Reinterpretation of the Belle II $B^+ \to K^+ \nu\bar{\nu}$ result

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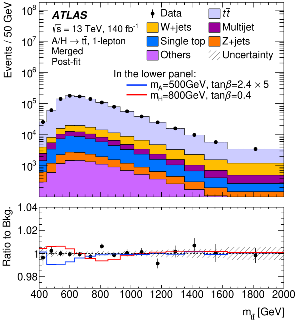
September 9, 2025

New measurement, new analysis?

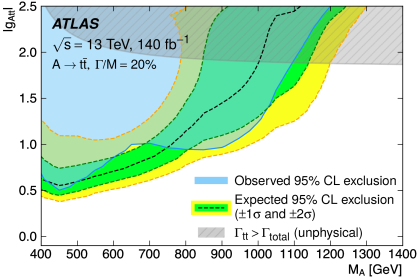
Measurements

Signal strength quantification



(Re)interpretations

Theoretical constraints



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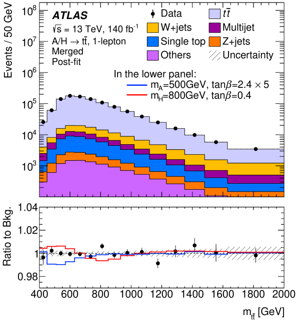
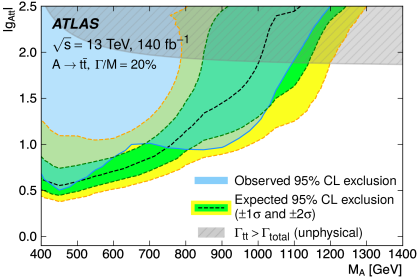
Measurements
Signal strength quantification



(Re)interpretations
Theoretical constraints



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Accessible analysis logic ➜ +62 % citations arXiv:2504.00256

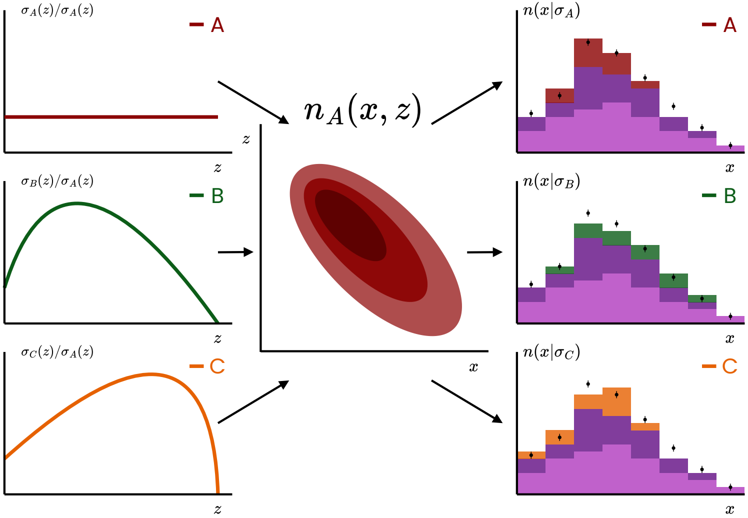
redist

A “novel” reinterpretation framework

EPJC 84, 693 (2024)

github.com/lorenzennio/redist

Templates from kinematic distributions

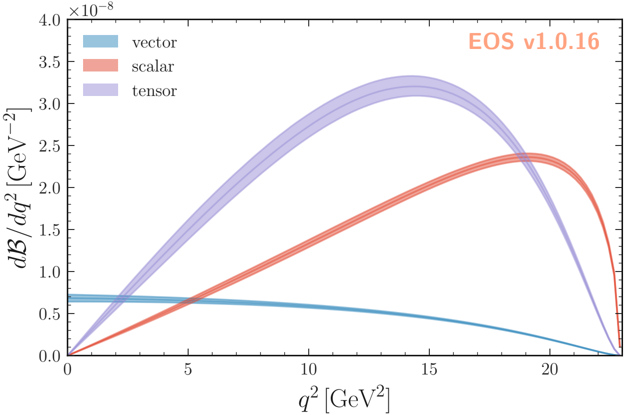


Constraining $b \to s\nu\nu$ weak effective field theory (WET) Wilson coefficients

$b \to s\nu\nu$ WET decay kinematics

$$\frac{d\mathcal{B}}{dq^2} = \alpha \left|C_{\mathrm{VL}} + C_{\mathrm{VR}}\right|^2 + \beta \left|C_{\mathrm{SL}} + C_{\mathrm{SR}}\right|^2 + \gamma \left|C_{\mathrm{TL}}\right|^2$$

SM contains only *vector* contribution.



$$q^2 = (p_B - p_K)^2$$

Validation through closure testing

- Build simple statistical model on MC data
- Inject new physics in MC data
- Infer a posterior in the Wilson coefficients

$$p(\theta|x) \propto p(x|\theta)p(\theta)$$

EPJC 84, 693 (2024)

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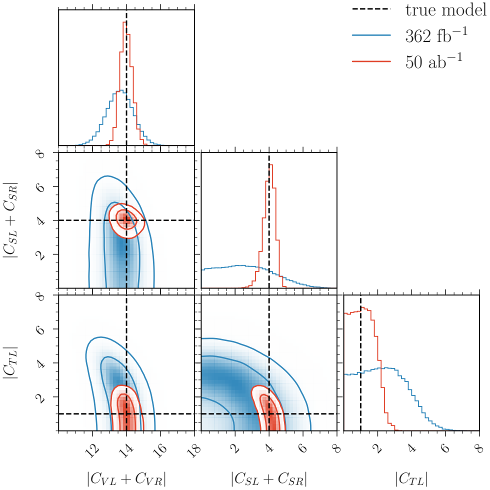
EPJC 84, 693 (2024)

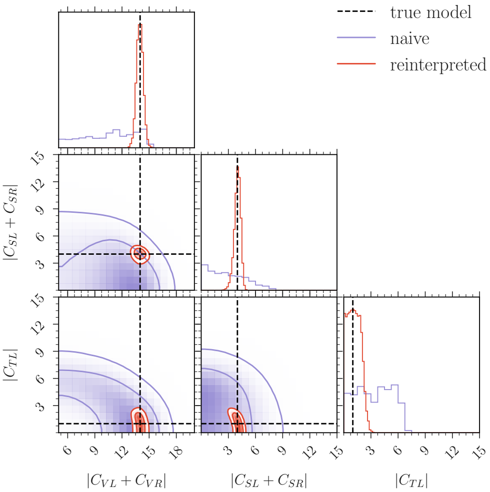


Necessity of reinterpretation

- Infer Wilson coefficients only from
$$\mathcal{B}(\{C_i\})/\mathcal{B}_{SM}$$
- Ignore effect of kinematic shape changes

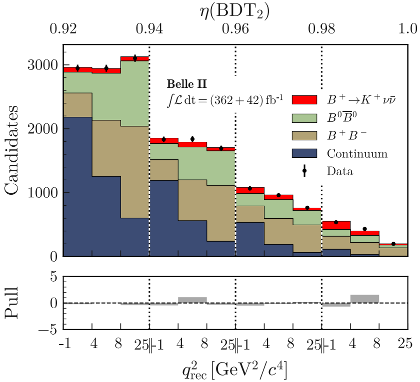
➜ **No constraining power!**

EPJC 84, 693 (2024)



Belle II has reported
“Evidence for $B^+ \to K^+\nu\bar{\nu}$ decays”

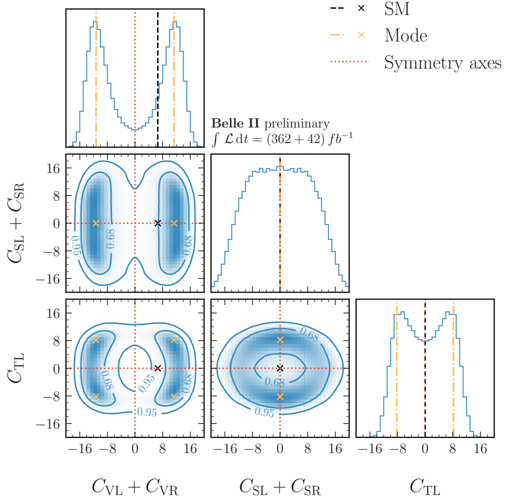
PRD 109.112006

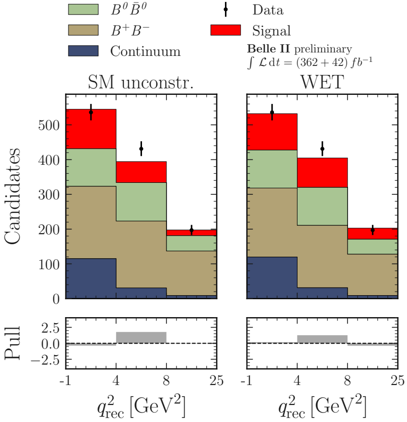


Based on $p(x|\mathrm{SM})$.

Implications for $p(x|\mathrm{NP})$?

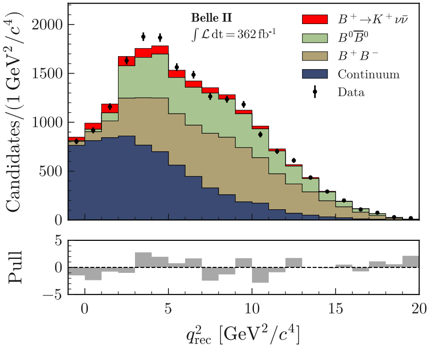
WET marginal posterior





Two-body scenarios

Potential $B^+ \to K^+ \nu\bar{\nu}$ signal contribution from $B^+ \to K^+ X$



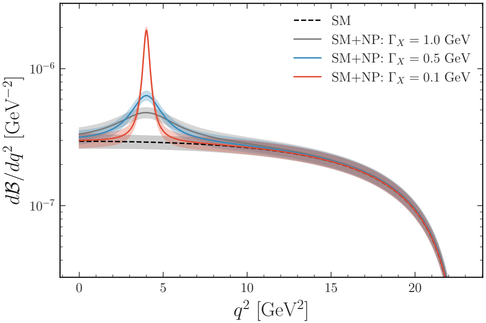
See arXiv:2311.14629, arXiv:2312.12507 for initial estimates.

Decay kinematics

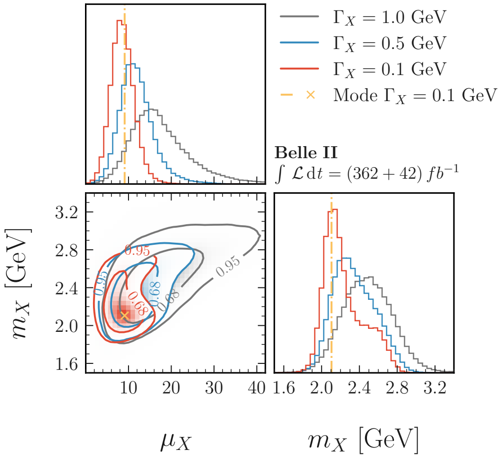
Model as sharp resonance peak,

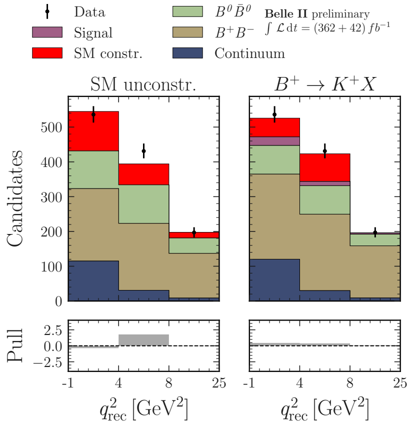
$$\frac{d\mathcal{B}}{dq^2} = \mu_{SM}\frac{d\mathcal{B}_{SM}}{dq^2} + \mu_X\, p_{BW}(q^2|m_X,\Gamma_X)\cdot 10^{-6},$$

$$p_{BW}(q^2|m_X,\Gamma_X) = \frac{k}{(q^2-m_X^2)^2+m_X^2\Gamma_X^2}$$



$B^+ \to K^+X$ marginal posterior





Main takeaways

♻ **Publishable, reinterpretable likelihood** for $B^+ \to K^+ \nu\bar{\nu}$

- **Bias-free BSM inference**, reproducibility, combinations, ...
- Precedent case for future Belle II analyses.

- **WET**: **first ever** CI for WET WCs, improved fit to data over SM
- $\boldsymbol{B^+ \to K^+ X}$: CI for m_X, μ_X, best fit to data among tested models
- **NEW**: $B^0 \to K^{*0}\tau^+\tau^-$ reinterp. ➜ Tim Würthner @ poster session

Reinterpretation enhances analysis impact and reach!

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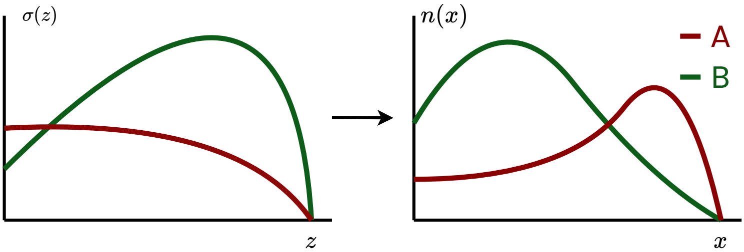
arXiv:2507.12393, EPJC 84, 693 (2024) / arXiv:2402.08417

Templates from kinematic predictions

$$n(x|\sigma) = \int dz\, L\, \varepsilon(x|z)\, \sigma(z) = \int dz\, n_\sigma(x,z)$$



$z(= q^2)$ – kinematic d.o.f.

x – reconstruction / fitting variable(s)

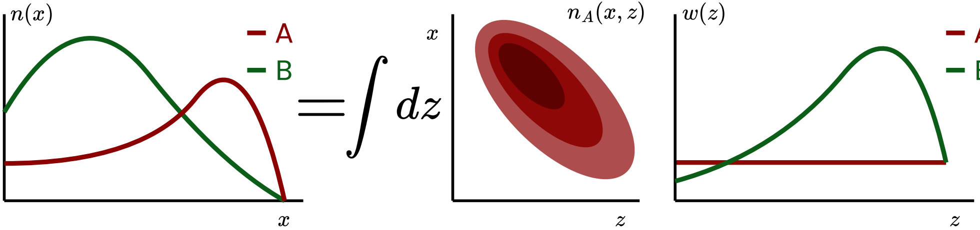
L – luminosity

$\varepsilon(x|z)$ – conditional efficiency

$n_\sigma(x,z)$ – joint number density

Reweight to new model

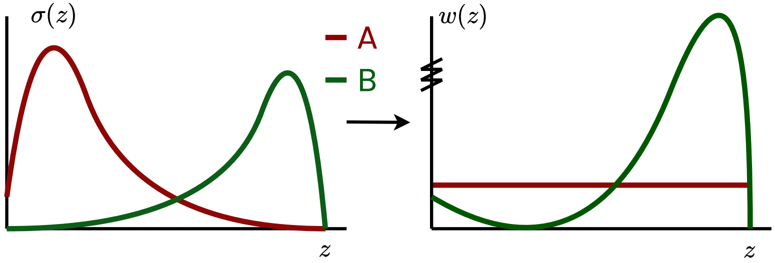
$$\boxed{n(x|B)} = \int dz\, L\, \varepsilon(x|z)\, \boxed{\sigma_B(z)} = \int dz\, L\, \varepsilon(x|z)\, \boxed{\sigma_A(z)}\, \frac{\boxed{\sigma_B(z)}}{\boxed{\sigma_A(z)}} = \int dz\, \underbrace{\boxed{n_A(x,z)}}_{\text{main object}}\, \boxed{w(z)}.$$



$$\boxed{p(x|n_A,\theta) = \text{model-agnostic likelihood}}$$

Method limitations

Substantial model changes ➜ large weights



Minimal requirement:

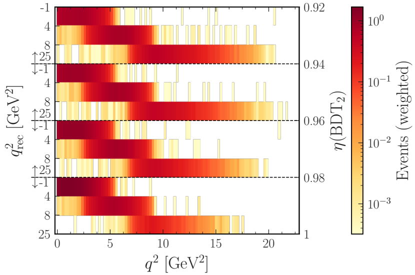
$$\mathrm{supp}(\sigma_B) \in \mathrm{supp}(\sigma_A)$$

Always possible to compare only in $\mathrm{supp}(\sigma_A)$

Joint number densities

Main object for reinterpretation, $n(x, z)$ ➜ **Essential for publication.**

ITA (plot for publication)



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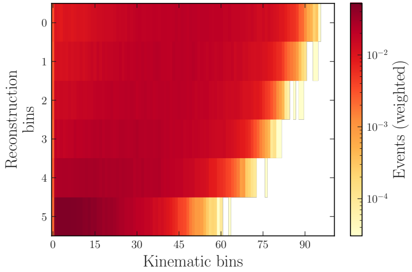


Kinematic binning: $q^2 = [-1, (0, 22.885, 100)]$ GeV^2

Weak Effective Theory for $B \to K\nu\bar{\nu}$

The effective Lagrangian is

$$\mathcal{L}^{WET} = \sum_{X=L,R} C_{\mathrm{VX}} \mathcal{O}_{\mathrm{VX}} + \sum_{X=L,R} C_{\mathrm{SX}} \mathcal{O}_{\mathrm{SX}} + C_{\mathrm{TL}} \mathcal{O}_{\mathrm{TL}} + \text{h.c.}$$

The $d = 6$ contributing operators in and beyond the SM are given by

$$\mathcal{O}_{\mathrm{VL}} = \left(\overline{\nu_L}\gamma_\mu \nu_L\right)\left(\overline{s_L}\gamma^\mu b_L\right) \qquad \mathcal{O}_{\mathrm{VR}} = \left(\overline{\nu_L}\gamma_\mu \nu_L\right)\left(\overline{s_R}\gamma^\mu b_R\right)$$

$$\mathcal{O}_{\mathrm{SL}} = \left(\overline{\nu_L^c}\nu_L\right)\left(\overline{s_R} b_L\right) \qquad \mathcal{O}_{\mathrm{SR}} = \left(\overline{\nu_L^c}\nu_L\right)\left(\overline{s_L} b_R\right)$$

$$\mathcal{O}_{\mathrm{TL}} = \left(\overline{\nu_L^c}\sigma_{\mu\nu}\nu_L\right)\left(\overline{s_R}\sigma^{\mu\nu} b_L\right)$$

arXiv:2111.04327

WET decay kinematics

$$\frac{d\mathcal{B}(B \to K\nu\bar{\nu})}{dq^2} = \frac{3G_F^2\alpha^2\tau_B}{32\pi^5 m_B^3}\left|V_{ts}^* V_{tb}\right|^2 \sqrt{\lambda_{BK}}\, q^2 \left[\frac{\lambda_{BK}}{24q^2}\left|f_+(q^2)\right|^2 \left|C_{\mathrm{VL}} + C_{\mathrm{VR}}\right|^2 + \frac{\left(m_B^2 - m_K^2\right)^2}{8\left(m_b - m_s\right)^2}\left|f_0(q^2)\right|^2 \left|C_{\mathrm{SL}} + C_{\mathrm{SR}}\right|^2 + \frac{2\lambda_{BK}}{3\left(m_B + m_K\right)^2}\left|f_T(q^2)\right|^2 \left|C_{\mathrm{TL}}\right|^2\right]$$

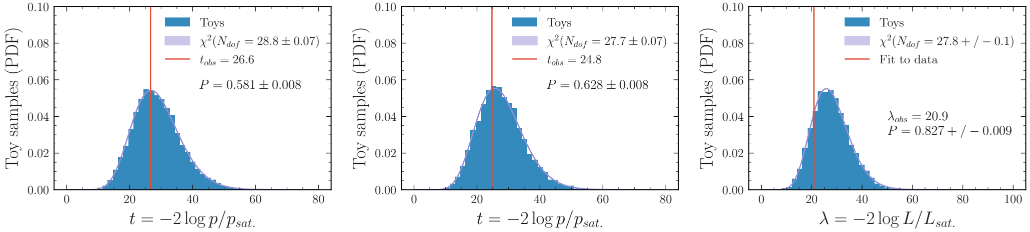
for $J^P = 0^-$ kaon states.
Note $q^2 \neq q^2_{rec}$.

arXiv:2111.04327

Goodness of fit

$$P_{\text{gof}} = \int_{t_{obs}}^{\infty} dt\, p(t, \quad t = -2 \ln \frac{p(n, a \mid \hat{\eta}, \hat{\chi})}{p_{\text{sat}}(n, a \mid \bar{\chi})},$$

SM WET $B^+ \to K^+ X$



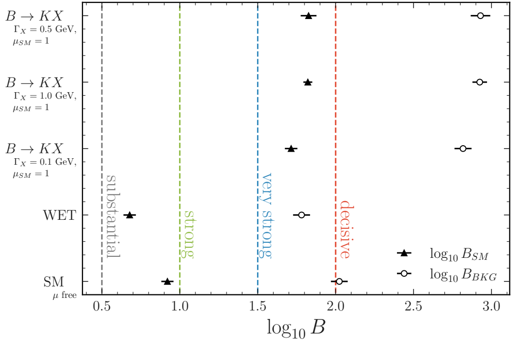
Averaged: Bayes factor

Compare the probabilities of the observed data being produced by a given model.

$$B_M = \frac{p(x|M')}{p(x|M)}$$

$$p(x|M) = \int d^n\theta \; p(x|\theta, M) \; p(\theta|M)$$

➜ $B^+ \to K^+X$ models dominate over SM and WET.

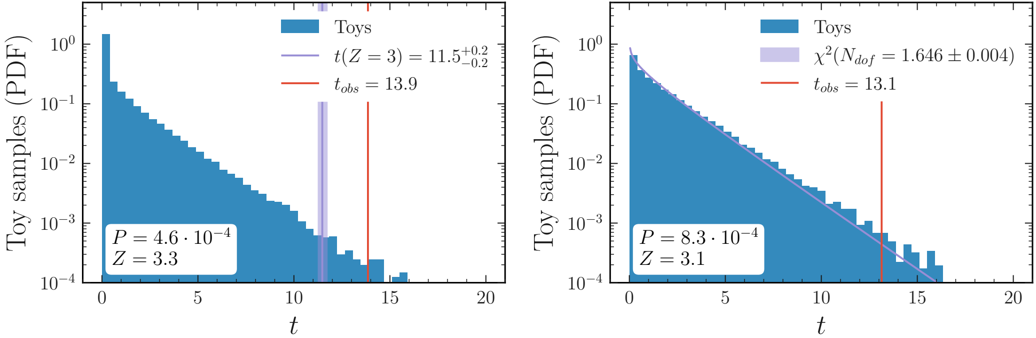


Pointwise: P-value

$$P = \int_{t_{obs}}^{\infty} dt\, p(t), \quad t = -2 \ln \frac{p(n, a \mid \eta = 0, \hat{\hat{\chi}})}{p(n, a \mid \hat{\eta}, \hat{\chi})},$$

WET vs. BKG

$B^+ \to K^+ X$ vs. SM+BKG



Prior sensitivity WET

Uniform priors

Parameters	Mode	68% HDI	95% HDI
$\lvert C_{\mathrm{VL}} + C_{\mathrm{VR}} \rvert$	11.3	[7.82, 14.6]	[1.86, 16.2]
$\lvert C_{\mathrm{SL}} + C_{\mathrm{SR}} \rvert$	0.00	[0.00, 9.53]	[0.00, 15.4]
$\lvert C_{\mathrm{TL}} \rvert$	8.21	[2.29, 9.62]	[0.00, 11.2]

Alternative priors

$$p(\eta_i) = \begin{cases} \mathcal{N}(\eta_i \mid \mu = C_i^{\mathrm{SM}}, \sigma = 20) & \eta_i \geq 0 \\ 0 & \eta_i < 0 \end{cases} \tag{1}$$

$$p(\eta_i) \propto \begin{cases} \eta_i & \eta_i \leq 30 \\ 0 & \eta_i > 30 \end{cases} \tag{2}$$

Alternative priors

Priors	Parameters	Mode	68% HDI	95% HDI
	$\lvert C_{\mathrm{VL}} + C_{\mathrm{VR}} \rvert$	11.4	[7.97, 14.6]	[2.21, 16.4]
(1)	$\lvert C_{\mathrm{SL}} + C_{\mathrm{SR}} \rvert$	0.00	[0.00, 9.16]	[0.00, 14.7]
	$\lvert C_{\mathrm{TL}} \rvert$	7.69	[1.54, 8.75]	[0.00, 11.0]
	$\lvert C_{\mathrm{VL}} + C_{\mathrm{VR}} \rvert$	11.6	[8.21, 14.0]	[4.17, 16.0]
(2)	$\lvert C_{\mathrm{SL}} + C_{\mathrm{SR}} \rvert$	8.93	[4.56, 12.6]	[1.27, 15.6]
	$\lvert C_{\mathrm{TL}} \rvert$	7.17	[3.89, 9.59]	[1.41, 11.7]

Prior sensitivity $B \to KX$

Alternative priors

$$p\left(\mu_X\right) = \begin{cases} \mathcal{N}(\mu_X|\mu = 0, \sigma = 20) & \mu_X \geq 0 \\ 0 & \mu_X < 0 \end{cases}, \tag{3}$$
$$p\left(m_X\right) = \mathcal{U}([1.5, 3.0]\ \text{GeV})$$

$$p\left(\mu_X\right) = \mathcal{U}([0, 24]),$$
$$p\left(m_X\right) \propto \begin{cases} m_X & m_X \leq 4.8\ \text{GeV} \\ 0 & m_X > 4.8\ \text{GeV} \end{cases} \tag{4}$$

Uniform priors

Param.	Mode	68% HDI	95% HDI
μ_X	9.15	[5.75, 11.0]	[3.44, 14.0]
m_X [GeV]	2.11	[1.96, 2.33]	[1.92, 2.69]

Alternative priors

Priors	Param.	Mode	68% HDI	95% HDI
(3)	μ_X	9.00	[5.67, 10.8]	[3.41, 13.8]
	m_X [GeV]	2.12	[1.97, 2.35]	[1.92, 2.69]
(4)	μ_X	9.04	[5.72, 11.0]	[2.99, 14.4]
	m_X [GeV]	2.11	[1.97, 2.39]	[1.92, 2.74]

A comparative scale

Naive reinterpretation

For each Wilson coefficient

$$\mu_{SM}\mathcal{B}_{SM} = \int dq^2 \frac{d\mathcal{B}_{WET}}{dq^2}$$

$$|C_{\mathrm{VL}} + C_{\mathrm{VR}}| = 14.2^{+1.9}_{-2.2}$$

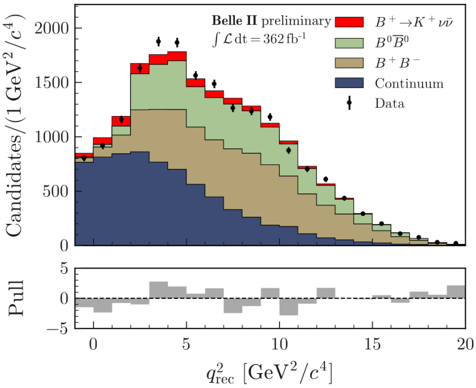
$$|C_{\mathrm{SL}} + C_{\mathrm{SR}}| = 8.38^{+1.12}_{-1.30}$$

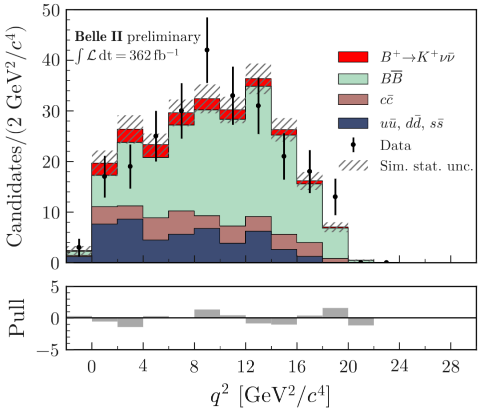
$$|C_{\mathrm{TL}}| = 6.93^{+0.93}_{-1.08}$$

Our results

Parameters	Mode	68% HDI
$\lvert C_{\mathrm{VL}} + C_{\mathrm{VR}}\rvert$	11.3	$[7.82,\ 14.6]$
$\lvert C_{\mathrm{SL}} + C_{\mathrm{SR}}\rvert$	0.00	$[0.00,\ 9.53]$
$\lvert C_{\mathrm{TL}}\rvert$	8.21	$[2.29,\ 9.62]$

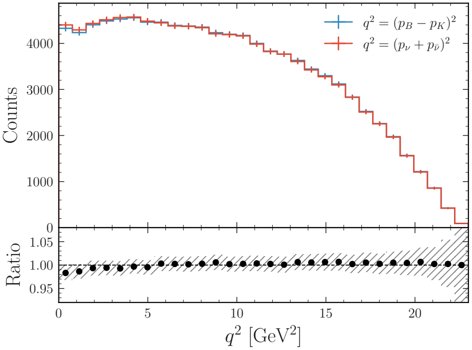
Motivation for $B \to KX$





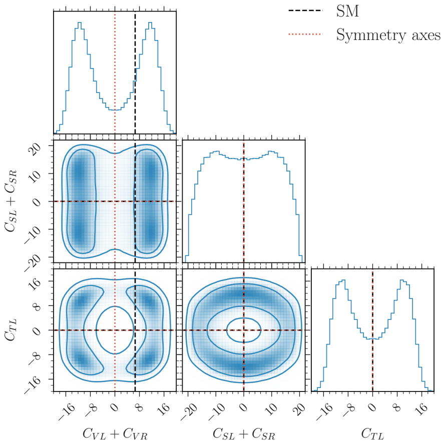
Excess around $3\ \mathrm{GeV}^2 < q^2_{rec} < 7\ \mathrm{GeV}^2$ for ITA best-fit projection.

Final state radiation



WET marginal posteriors (individual)

ITA



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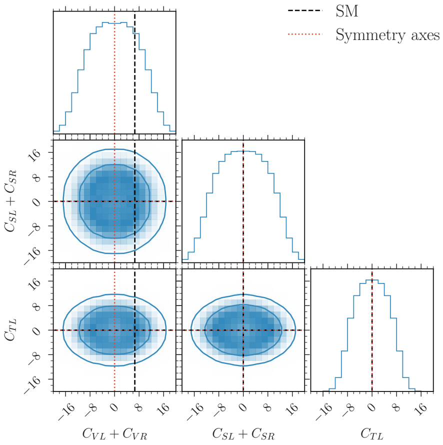
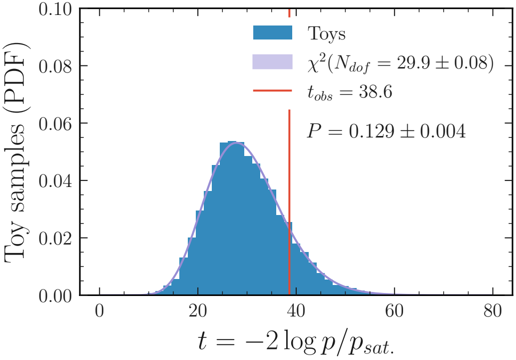


Goodness of fit: background only

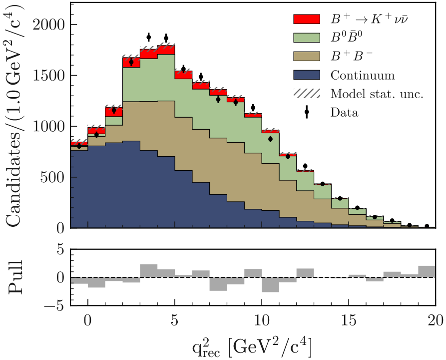
$$P_{gof} = \int_{\lambda_{obs}}^{\infty} d\lambda \, p(\lambda) = 1 - CDF_p(\lambda_{obs}),$$



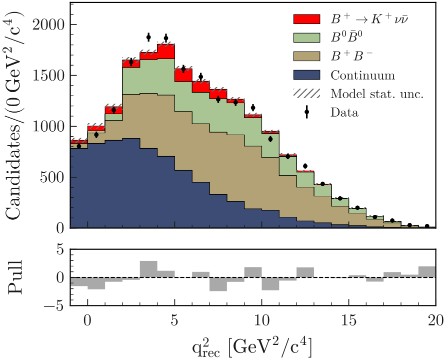
Test statistic PDF, $p(\lambda)$, from toy study, 10k toys

q^2_{rec} projections

WET



$B \to KX$



Since the $q^2_{rec.}$ resolution of the reconstruction

HistFactory model

Likelihood function for observed event counts n is

$$L(n, a|\eta, \chi) = \underbrace{\prod_{c \in \text{channels}} \prod_{b \in \text{bins}} \text{Pois}\left(n_{cb}|\nu_{cb}(\eta, \chi)\right)}_{\text{multiple channels}} \underbrace{\prod_{\chi \in \mathcal{X}} c_\chi \left(a_\chi|\chi\right)}_{\text{constraint terms}}$$

Expected number of events per channel per bin are

$$\nu_{cb}(\eta, \chi) = \sum_{s \in \text{samples}} \underbrace{\prod_{\kappa \in \boldsymbol{\kappa}} \kappa_{scb}(\eta, \chi)}_{\text{multiplicative modifiers}} \left(\nu^0_{scb}(\eta, \chi) + \underbrace{\sum_{\Delta \in \boldsymbol{\Delta}} \Delta_{scb}(\eta, \chi)}_{\text{additive modifiers}}\right).$$

Modifiers and constraints

Description	Modification	Constraint Term c_χ	Input
Uncorrelated Shape	$\kappa_{scb}(\gamma_b) = \gamma_b$	$\prod_b \mathrm{Pois}\left(r_b = \sigma_b^{-2} \middle\| \rho_b = \sigma_b^{-2}\gamma_b\right)$	σ_b
Correlated Shape	$\Delta_{scb}(\alpha) = f_p\left(\alpha \middle\| \Delta_{scb,\alpha=-1}, \Delta_{scb,\alpha=1}\right)$	$\mathrm{Gaus}\left(a = 0 \middle\| \alpha, \sigma = 1\right)$	$\Delta_{scb,\alpha=\pm1}$
Normalisation Unc.	$\kappa_{scb}(\alpha) = g_p\left(\alpha \middle\| \kappa_{scb,\alpha=-1}, \kappa_{scb,\alpha=1}\right)$	$\mathrm{Gaus}\left(a = 0 \middle\| \alpha, \sigma = 1\right)$	$\kappa_{scb,\alpha=\pm1}$
MC Stat. Uncertainty	$\kappa_{scb}(\gamma_b) = \gamma_b$	$\prod_b \mathrm{Gaus}\left(a_{\gamma_b} = 1 \middle\| \gamma_b, \delta_b\right)$	$\delta_b^2 = \sum_s \delta_{sb}^2$
Luminosity	$\kappa_{scb}(\lambda) = \lambda$	$\mathrm{Gaus}\left(l = \lambda_0 \middle\| \lambda, \sigma_\lambda\right)$	$\lambda_0, \sigma_\lambda$
Normalisation	$\kappa_{scb}(\mu_b) = \mu_b$		
Data-driven Shape	$\kappa_{scb}(\gamma_b) = \gamma_b$		

Custom modifiers