Precision Flavour Physics with Inclusive Penguin Decays

PRISMA+

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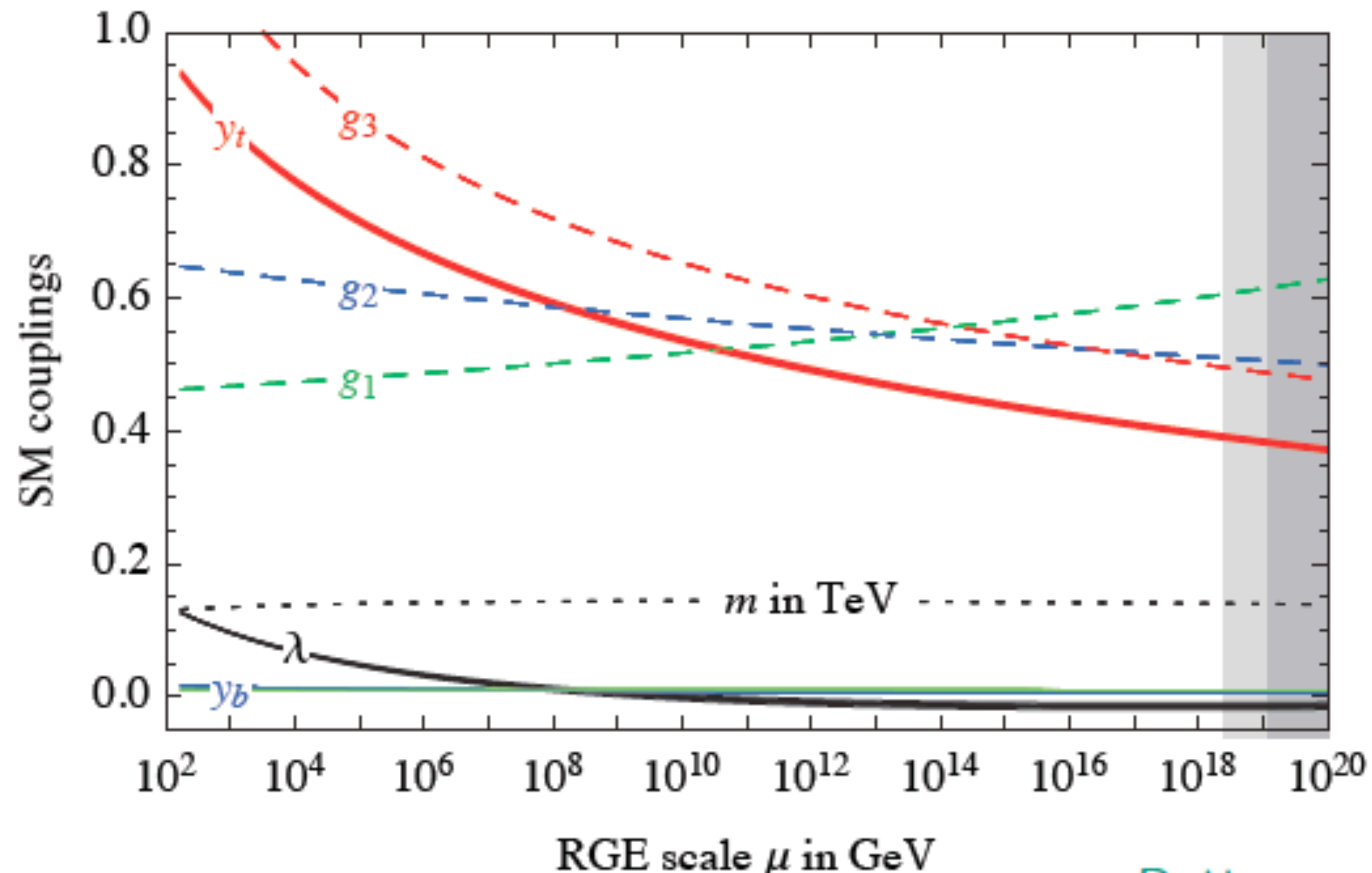
Prologue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)

- Neutrino masses (Dirac or Majorana masses ?)

- Baryon asymmetry of the Universe (new sources of CP violation needed)

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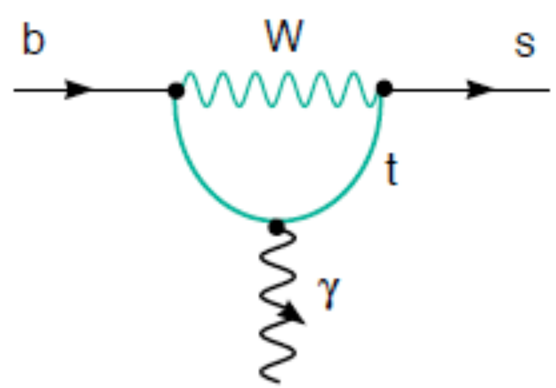
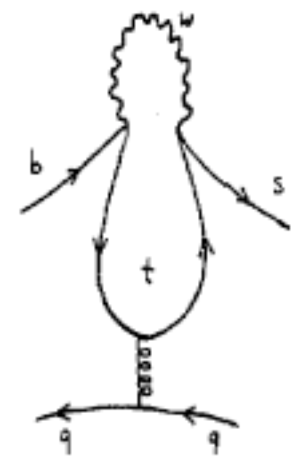
Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

Indirect exploration of higher scales via flavour

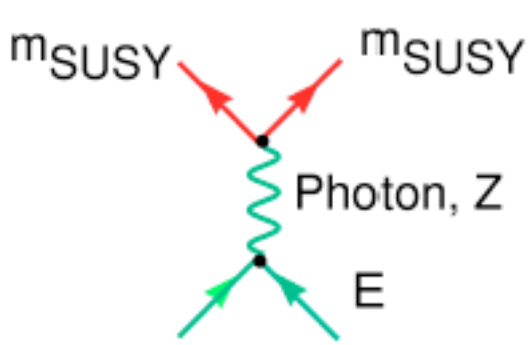
- Flavour changing neutral currrent processes like $b \to s\gamma$ or $b \to s\ell^+\ell^-$ directly probe the SM at the one-loop level.



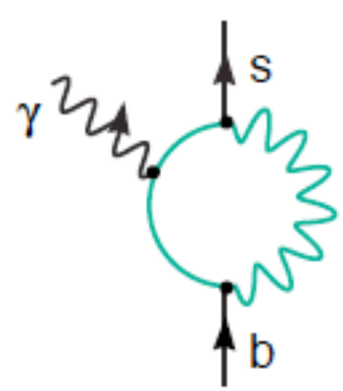


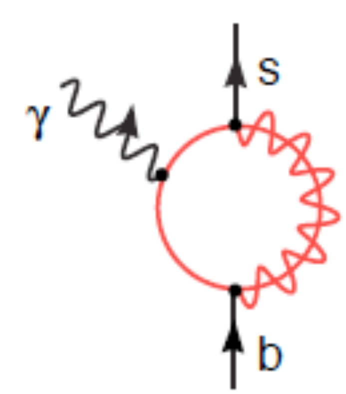
- Indirect search strategy for new degrees of freedom beyond the SM

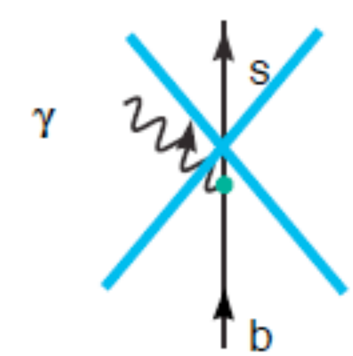
Direct:



Indirect:







Indirect exploration of higher scales via flavour

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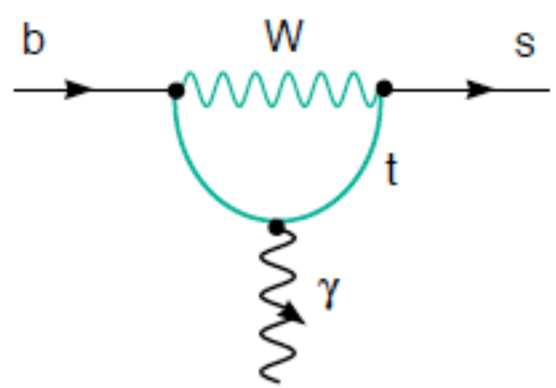

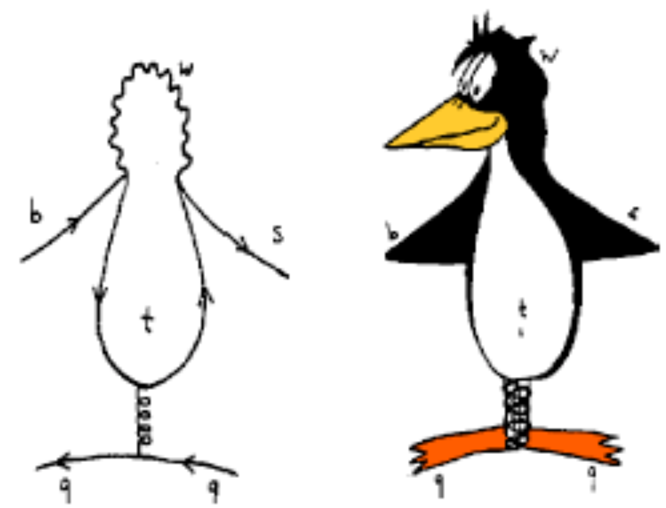

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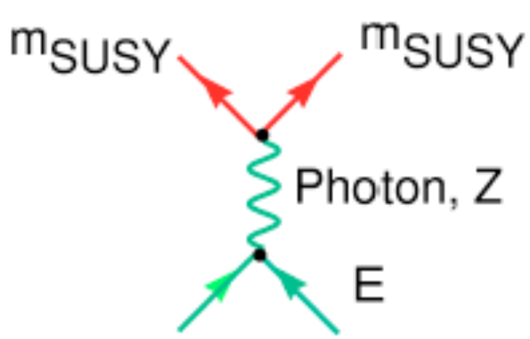

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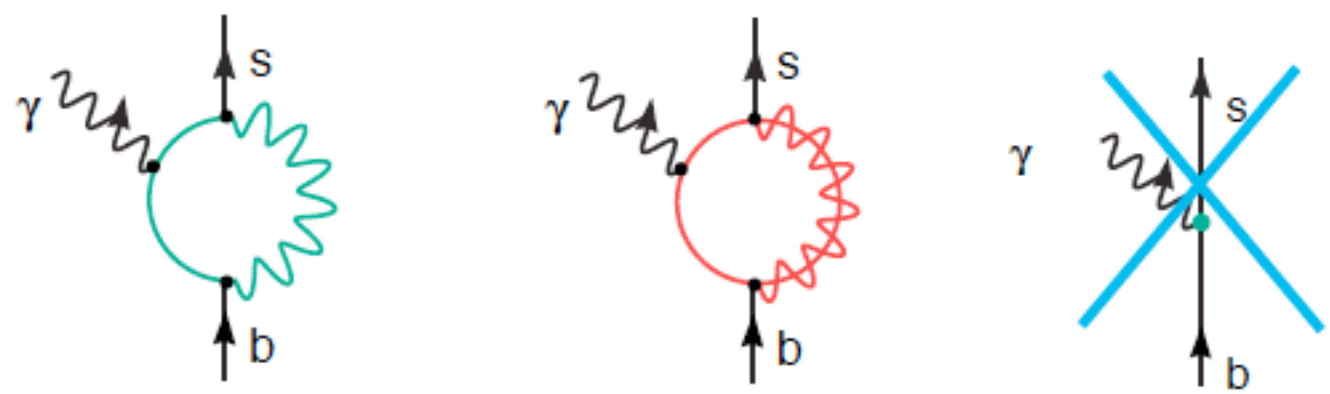
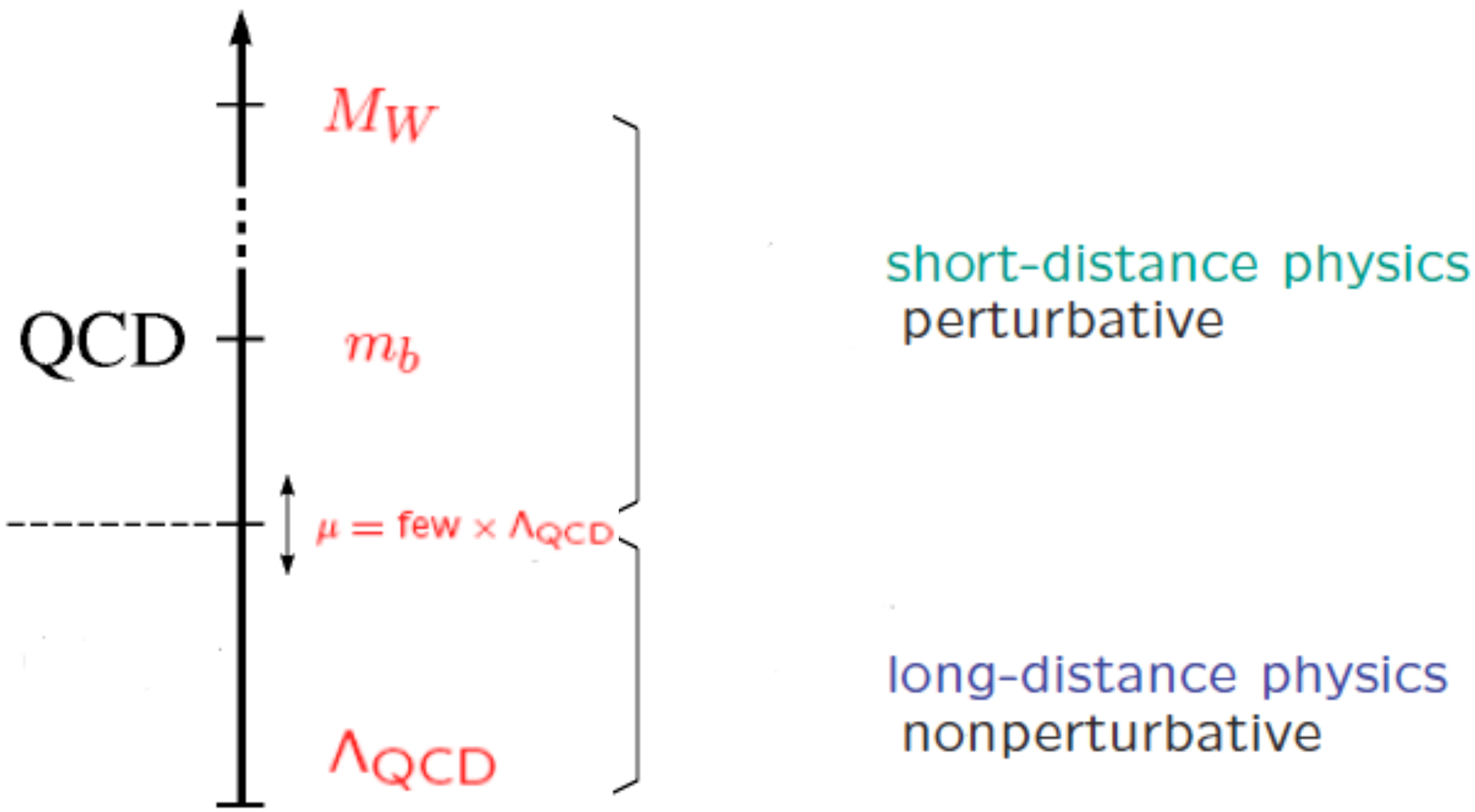


Ambiguity of new physics scale from flavour data

$$(\, C^i_{\mathrm{SM}}/M_W + C^i_{\mathrm{NP}}\,/\Lambda_{\mathrm{NP}}\,) \times \mathcal{O}_i$$

Theoretical Framework

Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy})\ \mathcal{O}_i(\mu)$

- $\mu^2 \approx M_{New}^2 >> M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

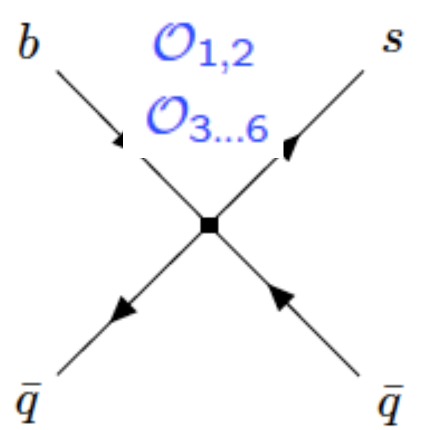
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

HQET, SCET, ...

Effective Weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \Bigg(\sum_{i=1\cdots 10,S,P} \Big(C_i(\mu)\mathcal{O}_i(\mu) + C_i'(\mu)\mathcal{O}_i'(\mu) \Big) \Bigg)$$

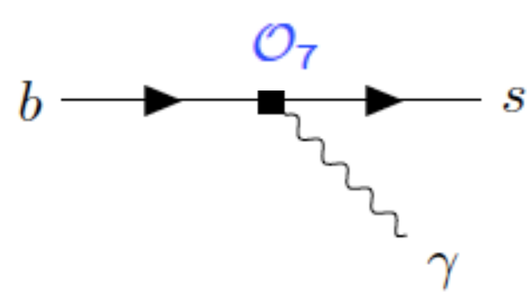
4-quark operators



$\mathcal{O}_{1,2} \propto (\bar{s}\Gamma_\mu c)(\bar{c}\Gamma^\mu b)$

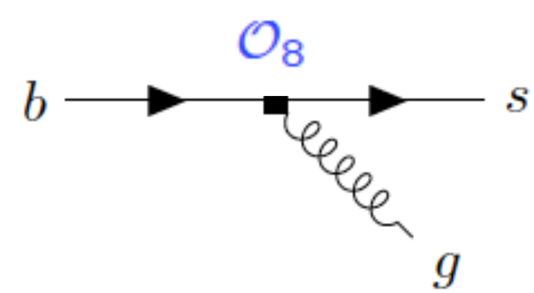
$\mathcal{O}_{3\ldots6} \propto (\bar{s}\Gamma_\mu b)\sum_q(\bar{q}\Gamma^\mu q)$

electromagnetic dipole operator



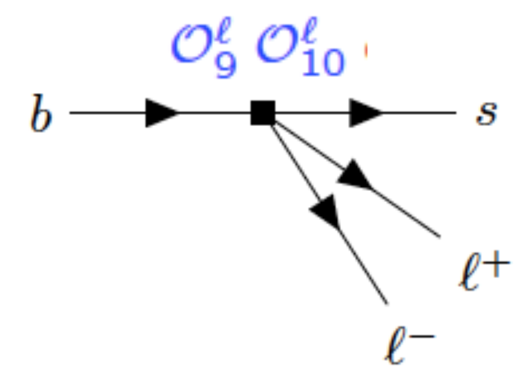
$\mathcal{O}_7 \propto (\bar{s}\sigma^{\mu\nu}P_R)F^a_{\mu\nu}$

chromomagnetic dipole operator



$\mathcal{O}_8 \propto (\bar{s}\sigma^{\mu\nu}T^a P_R)G^a_{\mu\nu}$

semileptonic operators



$\mathcal{O}_9^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu \ell)$

$\mathcal{O}_{10}^\ell \propto (\bar{s}\gamma^\mu b_L)(\bar{\ell}\gamma_\mu\gamma_5 \ell)$

In the SM: $C_7 = -0.29$ $\quad C_9 = 4.20$ $\quad C_{10} = -4.01$

New physics:

- Corrections to the Wilson coefficients: $C_i \to C_i^{\text{SM}} + \delta C_i^{\text{NP}}$
- Additional operators: Chirally flipped ($\mathcal{O}_i'$), (pseudo)scalar ($\mathcal{O}_S$ and $\mathcal{O}_P$)

Exclusive modes $B \to K^{(*)}\ell\ell$

Soft-collinear effective theory

$$\mathcal{T}_a^{(i)} = C_a^{(i)}\xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

– Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors

– Relations between formfactors in large-energy limit

– Limitation: insufficient information on power-suppressed Λ/m_b terms (breakdown of factorization: 'endpoint divergences')

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

Problem of nonfactorizable power corrections

- Crosscheck with $R_{\mu,e}$ ratios:

NP in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables (if there is a coherent picture)

Option out !

- Ongoing efforts: Estimate of power corrections based on analyticity

van Dyk et al.: arXiv:2011.09813, 2206.03797

Nico's Talk

- In the long run: Solution with refactorization techniques *

New developments in the SCET community

Neubert et al., arXiv:2009.06779

- Crosscheck of the anomalies via inclusive modes

*Caveat:
Many nonperturbative functions in exclusive modes at subleading order.

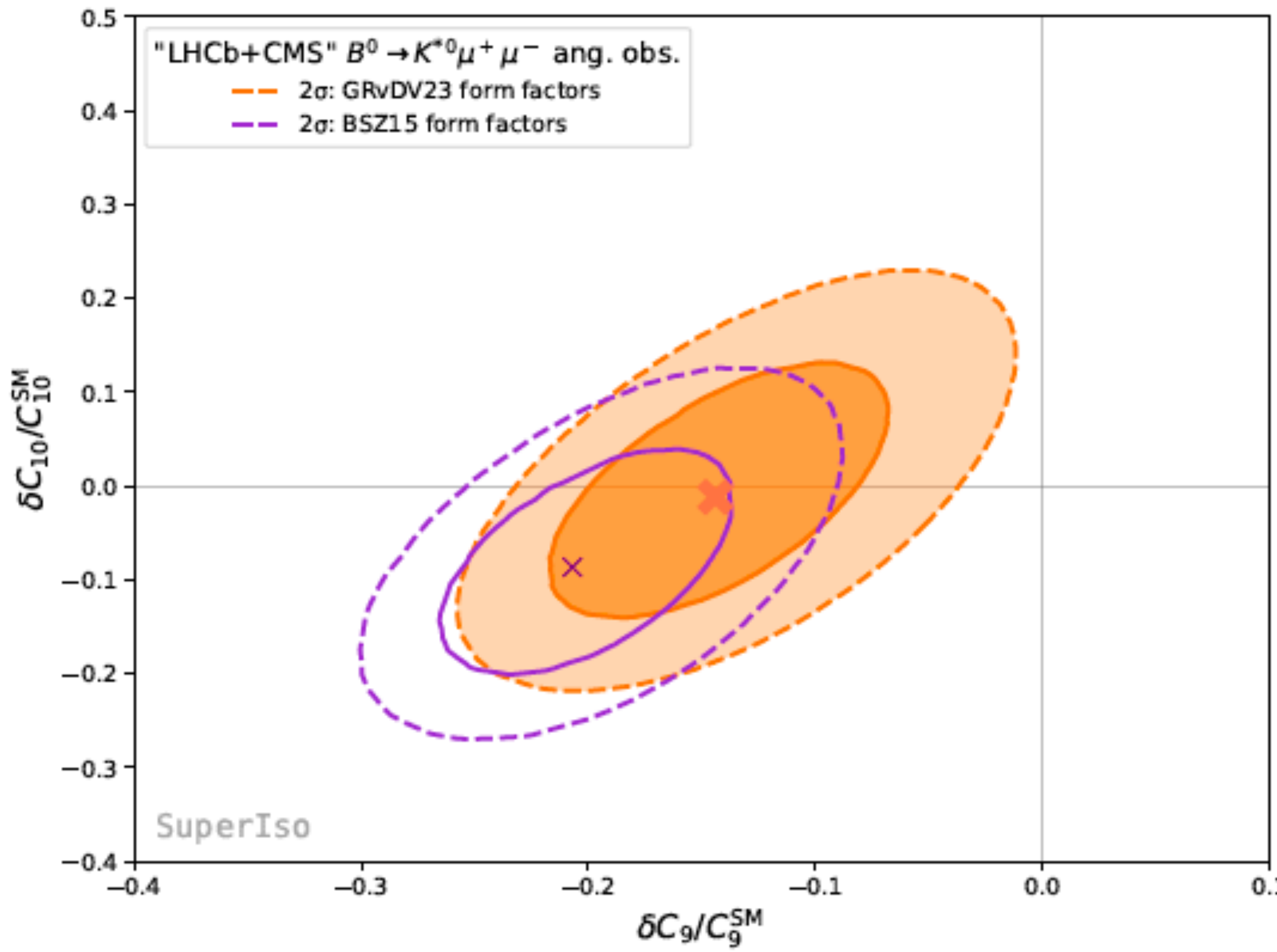
Side remarks on the $b \to s$ anomalies

Hurth, Mahmoudi, Monceaux, Neshatpour arXiv:2508.09986

Large ambiguity of the significance due to the local formfactors

The 1 and 2σ C.L. of the $\{C_9, C_{10}\}$, **without** $[6, 8]$ and $[6., 8.68]$ GeV^2 bins

Combined LHCb and CMS results, impact of the choice of the form factors



Orange: GRvDV23-FF (pull$_{SM}$: 2.8)
Purple contours : BSZ15-FF (pull$_{SM}$: 4.0)

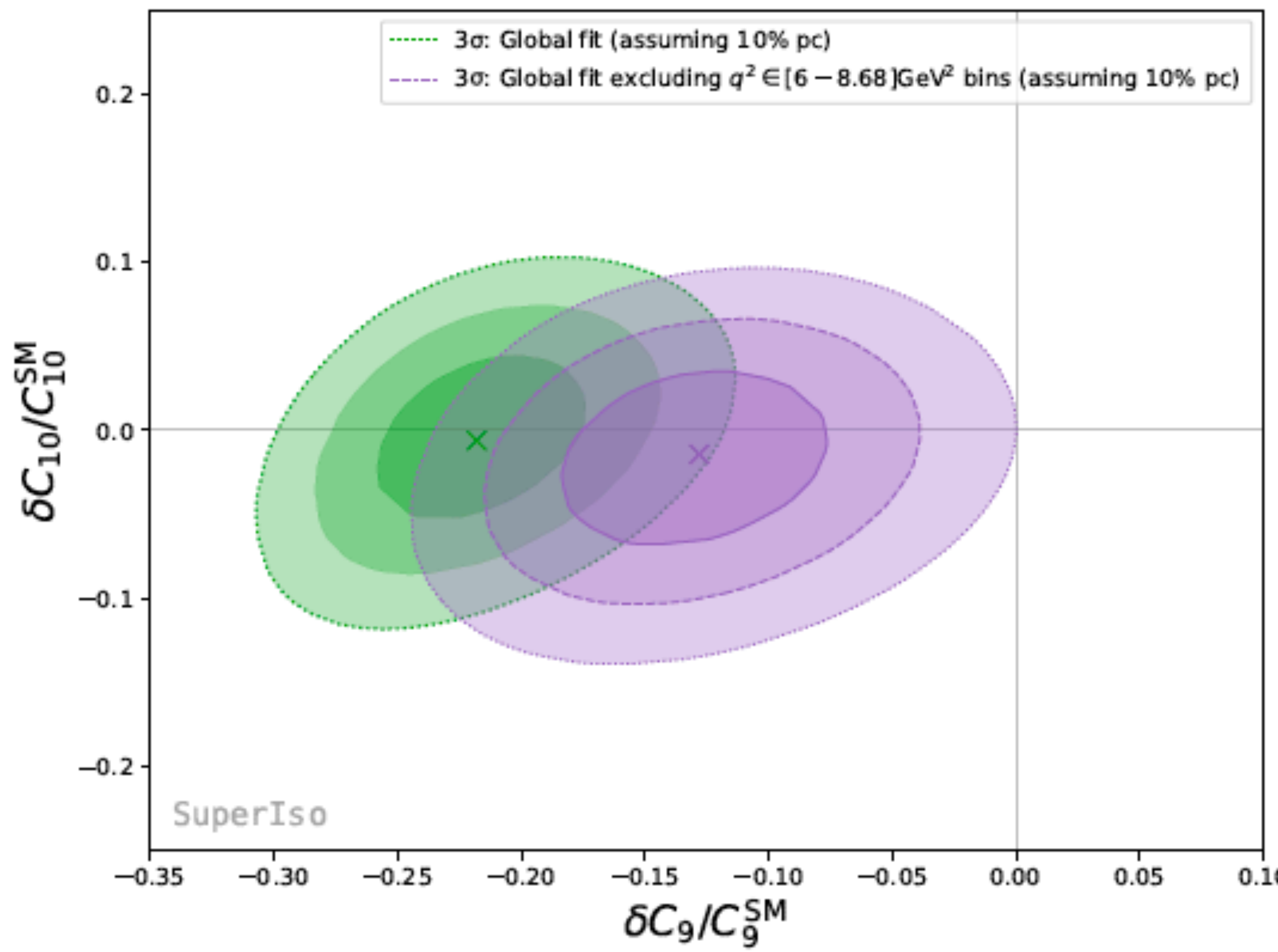
Side remarks on the $b \to s$ anomalies

Hurth, Mahmoudi, Monceaux, Neshatpour arXiv:2508.09986

Validity of QCDf /SCET questionable above the charm threshold (7 GeV^2)

Two-dimensional fit of $\{C_9, C_{10}\}$ to all observables

Assuming a 10% uncertainty to the leading-order non-factorisable QCDf amplitude



Purple : Without $q^2 \in [6, 8.68]$ GeV^2 bins (pull$_{SM}$: 3.0)
Green : With $q^2 \in [6, 8.68]$ GeV^2 bins (pull$_{SM}$: 5.6)

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

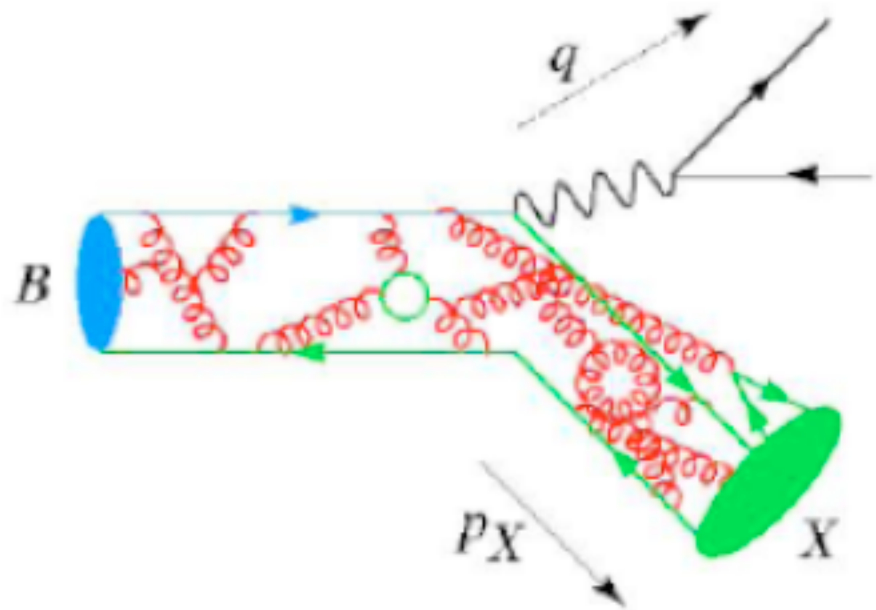
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \xrightarrow{m_b\to\infty} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990



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Old story:

- If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

Dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

$b \to s\gamma$: Benzke,Lee,Neubert,Paz,arXiv:1003.5012

$b \to s\ell\ell$: Benzke,Hurth,Turczyk,arXiv:1705.10366

Inclusive semi-leptonic penguins

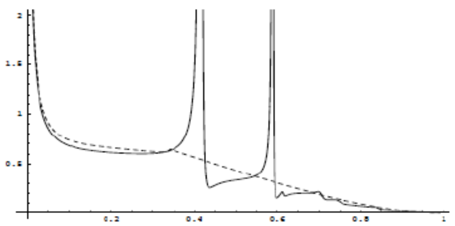
Complete angular analysis of inclusive $B \to X_s \ell\ell$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$



- Dependence on Wilson coefficients

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}\,C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}\,C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\,C_7\right|^2 + |C_{10}|^2\right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\rm em}\, log(m_b^2/m_\ell^2) \qquad q^2 = (p_{\ell+} + p_{\ell-}) \;\Rightarrow\; q^2 = (p_{\ell+} + p_{\ell-} + p_\gamma)$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- In the ratio of the inclusive $b \to s\ell\ell$ decay rate in the high-q^2 region and the semileptonic decay rate large part of the nonperturbative effects cancel out:

Ligeti,Tackmann, arXiv:0707.1694

$$R^{(\ell)}_{\rm incl}(q_0^2) = \frac{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}}$$

Intermezzo

Tensions in the inclusive high q^2 decay rate ??

Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612

$$R_{\rm incl}^{SM}(15) = \frac{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_s\bar{\ell}\ell)}{dq^2}}{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B\to X_u\bar{\ell}\nu)}{dq^2}} \quad x \quad \mathcal{B}(B\to X_u\bar{\ell}\nu)^{\rm exp}_{[15]} = (1.50\pm 0.24)\times 10^{-4}$$

Belle,arXiv:2107.13855

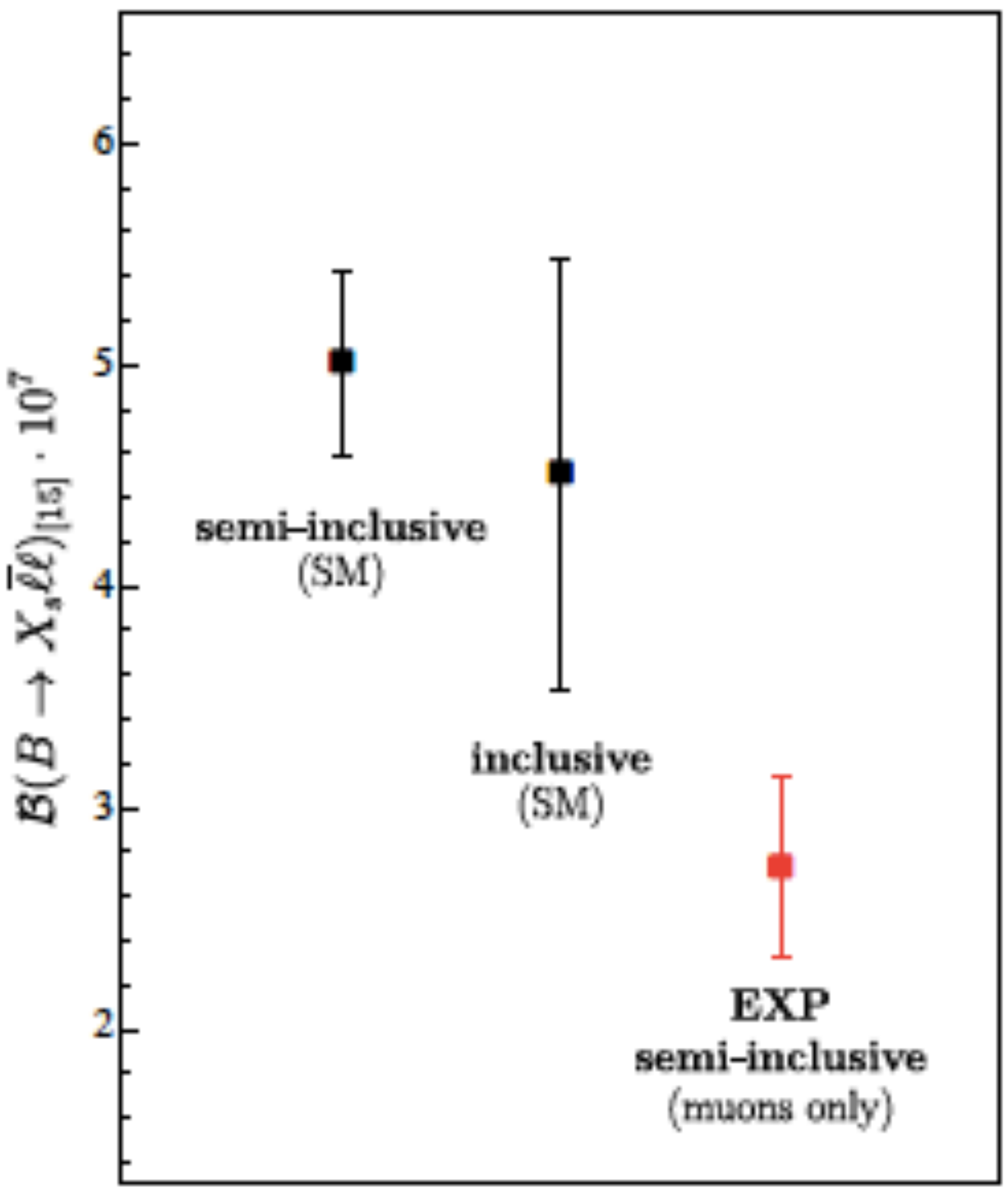
$$= \quad \text{"}\mathcal{B}(B\to X_s\bar{\ell}\ell)^{SM}_{[15]}\text{"} \quad \stackrel{!}{=} \quad \sum_i \mathcal{B}(B\to X_s^i\bar{\mu}\mu)^{\rm exp}_{[15]} = (2.74\pm 0.41)\times 10^{-7}$$

Isidori,Polonsky,Tinari,arXiv:2305.03076

- Experimental semi-inclusive rate is estimated by the sum of the $B\to K$ and $B\to K^*$ modes and a correction factor for the two-body final states $B\to K\pi$.

- Isidori et al. claim a tension up to 2σ – confirming analogous results in the exclusive modes.

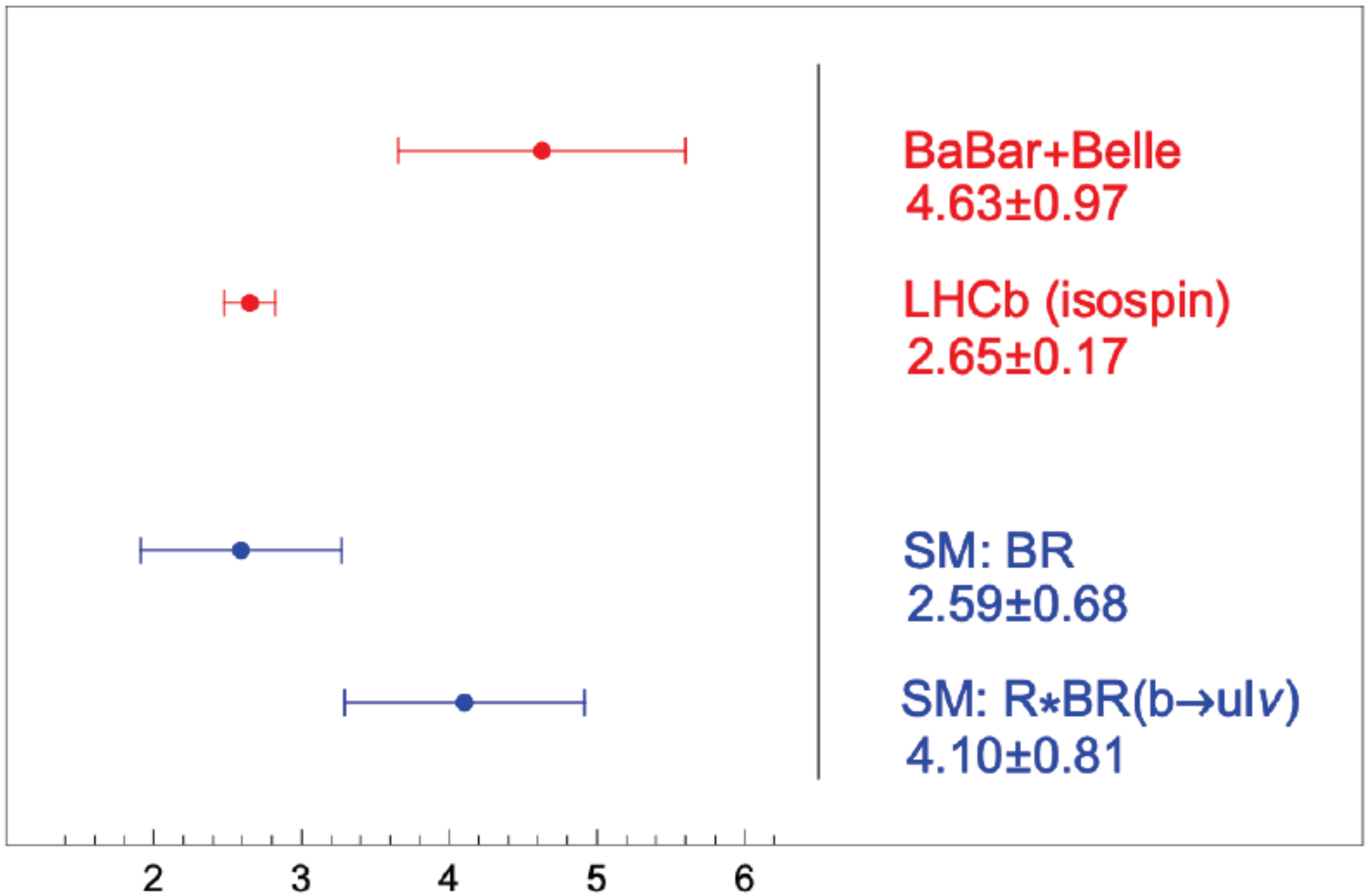
Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612



- We do not find any tension if we also consider our direct result for the branching $\mathcal{B}(B \to X_s \ell\ell)^{\text{SM}}_{[15]}$ and the Babar/Belle measurements.

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Talk by T.H. at FPCP23 and arXiv:2404.03517



$$\mathrm{BR}(B \to X_s \ell^+ \ell^-)[> 15] \times 10^7$$

- We find a slight tension between the two theoretical and also between the two experimental results. We have to be patient!

New Physics Reach of Semi-leptonic Penguin Decays

New physics sensitivity

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

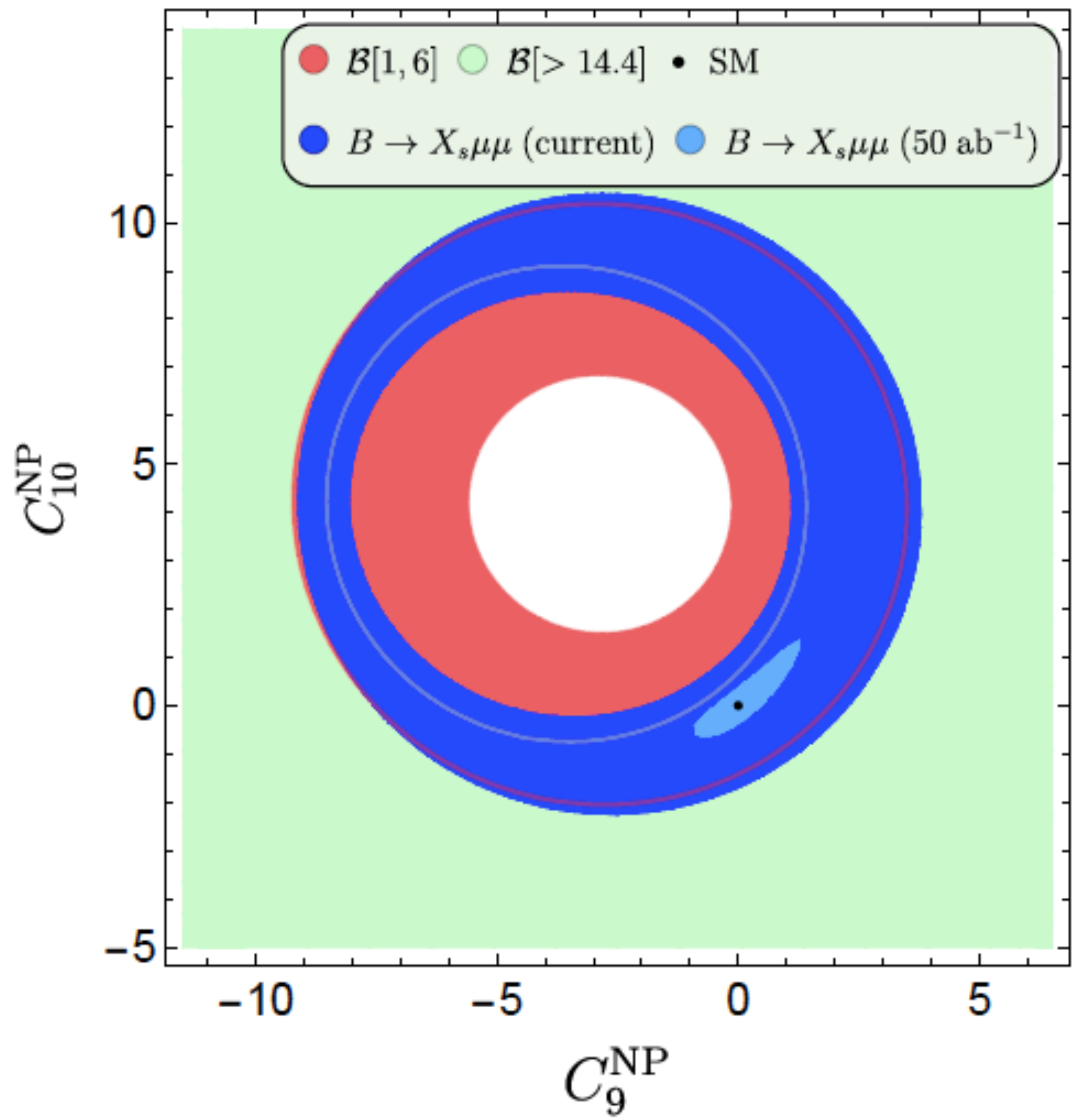
Constraints on Wilson coefficients C_9^{NP} and C_{10}^{NP}

that we obtain at 95% C.L. from present experimental data

(red low q^2, green high q^2)

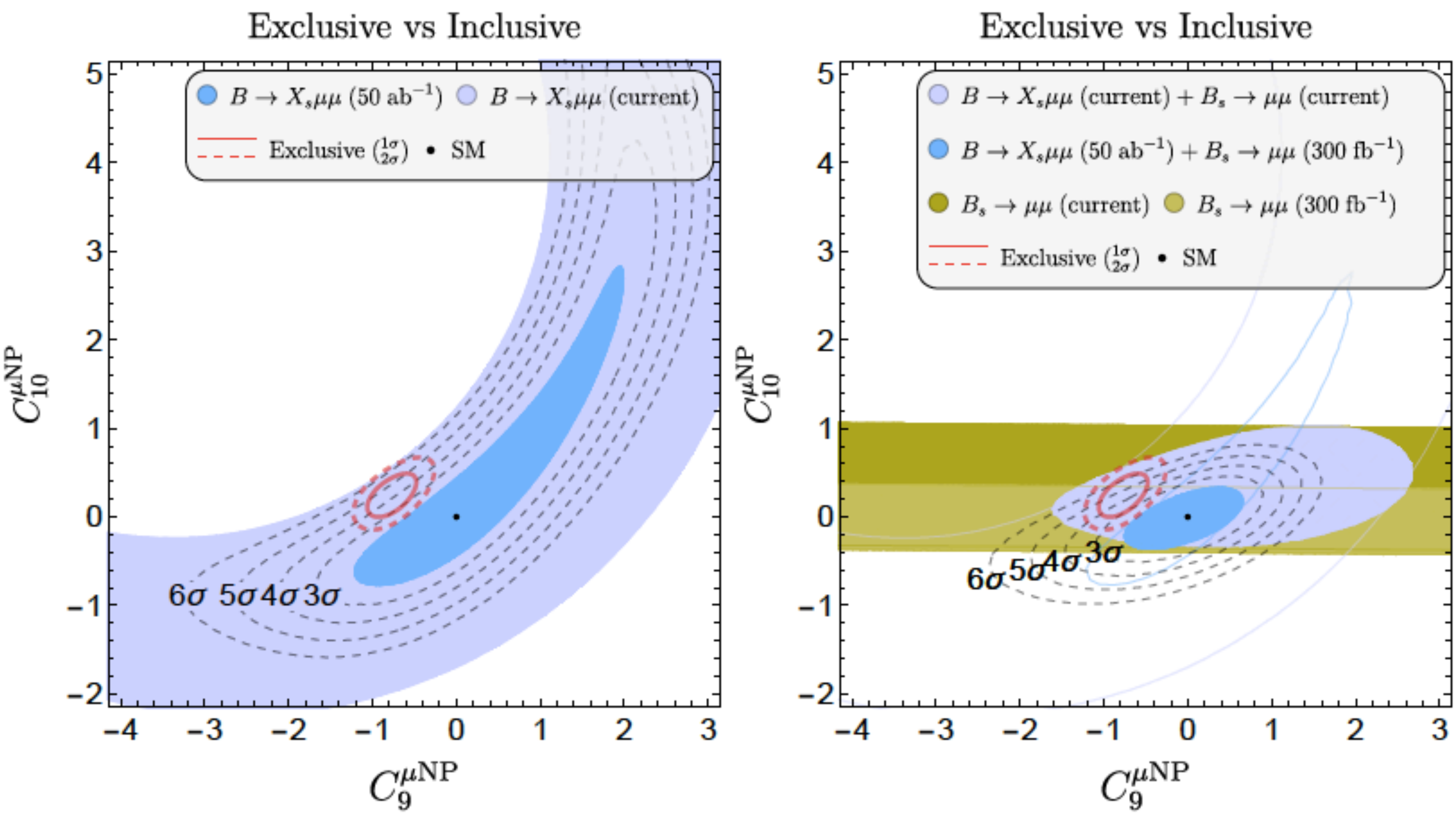
that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II

(light blue)



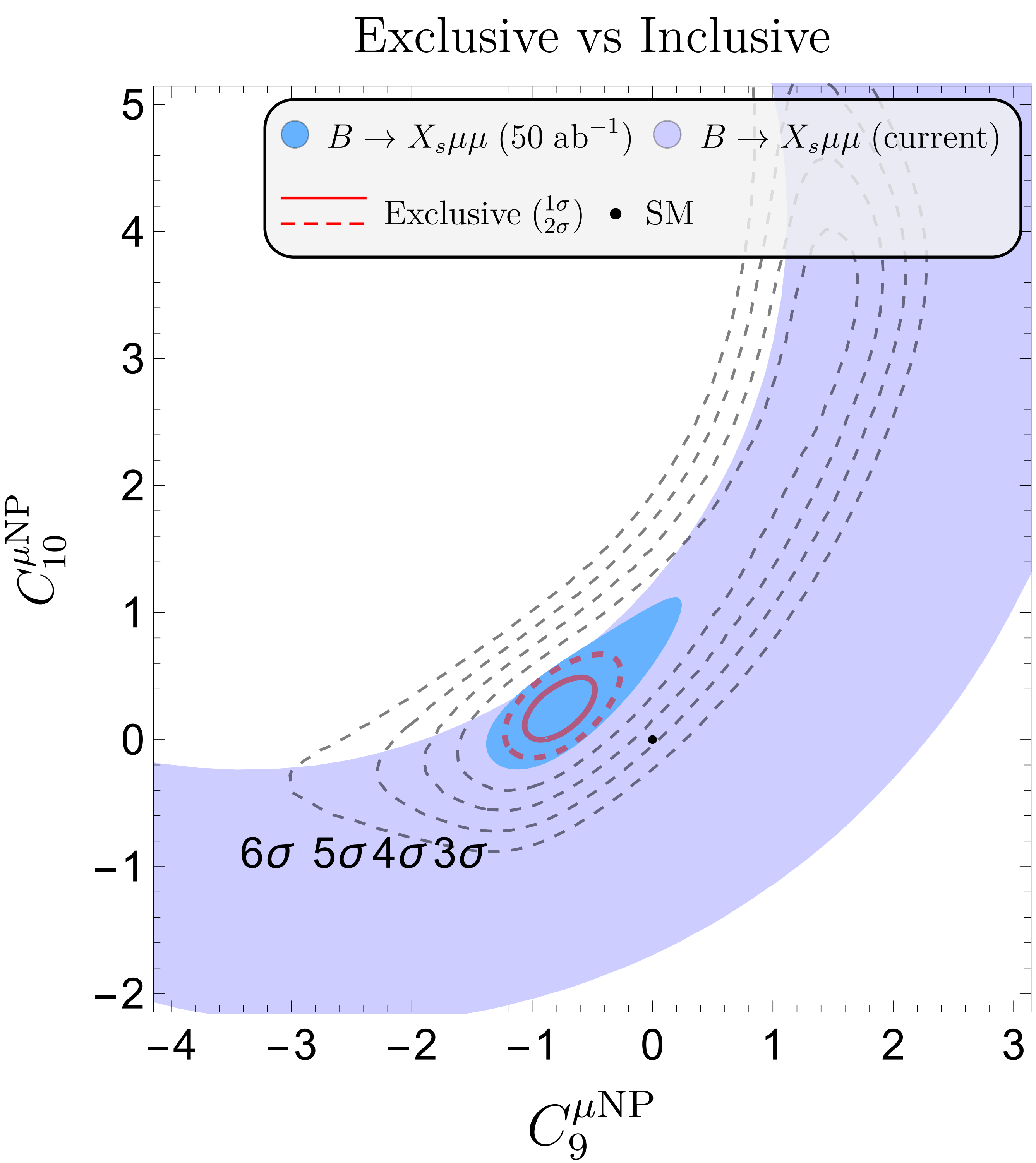
Assuming Belle II measures SM values

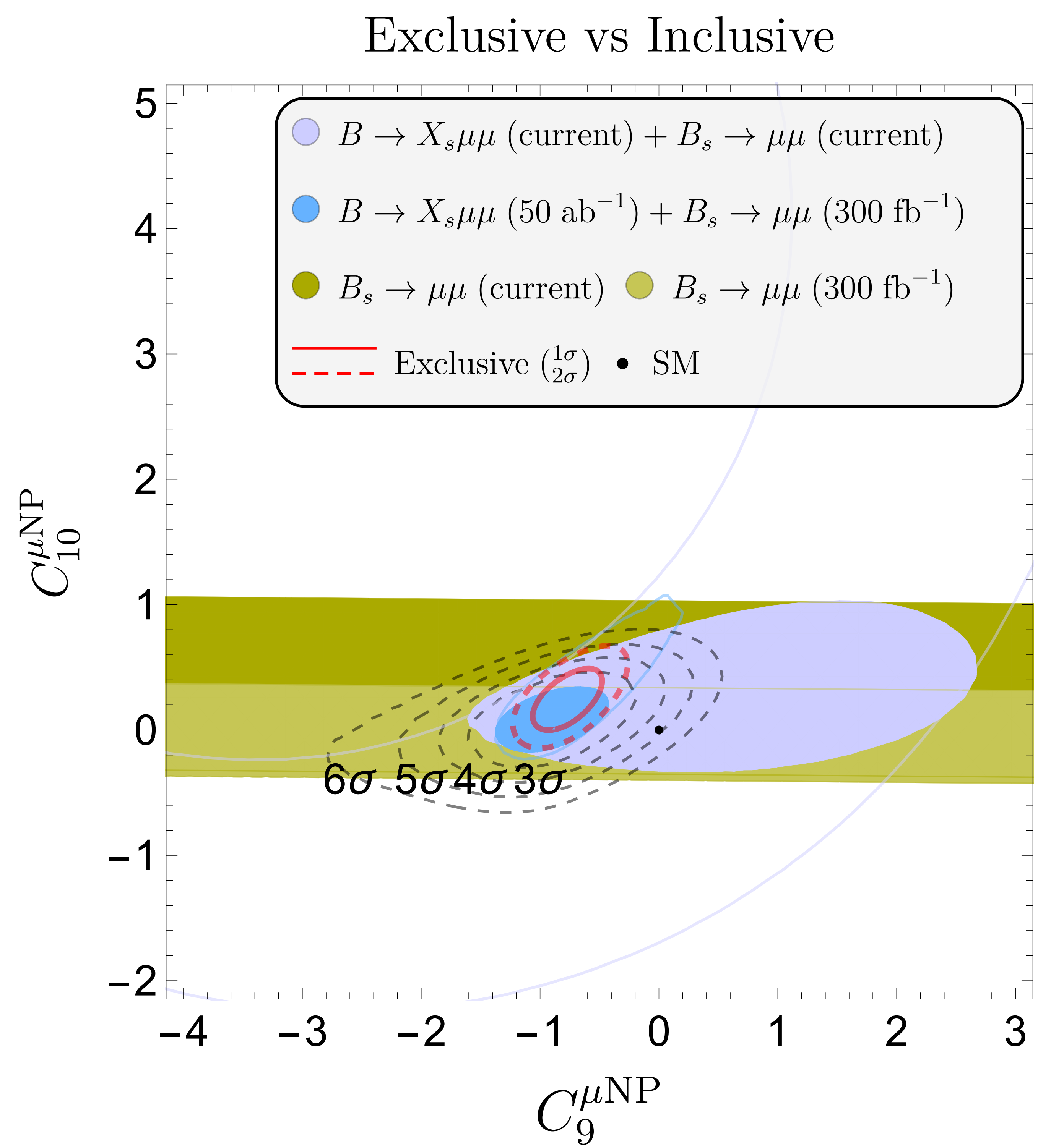
Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191



Assuming Belle II measures best fit point of exclusive fit

Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191





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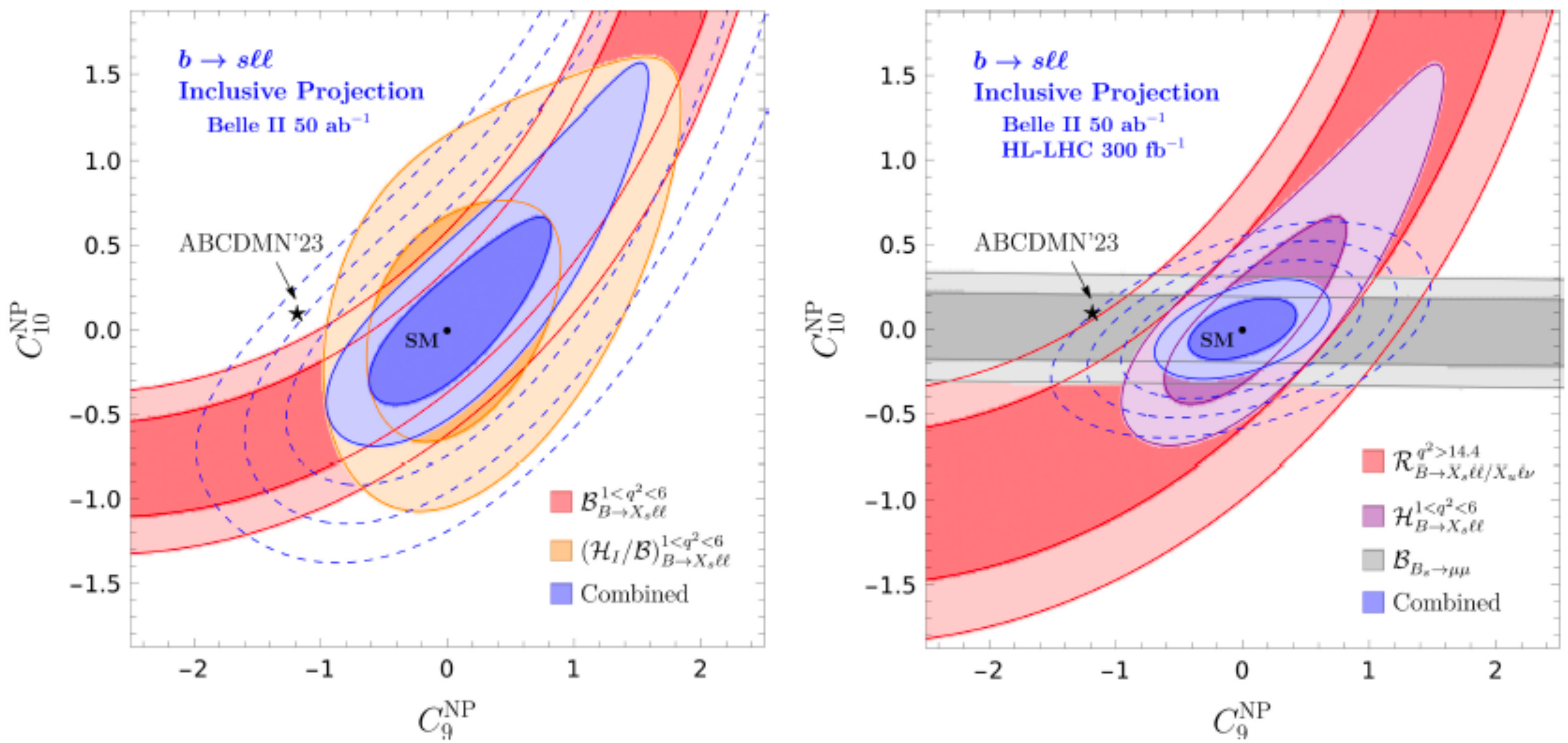
Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Update for post-R_K era arXiv:2404.03517



Belle-II Extrapolations

Akimasa Ishikawa, B2TiP

Error of Branching ratio $\bar{B} \to X_s \ell^+ \ell^-$

BF *(%)* (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
$\geq$ 14.4	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

Error of Normalized Forward-Backward-Asymmetry

AFBn (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
$\geq$ 14.4	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

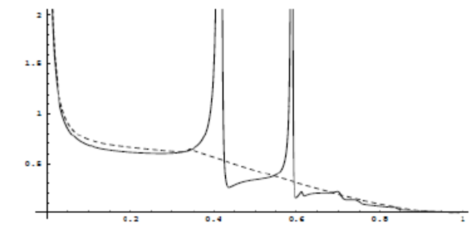
$B \to (\pi, \rho)\ell^+\ell^-$, semi-inclusive $\bar{B} \to X_d \ell^+\ell^-$ at 50/ab
(uncertainties like $\bar{B} \to X_s \ell^+\ell^-$ at 0.7/ab)

Nonlocal subleading contributions

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Benzke,Hurth,Turczyk,arXiv:1705.10366; Benzke,Hurth,arXiv:2006.00624

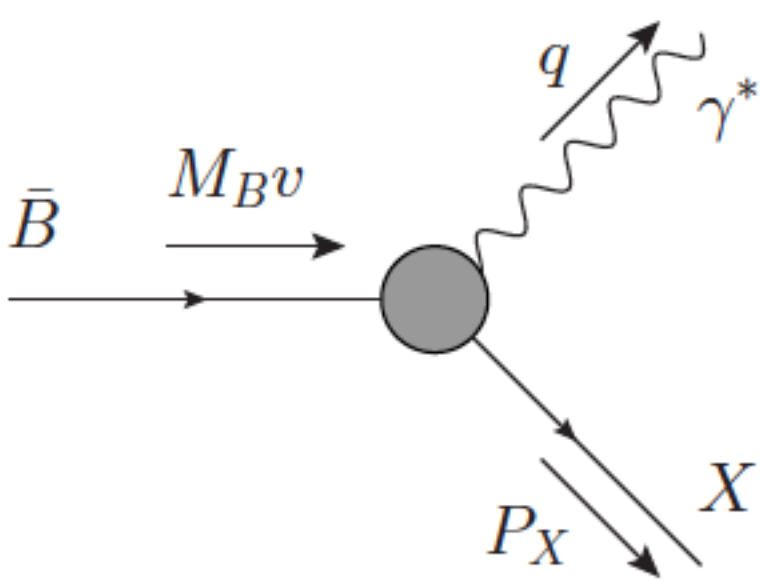
- Cuts in the dilepton mass spectrum necessary due to $c\bar{c}$ resonances



- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Kinematics: X_s is jetlike and $m_x^2 \le m_b \Lambda_{QCD}$ (shapefunction region)

- Multiscale problem $\Rightarrow$ SCET with scaling Λ_{QCD}/m_b



$$M_B^2 \sim m_b^2 \gg m_x^2 \sim m_b \Lambda_{QCD} \gg \Lambda_{QCD}^2$$

hard **hardcollinear** **soft** **modes**

Little calculation

- B meson rest frame $\quad q = p_B - p_X \qquad 2\, m_B\, E_X \; = m_B^2 + M_X^2 - q^2$

  X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

- $p_X^- p_X^+ = m_X^2 \qquad$ two light-cone components

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\mathrm{QCD}})$$

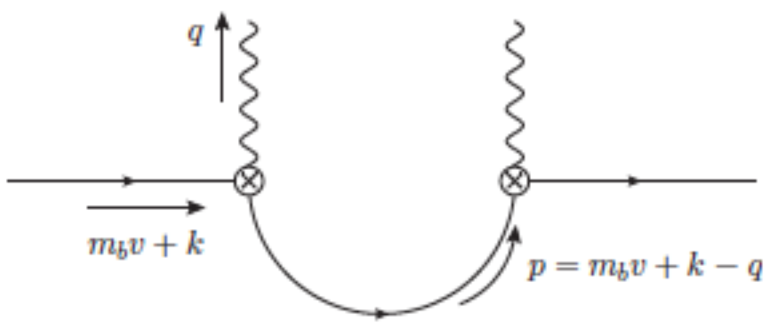
- $q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n}q = m_B - p_X^-$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

$$\lambda = \Lambda_{\mathrm{QCD}}/m_b \qquad m_X^2 \sim \lambda \Rightarrow \; m_b - n \cdot q \sim \lambda$$

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n\cdot q}\left(1 - \frac{n\cdot k}{m_b - n\cdot q} + \ldots\right)\frac{1}{m_b - \bar{n}\cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s\gamma$)

Factorization theorem $\quad d\Gamma \sim H \cdot J \otimes S$

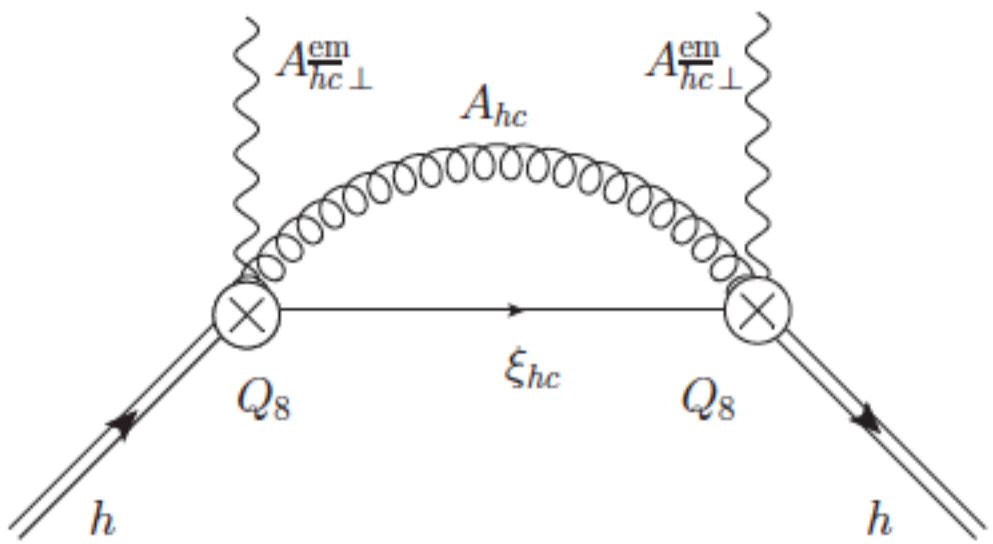
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

Example of **direct** photon contribution which factorizes $\quad d\Gamma \sim H \cdot j \otimes S$

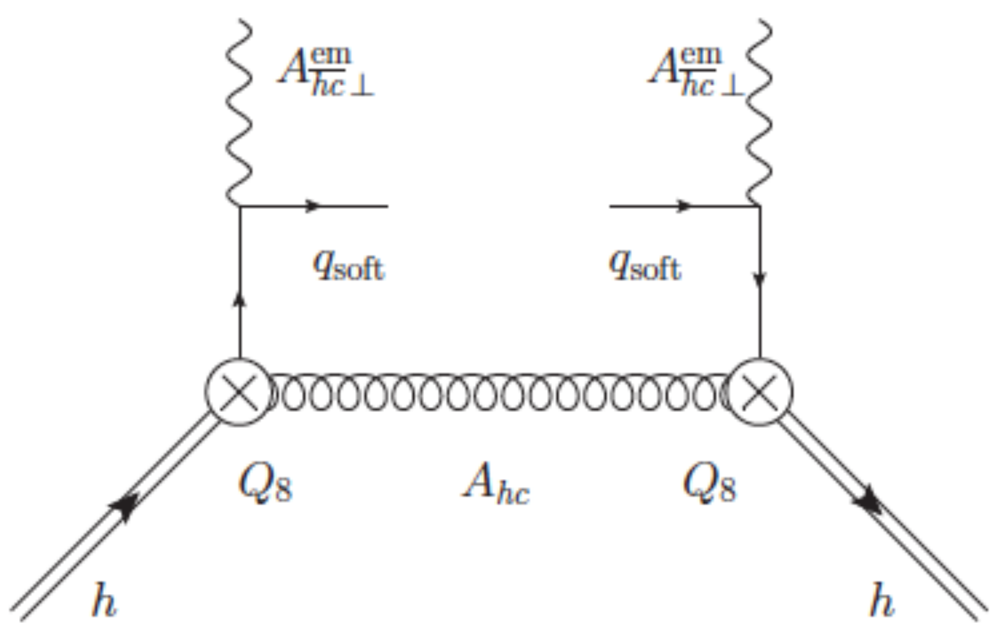


$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$

: Benzke,Lee,Neubert,Paz,arXiv:1003.5012



$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_1 and Q_7

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J}$$

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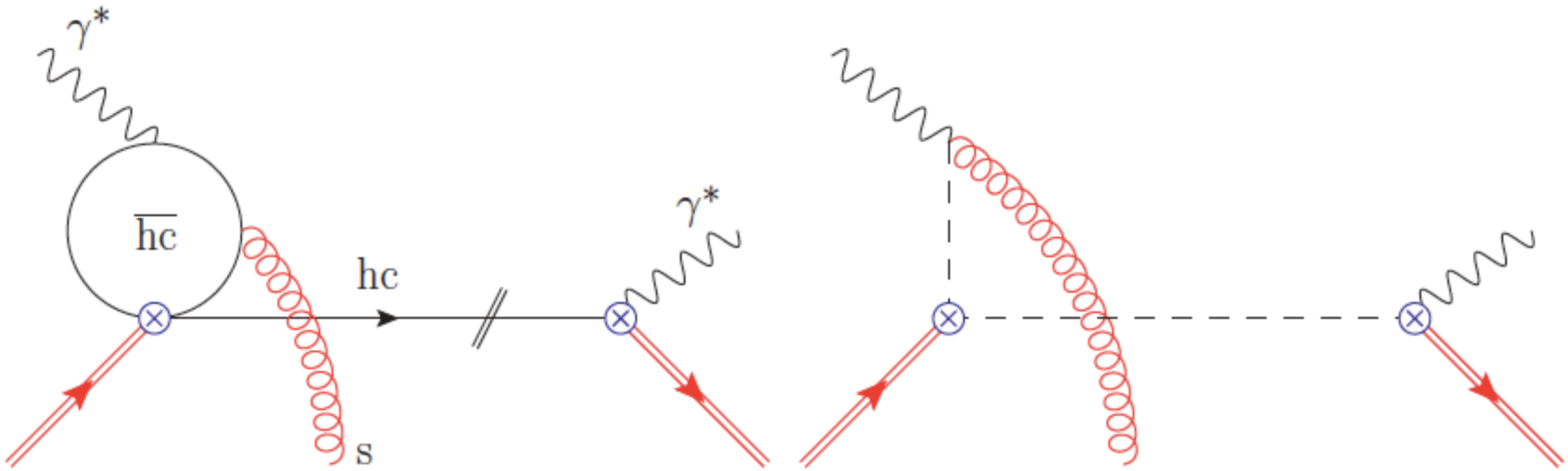

$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega+p_+)\int \frac{d\omega_1}{\omega_1+i\varepsilon}$$

$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-1\right)-(\bar{n}\cdot q+\omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)-1\right)\right.$$

$$\left.+\bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right)-G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q+\omega_1)}\right)\right)\right] g_{17}(\omega,\omega_1)$$

$$g_{17}(\omega,\omega_1)=\int\frac{dr}{2\pi}e^{-i\omega_1 r}\int\frac{dt}{2\pi}e^{-i\omega t}\frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots G_s^{\alpha\beta}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
  - PT invariance implies that soft functions are real
  - Moments of shape functions are related to HQET parameters
  - Soft functions have no significant structure outside the hadronic range
  - Values of soft functions are within the hadronic range
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$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^0 \, h_{17}(\omega_1, \mu) = 0.237 \pm 0.040 \, \mathrm{GeV}^2$$

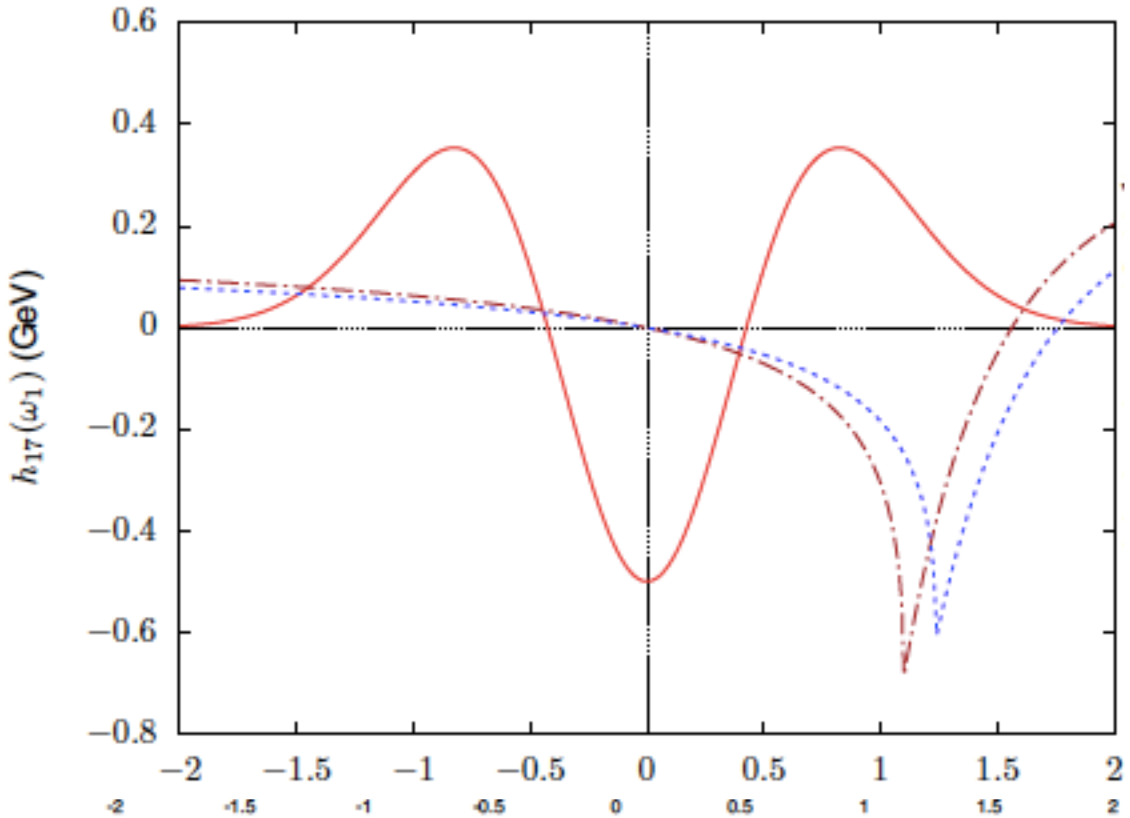
New input:
$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^2 \, h_{17}(\omega_1, \mu) = 0.15 \pm 0.12 \quad \mathrm{GeV}^4$$

Paz et al. arXiv:1908.02812

Updated result for $\bar{B} \to X_s\gamma$

Benzke,Hurth,arXiv:2006.00624

Charm dependence of jet function:

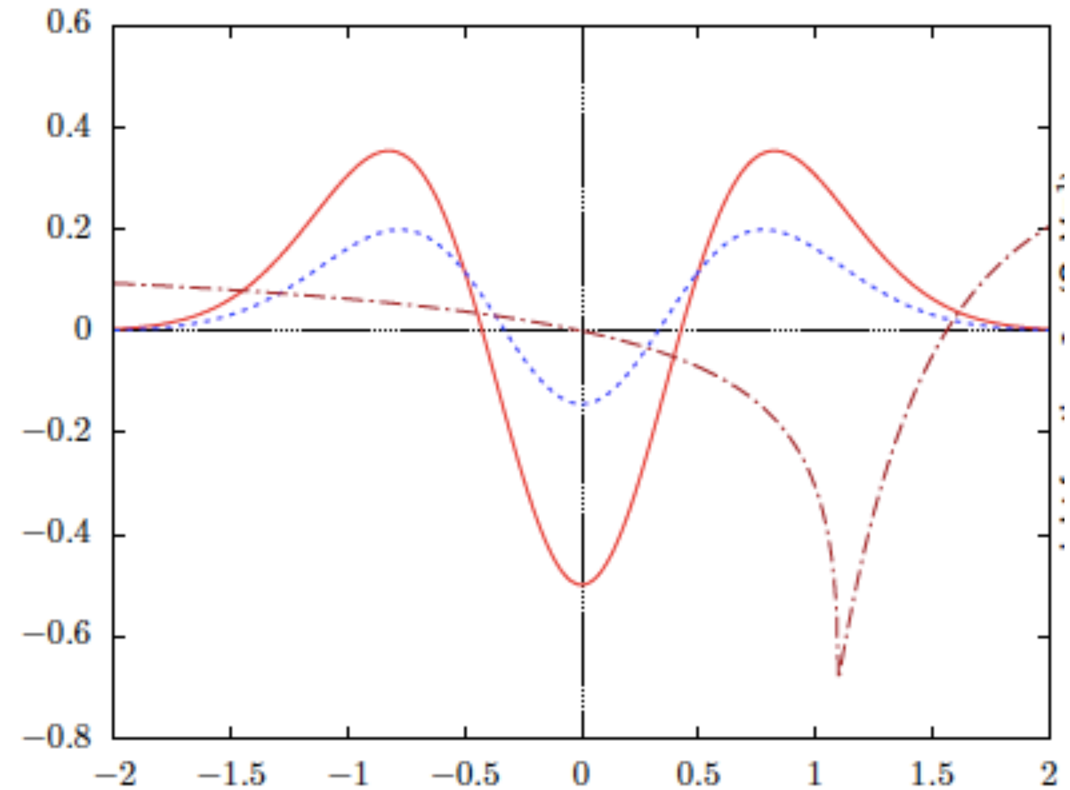


Benzke,Hurth,arXiv:2006.00624

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-3.7\%,\, 6.5\%]$$

Constraint on shape function:



Neubert et al., arXiv: 1003.5012

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-1.9\%,\, 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-5.2\%,\, 6.5\%]$$

(In addition: large scale dependence)

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \to X_s\gamma$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**
- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

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- **Task 1** For NLL analysis we have to establish a factorisation theorem.

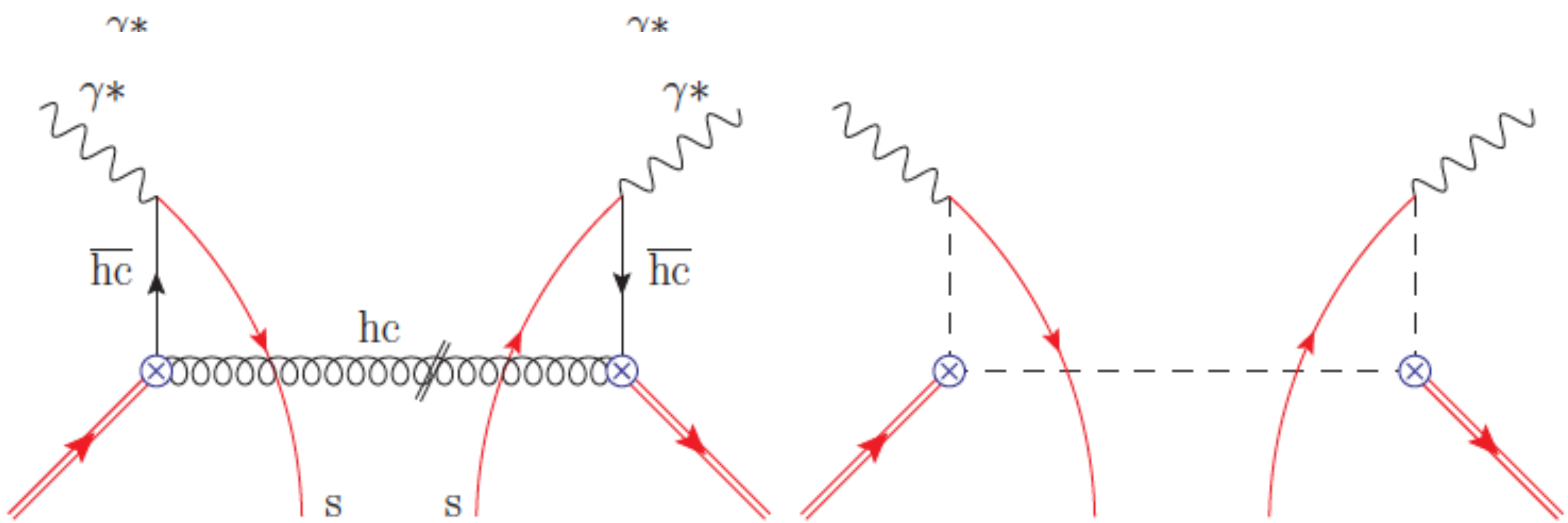
  Hurth,Szafron,arXiv:2301.01739

$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

- **Task 2** Various steps of the NLL analysis Bartocci,Böer,Hurth

- **Task 1** For **NLL analysis** we have to establish a factorisation theorem.

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\,\delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int\frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon}g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

- Subtlety in the Q_8-Q_8 contribution: convolution integral is UV divergent
  - This implies that there is no complete proof of the factorization formula yet.

Benzke,Lee,Neubert,Paz,arXiv:1003.5012

- **Task 1** For **NLL analysis** we have to establish a factorisation theorem.

Refactorisation in subleading $\bar{B} \to X_s\gamma$ Hurth,Szafron,arXiv:2301.01739

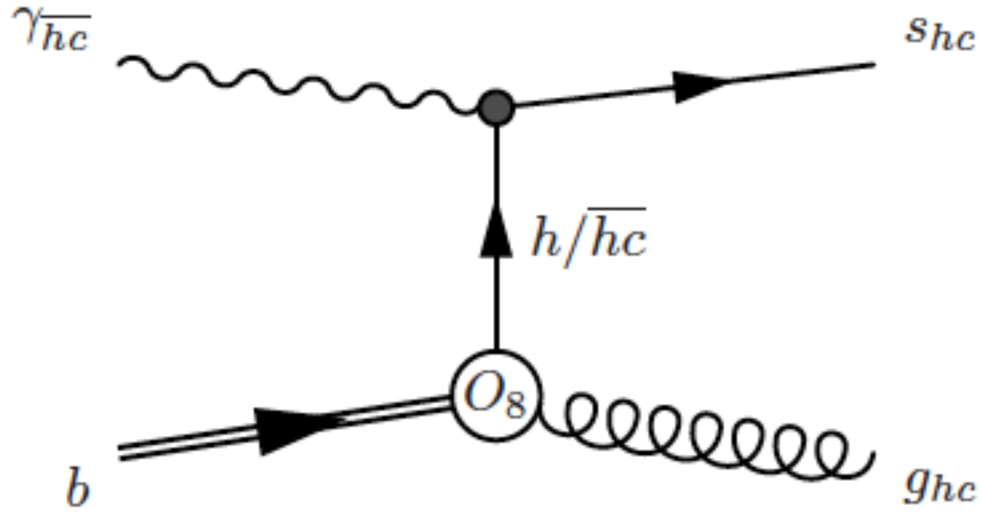
- Naive factorisation theorem with anti-hardcollinear Jet functions $\overline{J}$

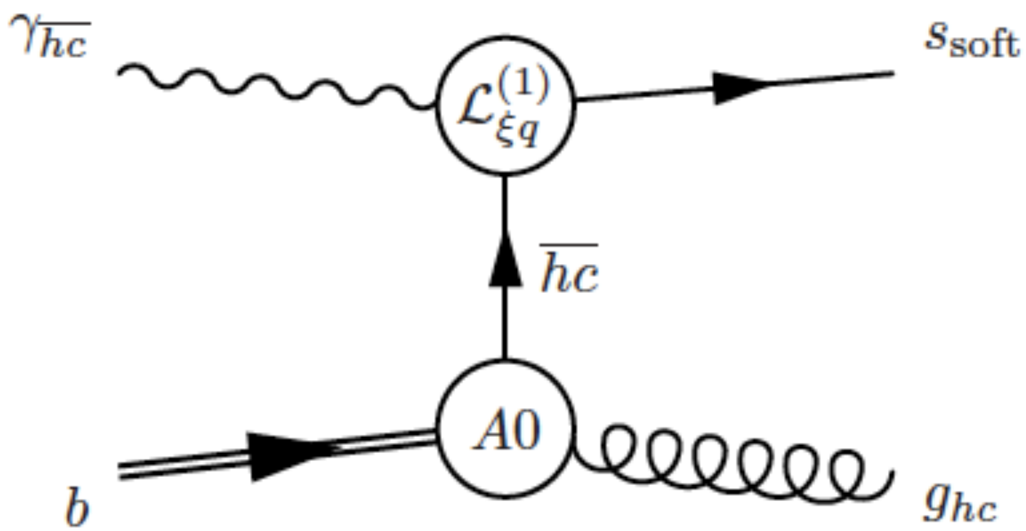
$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

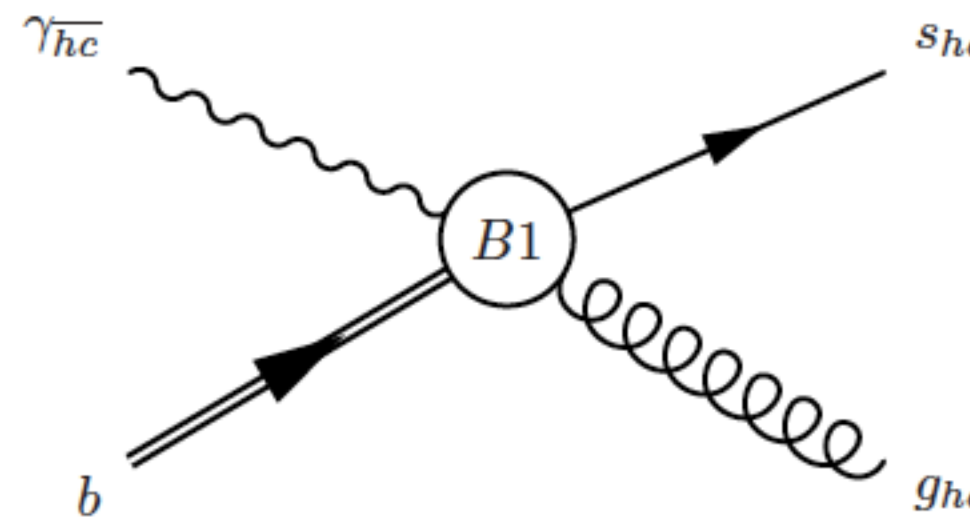
- Contribution of the gluon dipole operator does not factorise
- One can identify divergences in resolved *and* direct contribution in SCET-I as endpoint-divergences
- One can use refactorisation techniques developed in collider examples
  Neubert et al.,arXiv:2009.06779
- First QCD application with nonperturbative objects in flavour physics

Degeneracy in EFT leads to endpoint divergences

Hurth,Szafron,arXiv:2301.01739







$$\mathcal{O}_{8g}^{A0}(0) = \overline{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

$$\mathcal{O}_{8g}^{B1}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

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- **Task 2** Various steps of the NLL analysis   Bartocci,Böer,Hurth

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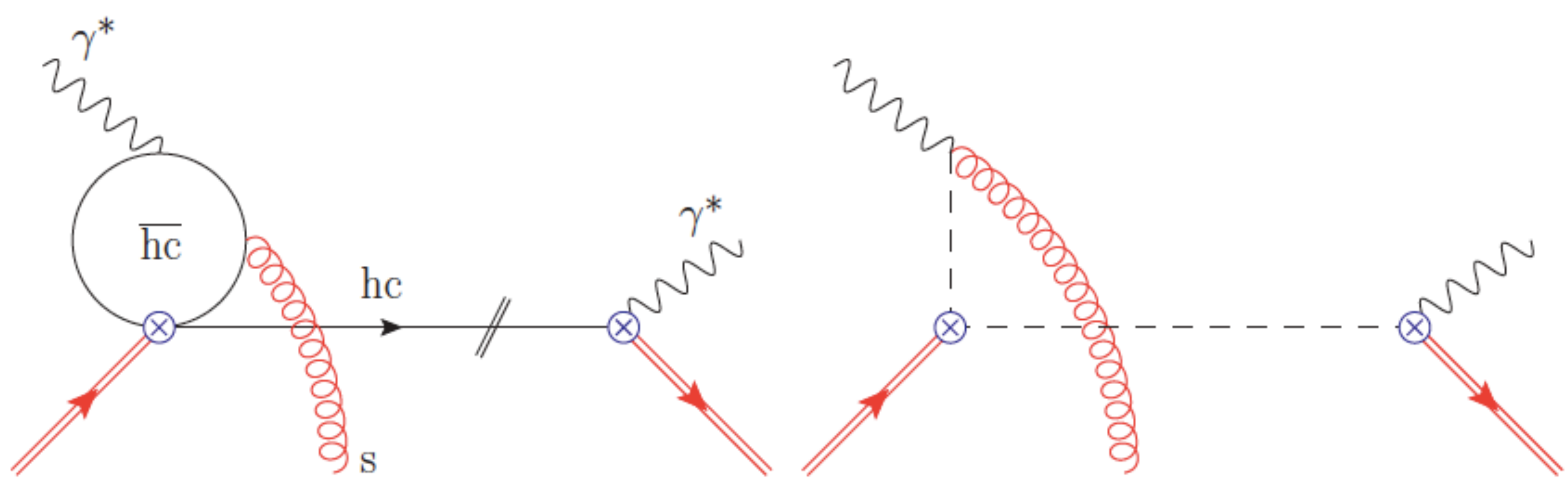
  - analysis of renormalisation properties of the soft function

$$\mathcal{S}_{ren}(\omega,\omega_1) = \int_{-\infty}^{\bar\Lambda} d\omega' \, Z_{\mathcal{S}}(\omega,\omega',\omega_1,\omega_1') \, \mathcal{S}_{bare}(\omega',\omega_1').$$



Bartocci,Böer,Hurth, arXiv:2411.16634

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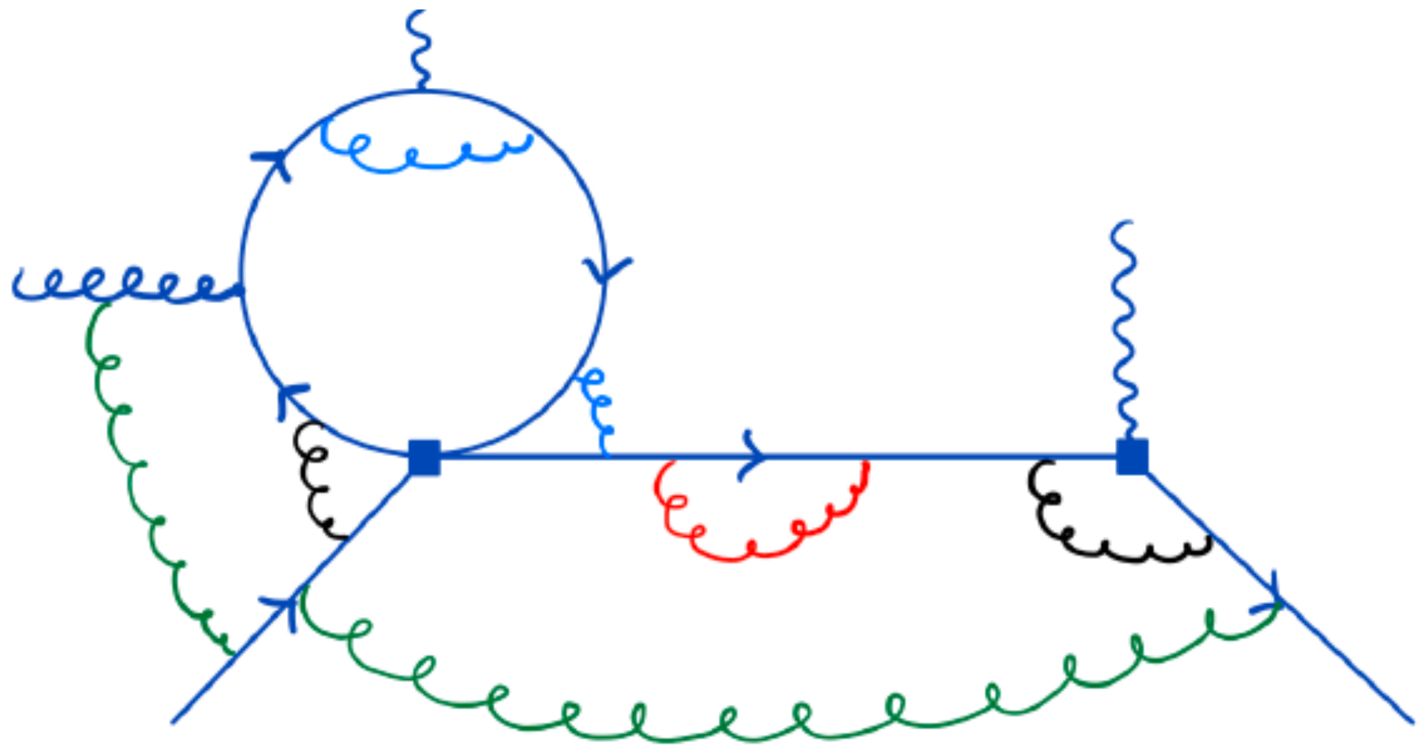

$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

In SCET, we can compute gauge invariant pieces separately.

- analysis of renormalisation properties of **the soft function** ✓
- α_s (two-loop) corrections to **anti-jet function**

  We already calculated all diagrams for $m_c \to m_u = 0$ ✓
- α_s corrections to **quark jet function** known ✓
- **hard** matching at order α_s known ✓
- use RG techniques to run various functions to a common scale.

  We already checked the pole cancellation for $m_c \to m_u = 0$ ✓

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- Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

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We use scale variation of the hard-collinear scale

$\mu_{\mathrm{hc}} \sim \sqrt{m_b\,\Lambda_{\mathrm{QCD}}}$ from $1.3\,\mathrm{GeV}$ to $1.7\,\mathrm{GeV}$ and get

$$1.14\,\mathrm{GeV} \le m_c \le 1.26\,\mathrm{GeV}$$

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Underestimation of the uncertainty due to the resolved contribution.

But used in recent $b \to s\gamma$ analysis. Misiak, Rehman, Steinhauser, arXiv:2002.01548v2

Summary

- In the post-R_K era we still have signifcant tensions in exclusive $b \to s$ angular observables and branching ratios.

- Inclusive semi-leptonic decays require Belle-II for full exploitation, but are theoretically very clean and allow for crosschecks of the present tensions.

- Nonlocal power corrections presently belong to the largest uncertainties in the inclusive modes (5%) $\bar{B} \to X_s\gamma$ and $\bar{B} \to X_s\ell\ell$ (up to 5%) (higher moments of shape functions and α_s corrections needed).

- Refactorisation techniques allow to solve the problem of endpoint divergences, in particular in subleading $\bar{B} \to X_s\gamma$.

Epilogue

- The days of "guaranteed" discoveries or no-lose theorems in particle physics are over, at least for the time being .....
- but the big questions of our field remain open (hierarchy problem. flavour, neutrinos, dark matter, baryogenesis,...)
- This simply implies that, more than for the past 30 years, future HEP's progress is to be driven by experimental exploration, possibly renouncing/reviewing deeply rooted theoretical bias.


EYGUIÈRES
EYGALIÈRES

Experimental flavour opportunities

- LHCb: allows for wide range of analyses, highlights: B_s mixing phase, angle γ, $B \to K^*\mu\mu$, $B_s \to \mu\mu$, $B_s \to \phi\phi$ then upgrades to 50 and 300fb^{-1}

- Dedicated kaon experiments J-PARC E14 and CERN P-326/NA62: rare kaon decays $K_L^0 \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$

- Super-B factory Belle-II at KEK (50ab^{-1})
  Belle-II is a Super Flavour factory: besides precise B measurements CP violation in charm, lepton flavour violating modes $\tau \to \mu\gamma$, ...

Spares 1

Refactorisation

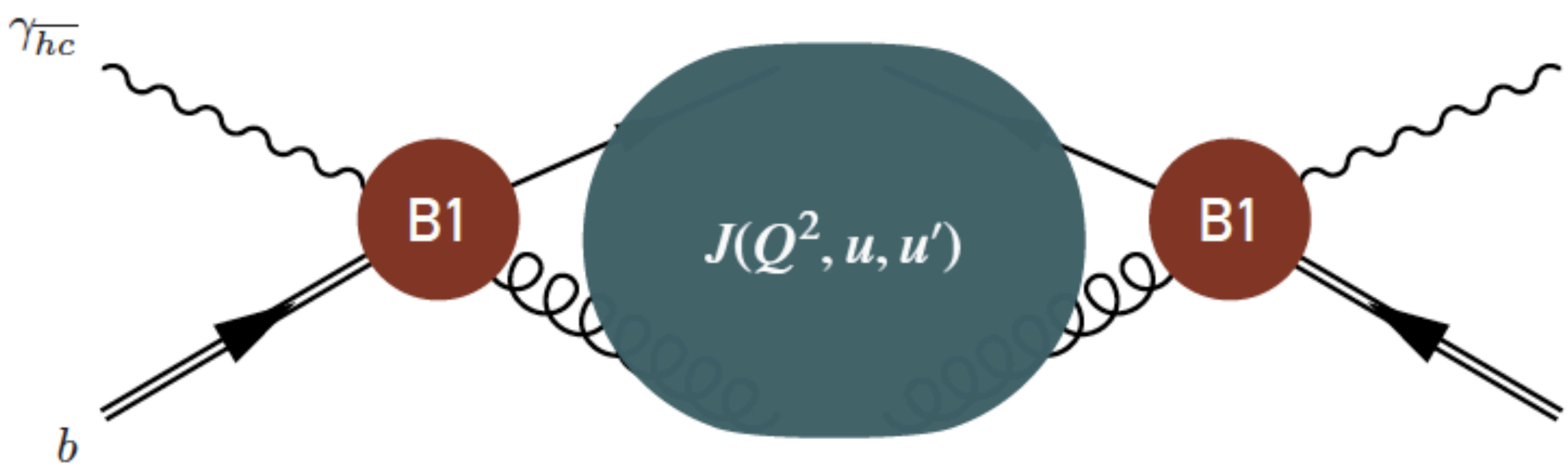
Hurth,Szafron,arXiv:2301.01739

Factorisation of direct contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, \mathbf{C^{B1}}\left(\mathbf{m_b}, \mathbf{u}\right) \int_0^1 du'\, \mathbf{C^{B1*}}\left(\mathbf{m_b}, \mathbf{u'}\right) \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J}\left(\mathbf{M_B}\left(\mathbf{p_+} + \omega\right), \mathbf{u}, \mathbf{u'}\right) \mathcal{S}\left(\omega\right)$$

$$\mathbf{J}\left(\mathbf{p^2}, \mathbf{u}, \mathbf{u'}\right) = \frac{(-1)}{2N_c}\,\frac{1}{2\pi} \int \frac{dt\,dt'}{(2\pi)^2}\, d^4x\; e^{-im_b(ut-u't')+ipx}$$

$$\mathrm{Disc}\left[\langle 0|\, tr\left[\frac{1+\not{v}}{2}(1-\gamma_5)\mathcal{A}_{hc\perp}(x)\,\gamma_\perp^\nu \chi_{hc}(t'\bar{n}+x)\,\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\mathcal{A}_{hc\perp}(0)\,(1+\gamma_5)\right] |0]\rangle\right]$$



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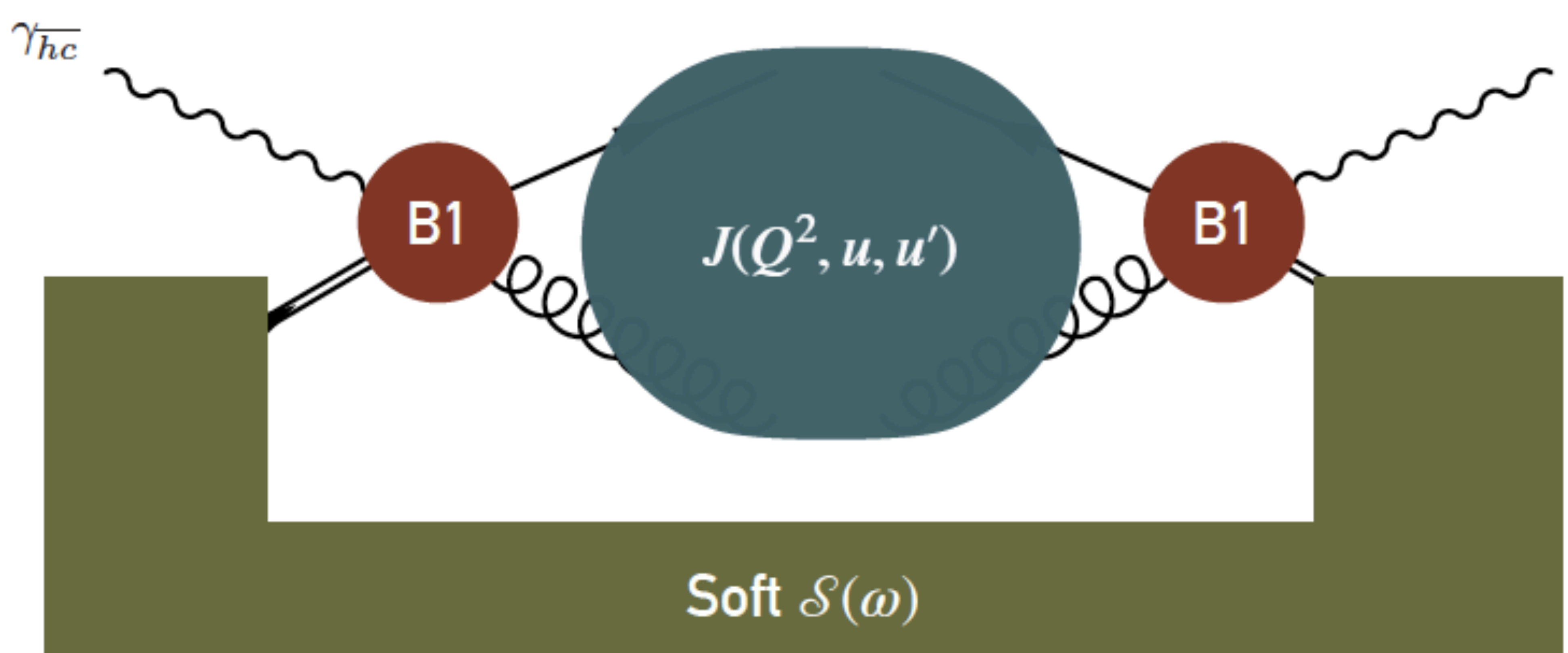
$$\mathcal{S}\left(\omega\right) = \frac{1}{2m_B} \int \frac{dt}{2\pi} e^{-i\omega t} \left\langle B\right| h\left(tn\right) S_n\left(tn\right) S_n^\dagger\left(0\right) h\left(0\right) \left|B\right\rangle$$



Endpoint divergence in direct contribution at leading order

Hard matching coefficients

$$\mathbf{C^{B1}_{LO}}\left(\mathbf{m_b}, \mathbf{u}\right) = (-1)\frac{\overline{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t C_{8g} = (-1)\frac{\overline{u}}{u}C^{A0}_{LO}\left(m_b\right)$$

convoluted with jet function

$$\mathbf{J}\left(\mathbf{p^2}, \mathbf{u}, \mathbf{u'}\right) = C_F\frac{\alpha_s}{4\pi\, m_b}\theta(p^2)\, A(\epsilon)\, \delta(u-u')u^{1-\epsilon}(1-u)^{-\epsilon}\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$$

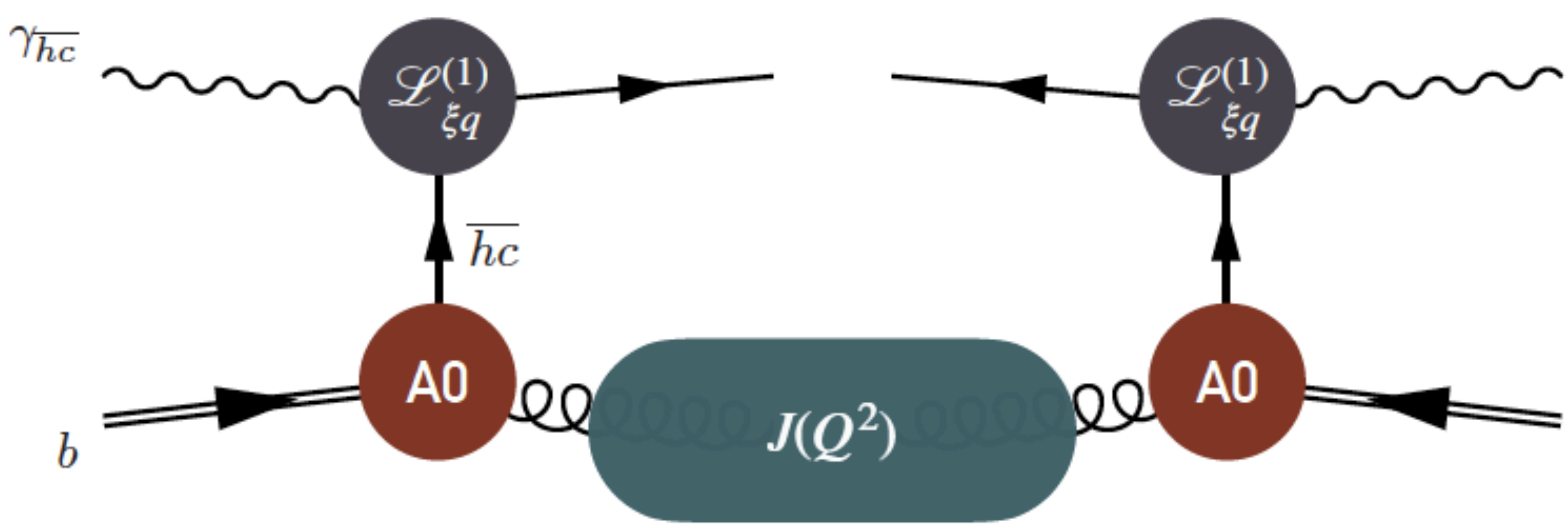
lead to endpoint divergence in the $u \to 0$ limit

$$\int_0^1 du\frac{1}{u}\int_u^1 du'\frac{1}{u'}u^{1-\epsilon}\delta(u-u') \sim \int_0^1 du\frac{1}{u^1+\epsilon}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

$$-g_s^2 \delta_{ab} g_\perp^{\mu\nu}\ \mathbf{J_g}(\mathbf{p^2}) = \frac{1}{2\pi i}\,\mathrm{Disc}\left[i\int d^4x e^{ipx} \langle 0|\, T\left[\mathcal{A}^{a\mu}_{hc\perp}(x), \mathcal{A}^{b\nu}_{hc\perp}(0)\right]|0\rangle\right]$$

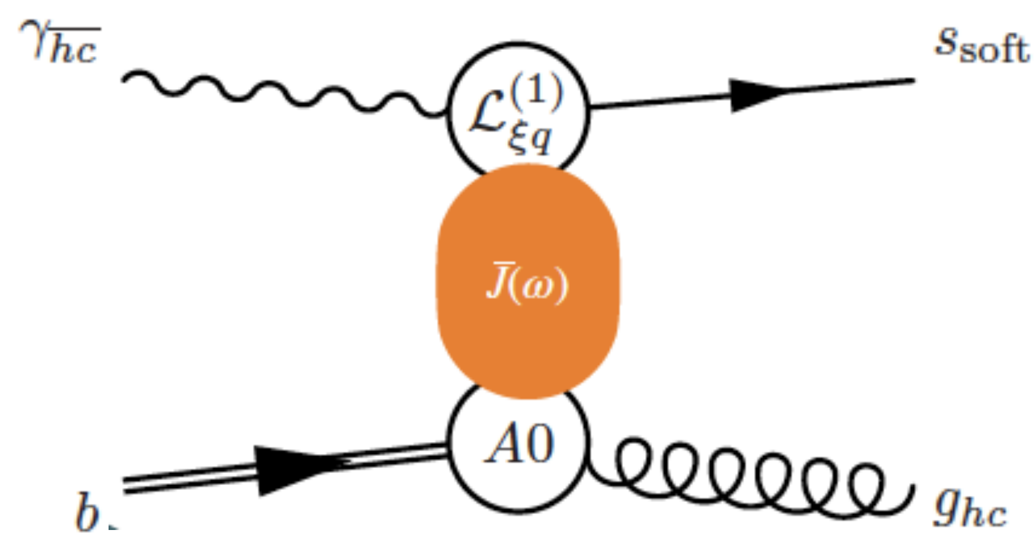


Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C}^{\mathbf{A0}}(\mathbf{m_b})\right|^2 \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J_g}(\mathbf{m_b}(\mathbf{p_+}+\omega)) \int d\omega_1 \int d\omega_2\, \overline{\mathbf{J}}(\omega_1)\, \overline{\mathbf{J}}^*(\omega_2)\, \mathcal{S}(\omega,\omega_1,\omega_2)$$

Anti-hardcollinear jet function $\overline{J}(\omega)$ is defined on the amplitude level.

$$O_{T\xi q} = i \int d^d x T\left[\mathcal{L}_{\xi q}(x), O_{8g}^{A0}(0)\right]$$



$$= \int d\omega \int \frac{dt}{2\pi} e^{-it\omega} \left[\overline{q_s}\right]_\alpha (tn) \left[\overline{\mathbf{J}}(\omega)\right]_{\alpha\beta}^{\mathbf{a}\,\nu\mu} Q_s\, \mathcal{B}^\nu_{\overline{hc}\perp}(0)\, \mathcal{A}^{\mu\, a}_{hc\perp}(0)\, [h(0)]_\beta$$

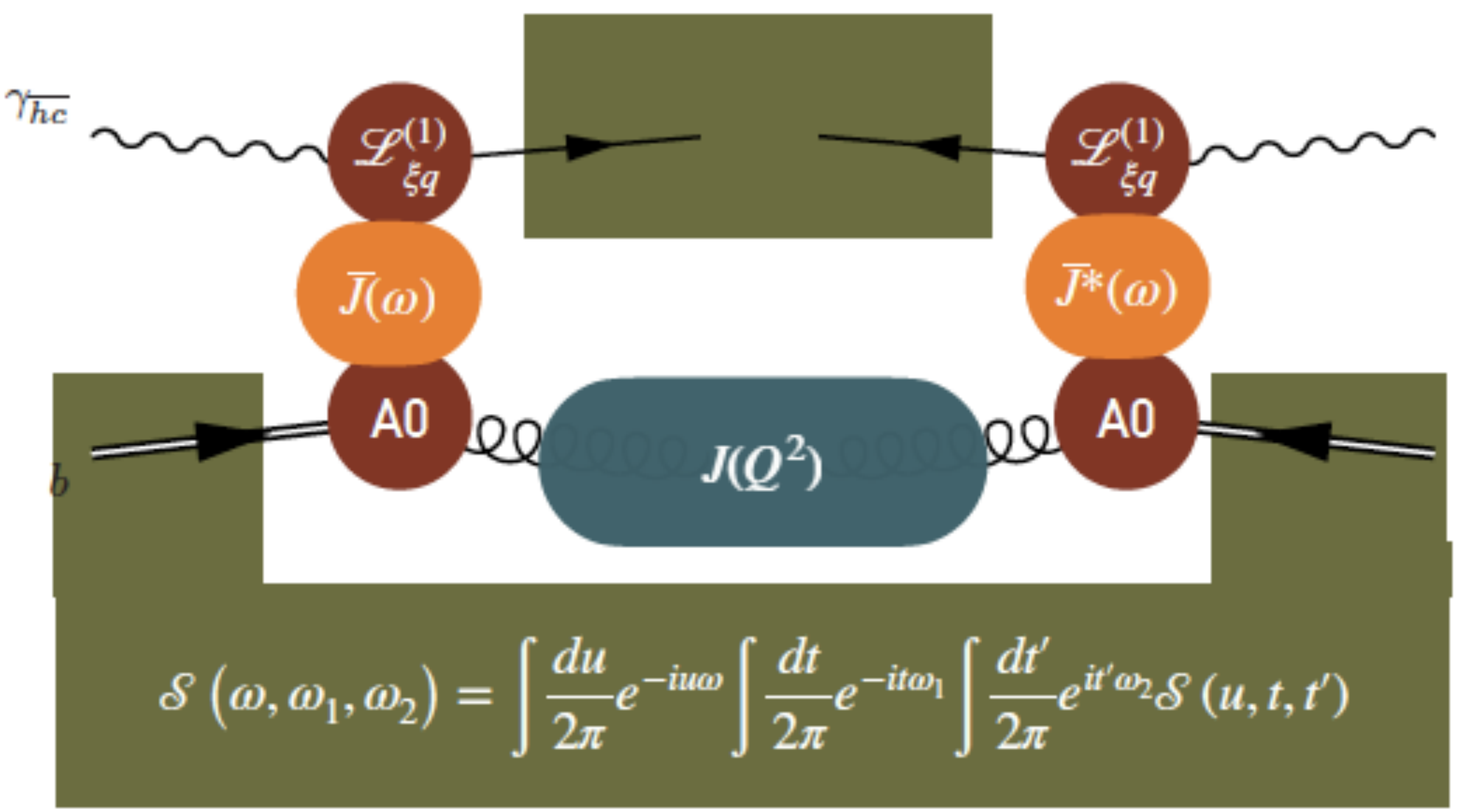
Decomposition to all orders: $\left[\overline{\mathbf{J}}(\omega)\right]_{\alpha\beta}^{\mathbf{a}\,\nu\mu} = \overline{J}(\omega)\, t^a \left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar{n}}\not{n}}{4}\right]_{\alpha\beta}$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

Operatorial definition of the soft function in position space $\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t'}\right)$

$$\mathcal{S}\left(\mathbf{u},\mathbf{t},\mathbf{t'}\right) = (d-2)^2 g_s^2 \langle B|\,\bar h\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right)t^a S_n^\dagger\left(un\right)\right] S_{\bar n}\left(un\right) S_{\bar n}^\dagger\left(t'\bar n+un\right)$$
$$\frac{\not{n}\not{\bar n}}{4} q_s\left(t'\bar n+un\right)\bar q_s\left(t\bar n\right)\frac{\not{\bar n}\not{n}}{4} S_{\bar n}\left(t\bar n\right) S_{\bar n}^\dagger\left(0\right)\left[S_n\left(0\right)t^a S_n^\dagger\left(0\right)\right]\left(1+\gamma_5\right)h\left(0\right)|B\rangle\;/\left(2m_B\right)$$



Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \mathbf{\bar J}\left(\omega_1\right) \mathbf{\bar J}^*\left(\omega_2\right) \mathcal{S}\left(\omega,\omega_1,\omega_2\right)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

$\omega_{1,2} \to \infty$ corresponds to $t, t' \to 0$ and $q_s(un) \to S_n(un) q_{hc}(un)$, $\bar{q}_s(0) \to q_{hc} S_n^{+}(0)$

$$\mathcal{S}\left(u,t,t'\right) = (d-2)^2 g_s^2 \langle B|\, \bar h\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right) t^a \cancel{S_n^\dagger}\left(un\right)\right] S_{\bar n}\cancel{\left(un\right) S_{\bar n}^\dagger}\left(t'\bar n + un\right)$$
$$\frac{\not{n}\not{\bar n}}{4} q_s \cancel{\left(t'\right.}\bar n + un\left.\right) \cancel{\bar q_s}\left(t\bar n\right) \frac{\not{n}\not{\bar n}}{4} S_{\bar n} \cancel{\left(t\bar n\right) S_{\bar n}^\dagger}\left(0\right)\left[\cancel{S_n}\left(0\right) t^a S_n^\dagger\left(0\right)\right]\left(1+\gamma_5\right) h\left(0\right)|B\rangle \,/\, \left(2m_B\right)$$

$$\mathcal{S}\left(u\right) = \langle B|\, \bar h\left(un\right) S_n\left(un\right)\, S_n^\dagger\left(0\right) h\left(0\right)|B\rangle \,/\, \left(2m_B\right)$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C}^{\mathbf{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+} + \omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
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More general:

Asymptotic $(\omega_1 \sim \omega_2 \leq \omega)$ soft function $\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$ is a convolution of a perturbabtive kernel K and the leading power soft function.

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = \int d\omega' K(\omega, \omega', \omega_1, \omega_2) \mathcal{S}(\omega')$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+}+\omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
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More general:
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$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = \int d\omega' K(\omega, \omega', \omega_1, \omega_2) \mathcal{S}(\omega')$$

Leading order in α_s:

$$\widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right) = C_F A(\epsilon) \frac{\alpha_s}{(4\pi)}\, \omega_1^{1-\epsilon} \delta(\omega_1 - \omega_2) \int_\omega^{\bar{\Lambda}} d\omega'\, \mathcal{S}(\omega') \left(\frac{(\omega' - \omega)}{\mu^2}\right)^{-\epsilon}$$

Refactorisation at leading order

$$\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0} = -\mathcal{N}\,\left|C_{LO}^{A0}\,(m_b)\right|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}\,A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega)\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}|_A^{\mathrm{asy}} = \mathcal{N}|C_{LO}^{A0}\,(m_b)\,|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}A(\epsilon)\,\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega')\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

One verifies that

$$\frac{d\Gamma}{dE_\gamma}|_A^{\mathrm{asy}} = (-1)\,\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0}$$

Refactorisation conditions can be formulated on the operator level

Express the fact that in the limits $u \sim u' \ll 1$ and $\omega_1 \sim \omega_2 \gg \omega$ the two terms of the subleading $\mathcal{O}_8 - \mathcal{O}_8$ contribution have the same structure.

- $[\![C^{B1}(m_b, u)]\!] = (-1)C^{A0}(m_b)\, m_b \overline{J}(u m_b)$
  ($[\![g(u)]\!]$ only denotes the leading term of a function $g(u)$ in the limit $u \to 0$)
- $\widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ corresponds to $\mathcal{S}(\omega, \omega_1, \omega_2)$ in the limit $\omega_1 \sim \omega_2 \gg \omega$
  (In this limit: $q_s \to q_{sc}$ and higher power corrections in $\omega/\omega_{1,2}$ are neglected)
- $\int_{-p_+}^{\overline{\Lambda}} d\omega\, [\![J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega)]\!] = \int_{-p_+}^{\overline{\Lambda}} d\omega\, J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$
  (In this limit $\chi_{hc} \to q_{sc}$, brackets indicate again that the $u \to 0$ *and* $u' \to 0$ limits)

The refactorisation relations are operatorial relations that guarantee the cancellation of endpoint divergences between the two terms to all orders in α_s.

Finally we show that refactorisation and renormalisation commute.

Refactorised (endpoint finite) factorisation theorem

We subtract the two asymptotic terms

$$0 = 2\mathcal{N} \left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\Lambda} d\omega J_g(m_b(p_+ + \omega)) \int_{m_b}^{\infty} d\omega_1 \overline{J}(\omega_1) \int_0^{\omega_1} d\omega_2 \overline{J}^*(\omega_2) \widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$$

$$+ 2\mathcal{N} \int_0^1 du \, [\![C^{B1}(m_b, u)]\!] \int_u^1 du' \, [\![C^{B1*}(m_b, u')]\!] \int_{-p_+}^{\overline{\Lambda}} d\omega \, [\![J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega)]\!]$$

with

$$[\![J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega)]\!] = J_g(m_b(p_+ + \omega)) \widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

$$[\![C^{B1}(m_b, u')]\!] = (-1) C^{A0}(m_b) \, m_b \overline{J}(u m_b)$$

from the all-order factorisation theorems we derived

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N} \left|C^{A0}(m_b)\right|^2 \int_{-\infty}^{\infty} d\omega_1 \overline{J}(\omega_1) \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}^*(\omega_2) \int_{-p_+}^{\overline{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \mathcal{S}(\omega, \omega_1, \omega_2)$$

$$+ 2\mathcal{N} \int_0^1 du C^{B1}(m_b, u) \int_u^1 du' C^{B1*}(m_b, u') \int_{-p_+}^{\overline{\Lambda}} d\omega J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega)$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N}\int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ J_g(m_b(p_+ + \omega)) \left|C^{A0}(m_b)\right|^2 \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}(\omega,\omega_1,\omega_2) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] + \int_0^1 du \int_u^1 du' \Big[C^{B1}_{LO}(m_b,u)\; C^{B1*}(m_b,u')\; J(m_b(p_+ + \omega), u, u')\,\mathcal{S}(\omega) - [\![C^{B1}(m_b,u)]\!]\,[\![C^{B1*}(m_b,u')]\!]\,[\![J(m_b(p_+ + \omega), u, u')\,\mathcal{S}(\omega)]\!]\Big]\Big\},$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$
\begin{aligned}
\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N} \int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ & J_g(m_b(p_+ + \omega)) \left|C^{A0}(m_b)\right|^2 \\
& \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}(\omega,\omega_1,\omega_2) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] \\
& + \int_0^1 du \int_u^1 du' \Big[C_{LO}^{B1}(m_b,u)\; C^{B1*}(m_b,u')\; J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega) \\
& - [\![C^{B1}(m_b,u)]\!]\, [\![C^{B1*}(m_b,u')]\!]\, [\![J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega)]\!] \Big] \Big\},
\end{aligned}
$$

Finally we show that refactorisation and renormalisation commute.

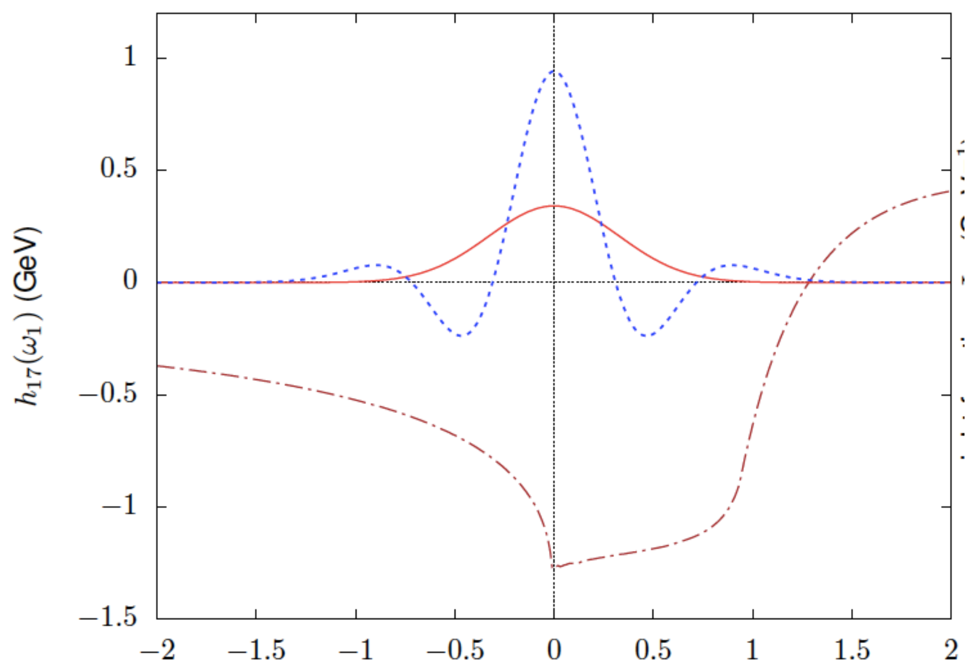
Spares II

Updated result for $\bar{B} \to X_s \ell\ell$

Benzke,Hurth,arXiv:2006.00624

Rather symmetric jet function $\to$

Various shape functions lead to very similar values of the convolution



arXiv:2006.00624

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\ell\ell} \in [+0.2\%, +2.6\%]$$

arXiv:1705.10366

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\ell\ell}|_{1/m_b} \in [-0.5\%, +3.4\%]$$

We find large scale dependence of the results in both penguins

$\Rightarrow$ α_s corrections desirable

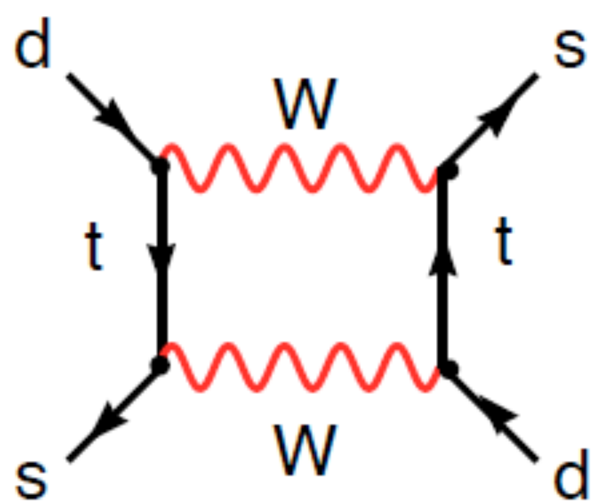
Numerical relevant contributions to $O(1/m_b^2)$

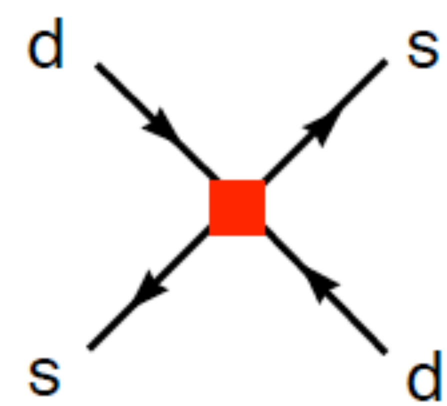
$\mathcal{F}_{19}$: $O(1/m_b^2)$ but $|C_{9/10}| \sim 13 |C_{7\gamma}|$

Flavour problem of New Physics or how FCNCs hide?

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \sum_i \frac{c_i^{New}}{\Lambda} \mathcal{O}_i^{(5)} + \ldots$$

- SM as effective theory valid up to cut-off-scale Λ
- $K^0 - \bar{K}^0$-mixing $\mathcal{O}^6 = (\bar{s}\,d)^2$: $c^{SM}/M_W^2 \times (\bar{s}\,d)^2 + c^{New}/\Lambda^2 \times (\bar{s}\,d)^2 \Rightarrow \Lambda > 10^4$ TeV





- Natural stabilisation of Higgs boson mass $\Rightarrow \Lambda \sim 1$TeV

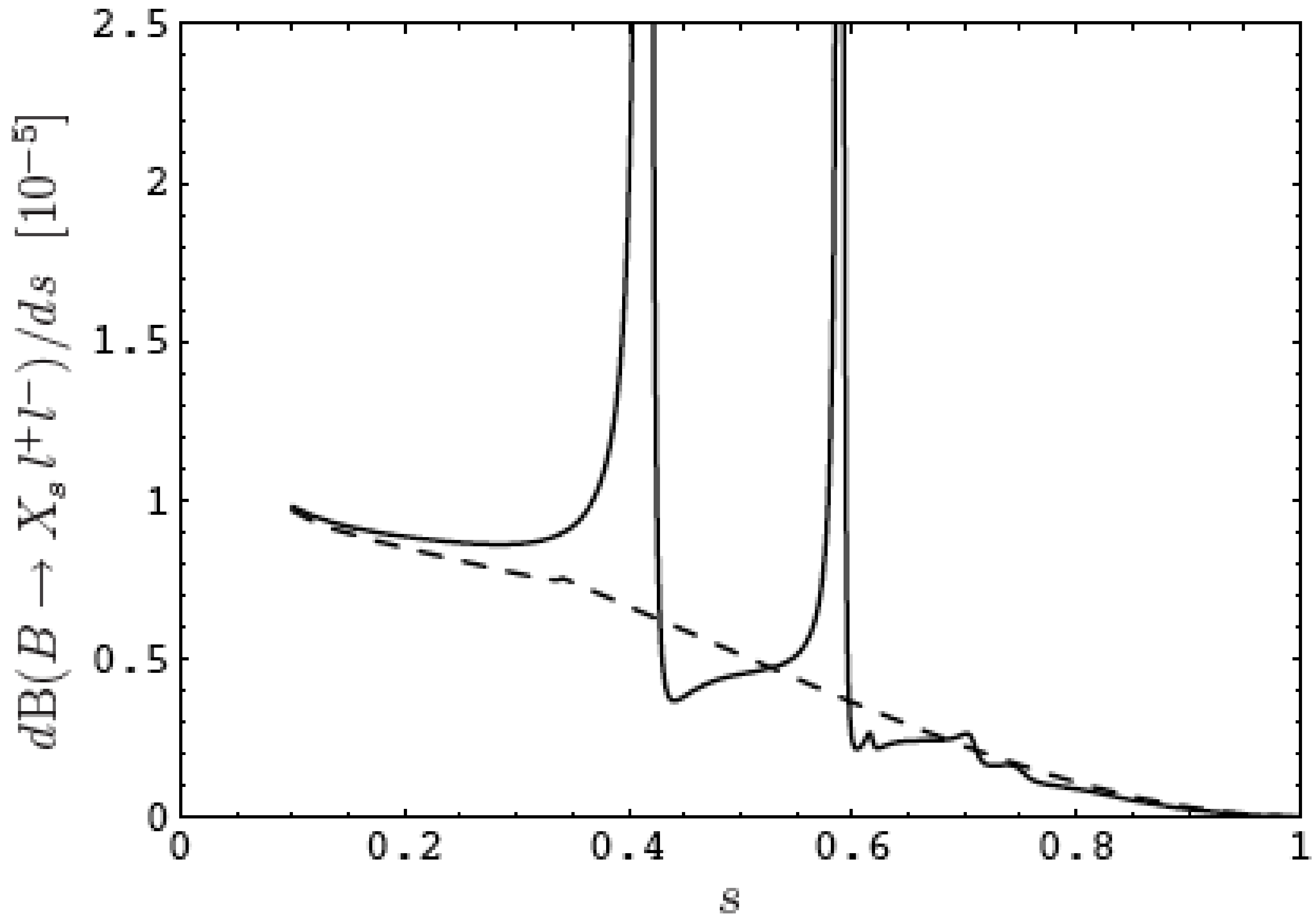
Ambiguity of new physics scale from flavour data

$$(\, C^i_{\mathrm{SM}}/M_W + C^i_{\mathrm{NP}}\,/\Lambda_{\mathrm{NP}}\,) \times \mathcal{O}_i$$

Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



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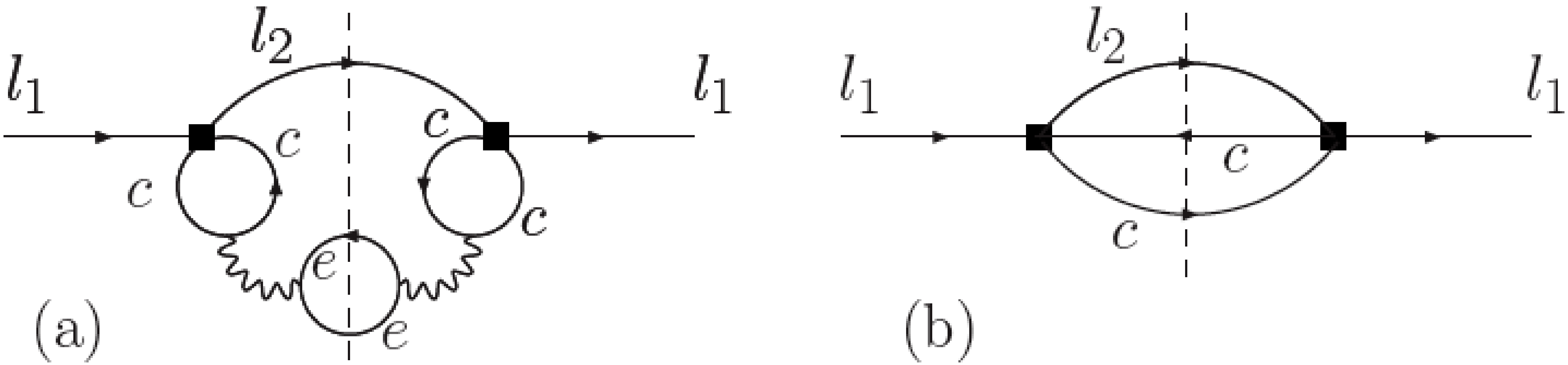


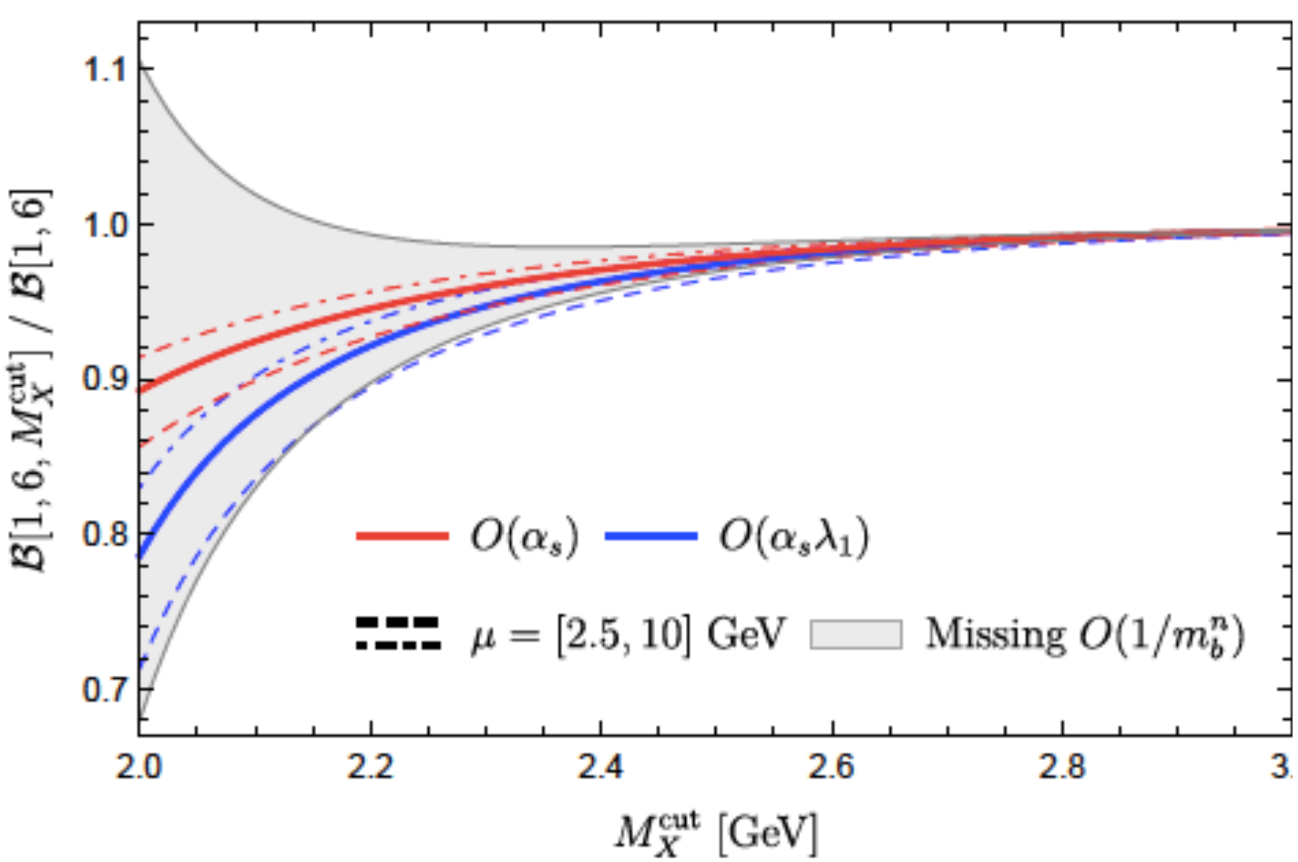
The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

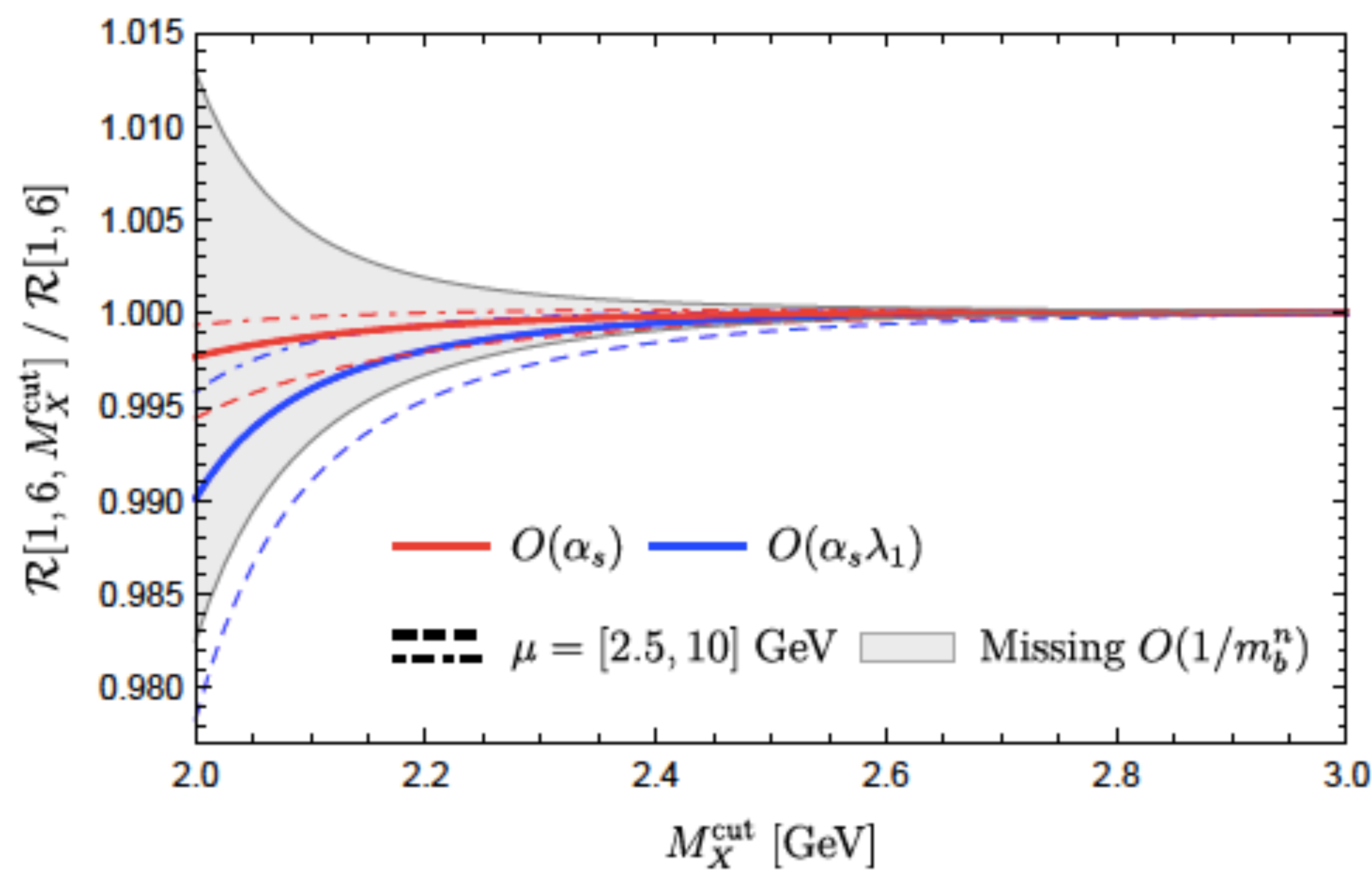
In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

Huber,Hurth,Jenkins,Lunghi arXiv 2306.03134

- We computed the fully differential distribution of $\bar{B} \to X_s \ell^+ \ell^-$ at $O(\alpha_s)$ in the OPE
- Also the three $\bar{B} \to X_s \ell^+ \ell^-$ angular observables, together with the $\bar{B} \to X_u \ell^- \nu$ branching fraction, all with the same hadronic mass cut
- We find effective Independence of the hadronic mass cut





Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to se^+\nu)e^-\bar{\nu}$)

- Previous SCET calculation with some simplifications and certain problems with SCET scaling (q assumed to be hard)
  Uncertainty due to subleading shape functions estimated to 5 – 10%

  Lee, Ligeti, Stewart, Tackmann hep-ph/0512191

  Lee, Tackmann arXiv:0812.0001

- New Strategy to minimise uncertainty

  Huber, Hurth, Jenkins, Lunghi arXiv 2306.03134

  – Calculation of cut dependence using OPE for mild hadronic cuts

  – Analyse breakdown of OPE via λ_1 power corrections

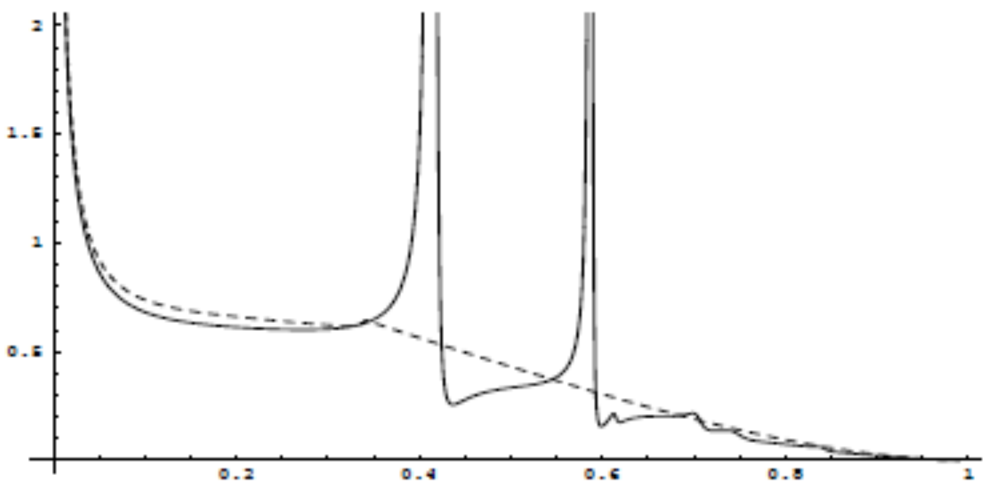
  – Try to interpolate betweeen SCET and OPE calculation

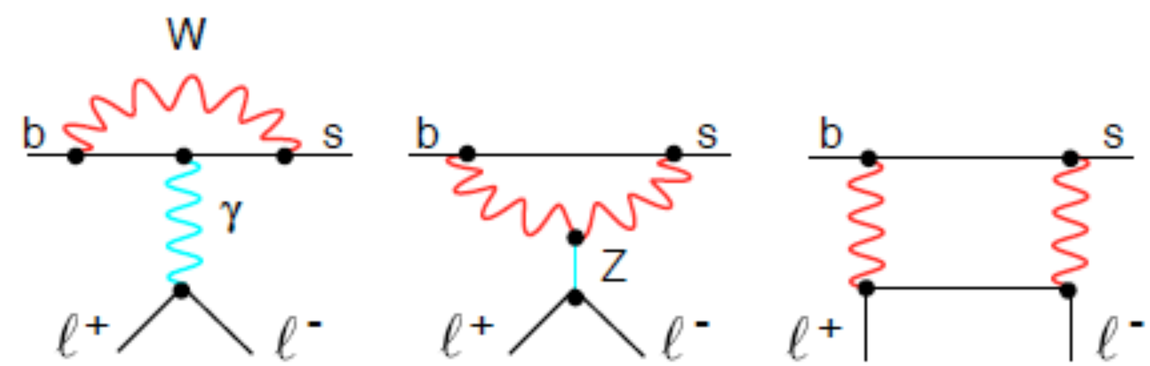
  – Use cut-independent ratios in OPE and SCET to analyse interpolation

Review of previous calculations for $B \to X_s \ell\ell$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dilepton mass spectrum necessary :
  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$





$\hat{s} = q^2/m_b^2$

- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: dilepton mass spectrum
  Asatryan,Asatrian,Greub,Walker,hep-ph/0204341
  Ghinculov,Hurth,Isidori,Yao,hep-ph/0312128

$$BR(\bar{B} \to X_s \ell^+ \ell^-)_{Cut:\ q^2 \in [1GeV^2, 6GeV^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \to X_s l^+ l^-)_{Cut:\ q^2 > 14.4GeV^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections $q^2 \in [1GeV^2, 6GeV^2]$

central value: -14%, perturbative error: $13\% \to 6.5\%$

- Further refinements:
  - Completing NNLL QCD corrections:
    Mixing into $\mathcal{O}_9$ ($+1\%$), NNLL matrixelement of $\mathcal{O}_9$ (-4%)
  - NLL QED two-loop corrections to Wilson coefficients
    -1.5% shift for $\alpha_{em}(\mu = m_b)$, -8.5% for $\alpha_{em}(\mu = m_W)$
    Bobeth,Gambino,Gorbahn,Haisch,hep-ph/0312090
  - QED two-loop corrections to matrix elements
    Large collinear logarithm $Log(m_b/m_\ell)$ which survive intregration
    if a restricted part of the dilepton mass spectrum is considered
    $+2\%$ effect in the low-q^2 region for muons, for the electrons
    the effect depends on the experimental cut parameters
    Huber,Lunghi,Misiak,Wyler,hep-ph/0512066 Huber,Hurth,Lunghi, arXiv:0712.3009

- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: forward-backward-asymmetry (FBA)
  Asatrian,Bieri,Greub,Hovhannisyan,hep-ph/0209006
  Ghinculov,Hurth,Isidori,Yao,hep-ph/0208088,hep-ph/0312128

$$A_{\rm FB} \equiv \frac{1}{\Gamma_{semilep}} \left(\int_0^1 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d(\cos\theta) \frac{d^2\Gamma}{dq^2 d\cos\theta} \right)$$

(θ angle between ℓ^+ and B momenta in dilepton CMS)

$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) GeV^2$$