BASICS OF LEPTON FLAVOR VIOLATION

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USEFUL REFERENCES

- some general introductions to flavor physics
 - Nir, 0708.1872, 1605.00433
 - Grossman, Tanedo, 1711.03624
 - JZ, 1903.05062

...

- on lepton flavor violation
 - Calibbi, Sirognelli, 1709.00294
 - Ardu, Pezzullo, 2204.08220

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FLAVOR IN THE SM QUARK SECTOR

- neutral currents are flavor conserving (at tree level)
 - photon, gluon, Z: have flavor (generation) universal interactions









LEPTONS

- first assume that neutrino masses are zero
- extremelly good approximation in
 - collider experiments, meson decays, charged lepton decays,...
 - in each of these: $E \gg m_{\nu}$

LEPTONS

- \Rightarrow in SM with massless ν no leptonic FCNCs
 - photon, Z: flavor (generation) universal interactions





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• Higgs has *flavor diagonal* interactions

proportional to lepton masses

• charged currents (W couplings) are *flavor universal*



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LEPTONS

• this means that for $m_{\nu} = 0$ in the SM • $Br(\mu^+ \to e^+ e^- e^+) = 0$ • $Br(\mu^+ \to e^+\gamma) = 0$ • $Br(\tau^+ \rightarrow \mu^+ \mu^- \mu^+) = 0$ • $Br(\tau^+ \rightarrow \mu^+ \rho^0) = 0$

LEPTONS

- agrees well with stringent experimental bounds in PDG
 - $Br(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$
 - $Br(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}$
 - $Br(\tau^+ \to \mu^+ \mu^- \mu^+) < 2.1 \times 10^{-8}$

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• $Br(\tau^+ \to \mu^+ \rho^0) < 1.2 \times 10^{-8}$

NEUTRINO MASSES

- however, neutrinos are not completely massless
 - at some level leptonic FCNCs will arise in the SM
- how much does $m_{\nu} \neq 0$ matter?
- in experiments we are interested in: not too much
 - corrections suppressed by $(m_{\nu}/E)^n \ll 1$
 - for instance for muon decays: $E \sim m_{\mu} \Rightarrow m_{\nu}/m_{\mu} < 10^{-9}$

NEUTRINO MASSES

- with *QUDL* field content m_{ν} forbidden in the SM
- two ways of introducing ν masses

• *Dirac neutrinos:* add RH neutrino fields ν_R , singlets under SM + $\mathcal{L}_{\text{Yukawa}} \supset -Y_{\nu}^{ij} \bar{L}_{L}^{i} H^{c} \nu_{R}^{j} + \text{h.c.}$ conserv. L 3×3 complex • *Majorana neutrinos:* m_{μ} from dimension 5 Weinberg operator, is $\Delta L = 2$ $\mathcal{L}_{\text{dim. 5}} \supset -\frac{1}{2} \frac{Y_{\nu}^{'ij}}{\Lambda} \left(\bar{L}_{L}^{ci} H^{c} \right) \left(H^{c*} L_{L}^{j} \right) + \text{h.c.}$ 3×3 symm., complex • counting of physical parameters slightly differs in the two cases in both cases weak (flavor) eigenstates are linear superpositions of mass eigenstates $\nu_{aL} = \sum_{i=1}^{2} U_{ai} \nu_{iL}, \quad a = e, \mu, \tau$ *i*=1 PMNS matrix KEK, Oct 17 2024 J. Zupan Basics of Lepton Flavor Violation

PMNS MATRIX

 canonical form of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times P$$

- *P* matrix takes the form:
 - P = 1 for Dirac neutrinos
 - $P = \text{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})$ for Majorana ν 's

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PMNS MATRIX

• assuming "normal ordering": $m_3 > m_2 > m_1$ $m_2^2 - m_1^2 \sim (10^{-3} \text{ eV})^2$ $m_3^2 - m_1^2 \sim (0.05 \text{ eV})^2$ $\sin \theta_{12} \sim \sin \theta_{23} \sim 0.5, \sin \theta_{13} \sim 0.15$ $\delta, \alpha_{12}, \alpha_{13} = ?$



$\mu \rightarrow e \gamma$ in the SM

- we already know that $\mu \rightarrow e\gamma$ vanishes for massless neutrinos
 - GIM mechanism very effective in LFV transitions
 - amplitude proportional to $A(\mu \rightarrow e\gamma) \propto m_{\nu}^2$

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U_{ek}^* m_{\nu_k}^2}{M_W^2} \right|^2$$

 $BR(\mu \to e\gamma) = 10^{-55} \div 10^{-54}$

- similar suppressions for $\mu \rightarrow 3e, \tau \rightarrow 3\mu, \mu \rightarrow e, \dots$
- for charged LFV transitions SM is well below experimental reach
 - if found, a clear signal of new physics

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Very small !!!



- for charged LFV transitions SM is well below experimental reach
 - if found, a clear signal of new physics

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SEARCHING FOR NEW PHYSICS

• LFV observables probe very high scales



• the rest of these lectures: focusing on the above observables

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OBSERVABLES

- CLFV transitions
 - $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \mu \rightarrow 3e, \mu \rightarrow e$ conv., ...
- searching for light new physics
- Higgs decays

• $h \rightarrow \tau \tau, h \rightarrow \mu \mu, h \rightarrow \tau \mu, ...$

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talk by Toshiyuki Iwamoto @ FPCP2020 cLFV experiments in the world



Mu₂e DeeMe, Production Detecto ransport COMET

Single e⁻ measurement: pulsed beam needed Many pion-induced backgrounds after proton pulse wait it out with 26 ns lifetime

Central Drift Ch

Coincidence measurement:

DC beam needed to minimize

backgrounds from accidental

LHCb/ATLAS/CMS

 $\tau \rightarrow 3\mu, \tau \rightarrow \mu\gamma$

sitrons (4 Ge

coincidences

CERN

BKG \propto (Rate)²



EXPERIMENTAL PROGRESS

steady experimental progress since 1940s



COMPLEMENTARY PROBES

• complete list of dim 6 CLFV operators

J.Z

	4-leptons operators		Dipole operators	-		
$egin{array}{c} Q_{\ell\ell} \ Q_{ee} \ Q_{\ell e} \end{array}$	$egin{aligned} &(ar{L}_L\gamma_\mu L_L)(ar{L}_L\gamma^\mu L_L)\ &(ar{e}_R\gamma_\mu e_R)(ar{e}_R\gamma^\mu e_R)\ &(ar{L}_L\gamma_\mu L_L)(ar{e}_R\gamma^\mu e_R) \end{aligned}$	$Q_{eW} \ Q_{eB}$	$egin{aligned} & (ar{L}_L\sigma^{\mu u}e_R) au_I\Phi W^I_{\mu u}\ & (ar{L}_L\sigma^{\mu u}e_R)\Phi B_{\mu u} \end{aligned}$	probed by		
2-lepton 2-quark operators						
$Q^{(1)}_{\ell q} \ Q^{(3)}_{\ell q} \ Q_{eq} \ Q_{\ell d} \ Q_{\ell d} \ Q_{\ell d}$	$egin{aligned} & (ar{L}_L \gamma_\mu L_L) (ar{Q}_L \gamma^\mu Q_L) \ & (ar{L}_L \gamma_\mu au_I L_L) (ar{Q}_L \gamma^\mu au_I Q_L) \ & (ar{e}_R \gamma^\mu e_R) (ar{Q}_L \gamma_\mu Q_L) \ & (ar{L}_L \gamma_\mu L_L) (ar{d}_R \gamma^\mu d_R) \ & (ar{e}_R \gamma_\mu e_R) (ar{d}_R \gamma^\mu d_R) \end{aligned}$	$egin{aligned} Q_{\ell u} \ Q_{eu} \ Q_{\ell edq} \ Q_{\ell edq} \ Q_{\ell equ} \ Q_{\ell equ} \ Q_{\ell equ}^{(1)} \ Q_{\ell equ}^{(3)} \ Q_{\ell equ}^{(3)} \end{aligned}$	$egin{aligned} &(ar{L}_L\gamma_\mu L_L)(ar{u}_R\gamma^\mu u_R)\ &(ar{e}_R\gamma_\mu e_R)(ar{u}_R\gamma^\mu u_R)\ &(ar{L}_L^a e_R)(ar{d}_RQ_L^a)\ &(ar{L}_L^a e_R)\epsilon_{ab}(ar{Q}_L^b u_R)\ &(ar{L}_i^a\sigma_{\mu u}e_R)\epsilon_{ab}(ar{Q}_L^b\sigma^{\mu u}u_R) \end{aligned}$	$\mu \to 3e$ $\mu \to e$		
	Lepton-Hig	ggs operators				
$egin{array}{llllllllllllllllllllllllllllllllllll$	$(\Phi^\dagger i \stackrel{\leftrightarrow}{D}_\mu \Phi) (ar{L}_L \gamma^\mu L_L) \ (\Phi^\dagger i \stackrel{\leftrightarrow}{D}_\mu \Phi) (ar{e}_R \gamma^\mu e_R)$	$Q^{(3)}_{\Phi\ell} \ Q_{e\Phi3}$	$(\Phi^\dagger i \stackrel{\leftrightarrow}{D}{}^I_\mu \Phi) (ar{L}_L au_I \gamma^\mu L_L) \ (ar{L}_L e_R \Phi) (\Phi^\dagger \Phi)$			

 $\mu \rightarrow e\gamma$

$\mu \rightarrow e \gamma$ EXPERIMENTAL RESULTS

- present best bound
 - MEG (2016): MEG coll., hep-ex/1605.05081 $Br(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}$
- future experiment (just started physics data taking)
 - MEG-II (~2025): $Br(\mu^+ \rightarrow e^+\gamma) < 6 \times 10^{-14}$

NEW PHYSICS EXAMPLES FOR $\mu \rightarrow e\gamma$

- any new states with FV couplings to SM leptons will contribute to $\mu \rightarrow e\gamma$
- a selection of examples
 - neutrino mass models
 - see-saw
 - loop generated neutrino masses
 - 2 Higgs Doublet Model
 - low energy supersymmetry
 - extra dimensional models







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 $\mu \rightarrow 3e$

• $\mu^+ \rightarrow e^+ e^- e^+$: tree level or one loop NP contribs. possible







- if NP heavy, can be integrated out
 - then the $\mu \rightarrow 3e$ transition described by an EFT with
 - dipole operators $\bar{\ell}^i_{\rm L} \sigma^{\mu\nu} \ell^j_{\rm B} F_{\mu\nu}$
 - operators

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• four fermion $(\bar{L}_L \gamma_\mu L_L) (\bar{L}_L \gamma^\mu L_L)$ $(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$ $(L_L \gamma_\mu L_L)(\bar{e}_R \gamma^\mu e_R)$

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DIPOLE LIMIT

- if NP such that the dipole contribution dominates
- then $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ rates are related



$$BR(\mu \to eee) \simeq \frac{\alpha}{3\pi} \left(\log \frac{m_{\mu}^2}{m_e^2} - 3 \right) \times BR(\mu \to e\gamma)$$

- in general all operators are present
 - the above operators mix under the RG

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$\mu \rightarrow e$ conversion

$\mu \rightarrow e$ conversion

• initial state: μ^- in 1s orbital



- a theory challenge: predictions require nuclear physics
- there is a small parameter $|\vec{q}| \sim \mathcal{O}(100 \,\text{MeV}) \ll m_N$
 - can use EFT techniques (non-relativistic EFT/chiral EFT)
 - MuonBridge code

Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

$\mu^- N \rightarrow e^- N$ CONVERSION

 results are quoted in terms of normalized conversion rate

 $R_{\mu e} = \operatorname{CR}(\mu N \to eN) \equiv \frac{\Gamma(\mu - e \text{ conversion})}{\Gamma(\text{nuclear capture})}$

 normalization to nuclear capture rate reduces theoretical uncertainties



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COMPLEMENTARY PROBES

• complete list of dim 6 CLFV operators

	4-leptons operators	Dipole operators				
$Q_{\ell\ell}$	$(\bar{L}_L \gamma_\mu L_L) (\bar{L}_L \gamma^\mu L_L)$ $(\bar{a}_D \gamma^\mu c_D) (\bar{a}_D \gamma^\mu c_D)$	Q_{eW}	$(\bar{L}_L \sigma^{\mu\nu} e_R) \tau_I \Phi W^I_{\mu\nu}$ $(\bar{L}_L \sigma^{\mu\nu} e_R) \Phi B$			
$Q_{\ell e}^{}$	$(ar{e}_R\gamma_\mu e_R)(ar{e}_R\gamma^\mu e_R) \ (ar{L}_L\gamma_\mu L_L)(ar{e}_R\gamma^\mu e_R)$	QeB	$(L_L o^* e_R) \Psi D_{\mu\nu}$	prohad by		
2-lepton 2-quark operators						
$\overline{Q^{(1)}_{\ell q}}$	$(ar{L}_L\gamma_\mu L_L)(ar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(ar{L}_L\gamma_\mu L_L)(ar{u}_R\gamma^\mu u_R)$	$\mu \rightarrow 3e$		
$Q_{\ell q}^{(3)}$	$(ar{L}_L\gamma_\mu au_I L_L)(ar{Q}_L\gamma^\mu au_I Q_L)$	Q_{eu}	$(ar{e}_R\gamma_\mu e_R)(ar{u}_R\gamma^\mu u_R)$	$\mu \rightarrow e$		
Q_{eq}	$(ar{e}_R\gamma^\mu e_R)(ar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(ar{L}_L^a e_R) (ar{d}_R Q_L^a)$			
$Q_{\ell d}$	$(ar{L}_L\gamma_\mu L_L)(ar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(ar{L}^a_L e_R) \epsilon_{ab} (ar{Q}^b_L u_R)$			
Q_{ed}	$(ar{e}_R\gamma_\mu e_R)(ar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(ar{L}^a_i\sigma_{\mu u}e_R)\epsilon_{ab}(ar{Q}^b_L\sigma^{\mu u}u_R)$			
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$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(ar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi3}$	$(ar{L}_L e_R \Phi)(\Phi^\dagger \Phi)$			

DIPOLE OPERATOR DOMINANCE

- simplified scenario assume the dipole operator dominates
- interesting to compare the reach of different experiments

$$\begin{split} \mathrm{BR}(\mu \to e e e) &\simeq \frac{\alpha}{3\pi} \bigg(\log \frac{m_{\mu}^2}{m_e^2} - 3 \bigg) \times \mathrm{BR}(\mu \to e \gamma) \,, \\ \mathrm{CR}(\mu \; \mathrm{N} \to e \; \mathrm{N}) &\simeq \alpha \times \mathrm{BR}(\mu \to e \gamma) \,. \end{split}$$


UPSHOT

- several different probes in rare muon decays
 - can probe different types of new physics
 - also disentangle different contributions
- significant improvements projected

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LFV IN 7 DECAYS

LFV 7 DECAYS

- several important differences relative to muons
- experimental:
 - τ lifetime is short \Rightarrow no "tau beams"



- need to be produced in $e^+e^- \rightarrow \tau^+\tau^-$ (Belle II) or in *pp* collisions (LHC)
- smaller experimental samples compared to muons
- τ is heavier, $m_{\tau} = 1.777$ GeV, many decay modes possible
- theoretical:
 - the models that lead to CLFV in muons tend to give CLFV tau decays
 - often couplings to 3rd generation are larger (motivated by flavor structure in the SM)

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FUTURE REACH

 significant improvements in the experimental reach expected

Akar et al., 1812.07638

example for tau: Belle 2 and HL-LHC reach



Number of τ pairs
~3.3 x 10⁵
~1 x 10 ⁷
~5 x 10 ⁸
~9 x 10 ⁸
~4.6 x 10 ¹⁰
~2.1 x 10 ¹⁰

E REACH

ements in the

n expected

Akar et al., 1812.07638

elle 2 and HL-LHC reach



NEW PHYSICS IN TAU DECAYS

- two categories of LFV tau decays
 - purely leptonic: $\tau \rightarrow \mu\gamma, \tau \rightarrow 3e, \tau \rightarrow 3\mu, \dots$

• NP can be purely leptophilic

• also involving hadrons:

 $\tau \to \mu \rho, \tau \to e \rho, \tau \to \mu K_{S'} \dots$

- NP needs to couple to both leptons and quarks
- the quark couplings may or may not be flavor violating
- comparison with FCNC muon decays
 - need concrete models to compare muon and tau decays

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- heavy new physics only part of the NP parameter space
- light particles: a window to high UV dynamics



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FLAVOR PORTAL

- example of a flavor portal: dim 5 op. $\partial_{\alpha} \varphi(\bar{e}\gamma^{\alpha}\gamma_{5}\mu)/f_{a} \Rightarrow Br(\mu \to e\varphi) \propto (m_{W}^{2}/f_{a}m_{\mu})^{2}$
- searching for $K \to \pi X$, $\mu \to eX$, $\pi \to X$ decays expect to reach very high UV scales



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FLAVOR PORTAL

- example of a flavor portal: dim 5 op. $\partial_{\alpha} \varphi(\bar{e}\gamma^{\alpha}\gamma_{5}\mu)/f_{a} \Rightarrow Br(\mu \to e\varphi) \propto (m_{W}^{2}/f_{a}m_{\mu})^{2}$
- searching for K → πX, μ → eX, π → X decays expect to reach very high UV scales



ALPS



$\mu \rightarrow 5e$

- if $\frac{m_{\mu}}{\Lambda} \phi(\bar{e}\mu)$ coupling \Rightarrow mediates $\mu \to e\phi$
 - if φ QCD axion \Rightarrow escapes the detector $\mu \rightarrow e + inv$
 - MEG-II, Mu3e, Mu2e-X, COMET-X can search for it
 - if φ can decay \Rightarrow sensitivity to even higher scales
 - example: $\mu \to 5e$ can probe $f_a \gtrsim 10^{13} \text{GeV}$



Hostert, Menzo, Pospelov, JZ, 2306.15631

$$\mu \rightarrow 5e$$

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HIGGS AS A PROBE OF FLAVOR

DUAL ROLE

- in the SM Higgs has a dual role
 - breaks electroweak symmetry and gives the masses to W, Z gauge bosons
 - same EWSB source gives the masses to the SM fermions
- how well have we tested this?



DUAL ROLE OF THE HIGGS



TESTING THE FLAVOR OF THE HIGGS

Nir, 1605.00433; JZ, 1903.05062

- several questions
 - proportionality $y_{ii} \propto m_i$
 - factor of proportionality

$$y_{ii}/m_i = \sqrt{2}/v$$

 diagonality (flavor violation)

$$y_{ij} = 0, \quad i \neq j$$

• reality (CP violation)
 $\operatorname{Im}(y_{ij}) = 0$

$$y_f^{\rm SM} = \sqrt{2}m_f/v$$



FLAVOR VIOLATING COUPLINGS

- in the SM Higgs couplings flavor diagonal
 - discovering flavor violating couplings mean New Physics
- for charged lepton final states accessible directly
 - from $h \rightarrow \tau \mu$, $h \rightarrow \tau e$

INDIRECT BOUNDS ON $h \rightarrow \tau \mu$

Harnik, Kopp, JZ, 1209.1397

see also Blankenburg, Ellis, Isidori, 1202.5704

indirect bounds from charged lepton FCNC transitions



FLAVOR VIOLATING COUPLINGS

- accessible directly for charged lepton final states
 - from $h \rightarrow \tau \mu$, $h \rightarrow \tau e$

FLAVOR VIOLATING COUPLINGS



INDIRECT BOUNDS ON $h \rightarrow e\mu$

Harnik, Kopp, JZ, 1209.1397

• indirect bounds especially severe for $h \rightarrow e\mu$

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- $Br(h \rightarrow e\mu) < 10^{-8}$ required to surpass the bound from $Br(\mu \rightarrow e\gamma)$
- caveat: could be cancellations in the loop



CONCLUDING REMARKS

- charged lepton flavor violating probes give us access to physics at very high scales
- both light and heavy NP of interest
- especially interesting in view of experimental anomalies involving muons

BACKUP SLIDES

QUARKS VS. LEPTONS

- when comparing quark and lepton sector of the Standard Model we observe:
- leptons of the same generation are lighter than quarks
 - smaller number of kinematically allowed decay modes for τ,
 μ than for t,b,c
 - e.g., $B^- \to \tau^- \bar{\nu}_{\tau}$ allowed, while $\tau^- \to B^- \nu_{\tau}$ is not
- quarks carry color \Rightarrow bound inside hadrons
 - lepton decays are simpler to predict
- "up" leptons' (= ν 's) mass \ll "down" leptons' (= ℓ ') mass
 - absolute neutrino masses not yet known
 - in many processes neutrino masses can be neglected



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LFV QCD AXION

- DFSZ-like model: 2HDM+S: $X_S = 1, X_{H_2} = 2 + X_{H_1}$
- flavor universal $U(1)_{PQ}$ charges in quark sector, nonuniversal in leptonic Yukawa coupl. to H_1 Yukawa coupl. to H_2

$$y_{e} = \begin{pmatrix} 0 & \mathbf{x} & \mathbf{x} \\ \mathbf{x} & 0 & 0 \\ \mathbf{x} & 0 & 0 \end{pmatrix}, \quad y'_{e} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \mathbf{x} & \mathbf{x} \\ 0 & \mathbf{x} & \mathbf{x} \end{pmatrix} \Rightarrow \text{ gives lepton FV coupl.s of axion}$$
$$y_{u} = \begin{pmatrix} \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \end{pmatrix}, \quad y_{d} = \begin{pmatrix} \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \\ \mathbf{x} & \mathbf{x} & \mathbf{x} \end{pmatrix} \Rightarrow \text{ axion-quark couplings flavor diagonal}$$

• hierarchy of entries external input

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LFV QCD AXION

two benchmarks, assume just 1-2 mixing



LEPTONIC FAMILON

- separate Froggatt-Nielsen U(1) for quarks and leptons
 - leptonic f_a scale assumed lighter \Rightarrow these couplings dominate

 $([L]_1, [L]_2, [L]_3) = (L, L, L),$

[Pure Anarchy].

[Hierarchy].

$$\Rightarrow$$
 RH ALP

 \Rightarrow LH and

RH couplings

• two benchmark charge assignments

 $([L]_1, [L]_2, [L]_3) = (L+2, L+1, L),$

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LEPTONIC FAMILON



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LEPTONIC FAMILON



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MAJORON

- majoron- PNGB due to spontaneous breaking of the lepton number
- neutrino masses $m_{\nu} \propto y_{\nu} y_{\nu}^T v^2 / m_N$
- majoron couplings, $C_{ij} \propto y_{\nu} y_{\nu}^{\dagger}$
- if m_{ν} suppressed by global U(1)
 - \Rightarrow majoron observable
 - "low energy see-saw"

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"low energy see-saw"

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NEW PHYSICS: SEE SAW EXAMPLE

- a simple example of new physics probed by $\mu \rightarrow e\gamma$
- a see-saw model for neutrino masses

• allow for Majorana mass term for ν_R

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{\nu}_R \partial \!\!\!/ \nu_R - \left(Y_{\nu} \overline{\nu}_R \widetilde{\Phi}^{\dagger} L_L + \frac{1}{2} M_R \overline{\nu}_R \nu_R^c + \text{h.c.} \right).$$

Dirac mass term \Rightarrow **mixing of** ν_L **and** ν_R

- mass spectrum consists of Majorana neutrinos
 - 3 heavy states, mostly ν_R with masses ~ M_R
 - 3 light neutrinos, mostly ν_L mass matrix

$$m_{\nu} = -\frac{v^2}{2} Y_{\nu}^T M_R^{-1} Y_{\nu}$$

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SEE SAW AND $\mu \rightarrow e\gamma$

- due to ν_L and ν_R mixing
 - PMNS matr. does not diagonalize the full $\nu_{L,R}$ mass matrix
 - the mixing matrix \mathscr{U} entering the $W \mathscr{C} \nu$ vertex is not unitary $\mathcal{U} = \left(1 - \frac{v^2}{2} V^{\dagger} M^{-2} V\right) U$ note: in m_{ν} we

$$\mathcal{U} = \left(1 - \frac{v^2}{2} Y_{\nu}^{\dagger} M_R^{-2} Y_{\nu}\right) U.$$

note: in m_{ν} we have Y_{ν}^T not Y_{ν}^{\dagger}

• modified prediction for $\mu \rightarrow e\gamma$

$$BR(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \frac{\left|\sum_{k} \mathcal{U}_{\mu k} \mathcal{U}_{ek}^{*} F(x_{k})\right|^{2}}{(\mathcal{U}\mathcal{U}^{\dagger})_{\mu\mu} (\mathcal{U}\mathcal{U}^{\dagger})_{ee}},$$

$$F(x_k) = \frac{10}{3} - x_k + \mathcal{O}\left(x_k^2\right).$$
$$x_k = \frac{m_{\nu_k}^2}{M_W^2}$$

- GIM mechanism no longer fully operational
- $Br(\mu \rightarrow e\gamma)$ not suppressed by light ν masses, can be larger

SEE SAW ANI

- due to ν_L and ν_R mixing
 - PMNS matr. does not diagonalize the f $U_{\mu k}$ mass
 - the mixing matrix \mathscr{U} entering the $W \mathscr{C} \nu$ vertex is not unitary $\mathcal{U} = \left(1 - \frac{v^2}{2}Y_{\nu}^{\dagger}M_R^{-2}Y_{\nu}\right)U.$ note: in m_{ν} we have Y_{ν}^T not Y_{ν}^{\dagger}

U

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'* ek

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HEAVY NEW PHYSICS

- if there is heavy NP, can be integrated out
 - results in SM Effective Field Theory (SMEFT)
 - renormalizable SM supplemented by higher dimensional operators

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{\Lambda} \sum_{a} C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2} \sum_{a} C_a^{(6)} Q_a^{(6)} + \dots$$

• $\mu \rightarrow e\gamma$ results in a dimension 6 operator

$$\mathcal{L} \supset -\frac{\sqrt{2}e\,v}{(4\pi\Lambda_{ij})^2}\,\bar{\ell}^i_{\rm L}\sigma^{\mu\nu}\ell^j_{\rm R}F_{\mu\nu} + \text{h.c.} ,$$

• exp. bounds imply that it is highly suppressed

$$\mu \to e\gamma \Rightarrow \Lambda_{21} \gtrsim 3500 \,\mathrm{TeV}$$

Greljo, Stangl, Thomsen, 2103.13991

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$\mu \rightarrow e \gamma$ experiments

- in muon rest frame e and γ are monochromatic
 - $E_e = E_{\gamma} \simeq m_{\mu}/2 \simeq 52.8 \,\mathrm{MeV}$
- convenient to perform experiments with stopped muons
 - use μ⁺ so that it does not get bound to nucleus, i.e., avoid the spread of line from decay in orbit
 - the measured process is thus $\mu^+ \rightarrow e^+ \gamma$
- muons are stopped in the thinnest possible targets
 - so that the e^+ do not loose energy when escaping
 - search for monochromatic e^+ line at the kinematical edge of SM $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ decay (the "Michele edge")
 - require coincidence with a photon of the same energy
 - energy resolution very important to reduce SM background
 - irreducible background is the SM decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \gamma$



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EXPERIMENTS

- also use stopped μ^+ so the lab frame is the muon rest frame
- $\mu^+ \rightarrow e^+ e^- e^+$ is a 3-body decay, so no mono-energetic particle

• maximal energy for each *e* is $E_{\text{max}} \simeq m_{\mu}/2$

- the signature is
 - 2*e*⁺ and 1*e*⁻ coming from common vertex (and nothing else)
 - their energy adds up to m_{μ}
- the main "irreducible" SM background $\mu^+ \rightarrow e^+ e^- e^+ \bar{\nu}_{\mu} \nu_e$ decay
 - two neutrinos appear as missing energy E_{inv}
 - need very precise energy measurement to make sure $E_{e^+} + E_{e^-} + E_{e^+} = m_\mu$

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present best bound

• SINDRUM (1988): $Br(\mu^+ \to e^+ e^- e^+) < 1.0 \times 10^{-12}$

- future
 - Mu3e: Phase 1 (~2025): $Br(\mu \to 3e) < 2 \times 10^{-15}$ Phase 2 (2030s): $Br(\mu \to 3e) \lesssim 10^{-16}$











 $< 2 \times 10^{-15}$ $\lesssim 10^{-16}$

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$\mu^- N \rightarrow e^- N$ CONVERSION

- experimentally $\mu \rightarrow e$ conversion offers many advantages over, e.g., $\mu \rightarrow e\gamma$
 - the only intrinsic bckgd is $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ decay in orbit
 - in $\mu^- N \rightarrow e^- N$ the e^- is at the kinematical edge of DIO



$\mu^- N \rightarrow e^- N$ CONVERSION

- present bound
 - SINDRUM-II (1993, 2006): $R_{\mu e} < 6.1(7.1) \times 10^{-13}$ on Ti (Au)

Physics Letters B 1993, 317, 631 Eur. Phys. J. C 2006, 47

- future (on C)
 - DeeMee: $R_{\mu e} \leq 1(0.2) \times 10^{-13}$ on C (SiC)
- future (on Al)
 - COMET Phase 1: $R_{\mu e} \lesssim 10^{-15}$
 - Mu2e & COMET Phase-II: $R_{\mu e} \lesssim 10^{-17}$
 - Mu2e-II: $R_{\mu e} \lesssim 10^{-18}$

SUPERSYMMETRIC SEE-SAW

- in general there are many flavor violating parameters even in the minimal SUSY see saw model
 - 124 from minimal SUSY SM (MSSM)
 - another 18 in the neutrino sector
- most of these related to SUSY breaking
 - the form of slepton and squark mass matrices
- focus on a very restricted case: constrained MSSM Antusch et al., hep-ph/0607263
 - SUSY breaking parameters are assumed to be flavor universal at the UV scale (=GUT scale)
- all LFV originates solely from the neutrino sector
 - some of the parameters are fixed by requiring to reproduce neutrino masses and PMNS, scanned over the rest

$$m_{\nu} = -\frac{v^2}{2} Y_{\nu}^T M_R^{-1} Y_{\nu}$$

• FV in slepton mass matrices from RGEs

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SUPERSYMMETRIC SEE-SAW

 the dominant LFV contribution comes from dipole operators ("photon penguin")



• the $\ell_i \to 3\ell_i$ are thus given by

Antusch et al., hep-ph/0607263

$$BR(l_j \to 3l_i) = \frac{\alpha}{3\pi} \left(\log \frac{m_{l_j}^2}{m_{l_i}^2} - \frac{11}{4} \right) \times BR(l_j \to l_i \gamma),$$

• because of restricted flavor structure there is also a relation between $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$



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FN SOLUTION TO THE FLAVOR PUZZLE

Froggatt, Nielsen, NPB 147, 277 (1979),...

- Large hierarchies in quark + lepton masses and in CKM matrix
 - can be addressed via horizontal $U(1)_{FN}$ symmetry
 - SM LH and RH fermions have different $U(1)_{FN}$ charges
 - hierarhical Higgs Yukawas after $U(1)_{\rm FN}$ broken via vev of scalar field, the flavon Φ
 - if $U(1)_{\rm FN}$ gauged there is an associated Z'



SPURION ANALYSIS



• effective Yukawas governed by flavon insertions (so that invariant under flavor symm.)

$$\mathcal{L}_{eff} \sim \left(\frac{\phi}{\Lambda_F}\right)^{\omega_{ij}} h \,\overline{q}_i u_j \qquad \epsilon \equiv \frac{\phi}{\Lambda_F}$$

- hierarchy from powers of small parameter ε
- FN mechanism involves
 - vector-like fermions + scalar flavon fields (no anomaly)
 - chiral fields at the end of the chains: in general anomalous $U(1)_{\rm FN}$
 - we show the results for an anomaly free $U(1)_{FN}$ (inverted FN) that is gauged

FLAVORFUL Z'

 for U(1)_{FN} benchmark, assuming anarching neutrino mass from Weinber op.



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• for U(1)_{FN} benchmark, as anarching neutrino mass





beam dumps

0.01

 10^{-5}

60

TAU DECAYS

- in this model tau decays less sensitive as discovery tool
- but essential to be measured in order to confirm the model



FLAVORFUL QCD AXION

• if QCD axion has $\partial_{\mu}a(\bar{d}\gamma^{\mu}\gamma_5 s)/f_a$ coupling $\Rightarrow K^+ \rightarrow \pi^+ a$ decay a very sensitive probe



