Revisiting $D^+ \to \pi^+\ell^+\ell^-$ in SM using LCSR

Anshika Bansal

(Work in progress with Alexander Khodjamirian and Thomas Mannel)

30/09/2024

UNIVERSITÄT SIEGEN

TP1 Theoretical Particle Physics

CPPS Center for Particle Physics Siegen

Talk at "BELLE II Germany Meeting 2024" @ DESY, Hamburg, Germany
(30/09/2024 - 02/10/2024)

Introduction

- $c \to u\ell^+\ell^-$ transition : FCNC transition $\Longrightarrow$ short distance effects are strongly suppressed in SM due to GIM mechanism.
- FCNCs in charm sector are enhanced in various BSM scenarios $\Longrightarrow$ considered to be a good indicator of New Physics.
- $D \to \pi\ell^+\ell^-$: Simplest decay mode to study $c \to u\ell^+\ell^-$.
  - Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.
  - A simple mechanism: $D \to \pi\ell^+\ell^- \approx D \to \pi V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V\to\mu^+\mu^-}$
$\rho^0(770)$	$(8.3 \pm 1.4) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.78 \pm 0.68) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
$\phi(1020)$	$(5.7 \pm 0.14) \times 10^{-3}$	$(2.85 \pm 0.19) \times 10^{-4}$	$(1.62 \pm 0.12) \times 10^{-6}$

[PDG]

Introduction

- $c \to u\ell^+\ell^-$ transition : FCNC transition $\Longrightarrow$ short distance effects are strongly suppressed in SM due to GIM mechanism.
- FCNCs in charm sector are enhanced in various BSM scenarios $\Longrightarrow$ considered to be a good indicator of New Physics.
- $D \to \pi\ell^+\ell^-$: Simplest decay mode to study $c \to u\ell^+\ell^-$.
  - Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.
  - A simple mechanism: $D \to \pi\ell^+\ell^- \approx D \to \pi V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V\to\mu^+\mu^-}$
$\rho^0(770)$	$(8.3 \pm 1.4) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.78 \pm 0.68) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
$\phi(1020)$	$(5.7 \pm 0.14) \times 10^{-3}$	$(2.85 \pm 0.19) \times 10^{-4}$	$(1.62 \pm 0.12) \times 10^{-6}$

[PDG]

- A QCD based study (to handle long distance effects) is desirable.

$\longrightarrow$

Available estimates are based on QCDf (for $D \to \rho\ell^+\ell^-$).
[T. Feldmann, B. Müller, D. Seidel, JHEP08 (2017) 105]

Later used for $D \to \pi\ell^+\ell^-$ (major focus on New Physics)
[A. Bharucha, D. Boito, C. Méaux, JHEP 04 (2021) 158]

Introduction

- $c \to u\ell^+\ell^-$ transition : FCNC transition $\Longrightarrow$ short distance effects are strongly suppressed in SM due to GIM mechanism.
- FCNCs in charm sector are enhanced in various BSM scenarios $\Longrightarrow$ considered to be a good indicator of New Physics.
- $D \to \pi\ell^+\ell^-$: Simplest decay mode to study $c \to u\ell^+\ell^-$.
  - Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.
  - A simple mechanism: $D \to \pi\ell^+\ell^- \approx D \to \pi V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V\to\mu^+\mu^-}$
$\rho^0(770)$	$(8.3 \pm 1.4) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.78 \pm 0.68) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
$\phi(1020)$	$(5.7 \pm 0.14) \times 10^{-3}$	$(2.85 \pm 0.19) \times 10^{-4}$	$(1.62 \pm 0.12) \times 10^{-6}$

[PDG]

- A QCD based study (to handle long distance effects) is desirable.

→

Available estimates are based on QCDf (for $D \to \rho\ell^+\ell^-$).
[T. Feldmann, B. Müller, D. Seidel, JHEP08 (2017) 105]

Later used for $D \to \pi\ell^+\ell^-$ (major focus on New Physics)
[A. Bharucha, D. Boito, C. Méaux, JHEP 04 (2021) 158]

- Other $D_{(s)} \to P\ell^+\ell^-$ channels ($P = \pi, K, \eta$)), Cabibbo favoured(CF) and doubly Cabibbo suppressed(DCS) are also interesting since they share long-distance dynamics (annihilation mechanism).

Effective Operators

- The effective Hamiltonian for $D \to \pi \ell^+ \ell^-$ (SCS)

suppressing factor

$V_{u\mathscr{D}} V^*_{c\mathscr{D}} \approx \lambda$ $\quad V_{ub} V^*_{cb} \approx \lambda^5$

$$\mathscr{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \lambda_{\mathscr{D}} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}} \right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i$$

$O_1^{\mathscr{D}} = (\bar{u}_L \gamma_\mu \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu c_L)$

$O_2^{\mathscr{D}} = (\bar{u}_L \gamma_\mu t^a \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu t^a c_L)$

$\ll C_{1,2} @ \mathcal{O}(m_c)$

WCs @ $\mu = 1.3$ GeV at NNLO : $C_1 = 1.034, C_2 = -0.633$

[Stefan de Boer, Bastian Müller, Dirk Siegel, JHEP 08 (2016)]

Effective Operators

- The effective Hamiltonian for $D \to \pi \ell^+ \ell^-$ (SCS)

suppressing factor

$V_{u\mathscr{D}} V^*_{c\mathscr{D}} \approx \lambda$

$V_{ub} V^*_{cb} \approx \lambda^5$

$$\mathscr{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \lambda_{\mathscr{D}} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}} \right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i$$

$O_1^{\mathscr{D}} = (\bar{u}_L \gamma_\mu \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu c_L)$

$O_2^{\mathscr{D}} = (\bar{u}_L \gamma_\mu t^a \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu t^a c_L)$

$\ll C_{1,2} @ \mathcal{O}(m_c)$

WCs @ $\mu = 1.3$ GeV at NNLO : $C_1 = 1.034, C_2 = -0.633$

[Stefan de Boer, Bastian Müller, Dirk Siegel, JHEP 08 (2016)]

- Hamiltonian in the GIM limit ($\lambda_b = 0$, $\lambda_d = -\lambda_s$):

$$\mathscr{H}_{eff}^{(\Delta_s=0, \lambda_b=0)} = \frac{4G_F}{\sqrt{2}} \lambda_d \left[C_1(O_1^d - O_1^s) + C_2(O_2^d - O_2^s) \right]$$

Effective Operators

- The effective Hamiltonian for $D \to \pi \ell^+ \ell^-$ (SCS)

suppressing factor

$V_{u\mathscr{D}} V^*_{c\mathscr{D}} \approx \lambda$ $\qquad$ $V_{ub} V^*_{cb} \approx \lambda^5$

$$\mathscr{H}^{\Delta S=0}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{\mathscr{D}=d,s} \lambda_{\mathscr{D}} \left[C_1(\mu) O_1^{\mathscr{D}} + C_2(\mu) O_2^{\mathscr{D}}\right] - \lambda_b \sum_{i=3}^{10} C_i(\mu) O_i$$

$O_1^{\mathscr{D}} = (\bar{u}_L \gamma_\mu \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu c_L)$

$O_2^{\mathscr{D}} = (\bar{u}_L \gamma_\mu t^a \mathscr{D}_L)(\bar{\mathscr{D}}_L \gamma^\mu t^a c_L)$

$\ll C_{1,2} @ \mathscr{O}(m_c)$

WCs @ $\mu = 1.3$ GeV at NNLO : $C_1 = 1.034, C_2 = -0.633$

[Stefan de Boer, Bastian Müller, Dirk Siegel, JHEP 08 (2016)]

- Hamiltonian in the GIM limit ($\lambda_b = 0$, $\lambda_d = -\lambda_s$):

$$\mathscr{H}^{(\Delta_s=0,\lambda_b=0)}_{eff} = \frac{4G_F}{\sqrt{2}} \lambda_d \left[C_1(O_1^d - O_1^s) + C_2(O_2^d - O_2^s)\right]$$

- The largest effect beyond GIM limit $\sim \lambda_b C_9$ ($C_9 = -0.488$)

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

The leptonic part

The hadronic part (hadronic matrix element)

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

The leptonic part

The hadronic part (hadronic matrix element)

$$\mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q) = i \int d^4x e^{iq.x} \langle \pi^+(p) | T \left\{ j_\mu^{em}(x), \mathscr{H}_{eff}^{(\Delta_s = 0, \lambda_b = 0)} \right\} | D^+(p+q) \rangle$$

$$= \left[(p \, . \, q) q_\mu - q^2 p_\mu \right] \mathscr{A}_{\mathscr{D}}^{D^+ \to \pi^+ \gamma^*}(q^2)$$

(Due to conservation of EM current)

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathscr{A}(D^+ \to \pi^+ \ell^+ \ell^-) = \left(\frac{16\pi\alpha_{em} G_F}{\sqrt{2}} \right) \lambda_d \frac{\bar{u}_\ell \gamma^\mu \nu_\ell}{q^2} \mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q)$$

The leptonic part

The hadronic part (hadronic matrix element)

$$\mathscr{A}_\mu^{D^+ \to \pi^+ \gamma^*}(p, q) = i \int d^4x e^{iq.x} \langle \pi^+(p) | T \left\{ j_\mu^{em}(x), \mathscr{H}_{eff}^{(\Delta_s = 0, \lambda_b = 0)} \right\} | D^+(p + q) \rangle$$

$$= \left[(p \,.\, q) q_\mu - q^2 p_\mu \right] \mathscr{A}_{\mathscr{D}}^{D^+ \to \pi^+ \gamma^*}(q^2)$$

(Due to conservation of EM current)

The non-local invariant amplitude : **dominated by long distance effects in the physical region of q^2.**

$(4m_\ell^2 < q^2 < (m_D - m_\pi)^2)$

Amplitude and Hadronic Matrix Element

- In the GIM limit ($\lambda_b = 0, \lambda_d = -\lambda_s$):,

$$\mathcal{A}(D^+ \to \pi^+\ell^+\ell^-) = \left(\frac{16\pi\alpha_{em}G_F}{\sqrt{2}}\right)\lambda_d \frac{\bar{u}_\ell\gamma^\mu\nu_\ell}{q^2}\mathcal{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q)$$

The leptonic part

The hadronic part (hadronic matrix element)

$$\mathcal{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q) = i\int d^4x e^{iq.x}\langle\pi^+(p)\,|\,T\left\{j_\mu^{em}(x), \mathcal{H}_{eff}^{(\Delta_s=0,\lambda_b=0)}\right\}|\,D^+(p+q)\rangle$$

$$= \left[(p\,.\,q)q_\mu - q^2 p_\mu\right]\mathcal{A}_{\mathcal{D}}^{D^+\to\pi^+\gamma^*}(q^2)$$

(Due to conservation of EM current)

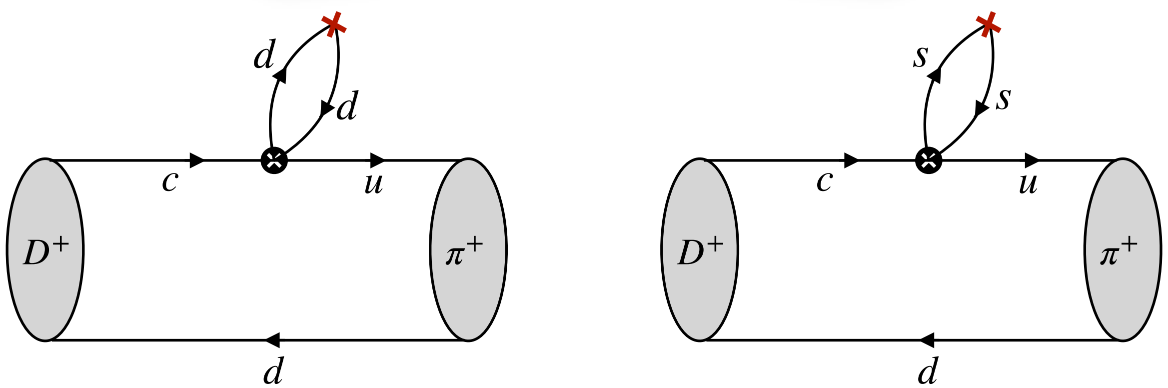
The non-local invariant amplitude : dominated by long distance effects in the physical region of q^2.

$(4m_\ell^2 < q^2 < (m_D - m_\pi)^2)$

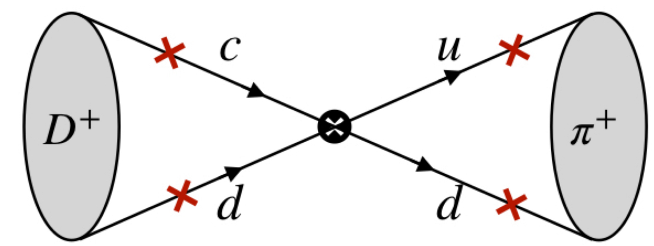
The object of our interest

Quark Topologies for $\mathscr{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q)$

Loop Topology
(Only possible in SCS decays)

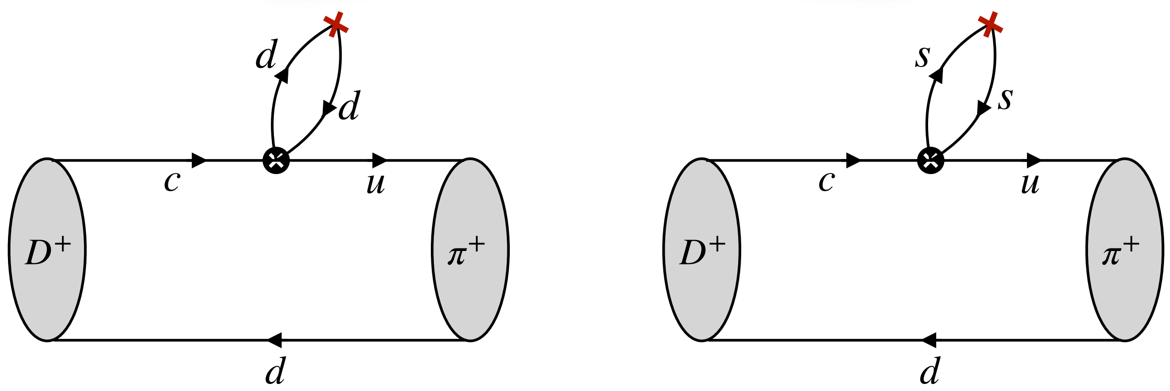


Annihilation Topology



Quark Topologies for $\mathscr{A}_{\mu}^{D^+\to\pi^+\gamma^*}(p,q)$

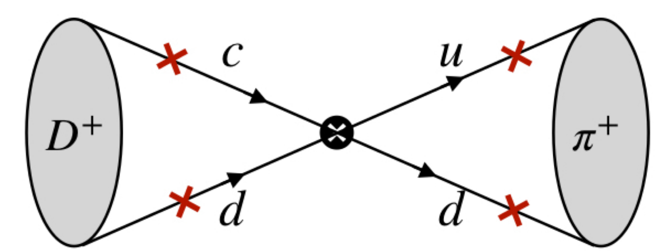
Loop Topology
(Only possible in SCS decays)



- In the $SU(3)_f$ (along with $\lambda_b \approx 0$), the two loops have complete GIM cancellation

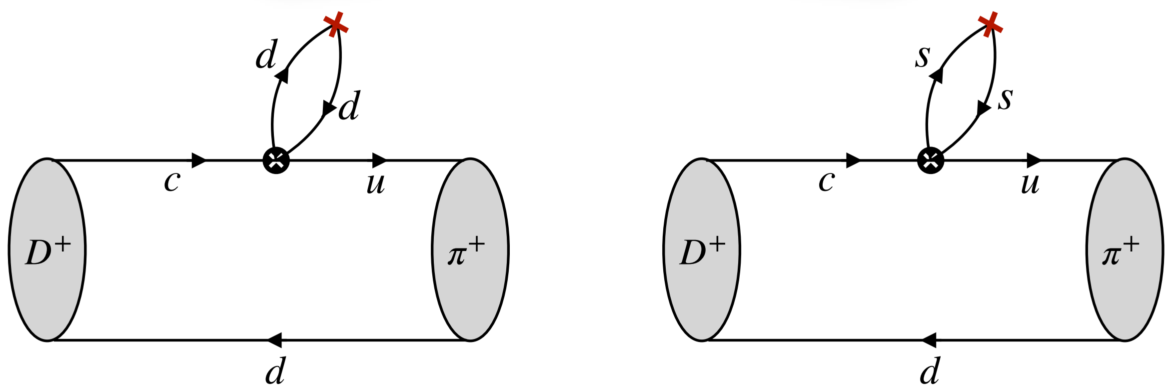
L-topology with have non-zero contribution only due to $m_s \neq m_d$: Expected to be small

Annihilation Topology



Quark Topologies for $\mathscr{A}_{\mu}^{D^+\to\pi^+\gamma^*}(p,q)$

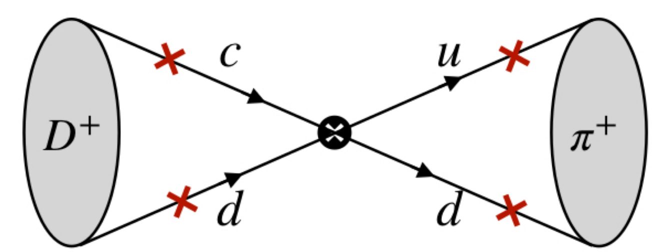
Loop Topology
(Only possible in SCS decays)



- In the $SU(3)_f$ (along with $\lambda_b \approx 0$), the two loops have complete GIM cancellation

L-topology with have non-zero contribution only due to $m_s \neq m_d$: Expected to be small

Annihilation Topology

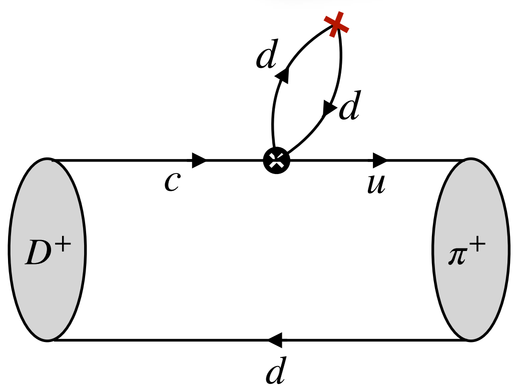


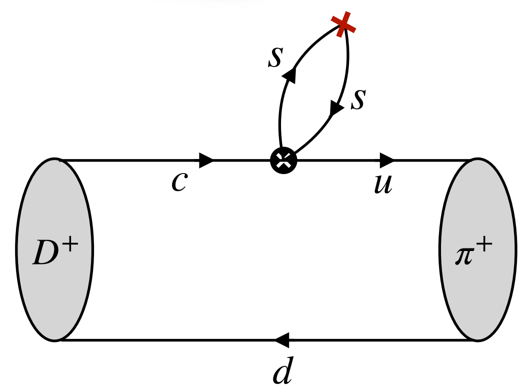
- Only one (d or s flavour) contribution : No GIM cancellation.

A-topology is the main contribution.

Quark Topologies for $\mathscr{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q)$

Loop Topology
(Only possible in SCS decays)

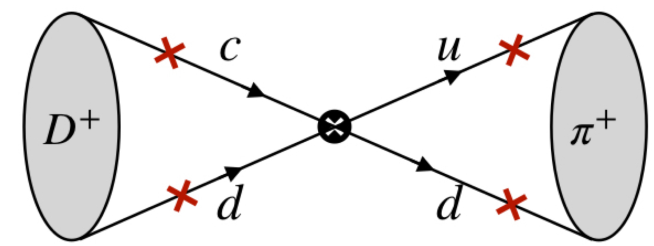




- In the $SU(3)_f$ (along with $\lambda_b \approx 0$), the two loops have complete GIM cancellation

L-topology with have non-zero contribution only due to $m_s \neq m_d$: Expected to be small

Annihilation Topology



- Only one (d or s flavour) contribution : No GIM cancellation.

A-topology is the main contribution.

- At NLO, there will be multiple diagrams with the exchange of virtual gluons : Out of the scope of the present study.

The use of U-spin

- Combining Two approximations: GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ and $SU(3)_{fl}$ limit, $m_s = m_{u,d}$
- The Hamiltonians of CF, SCS, and DSC modes form a U-triplet: **(Only annihilation topology)**

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L \gamma_\mu s_L)(\bar{d}_L \gamma^\mu c_L) \\ \frac{1}{\sqrt{2}} \left[(\bar{u}_L \gamma_\mu d_L)(\bar{d}_L \gamma^\mu c_L) - (\bar{u}_L \gamma_\mu s_L)(\bar{s}_L \gamma^\mu c_L) \right] \\ (\bar{u}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1, +1\rangle \\ -|1,0\rangle \\ |1, -1\rangle \end{pmatrix}$$

The use of U-spin

- Combining Two approximations: GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ and $SU(3)_{fl}$ limit, $m_s = m_{u,d}$
- The Hamiltonians of CF, SCS, and DSC modes form a U-triplet: **(Only annihilation topology)**

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu c_L) \\ \frac{1}{\sqrt{2}}\left[(\bar{u}_L\gamma_\mu d_L)(\bar{d}_L\gamma^\mu c_L) - (\bar{u}_L\gamma_\mu s_L)(\bar{s}_L\gamma^\mu c_L)\right] \\ (\bar{u}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ |1,-1\rangle \end{pmatrix}$$

- As j_μ^{em} is a U-singlet, the matrix element of interest:

$$\langle P^+ | j_\mu^{em}(x) O_1^{(U=1)} | D^+ \rangle \longrightarrow$$

Two ways to make a U-spin singlet

$$\langle P^+_{(U=1/2)} | j_\mu^{em}(x) O_1^{(U=1)} | D^+_{(U=1/2)} \rangle$$

$$\begin{pmatrix} |K^+\rangle = |u\bar{s}\rangle \\ |\pi^+\rangle = |u\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1/2,+1/2\rangle \\ -|1/2,-1/2\rangle \end{pmatrix}$$

$$\begin{pmatrix} |D_s^+\rangle = |c\bar{s}\rangle \\ |D^+\rangle = |c\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1/2,+1/2\rangle \\ -|1/2,-1/2\rangle \end{pmatrix}$$

$$\langle P^+_{(U=1)} | j_\mu^{em}(x) O_1^{(U=1)} | D^0 \rangle$$

$$|D^0\rangle = |c\bar{u}\rangle = |0,0\rangle$$

$$\begin{pmatrix} |K^0\rangle = |d\bar{s}\rangle \\ \frac{\sqrt{3}}{2}|\eta_8\rangle - \frac{1}{2}|\pi^0\rangle = \frac{1}{\sqrt{2}}|d\bar{d} - s\bar{s}\rangle \\ |\bar{K}^0\rangle = |s\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ -|1,-1\rangle \end{pmatrix}$$

The use of U-spin

- Combining Two approximations: GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ and $SU(3)_{fl}$ limit, $m_s = m_{u,d}$
- The Hamiltonians of CF, SCS, and DSC modes form a U-triplet: **(Only annihilation topology)**

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu c_L) \\ \frac{1}{\sqrt{2}}\left[(\bar{u}_L\gamma_\mu d_L)(\bar{d}_L\gamma^\mu c_L) - (\bar{u}_L\gamma_\mu s_L)(\bar{s}_L\gamma^\mu c_L)\right] \\ (\bar{u}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ |1,-1\rangle \end{pmatrix}$$

- As j_μ^{em} is a U-singlet, the matrix element of interest:

$$\langle P^+|j_\mu^{em}(x)O_1^{(U=1)}|D^+\rangle \longrightarrow$$

Two ways to make a U-spin singlet

$$\langle P^+_{(U=1/2)}|j_\mu^{em}(x)O_1^{(U=1)}|D^+_{(U=1/2)}\rangle$$

$$\begin{pmatrix} |K^+\rangle = |u\bar{s}\rangle \\ |\pi^+\rangle = |u\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1/2,+1/2\rangle \\ -|1/2,-1/2\rangle \end{pmatrix} \qquad \begin{pmatrix} |D_s^+\rangle = |c\bar{s}\rangle \\ |D^+\rangle = |c\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1/2,+1/2\rangle \\ -|1/2,-1/2\rangle \end{pmatrix}$$

$$\langle P^+_{(U=1)}|j_\mu^{em}(x)O_1^{(U=1)}|D^0\rangle$$

$$|D^0\rangle = |c\bar{u}\rangle = |0,0\rangle$$

$$\begin{pmatrix} |K^0\rangle = |d\bar{s}\rangle \\ \frac{\sqrt{3}}{2}|\eta_8\rangle - \frac{1}{2}|\pi^0\rangle = \frac{1}{\sqrt{2}}|d\bar{d} - s\bar{s}\rangle \\ |\bar{K}^0\rangle = |s\bar{d}\rangle \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ -|1,-1\rangle \end{pmatrix}$$

U-spin relations

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = -\mathcal{A}^{(D_s^+\to K^+\gamma^*)}(q^2) = \mathcal{A}^{(D_s^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to K^+\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\bar{K}^0\gamma^*)}(q^2) = \mathcal{A}^{(D^0\to K^0\gamma^*)}(q^2) = -\frac{1}{2}\mathcal{A}^{(D^0\to\pi^0\gamma^*)}(q^2) + \frac{\sqrt{3}}{2}\mathcal{A}^{(D^0\to\eta^0\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\eta_8\gamma^*)}(q^2) = -\sqrt{3}\mathcal{A}^{(D^0\to\pi^0\gamma^*)}(q^2)$$

$$\mathcal{A}^{(D^0\to\eta'\gamma^*)}(q^2) = 0$$

D^0, η': U-spin singlets.

- Measuring the CF modes, e.g. $D_s \to \pi^+\ell^+\ell^-$ will allow to disentangle this topology.

What do we know from Experiments?

- Upper bounds from PDG:

Decay mode	Cabibbo hierarchy	BR, exp. upper limit
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1\times10^{-6}(\ell=e)$ $6.7\times10^{-8}(\ell=\mu)$
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5\times10^{-7}(\ell=e)$ $5.4\times10^{-8}(\ell=\mu)$
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4\times10^{-5}(\ell=e)$ $2.6\times10^{-4}(\ell=\mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4\times10^{-6}(\ell=e)$ $1.8\times10^{-4}(\ell=\mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3\times10^{-6}(\ell=e)$ $5.3\times10^{-4}(\ell=\mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5\times10^{-6}(\ell=e)$ $1.8\times10^{-7}(\ell=\mu)$
$D_s^+ \to K^+\ell^+\ell^-$	SCS	$3.7\times10^{-6}(\ell=e)$ $1.4\times10^{-7}(\ell=\mu)$

[PDG]

What do we know from Experiments?

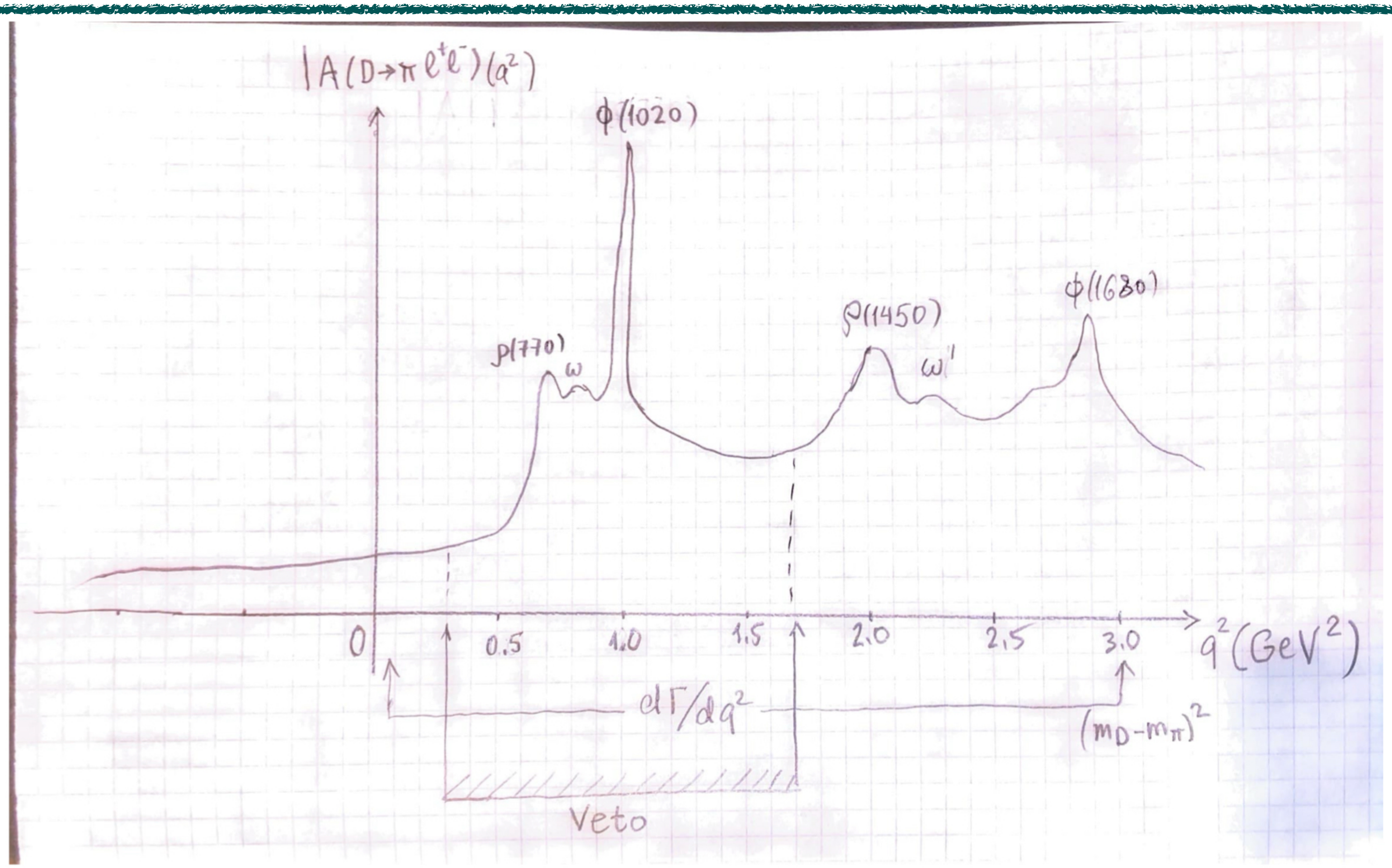
- Upper bounds from PDG:

Decay mode	Cabibbo hierarchy	BR, exp. upper limit
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1 \times 10^{-6}(\ell = e)$ $6.7 \times 10^{-8}(\ell = \mu)$
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5 \times 10^{-7}(\ell = e)$ $5.4 \times 10^{-8}(\ell = \mu)$
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4 \times 10^{-5}(\ell = e)$ $2.6 \times 10^{-4}(\ell = \mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3 \times 10^{-6}(\ell = e)$ $5.3 \times 10^{-4}(\ell = \mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5 \times 10^{-6}(\ell = e)$ $1.8 \times 10^{-7}(\ell = \mu)$
$D_s^+ \to K^+\ell^+\ell^-$	SCS	$3.7 \times 10^{-6}(\ell = e)$ $1.4 \times 10^{-7}(\ell = \mu)$

[PDG]

- Most recent upper bound on $(D^+ \to \pi^+\mu^+\mu^-)$: vetoing the resonance region. [LHCb, (JHEP06 (2021) 044)]

Can we really isolate resonances?

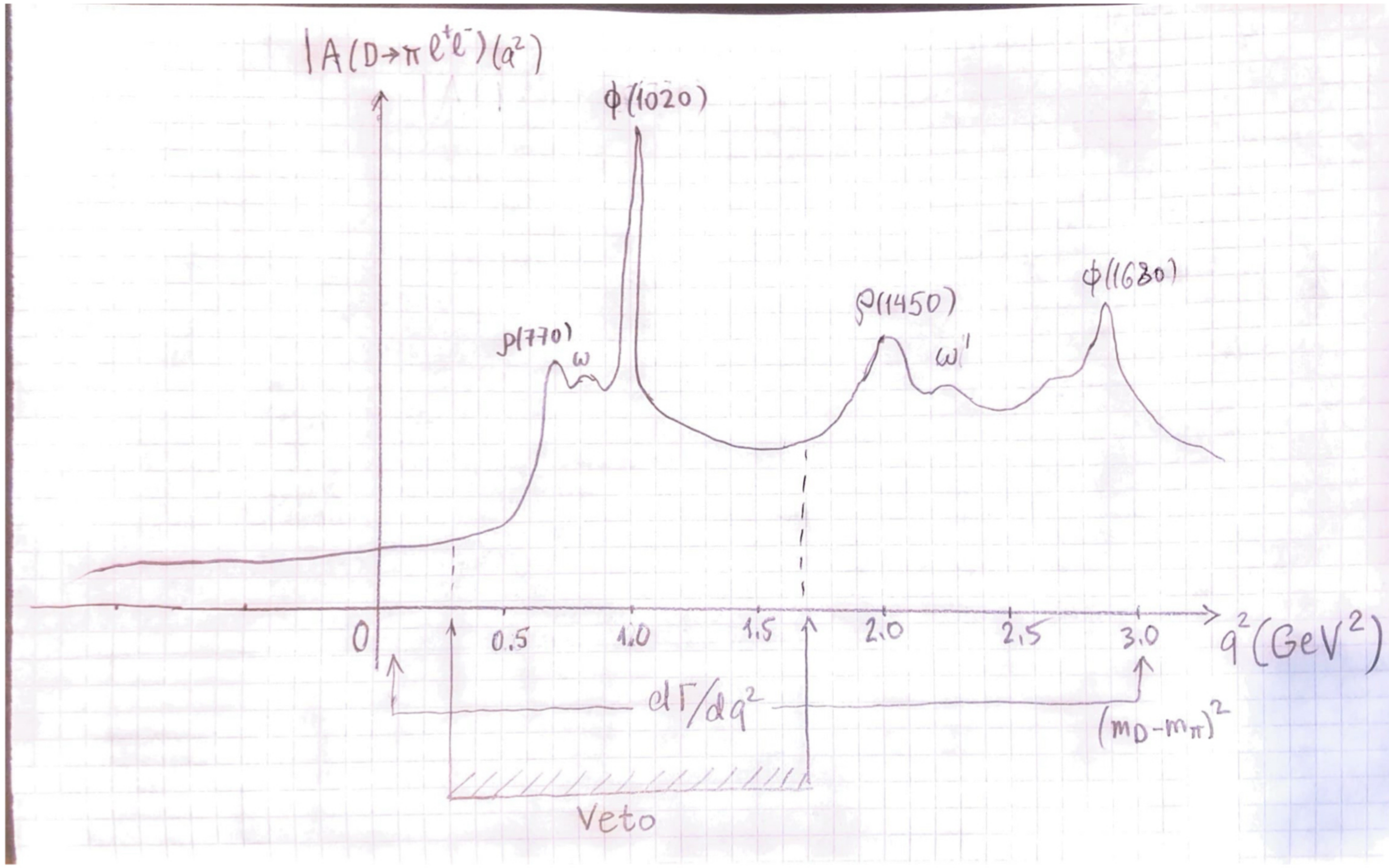


- The full amplitude represented via hadronic dispersion relation :

Decay constant — Amplitude for $D \to \pi V$ — Continuum and higher resonances

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

Can we really isolate resonances?



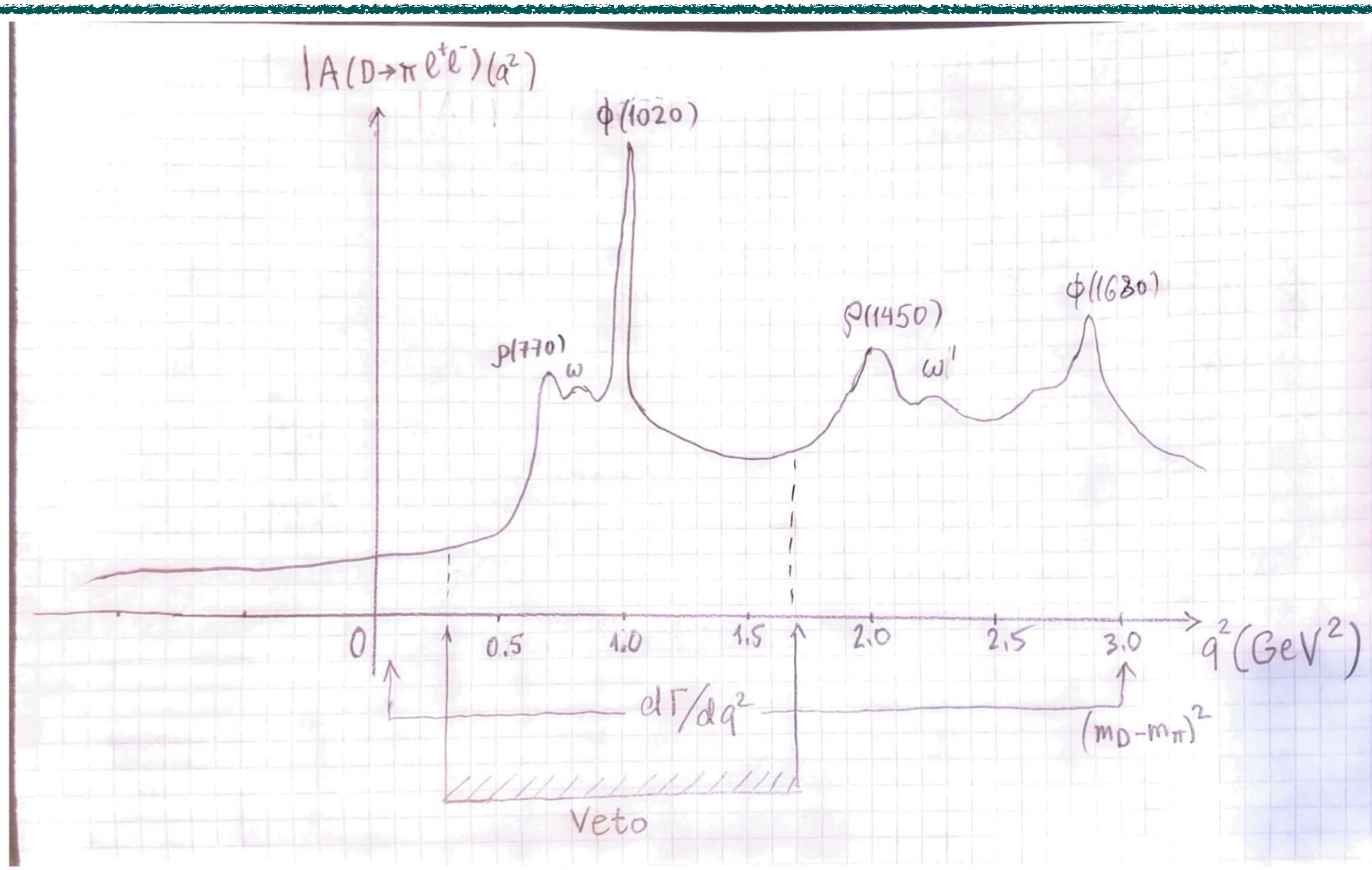
- The full amplitude represented via hadronic dispersion relation :

Decay constant — Amplitude for $D \to \pi V$ — Continuum and higher resonances

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

- Dispersion relation tells us: vetoing a certain q^2- region does not remove resonances from the amplitude.
- The radial excitations of ρ, ω, ϕ and the "tail" at $s > (m_D - m_\pi)^2$ are indispensable.

Can we really isolate resonances?



- The full amplitude represented via hadronic dispersion relation :

Decay constant — Amplitude for $D \to \pi V$ — Continuum and higher resonances

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

- Dispersion relation tells us: vetoing a certain q^2- region does not remove resonances from the amplitude.
- The radial excitations of ρ, ω, ϕ and the "tail" at $s > (m_D - m_\pi)^2$ are indispensable.

As, the experimental bounds are approaching theory predictions, it is important to revisit it within the Standard Model.

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.

,

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+\to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

,

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

Spectral density : too complicated to be parametrized

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+ \to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

,

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

Spectral density : too complicated to be parametrized

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+ \to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+ V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

can be parametrized using **z-parametrization** (valid below s_0^h)

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} = \sum_{k=0}^{K} a_k [z(q^2)]^k$$

with,

$$z(q^2) = \frac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$$

$a_k =$ Complex coefficients

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

Spectral density : too complicated to be parametrized

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+ \to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

can be parametrized using **z-parametrization** (valid below s_0^h)

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} = \sum_{k=0}^{K} a_k [z(q^2)]^k$$

with,

$$z(q^2) = \frac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$$

$a_k =$ Complex coefficients

- For $K = 2$, **9 unknown parameters**: $\phi_\rho, \phi_\omega, \phi_\phi, a_0, a_1, a_2$.

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

Spectral density : too complicated to be parametrized

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+ \to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

can be parametrized using **z-parametrization** (valid below s_0^h)

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} = \sum_{k=0}^{K} a_k [z(q^2)]^k$$

with,

$$z(q^2) = \frac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$$

$a_k =$ Complex coefficients

- For $K = 2$, **9 unknown parameters**: $\phi_\rho, \phi_\omega, \phi_\phi, a_0, a_1, a_2$.

Main idea :

Step-1: Compute $\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using Light Cone Sum Rules (valid only for $q^2 < 0$)
Step-2: Write the hadronic dispersion relation in terms of unknown phases and z-parameters (valid for all values of q^2).
Step-3: Match the LCSR results with the dispersion relation at $q^2 < 0$ and estimate the unknown parameters.
Step-4: Estimate $\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ in the physical region using dispersion relation.

Our methodology: LCSR-supported dispersion relation

- LCSR can provide estimates only in the spacelike region.
- The dispersion relation is valid for all values of q^2

Spectral density : too complicated to be parametrized

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + (q^2 - q_0^2)\Big[\sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{DV\pi}| e^{i\varphi_V}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{tot})} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)}\Big]$$

$$|A_{DV\pi}| = \left(\frac{8\pi BR(D^+\to V\pi^+)}{\tau(B) G_F^2 |V_{cs}|^2 |V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Follow from the valence quark content of V

can be parametrized using **z-parametrization** (valid below s_0^h)

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)} = \sum_{k=0}^{K} a_k [z(q^2)]^k$$

with,

$$z(q^2) = \frac{\sqrt{s_0^h - q^2} - \sqrt{s_0^h}}{\sqrt{s_0^h - q^2} + \sqrt{s_0^h}}$$

$a_k =$ Complex coefficients

- For $K = 2$, **9 unknown parameters**: $\phi_\rho, \phi_\omega, \phi_\phi, a_0, a_1, a_2$.

Main idea :

Step-1: Compute $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using Light Cone Sum Rules (valid only for $q^2 < 0$)
Step-2: Write the hadronic dispersion relation in terms of unknown phases and z-parameters (valid for all values of q^2).
Step-3: Match the LCSR results with the dispersion relation at $q^2 < 0$ and estimate the unknown parameters.
Step-4: Estimate $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ in the physical region using dispersion relation.

(Resembling partly the analysis of nonlocal effects in $B \to K^*\ell^+\ell^-$)

[A. Khodjamirian, T. Mannel, A. Pivovarov, Y. Wang, 1211.0234]

[A. Khodjamirian, A. V. Rusov, 1703.04765] , N. Gubernari, M. Rebound, D. van Dyk, J. Virto, 2011.09813

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a product of perturbatively calculated Hard scattering kernel and pion DAs)

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a product of perturbatively calculated Hard scattering kernel and pion DAs)

Dispersion Relation in D-meson channel
(Enables to relate the calculated correlation function to the sum over $D \to \pi\gamma^*$ hadronic matrix elements.)

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a product of perturbatively calculated Hard scattering kernel and pion DAs)

Dispersion Relation in D-meson channel
(Enables to relate the calculated correlation function to the sum over $D \to \pi\gamma^*$ hadronic matrix elements.)

Quark Hadron Duality
(Relates ground state hadronic matrix element in D-meson channel to the integral over perturbatively calculated correlation function)

A brief overview of LCSR method

TOOLS TO DERIVE LCSR

Light cone OPE
(Computing correlation function as a product of perturbatively calculated Hard scattering kernel and pion DAs)

Dispersion Relation in D-meson channel
(Enables to relate the calculated correlation function to the sum over $D \to \pi\gamma^*$ hadronic matrix elements.)

Quark Hadron Duality
(Relates ground state hadronic matrix element in D-meson channel to the integral over perturbatively calculated correlation function)

Borel Transformation
(To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

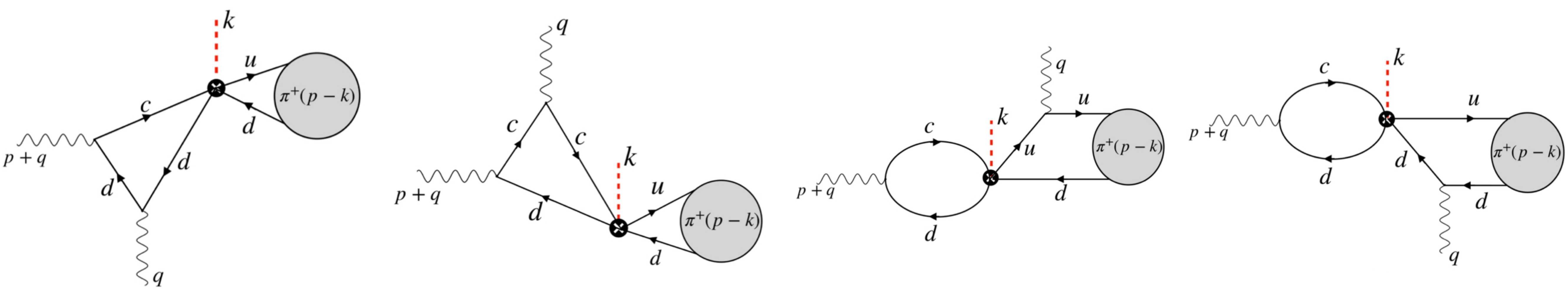
Weak Annihilation from LCSR

- The correlation function:

$$F_\mu(p,q,k) = -\,C_1 \int d^4x\, e^{iq\cdot x} \int d^4y\, e^{-i(p+q)\cdot y} \langle \pi^+(p-k)\,|\,T\{J_\mu^{em}(x)(\bar{u}_L\gamma_\nu d_L)(\bar{d}_L\gamma^\nu c_L)(0)J_5^D(y)\}\,|\,0\rangle$$

$$J_\mu^{em}(x) \to \sum_{q=u,d,c} Q_q \bar{q}(x)\gamma_\mu q(x) \qquad J_5^D(y) \to i m_c \bar{c}(y)\gamma_5 d(y)$$

Only O_1^d contributes. The O_2 contribution vanishes after Fierz transformation.



(a) (b) (c) (d)

Diagrams in terms of pion DAs

Weak Annihilation from LCSR

- The correlation function:

$$F_\mu(p,q,k) = -C_1 \int d^4x\, e^{iq\cdot x}\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\{J_\mu^{em}(x)(\bar{u}_L\gamma_\nu d_L)(\bar{d}_L\gamma^\nu c_L)(0)J_5^D(y)\}\,|\,0\rangle$$

$$J_\mu^{em}(x) \to \sum_{q=u,d,c} Q_q \bar{q}(x)\gamma_\mu q(x) \qquad J_5^D(y) \to i m_c \bar{c}(y)\gamma_5 d(y)$$

Only O_1^d contributes. The O_2 contribution vanishes after Fierz transformation.

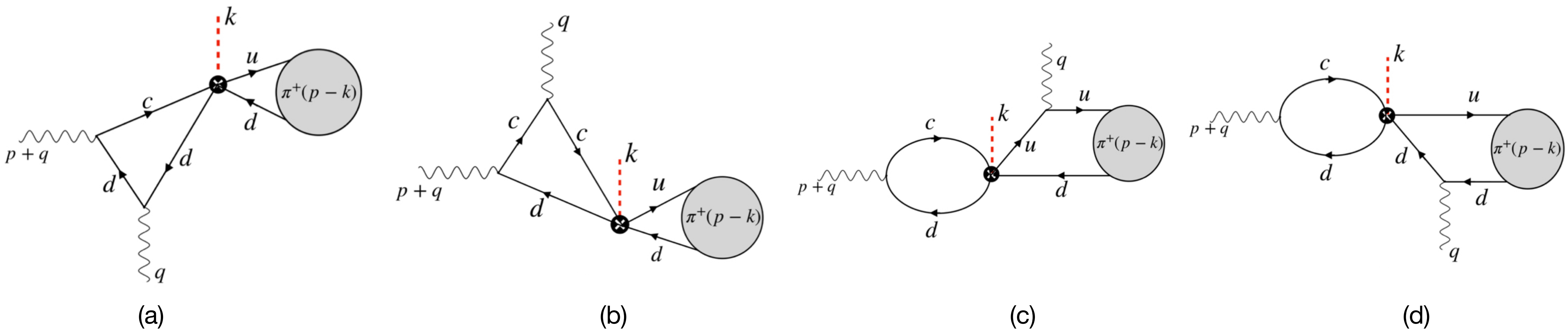
- The artificial momentum k is introduced at the four vertex to avoid parasitic contributions in the dispersion relation.

(Used before in LCSR analysis of $B \to 2\pi$ and $D \to 2\pi, K\bar{K}$)

[A. Khodjamirian, arXiv: hep-ph/0012271]

[A. Khodjamirian, M. Melcher, B. Melic, arXiv: hep-ph/0304179, hep-ph/0509049]

[A. Khodjamirian, A. A. Petrov, arXiv: 1706.07780]



Diagrams in terms of pion DAs

Weak Annihilation from LCSR

- The correlation function:

$$F_\mu(p,q,k) = -\,C_1\int d^4x\, e^{iq\cdot x}\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\{J^{em}_\mu(x)(\bar u_L\gamma_\nu d_L)(\bar d_L\gamma^\nu c_L)(0)J^D_5(y)\}\,|0\rangle$$

$$J^{em}_\mu(x) \to \sum_{q=u,d,c} Q_q\bar q(x)\gamma_\mu q(x) \qquad\qquad J^D_5(y) \to i m_c\bar c(y)\gamma_5 d(y)$$

Only O_1^d contributes. The O_2 contribution vanishes after Fierz transformation.

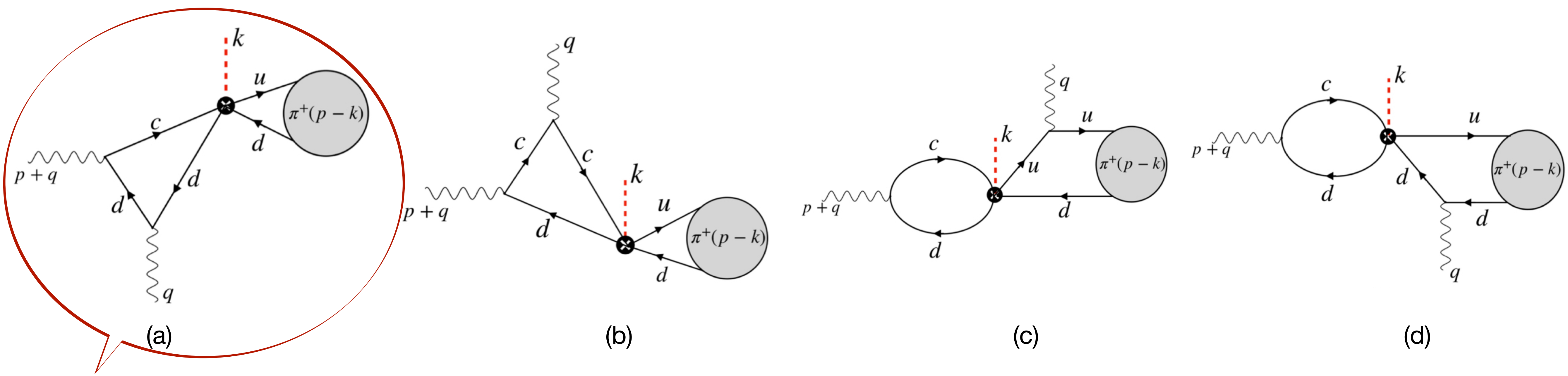
- The artificial momentum k is introduced at the four vertex to avoid parasitic contributions in the dispersion relation.

(Used before in LCSR analysis of $B\to 2\pi$ and $D\to 2\pi, K\bar K$)

[A. Khodjamirian, arXiv: hep-ph/0012271]

[A. Khodjamirian, M. Melcher, B. Melic, arXiv: hep-ph/0304179, hep-ph/0509049]

[A. Khodjamirian, A. A. Petrov, arXiv: 1706.07780]



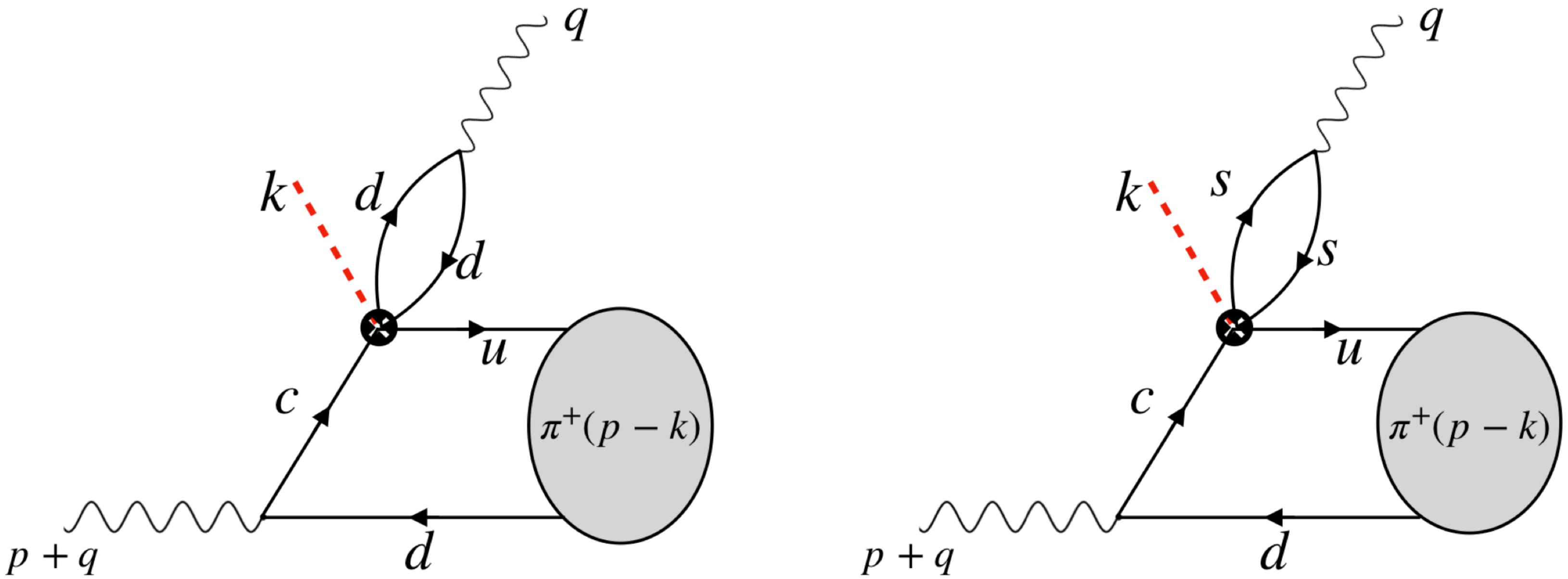
The only contribution considered in QCDf computations

Diagrams in terms of pion DAs

Loop diagram from LCSR

- The correlation function reads as:

$$\mathscr{F}_{\mu}^{(L)}(p,q,k) = -\left[(p\cdot q)q_\mu - q^2 p_\mu\right]\frac{1}{9}\left(C_1 + \frac{4}{3}C_2\right)\Pi^{(d-s)}(q^2)G((p+q)^2, q^2, P^2)$$

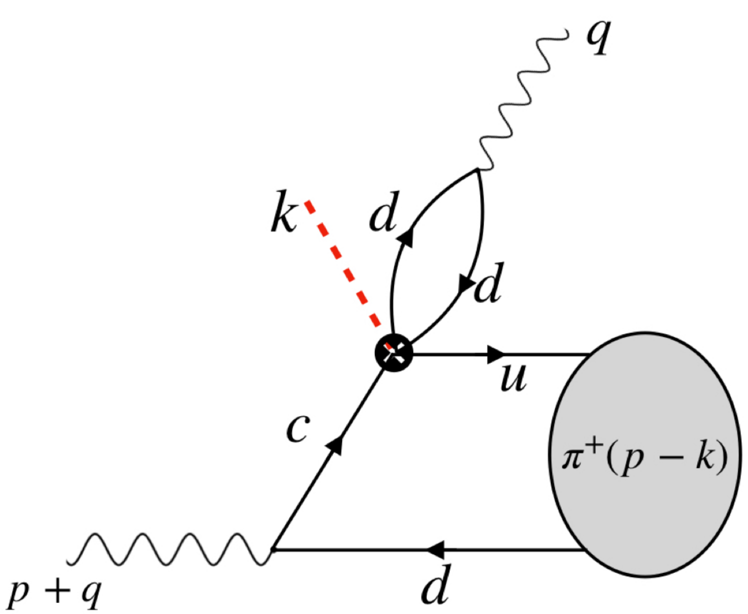


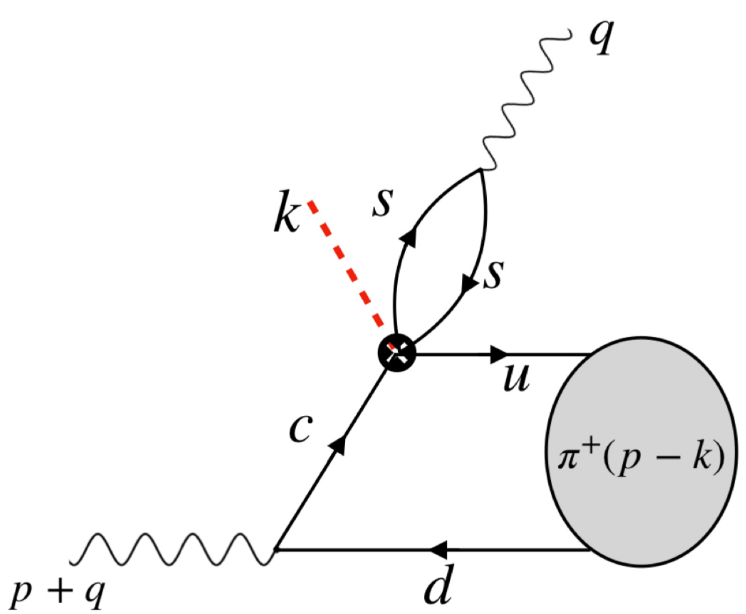
Loop diagram from LCSR

- The correlation function reads as:

$$\mathscr{F}_{\mu}^{(L)}(p,q,k) = -\left[(p\cdot q)q_{\mu} - q^2 p_{\mu}\right]\frac{1}{9}\left(C_1 + \frac{4}{3}C_2\right)\Pi^{(d-s)}(q^2)G((p+q)^2, q^2, P^2)$$

$$\Pi^{d}(q^2) - \Pi^{s}(q^2) \equiv \Pi^{(d-s)}(q^2) = \frac{3}{4\pi^2}\int_0^1 dx\, x\,(1-x)\log\left(\frac{m_s^2 - q^2 x(1-x)}{m_d^2 - q^2 x(1-x)}\right)$$





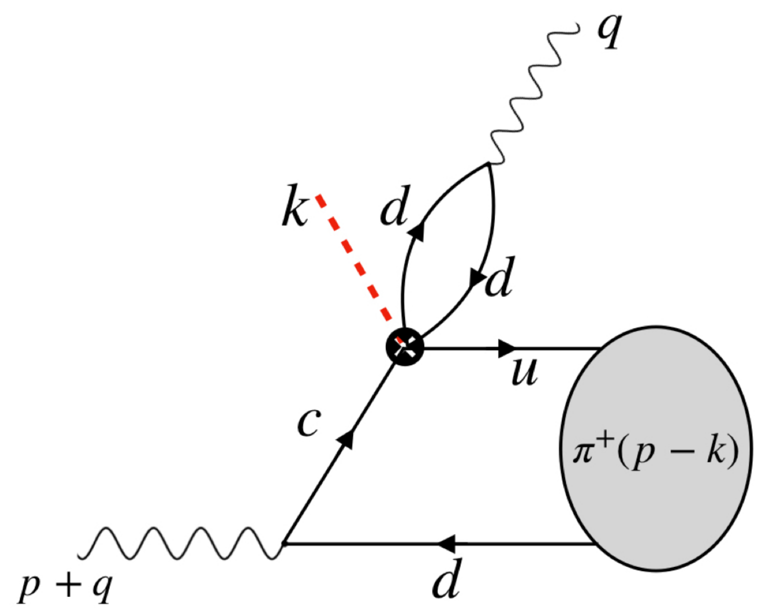
Loop diagram from LCSR

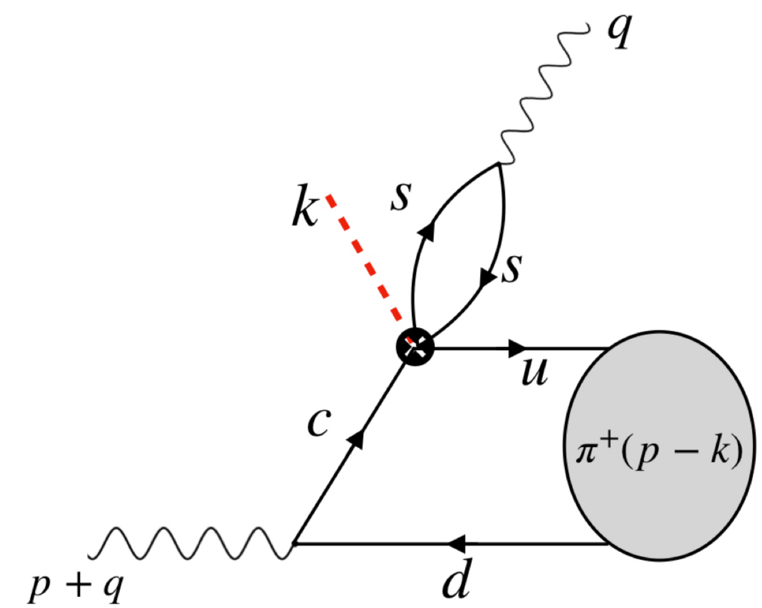
- The correlation function reads as:

$$\mathscr{F}_\mu^{(L)}(p,q,k) = -\left[(p\cdot q)q_\mu - q^2 p_\mu\right]\frac{1}{9}\left(C_1 + \frac{4}{3}C_2\right)\Pi^{(d-s)}(q^2)G((p+q)^2, q^2, P^2)$$

$$\Pi^d(q^2) - \Pi^s(q^2) \equiv \Pi^{(d-s)}(q^2) = \frac{3}{4\pi^2}\int_0^1 dx\, x\,(1-x)\log\left(\frac{m_s^2 - q^2x(1-x)}{m_d^2 - q^2x(1-x)}\right)$$

$$G_\rho(p,q,k) = i\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\Big\{\left(\bar{u}_L(0)\gamma_\rho c_L(0)\right)j_5^D(y)\Big\}\,|0\rangle$$





Loop diagram from LCSR

- The correlation function reads as:

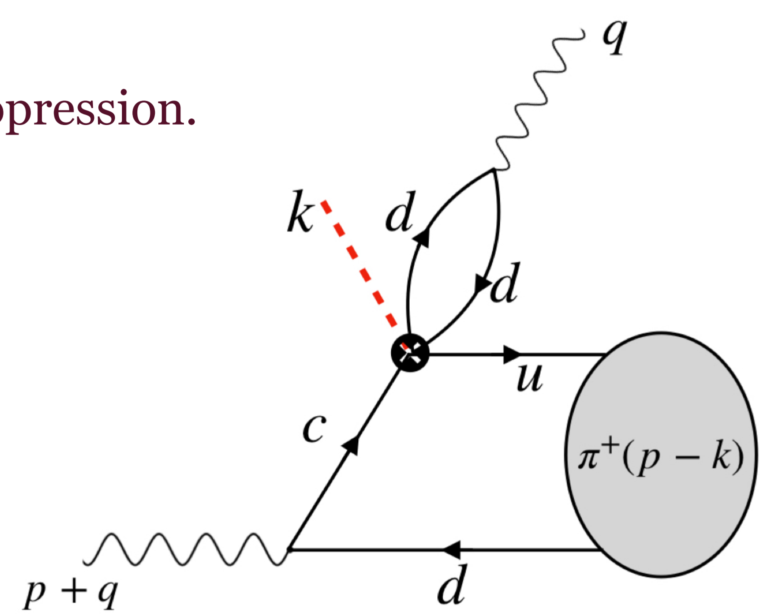
$$\mathcal{F}_{\mu}^{(L)}(p,q,k) = -\left[(p\cdot q)q_{\mu} - q^2 p_{\mu}\right]\frac{1}{9}\left(C_1 + \frac{4}{3}C_2\right)\Pi^{(d-s)}(q^2)G((p+q)^2, q^2, P^2)$$

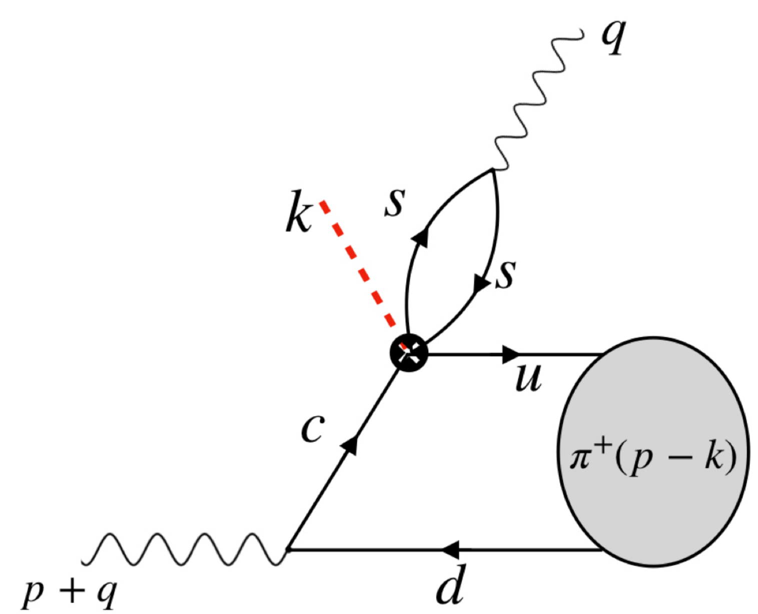
$$\Pi^{d}(q^2) - \Pi^{s}(q^2) \equiv \Pi^{(d-s)}(q^2) = \frac{3}{4\pi^2}\int_0^1 dx\, x\,(1-x)\log\left(\frac{m_s^2 - q^2x(1-x)}{m_d^2 - q^2x(1-x)}\right)$$

$$G_{\rho}(p,q,k) = i\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\Big\{\left(\bar{u}_L(0)\gamma_{\rho}c_L(0)\right)j_5^D(y)\Big\}\,|0\rangle$$

- Both WCs (C_1 and C_2) contribute in this case.

- The contribution is small due to GIM suppression.





LCSR Results

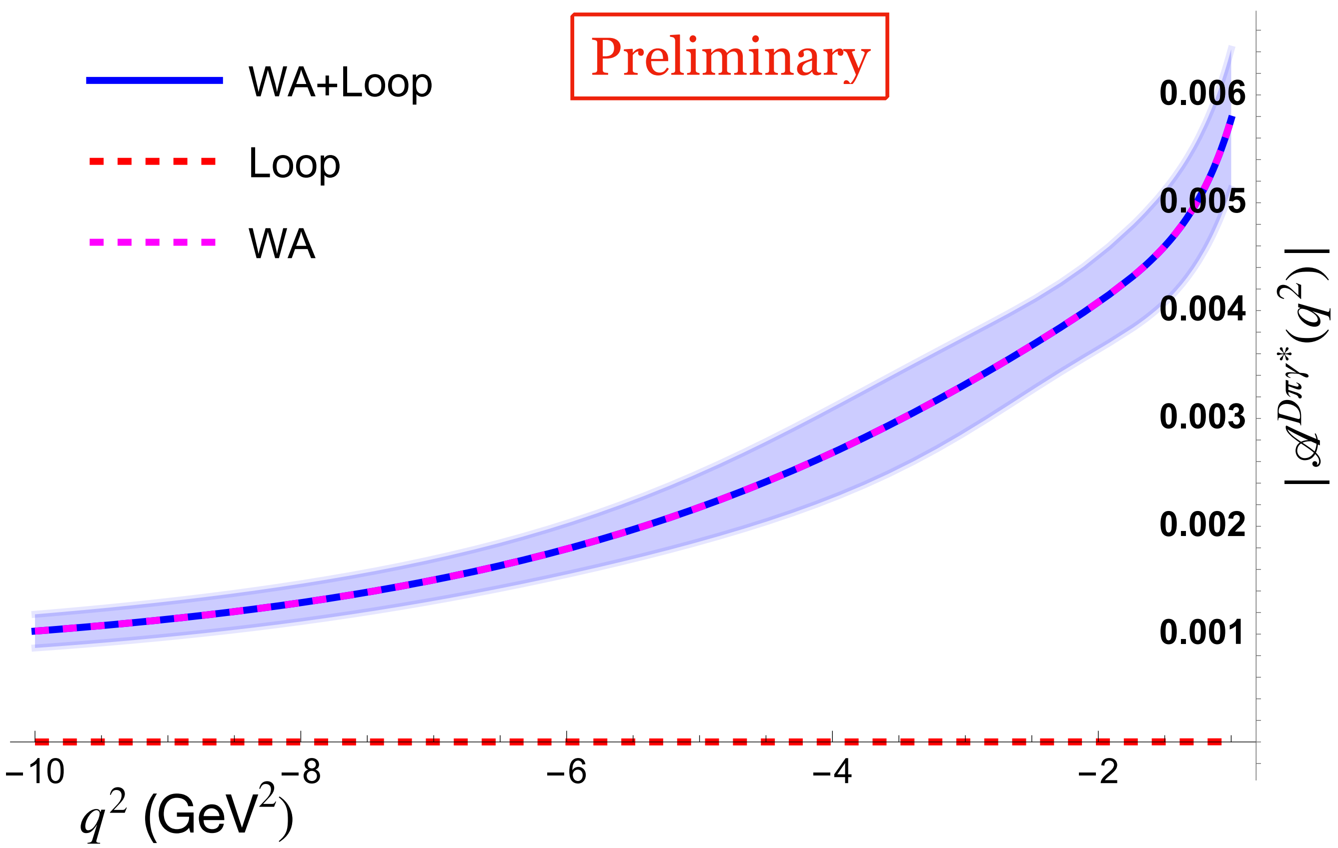
- The final sum rule read as (for $q^2 < 0$):

$$m_D^2 f_D \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) e^{-m_D^2/M^2} = \frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds e^{-s/M^2} \mathrm{Im}(F^{(OPE)}(s, q^2, m_D^2))$$

- M^2 (Borel parameter) and s_0^D (effective threshold) are the sum rule parameters taken to be:

$$M^2 = (4.5 \pm 1.0)\ \mathrm{GeV}^2$$

$$s_0^D = (4.95 \pm 0.35)\ \mathrm{GeV}^2$$



LCSR Results

- The final sum rule read as (for $q^2 < 0$):

$$m_D^2 f_D \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) e^{-m_D^2/M^2} = \frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds e^{-s/M^2} \text{Im}(F^{(OPE)}(s, q^2, m_D^2))$$

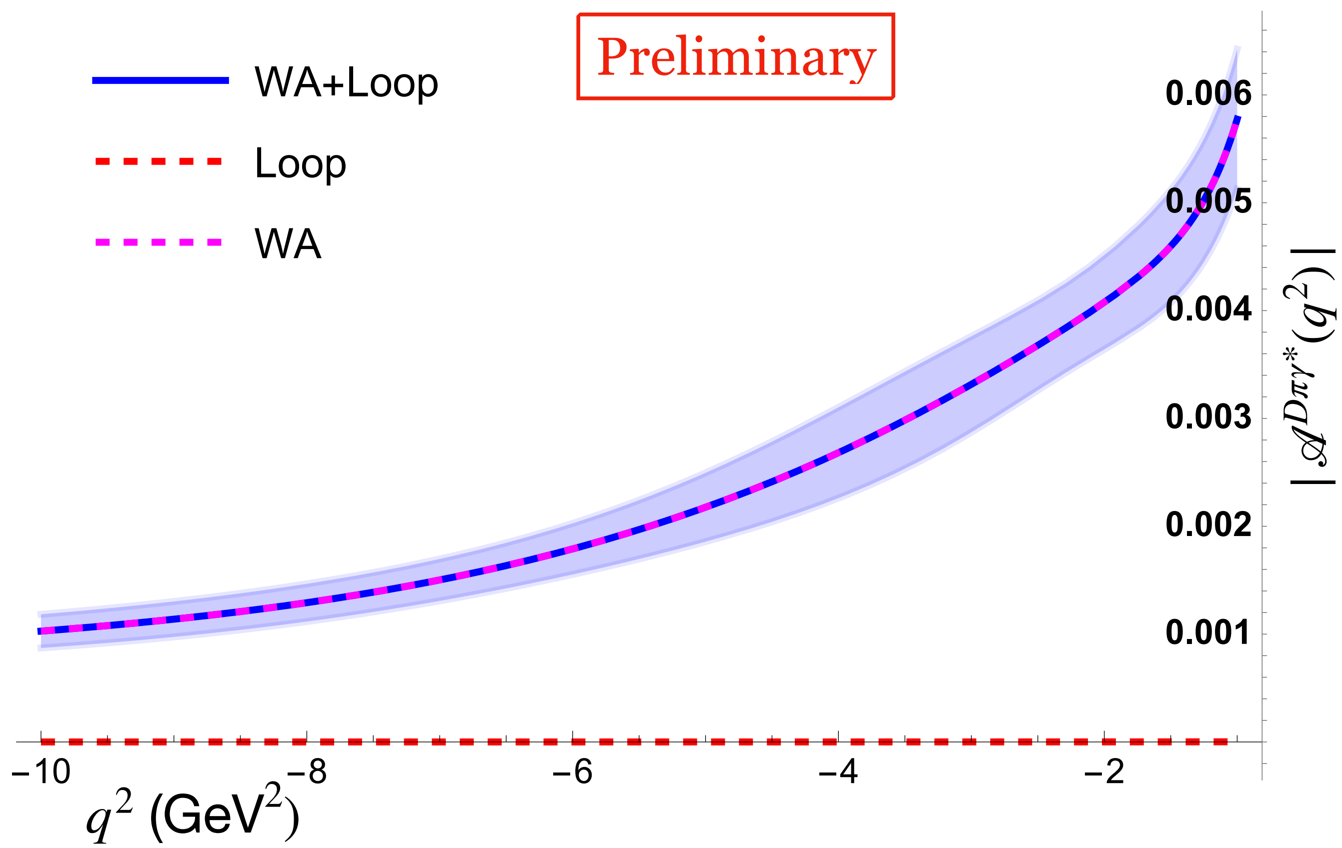
- M^2 (Borel parameter) and s_0^D (effective threshold) are the sum rule parameters taken to be:

$$M^2 = (4.5 \pm 1.0)\ \text{GeV}^2$$

$$s_0^D = (4.95 \pm 0.35)\ \text{GeV}^2$$

- F^{OPE} include contribution from twist-2 distribution amplitude (DA) of pion (using 2 Gegenbauer moments).

- The major source of calculated LCSR uncertainties are the uncertainties in s_0^D and the DA parameters.



LCSR Results

- The final sum rule read as (for $q^2 < 0$):

$$m_D^2 f_D \mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) e^{-m_D^2/M^2} = \frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds e^{-s/M^2} \mathrm{Im}(F^{(OPE)}(s, q^2, m_D^2))$$

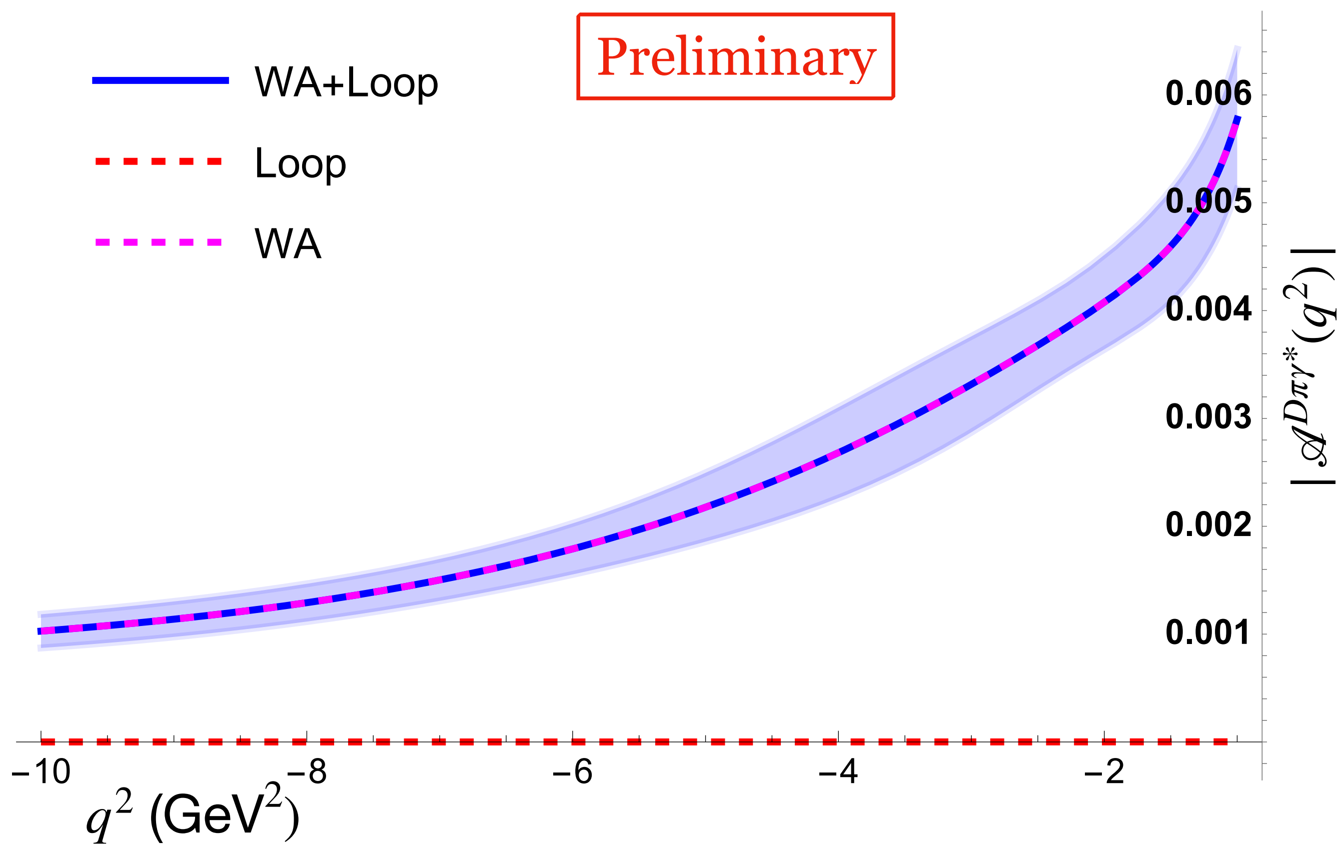
- M^2 (Borel parameter) and s_0^D (effective threshold) are the sum rule parameters taken to be:

$$M^2 = (4.5 \pm 1.0)\ \mathrm{GeV}^2$$

$$s_0^D = (4.95 \pm 0.35)\ \mathrm{GeV}^2$$

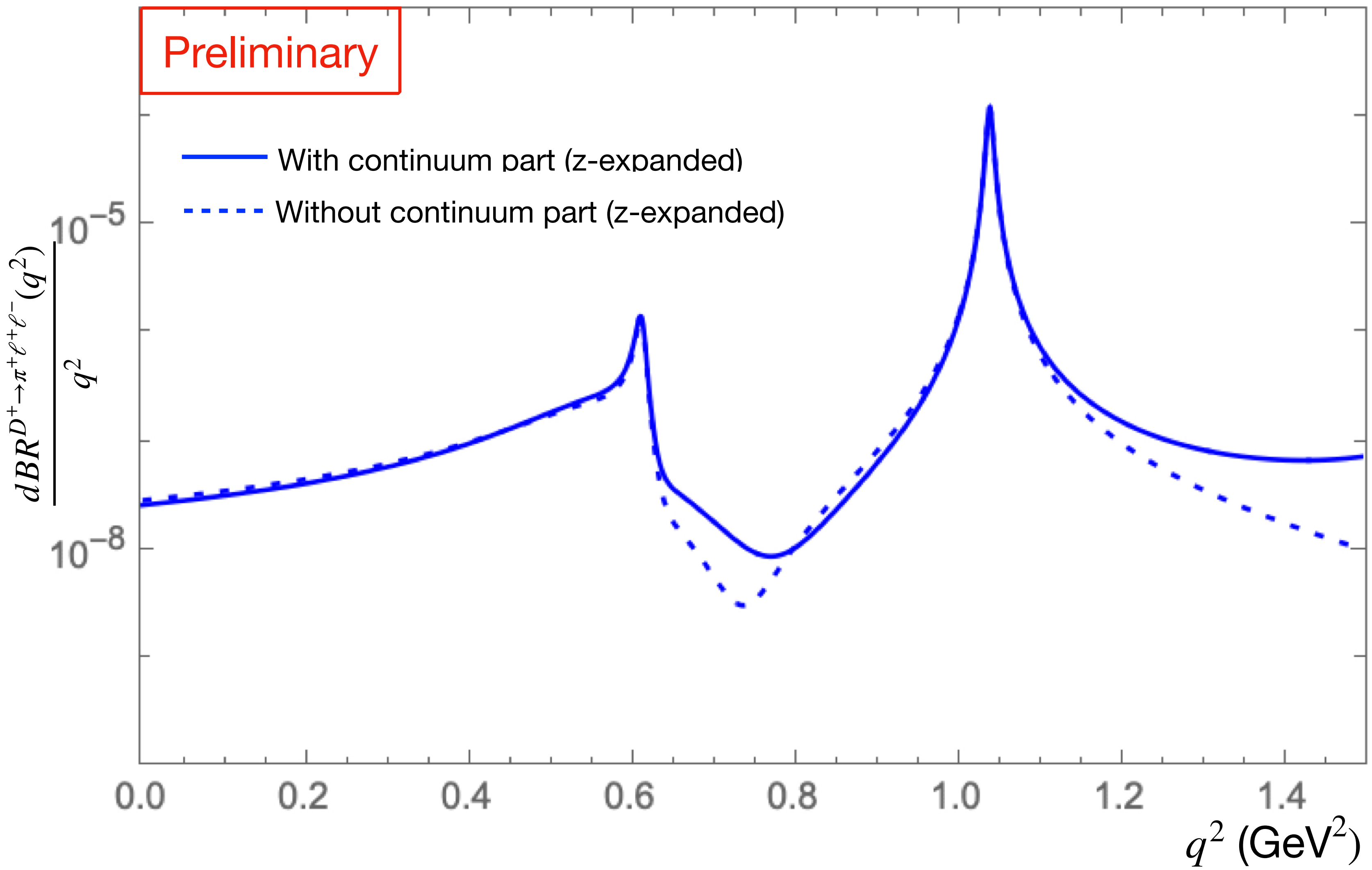
- F^{OPE} include contribution from twist-2 distribution amplitude (DA) of pion (using 2 Gegenbauer moments).

- The major source of calculated LCSR uncertainties are the uncertainties in s_0^D and the DA parameters.



- The contribution to the decay amplitude from O_9 varies from $\sim 1.5 \times 10^{-6}$ to $\sim 7.5 \times 10^{-6}$ at $0 < q^2 < (m_D - m_\pi)^2$: at least three order of magnitudes smaller than the WA+loop amplitude

Final Results

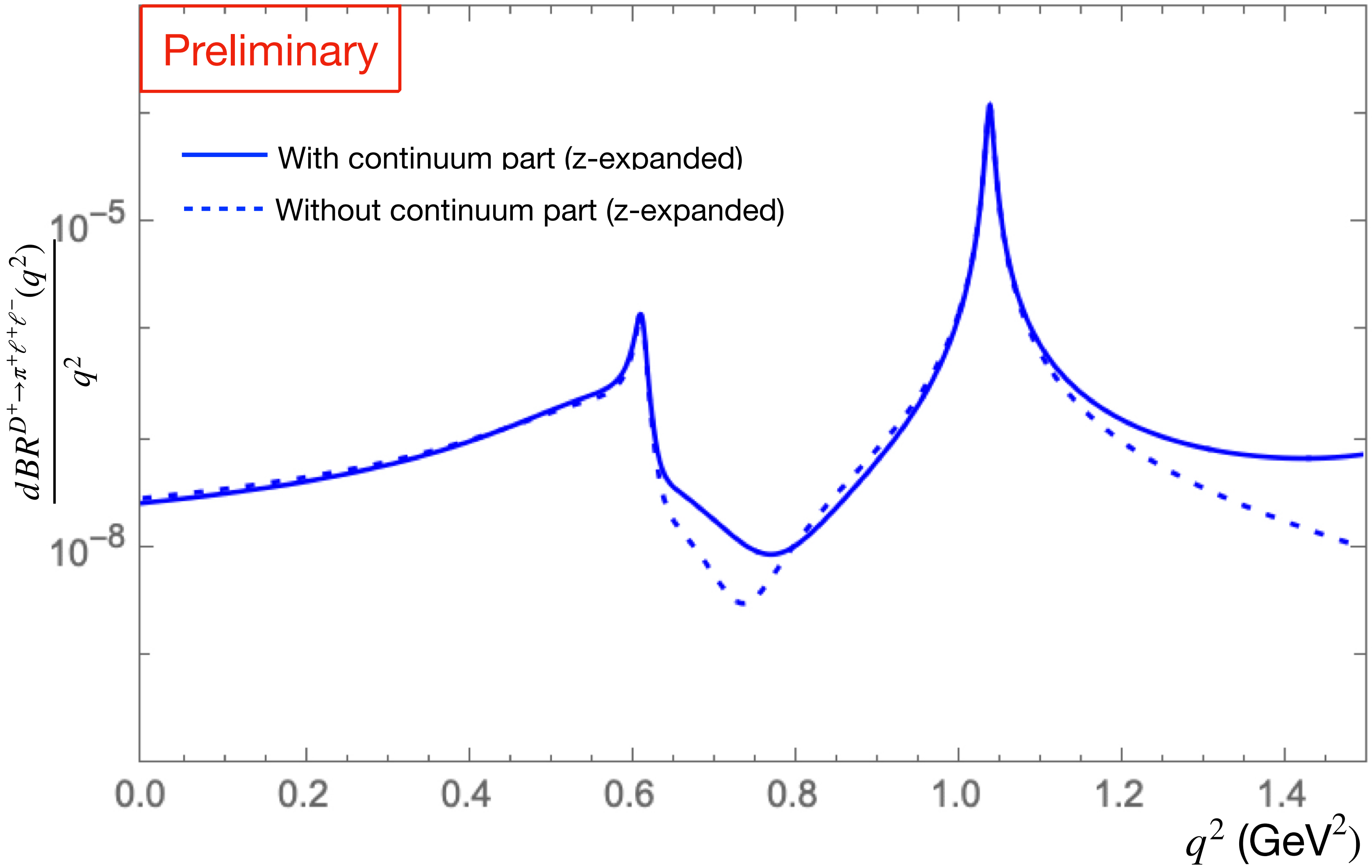


Uncertainties are yet to be computed

Figure: The results for the differential branching fraction using the dispersion relation with the fitted parameters

- The low q^2 region is generated by the "tail" of the resonances, the intermediate and high q^2 region is influenced by excited states.

Final Results



Uncertainties are yet to be computed

Figure: The results for the differential branching fraction using the dispersion relation with the fitted parameters

- The low q^2 region is generated by the "tail" of the resonances, the intermediate and high q^2 region is influenced by excited states.
- The low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$), integrated branching fraction $\sim 5.5 \times 10^{-9}$ ($\sim$ 2 times the QCDf predictions).

[A. Bharucha, D. Boito, C. Méaux, JHEP 04 (2021) 158]

Summary and Outlook

- In this work, we study the long distance effects in $D^+ \to \pi^+ \ell^+ \ell^-$ decays using LCSR supported dispersion relation.
- We found that the amplitude is mainly dominated by the weak annihilation topologies (loop and short distance contributions are tiny).
- We present the preliminary results for the differential decay width for $D^+ \to \pi^+ \ell^+ \ell^-$ decays.
- In low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$) (in GeV^2), integrated branching fraction is $\sim 5.5 \times 10^{-9}$ (preliminary).

Summary and Outlook

- In this work, we study the long distance effects in $D^+ \to \pi^+\ell^+\ell^-$ decays using LCSR supported dispersion relation.
- We found that the amplitude is mainly dominated by the weak annihilation topologies (loop and short distance contributions are tiny).
- We present the preliminary results for the differential decay width for $D^+ \to \pi^+\ell^+\ell^-$ decays.
- In low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$) (in GeV^2), integrated branching fraction is $\sim 5.5 \times 10^{-9}$ (preliminary).

- Work yet to be done:
  - Compute uncertainties in the branching fraction estimates.
  - Prediction for $D_s \to \pi \ell^+\ell^-$ (CF) modes as a byproduct by setting $m_s \neq 0$ in LCSR.

Summary and Outlook

- In this work, we study the long distance effects in $D^+ \to \pi^+\ell^+\ell^-$ decays using LCSR supported dispersion relation.
- We found that the amplitude is mainly dominated by the weak annihilation topologies (loop and short distance contributions are tiny).
- We present the preliminary results for the differential decay width for $D^+ \to \pi^+\ell^+\ell^-$ decays.
- In low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$) (in GeV^2), integrated branching fraction is $\sim 5.5 \times 10^{-9}$ (preliminary).

- Work yet to be done:
    - Compute uncertainties in the branching fraction estimates.
    - Prediction for $D_s \to \pi \ell^+\ell^-$ (CF) modes as a byproduct by setting $m_s \neq 0$ in LCSR.

- Future perspectives:
    - Perturbative and soft-gluon corrections to annihilation.
    - Estimates for other CF and SCS modes.
    - Varying resonance ansatz in the dispersion relation (including ρ', ω', ϕ').

Summary and Outlook

- In this work, we study the long distance effects in $D^+ \to \pi^+\ell^+\ell^-$ decays using LCSR supported dispersion relation.
- We found that the amplitude is mainly dominated by the weak annihilation topologies (loop and short distance contributions are tiny).
- We present the preliminary results for the differential decay width for $D^+ \to \pi^+\ell^+\ell^-$ decays.
- In low q^2 region ($(0.250)^2 \leq q^2 \leq (0.525)^2$) (in GeV^2), integrated branching fraction is $\sim 5.5 \times 10^{-9}$ (preliminary).

- Work yet to be done:
  - Compute uncertainties in the branching fraction estimates.
  - Prediction for $D_s \to \pi \ell^+\ell^-$ (CF) modes as a byproduct by setting $m_s \neq 0$ in LCSR.

- Future perspectives:
  - Perturbative and soft-gluon corrections to annihilation.
  - Estimates for other CF and SCS modes.
  - Varying resonance ansatz in the dispersion relation (including ρ', ω', ϕ').

- Important message for experimental analysis:

There is no way to isolate long distance effects in $D_{(s)} \to P\ell^+\ell^-$ decays by simply vetoing resonances, one need measurements of the differential decay rates in the whole q^2 region.

Thank you for your attention !!

Thank you for your attention !!

Back up!

What do we already know from theory: QCD factorization?

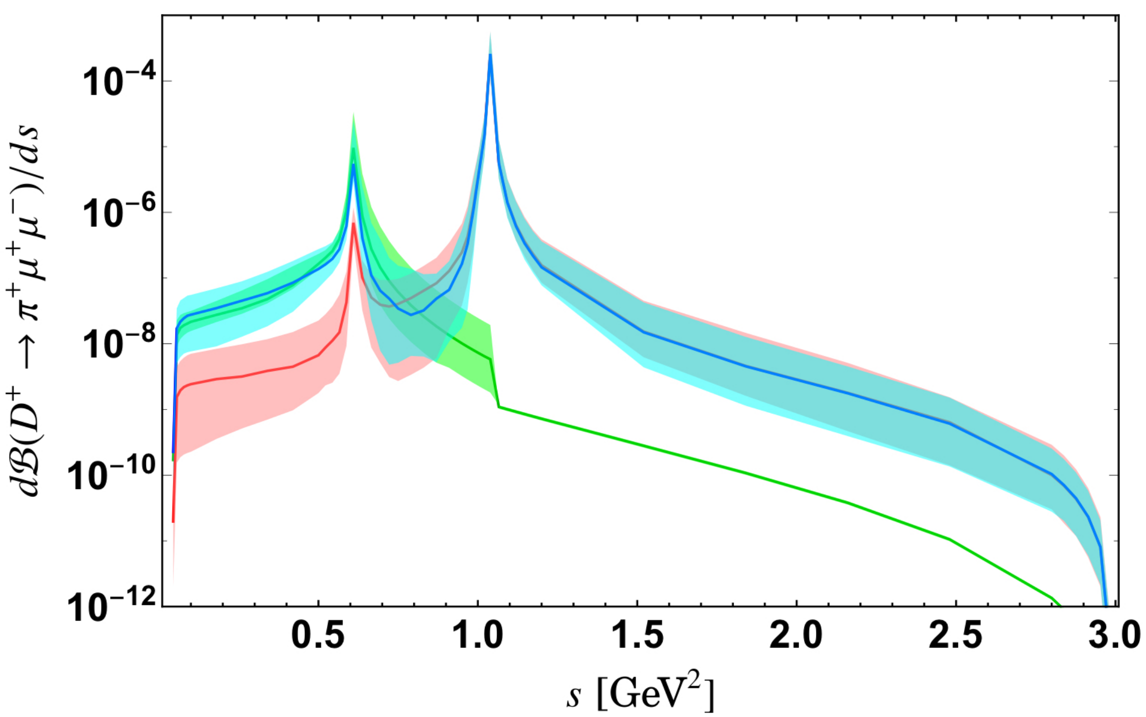
- The method was originally suggested for $B \to K^* \ell^+ \ell^-$.

- First use for charm decays in $D \to \rho \ell^+ \ell^-$:

  The loop topology diagram modified to include resonances. : Shifman model of loop-resonance duality

- Later, a similar method applied to $D \to \pi \ell^+ \ell^-$ (with the main focus on new physics).

$$\mathcal{B}(D^+ \to \pi^+ \mu^+ \mu^-)\Big|^{\mathrm{SM}}_{\mathrm{low}\, q^2} = (8.1^{+5.9}_{-6.1}) \times 10^{-9},$$
$$\mathcal{B}(D^+ \to \pi^+ \mu^+ \mu^-)\Big|^{\mathrm{SM}}_{\mathrm{high}\, q^2} = (2.7^{+4.0}_{-2.6}) \times 10^{-9},$$



- Open questions:
  - Includes only one of the four annihilation diagrams (emission from the initial d-quark) :
    - ✴ Other three diagrams turns out to be important.
  - $\dfrac{1}{m_c^2}$ corrections eg. from the use of D-meson distribution amplitudes:
    - ✴ Expected to be large (at least compared to the B-meson case).