

QED CORRECTIONS TO EXCLUSIVE LEPTONIC B DECAYS

Belle II Germany meeting
Sep. 9, 2025, Bonn, Germany

Max Ferré
JGU Mainz

Based on arXiv:2212.14430 [hep-ph] and work in progress [hep-ph]
In collab. w/ C. Cornella, M. König and M. Neubert (JGU Mainz)



Motivations

- Why leptonic B decays ? $\mathcal{B}(B^- \rightarrow \ell^- \bar{\nu}_\ell) = \tau_B G_F^2 f_B^2 |V_{ub}|^2 \frac{m_B m_\ell^2}{8\pi} \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2$
- Direct determination of the CKM element $|V_{ub}|$
 - **Chirality-suppressed** in the SM $\rightarrow$ powerful probe of (pseudo) scalar new physics
 - Testing flavor universality in charged current : Belle II will measure the $\ell = \tau, \mu$ channels at 5 – 6 % [Belle II Physics Book]. FCC-ee prospects are promising.

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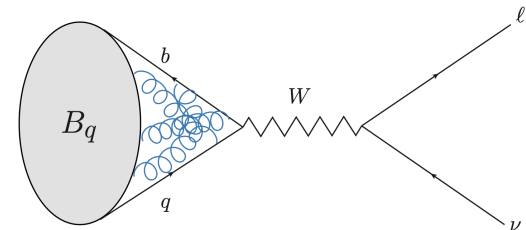
- Why QED corrections are needed?

- Pure hadronic effects are simple and well-understood:

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 b | B_q(p) \rangle = i f_{B_q} p^\mu$$

and f_{B_q} is known with $\mathcal{O}(1\%)$ precision : $f_{B_q} = 189.4 \pm 1.4 \text{ MeV}$ [FNAL/MILC 1712.09262]

- In the exclusive channel, with strong cuts on additional soft radiations, QED corrections can be **sizable** and compete with QCD uncertainties $\rightarrow$ need a **precise** estimation



QED corrections: Real and Virtual

- ▶ Electromagnetic corrections are sensitive to the lepton mass and the restriction on additional radiation, yielding large (double) logarithmic corrections

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- ▶ For a sufficiently tight cut on $E_\gamma^{\max}$, real emissions are **soft** enough to see the B -meson as a point like (pseudo) scalar $\rightarrow$ **eikonal approximation**
- ▶ Virtual corrections are unrestricted by such cuts, they live at higher scales and can resolve the partonic substructure of the B meson.

QED corrections: High and Low Scales

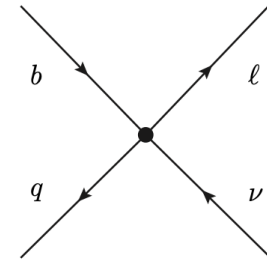
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- At energies harder than m_B , the weak effective Lagrangian captures all the hard corrections through the Wilson coefficients of the LEFT-operators,

$$\mathcal{L}_{\text{LEFT}} \supset -\frac{4 G_F^{(\mu)}}{\sqrt{2}} V_{ub} K_{\text{EW}}(\mu) \mathcal{O}_\ell^{V,LL} \longrightarrow$$



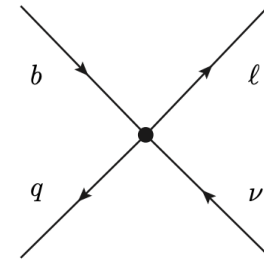
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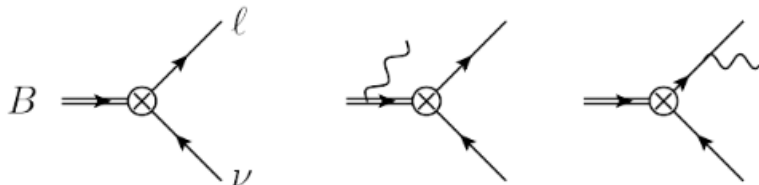
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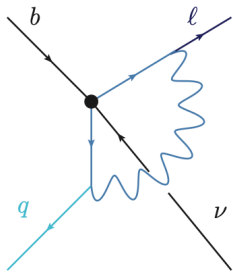
- Radiations softer than Λ_{QCD} sees the meson as point-like(*).



Effective Yukawa theory

QED corrections: Intermediate Scales

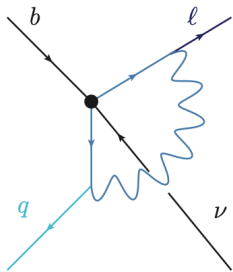
- Energetic virtual photons with $E_\gamma \sim m_B$ and $\vec{p}_\gamma \parallel \vec{p}_\ell$ can recoil against the **soft** spectator.



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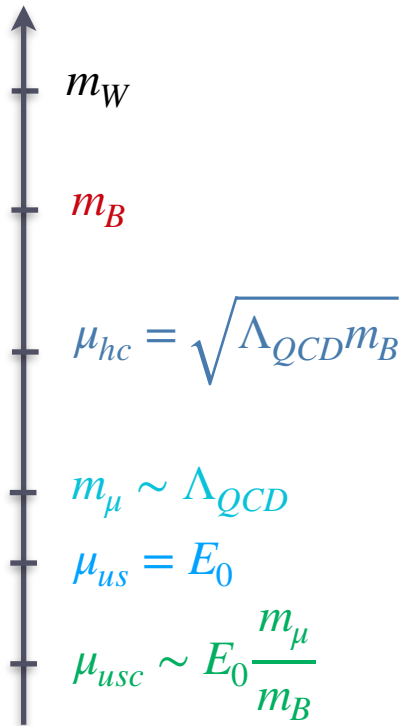
➔ momentum transfer at the intermediate virtuality $p_\gamma^2 \sim \Lambda_{QCD} m_B$

- ▶ The light partons are displaced on along the light cone, the hadronic **soft** currents become non-local → **Light-cone distributions** ($\phi_-(\omega), \phi_{3g}(\omega, \omega_g)$)
- ▶ When QED is switched on, these distributions are **no longer process-universal** → QED is sensitive to the direction of the **charged external states**

[Beneke et al (2108.05589)], [Beneke et al (2204.09091)]

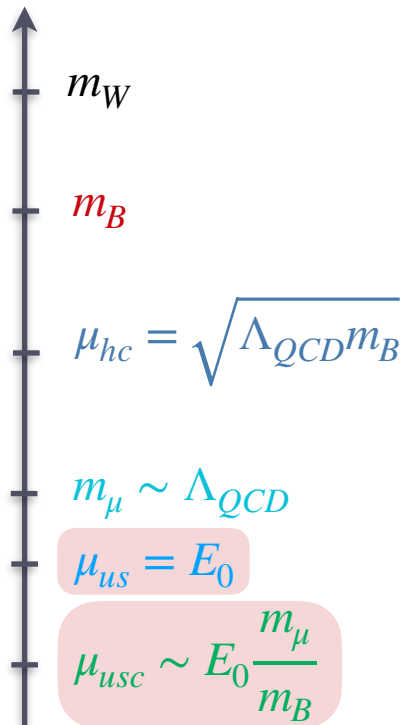
A multi-scale process

- Focusing on $\ell = \mu$ case, including QED effects introduce new scales (both static and dynamic) to which $B^- \rightarrow \mu^- \bar{\nu}_\mu$ is sensitive :



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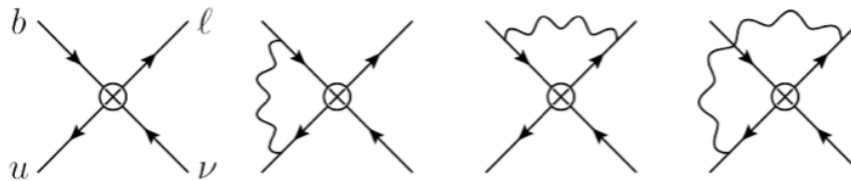
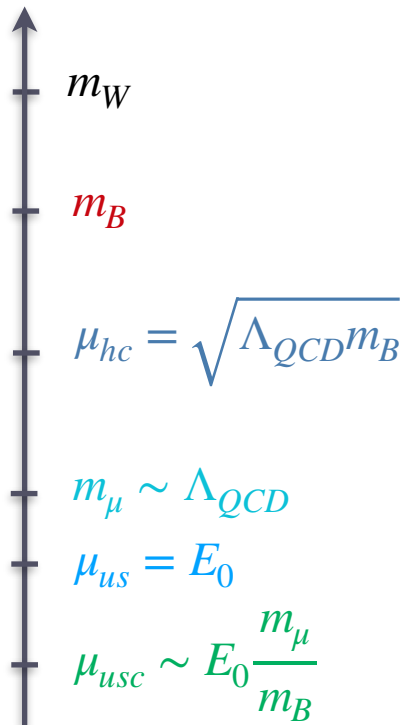
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Static scales dictated by the energy cut on the additional final radiation

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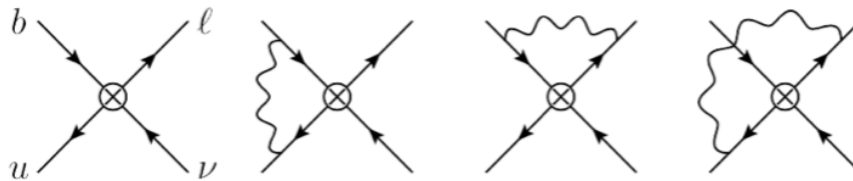
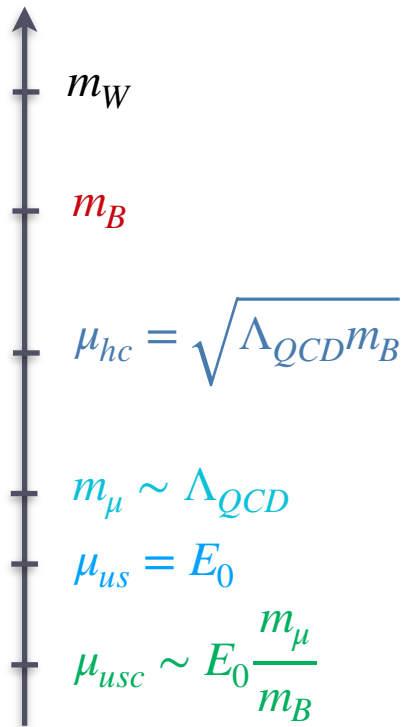
B-meson described as a superposition of Fock-states: $|\bar{u}b\rangle \oplus |\bar{u}bg\rangle \oplus \dots$

[Beneke et al(2019),JHEP10232]



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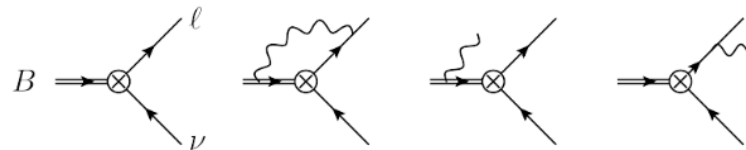
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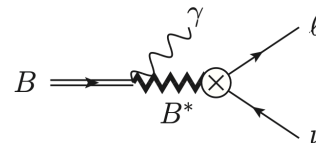
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B-meson described as a point-like pseudo-scalar boson...



[Dai et al (2022), Phys.Rev.D 105 3]

...(*)including its first vector excited state as off-shell intermediate propagator !



[Becirevic et al (0907.1845)]

The plan : running down with the scale !

Turning a multi-scale problem to a product of single scale objects by :

- ▶ Identifying the **appropriate effective description** at each scale.
- ▶ Performing a step-by-step **matching** between each EFT.
- ▶ Deriving a **factorization theorem** to break this multi-scale problem into a convolution of single-scale objects.
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In this talk we are going to embark on an EFT journey :

- ▶ Discussing the **(re)factorization** of the **virtual** amplitude in the partonic picture.
- ▶ Constructing the proper EFT valid at **low energy** to include the **real emissions**.
- ▶ Deriving the **factorization formula for the decay rate**.

LEFT $\rightarrow$ HQET $\otimes$ SCET_I

$$\mu \sim m_B$$

Power counting: $\lambda = \frac{m_\ell}{m_B} \sim \frac{\Lambda_{QCD}}{m_B}$

Relevant scalings : $p^\mu = (\bar{n} \cdot p, n \cdot p, p_\perp)$

$$p \sim (1, \lambda^2, \lambda) \text{ « collinear »,}$$

$$p^2 \sim m_\ell^2 \sim \mathcal{O}(\lambda^2)$$

$\rightarrow$ given by the lepton virtuality

$$p \sim (1, \lambda, \sqrt{\lambda}) \text{ « hard-collinear »,}$$

$$p^2 \sim \Lambda_{QCD} m_B \sim \mathcal{O}(\lambda)$$

$\rightarrow$ soft and collinear quark X-talk

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$$\ell(x) \rightarrow \left(1 + \frac{1}{i\bar{n} \cdot D_s} (i \not{D}_\perp + m_\ell) \frac{\not{n}}{2} \right) \xi_C^{(\ell)}(x) + \left(1 + \frac{1}{i\bar{n} \cdot D_s} e Q_q A_{C\perp} \frac{\not{n}}{2} \right) \ell_s(x)$$

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SCET_I operators

$$\mathcal{O}_{\ell}^{V,LL} = H_i^A O_i^A + \int_0^1 dy \left[H_i^B(y) O_i^B(y) + H_i^C(y) O_i^C(y) \right]$$

We build our SCET_I basis with the following power counting :

$$h_v, q_s \sim \mathcal{O}(\lambda^{3/2}) \quad \chi_C \sim \mathcal{O}(\lambda^{1/2}) \quad \mathcal{A}_C^\perp \sim \mathcal{O}(\lambda^{1/2}) \quad \chi_c \sim \mathcal{O}(\lambda) \quad \mathcal{A}_c^\perp \sim \mathcal{O}(\lambda) \quad \chi_{\bar{c}} \sim \mathcal{O}(\lambda)$$

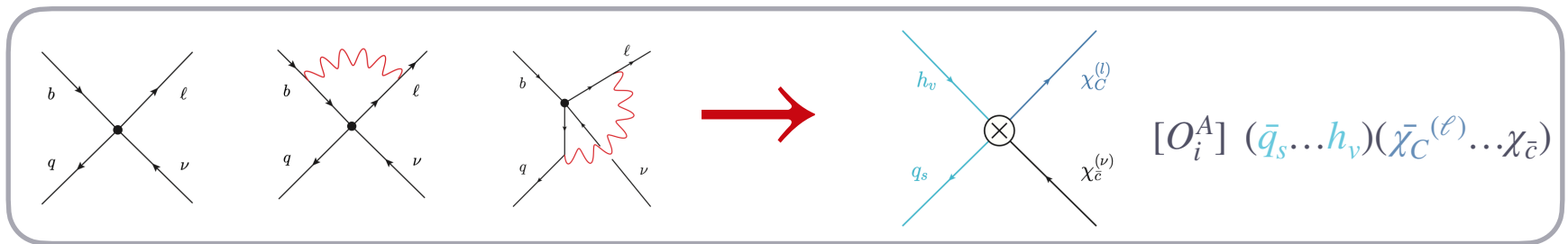
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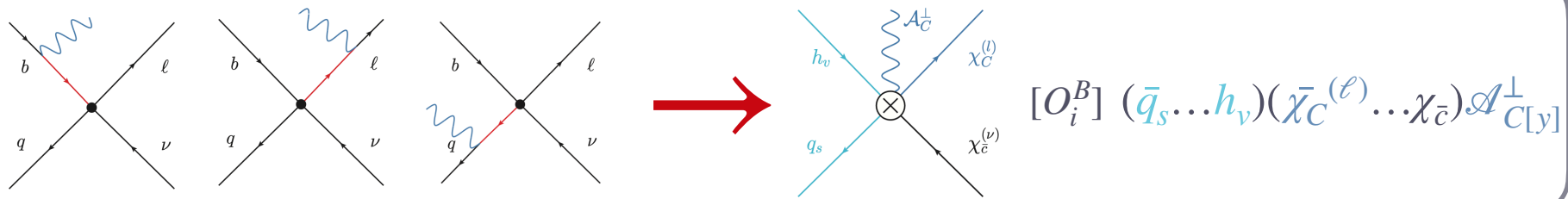
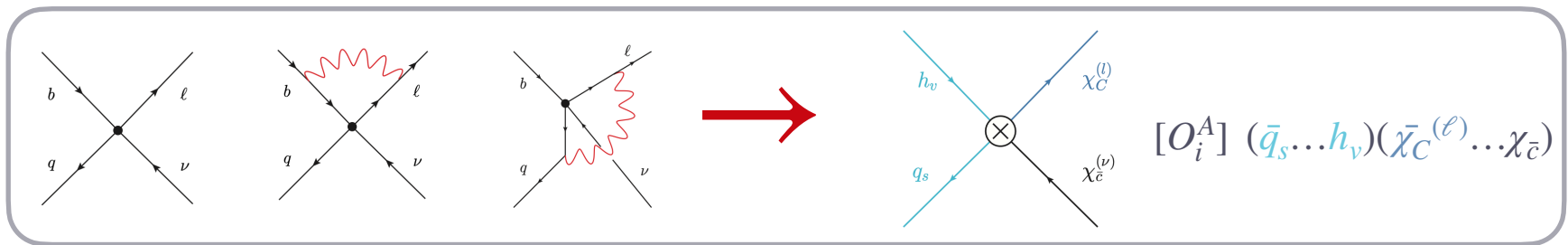


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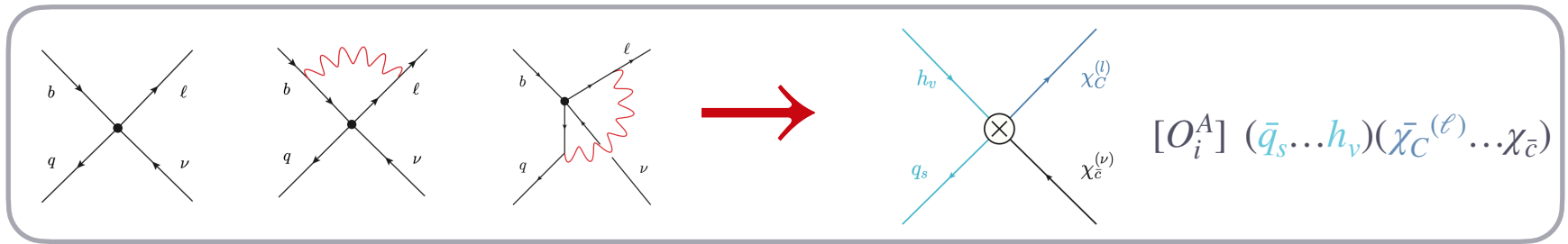


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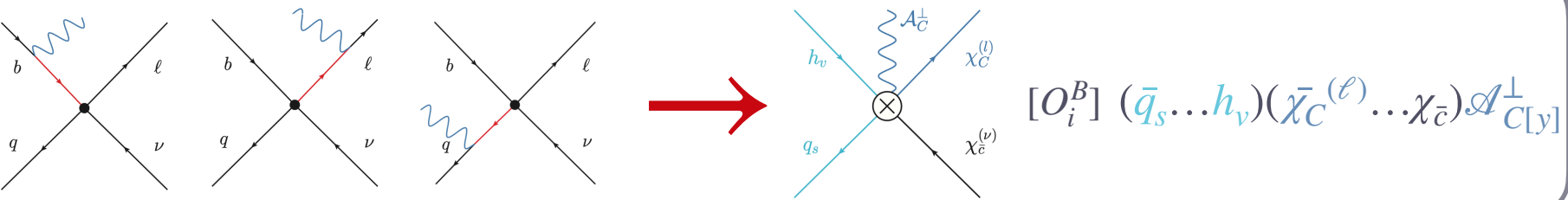
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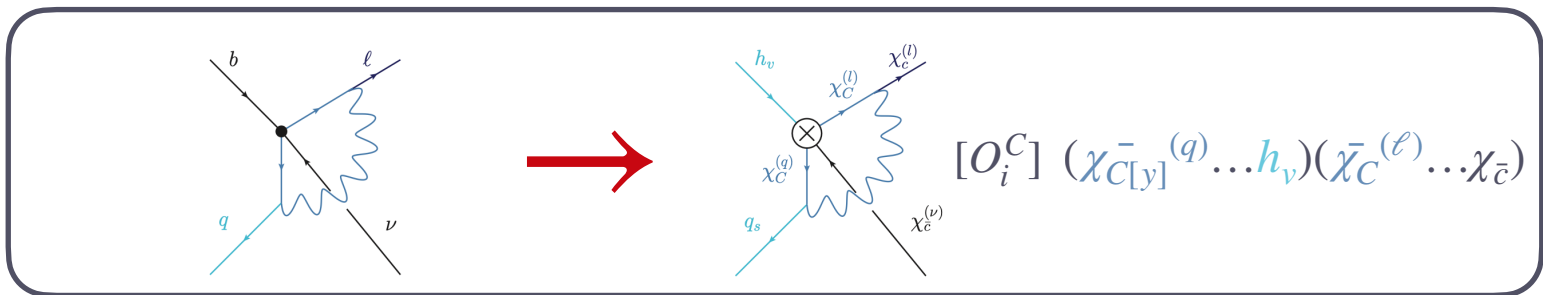
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$$[O_i^A] (\bar{q}_s \dots h_v) (\bar{\chi}_C^{(\ell)} \dots \chi_{\bar{c}})$$



$$[O_i^B] (\bar{q}_s \dots h_v) (\bar{\chi}_C^{(\ell)} \dots \chi_{\bar{c}}) \mathcal{A}_{C[y]}^\perp$$



$$[O_i^C] (\chi_{\bar{C}[y]}^{(q)} \dots h_v) (\bar{\chi}_C^{(\ell)} \dots \chi_{\bar{c}})$$

$$\text{SCET}_I \rightarrow \text{SCET}_{II} \quad \mu \sim \mu_{hc} \sim \sqrt{\Lambda_{QCD} m_B}$$

- At $\mu \sim \mu_{hc}$, we lower the virtuality removing **hard-collinear** modes $\rightarrow$ pure SCET_{II} construction where **collinear** and **soft** carry the same virtuality :

$$p_c \sim (1, \lambda^2, \lambda), \quad p_s \sim (\lambda, \lambda, \lambda), \quad p_c^2 \sim p_s^2 \sim \mathcal{O}(\lambda^2)$$

- Integrating out **hard-collinear** propagators introduces **non-localities** even in **soft** product :

$$\frac{1}{n \cdot \partial} q_s, \left(\frac{1}{n \cdot \partial} \mathcal{G}_s^\perp \right) \left(\frac{1}{n \cdot \partial} q_s \right), \dots$$

$\rightarrow$ Contains more fields but are of the same order !

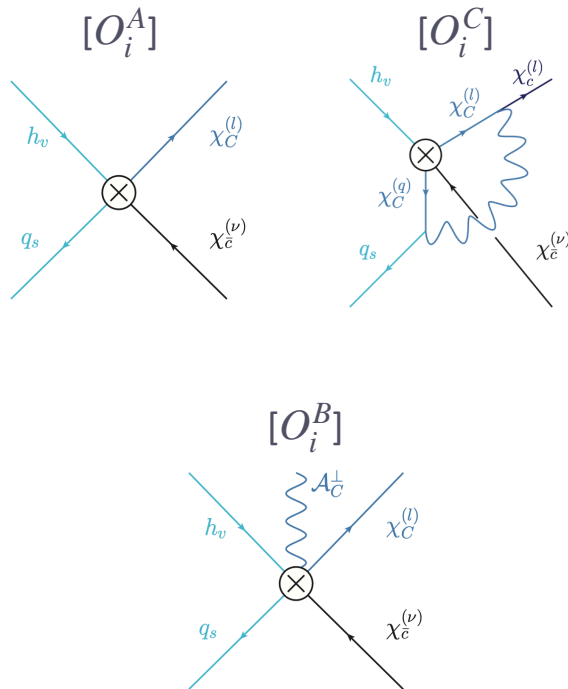
- Contrary to the $B_s \rightarrow \mu^+ \mu^-$ case, those structure-dependent contributions to $B^- \rightarrow \mu^- \bar{\nu}_\mu$ carry the same suppression as the tree level result and do not overcome the chiral suppression.

[Beneke, Bobeth, Szafron 2017, 2019]

SCET_{II} basis

$$O_i^X(\{y\}) = \sum_{j,X'} J_{O_i^X \rightarrow Q_j^{X'}}(\{y\}, \{x\}, \{\omega\}) \otimes_{x,\omega} Q_j^{X'}(\{x\}, \{\omega\})$$

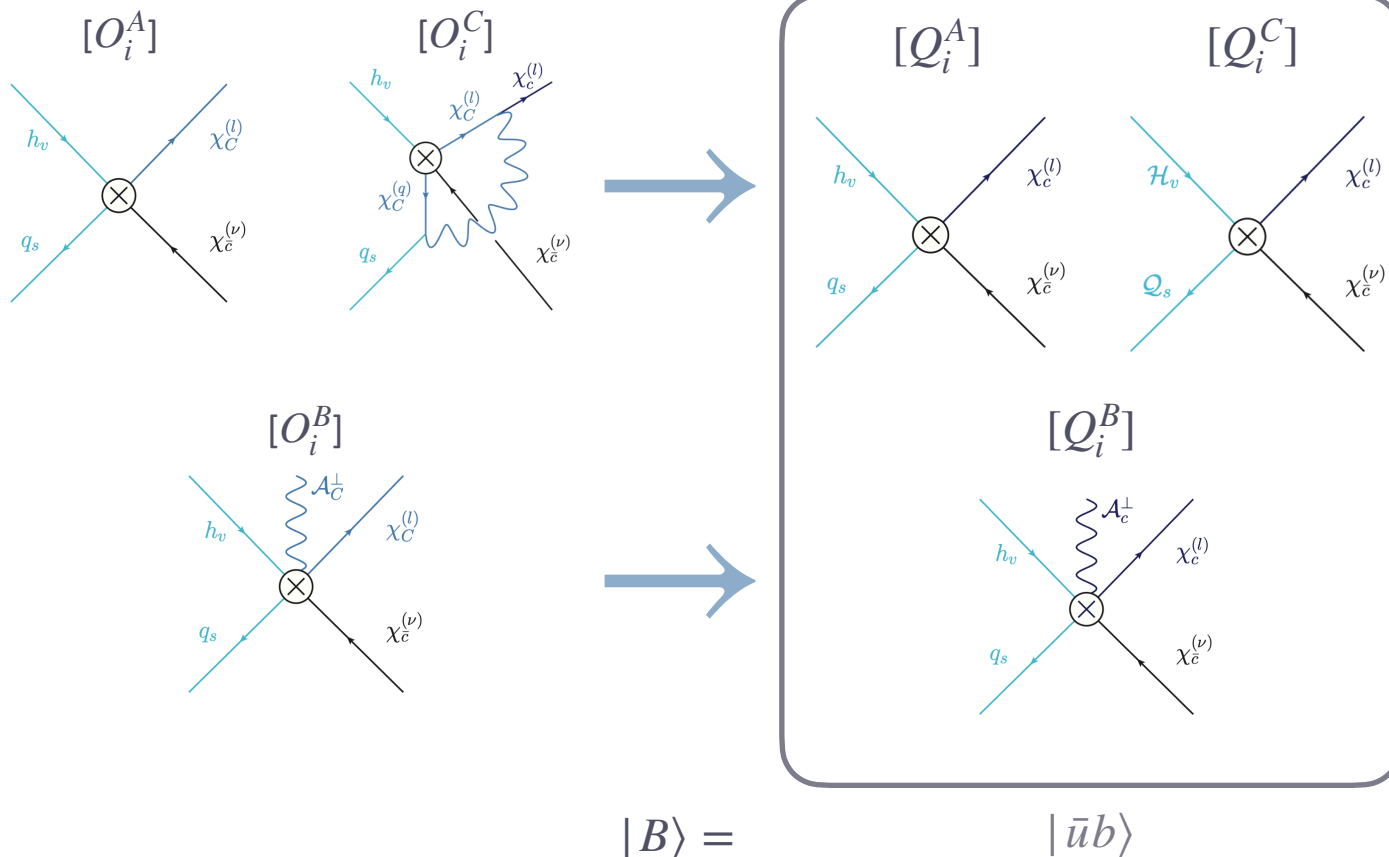
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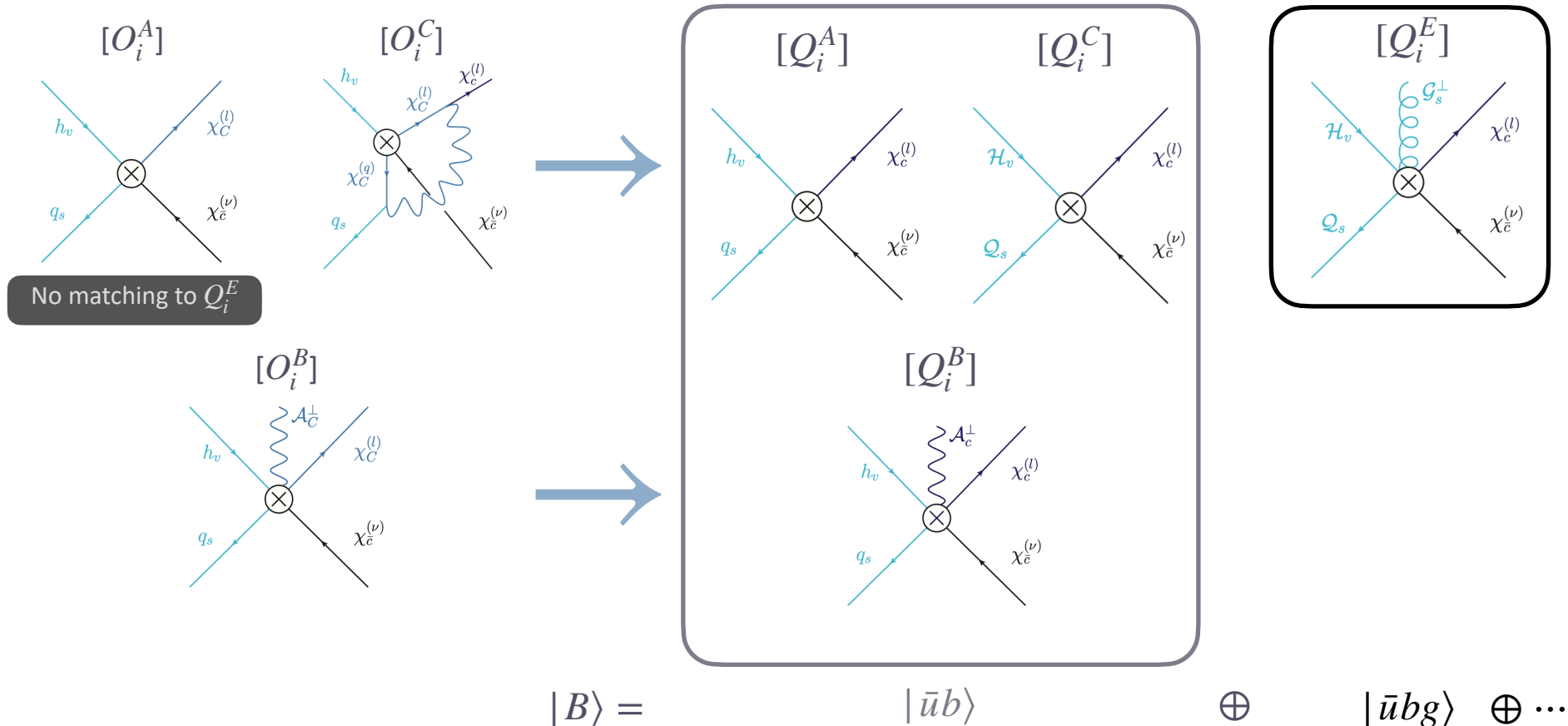
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Refactorization

- The matrix element of C-type SCET I operators involve an **endpoint-divergent** integral → the theory fails to properly separate **hard-collinear modes** with low energy (hard-collinear fraction $y \sim \mathcal{O}(\lambda)$) from **soft modes**.

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- ▶ This region can be identified with the energetic limit of the soft region → we rearrange terms between the two (**RBS scheme**) [Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang 2020; Liu, Neubert, Schnubel, Wang 2022; Beneke et al. 2022]

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$$H_1^C(y) J_{O_1^C \rightarrow Q_1^C}(y, \omega, \Lambda) = H_1^C(y) J_{O_1^C \rightarrow Q_1^C}^{bare}(y, \omega)$$

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$$0 < \eta = \Lambda/m_B \leq 1 : \text{arbitrary scale}$$

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[Cornella, König, Neubert 2023]

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New hadronic parameter !

[Cornella, König, Neubert 2023]

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Factorization formula (virtual corrections)

- Taking SCET_{II} matrix elements, we obtain a factorisation theorem for the virtual corrections :

$$\begin{aligned}\mathcal{A}_{\text{virt}} &= \langle \ell \nu | \mathcal{L}_{\text{SCET II} \otimes \text{HQET}} | B \rangle \\ &= \sum_{i,X} H_i^X(\{y\}) \otimes_y \sum_{j,X'} J_{O_i^X \rightarrow Q_j^{X'}}(\{y\}, \{x\}, \{\omega\}) \otimes_{x,\omega} \langle Q_j^{X'}(\{x\}, \{\omega\}) \rangle\end{aligned}$$

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- SCET_{II} operators factorizes into non perturbative **soft hadronic contributions** and purely leptonic perturbative collinear functions :

$$\langle Q_i^{X'}(\{x\}, \{\omega\}) \rangle = S_i^{X'}(\{\omega\}) \underbrace{\left\langle B \left| Y_n^{(\ell)\dagger} \frac{\phi_\nu}{\sqrt{2m_B}} \right| 0 \right\rangle}_{R^{(\ell,B)}} K_i^{X'}(\{x\}) \frac{m_\ell}{m_B} \langle \ell \nu | \bar{h}_n P_L \nu_{\bar{c}} | 0 \rangle$$

- They define matching coefficients to the low energy theory hadronic and leptonic currents.

Virtual amplitude at NLO QED

$$\mathcal{A}_{\text{virtual}} = i\sqrt{2} G_F K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_B} \sqrt{m_B} F_-(\Lambda, \mu) \bar{u}(p_\ell) P_L v(p_\nu) \sum_j \mathcal{M}_j(\mu),$$

$$\begin{aligned} \mathcal{M}_{2p}(\mu) = & 1 + \frac{C_F \alpha_s}{4\pi} \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right] \\ & + \frac{\alpha}{4\pi} \left\{ Q_b^2 \left[-\frac{3}{2} \ln \frac{\mu^2}{m_b^2} - 2 \right] \right. \\ & - Q_\ell^2 \left[\frac{1}{\epsilon_{\text{IR}}} \left(2 + \ln \frac{m_\ell^2}{m_B^2} \right) - \frac{1}{2} \ln^2 \frac{\mu^2}{m_\ell^2} + \frac{3}{2} \ln \frac{\mu^2}{m_\ell^2} - \frac{\pi^2}{12} + 2 \right] \\ & - Q_b Q_\ell \left[\frac{1}{2} \ln^2 \frac{\mu^2}{m_B^2} - 2 \ln \frac{\mu^2}{m_B^2} + \ln \frac{\mu^2}{m_\ell^2} - 1 + \frac{\pi^2}{12} \right] \\ & \left. + Q_\ell Q_u \left[-5 - 2 \ln \frac{\mu^2}{m_\ell^2} + \int_0^\infty d\omega \phi_-(\omega, \mu) \left(\ln \frac{\mu^2}{\omega \sqrt{m_B} \Lambda} \ln \frac{\Lambda^2}{m_B^2} - 2 \ln \frac{\mu^2}{\omega \sqrt{m_B} \Lambda} + \ln \frac{\Lambda}{m_B} - 2 - \frac{\pi^2}{3} \right) \right] \right\}, \end{aligned}$$

$$\mathcal{M}_{3p}(\mu) = -\frac{\alpha}{\pi} Q_\ell Q_u \left(1 + \ln \frac{\Lambda}{m_B} \right) \times \int_0^\infty d\omega \int_0^\infty d\omega_g \frac{\phi_3(\omega, \omega_g)}{\omega_g} \left[\frac{1}{\omega_g} \ln \left(1 + \frac{\omega_g}{\omega} \right) - \frac{1}{\omega + \omega_g} \right].$$

Low-energy theory : bHLET $\otimes$ HSET $\mu < \Lambda_{QCD} \sim m_\mu$

- Below $\mu \sim \Lambda_{QCD}$, quarks **hadronize** and the meson can be described by a charged scalar of mass m_B . We can describe it with a **heavy scalar (HS)** :

$$\Phi_B(x) \rightarrow \frac{e^{-im_B(v \cdot x)}}{\sqrt{2m_B}} \varphi_v(x)$$

- Since $\Lambda_{QCD} \sim m_\mu$, also the muon becomes infinitely heavy. We describe it as a **boosted heavy lepton field (bHL)** :

$$\chi_c^{(\ell)}(x) = e^{-im_\ell(v_\ell \cdot x)} h_n(x)$$

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- We match the resulting low energy EFT by taking the hadronic matrix element :

$$\langle \ell \nu | \mathcal{L}_{SCET_H \otimes HQET} | B \rangle = \langle \ell \nu | \mathcal{L}_{bHLET \otimes HSET} | B \rangle$$



A theory of Wilson lines ?

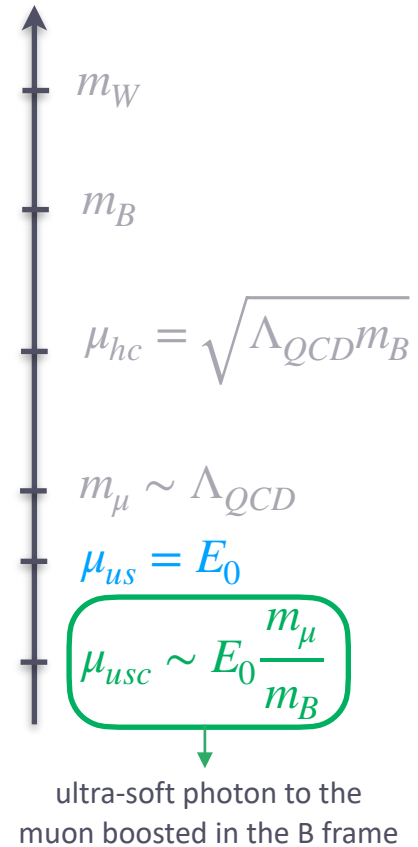
- We can decouple the interactions of the B and the muon with **ultrasoft** and **ultrasoft-collinear** photons :

$$Y_n^{(\ell)\dagger}(x_-)\varphi_v(x) = Y_n^{(\ell)\dagger}(x_-)Y_v^{(B)}(x)C_{\tilde{n}}^{(B)}(x_+)\varphi_v^0(x) \quad h_n(x) = C_{v_\ell}^{(\ell)}(x)h_n^0(x)$$

$$Y_r^{(i)}(x) = \mathcal{P} \exp \left\{ ieQ_i \int_{-\infty}^0 ds r \cdot A_{us}(x+sr) \right\} \quad C_r^{(i)}(x) = \mathcal{P} \exp \left\{ ieQ_i \int_{-\infty}^0 ds r \cdot A_{usc}(x+sr) \right\}$$

- Real corrections can be expressed as matrix elements of these Wilson lines, convoluted with the **measurement function** implementing the experimental radiation veto

$$R(E_0, \mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc} \theta\left(\frac{E_0}{2} - \omega_{us} - \omega_{usc}\right) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$



Factorization formula...

- ▶ Using this framework, we can write the **factorization formula** for $B^- \rightarrow \mu^- \bar{\nu}_\mu$ at the level of the decay rate.
- ▶ **Real emissions** are factorized at the level of the **decay rate**. **Ultrasoft/ultrasoft-collinear** logarithms can be **resummed** in Laplace space ($\sim \mathcal{O}(10\%)$ corrections).
- ▶ The **virtual corrections** already factorize at the **amplitude level** and appear as an effective **Yukawa coupling** containing the **hard/hard-collinear** logarithms (which could be resummed but lead to a $\sim \mathcal{O}(1\%)$ correction).

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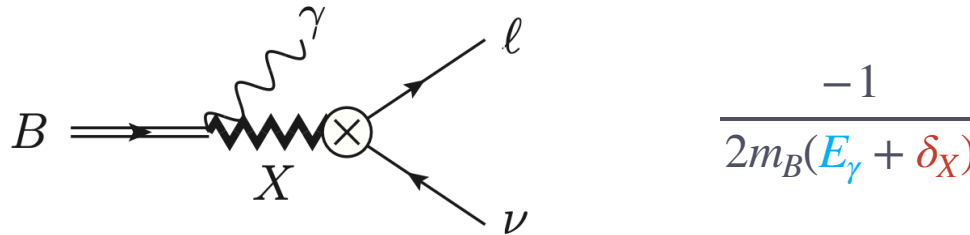
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- Below Λ_{QCD} , the initial B meson can emit a photon and transition into an excited state X :

[Becirevic et al (0907.1845)]



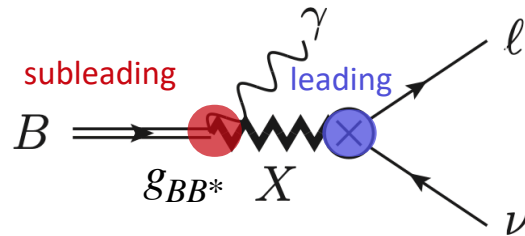
- For $E_0 \lesssim \Lambda_{QCD}$, only the B^* needs to be included (**heavy-spin symmetry**), contributions from higher states being power-suppressed → **Pole-dominance approximation**

$$\delta_X = \frac{m_X^2 - m_B^2}{2m_B} \lesssim E_\gamma^{\max} = \frac{E_0}{2} \quad \delta_{B^*} \sim \mathcal{O}(\lambda^2) \quad \delta_{B_1'}, \delta_{B_2^*} \sim \mathcal{O}(\lambda)$$

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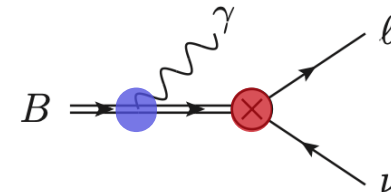
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- The $B \rightarrow B^*\gamma$ interaction carries the **same power suppression** as the effective **Yukawa**.



- g_{BB^*} is **unobserved**, has to be estimated by a mixture of **QCD sum rules**, **quark models** and **lattice QCD**.

Factorization formula.... now complete !

$$\Gamma = \Gamma_0 \left(\underbrace{|y_B(\mu)|^2}_{\text{Non-radiative}} \underbrace{R(E_0, \mu)}_{\text{Radiative}} + \underbrace{|y_B^*|^2 R_{B^*}(E_0)}_{\text{Structure-dependent radiative corrections}} \right)$$

Non-radiative Radiative Structure-dependent radiative corrections

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for various values of $E_\gamma^{max} = E_0/2$:

Tree level: $\mathcal{B}_0(B^- \rightarrow \mu^- \bar{\nu}_\mu) = (3.71 \pm 0.29_{V_{ub}} \pm 0.05_{f_B}) \cdot 10^{-7}.$

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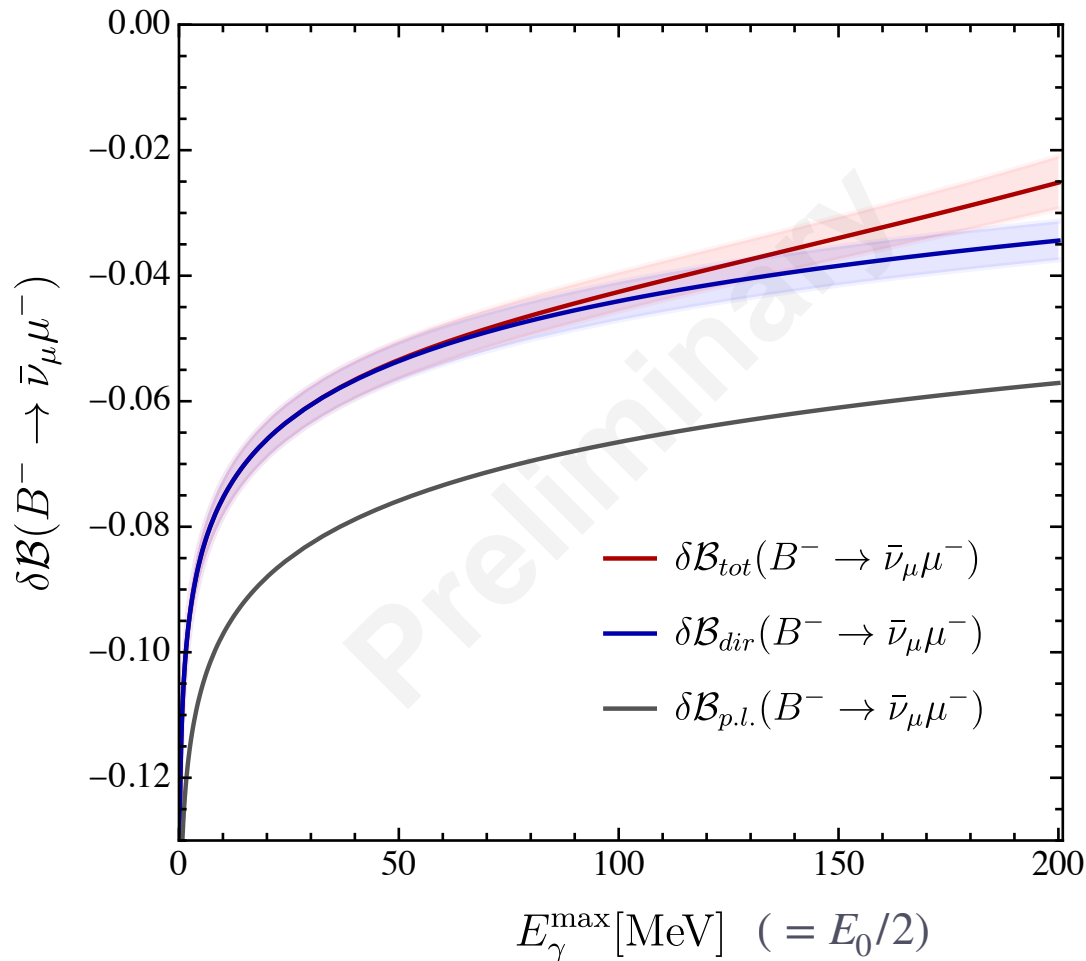
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Precision for the extraction of V_{ub}

$$\mathcal{B} = \mathcal{B}_0 \left(|y_B(\mu)|^2 R(E_0, \mu) + |y_B^*|^2 R_{B^*}(E_0) \right)$$

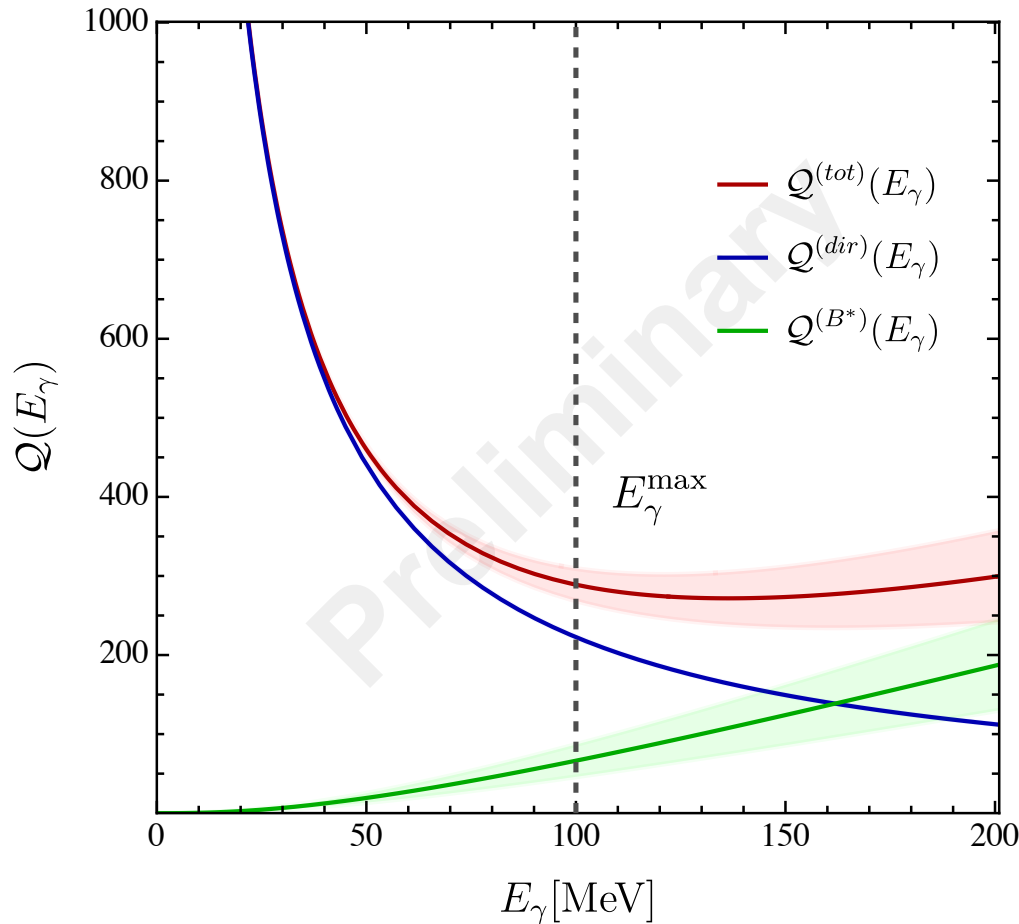


$$R(E_0, \mu) \sim \left(\frac{E_0^2 m_\ell}{\mu^2 m_B} \right)^{\frac{\alpha Q_B^2}{2\pi}} \left(\ln \frac{m_B^2}{m_\ell^2} - 2 \right)$$

$$R_{B^*}(E_0) \sim E_0^2$$

Fully inclusive differential rate

$$Q^{(A)}(E_\gamma) = \frac{4\pi}{\alpha\mathcal{B}_0} \frac{d\mathcal{B}^{(A)}(E_\gamma)}{dE_\gamma}, \quad A = \{\text{tot}, \text{dir}, B^*, \}.$$



$$Q^{(dir)}(E_\gamma) \sim E_\gamma^{-1 + \frac{\alpha Q_B^2}{\pi} \left(\ln \frac{m_B^2}{m_\ell^2} - 2 \right)}$$

$$Q^{(B^*)}(E_\gamma) \sim \frac{E_\gamma^3}{(E_\gamma + \delta_{B^*})^2}$$

Conclusions

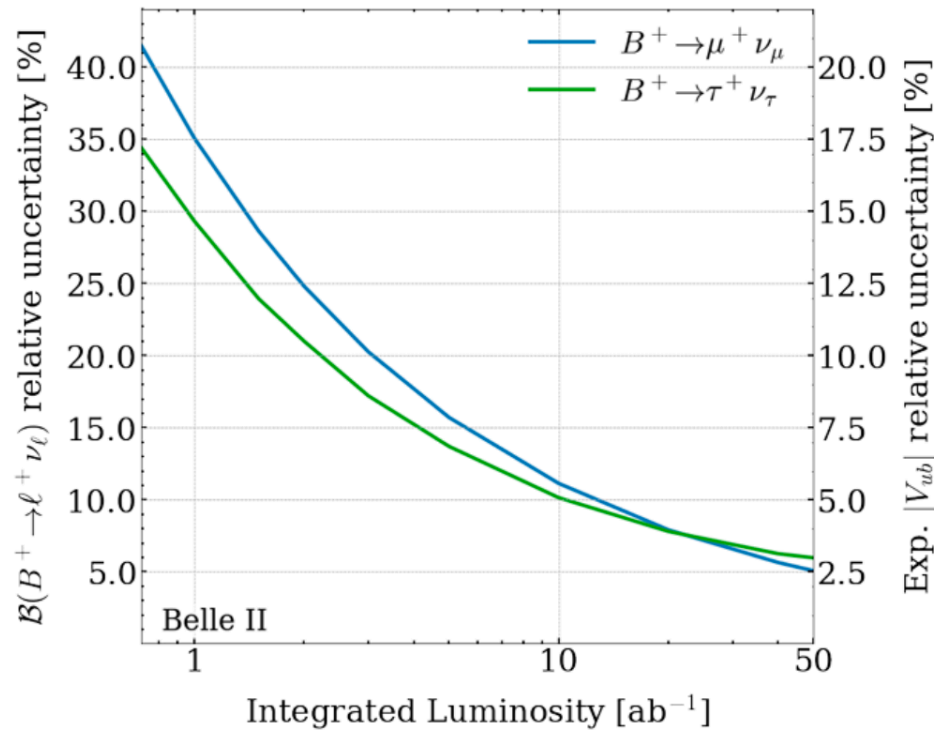
- ▶ The **exclusive leptonic decay** $B^- \rightarrow \mu^- \bar{\nu}_\mu$, is an important channel for test of physics in **SM and beyond**. Since future experimental accuracy calls for percent precision in the theory prediction, QED corrections are needed.
- ▶ On top of **large logarithms** of lepton mass and photon cuts, we find (single logarithmic) **structure-dependent** effects in the virtual & real corrections:
 - virtual: **hard-collinear** photons between the **lepton** and light **spectator quark**
 - real: B^* contribution; gets **more important** for a looser cut.
- ▶ These corrections introduce **new uncertainties**: they probe the inner structure of the B meson & introduce **new hadronic parameters** which need to be estimated (Sum rules, lattice determination ?)
- ▶ These EFT methods can be adapted to tackle **other leptonic decays** ($D, D_s, \dots$) and need to be extended for **semi-leptonic channels** ($B \rightarrow \pi \ell \nu, B \rightarrow D \ell \nu, \dots$), crucial for the extraction of $|V_{ub}|$ and $|V_{cb}|$.

Thanks for listening !



Backup-slides

Belle II projections



[Belle II Physics Book]

Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \rightarrow \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \rightarrow \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

SCET_{II} basis

► SCET_{II} operators contributing at $\mathcal{O}(\alpha)$:

$$\mathcal{H}_v = Y_n^{(q)\dagger} Y_n^{(\ell)\dagger} h_v \quad , \quad \mathcal{G}_s^\mu = Y_n^\dagger (iD_s^{(G)\mu} Y_n)$$

$$\mathcal{Q}_s = Y_n^{(q)\dagger} q_s \quad \omega = n \cdot p_q \quad , \quad \omega_g = n \cdot p_g$$

[A]

$$Q_1^A(\Lambda) = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \left[1 - \theta_T \left(-\frac{i\bar{n} \cdot \overleftarrow{D}_s}{\bar{n} \cdot v} - \Lambda \right) \right] \not{n} P_L h_v Y_n^{(\ell)\dagger} \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

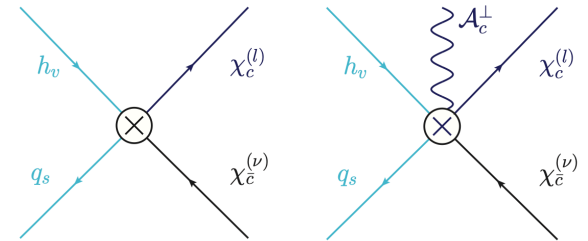
$$Q_2^A = \frac{m_\ell (\bar{n} \cdot v)^2}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \not{n} P_L h_v Y_n^{(\ell)\dagger} \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

[B]

$$Q_1^B(x) = \frac{1}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \not{n} P_L h_v \right) \left(\bar{\chi}_c^{(\ell)} \not{A}_{c[x]}^\perp P_L \chi_{\bar{c}}^{(\nu)} \right)$$

$$Q_2^B(x) = \frac{m_\ell (\bar{n} \cdot v)^2}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{q}_s \not{n} P_L h_v \right) \left(\bar{\chi}_c^{(\ell)} \not{A}_{c[x]}^\perp P_L \chi_{\bar{c}}^{(\nu)} \right)$$

« Local operators »



[C]

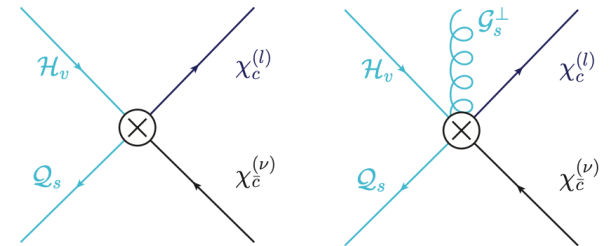
$$Q_1^C(\omega) = \frac{m_\ell}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_{s[\omega]} \not{n} P_L \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

$$Q_2^C(\omega) = \frac{m_\ell (\bar{n} \cdot v)^2}{(\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_{s[\omega]} \not{n} P_L \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

[E]

$$Q_2^E(\omega, \omega_g) = \frac{m_\ell (\bar{n} \cdot v)^2}{\omega (\bar{n} \cdot \mathcal{P}_c)} \left(\bar{\mathcal{Q}}_{s[\omega]} \mathcal{G}_{s[\omega_g]}^{\perp\mu} \not{n} P_R \mathcal{H}_v \right) \left(\bar{\chi}_c^{(\ell)} P_L \chi_{\bar{c}}^{(\nu)} \right)$$

« Non-local operators »



Generalized decay constant

- The QED corrected **soft function** of local operators define a **generalized decay constant** :

$$S_1^A(\Lambda) \equiv \frac{\langle 0 | \bar{q}_s \left[1 - \theta_T \left(-\frac{i\vec{n} \cdot \vec{D}_s}{\vec{n} \cdot v} - \Lambda \right) \right] \not{n} P_L h_v Y_n^{(\ell)\dagger} | B \rangle}{R^{(\ell, B)}} \\ = -\frac{i\sqrt{m_B}}{2} F_-(\Lambda, \mu)$$

$$R^{(\ell, B)} = \langle 0 | Y_{v_B}^{(B)} Y_n^{(l)\dagger} | 0 \rangle = 1 \text{ (OS)}$$

$$Y_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_s(x + sr) \right\}$$

[Beneke et al (2108.05589)]

- For $\alpha \rightarrow 0$, F_- reduces to the standard HQET decay constant :

$$\sqrt{m_B} f_B \left[1 - \frac{C_F \alpha_s}{4\pi} \left(3 \ln \frac{m_b}{\mu} - 2 \right) + \mathcal{O}(\alpha_s^2) \right] = F_-(\Lambda, \mu) \Big|_{\alpha \rightarrow 0} = F_{\text{QCD}}(\mu)$$

Generalized decay constant

► $F_-(\Lambda, \mu)$ is an unknown non-perturbative parameter following evolution equations for Λ and μ :

$$F_-(\Lambda, \mu) = \exp \left[\frac{\alpha Q_\ell Q_u}{\pi} \left(M_{\phi_-}(\Lambda, \mu) + M_{\phi_3}(\mu) \ln \frac{\Lambda_0}{\Lambda} \right) \right] F_-(\Lambda_0, \mu)$$

$$M_{\phi_-}(\Lambda, \mu) = \frac{1}{4} \int d\omega \phi_-(\omega, \mu) \left(\ln^2 \frac{\mu^2}{\Lambda \omega} - \ln^2 \frac{\mu^2}{\Lambda_0 \omega} + 2 \ln \frac{\Lambda_0}{\Lambda} \right), \quad \text{two-particle LCDA}$$

$$M_{\phi_3}(\mu) = \iint d\omega d\omega_g \frac{\phi_3(\omega, \omega_g, \mu)}{\omega_g} \left(\frac{1}{\omega + \omega_g} + \frac{1}{\omega_g} \ln \frac{\omega}{\omega + \omega_g} \right). \quad \text{three-particle LCDA}$$

$$F_-(\Lambda, \mu) = \exp \left[\frac{\alpha}{2\pi} Q_\ell Q_u \left(\ln^2 \frac{\mu}{\Lambda} - \ln^2 \frac{\mu_0}{\Lambda} \right) \right] F_-(\Lambda, \mu_0)$$

► Need to be computed at a single value of Λ and μ . Lattice determination ? QCD SR estimate ?

Generalized decay constant

- $F_-(\Lambda, \mu)$ is an unknown non-perturbative parameter following evolution equations for Λ and μ :

$$F_-(\Lambda, \mu) = \exp \left[\frac{\alpha Q_\ell Q_u}{\pi} \left(M_{\phi_-}(\Lambda, \mu) + M_{\phi_3}(\mu) \ln \frac{\Lambda_0}{\Lambda} \right) \right] F_-(\Lambda_0, \mu)$$

$$F_-(\Lambda, \mu) = \exp \left[\frac{\alpha}{2\pi} Q_\ell Q_u \left(\ln^2 \frac{\mu}{\Lambda} - \ln^2 \frac{\mu_0}{\Lambda} \right) \right] F_-(\Lambda, \mu_0)$$

- To estimate the uncertainty linked to additional QED effects, we define:

$$F_-(\Lambda_0, \mu_0) = F_{\text{QCD}}(\mu) \left(1 + \frac{\alpha}{4\pi} f^{(1)}(\Lambda_0, \mu_0) \right)$$

- We assign an $\mathcal{O}(1)$ parametric uncertainty to $f^{(1)}(\Lambda_0, \mu_0)$ at $\Lambda_0 = \mu_0 = \mu_s = 1.4 \text{ GeV}$, where this quantity is free from logarithmic enhancements.

Coft Wilson-lines cancellation

- One can also define $F_-(\Lambda, \mu)$ using **time-like Wilson-lines** :

$$-\frac{i\sqrt{m_B}}{2}F_-(\Lambda, \mu) \equiv \frac{\left\langle 0 \left| \bar{q}_s \left[1 - \theta_T \left(-\frac{i\vec{n} \cdot \vec{D}_s}{\vec{n} \cdot v} - \Lambda \right) \right] \not{n} P_L h_v \mathcal{S}_{v_\ell}^{(l)\dagger} \right| B^- \right\rangle}{\langle 0 | \mathcal{S}_v^{(B)} \mathcal{S}_{v_\ell}^{(l)\dagger} | 0 \rangle}$$

- Those time-like WL can be splitted between **soft** and **soft-collinear (coft)** contributions :

$$\mathcal{S}_v^{(l)}(x) = Y_v^{(B)}(x) C_{\vec{n}}^{(B)}(x_+) = Y_v^{(B)}(x) C_{\vec{n}}^{(b)}(x_+) C_{\vec{n}}^{(q)\dagger}(x_+) \quad \mathcal{S}_{v_\ell}^{(l)}(x) = Y_n^{(\ell)}(x_-) C_{v_\ell}^{(\ell)}(x)$$

- After **soft-collinear** decoupling, $h_v \rightarrow C_{\vec{n}}^{(b)} h_v$, $q_s \rightarrow C_{\vec{n}}^{(q)} q_s$

contributions from the **coft Wilson-lines** cancel in the ratio :

$$-\frac{i\sqrt{m_B}}{2}F_-(\Lambda, \mu) = \frac{\left\langle 0 \left| \bar{q}_s \left[1 - \theta_T \left(-\frac{i\vec{n} \cdot \vec{D}_s}{\vec{n} \cdot v} - \Lambda \right) \right] \not{n} P_L h_v Y_n^{(\ell)\dagger} \right| B^- \right\rangle}{\langle 0 | Y_v^{(B)} Y_n^{(\ell)\dagger} | 0 \rangle}$$

A theory of Wilson lines ?

$$Y_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_{us}(x + sr) \right\} \quad C_r^{(i)}(x) = \mathcal{P} \exp \left\{ ie Q_i \int_{-\infty}^0 ds r \cdot A_{usc}(x + sr) \right\}$$

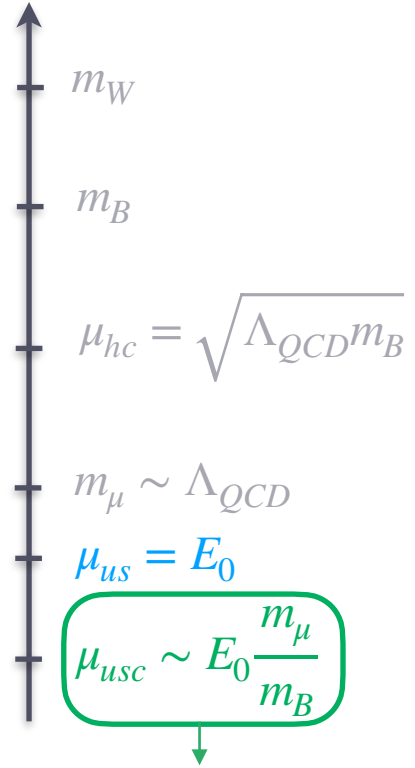
- We can decouple the interactions of the B and the muon with **ultrasoft** and **ultrasoft-collinear** photons :

$$Y_n^{(\ell)\dagger}(x_-) \varphi_v(x) = Y_n^{(\ell)\dagger}(x_-) Y_v^{(B)}(x) C_{\bar{n}}^{(B)}(x_+) \varphi_v^0(x) \quad h_n(x) = C_{v_\ell}^{(\ell)}(x) h_n^0(x)$$

- Real corrections can be expressed as matrix elements of Wilson lines :

$$W_s(\omega_{us}, \mu) = \left[\sum_{n_{us}=0}^{\infty} \prod_{i=1}^{n_{us}} \int d\Pi_i(q_i) \right] \left| \langle n_{us} \gamma_{us}(q_i) | Y_v^{(B)} Y_n^{\dagger(\ell)} | 0 \rangle \right|^2 \delta(\omega_{us} - q_0^{us}) \quad , \quad q_0^{us} = \sum_i q_{0i}$$

$$W_{usc}(\omega_{usc}, \mu) = \left[\sum_{n_{usc}=0}^{\infty} \prod_{j=1}^{n_{usc}} \int d\Pi_j(q_j) \right] \left| \langle n_{usc} \gamma_{usc}(q_j) | C_{\bar{n}}^{(B)} C_{v_\ell}^{\dagger(\ell)} | 0 \rangle \right|^2 \delta(\omega_{usc} - q_0^{usc}) \quad , \quad q_0^{usc} = \sum_j q_{0j}$$



Soft photon to the muon
boosted in the B frame

- These radiative functions are convoluted with the **measurement function** implementing the experimental radiation veto

$$S(E_0, \mu) = \int_0^\infty d\omega_s \int_0^\infty d\omega_{sc} \theta\left(\frac{E_0}{2} - \omega_{us} - \omega_{usc}\right) W_{us}(\omega_{us}, \mu) W_{usc}(\omega_{usc}, \mu)$$

$HH\chi Pt$ (Heavy Hadron Chiral Perturbation theory)

- Using approximate heavy spin symmetry, the B and B^* can be put into a superfield:

$$H = \frac{1 + \not{v}}{2}(\not{\phi}_v - \not{\varphi}_v \gamma_5) \quad \text{where} \quad \begin{aligned} H &\rightarrow S H \quad \text{under } SU(2)_v \\ \bar{H} &\rightarrow \bar{H} S^{-1} \end{aligned}$$

[Wise (PhysRevD.45.R2188),
Boyd et al. (9402340),...]

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$$\bar{\Lambda} = m_B - m_b \sim \mathcal{O}(\Lambda_{QCD})$$

- Expanding over $\zeta = \frac{E_0}{\bar{\Lambda}}$, we can derive an EFT for the heavy meson valid for $E_0 \lesssim \bar{\Lambda}$:

$$\mathcal{L}_H = -\frac{1}{2} \text{Tr}[\bar{H} i(\not{v} \cdot \partial) H]$$

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$$\bar{H} \rightarrow \bar{H} S^{-1}$$

$$\Sigma_s^{\mu\nu} \rightarrow S \Sigma_s^{\mu\nu} S^{-1} \quad \bar{\Lambda} = m_B - m_b \sim \mathcal{O}(\Lambda_{QCD})$$

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- $SU(2)_v$ breaking effects arise via the chromomagnetic/magnetic operator

$$\frac{g}{4m_B} (\bar{h}_v \sigma^{\mu\nu} h_v) G_{\mu\nu} \quad \text{giving rise to the spurion :} \quad \frac{\Sigma_s^{\mu\nu}}{m_B} = \frac{1 + \not{v}}{2} \frac{\sigma_s^{\mu\nu}}{m_B} \frac{1 + \not{v}}{2}$$

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- Subleading $B \rightarrow B^* \gamma$ and $B^* \rightarrow B \gamma$ interactions at $\mathcal{O}(\zeta)$ are described by a single $SU(2)_v$ conserving operator :

$$c_{B^* B^* \gamma} = c_{B B^* \gamma} = c_2 = g_{B B^*} \bar{\Lambda}$$

$HH\chi Pt$ (Heavy Hadron Chiral Perturbation theory)

- Using approximate **heavy spin symmetry**, the B and B^* can be put into a **superfield**:

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$$\begin{aligned} \mathcal{L}_H &= -\frac{1}{2} \text{Tr}[\bar{H} i(\not{v} \cdot \partial) H] + \frac{c_1 \bar{\Lambda}^2}{m_B} \text{Tr}[\bar{H} \Sigma_s^{\mu\nu} H \sigma_{\mu\nu}] + \frac{ec_2}{8\bar{\Lambda}} \text{Tr}[H \sigma_{\mu\nu} \bar{H}] F_{us}^{\mu\nu} \\ &= \phi_v^\dagger (i\not{v} \cdot \partial) \phi_v - \rho_{v\mu}^\dagger (i\not{v} \cdot \partial) \rho_v^\mu + \delta_{B^*} \rho_{v\mu}^\dagger \rho_v^\mu \\ &\quad + \frac{ec_{B^*B\gamma}}{2\bar{\Lambda}} \left(\phi_v \not{v}^\mu \rho_v^{\dagger\nu} \tilde{F}_{\mu\nu}^{us} + \text{h.c.} \right) + \frac{iec_{B^*B^*\gamma}}{2\bar{\Lambda}} \rho_v^\mu \rho_v^{\dagger\nu} F_{\mu\nu}^{us} \end{aligned}$$

with : $c_{B^*B^*\gamma} = c_{BB^*\gamma} = c_2 = g_{BB^*} \bar{\Lambda} \quad ; \quad \delta_{B^*} = -16c_1 \frac{\bar{\Lambda}^2}{m_B^2}$