Precision tests of the Standard Model with Tau physics

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IFIC Valencia/Indiana University

2024 Belle II Physics Week
«Tau and dark sector with Belle II»
October 14 - 17, 2024

Outline :

1. Introduction and Motivation
   Tau lepton as a clean laboratory for Electroweak Interactions and QCD

2. Leptonic Tau decays

3. Hadronic Tau decays:
   - Cabbibo anomaly and V_{us}
   - CP violation in Tau decays

4. Anomalous magnetic moment of the muon

5. Lepton Flavour Violation

6. Conclusion and outlook

1. Introduction and Motivation

1.1 τ-decays

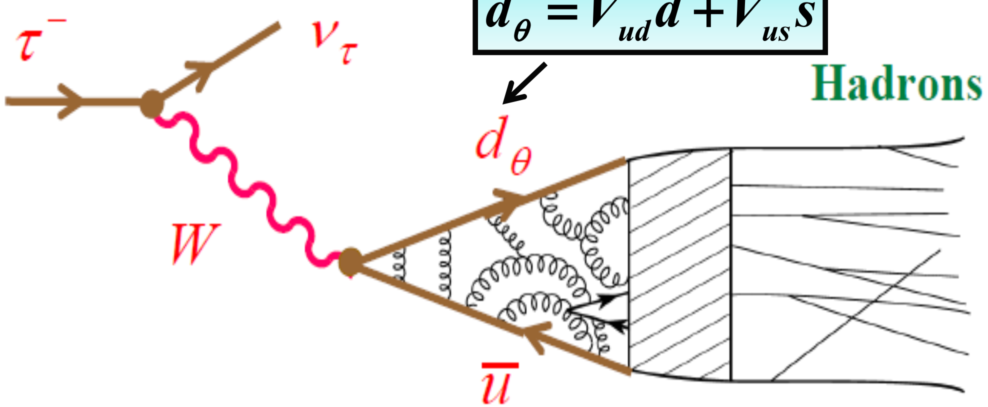
- τ lepton discovered in 1976 by M. Perl et al. (SLAC-LBL group)

  PDG'22

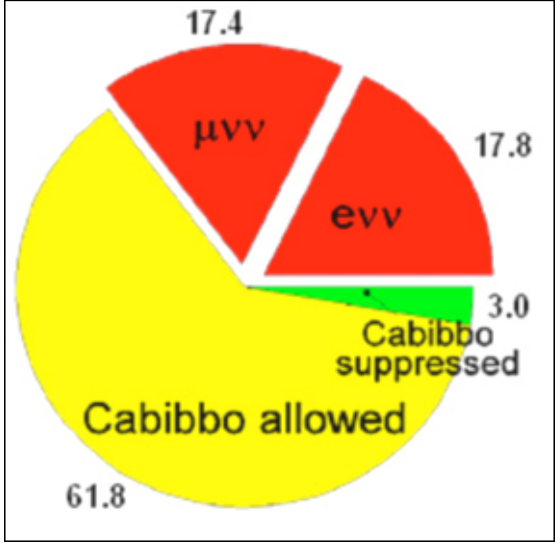
  - Mass : $\boxed{m_\tau = 1.77686(12)\ \mathrm{GeV}}$
  - Lifetime : $\boxed{\tau_\tau = 2.903(5)\cdot 10^{-13}\, s}$

- The only lepton heavy enough to decay into hadrons :
  lots of semileptonic decays !

$$\boxed{d_\theta = V_{ud}\, d + V_{us}\, s}$$



Including QED & QCD corrections:



PDG'14

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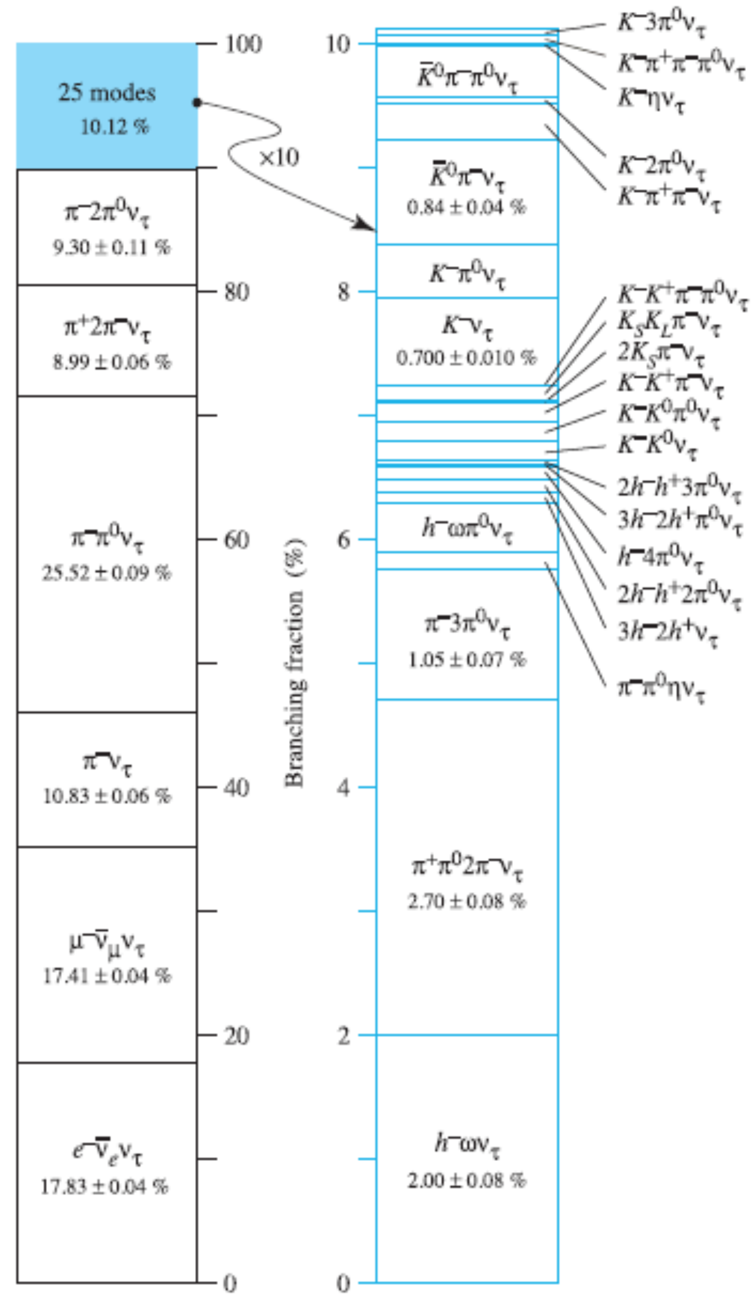
PDG'22

- The only lepton heavy enough to decay into hadrons : lots of semileptonic decays !

  ⇨ Very rich phenomenology

  Test of QCD and EW interactions

- For the tests:
  - Precise measurements needed
  - Hadronic uncertainties under control



1.2 Experimental situation

- A lot of progress in tau physics since its discovery on all the items described before ⇨ important experimental efforts from *LEP*, *CLEO, B factories: Babar, Belle, BES-III, now Belle II*

  ⇨ More to come from STCF, Tera-Z

- But τ physics has still potential "*unexplored frontiers*"

  ⇨ deserve future exp. & th. efforts

Experiment	Number of τ pairs
LEP	$\sim 3\times10^{5}$
CLEO	$\sim 1\times10^{7}$
BaBar	$\sim 5\times10^{8}$
Belle	$\sim 9\times10^{8}$
Belle II	$\sim 4.6\times10^{10}$

1.3 τ lepton as a unique probe of new physics

- Unique probe of *Lepton Universality* and *Charged Lepton Flavour Violation*
  No SM background
  Indirect probe of flavor-violating NP occurring at energies not directly accessible at accelerators:
  - Kaon physics: $$\boxed{\frac{s\bar{d}s\bar{d}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^5 \text{ TeV}}$$
    [ε_K]
  - Tau physics: $$\boxed{\frac{\tau\bar{\mu}f\bar{f}}{\Lambda^2} \quad \Rightarrow \quad \Lambda \gtrsim 10^2 \text{ TeV}}$$
    [$\tau \rightarrow \mu\gamma$]

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  Ex: $h \to \tau\tau$, LFV in $h \to \tau\mu$

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- Studying its hadronic decays: *inclusive* & *exclusive*
  - Unique probe of some of the *fundamental SM parameters*

    ⇨ α_S, $|V_{us}|$, m_s

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    ⇨ α_S, $|V_{us}|$, m_s
  - Ideal set-up for the "R&D" of theory tools about *non perturbative* & *perturbative dynamics*: OPE, Chiral Perturbation Theory, Resonances, large N_c, dispersion relations lattice QCD, etc…

    ⇨ improve our understanding of the SM and QCD at low energy

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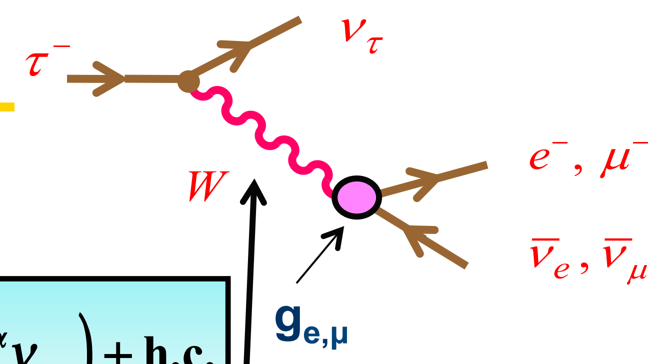
  - Inputs for the *muon g-2*

- In the following, some selected examples

2. Leptonic Tau decays

See talk yesterday
by Soeren Prell and P. Roig

2.1 Lepton Universality



- Description of the weak interactions:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L V_{CKM} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \mathrm{h.c.}$$

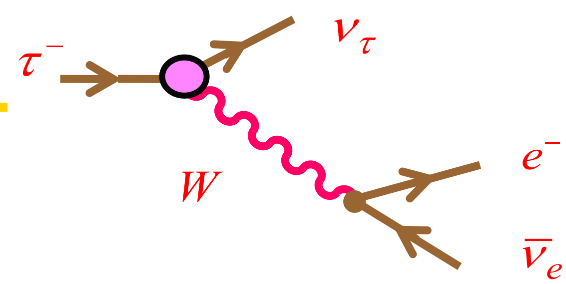
$$\frac{g^2}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \frac{g^2}{M_W^2} = 4\sqrt{2}\, G_F$$

- Lepton Universality in the SM:

$\boxed{g_\tau = g_\mu = g_e \equiv g}$: the electroweak coupling

- Use Tau physics to test lepton universality

2.1 Lepton Universality



- The leptonic decay width:

$$\Gamma(\tau \to \nu_\tau\, l\, \bar{\nu}_l) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\, f(m_l^2/m_\tau^2)\, \left(1+\delta_{\mathrm{RC}}\right)$$

Experimental inputs:

$\Gamma\left(\tau_{l3}\right)$ Rates with well-determined treatment of radiative decays

- Branching ratios
- Tau lifetimes

Inputs from theory: *Marciano'88*

δ_{RC} Radiative corrections

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

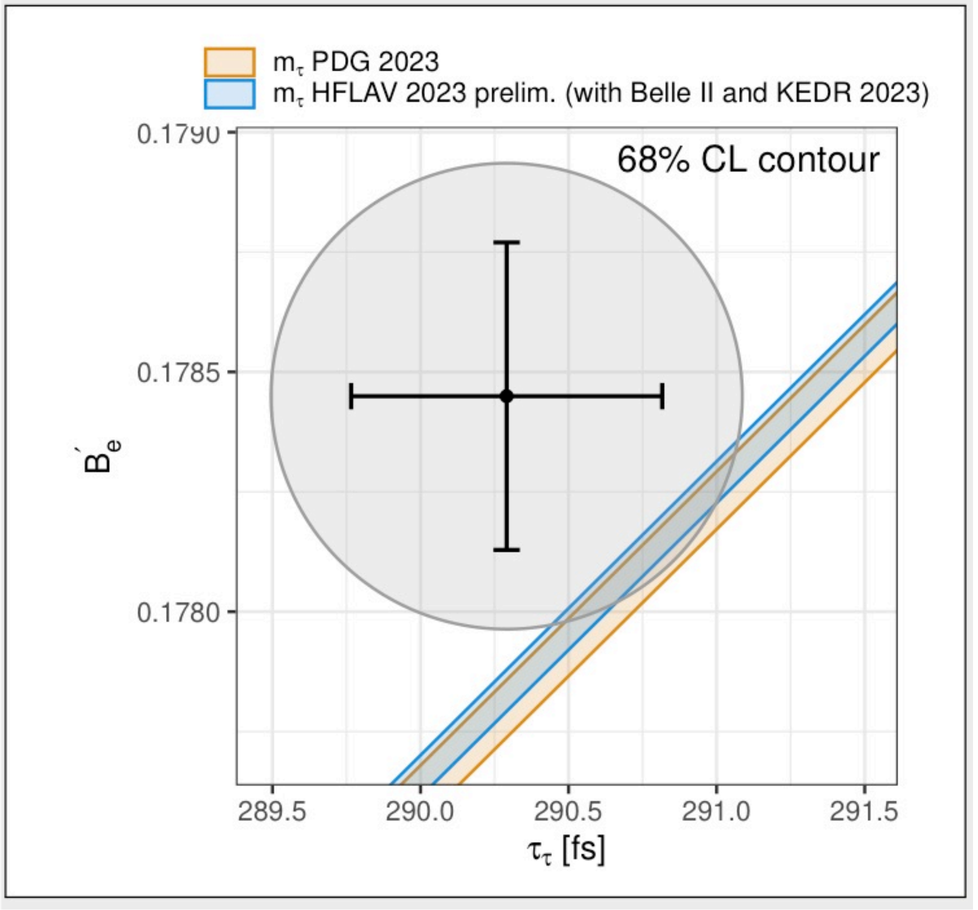
$$Br(\tau \to \nu_\tau \ell \bar{\nu}_\ell) = \frac{\Gamma\left(\tau \to \nu_\tau \ell \bar{\nu}_\ell\right)}{\Gamma_{tot}} = \tau_\tau * \Gamma\left(\tau \to \nu_\tau \ell \bar{\nu}_\ell\right)$$

- Test of τ/μ universality:

2.1 Lepton Universality

$$B'(\tau \to e\bar{\nu}\nu) \approx B(\mu \to e\bar{\nu}\nu)\,\frac{m_\tau^5}{m_\mu^5}\frac{\tau_\tau}{\tau_\mu}$$

Good agreement with LU



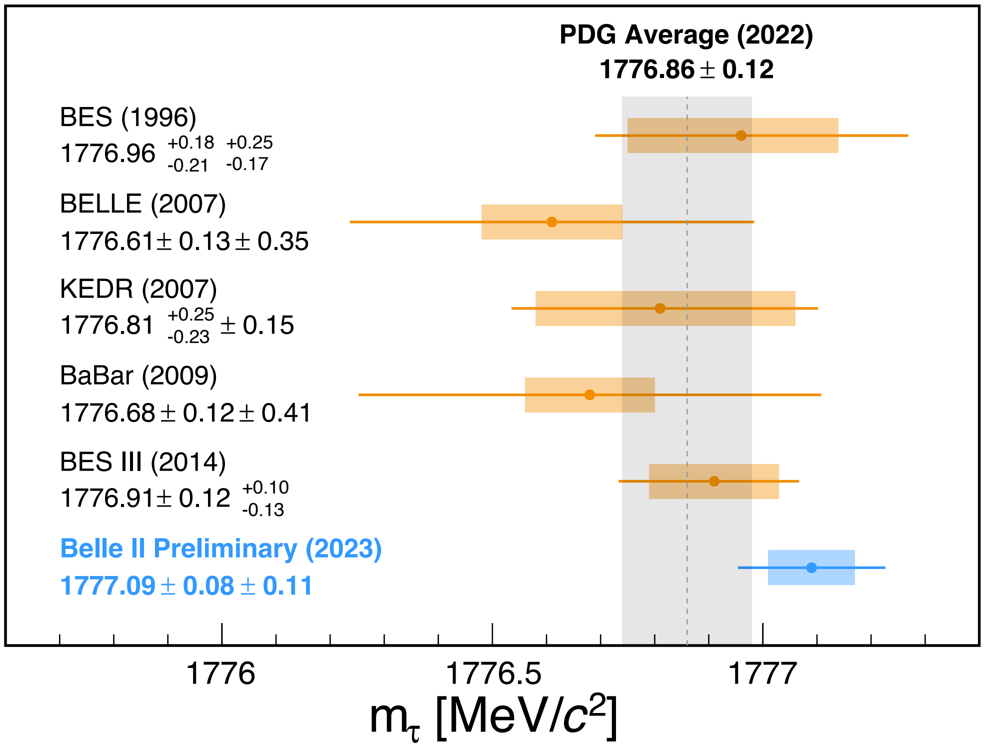
A. Lusiani@ICHEP'24

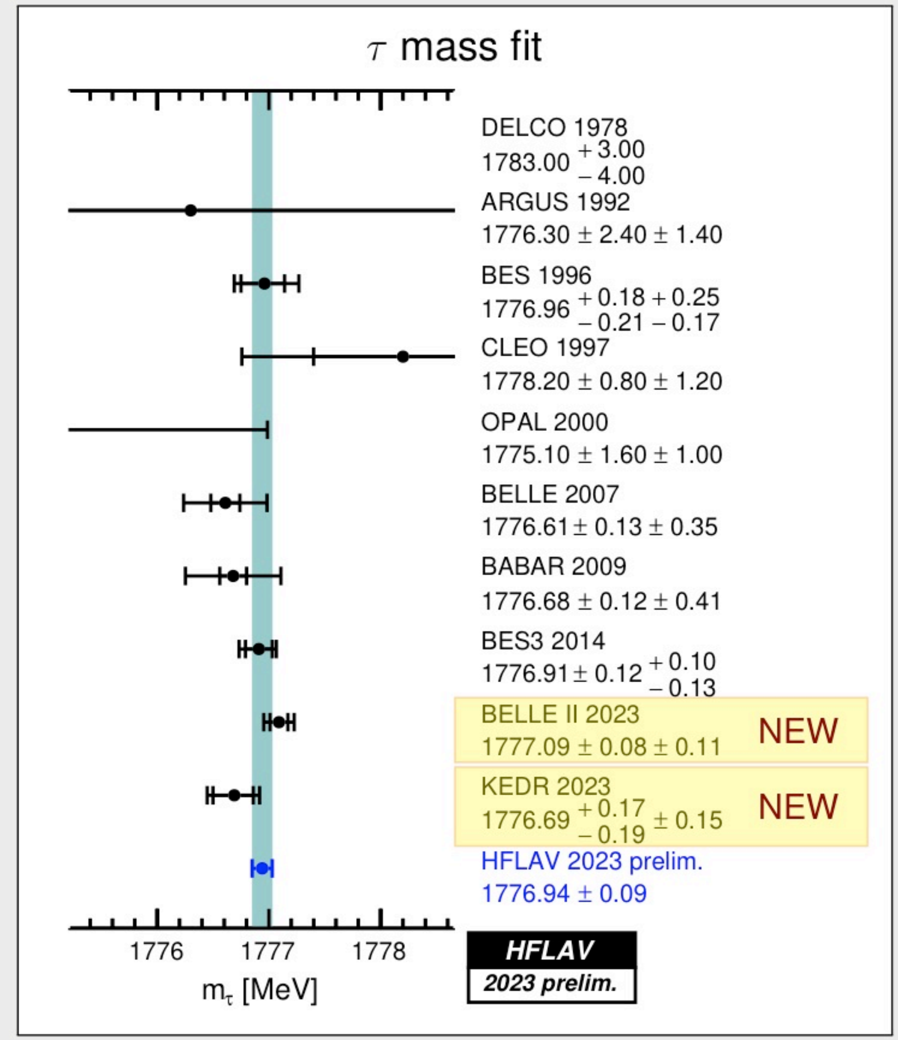
2.1 Lepton Universality

- New inputs from Belle II

A. Lusiani@ICHEP'24

arXiv:2305.19116

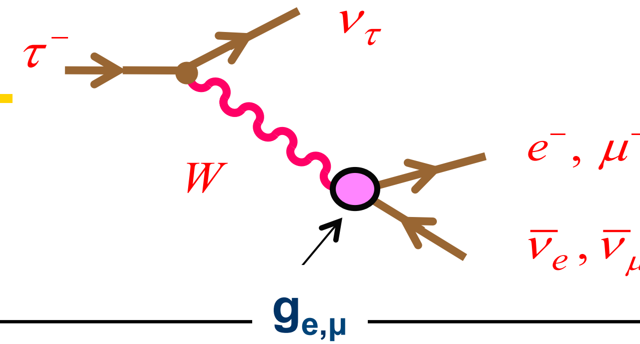




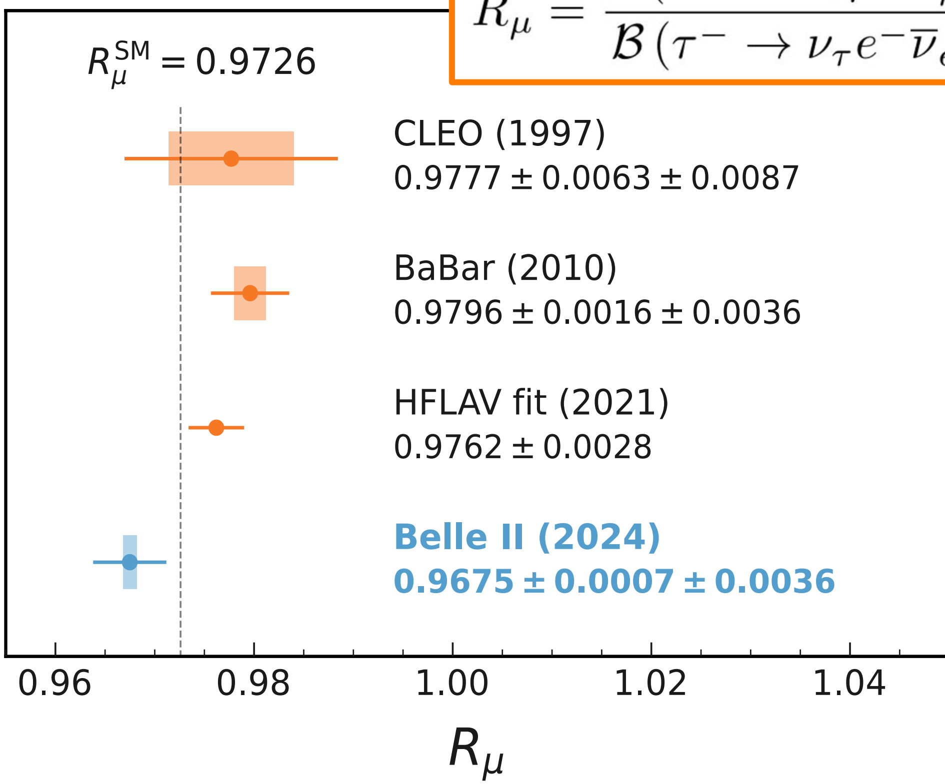
$$m_\tau = 1.77694(9)\ \text{GeV}$$

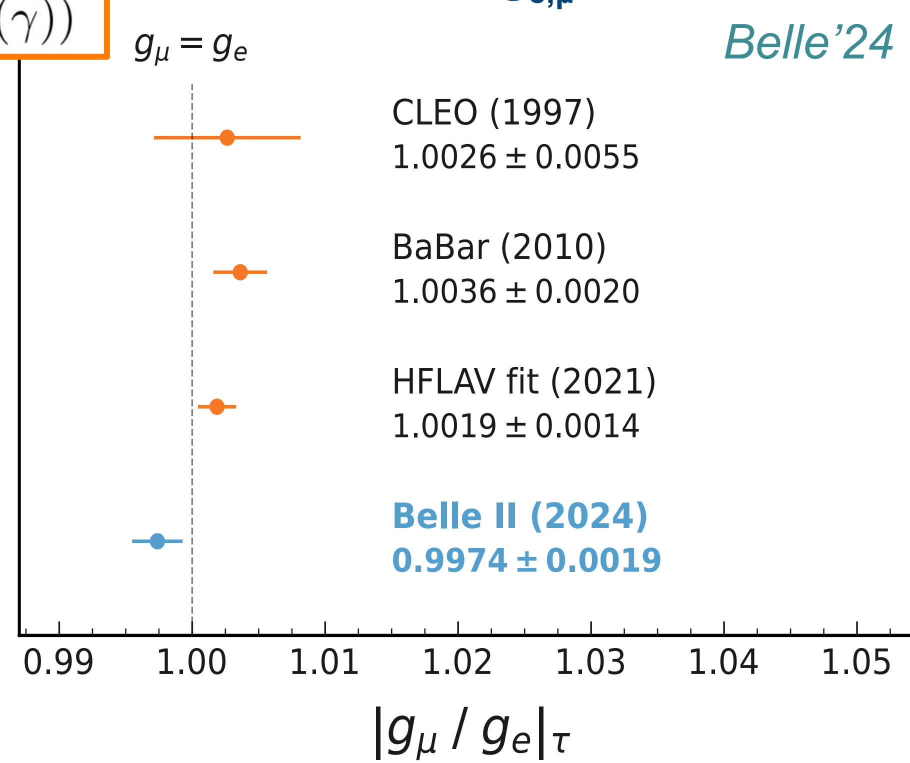
2.1 Lepton Universality

- Test of μ/e universality: New inputs from Belle II



$$R_\mu = \frac{\mathcal{B}\left(\tau^- \to \nu_\tau \mu^- \overline{\nu}_\mu(\gamma)\right)}{\mathcal{B}\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e(\gamma)\right)}$$

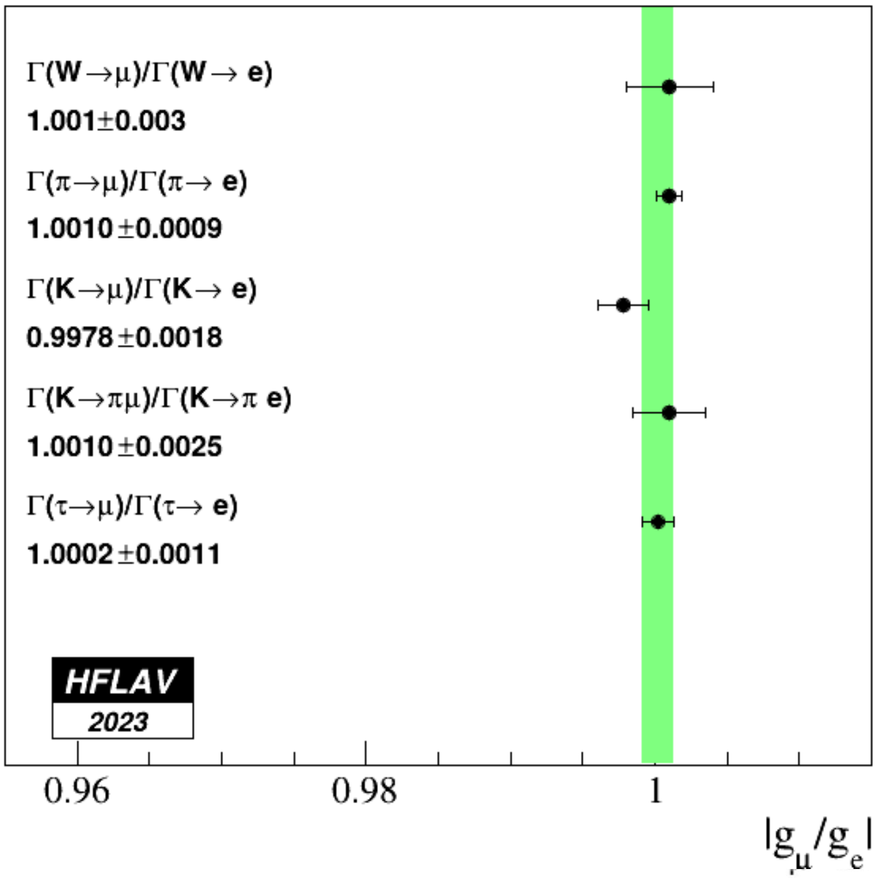


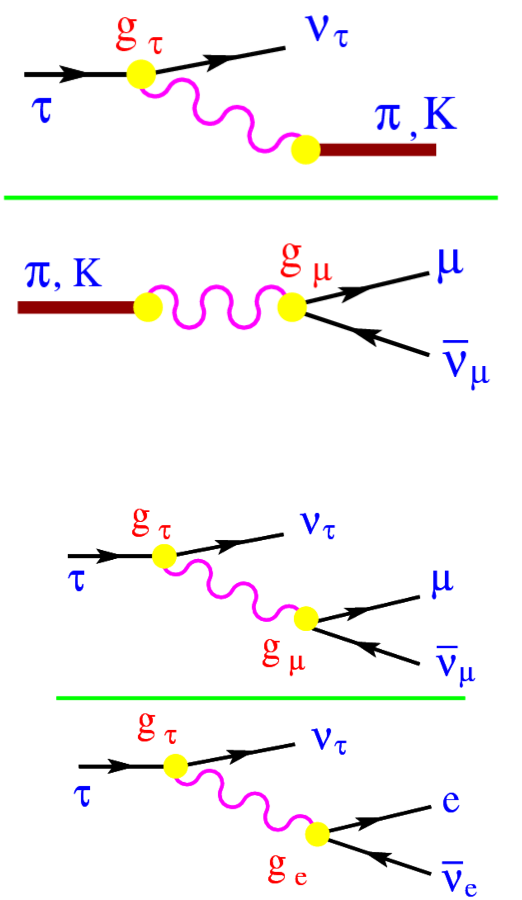


$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau\mu} f_{\tau e} R^{\tau e}}{\mathcal{B}_{\tau e} f_{\tau\mu} R^{\tau\mu}}} = 1.0002 \pm 0.0011$$

2.1 Lepton Universality

- Test of μ/e universality: Courtesy of S. *Banerjee*



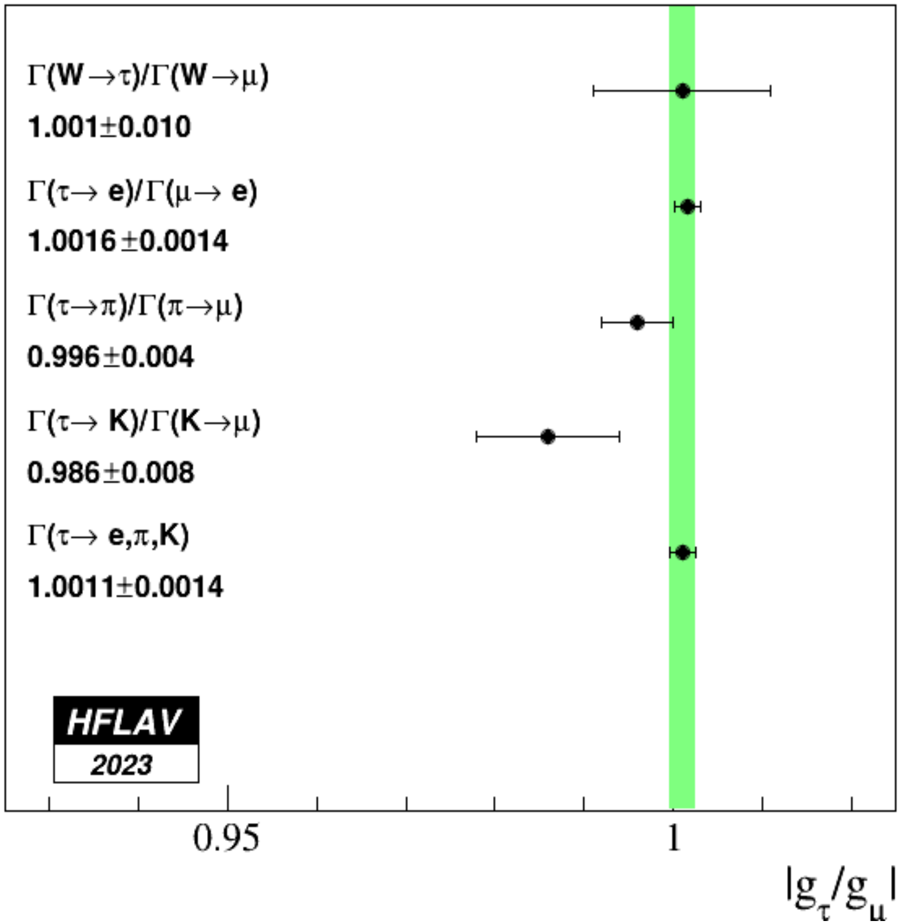


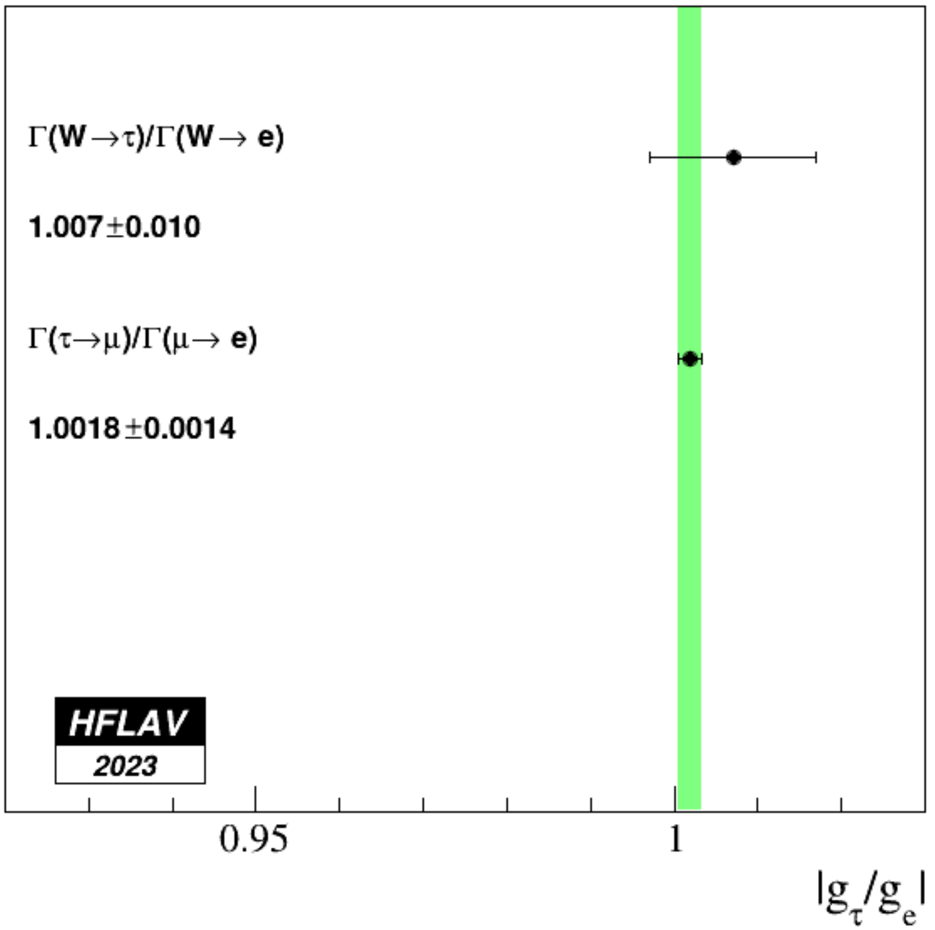
- Tested at *0.13%* from Tau leptonic Brs!

2.1 Lepton Universality

- What about the *third family*?

Courtesy of S. *Banerjee*





Universality tested at 0.14% level and excellent agreement with new data from CMS and ATLAS

2.3 Lorentz structure of leptonic Tau decays

- For constraints on the *Lorentz structure*:

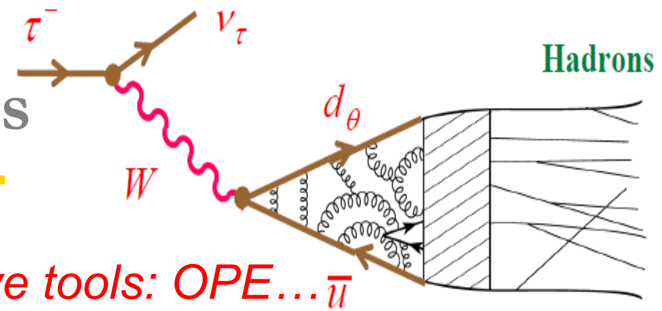
  ⇨ Michel parameters

  Important activity in Belle ⇨ see talks by *Soeren Prell* and *P. Roig*

3. Hadronic τ-decays

See talk by *S. Banerjee, P. Roig* and *M. Bruno*

3.1 Test of QCD and EW interactions



- Inclusive τ-decays : full hadron spectra, *perturbative tools: OPE…*

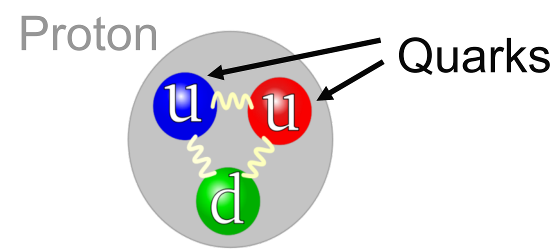
$\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fundamental SM parameters: $\alpha_s(m_\tau)$, $|V_{us}|$, m_s
QCD studies

- Exclusive τ-decays : specific hadron spectrum, *non perturbative tools*

$\boxed{\tau \to (PP, PPP, \ldots)\nu_\tau}$ ⇨ Study of ffs, resonance parameters (M_R, Γ_R)
Hadronization of QCD currents

3.2 Tools

- Hadronic Physics: Interactions of quarks at low energy



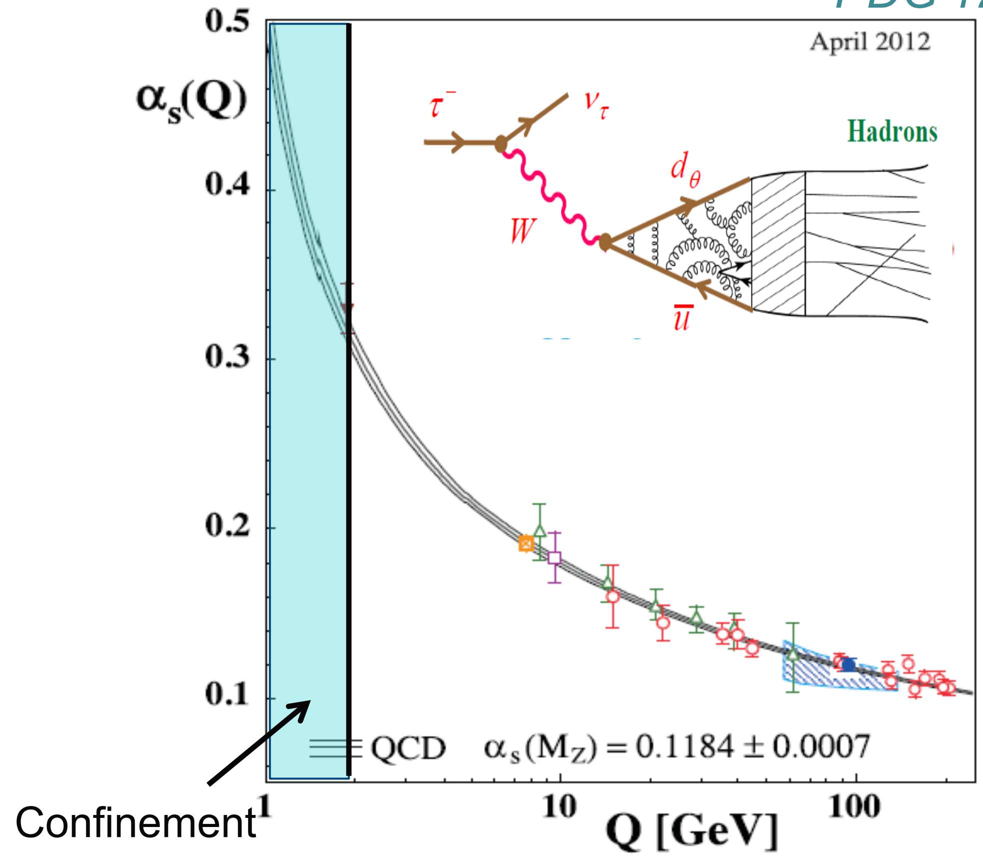
- Low energy (Q <~1 GeV), long distance: *α_S becomes large !*

  ⇨ *Non-perturbative QCD*

- A perturbative expansion in the usual sense fails

- Use of alternative approaches, expansions…: e.g.
  - Effective field theory
    Ex: ChPT for light quarks
  - Dispersion relations
  - Numerical simulations on the lattice

PDG'12



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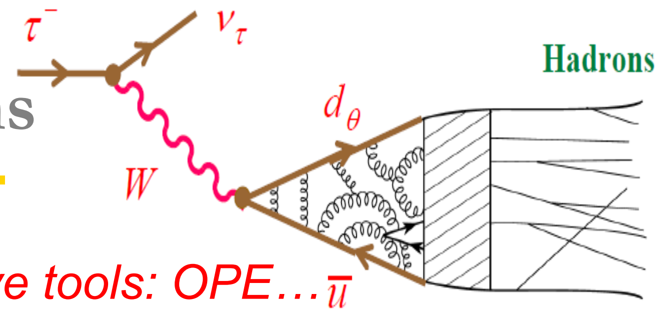

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Hadronization of QCD currents

- τ decays: tool to search for New Physics in inclusive and exclusive decays :

⇨ Unitarity test, CPV, LFV, EDMs, etc.

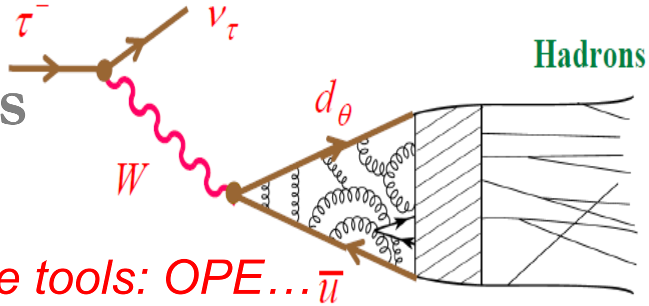
Test of unitarity $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1}$

$0^+ \to 0^+$
β decays

K_{l3} decays
or τ decays

Negligible
(B decays)

3.2 Test of QCD and EW interactions



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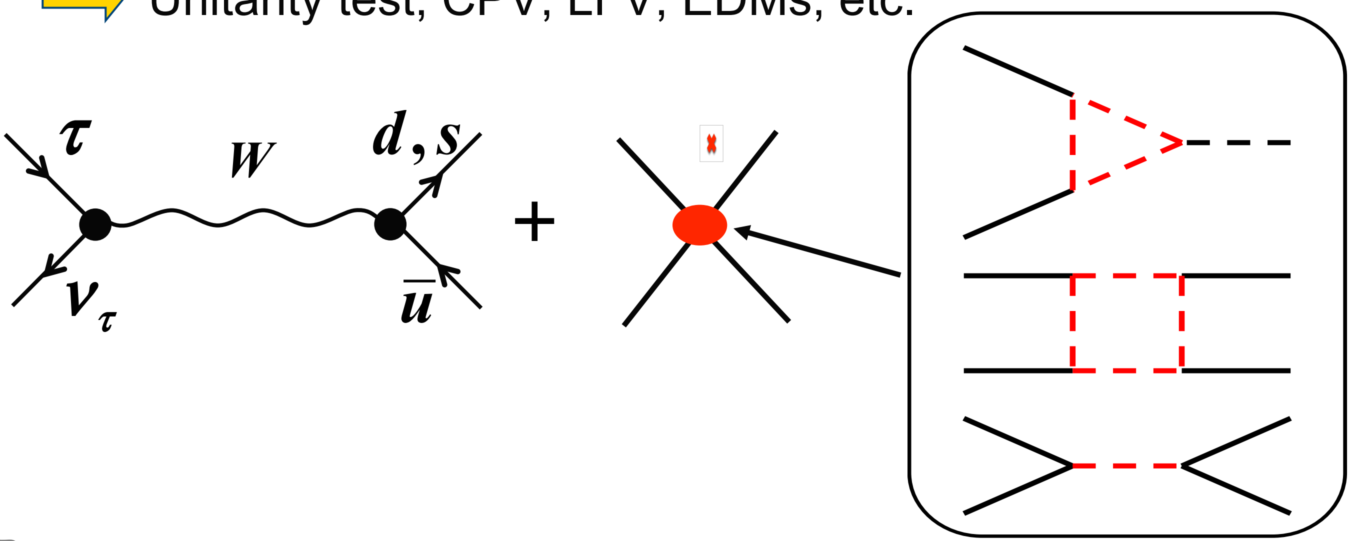


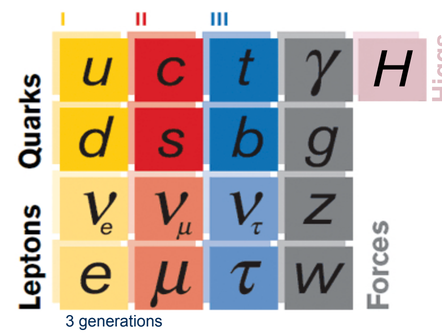
SUSY loops,
Leptoquarks,
Z', Charged Higgs,
Right-Handed
Currents,….

3.3 Test of the Standard Model: V_{us} and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model

    Description of the weak interactions:

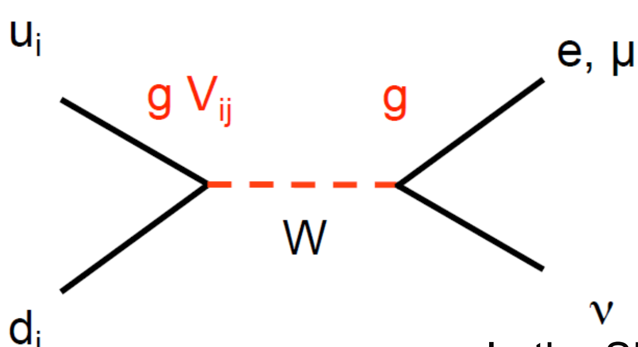
$$\boxed{\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L \underbrace{V_{CKM}}_{\text{Unitary matrix}} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}}$$



  - Check unitarity of the first row of the CKM matrix:

    Cabibbo Universality: $\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \stackrel{?}{=} 1}$

    Negligible ~2x10^{-5}
    (B decays)



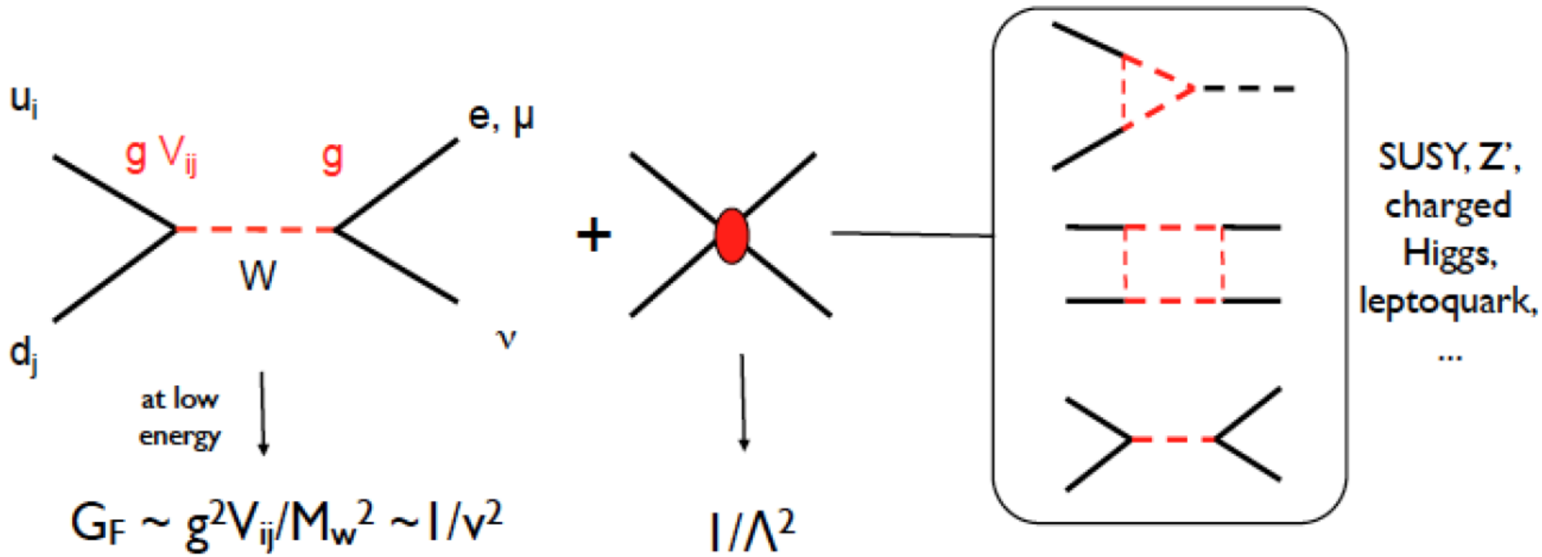
$$|V_{ud}| = \cos\theta_C \quad \text{and} \quad |V_{us}| = \sin\theta_C$$

In the SM: W exchange ⇨ V – A structure only

3.3 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

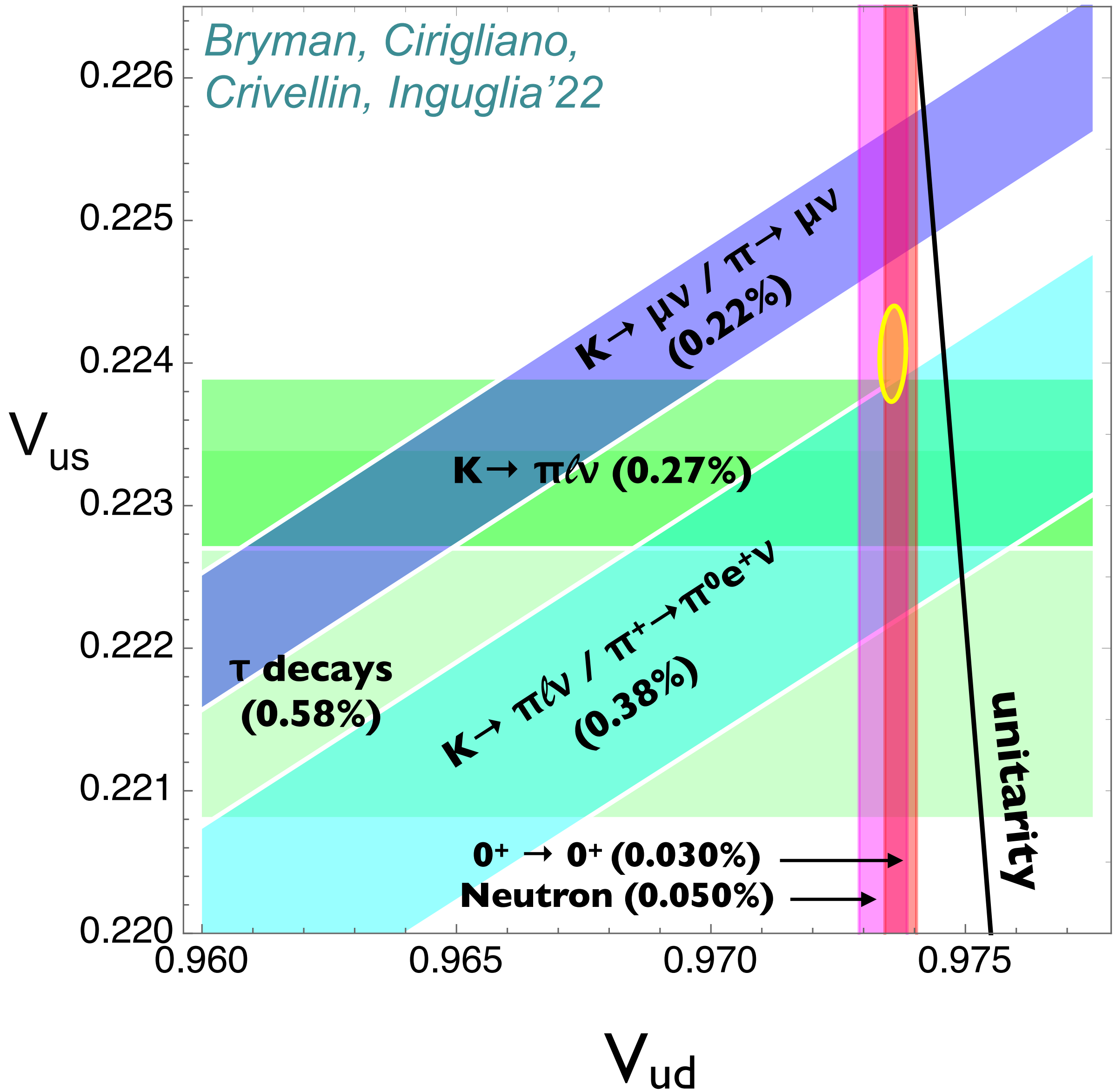
$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$



BSM effects : $\boxed{\Delta \sim \frac{c_n}{g^2}\frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \longleftrightarrow \Lambda \sim 1\text{-}10 \text{ TeV}}$

3.4 Cabibbo angle anomaly

Moulson & E.P.@CKM2021



$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{CKM} = -0.0018(6)$

-2.7σ

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 + \Delta_{CKM}$$

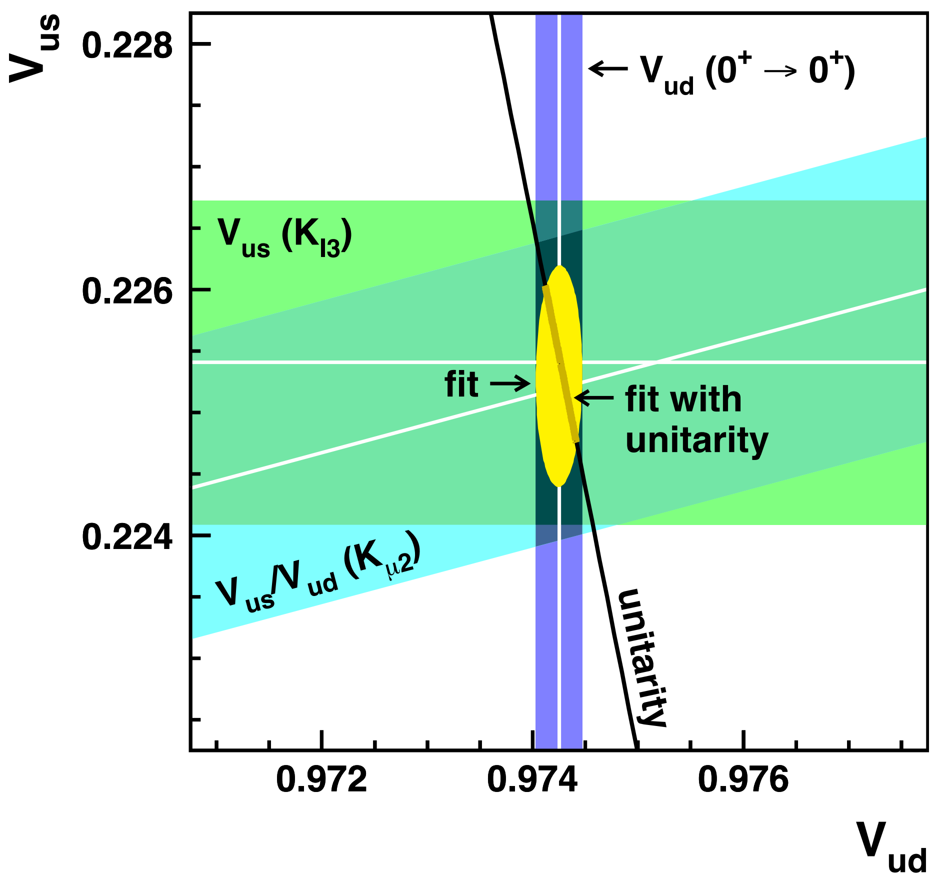
Negligible ~2×10^{-5} (B decays)

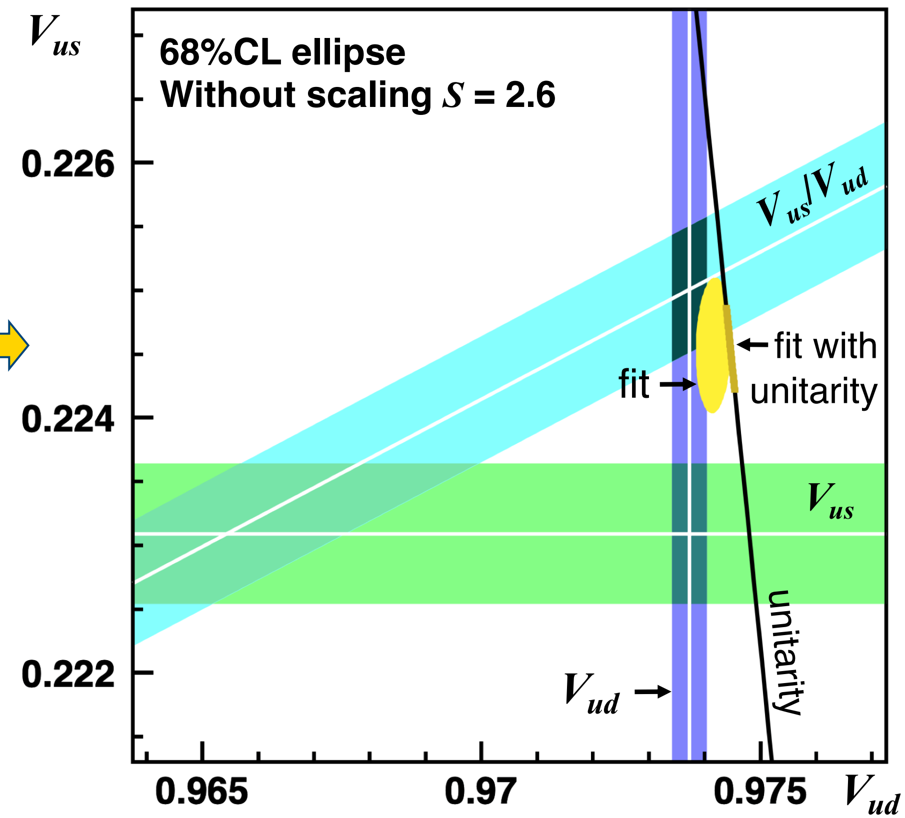
Why this anomaly?

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021





Changes on V_{us} and V_{ud} since 2011

- Almost no change on the experimental side since 2011

Flavianet Kaon WG: *Antonelli et al'11*

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$

Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

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Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from *lattice QCD* for V_{us} and V_{us}/V_{ud} extraction from Kaon decays

FLAG'21

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Flavianet Kaon WG: *Antonelli et al'11*

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Hadronic matrix element

Radiative corrections

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  - Impressive progress on hadronic matrix element computations from lattice QCD for V_{us} and V_{us}/V_{ud} extraction from Kaon decays
  - Radiative corrections from dispersive methods for V_{ud} extraction

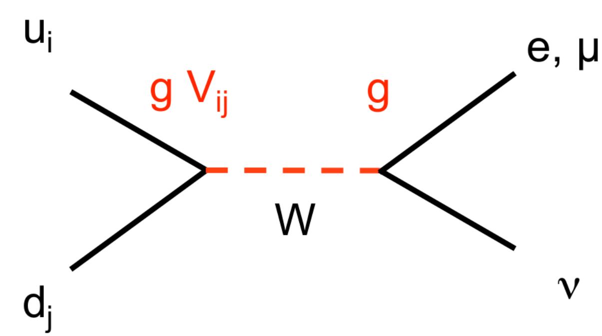
Seng et al.'18'19, Gorshteyn'18, Cirigliano et al.'22,'24

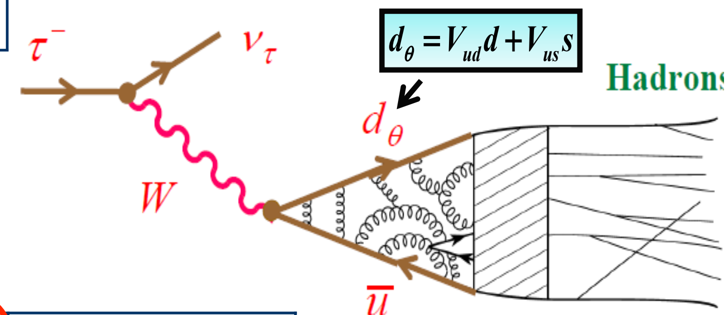
Can Tau physics help?

Path to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$





- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

V_{us} from $\tau \to K \nu_\tau$ / $\tau \to \pi \nu_\tau$ decays

- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

$$\boxed{\frac{\Gamma(\tau \to K\nu[\gamma])}{\Gamma(\tau \to \pi\nu[\gamma])} = \frac{\left(1-m_{K^\pm}^2/m_\tau^2\right)}{\left(1-m_{\pi^\pm}^2/m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \left(1+\delta_{\mathrm{LD}}\right)}$$

- Main input hadronic input: f_K/f_π as for Kaon physics

From Tau physics: V_{us}/V_{ud} = 0. 2289$(18)_{exp}(4)_{lat}$ *HFLAV'23* *-2.1σ* away from unitarity

to be compared to V_{us}/V_{ud} = 0. 2311$(3)_{exp}(4)_{lat}$ ⇨ Need important exp. improvement !

Inclusive determination of V_{us}

- From τ decays (crossed channel)

V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

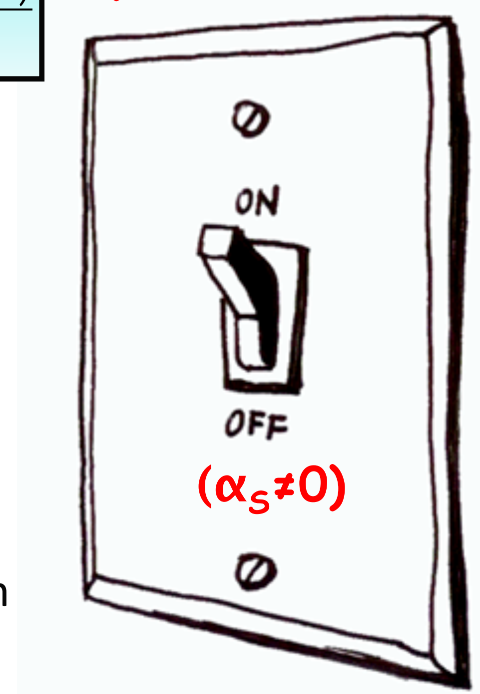
–

Inclusive determination of V_{us}

- With QCD on: $$\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$$

QCD switch



- Use OPE: $$R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1 + \delta_P + \delta_{NP}^{ud}\right)$$

$$R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1 + \delta_P + \delta_{NP}^{us}\right)$$

- $$\delta R_\tau \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0238(33)$$ *Gamiz et al'07, Maltman'11*

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

HFLAV'23

$R_{\tau,S} = 0.1615(28)$

$R_{\tau,NS} = 3.4650(84)$

$|V_{ud}| = 0.97373(32)$

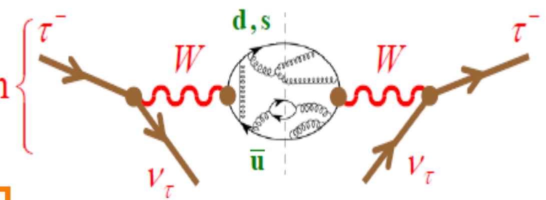
⇨ $$|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0010_{\text{th}}$$

-3.7σ away from unitarity!

A. Lusiani@Tau'23

Calculation of the QCD corrections

- Calculation of R_τ:

$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}$ 

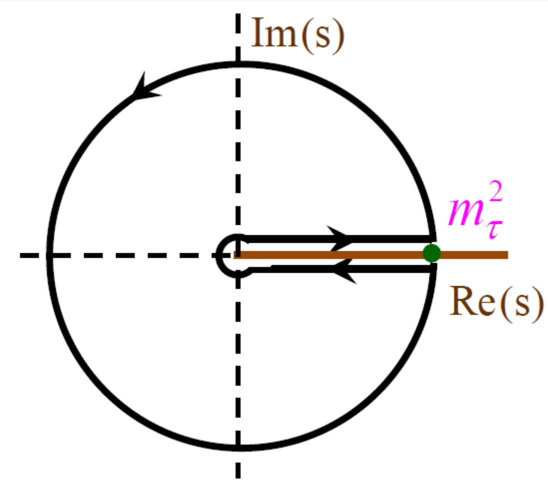
$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇒ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O = D} C^{(J)}(s,\mu)\, \langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

Operator Product Expansion

$$\Pi^{(J)}(s) = \sum_{D=0,2,4...} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathrm{C}^{(J)}(s,\mu) \left\langle O_D(\mu) \right\rangle$$

Wilson coefficients

Operators

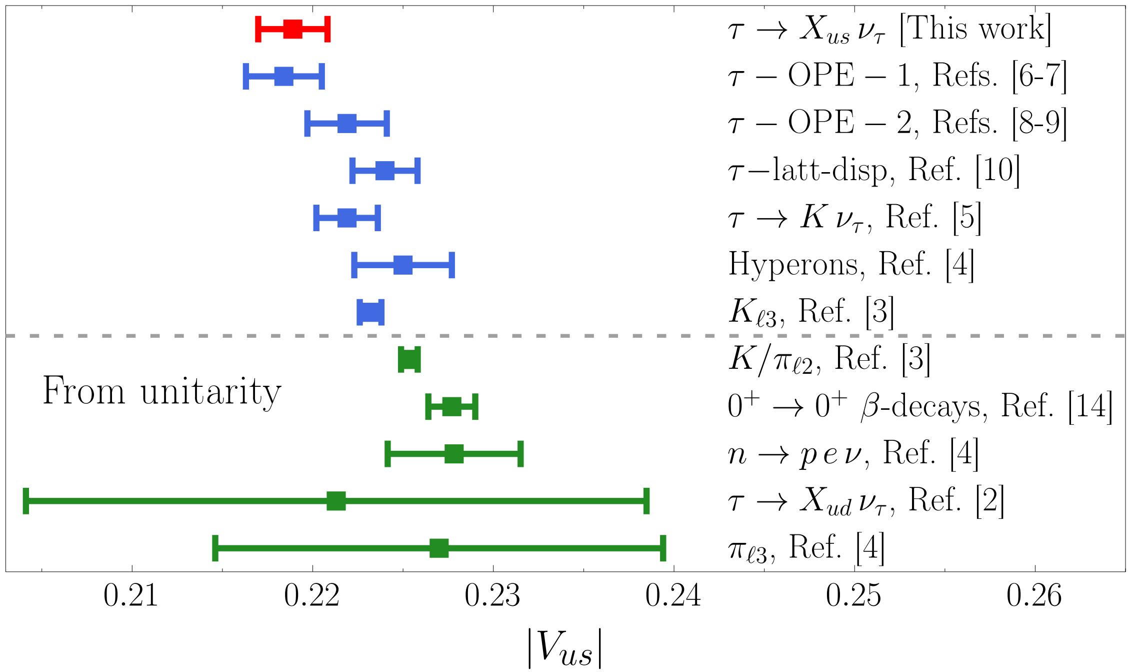
μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, supposed to be small…

$\Rightarrow$ $\boxed{R_{\tau,V}(s_0) = \frac{3}{2} \left| V^{ud} \right|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V} \right)}$ similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

Inclusive determination of V_{us}

- See recent lattice work by *ETMC'24*



$$\boxed{|V_{us}|_{\tau-\mathrm{latt-incl}} = 0.2189(7)_{\mathrm{th}}(18)_{\mathrm{exp}}}$$

3.4 V_{us}: summary

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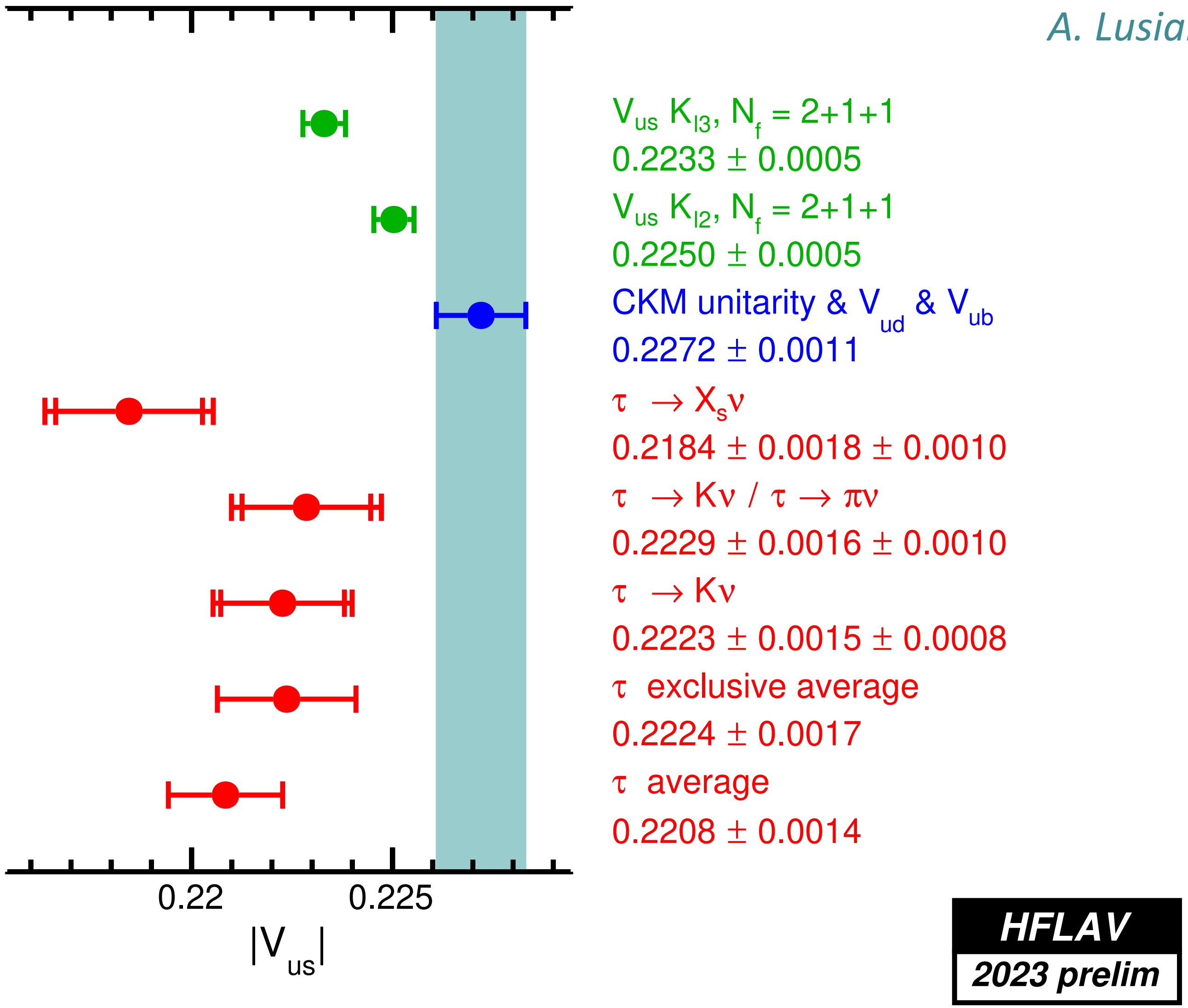
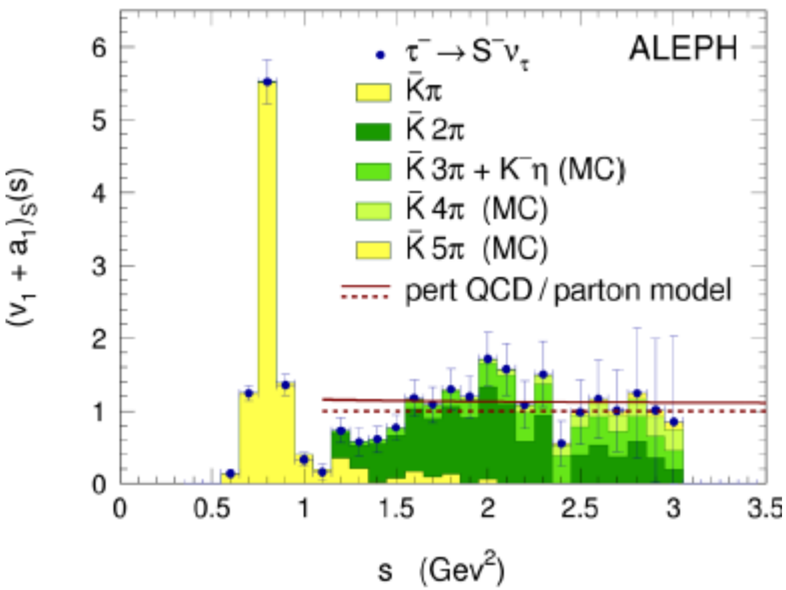
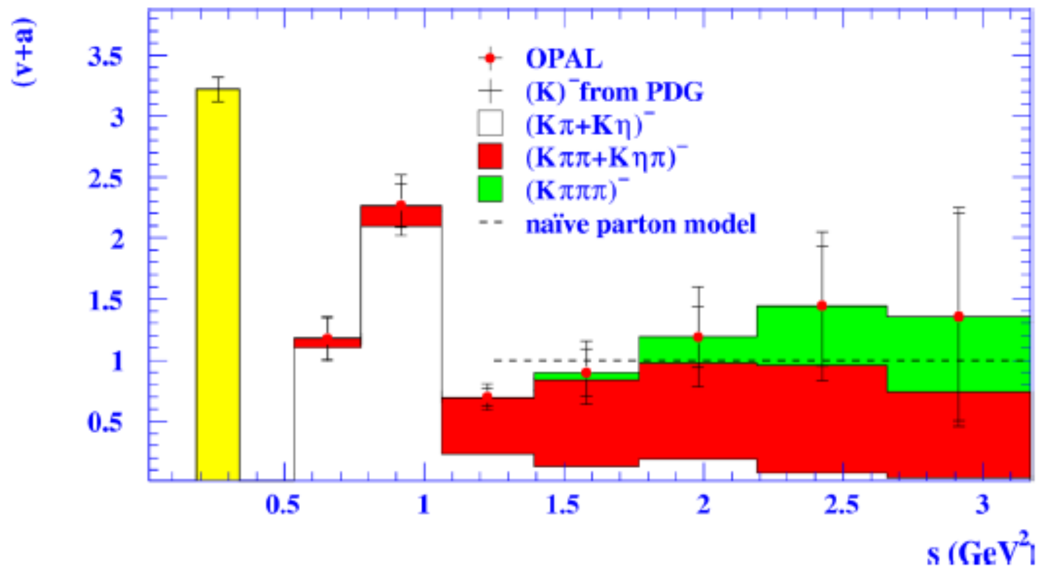


3.5 Prospects : τ strange Spectral functions

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

Smaller τ → K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

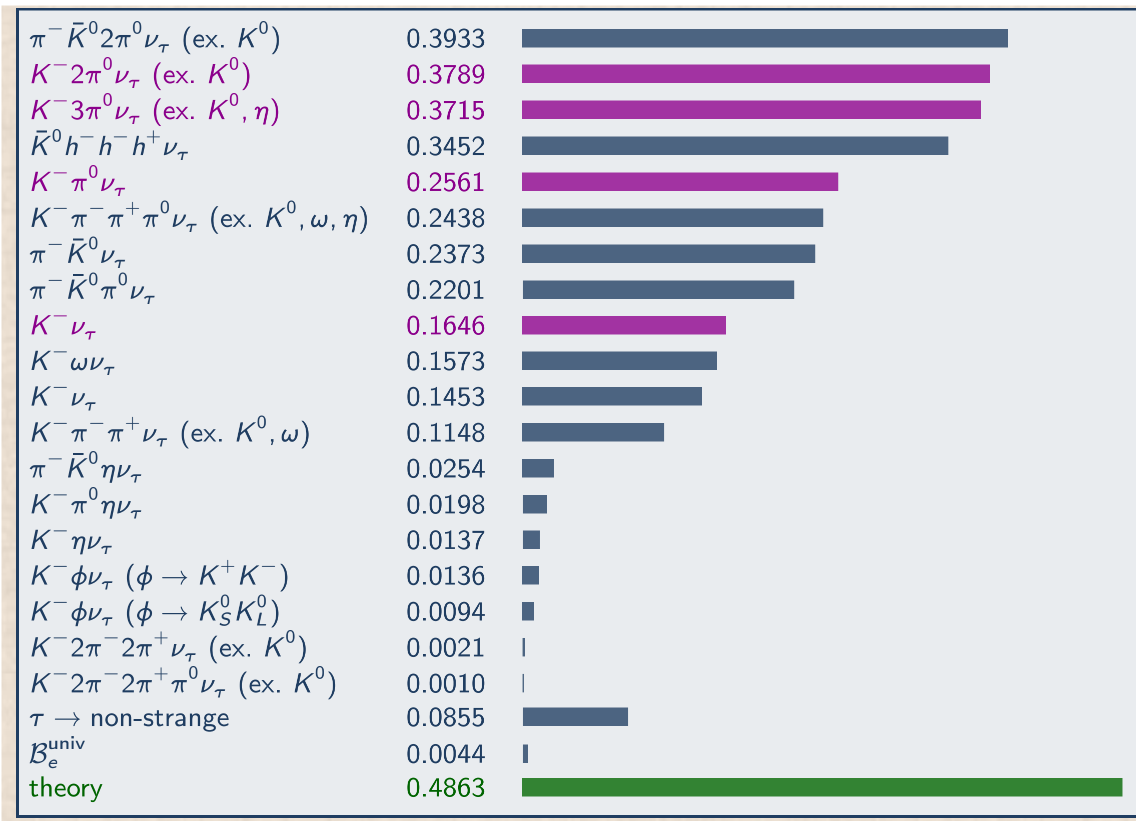
$$\boxed{R_\tau^S\Big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_\tau^S\Big|_{\text{new}} = 0.1615(28)}$$

$$\boxed{\left|V_{us}\right|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{\left|V_{us}\right|_{\text{new}} = 0.2176 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

3.5 Prospects : τ strange BRs

- Very interesting quantity to extract V_{us}: QCD part completely independent from form factors or decay constants ⇨ Use OPE

- Experimentally very challending since all Brs need to be measured



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⇨ *Belle II @ 50 ab^{-1}* + Creativity from young physicists

3.5 Prospects V_{us} from $\tau \to K\pi\nu_\tau$

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

$$\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$$

Hadronic matrix element: Crossed channel from $K \to \pi l \nu_l$

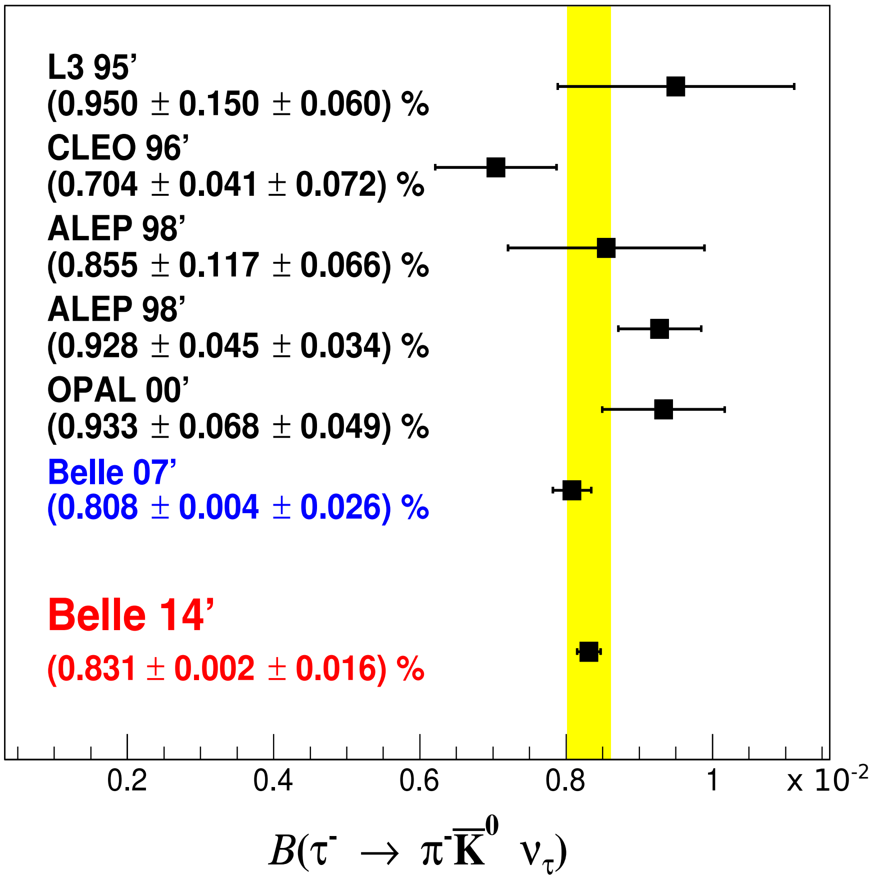
$$\boxed{\left\langle \mathrm{K}\pi \right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left| 0 \right\rangle = \left[\left(p_K - p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu \right] \underbrace{f_+(s)}_{\text{vector}} + \frac{\Delta_{K\pi}}{s}\left(p_K + p_\pi\right)_\mu \underbrace{f_0(s)}_{\text{scalar}}}$$

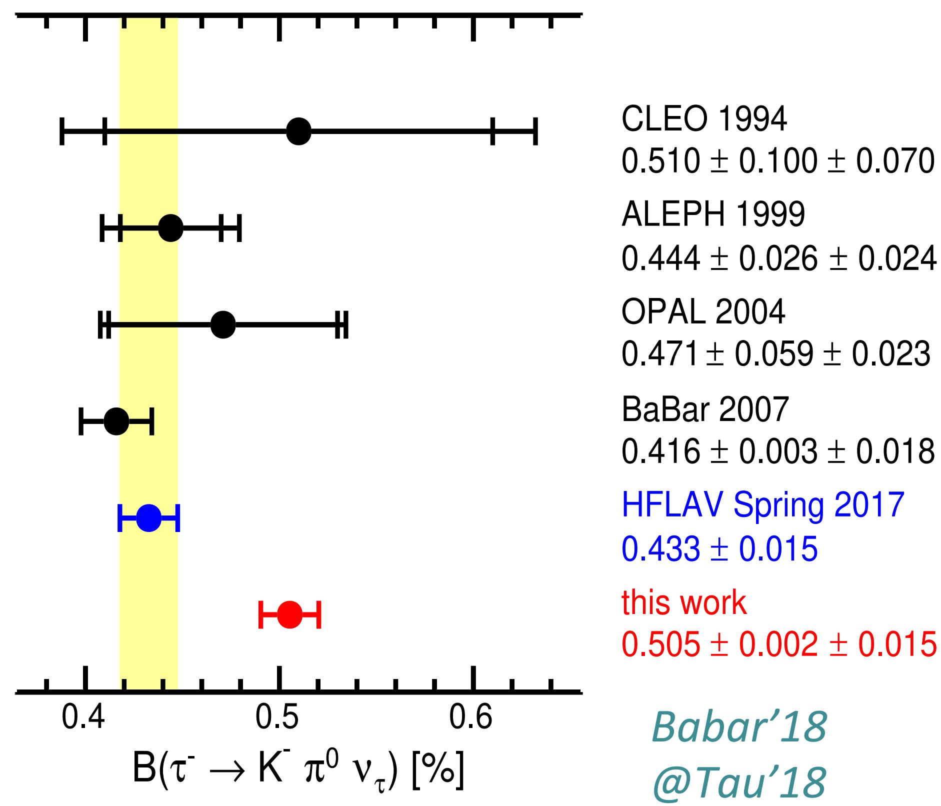
with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(s) = \dfrac{f_{0,+}(s)}{f_+(0)}$

⇨ Use a *parametrization* to fit the form factors

Experimental Situation

- $\tau \rightarrow K\pi\nu_\tau$: Brs measured by Belle and BaBar as well as spectrum but only Belle one publicly available





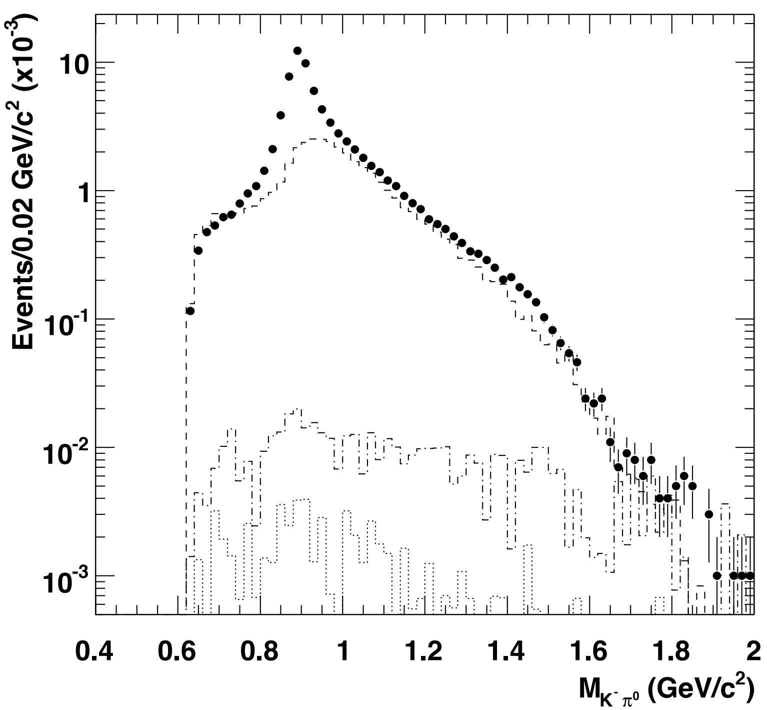
Babar'18 @Tau'18

Experimental Situation

- $\tau \rightarrow K\pi\nu_\tau$: Brs measured by Belle and BaBar as well as spectrum but only Belle one publicly available

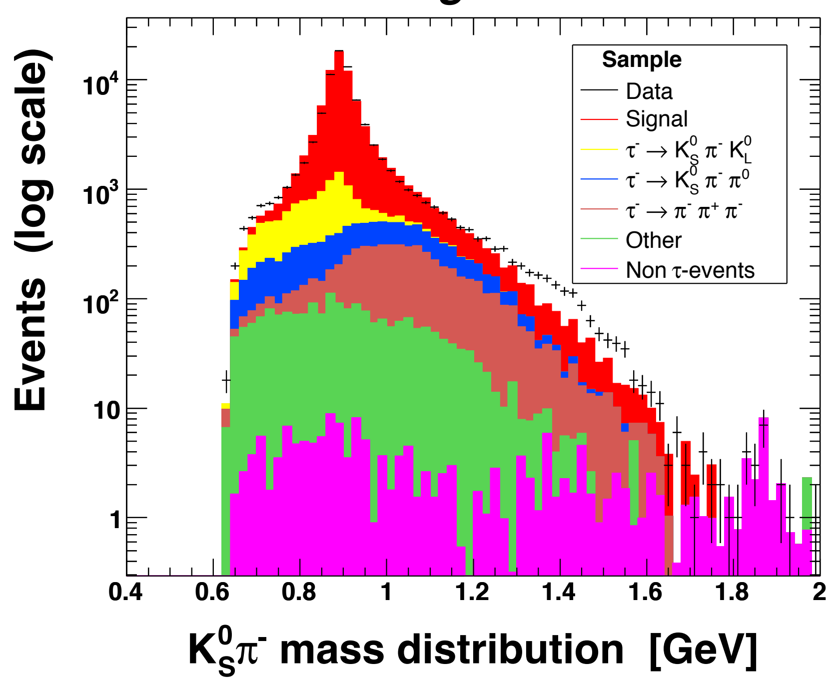
Babar'07

$\tau^- \rightarrow K^-\pi^0\nu_\tau$



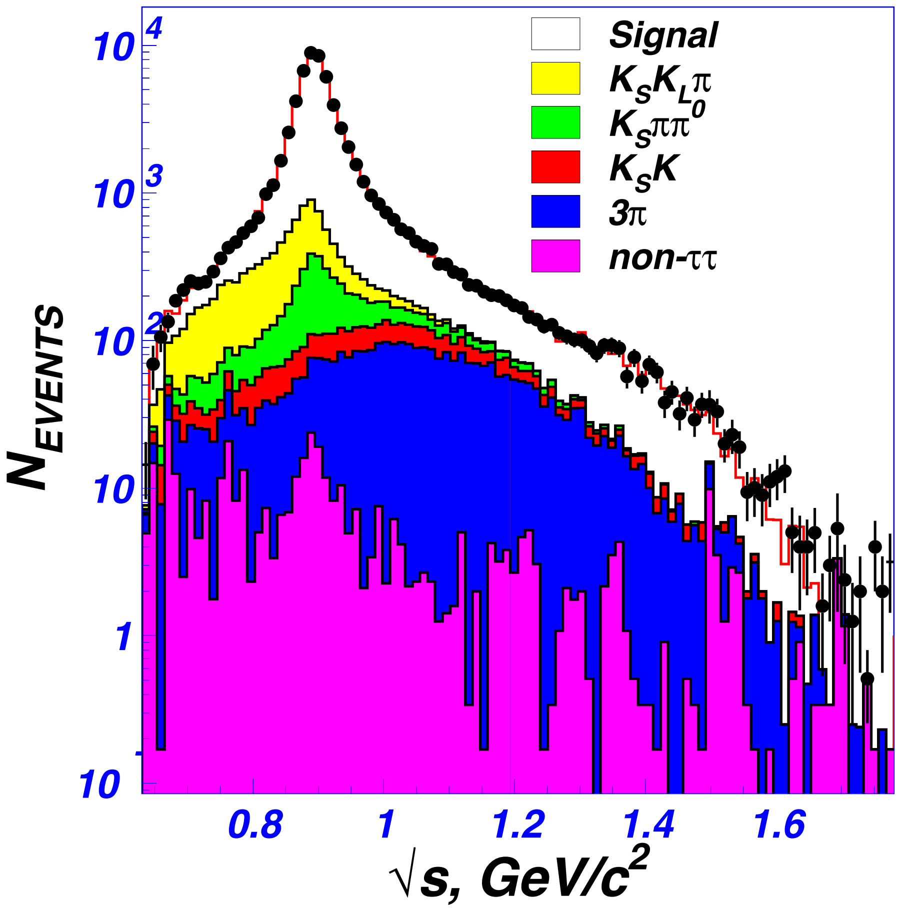
Babar'07

$\tau^- \rightarrow K^0_S\pi^-\nu_\tau$



Experimental Situation

- $\tau \rightarrow K\pi\nu_\tau$: Brs measured by Belle and BaBar as well as spectrum but only Belle one publicly available

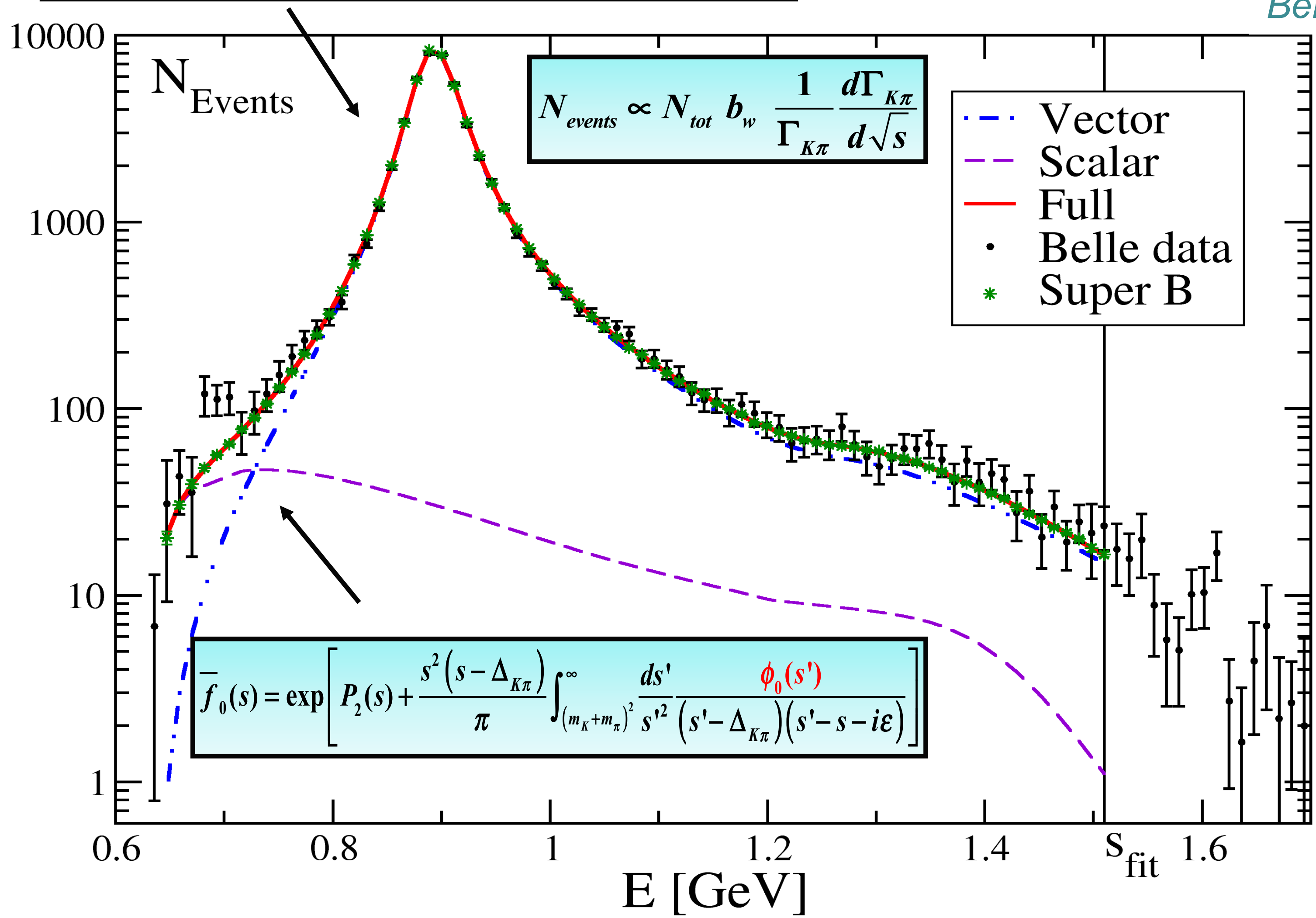


Belle'07

Fit to the $\tau \rightarrow K\pi\nu_\tau$ decay data + K_{l3} constraints

$$\bar{f}_+(s) = \exp\left[P'_2(s) \; + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^3} \frac{\phi_+(s')}{(s'-s-i\varepsilon)} \right]$$

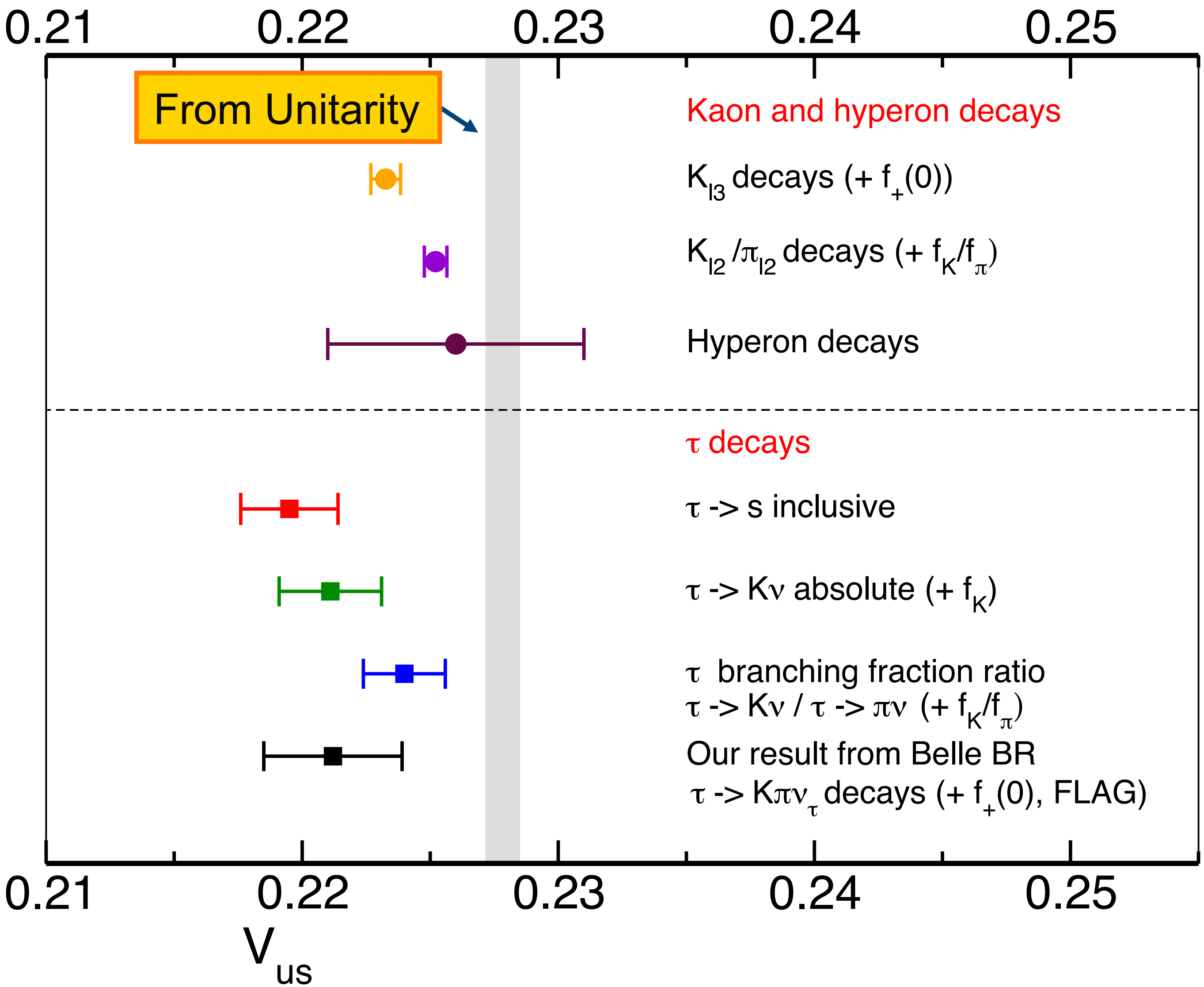
Bernard, Boito, E.P.'11
Antonelli, Cirigliano, Lusiani, E.P.'13
Bernard'14



$$\bar{f}_0(s) = \exp\left[P_2(s) + \frac{s^2\left(s-\Delta_{K\pi}\right)}{\pi}\int_{(m_K+m_\pi)^2}^{\infty} \frac{ds'}{s'^2} \frac{\phi_0(s')}{\left(s'-\Delta_{K\pi}\right)\left(s'-s-i\varepsilon\right)} \right]$$

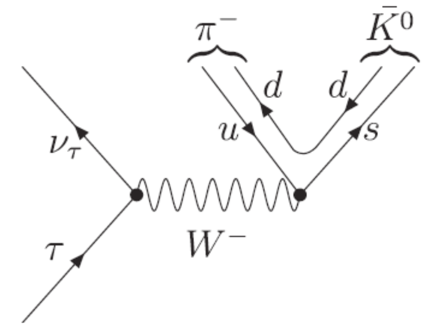
V_{us} from Tau decays

$\left|V_{ud}\right| = 0.97373(31)$



CPV in tau decays

$\tau \to K\pi\nu_\tau$ CP violating asymmetry



- $$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

  $= |p|^2 - |q|^2$ $\boxed{\approx (0.36 \pm 0.01)\%}$ in the SM

  Bigi & Sanda'05

  Grossman & Nir'11

$$|K_S^0\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$

$$|K_L^0\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

$$\boxed{\langle K_L | K_S\rangle = |p|^2 - |q|^2 \simeq 2\,\mathrm{Re}\left(\varepsilon_K\right)}$$

- Experimental measurement : *BaBar'11*

  $\boxed{A_{Q\,\exp} = \left(-0.36 \pm 0.23_{\text{stat}} \pm 0.11_{\text{syst}}\right)\%}$ $\Rightarrow$ $\boxed{2.8\sigma}$ from the SM!

- CP violation in the tau decays should be of opposite sign compared to the one in D decays in the SM

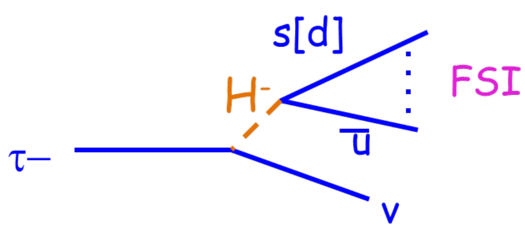
  Grossman & Nir'11

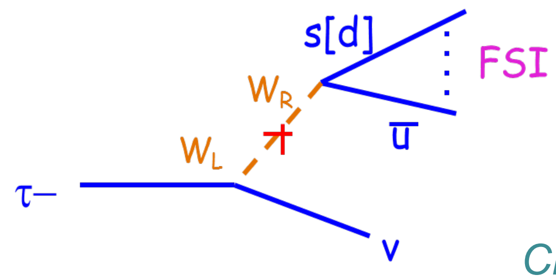
$$\boxed{A_D = \frac{\Gamma\left(D^+ \to \pi^+ K_S^0\right) - \Gamma\left(D^- \to \pi^- K_S^0\right)}{\Gamma\left(D^+ \to \pi^+ K_S^0\right) + \Gamma\left(D^- \to \pi^- K_S^0\right)}} = \left(-0.54 \pm 0.14\right)\%$$

Belle, Babar, CLOE, FOCUS

$\tau \to K\pi\nu_\tau$ CP violating asymmetry

- New physics? Charged Higgs, W_L-W_R mixings, leptoquarks, tensor interactions (*Devi, Dhargyal, Sinha'14*)?





Bigi'Tau12

Very difficult to explain! Incompatible with other flavor data using EFT

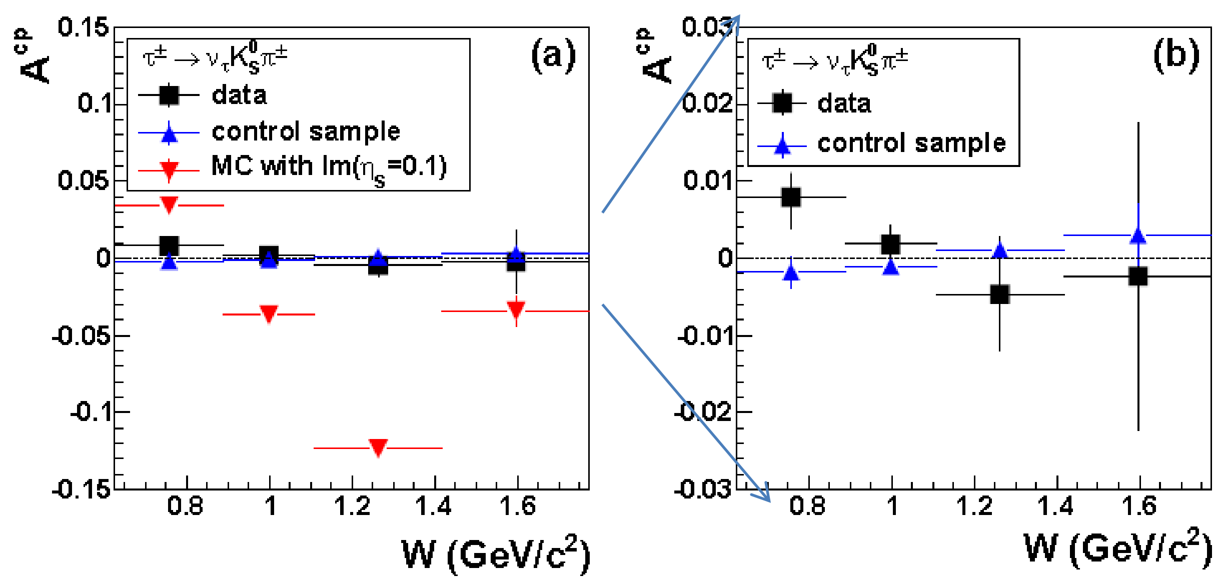
Cirigliano et al'18, Rendón et al'19

- Problem with this measurement? ⇨ It would be great to have other experimental measurements from *Belle II*

Belle'11

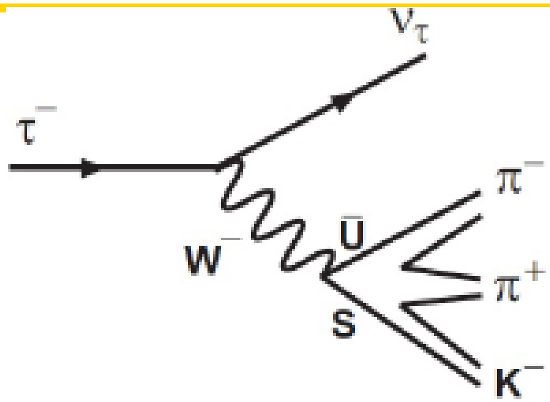
- Measurement of the direct contribution of NP in the angular CP violating asymmetry done by *CLEO* and *Belle*

  ⇨ Belle does not see any asymmetry at the *0.2 - 0.3% level*



Three body CP asymmetries

- Ex: $\tau \to K\pi\pi\nu_\tau$



- A variety of CPV observables can be studied :
$\tau \to K\pi\pi\nu_\tau, \tau \to \pi\pi\pi\nu_\tau$ rate, angular asymmetries, triple products,….

e.g., Choi, Hagiwara and Tanabashi'98
Kiers, Little, Datta, London et al.,'08
Mileo, Kiers and, Szynkman'14

Same principle as in charm, *see Bevan'15*

Difficulty : Treatement of the hadronic part
Hadronic final state interactions have to be taken into account!
⇒ Disentangle weak and strong phases

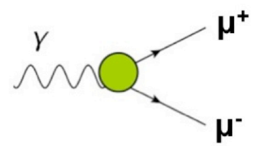
- More form factors, more asymmetries to build but same principles as for 2 bodies

4. Role of Tau Physics in anomalous magnetic moment of the muon

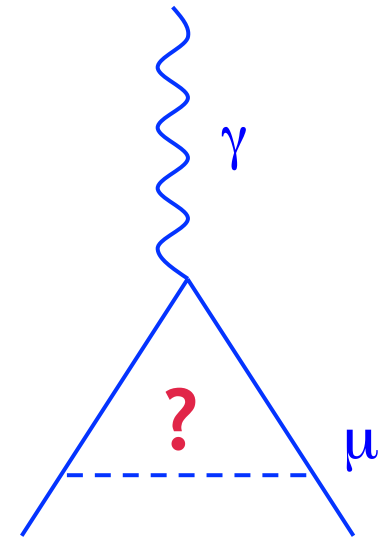
4.1 Anomalous magnetic moment of the muon

$$a_\mu = \frac{(g-2)_\mu}{2}$$

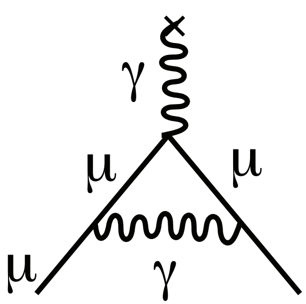
Anomalous magnetic moment

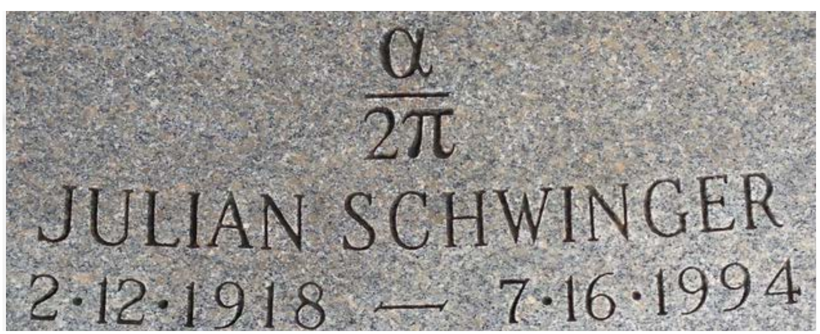


- The gyromagnetic factor of the muon is modified by loop contribution
- Predicted by Dirac to be 2
- Schwinger computed the first order correction



QED

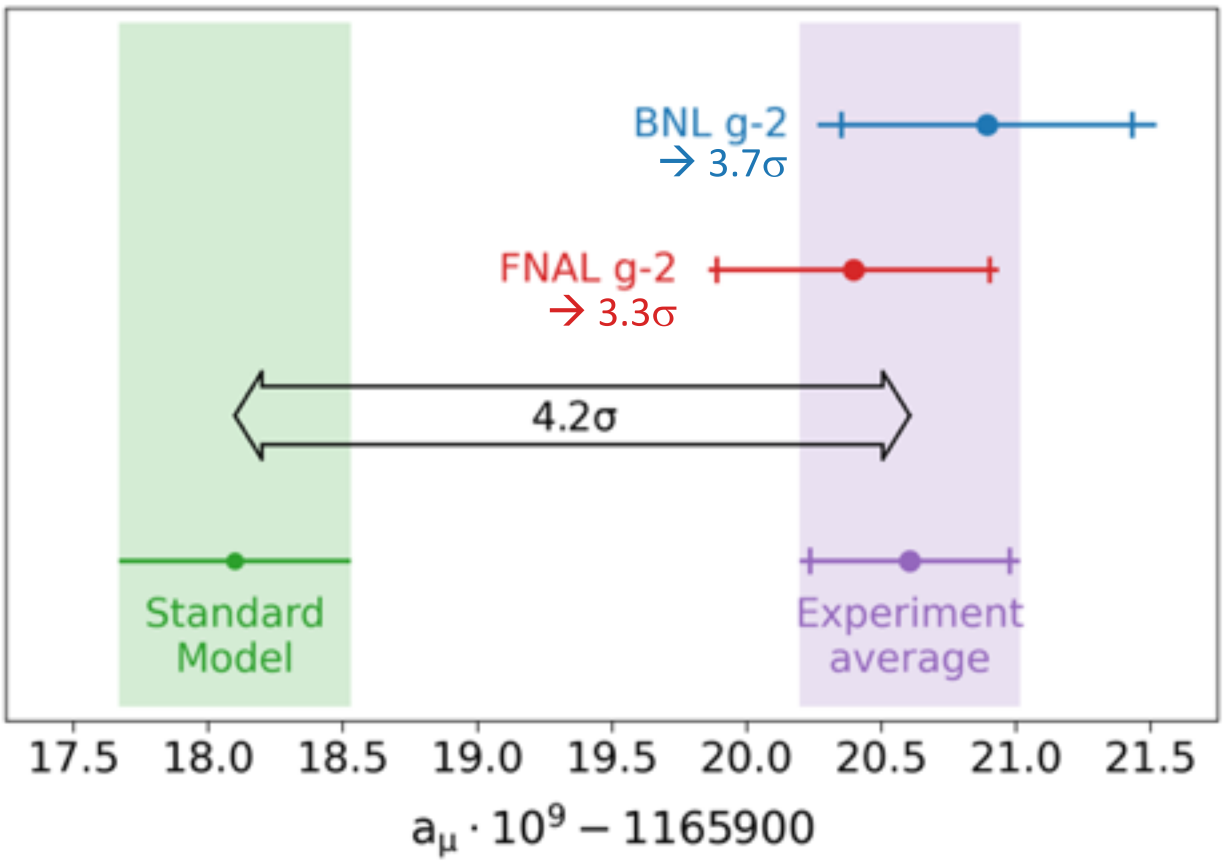




4.1 Anomalous magnetic moment of the muon

FNAL g-2
Chris Polly'21

- In 2021 $a_\mu(SM) = 0.00116591810(43) \rightarrow 368$ ppb



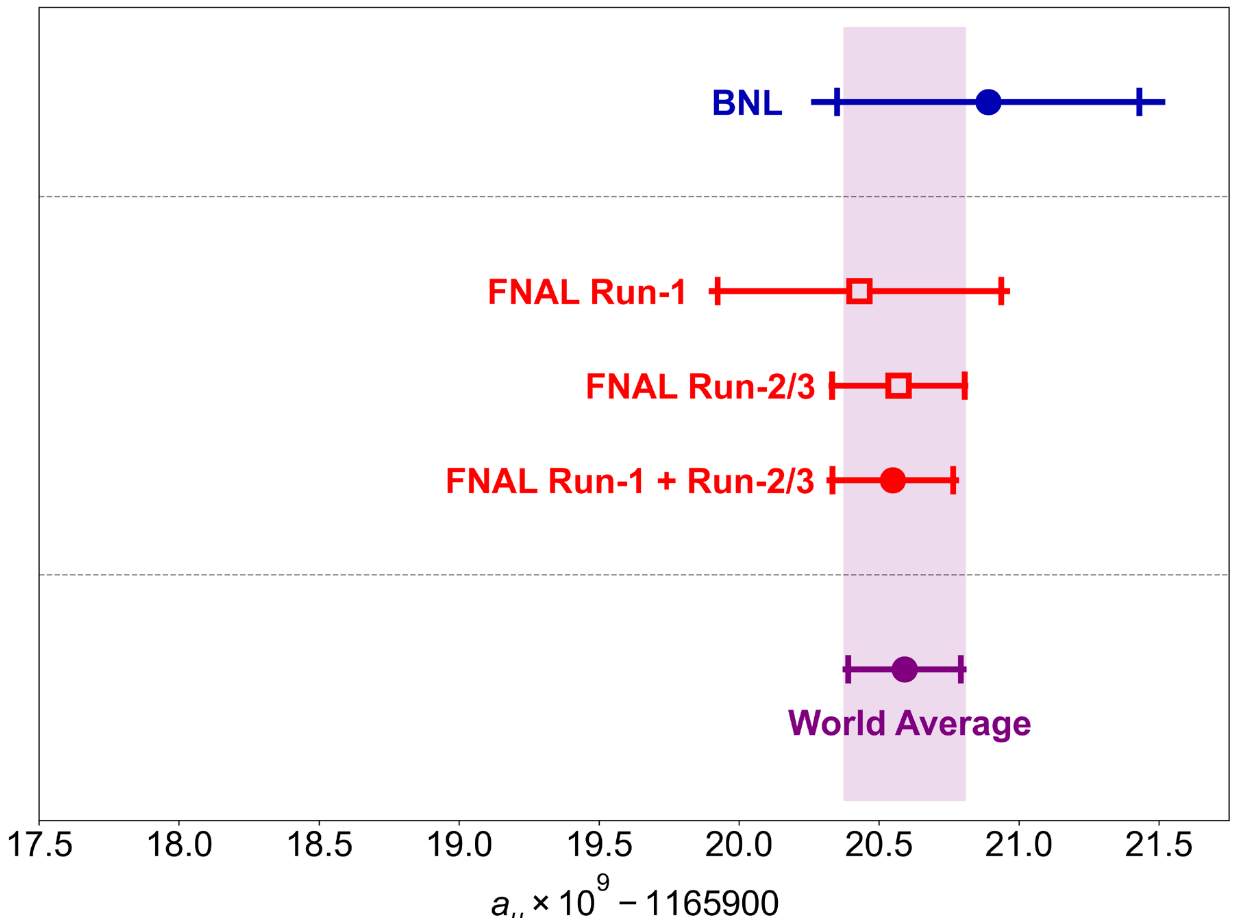
- Individual tension with SM
  - BNL: 3.7σ
  - FNAL: 3.3σ

$$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59) \rightarrow 4.2\sigma$$

4.1 Anomalous magnetic moment of the muon

FNAL g-2
James Mott'23

- In 2023 **a_μ(FNAL) = 0.00 116 592 055(24) [203 ppb]**

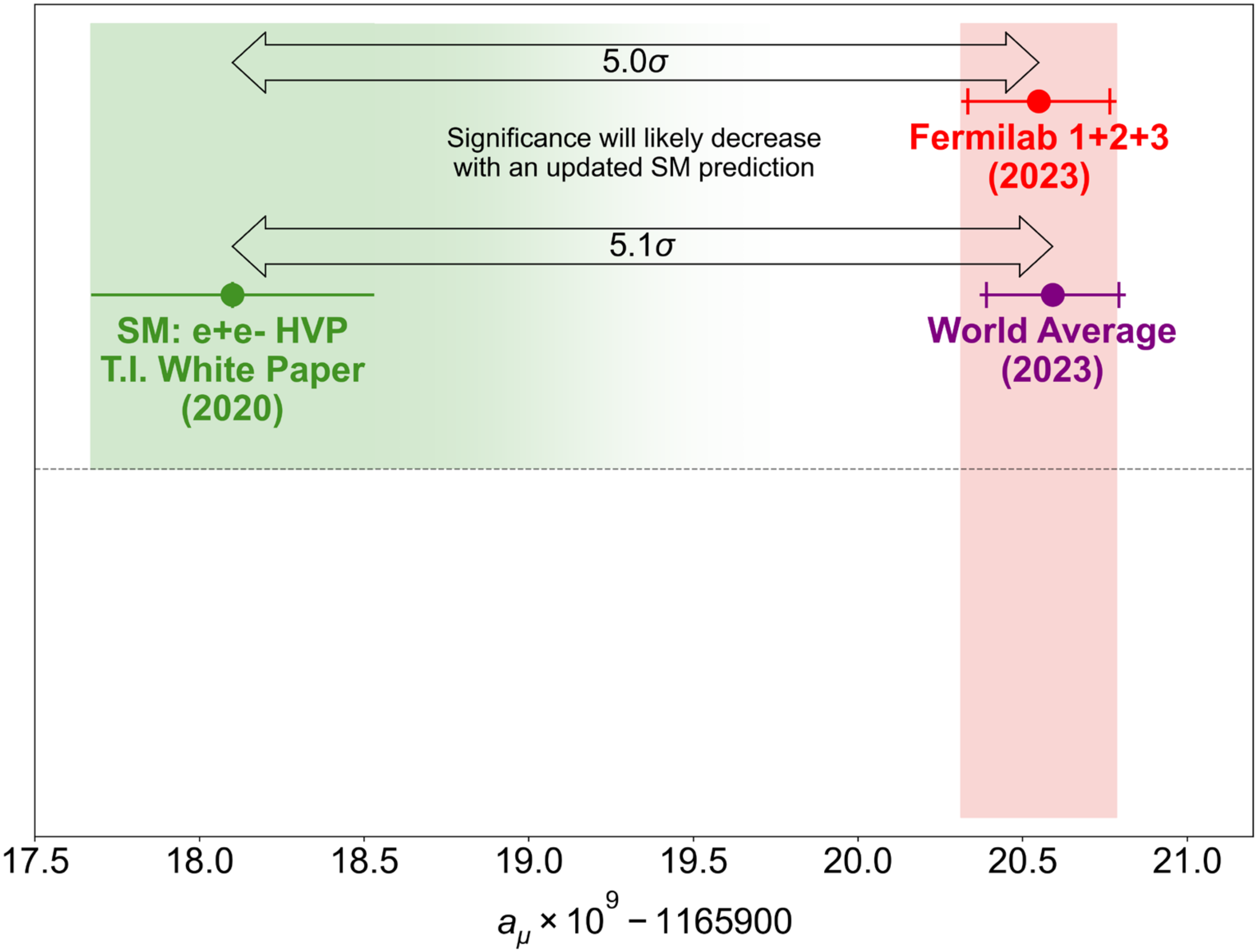


a_μ(Exp) = 0.00 116 592 059(22) [190 ppb]

4.1 Anomalous magnetic moment of the muon

FNAL g-2
James Mott'23

- In 2023



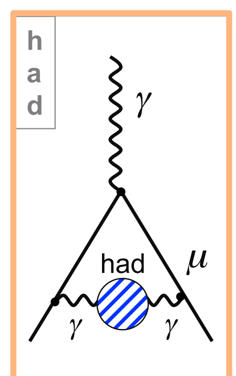
4.2 Confronting measurement and prediction

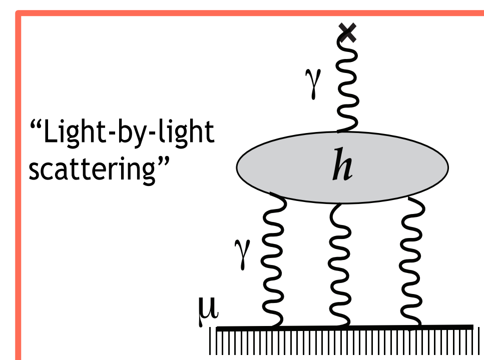
- Theoretical Prediction:

Colangelo et al.
Snowmass 2022

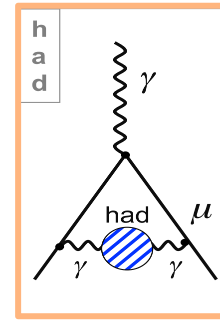
Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	693.1 ± 4.0
HVP(higher order) (*)	-8.59 ± 0.07
HLBL	9.2 ± 1.8
EW	15.4 ± 0.1
Total	11659181.0 ± 4.3

- Important contribution comes from virtual hadrons in the loop!
- Tackled using :
  - Models
  - Dispersion Relations
  - Lattice QCD



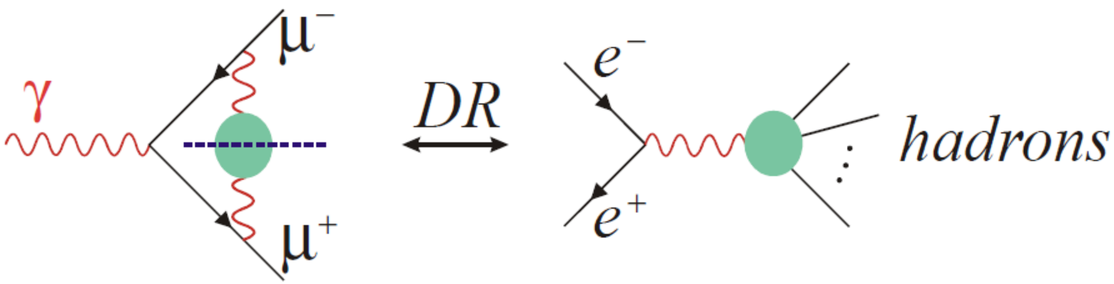


Model independent determination of HVP



- Hadronic contribution cannot be computed from first principles due to low-energy hadronic effects

- Use analyticity + unitarity ⇒ real part of photon polarisation function from *dispersion relation* over *total hadronic cross section data*



$$R_V(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

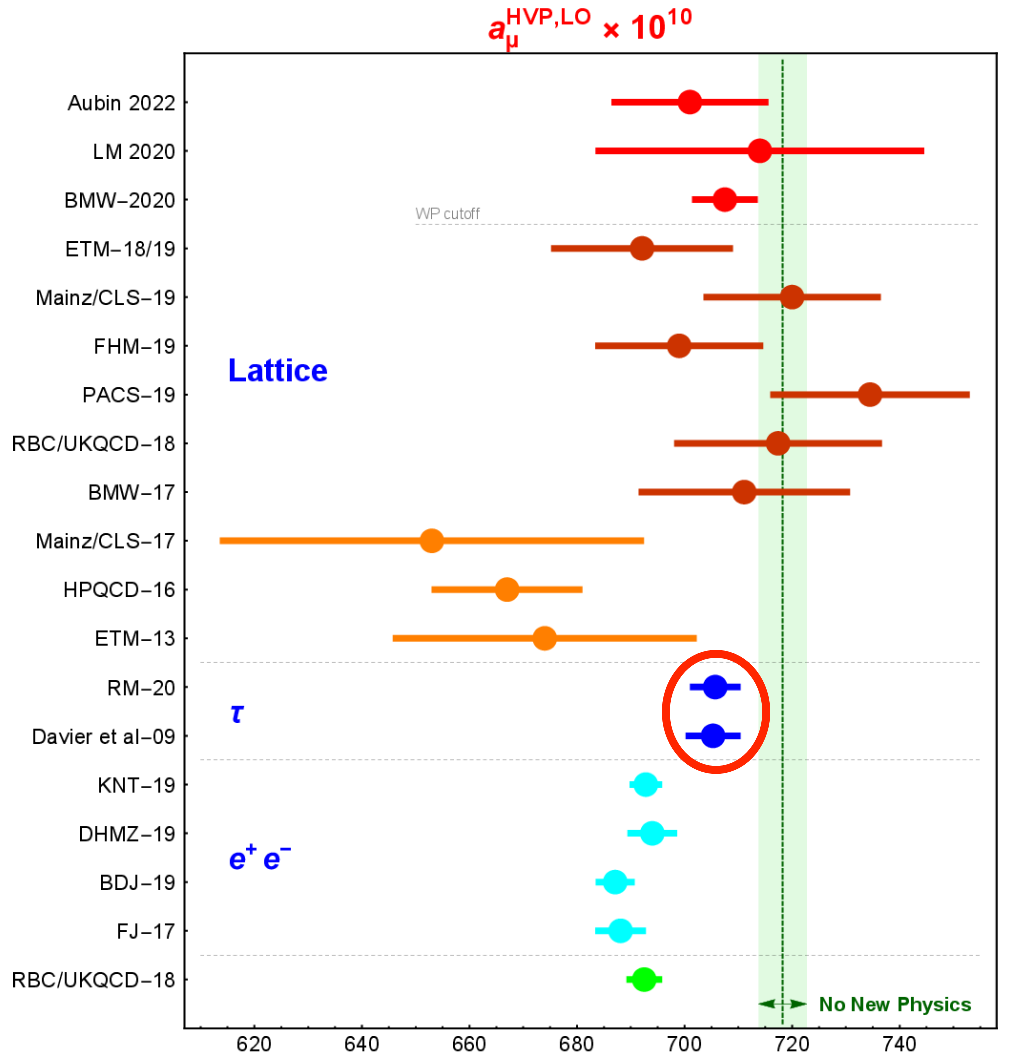
- Leading order hadronic vacuum polarization :

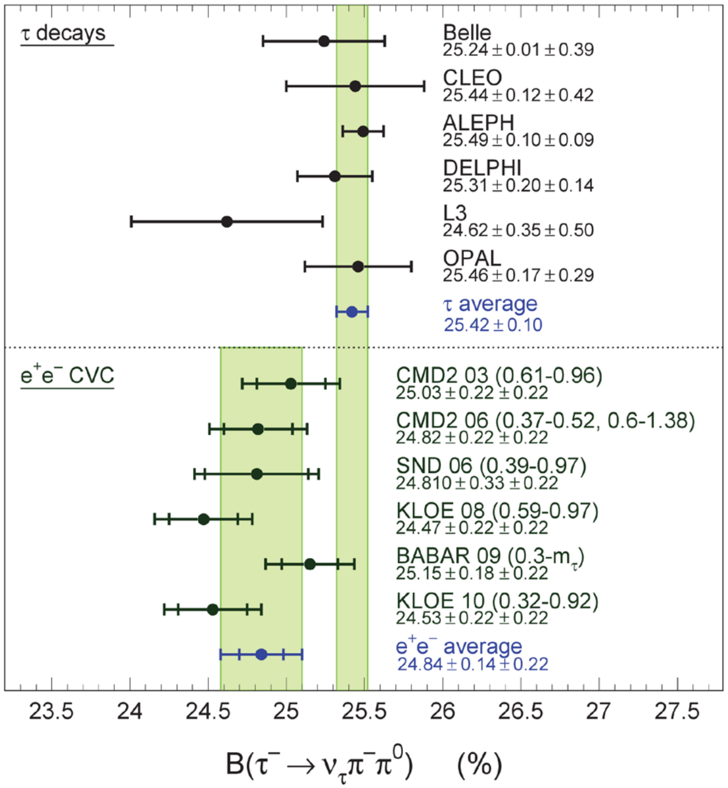
$$a_\mu^{had,LO} = \frac{\alpha^2 m_\mu^2}{(3\pi)^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s^2} R_V(s)$$

- Low energy contribution dominates : ~75% comes from s < (1 GeV)^2
  ⇒ ππ contribution extracted from data

4.3 Can τ help?

From T. Pich



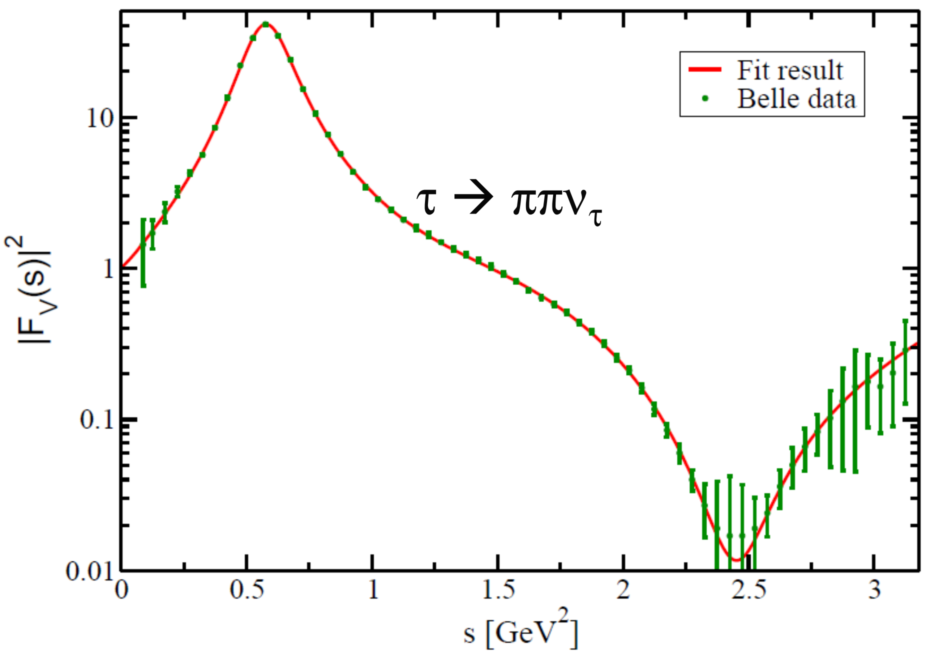


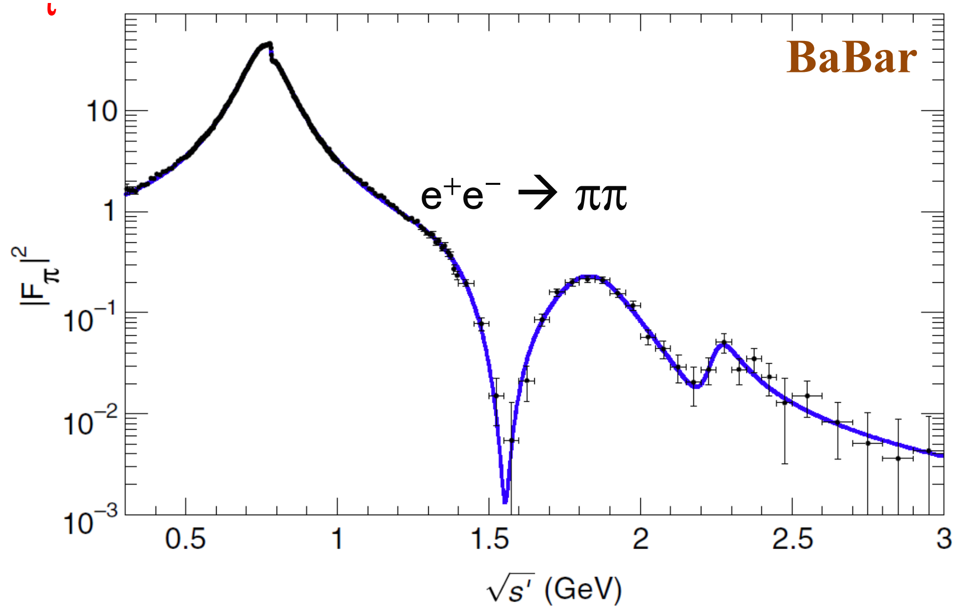
Need to estimate IB corrections to go from $\pi^-\pi^0$ to $\pi^-\pi^+$

Cirgliano et al.'01,'02
Miranda & Roig'20,

See talk by M. Bruno

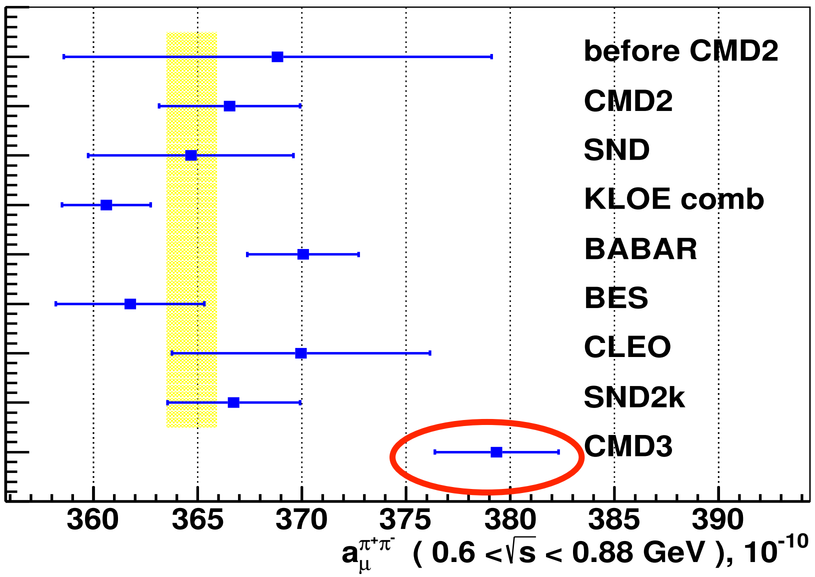
ππ form factor



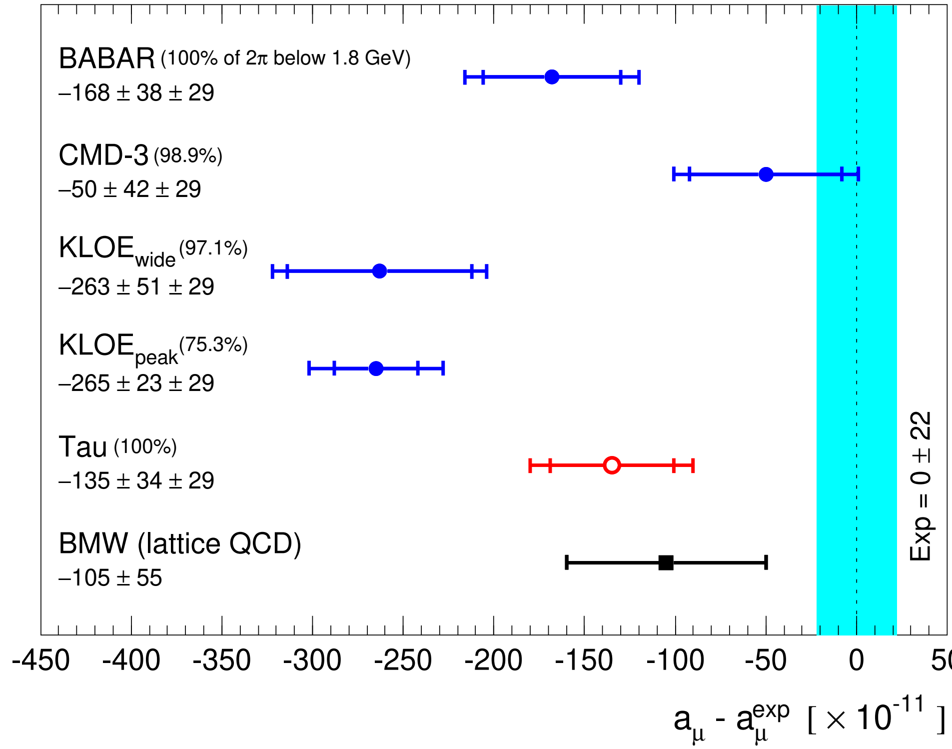


4.3 Can τ help?

CMD-3



Davier et al.'24



In view of the differences in $e^+e^- \rightarrow \pi^-\pi^+$ measurements

- $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ measurements are very useful!
- One can also look at Tau g-2, see talk by *M. Hoferichter*

5. Lepton Flavour Violation

See talks by *L. Calibbi, J. Zupan,*
M. Ardu, O. Sumensari

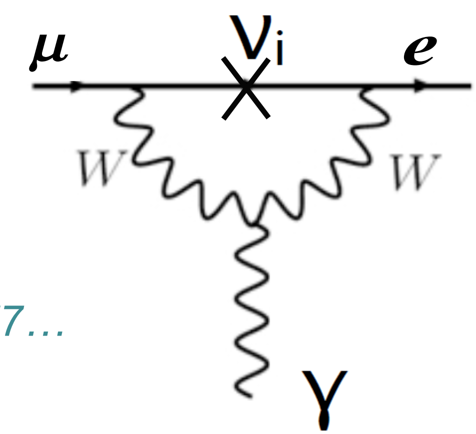
5.1 Introduction and Motivation

- Lepton Flavour Violation is an « accidental » symmetry of the SM (m_ν=0)

- In the *SM* with massive neutrinos effective CLFV vertices are tiny due to GIM suppression ⇨ *unobservably small rates!*

  E.g.: $\boxed{\mu \to e\gamma}$

$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M^2_W}\right|^2 < 10^{-54}}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77…

$$\left[Br(\tau \to \mu\gamma) < 10^{-40}\right]$$

- Extremely *clean probe of beyond SM physics*

4.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

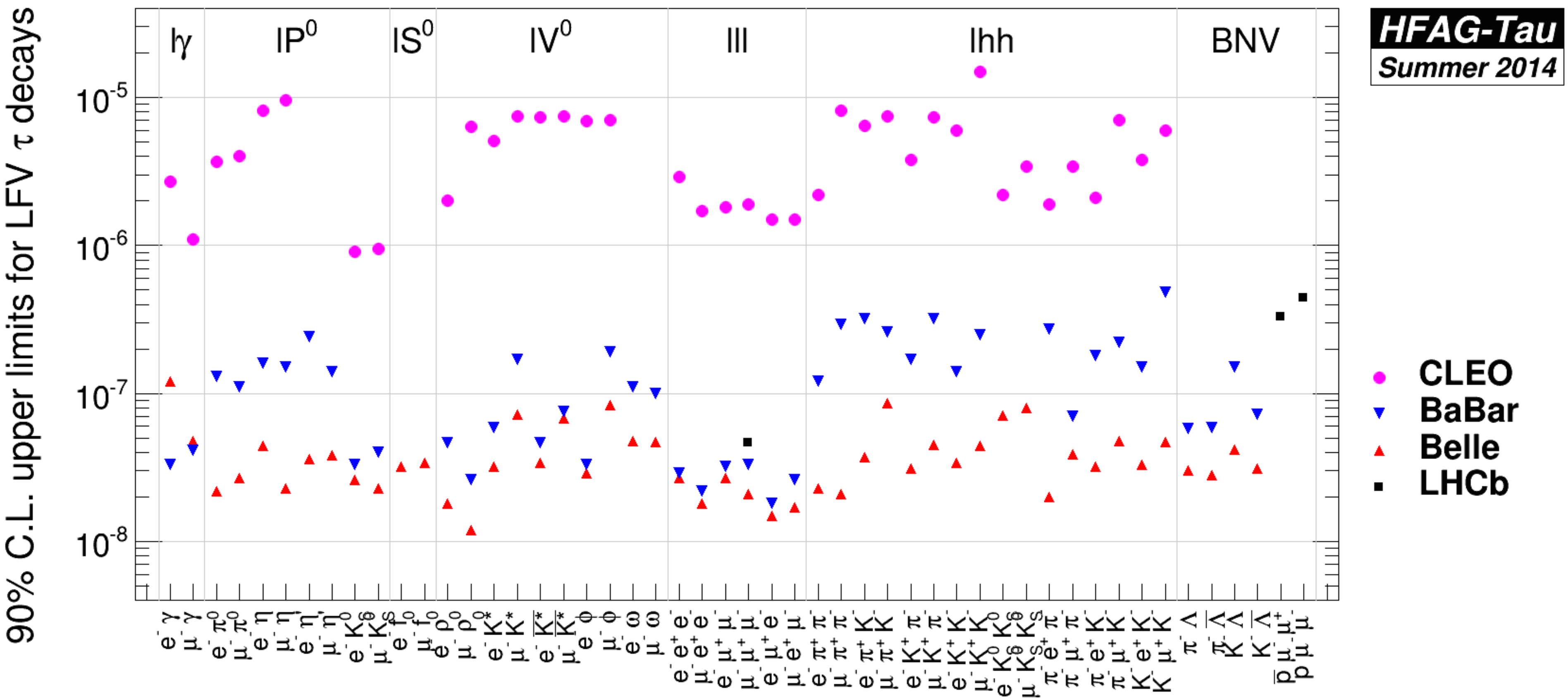
Talk by D. Hitlin @ CLFV2013

		$\tau \to \mu\gamma$	$\tau \to \ell\ell\ell$
SM + ν mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10^{-10}	10^{-7}
SM + heavy Maj ν_R	Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA + Seesaw	Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013	10^{-7}	10^{-9}

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

5.2 CLFV processes: tau decays

- Several processes: $\tau \to \ell\gamma,\ \tau \to \ell_\alpha \bar{\ell}_\beta \ell_\beta,\ \tau \to \ell Y$ ← $P,\ S,\ V,\ P\bar{P},\ldots$



- 48 LFV modes studied at Belle and BaBar

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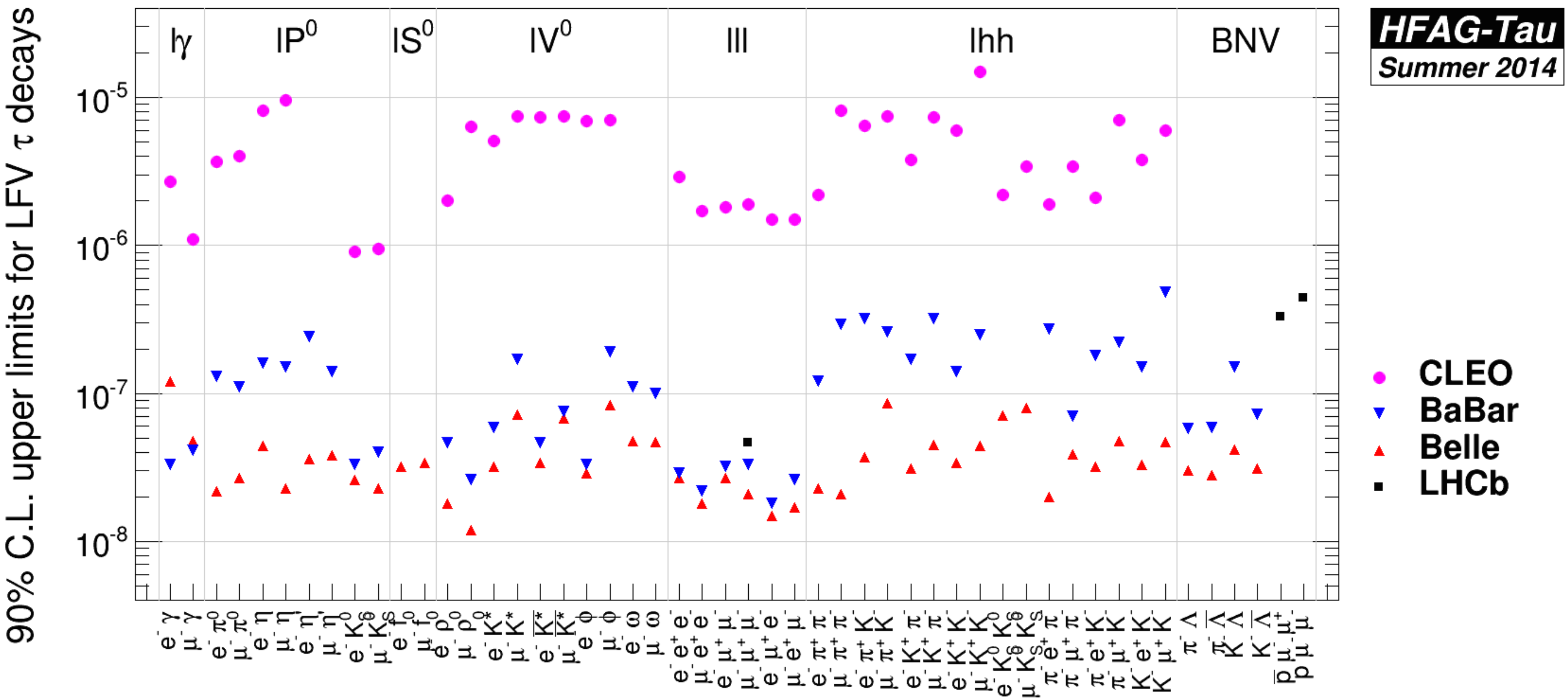

- Expected sensitivity 10^{-9} or better at *LHCb, Belle II?*

CLFV processes: tau decays

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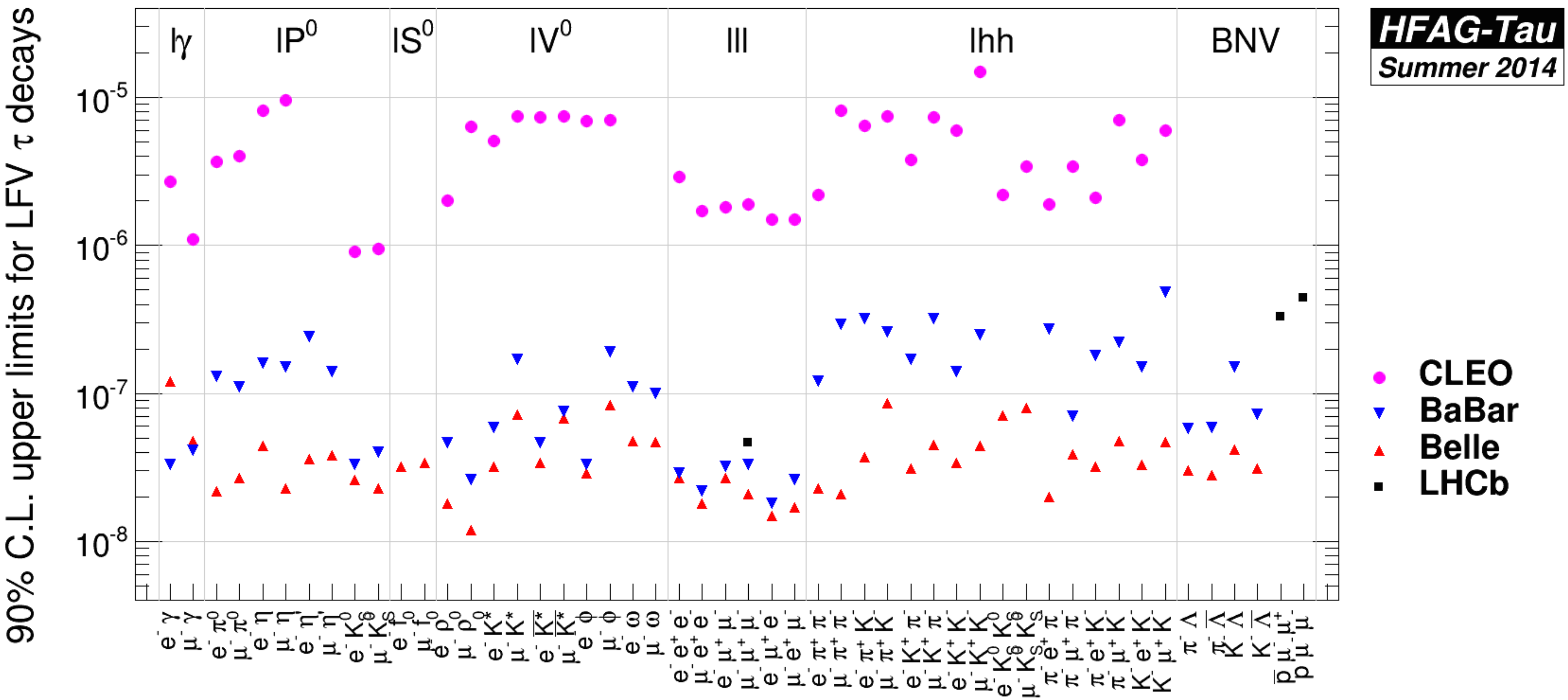

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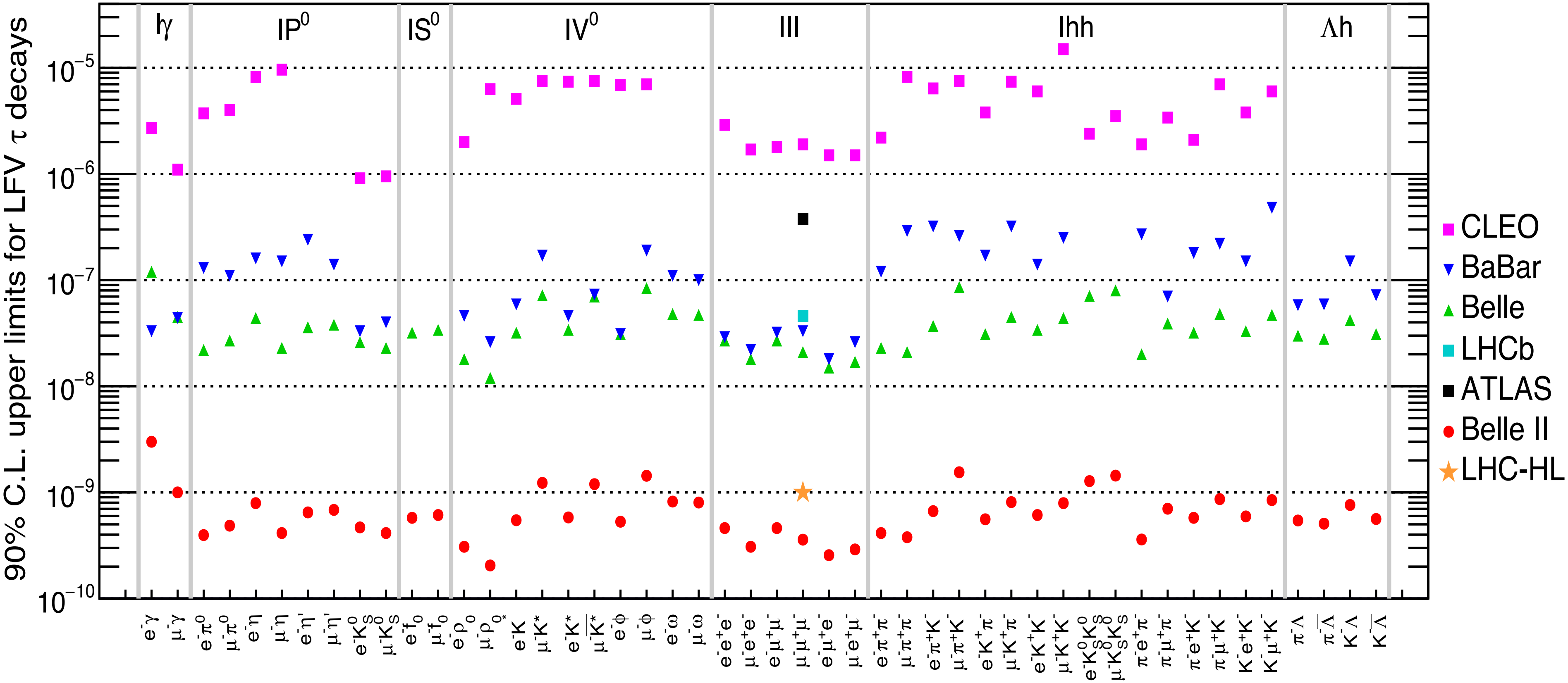

- Expected sensitivity 10^{-9} or better at *LHCb, Belle II, HL-LHC?*

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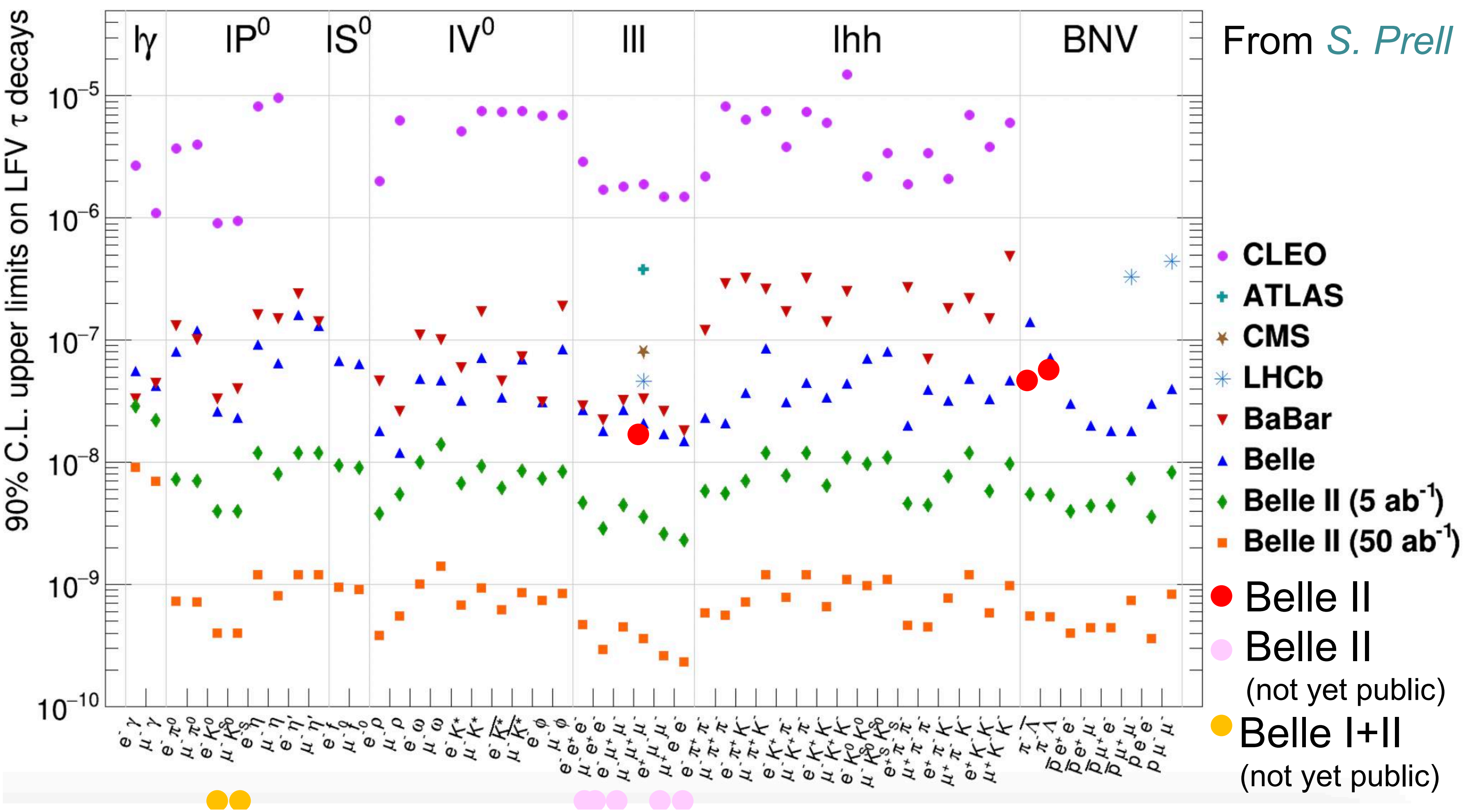


5.3 Effective Field Theory approach

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots$$

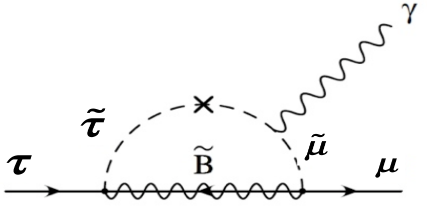
See e.g.
Black, Han, He, Sher'02
Brignole & Rossi'04
Dassinger et al.'07
Matsuzaki & Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov & Zhuridov'14
Cirigliano, Celis, E.P.'14

- Build all D>5 LFV operators:

  - Dipole:

$$\boxed{\mathcal{L}_{eff}^{D} \supset -\frac{C_D}{\Lambda^2} m_\tau \bar{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}}$$

e.g.



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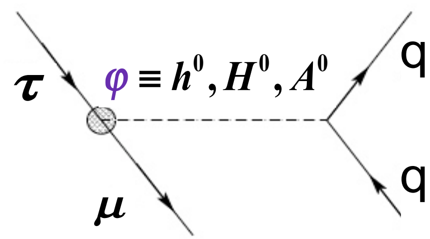
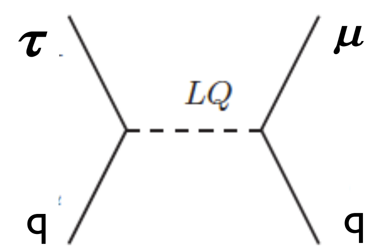
  - Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\boxed{\mathcal{L}_{eff}^{S,V} \supset -\frac{C_{S,V}}{\Lambda^2} m_\tau m_q G_F \bar{\mu}\, \Gamma P_{L,R} \tau\ \bar{q} \Gamma q}$$

e.g.



$\boxed{\Gamma \equiv 1}$



$\boxed{\Gamma \equiv \gamma^\mu}$

.

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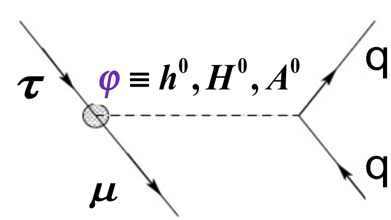
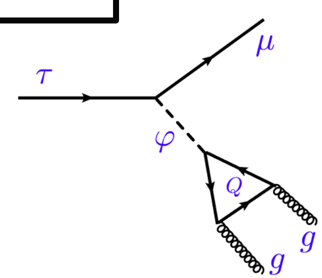
  - Integrating out heavy quarks generates *gluonic operator*

$$\boxed{\frac{1}{\Lambda^2}\bar{\mu}P_{L,R}\tau Q\bar{Q}} \rightarrow \boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu}P_{L,R}\tau\ G_{\mu\nu}^{a} G_{a}^{\mu\nu}}$$

.





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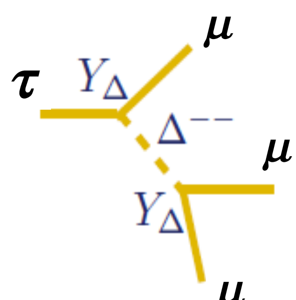
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C^{4\ell}_{S,V}}{\Lambda^2} \bar{\mu}\, \Gamma P_{L,R} \tau\, \bar{\mu}\, \Gamma P_{L,R} \mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

e.g.



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  - Lepton-gluon (Scalar, Pseudo-scalar): $\boxed{\mathcal{L}_{eff}^{G} \supset -\frac{C_G}{\Lambda^2} m_\tau G_F \bar{\mu} P_{L,R} \tau\ G_{\mu\nu}^a G_a^{\mu\nu}}$
  - 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\boxed{\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \bar{\mu}\,\Gamma P_{L,R} \tau\ \bar{\mu}\,\Gamma P_{L,R} \mu}$

$\boxed{\Gamma \equiv 1, \gamma^\mu}$

- Each UV model generates a *specific pattern* of them

5.4 Model discriminating power of Tau processes

- Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$O^{4\ell}_{S,V}$	✓	–	–	–	–	–
O_D	✓	✓	✓	✓	–	–
O^q_V	–	–	✓ (I=1)	✓(I=0,1)	–	–
O^q_S	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
O^q_A	–	–	–	–	✓ (I=1)	✓ (I=0)
O^q_P	–	–	–	–	✓ (I=1)	✓ (I=0)
$O_{G\tilde{G}}$	–	–	–	–	–	✓

- The notion of "*best probe*" (process with largest decay rate) is *model dependent*

- If observed, compare rate of processes ⇨ key handle on *relative strength* between operators and hence on the *underlying mechanism*

5.4 Model discriminating power of Tau processes

Celis, Cirigliano, E.P.'14

- Summary table:

	$\tau \to 3\mu$	$\tau \to \mu\gamma$	$\tau \to \mu\pi^+\pi^-$	$\tau \to \mu K\bar{K}$	$\tau \to \mu\pi$	$\tau \to \mu\eta^{(\prime)}$
$\mathrm{O}_{\mathrm{S,V}}^{4\ell}$	✓	–	–	–	–	–
O_{D}	✓	✓	✓	✓	–	–
$\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$	–	–	✓ (I=1)	✓(I=0,1)	–	–
$\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$	–	–	✓ (I=0)	✓(I=0,1)	–	–
O_{GG}	–	–	✓	✓	–	–
$\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$	–	–	–	–	✓ (I=1)	✓ (I=0)
$\mathrm{O}_{\mathrm{G\tilde{G}}}$	–	–	–	–	–	✓

- In addition to leptonic and radiative decays, *hadronic decays* are very important ⇒ sensitive to large number of operators!

- But need reliable determinations of the hadronic part: *form factors* and *decay constants* (e.g. f_η, $f_{\eta'}$)

6. Conclusion and outlook

Conclusion and outlook

- Tau physics is a very rich field: test QCD and EW, etc..

- Several interesting anomalies: V_{us}, CPV in $\tau \to K\pi\nu_\tau$, g-2
  ⇨ Tau physics can help

- Important experimental activities: Belle, BaBar, LHCb, BESIII, VEPP
  ⇨ *Belle II*

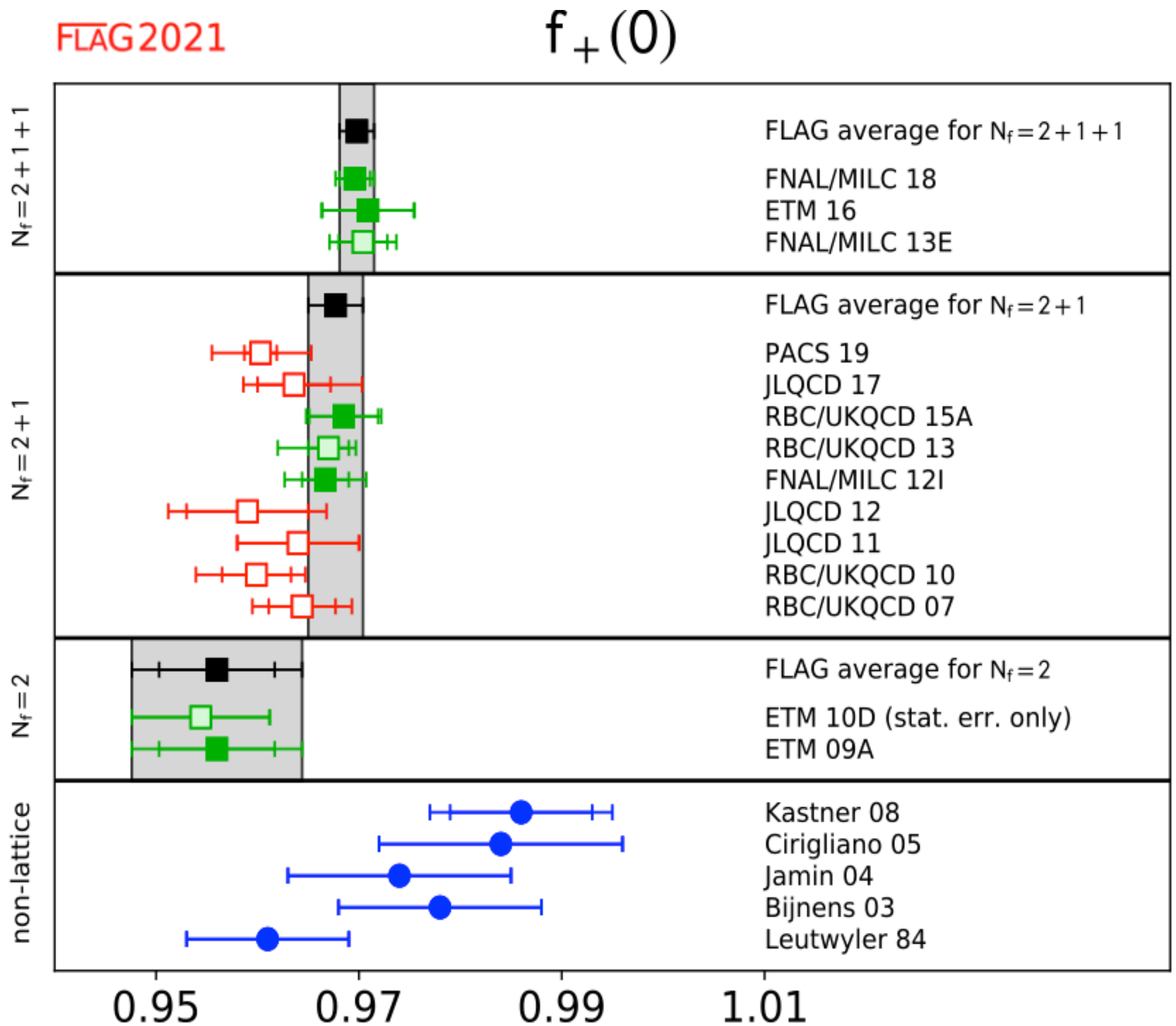
- Intense theoretical activities : QCD, new physics

- A lot of *very interesting physics* remains to be done in the tau sector!

7. Back-up

2.2 $f_+(0)$ from lattice QCD

- Recent progress on Lattice QCD for determining $f_+(0)$



$$f_+(0)^{FLAG21}_{N_f=2+1+1} = \mathbf{0.9698(17)}$$

0.18% uncertainty

to be compared to

$$f_+(0)^{FLAG16}_{N_f=2+1+1} = \mathbf{0.9704(32)}$$

$$f_+(0)^{2010}_{N_f=2+1} = \mathbf{0.959(50)}$$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

➡ Lattice uncertainties at the same level as exp.

-3.2σ away from unitarity!

2011: $V_{us} = 0.2254(5)_{exp}(11)_{lat}$ → $V_{us} = 0.2231(4)_{exp}(4)_{lat}$

V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\rm EM}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

Di Carlo et al.'19

- Main input hadronic input: f_K/f_π

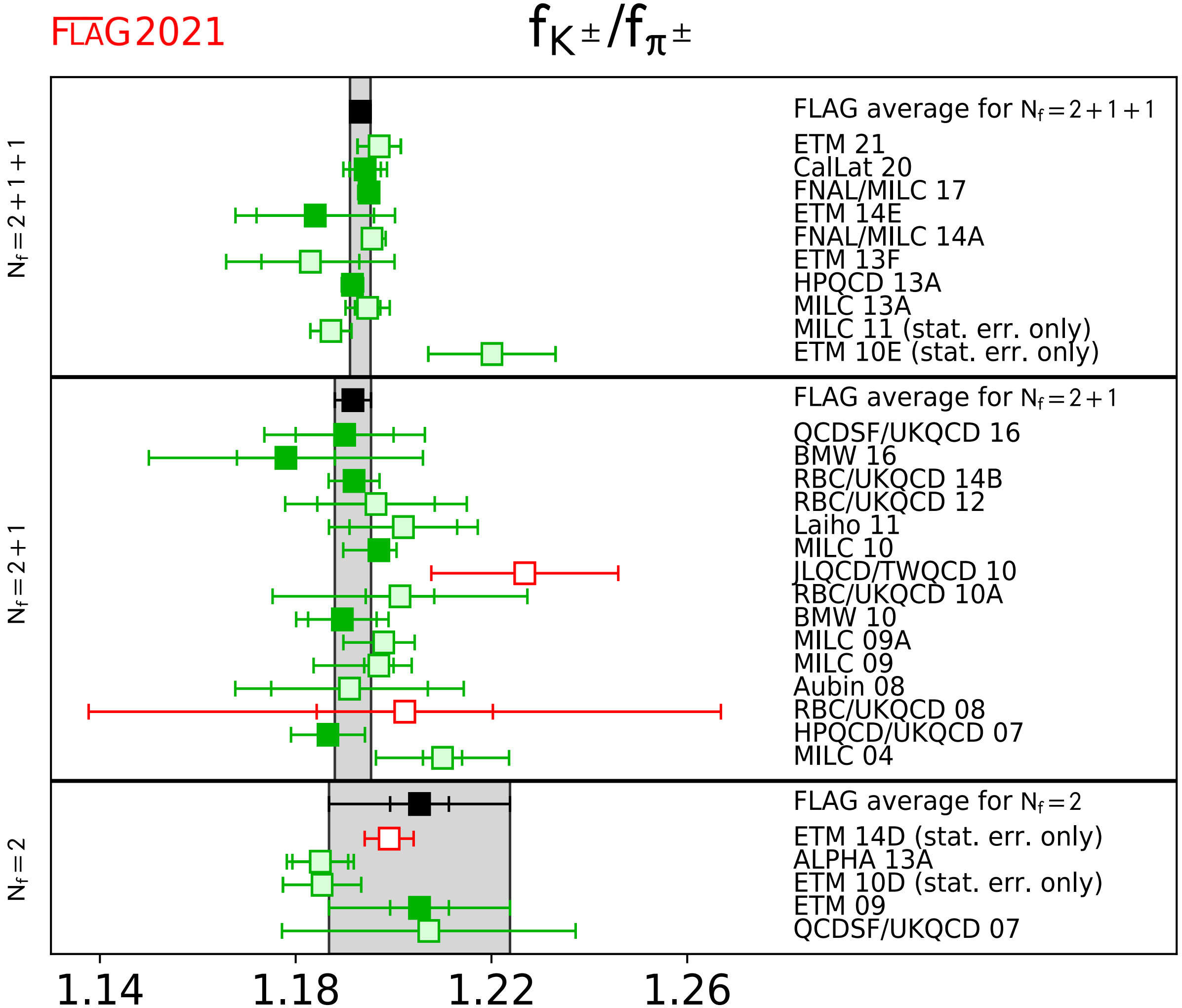
- In 2011: V_{us}/V_{ud} = 0.2312(4) $_{\rm exp}$(12)$_{\rm lat}$

- In 2021: V_{us}/V_{ud} = 0. 2311(3)$_{\rm exp}$(4)$_{\rm lat}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

-1.8σ away from unitarity

2.2 f_K/f_π from lattice QCD

Progress since 2018: ➡ new results from *ETM'21* and *CalLat'20*



Now Lattice collaborations include SU(2) IB corr.
For N_f=2+1+1, FLAG2021

$$f_{K^+}/f_{\pi^+} = 1.1932(21)$$

0.18% uncertainty

Results have been stable over the years

For average substract IB corr.

$$f_K/f_\pi = 1.1967(18)$$

In 2011: $f_K/f_\pi = 1.193(6)$

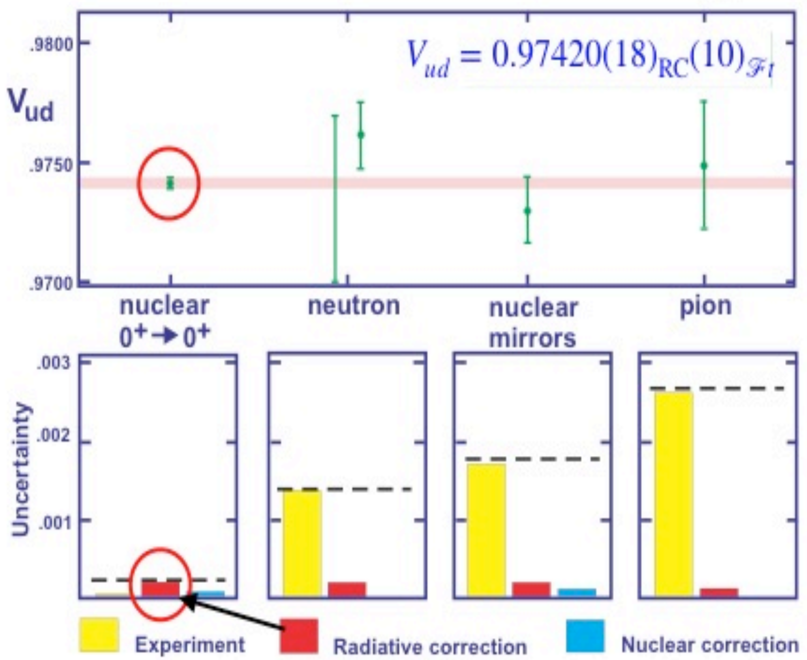
➡ V_{us}/V_{ud} = 0. $23108(29)_{exp}(42)_{lat}$

2.3 $|V_{ud}|$ from $0^+ \rightarrow 0^+$ superallowed β decays

See Talk by Misha Gorshteyn @CKM2021

PDG 2018:

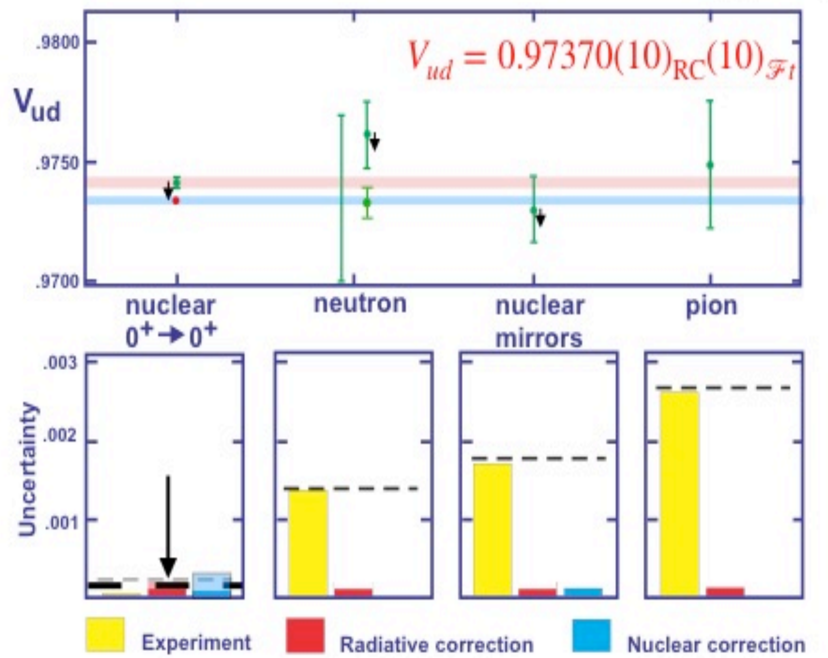
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

Figure adapted from J. Hardy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



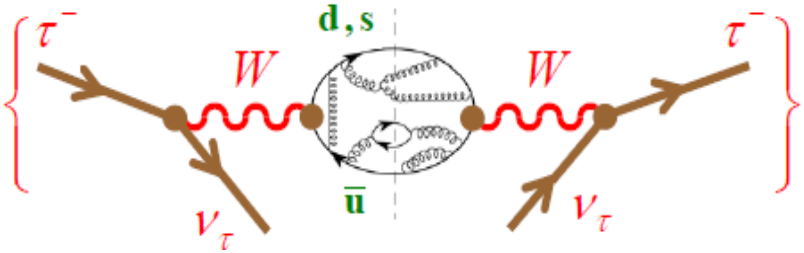
Recent improvement on the theoretical RCs +Nuclear Structure Corrections

⇨ Use of a data driven dispersive approach *Seng et al.'18'19, Gorshteyn'18*

Inclusive τ-decays

Braaten, Narison, Pich'92

$\Gamma_{\tau \to \nu_\tau + \mathrm{had}} \sim \mathrm{Im}$



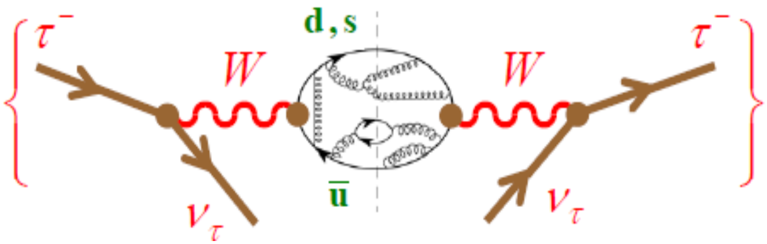
- Quantity of interest : $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \textbf{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \quad \right\}$$



$$\Rightarrow \quad \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

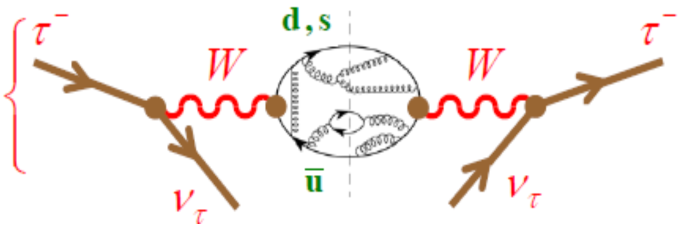
$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left(\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right) + |V_{us}|^2 \left(\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right)$$

J

$$\Pi^{\mu\nu}_{ij,V/A}(q) = \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{ij,V/A}(q^2) + q^\mu q^\nu\,\Pi^{(0)}_{ij,V/A}(q^2)$$

3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ: $\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\left\{ \right.$  $\left. \right\}$

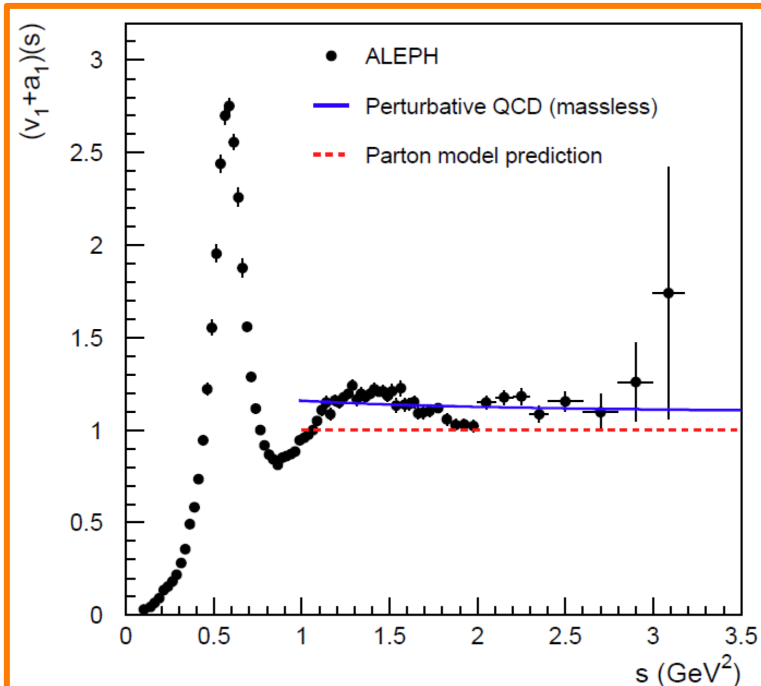
$$\Rightarrow \quad \boxed{R_\tau(m_\tau^2) = 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon)+\text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]}$$

- Spectral functions:

$$\text{Im}\Pi^{(1)}_{\bar{u}d,V/A}(s) = \frac{1}{2\pi}v_1/a_1(s)$$

- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system

$\Rightarrow$ mix of non-perturbative and perturbative effects



Measurements

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} = ?$$

- Decomposition as a function of observed and separated final states:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

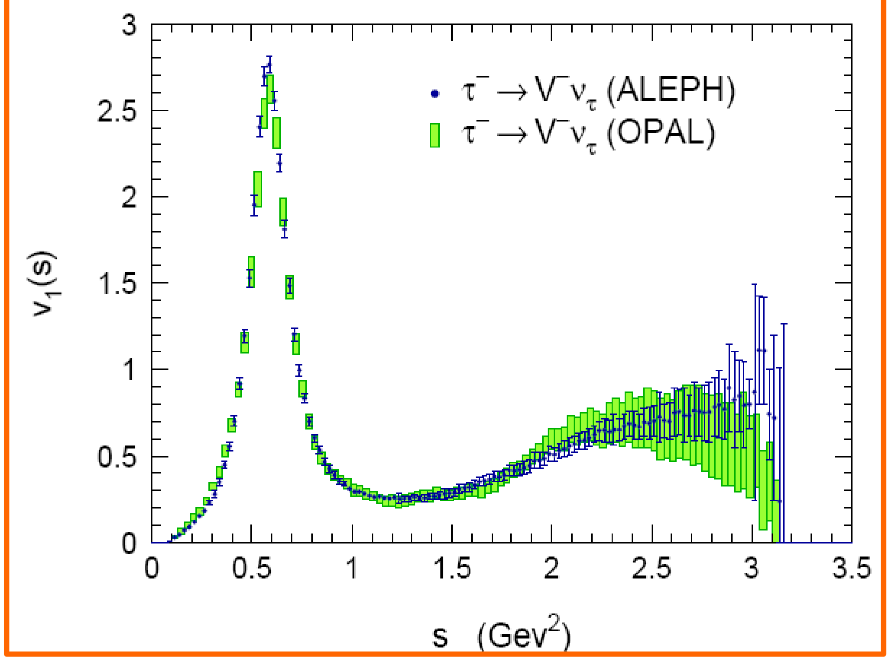
$R_{\tau,V} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{v,s=0}}$

(even number of pions)

$R_{\tau,A} \Rightarrow \tau^- \to \nu_\tau + h_{A,s=0}$

(odd number of pions)

$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$



Measurements

- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} = ?$$

- Decomposition as a function of observed and separated final states:

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

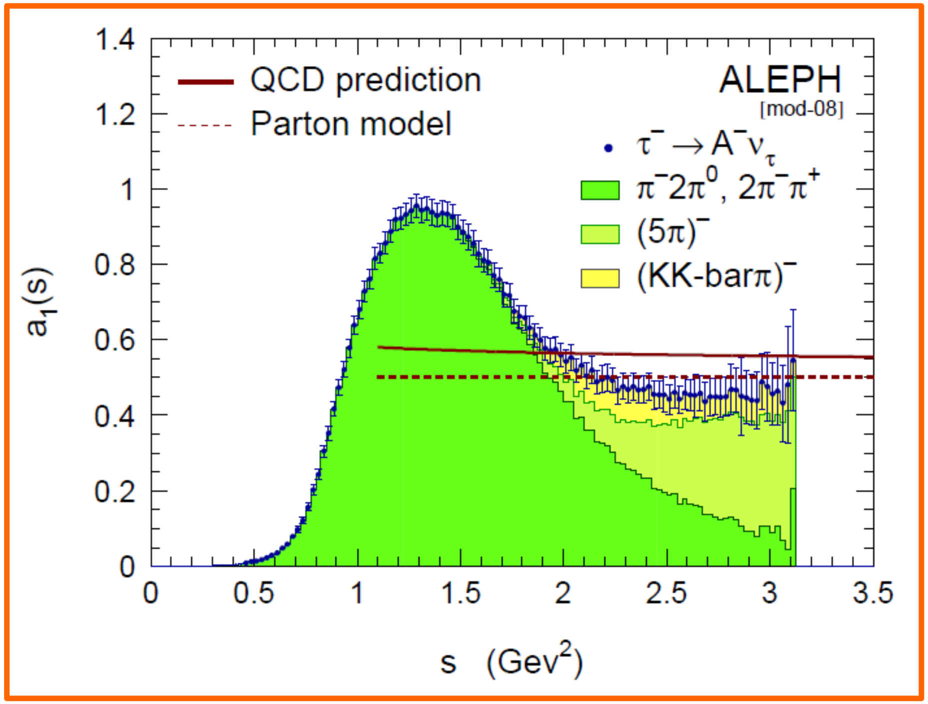
$R_{\tau,V} \Rightarrow \tau^- \to \nu_\tau + h_{v,s=0}$

(even number of pions)

$R_{\tau,A} \Rightarrow \boxed{\tau^- \to \nu_\tau + h_{A,s=0}}$

(odd number of pions)

$R_{\tau,S} \Rightarrow \tau^- \to \nu_\tau + h_{V+A,s=1}$



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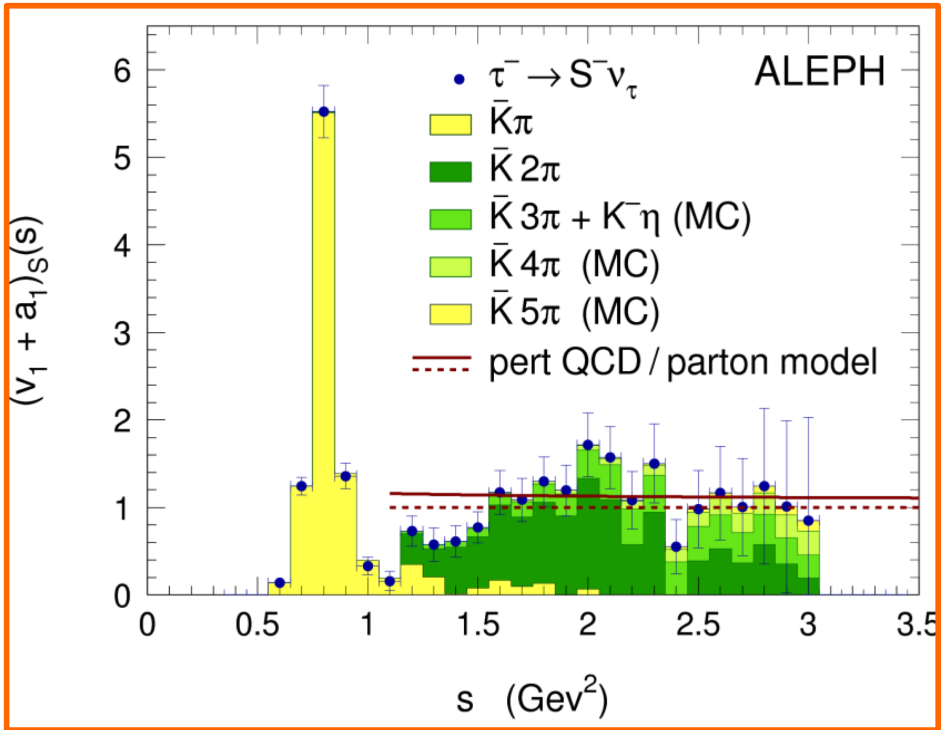
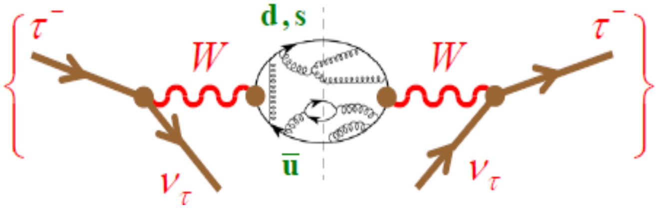


3.2 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

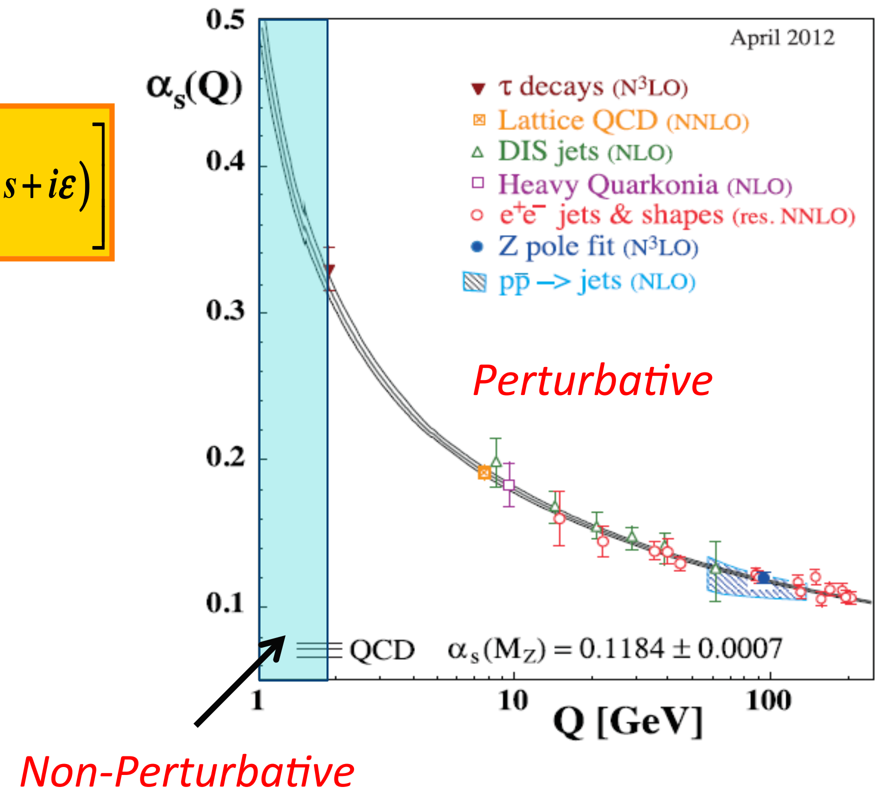
$\Gamma_{\tau \to \nu_\tau + \text{had}} \sim \text{Im}$



$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\text{Im}\,\Pi^{(1)}(s+i\varepsilon) + \text{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

- We are in the *non-perturbative* region: we do not know how to compute!

- Trick: use the analytical properties of Π!



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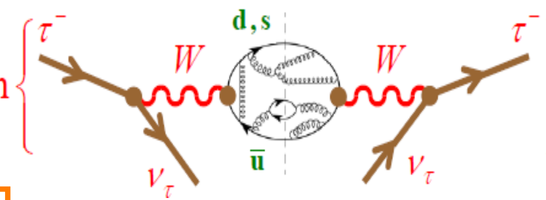

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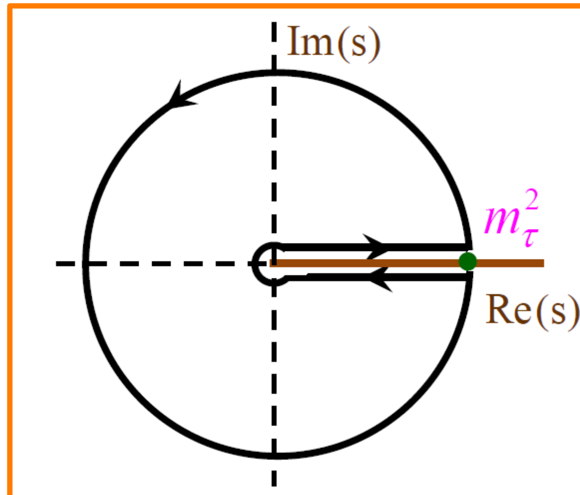
Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇨ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} C^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

3.3 Operator Product Expansion

$$\Pi^{(J)}(s) = \sum_{D=0,2,4...} \frac{1}{(-s)^{D/2}} \sum_{\dim O = D} \mathrm{C}^{(J)}(s,\mu) \left\langle O_D(\mu) \right\rangle$$

Wilson coefficients

Operators

μ separation scale between short and long distances

- D=0: Perturbative contributions
- D=2: Quark mass corrections
- D=4: Non perturbative physics operators, $\left\langle \frac{\alpha_s}{\pi} GG \right\rangle$, $\left\langle m_j \bar{q}_i q_i \right\rangle$
- D=6: 4 quarks operators, $\left\langle \bar{q}_i \Gamma_1 q_j \bar{q}_j \Gamma_2 q_i \right\rangle$
- D≥8: Neglected terms, supposed to be small…

$\Rightarrow$ $$\boxed{R_{\tau,V}(s_0) = \frac{3}{2} \left| V^{ud} \right|^2 S_{EW} \left(1 + \delta^{(0)} + \sum_{D=2,4..} \delta^{(D)}_{ud,V} \right)}$$

similar for $R_{\tau,A}(s_0)$ and $R_{\tau,S}(s_0)$

Perturbative Part

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ Marciano &Sirlin'88, Braaten & Li'90, Erler'04

- Perturbative part (D=0):

$$a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$$

$$\boxed{\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%}$$

Baikov, Chetyrkin, Kühn'08

Non-perturbative part

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

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Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^S\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇨ Use of weighted distributions

Ex: In the non-strange sector: $\boxed{\delta_{NP}^{NS} = -0.0064(13)}$ *Davier et al.'14*

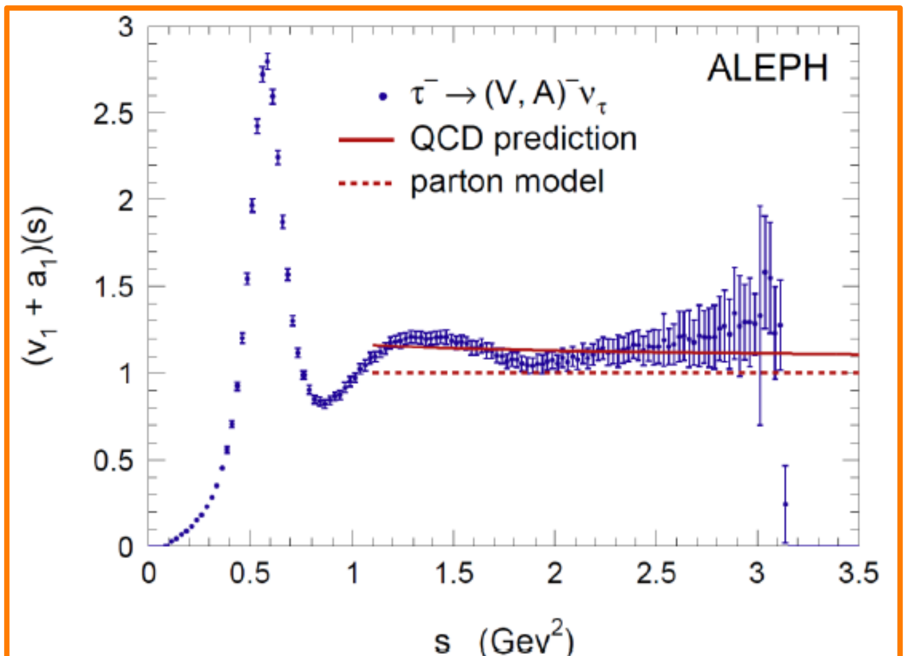
Non-Perturbative part

Le Diberder&Pich'92

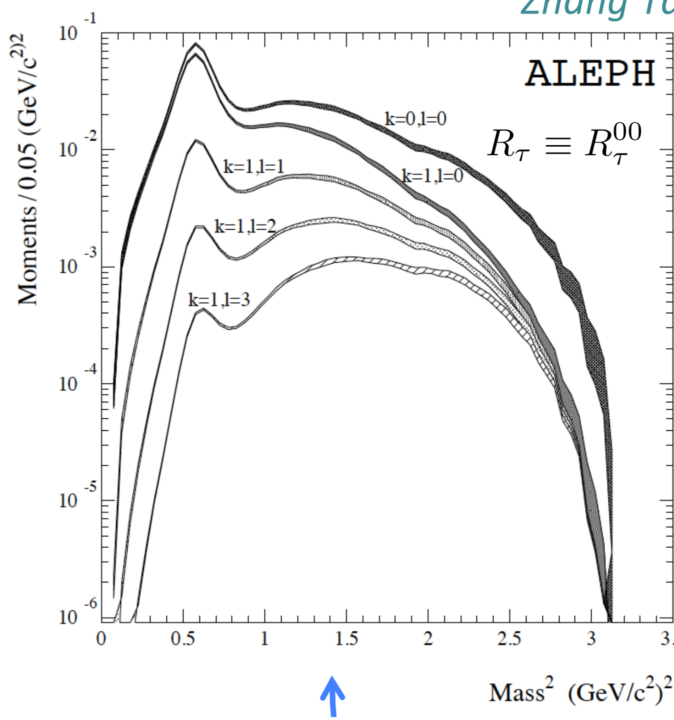
- D ≥ 4: Non perturbative part, not known, *fitted from the data*
  ⇒ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



3.3 Exclusive hadronic processes

- For the exclusive hadronic processes $\tau \rightarrow H\nu_\tau$:

$$\boxed{M\left(\tau \rightarrow H\nu_\tau\right) = \frac{G_F}{\sqrt{2}} V_{CKM} \bar{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma^5\right) u_\tau H_\mu}$$

- The hadronic matrix element : $\boxed{H_\mu = \langle H | \left(V_\mu - A_\mu\right) e^{iL_{QCD}} | 0 \rangle = \left(Lorentz\ \mathrm{struct.}\right)^i_\mu F_i\left(q^2\right)}$

- Experimental measurement : decay rate

$$\boxed{d\Gamma\left(\tau \rightarrow H\nu_\tau\right) = \frac{G_F^2}{4m_\tau} \left|V_{CKM}\right|^2 L_{\mu\nu} H^{\mu\nu} dP_S}$$

- Challenge : determination of the form factors to extract SM parameters or NP

SM param., NP

parametrization of the ffs

ChPT + Analyticity + Unitarity
Dispersion Relations
Models

Experimental Data
TAUOLA etc

FFs: masses,
widths,couplings

3.3 Exclusive hadronic processes

Experimental situation :

- $\tau \to PP\nu_\tau$

$$\begin{cases} \pi^-\pi^0, K^-K^0 \\ K^-\pi^0, \bar{K}^0\pi^- \\ \eta \text{ modes} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

- $\tau \to PPP\nu_\tau$

$$\begin{cases} \pi\ \pi\ \pi \\ KK\pi \\ K\pi\pi \\ \eta \text{ modes} \\ \mathrm{KKK} \end{cases}$$

Branching fractions
Spectrum

Branching fractions

ALEPH, CLEOIII, OPAL
Belle, BaBar

- $\tau \to\!> 3P\nu_\tau$

Theoretical situation

Parametrization using

ChPT + Analyticity + Unitarity
Dispersion relations on the market

⇨ Reasonably good control

Parametrization using
ChPT + Analyticity + Unitarity+
Resonances

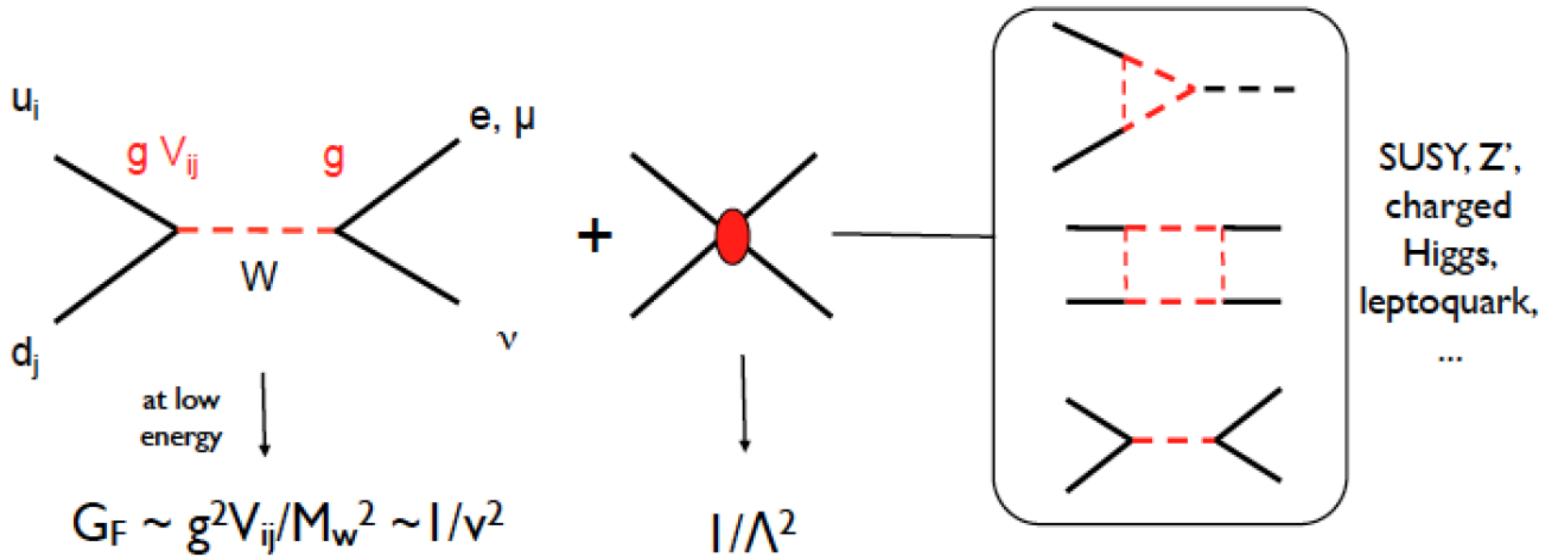
⇨ Much more difficult and model dependent

⇨ Poor knowledge

1.2 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$

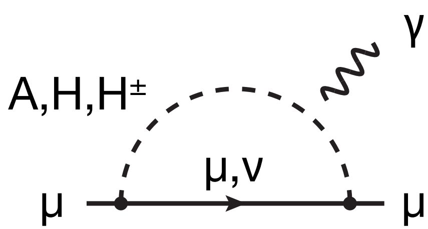


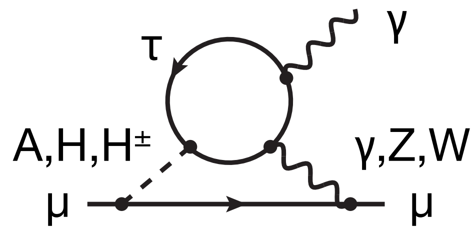
Grossman, E.P., Schacht'20

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

2.2 Lepton universality & NP

- The lepton universality tests give strong constraints on type-X (lepton-specific) 2HDMs ⇒ Model favoured to explain the g-2 discrepancy



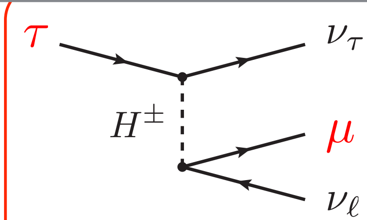


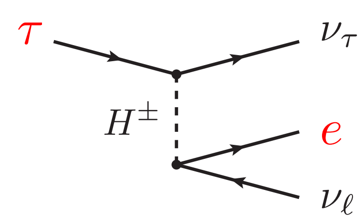
M. Endo@b2tip'15

Barr-Zee contribution with A enhances muon g-2

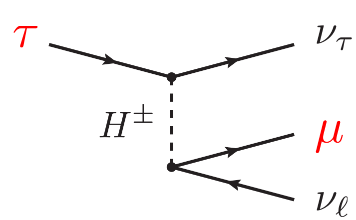
⇒ Contribution to LU : Large Negligible

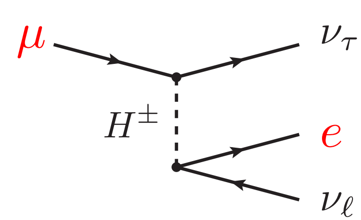
$$\frac{g_\mu}{g_e} = \frac{1+\Delta_{\tau\to\mu}}{1+\Delta_{\tau\to e}} = 1.0018 \pm 0.0014$$





$$\frac{g_\tau}{g_e} = \frac{1+\Delta_{\tau\to\mu}}{1+\Delta_{\mu\to e}} = 1.0029 \pm 0.0015$$





[HFAG'14]

Note: $Y_{\tau,\mu} \gg Y_e$

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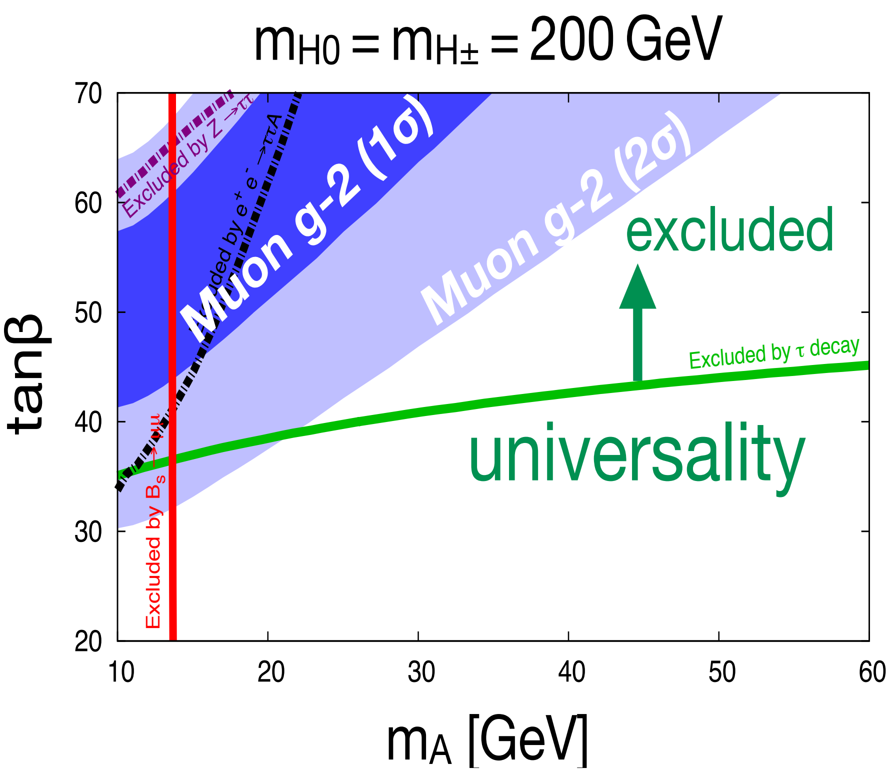
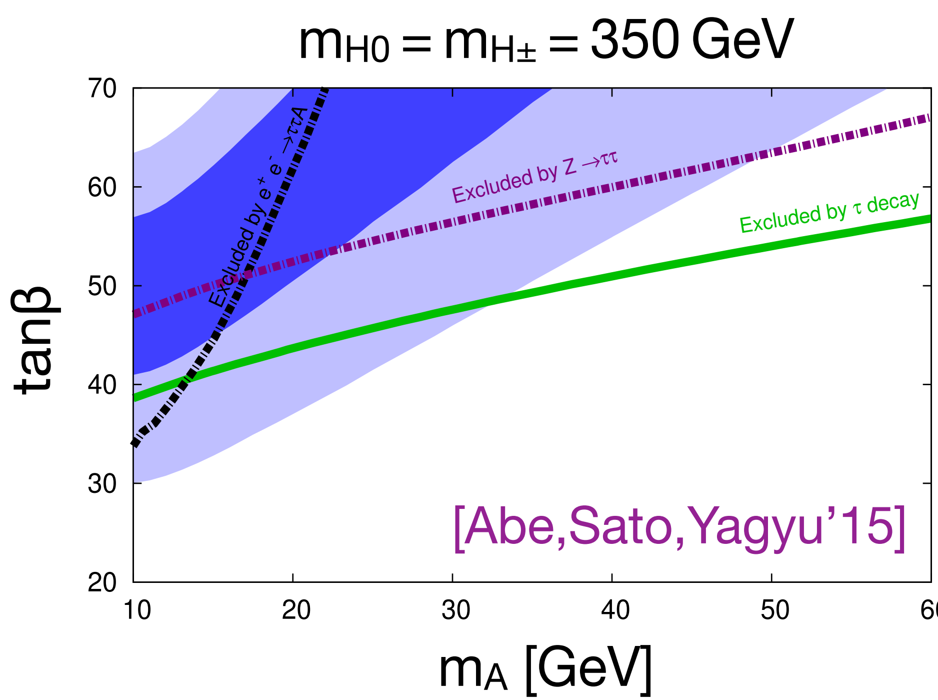
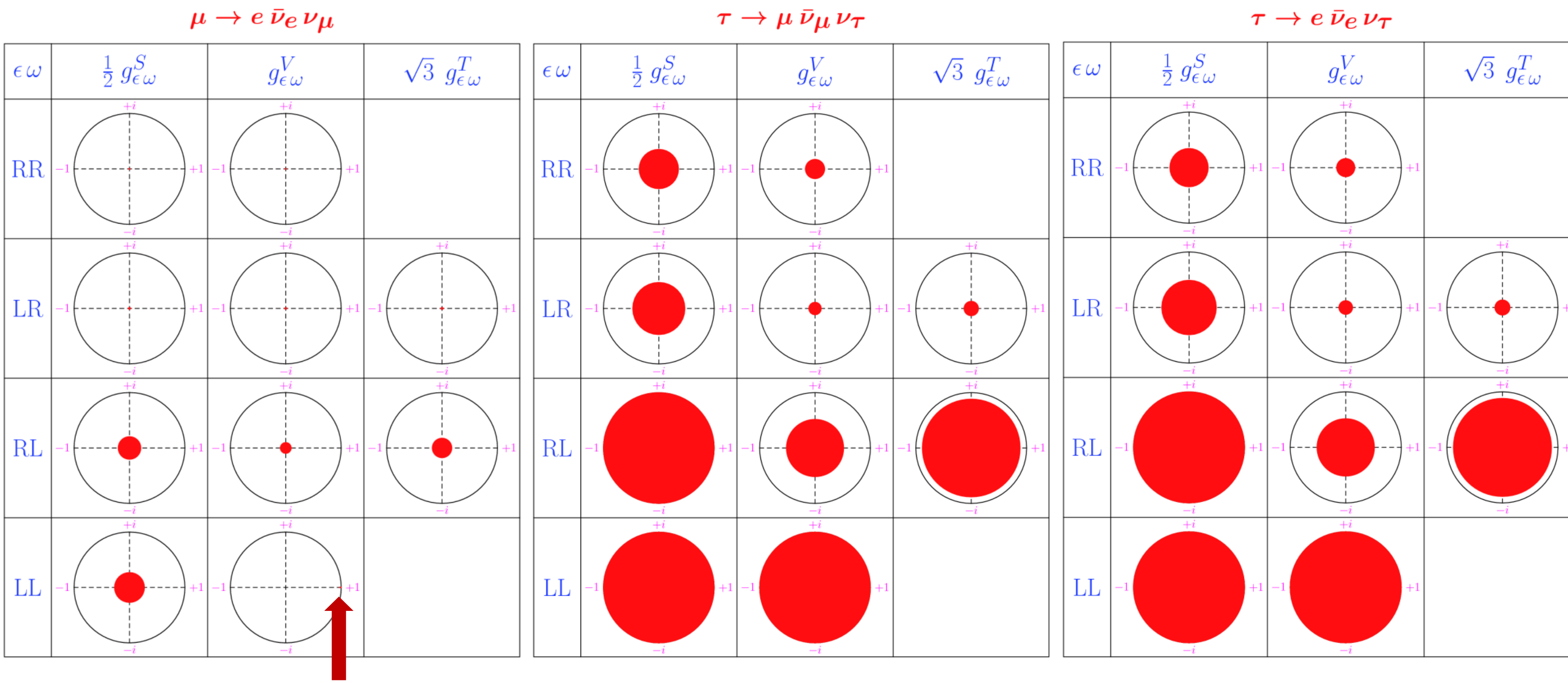
M. Endo@b2tip'15





Effective Hamiltonian: $$\mathcal{H} = 4\,\frac{G_{\ell'\ell}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{\ell'_\epsilon}\Gamma^n (\nu_{\ell'})_\sigma\right] \left[\overline{(\nu_\ell)_\lambda}\Gamma_n \ell_\omega\right]$$

Normalization: $$\Gamma \propto \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) + 3\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2\right) \equiv 1$$



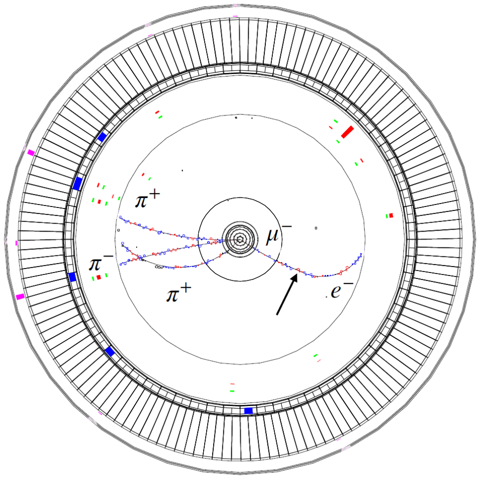
$|g^V_{LL}| > 0.960$ (90% CL)

High-precision τ data needed!

Probability to decay into a right-handed muon:

$$Q_{\mu_R} = Q_{RR} + Q_{RL} = \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2\right) + 3|g^T_{RL}|^2 + |g^V_{RR}|^2 + |g^V_{RL}|^2 = \frac{1}{2}\left(1-\xi'\right)$$

MC $\tau^+\tau^-$



Tiny probability of muon decaying inside the detector compensated by huge statistics

$$\xi' = 0.22 \pm 0.94 \pm 0.42$$ Belle, 2303.10570

$$\Rightarrow \quad Q_{\mu_R} \leq 1.23 \quad (90\%\ \mathrm{CL})$$

Not yet constraining. Error dominated by statistics…

1.1 The triumph of the SM and quest for NP

- New era in particle physics :
  ⇨ *success of the Standard Model*: a successful theory of microscopic phenomena with *no intrinsic energy limitation*

- *Shall we continue to test the Standard Model and search for New Physics?*

  Yes! ⇨ Despite its phenomenological successes, the SM has some *deep unsolved* problems:
  - hierarchy problem
  - flavour pattern
  - dark-matter, etc.…
  - Strong interaction not so well understood: confinement, etc

1.2 Quest for New Physics

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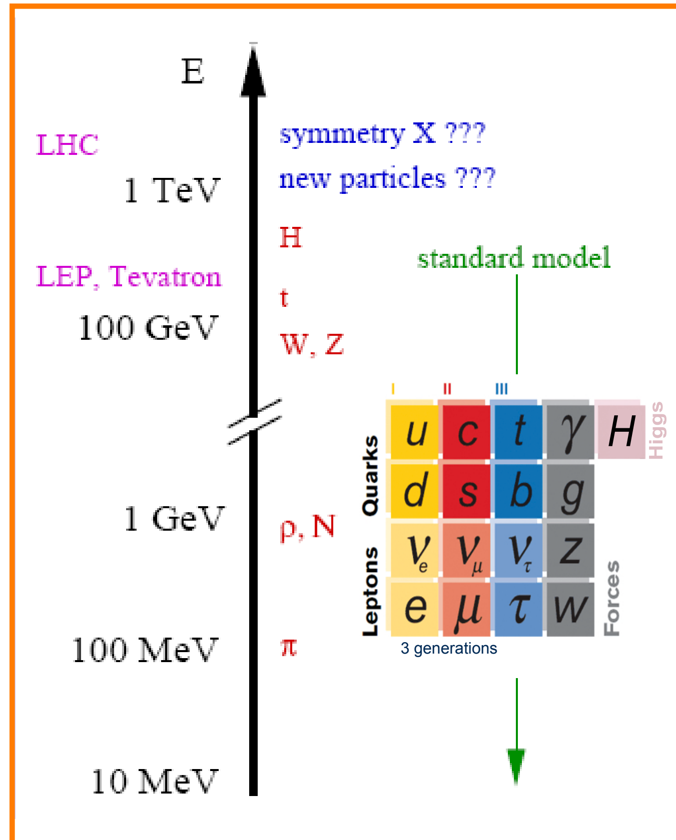
  - hierarchy problem
  - flavour pattern
  - dark-matter, etc.…
  - Strong interaction not so well understood: confinement etc

- Consider the SM as as an *effective theory*, i.e. the limit *–in the accessible range of energies and effective couplings–* of a more fundamental theory, with
  - new degrees of freedom
  - new symmetries



1.2 Quest for New Physics

- *Where do we look?* Everywhere!

  ⇨ search for New Physics with a *broad search strategy* given the lack of clear indications on the SM-EFT boundaries (*both in terms of energies and effective couplings*)

  ⇨ Key unique role of *Tau physics*