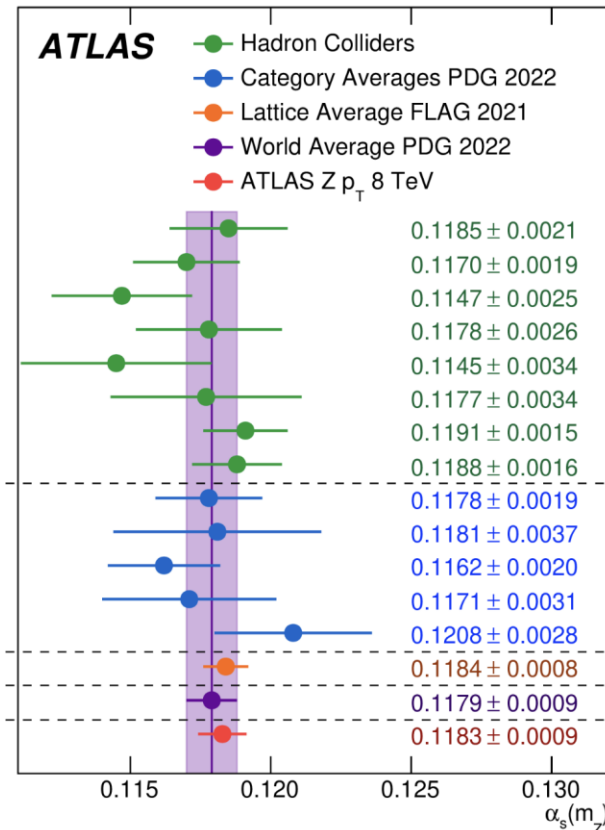
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- *“Physics with tau leptons”, Achim Stahl, Springer Tracts in Modern Physics (1999)*
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- *“The Physics of the B factories”, A. Bevan, B. Golob, T. Mannel, S. Prell, and B. Yabsley (eds.), Eur. Phys. J. C74 (2014) 3026.*
- *“The Belle II Physics Book”, Belle II Collaboration, E. Kou et al., Prog. Theor. Exp. Phys. (2019)*

Back-up slides

Strong coupling constant α_s



ATLAS ATEEC
 CMS jets
 H1 jets
 HERA jets
 CMS $t\bar{t}$ inclusive
 Tevatron+LHC $t\bar{t}$ inclusive
 CDF $Z p_T$
 Tevatron+LHC W, Z inclusive
 τ decays and low Q^2
 $Q\bar{Q}$ bound states
 PDF fits
 e^+e^- jets and shapes
 Electroweak fit
 Lattice
 World average
 ATLAS $Z p_T$ 8 TeV



α_s can be determined from τ hadronic width and spectral moments

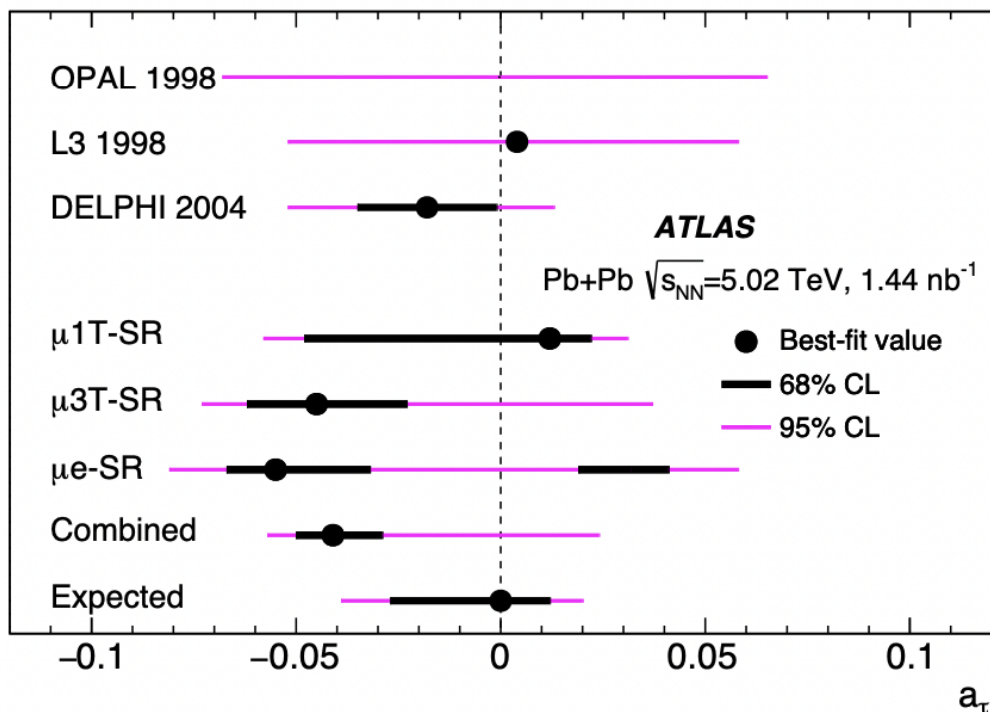
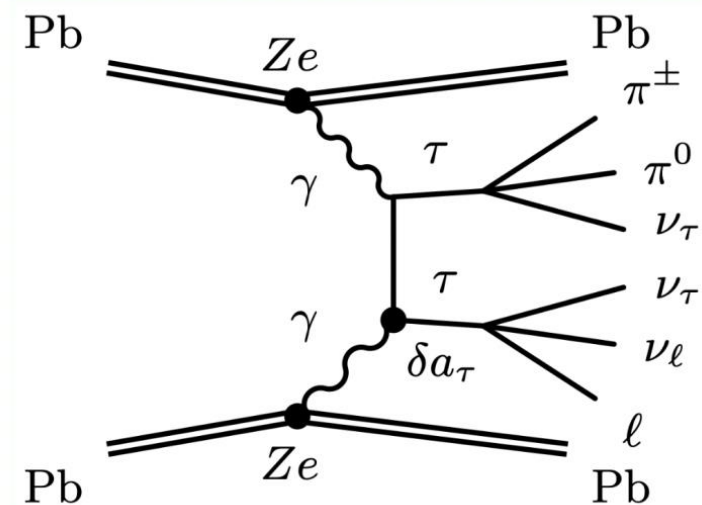
Last measurements from LEP & CLEO

→ Very precise measurement from ATLAS at LHC

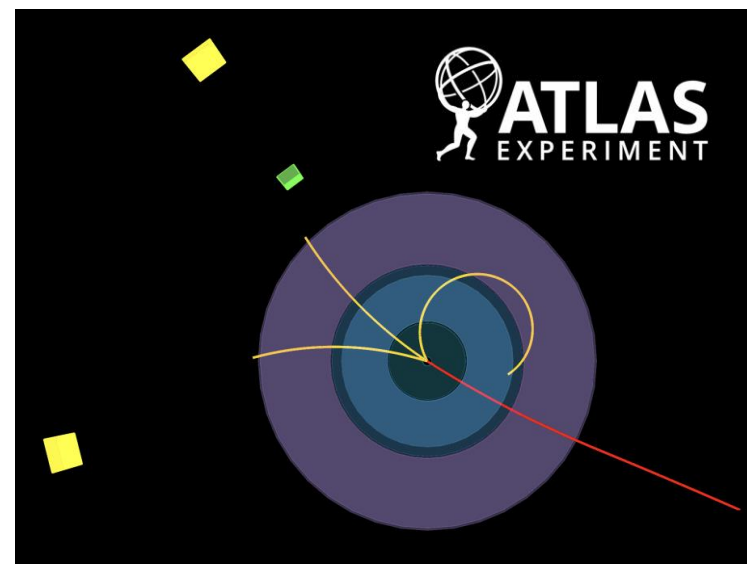
Tau g-2

- *Photoproduction cross-section of tau pairs depends on a_τ*
 - *ATLAS result has similar precision to DELPHI result; ALICE analysis is in progress*
- *Also possible at Belle II (pol. beams help)*

Ultra peripheral Pb-Pb collisions



1-loop QED, Schwinger term
 $\alpha/2\pi = 0.0012$



Tau electric dipole moment

- *New measurement of tau **EDM** from Belle using spin correlations*
 - *Expect to improve to $(1 - 2) \times 10^{-19}$ ecm with improved technique and Belle II data,*
 $\text{Re}(d_\tau) = (-0.62 \pm 0.63) \times 10^{-17}$ ecm,
 $\text{Im}(d_\tau) = (-0.40 \pm 0.32) \times 10^{-17}$ ecm.

