Lepton Flavour Violation at Belle II
(with EFTs)

Belle II Physics Week - 2024

Marco Ardu
Univ. Valencia & IFIC
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Vniversitat de València
CSIC
Consejo Superior de Investigaciones Científicas
IFIC
Institut de Física Corpuscular

Neutrino masses imply Lepton Flavour Violation

The Standard Model Lagrangian (without right-handed neutrinos) is accidentally invariant under a phase rotation of each lepton flavor $U(1)_{L_\alpha}$

$$\ell_\alpha = \begin{pmatrix} \nu_\alpha \\ \alpha_L \end{pmatrix}, e_\alpha = \alpha_R \quad \text{with} \quad \alpha = e, \mu, \tau$$

$$U(1)_{L_\alpha} : \begin{cases} \ell_\alpha \to e^{i\chi_\alpha}\ell_\alpha \\ e_\alpha \to e^{i\chi_\alpha}e_\alpha \end{cases}$$

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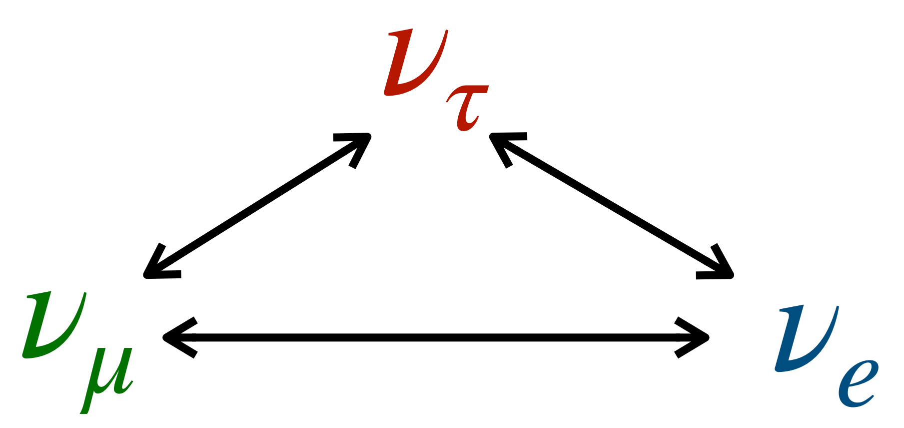
Neutrino masses break all symmetries



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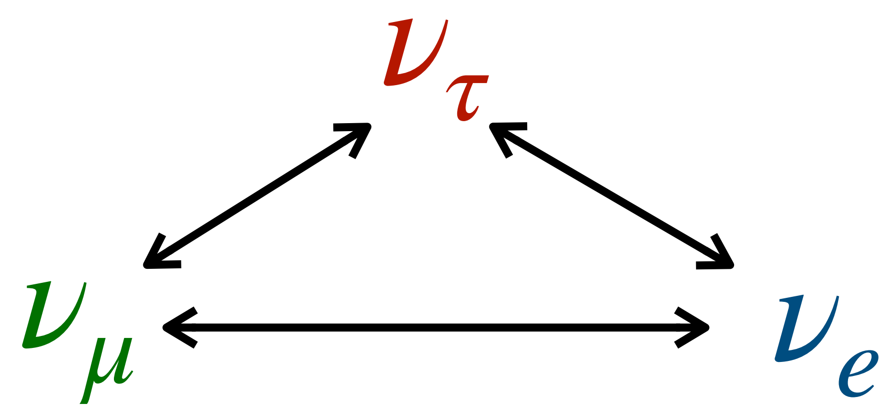
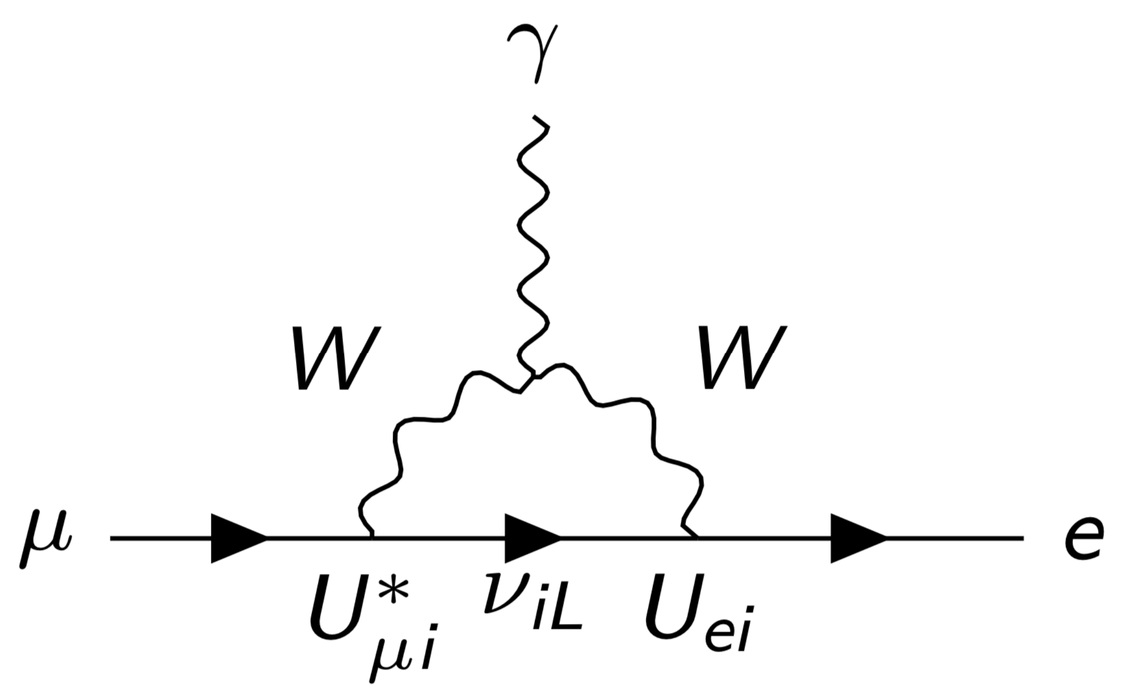


Since there is no symmetry that forbids it, lepton flavour violation in the charged sector is inevitable:

$$\mu^\pm \to e^\pm \gamma \qquad \tau^\pm \to e^\pm e^+ e^- \qquad h \to \tau^\pm \mu^\mp \ldots$$

must happen, but at what rates?

Charged Lepton Flavour Violation (LFV)



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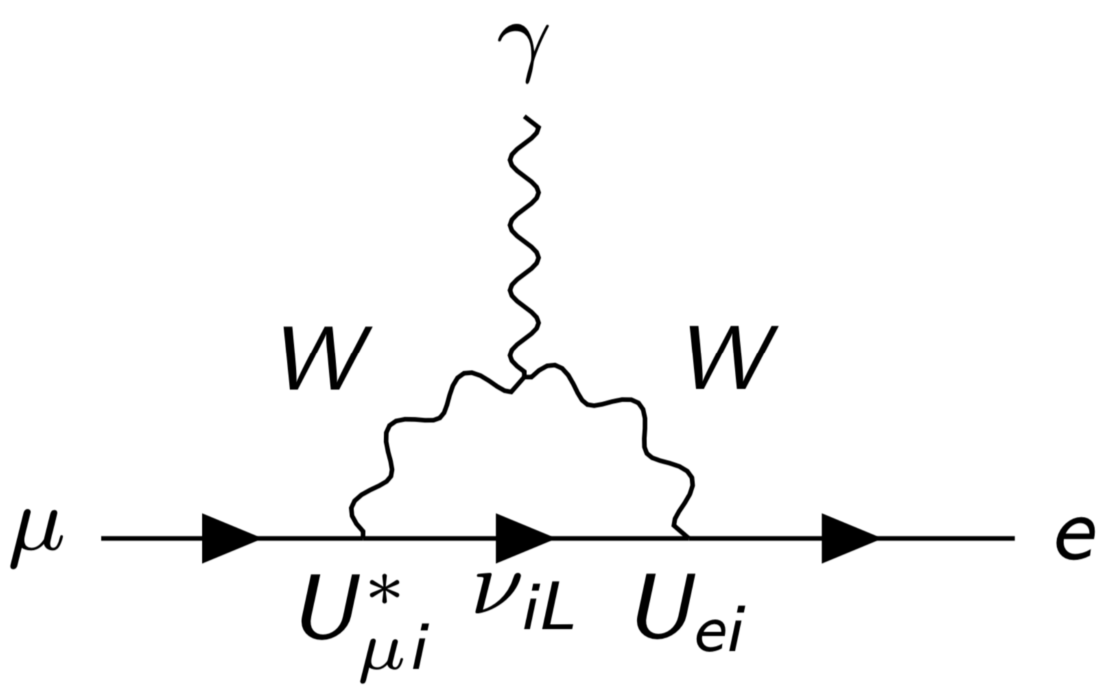

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- An observation of LFV would be a clear signature of new physics

- It could shed light on the mechanism behind neutrino masses (and potentially on the baryon asymmetry if generated via leptogenesis?)

- Many models that address unresolved puzzles (independently from neutrino masses) predict potentially observable LFV signals

Experimental searches at Belle-II

- Meson decays

Snowmass tau CLFV, 2203.14919

$B^0 \to e^\pm \tau^\mp$	2.8×10^{-5}	BaBar [110]
$B^0 \to \mu^\pm \tau^\mp$	2.2×10^{-5}	BaBar [110]
	1.2×10^{-5}	LHCb [62]
$B^+ \to \pi^+ e^\pm \tau^\mp$	7.5×10^{-5}	BaBar [111]
$B^+ \to \pi^+ \mu^\pm \tau^\mp$	7.2×10^{-5}	BaBar [111]
$B^+ \to K^+ e^\pm \tau^\mp$	3.0×10^{-5}	BaBar [111]
$B^+ \to K^+ \mu^\pm \tau^\mp$	4.8×10^{-5}	BaBar [111]
$B^+ \to K^+ \mu^- \tau^+$	3.9×10^{-5}	LHCb [63]
$B_s^0 \to \mu^\pm \tau^\mp$	3.4×10^{-5}	LHCb [62]
$\Upsilon(1S) \to e^\pm \tau^\mp$	2.7×10^{-6}	Belle [112]
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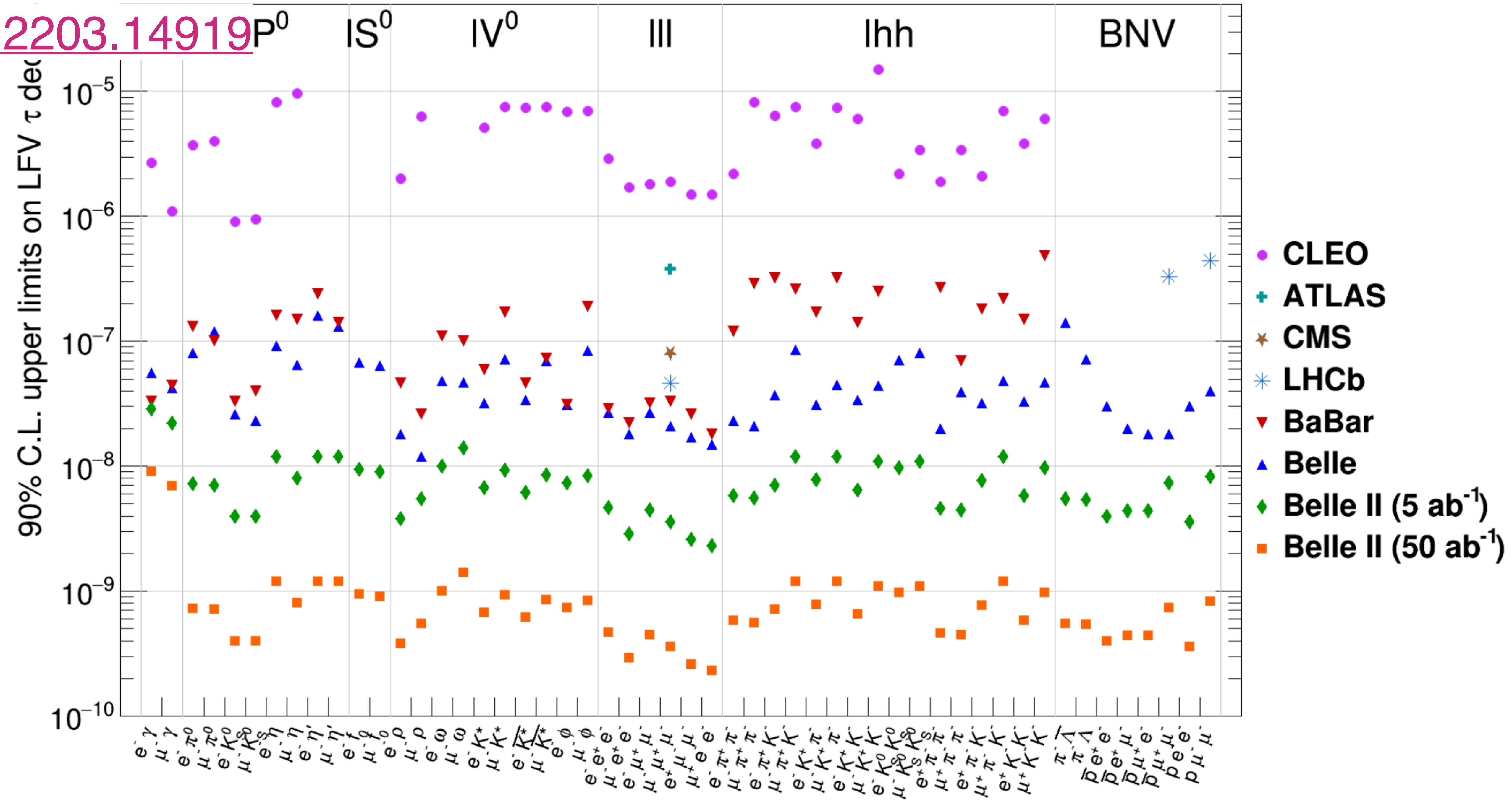


$B^0 \to e\mu$	1.0×10^{-9}
$B^+ \to \pi^+ e\mu$	9.2×10^{-8}
$B_s \to e\mu$	5.4×10^{-9}
$B^+ \to K^+ e\mu$	1.8×10^{-8}
$B^0 \to K^{*0} e\mu$	1.0×10^{-8}
$B_s \to \phi e\mu$	1.6×10^{-8}

Becirevic et al 2407.19060

- $\tau \to l$ decays

Not covered here, see L. Calibbi's talk

$$\tau \to eX \lesssim 10^{-3} - 10^{-2} \times Br(\tau \to e\nu\nu)$$

$$\tau \to \mu X \lesssim 10^{-2} - 10^{-3} \times Br(\tau \to \mu\nu\nu)$$

Belle-II, 2212.03634

$\tau \to l$ transitions

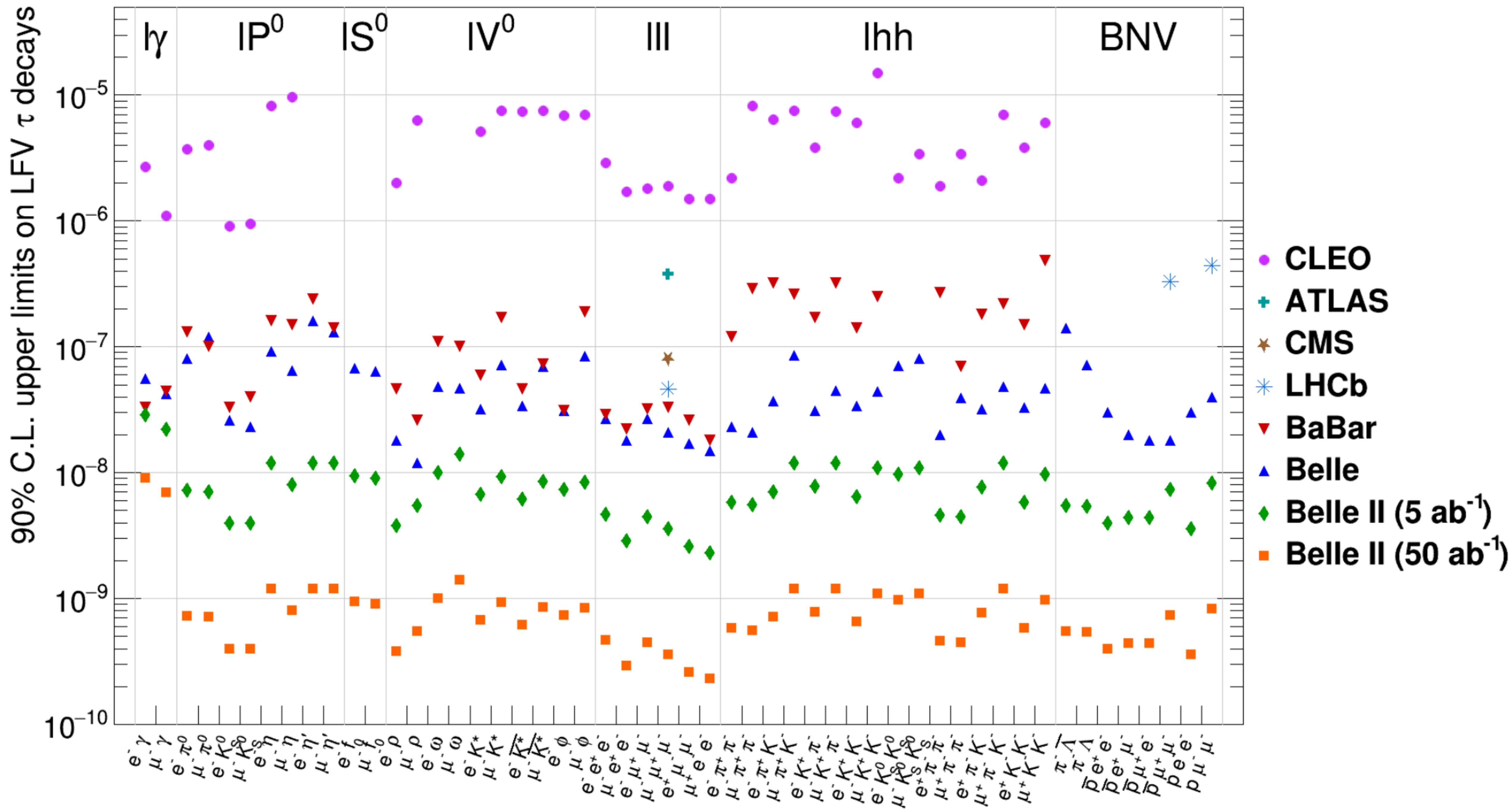
- The sensitivities of $\tau \to l$ processes are $Br(\tau \to l) \lesssim 10^{-8} \to 10^{-10}$ (**Belle-II,** Belle, LHC(b), BaBar)

- If we see $\tau \to l$ in the near future, it should be relatively large

- The big phase available means there is a plethora of different channels (possible to overconstrain models = distinguish them)

- Belle-II has the best projections in all channels

- High energy probes are sometimes competitive with the decays



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Lepton Flavour Triality Ma, 1006.3524

$$\ell_\alpha \to \left(e^{i\frac{2\pi}{3}}\right)^{T_\alpha} \ell_\alpha$$

$T_e = 1$

$T_\mu = 2$

$T_\tau = 3$

$\mu^- \to e^- \gamma$

$T_\mu = 2 \quad T_e = 1$

$\Delta T \neq 0$

$\tau^- \to \mu^+ e^- e^-$

$3 \quad -2 \quad +1 \quad +1$

$\Delta T = 0 \mod 3$

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$\mu^- \to e^- \gamma$: $T_\mu = 2$, $T_e = 1$, $\Delta T \neq 0$

$\tau^- \to \mu^+ e^- e^-$: 3, -2, $+1$, $+1$, $\Delta T = 0 \mod 3$

- In general, new states that dominantly couple with third generation fermions may lead to larger LFV involving taus

B meson decays

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- The SM has an approximate $U(2)^5$ symmetry (because only third generation Yukawas are large)

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Barbieri+Isidori 2312.14004
Greljo+Isidori 2406.01696

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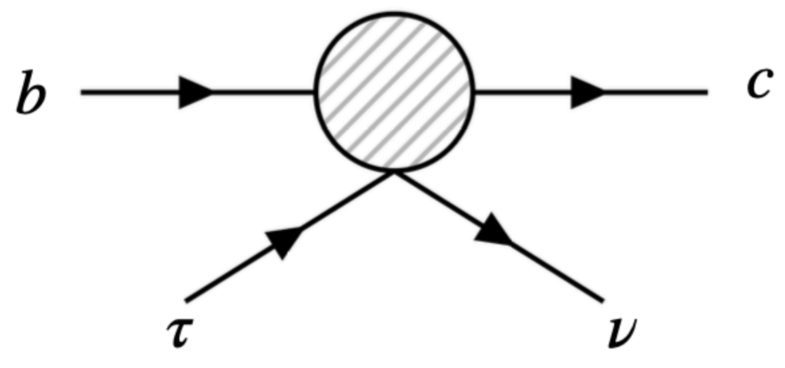

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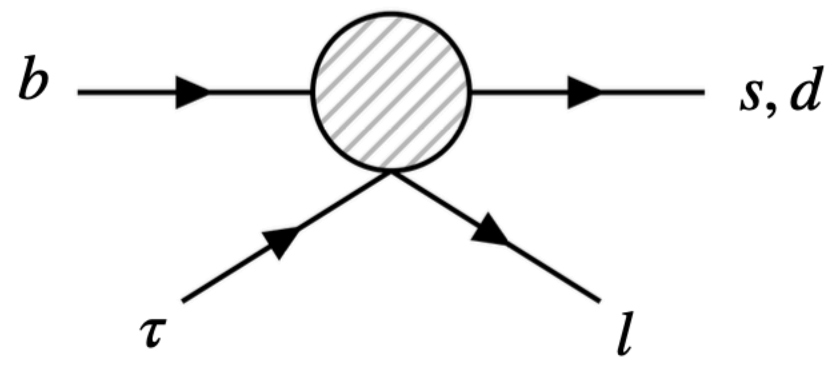


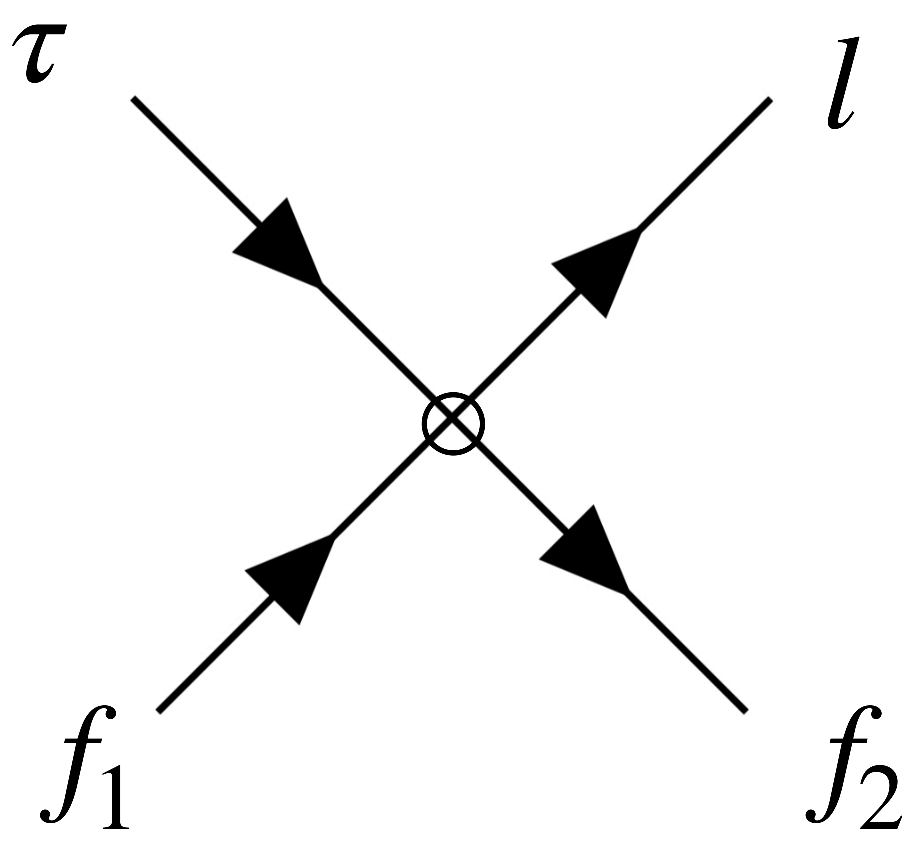
?

A jungle of possible models that give LFV…

Effective Field Theories

- If LFV New Physics is heavy* $(\Lambda \gtrsim \text{few TeV})$, it can be parametrised in terms of non-renormalizable operators

*See L. Calibbi talk for light new physics



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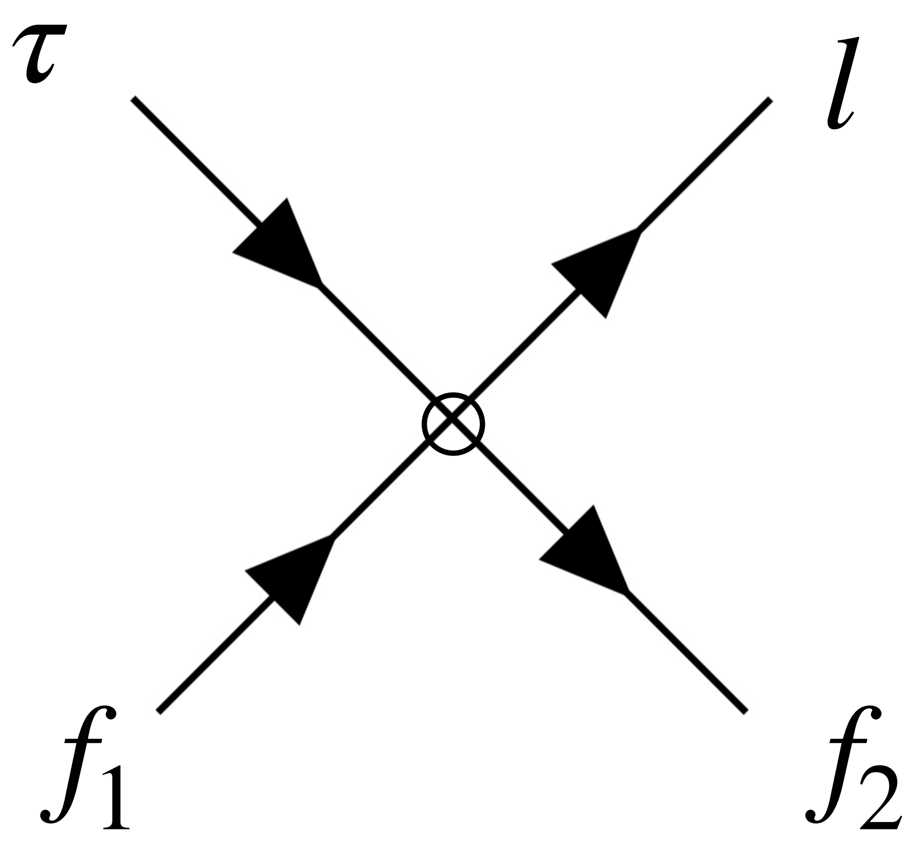

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- Add to the Lagrangian the relevant contact interactions (non-renormalizable operators) compatible with the symmetries

$$\mathscr{L}_{\text{EFT}} = \mathscr{L}_{d\leq 4} + \sum_{n>4} \frac{C_n \mathcal{O}_n}{\Lambda^{n-4}}$$

and calculate observables…

Outline

- Leptonic τ decays ($\tau \to l_i\gamma$, $\tau \to l_i\bar{l}_k l_k$, $\tau \to \bar{l}_i l_k l_k$)
- Semi-leptonic τ decays (ex: $\tau \to \pi l_i$)
- Meson decays
- Conclusion

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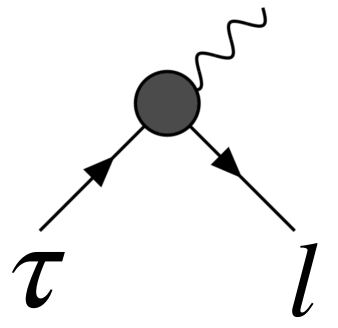
LFV Radiative decay: branching ratio



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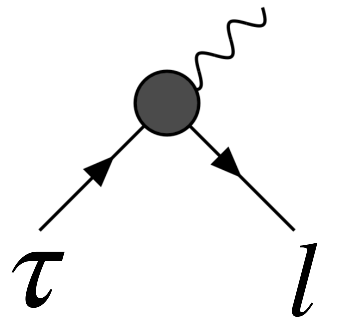

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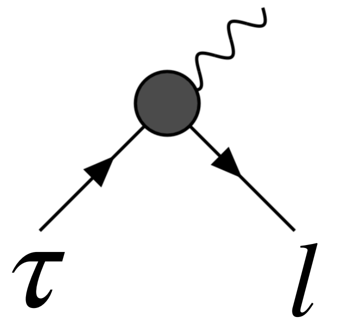

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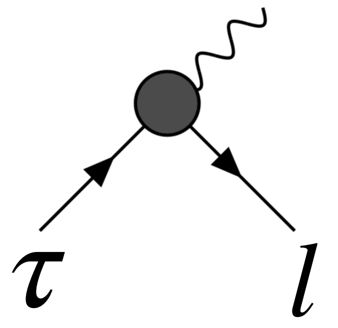

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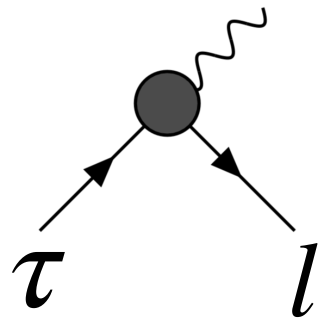

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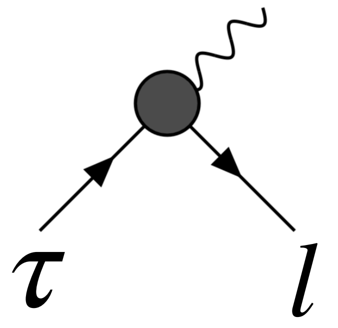

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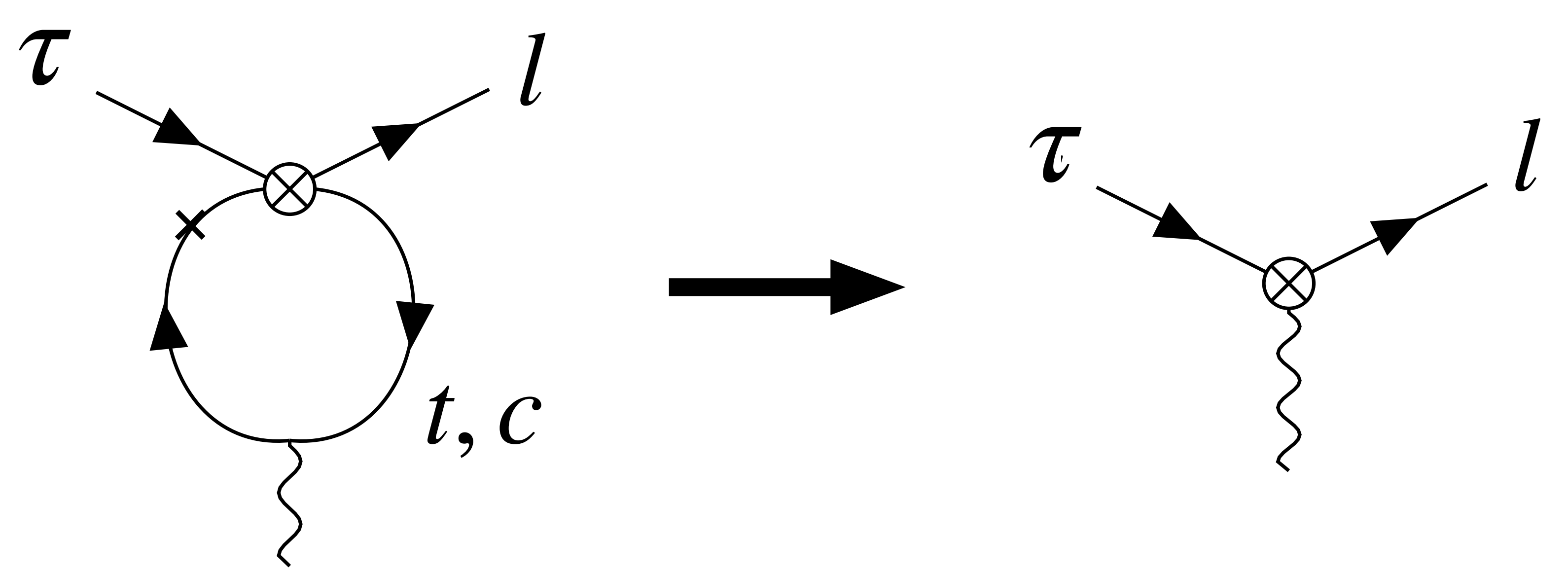
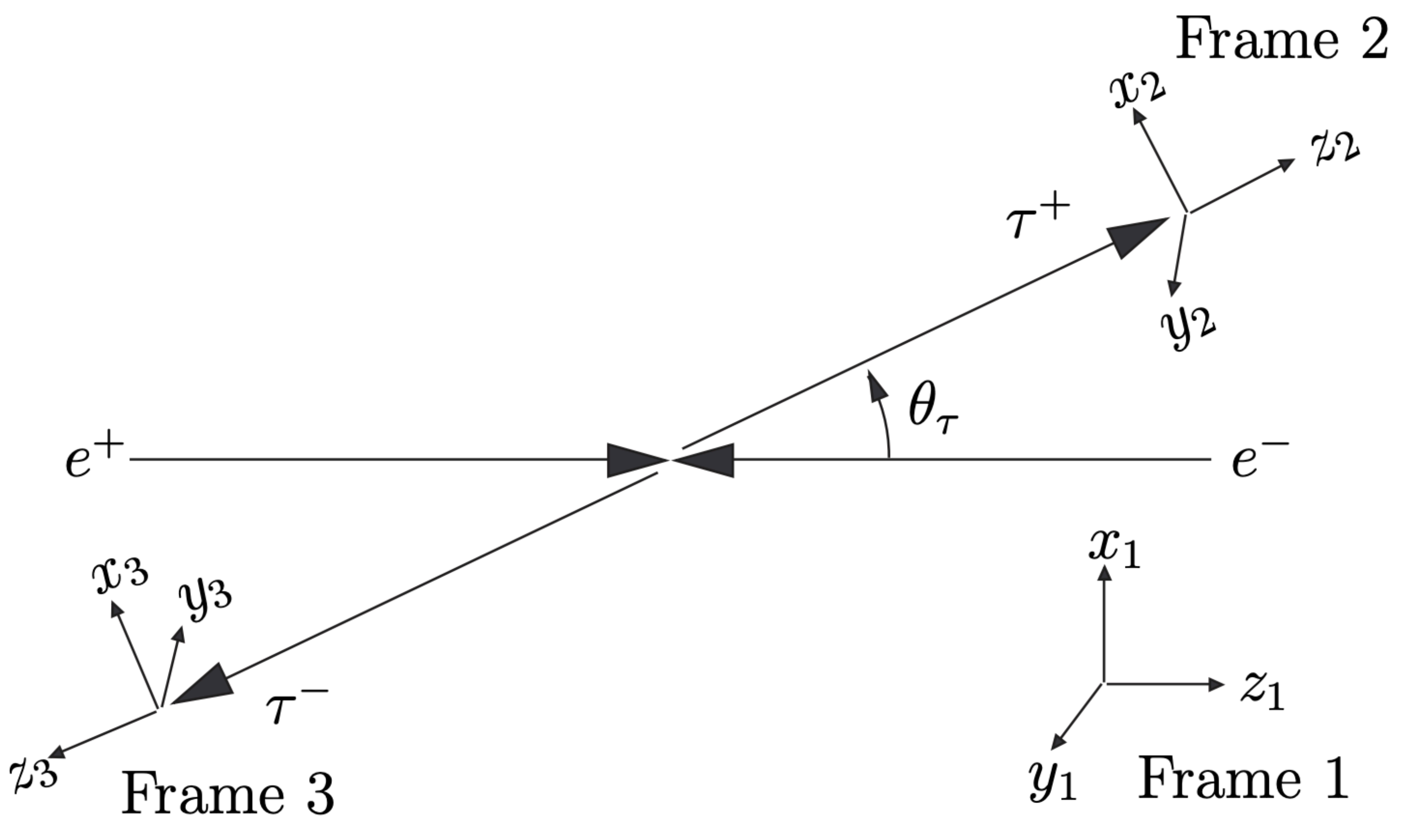
$C^{qq}_{T,X}(\bar{l}\sigma P_X\tau)(\bar{q}\sigma P_X q)$



Kitano, Okada hep-ph/0012040

LFV Radiative decay: distinguishing chiralities



$$dR_b^{\tau^+\to l^+\gamma} = \frac{d\Omega_l}{4\pi}\frac{1}{\Gamma}\frac{2}{\pi}G_{\mathrm{F}}^2 m_\tau^5\left(\left|C_{D,L}^{l\tau}\right|^2 - \left|C_{D,R}^{l\tau}\right|^2\right)\begin{pmatrix}\sin\theta_{l^+}\cos\phi_{l^+}\\ \sin\theta_{l^+}\sin\phi_{l^+}\\ \cos\theta_{l^+}\end{pmatrix}$$

- Angles in Frame 2, and taking the normalization $\Lambda = v$ for the dipoles

$$dR_a^{\tau^-\to l^-\bar{\nu}\nu} = \frac{d\Omega_l}{4\pi}dx\frac{1}{\Gamma}\frac{G_{\mathrm{F}}^2 m_\tau^5}{192\pi^3}2x^2(1-2x)\begin{pmatrix}\sin\theta_{l^-}\cos\phi_{l^-}\\ \sin\theta_{l^-}\sin\phi_{l^-}\\ \cos\theta_{l^-}\end{pmatrix}$$

- Angles in Frame 3 and $x = (2E_{l^-})/m_\tau$

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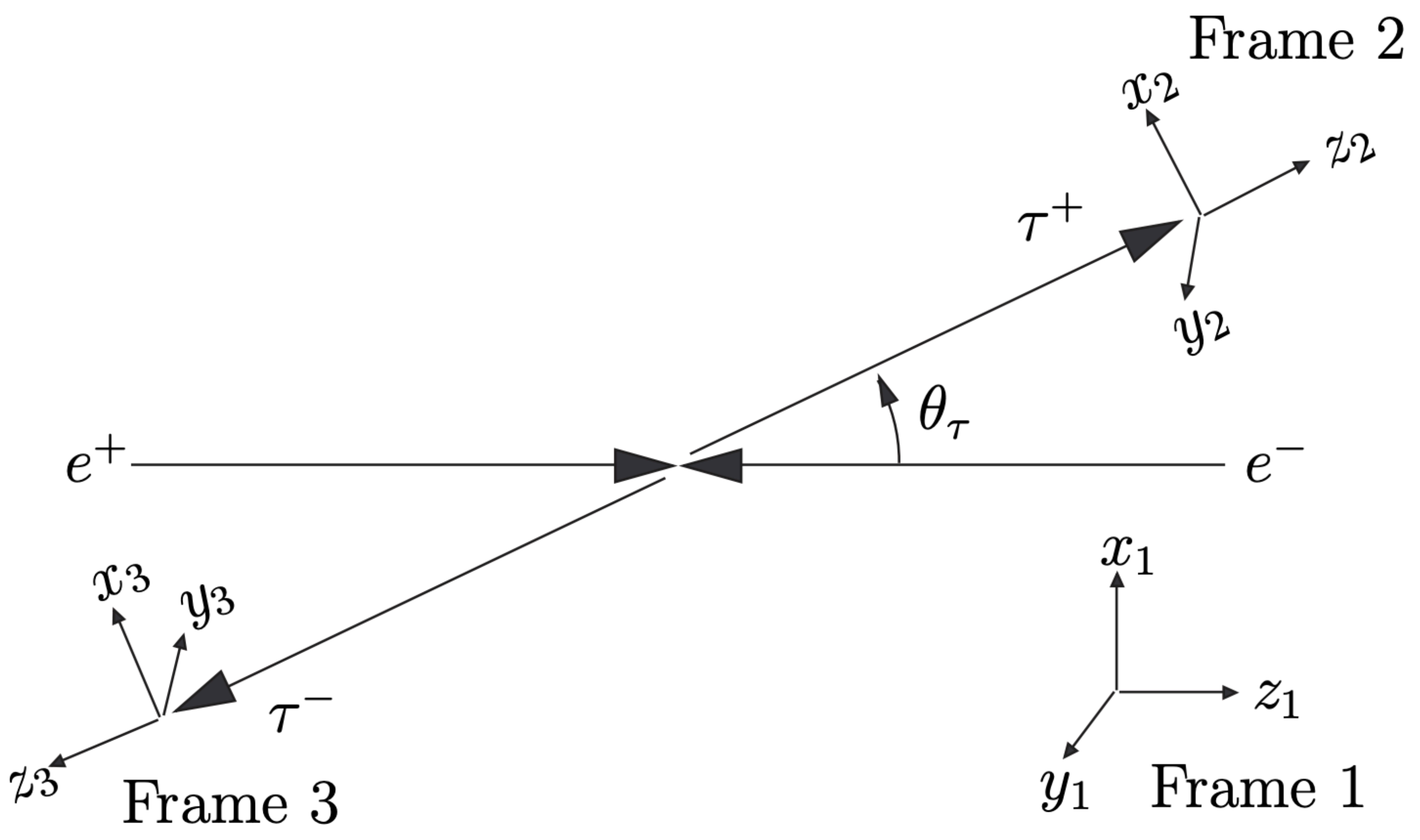

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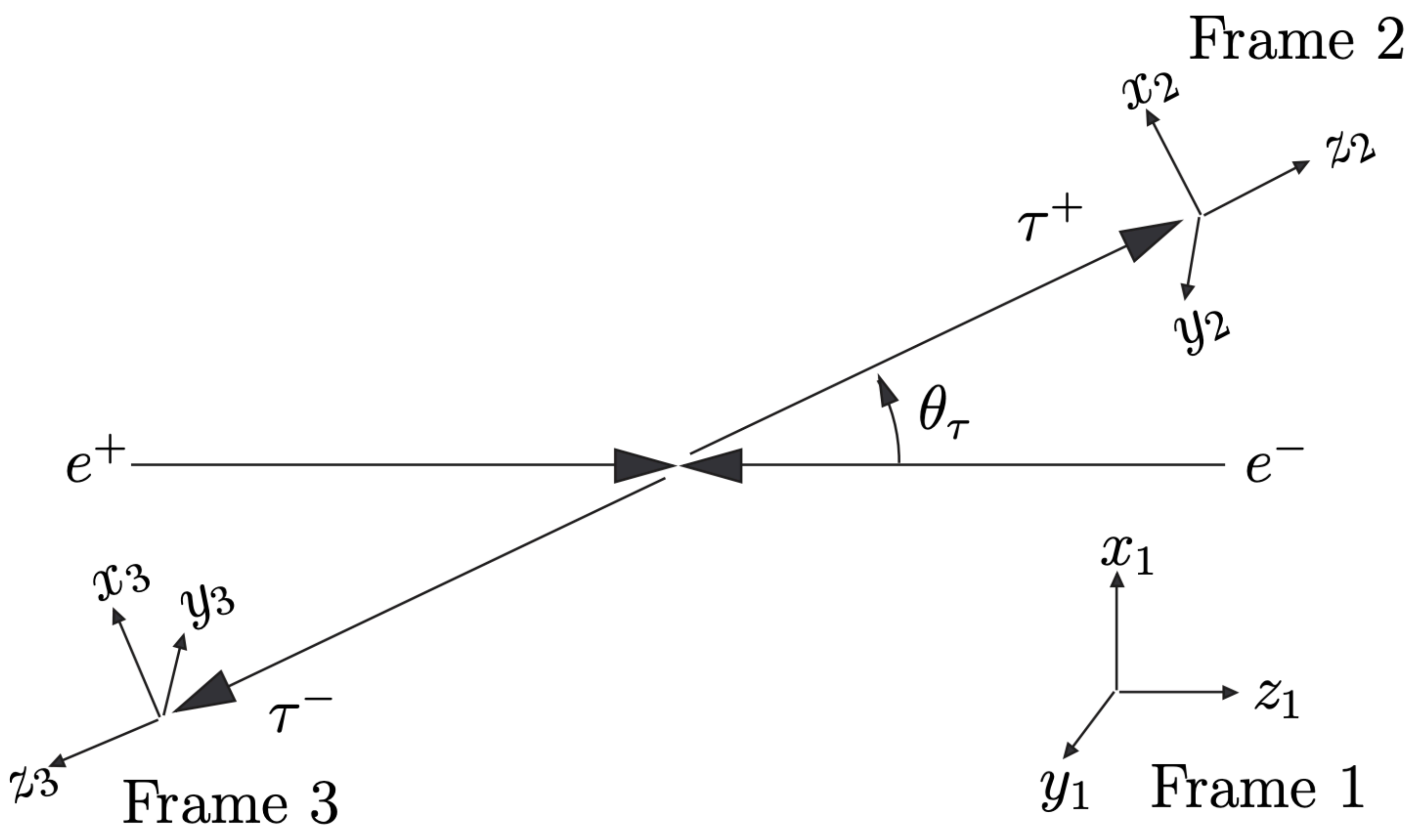
- The lepton angular distribution (P asymmetry) can distinguish between left-handed and right-handed dipoles

$$d\sigma\left(e^+e^- \to \tau^+\tau^- \to l^+\gamma + l^-\bar{\nu}\nu\right)$$
$$= \sigma\left(e^+e^- \to \tau^+\tau^-\right) B\left(\tau^+ \to l^+\gamma\right) B\left(\tau^- \to l^-\bar{\nu}\nu\right)\frac{d\cos\theta_{l^+}}{2}\frac{d\cos\theta_{l^-}}{2}dx2x^2$$
$$\times\left\{3 - 2x - \frac{s-2m_\tau^2}{s+2m_\tau^2}(1-2x)A_P\cos\theta_{l^+}\cos\theta_{l^-}\right\}$$

LFV Radiative decay: distinguishing chiralities

Kitano, Okada hep-ph/0012040



$$dR_b^{\tau^+\to l^+\gamma} = \frac{d\Omega_l}{4\pi}\frac{1}{\Gamma}\frac{2}{\pi}G_F^2 m_\tau^5\left(\left|C_{D,L}^{l\tau}\right|^2 - \left|C_{D,R}^{l\tau}\right|^2\right)\begin{pmatrix}\sin\theta_{l^+}\cos\phi_{l^+}\\ \sin\theta_{l^+}\sin\phi_{l^+}\\ \cos\theta_{l^+}\end{pmatrix}$$

- Angles in Frame 2, and taking the normalization $\Lambda = v$ for the dipoles

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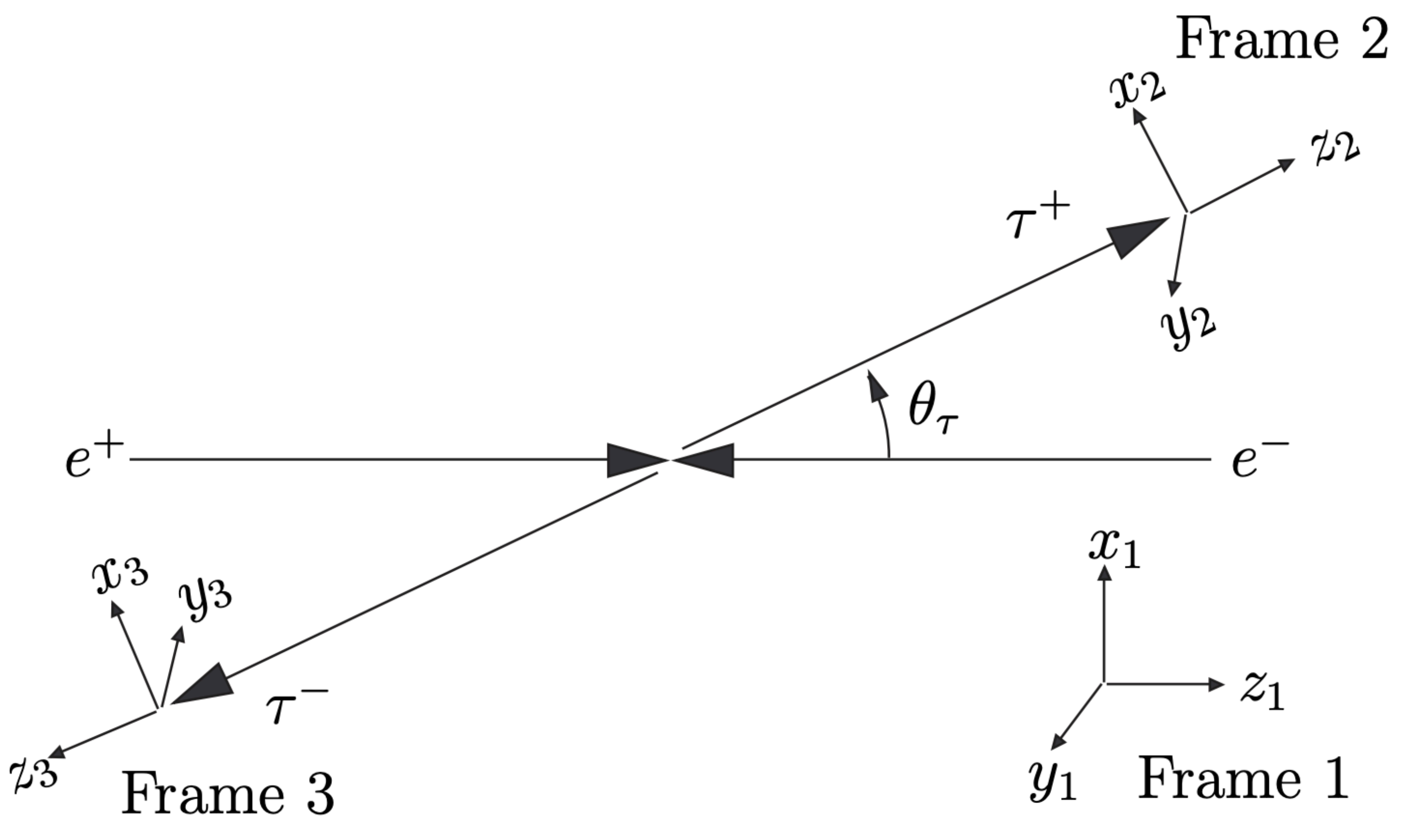

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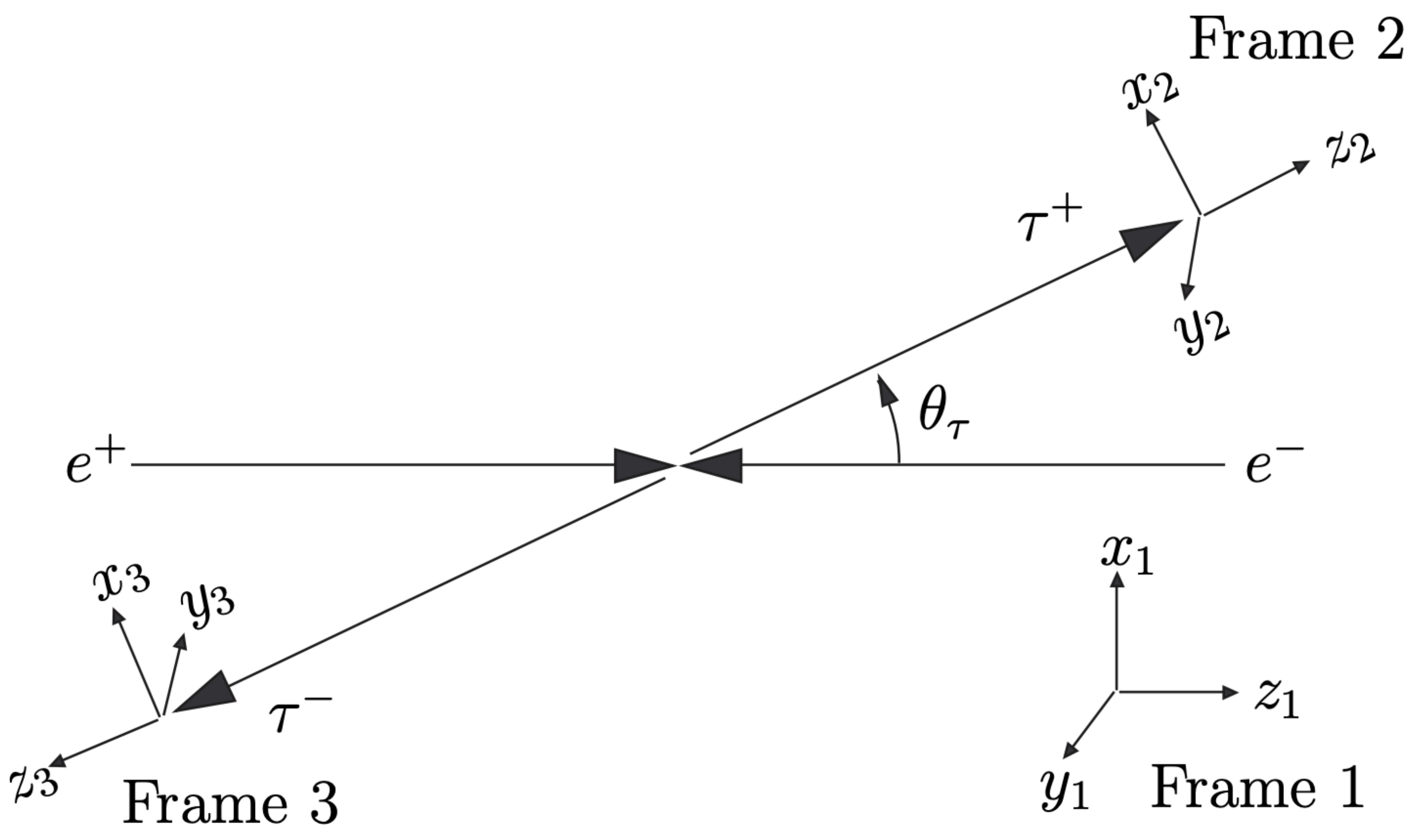

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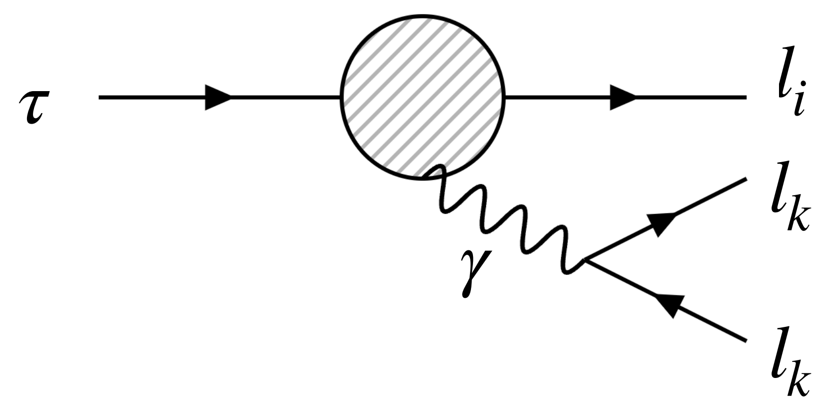
Need to look at the both τ^-/τ^+ angular distributions, if not polarised

LFV three body decays $(\Delta F = 1)$

- All decays with only one flavour changing current: $\tau \to \mu\bar{\mu}\mu$, $\tau \to \mu\bar{e}e$, $\tau \to e\bar{e}e$, $\tau \to e\bar{\mu}\mu$



$+$

Can be neglected because of $\tau \to l\gamma$

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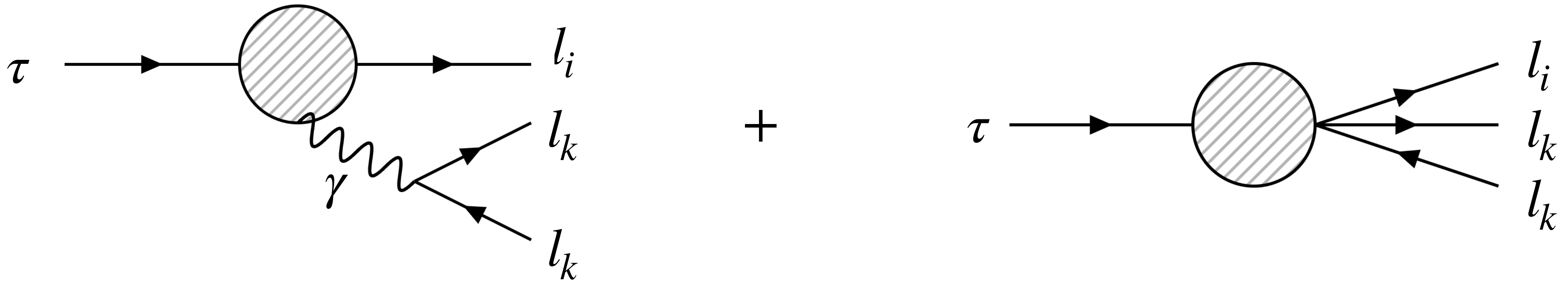

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Can include four-lepton scalars, vectors and tensors*

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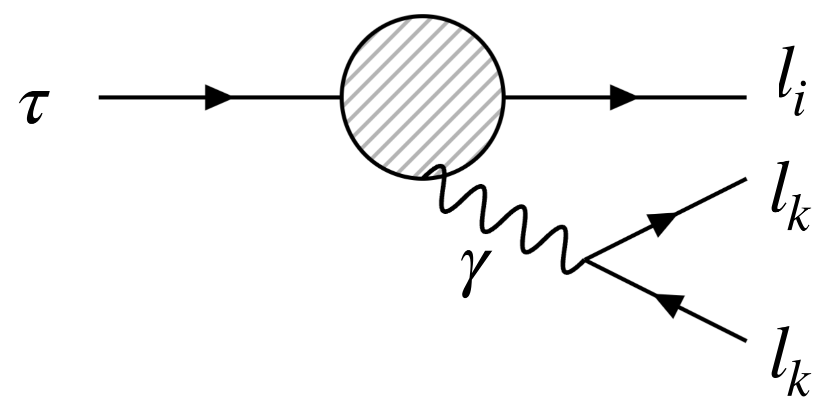

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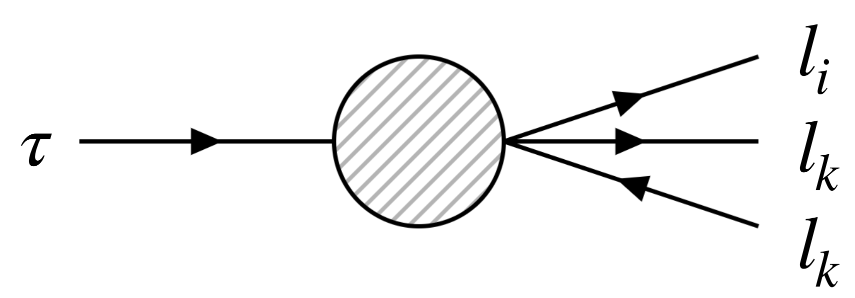

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four-lepton scalars are Yukawa suppressed or at dimension eight

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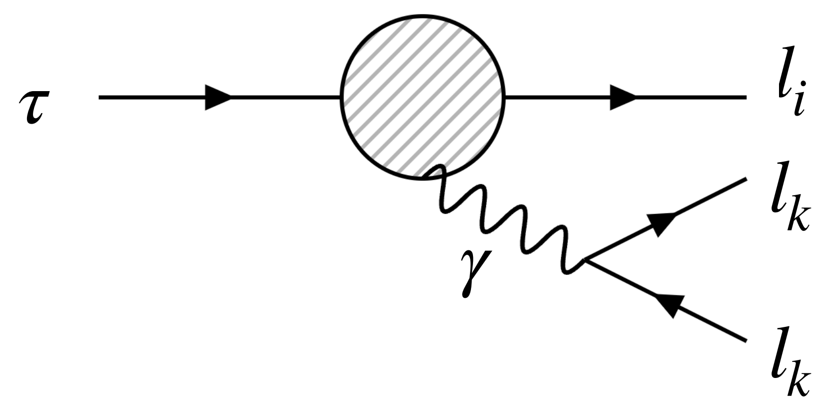

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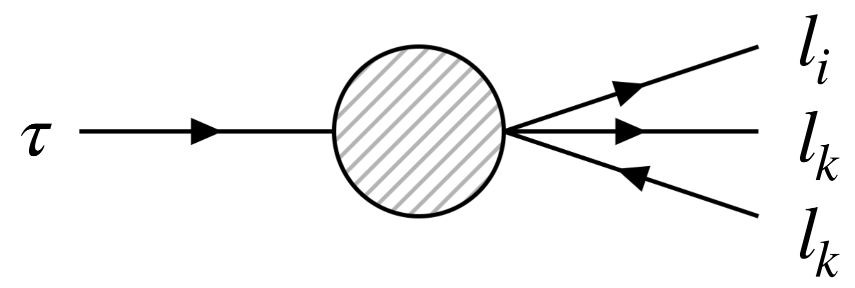

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$$\frac{Br(\tau\to\mu\mu\mu)}{Br(\tau\to\mu\bar{\nu}_\mu\nu_\tau)} = \left(\frac{v}{\Lambda}\right)^4\left[2\,|C_{V,LL}+4eC_{D,R}|^2 + |C_{V,LR}+4eC_{D,R}|^2 + |C_{S,R}|^2/8 + (64\log(m_\tau/m_\mu)-136)\,|eC_{D,R}|^2 + L\leftrightarrow R\right]$$

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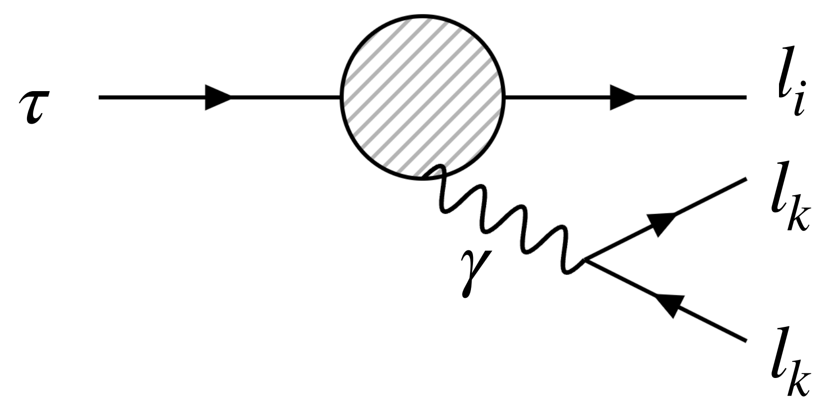

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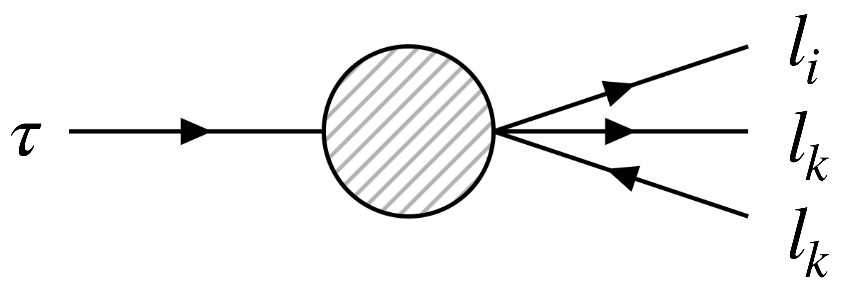

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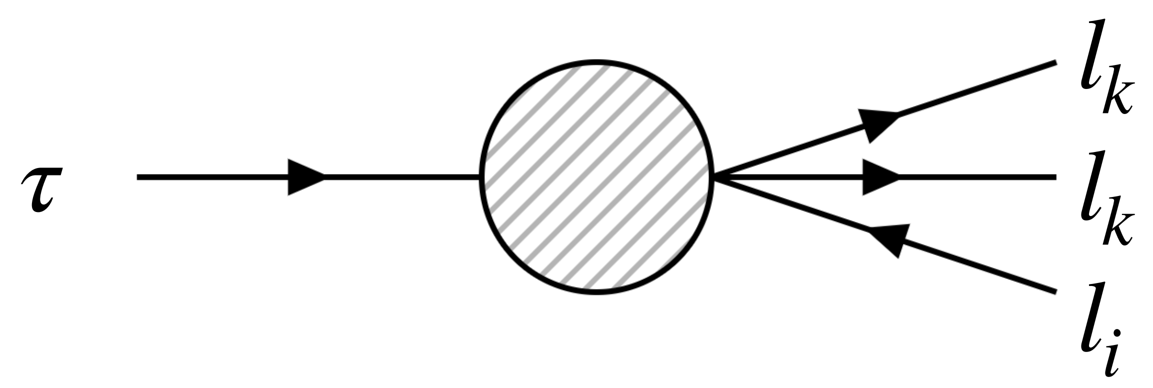
$$\frac{Br(\tau\to\mu\mu\mu)}{Br(\tau\to\mu\bar{\nu}_\mu\nu_\tau)} \lesssim 1.5\times10^{-7} \quad\to\quad \frac{v^2}{\Lambda^2}\left(C_{D,X}\quad C_{V,XX}\quad C_{V,XY}\quad C_{S,X}\right) \lesssim \left(8.3\times10^{-5}\quad 2.4\times10^{-4}\quad 3.4\times10^{-4}\quad 9.7\times10^{-4}\right)$$

LFV three body decays ($\Delta F = 2$)

- All decays with two flavour changing currents: $\tau \to \bar{\mu}ee$, $\tau \to \bar{e}\mu\mu$,



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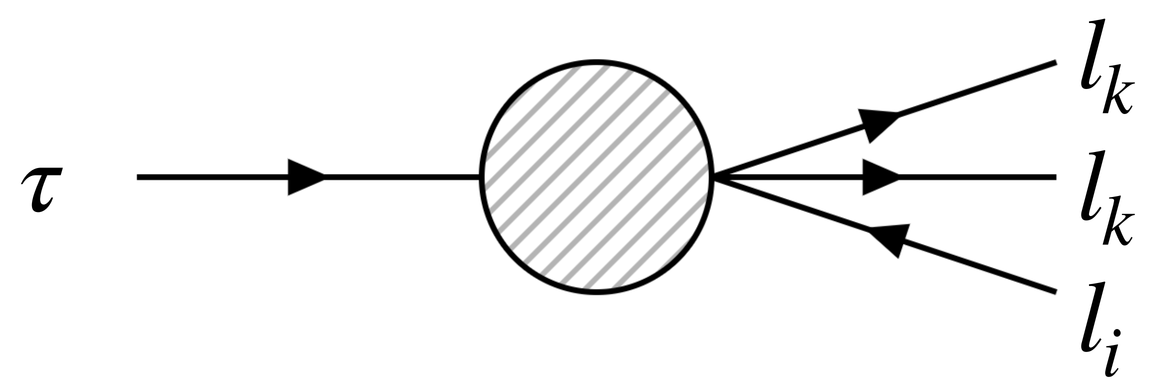

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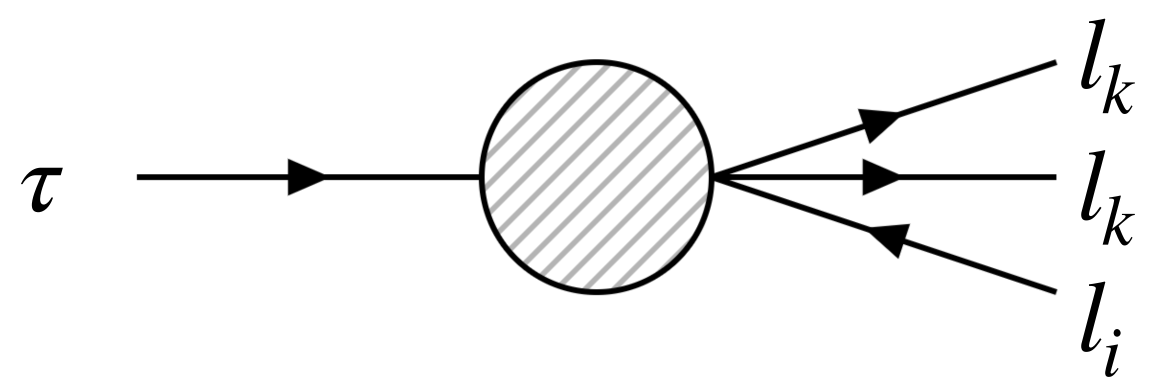

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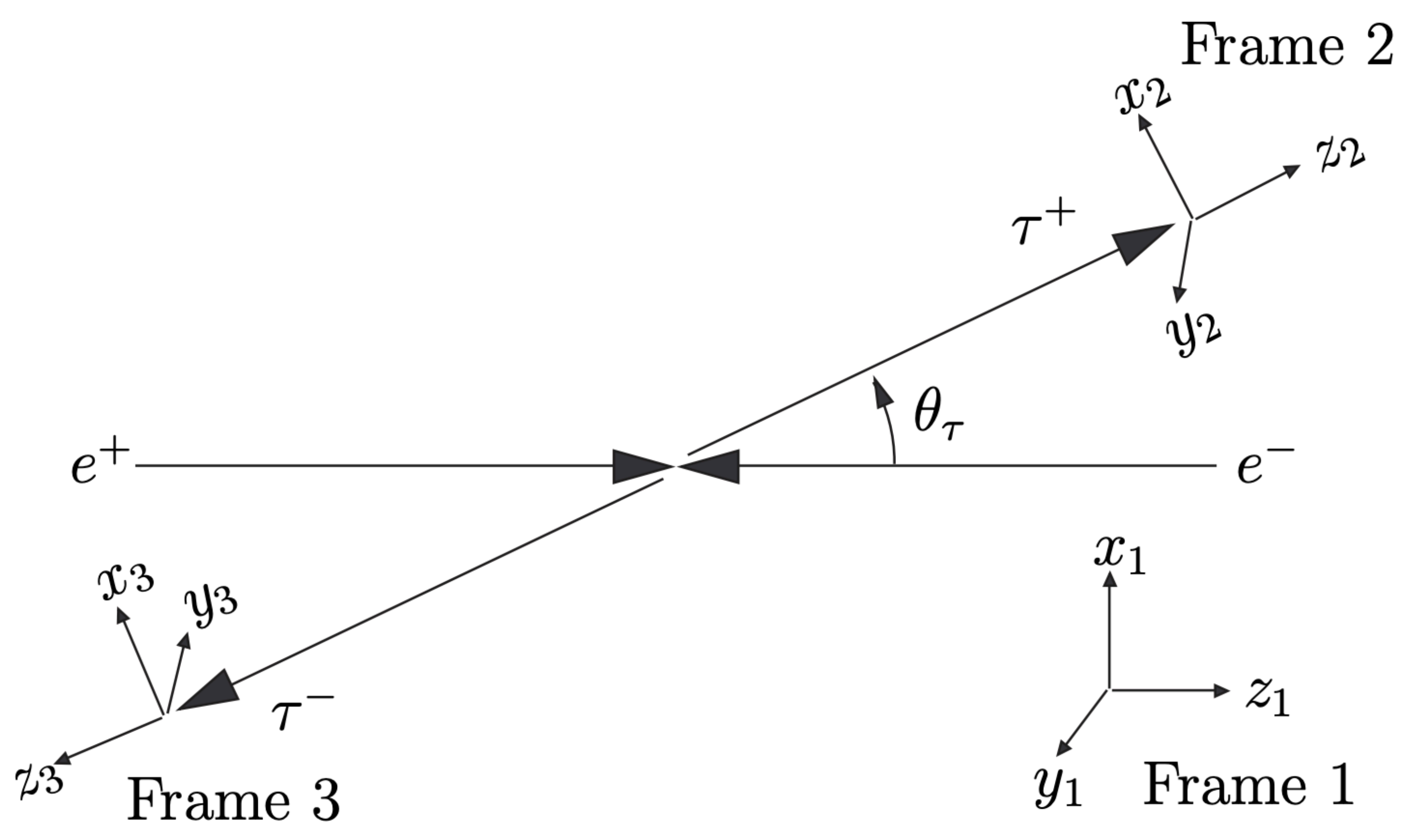
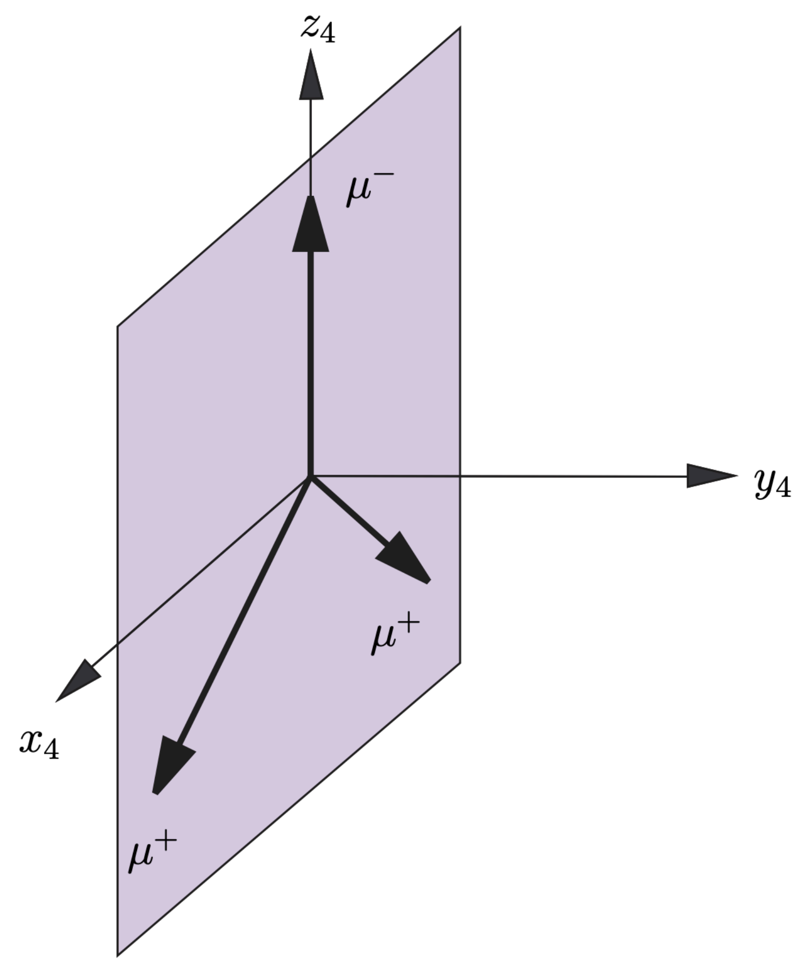
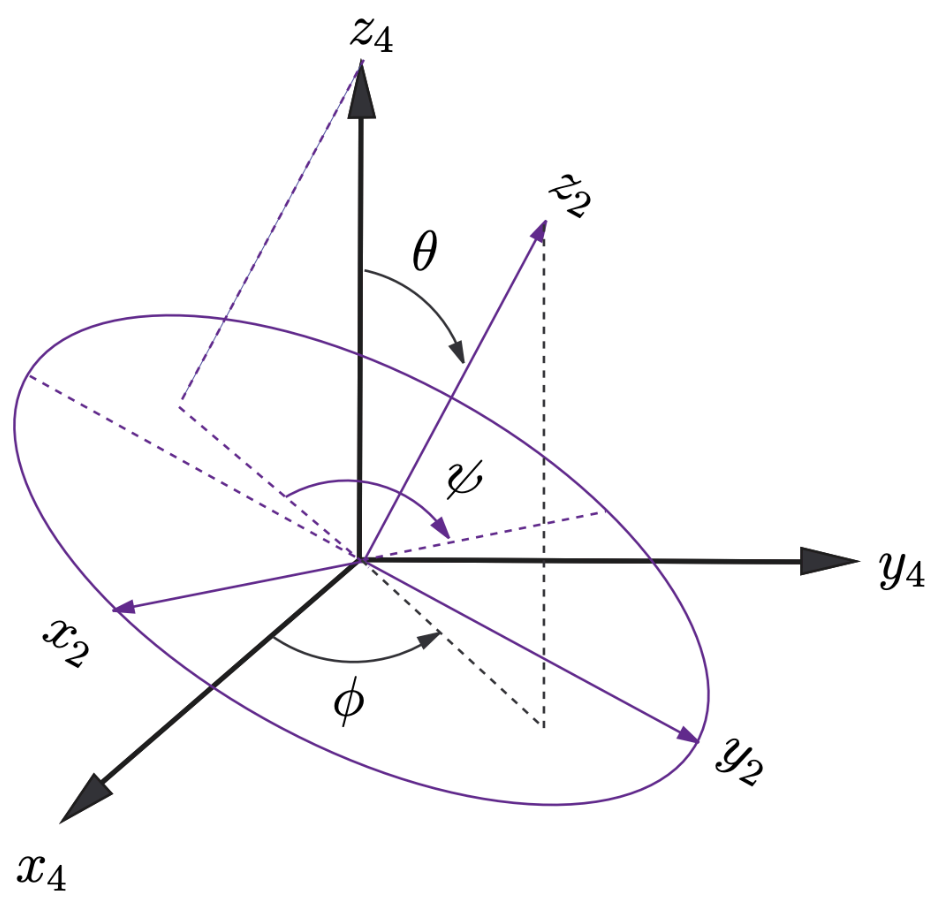
Three body decay: lepton angular asymmmetries

Kitano, Okada hep-ph/0012040







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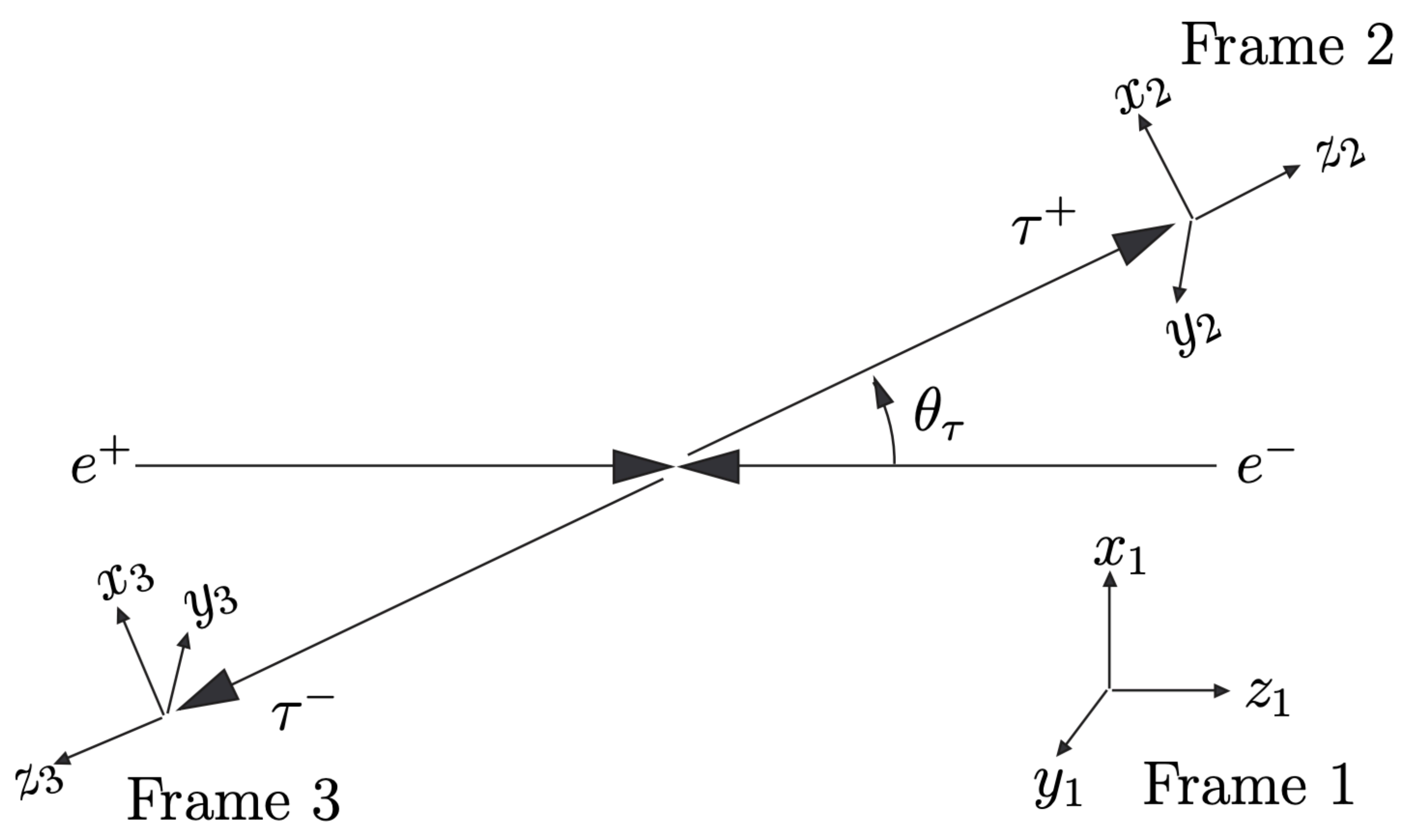
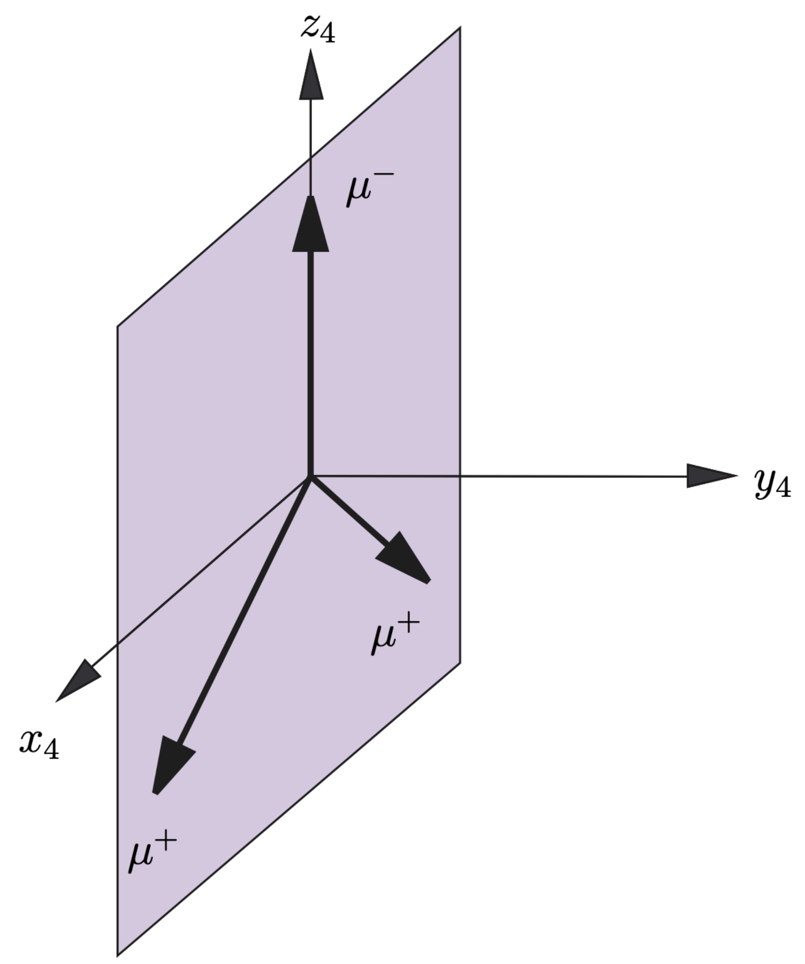
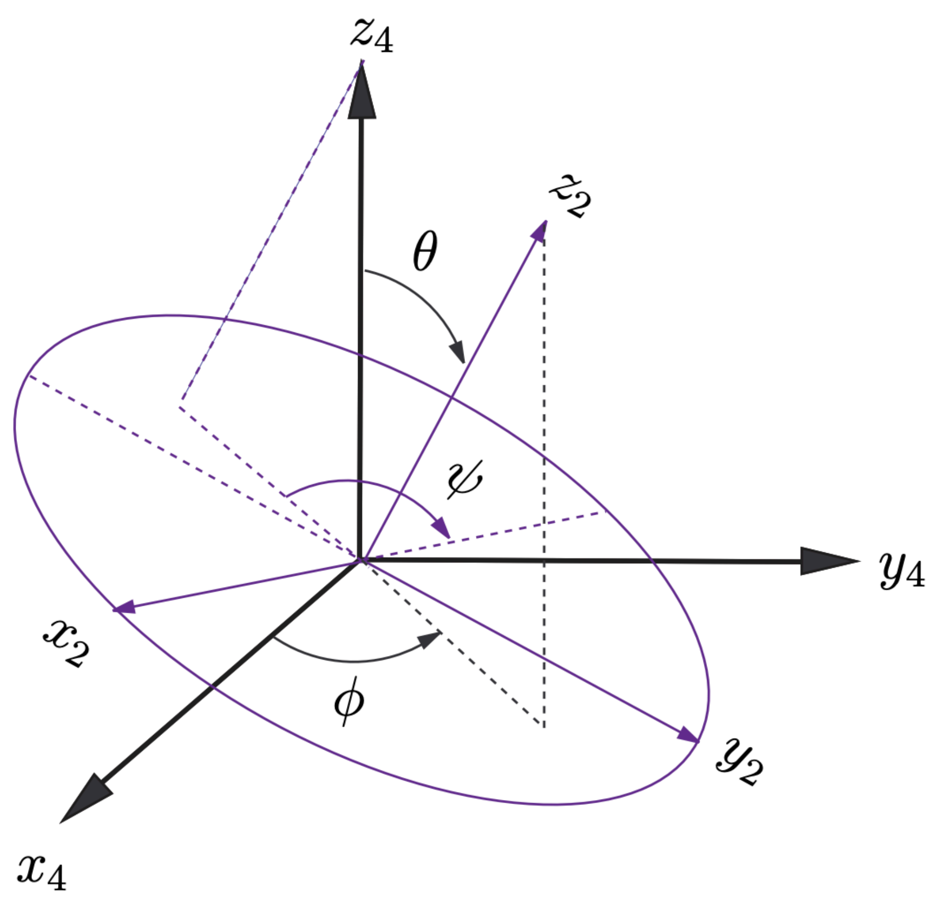


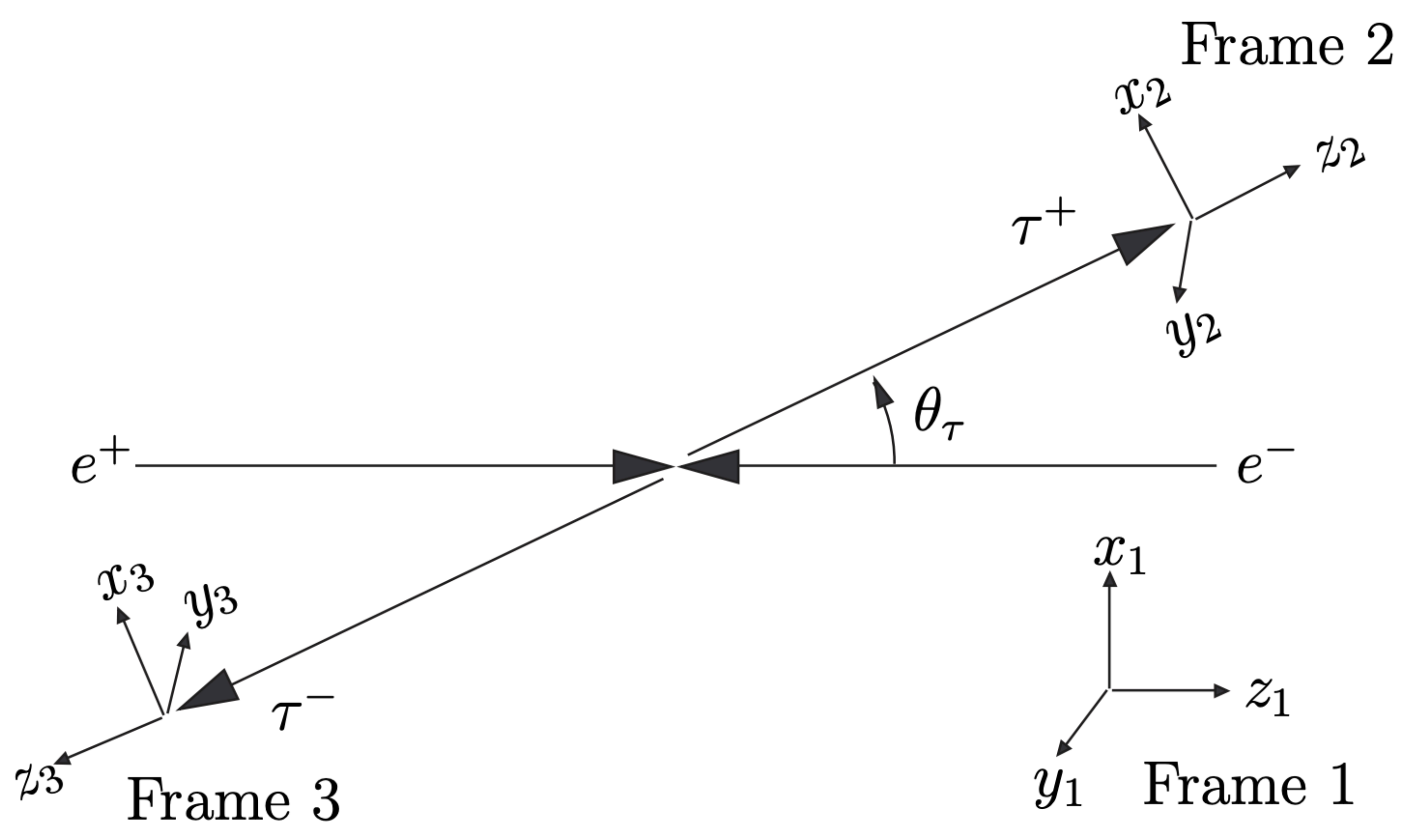


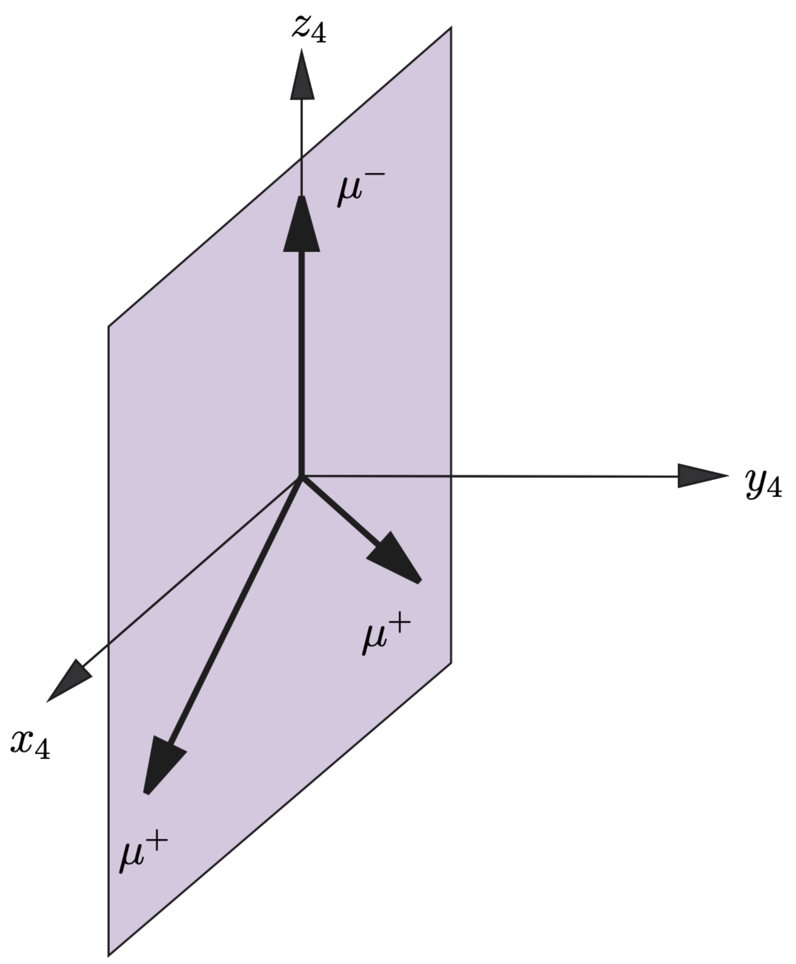


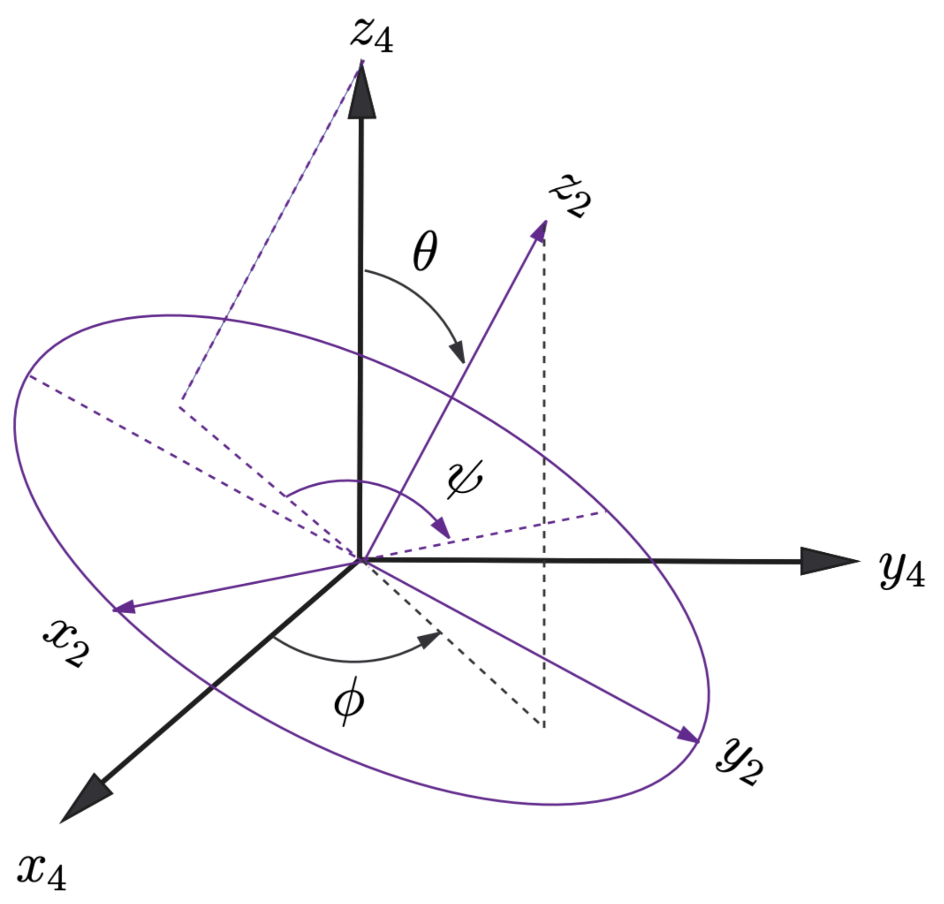
$$
\begin{aligned}
& d\sigma\left(e^+e^- \to \tau^+\tau^- \to \mu^+\mu^+\mu^- + \pi^-\nu\right) \\
& = \sigma\left(e^+e^- \to \tau^+\tau^-\right) B\left(\tau^- \to \pi^-\nu\right) \left(\frac{m_\tau^5 G_F^2}{128\pi^4}/\Gamma\right) \frac{d\cos\theta_\pi}{2} dx_1 dx_2 d\cos\theta d\phi \\
& \times \left[X - \frac{s-2m_\tau^2}{s+2m_\tau^2}\{Y\cos\theta + Z\sin\theta\cos\phi + W\sin\theta\sin\phi\}\cos\theta_\pi\right]
\end{aligned}
$$

Three body decay: lepton angular asymmmetries

[Kitano, Okada hep-ph/0012040]







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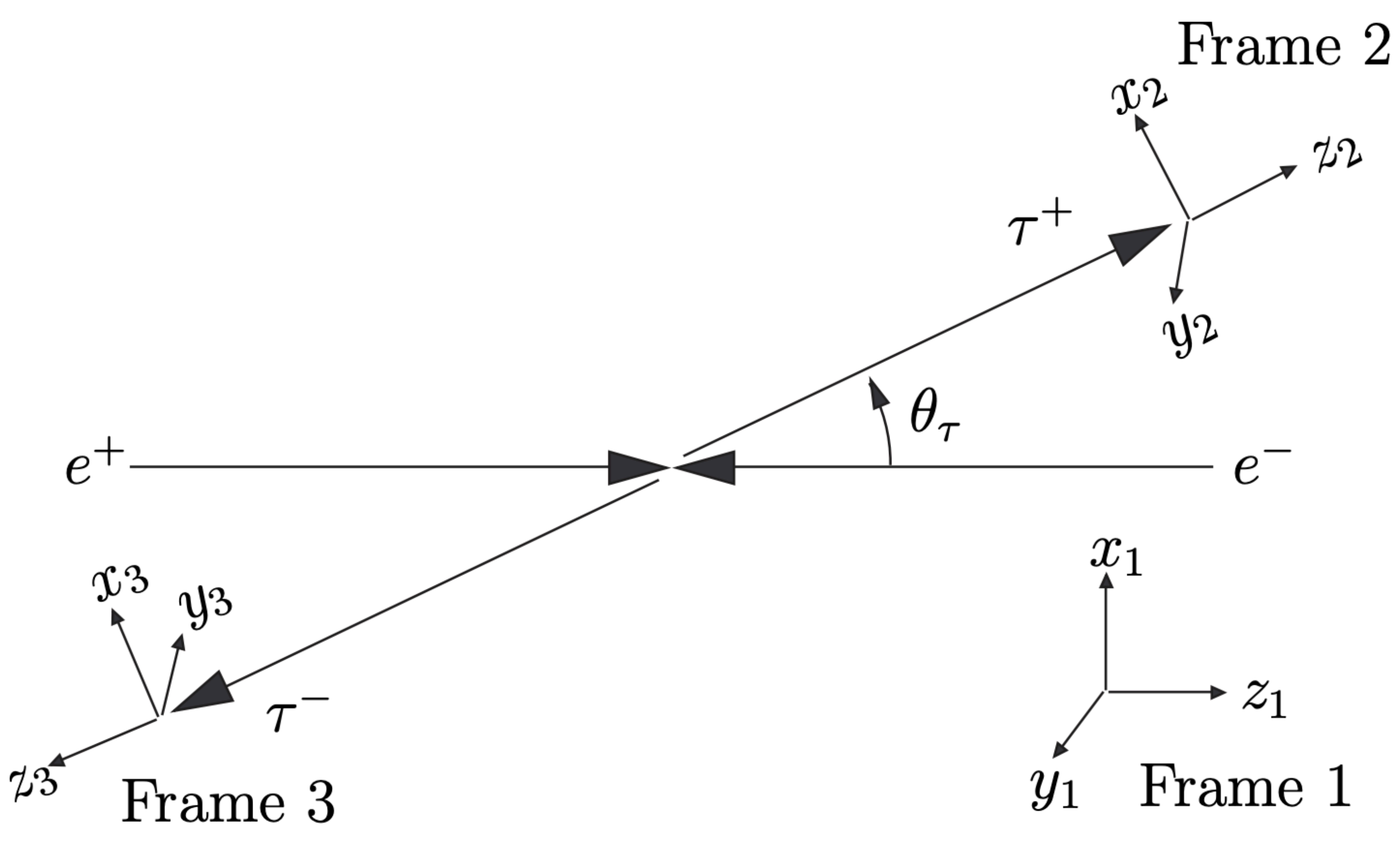
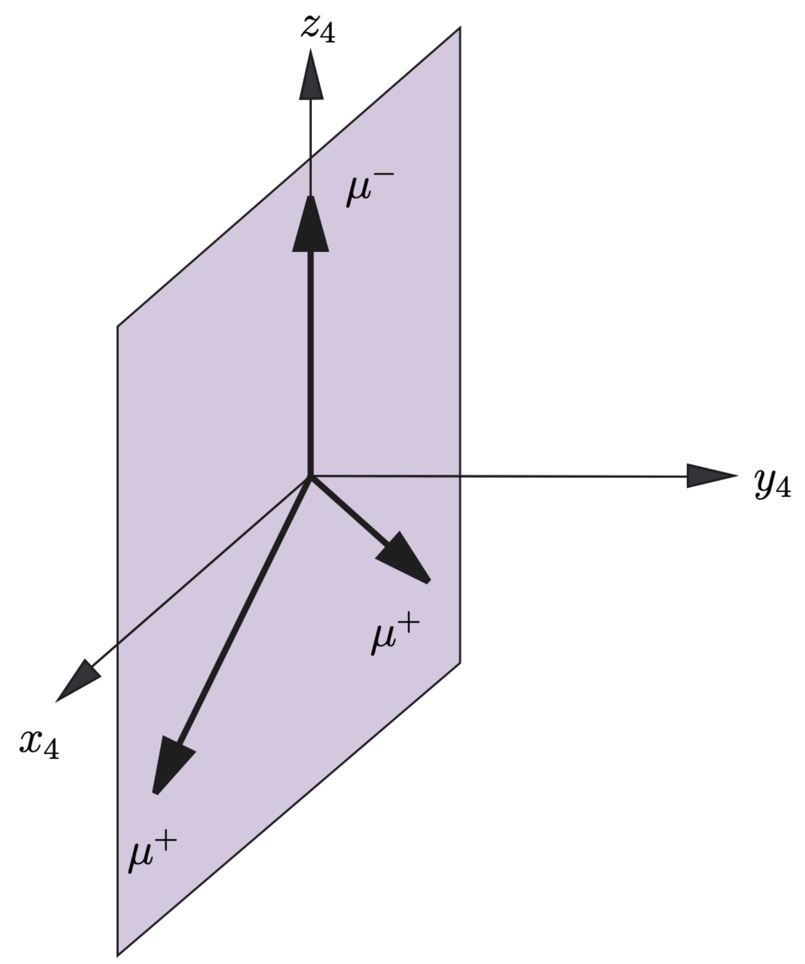
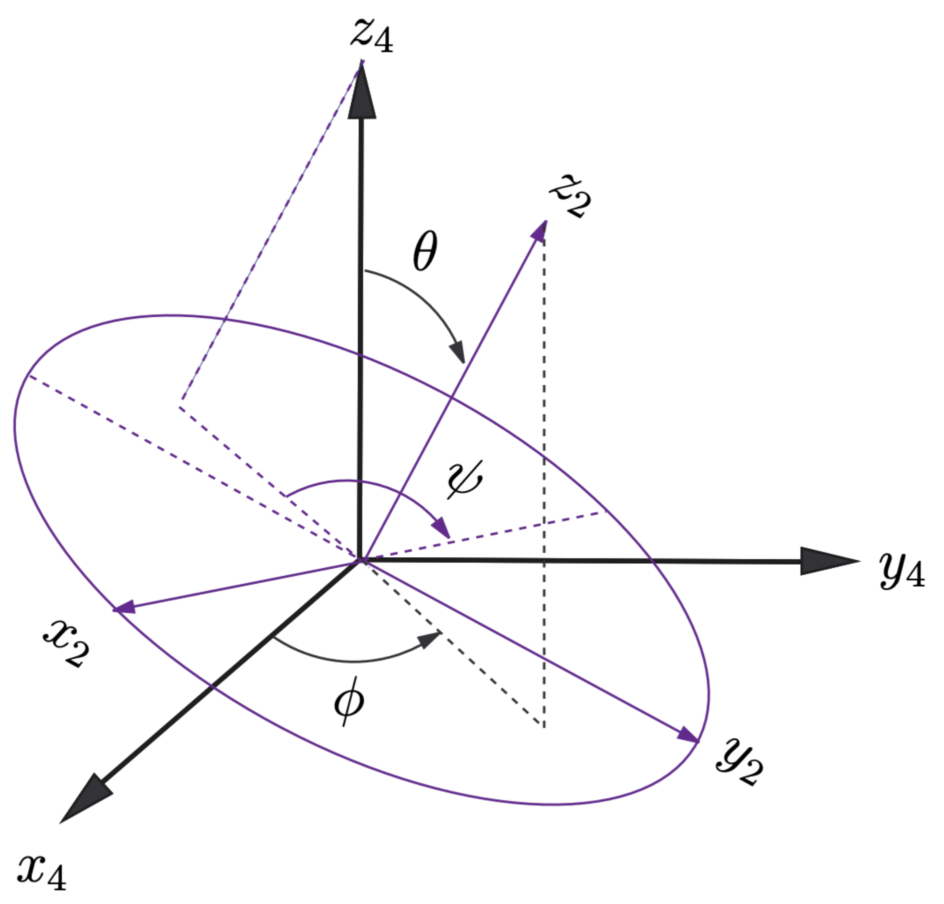
Can distinguish $C_{V,LX}$, $C_{V,LX}$, $C_{S,R}$ **from** $C_{V,RX}$, $C_{V,RX}$, $C_{S,L}$ **but not scalars from vectors**

Three body decay: lepton angular asymmmetries

Kitano, Okada hep-ph/0012040







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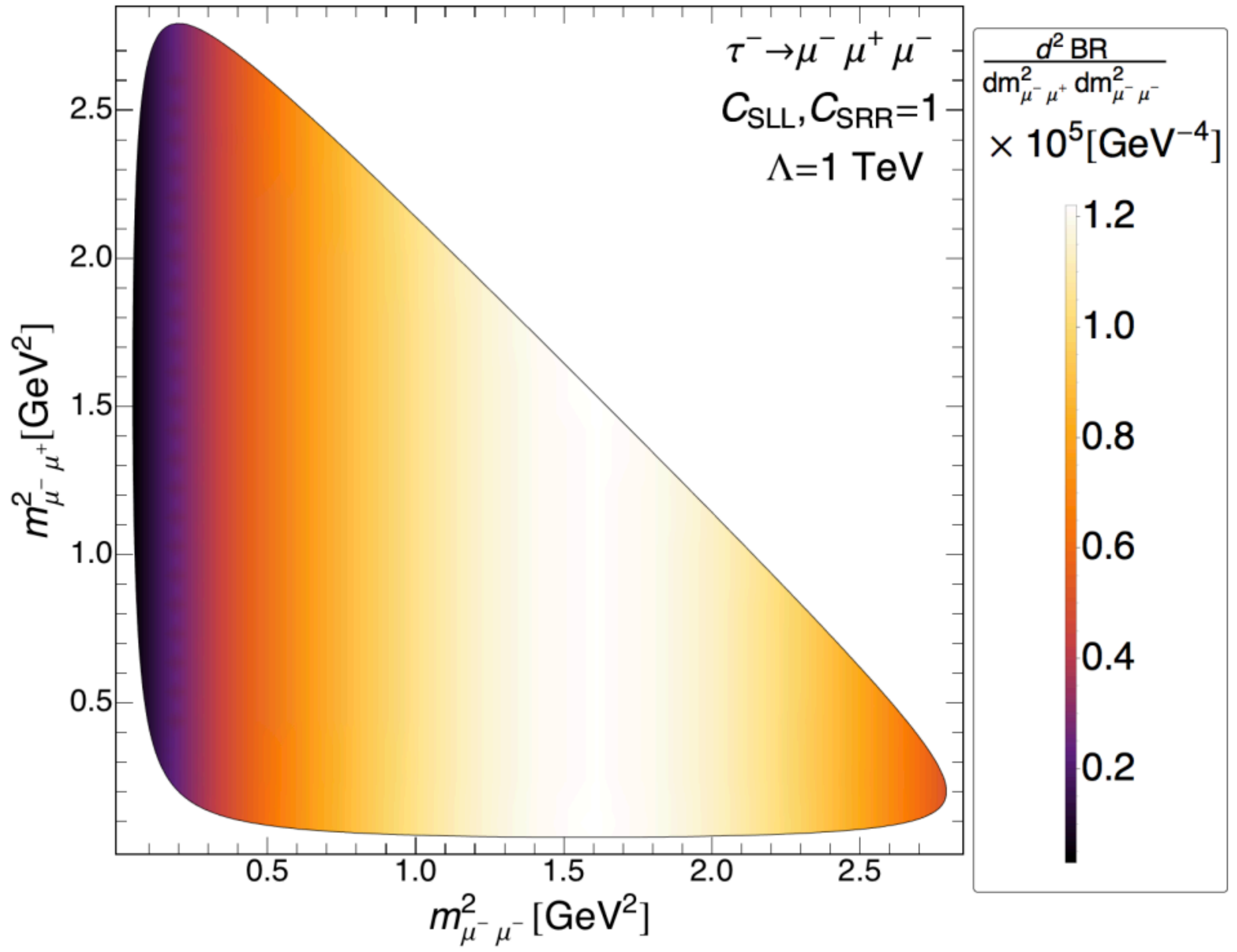
T asymmetry related to CP violation in LFV decay (phase between dipoles and vectors)

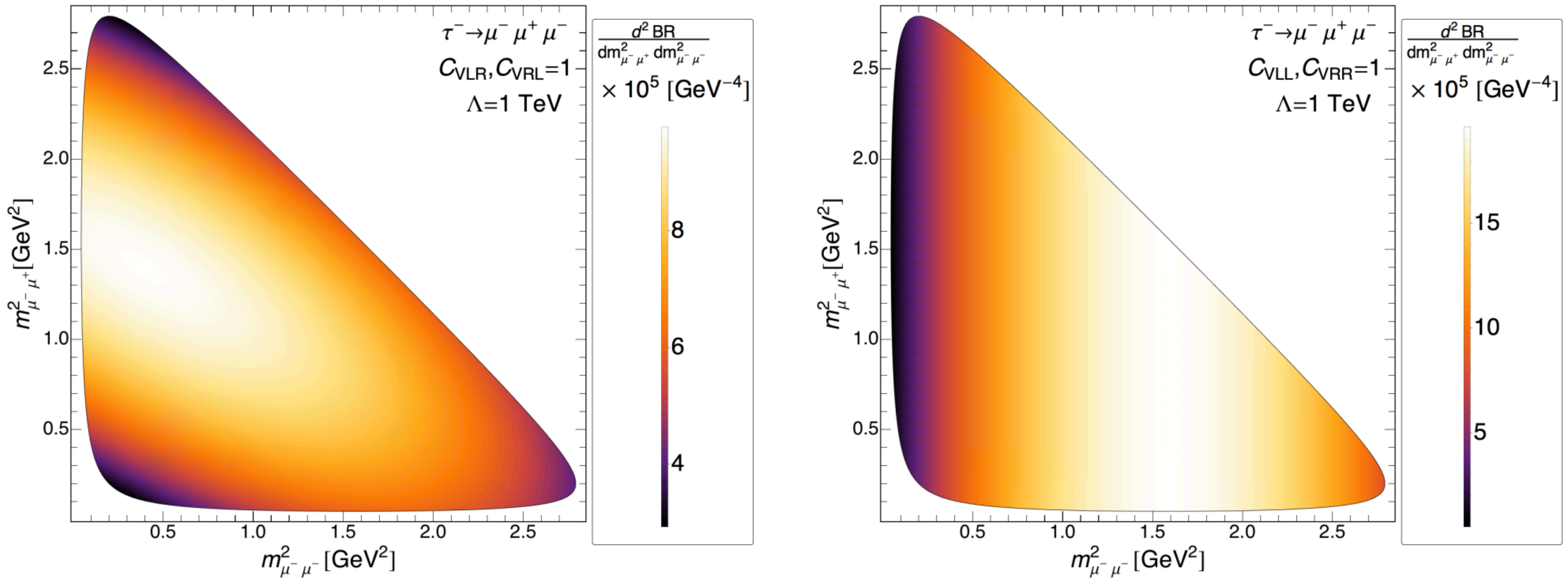
Three body decay: Dalitz plots

[Celis, Passemar, Cirigliano 1403.5781]

- Dalitz plots could also assist in distinguishing operators



Scalars



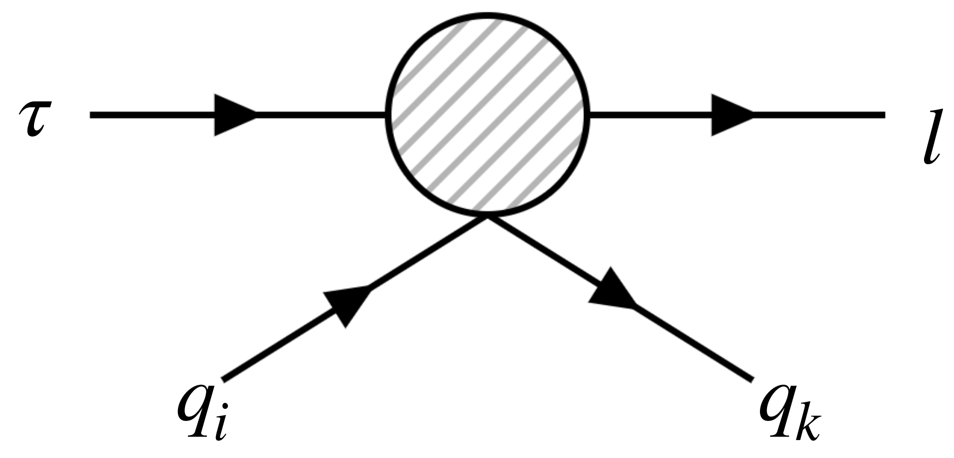
Vectors

Outline

- Leptonic decays ($\tau \to l_i\gamma$, $\tau \to l_i\bar{l}_k l_k$, $\tau \to \bar{l}_i l_k l_k$)
- Semi-leptonic decays (ex: $\tau \to \pi l_i$)
- Complementarity with other experiments
- Conclusion

Semi-leptonic τ LFV decays

- Various decay channels probing LFV interactions between τ flavoured currents and quarks



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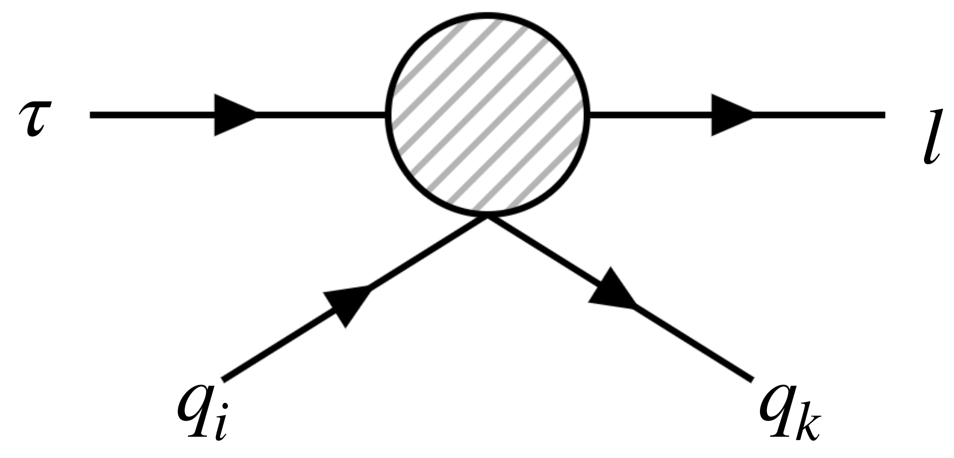

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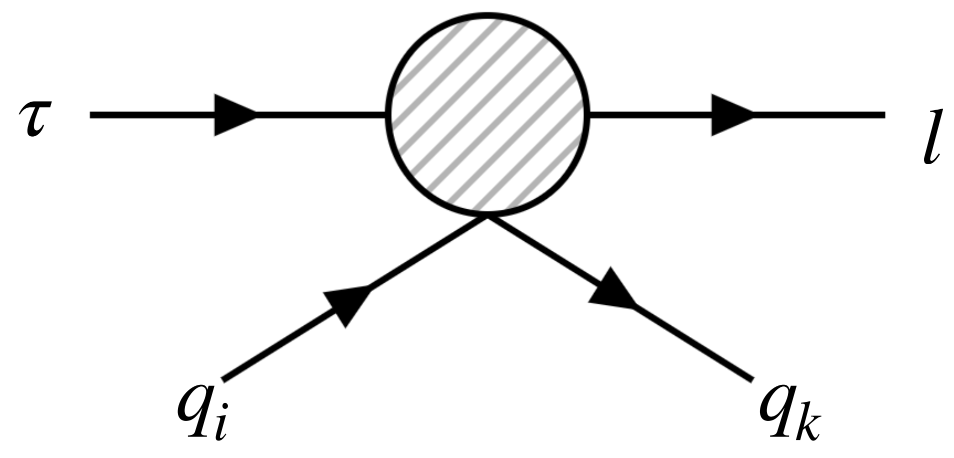

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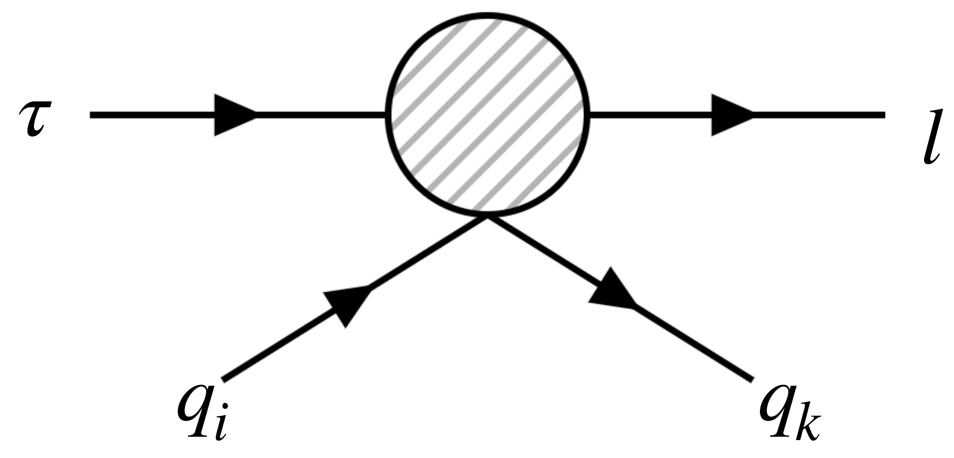

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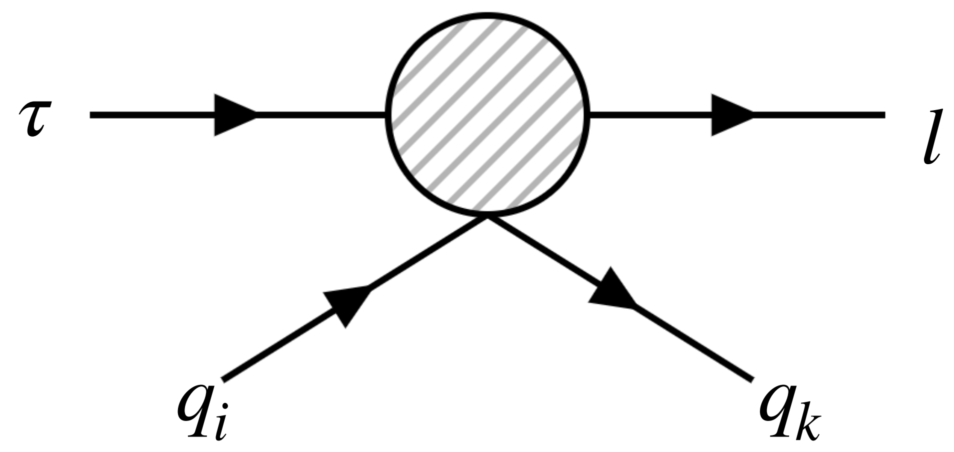

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For a recent EFT analysis see Plakias, Sumensari 2312.14070

Semi-leptonic τ LFV decays

Black et al. hep-ph/0206056

Davidson 2010.00317

- Rate predictions depend on the hadronic matrix elements

$$\left\langle 0 \left| 1/2 \left(\bar{u}\gamma^\alpha\gamma_5 u - \bar{d}\gamma^\alpha\gamma_5 d\right) \right| \pi^0(P) \right\rangle = iP^\alpha f_\pi$$

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$$\frac{Br\left(\tau \to l\pi_0\right)}{Br(\tau \to l\bar{\nu}\nu)} = \frac{3\pi^2 f_\pi^2}{m_\tau^2}\frac{v^4}{\Lambda^4}\left| C^{l\tau uu}_{V,XR} - C^{l\tau uu}_{V,XL} - C^{l\tau dd}_{V,XR} + C^{l\tau dd}_{V,XL}\right|^2$$

$$+24\pi^2\left(\frac{m_{\pi_0}}{m_\tau}\right)^4\left(\frac{f_\pi}{m_u + m_d}\right)^2\frac{v^4}{\Lambda^4}\left| C^{l\tau uu}_{S,XR} - C^{l\tau uu}_{S,XL} - C^{l\tau dd}_{S,XR} + C^{l\tau dd}_{S,XL}\right|^2$$

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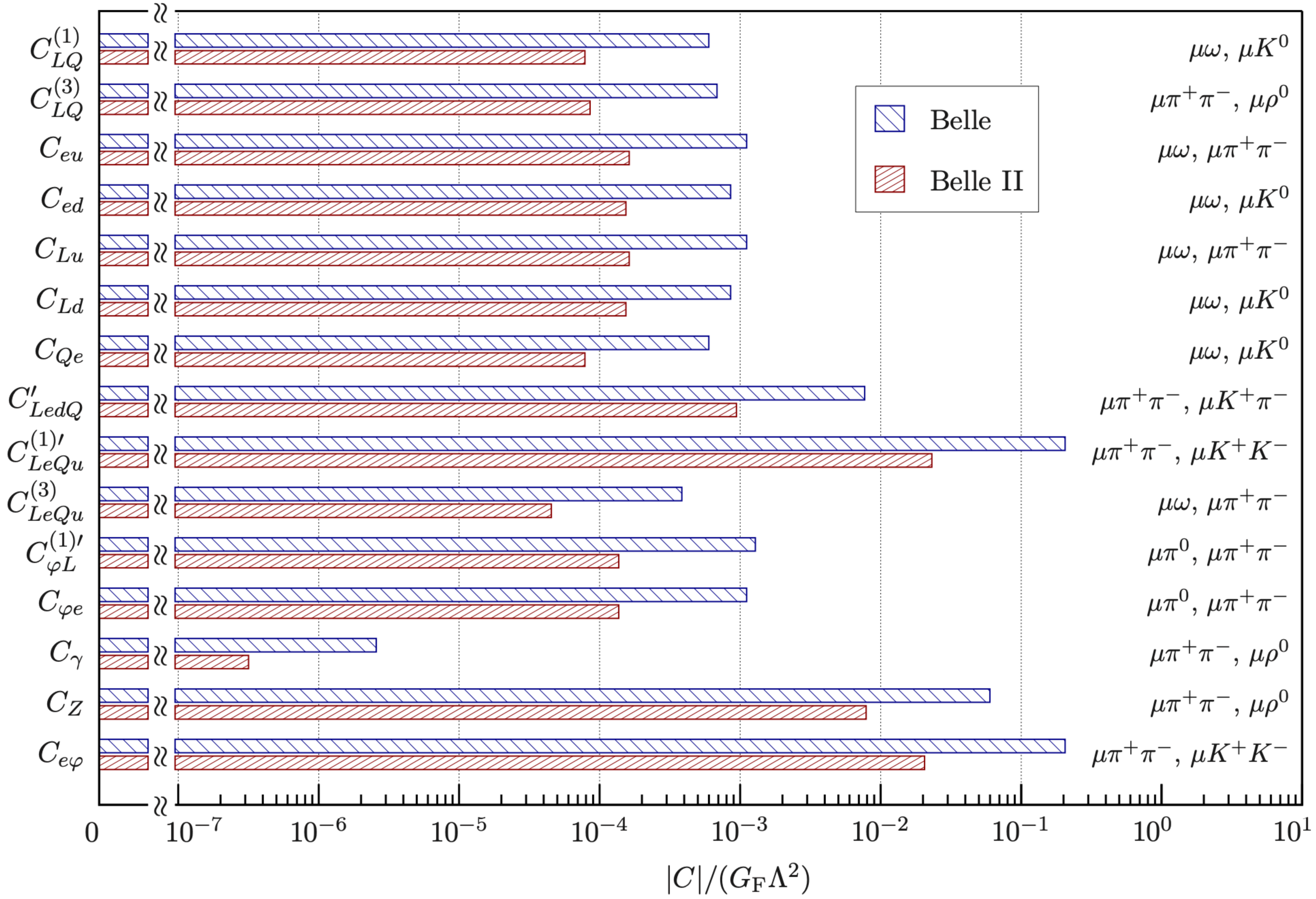
- Sensitive to all vector that can mix with the axial current at one-loop, and also marginally to tensors that can mix with the pseudoscalar current. QCD running is relevant to get numbers right!

Semi-leptonic τ LFV decays

- Effective coefficients probed by τ LFV decays (dimension six SMEFT operators)

Snowmass tau CLFV, 2203.14919



Differential distributions to distinguish operators

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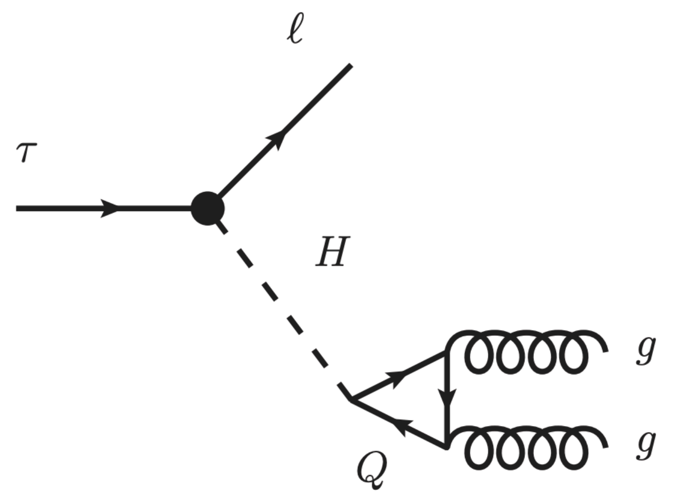

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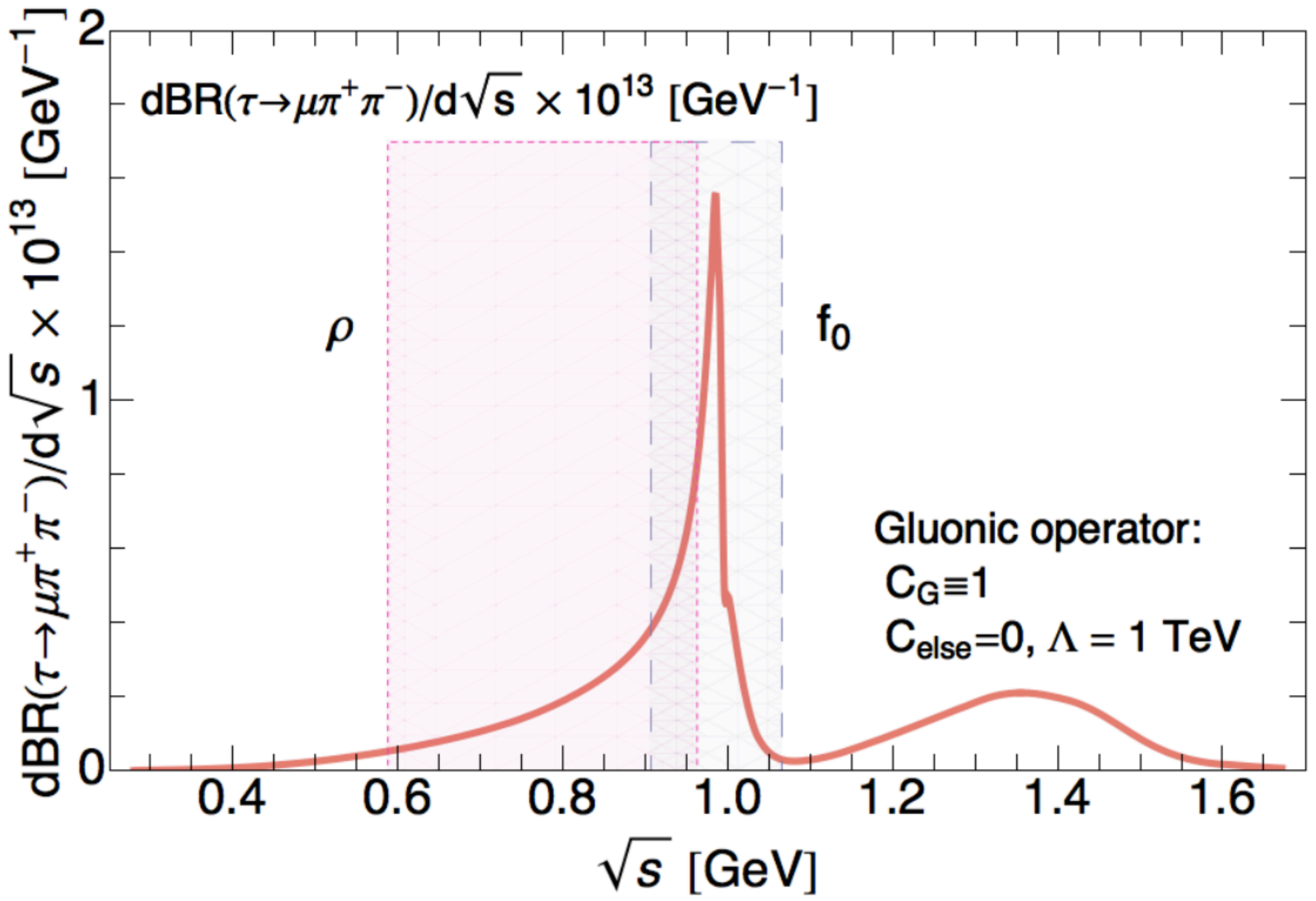

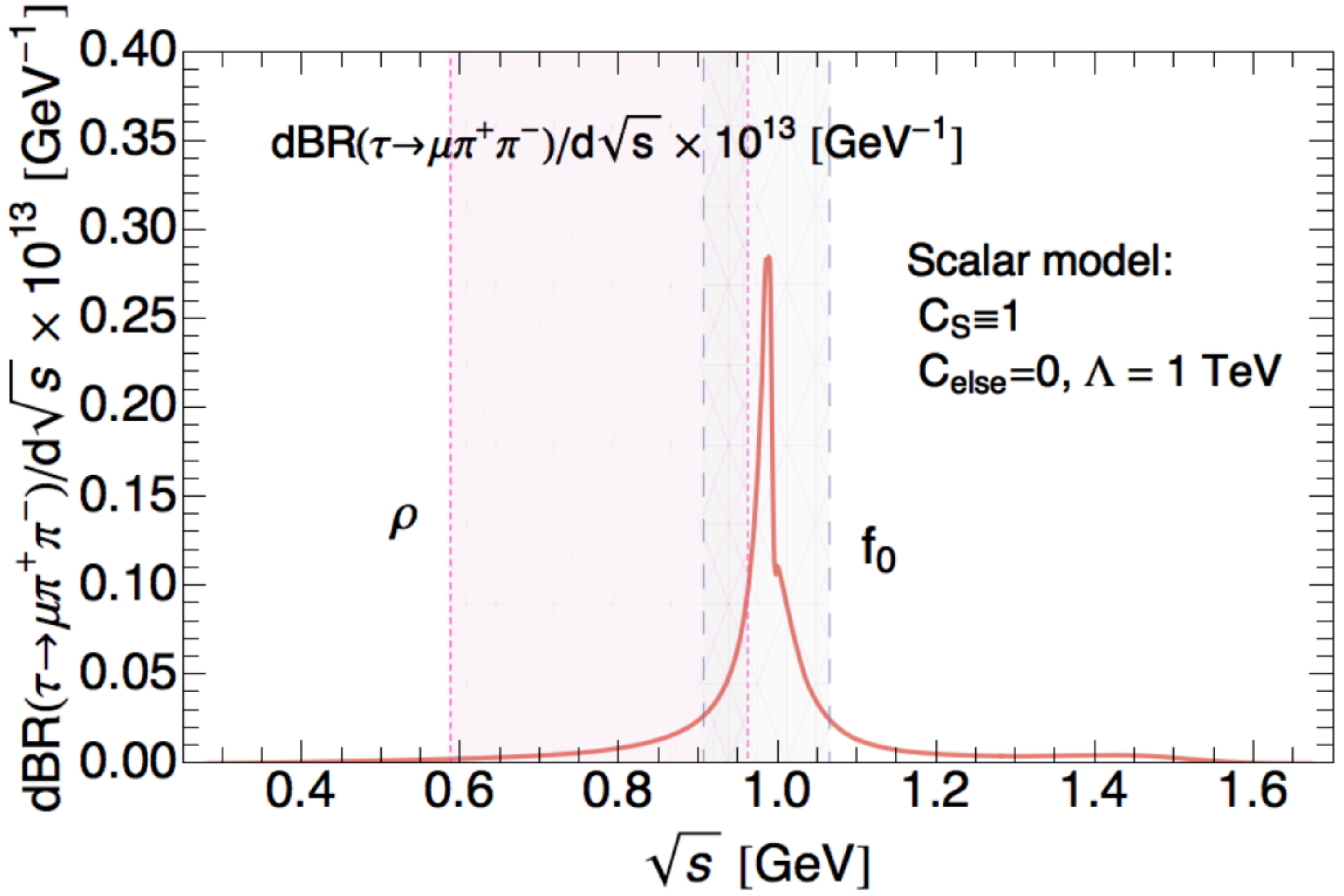

Celis, Passemar, Cirigliano 1403.5781

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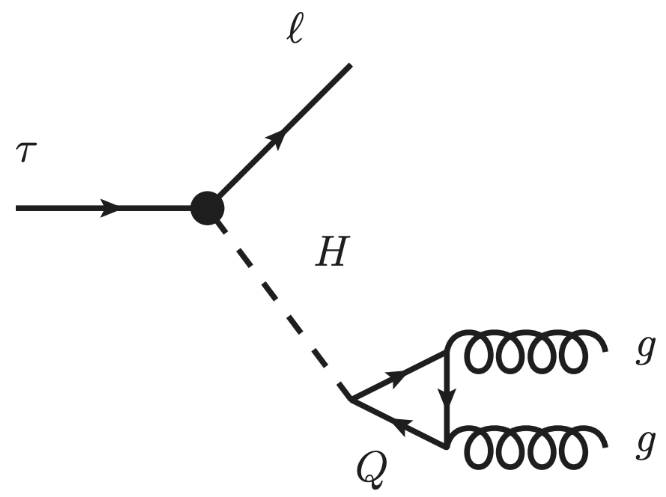

Outline

- Leptonic τ decays ($\tau \to l_i \gamma$, $\tau \to l_i \bar{l}_k l_k$, $\tau \to \bar{l}_i l_k l_k$)
- Semi-leptonic τ decays (ex: $\tau \to \pi l_i$)
- Meson decays
- Conclusion

EFT for B meson decays Becirevic et al 2407.19060

- Rate predictions in terms of the EFT operator coefficients (short-distance) and hadronic matrix element (QCD dynamic) with b-s

$$\mathcal{B}(B_q \to \ell_i^- \ell_j^+) = \frac{\tau_{B_q}}{64\pi^3} \frac{\alpha_{\rm em}^2 G_F^2}{m_{B_q}^3} f_{B_q}^2 |V_{tb} V_{tq}^*|^2 \lambda^{1/2}(m_{B_s}, m_{\ell_i}, m_{\ell_j})$$
$$\times \left\{ \left[m_{B_q}^2 - (m_{\ell_i} + m_{\ell_j})^2\right] \left| (C_9^{q\,ij} - C_{9'}^{q\,ij})(m_{\ell_i} - m_{\ell_j}) + (C_S^{q\,ij} - C_{S'}^{q\,ij}) \frac{m_{B_q}^2}{m_b + m_q} \right|^2 \right.$$
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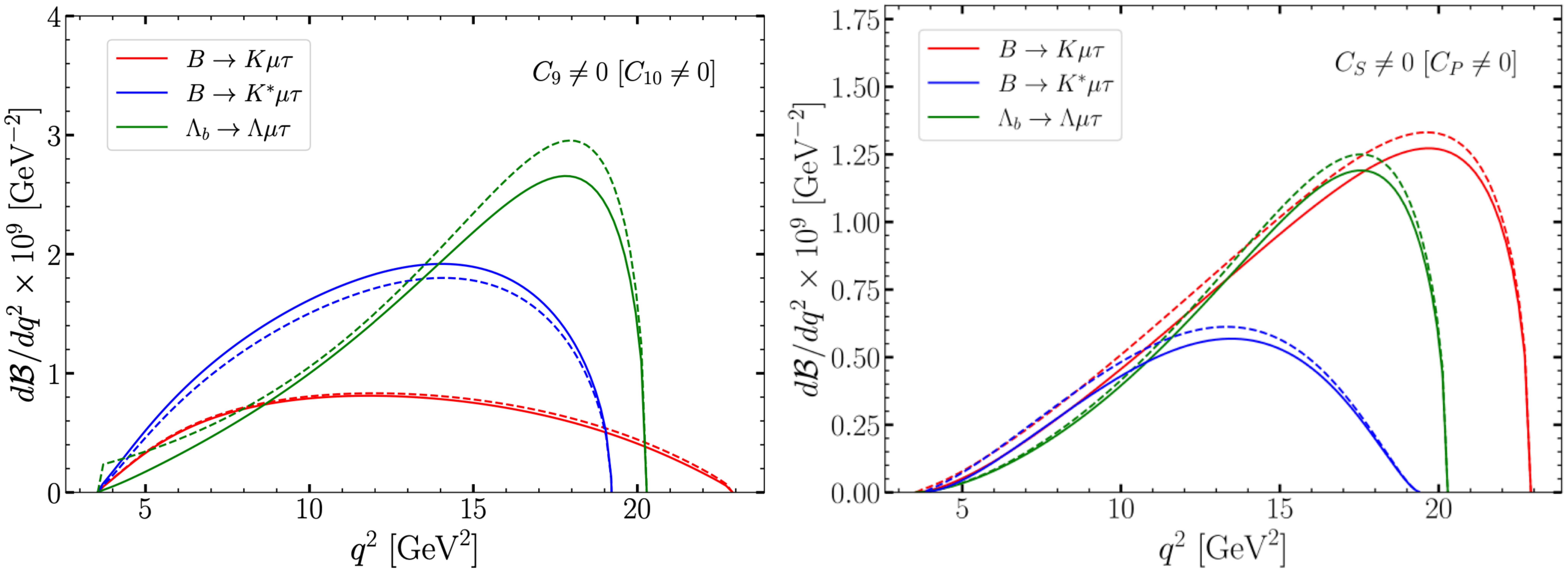
- Similar for semi-leptonic decays (more hadronic matrix elements in general)

$$B \to K^{(*)}\ell_i \ell_j \qquad B \to \pi \ell_i \ell_j \qquad \Lambda_b \to \Lambda \ell_i \ell_j$$

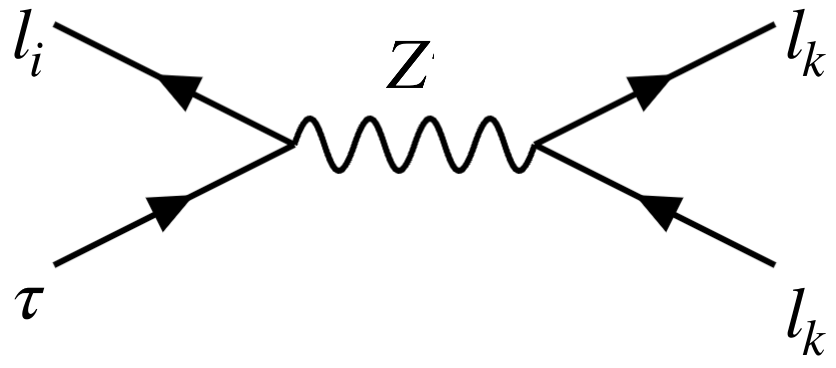
Differential distributions

Becirevic et al 2407.19060



Complementarity with other experiments

Complementarity: Z decays



- If the τ decays happen via Z LFV couplings, they could be probed by $Z \to \tau l_i$ searches

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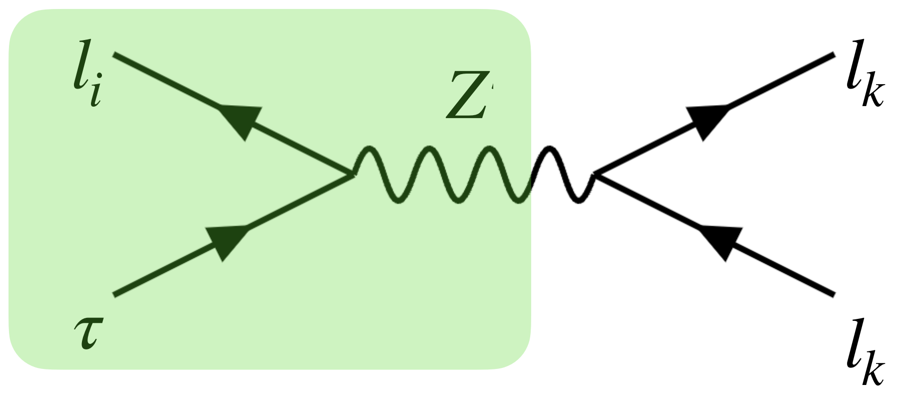

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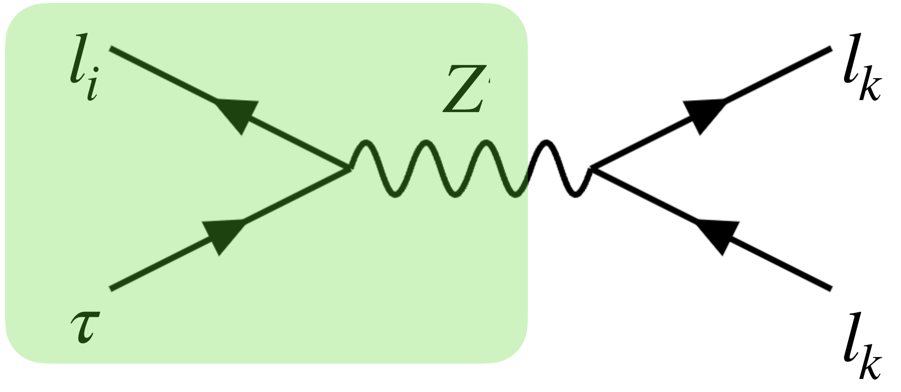

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$$\mathrm{BR}(Z \to \tau e) < 5.0 \times 10^{-6}$$
$$\mathrm{BR}(Z \to \tau \mu) < 6.5 \times 10^{-6}$$

LHC current bounds

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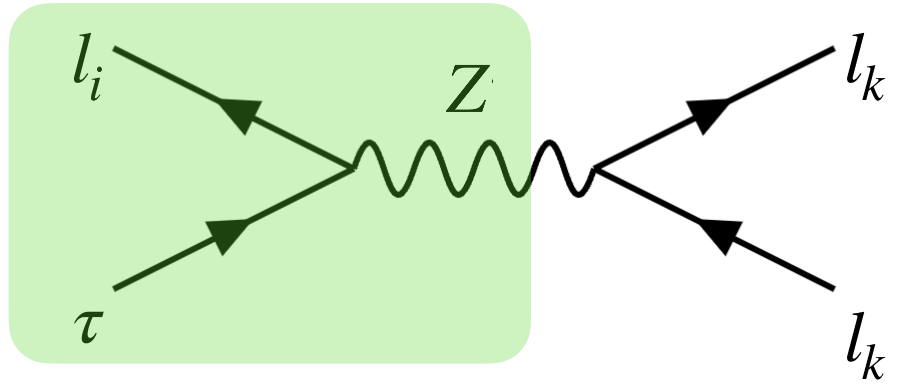

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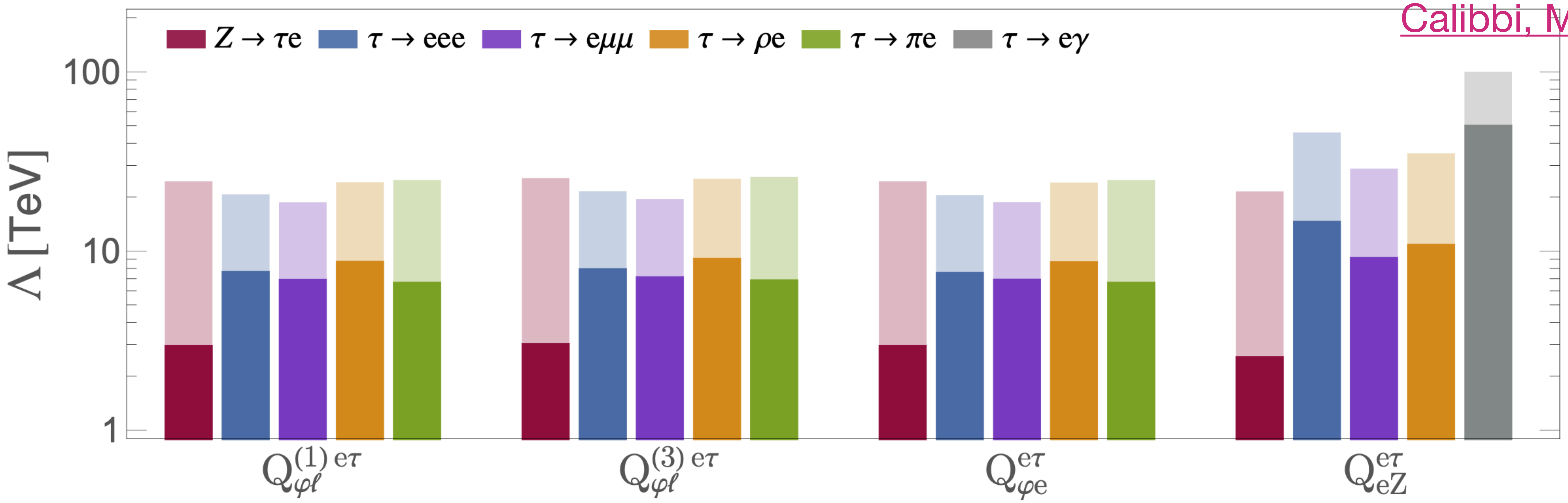
LHC current bounds

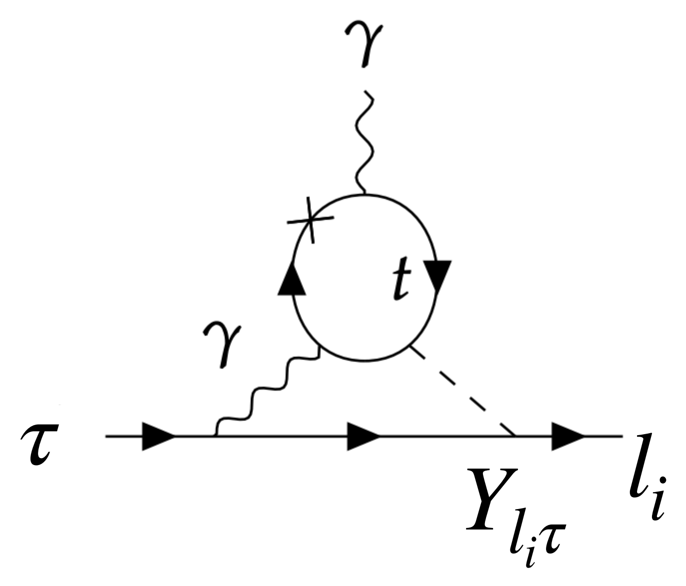
- Expect a huge number of Z at the FCC-ee = can compete with the sensitivities of Belle-II for the LFV decays

Calibbi, Marcano, Roy 2107.10273

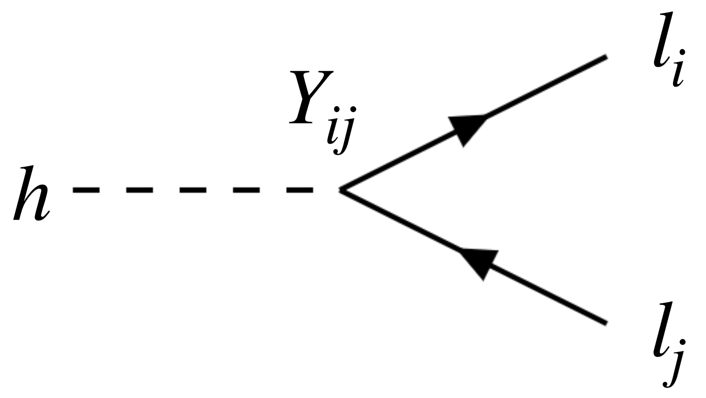


Similar for $\tau \to \mu$

Complementarity: Higgs decays



vs



$\mathrm{BR}(h \to \tau e) < 0.20\,\%$

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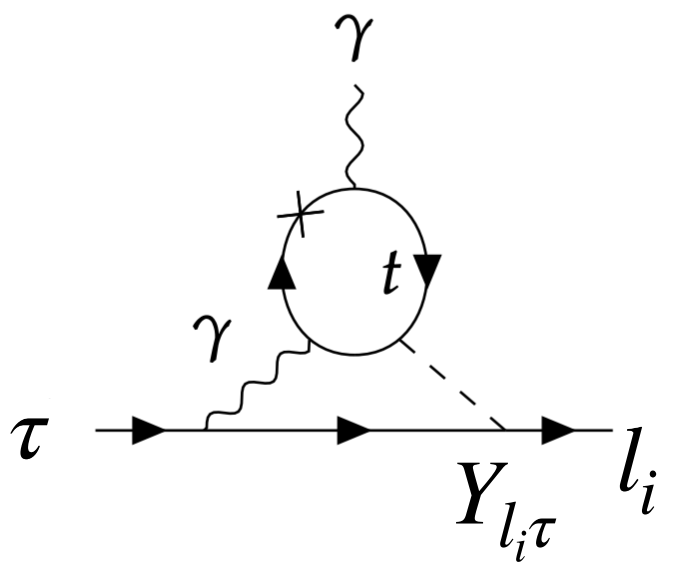
- If the τ decays happen via Higgs LFV couplings, they could be probed by $h \to \tau l_i$ searches

Complementarity: Higgs decays



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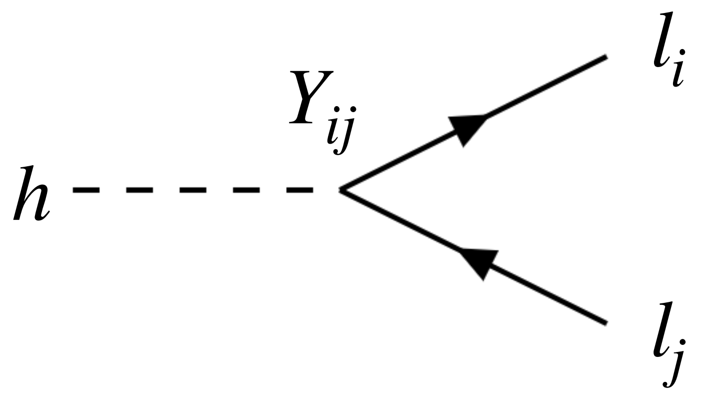

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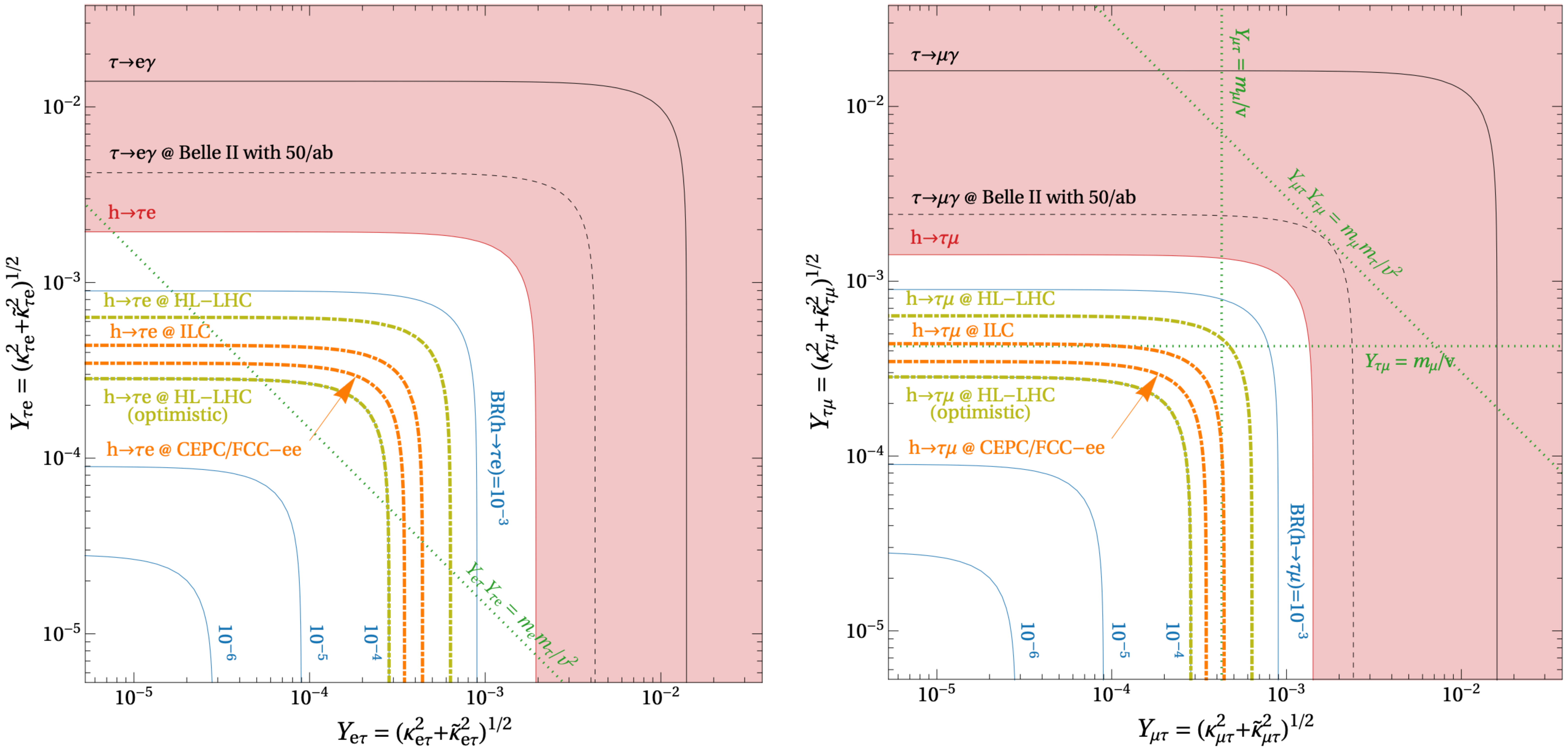


Atlmannshofer et al. 2205.10576

Conclusion

- LFV is New Physics that must exist because we see it in neutrino oscillations, and could be just around the corner

- τ LFV is interesting because:

    A. If observed, the new interactions should be relatively large

    B. There are numerous processes that one can look for in τ decays because of the large phase space **(Belle-II will have the best sensitivities for these processes)**

- We can investigate τ LFV in the EFT framework by assuming heavy new states. Generally, experiments are sensitive to $\tau \to l_i$ Wilson coefficients if the New Physics scale is around $\Lambda \sim 10$ TeV

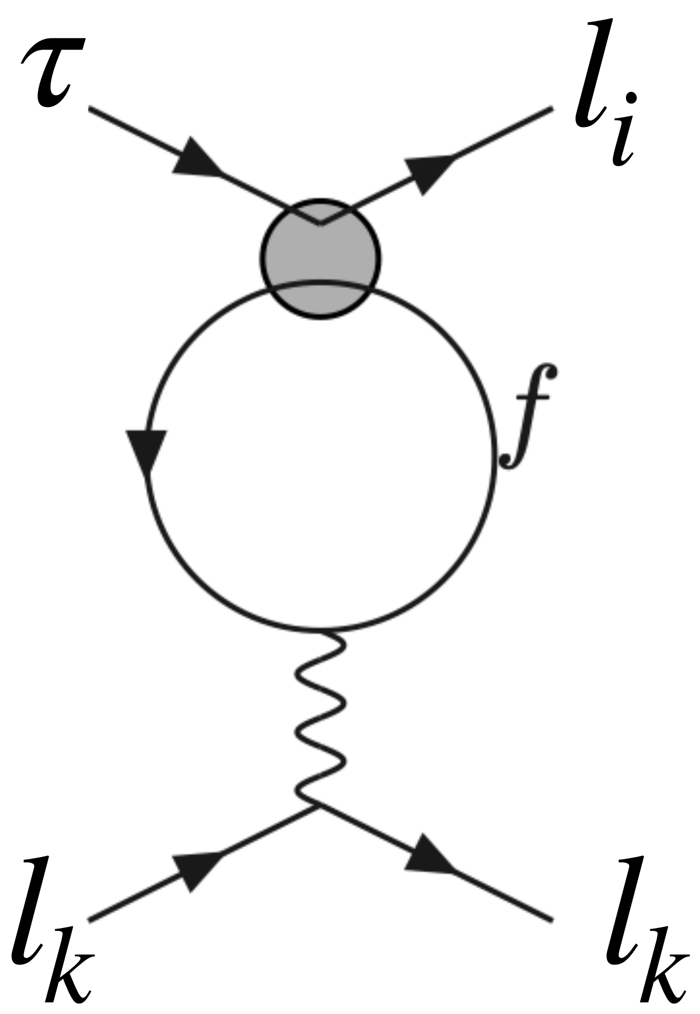
- LFV meson/baryon decays with b quarks are theoretically interesting and Belle-II will also improve the branching ratio sensitivities

- The multitude of processes, together with Dalitz plots, angular and kinematical distributions, allow for a detailed knowledge of the EFT coefficients, with a promising potential to pinpoint particular models

Back-up

Leptonic three body decay: one-loop RGEs

- QED penguin can mix any $\tau \to l$ vector with the $\Delta F = 1$ four-lepton vector involved in the tree-level process, leading to a sensitivity to all vectors for NP scales $\Lambda \sim \text{few TeV}$ and $\mathcal{O}(1)$ coefficients



$$C^{l_i \tau l_k l_k}_{V,XY} \sim q_f \frac{\alpha}{\pi} \log\left(\frac{\Lambda}{m_\tau}\right) C^{l_i \tau f f}_{V,XZ}$$

SMEFT basis dimension six

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D^\mu H\right)^* \left(H^\dagger D_\mu H\right)$

$5: \psi^2 H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma^\mu l_r)(\bar{l}_s \gamma_\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma^\mu q_r)(\bar{q}_s \gamma_\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma^\mu \tau^I q_r)(\bar{q}_s \gamma_\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma^\mu l_r)(\bar{q}_s \gamma_\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma^\mu \tau^I l_r)(\bar{q}_s \gamma_\mu \tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma^\mu e_r)(\bar{e}_s \gamma_\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma^\mu u_r)(\bar{u}_s \gamma_\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma^\mu d_r)(\bar{d}_s \gamma_\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma^\mu e_r)(\bar{u}_s \gamma_\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma^\mu e_r)(\bar{d}_s \gamma_\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma^\mu u_r)(\bar{d}_s \gamma_\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma^\mu T^A u_r)(\bar{d}_s \gamma_\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma^\mu l_r)(\bar{e}_s \gamma_\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma^\mu l_r)(\bar{u}_s \gamma_\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma^\mu l_r)(\bar{d}_s \gamma_\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma^\mu q_r)(\bar{e}_s \gamma_\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma^\mu q_r)(\bar{u}_s \gamma_\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma^\mu T^A q_r)(\bar{u}_s \gamma_\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma^\mu q_r)(\bar{d}_s \gamma_\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma^\mu T^A q_r)(\bar{d}_s \gamma_\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk} (\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk} (\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$

$8: (\not{B}) + \text{h.c.}$	
Q_{duql}	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} (d_p^\alpha C u_r^\beta)(q_s^{j\gamma} C l_t^k)$
Q_{qque}	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} (q_p^{j\alpha} C q_r^{k\beta})(u_s^\gamma C e_t)$
Q_{qqql}	$\epsilon_{\alpha\beta\gamma} \epsilon_{mn} \epsilon_{jk} (q_p^{m\alpha} C q_r^{j\beta})(q_s^{k\gamma} C l_t^n)$
Q_{duue}	$\epsilon_{\alpha\beta\gamma} (d_p^\alpha C u_r^\beta)(u_s^\gamma C e_t)$

Hadronic matrix elements

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$$[\, i\,\overline{q}_i\,\gamma_5\,q_j \;\to\; P\,] \;\simeq\; 2\,B_0\,F\,\Omega_P^{(1)}(ij) \,+\, 2\,\frac{B_0}{F}\,\frac{d_m^2}{M_P^2}\,m_K^2\,\Omega_P^{(2)}(ij)\,,$$

$$[\,\overline{q}_i\,\gamma_\mu\,\gamma_5\,q_j \;\to\; P\,] \;\simeq\; -i\,2\,F\,\Omega_A^{(1)}(ij)\,p_\mu\,,$$

$$[\,\overline{q}_i\,\gamma_\mu\,q_j \;\to\; V\,] \;\simeq\; -2\,F_V\,M_V\,\Omega_V^{(1)}(ij)\,\varepsilon_\mu\,,$$

$$[\,\overline{q}_i\,\sigma_{\mu\nu}\,q_j \;\to\; V\,] \;\simeq\; i\,2\,\frac{T_V}{M_V}\,\Omega_T^{(1)}(ij)\,(\,p_\mu\,\varepsilon_\nu \,-\, p_\nu\,\varepsilon_\mu\,)\;,$$

$$\begin{aligned}[\,\overline{q}_i\,q_j \;\to\; P_1\,P_2\,] \simeq\; & 2\,B_0\,\Omega_S^{(1)}(ij)\left[1+4\,\frac{L_5^{\mathrm{SD}}}{F^2}\,\left(s-m_1^2-m_2^2\right)\right] + \,2\,\frac{B_0}{F^2}\,\frac{d_m^2}{M_P^2}\,m_K^2\,\Omega_S^{(2)}(ij) \\ & + \frac{B_0}{F^2}\,c_m\sum_S \frac{\Omega_S^{(3)}(ij)}{s\,-\,M_S^2}\left[c_d\,\Omega_S^{(4)}\,\left(\,s\,-\,m_1^2\,-\,m_2^2\,\right) \,+\, 2\,c_m\,m_K^2\,\Omega_S^{(5)}\right] \\ & + \frac{1}{3}\,\frac{B_0}{F^2}\,\gamma\sum_T \frac{\Omega_T^{(2)}(ij)}{M_T^4}\,\Big\{ g_T\,\Omega_T^{(3)}\,\left[\,(m_1^2-m_2^2)^2 \,+\, M_T^2\,(m_1^2+m_2^2)\right. \\ & \left. -\; s\,(M_T^2+s)\right] \,+\, 2\,(2M_T^2+s)\,\left[\beta\,\Omega_T^{(4)}(m_1^2+m_2^2-s)\;-\;2\,\gamma\,m_K^2\,\Omega_T^{(5)}\right]\Big\}\,,\end{aligned}$$

$$\begin{aligned}[\,\overline{q}_i\,\gamma_\mu\,q_j \;\to\; P_1\,P_2\,] \simeq\; & \left[2\,\Omega_V^{(2)}(ij) \,+\, \sqrt{2}\,\frac{F_V\,G_V}{F^2}\sum_V \frac{s}{M_V^2\,-\,s}\,\Omega_V^{(1)}(ij)\,\Omega_V^{(3)}\right](\,p_1\,-\,p_2\,)_\mu \\ & + \left[\sqrt{2}\,\frac{F_V\,G_V}{F^2}\,(m_2^2-m_1^2)\sum_V \frac{\Omega_V^{(1)}(ij)\,\Omega_V^{(3)}}{M_V^2\,-\,s}\right](\,p_1\,+\,p_2\,)_\mu\,,\end{aligned}$$

$$[\,\overline{q}_i\,\sigma^{\mu\nu}\,q_j \;\to\; P_1\,P_2\,] \simeq \frac{i}{F^2}\left[-\Lambda_2^{\mathrm{SD}}\,\Omega_T^{(6)}(ij) \,+\, 2\,\sqrt{2}\,G_V\,T_V\sum_V \frac{\Omega_T^{(1)}(ij)\,\Omega_V^{(3)}}{M_V^2\,-\,s}\right](\,p_1^\mu\,p_2^\nu \,-\, p_1^\nu\,p_2^\mu\,)\,.$$