



Istituto Nazionale di Fisica Nucleare  
SEZIONE DI TORINO



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# *From PID detectors to PID variables*

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*Belle II starter kit  
KEK, February 1<sup>st</sup> 2020*

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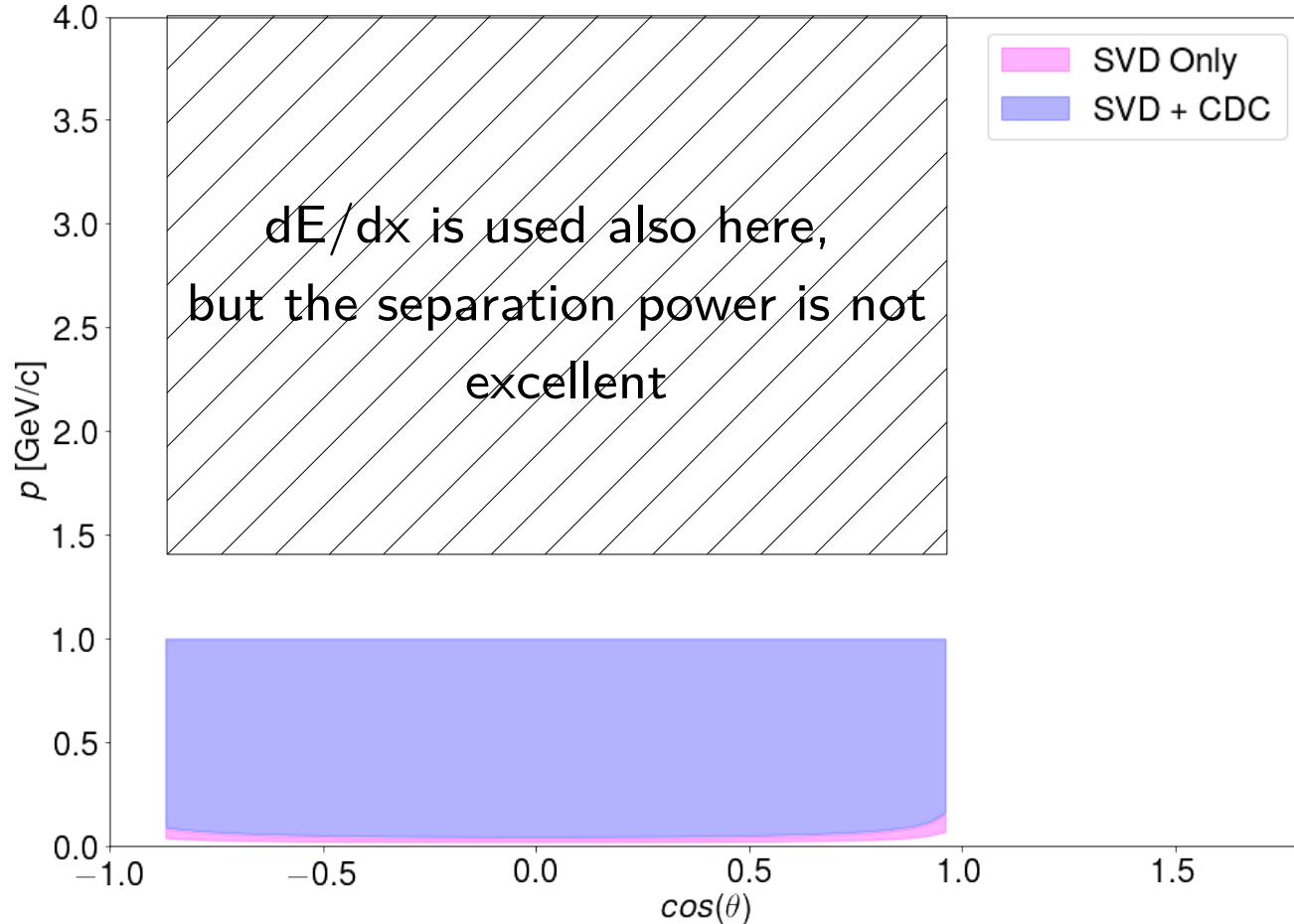
*INFN – Sezione di Torino*

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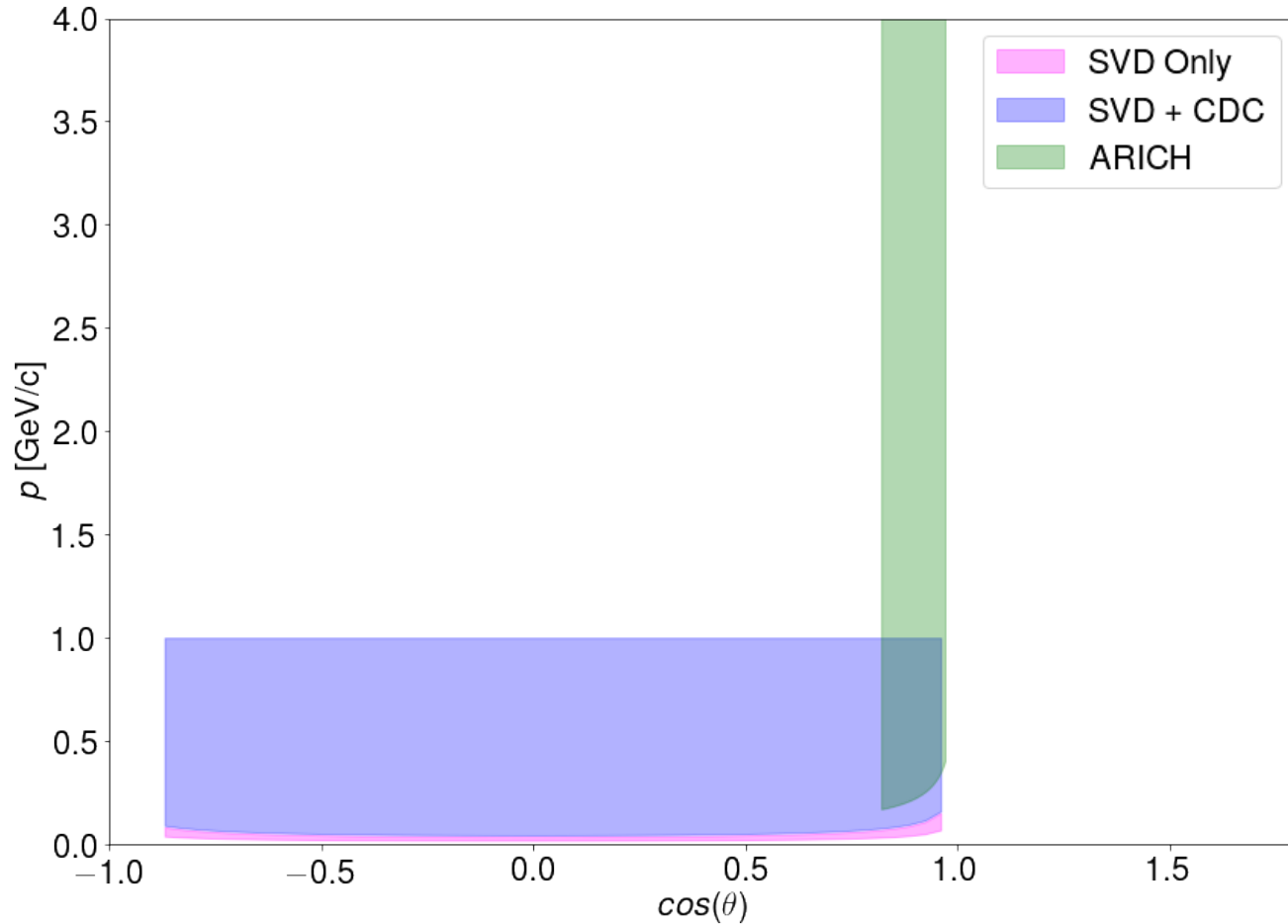
*Side A: PID likelihoods*

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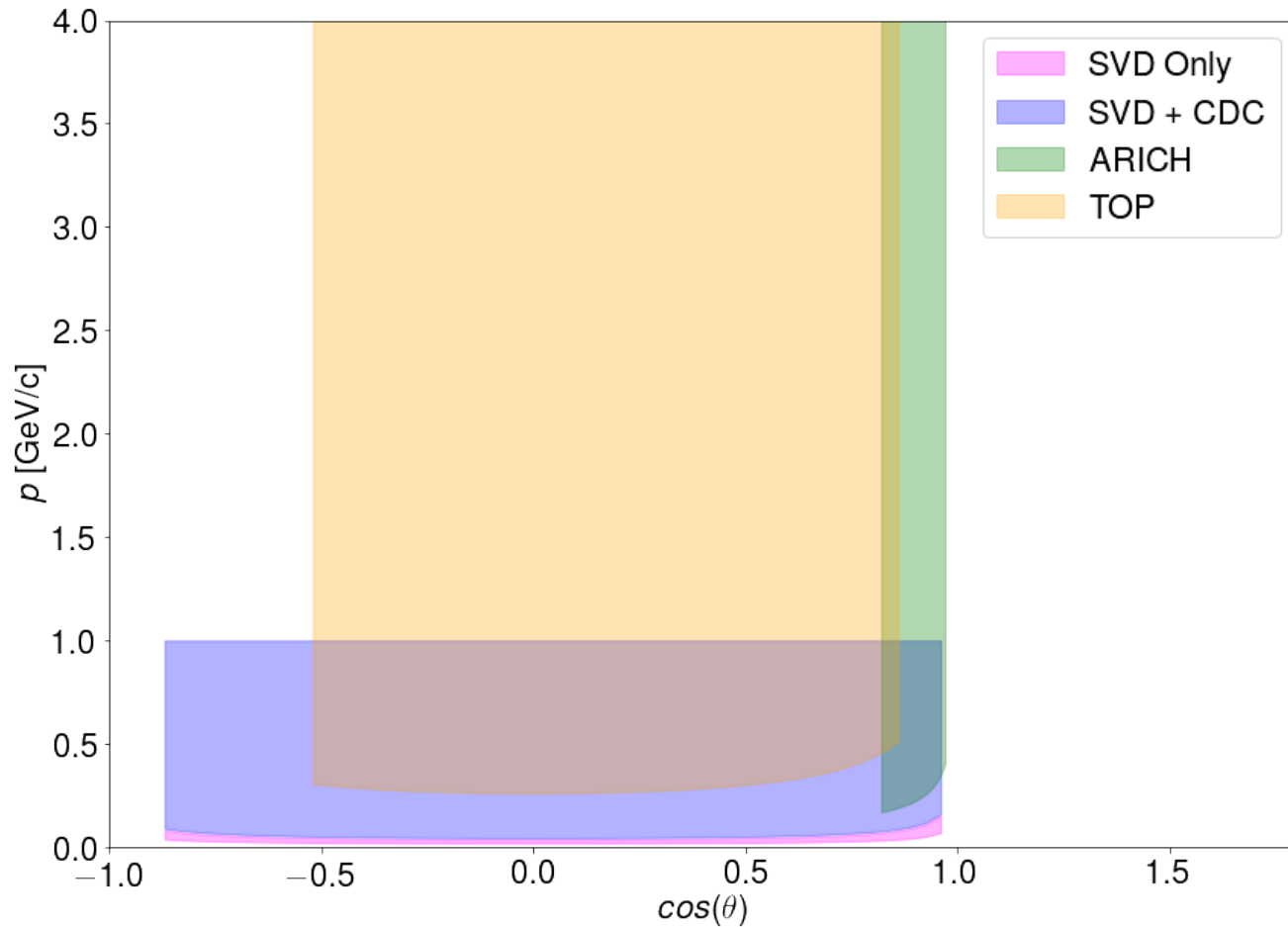
# $dE/dx$ : where does it matter the most



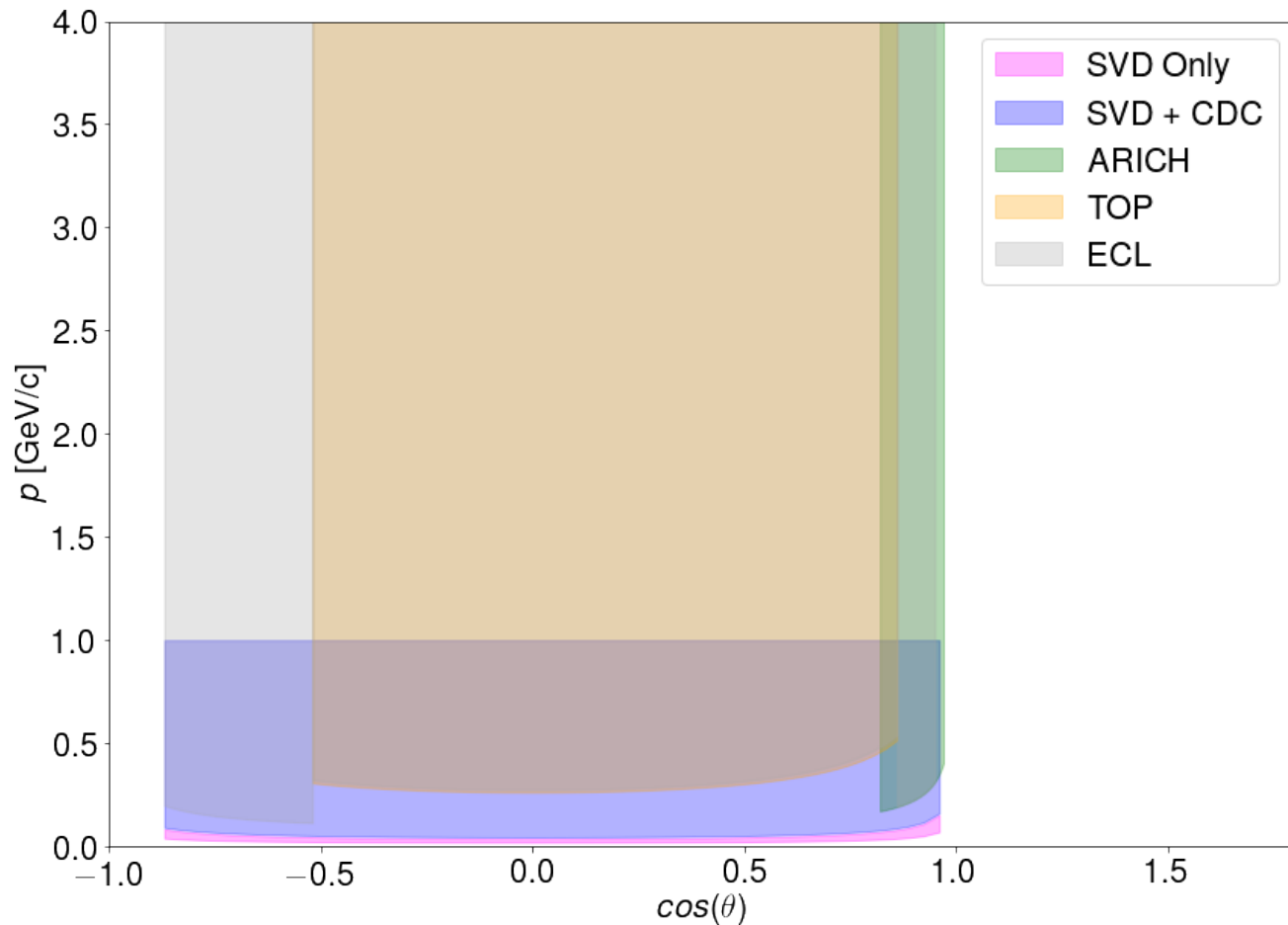
*For illustration only. Do not use this plot to get some serious number*



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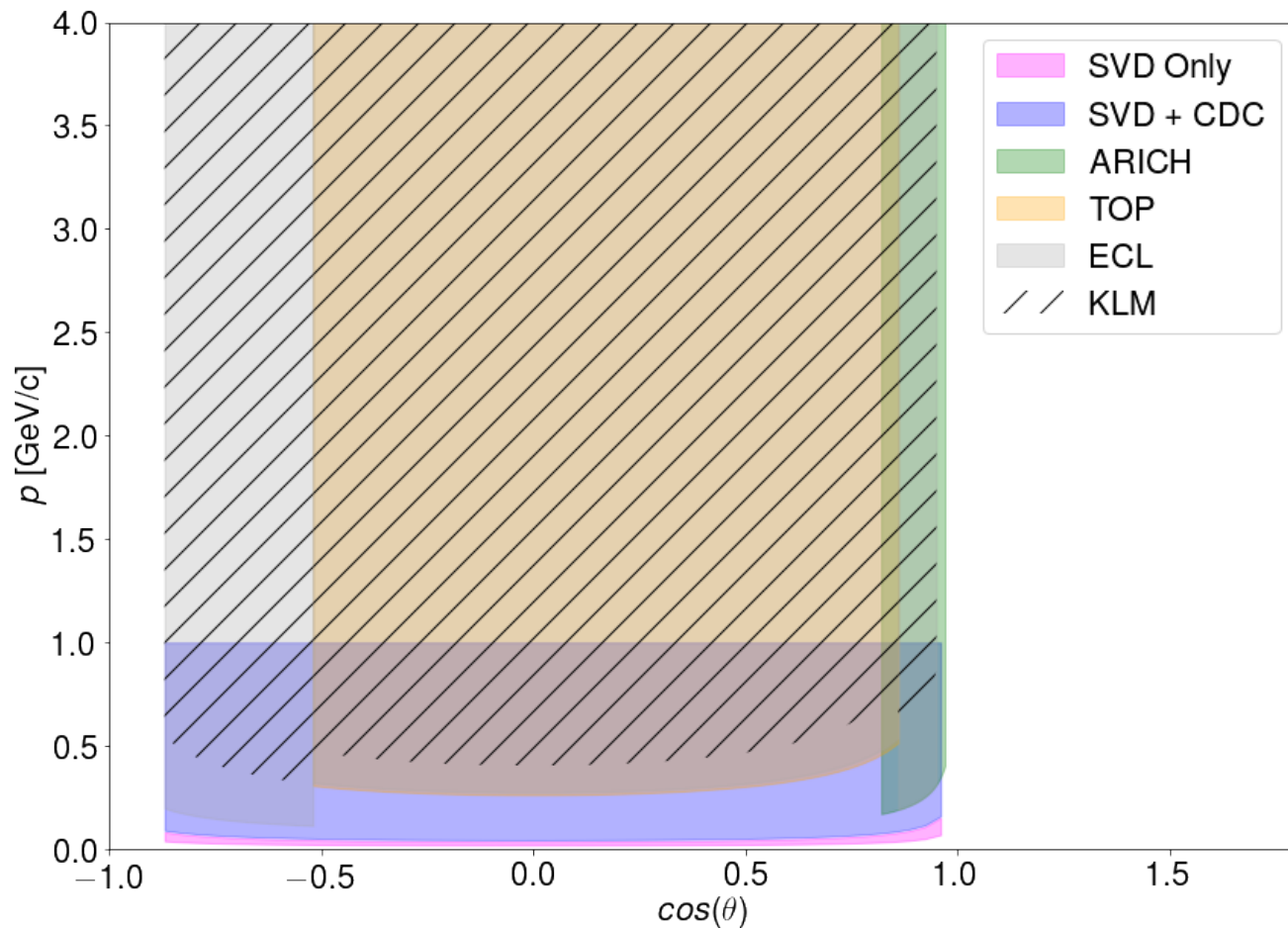


*For illustration only. Do not use this plot to get some serious number*



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The ECL dominates the electron identification



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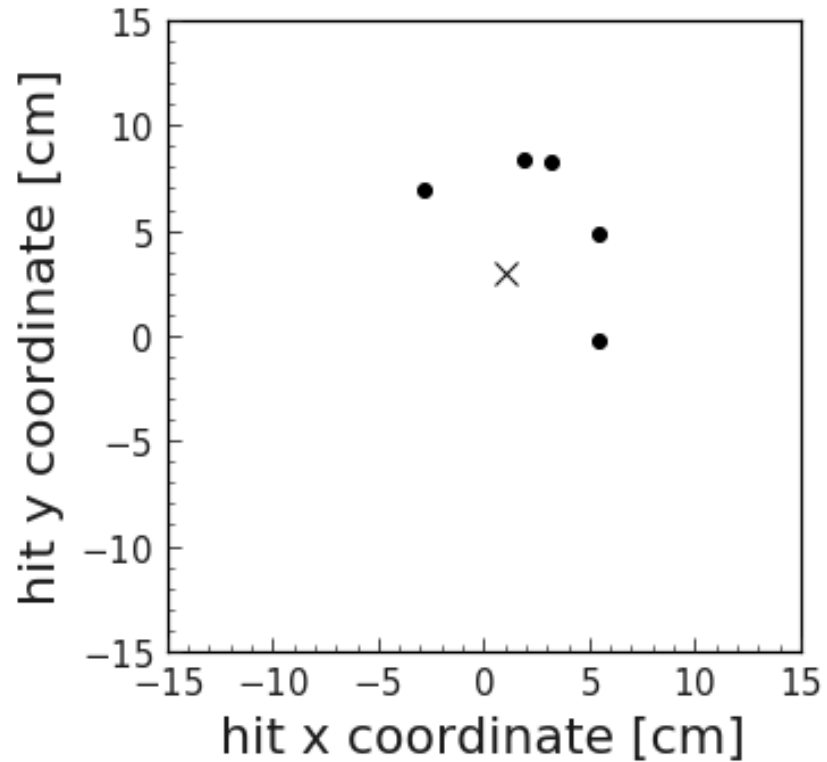
The KLM dominates the muon identification

***How can we combine in a coherent way all the signals from the sub-detectors?***

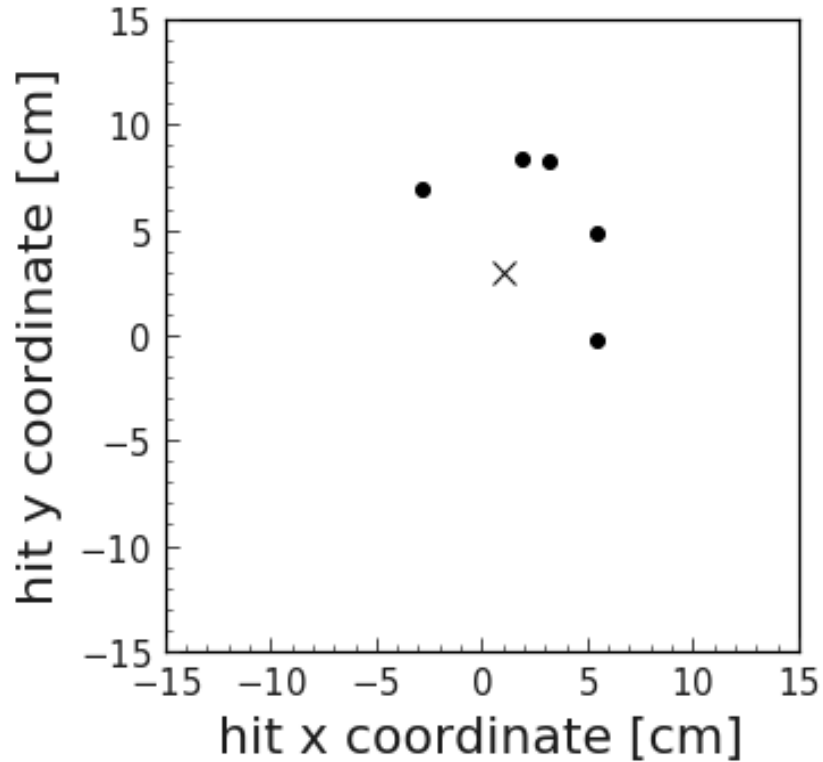
- 1) Each detector fits the distribution of its hits with six PDFs (one per species)



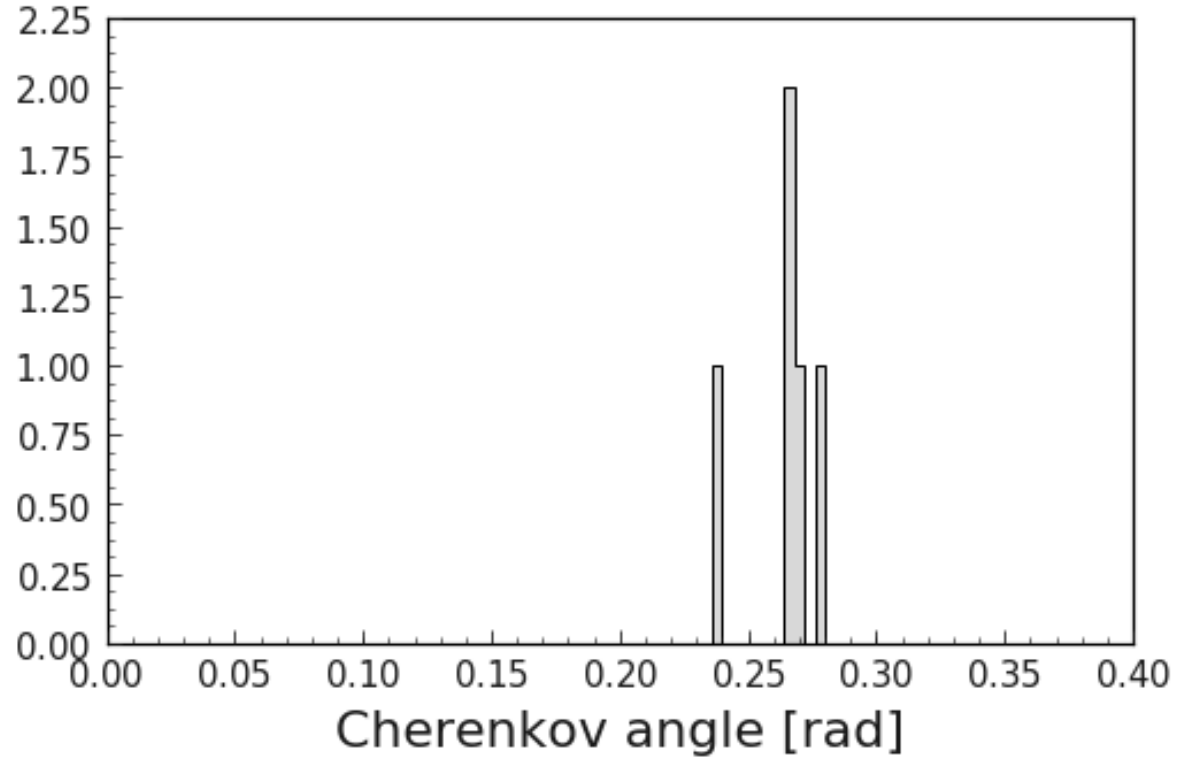
## Detector level



*Detector level*



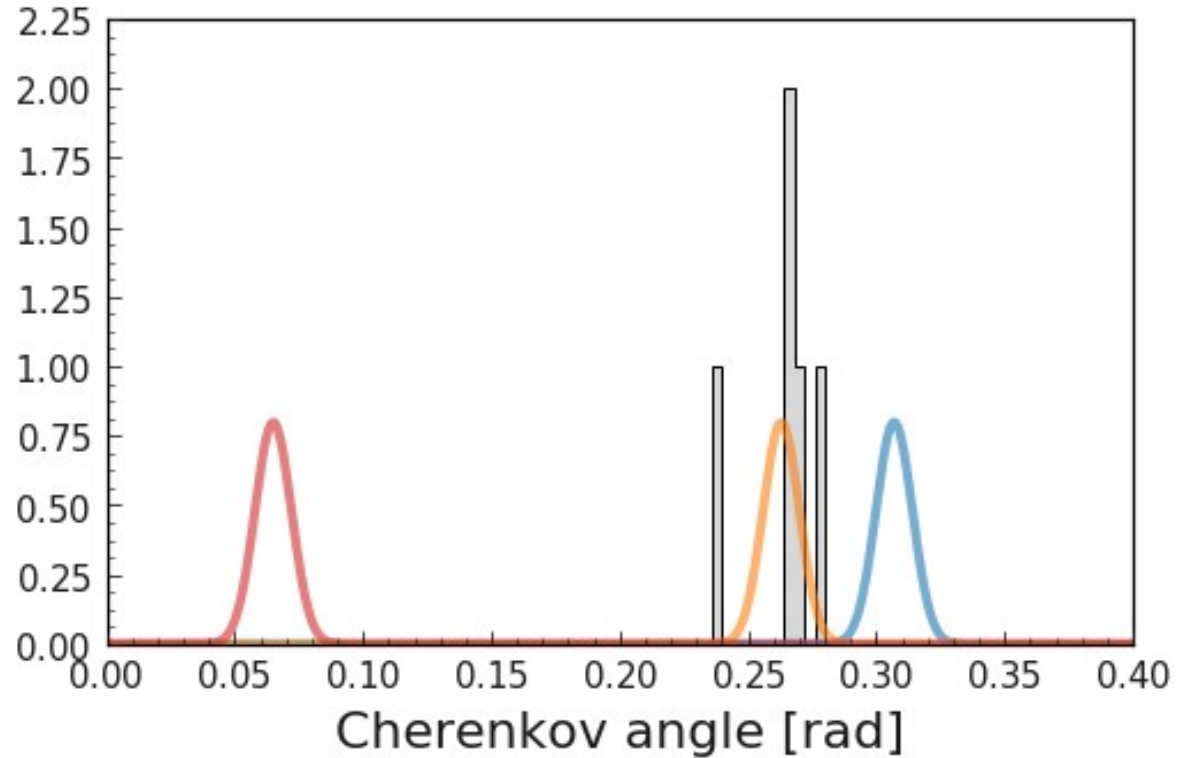
*Reconstruction level*



## Reconstruction level

Compare the observed distro with the expected one for each particle type

pi, K and p here. Which is which?



## Reconstruction level

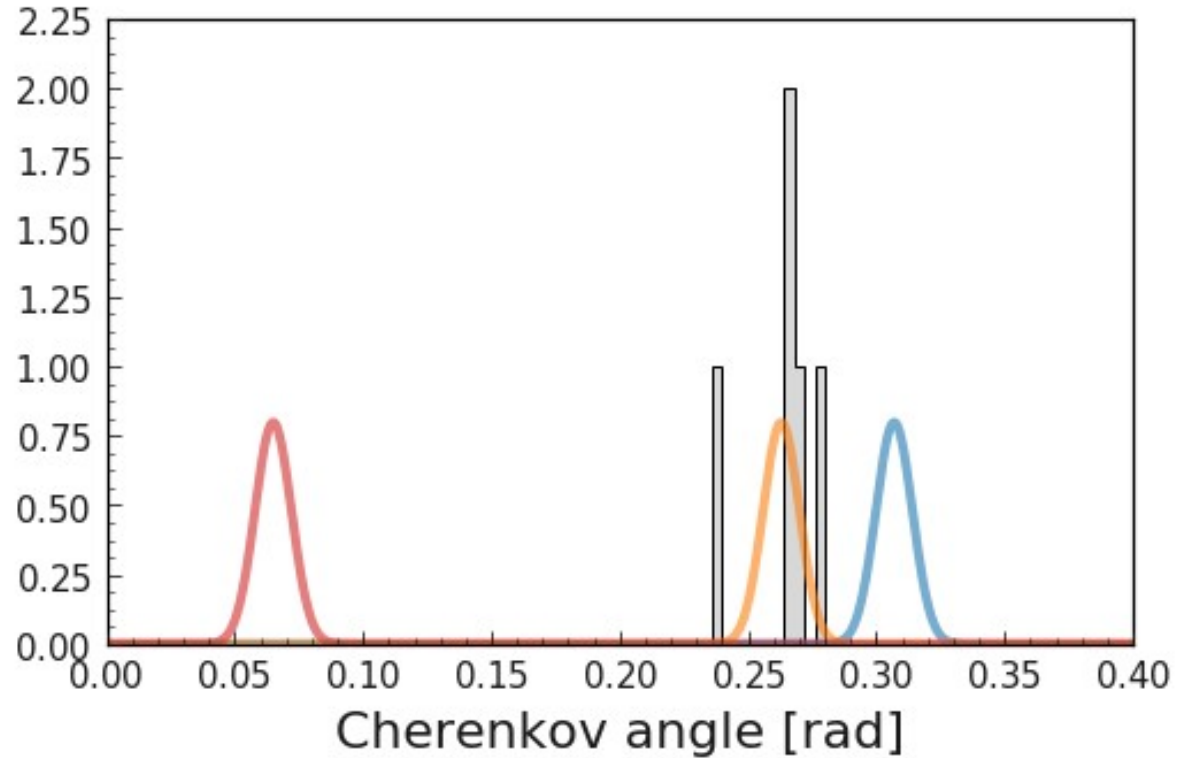
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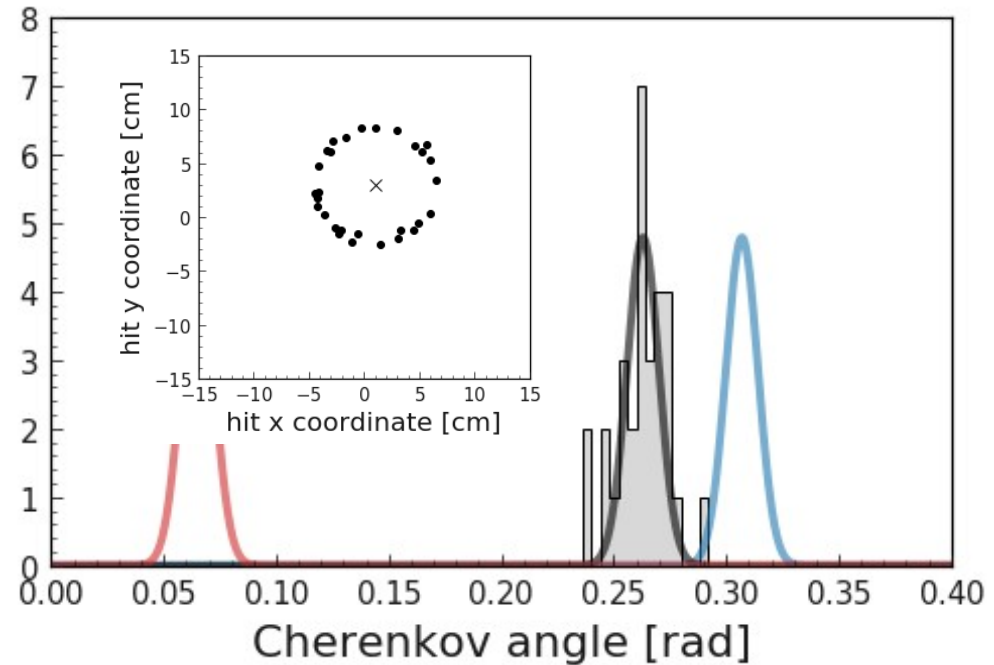
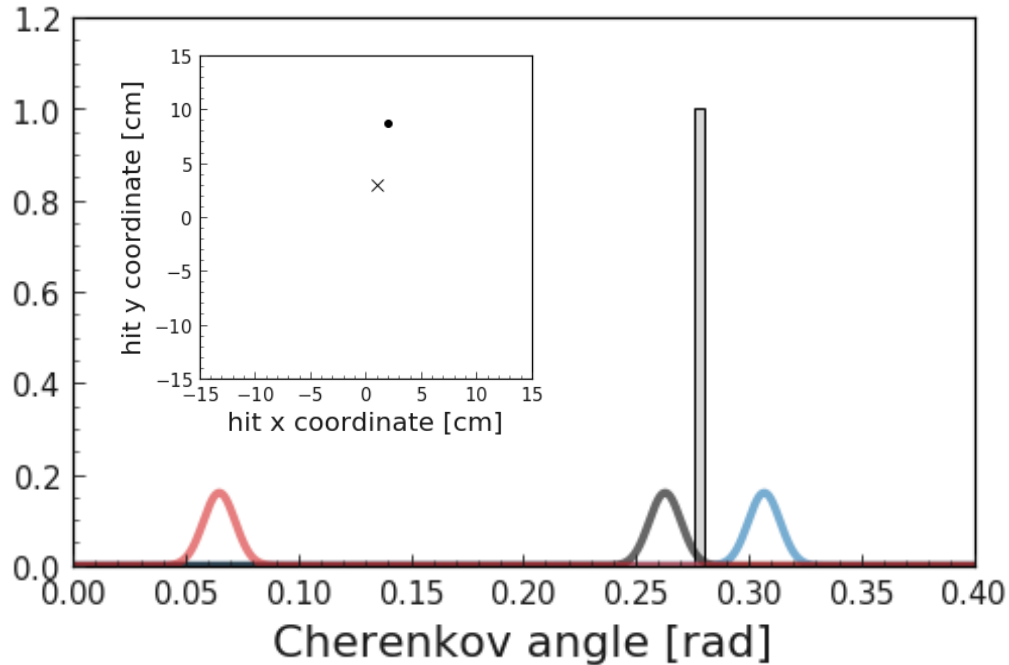
$$LL(\pi) = -165$$

$$LL(K) = -28$$

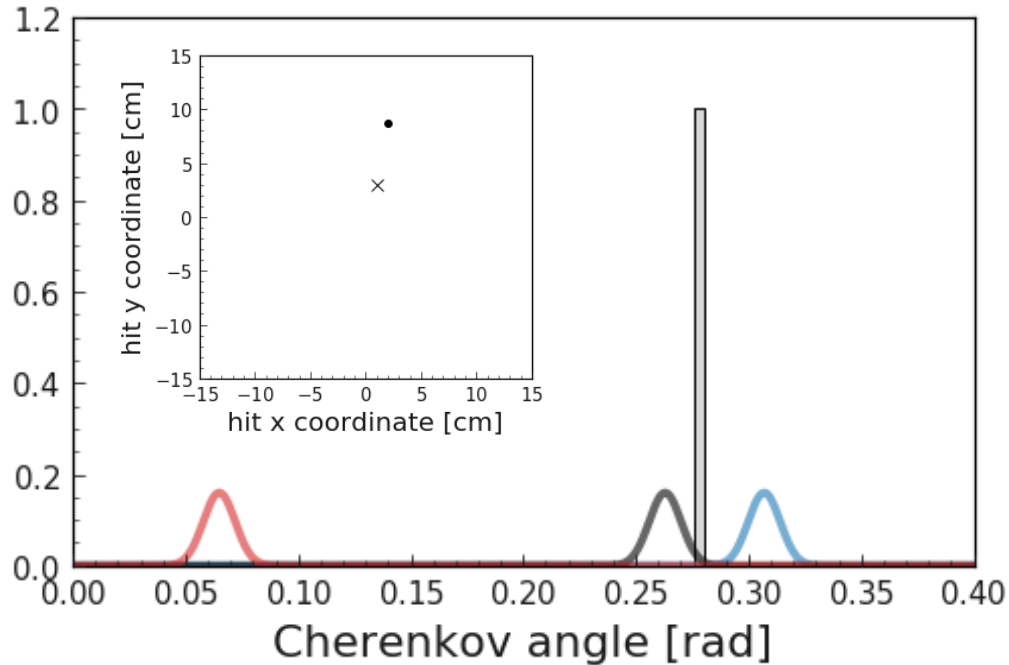
$$LL(p) = -1805$$



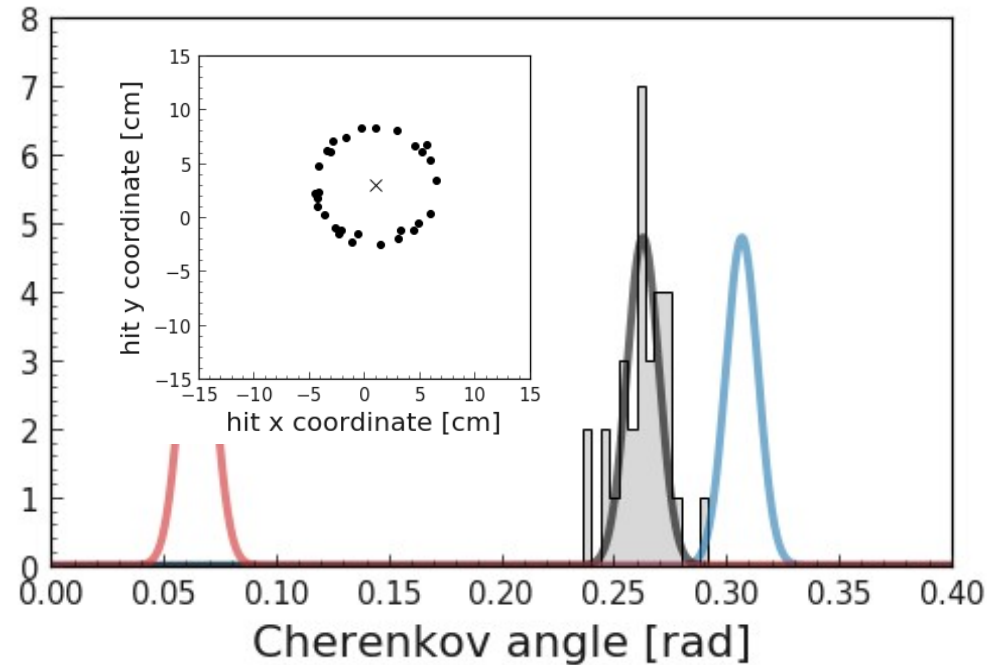
Will a kaon always have a high kaon LL?



Will a kaon always have a high kaon LL?



$LL(K) = -6$



$LL(K) = -135$

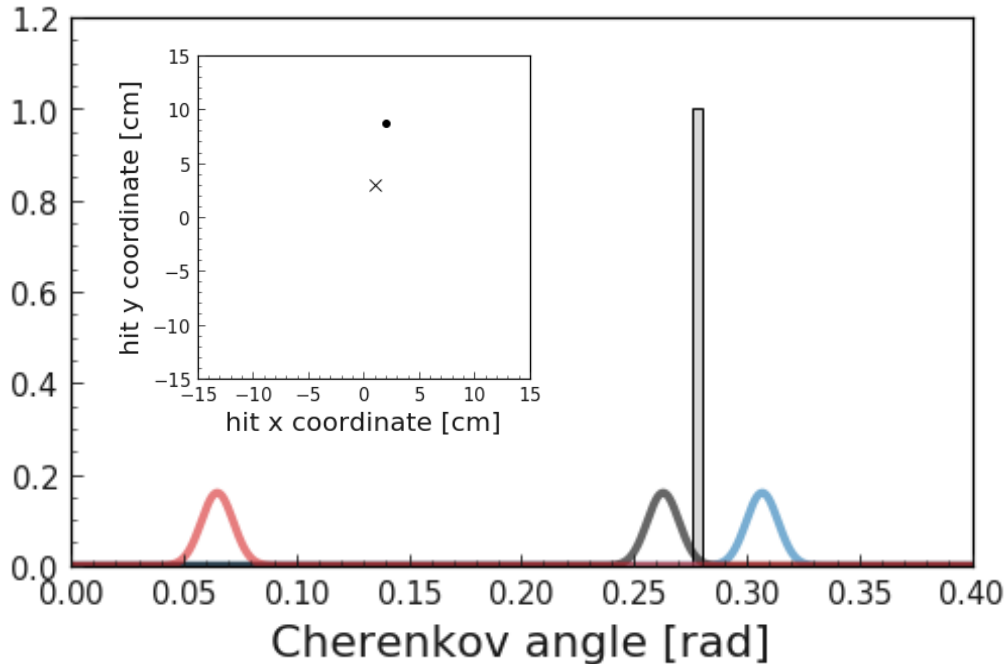
## *How do we compare different hypotheses?*

→ The better the mass hypothesis fits the data, the larger the likelihood is.

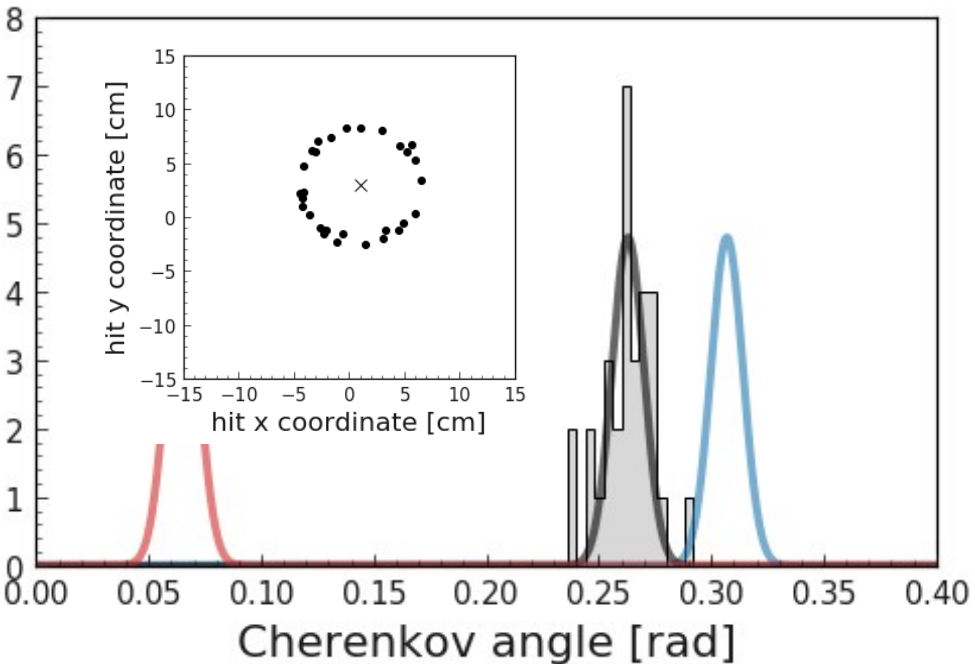
→ The most basic comparison is the **Log-likelihood difference**

$$\Delta LL = \log L_A - \log L_B$$

→  $\Delta LL$  tells you which one of two hypotheses is the most likely



$DLL(K,\pi) = 4.6$



$DLL(K,\pi) = 590$



***How can we combine in a coherent way all the signals from the sub-detectors?***

- 1) Each detector fits the distribution of its hits with six PDFs (one per species)
- 2) The outcome of each fit is quantified in a (Log)-likelihood value

***How can we combine in a coherent way all the signals from the sub-detectors?***

- 1) Each detector fits the distribution of its hits with six PDFs (one per species)
- 2) The outcome of each fit is quantified in a (Log)-likelihood value
- 3) The for each mass hypothesis, we sum the LogLikelihoods of the sub-detectors to construct **a single particle likelihood**

$$\log \mathcal{L}_\pi = \log \mathcal{L}_\pi^{\text{SVD}} + \log \mathcal{L}_\pi^{\text{CDC}} + \log \mathcal{L}_\pi^{\text{TOP}} + \log \mathcal{L}_\pi^{\text{ARICH}} + \log \mathcal{L}_\pi^{\text{ECL}} + \log \mathcal{L}_\pi^{\text{KLM}}$$

# *Weighting the detector info*

---

Imagine two detectors D1 and D2

→ D1 can separate  $\pi$  and  $\mu$

→ D2 cannot

***Should we weight the subdetectors in the combined LL?***

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$$\Delta \log L(\pi, \mu) = \log L(\pi) - \log L(\mu)$$

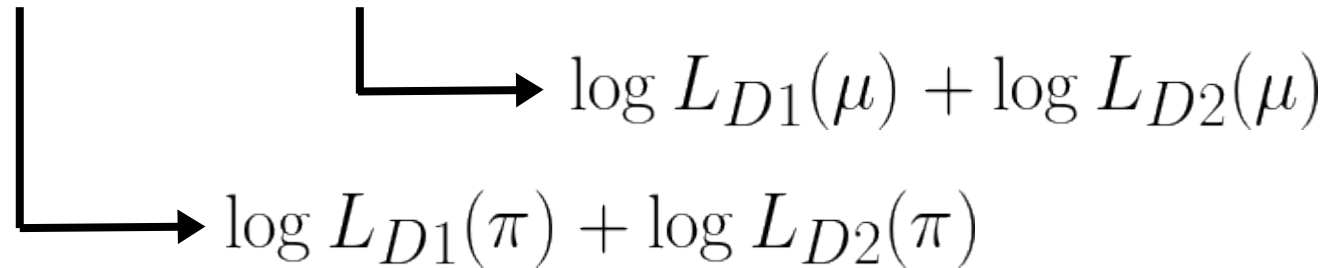
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The diagram shows the decomposition of the log-likelihood difference into detector-specific terms. A vertical line descends from the left side of the equation, with two horizontal arrows pointing to the right. The top arrow points to the expression  $\log L_{D1}(\mu) + \log L_{D2}(\mu)$ , and the bottom arrow points to the expression  $\log L_{D1}(\pi) + \log L_{D2}(\pi)$ .

# Weighting the detector info

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***Should we weight the subdetectors in the combined LL?***

Imagine two detectors D1 and D2

→ D1 can separate  $\pi$  and  $\mu$

→ D2 cannot

$$\Delta \log L(\pi, \mu) = \log L(\pi) - \log L(\mu)$$

$$\Delta \log L(\pi, \mu) = \Delta \log L_1(\pi, \mu) + \Delta \log L_2(\pi, \mu)$$

Question: Does D2 contribute?

*Should we weight the subdetectors in the combined LL?*

**NO, Likelihoods are “self-weighting”**

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*Side B: PID probabilities*

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DLL is a powerful tool to understand a detector's performance

However, how do you quantify the "PID level" of a particle in an understandable way?

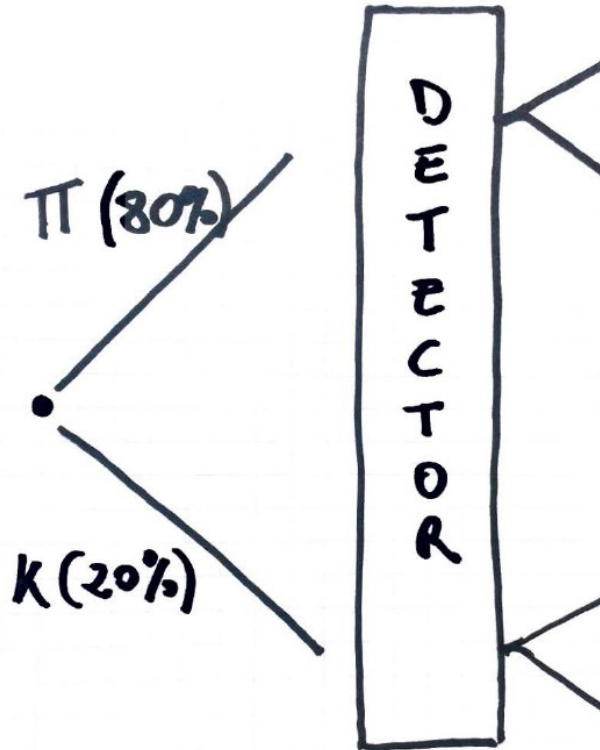
DLL is a powerful tool to understand a detector's performance

However, how do you quantify the “PID level” of a particle in an understandable way?

PID is inherently a Bayesian problem.

I observe a “kaon-like” signal, and want to know what's the probability for that signal to be really generated by a kaon

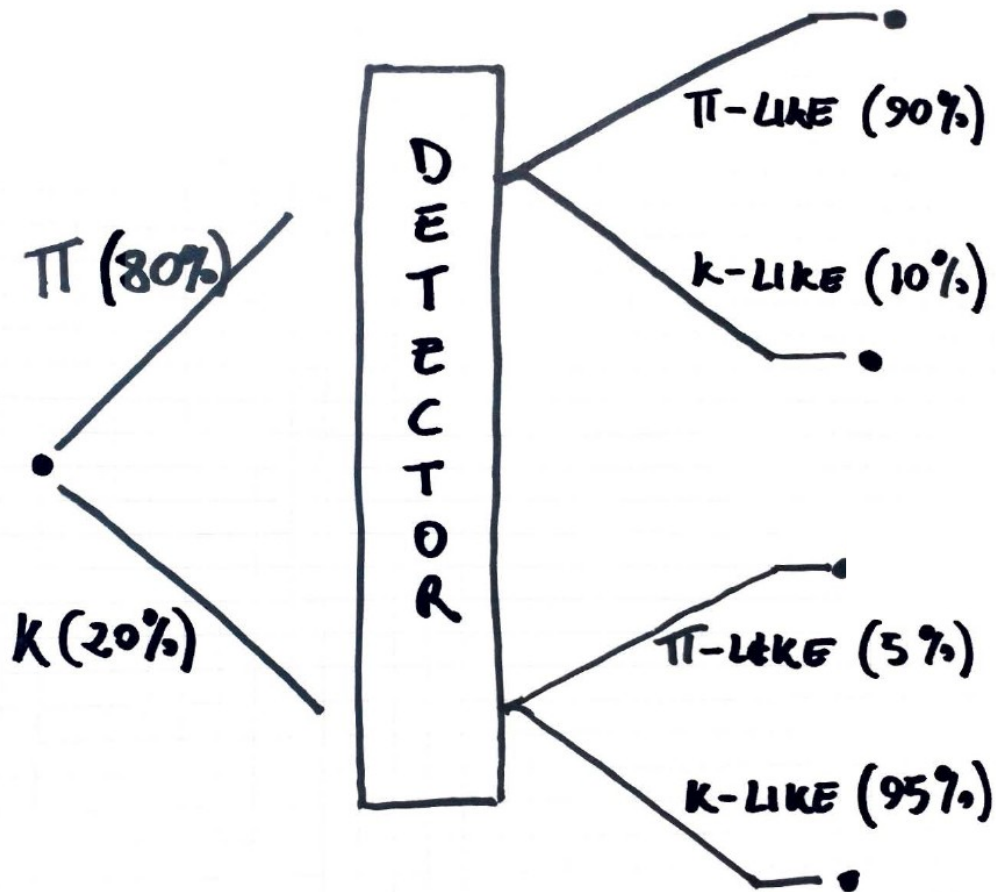
A very simple example: a magic universe where only pions and kaons exist.



Let's assume that our data sample contains 20% kaons and 80% pions (how do we know it? )

# From likelihoods to probabilities

A very simple example: a magic universe where only pions and kaons exist.

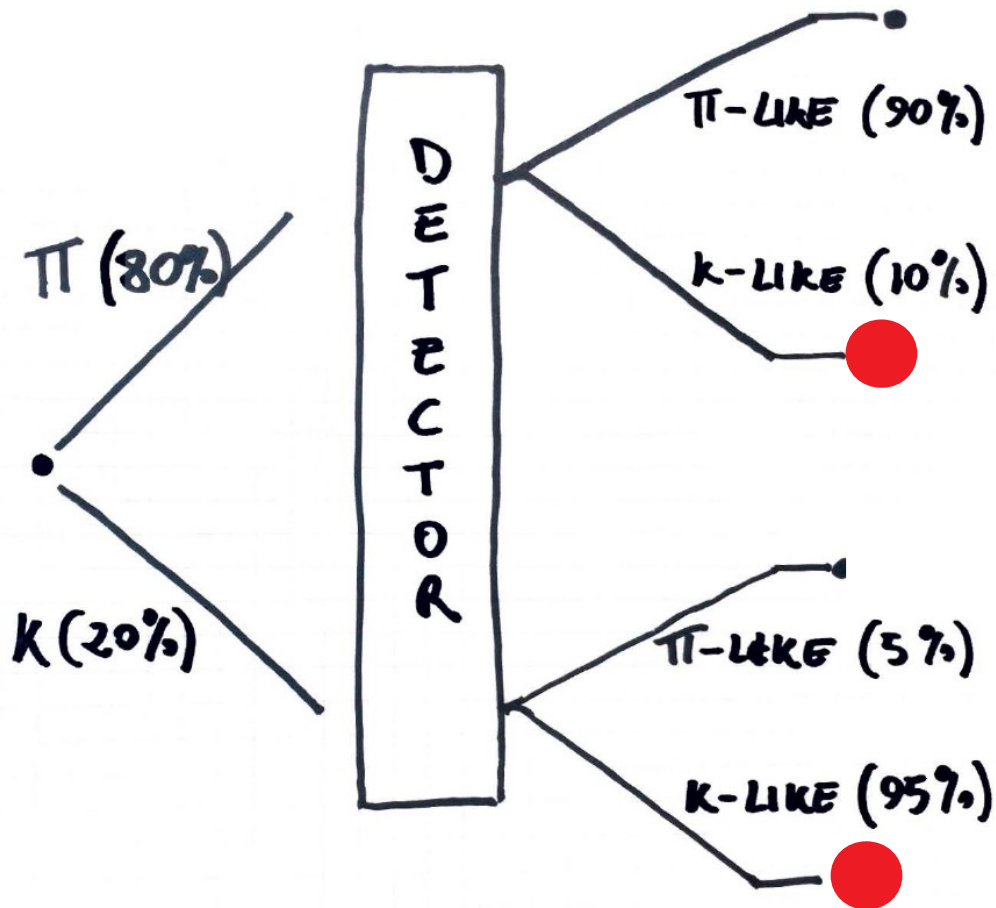


Let's assume that our data sample contains 20% kaons and 80% pions (how do we know it? )

The detector has a certain probability of assigning pion or kaon ID, depending on the original particle

# From likelihoods to probabilities

A very simple example: a magic universe where only pions and kaons exist.

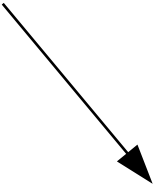


We observe a “kaon-like” signal.  
What’s the probability for that particle to be a kaon?

A very simple example: a magic universe where only pions and kaons exist.

We observe a “kaon-like” signal.  
What’s the probability for that  
particle to be a kaon?

Posterior probability


$$P(\text{S is from K}) = \frac{P(\text{K gives S}) \cdot P(K)}{P(\text{K gives S}) \cdot P(K) + P(\pi \text{ gives S}) \cdot P(\pi)}$$

Prior probability

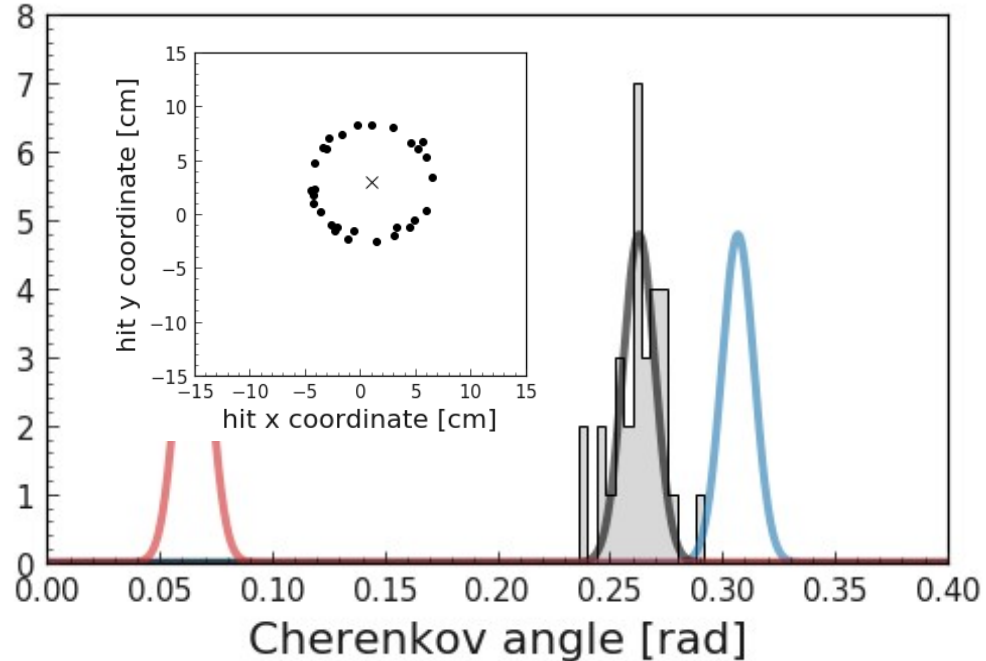


The likelihood value is actually a proxy (i.e. is proportional) exactly to the conditional probability!

$$P(\text{S is from K}) = \frac{L(K) \cdot P(K)}{L(K) \cdot P(K) + L(\pi) \cdot P(\pi)}$$

**Belle II default PID variables are posterior probabilities**

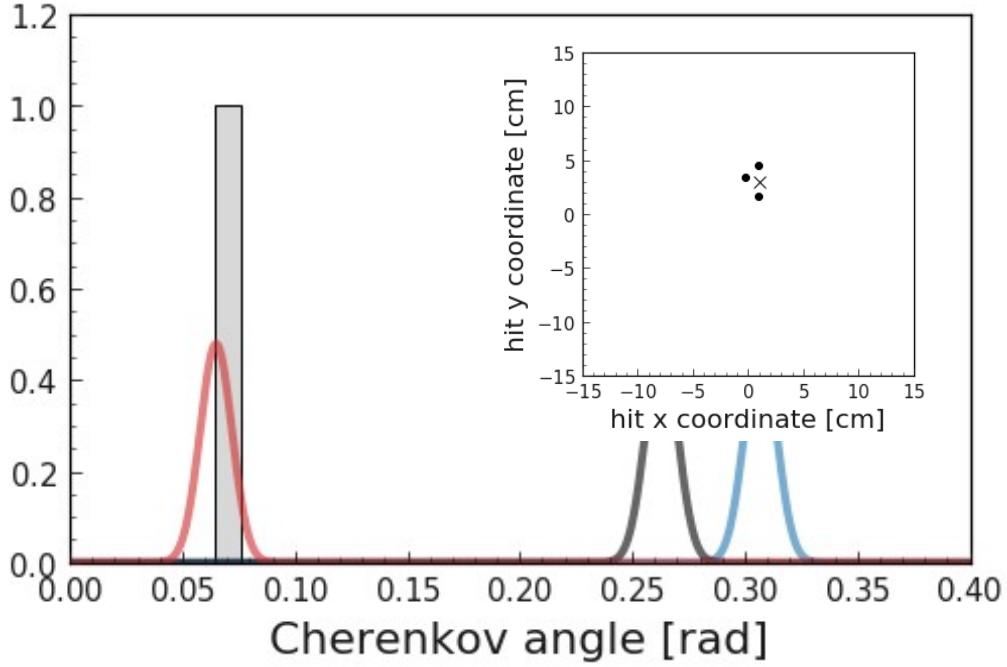
?



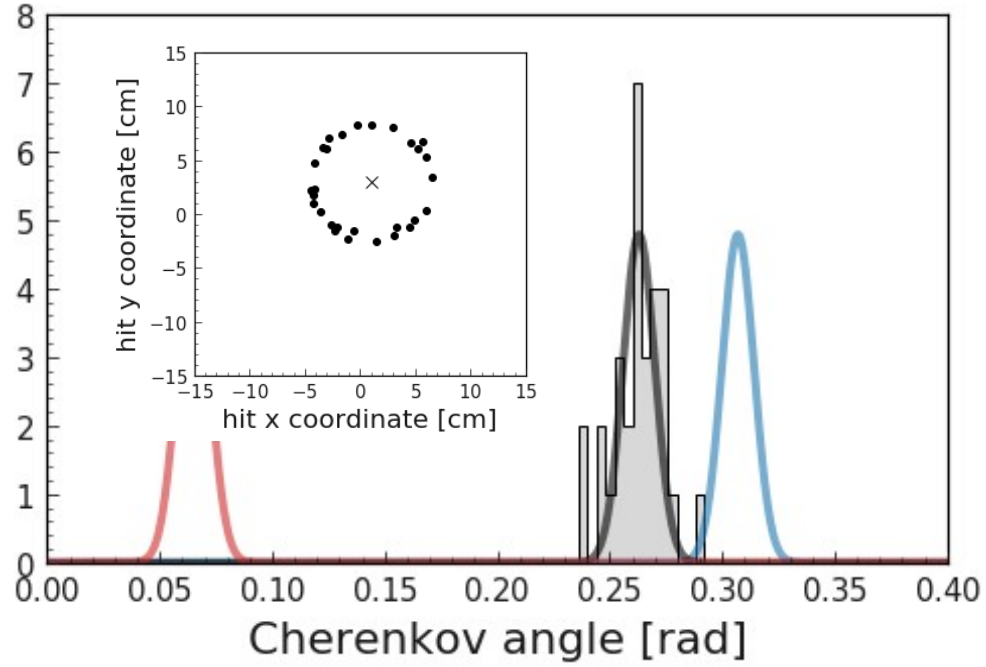
$DLL(K,\pi) = 568$   
 $P(K, \pi) = 1$

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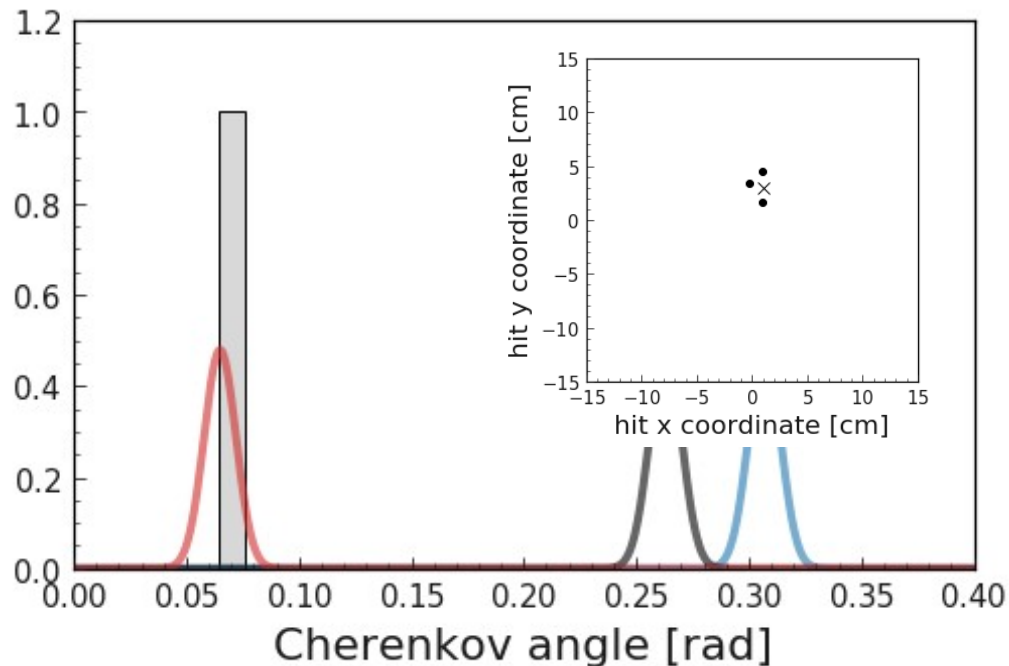


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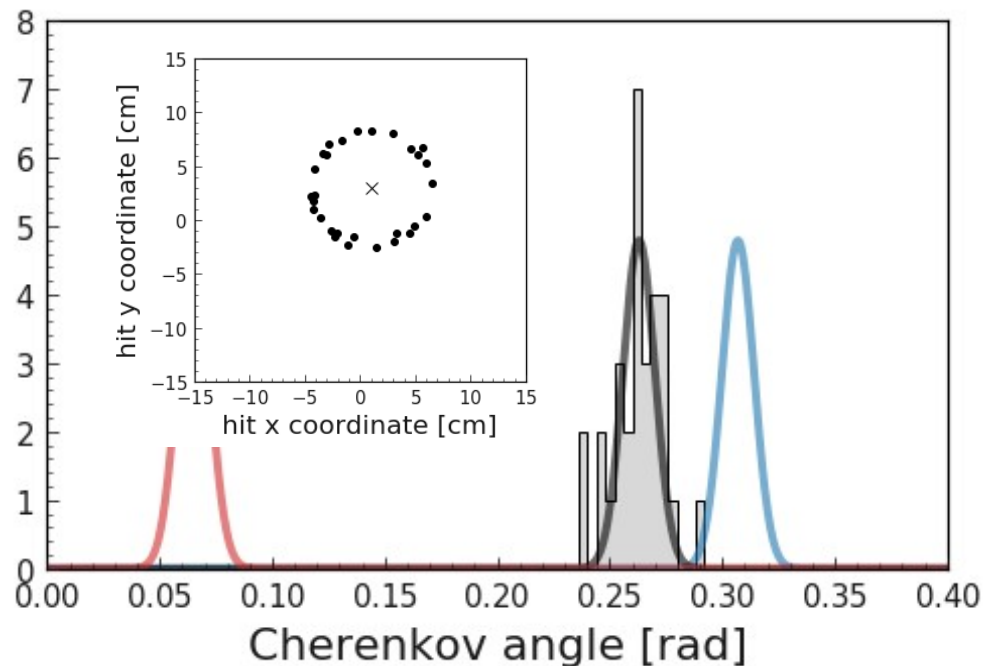
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 $P(K, \pi) = 1$

What is going on here?



$$DLL(K, \pi) = 2375$$

$$P(K, \pi) = 1$$



$$DLL(K, \pi) = 612$$

$$P(K, \pi) = 1$$

# “Global” and “Binary” PID

“Binary PID” is a special case of the “global PID”

$$Pid(K, \pi) = \frac{L(K)P(K)}{L(K)P(K) + L(\pi)P(\pi)}$$

$$Pid(K) = \frac{L(K)P(K)}{\sum_{i=e,\mu,\pi,K,p,d} L(i)P(i)}$$

Can you see what the only difference is?

Likelihood values are meaningless without a reference

PID probabilities are meaningless without a prior scheme

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*Bonus track: using PID*

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## Basic variables:

```
electronID, muonID, pionID, kaonID, protonID, deuteronID  
pidPairChargedBDTScore(pdgCodeHyp, pdgCodeTest)
```

Can you find the documentation yourself?

## Basic variables:

```
electronID, muonID, pionID, kaonID, protonID, deuteronID  
pidPairChargedBDTScore(pdgCodeHyp, pdgCodeTest)
```

## “Expert” variables

```
pidLogLikelihoodValueExpert(pdgCode, detectorList)  
pidDeltaLogLikelihoodValueExpert(pdgCode1, pdgCode2, detectorList)  
pidPairProbabilityExpert(pdgCodeHyp, pdgCodeTest, detectorList)  
pidProbabilityExpert(pdgCodeHyp, detectorList)
```

Few metrics are used to characterize the performances of a PID detector

→ **Efficiency: ability to correctly assign the ID**

$$\varepsilon(K) = N(K \text{ identified as } K) / N(\text{real } K)$$

Equal, by definition, to the “probability of a kaon to be called kaon”



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→ **Mis-ID probability: ability not to assign the incorrect ID**

$$\text{Mis-ID}(K) = N(\text{non-}K \text{ identified as } K) / N(\text{non } K)$$

Equal, by definition, to the “probability for a non-kaon to be called kaon”

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→ **Fake rate: fraction of particles with the wrong ID**

$$F(K) = N(\text{non-}K \text{ identified as } K) / N(\text{identified as } K)$$

Equal, by definition, to the “fraction of non-kaons in my collection of kaons”

The fake rate is (to a certain extent) a Bayesian idea

*Given that I have something that looks like a kaon, what are the chances for this to really be a kaon and not a pion?*

**Let's assume:**

Mis-ID probability  $\pi \rightarrow \mathbb{K} \sim 1\%$

Kaon efficiency  $\sim 100\%$

2% of kaons and 98% pions in the data

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Bayes theorem:

$$P(\text{my "kaon" is kaon}) = 1 \times 0.02 / (1 \times 0.02 + 0.01 \times 0.98) \sim 67\%$$

**Fake rate = 33%**

## **1) PID variables are probabilities**

- Bayes theorem with Likelihoods are conditional probabilities
- Priors are constant (for now)

## **2) Don't confuse fake rate with mis-ID probability**

## **3) Things will improve in future**

- Priors will be implemented
- ML to properly deal with high order correlations