Introduction to Wilson Coefficients

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Standard Model at a Glance

- Standard Model describes our Universe at the most fundamental level
- It is an elegant model that describes the fundamental particles and how they interact via three of the four fundamental forces of nature
- It is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$
  - Strong Int. (QCD) : $SU(3)_C \Longrightarrow N^2 - 1 = 8$ (gluons)
  - Weak Int. : $SU(2)_L \Longrightarrow N^2 - 1 = 3$ $(W^{\pm}, Z)$
  - EM Int. : $U(1)_Y \Longrightarrow$ One generator (γ)
- However the gauge Invariance : No mass terms to the mediating particles ($W^{\pm}$ and Z) and the fermions.
- The masses can be generated by introducing a scalar field (Higgs) which breaks the symmetry spontaneously :

$$SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{EM}}$$

Quick overview of SM

- A model of elementary particles and their interactions is defined by three ingredients:
  - The symmetries of the Lagrangian
  - The representations of fermions and scalars
  - The pattern of spontaneous symmetry breaking.
- The Standard Model is defined as follows:
- The Gauge Symmetry is: $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$
- There are 3-fermion generations, each with 5 representations:

$$
\begin{array}{lll}
Q^I_{Li}(3, 2 + 1/6), & U^I_{Ri}(3, 1, +2/3), & D^I_{Ri}(3, 1, -1/3), \\
L^I_{Li}(1, 2, -1/2), & E^I_{Ri}(1, 1, -1), &
\end{array}
$$

- There is a single scalar representation $\phi(1, 2, +1/2)$
- The scalar ϕ assumes a VEV, $\langle \phi \rangle = (0, v/\sqrt{2})^T$, so that the gauge group is spontaneously broken

$$G_{SM} \to SU(3)_C \times U(1)_{EM}$$

SM Lagrangian

- The most general renormalizable Lagrangian:

$$\mathcal{L}_{SM} = \mathcal{L}_{Kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- The kinetic terms to maintain the gauge inv, i.e., replace the ordinary derivative by covariant derivative:

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y$$

- For example, for left-handed quarks

$$\mathcal{L}_{Kin} = i\overline{Q_{Li}^I}\gamma^\mu \left(\partial^\mu + \frac{i}{2} g_s G_a^\mu \lambda_a + \frac{i}{2} gW_b^\mu \tau_b + \frac{i}{6} g' B^\mu Y\right) Q_{Li}^I$$

- For left-handed lepton doublets

$$\mathcal{L}_{Kin} = i\overline{L_{Li}^I}\gamma^\mu \left(\partial^\mu + \frac{i}{2} gW_b^\mu \tau_b - ig' B^\mu Y\right) L_{Li}^I$$

- These parts of the Interaction Lagrangian are always CP conserving and involve three parameters.

SM Lagrangian

- The Higgs potential, which describes the scalar self-interaction

$$\mathcal{L}_{Higgs} = \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

- For SM scalar sector, there is a single doublet, this part of the $\mathcal{L}$ is also CP conserving, this part involves two parameters, i.e., μ and λ or Higgs mass and the VEV
- Yukawa Lagrangian for the leptonic part

$$\mathcal{L}_{Yuk}^{lepton} = Y_{ij}^{e} \overline{L_{Li}^{I}} \phi E_{Rj}^{I} + h.c.$$

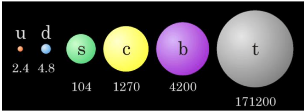
- After the Higgs acquires the VEV, these terms lead to charged lepton masses. (three parametes in this sector)
- $\mathcal{L}_{Yuk}$ for the Quark part is

$$\mathcal{L}_{Yuk}^{quark} = Y_{ij}^{d} \overline{Q_{Li}^{I}} \phi D_{Rj}^{I} + Y_{ij}^{u} \overline{Q_{Li}^{I}} \tilde{\phi} U_{Rj}^{I} + h.c.$$

- This is the part where quarks masses and flavour arises.
- The Yukawa ints for the quarks are described by ten physical parameters. They can be chosen to be the six quark masses and the four parameters of the CKM matrix.

Importance of Flavour Physics

- Flavour Physics encompasses many of the open questions of the Standard Model
- Why there are 3-generations of quarks with hierarchical masses



- Why the Quark and Lepton mixing matrices are so different
- Most importantly, Flavour Physics serves as a tool to discover New Physics beyond the SM.
- Three Pillars of Flavour Physics:
  - The CKM mixing matrix and the Unitarity Triangle
  - Neutral Meson Mixing ($M^0 - \overline{M^0}$)
  - Rare decays: Flavour Changing Neutral Current transitions ($b \to s, d$)

Key Ingredient of Flavour Physics: CKM Matrix

- The unitary CKM matrix $V_{\rm CKM}$ relates the weak eigenstates of d-type quarks to the corresponding mass eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

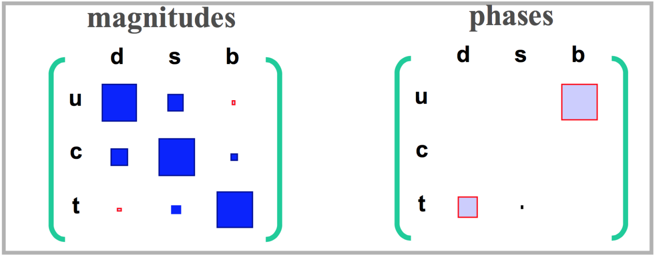
- Standard parametrization :

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Wolfenstein parametrization of CKM matrix is:

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \qquad \lambda = 0.22.$$

Hierarchical Nature of CKM matrix



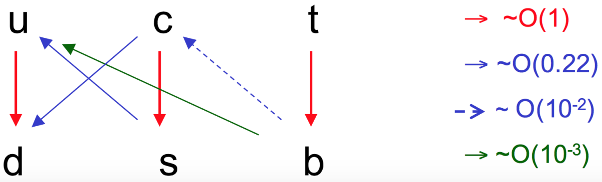
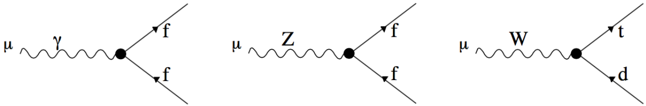


Illustration of CKM Matrix

- In the SM, we have three generations of quarks

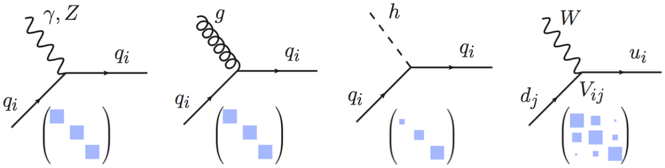
$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

- The electroweak interactions are described by Charged Current (CC) mediated by $W_\mu^\pm$ or Neutral Current (NC) transitions mediated by A_μ and Z_μ



- Flavour Changing Neutral Currents (FCNCs) are processes in which the quark flavour changes, while the quark charge stays the same.

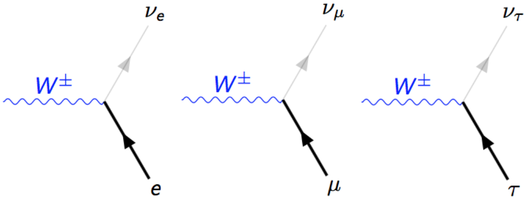
- The flavor sector is that part of the Standard Model which arises from the interplay of quark weak gauge couplings and quark-Higgs couplings.



- From a theorist's perspective, the aim flavor physics is to search for those places where SM predictions are clean enough, so that effects from physics BSM could be recognized if indeed they occur.
- In other words : The objective is to critically test the SM and look for New Physics beyond it.

Lepton Flavour Universality a key ingredient of SM

- In the SM, the couplings of the gauge bosons to leptons are independent of the lepton flavour
- Equal couplings of the W and Z bosons to electrons, muons and taus



- For Z boson, this has been checked at 2×10^{-3} level of accuracy at LEP
- For W boson, the τ BR is 2.6σ above $\langle e, \mu \rangle$ which are equal to about 2% precision level

$$\frac{2\Gamma(W \to \tau\nu)}{[\Gamma(W \to \mu\nu) + \Gamma(W \to e\nu)]}\Big|_{\mathrm{LEP}} = 1.066 \pm 0.025$$

Charged current and Neutral Current Interactions

- The CC interaction is given as

$$-\frac{g}{\sqrt{2}}\bar{\nu}_{eL}W^{\mu}\gamma_{\mu}e_L + \mathrm{h.c.}$$

- Only LH fields take part in CC interactions. Therefore the W interaction violate parity
- The $Wl\nu$ interaction is universal
- The interaction Lagrangian is

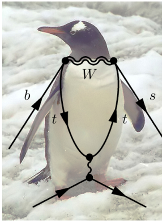
$$\mathcal{L}_{\rm int} = \frac{g}{\cos\theta_W}(T_3 - \sin^2\theta_W Q)\bar{\psi}\gamma^{\mu}\psi Z_{\mu} + eQ\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$

- We define: $Q = T_3 + Y, \qquad e = g\sin\theta_W$
- Photon coupling is parity invariant
- Z couples to both LH and RH fermions but in a parity violating way

Possible ways to search for New Physics

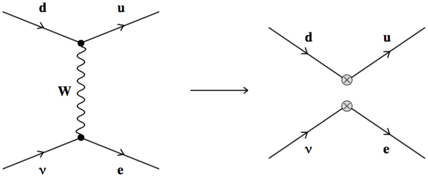
- Searches for NP signature can be performed in two ways
- The first one is through direct production of new particles in colliders. The key ingredient for this is so-called "relativistic path" is the amount of energy available in collision, which drives the maximum range that can be probed
- The second method, the so-called "quantum path" exploits the presence of virtual states in the decays of SM particles.

- Due to QM, the intermediate states can be much heavier than the initial and final particles and can affect the decay rate as well as the angular distributions.



Phenomenology of Weak decays of heavy hadrons

- The basic starting point for the phenomenology of weak decays of hadrons is the weak Hamiltonian
$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\mathrm{CKM}} C_i(\mu) O_i$$
- O_i are the local operators which govern the decays in question
- C_i are the Wilson coefficients describe the strength with which a given operator enters the Hamiltonian
- In the simplest case of β decay $\mathcal{H}_{eff}$ takes the familiar form
$$\mathcal{H}^{\beta}_{eff} = \frac{G_F}{\sqrt{2}} V_{ud}[\bar{u}\gamma_\mu(1-\gamma_5)d \otimes \bar{e}\gamma^\mu(1-\gamma_5)\nu_e]$$



Weak decays of muons

- Amplitude for $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ is

$$\mathcal{A} = -\frac{1}{8}\frac{g^2}{k^2 - M_W^2}[\bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu][\bar{e}\gamma_\mu(1-\gamma_5)\nu_e]$$

$$\Longrightarrow \frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu][\bar{e}\gamma_\mu(1-\gamma_5)\nu_e], \quad \text{in the limit } \mathrm{k}^2 \ll \mathrm{M}_\mathrm{W}^2$$

- $\mathcal{B}(\mu \to e\nu_\mu\bar{\nu}_e) \sim 100\% \Longrightarrow$ decay width of muon used to evaluate G_F

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_\mu^2}\right)$$

$$\tau_\mu^{\mathrm{theo}} = 2.18776 \times 10^{-6}\mathrm{s}, \qquad \tau_\mu^{\mathrm{exp}} = 2.1969811(22) \times 10^{-6}\mathrm{s}$$

Weak decays of muons

- Including 1-loop EW corrections

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]\left(1 + \frac{3}{5}\frac{m_\mu^2}{m_\mu^2}\right)$$

$$\tau_\mu^{\rm theo} = 2.19699 \times 10^{-6}\mathrm{s}, \qquad \tau_\mu^{\rm exp} = 2.1969811(22) \times 10^{-6}\mathrm{s}$$

- Theory and Expt. data are in perfect agreement

Weak decays of tau

- τ being the heaviest lepton decays to both quarks and leptons: $\tau \to \ell \nu_\tau \bar{\nu}_\ell$ and $\tau \to \nu_\tau (\pi^-, K^-)$
- Total decay width of τ is

$$\Gamma_\tau^{\mathrm{tot}} = \frac{G_F^2 m_\tau^5}{192\pi^3} \left[f(m_f^2/m_\tau^2) + \mathrm{h.o} \right]$$

$$\tau_\mu^{\mathrm{theo}} = 3.26707 \times 10^{-13}\mathrm{s}, \qquad \tau_\mu^{\mathrm{exp}} = 2.906 \times 10^{-13}\mathrm{s}$$

- Gluon exchange within the quarks.
- QED effects are under control but not QCD
- Need to know the dynamics of loop calculations

Effective Field Theory Approach

- An EFT is defined by an effective action, which in turn is completely specified by the following three ingredients:
- Degrees of freedom: The first step of building an EFT is to figure out what are the degrees of freedom that are relevant to describe the physical system. These are the variables that will appear in the effective action.
- Symmetries: The second step in building an EFT consists in identifying the symmetries that constrain the form of the effective action, and therefore the dynamics of the system.
- Any term compatible with the symmetries of the system should in principle be included in the effective action. As a result, effective actions generically contain an infinite number of terms.

- Expansion Parameter The key to handling an action with an infinite number of terms lies in the fact that all EFTs feature one or more expansion parameters.
- Observable quantities are calculated in perturbation theory as series in these small parameters.
- This ensures that only a finite number of terms contribute at any given order in perturbation theory
- These three elements are fairly easy to state, but ensuring that they are properly implemented in an EFT can be a subtle matter.

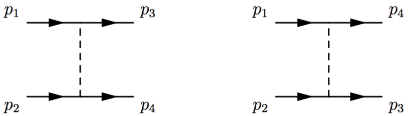
Toy Model for Effective Field Theory

- Let's consider a toy model of the interaction between fermion (ψ) and pseudoscalar meson (ϕ)
- The action is given as

$$S = \int d^4x \left(\frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \frac{1}{2} M^2 \phi^2 + \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + iy\phi\bar{\psi}\gamma^5\psi + \frac{\lambda}{4!}\phi^4 \right)$$

- This action is invariant under parity provided ϕ is a pseudoscalar, i.e. it transforms like $\phi \to -\phi$ under a parity transformation.
- A cubic self-interaction for ϕ would not be invariant and has therefore been omitted.
- We will consider a region of parameter space where $y, \lambda \ll 1$, so that we can work in perturbation theory. Moreover, we will assume that the scalar is much heavier than the fermion, i.e. $M \gg m$.
- Suppose we are interested in describing a simple process such as the elastic scattering of two fermions

Toy Model for Effective Field Theory



$$i\mathcal{M} = y^2 \left[(\bar{u}_3(p_3)\gamma^5 u_1(p_1)) \frac{-i}{(p_3 - p_1)^2 + M^2} (\bar{u}_4(p_4)\gamma^5 u_2(p_2)) - (3 \leftrightarrow 4) \right]$$

- If the typical energy E of the fermions involved in this process is: $m \ll E \ll M$, the scalar propagator can be expanded in powers of momentum transfer, which gives

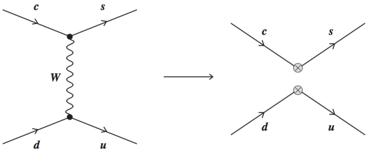
$$i\mathcal{M} = i\frac{y^2}{M^2} \left[(\bar{u}_3(p_3)\gamma^5 u_1(p_1))(\bar{u}_4(p_4)\gamma^5 u_2(p_2)) - (3 \leftrightarrow 4) \right] + \mathcal{O}(E^2/M^2)$$

Toy Model for Effective Field Theory

- As long as E^2/M^2 is smaller than the required precision, the full or effective theories will yield almost the same result.
- The ratio E/M controls how well our EFT approximates the full theory, which is the expansion parameter.
- At energies $E > M$, our EFT ceases to be useful and we need to resort to the full theory.

Basic Idea of Wilson Coefficients

- Consider the quark level transition $c \to s u \bar{d}$



- The amplitude can be given as

$$\begin{aligned}\mathcal{M} &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{1}{1 - k^2/M_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} \\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \mathcal{O}(k^2/M_W^2)\end{aligned}$$

- This result may also be obtained from

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + \text{higher order operators}$$

- Higher order operators typically involve the derivative terms, correspond to $\mathcal{O}(k^2/M_W^2)$.

Basic Idea

- Neglecting the higher- dimensional operators and keeping only operators with dim-6

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} C O, \qquad O = (\bar{s}c)_{V-A}(\bar{u}d)_{V-A}$$

- This simple example illustrates the basic idea of OPE: Product of two CC operators expanded into a series of local operators (O), weighted by C, the Wilson coefficients.
- When QCD effects are taken into account

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_1(\mu) O_1 + C_2(\mu) O_2 \right]$$

$$O_1 = (\bar{s}_\alpha c_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A} \qquad O_2 = (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{u}_\beta d_\beta)_{V-A}$$

- A new operator with same flavor but different color structure appeared due to the color algebra

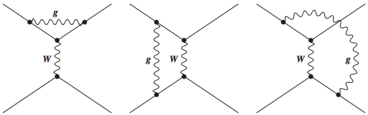
$$t^a_{\alpha\beta} t^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$$

Calculation of Wilson Coefficients

- $C_{1,2}$ can be determined matching the full and effective theory amplitudes

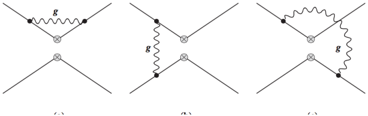
$$\mathcal{M}_{\rm full} = \mathcal{M}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs}^{*} V_{ud} \left[C_1 \langle O_1 \rangle + C_2 \langle O_2 \rangle \right]$$

- Full Theory (all particles appear as dynamical degrees of freedom), Effective Theory (Integrating out the heavy field W)
- The full amplitude can be calculated from the diagrams:



Calculation of Wilson Coefficients

- The matrix elements can be calculated $\mathcal{O}(\alpha_s)$



- Thus we obtain

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right), \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right)$$

- These results are valid only for $\mu = \mathcal{O}(M_W)$. For $\mu \ll M_W$, we have sum the large logarithms to all order of perturbation theory, which can be done using the RGE for $C(M_W)$.

Summary of the basic Formalism

- The ultimate goal: Evaluation of weak-decay amplitudes involving hadrons in the framework of a low-energy effective theory

$$\langle \mathcal{H}_{\rm eff} \rangle = \frac{G_F}{\sqrt{2}} V_{\rm CKM} \sum_i C_i(\mu) O_i(\mu) = \frac{G_F}{\sqrt{2}} V_{\rm CKM} \langle \vec{O}_i^T(\mu) \rangle \vec{C}(\mu)$$

- Step 1: Calculate $C(\mu_W)$ to the desired order of α_s. Since $\ln(\mu_W/M_W)$ are not large, can be done in ordinary perturbation theory
- Step 2: Evolve the coefficients from μ_W to low-energy scale μ using RGE

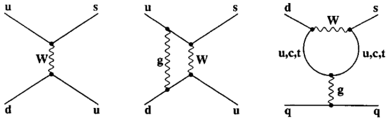
$$\vec{C}(\mu) = U(\mu, \mu_W) \vec{C}(\mu_W)$$

- Step 3: Calculate the hadronic matrix elements $\langle \vec{Q}(\mu) \rangle$ by some non-perturbative method.

Effective Hamiltonian for $\Delta B = \Delta S = 1$

- The effective Hamiltonian is given as

$$\mathcal{H}_{\rm eff}(\Delta B = 1) = \frac{G_F}{\sqrt{2}}\left[\lambda_{u,c}(C_1(\mu_b)O_1^{u,c} + C_2(\mu_b)O_2^{u,c}) - \lambda_t \sum_{i=3}^{6} C_i(\mu_b)O_i\right]$$



- The Wilson Coefficients evaluated at M_W scale are given as

$$C_1(M_W) = \frac{11}{2}\frac{\alpha_s(M_W)}{4\pi}, \quad C_2(M_W) = 1 - \frac{11}{6}\frac{\alpha_s(M_W)}{4\pi},$$
$$C_{3,5}(M_W) = -\frac{\alpha_s(M_W)}{24\pi}\tilde{E}_0(x_t), \quad C_{4,6}(M_W) = -\frac{\alpha_s(M_W)}{8\pi}\tilde{E}_0(x_t)$$

Renormalization Group Equation

- These Wilson coefficients will be evolved to b scale using RGE

$$\vec{C}(\mu_b) = \hat{U}_5(\mu_b, M_W)\vec{C}(M_W)$$

with the final results

$$\begin{aligned} C_1(\mu_b) &= -0.184, \quad C_2(\mu_b) = 1.078, \quad C_3(\mu_b) = -0.013, \\ C_4(\mu_b) &= -0.035, \quad C_5(\mu_b) = -0.009, \quad C_6(\mu_b) = -0.041 \end{aligned}$$

Flavor Changing Neutral Current (FCNC)

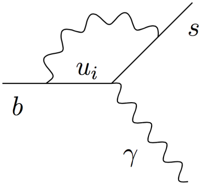
- FCNCs are highly suppressed, e.g., $K \to \mu\nu$ vs. $K_L \to \mu\mu$
- In the SM, there are no FCNC at the tree level
- In the SM there are four neutral bosons (g, γ, Z, h) and their couplings are diagonal.
- The photon, gluon and Z couplings are also universal, while the Higgs couplings are proportional proportional to the masses

FCNC at one loop

- FCNCs are suppressed in the SM, i.e., there is no tree level exchange
- The suppression factors are: CKM factors and Mass factors/GIM Mechanism
- The Loop factor is Universal. For an example consider the $b \to s\gamma$ process

The amplitude for this process is

$$\mathcal{A}(b \to s\gamma) \propto V_{ib} V_{is}^{*} f(m_i)$$



- CKM unitarity $\Longrightarrow$ m_i independent term in $f(m_i)$ vanishes.
- The amplitude must depend on mass

FCNC at one loop

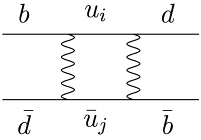
$$\mathcal{A}(b \to s\gamma) \propto V_{ib} V_{is}^* f(m_i)$$

- For small $x_i = m_i^2/m_W^2$: $\mathcal{A} \sim x_i$ or $x_i \ln x_i$
- In s decays this gives m_c^2/m_W^2 extra suppression and for charm it gives m_s^2/m_W^2 suppression
- However, for b decays it is not important as $m_t \sim m_W$

Box diagram

- We know that the neutral mesons mix with the box diagrams
- There are four such mesons: $K(\bar{s}d)$, $B(\bar{d})$, $B_s(\bar{b}s)$, $D(c\bar{u})$
- Question is: Why not π^0 or the excited mesons, e.g. K^{*0}

$$\Delta M \propto \sum_{i,j} V_{is} V_{id}^* V_{js} V_{jd}^* f(m_i, m_j)$$



- The constant tem vanises due to unitarity (GIM)
- To leading order $f \sim m_i^2/m_W^2$
- K mixing : m_c^2/m_W^2, D mixing : m_c^2/m_W^2, and no suppression for B/B_s mesons
- Meson mixing is FCNC process with : Loop Suppression, CKM Suppression and GIM suppression

Contributions of Loop Diagrams : $b \to s$ transition

- The contributions from FCNC one-loop diagrams in 't Hooft-Feynman gauge:

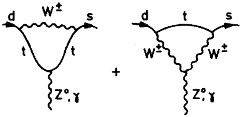
$$\mathrm{Box}(\Delta B = 2) = \lambda_i^2 \frac{G_F^2}{16\pi^2} m_W^2 S_0(x_i) (\bar{b}s)_{V-A} (\bar{b}s)_{V-A}$$

$$\mathrm{Box}(b \to s\nu\bar{\nu}) = \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} [-4B_0(x_i)] (\bar{b}s)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$\mathrm{Box}(b \to s\mu\bar{\mu}) = \lambda_i \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} [B_0(x_i)] (\bar{b}s)_{V-A} (\bar{\mu}\mu)_{V-A}$$

$$\bar{b}Zs = i\lambda_i \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} m_Z^2 \frac{\cos\theta_W}{\sin\theta_W} C_0(x_i) \bar{b}\gamma_\mu (1-\gamma_5) s$$

$$\bar{b}\gamma s = -i\lambda_i \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} D_0(x_i)\, \bar{b}(q^2\gamma_\mu - q_\mu \not{q})(1-\gamma_5) s$$



Basic Functions

- The loop functions are:

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3}{2(1-x_t)^3}\ln x_t$$

$$B_0(x_t) = \frac{1}{4}\left[\frac{x_t}{1-x_t} + \frac{x_t \ln x_t}{(x_t-1)^2}\right]$$

$$C_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-6}{x_t-1} + \frac{3x_t+2}{(x_t-1)^2}\ln x_t\right]$$

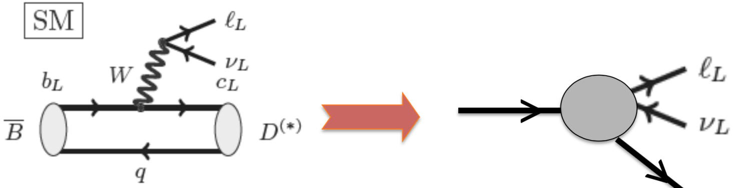
$$D_0(x_t) = -\frac{4}{9}\ln x_t + \frac{-19x_t^3+25x_t^2}{36(x_t-1)^3} + \frac{x_t^2(5x_t^2-2x_t-6)}{18(x_t-1)^4}\ln x_t$$

Summary of the Effective Vertices

- Depend on the masses of internal quarks/leptons and consequently are calculable functions of $x_i = m_i^2/m_W^2$
- Depend on elements of the CKM matrix elements
- The dependences of a given vertex on the CKM factors and the masses of internal fermions govern the strength of the vertex in question.

Effective Hamiltonian for B meson decays

- Formulation of weak decays in terms of effective Hamiltonians is very suitable for the inclusion of new physics effects.
- B-meson decays, is governed by Feynman diagrams with W, Z- and top quark exchanges
- Such diagrams with full W, Z and t-propagators represent the situation at very short distance scales $\mathcal{O}(m_W, m_Z, m_t)$, whereas the true picture of a decaying hadron with masses $\mathcal{O}(m_b)$ more properly described by effective point-like vertices and are represented by the local operators O_i.
- The Wilson coefficients C_i can then be regarded as coupling constants associated with these effective vertices.



Effective Hamiltonian for B meson decays

- $\mathcal{H}_{\mathrm{eff}}$ is simply a series of effective vertices multiplied by effective coupling constant C_i (OPE)

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{\mathrm{CKM}} C_i(\mu) O_i$$

- $C_i(\mu)$ summarize the contributions from scales higher than μ and can be calculated in perturbation theory as long as μ is not too small.
- Includes contribution from heavy particles : W, Z, t or new BSM particles
- $C_i(\mu)$ depend generally on m_t dependence can be found by evaluating so-called box and penguin diagrams
- The value of μ can be chosen arbitrarily.
- It is customary to choose μ to be of the order of the mass of the decaying hadron, i.e., $\mathcal{O}(m_b)$ for B decays

Phenomenology of Weak decays of heavy hadrons

- After constructing the effective Hamiltonian we can proceed to evaluate the decay amplitudes.
- An amplitude for a decay of a given meson say $B \to M_1 M_2$ is simply given by

$$A(B \to M_1 M_2) = \langle M_1 M_2 | \mathcal{H}_{eff} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle M_1 M_2 | Q_i(\mu) | B \rangle$$

- The essential virtue of OPE : It allows to separate the problem of calculation of $A(B \to M_1 M_2)$ into two distinct parts :
  - The short-distance or perturbative calculation of the couplings
  - The long-distance calculation of the matrix elements $\langle Q_i(\mu) \rangle$
- Clearly, in order to calculate the amplitude $A(B \to M_1 M_2)$, the matrix elements $\langle Q_i(\mu) \rangle$ have to be evaluated.

- Non-perturbative methods like QCD Sum rules, Light Cone Sum Rules, ChPT, HQET etc are used for $\langle Q_i \rangle$
- Needless to say, all these non-perturbative methods have some limitations.
- The fact that in most cases the matrix elements$\langle Q_i \rangle$ cannot be reliably calculated at present, is very unfortunate.
- One of the main goals of the experimental studies of weak decays is the determination of the CKM factors V_{CKM} and the search for the physics beyond the Standard Model.
- Without a reliable estimate of $\langle Q_i \rangle$ this goal cannot be achieved
- One of such schemes, the factorization scheme for matrix elements: $\langle M_1 M_2 | J^\mu J_\mu^\dagger | B \rangle = \langle M_1 | J^\mu | B \rangle \langle M_2 | J_\mu^\dagger | 0 \rangle$

Form Factors

- Let us consider a simple example of β decay process : $n \to pe\bar{\nu}_e$.
- We focus only on the vector current, the relevant matrix element is

$$\langle p(p')|\bar{d}\gamma^\mu u|n(p)\rangle$$

  which we can not calculate from the first principle.
- We parametrize this by writing out the most general linear combination of kinematic variables satisfying the Lorentz symmetry

$$\langle p(p')|\bar{d}\gamma^\mu u|n(p)\rangle \sim ap^\mu + bp'^\mu$$

  where a and b are form factors.
- These can depend only on Lorentz scalars and there are three of these available (p^2, p'^2 and $p \cdot p'$).
- The first two are just masses and are not dynamical (they do not change when we change the momenta). The third one is a dynamical Lorentz scalar

Form Factors

- It is convenient to write in a different momentum basis by defining the momentum transfer

$$q = p - p' \implies q^2 = p^2 + p'^2 - 2p \cdot p'$$

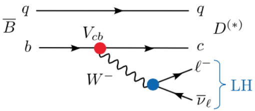
- Analogously, we can write the form factors, e.g., for $B \to D$ transition:

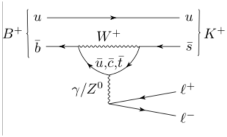
$$\langle D(p_D)|\bar{c}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)(p_D + p_B)^\mu + f_-(q^2)q^\mu$$

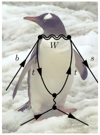
- These form factors are determined from Lattice Calculation or using some quark models, Light cone sum rule etc.

Charged Current vs. Neutral Current

- Charged currents are mediated by $W^{\pm}$ and Neutral currents are by Z. However, flavor changing neutral currents are suppressed in the SM and occur at one-loop level







- A glimpse at the PDG reveals that the probabilities for the charged currents lead to the dominant weak decays, while the FCNC induced decays are extremely suppressed.

	Charged Currents		Neutral Current
$b \to c\ell\bar{\nu}_\ell$:	$\mathcal{B}(B \to D^0\ell\bar{\nu}_\ell)$=2.3%	$b \to s\ell^+\ell^-$:	$\mathcal{B}(B \to K^*\ell^+\ell^-) = 5 \times 10^{-7}$
$c \to s\ell\bar{\nu}_\ell$:	$\mathcal{B}(D \to K\mu\bar{\nu}_\mu)$=9%	$c \to u\ell^+\ell^-$:	$\mathcal{B}(D \to \pi\ell^+\ell^-) < 1.8 \times 10^{-4}$
$s \to u\mu\bar{\nu}_\mu$:	$\mathcal{B}(K \to \mu\bar{\nu}_\nu)$=64%	$s \to d\ell^+\ell^-$:	$\mathcal{B}(K_L \to \mu^+\mu^-) = 7 \times 10^{-9}$

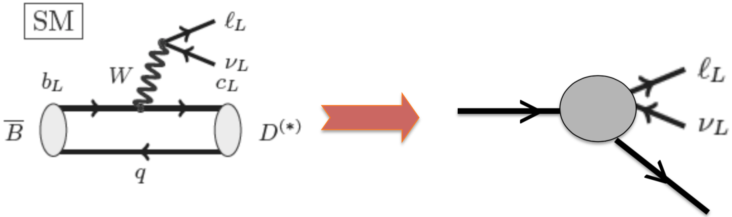
Effective Field Theory Approach for $b \to c\tau^- \bar{\nu}_\tau$

- The effective Hamiltonian responsible for the CC $b \to c\tau\bar{\nu}_l$ quark level transitions is

$$\mathcal{H}_{\rm eff}^{\rm CC} = \frac{4G_F}{\sqrt{2}} V_{cb} \Big[\left(\delta_{l\tau} + C^l_{V_L} \right) \mathcal{O}^l_{V_L} + C^l_{V_R}\mathcal{O}^l_{V_R} + C^l_{S_L}\mathcal{O}^l_{S_L} + C^l_{S_R}\mathcal{O}^l_{S_R} + C^l_T\mathcal{O}^l_T \Big]$$

- The corresponding dimension-six effective operators are given as

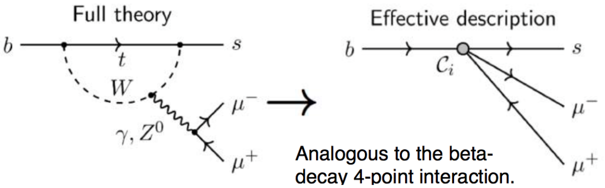
$$\begin{aligned}
\mathcal{O}^l_{V_L} &= (\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_{lL}), & \mathcal{O}^l_{V_R} &= (\bar{c}_R\gamma^\mu b_R)(\bar{\tau}_L\gamma_\mu\nu_{lL}), \\
\mathcal{O}^l_{S_L} &= (\bar{c}_R b_L)(\bar{\tau}_R\nu_{lL}), & \mathcal{O}^l_{S_R} &= (\bar{c}_L b_R)(\bar{\tau}_R\nu_{lL}), \\
\mathcal{O}^l_T &= (\bar{c}_R\sigma^{\mu\nu} b_L)(\bar{\tau}_R\sigma_{\mu\nu}\nu_{lL})
\end{aligned}$$



Effective Field Theory Approach for $b \to s\ell\ell$

- Compared to $b \to c\ell\nu_\ell$, $b \to s\ell\ell$ transitions are richer due to large no of observables
- The effective Hamiltonian describing $b \to s\ell^+\ell^-$ process

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i + \sum_{i=7,9,10,S,P} \left(C_i(\mu)\mathcal{O}_i + C_i'(\mu)\mathcal{O}_i' \right) \right] .$$



Effective Lagrangian for $b \to s\ell^-\ell^+$

- The effective Hamiltonian mediating the NC leptonic/semileptonic $b \to s\ell^+\ell^-$

$$\mathcal{H}_{\rm eff}^{\rm NC} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left[\sum_{i=1}^{6} C_i(\mu)\mathcal{O}_i + \sum_{i=7,9,10,S,P} \Big(C_i(\mu)\mathcal{O}_i + C_i'(\mu)\mathcal{O}_i' \Big) \right] .$$

where $\mathcal{O}_i$'s are the dimension-six operators:

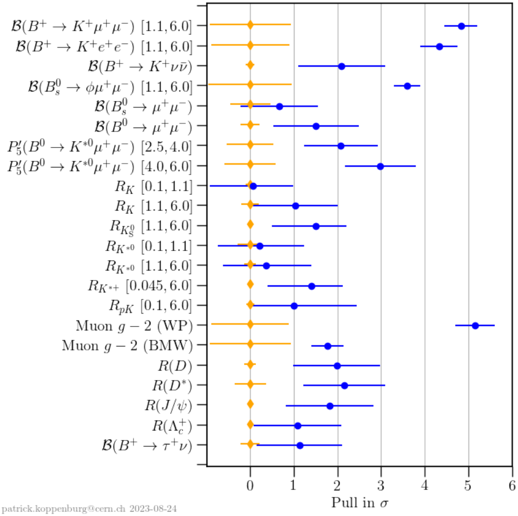
$$\mathcal{O}_7^{(\prime)} = \frac{\alpha_{\rm em}}{4\pi} \left[\bar{s}\sigma_{\mu\nu}\big(m_s P_{L(R)} + m_b P_{R(L)}\big) b \right] F^{\mu\nu},$$

$$\mathcal{O}_9^{(\prime)} = \frac{\alpha_{\rm em}}{4\pi} \big(\bar{s}\gamma^\mu P_{L(R)} b\big)\big(\bar{\ell}\gamma_\mu \ell\big) , \qquad \mathcal{O}_{10}^{(\prime)} = \frac{\alpha_{\rm em}}{4\pi} \big(\bar{s}\gamma^\mu P_{L(R)} b\big)\big(\bar{\ell}\gamma_\mu\gamma_5 \ell\big) ,$$

$$\mathcal{O}_S^{(\prime)} = \frac{\alpha_{\rm em}}{4\pi} \big(\bar{s} P_{L(R)} b\big)\big(\bar{\ell}\ell\big) , \qquad \mathcal{O}_P^{(\prime)} = \frac{\alpha_{\rm em}}{4\pi} \big(\bar{s} P_{L(R)} b\big)\big(\bar{\ell}\gamma_5 \ell\big) ,$$

- The primed as well as (pseudo)scalar operators are absent in the SM and can be generated only in the BSM theories.

List of Anomalies



Summary

- Current anomalies in the flavour sector provide an ideal platform to look for New Physics.
- They have huge impact on model building and also in the searches new particle like Leptoquarks and Z' .
- Building a viable model which accommodates the observed B anomalies and consistent with all other measured flavor observables is the need of the hour.

Thank you for your attention!