Hadron Spectroscopy from lattice QCD simulations

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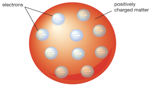
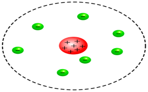
Timeline of Quantum ElectroDynamics

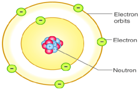
- Thomson's atomic model 1890s.



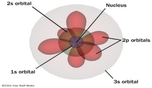
- Rutherford's atomic model 1910s.



- Bohr's atomic model 1910s.



- Schrödinger's atomic model 1920s.



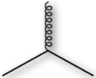
Lamb's shift and Quantum ElectroDynamics.

Importance of spectroscopy in the development of the theory.

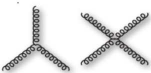
Quantum ChromoDynamics

$$L_{QCD} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{\alpha=1}^{N_f} \bar{\psi}_\alpha(i\gamma^\mu D_\mu - m_\alpha)\psi_\alpha = L_g[U] + L_q[\bar{\psi},\psi,U]$$

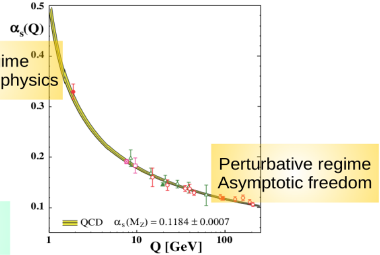
where $D_\mu = \partial_\mu - ig\sum_{i=1}^{8} \lambda^a A^a_\mu,$ and $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$



Quark-gluon vertex



Gluon-gluon vertices.
Not present in QED



Hadronic regime
Nonperturbative physics

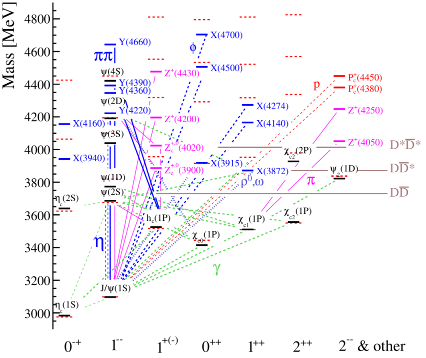
Perturbative regime
Asymptotic freedom

Perturbative approaches fail in the hadronic regime.

Nonperturbative approaches required for first principles investigation: **Lattice QCD**

Charmonium spectrum

- Rich experimental spectrum with several prospects.
  Exotics ...
- Comparison with simple minded model of hadrons.
  Mesons $\sim$ $\bar{q}q$; Baryons $\sim$ qqq
- Model: Quantum mechanical system with a potential inspired from properties of strong interactions.
  Godfrey, (Isgur), ... 2015



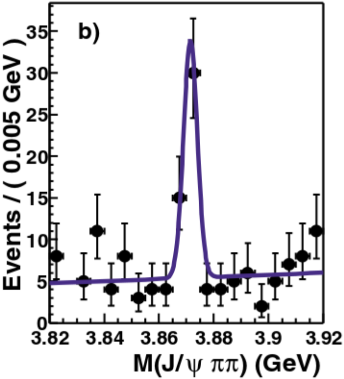
Olsen *et al* 2017

A handful of models and several different predictions.

Need for hadron spectroscopy in QCD from first principles

Experimental facts : X(3872)

- first observed in Belle 2003
  (Belle PRL 2003)
- Quantum numbers, $J^{PC} = 1^{++}$
  (LHCb, 2013)
- Appears within 1 MeV below
  $D^0 \bar{D}^{*0}$ threshold.
- Preferred strong decay modes
  $J/\psi\ \omega$ and $J/\psi\ \rho$



- The isospin still uncertain
  - nearly equal branching fraction to $J/\psi\ \omega$ and $J/\psi\ \rho$ decays.
  - No charge partner candidates observed.

Path integrals in Minkowski space

Path Integral:

$$Z = \int D\phi(x) e^{-iS[\phi(x)]},$$

where $S[\phi(x)] = \int d^4x \ L[\phi(x)]$.

$e^{-iS[\phi(x)]}$ factor $\equiv$ Boltzmann factor in statistical mechanics.

Correlation functions:

$$\langle 0|\hat{O}|0\rangle = \int D\phi(x) O[\phi] e^{-iS[\phi(x)]}$$

The spectral information in two point correlations:

$$\begin{aligned}\langle 0|\hat{\phi}(t)\hat{\phi}(0)|0\rangle &= \langle 0|e^{i\hat{H}t}\hat{\phi}(0)e^{-i\hat{H}t}\hat{\phi}(0)|0\rangle \\ &= e^{iE_{vac}t}\langle 0|\hat{\phi}(0)e^{-i\hat{H}t}\sum_n |n\rangle\langle n|\hat{\phi}(0)|0\rangle \\ &= \sum_n e^{-i\Delta E_n t}\langle 0|\hat{\phi}(0)|n\rangle\langle n|\hat{\phi}(0)|0\rangle\end{aligned}$$

Path integrals in Euclidean space

Wick rotation in time ($t \to -i\tilde{t}$):

$$-iS \;=\; -i \int dx^3 dt \; L \;\to\; -\int dx^3 d\tilde{t} \; \tilde{L} \;=\; -\tilde{S}$$

Path Integral:

$$\tilde{Z} = \int D\phi(x) e^{-\tilde{S}[\phi(x)]}$$

Positive weight factor: $e^{-\tilde{S}[\phi(x)]}$ [probability]

Importance Sampling:

- random sampling of configurations $\{\phi\}$.

$$\langle O \rangle \approx \overline{O} = \sum O[\phi] e^{-\tilde{S}[\phi(x)]} / \sum e^{-\tilde{S}[\phi(x)]}$$

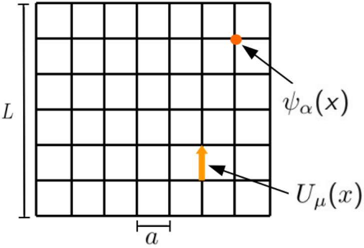
- importance sampling of configurations $\{\phi\}$.
  sample the configuration space with probability $e^{-\tilde{S}[\phi(x)]}$.

$$\langle O \rangle \approx \overline{O} = \sum O[\phi]/N$$

Lattice QCD: theoretical aspects

LQCD : A non-perturbative, gauge invariant regulator for the QCD path integrals.

- Quark fields $\psi_\alpha(x)$ on lattice sites
- Gauge fields as parallel transporters U_μ
  Lives in the links. $U_\mu(x) = e^{igaA_\mu(x)}$
- $\bar{\psi}^i_\alpha(x)[U_\mu(x)]_{ij}\psi^j_\alpha(x + a\hat{\mu})$ is gauge invariant.
- Lattice spacing : UV cut off
- Lattice size : IR cut off



Employ MC Monte Carlo importance sampling methods on Euclidean metric for numerical studies.

Correlation functions

- Aim : to extract the physical states of QCD.
- Example case: mass of a pseudoscalar meson (pion)
  The simplest interpolating current: $\bar{\psi}\gamma_5\psi$
- Euclidean two point current-current correlation functions

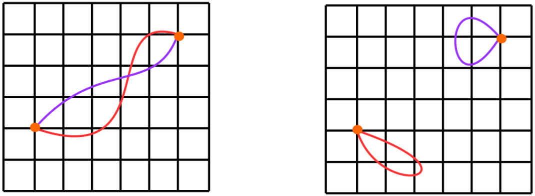
$$\begin{aligned} C(t) &= \langle 0|[\bar{\psi}\gamma_5\psi](t)[\bar{\psi}\gamma_5\psi](0)|0\rangle \\ &= \langle 0|e^{Ht}[\bar{\psi}\gamma_5\psi](0)e^{-Ht}[\bar{\psi}\gamma_5\psi](0)|0\rangle \\ &= \sum_n e^{-E_n t}\langle 0|\bar{\psi}\gamma_5\psi(0)|n\rangle\langle n|\bar{\psi}\gamma_5\psi(0)|0\rangle \\ &= \sum_n |Z_n|^2 e^{-E_n t} \end{aligned}$$

Computing correlation functions: mesons

$$C(t) = \langle 0|[\bar{\psi}\gamma_5\psi](x,t)[\bar{\psi}\gamma_5\psi](0,0)|0\rangle$$

$$= \int DU\ tr[\gamma_5\ M^{-1}_{xt,00}\ \gamma_5\ M^{-1}_{00,xt}]\ det(M)\ e^{-S_g[U]}$$

$$- \int DU\ tr[\gamma_5\ M^{-1}_{xt,xt}\ \gamma_5\ M^{-1}_{00,00}]\ det(M)\ e^{-S_g[U]}$$



On the importance sampled ensemble this amounts to computing

$$\frac{1}{N}\sum\ tr[\gamma_5\ M^{-1}_{xt,00}\ \gamma_5\ M^{-1}_{00,xt}] -\ tr[\gamma_5\ M^{-1}_{xt,xt}\ \gamma_5\ M^{-1}_{00,00}]$$

The sum is over the ensemble [N: # configurations in the ensemble].

Extraction of the mass spectrum

$C(t) = \sum_n |Z_n|^2 e^{-E_n t}$, which at large times, $C(t) \to |Z_0|^2 e^{-E_0 t}$

The operator can in principle couple with all the states that have its q. #s.
The strength of coupling Z_n determines the quality of signal.

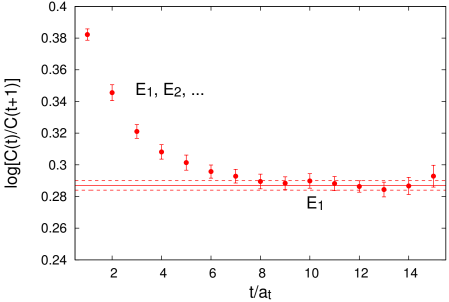
Effective mass defined as $m_{eff} = \frac{1}{dt}\log[\frac{C(t)}{C(t+dt)}]$
Mass extraction: Fit to $C(t)$ across multiple time slices.

Ground states: Single exponential fit forms

Excited states: Multi-exponential fit forms:
Stability of fits!

Limited # time slices to extract excited state energies from multi-exponential fits.

Extraction of energy degenerate states is impossible this way.



Correlation matrices $C_{ji}(t)$ and GEVP

- Instead let us build a matrix of correlatiion functions:

$$C_{ji}(t) = \langle 0|\Phi_j(t)\bar{\Phi}_i(0)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2E_n} e^{-E_n(t)}$$

  where $\Phi_j(t)$ and $\bar{\Phi}_i(0)$ are the desired interpolating operators.
  $Z_j^n = \langle 0|\Phi_j|n\rangle$ are the operator-state overlaps.

- $C_{ji}(t)$ is Hermitian by construction. The eigensystem is automatically orthogonal. The eigenvalues representing the evolution of physical states.

- Solving the generalized eigenvalue problem for $C_{ji}(t)$. C. Michael (1985)

$$C_{ji}(t)v_j^{(n)}(t_0) = \lambda^{(n)}(t, t_0)C_{ji}(t_0)v_j^{(n)}(t_0)$$

- The m principal correlators given by eigenvalues behave as

$$\lambda_n(t, t_0) \sim e^{-E_n(t-t_0)}(1 + \mathcal{O}(e^{-\partial E(t-t_0)})).$$

- Eigenvectors related to the operator state overlaps

$$Z_j^n = \langle 0|\Phi_j|n\rangle \propto v_j^{(n)}(t_0)$$

The interpolating operators: Example

Let us focus on the meson sector.

The simplest operators are local fermion bilinears:

$$
\begin{aligned}
0^{-+} &\sim \bar{\psi}\gamma_5\psi \\
1^{--} &\sim \bar{\psi}\gamma_i\psi \\
0^{++} &\sim \bar{\psi}\psi \\
1^{++} &\sim \bar{\psi}\gamma_5\gamma_i\psi \\
1^{+-} &\sim \bar{\psi}\gamma_i\gamma_j\psi\epsilon_{ijk}
\end{aligned}
$$

No local fermion bilinear for $J^{PC} = 1^{-+}$, which is a quark model exotic q. #.

No higher spin local operators to extract orbital excitations.

Non-local operators:

Either involving displacements or using discrete derivatives.

Forward derivative: $\overrightarrow{D}_i\psi_x = \psi_{x+ai} - \psi_x$

Backward derivative: $\overleftarrow{D}_i\psi_x = \psi_x - \psi_{x-ai}$

Symmetric derivative: $\overleftrightarrow{D}_i = \overleftarrow{D}_i - \overrightarrow{D}_i$

A simple derivative operator: $\psi\overleftrightarrow{D}_i\psi$ $[J^{PC} = 1^{--}]$

Other possible operators:

Multi-meson operators, diquark-antidiquark operators, baryon-antibaryon-like operators, ...

Lattice systematics

Fermion related systematics

Unphysically heavy light quark masses: Chiral extrapolation
Tuning errors: strange, charm and bottom quark masses.
Discretization errors in heavy quark systems.

Non-zero lattice spacing

All calculations performed at finite non-zero lattice spacing.
Need for continuum extrapolation.

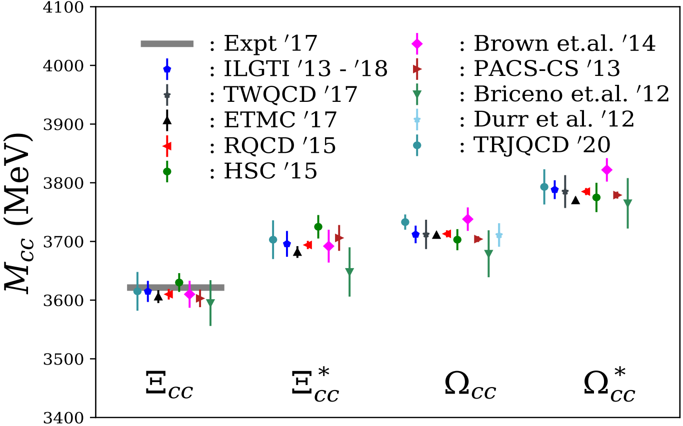
Finite volume

All calculations performed at finite physical lattice extent.
Need for infinte volume extrapolation.
Scattering and resonances: Need for multi-hadron operators, Quantization conditions, ...

Other systematics

Scale setting errors, effects from charm and bottom sea quenching, action specific uncertainties, mixed action effects, QED and strong isospin breaking effects, ...

Doubly charm baryons: An example

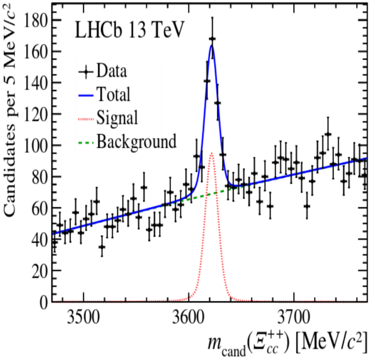


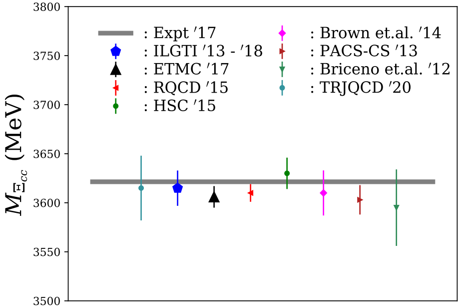
Another calculation of heavy baryon masses: QCDSF-UKQCD 1711.02485.
Heavy baryon mass splittings : BMW Science**347** 1452 '15

Early quenched lattice calculations : Lewis *et al.* '01; Mathur *et al.* '02; Flynn *et al.* '03
Dynamical (light quark) investigations : Liu *et al.* '10

Summary of results: MP Lattice 2018, Charm 2020, FTCF2024;
Mai, Meißner, Urbach, Physics Reports 2023

The first doubly charm baryon : Ξ_{cc}





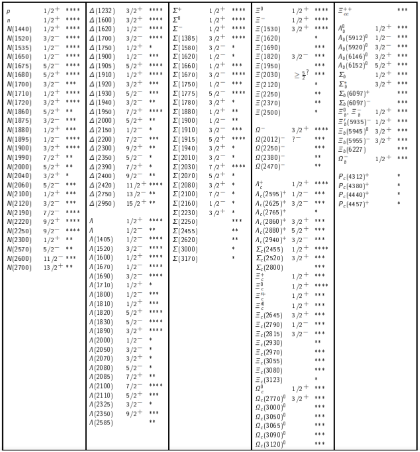
Ξ_{cc} isospin splitting (LQCD), 2.16(11)(17) MeV : BMW Science**347** 1452 '15
SELEX measurement (3519 MeV) : Mattson *et al.* PRL**89** 112001 '02

All lattice calculations disfavors SELEX peak to be a doubly charm baryon.

Family of strong interacting particles

There is a big family of particles observed in nature, of which nucleon is just a member.

Baryons



p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

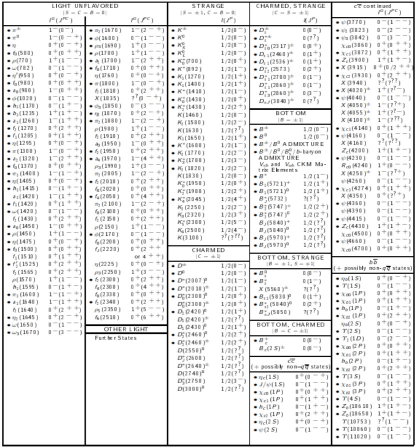
$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
Λ	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2160)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c^{\prime +}$	$1/2^+$	***
$\Xi_c^{\prime 0}$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		**
$\Xi_c(2970)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***

Ξ_{cc}^{++}		***
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6227)$		***
Ω_b^-	$1/2^+$	***
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*

Mesons



LIGHT UNFLAVORED
$|S = C = B = 0|$

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$

	$I^G(J^{PC})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE
$|S = \pm 1, C = B = 0|$

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED
$|C = \pm 1|$

• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE
$|C = S = \pm 1|$

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
$D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM
$|B = \pm 1|$

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE
$|B = \pm 1, S = \mp 1|$

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED
$|B = C = \pm 1|$

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\overline{c}$
(+ possibly non-$q\overline{q}$ states)

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$

$c\overline{c}$ continued

	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

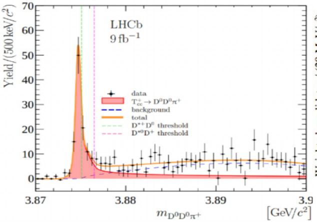
$b\overline{b}$
(+ possibly non-$q\overline{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

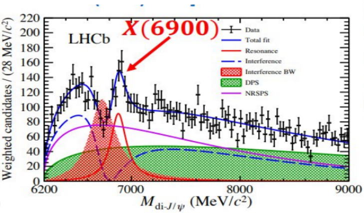
Summary tables taken from Particle Data Group website (2021): pdg.lbl.gov/

Going beyond 'stable' hadrons at the core of understanding strong interactions.

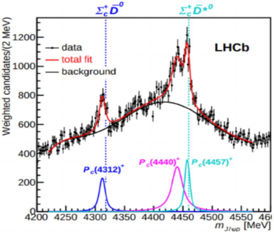
Beyond baryons and mesons in experiments



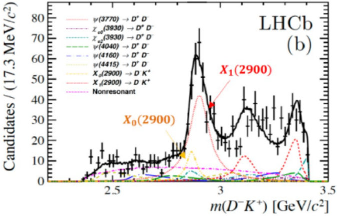
T_{cc} LHCb 2021



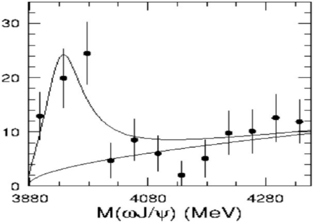
X(6900) LHCb 2020



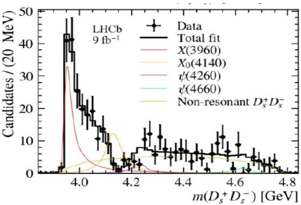
P_c LHCb 2019



X(2900) LHCb 2020



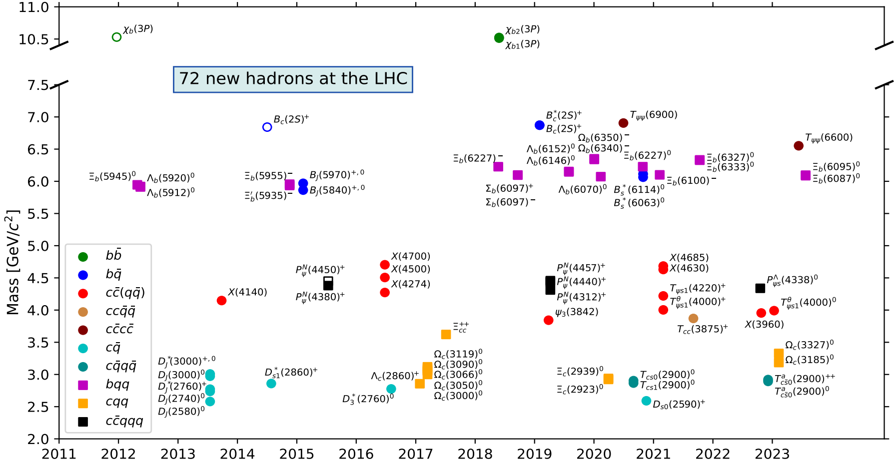
X(3915) Belle 2005



X(3960) LHCb 2021

See a recent talk by Liming Zhang here

Summary of LHCb discoveries

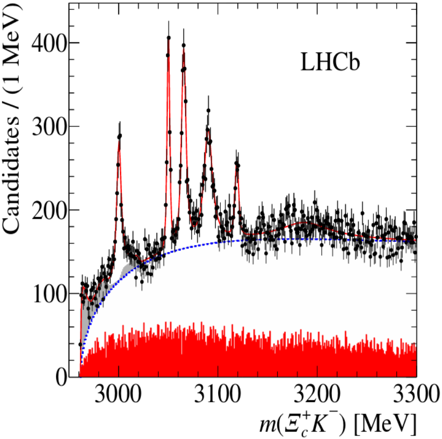


patrick.koppenburg@cern.ch 2023-08-16

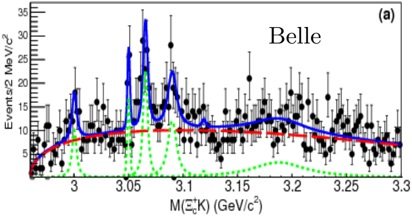
https://www.nikhef.nl/∼pkoppenb/particles.html

See a recent talk by Liming Zhang here

LHCb discovery of excited Ω_c^0 baryons



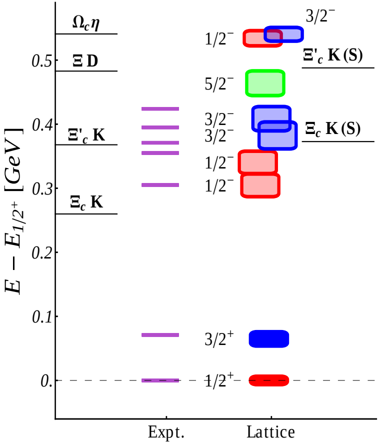
Resonance	Energy	Width	Q.no.
Ω_c^0	2695(2)	-	$1/2^+$
$\Omega_c^0(2770)$	2766(2)	-	$3/2^+$
$\Omega_c^0(3000)$	3000(1)	4.5(1)	?
$\Omega_c^0(3050)$	3050(1)	1(-)	?
$\Omega_c^0(3066)$	3066(1)	3.5(-)	?
$\Omega_c^0(3090)$	3090(1)	8.7(1)	?
$\Omega_c^0(3119)$	3119(1)	1(1)	?



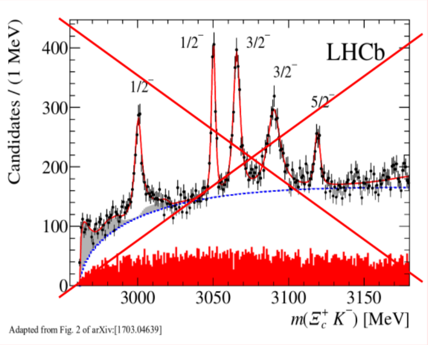
Aaij *et al.* (LHCb) PRL**118** 182001 '17

Confirmation by Belle : Yelton *et al.* (Belle) PRD**97** 051102 '18

Excited states, quantum number assignment and falsification



Excluded by 3.6σ



Adapted from Fig. 2 of arXiv:[1703.04639]

to be revisited!

MP and Mathur 2017 PRL and other pheno predictions.

LHCb PRD 104, L091102 (2021)

On the lattice, strong decays are ignored, and there remain various unattended systematics.

Computational requirements: a major bottleneck.

See my talk at ICNFP 2024 for more: click here.

Excited state spectroscopy from lattice: Single hadron approach

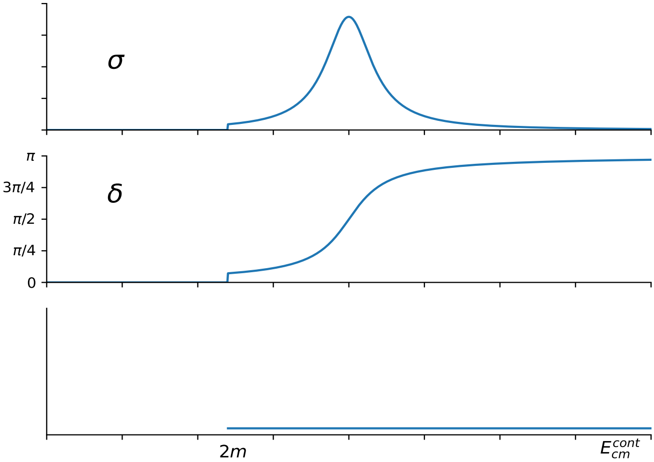
- Large basis of carefully constructed hadron interpolators
  Mesons : Liao & Manke hep-lat/0210030 '02; Thomas (HSC) PRD**85** 014507 '12
  Baryons : Basak *et al.* (LHPC) PRD**72** 074501, PRD**72** 094506 '05
  Morningstar *et al.* PRD**88** 014511 '13.
- Matrix of correlation functions & Variational study
  Dudek *et al.* PRD**77** 034501 '08, Michael NPB**259** 58 (1985)
- Established and practised by many groups

  Relatively old summary. Many more in the recent years

  Light mesons : Dudek *et al.* (HSC) PRL**103** 262001 '09, PRD**82** 034508 '10
  Dudek *et al.* (HSC) PRD**85** 014507 '12
  Light baryons : Bulava *et al.* (HSC) PRD**82** 014507, '10
  Edwards *et al.* (HSC) PRD**84** 074508 '11, PRD**87** 054506 '13
  Heavy mesons : Liu *et al.* (HSC) JHEP**1207** 126; Moir *et al.* JHEP**1305** 021
  Cheung *et al.* JHEP**1612** 089; Mohler *et al.* PRD**87** 034501 '13
  Bali *et al.* PRD**84** 094506 '11; Wurtz *et al.* PRD**92** 054504 '15
  Heavy baryons : Meinel, PRD**85** 114510 '12
  MP *et al.* (HSC) PRD**90** 074504 '14, PRD**91** 094502 '15
  MP & Mathur (HSC) PRL**119** 042001 '17, 1508.07168.
- Single hadron approach. Naive expectation : correct up to $\mathcal{O}(\Gamma)$

The challenge on lattice: Resonances in the infinite volume continuum

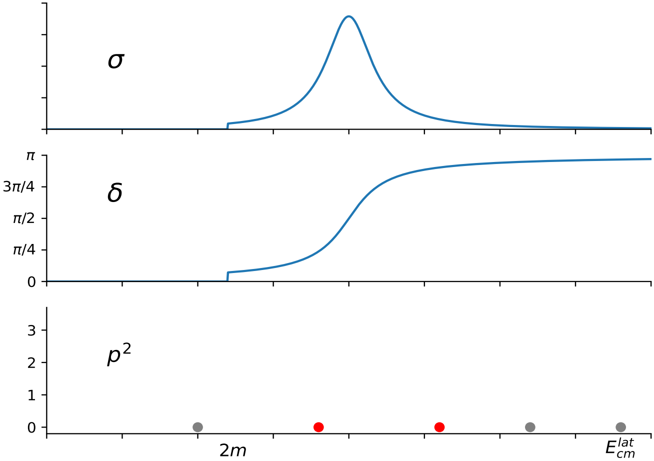
Scattering cross sections, phase shifts, branch cuts, Riemann sheets.



Schematic picture for illustration. Should not be taken quantitatively.

Resonances on the lattice (elastic) : ??

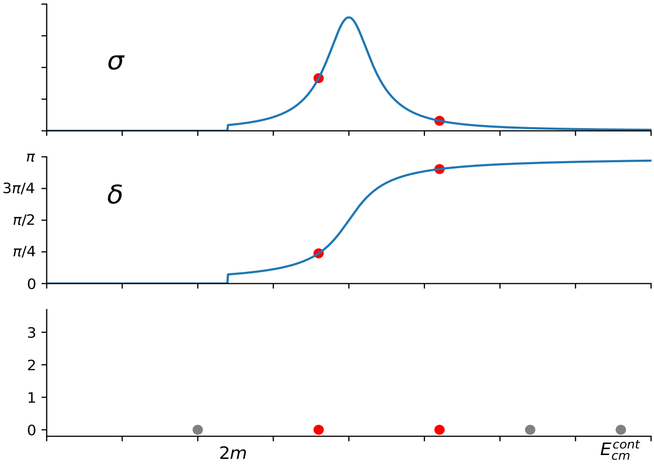
Discrete spectrum: No branch cuts, no Riemann sheets, no resonances!



Maiani-Testa no-go theorem [1990]

Resonances on the lattice (elastic) : Lüscher (1991)

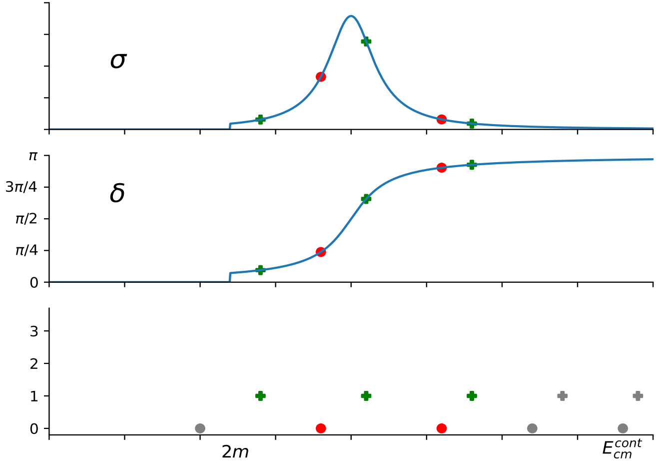
Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Lüscher [1991]

Resonances on the lattice (elastic) : Lüscher (1991)

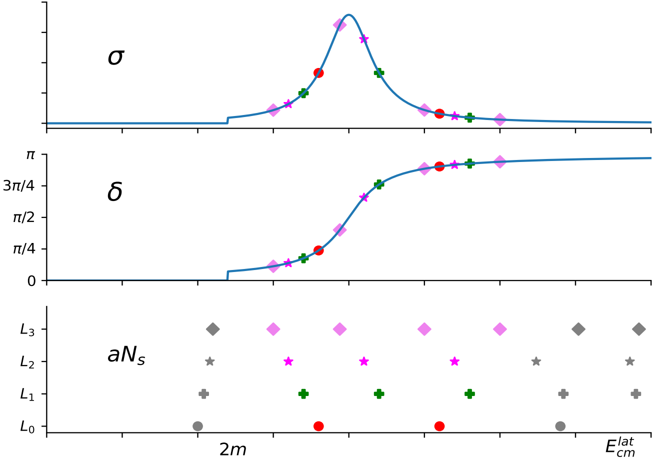
Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Different inertial frames can be utilized to extract more information

Resonances on the lattice (elastic) : Lüscher (1991)

Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Multiple physical volumes can also be utilized to extract more information.

For generalizations of Lüscher framework, *c.f.* Briceño, Hansen 2014-15

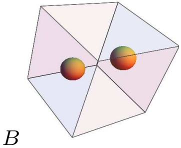
Finite volume spectrum and infinite volume physics

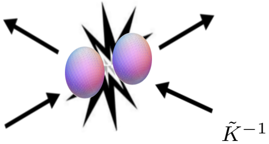
- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold.

Maiani-Testa 1990

- Non-interacting two-hadron levels are given by
  $$E(L) = \sqrt{m_1^2 + \boldsymbol{k}_1^2} + \sqrt{m_2^2 + \boldsymbol{k}_2^2} \text{ where } \boldsymbol{k}_{1,2} = \tfrac{2\pi}{L}(n_x, n_y, n_z).$$
- Switching on the interaction: $\boldsymbol{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\boldsymbol{k}_{1,2} = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$.
- Lüscher's formalsim: **finite volume level shifts $\Leftrightarrow$ infinite volume phase shifts.**

Lüscher 1991





- Generalizations of Lüscher's formalism: *c.f.* Briceño 2014
  Quite complex problem: inelastic resonances $(R \to H_1H_2, H_3H_4)$
  Quantization condition is a determinant equation: $Det(B(L,k^2) - \tilde{K}^{-1}(k^2)) = 0$
  becomes an underconstrained problem with only few energy levels at hand.

Extensions and other methods

- ✿ Extensions within and beyond elastic scattering :
  - different inertial frames, boundary conditions
  - multiple scattering channels
  - particles with different identities Briceño 1411.6944; Hansen 1511.04737
  - 2-particle scattering in finite volume code: https://github.com/cjmorningstar10/TwoHadronsInBox
  - 3-particle scattering : Hansen, Sharpe, Lopez, Mai, Döring, Rusetsky, ...
- ✿ HALQCD method :
  - Determine the potential between scattering particles
  - Extract resonance information solving Schrödinger equation.

  Ishii *et al.* PRL**99** 022001 '07; PLB**712** 437 '12
- ✿ finite volume Hamiltonian EFT / Quantization condition in plane wave basis :
  - Constrain free parameters of the Hamiltonian based on lattice spectrum
  - Solve for EVP to extract resonance information.

  Hall *et al.* PRD**87** 094510 '12

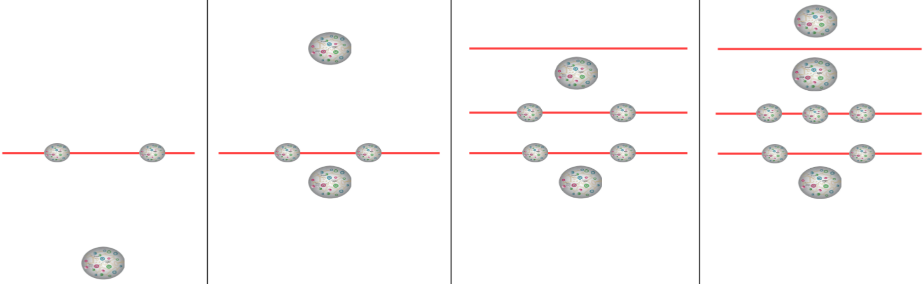
  Meng & Epelbaum JHEP**10** (2021) 051

  Mai & Döring Eur.Phys.J.A**53** (2017) 12, 240
- ✿ Optical potential :

  Agadjanov *et al.* JHEP**06** 043 '16 [HSI]

  Hammer, Pang, Rusetsky, JHEP**1709** 109

Complexity in Hadron spectroscopy



Straightforward

Deeply bound;
Strong decay stable;

$\pi, K, D, p, n, \Lambda, \Xi_{cc}, \ldots$

Exponential volume corrections
$[E_\infty - E_L \propto e^{-mL}]$

Relatively Easy

Shallow bound states;
Elastic resonances;
Only two body decays.

Deuteron,
$\Delta, \rho, D^*, D^*_{s0}, D_{s1}, \ldots$

Difficult

Inelastic resonances;
Multiple two-body final states;
Most hadrons.

Additionally three-body decays.

Quite complex

Inelastic resonances;
Multiple final state configs;
Most hadrons.

XYZTPs, glueballs, nuclei, ...

Power law volume corrections **$\Delta E \propto \frac{a_0}{L^3} + O(\frac{1}{L^4})]$**
Need a rigorous finite-volume amplitude analysis.

Scattering amplitude parametrization

- Scattering amplitude: $S = 1 + i\frac{4k}{E_{cm}}t$
- For an elastic scattering, and assuming only S-wave,

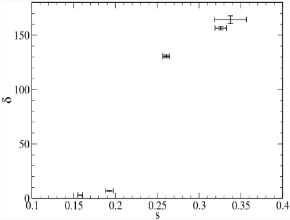
$$t^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k.cot\delta(k)$$

(virtual/bound) state constraint below threshold: $k.cot\delta(k) = (+/-)\sqrt{-k^2}$

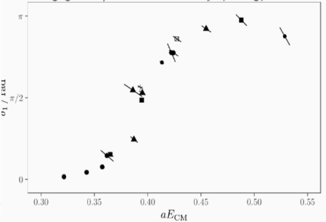
- Lüscher's prescription: $k.cot\delta(k) = B(L, k^2)$: a known mathematical function.
  k^2 is determined from each extracted finite volume energy splittings.
- Parametrize $k.cot\delta(k)$ as different functions of k.
  Effective Range Expansion (ERE): $k.cot\delta(k) = a_0^{-1} + 0.5r_0k^2 + \beta_i k^{2i+4}$.
  The best fits determined to represent the energy dependence.
- For multichannel processes, $\tilde{K}^{-1}(k^2)$ and $B(L, k^2)$ become matrices,
  the Quantization conditions become a matrix determinant equation, each energy level gives a constraint, and each $\tilde{K}^{-1}$-matrix element* needs to be parametrized.

$$Det(\tilde{K}^{-1}(k^2) - B(L, k^2)) = 0$$

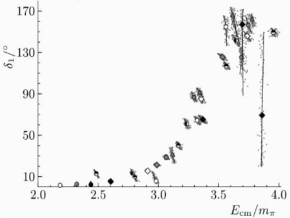
Vanilla resonance: $\rho \to \pi\pi$



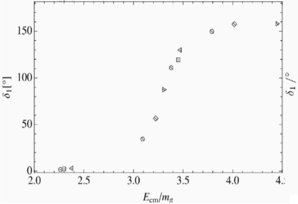
266 MeV, Lang et al. 1105.5636



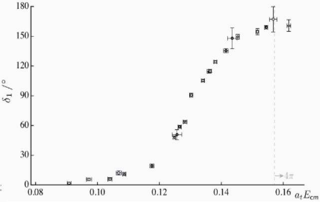
315 MeV, Werner et al. 1907.01237



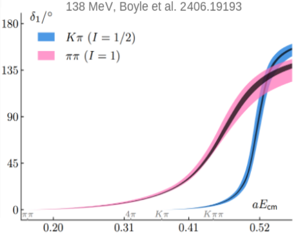
220 MeV, Andersen et al. 1808.05007



226 MeV, Guo et al. 1605.03993



236 MeV, Wilson et al. 1507.02599

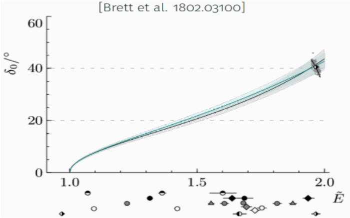


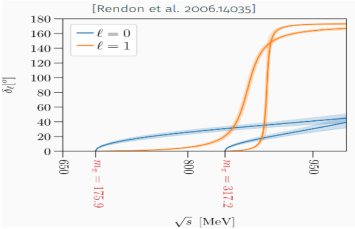
138 MeV, Boyle et al. 2406.19193

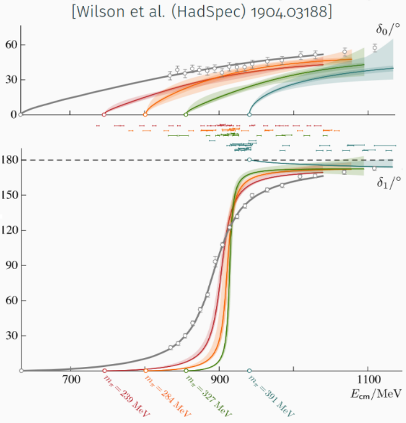
Incomplete list of lattice calculations

See my talk at ICNFP 2024 for more: click here.

Another vanilla resonance: $K^* \to K\pi$







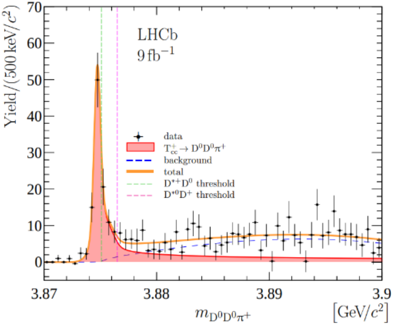
K^* and κ resonances

$K\pi$ atoms at DIRAC experiment 1605.06103

Incomplete list of lattice calculations

See my talk at ICNFP 2024 for more: click here.

Doubly heavy tetraquarks: T_{cc}^{+}



LHCb: 2109.01038, 2109.01056

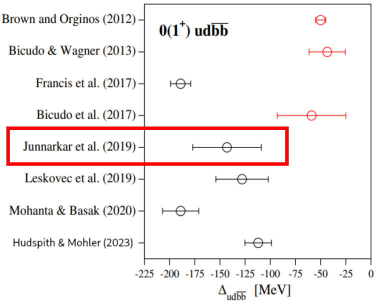
$$\delta m \equiv m_{\mathrm{T}_{\mathrm{cc}}^{+}} - (m_{\mathrm{D}^{*+}} + m_{\mathrm{D}^0})$$

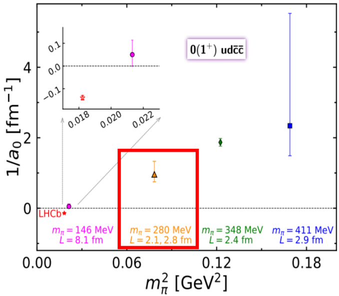
$$\begin{aligned} \delta m_{\mathrm{pole}} &= -360 \pm 40\,^{+4}_{-0}\ \mathrm{keV}/c^2\,, \\ \Gamma_{\mathrm{pole}} &= 48 \pm 2\,^{+0}_{-14}\,\mathrm{keV}\,. \end{aligned}$$

- The doubly charmed tetraquark T_{cc}^{+}, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0 D^+ \pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.

c.f. Ader&Richard PRD25(1982)2370

Doubly heavy tetraquarks using lattice QCD, T_{bb} and T_{cc}: $I(J^P) = 0(1^+)$





- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100 MeV)$.

  Fig: Hudspith&Mohler 2023

  Red box: ILGTI work on T_{QQ} tetraquarks: Junnarkar, Mathur, MP PRD 2019

- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100 keV)$.

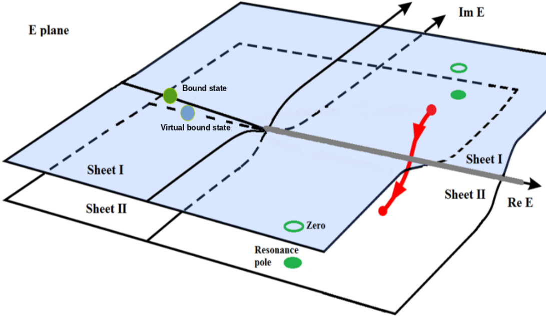
  Fig: Lyu *et al.*PRL 2023

  Red box: T_{cc} (RQCD) [PRL 2022] and its quark mass dependence [2402.14715].

- Several recent calculations in the bottom-charm tetraquark sector.
  A summary of different lattice investigations →

  see review by Pedro Bicudo, 2212.07793

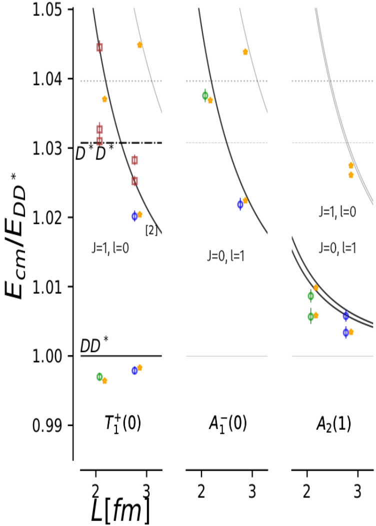
Virtual/bound states

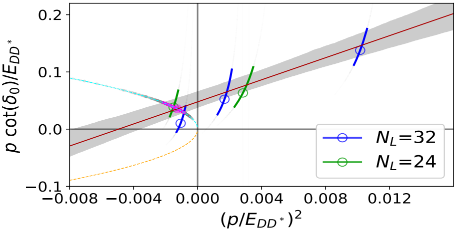


- $T \propto (p cot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$. Virtual bound state is a pole in T with $p = -i|p|$.

- An example for virtual bound state: spin-singlet dineutron.

DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with an ERE: T_{cc}^+





MP, Prelovsek PRL 2022

$m_\pi \sim 280$ MeV

- Fit quality:
  $$\chi^2/d.o.f. = 3.7/5.$$
- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}
- Binding energy:
  $$\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2}) \text{ MeV}.$$
- First evaluation of the DD^* amplitude in T_{cc} channel.

+/g refers to positive parity, -/u refers to negative parity.

Pion exchange interactions/left-hand cut: ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)

  $m_\pi > m_{D^*} - m_D \quad \Rightarrow \quad D^* \to D\pi$ is kinematicaly forbidden.

  $2 \to 2$ Generalized LQC: does not subthreshold lhc effects.

  Raposo&Hansen 2311.18793, Dawid *et al* 2303.04394, Hansen *et al* 2401.06609

  See recent talks by Hansen and Lopez

  ERE convergence fails at the nearest singularity.

  Left-hand cut in the DD^* system close below the DD^* threshold.

  Du *et al* 2303.09441[PRL]

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):

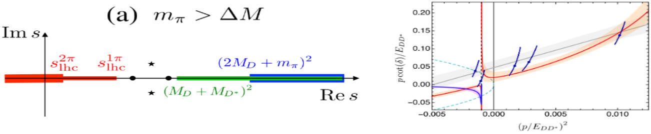
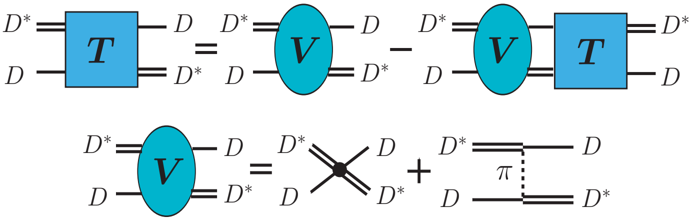


Figure taken from Du *et al* 2303.09441[PRL]

Long range pion exchange interactions: the origin of left-hand singularity and cut.
Fits with a potential that incorporates the one pion exchange:

Virtual bound states ⇒ Virtual resonances

Solving Lippmann-Schwinger Equation for the DD^* amplitude



- The potential: a sum of short range and long range interactions

$$V(\boldsymbol{p},\boldsymbol{p}') = V_{\rm CT}(p,p') + V_\pi^S(p,p') \quad \text{with} \quad V_{\rm CT}(p,p') = 2c_0 + 2c_2(p^2 + p'^2) + \mathcal{O}(p^4, p'^4)$$

- The scattering amplitude $\qquad T^{-1} \propto p\cot\delta_0 - ip$

- The pion decay constant f_π and $DD^*\pi$ coupling g_c
  at $m_\pi \sim 280$ MeV following the 1-loop χPT.

Du *et al* 2303.09441[PRL]

Collins, Nefediev, MP, Prelovsek 2402:14715

One-pion exchange interaction/left-hand cut

- OPE from the lowest order NR Lagrangian

$$\mathcal{L} = \frac{g_c}{2f_\pi}\boldsymbol{D}^{*\dagger}\cdot\boldsymbol{\nabla}\pi^a\tau^a D + h.c. \ . \quad \Rightarrow \quad V_\pi(\boldsymbol{p},\boldsymbol{p}') = 3\left(\frac{g_c}{2f_\pi}\right)^2 \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u-m_\pi^2}$$

Fleming *et al.* hep-ph/0703168, Hu&Mehen hep-ph/0511321

- Upon S-wave projection, we have

$$V_\pi^S(p,p) = \frac{g_c^2}{4f_\pi^2}\left[\frac{m_\pi^2-q_0^2}{4p^2}\ln\left(1+\frac{4p^2}{m_\pi^2-q_0^2}\right)-1\right]$$

Logarithmic function branch cut → infinite set of Riemann sheets

- With the finite branch point at

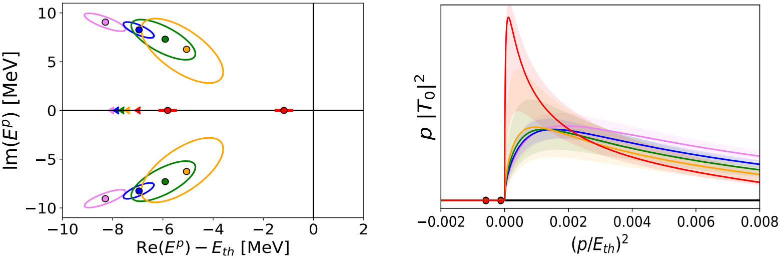
$$\boldsymbol{p^2_{\mathrm{lhc}} = \frac{1}{4}(q_0^2 - m_\pi^2) < 0} \text{ for all lattice setups.}$$

with $q_0 \simeq m_{D^*} - m_D$, where the $D^{(*)}$-meson recoil terms are ignored.

Du *et al.* 2303.09441[PRL]

- Consequences:
  - Complex phase shifts below the lhc.
  - Modified near-threshold energy dependence.

Pole positions and scattering rate [EFT]



- Subthreshold resonance pole pair moving towards the real axis with increasing m_c.

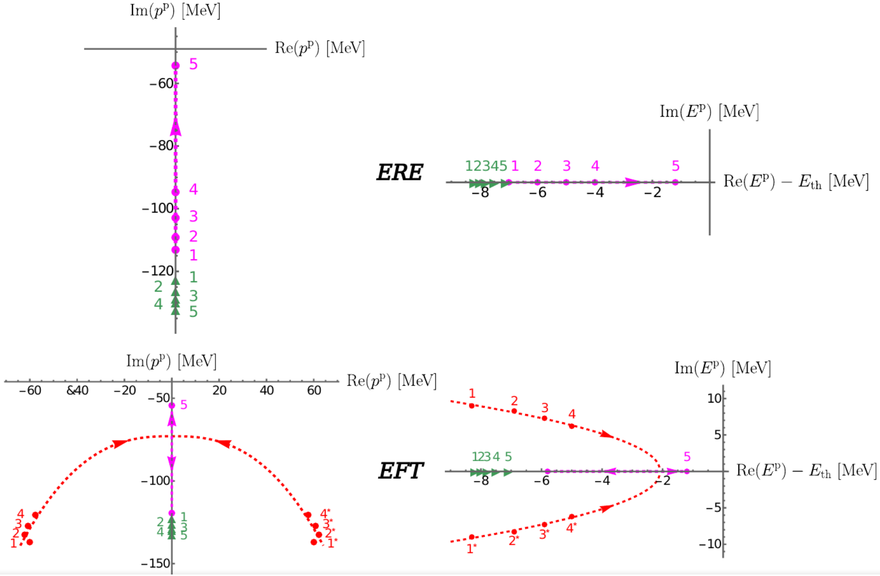
- Collide on the real axis below threshold and turn back-to-back.
  At the heaviest m_c: virtual bound poles [in Red]

- With increasing m_c, subthreshold resonance poles evolves to become
  a pair of virtual bound poles.

- Enhancement in the DD^* scattering rate ($p|T_0|^2$).

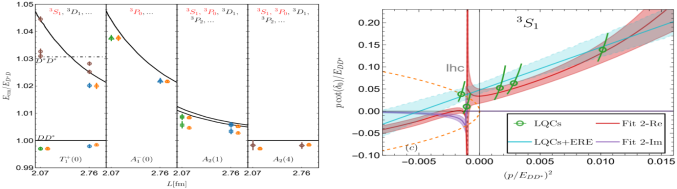
Collins, Nefediev, MP, Prelovsek 2402:14715

Pole trajectory of T_{cc}^+: ERE Vs EFT



Work around to LQC: A plane-wave approach and modified LQC

- An effective field theory incorporating OPE with a plane wave basis expansion.



Lu Meng *et al* arXiv:2312.01930

Virtual bound states ⇒ Virtual resonances [m_π ∼280 MeV]

- Modified 3-particle (Lüscher) Quantization Condition:

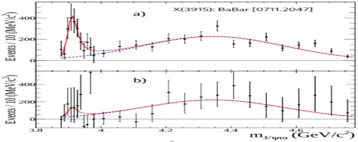
Hansen, Romero-Lopez, Sharpe, 2401.06609, Raposo, Hansen, 2311.18793

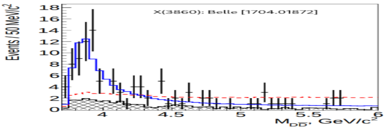
Dawid, Lopez, Sharpe 2409.17059, See a recent talk by Dawid here

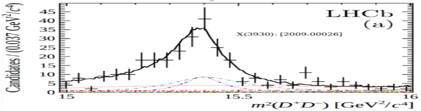
A rigorous procedure, but demands multiple lattice inputs.

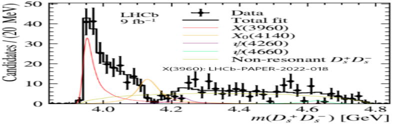
- $D\pi$ finite volume spectrum up to the $D\pi\pi$ threshold.
- Isovector DD finite volume spectrum up to the $DD\pi$ threshold.
- Isoscalar $DD\pi$ finite volume spectrum up to the $DD\pi\pi$ threshold.

Scalar charmonium-like states









- Several likely related features, $X(3915)$, $X(3930)$, $X(3960)$.
  Proximity to the $\bar{D}_sD_s$ threshold: Possible hidden strange content [$\mathbf{cs\bar{c}\bar{s}}$]
  $\Rightarrow$ narrow width from $\bar{D}D$

- Several phenomenological studies supporting this:

  Lebed Polosa 1602.08421, Chen *et al* 1706.09731, Bayar *et al* 2207.08490

- Another feature named as X(3860) observed by Belle. No evidence from LHCb.

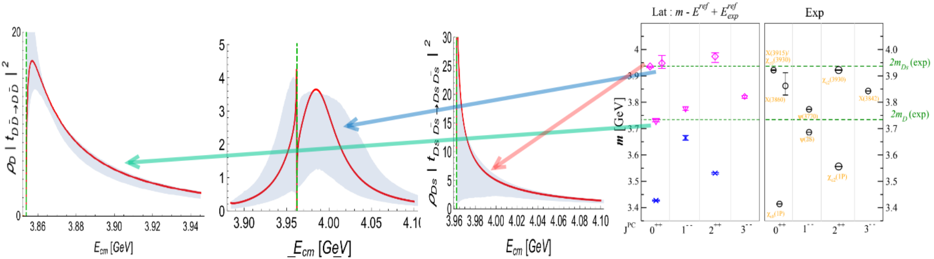
- Yet unknown $\bar{D}D$ bound state, predicted by models.

  Gamermann *et al* 0612179, Hidalgo-Duque *et al* 1305.4487, Baru *et al* 1605.09649

- Such a $\bar{D}D$ bound state is supported by re-analysis of the exp. data.

  Danilkin *et al* 2111.15033, Ji *et al* 2212.00631.

Charmonium-like resonances and bound states on the lattice



- ✿ First extraction of coupled $\bar{D}D$-$\bar{D}_sD_s$ scattering amplitude. $[\mathbf{\bar{c}c},\ \mathbf{\bar{c}c\bar{q}q};\ \mathbf{q} \to \mathbf{u}, \mathbf{d}, \mathbf{s}$, and $\mathbf{I} = \mathbf{0}]$.
- ✿ Lattice QCD ensembles : CLS Consortium

  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $m_D \sim$ 1927 MeV, $a \sim 0.086$ fm
- ✿ In addition to conventional charmonium states, we observe candidates for three excited scalar charmonium states
  - ⇒ a yet unobserved shallow $\bar{D}D$ bound state.
  - ⇒ a $\bar{D}D$ resonance possibly related to X(3860).
  - ⇒ a narrow resonance just below and with large coupling to $\bar{D}_sD_s$ threshold.
    possibly related to X(3960) / X(3930) / X(3915).
- ✿ Our (RQCD) recent publications on charmonium:

Collins, Mohler, MP, Piemonte, Prelovsek 2111.02934, 2011.02541, 1905.03506.

Summary

- We have a handful of hadrons, with a large set of them still demanding an understanding based on first principles. The list is proliferating with those several experimental efforts across globe.
- Lattice QCD, being a suitable nonperturbative framework, has been used to study several of these hadrons.
- Made a 'very' brief outline of how hadron masses are extracted and how resonances are studied in a finite volume.
- Presented a selected examples of lattice investigations, particularly addressing shallow bound states, near threshold poles and conventional resonances.
- Many hadronic states remain unaddressed and several remaining challenges even before addressing lattice systematics. Formalisms accounting three body dynamics. New ideas to access highly excited states. ...
- Quark mass dependence as a probe to understand the nature of resonances.
  Heavy hadron sector serving as an excellent test bed.
- Lattice systematics: Need for huge computation resources.

Thank you