FLAVOUR MODEL BUILDING: CONSEQUENCES IN THE QUARK AND HIGGS SECTORS

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THE FLAVOUR PROBLEM

- What happens in the quark sector?
    - Textures + symmetries
    - What happens in the scalar sector?
- A (very brief) S3 example multi-Higgs example
  - quarks and Higgs sectors

FLAVOUR

- Interactions that distinguish between flavours

- why 3 generations?
- why those masses?
- why the gap between neutral and charged fermions
- why the difference between mixing matrices?
- why that amount of CP violation?
- …

- *Fermion masses*
- *Mixing*
- *CP violation*

Lead to discoveries

- $\Gamma(K_L \to \mu^+ \mu^-)/\Gamma(K^+ \to \mu^+ \nu) \to$ *charm quark*
- $\Delta m_K \to$ *charm mass*
- $\Delta m_B \to$ *top mass*
- $\varepsilon_K \to$ *third generation*
- ν *oscillation* $\to$ ν *mass*

Connections to new/unknown physics

- *Dark matter*
- *Baryogenesis*
- *Leptogenesis*
- *EW phase transition*
- *??*

Nir, CERN–LATAM School HEP (2015)

SOME ASPECTS OF THE FLAVOUR PROBLEM

- Quark and charged lepton masses very different, very hierarchical

$$m_u : m_c : m_t \sim 10^{-6} : 10^{-3} : 1$$

$$m_d : m_s : m_b \sim 10^{-4} : 10^{-2} : 1$$

$$m_e : m_\mu : m_\tau \sim 10^{-5} : 10^{-2} : 1$$

- Neutrino masses unknown, only difference of squared masses.
- Type of hierarchy (normal or inverted) also unknown
- Higgs sector under study

- Quark mixing angles

$$\theta_{12} \approx 13.0^o$$
$$\theta_{23} \approx 2.4^o$$
$$\theta_{13} \approx 0.2^o$$

- Neutrino mixing angles

$$\Theta_{12} \approx 33.8^\circ$$
$$\Theta_{23} \approx 48.6^\circ$$
$$\Theta_{13} \approx 8.6^\circ$$

- Small mixing in quarks, large mixing in neutrinos. Very different
- Is there an underlying symmetry?

?

The matter particles

$$Q_{Li}(3,2)_{+1/6},\quad U_{Ri}(3,1)_{+2/3},\quad D_{Ri}(3,1)_{-1/3},\quad L_{Li}(1,2)_{-1/2},\quad E_{Ri}(1,1)_{-1}\quad (i=1,2,3)$$

$$\phi(1,2)_{+1/2}.$$

The scalar

The Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{cin}} + \cancel{\mathcal{L}_{\psi}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\phi}$$

The fields strengths

$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f_{abc} G_b^\mu G_c^\nu$$
$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g \epsilon_{abc} W_b^\mu W_c^\nu$$
$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

The covariant derivative

U(1) charges

$$D^\mu = \partial^\mu + i g_s G_a^\mu L_a + i g W_b^\mu T_b + i g' B^\mu Y$$

group generators

The Yukawa interactions

$$\mathcal{L}_{\text{Y}}^{\text{ME}} = Y_{ij}^d \overline{Q_{Li}} \phi D_{Rj} + Y_{ij}^u \overline{Q_{Li}} \tilde{\phi} U_{Rj} + Y_{ij}^e \overline{L_{Li}} \phi E_{Rj} + \text{h.c.}.$$

$$\tilde{\phi} = i\tau_2 \phi^\dagger$$

The electroweak sector of the SM

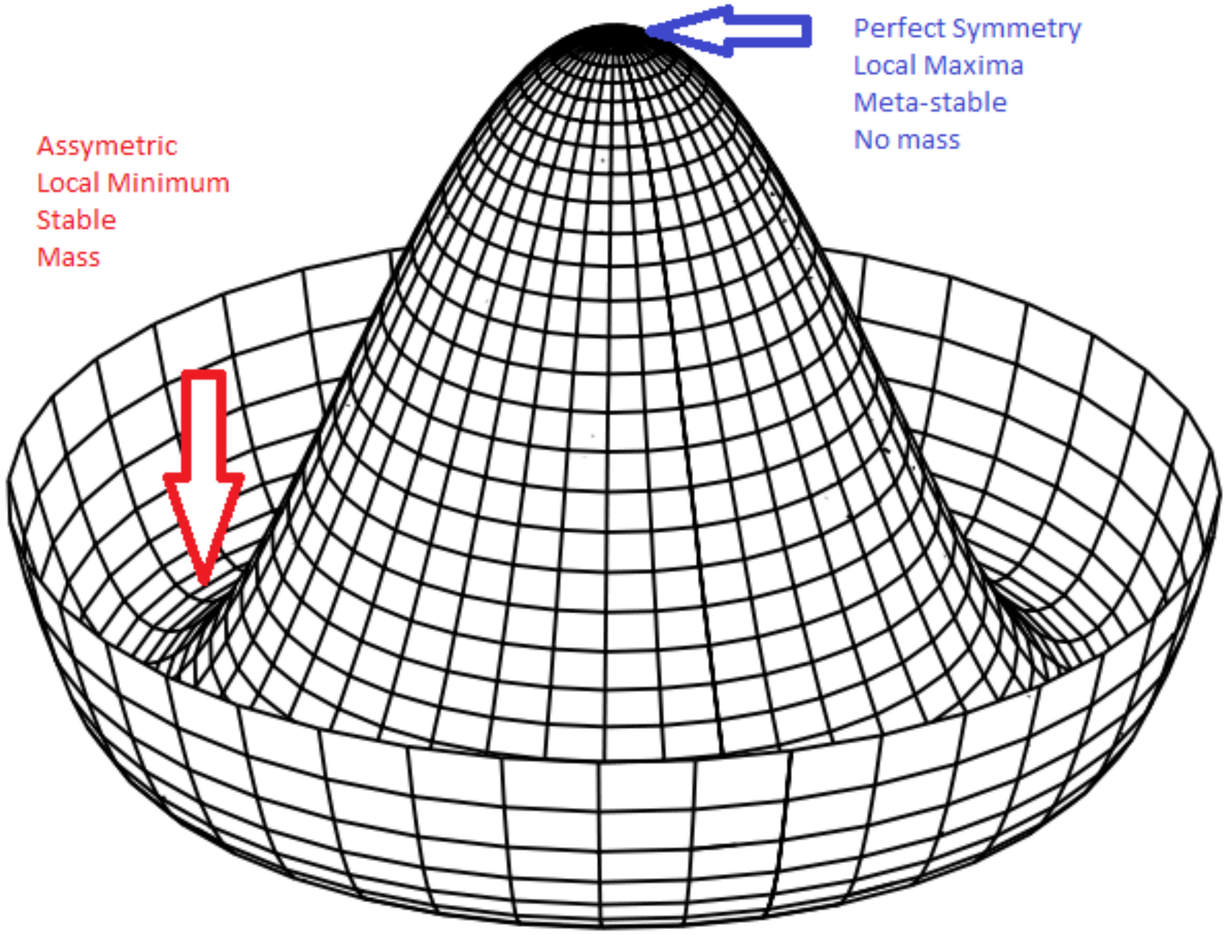
HIGGS POTENTIAL

$$\mathcal{L}_{\phi}^{\text{ME}} = -\mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi \right)^2$$

$\mu^2 < 0 \, , \lambda > 0$

$$v^2 = -\frac{\mu^2}{\lambda}$$

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$



$SU(2)xU(1) \rightarrow U(1)_{EM}$

QUARKS AND HIGGS INTERRELATED

- Yukawa couplings: several orders of magnitude of difference, strong hierarchy

$$\mathcal{L}_{\mathrm{Y}}^{\mathrm{ME}} = Y_{ij}^{d}\overline{Q_{Li}}\phi D_{Rj} + Y_{ij}^{u}\overline{Q_{Li}}\tilde{\phi} U_{Rj} + Y_{ij}^{e}\overline{L_{Li}}\phi E_{Rj} + \text{h.c.}.$$

Also neutrinos, but they could acquire mass other ways.

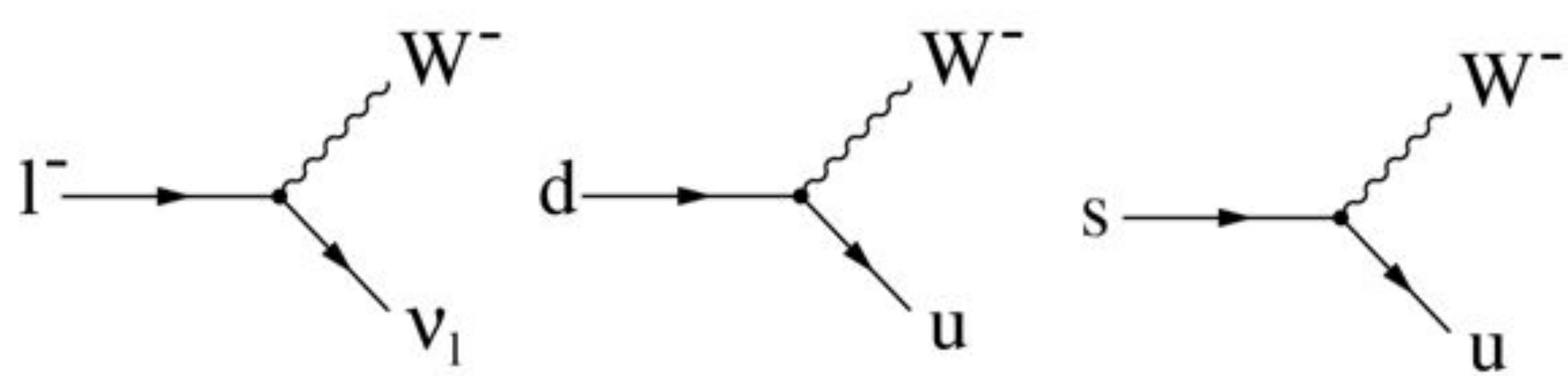
- Higgs sector:

$$\mathcal{L}_{\phi}^{\mathrm{ME}} = -\mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2 \qquad v^2 = -\frac{\mu^2}{\lambda}$$

  - hierarchy problem (quadratic radiative corrections)
  - limits to perturbative unitarity
  - Why $M_{\mathrm{Higgs}} \sim 125$ GeV?

CHARGED CURRENT INTERACTIONS

- Quarks change flavour through charged current interactions
- CP violation in the weak interactions
- Coupling is complex
- On



- Flavour changing neutral currents greatly suppressed

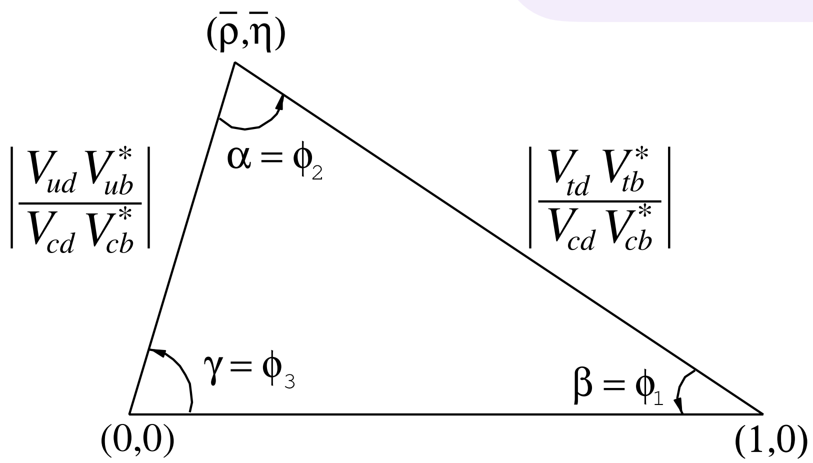
$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$V_{\rm CKM}$ very well determined

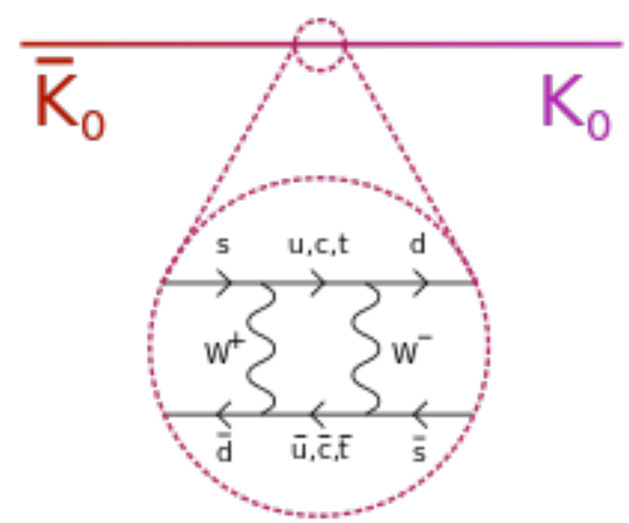
PDG 2023

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$



$$J = (3.08^{+0.15}_{-0.13}) \times 10^{-5}$$

$$\sin\theta_{12} = 0.22500 \pm 0.00067\,, \qquad \sin\theta_{13} = 0.00369 \pm 0.00011\,,$$
$$\sin\theta_{23} = 0.04182^{+0.00085}_{-0.00074}\,, \qquad \delta = 1.144 \pm 0.027\,.$$



K, B, B_S, D processes can be used to study new physics

FCNCs very sensitive to BSM

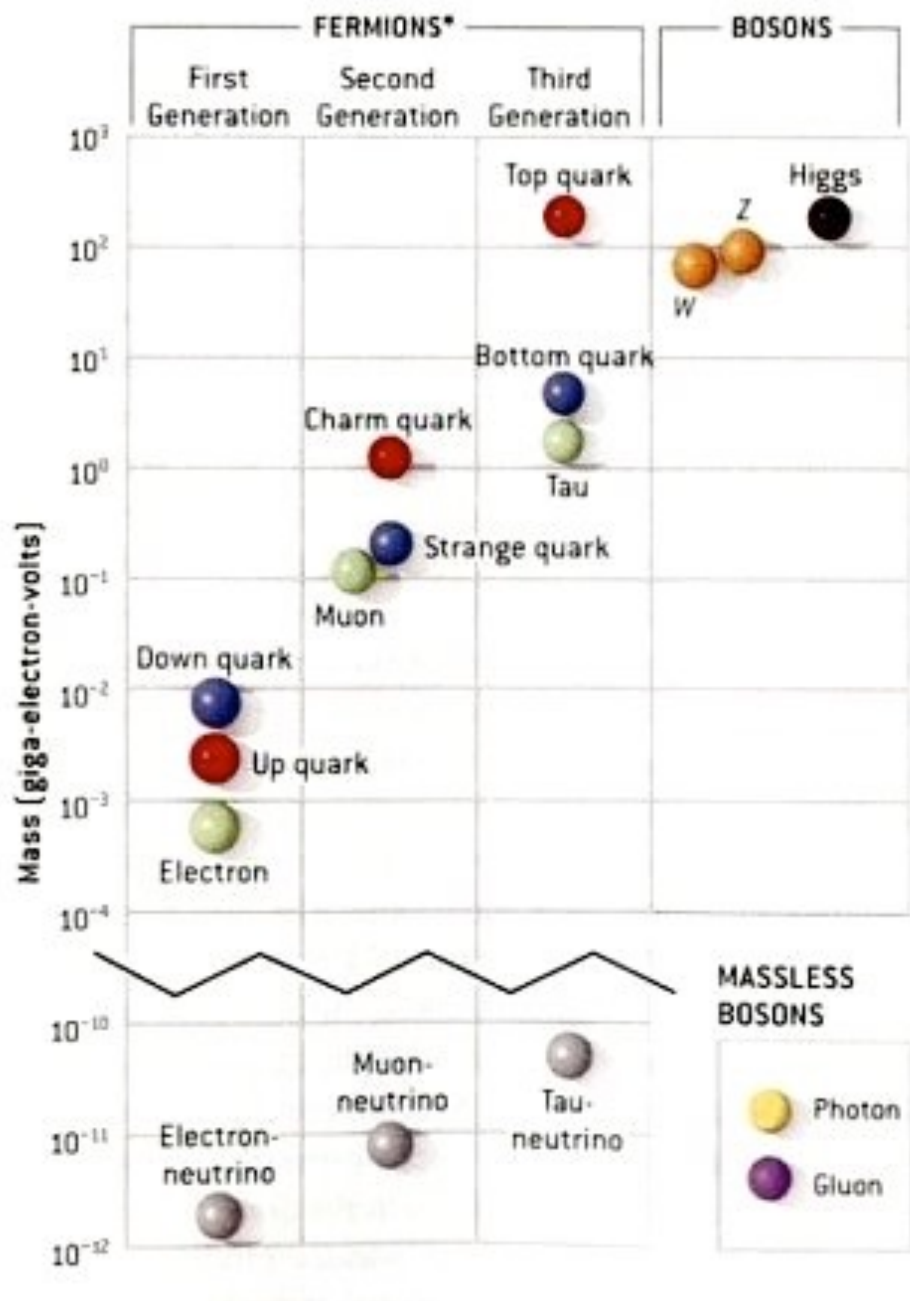
FERMION AND SCALAR SECTORS

- Free parameters in quarks:
  6 masses -> Yukawa couplings
  3 mixing angles
  CP violating phase
- Unitarity —> Jarlskog invariants

- Free parameters in neutrinos:
  6 masses
  3 mixing angles
  CP violating phase
  2 Majorana phases
- Unitarity? —> Also Jarlskog invariants

Plus Higgs vev

FERMION MASSES



picture: universe-review.ca/F15-particle01.htm

- Fermion masses are vastly different
- In the SM neutrinos are massless
- But they do have mass…

NEUTRINOS HAVE MASS

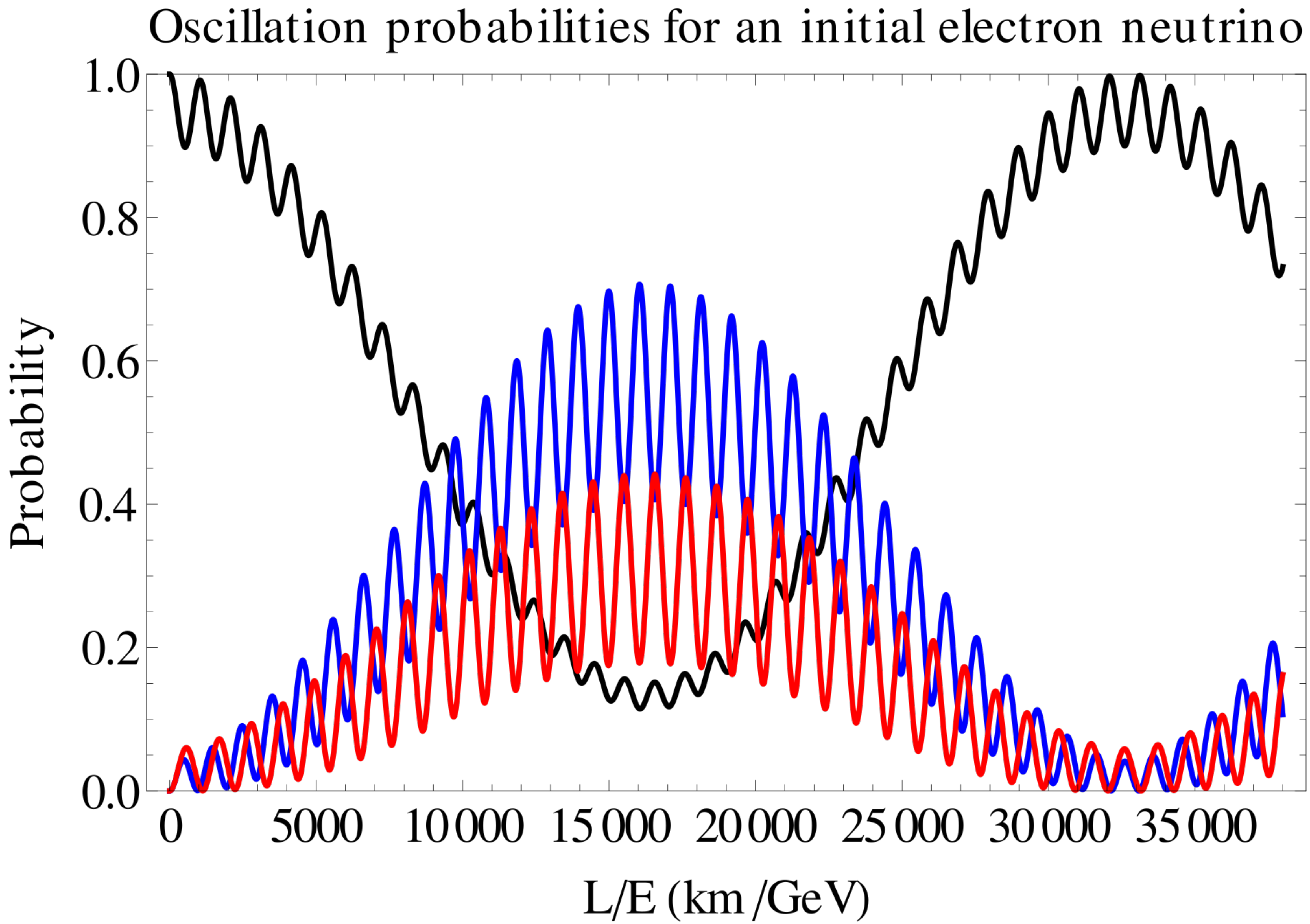
- Neutrinos also mix… they are created/destroyed in flavor eigenstates, but propagate in mass eigenstates:

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\nu_\mu = \nu_2 \cos\theta + \nu_3 \sin\theta$$

$$\nu_\tau = -\nu_2 \sin\theta + \nu_3 \cos\theta$$

- If ν is created in a particular state, after propagating for some time it will be a superposition of states
- The masses are different, they have different frequencies, and phases will develop



black = ν_e, *blue* = ν_μ, *red* = ν_τ

Wikipedia

NEUTRINOS HAVE MASSES…

- In SM neutrinos are massless… in real life they aren’t
- Neutrinos could be their own anti-particles
- Large Majorana masses M plus Higgs induced Dirac masses m generates large neutrino hierarchy
- Best/popular explanation for their tiny mass comes from see-saw mechanism

$$\mathcal{M} = \begin{bmatrix} m_M^L & m_D \\ m_D & m_M^R \end{bmatrix} \quad \tilde{\mathcal{M}} = \begin{bmatrix} m_\nu & 0 \\ 0 & M \end{bmatrix}$$

PMNS MATRIX

$$U = \begin{vmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{vmatrix} \begin{vmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{vmatrix} \begin{vmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

↑ atmospheric ↑ reactor ↑ solar

- Neutrinos also mix → neutrino oscillations
- Pontecorvo-Maki-Nakagawa-Sakata matrix
- Three mixing angles and a phase: atmospheric Θ_{23}, solar Θ_{12} and reactor Θ_{13}
- Possible also Majorana phases
- Only determined squared mass differences

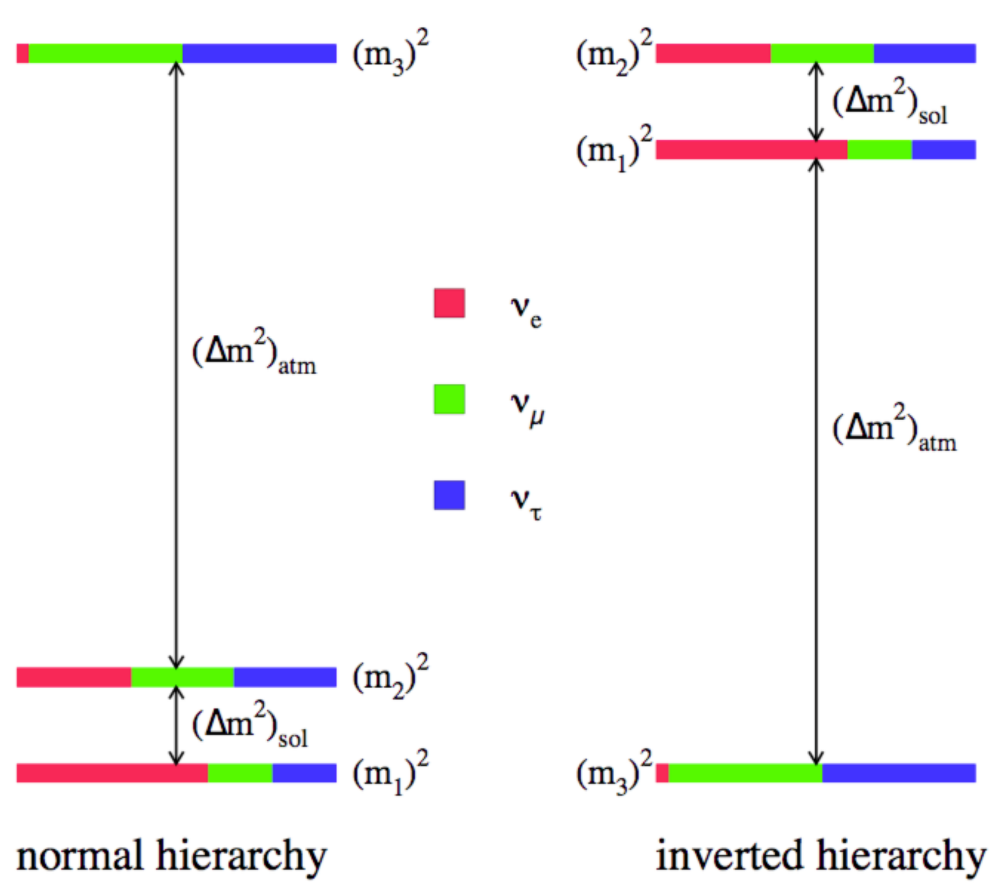
$$\Delta m_{ij}^2 = m_i^2 - m_j^2$$

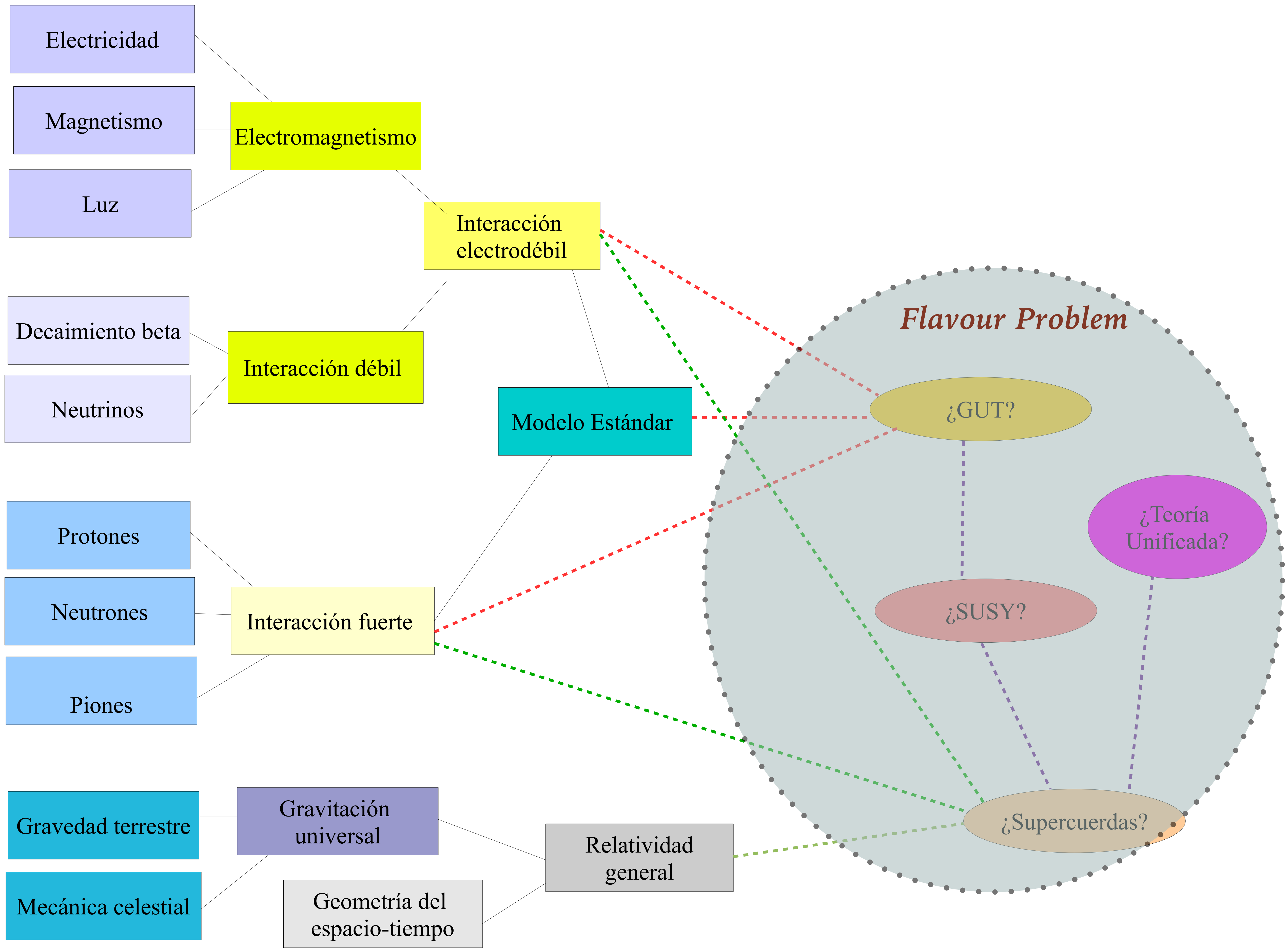
2 POSSIBILITIES

- PMNS matrix

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}.$$

- 2 possible orderings/hierarchies of masses





Electricidad
Magnetismo
Luz
Electromagnetismo
Interacción electrodébil
Decaimiento beta
Neutrinos
Interacción débil
Modelo Estándar
Protones
Neutrones
Piones
Interacción fuerte
Gravedad terrestre
Mecánica celestial
Gravitación universal
Geometría del espacio-tiempo
Relatividad general
Flavour Problem
¿GUT?
¿Teoría Unificada?
¿SUSY?
¿Supercuerdas?

FLAVOUR SYMMETRIES

- Flavour symmetries: continuous or discrete?

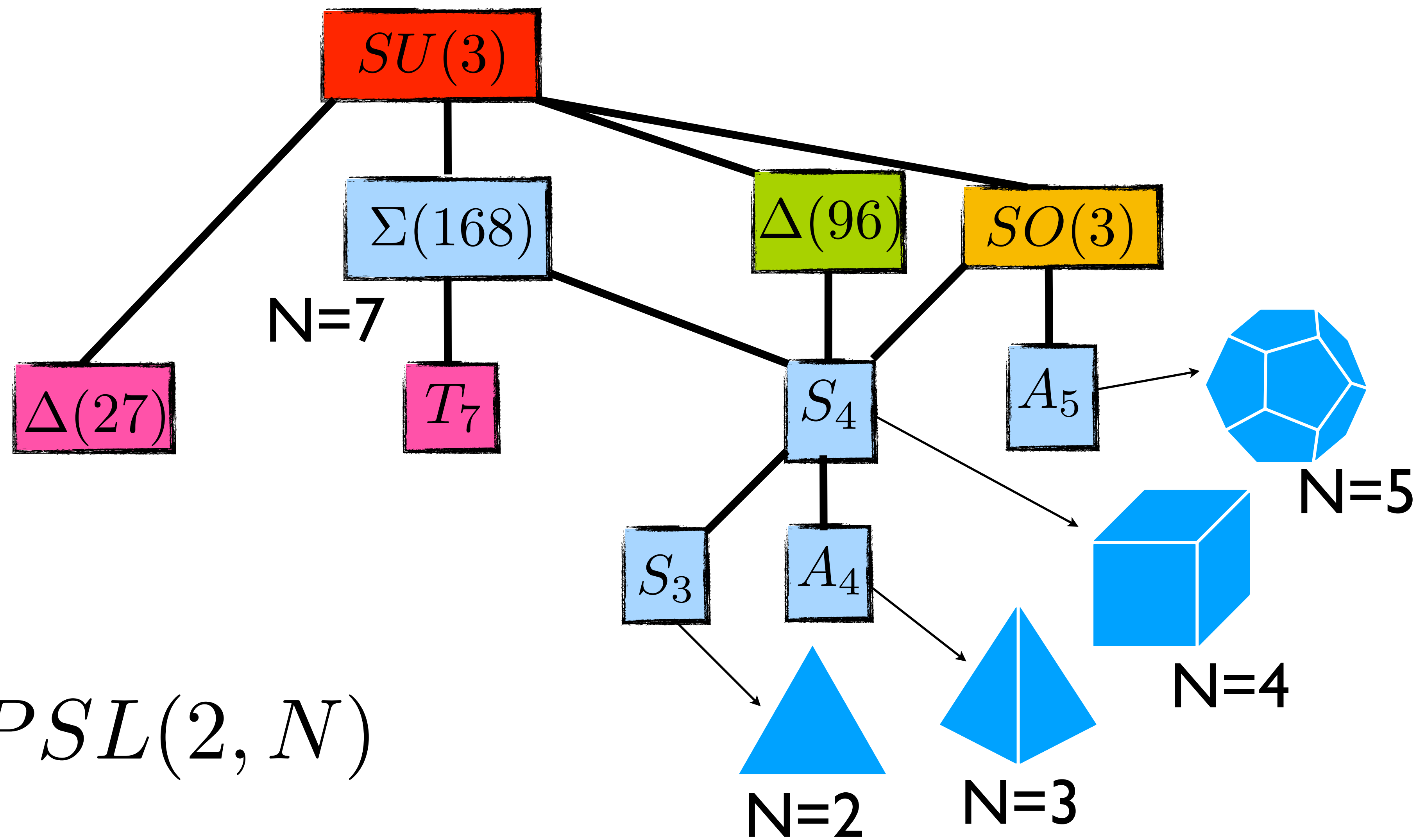
discrete
could lead to domain walls

continuous
breaking may give massless
Goldstone bosons

- At low energies now discrete preferred. Could be:
  - Residual symmetry from breaking from continuous one
  - From the breaking of a larger discrete group
  - Discrete from the "beginning"

- H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, 1003.3552
- S. F. K., A. Merle, S. Morisi, Y. Shimizu, and M. Tanimoto, 1402.4271

Non-Abelian Family Symmetry



$PSL(2, N)$

from Steve King's talk at Modular Invariance Approach to the lepton and quark flavor problem, Mainz, May 2024

MASS MATRICES TEXTURES — TEXTURE ZEROES

- Zeroes in the mass matrices —> less parameters, underlying symmetries: Fritzsch

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & 0 & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

This version excluded already

hierarchical A»|B|»|C|

- In SM and extensions (no FC right-handed currents) is always possible to simultaneously the Mu and Md to Hermitian or NNI textures

- NNI

$$M'_q = \begin{pmatrix} 0 & C_q & 0 \\ C'_q & 0 & B_q \\ 0 & B'_q & A_q \end{pmatrix}$$

B' ≠ B, C' ≠ C

- For any Hermitian 3x3 Mu, Md always possible to change basis to (1,3)=(3,1)=0

Xing Phys.Rept. 854 (2020)

MORE ON TEXTURES

- Add zeroes? Use Z_N, arbitrary but effective
- Better, theoretical motivation
- Use invariants, calculate mass ratios —> V_{CKM}
- What works? up and down sector same structure, coming from same dynamics
- Best type of texture with current data

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B'_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

$A \gg |B| \gg |B'| \gg |C|$
$A > 0$, B' real

ALLOWED TEXTURES

Table 14: The five phenomenologically viable five-zero textures of Hermitian quark mass matrices.

	I	II	III	IV	V
$M_u =$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & 0 \\ 0 & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & 0 & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & 0 \\ D_u^* & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & B_u \\ D_u^* & B_u^* & A_u \end{pmatrix}$
$M_d =$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$

Above textures first found by Ramond et al (1993), work today if not strongly hierarchical.

- ➤ But so far the best one is:

$$M_q = \begin{pmatrix} 0 & C_q & 0 \\ C_q^* & B'_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

From Z. Xing, Phys.Rept. 854 (2020)

TEXTURES AT HIGH ENERGIES

- Usually express mass matrices as mass ratios → they remain stable below eW scale, but renormalize above it, depending on model

- From high to low energies they get renormalized as,

$$M_{\rm u}(\Lambda_{\rm EW}) \simeq \gamma_{\rm u} \left[\begin{pmatrix} 0 & C_{\rm u} & 0 \\ C_{\rm u}^* & B'_{\rm u} & B_{\rm u} I_t^{C_{\rm u}^{\rm u}} \\ 0 & B_{\rm u}^* I_t^{C_{\rm u}^{\rm u}} & A_{\rm u} I_t^{C_{\rm u}^{\rm u}} \end{pmatrix} + \frac{I_t^{C_{\rm u}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & |B_{\rm u}|^2 & B_{\rm u} B'_{\rm u} \\ 0 & B_{\rm u}^* B'_{\rm u} & 0 \end{pmatrix} \right]$$

$$M_{\rm d}(\Lambda_{\rm EW}) \simeq \gamma_{\rm d} \left[\begin{pmatrix} 0 & C_{\rm d} & 0 \\ C_{\rm d}^* & B'_{\rm d} & B_{\rm d} \\ 0 & B_{\rm d}^* I_t^{C_{\rm d}^{\rm u}} & A_{\rm d} I_t^{C_{\rm d}^{\rm u}} \end{pmatrix} + \frac{I_t^{C_{\rm d}^{\rm u}} - 1}{A_{\rm u}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_{\rm u} B_{\rm d}^* & A_{\rm d} B_{\rm u} \\ 0 & B_{\rm u}^* B'_{\rm d} & B_{\rm u}^* B_{\rm d} \end{pmatrix} \right]$$

I's are the one-loop corrections, γ anomalous dimensions, C's coefficients in the running

- Textures remain, coefficients change, for MSSM there is dependence on soft breaking terms

WHAT ABOUT THE HIGGS SECTOR? ORIGIN OF FLAVOUR PROBLEM(S)?

- One Higgs field: “takes care” of all masses, might be too much
- More Higgs fields:
  more doublets, absolutely necessary in SUSY models, always in pairs
  2HDM without SUSY
  3HDM also studied
- More scalars: potential more complicated breaking of flavour symmetry at low energies… either by “hand” or spontaneously
- Where does the flavour symmetry breaking come from?

N-HIGGS DOUBLET MODELS — NHDM

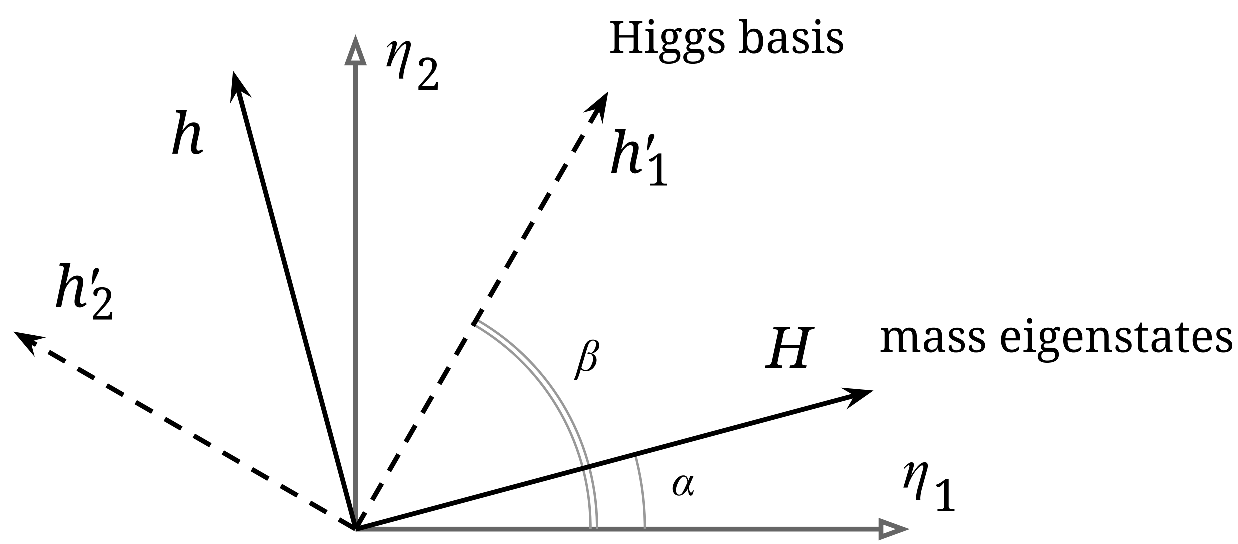
- Add more complex electroweak doublets
  All with same hyper charge Y=1

$$V(\phi) = Y_{ij}\phi_i^\dagger\phi_j + Z_{ijkl}(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l)\,.$$

- $N^2 + N^2(N^2+1)/2$ real parameters:
  12 for 2HDM, 54 for 3HDM…
- Potential must be bounded by below, no charge or colour breaking minima
- Must respect unitarity bounds
- Can have CP breaking minima $\rightarrow$ baryogenesis (or disaster)

BASIS, FLAVOUR BASIS

- Convenient to rotate to Higgs basis, vev all in first doublet
- Goldstone bosons in first one, physical Higgses in the rest



Ivanov, Prog.Part.Nucl.Phys. 95 (2017)

- N-1 pairs of charged Higgses, 2N-1 neutral scalars (odd and even)
- Suitable basis for studying phenomenology, e.g. FCNCs

MULTI-HIGHS MODEL AND FLAVOUR SYMMETRIES

- 2HDM widely studied, several studies on 3HDM (Branco et al,; King et al, *JHEP* 01 (2014) 052 al, 2014)
- Minimization of scalar potential must be performed. Sometimes vev alignments are chosen by hand, e.g. v1»v2» v3 → maybe only local minima
- Extra Higgs doublets and discrete symmetries → continuous symmetries
- Also usually after minimization of the potential there are residual symmetries → unphysical quark sector, either degenerate masses, zero masses or zeroes in V_{CKM}
  - S_3, S_4, A_4, $\Delta(54)$ all have residual symmetries in 3HDM
- If soft breaking performed, stability and unitarity conditions must be recalculated
- Connection with dark matter, inert scalars vev=0

Ivanov, Prog.Part.Nucl.Phys. 95 (2017)

MORE SCALARS

- Add singlets, same considerations as before
- Flavons: responsible for family symmetry breaking at high energies, Froggatt-Nielsen mechanism
- Scalars can be used for a number of other purposes: inflation, dark matter, dark energy, phase transitions
  - Is there evidence for new scalars?
    95 GeV?   CMS ~ 2.9 sigma
    150 GeV? multilepton anomalies
    650 GeV? CMS ~ 3.8 sigma
    All of them???
- Not significative, but persistent…

INTERPLAY BETWEEN FLAVOUR AND ASTROPARTICLE PHYSICS

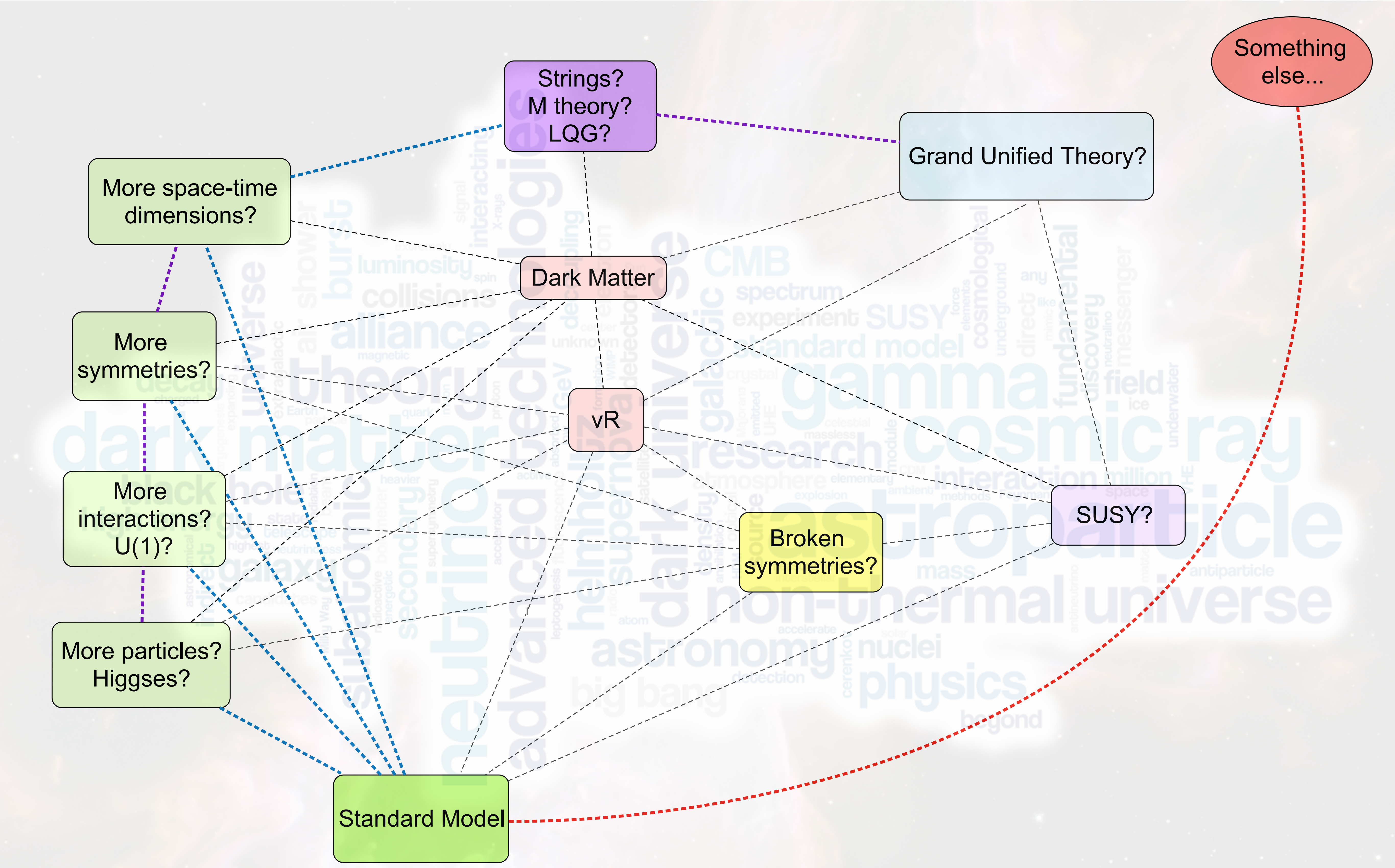
- Dark matter candidates:

scalars:
exotic Higgses, axion-like particles,
KK particles,

fermions:
right-handed neutrinos,
neutralinos, KK particles…

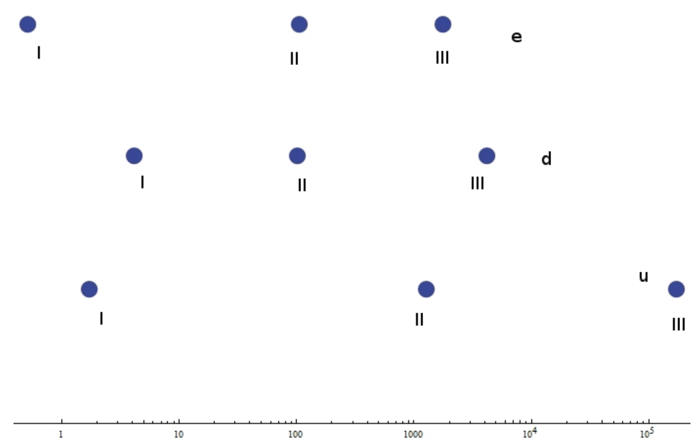
Related to flavour,
Constrained by symmetries

- CP violation: baryogenesis, leptogenesis
- g-2: many extensions attempt explanation. LHC and DM experiments constrain it
- Effective field theory approach (κ formalism) helps constrain new processes

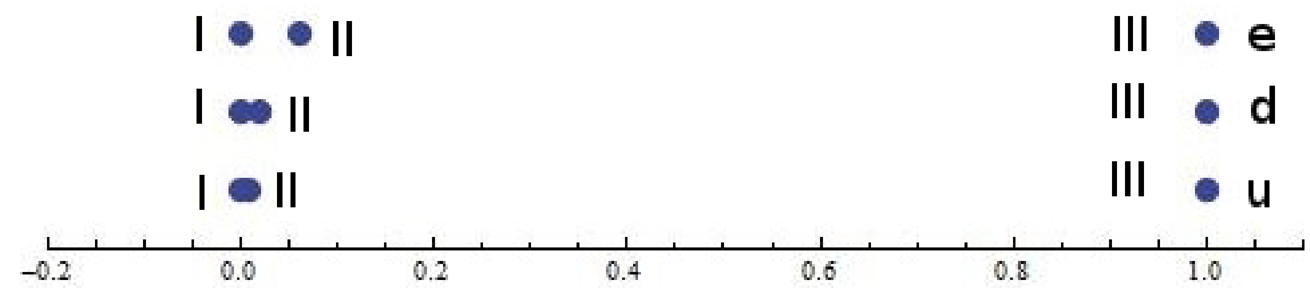

Strings?
M theory?
LQG?
Something else...
Grand Unified Theory?
More space-time dimensions?
Dark Matter
More symmetries?
vR
More interactions? U(1)?
Broken symmetries?
SUSY?
More particles? Higgses?
Standard Model

HOW DO WE CHOOSE A FLAVOUR SYMMETRY?

- Several ways:
- Look for inspiration in a high energy extension of SM, i.e. strings or GUTs
- Look at low energy phenomenology
- At some point they should intersect…
- In here:
    - Find the smallest flavour symmetry suggested by data
    - Explore how generally it can be applied (universally)
    - Follow it to the end
    - Compare it with the data



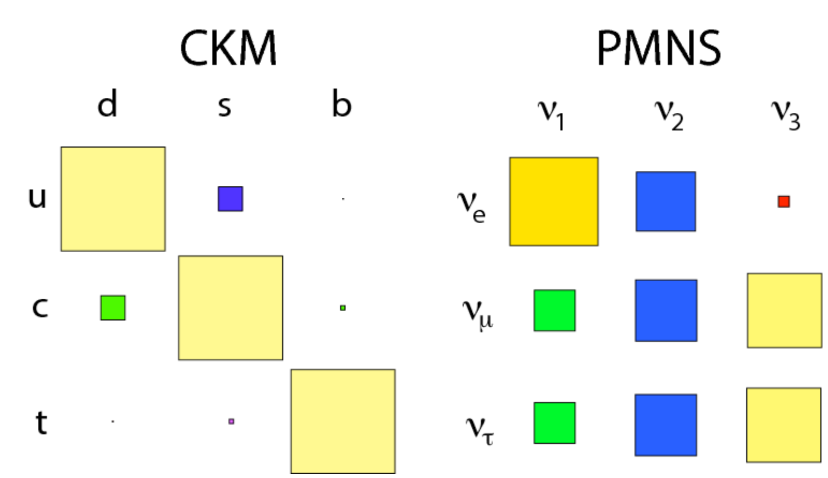
Plot of mass ratios



Logarithmic plot of quark masses

$$\begin{bmatrix} |V_{\rm ud}| & |V_{\rm us}| & |V_{\rm ub}| \\ |V_{\rm cd}| & |V_{\rm cs}| & |V_{\rm cb}| \\ |V_{\rm td}| & |V_{\rm ts}| & |V_{\rm tb}| \end{bmatrix} \approx \begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix},$$

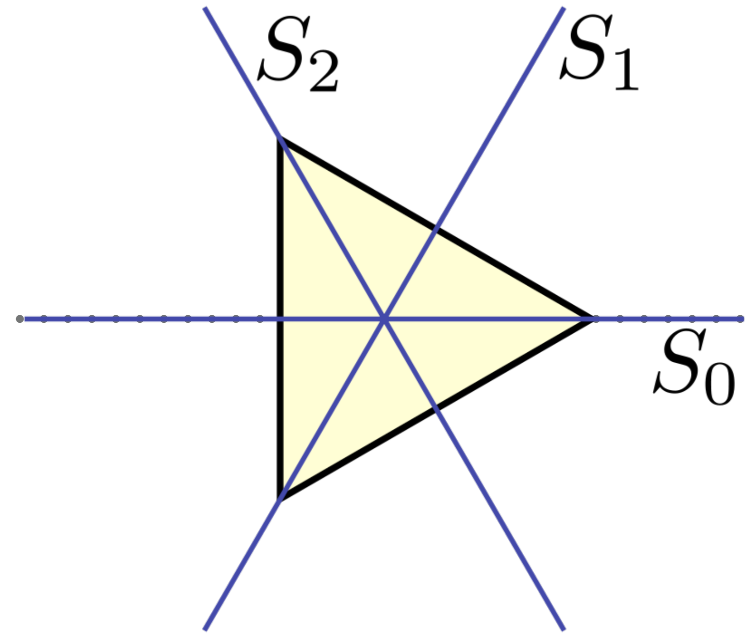
Suggests a 2⊕1 structure



3HDM

- Without symmetry $\Longrightarrow$ 54 real parameters in potential
- Complemented with additional symmetry(ies)
- Studies started in the 70's, hope to find global symmetry that explains the mass and mixing patterns
- The first symmetries to be added were the permutational groups S3 and S4
- Different modern versions of these models exist

S3



- Smallest non-Abelian discrete group
- Permutation symmetry of three objects; reflections and rotations that leave an equilateral triangle invariant
- Has irreducible representations, 2, 1_S d 1_A
- 3 right handed neutrinos
- 3 Higgs doublets

- We apply the symmetry “universally” to quarks, leptons and Higgs-es
  - First two families in the doublet
  - Third family in symmetric singlet
- Treat scalars and fermions simultaneously

A sample of S3 models

S. Pakvasa et al, Phys. Lett. 73B, 61 (1978)

E. Derman, Phys. Rev. D19, 317 (1979)

D. Wyler, Phys. Rev. D19, 330 (1979)

R. Yahalom, Phys. Rev. D29, 536 (1984)

Y. Koide, Phys. Rev. D60, 077301 (1999)

A. Mondragon et al, Phys. Rev. D59, 093009, (1999)

J. Kubo, A. Mondragon, MM et al, Prog. Theor. Phys. 109, 795 (2003)

J. Kubo et al, Phys. Rev. D70, 036007 (2004)

S. Chen, M. Frigerio and E. Ma, Phys. Rev. D70, 073008 (2004)

A. Mondragon et al, Phys. Rev. D76, 076003, (2007)

S. Kaneko et al, hep-ph/0703250, (2007)

S. Chen et al, Phys. Rev. D70, 073008 (2004)

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Just a sample, there are many more…
I apologize for those not included

PREDICTIONS, ADVANTAGES?

- Possible to reparametrize mixing matrices in terms of mass ratios, successfully
- CKM has NNI and Fritzsch textures
- PMNS → fix one mixing angle, predictions for the other two within experimental range
- Reactor mixing angle $\theta_{13} \neq 0$
- Some FCNCs suppressed by symmetry
- Higgs potential has 8 couplings
- Underlying symmetry in quark, leptons and Higgs → residual symmetry of a more fundamental one?
- Lots of Higgses: 3 neutral, 4 charged, 2 pseudoscalars
- Further predictions will come from Higgs sector: decays, branching ratios

A. Mondragón, M. M., F. González, E. Peinado, U. Saldaña, O. Félix, E. Rodríguez, A. Pérez, H. Reyes...; Das, Dey et al; Teshima et al;

FERMION MASSES

- The Lagrangian of the model

$$\mathcal{L}_Y = \mathcal{L}_{Y_D} + \mathcal{L}_{Y_U} + \mathcal{L}_{Y_E} + \mathcal{L}_{Y_\nu},$$

- The general form of the fermion mass matrices in the symmetry adapted basis is

$$\mathbf{M} = \begin{pmatrix} m_1 + m_2 & m_2 & m_5 \\ m_2 & m_1 - m_2 & m_5 \\ m_4 & m_4 & m_3 \end{pmatrix}.$$

where $m_{1,3} = Y_{1,3}v_3$ and $m_{1,2,4,5} = Y_{1,2,4,5}$ (v_1 or v_2)

QUARKS

without taking into account minimization conditions

3HDM: $G_{SM} \otimes S_3$

	ψ_L^f	ψ_R^f	Mass matrix	Possible mass textures	
A	$\mathbf{2}$, $1_{\mathbf{S}}$	$\mathbf{2}$, $1_{\mathbf{S}}$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_6^f \\ \mu_4^f & \mu_1^f - \mu_2^f & \mu_7^f \\ \mu_8^f & \mu_9^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & \mu_2^f sc\,(3-t^2) & 0 \\ \mu_2^f sc\,(3-t^2) & -2\mu_2^f c^2\,(1-3t^2) & \mu_7^f/c \\ 0 & \mu_7^{f*}/c & \mu_3^f - \mu_1^f - \mu_2^f c^2(1-3t^2) \end{pmatrix}$	
$A^{'}$				$\begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2^f & 0 \\ \frac{2}{\sqrt{3}}\mu_2^f & 0 & \frac{2}{\sqrt{3}}\mu_7^f \\ 0 & \frac{2}{\sqrt{3}}\mu_9^f & \mu_3^f - \mu_1^f \end{pmatrix}$	*NNI*
B	$\mathbf{2}$, $1_{\mathbf{A}}$	$\mathbf{2}$, $1_{\mathbf{A}}$	$\begin{pmatrix} \mu_1^f + \mu_2^f & \mu_4^f & \mu_7^f \\ \mu_4^f & \mu_1^f - \mu_2^f & -\mu_6^f \\ -\mu_9^f & \mu_8^f & \mu_3^f \end{pmatrix}$	$\begin{pmatrix} 0 & -\mu_4^f c^2\,(1-3t^2) & 0 \\ -\mu_4^f c^2\,(1-3t^2) & 2\mu_4^f sc\,(3-t^2) & -\mu_6^f/c \\ 0 & -\mu_6^{f*}/c & \mu_3^f - \mu_1^f + \lvert\mu_4^f sc(3-t^2) \end{pmatrix}$	
$B^{'}$				$\begin{pmatrix} 0 & -2\mu_4^f & 0 \\ -2\mu_4^f & 0 & -2\mu_6^f \\ 0 & 2\mu_8^f & \mu_3^f - \mu_1^f \end{pmatrix}$	*NNI*

Table 2: Mass matrices in S_3 family models with three Higgs $SU(2)_L$ doublets: H_1 and H_2, which occupy the S_3 irreducible representation $\mathbf{2}$, and H_S, which transforms as $1_{\mathbf{S}}$ for the cases when both the left- and right-handed fermion fields are in the same assignment. The mass matrices shown here follow a normal ordering of their mass eigenvalues (m_1^f, m_2^f, m_3^f). We have denoted $s = \sin\theta$, $c = \cos\theta$ and $t = \tan\theta$. The third column of this table corresponds to the general case, while the fourth column to a case where we have rotated the matrix to a basis where the elements $(1,1)$, $(1,3)$ and $(3,1)$ vanish. The primed cases, A' or B', are particular cases of the unprimed ones, A or B, with $\theta = \pi/6$ or $\theta = \pi/3$, respectively.

Mass matrices reproduce the NNI or the Fritzsch forms (rotation + shift)

F. González, AM, MM et al, Phys.Rev. D88 (2013) 096004

HIGGS SECTOR – TESTS FOR THE MODEL

General Potential:

$$
\begin{aligned}
V \;=\;& \mu_1^2\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2\left(H_s^\dagger H_s\right) + a\left(H_s^\dagger H_s\right)^2 + b\left(H_s^\dagger H_s\right)\left(H_1^\dagger H_1 + H_2^\dagger H_2\right) \\
&+c\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 + d\left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk}\left(\left(H_s^\dagger H_i\right)\left(H_j^\dagger H_k\right) + h.c.\right) \\
&+f\left\{\left(H_s^\dagger H_1\right)\left(H_1^\dagger H_s\right) + \left(H_s^\dagger H_2\right)\left(H_2^\dagger H_s\right)\right\} + g\left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)^2\right\} \\
&+h\left\{\left(H_s^\dagger H_1\right)\left(H_s^\dagger H_1\right) + \left(H_s^\dagger H_2\right)\left(H_s^\dagger H_2\right) + \left(H_1^\dagger H_s\right)\left(H_1^\dagger H_s\right) + \left(H_2^\dagger H_s\right)\left(H_2^\dagger H_s\right)\right\} \qquad (1)
\end{aligned}
$$

Derman and Tsao (1979); Sugawara and Pakwasa (1978); Kubo et al (2004); Felix-Beltrán, Rodríguez-Jáuregui, M.M (2009); Das and Dey (2014), Barradas et al (2014); Costa, Ogreid, Osland and Rebelo (2016), etc

- *The minimum of potential can be parameterised in spherical coordinates, two angles and v*
- *Minimisation fixes* $v_1^2 = 3v_2^2$
- *e = 0 massless scalar, residual continuous S2 symmetry*
- *Conditions for normal vacuum already studied, also for CP breaking ones*
  Felix-Beltrán, Rodríguez-Jáuregui, M.M (2007); Barradas et al (2015); Costa et al (2016)

$$v_1 = v\cos\varphi\sin\theta, \quad v_2 = v\sin\varphi\sin\theta \quad v_3 = v\cos\theta.$$

$$\tan\varphi = 1/\sqrt{3} \quad \Rightarrow \quad \sin\varphi = \frac{1}{2} \quad \& \quad \cos\varphi = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{2v_2}{v_3} \quad \Rightarrow \quad \sin\theta = \frac{2v_2}{v} \quad \& \quad \cos\theta = \frac{v_3}{v}$$

RESIDUAL Z2 SYMMETRY

- After eW symmetry breaking, S3 breaks -> residual Z2 symmetry
  Das and Dey (2014), Ivanov (2017)
- h0 decoupled from gauge bosons
- There are 2 “alignment” limits 👀
  - H2 is the SM Higgs → H1 decoupled from gauge bosons
  - H1 is the SM Higgs → H2 decoupled from gauge bosons
    mH2 < mH1
- Z2 parity:

  h_0, A_1, $H_1^\pm$ parity -1,
  H_1, H_2 parity +1
  $H_2^\pm$, A_2 parity +1

  Das and Dey (2014)
- This forbids certain couplings

MASSES — TREE LEVEL — ALIGNMENT LIMITS

- Scenario A, H2 SM Higgs
  - Upper bound for masses
    mh0 ≲ 900 GeV , mH1 ≲ 3 TeV
    mA1 ≲ 1 TeV, mA2 ≲ 3 TeV
    mH1 ≲ 1 TeV, mH2 ≲ 3 TeV
  - Taking (α-θ) 1% lowers mH1, mA2, MH2 ≲ 1 TeV
- **Allows for a neutral scalar lighter than SM Higgs**
  h0 in this case
- Some of scalar masses are almost degenerate → good for oblique parameters

Gómez-Bock, MM, Perez-Martínez (2022)

EXACT ALIGNMENT LIMIT A

- In the exact alignment limit A (SM Higgs the lightest scalar)

$$\sin(\alpha-\theta) = 1,\ \cos(\alpha-\theta) = 0,$$

- "Our" SM Higgs trilinear and quartic couplings reduce exactly to SM ones

$$g_{H_2H_2H_2} = \frac{1}{v\, s_{2\theta}}\left[m_{H_2}^2 s_\alpha s_\theta\right] = \frac{1}{2v}\frac{s_\alpha}{c_\theta} m_{H_2}^2 = \frac{m_{H_2}^2}{2v} \equiv \lambda_{SM}.$$

$$g_{H_1H_1H_1} = \frac{1}{v\, s_{2\theta}}\left[\frac{1}{9c_\theta^2}m_{h_0}^2 - s_\theta^2 m_{H_1}^2\right] = \frac{1}{v\, s_{2\theta}c_\theta^2}\left[\frac{1}{9}m_{h_0}^2 - \frac{1}{2}s_{2\theta}m_{H_1}^2\right].$$

;

$$g_{H_2H_2H_2H_2} = \frac{1}{2v^2 s_{2\theta}^2} m_{H_2}^2(-s_\theta^3 c_\theta - c_\theta^3 s_\theta)^2 = \frac{m_{H_2}^2}{8v^2}.$$

$$g_{H_2H_2h_0h_0} = \frac{1}{v^2 s_{2\theta}}\left(\frac{1}{6}m_{h_0}^2 3 s_{2\theta} + \frac{1}{4}m_{H_2}^2 s_{2\theta}\right) = \frac{1}{4v^2}(2m_{h_0}^2 + m_{H_2}^2)\ .$$

CONSTRAINTS ON SCALARS

- Constraints are imposed over the parameter space:
  - Vacuum stability and unitarity conditions
  - SM Higgs boson mass within 125 ± 3 GeV
- We recover SM Higgs boson properties, trilinear and quartic couplings are the same, extra heavier scalars, bounded from above and below
- BUT residual Z2 symmetry: 😵

$$M_q = \begin{pmatrix} x & 0 & 0 \\ 0 & x & x \\ 0 & x & x \end{pmatrix}$$

4HDM –S3 WITH DM

- We add another doublet, inert, to have a DM candidate. We assign it to the 1^A, and thus “saturate” the irreps
- First two generations in a flavour doublet, third in a singlet, extra anti-symmetric singlet is inert → DM candidates
- A lot of Higgses (13), but the good features of 3H-S3 remain
  Quark and lepton sectors remain unchanged
  DM candidate in inert sector
- Add a Z2 symmetry to prevent the DM candidate to decay
- S3 symmetry constrains strongly the allowed couplings

C. Espinoza, E. Garcés, M.M., H. Reyes (2019)

IN YUKAWA SECTOR

- The Higgs Z2 symmetry will lead to zeroes in the CKM and PMNS matrices 😱 Das, Dey, Pal (2015), Ivanov (2017)
- To recover the good features of the symmetry:
  - Add S3 singlet Brown, Deshpande,Sugawara, Pakwasa (1984)
  - Break very softly the S3 symmetry with mass terms, recover original structure e.g., Kubo, Okada, Sakamaki (2004), Das, Dey, Pal (2015)
  - Consider CP violation Costa, Ogreid, Osland, Rebelo(2014,2021)
  - Higher order interactions
  - Second B-L sector at high scale with small interaction Gómez-Izquierdo, MM (2018)
- Combinations of the above: all introduce more parameters

RECAP

- Flavour problem: one of the most important open problems in HEP
- Has served as guidance for discoveries
- Far reaching consequences in particles and astroparticle physics
  work them out!
- Flavour symmetries:
    - Might give insight into what lies ahead,
      either top-down or bottom up
    - Important to look both at **fermionic** and **scalar sector simultaneously** (surprises, pleasant and not, might appear)
- Where do the Yukawa couplings come from? Why those?

THANKS!