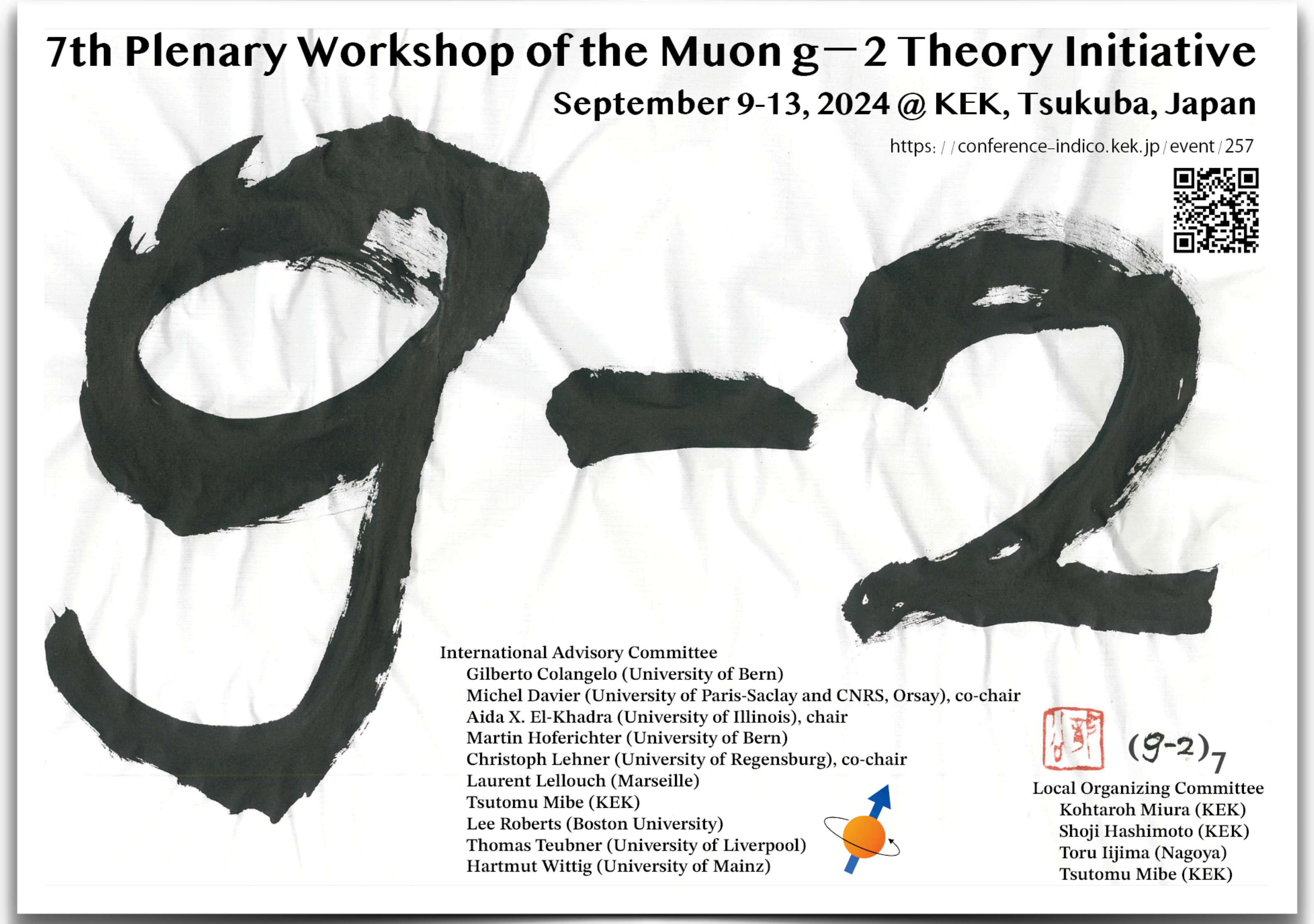
Belle II Input to HVP[1]

Seventh Plenary Workshop of the Muon g-2 Theory Initiative



- Introduction
- ISR method and trigger
- $e^+e^- \to \pi^+\pi^-(\gamma)$ status
- $e^+e^- \to \pi^+\pi^-\pi^0(\gamma)$ result
- Summary

1. Hadronic Vacuum Polarization

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Sept. 09, 2024

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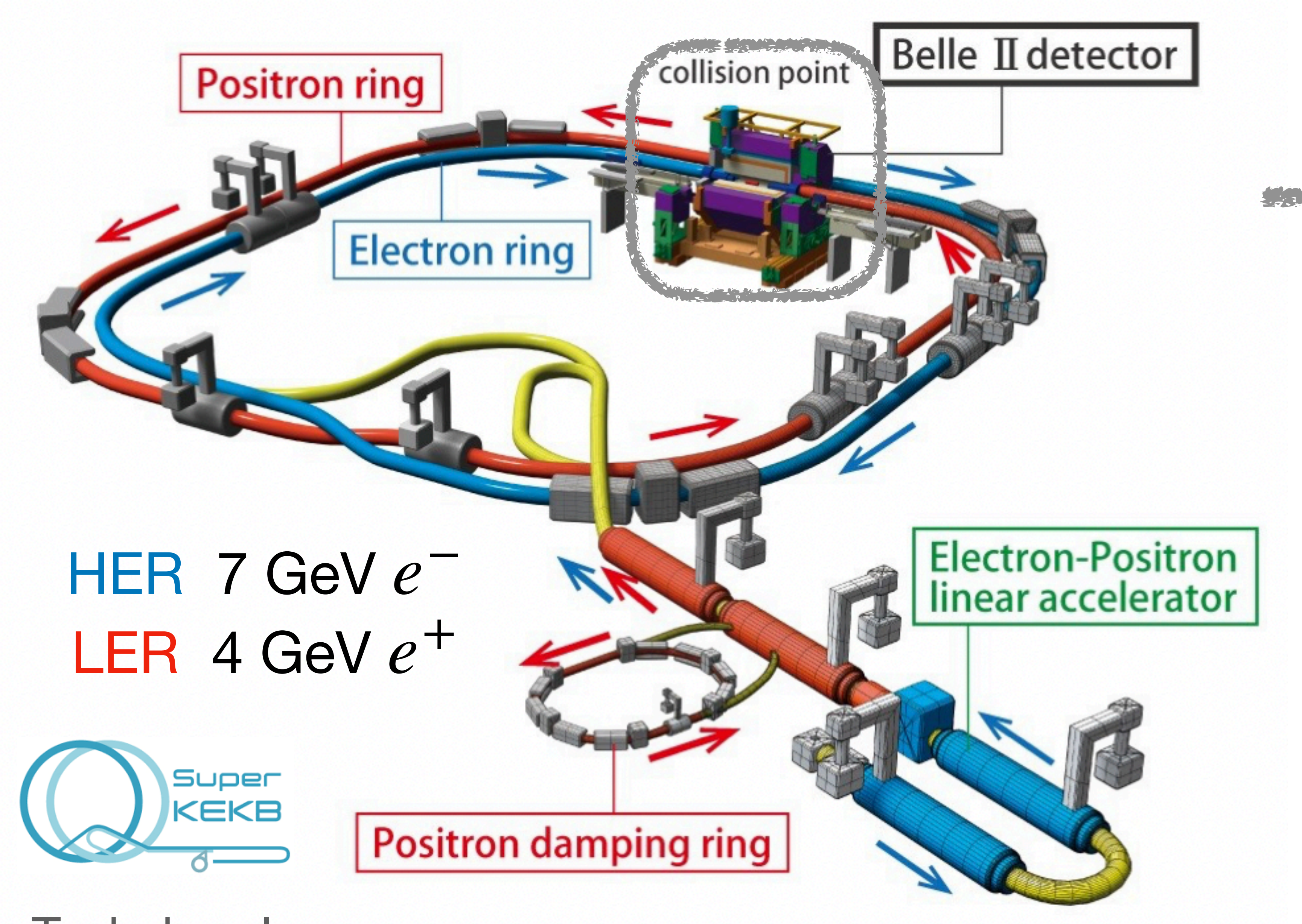
Introduction

SuperKEKB

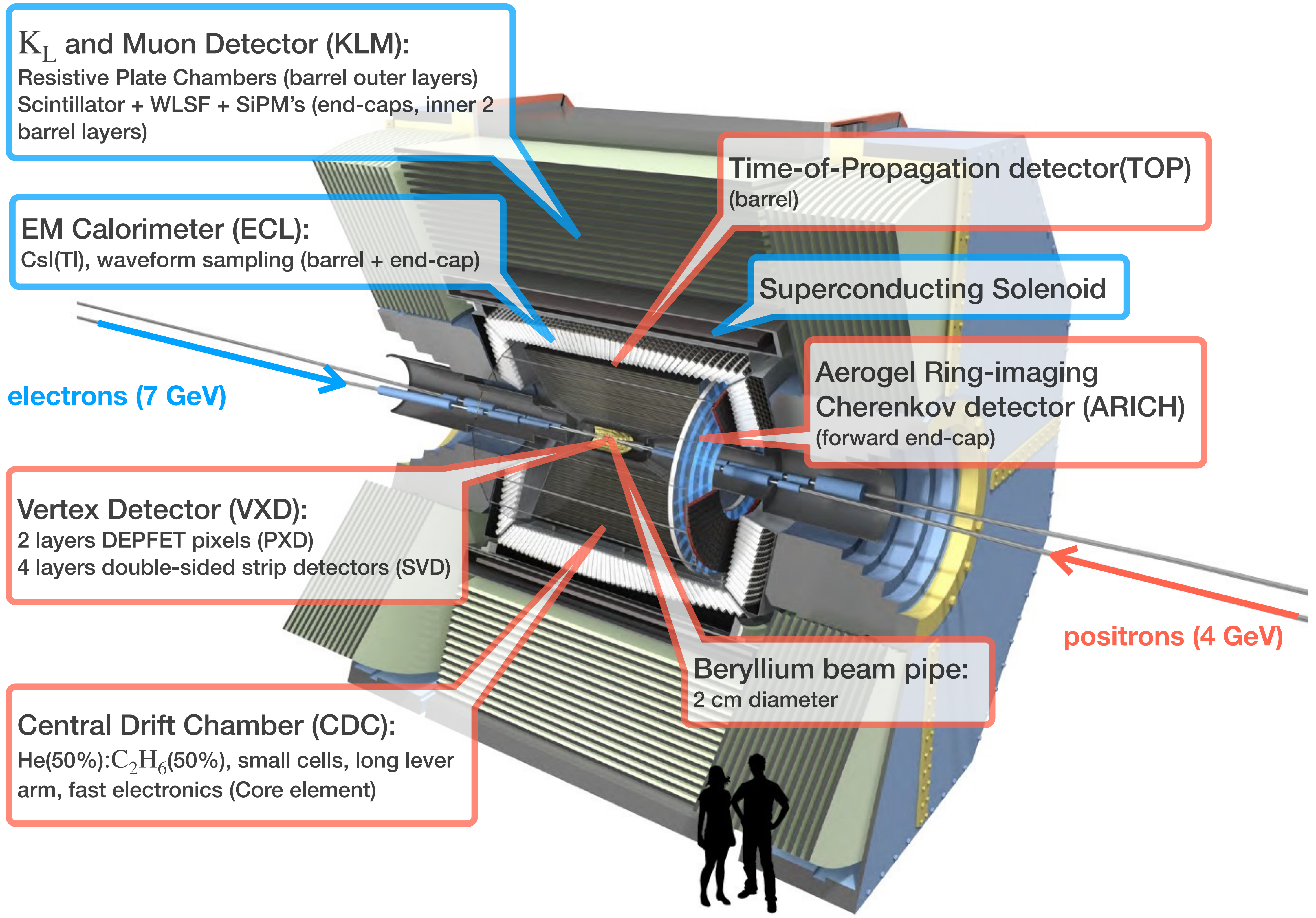
- **Asymmetric**-energy e^+e^- collider
- $E_{cm} = M_{\Upsilon(4S)} \approx 10.58$ GeV, B factory
- Goal: $L_{peak} = 6 \times 10^{35}\,cm^{-2}s^{-1}$
  - **Nano-beam** scheme and **increased currents**
  - $4.7 \times 10^{34}\,cm^{-2}s^{-1}$ (**June 2022**, world record)



Tsukuba, Japan

Belle II

- Target L_{int}: $50\,ab^{-1}$
  - Physics data taking with full setup in March 2019
  - $531\,fb^{-1}$ has been recorded by July 2024
- Upgraded detectors, trigger and DAQ **vs** Belle



Introduction

Muon g-2 and HVP

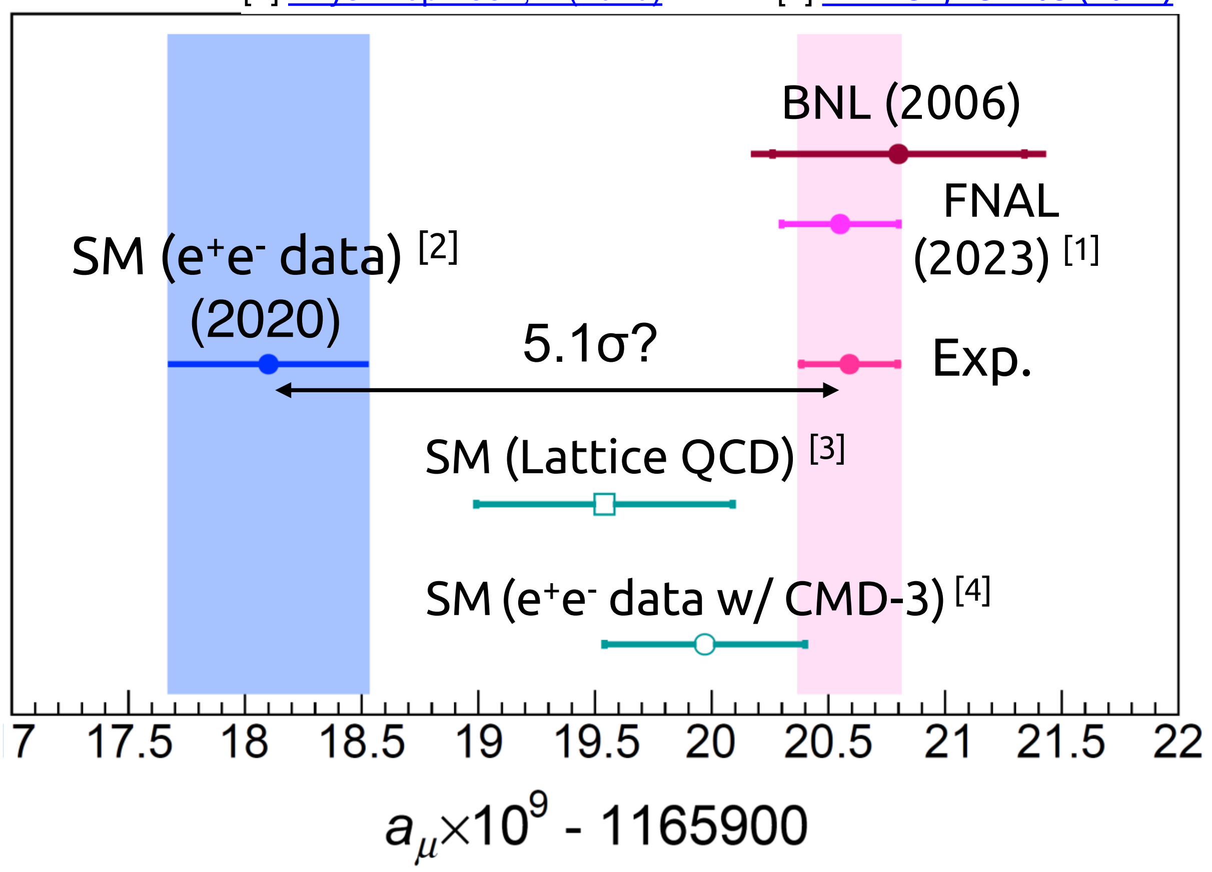
- Anomalous magnetic moment of muon in SM deviates from direct measurement by 5σ or $1\text{-}2\sigma$?

$$a_\mu^{\mathrm{SM}} \equiv (g_\mu - 2)/2 = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLBL}}$$

- Uncertainty is dominated (>80%) by the **leading order** (LO) **Hadronic Vacuum Polarization** (HVP)
  - Can be calculated by either **Lattice QCD** or
  - **Dispersion integral** over the bare cross section $\sigma^0(s)$ of $e^+e^- \to \mathrm{hadrons}(\gamma)$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty} \frac{K(s)}{s} R(s)ds$$

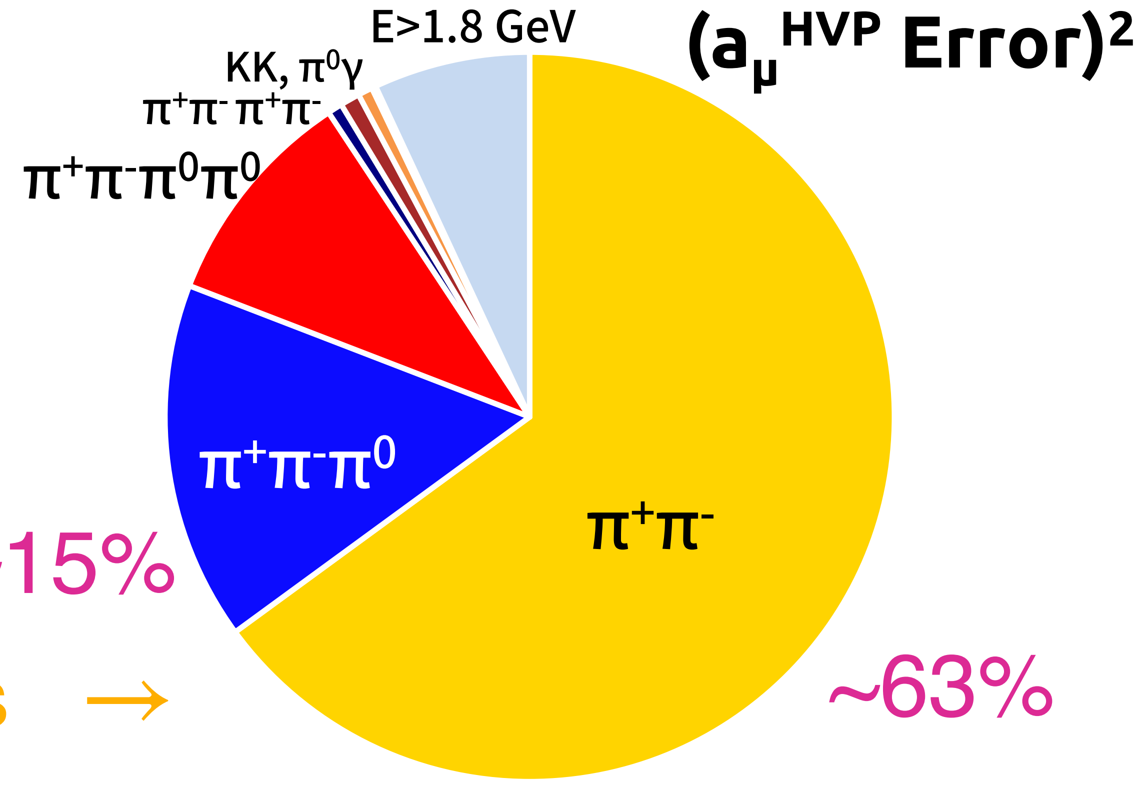
[1] PRL 131, 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)
[3] Nature 593, 7857 (2021)
[4] PRL 132, 231903 (2024)

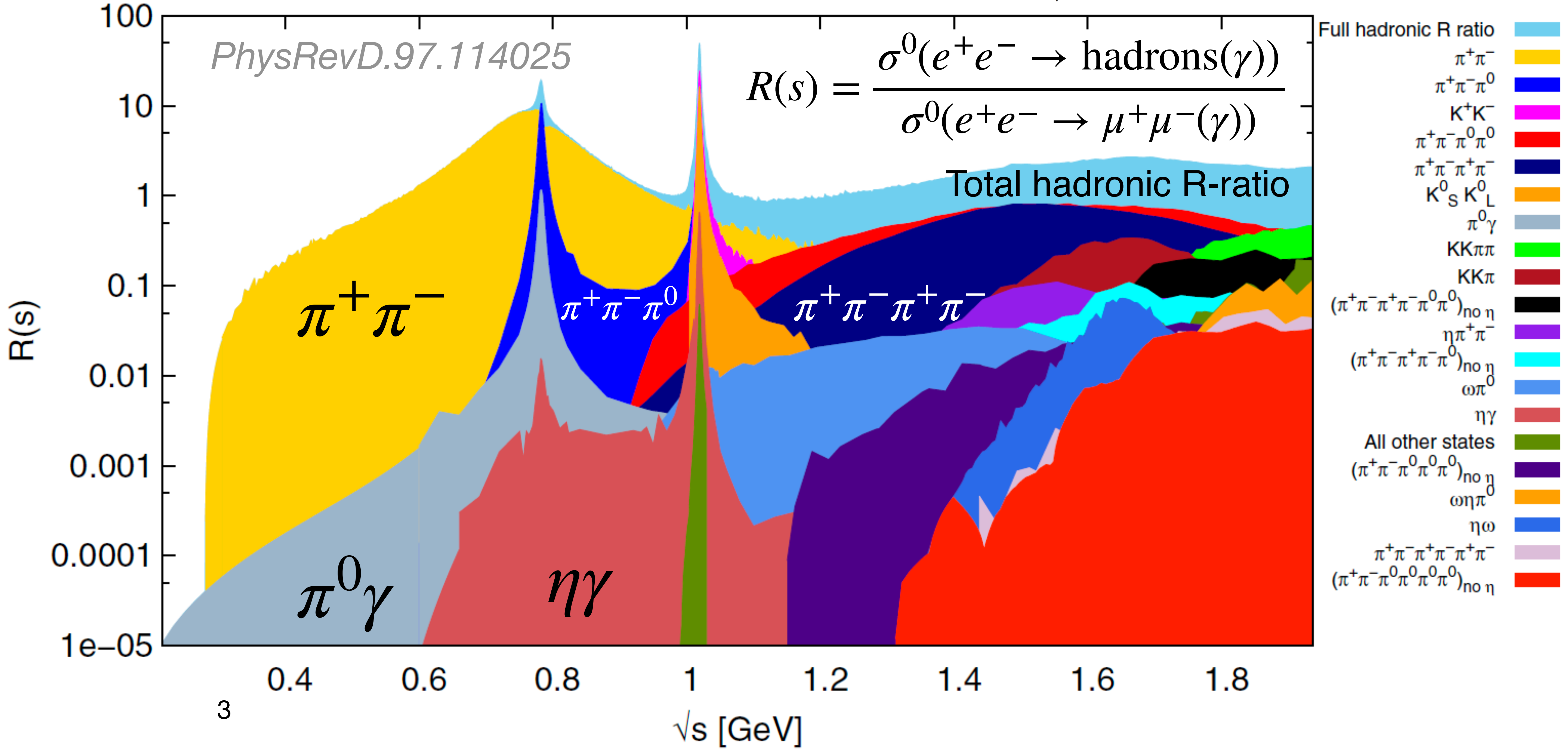


Belle II status

Measured →
arXiv:2404.04915

Work in progress →



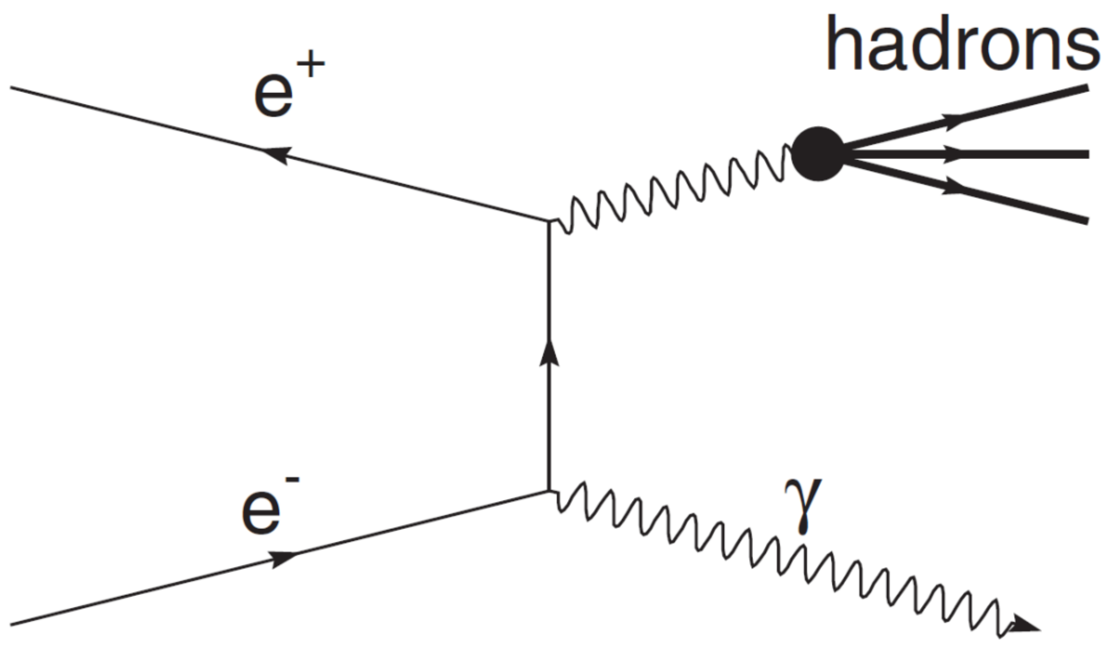


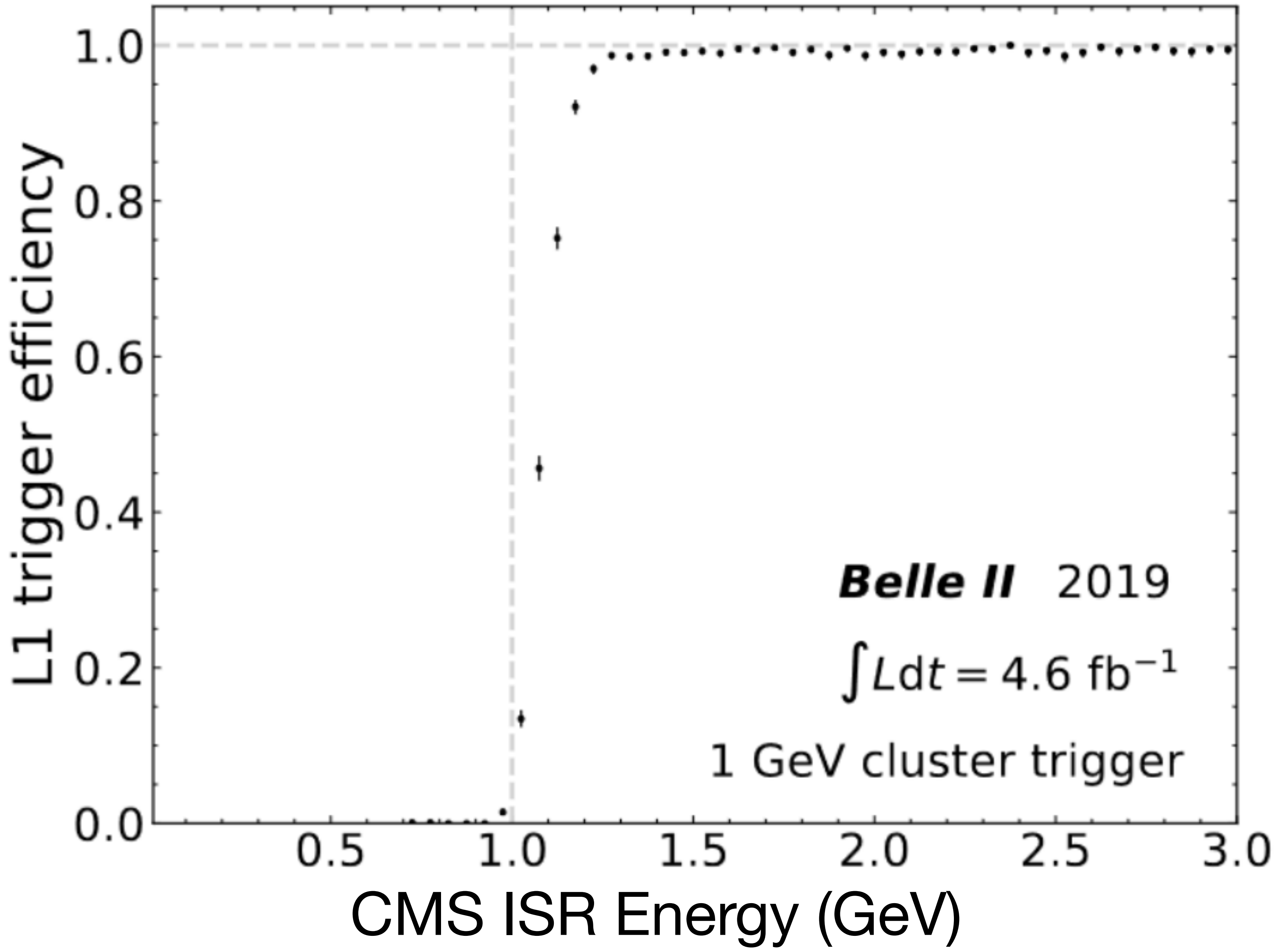
ISR method and trigger in Belle II

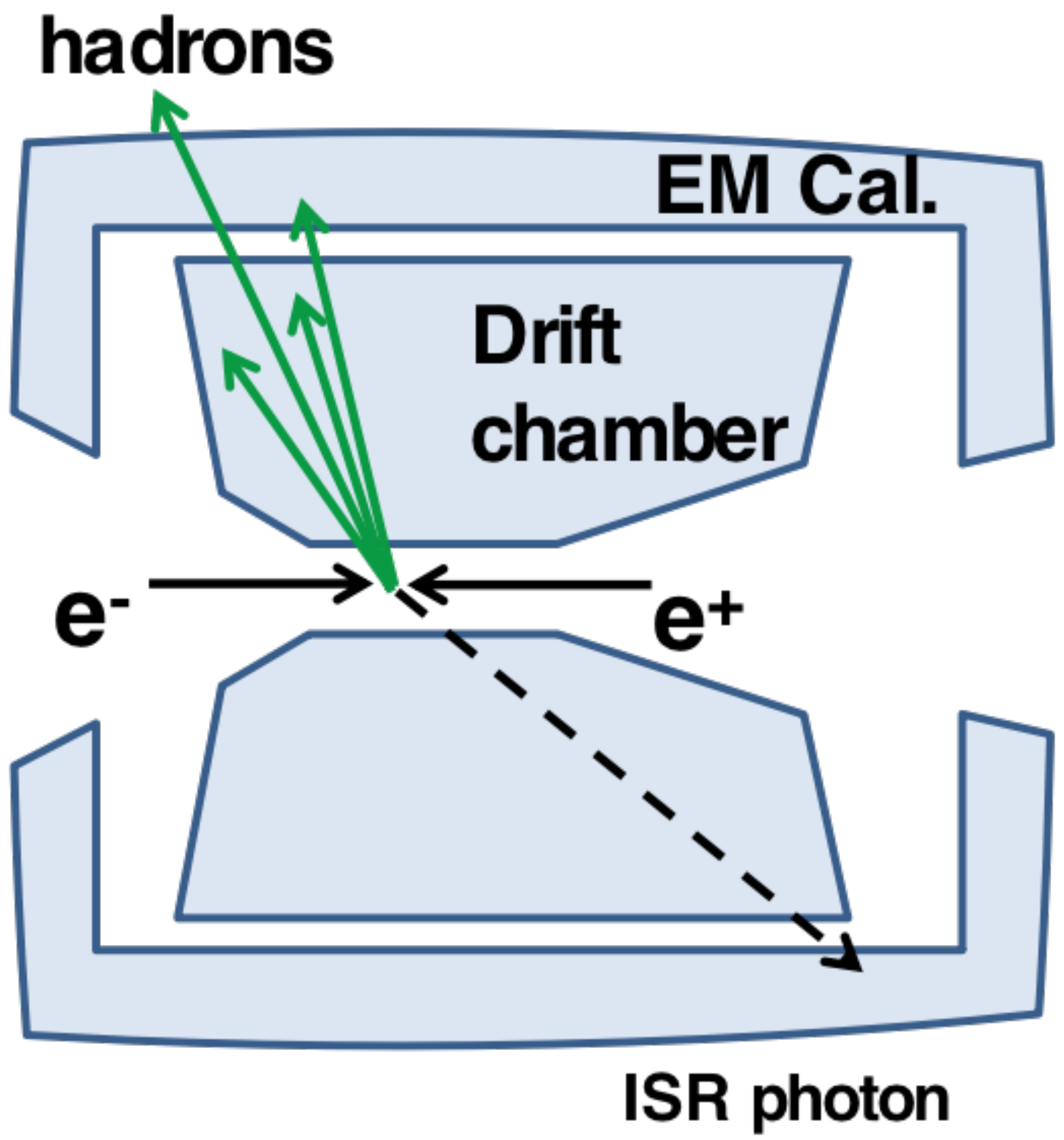
Scan over masses of the hadronic system via initial state radiation (ISR)

- Fixed center-of-mass energy $\sqrt{s} \approx 10.58$ GeV
- Scan $s' = (1 - 2E^*_\gamma/\sqrt{s})s$, E^*_γ is the ISR photon energy in c.m.s.
- Efficient **L1 trigger for ISR** events using ECL (cluster energy $\geq$ 2.0 GeV)
  - Studied with independent track trigger for **μμγ**: 99.9% in barrel region

  → 0.1% uncertainty   Not possible with Belle data !



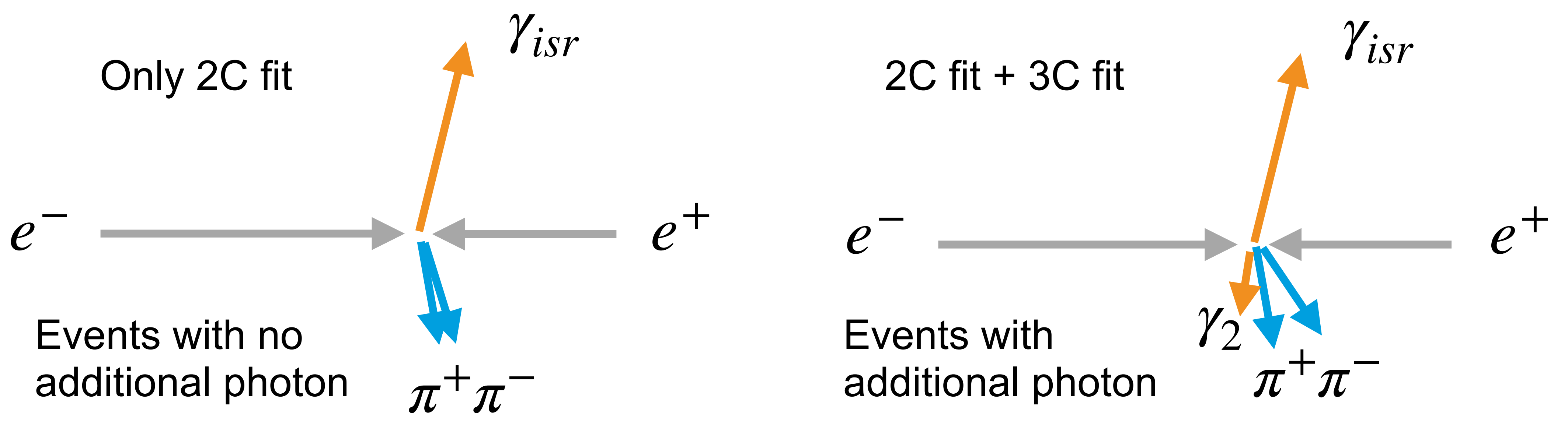


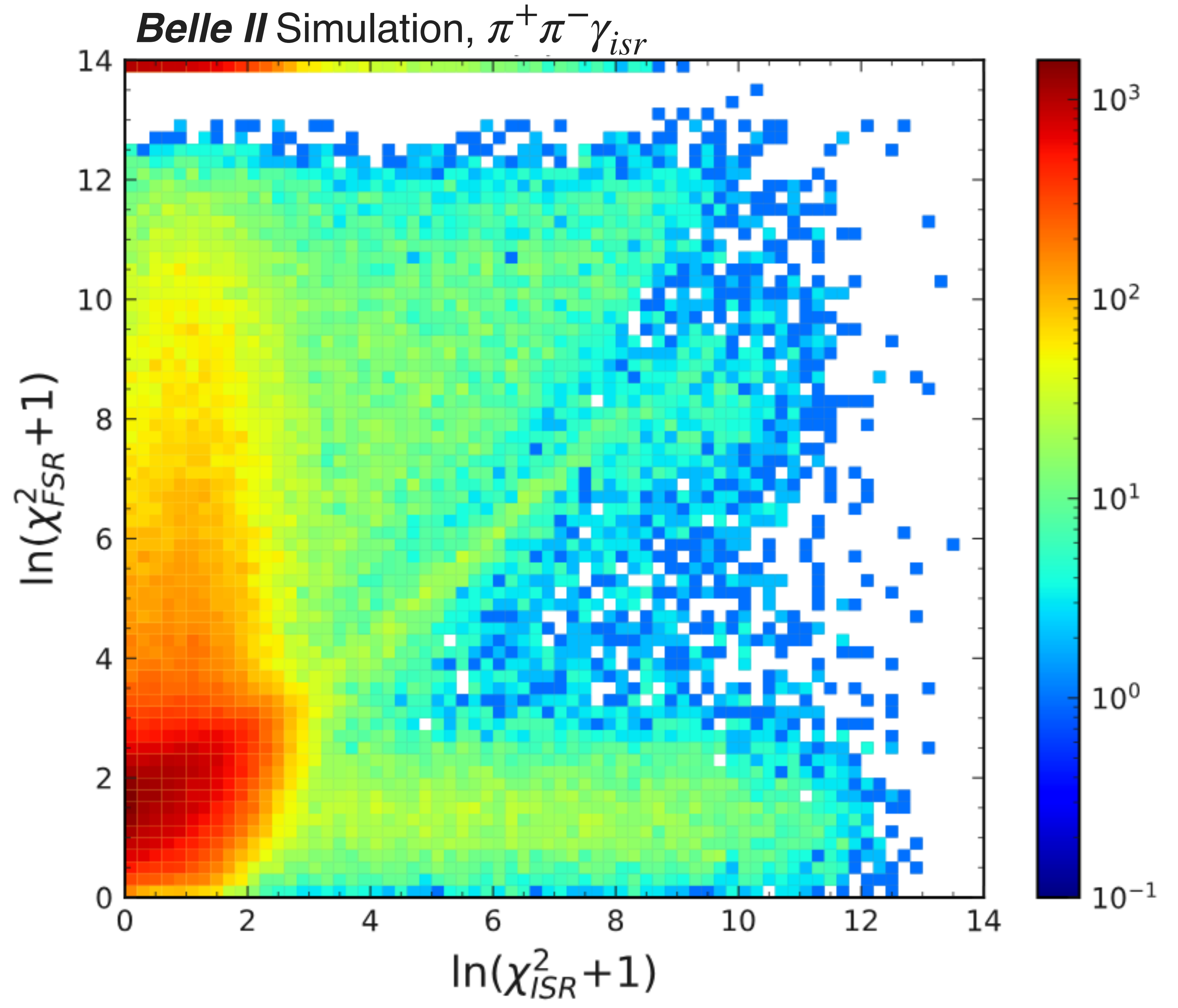


Status of $e^+e^- \to \pi^+\pi^-(\gamma)$ measurement

Following BaBar's approach [Phys. Rev. D 86, 032013]

- Reconstruction for **R-ratio** measurement
  - 1 hard photon + 1 optional photon
  - 2 tracks w/o particle identification (PID) in preselection
- Double kinematic fits for selecting signal events and disentangling QED corrections:
  - **2C "ISR" fit** for **all events after preselection**
    - 3 measured particles: 2 tracks and γ_{isr}
      - ISR energy not used
    - Assume 1 unmeasured photon (**ISR**) along beam directions
  - **3C "FSR" fit** only for **events with γ_2 reconstructed**
    - 4 measured particles: 2 tracks, γ_{isr} and γ_2
      - ISR energy not used
- **PID** to separate μμ/KK/ππ





Status of $e^+e^- \to \pi^+\pi^-(\gamma)$ measurement

Following BaBar's approach [Phys. Rev. D 86, 032013]

- Data set : 424 fb^{-1} (taken in Run1)
- Target precision: 0.5%
- **Successful sanity check** with < 2 fb^{-1} data
  - Good Data/MC ratio using preliminary selections
  - Confirmed high trigger efficiency for $\pi^+\pi^-\gamma_{ISR}(\gamma)$ events
- Single track **inefficiency** and **correlated track loss** have been studied with **MC**
  - Good agreement between the data-driven approach and the MC truth based one
- PID performance is being studied with "probe and tag" method

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Analysis overview

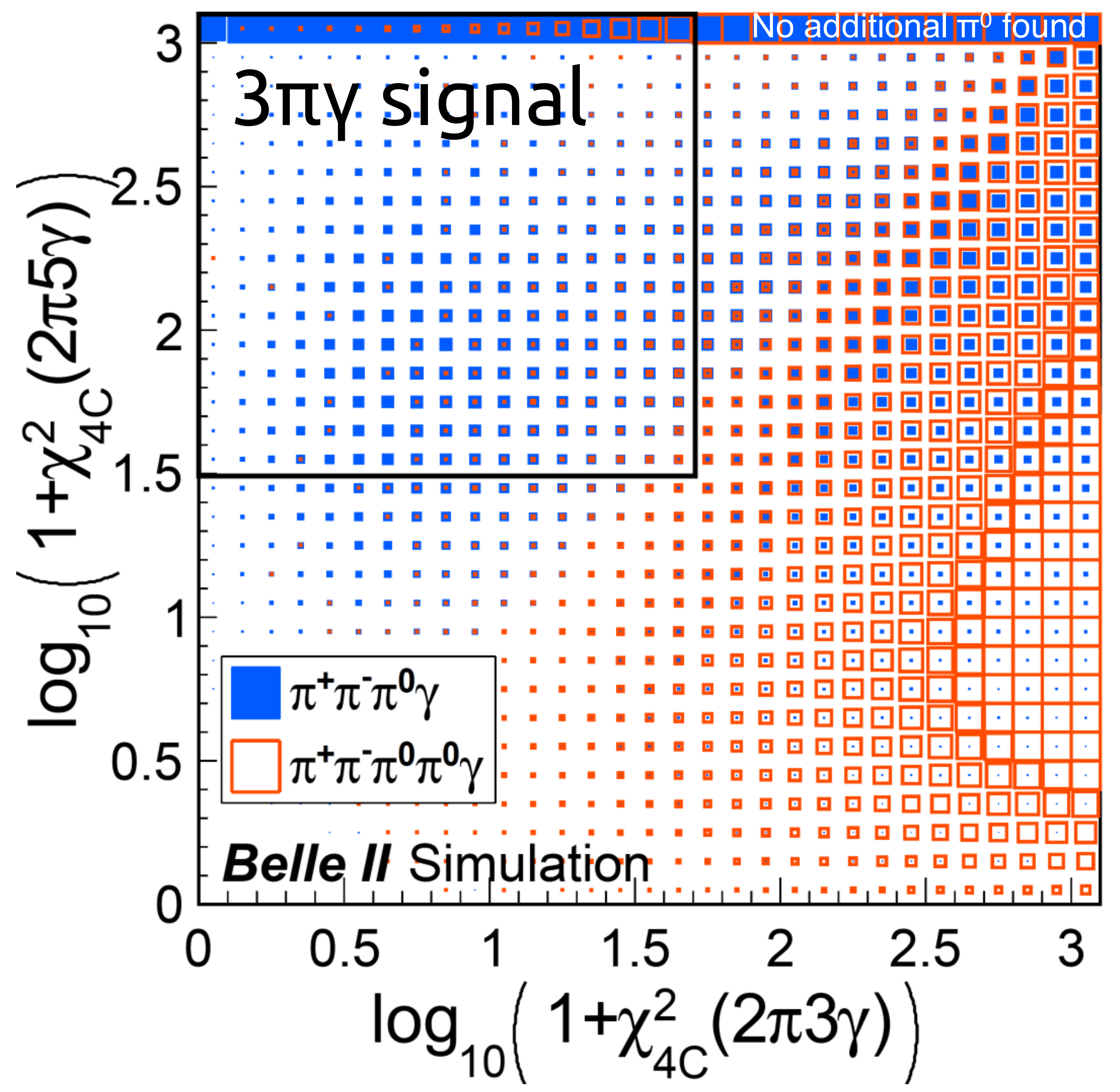
- Data set : 191 fb^{-1}

$$\sigma_{3\pi}(\mathrm{M}_{3\pi}) = \frac{\mathrm{N}_{\mathrm{signal}}}{\epsilon(\mathrm{M}_{3\pi}) \cdot \mathrm{L}_{\mathrm{eff}}(\mathrm{M}_{3\pi})}$$

- $\sqrt{s'}$ range: 0.62 to 3.5 GeV
- Robust event selection to extract $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{isr}$
  - Background determination and suppression (≤1% background at ω)
- Precise determination of the efficiency with ≤1% precision
- **Unfolding** the spectrum to mitigate detector resolution effects
- **Blind analysis**: all selections and corrections are determined with MC and control samples

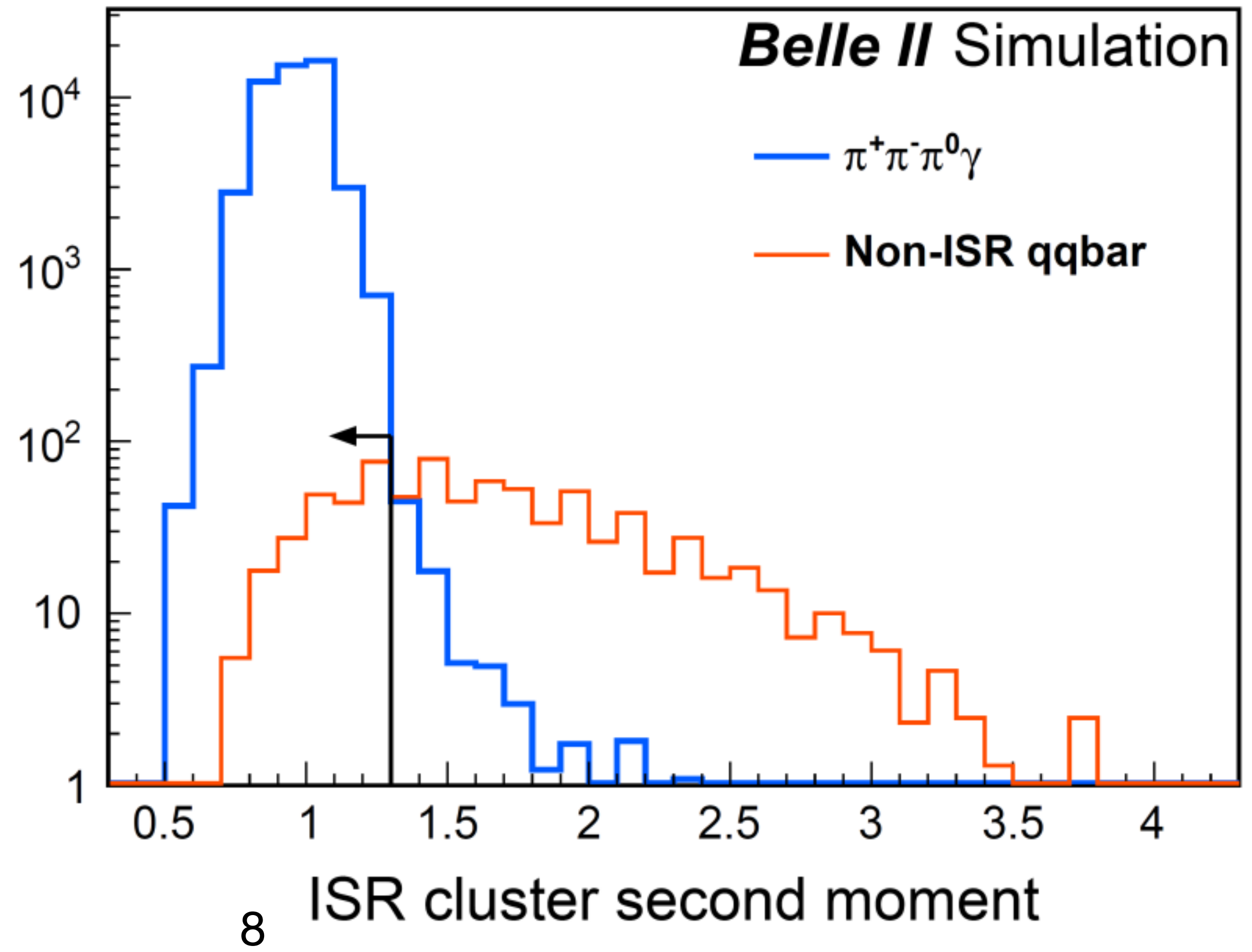
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

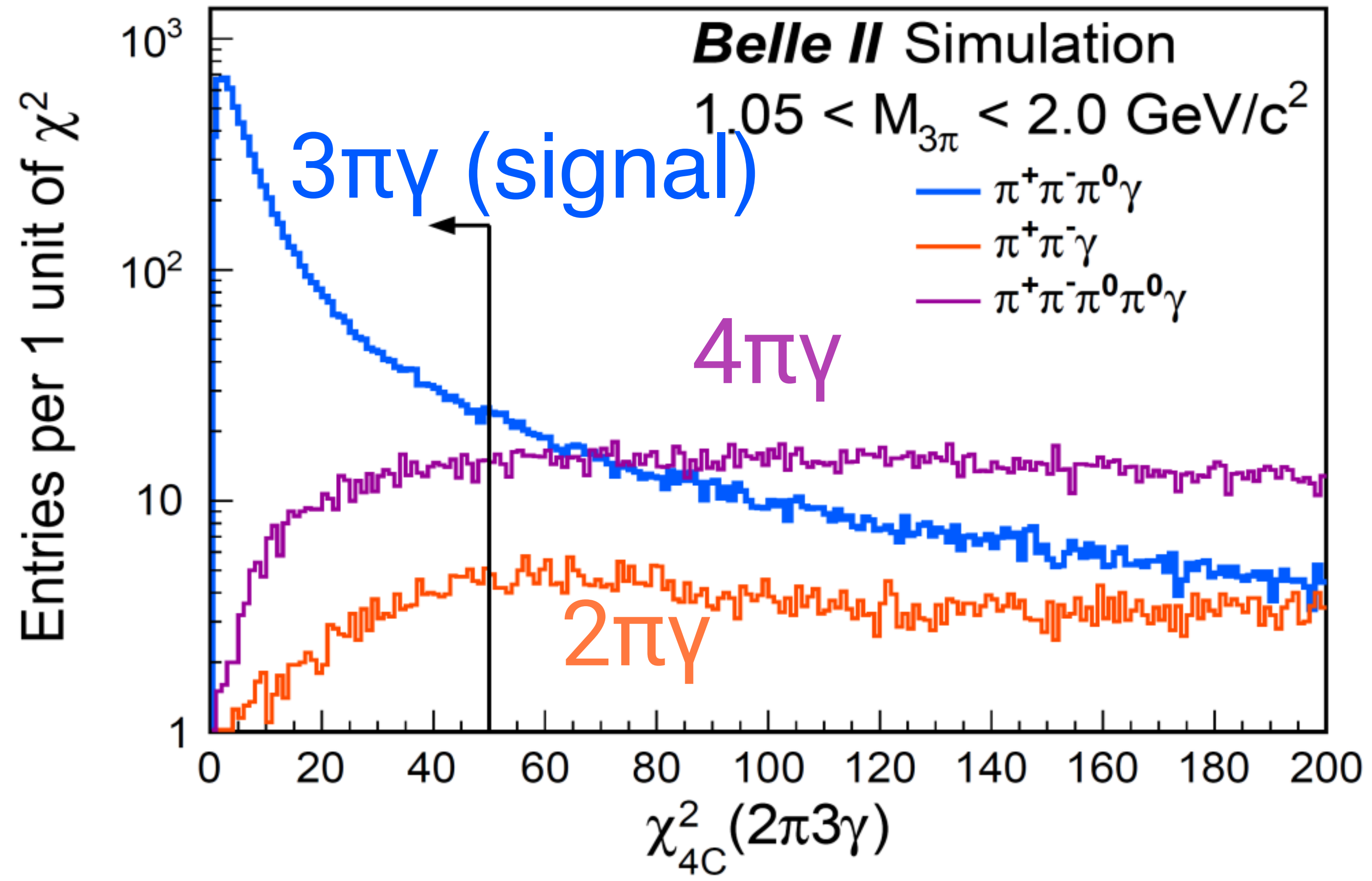
Event selection



- Reconstruct 2 tracks + 3 photons:
  $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{isr} \to \pi^+\pi^-\gamma\gamma\gamma_{isr}$
  - ISR photon: $\mathrm{E^{CMS}} > 4\,\mathrm{GeV}$ in ECL barrel region
  - $\pi^\pm$ from the IP with $\mathrm{p_T} > 0.2\,\mathrm{GeV}$, π/e ID > 0.1, π/K ID > 0.1
  - π^0: $\mathrm{E}_\gamma > 0.1\,\mathrm{GeV}$, $\mathrm{M}_{\gamma\gamma} < 1\,\mathrm{GeV}$, wide range for π^0 mass fit
- $\mathrm{M^2_{recoil}}(\pi^+\pi^-) > 4\,\mathrm{GeV^2/c^4}$ against non-π^0 events: $e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
- Four-momentum kinematic fit (**4C-Kfit**)
  - Constrain to initial e^+e^-
  - $\chi^2_{4C}(2\pi3\gamma) \leq 50$ and $\chi^2_{4C}(2\pi5\gamma) > 30$
- Suppress non-ISR background
  - $\mathrm{M}_{\pi^\pm\gamma_{isr}}$, $M_{\gamma_{isr}\gamma}$ and ECL cluster shape

High $p_{\rho^\pm\to\pi^\pm\pi^0}$ *High p_{π^0}*

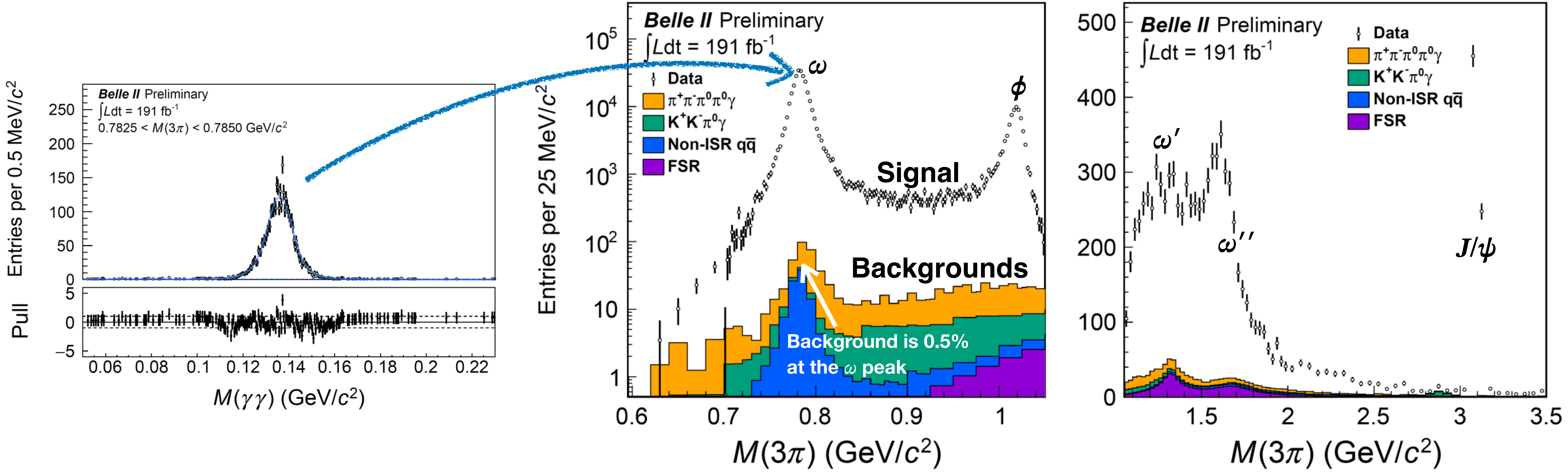




Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Signal extraction

- Fitting $M_{\gamma\gamma}$ spectrum in each $M_{3\pi}$ bin to extract π^0 signal
- Residual background estimated with **data-MC correction** factors



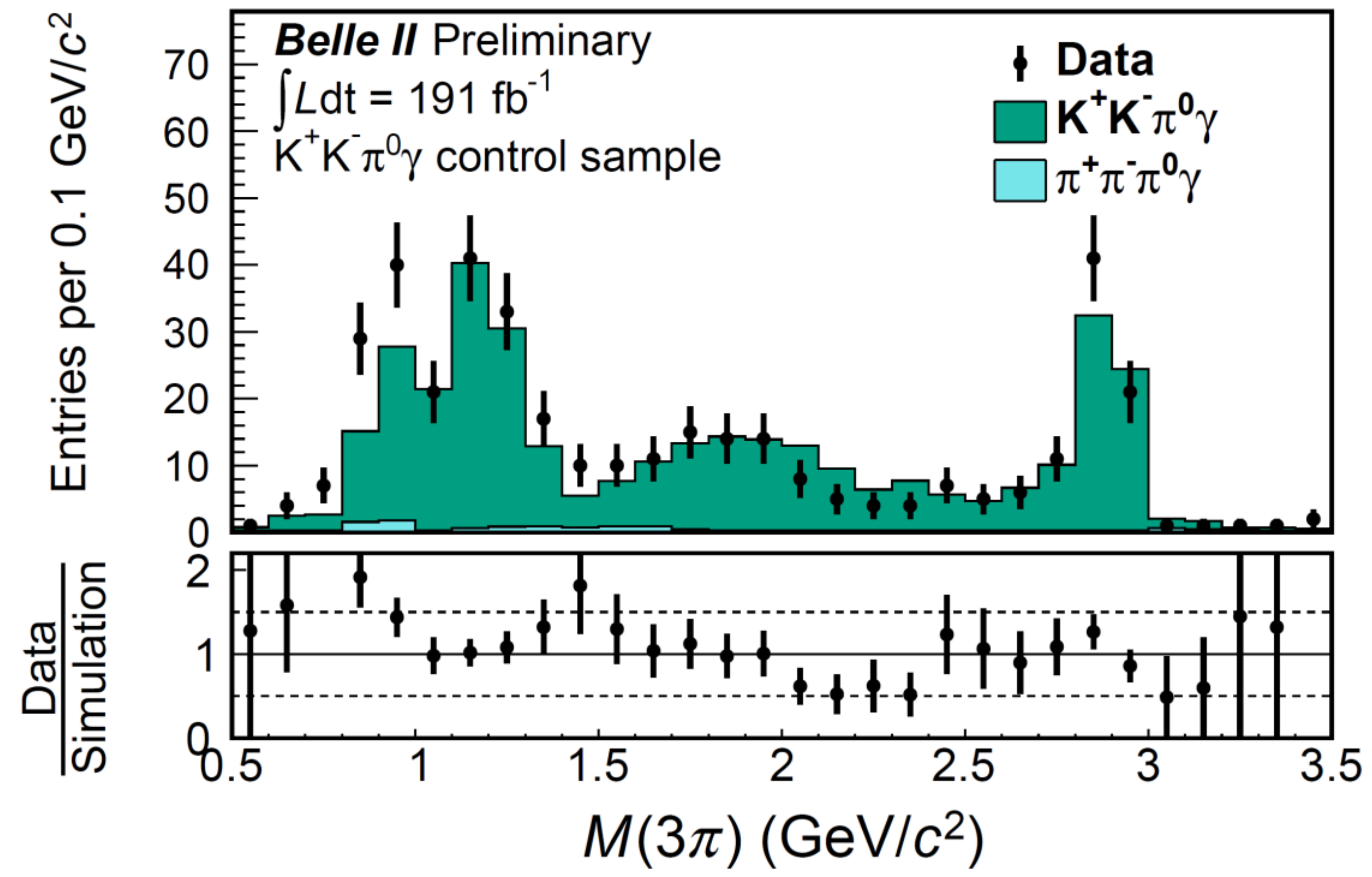
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

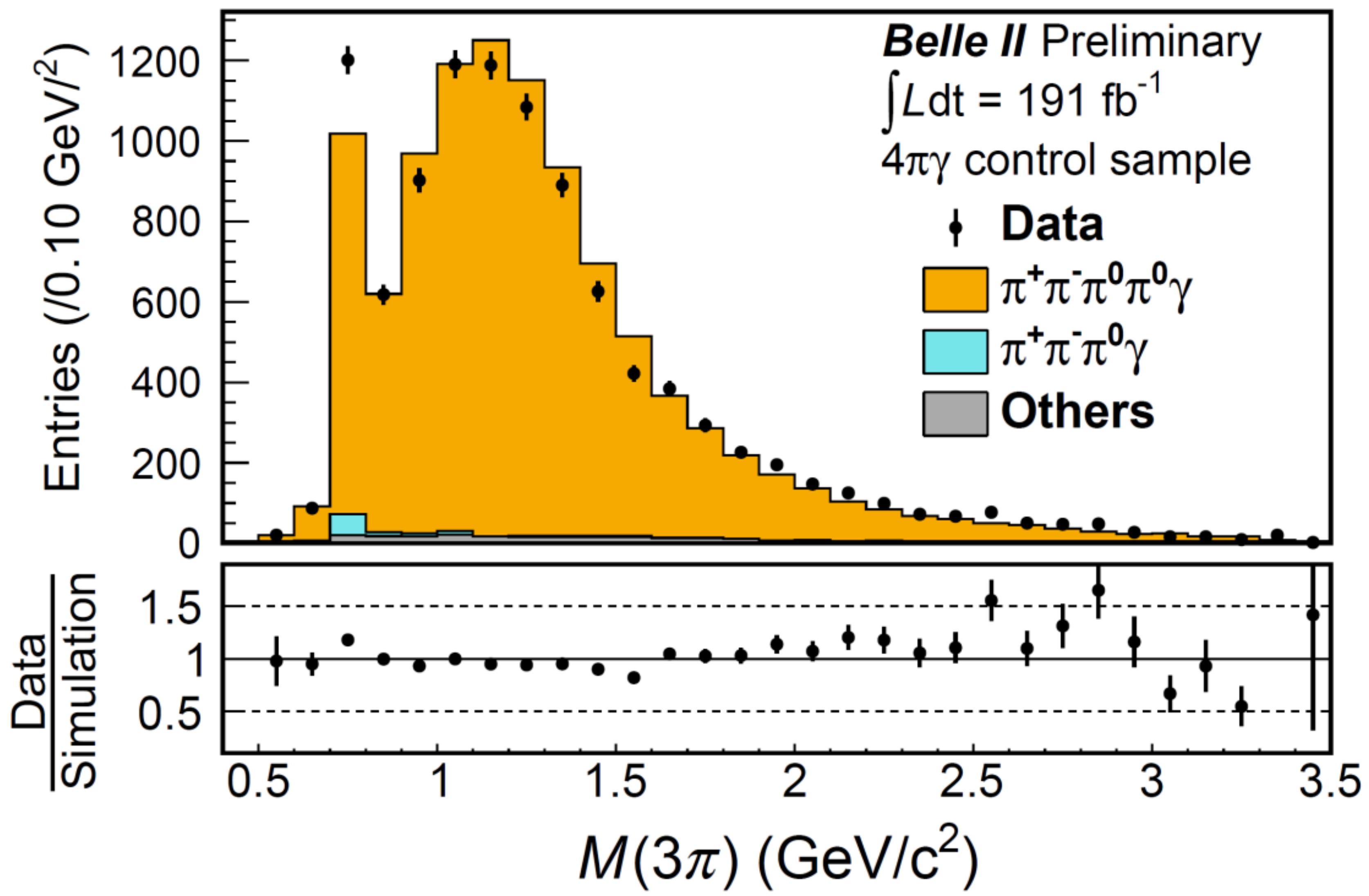
Background estimation and validation

Background enhanced data as a control sample to determine a **mass-dependent** data-MC **scale factor** :

$$N^{\text{data}}_{\text{Signal}} = N^{\text{MC}}_{\text{Signal}} \cdot \frac{N^{\text{data}}_{\text{Control}}}{N^{\text{MC}}_{\text{Control}}}$$

- $e^+e^- \to K^+K^-\pi^0\gamma$: Invert π/K-ID L(π/K) > 0.1 ⇒ L(π/K) < 0.1
- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2_{4\pi\gamma}$< 30
- Non-ISR $q\bar{q}$: 0.10 < $M_{\gamma_{isr}\gamma}$< 0.17 GeV or large cluster second moment

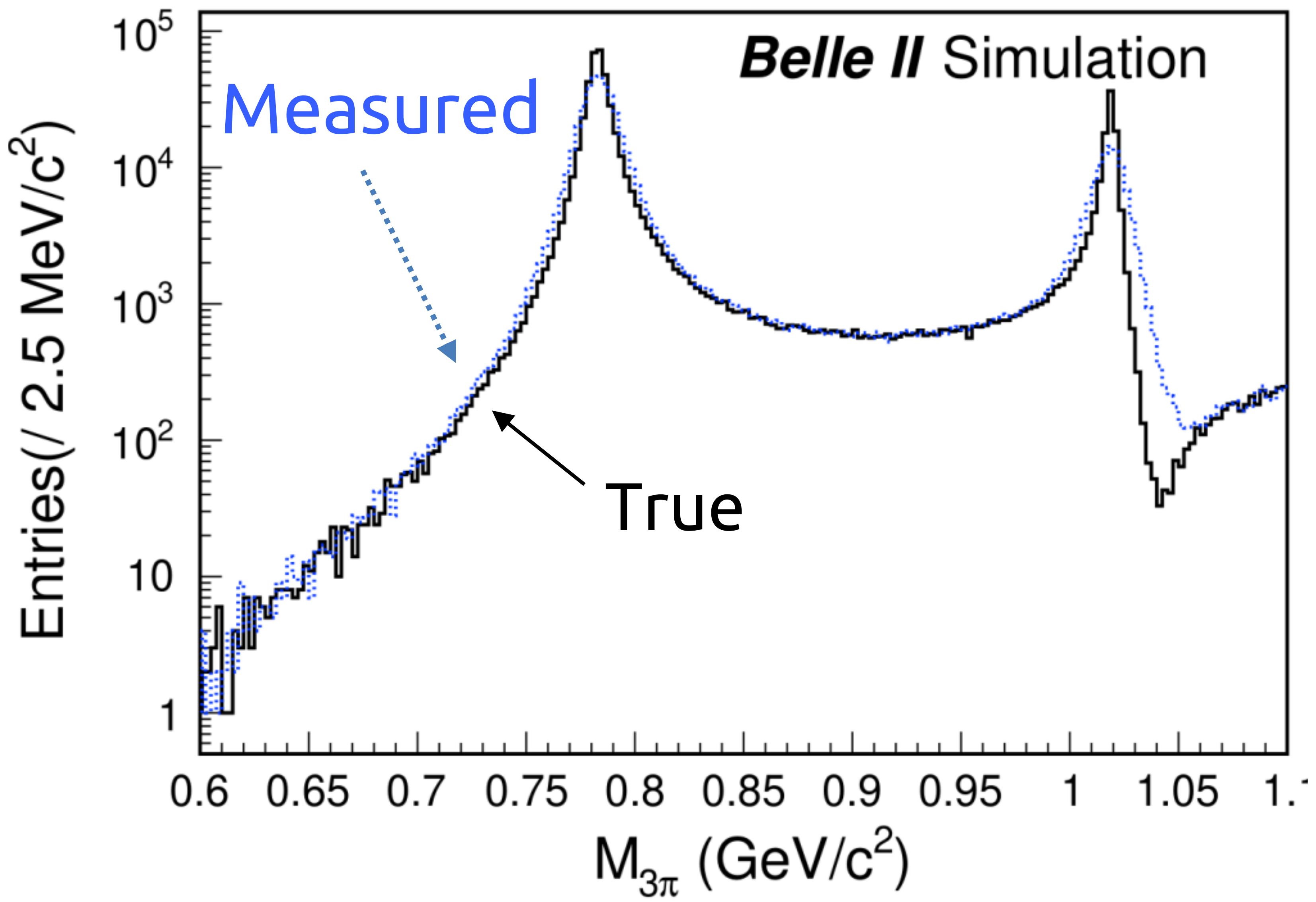


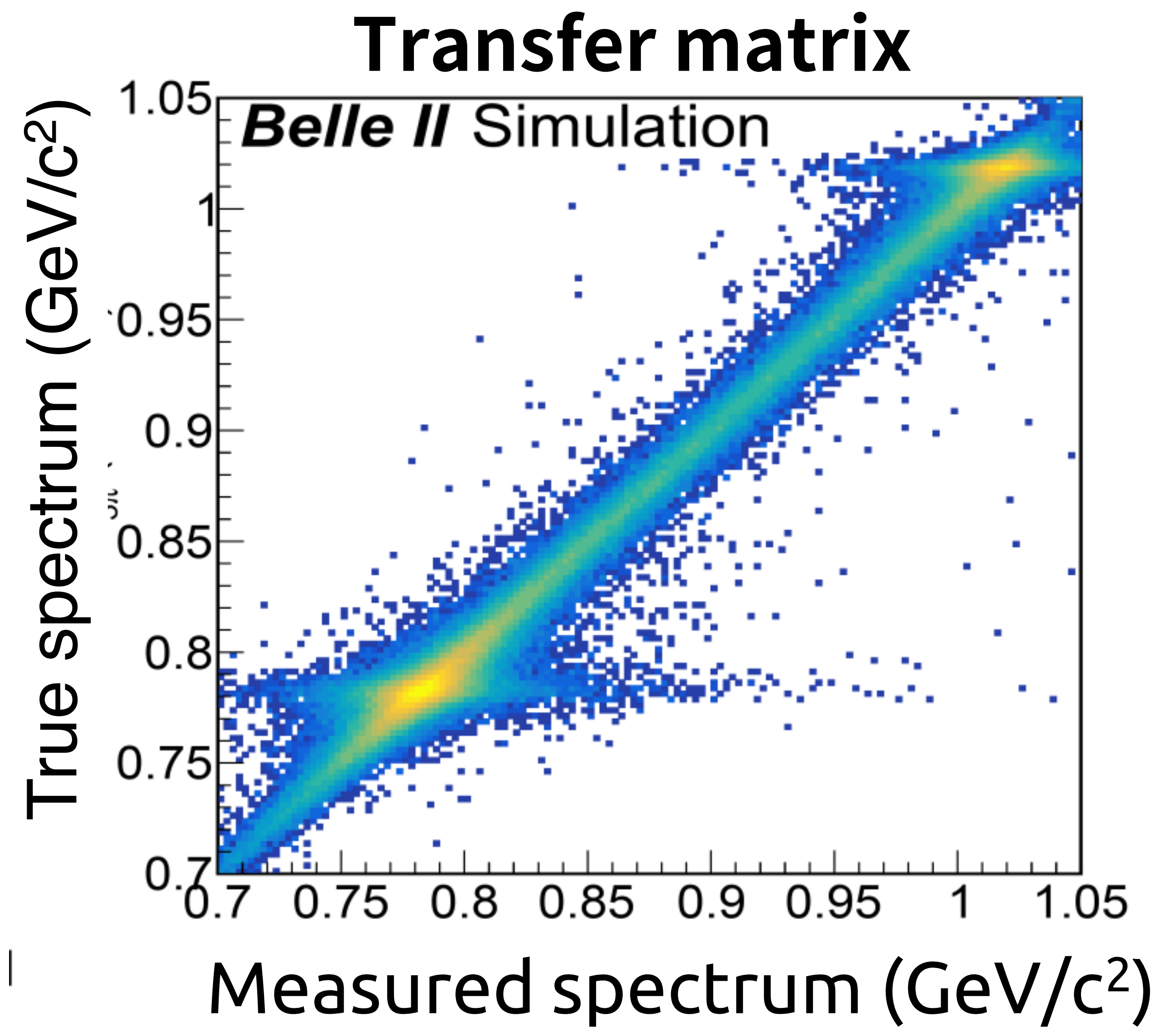


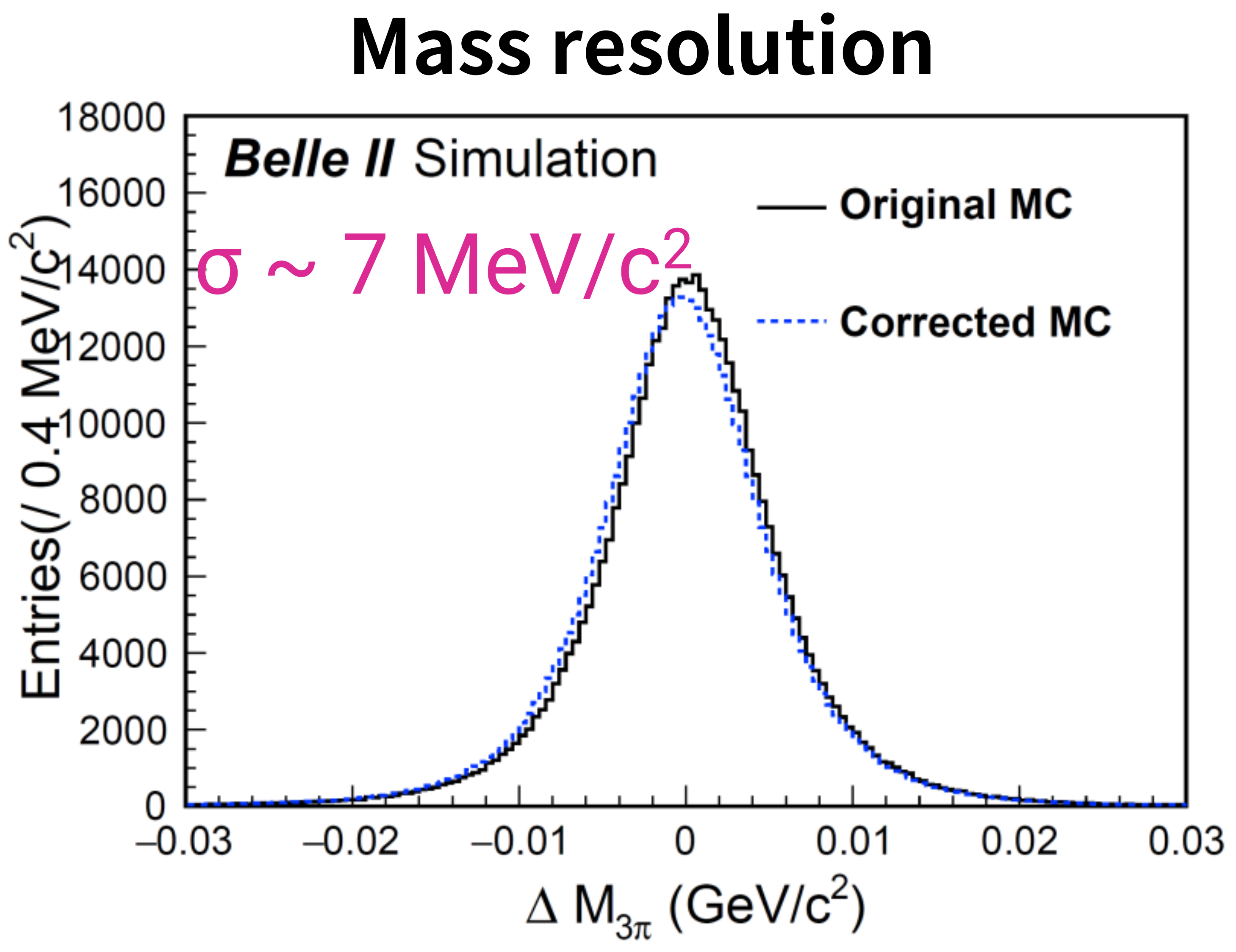
Measurement of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section

Unfolding to mitigate the effect of detector resolution

- Typical **mass resolution**: ~ 7-10 $\mathrm{MeV/c^2}$
- **Data-MC difference** of mass bias and detector resolution is studied with narrow peaks at ω, Φ, and **J/ψ** in data
  - Correct MC by 1 $\mathrm{MeV/c^2}$ for resolution and 0.5-1.5 $\mathrm{MeV/c^2}$ for mass shift





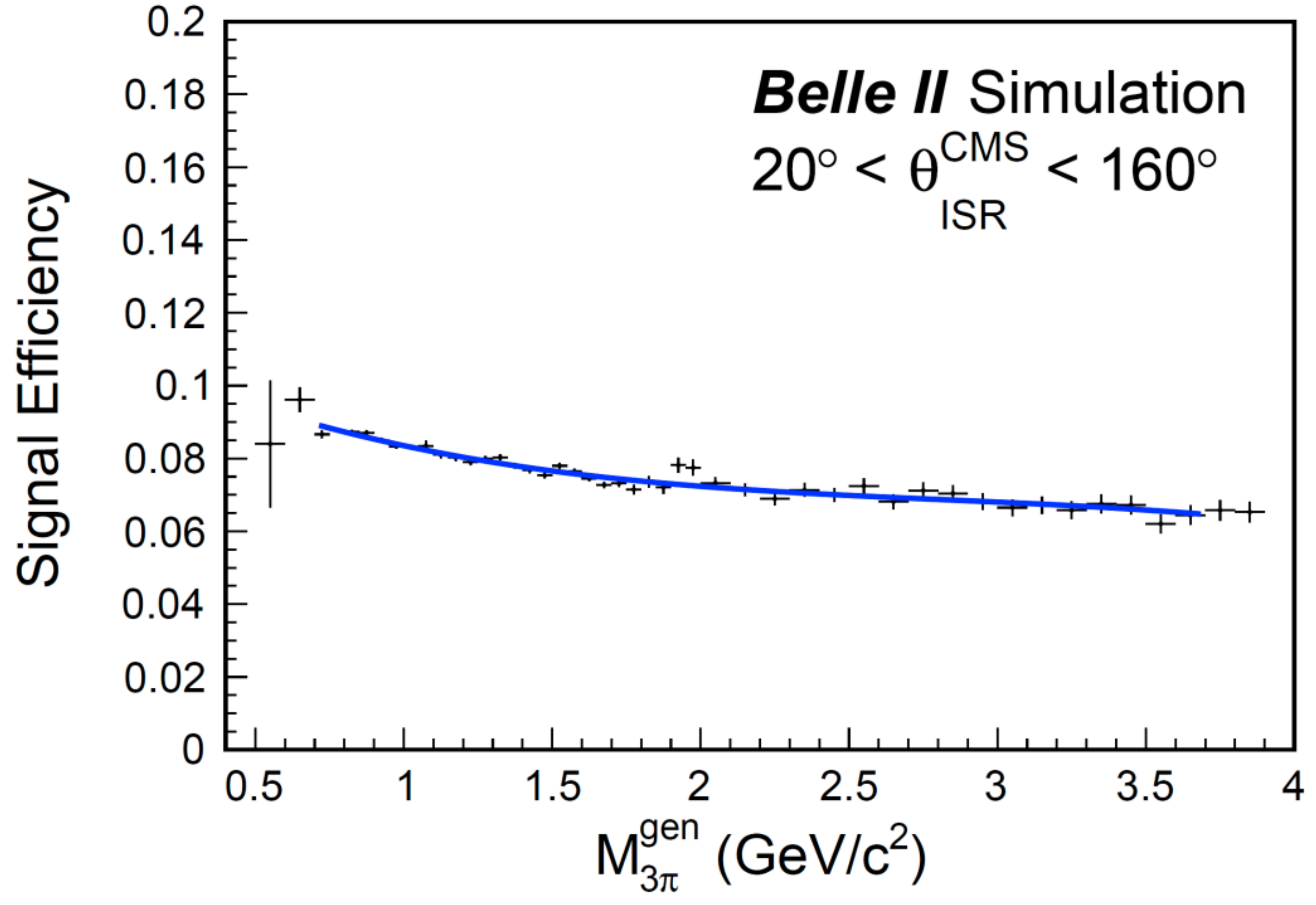


Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Signal efficiency and data-MC corrections

Efficiency $\epsilon = \epsilon_{MC}\prod_i (1+\eta_i)$, Data-MC **correction** η_i ~ O(1)%

- Signal efficiency is estimated with **MC** of **10 x larger** statistics
- Data-MC correction factors are studies with **data-driven methods** and different **control samples**

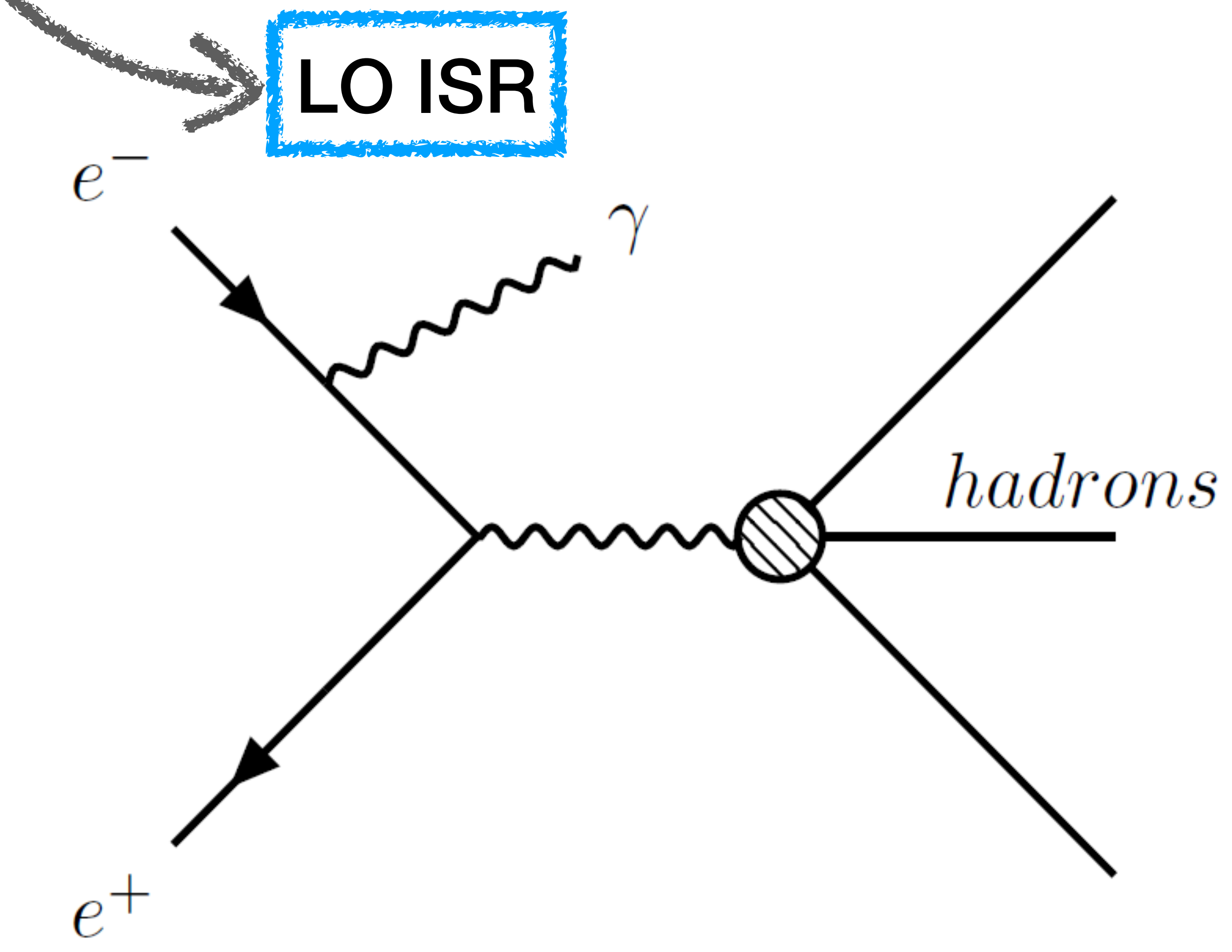


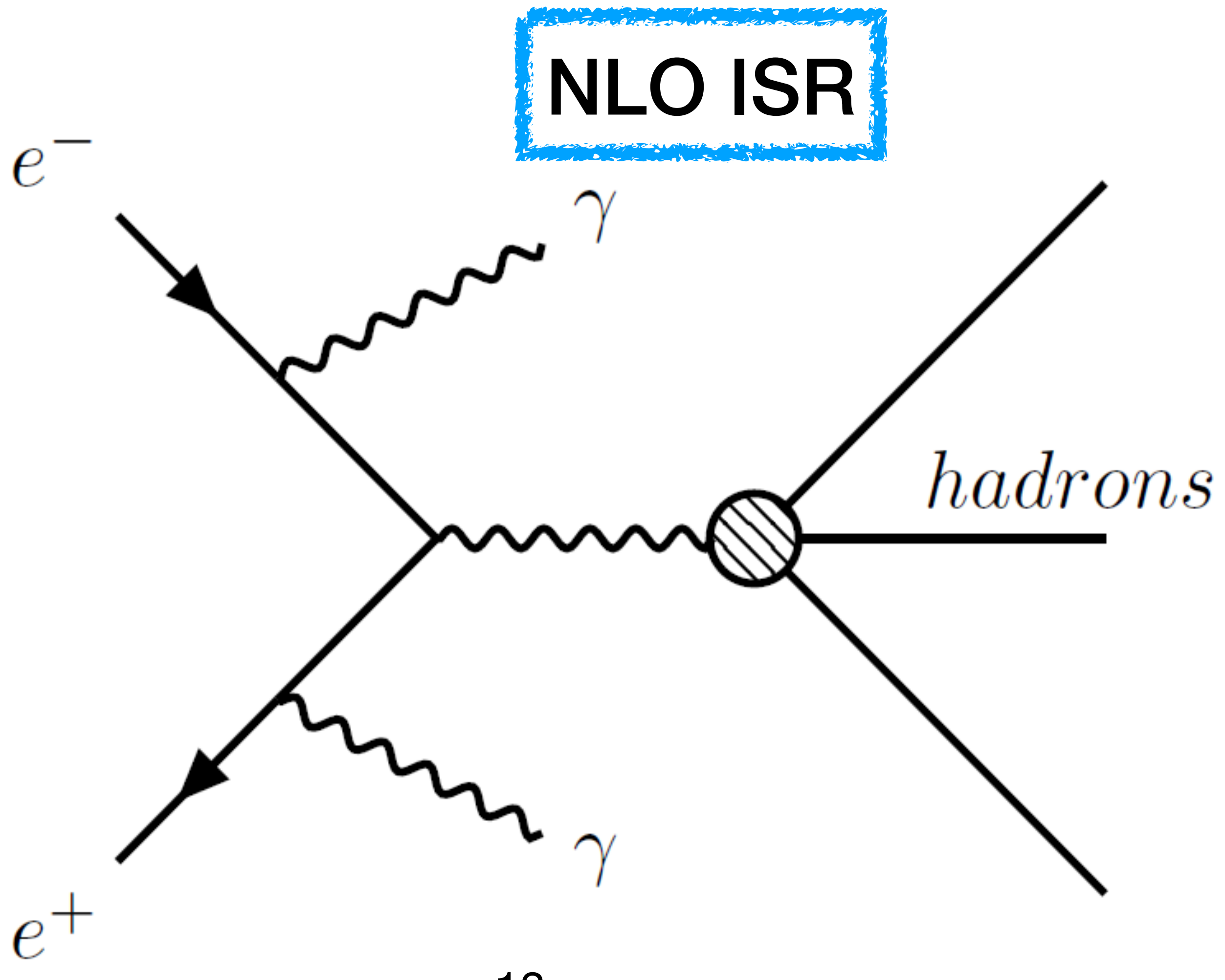
Sources	Efficiency correction η_i (%)
Trigger	-0.1±0.1
ISR photon detection	0.2±0.7
Tracking	-1.4±0.8
π^0 detection	-1.4±1.0
Background suppression	-1.9±0.2
χ^2 distribution	0.0±0.6
Total correction	-4.6±2.0

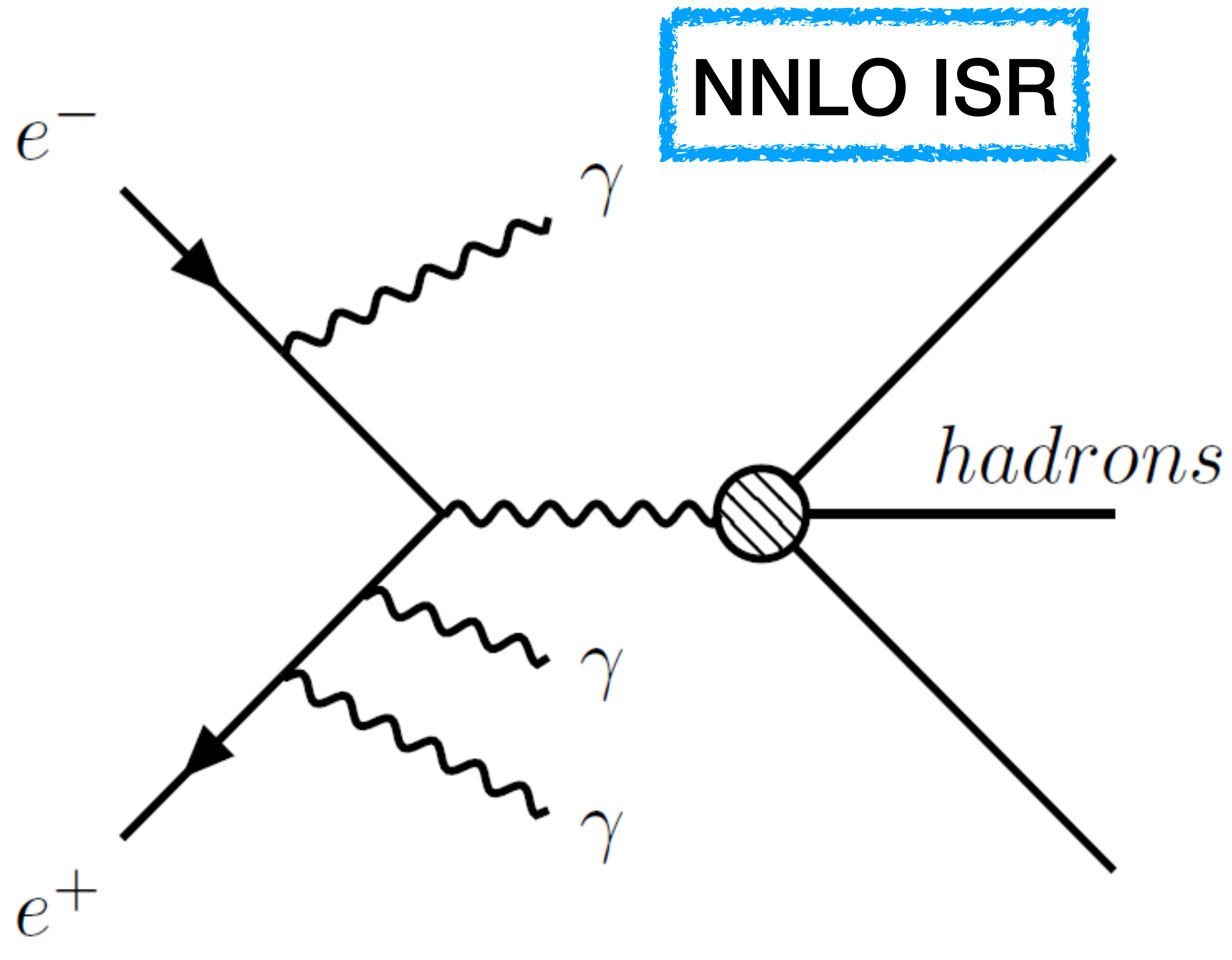
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Higher-order ISR effects

- Signal in this analysis: single ISR emission
  - In reality: There are processes with multiple ISR photon emissions
- Two effects of the existence of multiple ISR photons
  - Effective integrated luminosity L_{eff} (radiative correction): **0.5% unc.**
  - χ^2 selection efficiency due to ISR photon calculation in generator: **1.2% unc.**







Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Systematic uncertainty

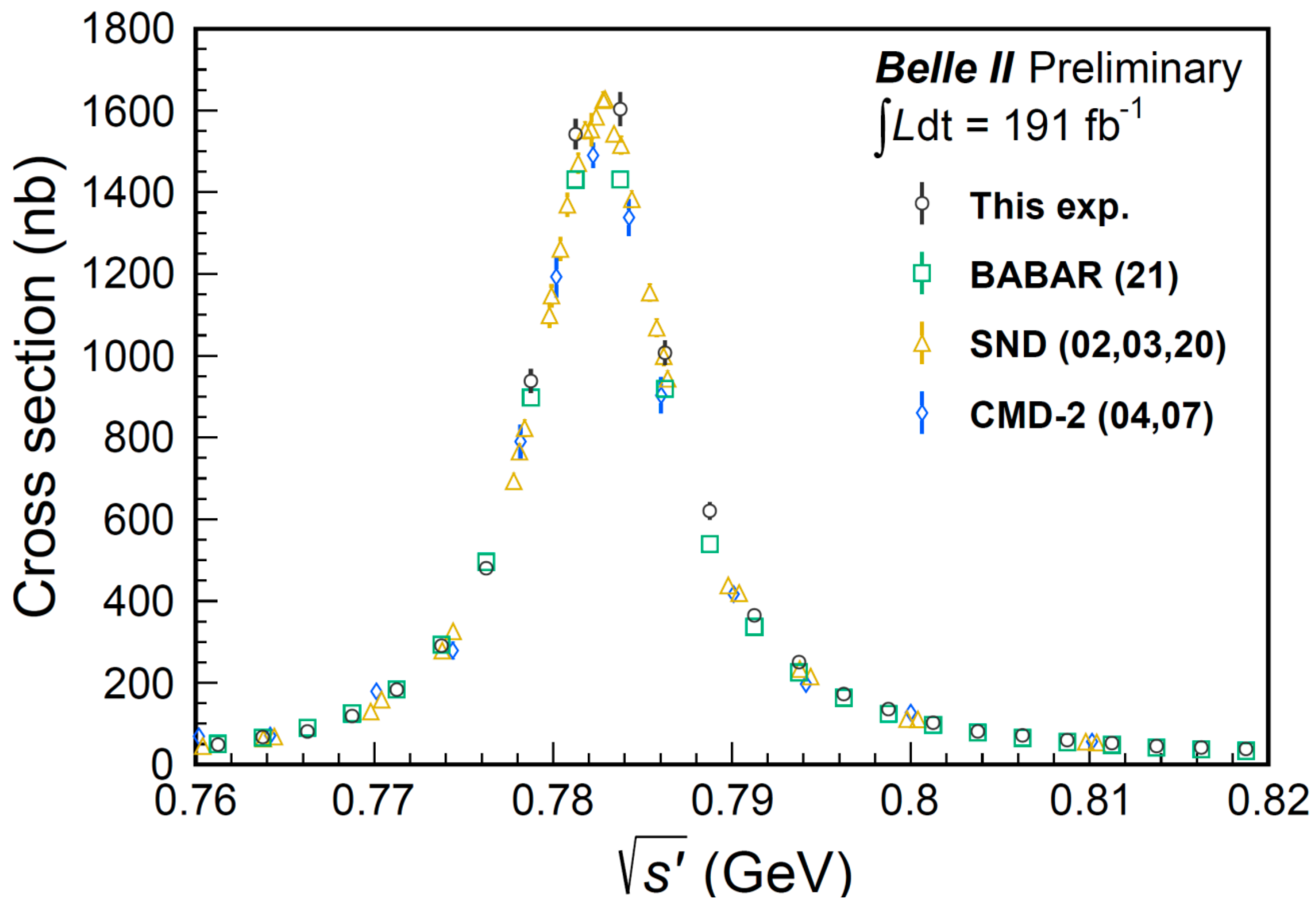
- **Luminosity** is measured with **Bhabha events**
- **Major systematic** uncertainty from MC generator and π^0 efficiency

Source	Systematic uncertainty (%)	
	$\sqrt{s} < 1.05$ GeV²	$\sqrt{s} > 1.05$ GeV
Trigger efficiency	0.1	0.2
ISR photon efficiency	0.7	0.7
Tracking efficiency	0.8	0.8
π^0 efficiency	1.0	1.0
χ^2 criteria efficiency	0.6	0.3
Background suppression efficiency	0.2	1.9
MC generator (due to missing NNLO MC)	1.2	1.2
Radiative correction	0.5	0.5
Integrated luminosity	0.6	0.6
Total systematics	2.2	2.8

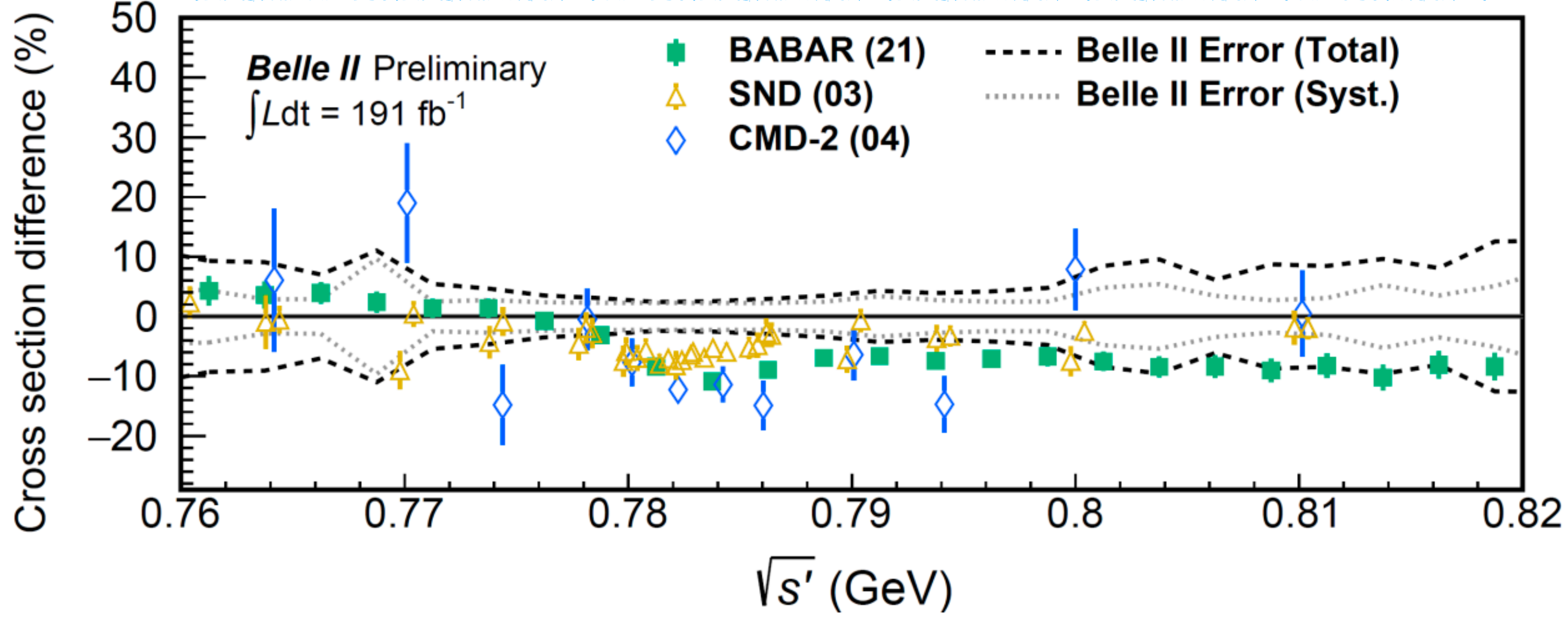
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Results: cross section at the ω resonance

- **ω** resonance has a large cross section and contributes largely to $a_\mu(3\pi)$
- Our result is **5-10%** higher than BaBar, SND, and CMD-2



Cross section differences compared to ours

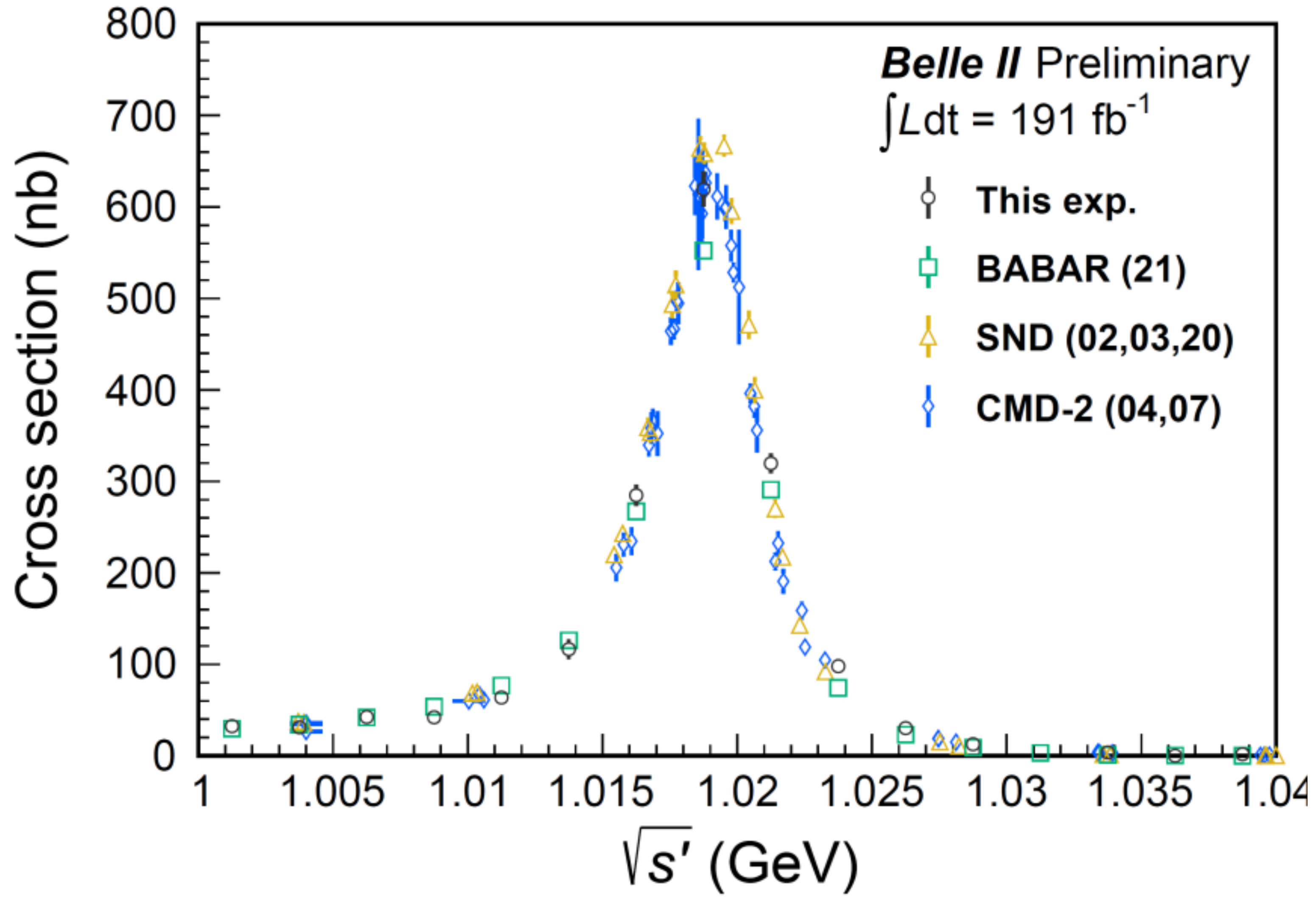


Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

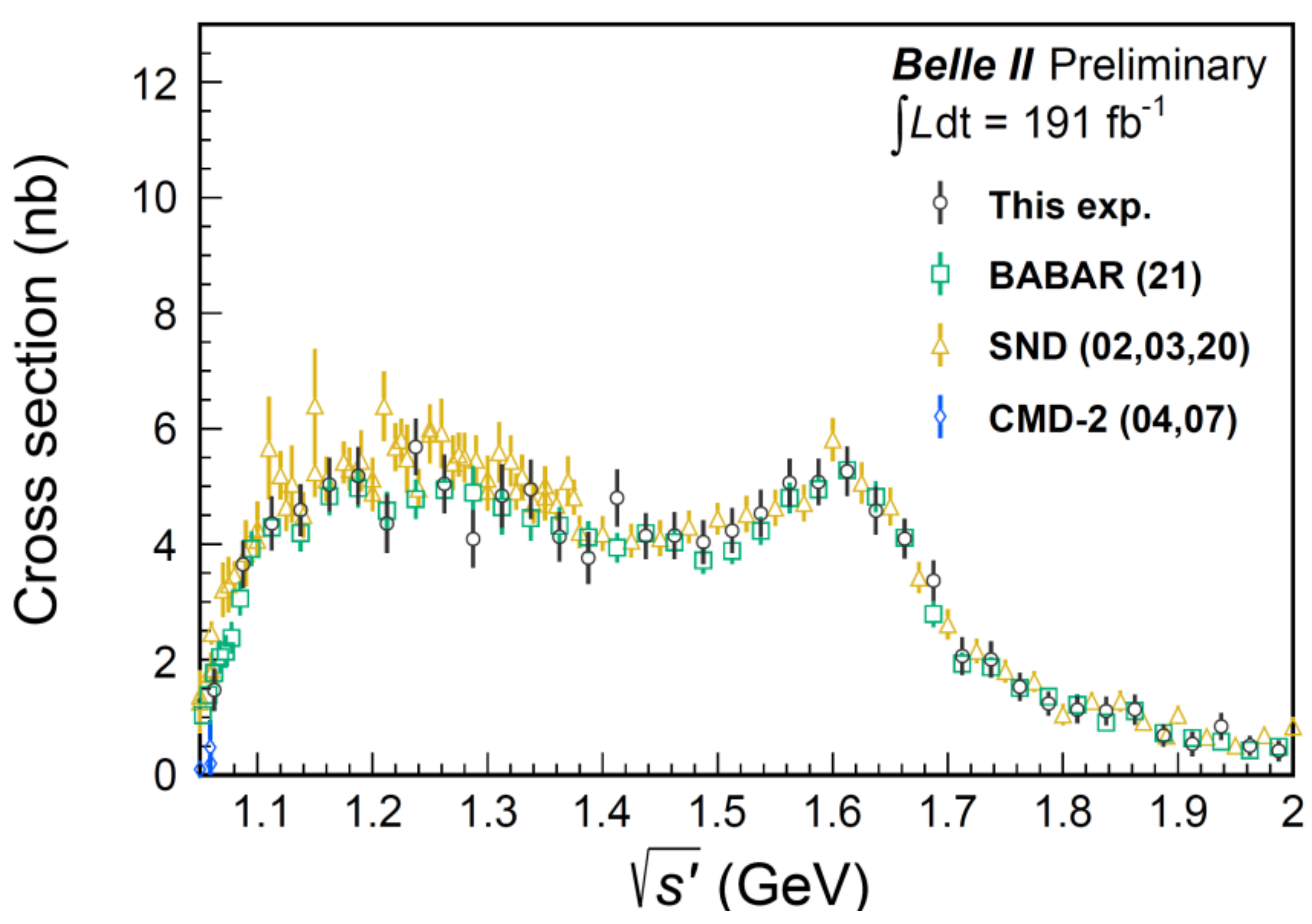
Results: cross section at higher energy

- Good agreement with BaBar's result

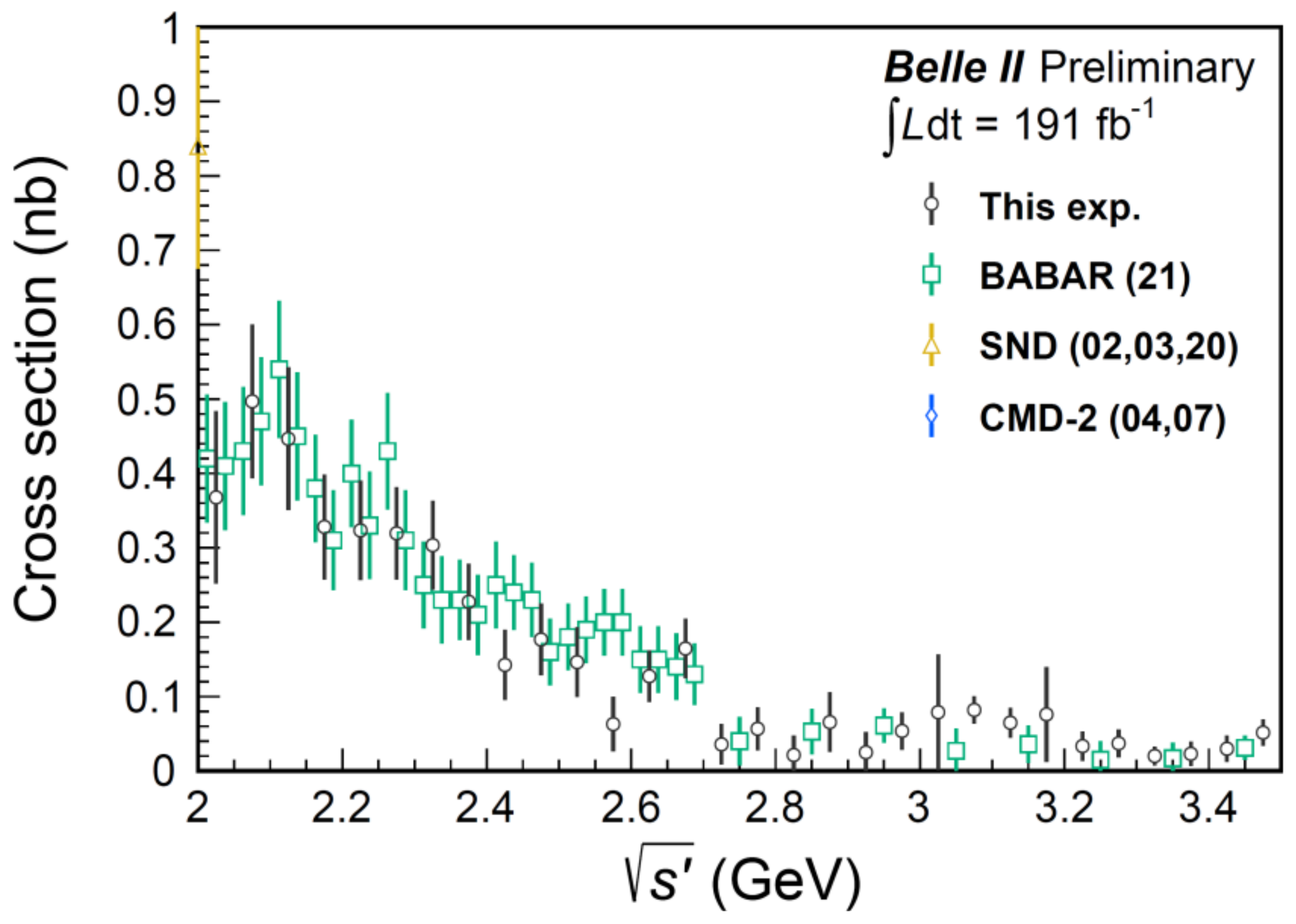
Φ(1020)



1.05-2.00 GeV



2.00-3.50 GeV



Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Results: 3π contribution to $a_\mu^{\text{LO,HVP}}$

- Using our result:

$$a_\mu^{\text{LO,HVP},3\pi}(0.62\text{-}\ 1.8\ \text{GeV}) = \left(48.91 \pm 0.25_{\text{stat}} \pm 1.07_{\text{syst}}\right) \times 10^{-10}$$

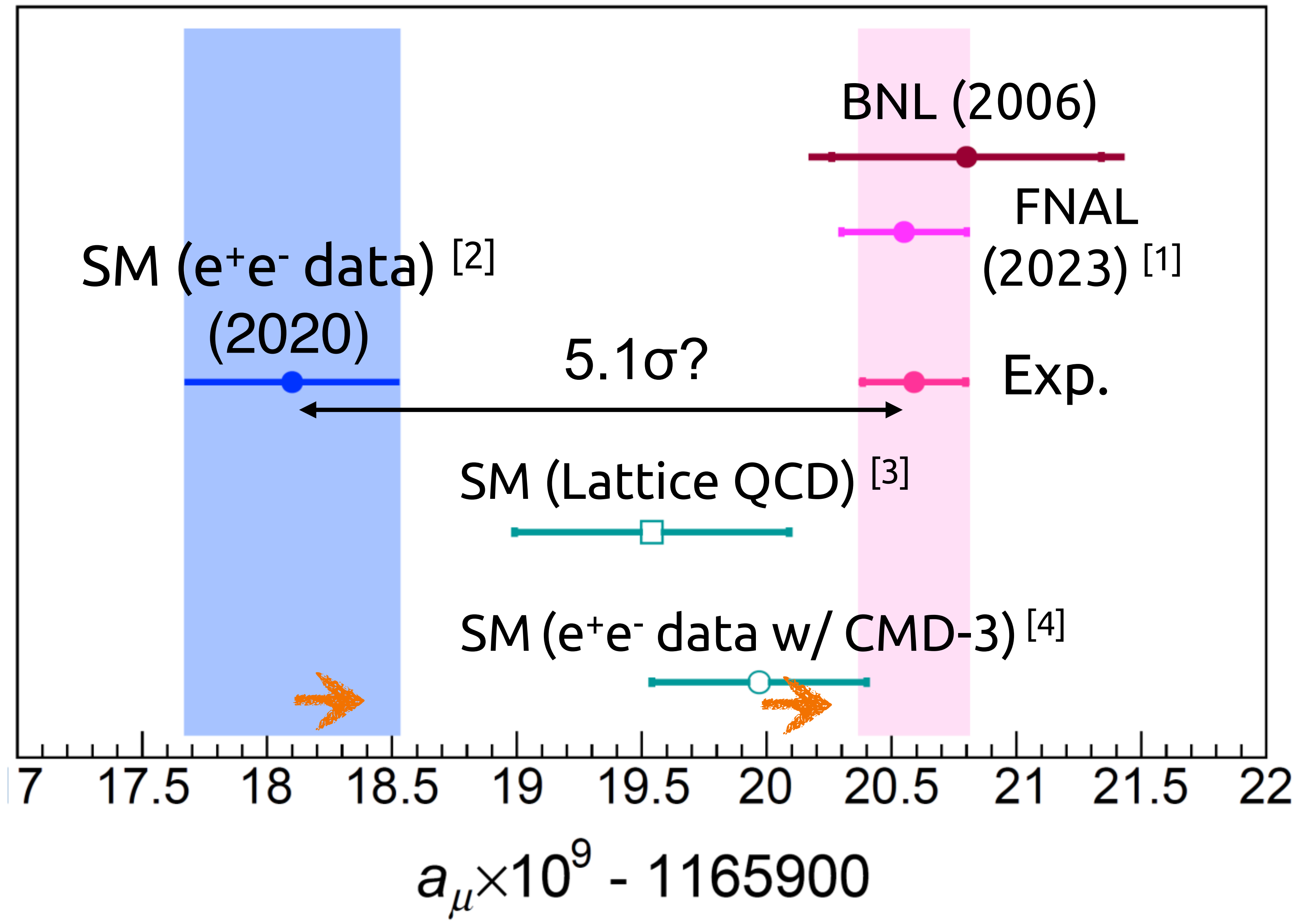
	$a_\mu(3\pi)\times10^{10}$	Difference×10^{10}
BABAR alone [PRD 104, 11 (2021)]	45.86 ± 0.14 ± 0.58	3.2±1.3 (6.9%)
Global fit* [JHEP 08, 208 (2023)]	45.91 ± 0.37 ± 0.38	3.0±1.2 (6.5%)

* Not includes BESIII preliminary result [arXiv:1912:11208]

- 6.5% higher than the **global fit** result with $2.5\,\sigma$ significance
- The difference, 3×10^{-10}, corresponds to **10% of**

$$\Delta a_\mu = a_\mu(\text{Exp}) - a_\mu(\text{SM}) = 25 \times 10^{-10}$$

WP2020



Summary

- **High** ISR trigger efficiency at Belle II
  - studied with **orthogonal trigger** lines
- $e^+e^- \to \pi^+\pi^-(\gamma)$ study is **ongoing**
- Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section
  - **Submitted** to PRD [arXiv:2404.04915]
  - First cross-section measurement for a_μ^{HVP} using the ISR method at Belle II
  - Systematic uncertainty of **2.2% at ω**
  - Our $a_\mu^{LO,HVP}(3\pi)$ is about $2.5\,\sigma$ larger than BaBar's and the global fit
  - **NNLO QED generators** are crucial for further improvement

Systematic uncertainty in $a_\mu^{LO,HVP}(3\pi)$

Source	Systematic uncertainty (%)
Efficiency corrections	1.63
Monte Carlo generator	1.20
Integrated luminosity	0.64
Simulated sample size	0.15
Background subtraction	0.02
Unfolding	0.12
Radiative corrections	0.50
Vacuum polarization corrections	0.04
Total	2.19

Thanks!

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Cross section calculation

Unfolded signal spectrum

Cross section

$$\sigma_{ee\to 3\pi}(M_i(3\pi)) = \frac{N_{\mathrm{unfolded},i}}{\varepsilon(M_i(3\pi)) \cdot L_{\mathrm{eff}}(M_i(3\pi)) \cdot r_{\mathrm{rad}}}$$

3π mass at i-th bin

Radiative correction

Effective luminosity

Corrected Efficiency

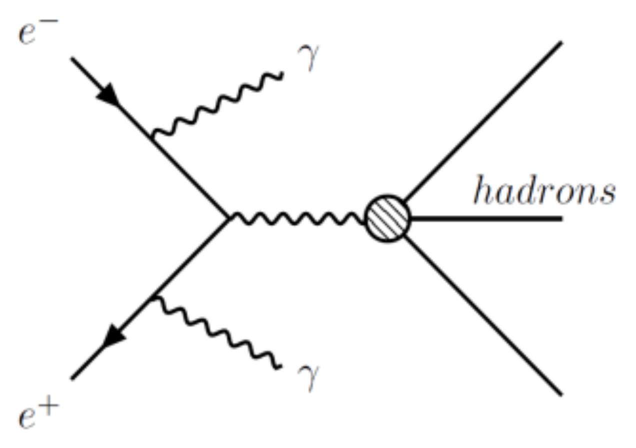
$r_{rad} = 1.0080 \pm 0.0007$

Correction is <1 %.

L_{eff}

$$\frac{\mathrm{d}L_{\mathrm{eff}}}{\mathrm{d}\sqrt{s'}} = L_{\mathrm{int}}\frac{2\sqrt{s'}}{s}\int_{\theta^*_{\mathrm{min}}}^{\pi-\theta^*_{\mathrm{min}}} W(s,s',\theta')\sin\theta'\mathrm{d}\theta'$$
$$= L_{\mathrm{int}}\frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2+s'^2}{s(s-s')}\ln\frac{1+C}{1-C} - \frac{s-s'}{s}C\right), \quad (4)$$

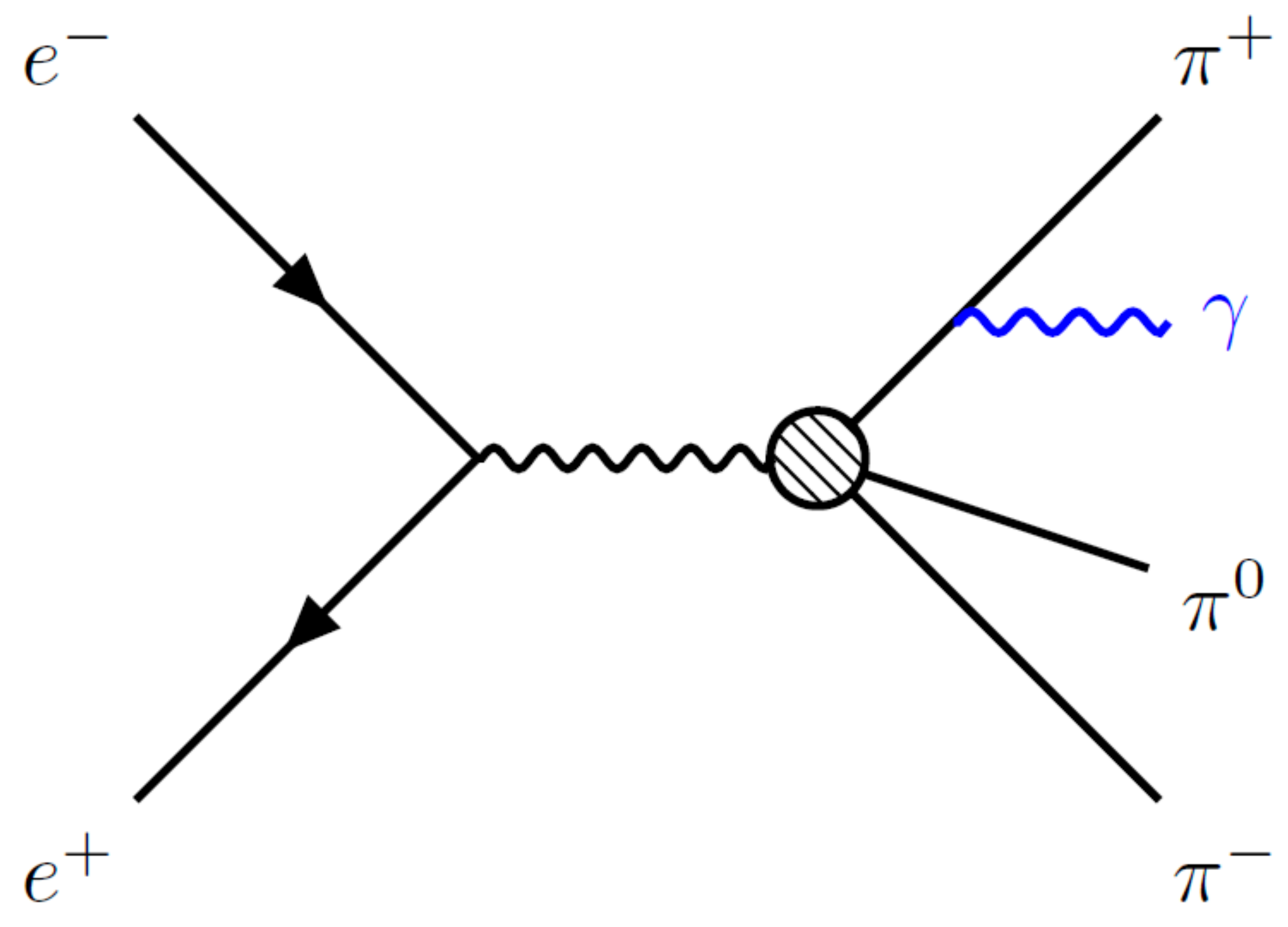
where L_{int} is the integrated luminosity of the data set, θ^*_{min} is the minimum polar angle of an ISR photon in the c.m. frame, and C is $\cos\theta^*_{\mathrm{min}}$.



Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

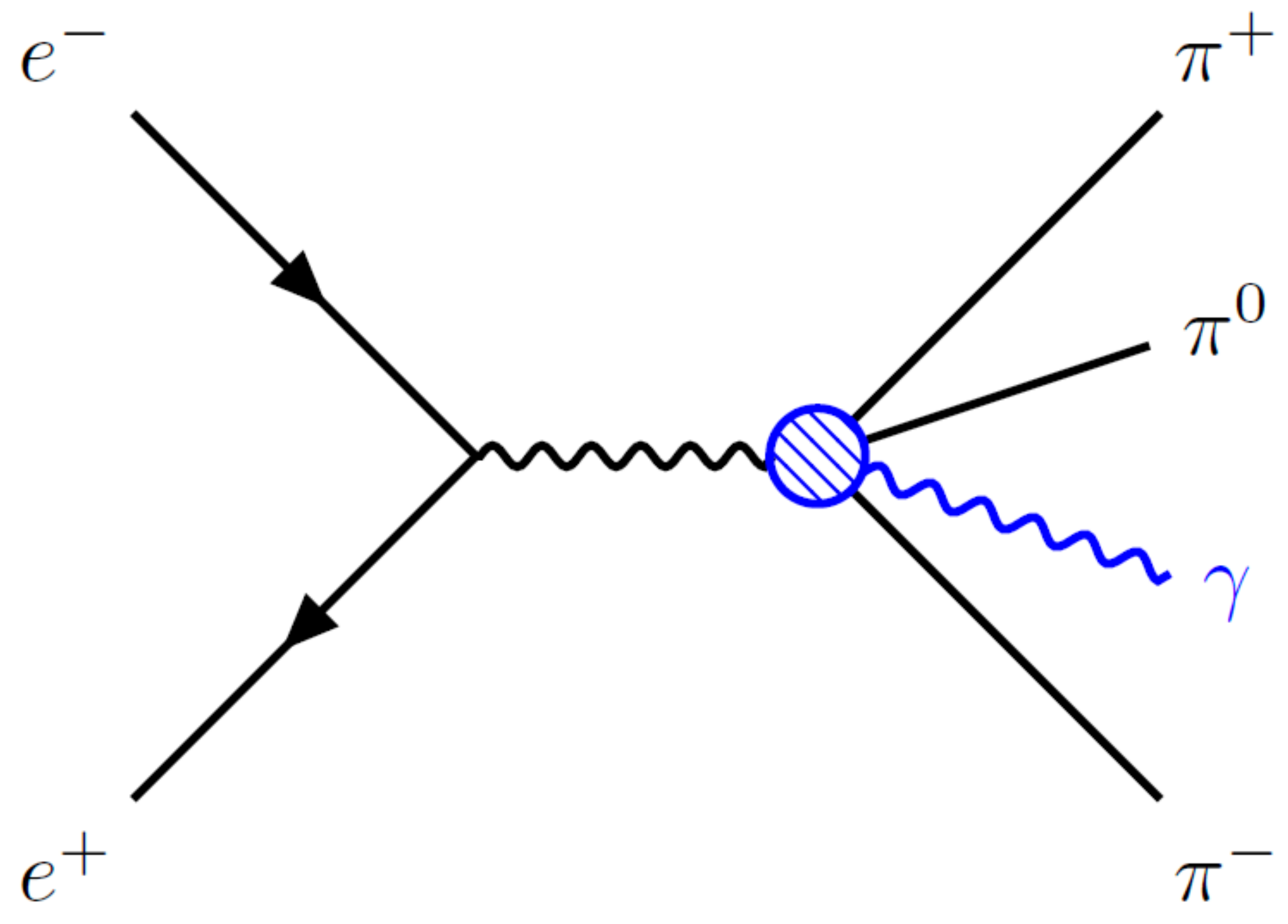
Final-state radiation background

- Difficult to reject FSR events or extract control samples
- Estimate FSR using pQCD prediction based on BaBar's [PhysRevD.104.112003]



FSR emission from final-state pions

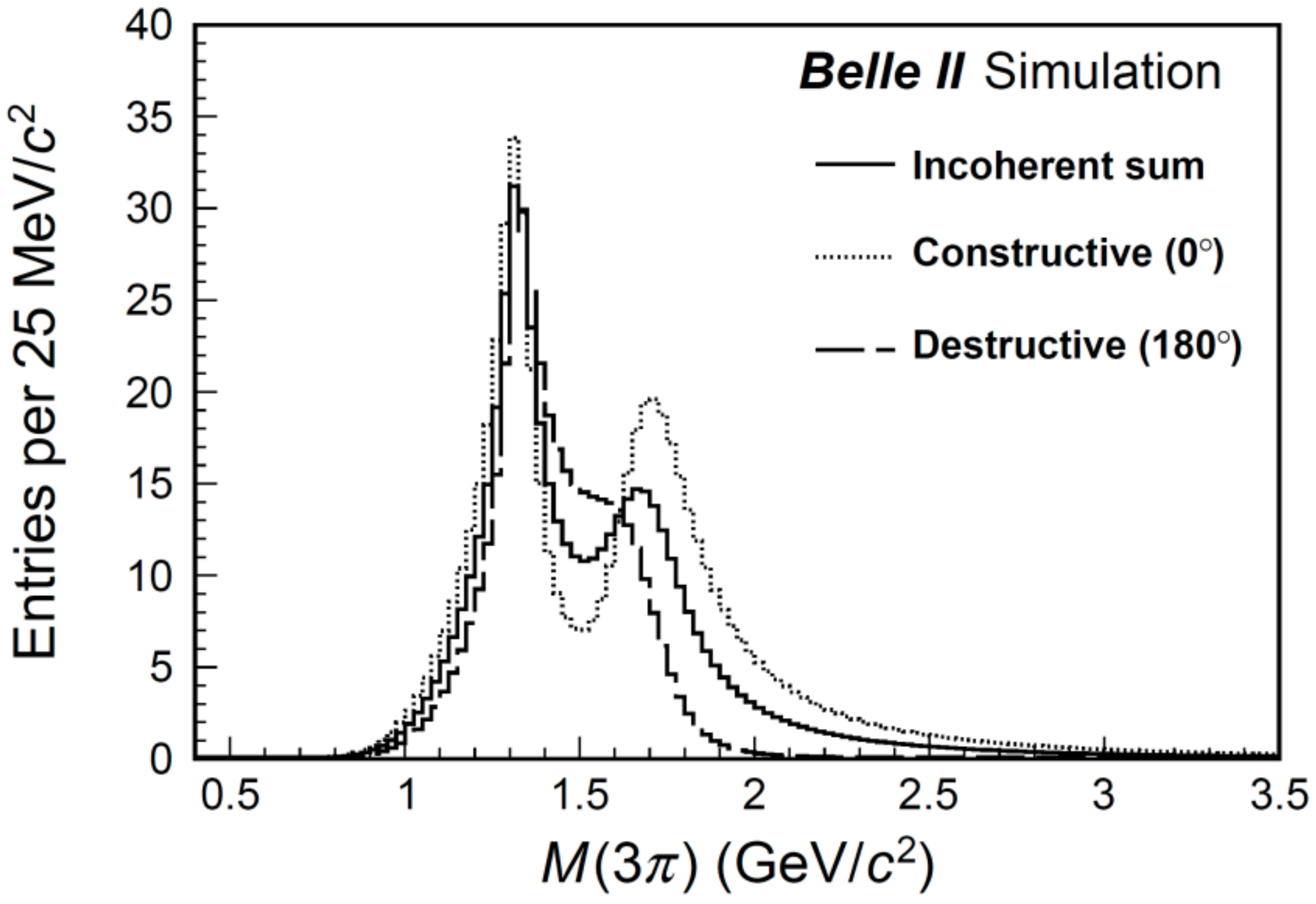
~ 0.001fb → < 1 event occur



FSR emission from the quark legs

$e^+e^- \to M\gamma_{FSR} \to \pi^+\pi^-\pi^0\gamma_{FSR}$,

$M = \eta, a_1(1260), a_2(1320), a_1(1640), a_2(1700), a_1(1930), a_2(2030)$

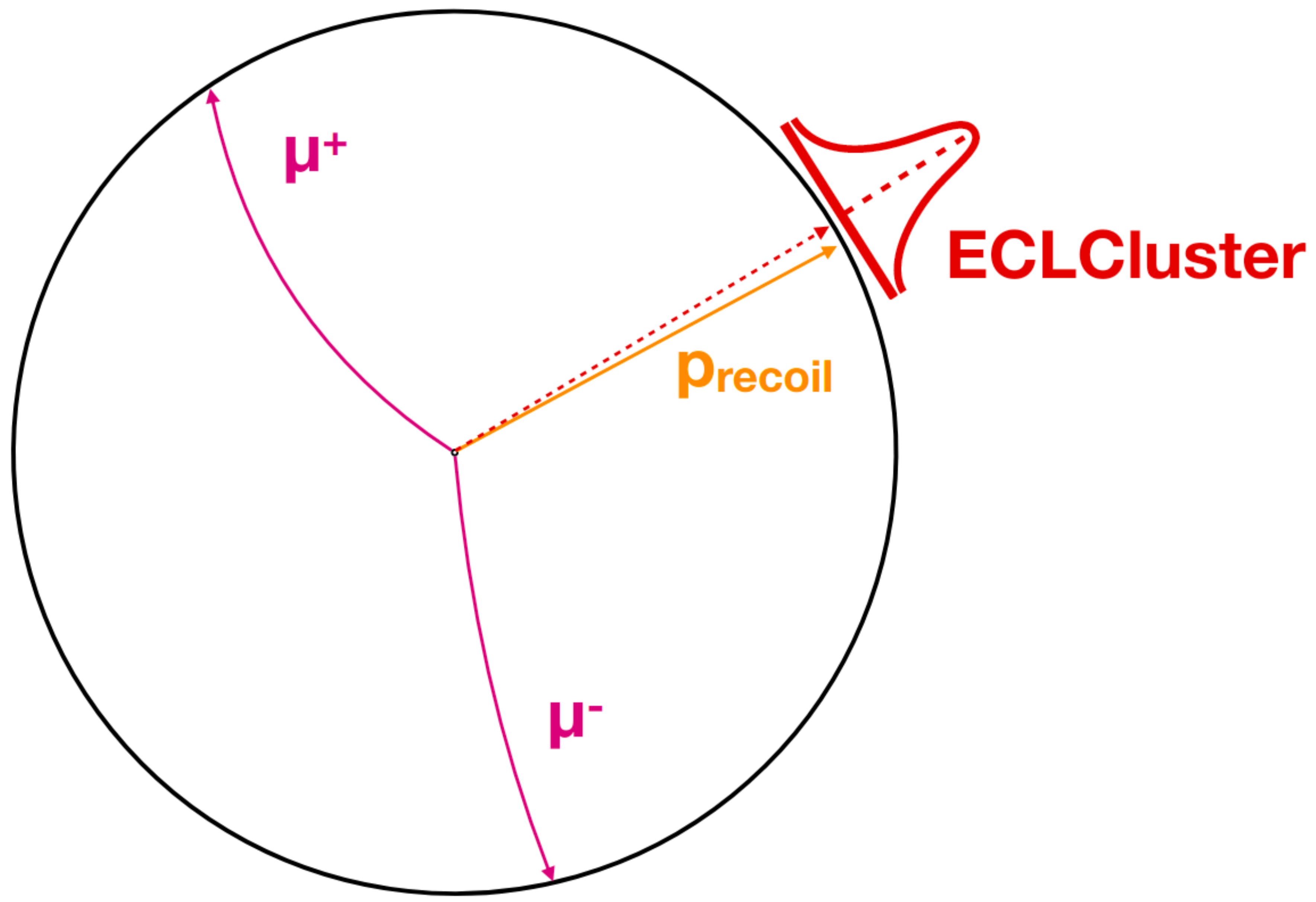


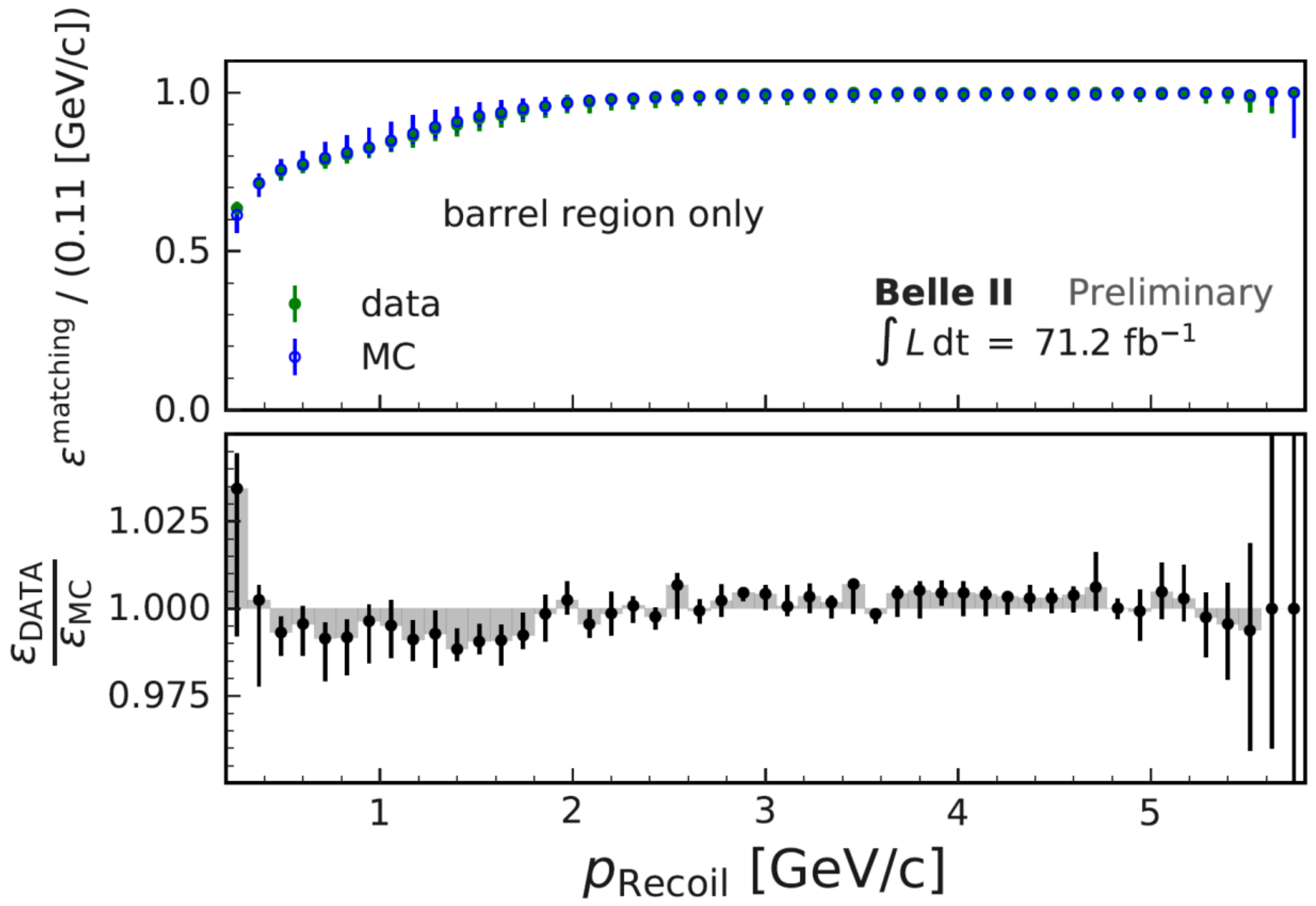
Considered in systematic uncertainty

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

ISR photon detection efficiency

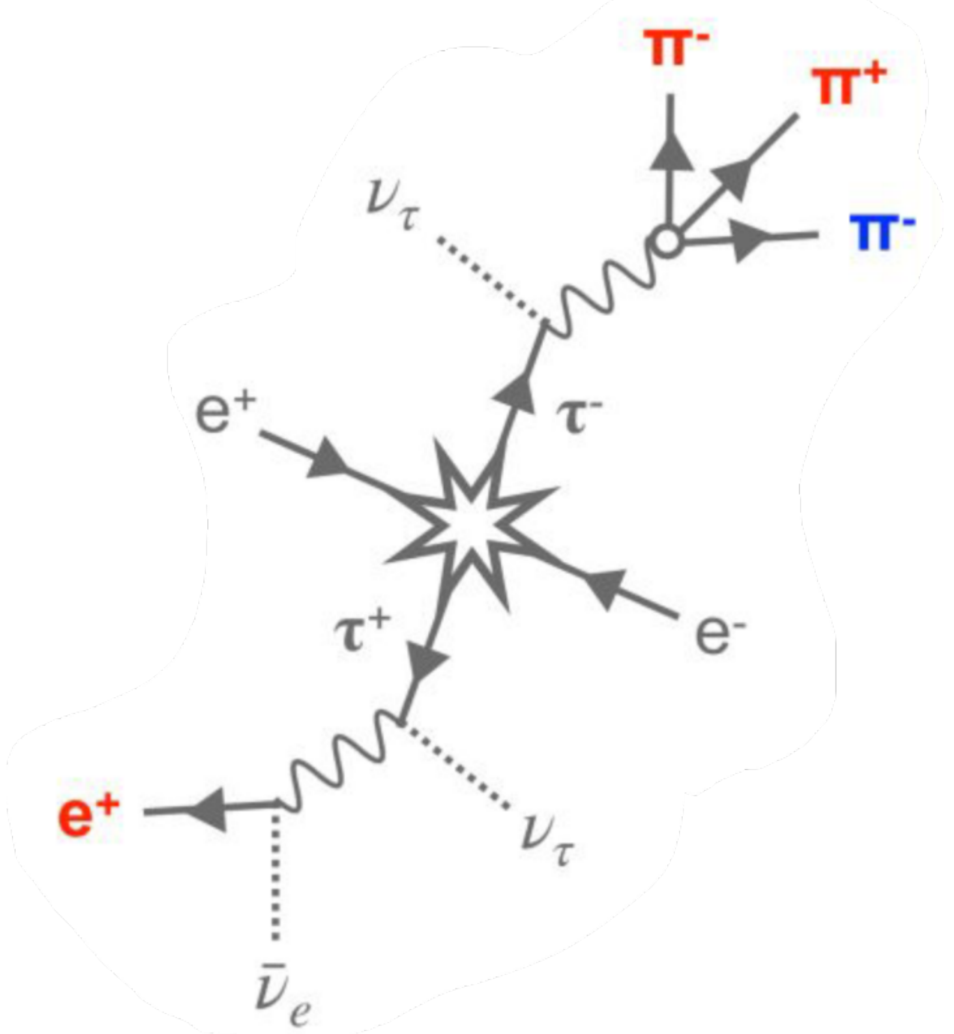
- Measured using $e^+e^- \to \mu^+\mu^-\gamma$ events
  - Matching a ECL cluster with missing momentum of the dimuon system
  - Good data-MC agreement → 0.7% systematic uncertainty



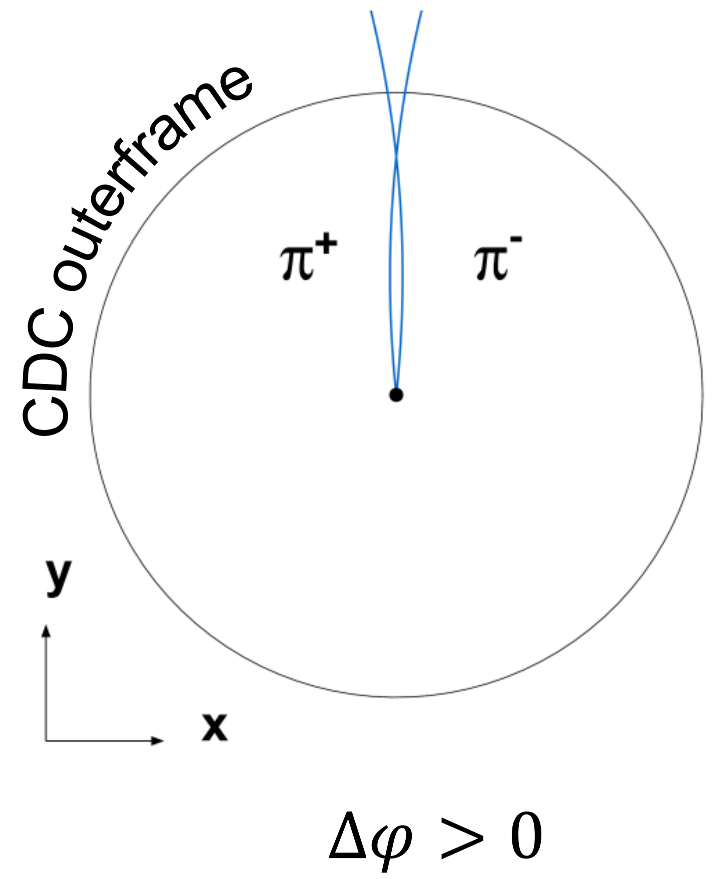


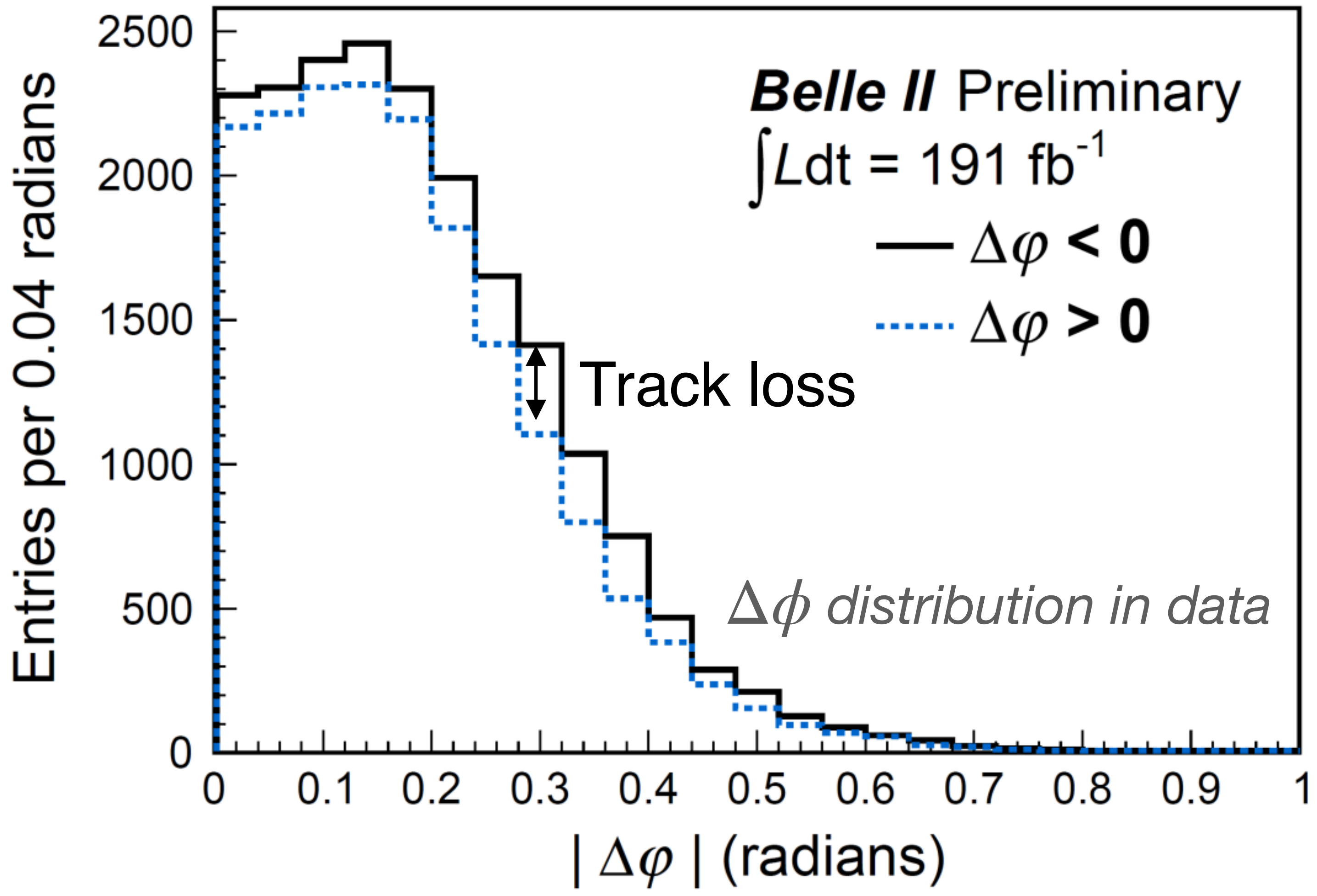
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

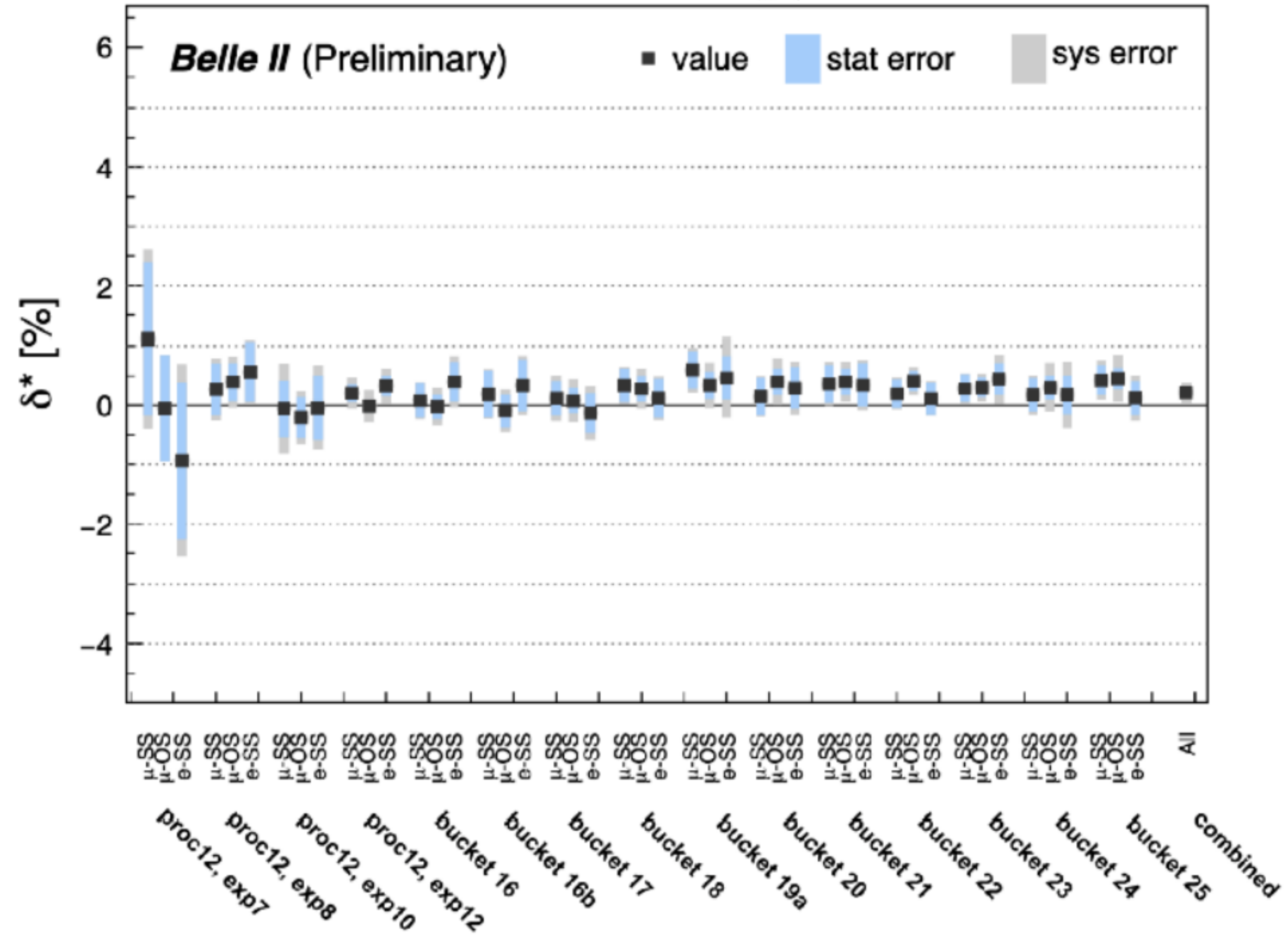
Tracking efficiency



- Studied with $e^+e^- \to \tau^+\tau^-$ process (1x3 prong)
  - Tag $\pi^+\pi^-e^\pm$ **or** $\pi^+\pi^-\mu^\pm$ and prob $\pi^\mp$
  - Small data-MC discrepancy → 0.3% uncertainty per track
- Correlated track loss due to shared hits in CDC is confirmed with $e^+e^- \to \pi^+\pi^-\pi^0\gamma$
  - Define $\Delta\phi \equiv \phi_{\pi^+} - \phi_{\pi^-}$, inefficiency: $f = \dfrac{N(\Delta\phi < 0) - N(\Delta\phi > 0)}{2N(\Delta\phi < 0)}$
    - 5% in data and 4% in MC
- Total correction factor for tracking: -1.4 ± 0.8 %
  - including dependency on no. of CDC hits and duplicated tracks







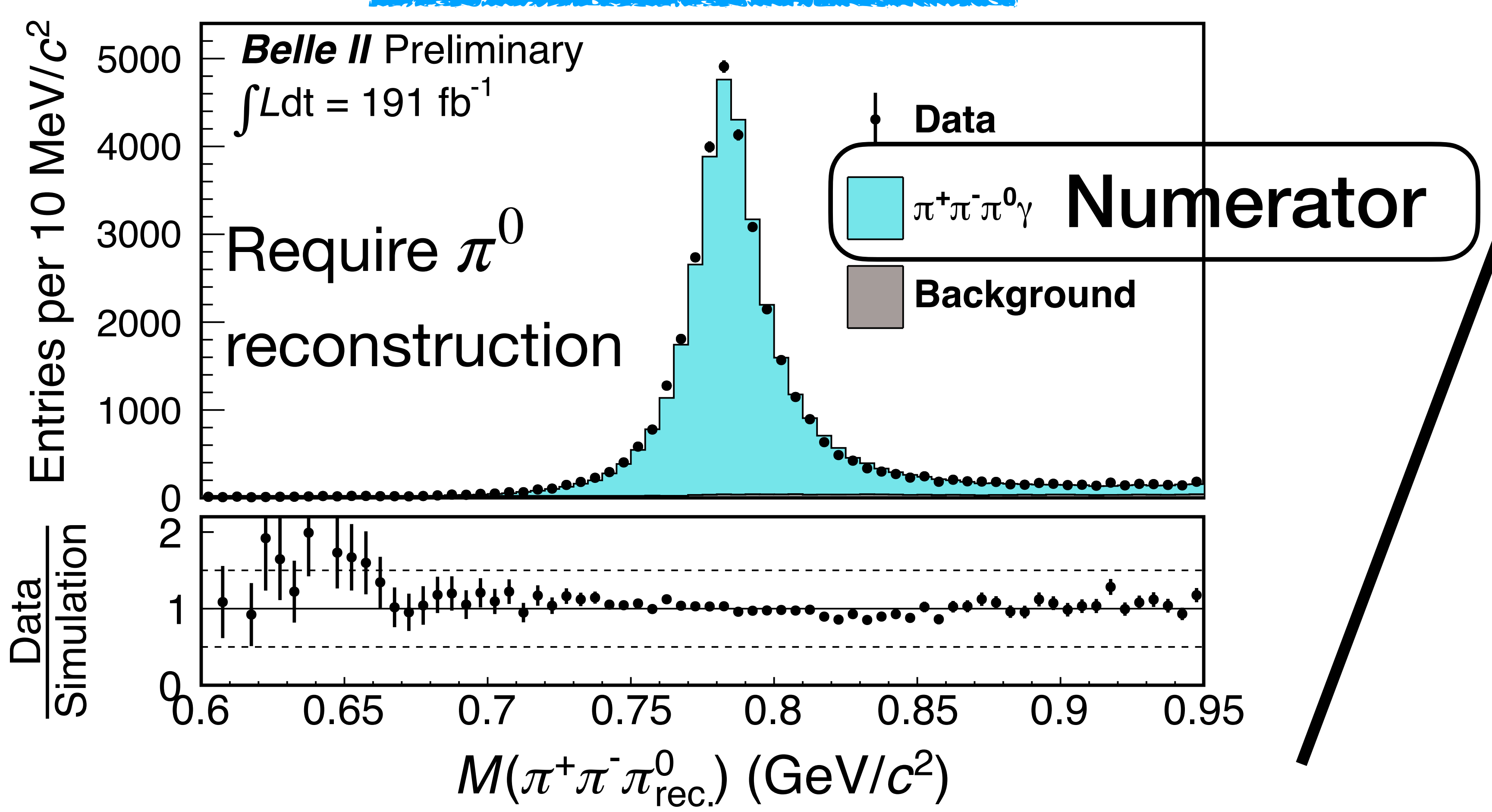
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

π^0 efficiency

- Estimated using the exclusive process $e^+e^- \to \omega\gamma_{isr} \to \pi^+\pi^-\pi^0\gamma_{isr}$
  - Reconstruct only $\pi^+\pi^-\gamma_{isr}$, and constrain their recoil with π^0 mass (1C recoil fit) → counting $\omega \to \pi^+\pi^-\pi^0_{rec}$ as denominator
    - Events with successful π^0 reconstruction as numerator

Full reconstruction

$\epsilon_{\pi^0} =$



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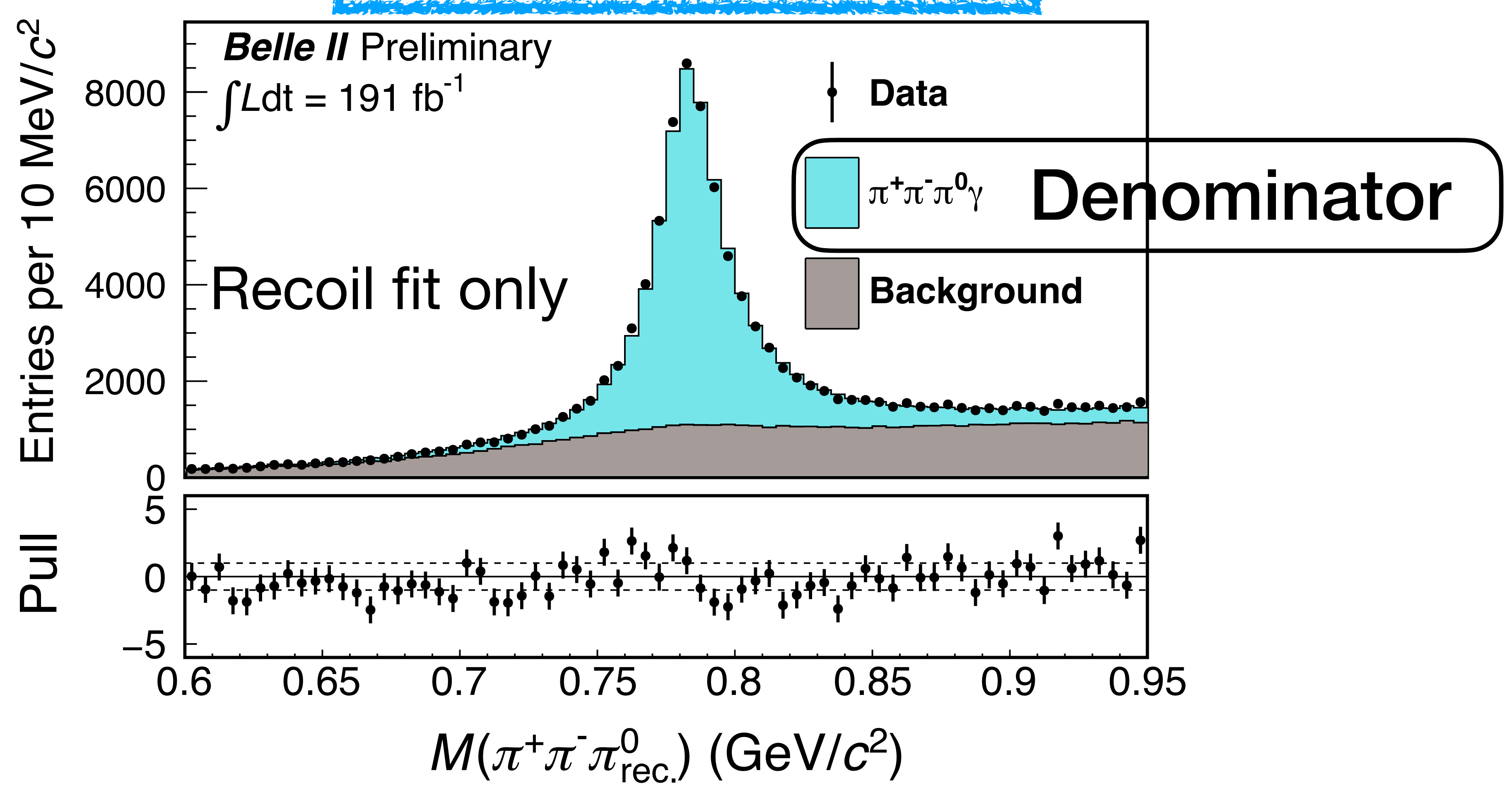
Partial reconstruction



- ϵ_{π^0} is studied in data and MC respectively: Data/MC ratio = $0.986 \pm 0.006_{stat}$
- Related systematic uncertainty is 1.0% by varying M(γγ) signal pdf, background pdfs, and selections

Measurement of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section

Background suppression efficiency

- Estimated by the ratio of signal yield before/after the suppression criteria
- Using ω and Φ, J/ψ resonances of good signal-to-noise ratio
- In $\mathrm{M}_{3\pi} < 1.05\,\mathrm{GeV/c^2}$, efficiency is (89.5±0.2)% for data
  - $\epsilon_{\mathrm{data}}/\epsilon_{\mathrm{MC}} - 1$ = (-1.90±0.20)%
- In $\mathrm{M}_{3\pi} > 1.05\,\mathrm{GeV/c^2}$, no. of J/ψ events is obtained by fitting $\mathrm{M}_{3\pi}$
  - $\epsilon_{\mathrm{data}}/\epsilon_{\mathrm{MC}} - 1$ = (-1.78±1.85)%

statistical errors in the sample

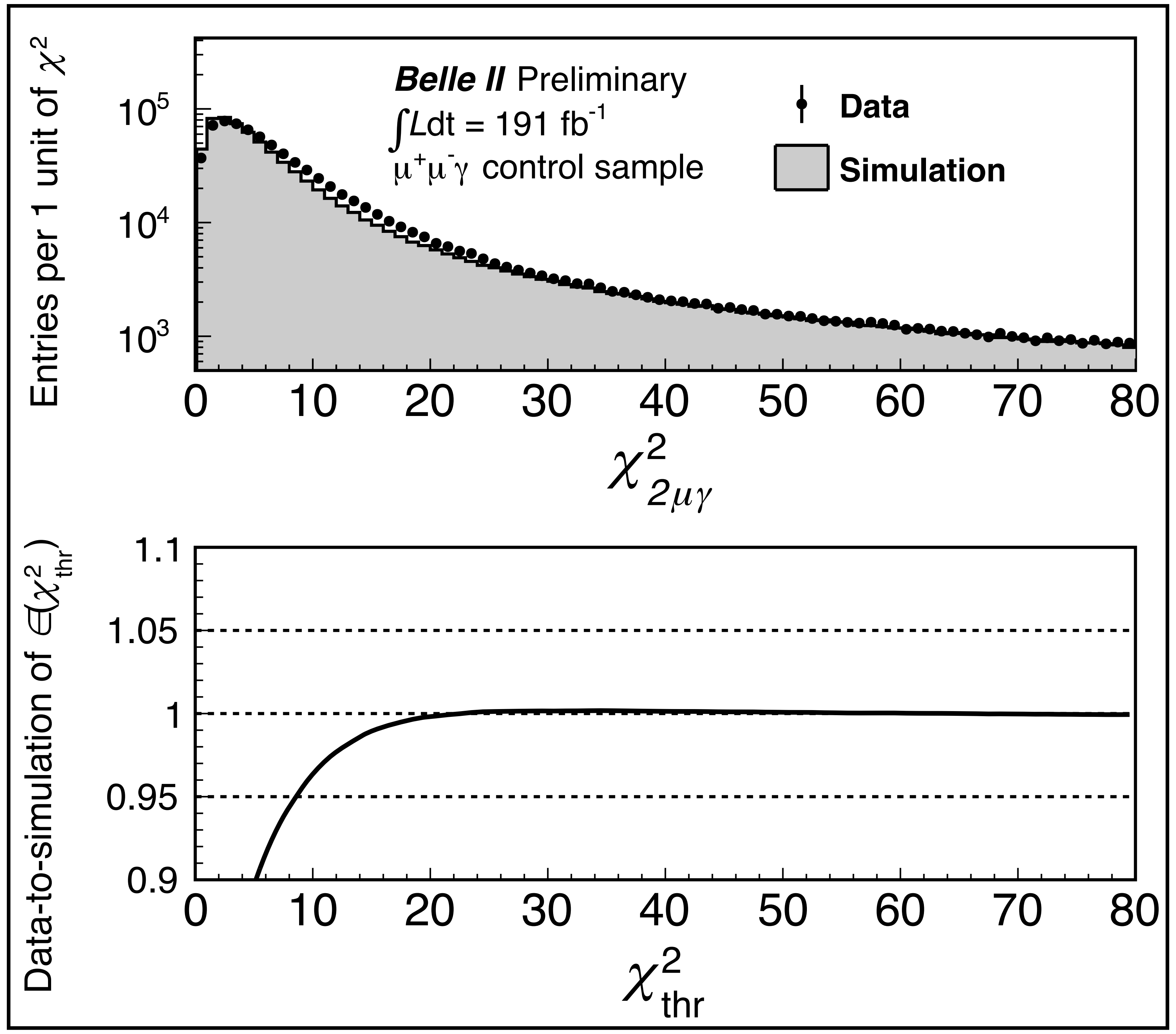
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Kinematic χ^2 selection efficiency

- Studied with $e^+e^- \to \mu^+\mu^-\gamma$
- Check effects from differences in vertex, momentum and energy of ISR and tracks
  - Agreement confirmed within ±0.6% uncertainty

$$\varepsilon(\chi^2_{\text{thr}}) = \frac{N(\chi^2 < \chi^2_{\text{thr}})}{N_{\text{all}}}$$

$$\frac{\varepsilon_{\text{data}}(\chi^2_{\text{thr}})}{\varepsilon_{\text{MC}}(\chi^2_{\text{thr}})}$$



- Multi-ISR photon is discussed on next slides

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Higher-order ISR effects: radiative correction

- Leading order (LO) ISR luminosity with L_{int} = 191/fb is given by:

$$L_{eff} = \frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2+s'^2}{s(s-s')}\ln\frac{1+\cos\theta}{1-\cos\theta} - \frac{s-s'}{s}\cos\theta\right)L_{int}$$

- **Radiative correction** is the ratio of the ISR emission probability including higher-order effects (LO+NLO+…) to LO
- Higher order (LO+NLO) effects calculated by PHOKHARA
  - Give us radiative correction of 1.008-1.013 depending on hadronic energy √s'

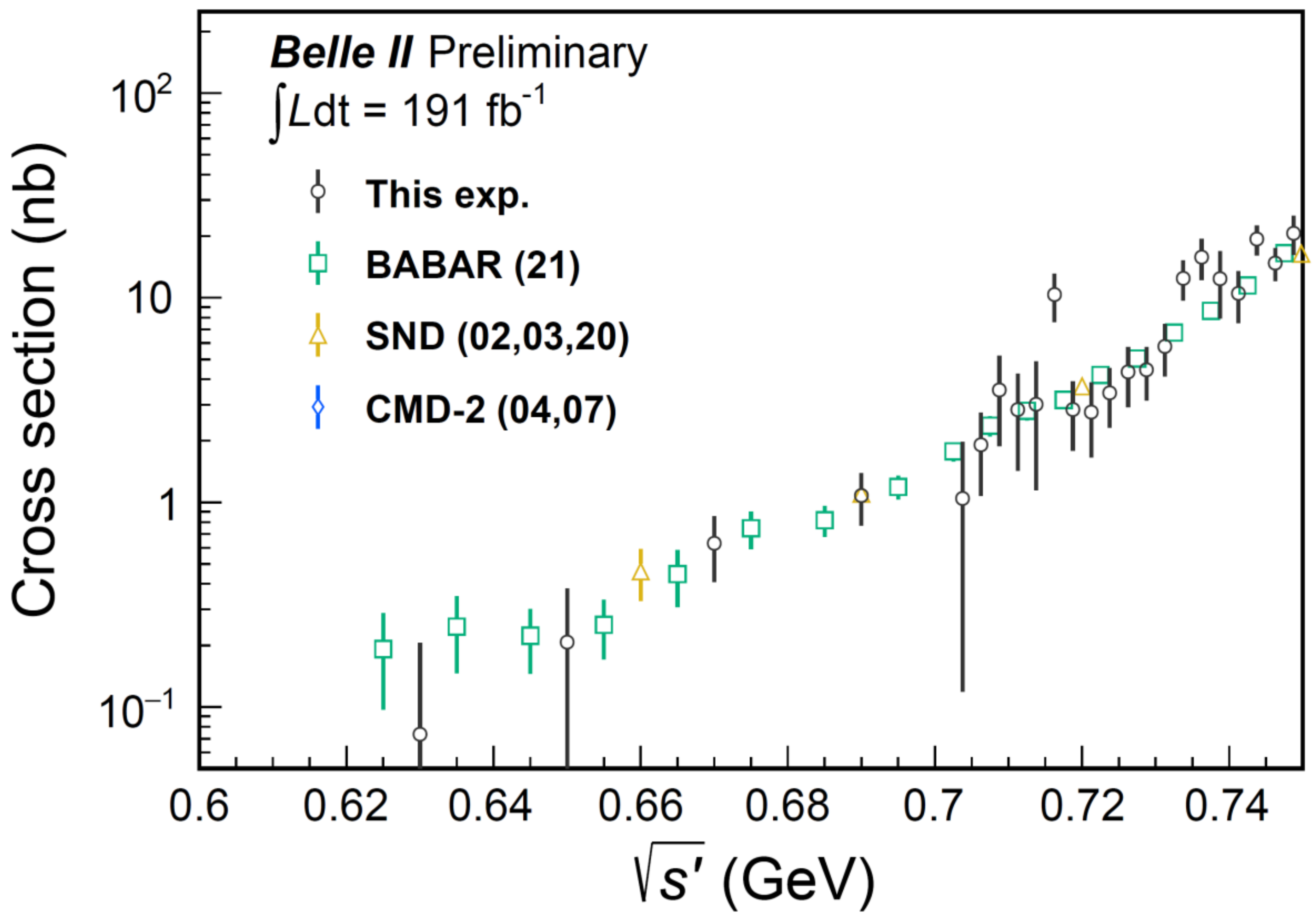
Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

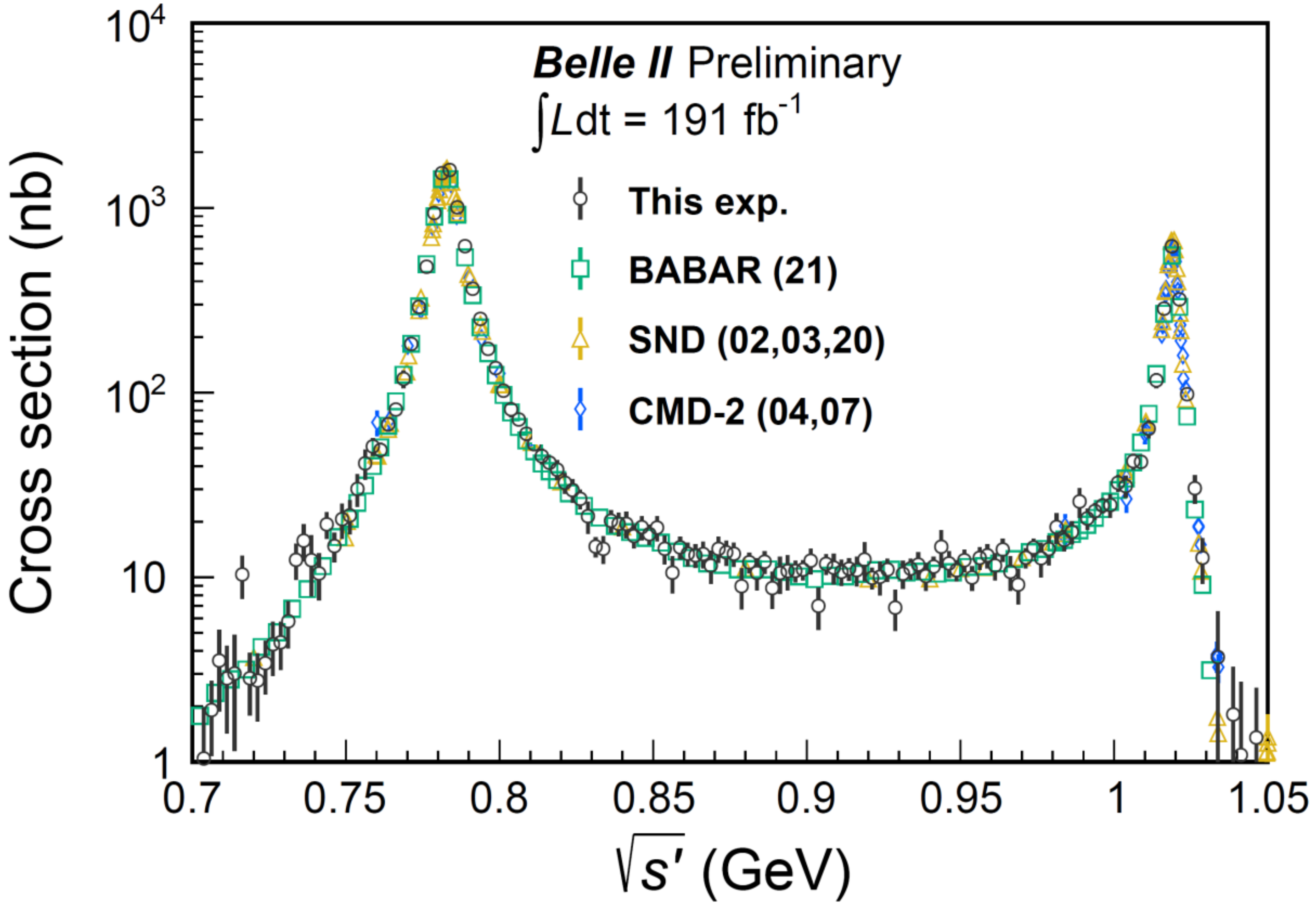
Higher-order ISR effects: χ^2 efficiency

- 20% excess of the fraction of NLO (two ISR) events on PHOKHARA is reported by BaBar [PhysRevD.108.L111103]
  - Also confirmed with Belle II data
  - Our χ^2 selection rejects most NLO events → efficiency change
    - Estimated with MC only: χ^2 efficiency is **underestimated** by **(2.4±0.7)%**
- NNLO (three ISR) is not included in the generator
  - (3.4±0.4)% observed by BABAR
  - Influence to this analysis: efficiency **overestimation** by **1.9%**
- No correction is applied to our result, but
  - **1.2% systematic uncertainty** is assigned as **MC generator derived error**
    - 0.7% (error from NLO excess) ⊕ 0.95% (half of NNLO effect) = 1.2%

Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Results: cross section below 1.05 GeV





Measurement of $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

Comparison with BaBar 2021 measurement

- In quite a few respects, this analysis follows BaBar's method
- Systematic uncertainty is still nearly twice as large
  - NNLO generator is needed

	Belle II	BABAR (2021)
Dataset	191 fb^{-1}	469 fb^{-1}
Combinatorial γγ background	M(γγ) fit	Negligibly small(?)
ISR energy in kinematic fit	Used	Unused
Generator	PHOKHARA	AfkQed
Generator uncertainty	1.2%	-
Detection efficiency uncertainty	1.6%	1.1%
Integrated luminosity	0.6%	0.3%
Total systematic uncertainty for $a_\mu(3\pi)$	2.2%	1.3%