2504.05209

Exploring Hadronic B decays through $SU(3)_F$ symmetry and factorizable corrections

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In collaboration with: **Keri Vos (Maastricht University - Nikhef)**

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Why looking at

B mesons decaying hadronically?

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Large phase space allows for **many** decay modes

more than 500 for the B^+!!

Decays happen through the weak interaction

powerful for studying
CP Violation

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powerful for studying **CP Violation**

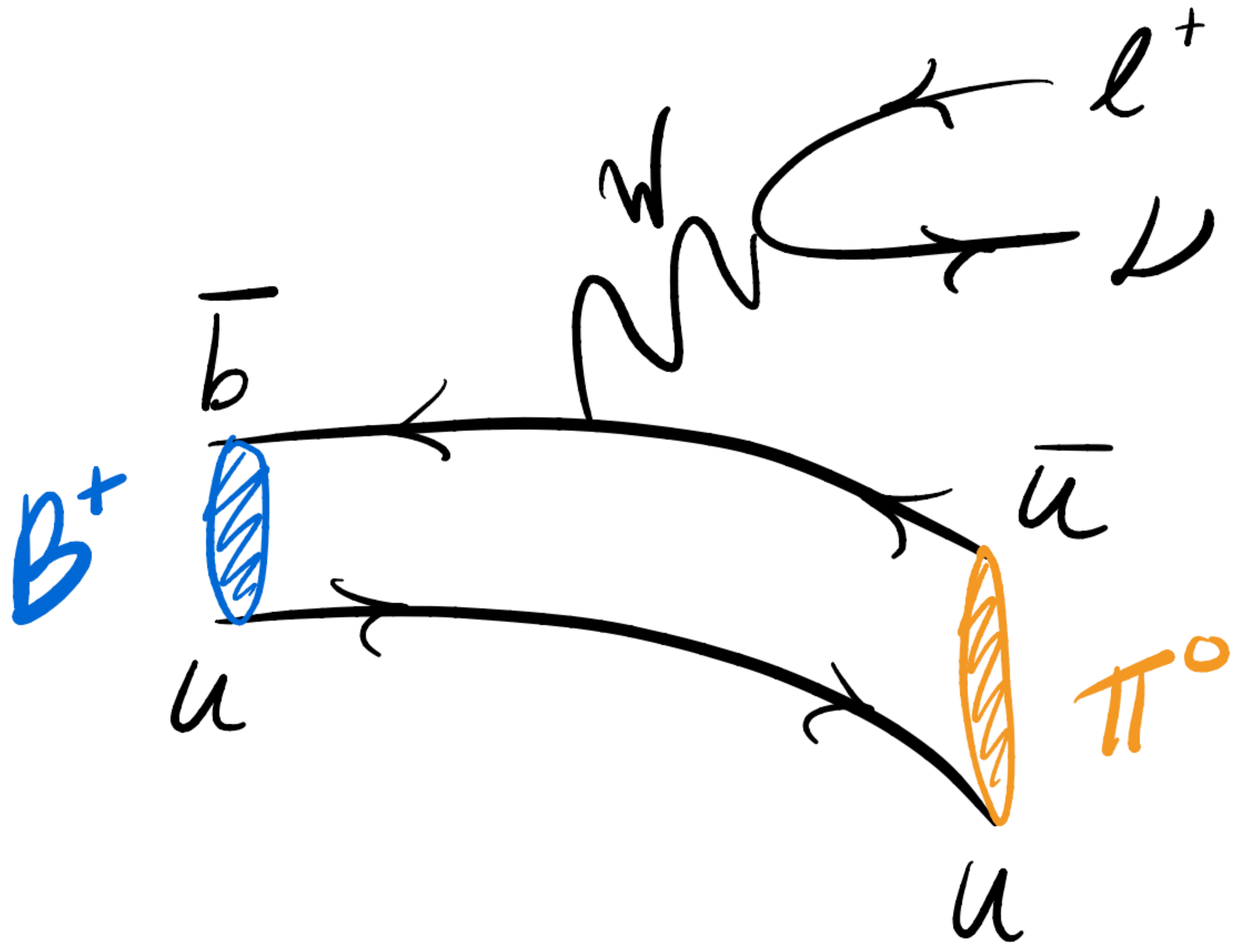
Plenty of experimental data available, specially **CP asymmetries**

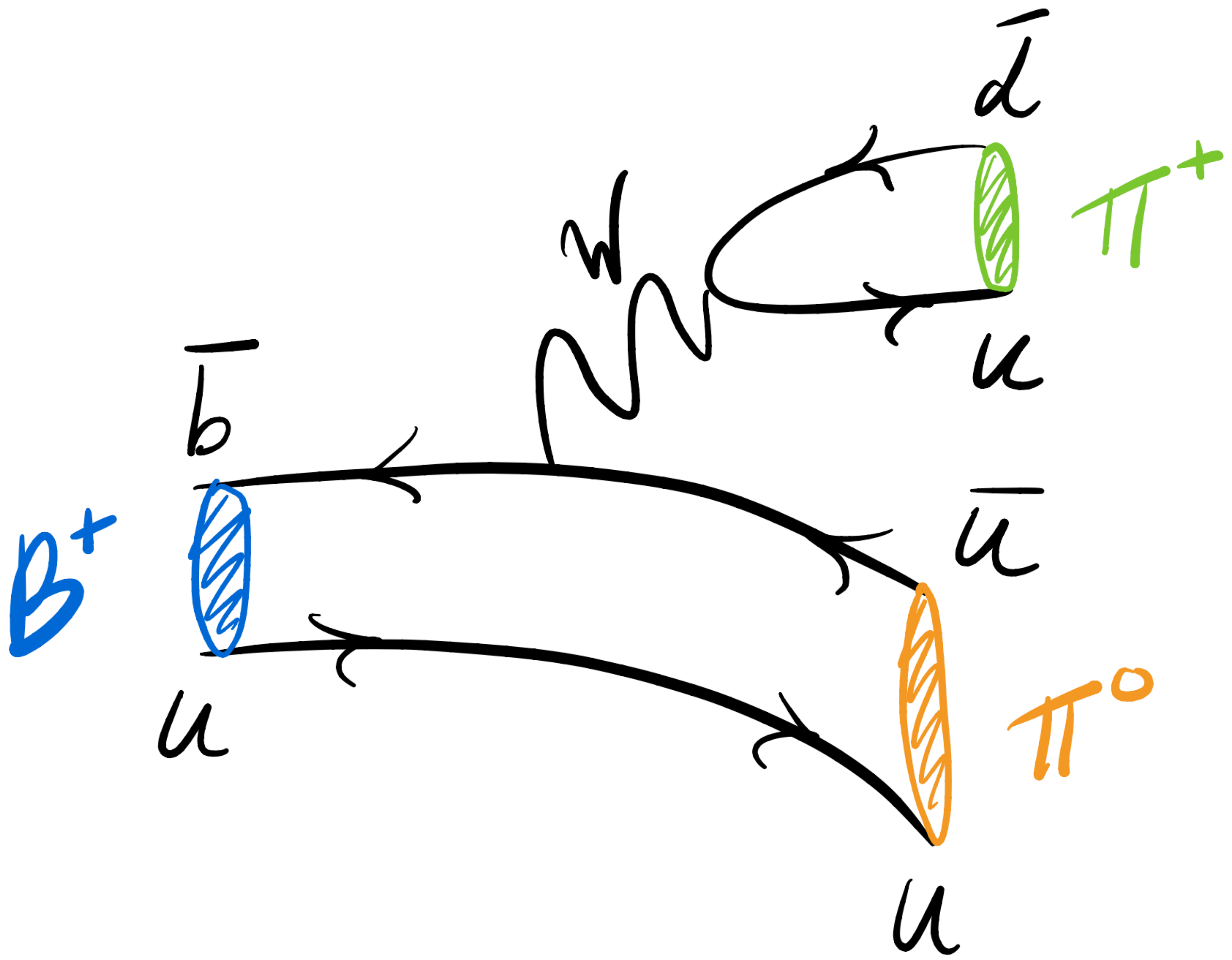
Purely **perturbative** techniques **no longer valid**

Current predictions governed by **uncertainties**

What is so complicated?

Semileptonic vs. **Hadronic**





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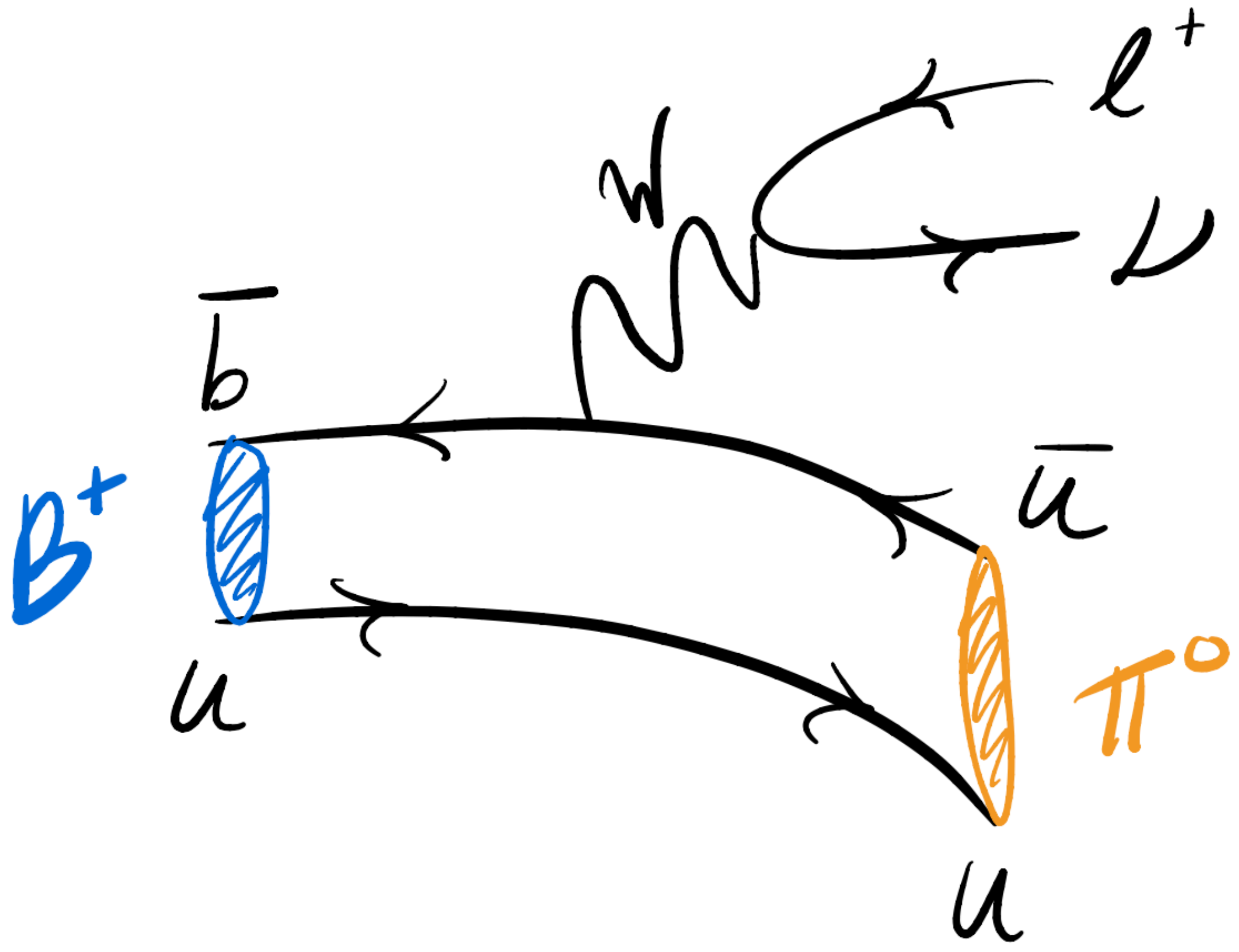
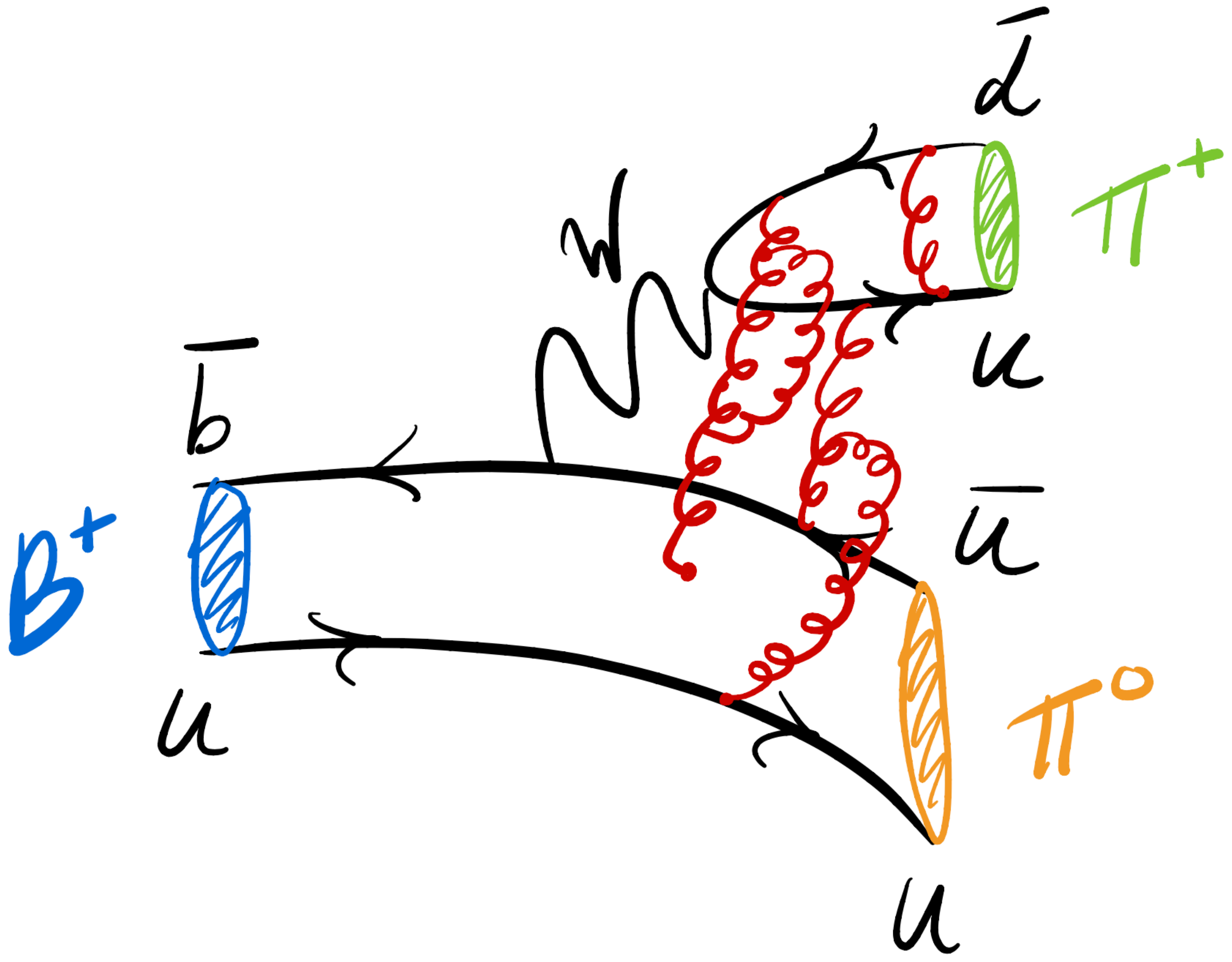




Leptonic and hadronic parts **factorize**

Strong interaction **confined** to the $B \to P$ transition

Non-perturbative interactions between the final state hadrons

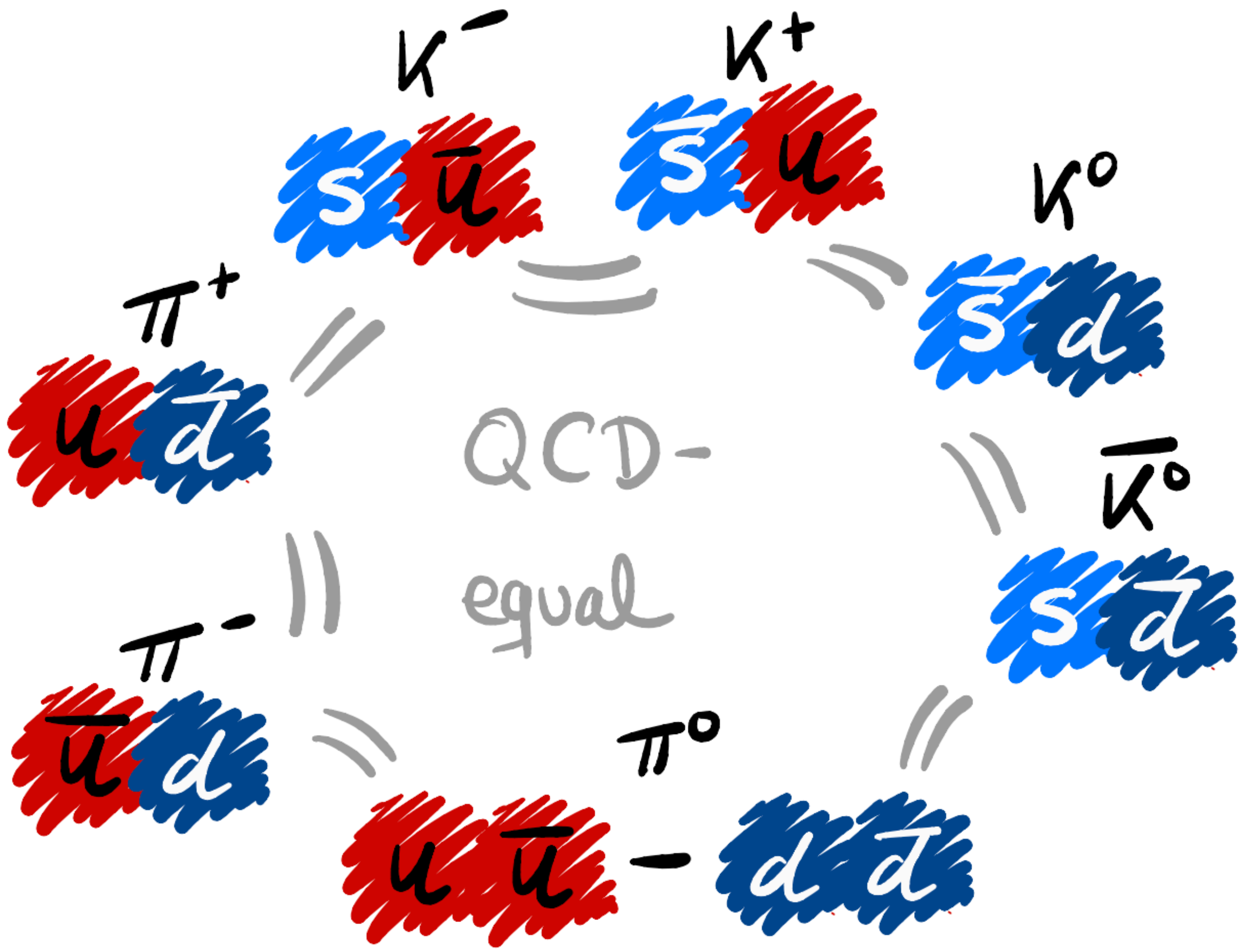
There is currently **no strict theoretical approach** possible

How can we describe $B \to PP$ decays?

1. $SU(3)$ Flavor Symmetry

Assume quarks up, down and strange are degenerate and massless under the **strong interaction**

Under $SU(3)_F$ symmetry, all 16 $B \to PP$ modes are **related**, with $P = \pi, K$ because all interact the same way (under QCD)



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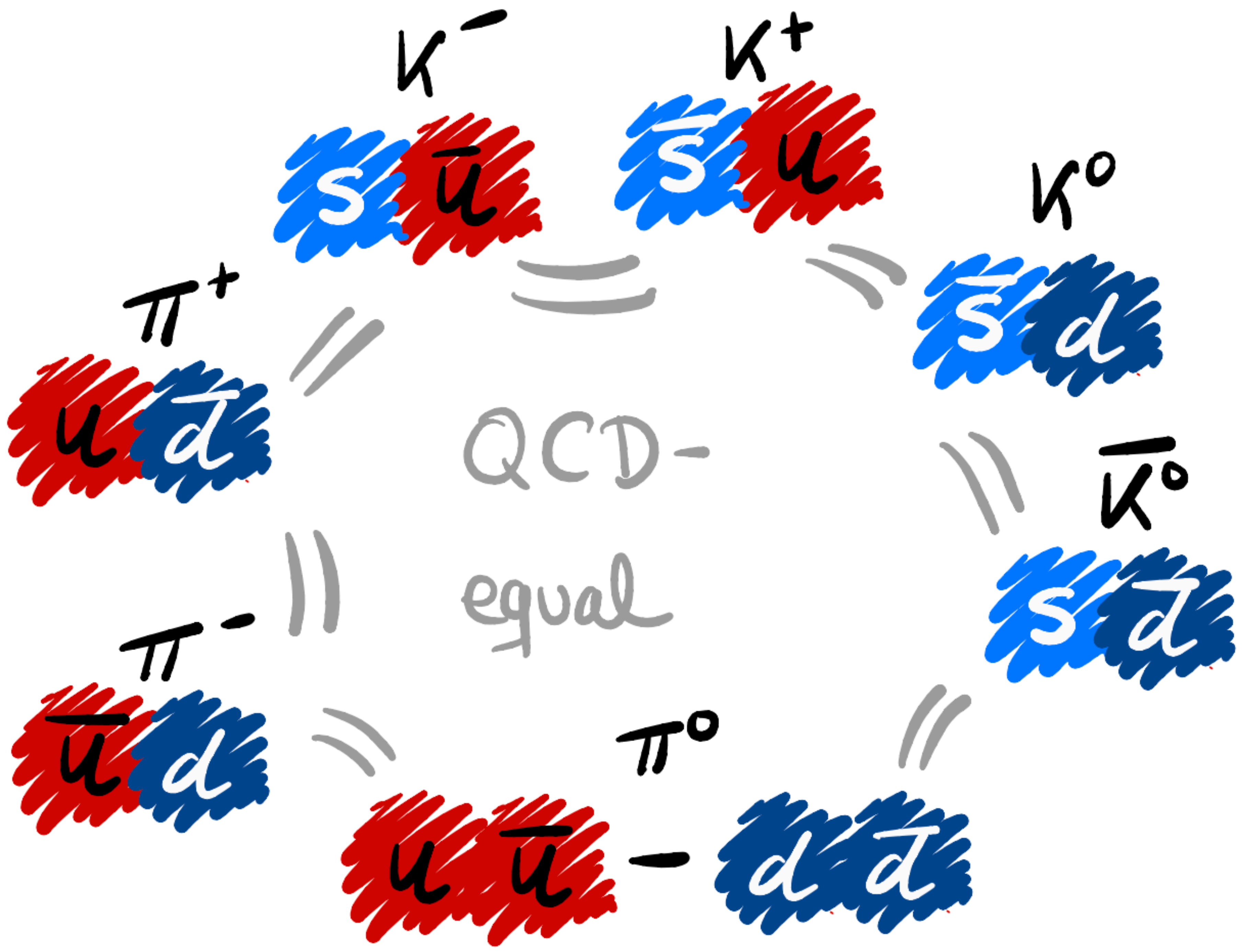

Note! This symmetry is broken in nature $m_u \neq m_d \neq m_s$ but it is a useful approximation

2. Topological parametrization

Parametrize all $B \to PP$ decays in terms of topological coefficients

Each coefficient represents a different topology

Equivalent to $SU(3)_F$ irreducible rep. See He, Wang 1803.04227v1

Any two-body B decay can be expressed as: $A(B \to PP) = \lambda_u^{(q)} A_u + \lambda_c^{(q)} A_c + \lambda_t^{(t)} A_t$

$$\lambda_i^{(q)} = V_{ib}^* V_{iq}$$

$$q = d, s$$

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CKM unitarity!

$$\lambda_u^{(q)} + \lambda_c^{(q)} + \lambda_t^{(q)} = 0$$

$$\lambda_i^{(q)} = V_{ib}^* V_{iq}$$

$$q = d, s$$

For every **tree** topology contributing to a decay we have its **penguin** counterpart:

$$A(B \to PP) = \lambda_u^{(q)} \mathcal{T} + \lambda_c^{(q)} \mathcal{P}$$

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Tree amplitude

$$\mathscr{T} = \mathbf{T}\ B_i (M)^i_j \bar{H}^{jl}_k (M)^k_l + \mathbf{C}\ B_i (M)^i_j \bar{H}^{lj}_k (M)^k_l + \mathbf{A}\ B_i \bar{H}^{il}_j (M)^j_k (M)^k_l$$
$$+\mathbf{E}\ B_i \bar{H}^{li}_j (M)^j_k (M)^k_l + \mathbf{T_P} B_i (M)^i_j (M)^j_k \bar{H}^{lk}_l + \mathbf{T_{PA}} B_i \bar{H}^{li}_l (M)^j_k (M)^k_j$$

Penguin amplitude

$$\mathscr{P} = \mathbf{P_T}\ B_i (M)^i_j \tilde{H}^{jl}_k (M)^k_l + \mathbf{P_C}\ B_i (M)^i_j \tilde{H}^{lj}_k (M)^k_l + \mathbf{P_A}\ B_i \tilde{H}^{li}_l (M)^j_k (M)^k_j$$
$$+\mathbf{P_{TE}}\ B_i \tilde{H}^{ji}_k (M)^k_l (M)^l_j + \mathbf{P}\ B_i (M)^i_j (M)^j_k \tilde{H}^k + \mathbf{P_{TA}}\ B_i \tilde{H}^{il}_j (M)^j_k (M)^k_l$$

B-meson vector

$$B_i = (B^+, B^0, B^0_s)$$

Pseudo-scalar meson octet + singlet

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \bar{K}^0 \\ K^+ & K^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix} + \begin{pmatrix} \frac{\eta_0}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{\eta_0}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{\eta_0}{\sqrt{3}} \end{pmatrix}$$

Flavor tensor

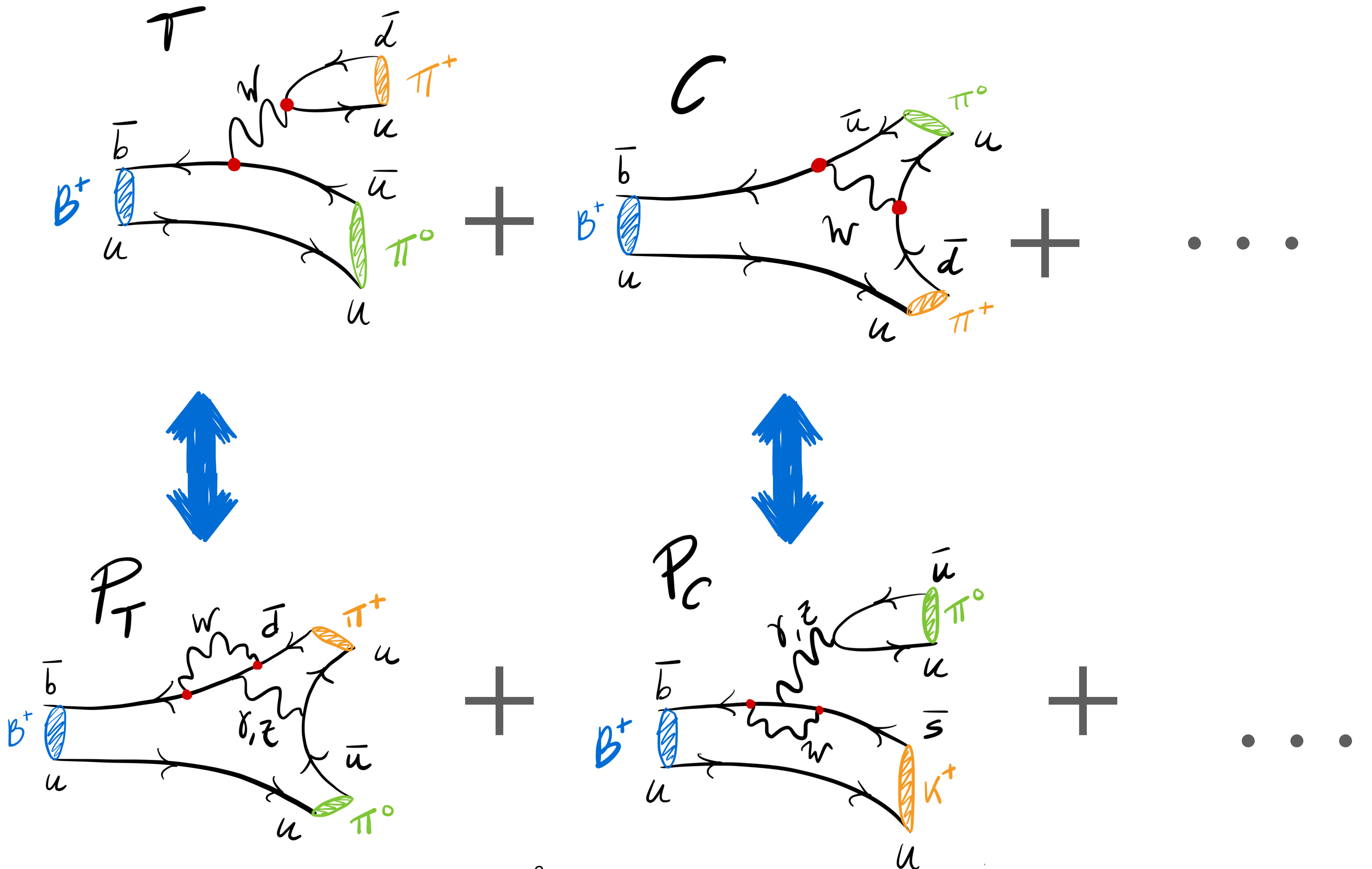
$$H^{j,k}_i$$

See Huber and Tetlalmatzi-Xolocotzi 2111.06418

2. Topological parametrization

Tree amplitude
$\mathscr{T} \sim V_{ub}^{*} V_{uq}$

Penguin amplitude
$\mathscr{P} \sim V_{cb}^{*} V_{cq}$



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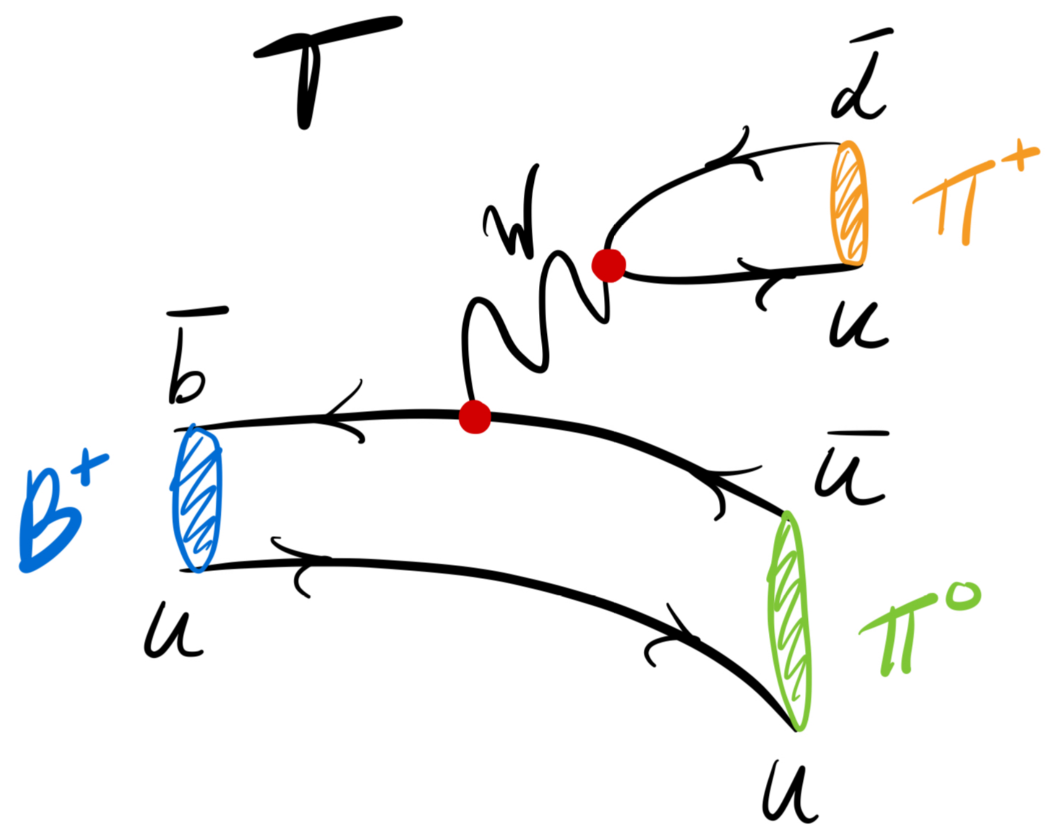
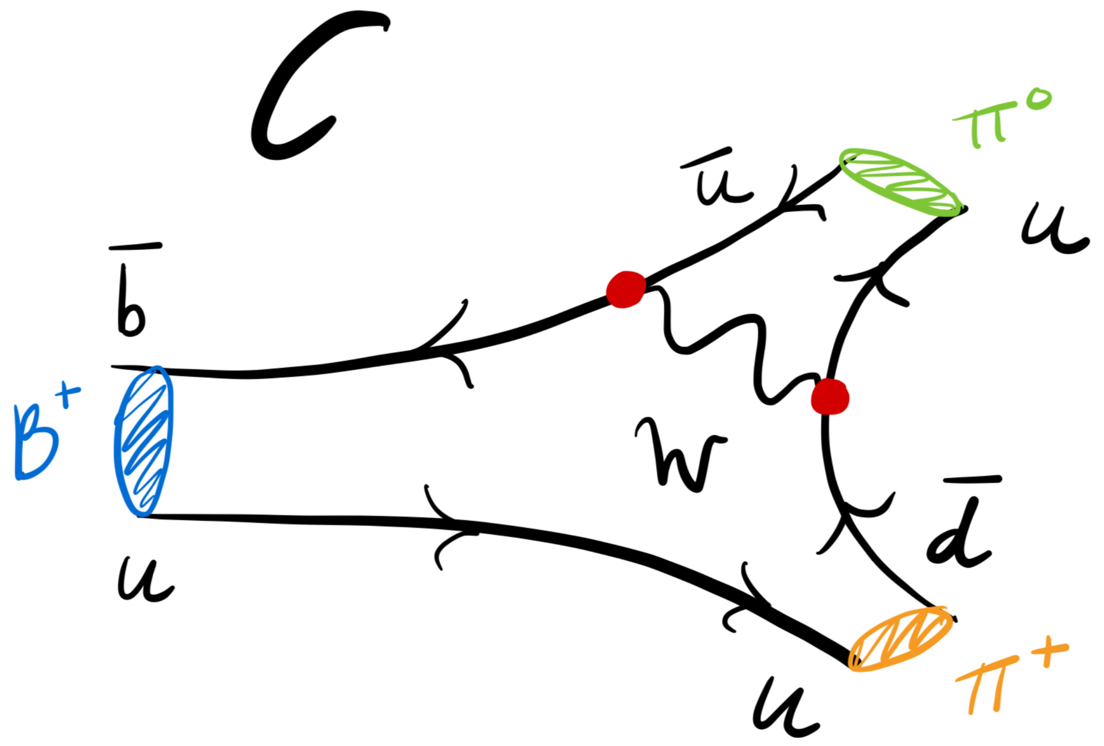
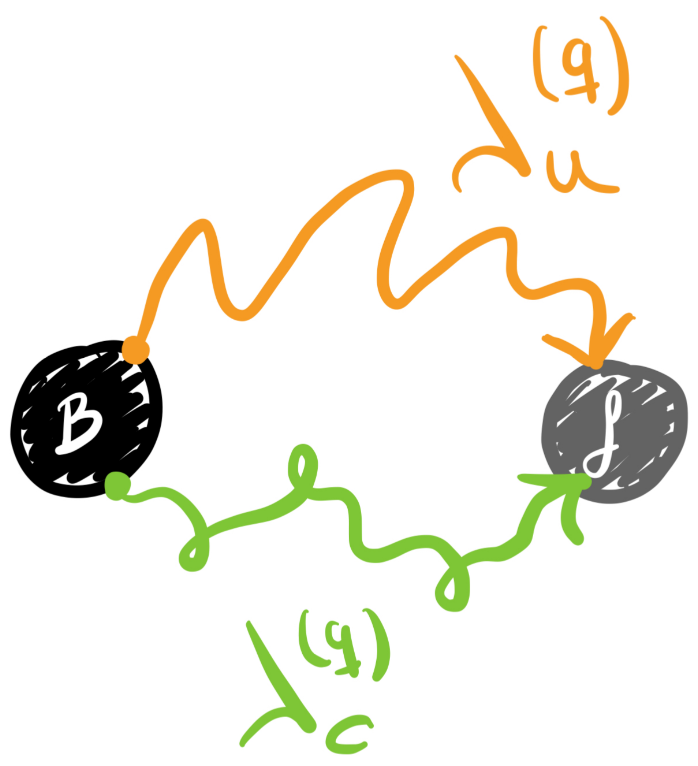
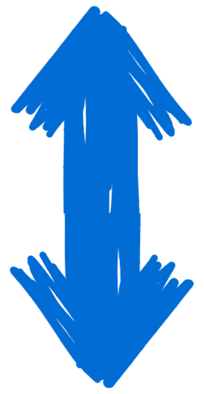






Different CKM structure
necessary for CP Violation



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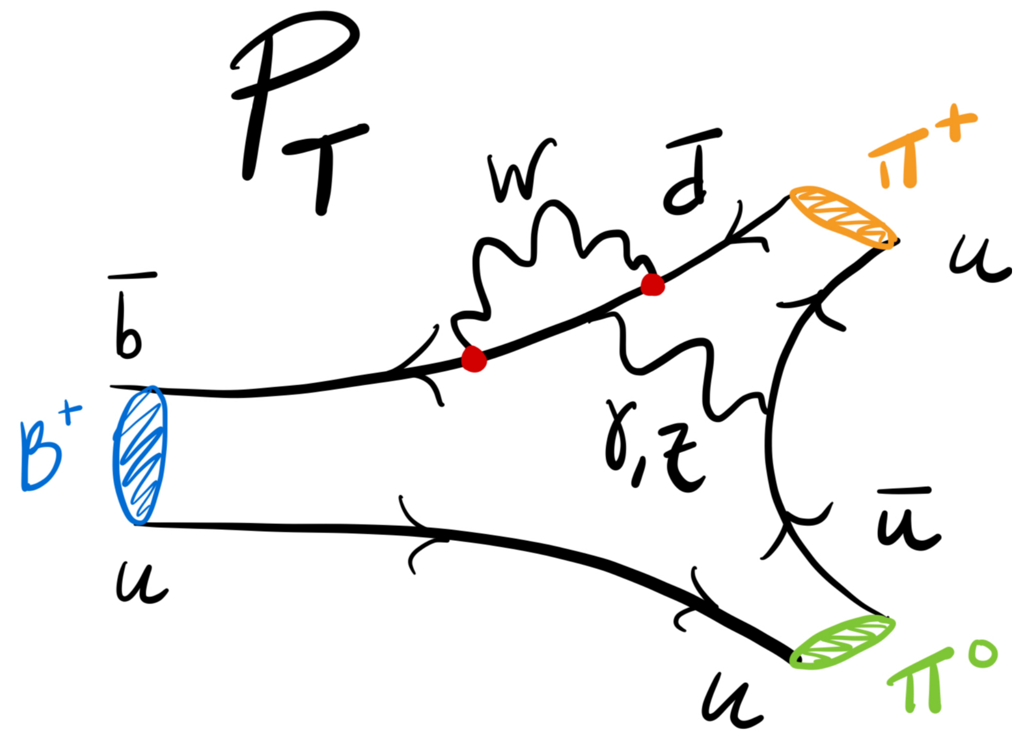

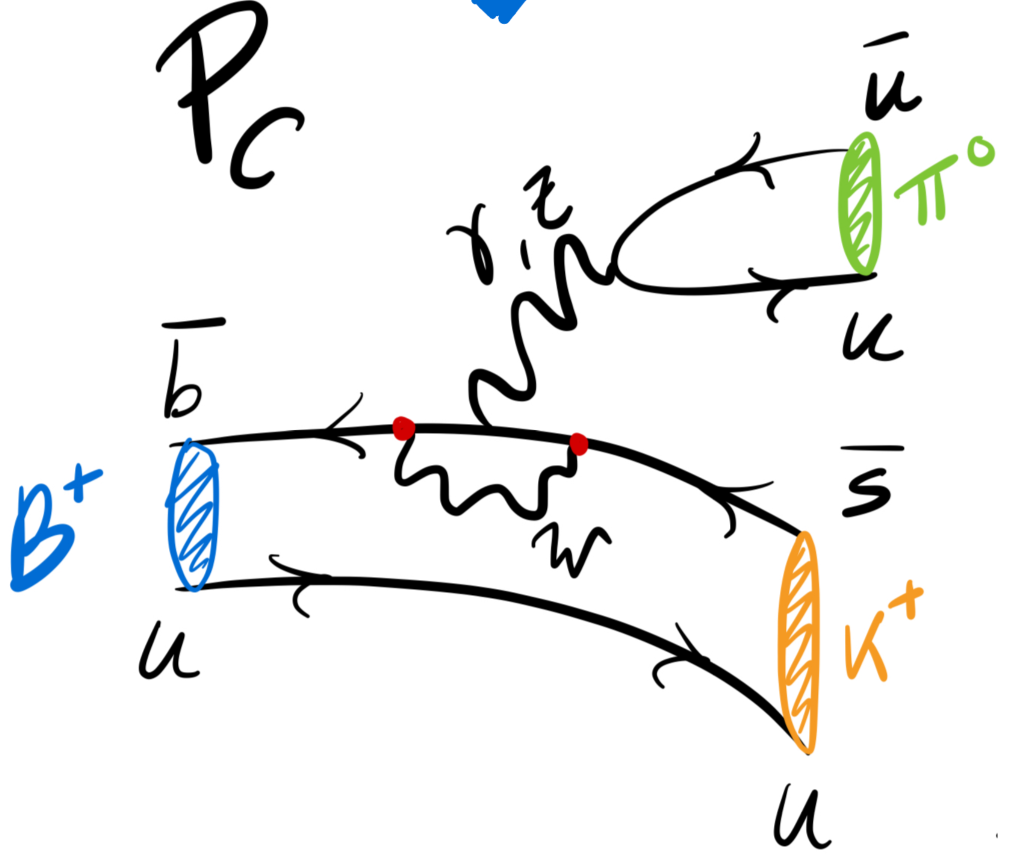

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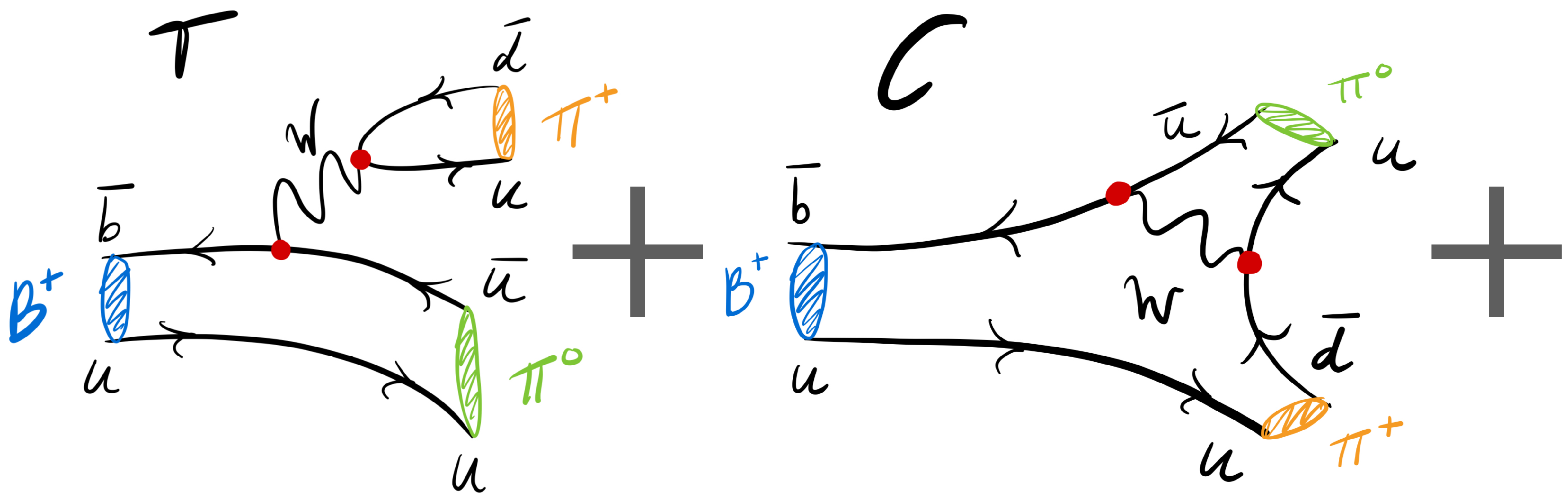
We can relate same coefficients in different decays:

$$B^+ \to \pi^0 \pi^+$$



$$+ \dots$$

$$A(B^+ \to \pi^0 \pi^+) = V_{ub}^* V_{ud} (T + C + \dots)$$

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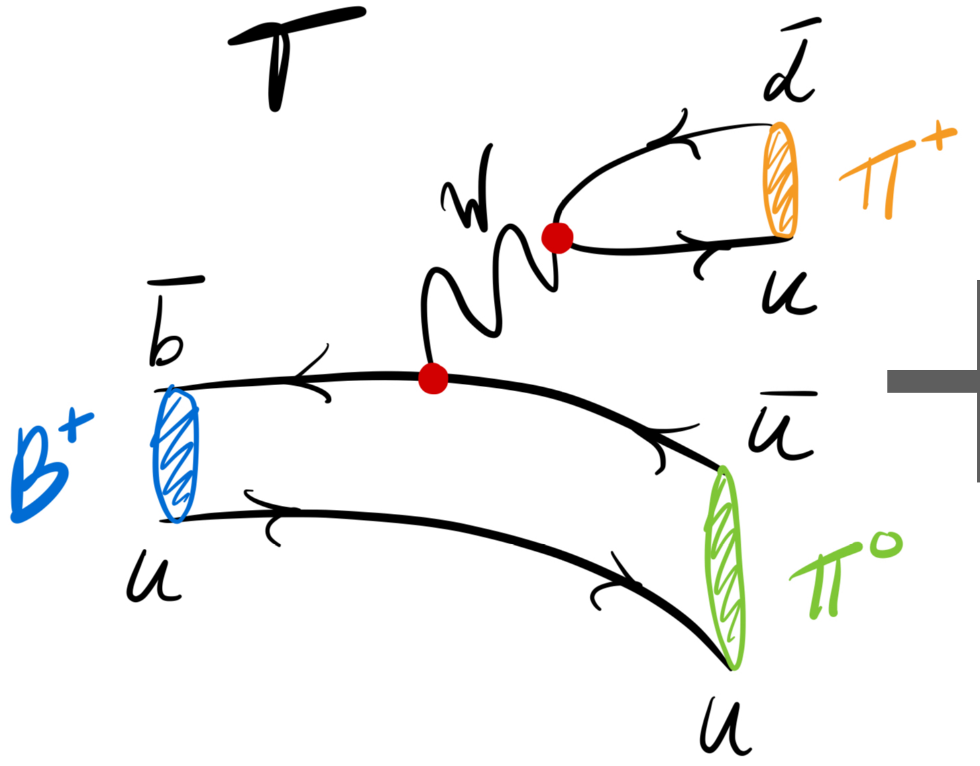

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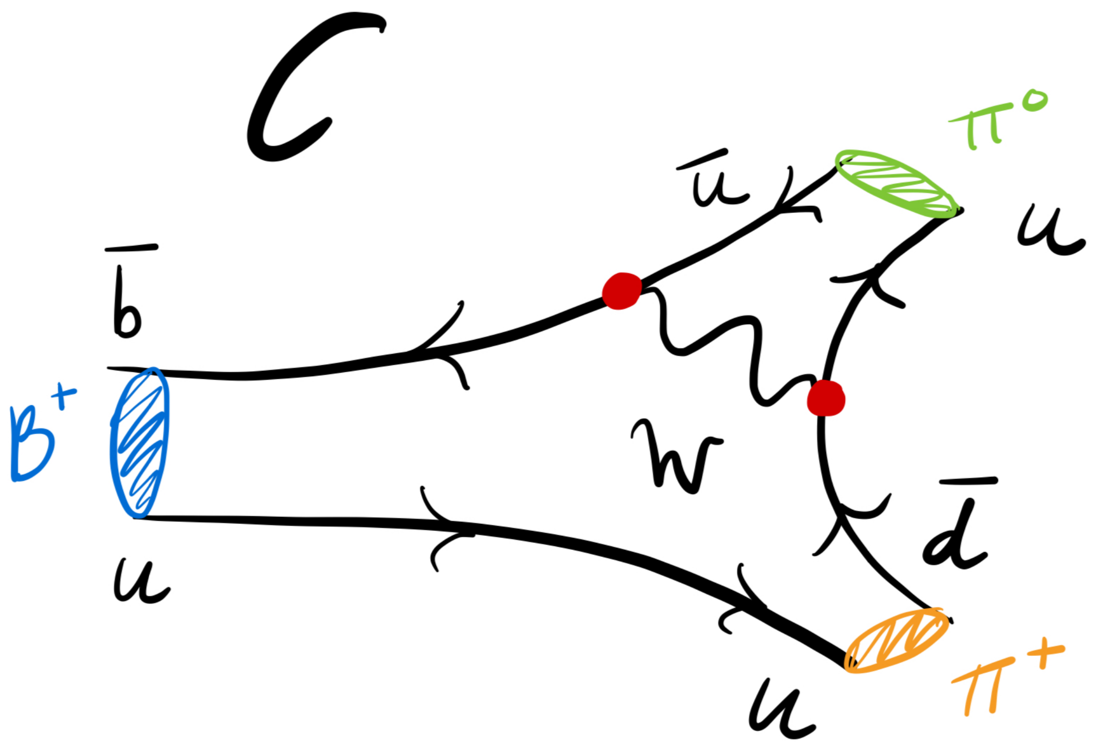

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$$A(B^+ \to \pi^0 \pi^+) = V_{ub}^* V_{ud} (T + C + \dots)$$

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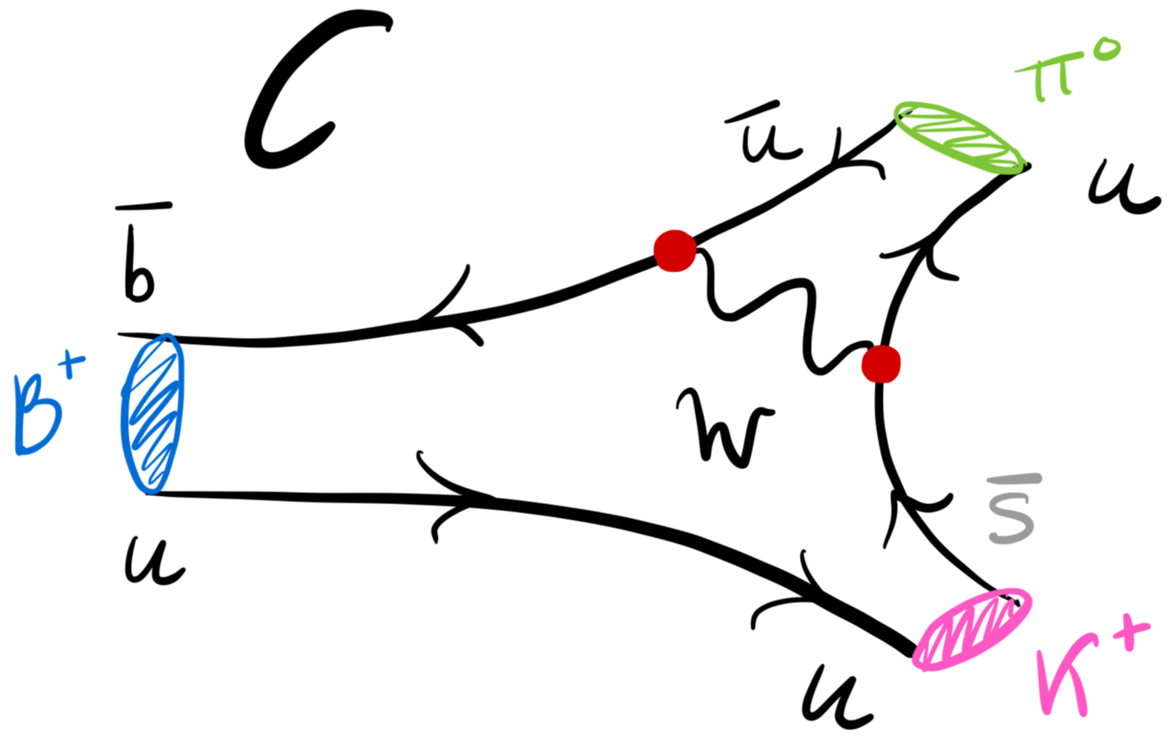

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Same C
Under $SU(3)_F$

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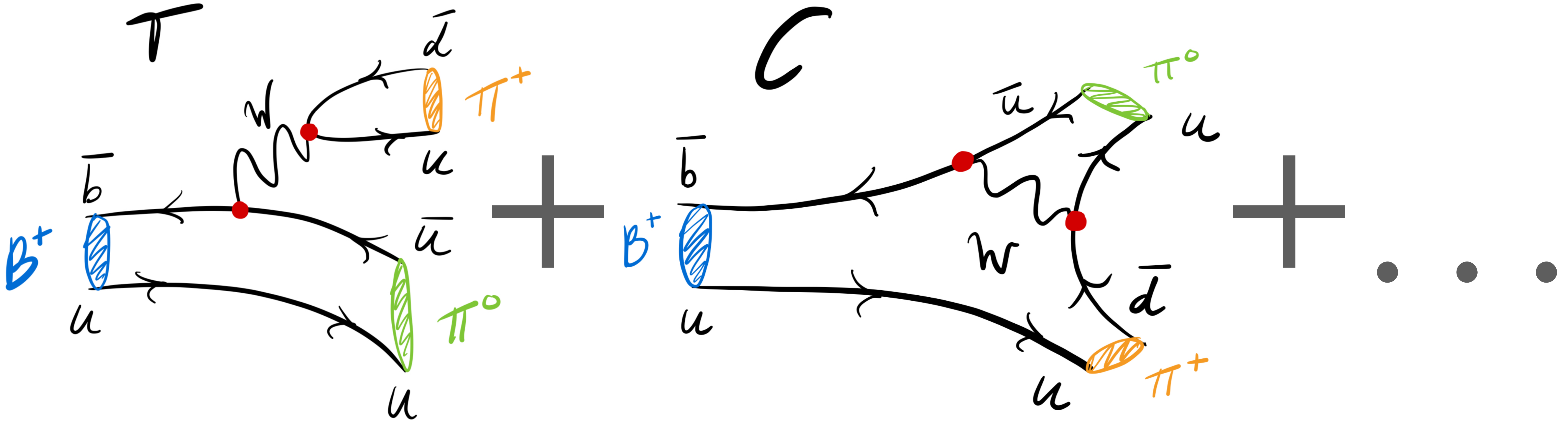

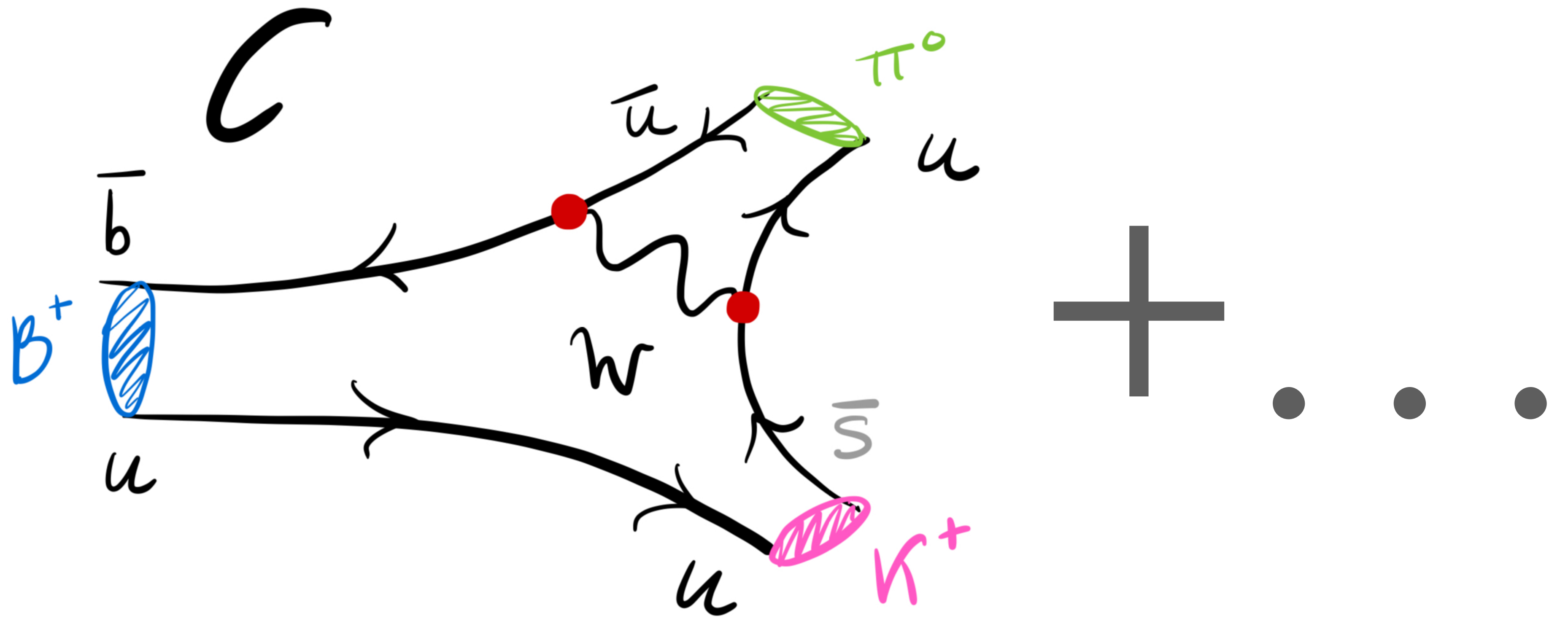

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3. Extract the Coefficients from Experimental Data

Express **observables** in terms of the amplitudes under **topological parameterization**

+

Experimental results from 16 decay modes

=

Fit the values for the **topological coefficients** in $SU(3)_F$ symmetry

For other SU(3) analysis see Huber and Tetlalmatzi-Xolocotzi 2111.06418, Berthiaume et al 2311.18011

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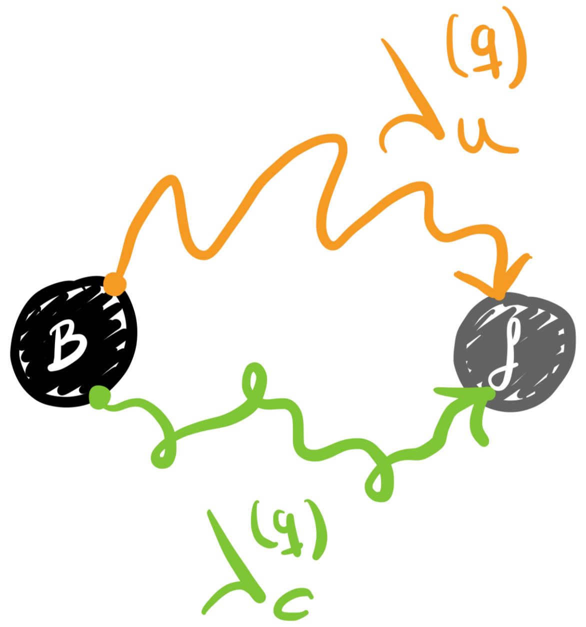
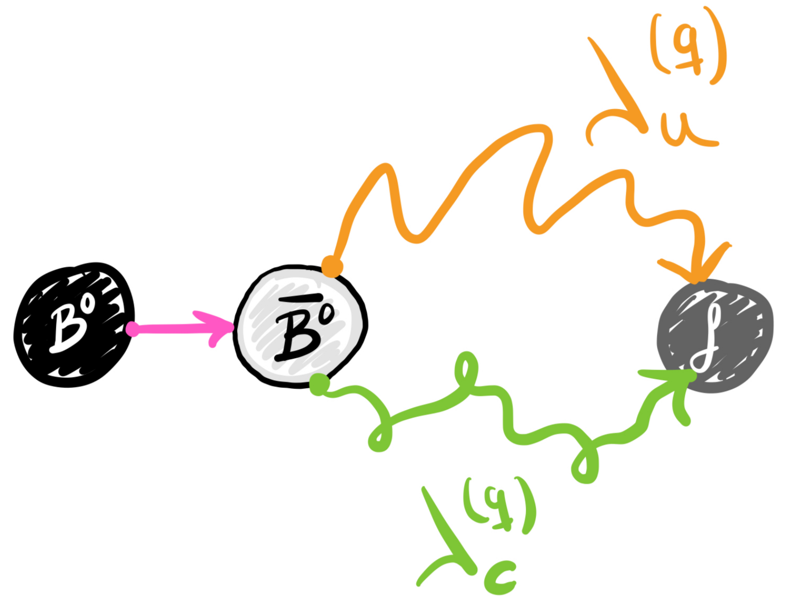
Branching ratios

Direct CP asymmetry: $\mathscr{A}_{CP}^{dir}$



Mixing-induced CP asymmetry: $\mathscr{A}_{CP}^{mix}$



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4. Fit predictions

The fit gives a $\chi^2 \simeq 32.3$ for 15 degrees of freedom, and a **p-value of 0.58%**. The fit is not satisfactory and we conclude that $\mathbf{SU(3)_F}$ **cannot describe the experimental data**

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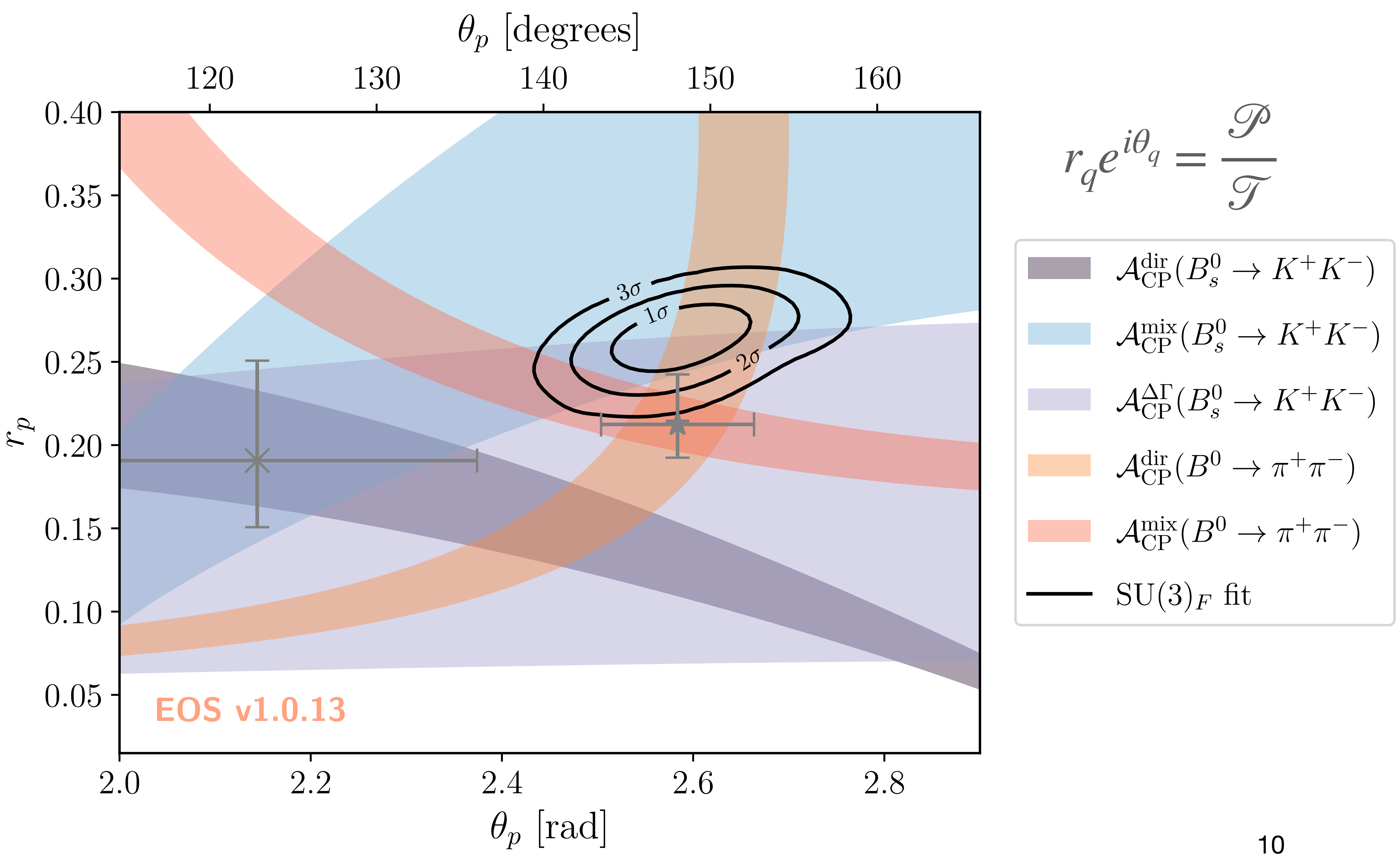

$B_s^0 \to K^+K^-$ and $B^0 \to \pi^+\pi^-$ are U-spin partners (same topologies, different CKM elements).

Experimental data show $SU(3)_F$ breaking

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Is it possible to do a fit with no assumption on $SU(3)_F$ symmetry? **No, too many parameters**

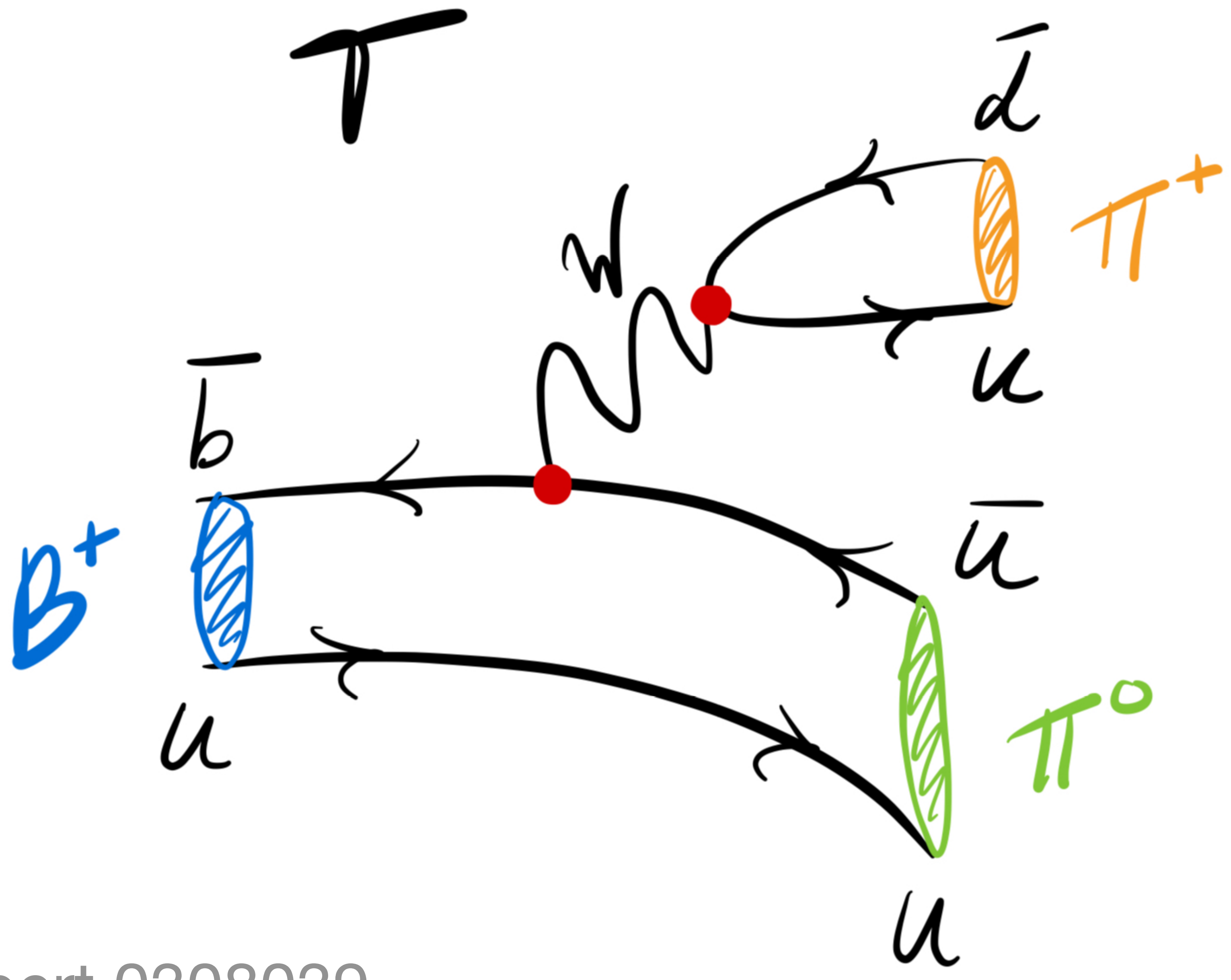
Is it possible to include **some** $SU(3)_F$ symmetry breaking without increasing dramatically the number of parameters? **Yes!**

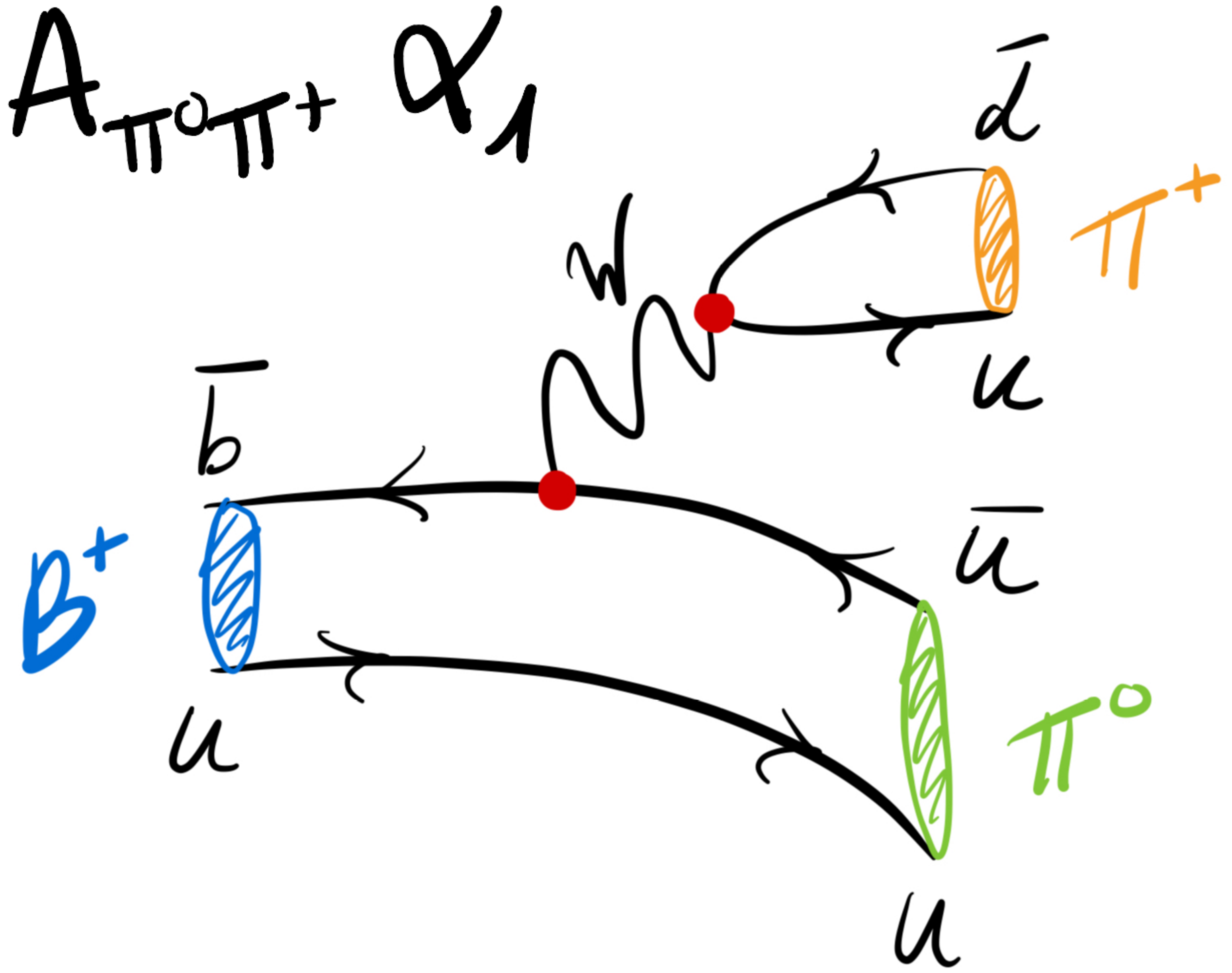
Factorizable $SU(3)_F$ breaking

Factorizable $SU(3)$ breaking

Factorizable $SU(3)_F$ breaking allows us to account for the different masses of the mesons without adding (almost) any new coefficient

$SU(3)_F$ **symmetry:** T → **Factorizable** $SU(3)_F$ **breaking:** $A_{M_1M_2}\ \alpha_1$





$$A_{\pi^0\pi^+} = M_{B^+}^2\, F_0^{B^+\to\pi^0}(m_{\pi^+}^2)\, f_{\pi^+}$$

Beneke, Neubert 0308039,
Beneke, Buchalla, Neubert, Sachrajda 0006124

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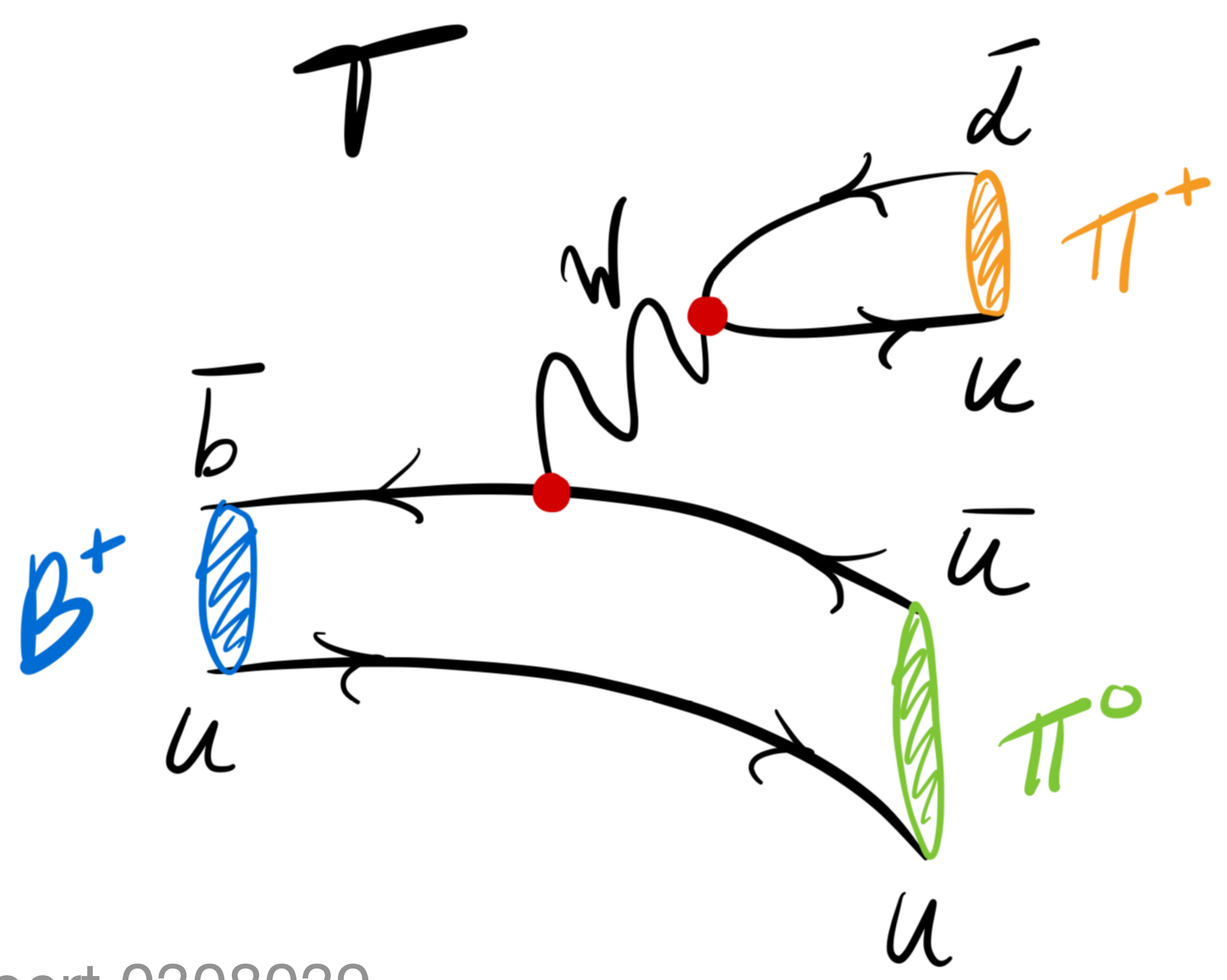

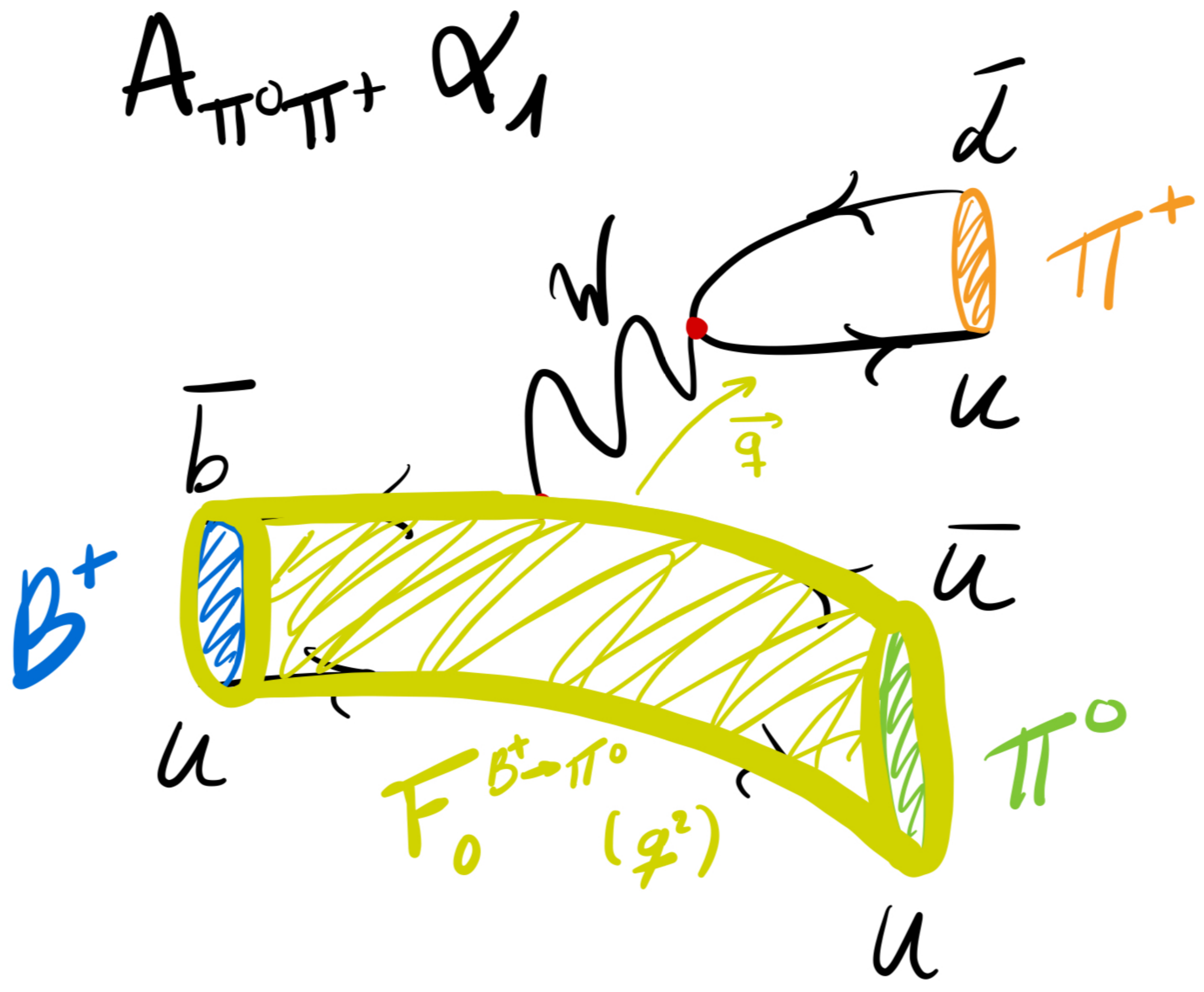

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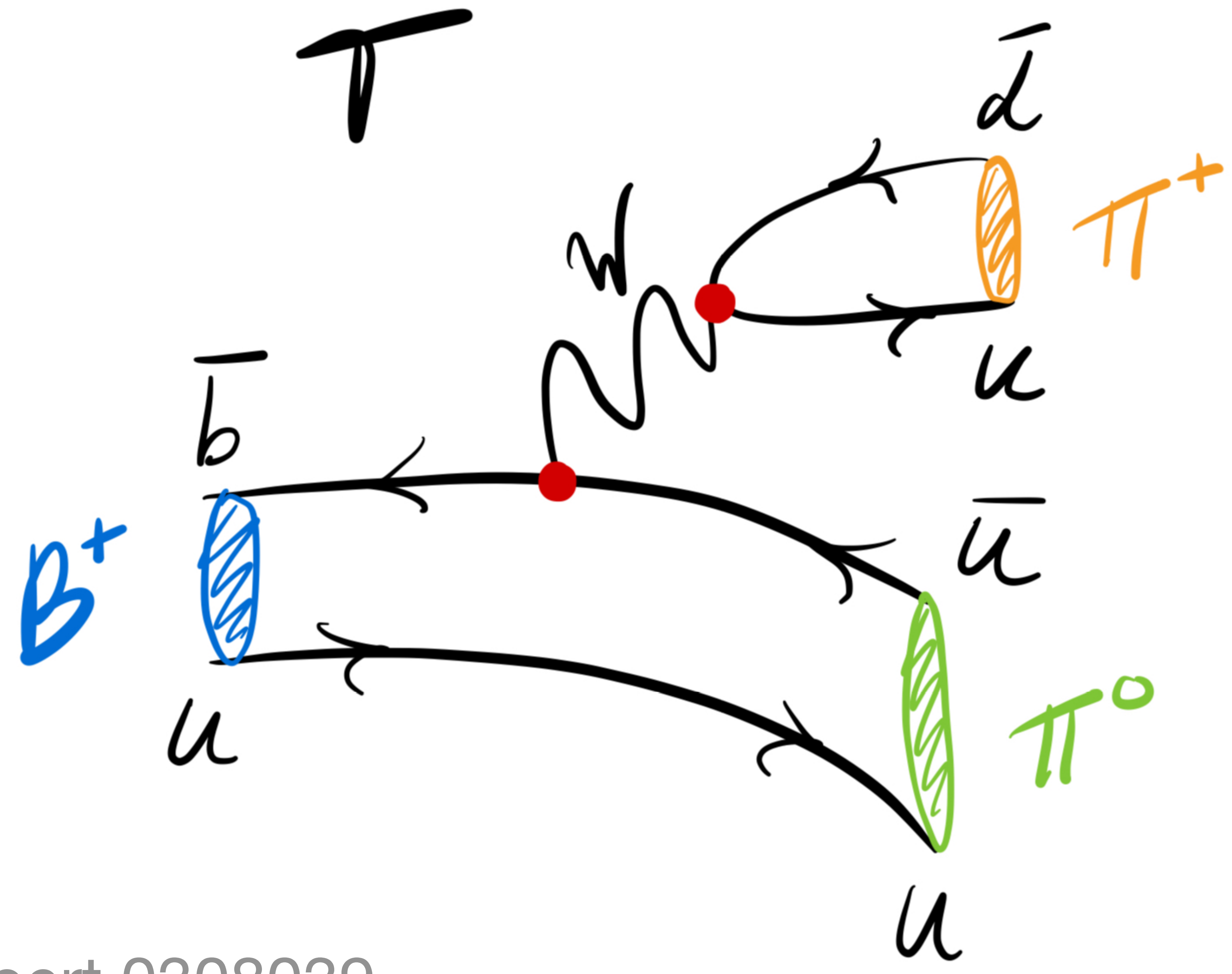

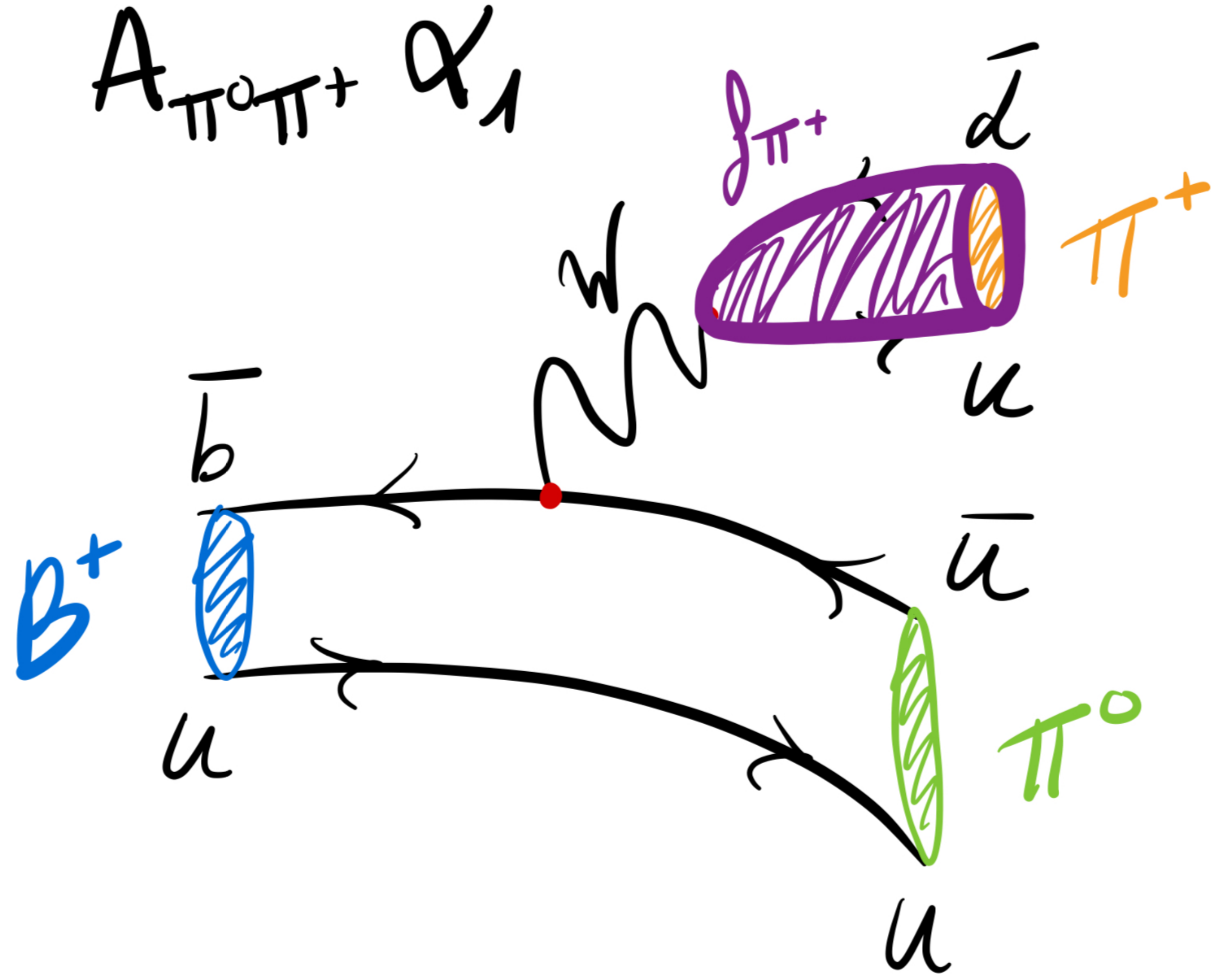

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$A_{M_1M_2}$: $SU(3)$ **breaking**, but **known**! (Fixed, no new coefficients)

α_1 : $SU(3)$ **symmetric**, fitted from experimental data

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$$C \Rightarrow A_{M_1M_2}\tilde{\alpha}_2$$

$$P_C \Rightarrow A_{M_1M_2}\tilde{\alpha}^c_{3,EW}$$

$$+\ldots \qquad +\ldots$$

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Inspired by QCD Factorization, more on that later!

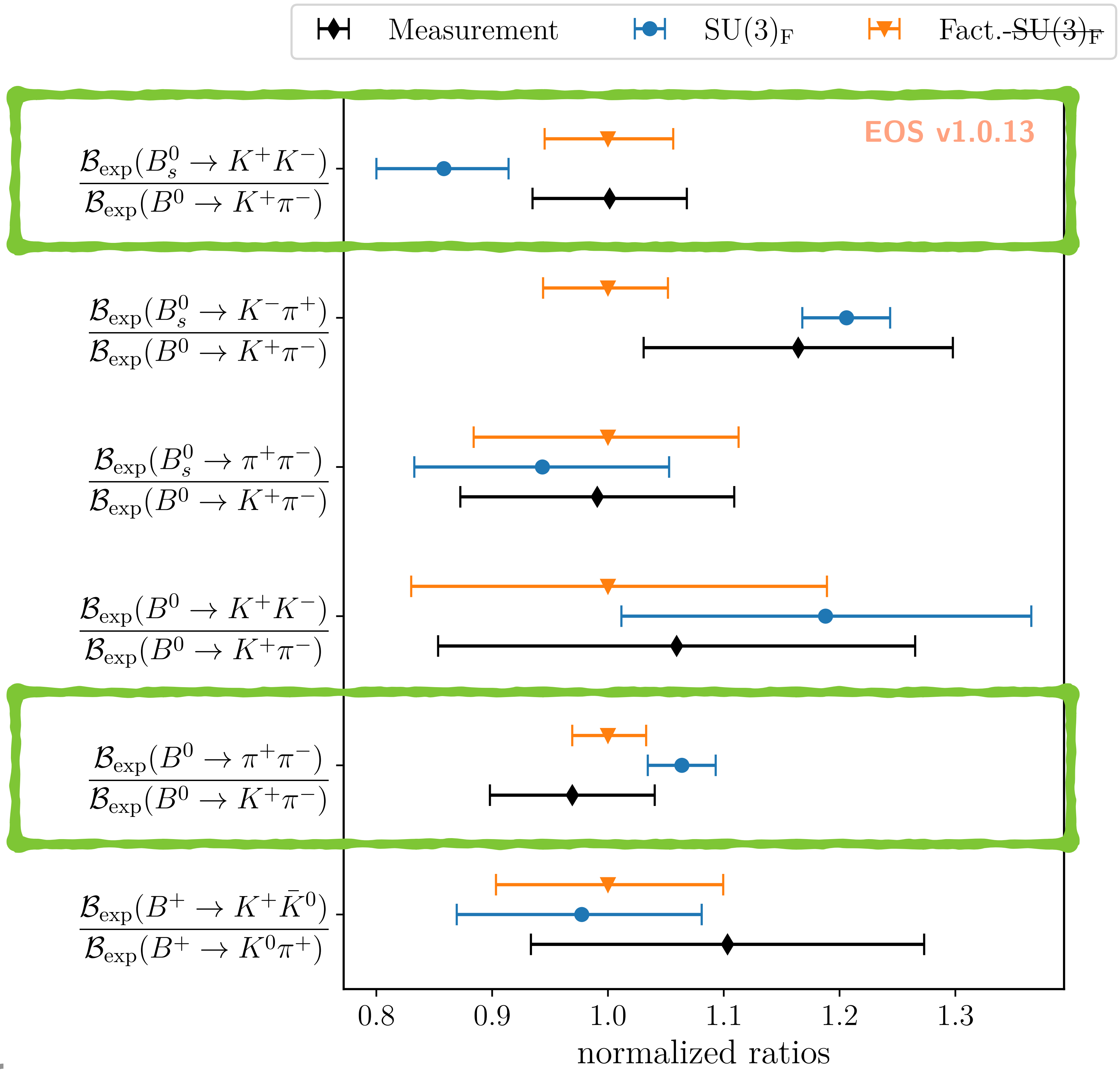
2111.15428

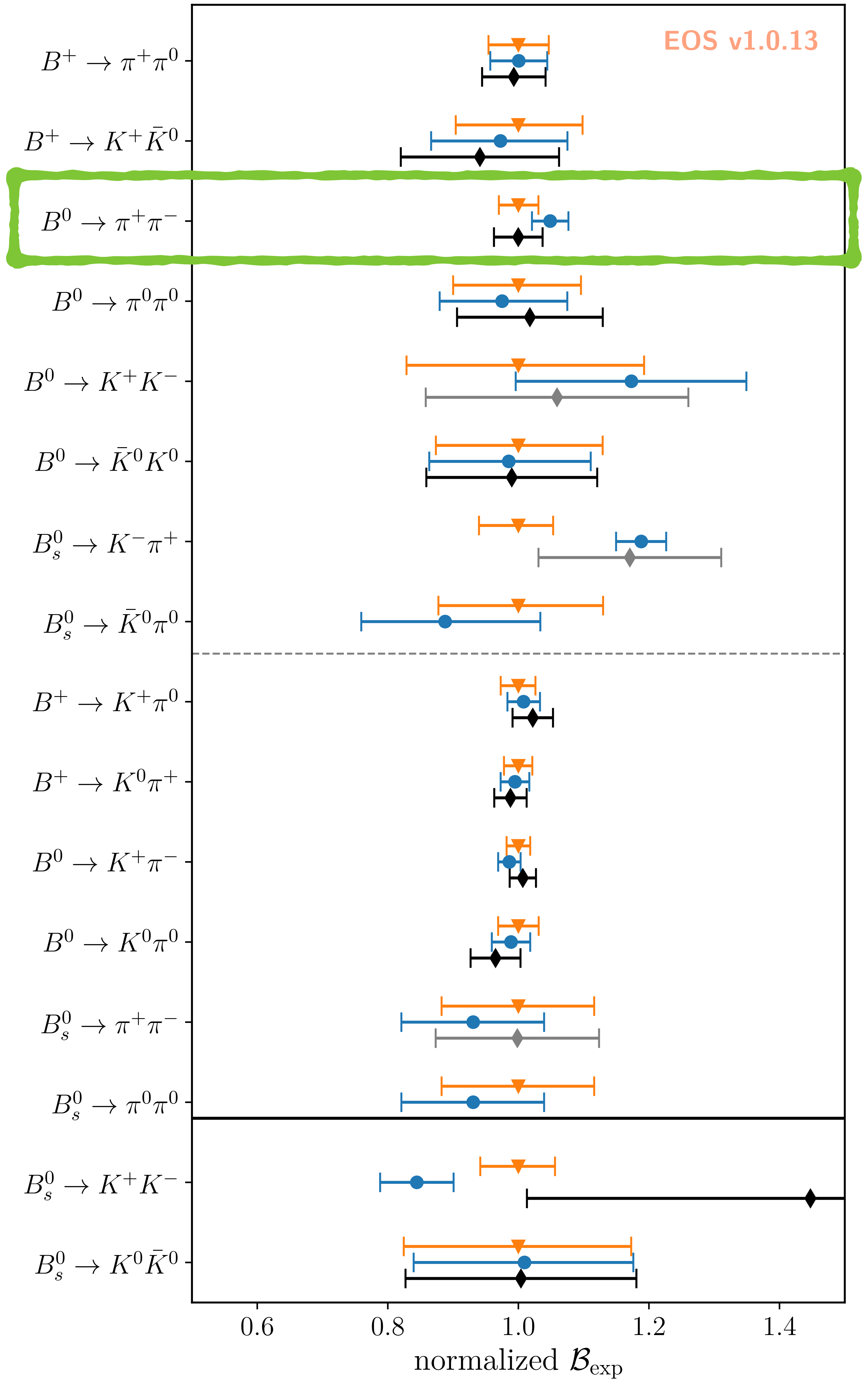
Results: Factorizable $SU(3)$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

Factorizable $SU(3)_F$ breaking describes data almost perfectly, with a **p-value of 31%**

Fact. $SU(3)_F$ improves predictions compared to full $SU(3)_F$





Experimental data from LHCb, Belle II and BaBar

2111.15428

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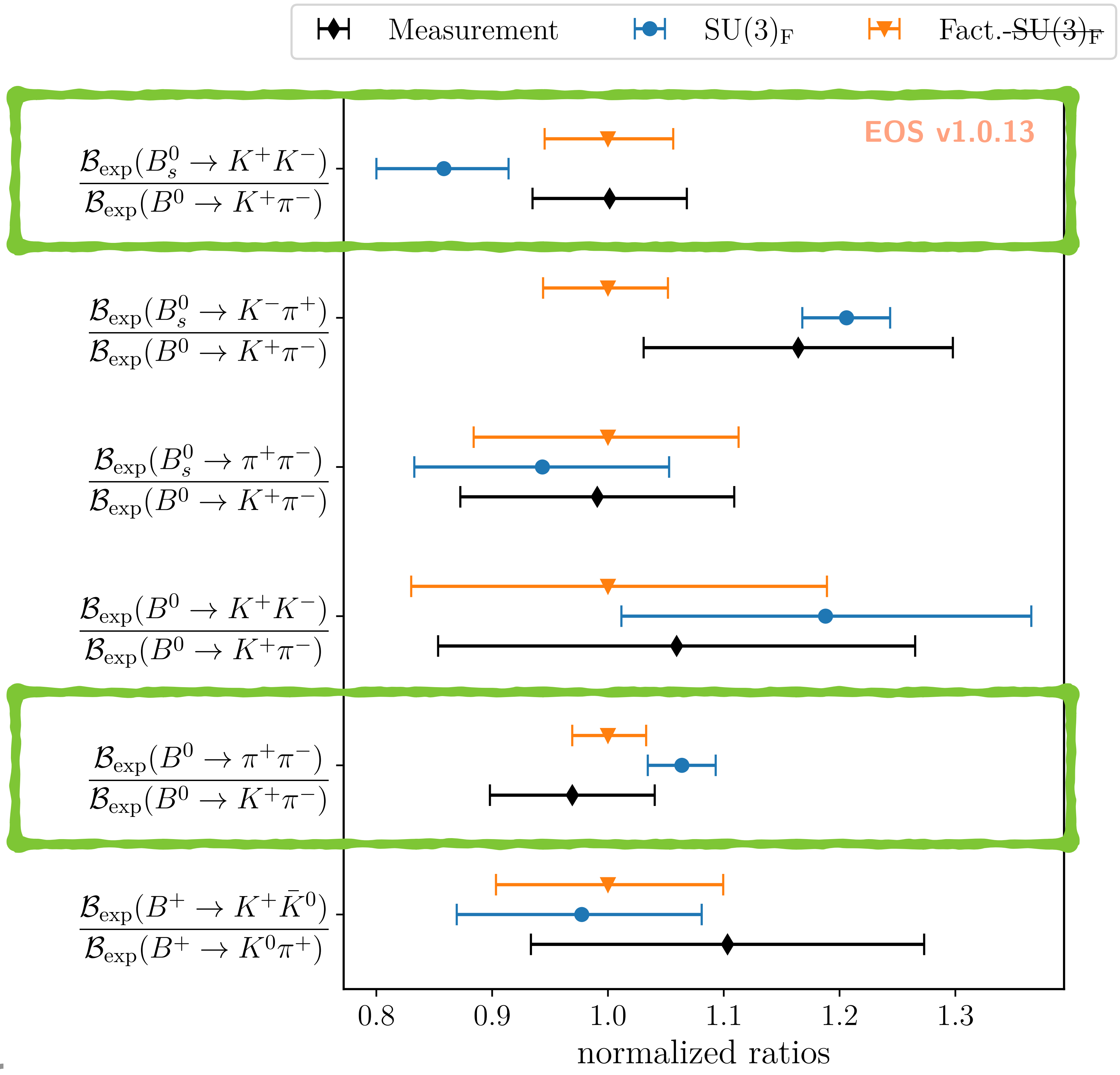

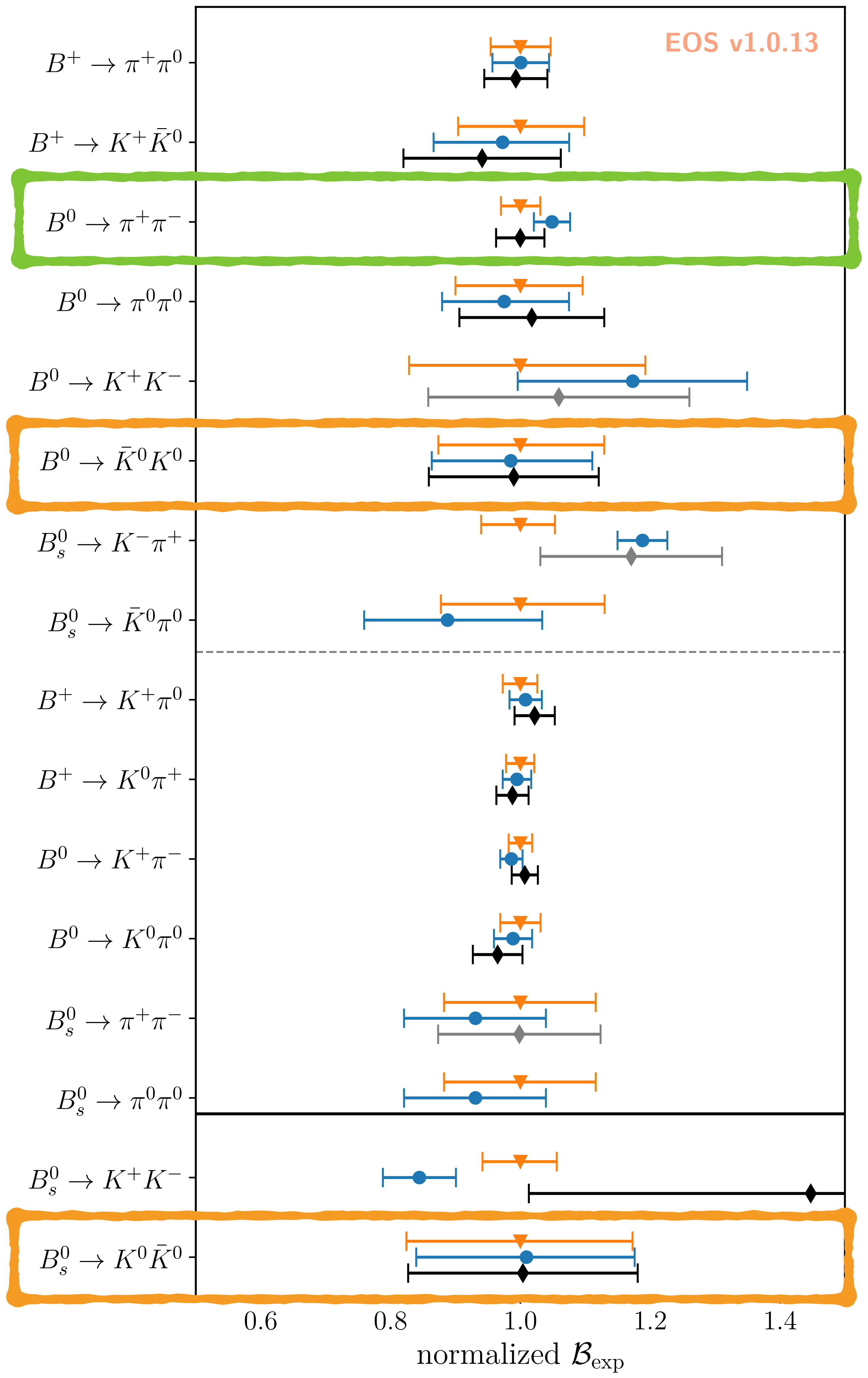

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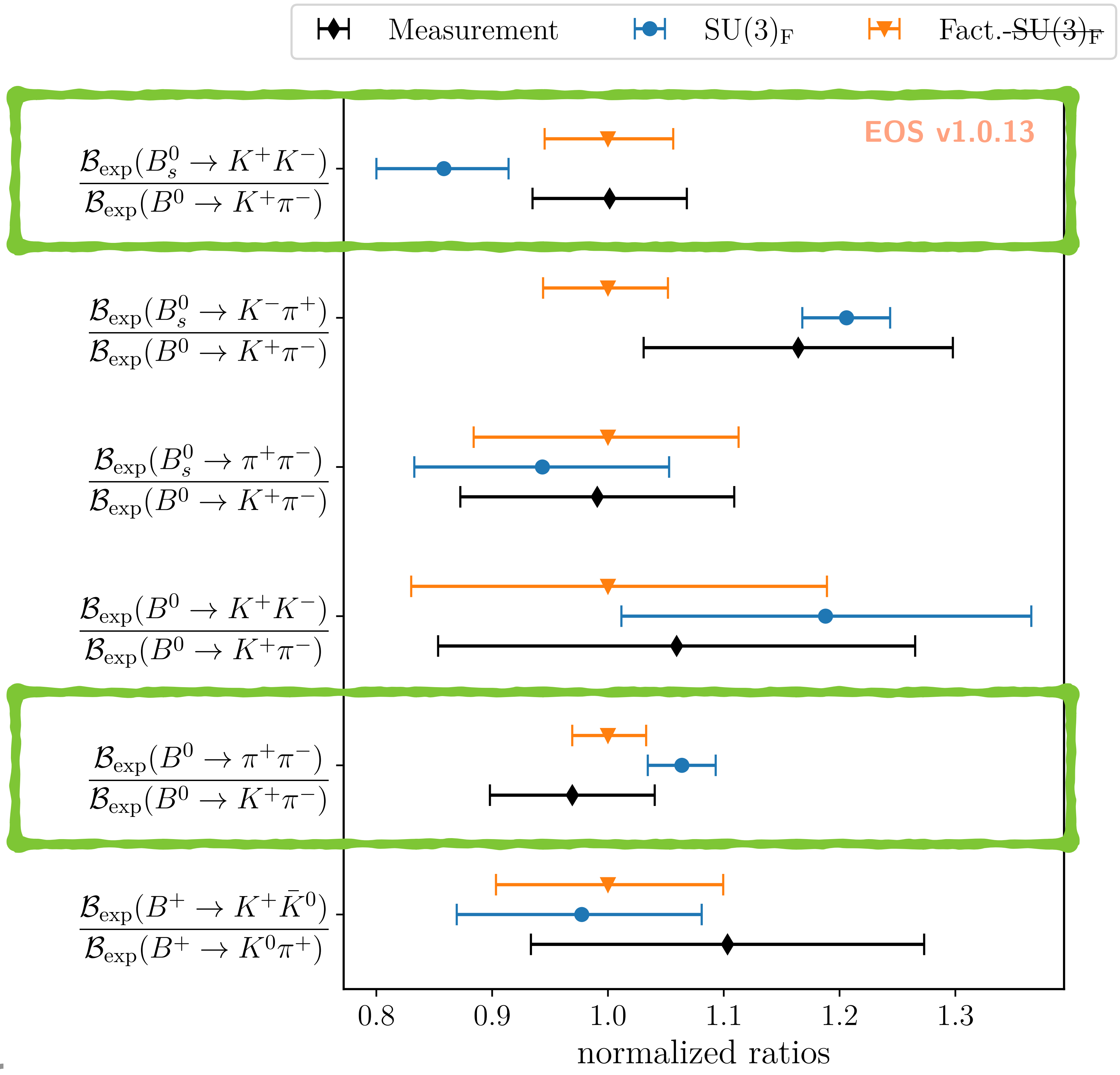
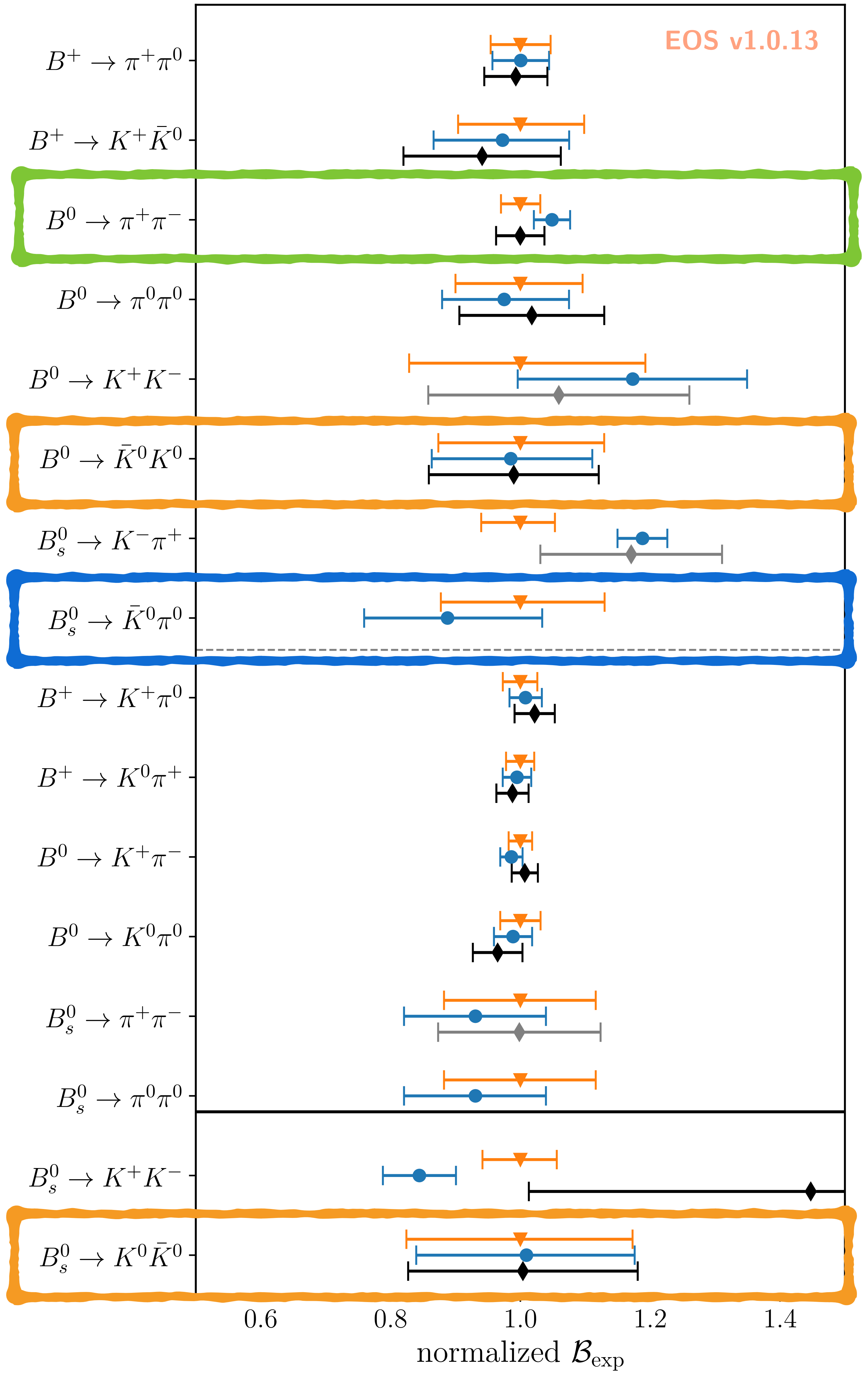
Obtain predictions for decays that have not been measured yet





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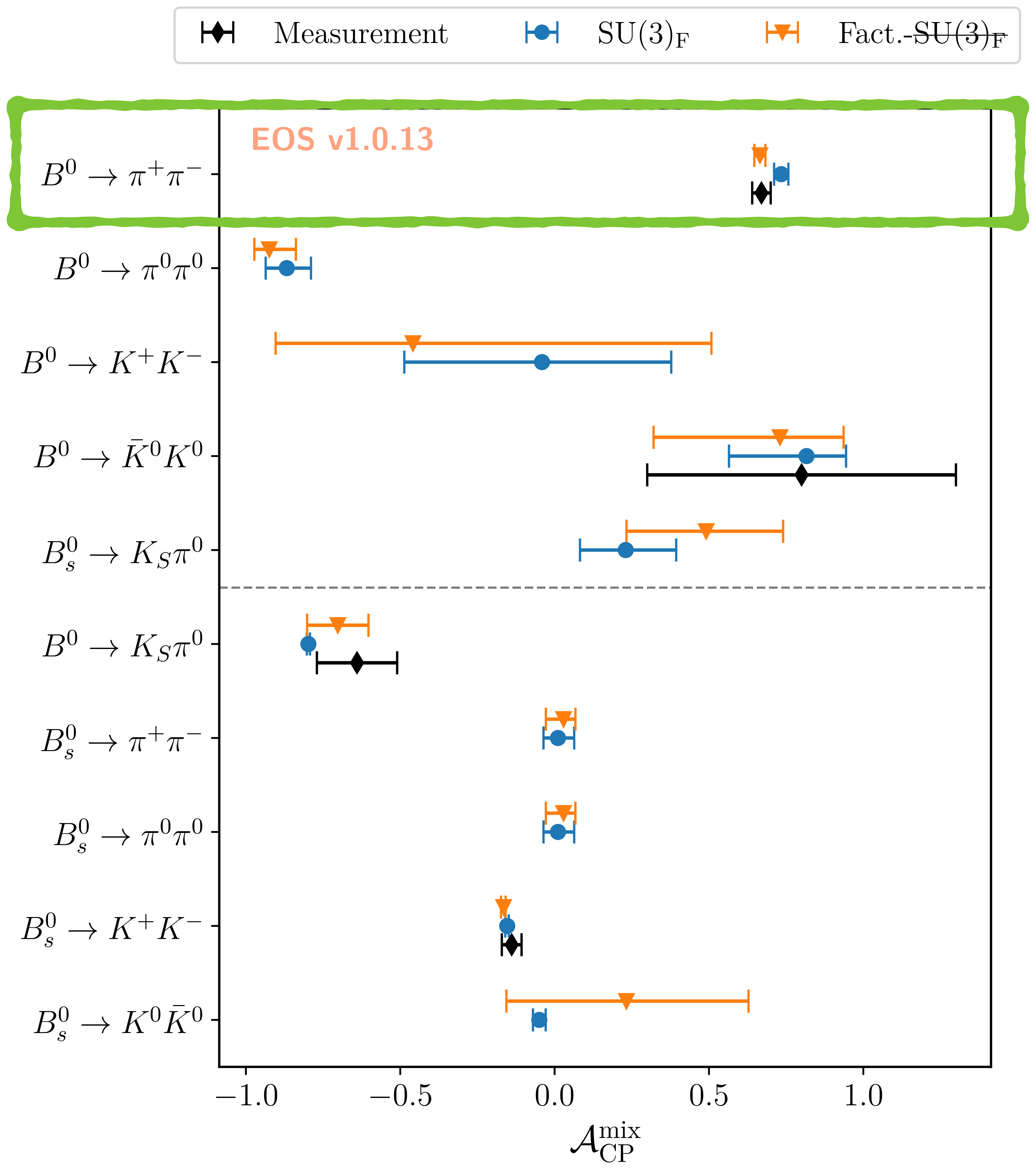

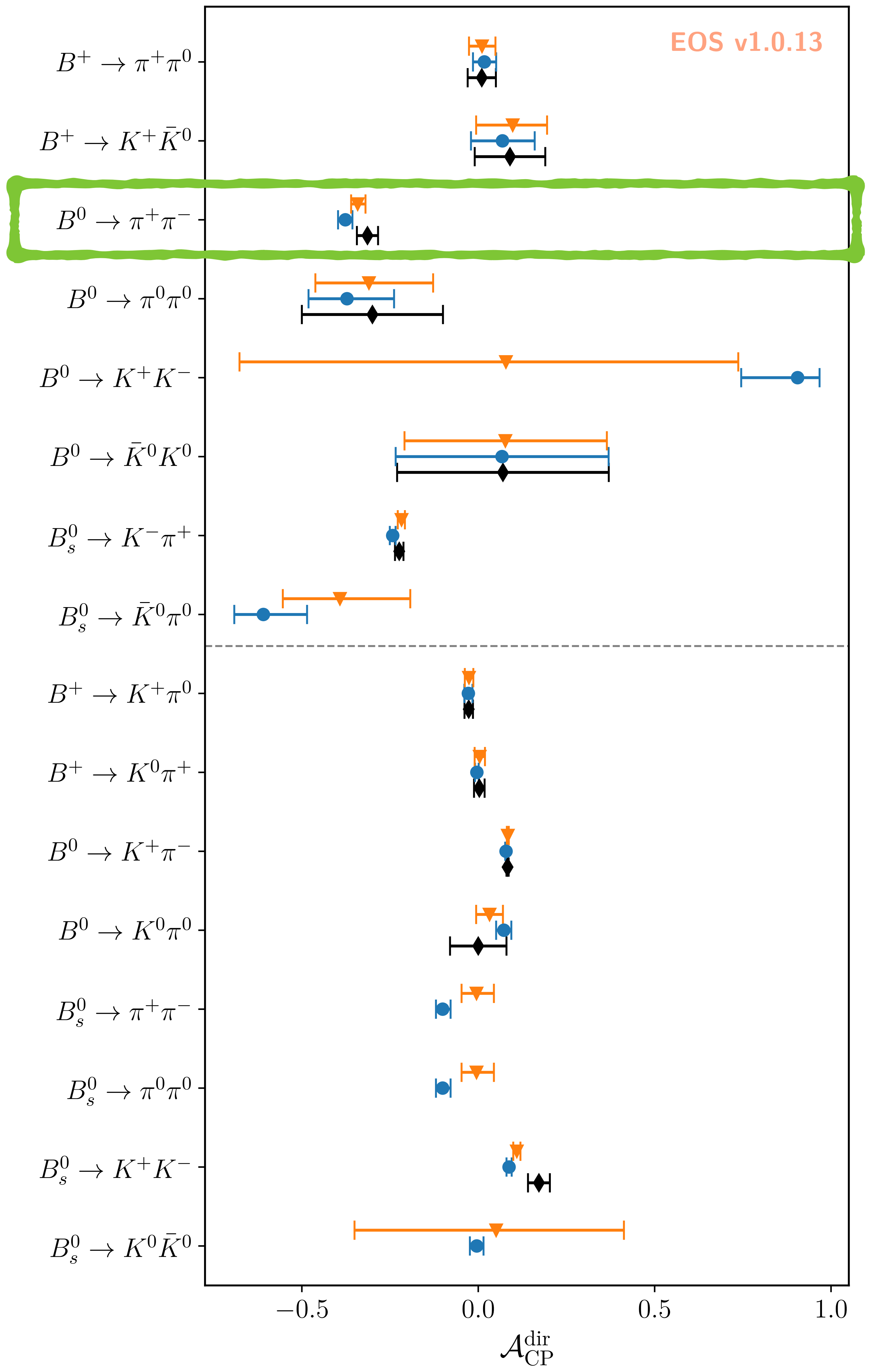

Experimental data from LHCb, Belle II and BaBar

2111.15428

Results: Factorizable $SU(3)$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

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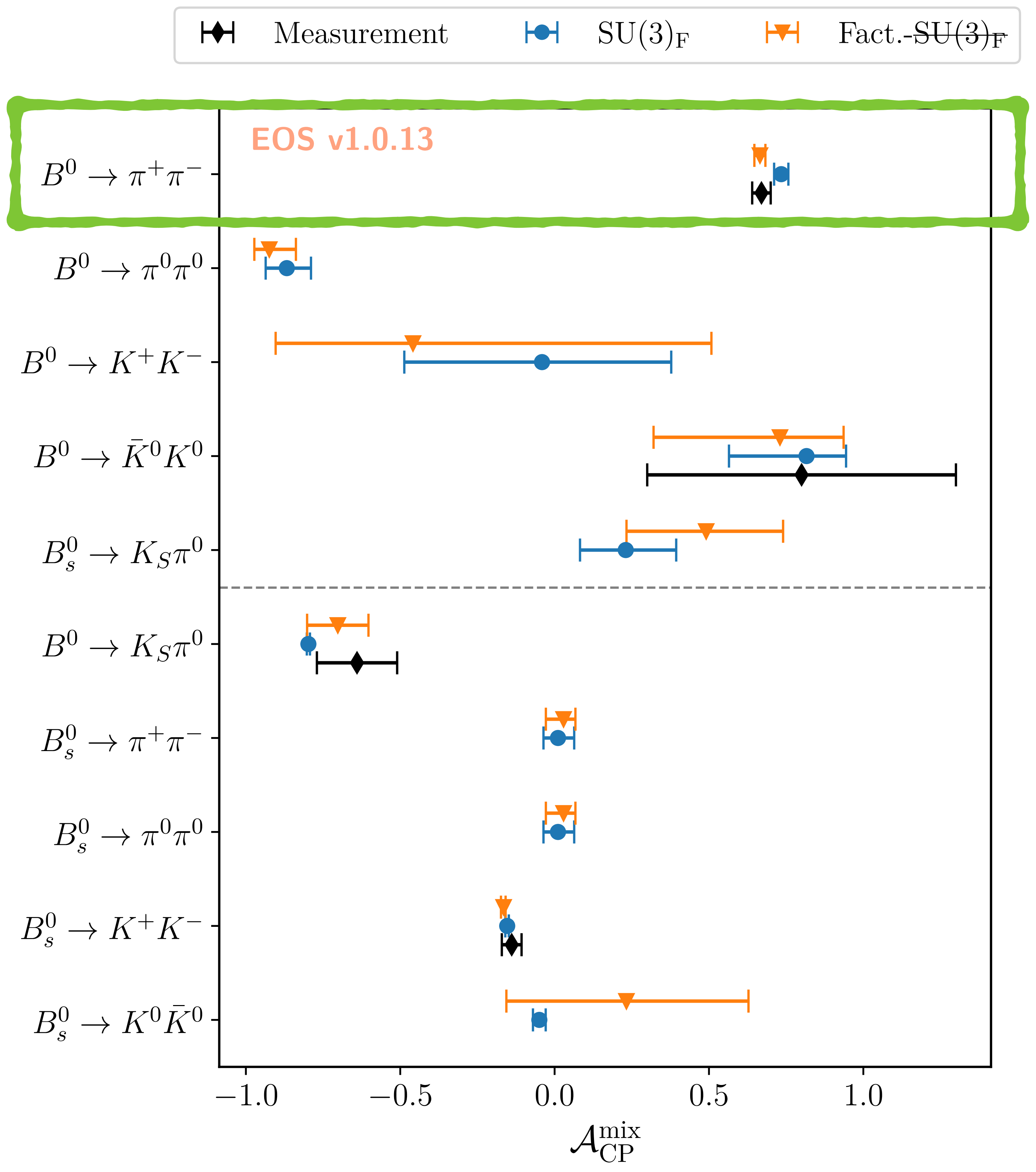

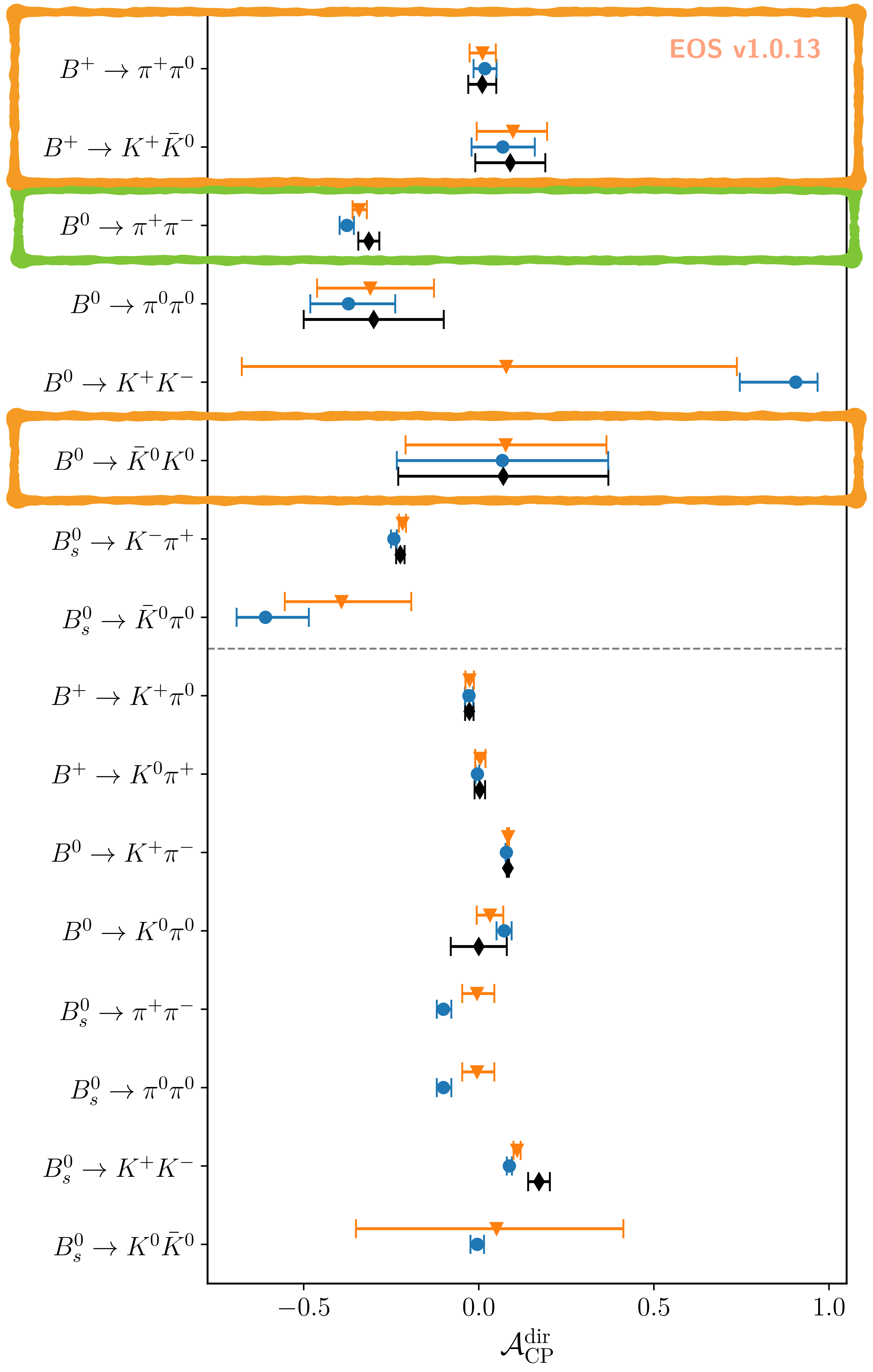

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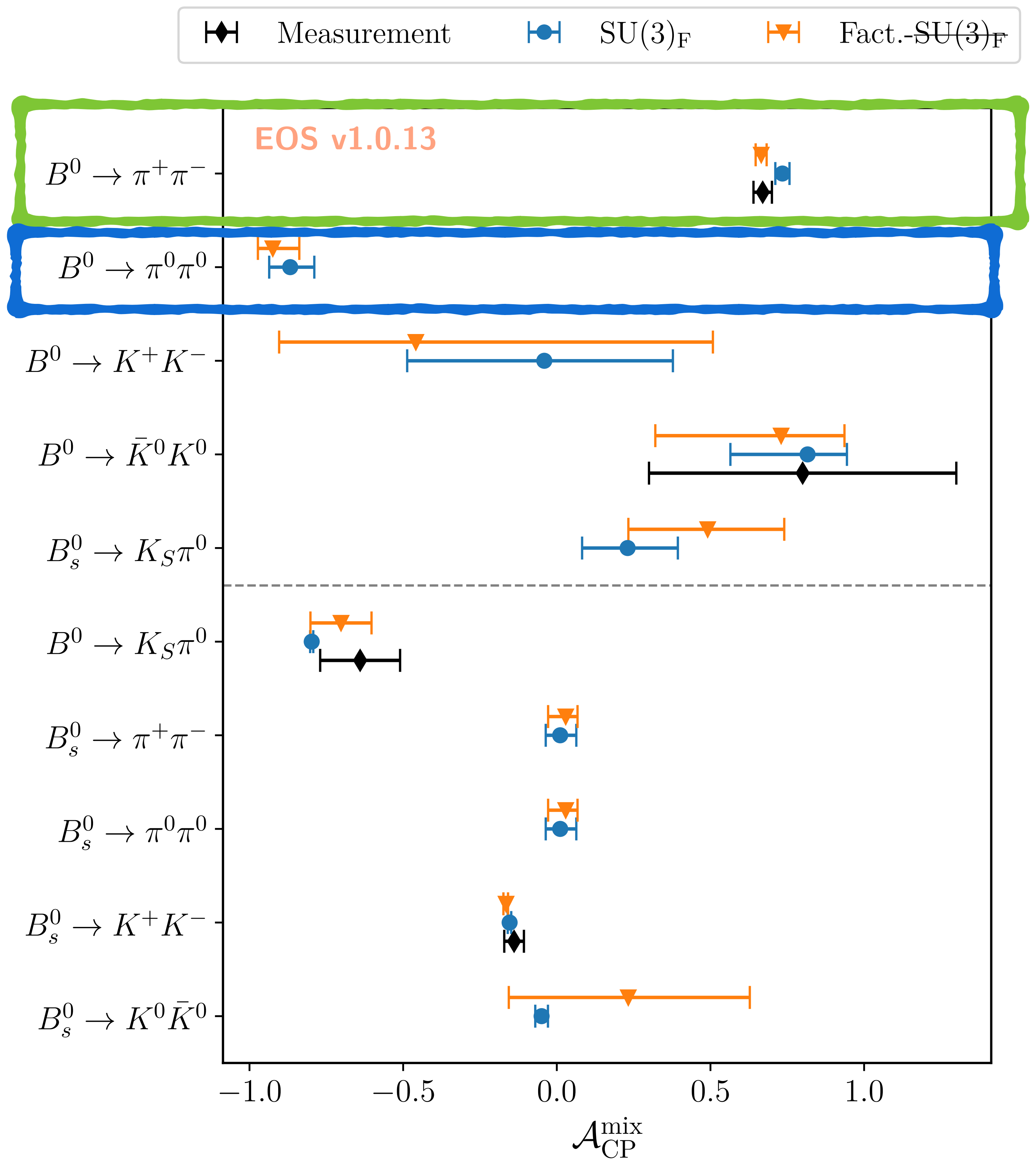
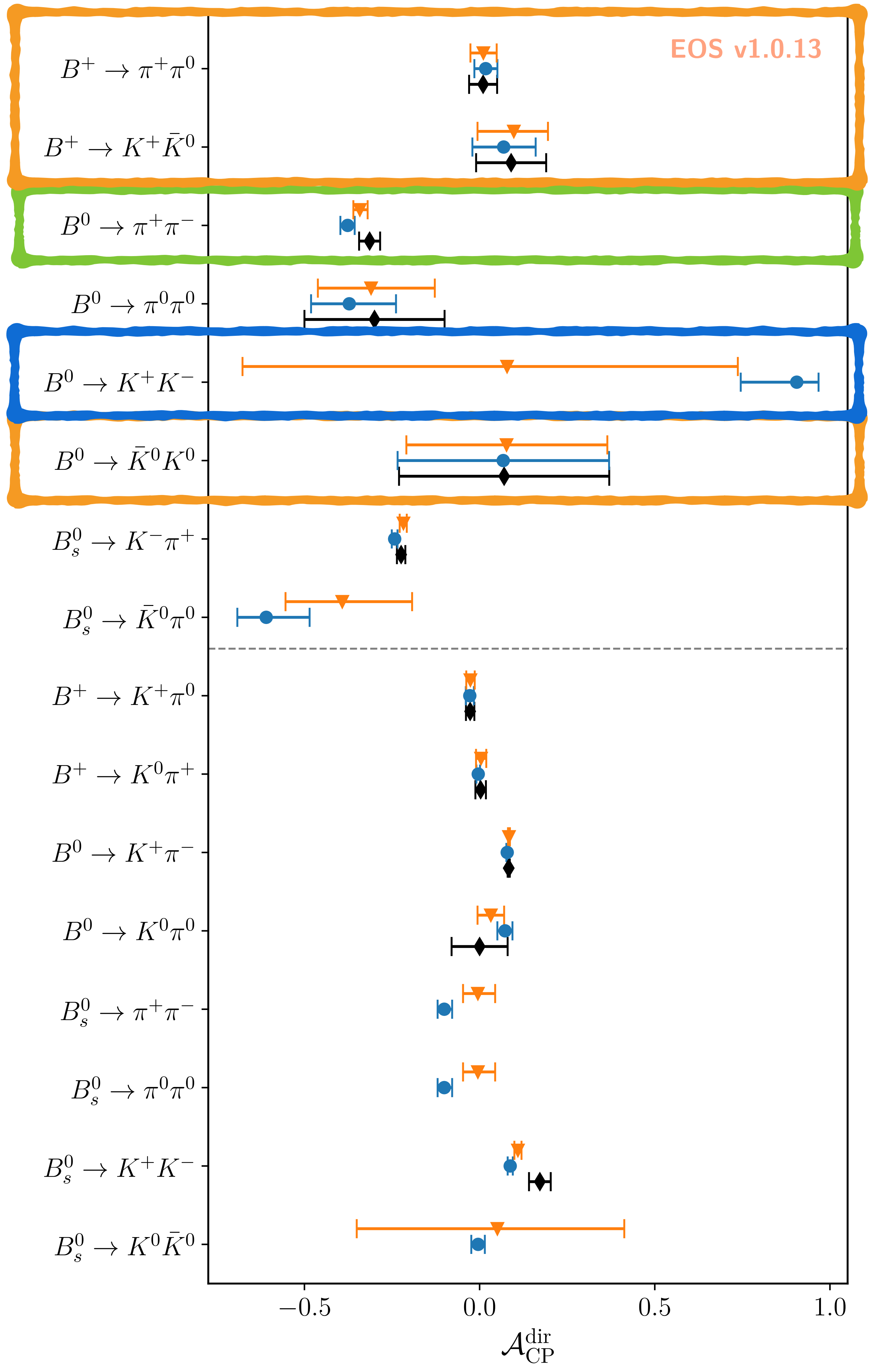
- Certain predictions saturate experimental uncertainty
- Obtain predictions for decays that have not been measured yet





Experimental data from LHCb, Belle II and BaBar

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Results: Factorizable $SU(3)$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

EOS

Factorizable $SU(3)_F$ breaking describes data very accurately. Small tensions in $\mathscr{A}_{\rm CP}(B_s^0 \to K^+K^-)$, $\mathscr{A}_{\rm CP}^{\rm mix}(K_S^0\pi^0)$ and $\mathscr{A}_{\rm CP}^{\rm dir}(B^0 \to \pi^+\pi^-)$ that do not exceed 1.5σ

Updates / new measurements would reduce the overall uncertainty in the predictions: $B_{(s)}^0 \to K^0\bar{K}^0, B^0 \to K^+K^-, \ldots$

What about the coefficients? Can we improve our understanding of QCD?

What do we learn about QCD?

Factorizable $SU(3)_F$ breaking vs QCDF

Beneke, Neubert 0308039,
Beneke, Buchalla, Neubert, Sachrajda 0006124

Fact. $SU(3)_F$ breaking parametrisation is inspired by QCD Factorization:

QCDF	Fact. $\cancel{SU(3)}_F$
Method to calculate theoretically amplitudes for hadronic processes at higher orders	Purely data-driven

- Expand in orders of α_s and $1/m_b$
- Use the large mass of the b-quark to factorize the final state. Then correct for gluon interactions
- Divergences show up in higher order and annihilation terms, for which only a model can be used

Coefficient results: Fact. $SU(3)_F$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

- Due to high correlations, only certain combinations can be determined with a reasonable uncertainty: **individual coefficients** are mostly **unconstrained**

2111.15428

Coefficient results: Fact. $SU(3)_F$ breaking

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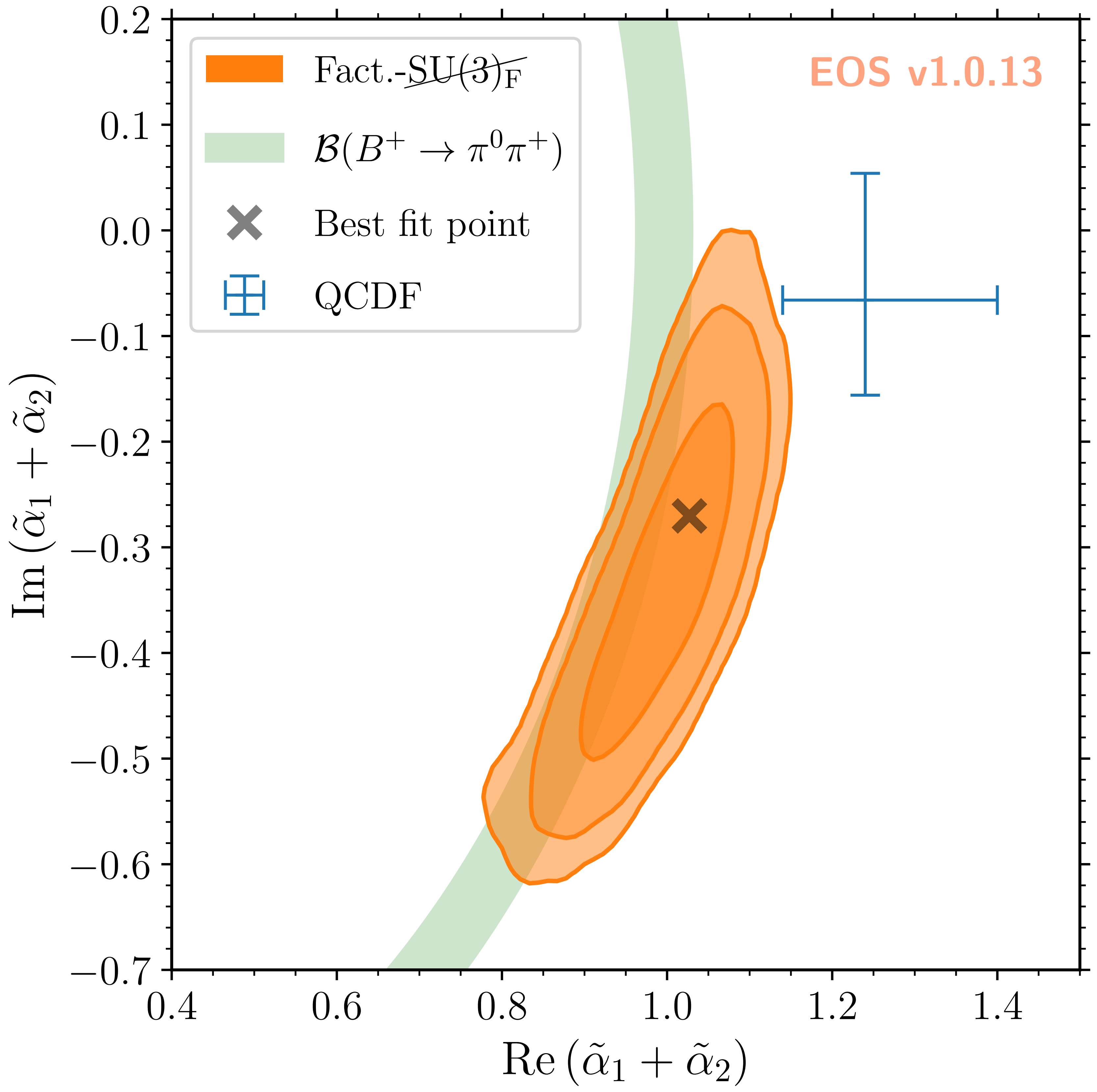
- Due to high correlations, only certain combinations can be determined with a reasonable uncertainty: **individual coefficients** are mostly **unconstrained**

→ $\tilde{\alpha}_1 + \tilde{\alpha}_2$



Fixing $\mathrm{Arg}[\tilde{\alpha}_1 + \tilde{\alpha}_4^u] = 0$

QCDF Point: Beneke, Huber, Xin-Qiang Li 0911.3655

Coefficient results: Fact. $SU(3)_F$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

EOS

- Due to high correlations, only certain combinations can be determined with a reasonable uncertainty: **individual coefficients** are mostly **unconstrained**

$$\Rightarrow \left|\frac{\alpha^c_{4,EW}}{\tilde{\alpha}_1}\right| \sim \left|\frac{\alpha^c_{3,EW}}{\tilde{\alpha}_2}\right| \sim \left|\frac{\tilde{\alpha}^c_4}{\tilde{\alpha}^u_4}\right| \sim [10^{-4}, 10^{-2}]$$

Data is consistent with small EW corrections!

Surprisingly $\tilde{\alpha}^c_4 \ll \tilde{\alpha}^u_4$ (QCDF prediction: $\tilde{\alpha}^c_4 \simeq \tilde{\alpha}^u_4$)

2111.15428

Coefficient results: Fact. $SU(3)_F$ breaking

Burgos Marcos, Reboud, Vos 2504.05209

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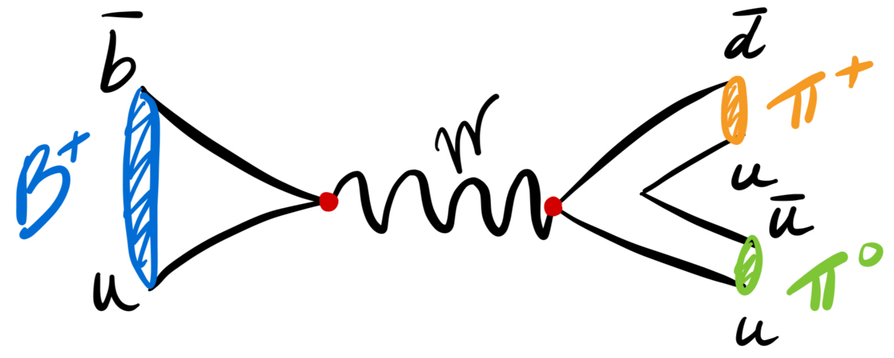
→ QCDF predictions for **annihilation** modes (b coefficients) are **model dependent**



→ In progress: detailed comparison with QCDF. Can we improve the modelling?

Conclusions

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- $SU(3)$ **flavor symmetry** is a useful approximation that allows us to make predictions on **hadronic decays**, for which we do not have a strict theoretical framework

- However, full $SU(3)_F$ **cannot describe experimental data successfully**

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- Factorizable $SU(3)_F$ breaking can be included with (almost) **no extra coefficients**
- In this approach, data can be explained **almost perfectly**, with a p-value of 31%.
- Certain relations ($\tilde{\alpha}_1 + \tilde{\alpha}_2$, $\left|\frac{\tilde{\alpha}_4^c}{\tilde{\alpha}_4^u}\right|$), do not agree with QCDF predictions

Conclusions

- $SU(3)$ **flavor symmetry** is a useful approximation that allows us to make predictions on **hadronic decays**, for which we do not have a strict theoretical framework
- However, full $SU(3)_F$ **cannot describe experimental data successfully**
- Factorizable $SU(3)_F$ breaking can be included with (almost) **no extra coefficients**
- In this approach, data can be explained **almost perfectly**, with a p-value of 31%.
- Certain relations ($\tilde{\alpha}_1 + \tilde{\alpha}_2, \left|\frac{\tilde{\alpha}_4^c}{\tilde{\alpha}_4^u}\right|$), do not agree with QCDF predictions
- Experimental updates in modes like $B^0 \to K^0\bar{K}^0$ or $B^0 \to \pi^0\pi^0$ could improve significantly our predictions and solve the strong correlations between coefficients
- Detailed comparison with QCDF and inclusion of η, η' mesons in progress

Backup

$\eta - \eta'$: Flavor vs. Mass basis

In addition to pions and kaons, the pseudo scalar meson spectrum also includes η_8 and η_1

$$\eta_8 = \frac{uu + dd - 2\cdot ss}{\sqrt{6}} \qquad \eta_1 = \frac{uu + dd + ss}{\sqrt{3}}$$

However, the mesons observed in experiments, η and η', are a mixture between these two

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \end{pmatrix}$$

$SU(3)$ limit in the $\eta - \eta'$ system

- In full $SU(3)$ flavor symmetry, $\theta = 0$ (therefore $\eta = \eta_8$ and $\eta' = \eta_1$) and η_8 meson is massless (like the pions and kaons.

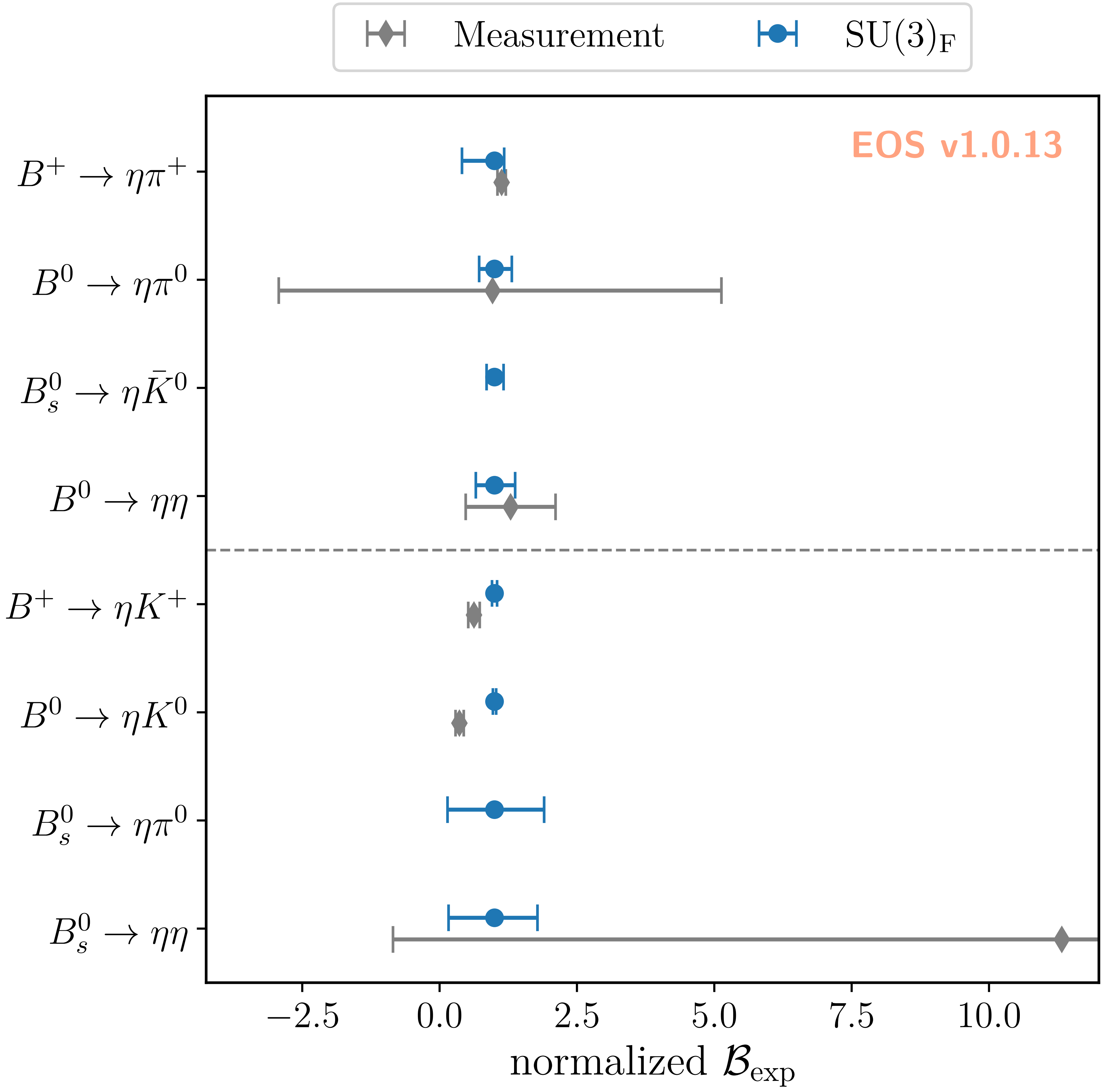
- This is **not** the case for η_1, which remains massive even if the masses of the u, d and s are zero.

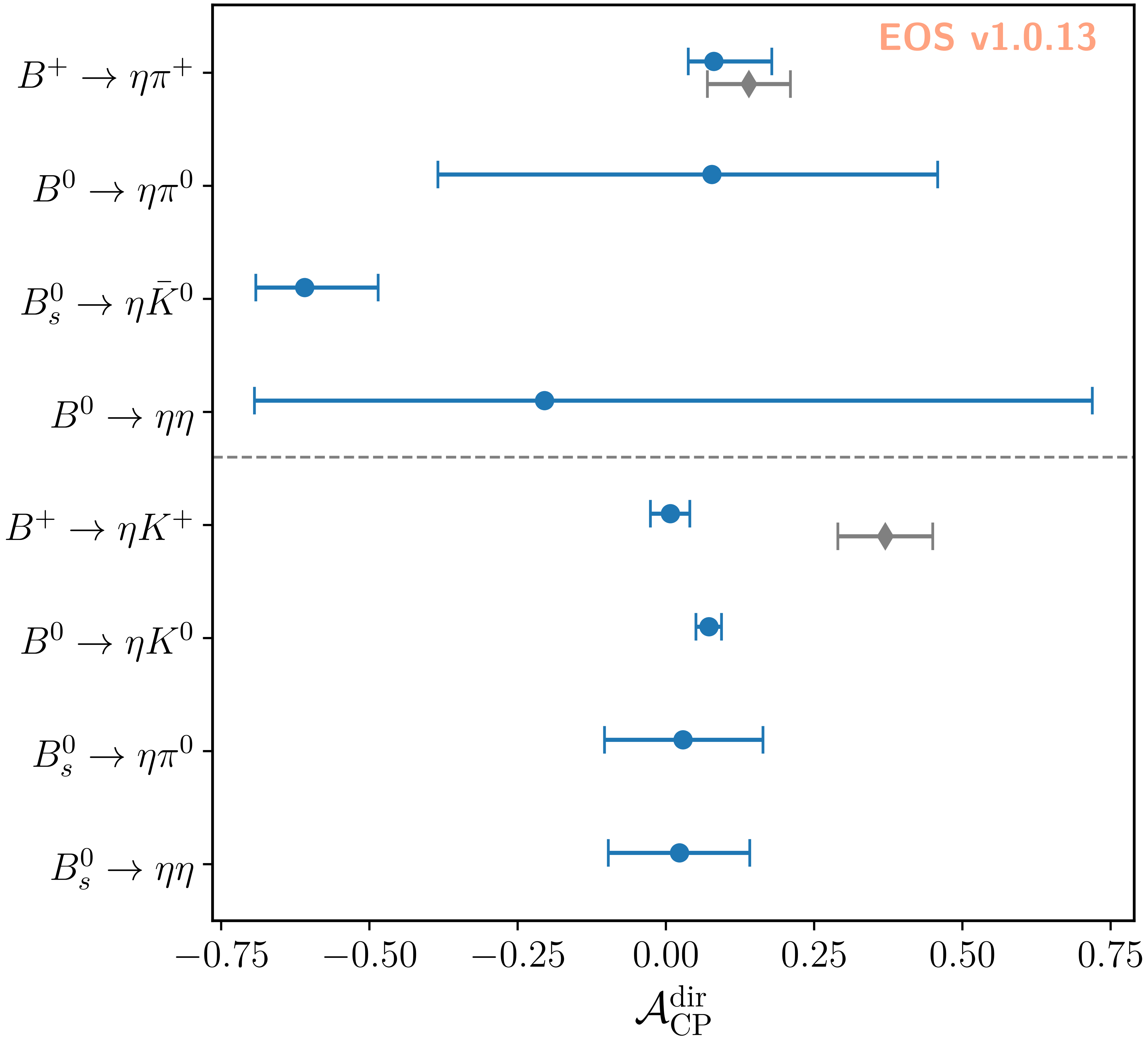
- In experiments, it is observed non-negligible mixing, $\theta \simeq -19°$: η receives contributions form η_1

- $SU(3)$ breaking is needed to describe the $\eta - \eta'$ system with a certain level of accuracy

- Fact. $SU(3)_F$ breaking in $B \to \eta h$ would require inclusion of singlet coefficients and decoupling of form factors and decay constant into singlet and octet contributions

$SU(3)_F$ postdictions for the η system





See 2311.18011

1. $SU(3)$ Flavor Symmetry

$q = u, d, s$

$D = d, s$

1. Fundamental representation of $SU(3)_F$, q:

$$\mathbf{3} = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

2. Transformation of each element in $\langle PP | \mathcal{O}_i | B \rangle$

- Light meson $P = q\bar{q}$: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$
- B meson $\bar{b}q$: $\mathbf{3}$
- Hamiltonian operators $\mathcal{O}_i$: $\bar{b} \to \bar{D}u\bar{u}$ and $\bar{b} \to \bar{D}$: $\bar{\mathbf{3}}, \mathbf{6}$ and $\overline{\mathbf{15}}$

How do we construct a matrix element $\langle PP | \mathcal{O}_i | B \rangle$ using $SU(3)_F$?

3. Representation of final state:

$$\langle PP |$$

$$(\mathbf{8} \otimes \mathbf{8})_s = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}$$

$$\mathcal{O}_i | B \rangle$$

- $\bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}_1$
- $\mathbf{6} \otimes \mathbf{3} = \mathbf{8}_2 \oplus \cancel{\mathbf{10}}$
- $\overline{\mathbf{15}} \otimes \mathbf{3} = \mathbf{8}_3 \oplus \cancel{\mathbf{10}} \oplus \mathbf{27}$

4. Independent parameters

5 components for "tree"-like, 5 for "penguin" like:

10 independent coefficients in any parameterisation

Fact. $SU(3)_F$ Coefficients

Beneke, Neubert 0308039,

$$\sum_{p=u,c} A_{M_1 M_2} \Big\{ \boldsymbol{B M_1} \left(\alpha_1 \boldsymbol{U}_p + \alpha_4^p + \alpha_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_2} \boldsymbol{\Lambda}_p$$
$$+ \boldsymbol{B M_1 \Lambda}_p \cdot \mathrm{Tr}\left[\left(\alpha_2 \boldsymbol{U}_p + \alpha_3^p + \alpha_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_2}\right]$$
$$+ \boldsymbol{B} \left(\beta_2 \boldsymbol{U}_p + \beta_3^p + \beta_{3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_1 M_2 \Lambda}_p$$
$$+ \boldsymbol{B \Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_1 \boldsymbol{U}_p + \beta_4^p + b_{4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_1 M_2}\right]$$
$$+ \boldsymbol{B} \left(\beta_{S2} \boldsymbol{U}_p + \beta_{S3}^p + \beta_{S3,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_1 \Lambda}_p \cdot \mathrm{Tr} \boldsymbol{M_2}$$
$$+ \boldsymbol{B \Lambda}_p \cdot \mathrm{Tr}\left[\left(\beta_{S1} \boldsymbol{U}_p + \beta_{S4}^p + b_{S4,\mathrm{EW}}^p \hat{\boldsymbol{Q}}\right) \boldsymbol{M_1}\right] \cdot \mathrm{Tr} \boldsymbol{M_2} \Big\}$$

The end-point divergencies can be parameterised like:

$$X_H^M = \int_0^1 \frac{dy}{1-y} \Phi_p^M = \left(1 + \rho_H e^{i\phi_H}\right) \ln \frac{m_B}{\Lambda_{QCD}}$$

Redefinitions in Fact. $SU(3)_F$

The parametrisation formula show redundancy in the number of coefficients:

$$\tilde{\alpha}_1 \equiv \alpha_1 + \frac{3}{2}\alpha^u_{4,EW}, \tilde{\alpha}_2 \equiv \alpha_2 + \frac{3}{2}\alpha^u_{3,EW}$$

$$\tilde{\beta}_1 \equiv \beta_1 + \frac{3}{2}\beta^u_{4,EW}, \tilde{\beta}_2 \equiv \beta_2 + \frac{3}{2}\beta^u_{3,EW},$$

$$\tilde{\alpha}^r_4 \equiv \alpha^r_4 + \beta^r_3 - \frac{1}{2}\left(\alpha^r_{4,EW} + \beta^r_{3,EW}\right), \tilde{\beta}^r_4 \equiv \beta^r_4 - \frac{1}{2}\beta^r_{4,EW},$$

Certain coefficients have been calculated up to NNLO in QCDF (See 0911.3655, 1507.03700)

$$\alpha_1 = 1.000^{+0.029}_{-0.069} + 0.011^{+0.023}_{-0.050}i$$

$$a^u_4 = -\left(2.46^{+0.49}_{-0.24} + 1.94^{+0.32}_{-0.20}i\right) \times 10^{-2}$$

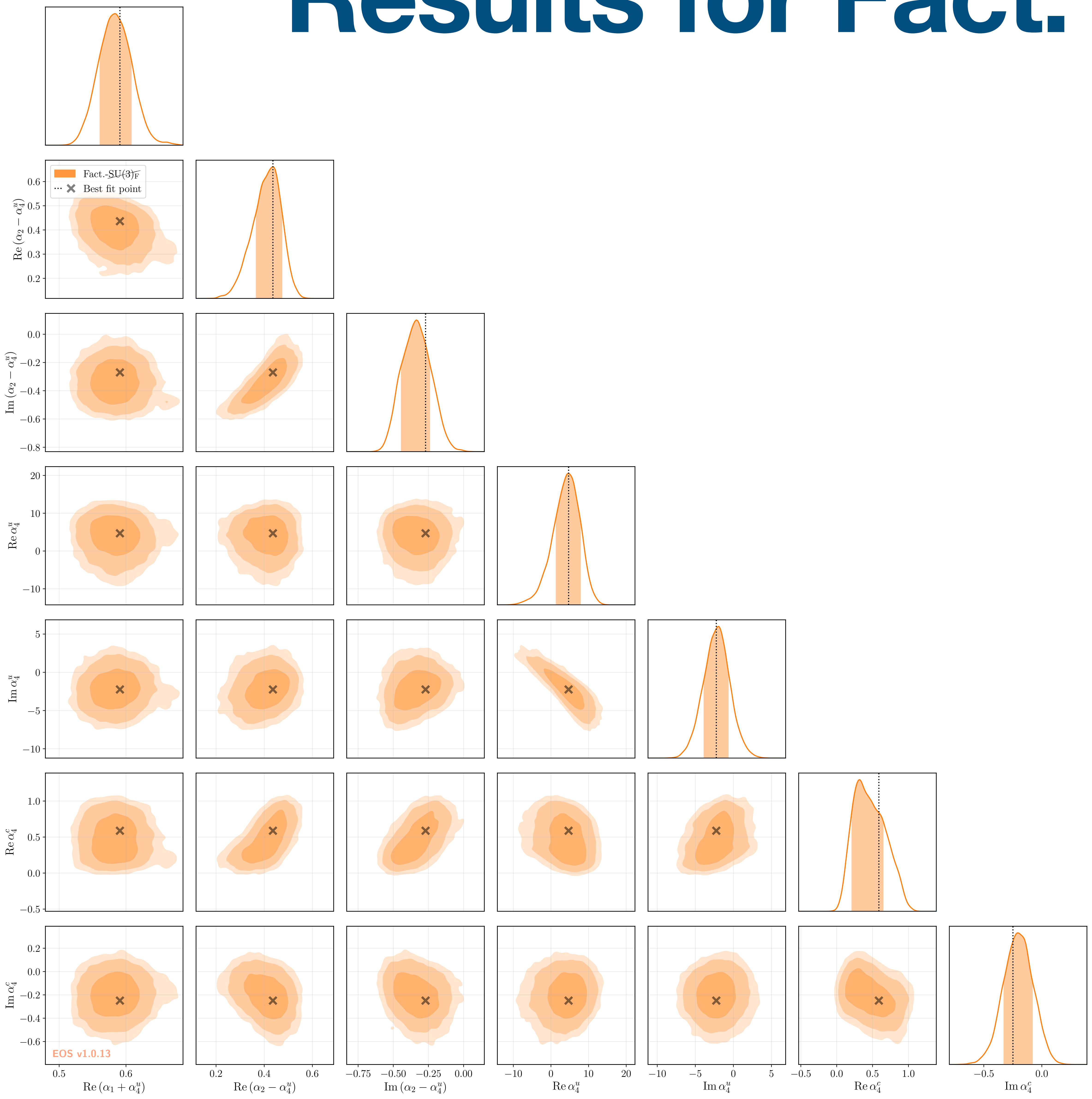
$$\alpha_2 = 0.240^{+0.217}_{-0.125} - 0.077^{+0.115}_{-0.078}i$$

$$a^c_4 = -\left(3.34^{+0.43}_{-0.27} + 1.05^{+0.45}_{-0.36}i\right) \times 10^{-2}$$

Fact. $SU(3)_F$ Coefficients

$b \to d$ **decay**	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_4^u$	$\tilde{b}_2$	$\tilde{b}_1$	$\tilde{b}_4^u$	$b \to s$ **decay**	$\tilde{\alpha}_1$	$\tilde{\alpha}_2$	$\tilde{\alpha}_4^u$	$\tilde{b}_2$	$\tilde{b}_1$	$\tilde{b}_4^u$
$B^+ \to \pi^0\pi^+$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0	$B^+ \to \pi^0 K^+$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	0
$B^+ \to \pi^+\pi^0$	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	$B^+ \to K^+\pi^0$	0	$\frac{1}{\sqrt{2}}$	0	0	0	0
$B^+ \to \bar{K}^0 K^+$	0	0	0	0	0	0	$B^+ \to K^0\pi^+$	0	0	0	0	0	0
$B^+ \to K^+\bar{K}^0$	0	0	1	1	0	0	$B^+ \to \pi^+ K^0$	0	0	1	1	0	0
$B^0 \to \pi^+\pi^-$	0	0	0	0	1	1	$B_s \to K^+K^-$	0	0	0	0	1	1
$B^0 \to \pi^-\pi^+$	1	0	1	0	0	1	$B_s \to K^-K^+$	1	0	1	0	0	1
$B^0 \to \pi^0\pi^0$	0	-1	1	0	1	2	$B_s \to \pi^0\pi^0$	0	0	0	0	1	2
$B^0 \to K^+K^-$	0	0	0	0	1	1	$B_s \to \pi^+\pi^-$	0	0	0	0	1	1
$B^0 \to K^-K^+$	0	0	0	0	0	1	$B_s \to \pi^-\pi^+$	0	0	0	0	0	1
$B^0 \to K^0\bar{K}^0$	0	0	1	0	0	1	$B_s \to \bar{K}^0 K^0$	0	0	1	0	0	1
$B^0 \to \bar{K}^0 K^0$	0	0	0	0	0	1	$B_s \to K^0\bar{K}^0$	0	0	0	0	0	1
$B_s \to \pi^+K^-$	0	0	0	0	0	0	$B^0 \to K^+\pi^-$	0	0	0	0	0	0
$B_s \to K^-\pi^+$	1	0	1	0	0	0	$B^0 \to \pi^-K^+$	1	0	1	0	0	0
$B_s \to \pi^0\bar{K}^0$	0	0	0	0	0	0	$B^0 \to \pi^0 K^0$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	0
$B_s \to \bar{K}^0\pi^0$	0	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	0	0	$B^0 \to K^0\pi^0$	0	$\frac{1}{\sqrt{2}}$	0	0	0	0

Results for Fact. $SU(3)_F$ coefficients



$$\begin{cases} \tilde{\alpha}_1 + \tilde{\alpha}_4^u = 0.584 \pm 0.24 \\ \dfrac{3}{2}\alpha^c_{4,\mathrm{EW}} + \tilde{\alpha}_4^c = -\left(0.102 \pm 0.001\right) + \left(0.044 \pm 0.002\right) i \end{cases}$$

$$\begin{cases} \tilde{\alpha}_2 - \tilde{\alpha}_4^u = \left(0.414^{+0.053}_{-0.065}\right) - \left(0.34 \pm 0.11\right) i \\ \dfrac{3}{2}\alpha^c_{3,EW} - \tilde{\alpha}_4^c = \left(0.141^{+0.031}_{-0.024}\right) - \left(0.059 \pm 0.010\right) i \end{cases}$$

$$\begin{cases} \tilde{b}_1 + 2\tilde{b}_4^u = -\left(3.7^{+9.6}_{-8.2}\right) + \left(13.7^{+6.9}_{-11.9}\right) i \\ \dfrac{3}{2}b^c_{4,EW} + 2\tilde{b}_4^c = -\left(1.5^{+1.4}_{-1.5}\right) + \left(11.0^{+4.6}_{-5.2}\right) i \end{cases}$$

Theoretical vs. Experimental Branching Ratio

Experimental Branching ratio is given by:

$$\langle\Gamma(B_s(t)\to f)\rangle \equiv \Gamma(B_s^0(t)\to f)+\Gamma(\bar{B}_s^0(t)\to f) = R_{\mathrm{H}}^f e^{-\Gamma_{\mathrm{H}}^{(s)}t}+R_{\mathrm{L}}^f e^{-\Gamma_{\mathrm{L}}^{(s)}t},$$

Theoretical Branching ratio is given by:

$$\langle\Gamma(B_s(t)\to f)\rangle\,|_{t=0} \equiv \Gamma(B_s^0\to f)+\Gamma(\bar{B}_s^0\to f)$$

Relation between Theoretical and Experimental BR (effects up to $\mathcal{O}(7\%)$):

$$\mathrm{BR}\left(B_s\to f\right)_{\mathrm{theo}} = \left[\frac{1-y_s^2}{1+\mathcal{A}_{\Delta\Gamma}^f\, y_s}\right]\mathrm{BR}\left(B_s\to f\right)_{\mathrm{exp}}$$

Role of interference in CP violation

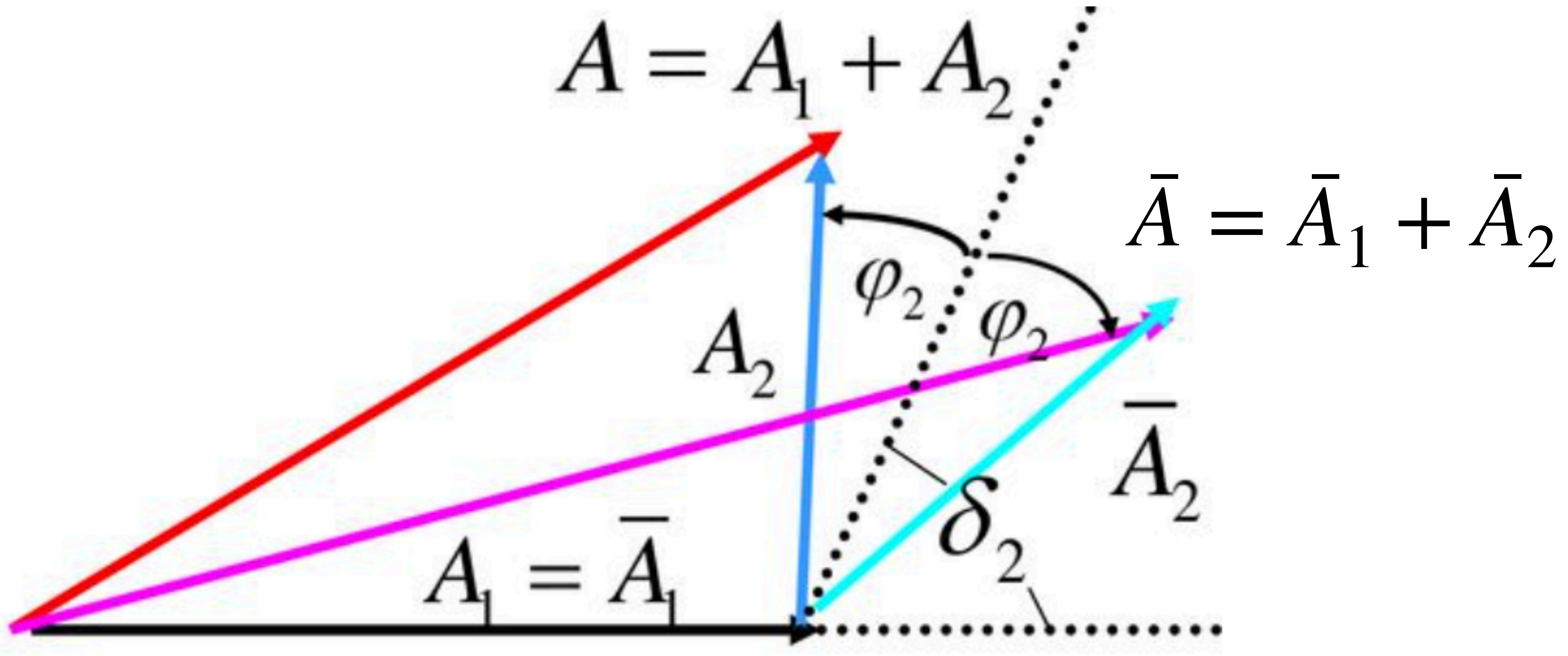
Consider the amplitude of a certain process $B \to f$. CP violation arises if for a certain observable $\mathcal{O}$ we have $\mathcal{O}(B \to f) \neq \mathcal{O}(\bar{B} \to \bar{f})$

Observables depend only on the **modulus** of the amplitude

Only weak phases are shifted when they are CP conjugated

We can define de amplitude of the process as $A = A_1 + A_2$

CP violation only appears if A_1 and A_2 have a relative weak phase, ϕ_2, and strong phase, δ_2



$$A_1 = |A_1|$$

$$A_2 = e^{i\delta_2}e^{i\phi}|A_2|$$