

$B \rightarrow K + a$ decays: which framework leads to consistent exclusion plots?

Xiyuan Gao

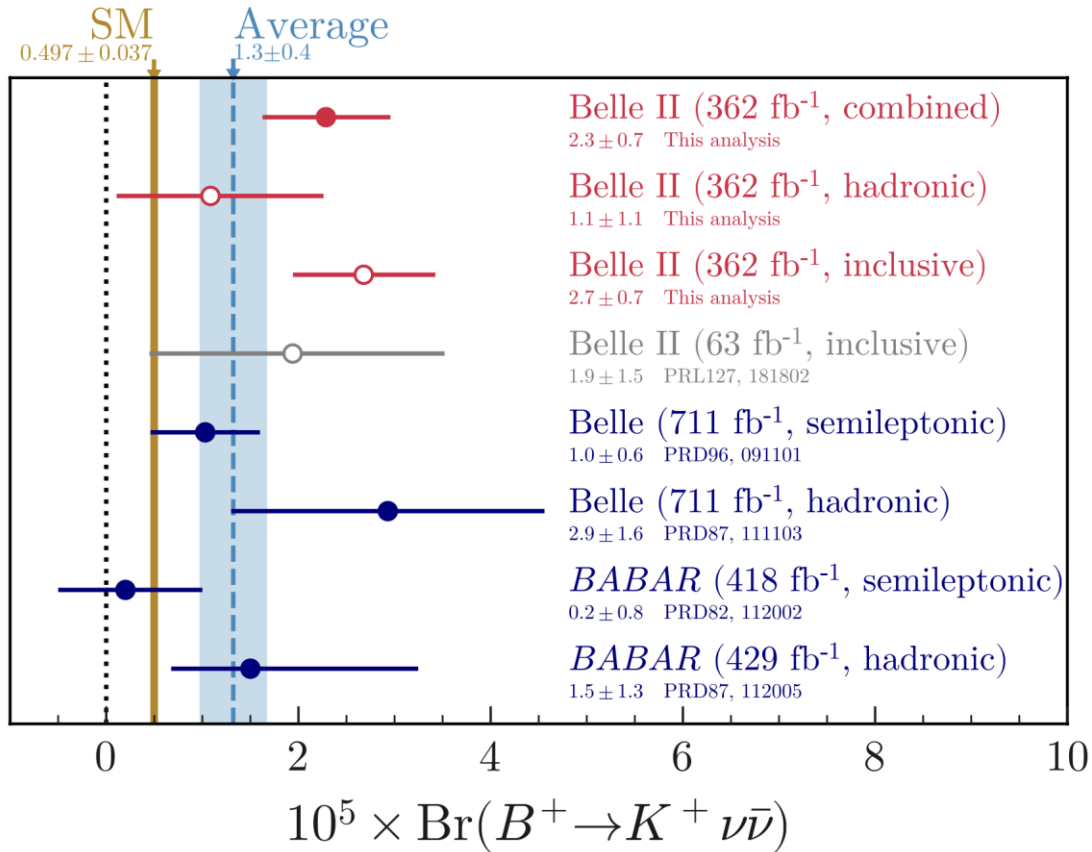
*Institut für Theoretische Teilchenphysik (TTP),
Karlsruher Institut für Technologie (KIT), Germany*

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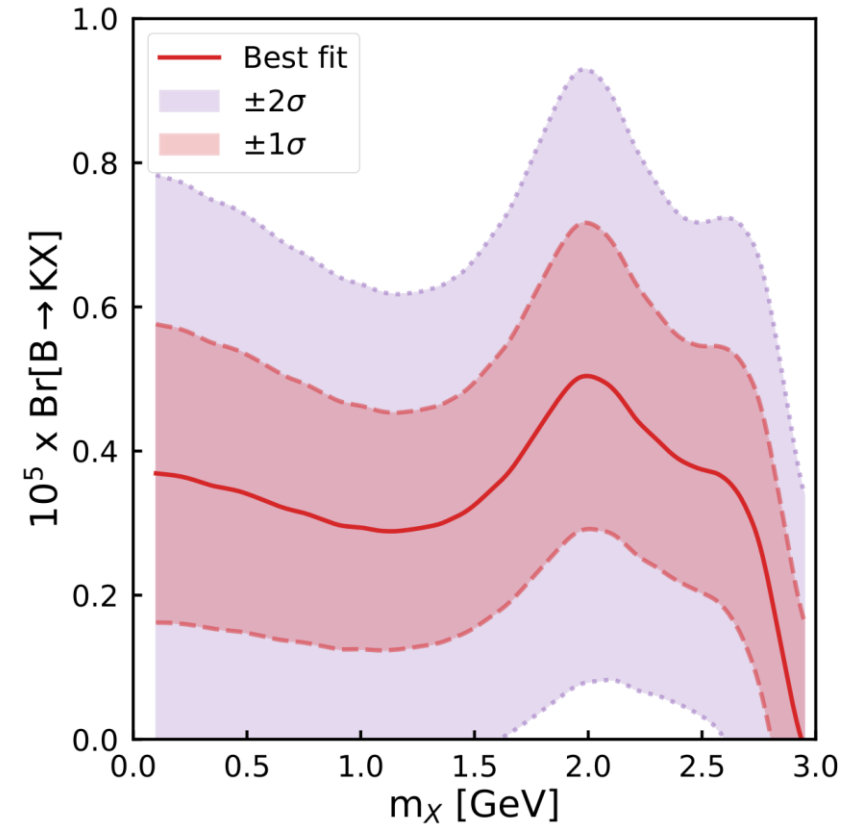
Together with Ulrich Nierste

The Motivation

Recent Belle-II $B \rightarrow K + \text{inv.}$ $\sim 3\sigma$ localized excess.



Belle-II 2311.14647



Altmannshofer et.al 23'

The Motivation

Puzzle: lightness of a :

- Fine-tuning and implicit naturalness.
- A pseudo-Goldstone boson: Peccei, Quinn 77'
 - Massless a from spontaneous broken global symmetry.
 - $a \sim 2$ GeV: global symmetry is never exact.

$$\mathcal{L}_a = \frac{\partial_\mu a}{2f_a} C_{bs}^V \bar{s} \gamma^\mu b + \text{h.c.}$$

But, why? Testable?

$$F_{bs}^V \equiv 2f_a/C_{bs}^V = 3.1_{-0.5}^{+1.0} \times 10^8 \text{ GeV is needed}$$

The Motivation

(Specific) Axion-EFT:

Batell, Pospelov, Ritz. 11'; Izaguirre, Lin, Shuve. 17'; Choi, Im, Park, Yun. 17'; Aloni, Soreq, Williams. 19'; Gavela, Houtz, Quilez, Del Rey, Sumensari, 19'; Chakraborty, Kraus, Loladze, Okui, Tobioka. 21'; Bauer, Neubert, Renner, Schnubel, Thamm, 21; Calibbi, Li, Mukherjee, Schmidt. 25'; Camalich, Ziegler. 25'; and more...

$$\mathcal{L} \supset \sum_{q=t,b} \frac{c_q}{f} \bar{q} \gamma^\mu \gamma_5 q \partial_\mu a, \quad \text{or} \quad \mathcal{L} \supset \frac{c_g}{f} a G^{\mu\nu} \tilde{G}_{\mu\nu}.$$

- Elephant in the room:

$$\text{Br}(B \rightarrow Ka) \sim \left(\frac{1}{f} \ln \frac{\Lambda_{UV}}{m_t} \right)^2$$

- Physically not divergent.
- But still need a Λ_{UV} for plots.
- UV completion required.

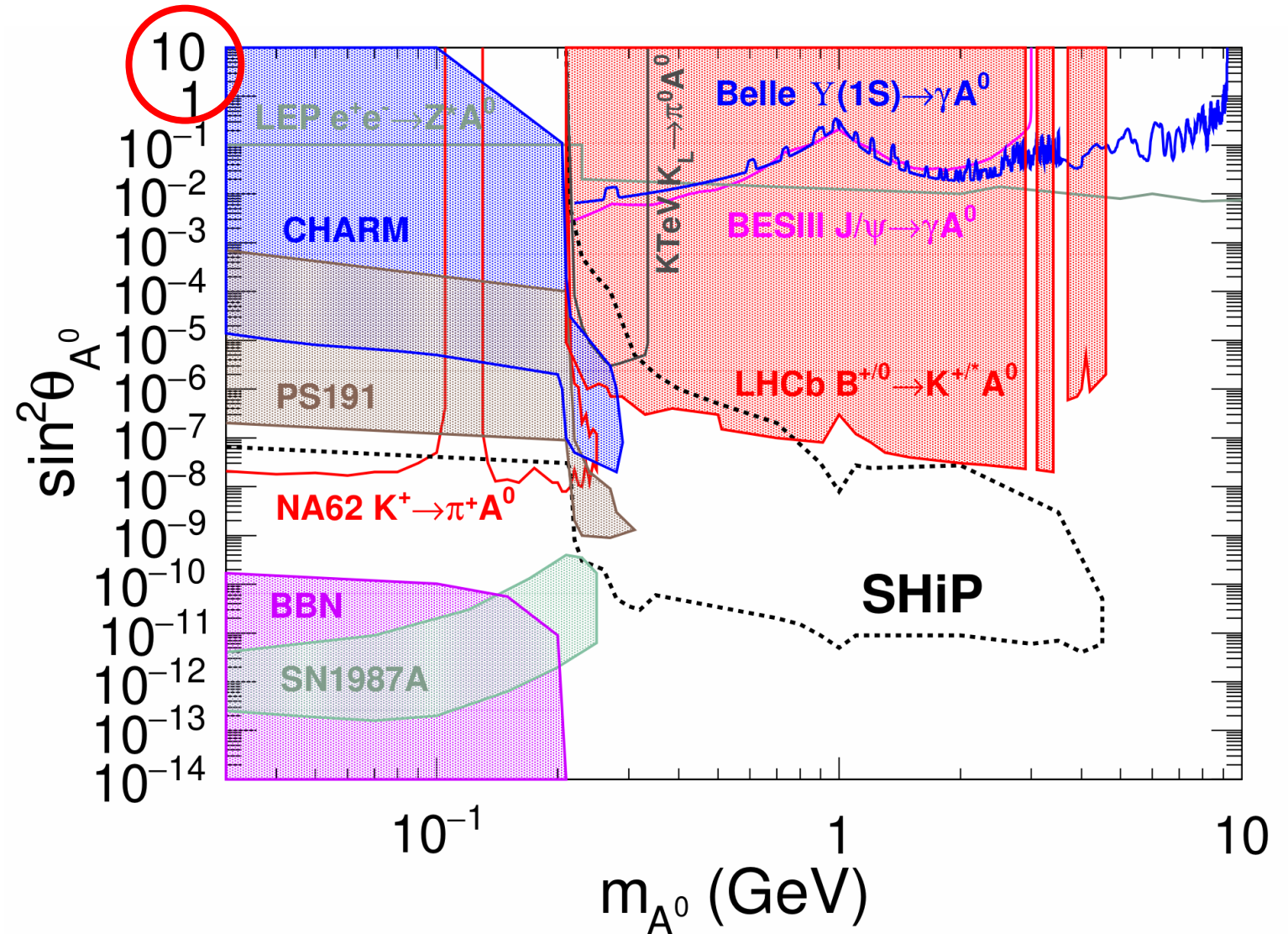
No bootstrap!

The Motivation

A typical exclusion plot:

Belle II, 2022. Phys.
Rev. Lett. 128, 081804

- θ_A : mix with whom?
- Mixing picture make sense?
- Where is Λ_{UV} ?



$B \rightarrow Ka$ with UV completion

DFSZ model: a minimal benchmark.

Dine, Fischler, Srednicki. 81'; Zhitnitsky. 80'

SM with 2HDM + complex singlet + Global $U(1)_{PQ}$

$$V_{\Phi} = \tilde{V}_{\text{moduli}}(|\Phi_u|, |\Phi_d|, |\Phi_u \Phi_d|, |\Phi_s|) + \lambda \Phi_s^2 \Phi_u \Phi_d^{\dagger} + \text{h.c.},$$

$$\mathcal{L}_Y = Y_u \overline{Q}_L u_R \Phi_u + Y_d \overline{Q}_L d_R \tilde{\Phi}_d + Y_e \overline{L}_L e_R \tilde{\Phi}_u + \text{h.c.}$$

$$\Phi_{\alpha} = \begin{pmatrix} \phi_{\alpha}^{-} \\ v_{\alpha} + (\rho_{\alpha} + i\eta_{\alpha})/\sqrt{2} \end{pmatrix}, \quad \alpha = u, d, \quad \Phi_s = f + \frac{r_0 + ia_0}{\sqrt{2}}.$$

$$\mathcal{A}(b \rightarrow sa) = -\sin \theta \times \mathcal{A}(b \rightarrow sA).$$

Mixing angle θ in $a - A$ mass term.

A , the CP-odd scalar of 2HDM.

Massless when $\lambda = 0$ (PQWW)

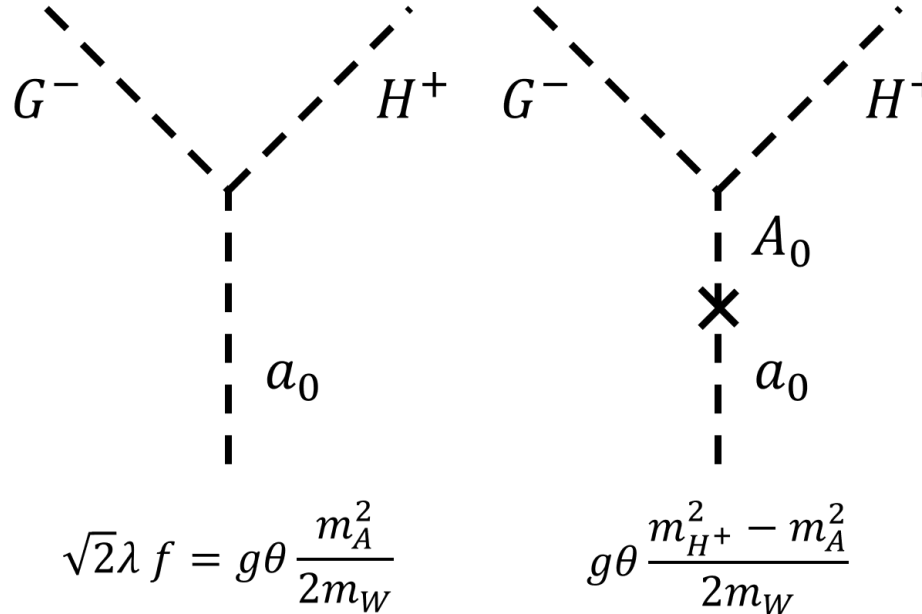
Freytsis, Ligeti, Thaler. 10'

$B \rightarrow Ka$ with UV completion

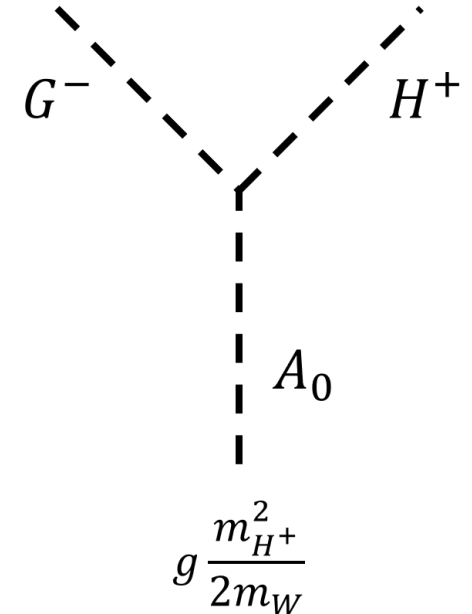
When $\lambda \neq 0$:

- Massive A_0 . Off-shell, unphysical $b \rightarrow sA$ amplitude.
- Mixing picture is **wrong**, however, mixing formula **correct**.
- Somehow similar to: Kachanovich, Nierste, Nisandzic. 20'. Higgs portal;

DFSZ



PQWW



$$\mathcal{A}(b \rightarrow sa)_{\text{DFSZ}} \neq -\sin \theta \times \mathcal{A}(b \rightarrow sA_0)_{\text{DFSZ}},$$

$$\mathcal{A}(b \rightarrow sA_0)_{\text{DFSZ}} \neq \mathcal{A}(b \rightarrow sA_0)_{\text{PQWW}},$$

But two changes cancel each other:

$$\mathcal{A}(b \rightarrow sa)_{\text{DFSZ}} = -\sin \theta \times \mathcal{A}(b \rightarrow sA_0)_{\text{PQWW}}$$

$B \rightarrow Ka$ with UV completion

λ only disappears at 1-loop.

The complete result when $m_H \gg m_W$:

$$\mathcal{H}_{\text{eff}} = \theta \frac{g^3 V_{ts}^* V_{tb}}{128\pi^2} \frac{m_t^2}{m_W^3} \left(X_1 \frac{1}{\tan \beta} + X_2 \frac{1}{\tan^3 \beta} + X_3 \frac{\lambda}{16\pi^2} \right) \bar{s} \gamma^\mu P_L b \partial_\mu a.$$

$\nearrow$ decouple
 $\nwarrow$ GIM
 $\uparrow$ Z_2
 $\nwarrow$ Lorentz

$$X_1 = -\ln \frac{m_H^2}{m_t^2} + \frac{3m_W^4}{(m_t^2 - m_W^2)^2} \ln \frac{m_t^2}{m_W^2} + \frac{3(m_t^2 - 2m_W^2)}{m_t^2 - m_W^2},$$

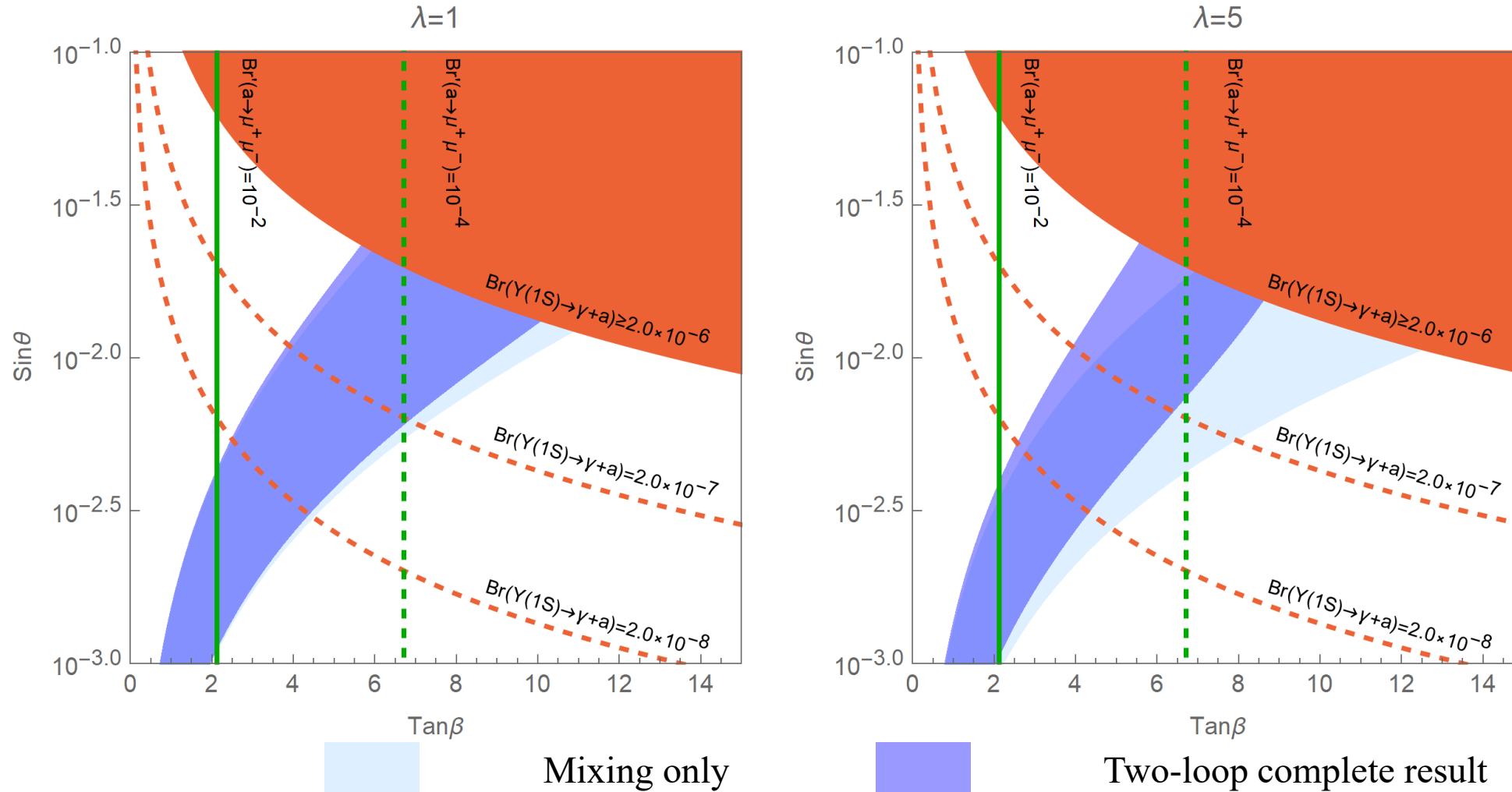
$$X_2 = 0,$$

$$X_3 = \ln \frac{m_H^2}{m_t^2} + \frac{6m_W^2}{m_t^2 - m_W^2} \ln \frac{m_t^2}{m_W^2} + \frac{1}{2}.$$

- X_1, X_2 are consistent with the mixing formula.
Known for more than 40 years.
Hall, Wise. 81'
- X_3 is **new**, arising at a **two-loop**.
Enhanced when $\tan \beta$ is large.
See back-up for 2-loop diagrams.

$B \rightarrow Ka$ with UV completion

Mixing formula needs refinement:



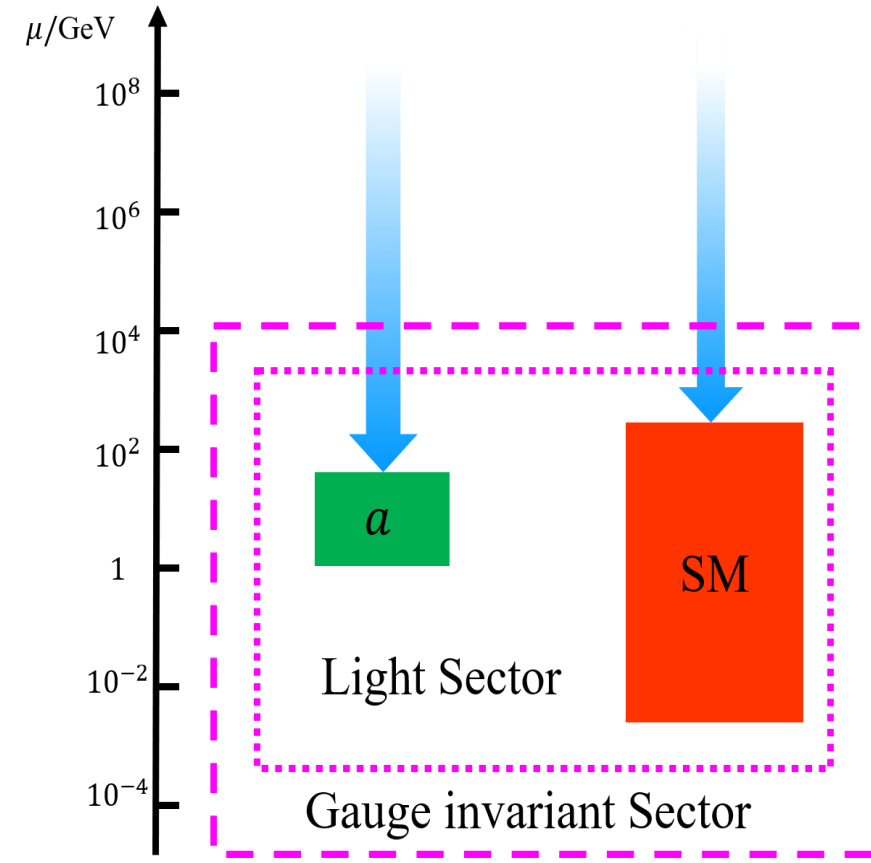
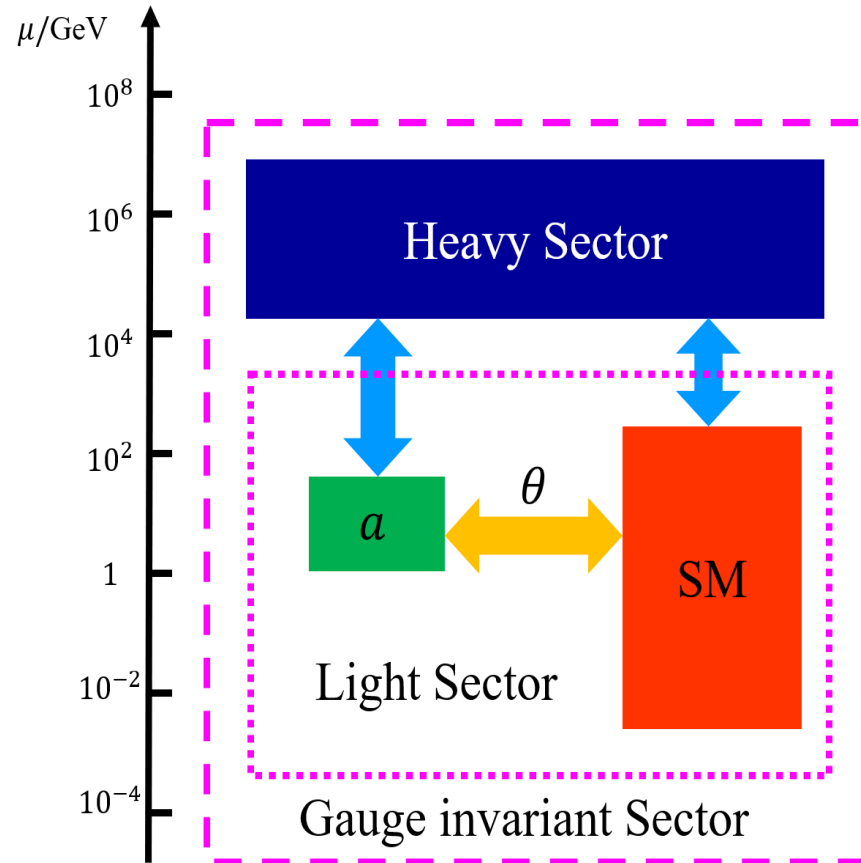
Reflections: Apparent Non-decoupling

Among the three loop factors, only X_2 is suppressed by m_H^2 .

$$\mathcal{H}_{eff} \sim \ln m_H ,$$

but $\theta = 2 \frac{\lambda f v}{m_H^2} .$

- Dimensionless θ ,
Wilson picture fails?
- The Light theory is
not Gauge invariant
when $\theta \neq 0$.



Reflections: Apparent Non-decoupling

necessary requirement for the decoupling of heavy particles: *at each stage of symmetry breaking the light subtheory has to be renormalizable*. In other words, whenever we isolate the heavy sector of the theory, the gauge invariance of the remaining light sector should not be broken.

Senjanovic, Sokorac, 79'

Important lesson, if we:

- Change SM (data driven),
- Do Bottom-up analysis.

The result may be **incomplete**.
Must check **Gauge invariance**.

Example:

$\mu \rightarrow e\gamma$ in a 2HDM: $c_{\alpha\beta}$

Chang, Hou, Keung. 93';

Gunion, Haber. 03';

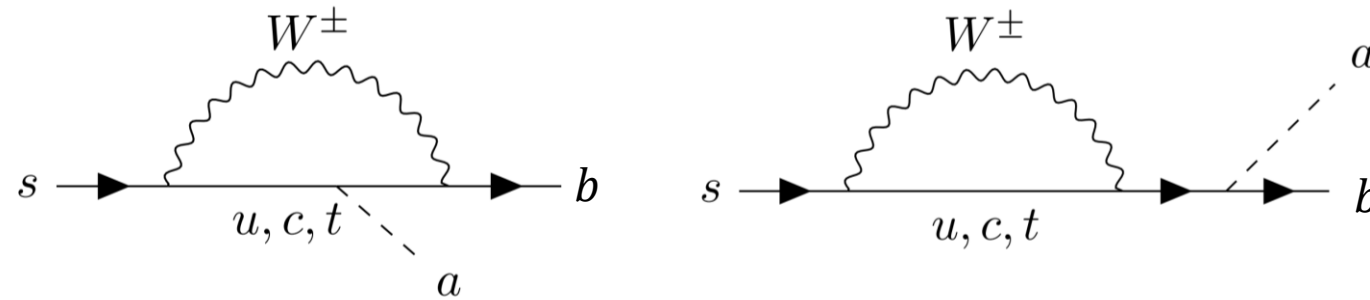
Davidson. 16'; Altmannshofer, et al. 20'.

Reflections: Apparent Non-decoupling

The EFT must reveal **apparent non-decoupling**:

SM loop:

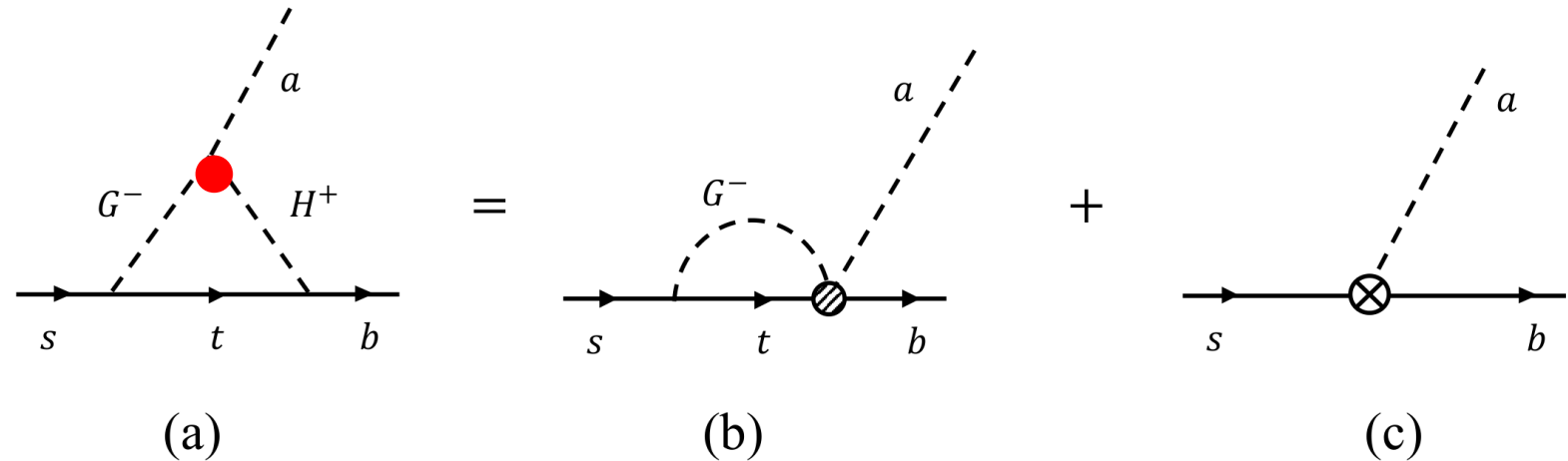
e.g. Alonso-
Alvarez, et al. 21'



2HDM loop:

$G^- H^+ a$ vertex:

$$\bullet \quad g\theta \frac{m_H^2}{2m_W}$$



Unsuppressed
 $\ln m_H$

Divergent:
Captured by EFT?

Counter-term

Reflections: Apparent Non-decoupling

Some basis give the wrong result:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + ia \sum_{q=t,b} c_q \bar{q} \gamma_5 q.$$

Renormalizable, but not SU(2)×U(1) invariant

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{SM}} + i \frac{a}{v} \left(c_b \bar{Q}_L b_R \tilde{H}_u + c_t \bar{Q}_L t_R H_u + \text{h.c.} \right) \\ &= \mathcal{L}_{\text{SM}} + ia \sum_{q=t,b} c_q \bar{q} \gamma_5 q + i \frac{a}{v} \left[c_b V_{tb} \bar{t}_L b_R G^+ \right. \\ &\quad \left. + c_t (V_{tb}^* \bar{b}_L t_R G^- + V_{ts}^* \bar{s}_L t_R G^-) + \text{h.c.} \right] + \dots \\ &= \mathcal{L}_{\text{SM}} + \sum_{\psi_L = Q_L, t_R^c, b_R^c} \frac{c_\psi}{f} \bar{\psi}_L \gamma^\mu \psi_L \partial_\mu a + \dots \end{aligned}$$

SU(2)×U(1) invariant, but not renormalizable

The **gauge invariant** EFT reproduces the correct **leading-log** term.

Beyond leading-log, the **counter term** is a **definition**.

Conclusions: consistent plots?

- UV model:
 - Need to specify small λ .
 - Or add a two-loop enhanced correction.
- EFT:
 - $SU(2) \times U(1)$ hidden at low scale.
 - Explicitly pick the gauge invariant basis.

Thanks

Phenomenology: Explaining the Belle II excess

What we need?

$$\text{Br}(B \rightarrow Ka) \times \text{Br}(a \rightarrow \text{invisible}) = (1 \sim 9) \times 10^{-6},$$

$$\text{Br}(B \rightarrow Ka) \times \text{Br}(a \rightarrow \text{visible}) \lesssim \text{exclusive search limits.}$$

- $a \rightarrow \chi\chi$ gives invisible signals.
- $a \rightarrow \mu^+\mu^-$ constrains small $\tan\beta$.
- $\Upsilon \rightarrow \gamma a$ constrains large $\tan\beta$.

Need to extend DFSZ
and couple a to a dark
sector.

$B \rightarrow Ka$ with UV completion

Some technical details on two-loop calculation:

- Thousands of diagrams at two loop, but only a few are relevant to X_3 .
- Master integrals are well-known.
Davydychev, Tausk. 92'; Nierste. 95'
- Cross checked within arbitrary gauge.
Box+Penguin=Gauge inv

