Semileptonic Decays in a Nutshell

Belle II US Summer Workshop 2025

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Belle II

More Resources :

Reviews (RMPs) on the subject :

Mannel, Dingfelder

Richman, Burchat

Bernlochner, Robinson, Franco Sevilla, Wormser

Attached to the agenda :-)

In addition: some notes on $B \to D\ell\bar{\nu}_\ell$ are also attached

Semileptonic decay rate for $B \to D\,\ell\,\bar{\nu}_\ell$

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1 Overview

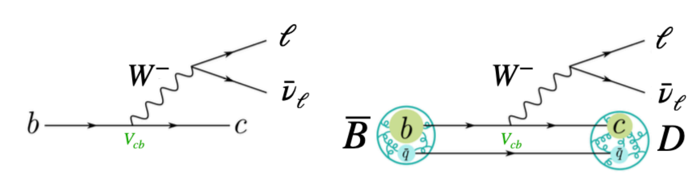
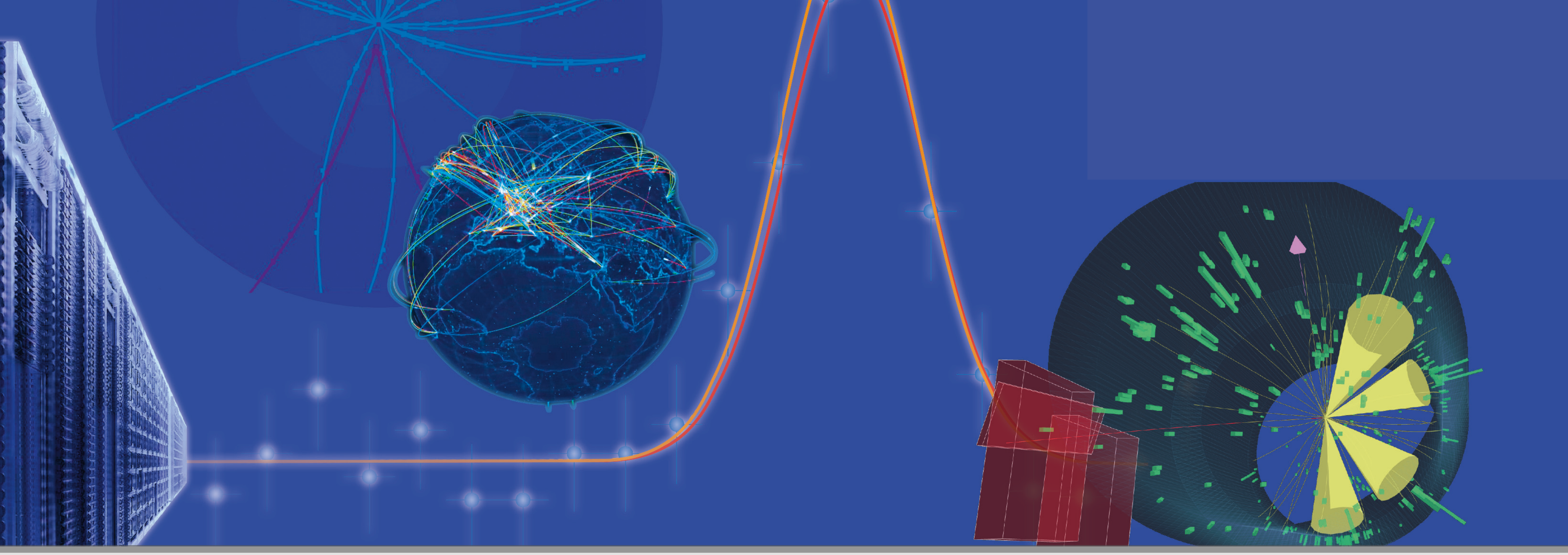
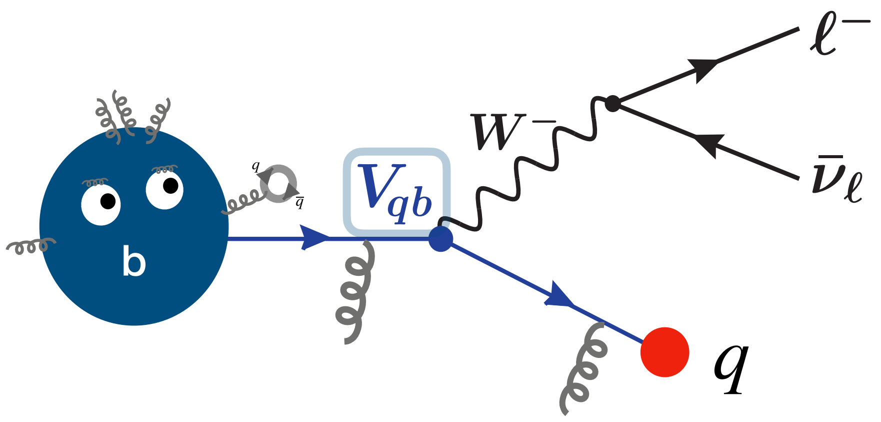


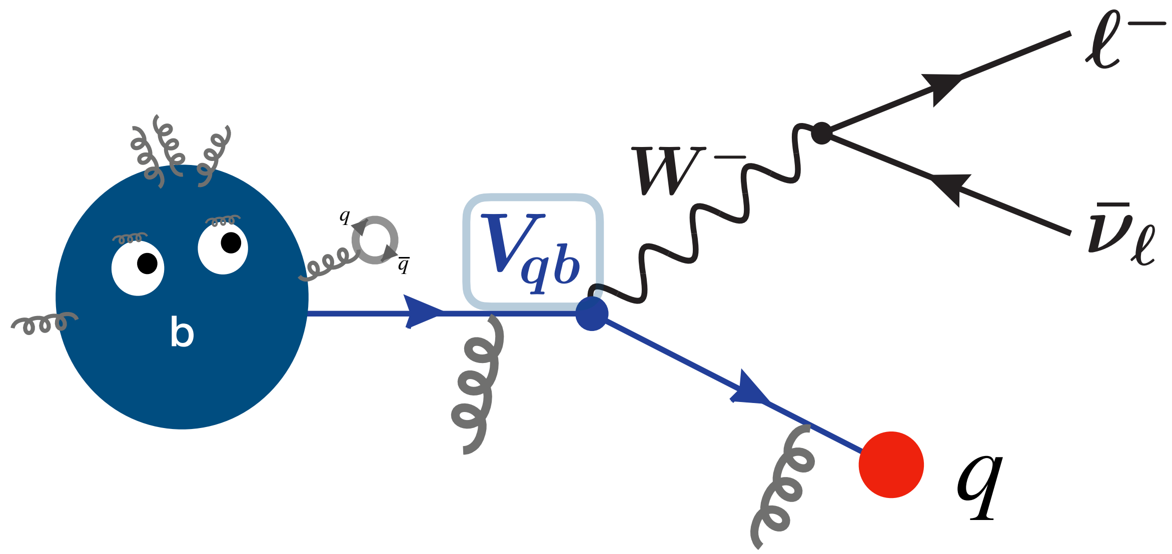
Fig. 1: Quark and parton-level decay of $B \to D\,\ell\,\bar{\nu}_\ell$ are shown.



1) Overview

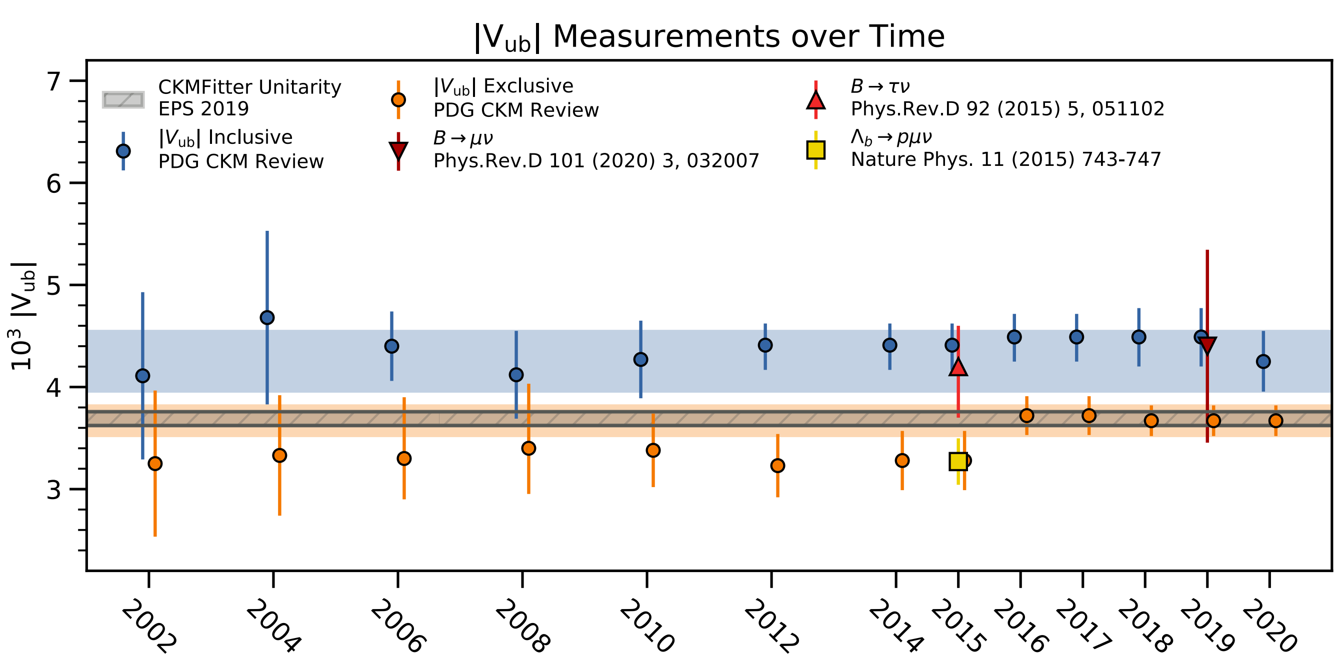


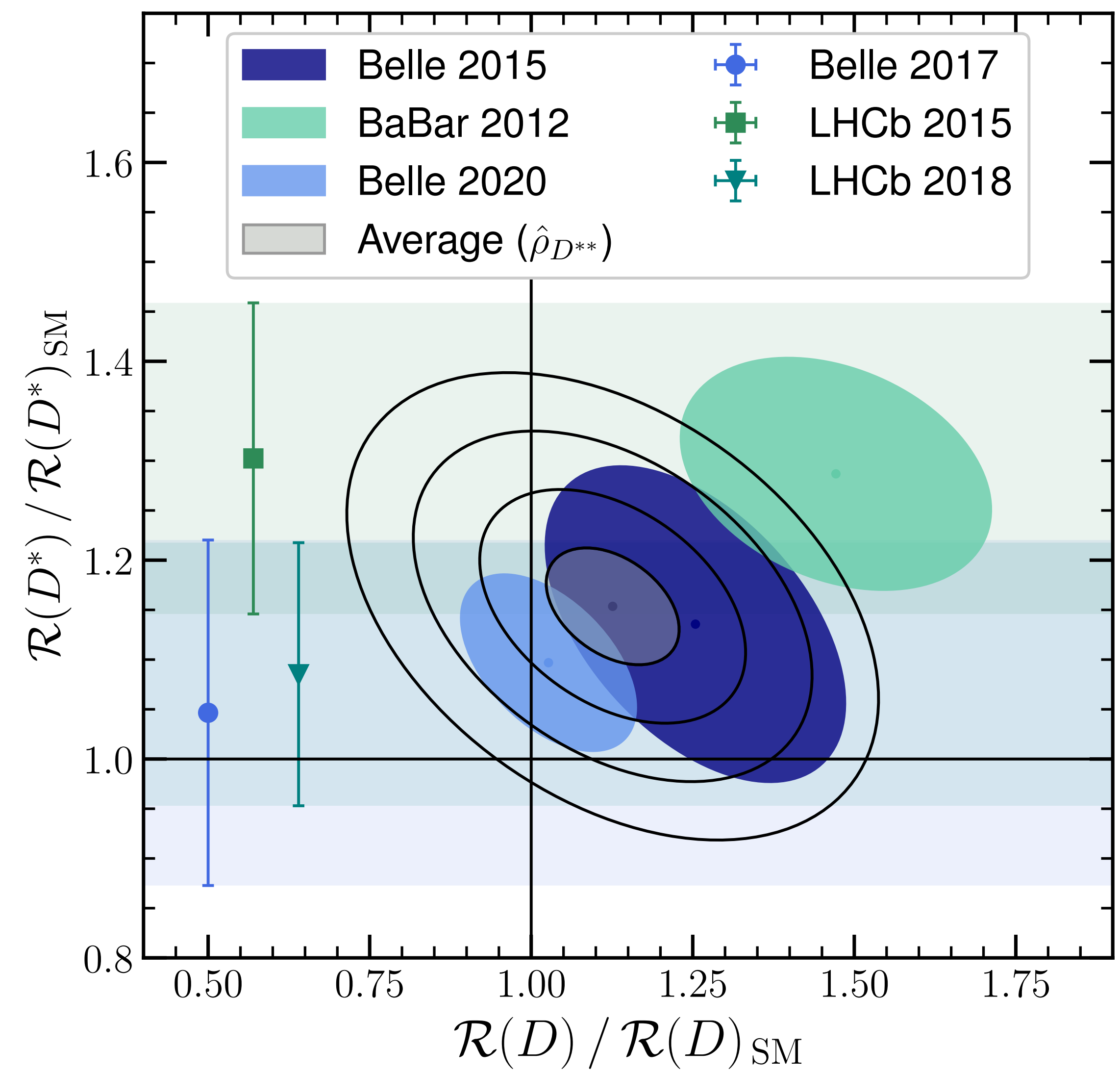
Let’s take a deep dive



They look **cute**, but
that could be **deceiving** …

… they are responsible for some of the long-standing **discrepancies** since about a decade





Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

$$
\begin{array}{c} \\ u \\ c \\ t \end{array}
\begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}
$$

CKM Matrix

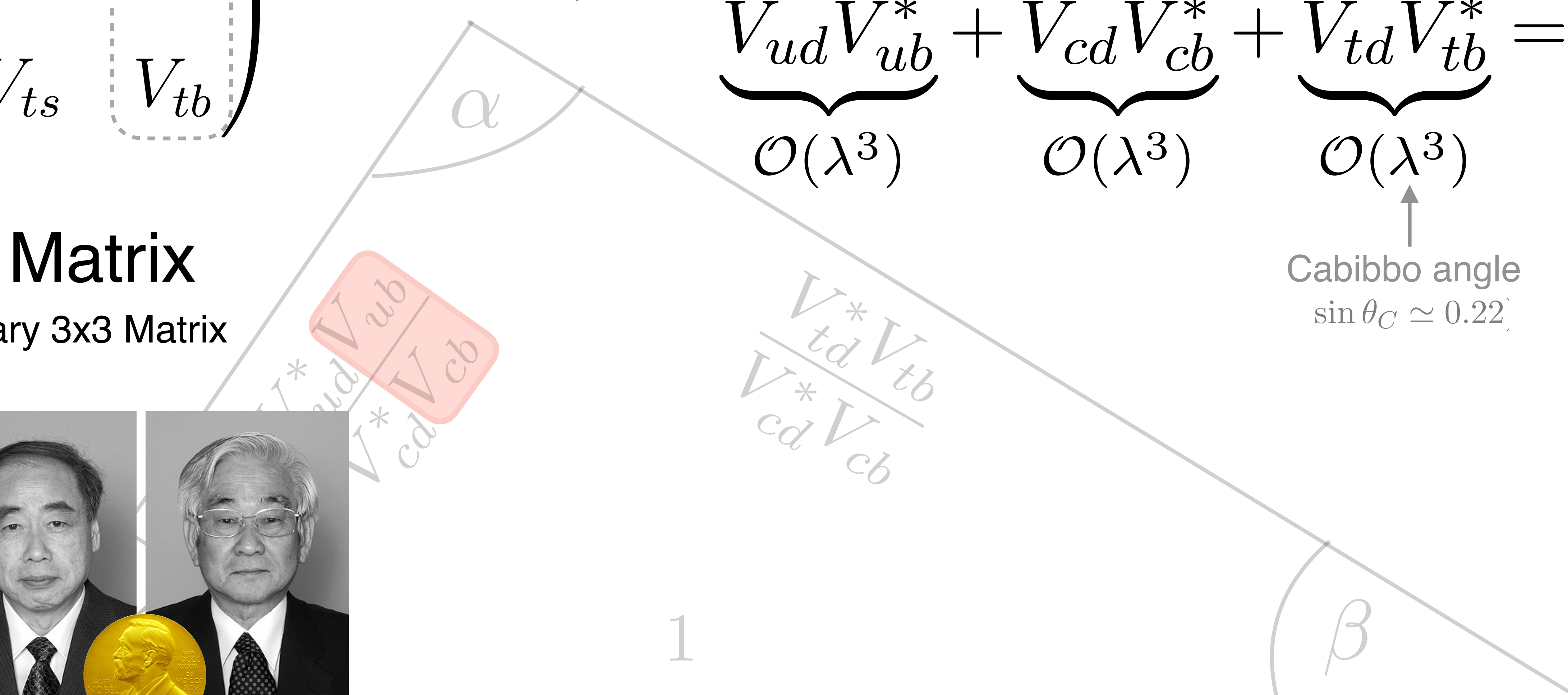
SM: Unitary 3x3 Matrix

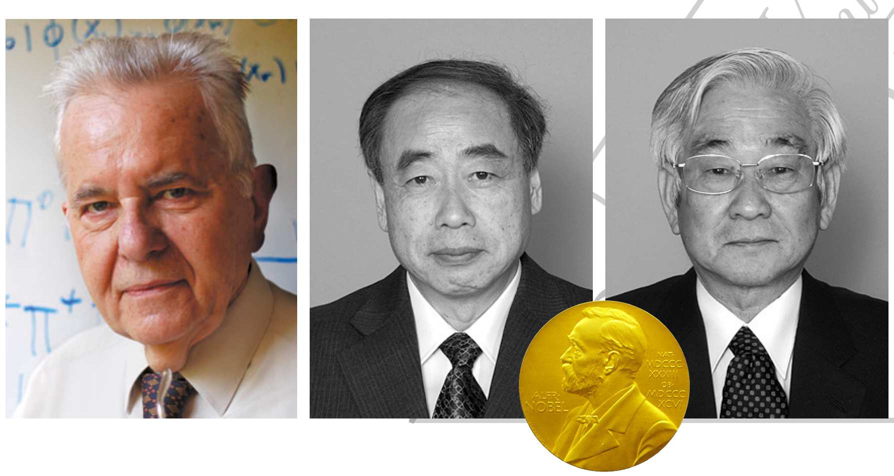
Over constrain Unitarity condition
→ Potent test of Standard Model

Unitarity
$CC^\dagger = 1$

$$
\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0
$$

Cabibbo angle
$\sin\theta_C \simeq 0.22$

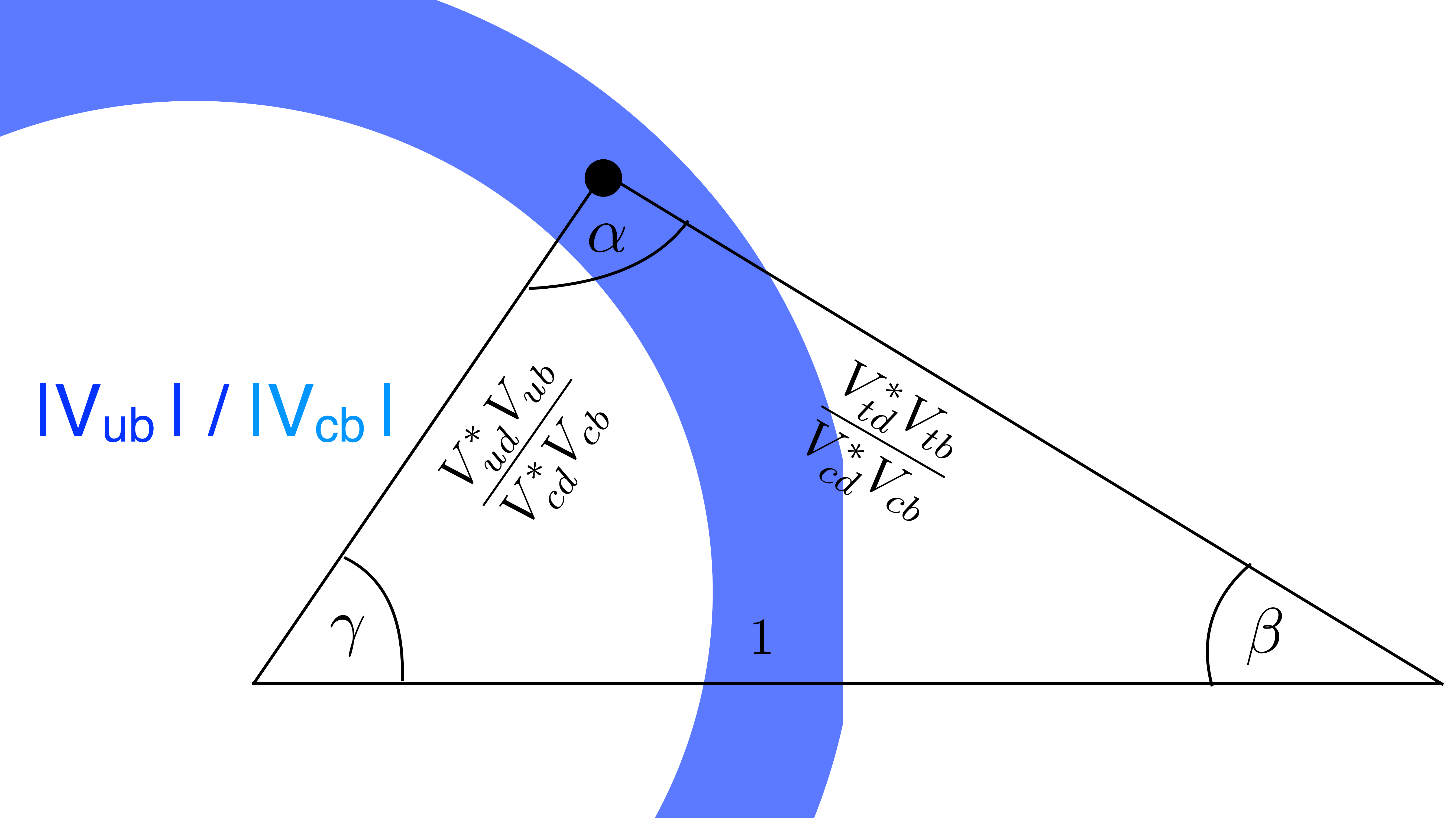




Nobel prize 2008

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

Over constrain Unitarity condition
→ Potent test of Standard Model



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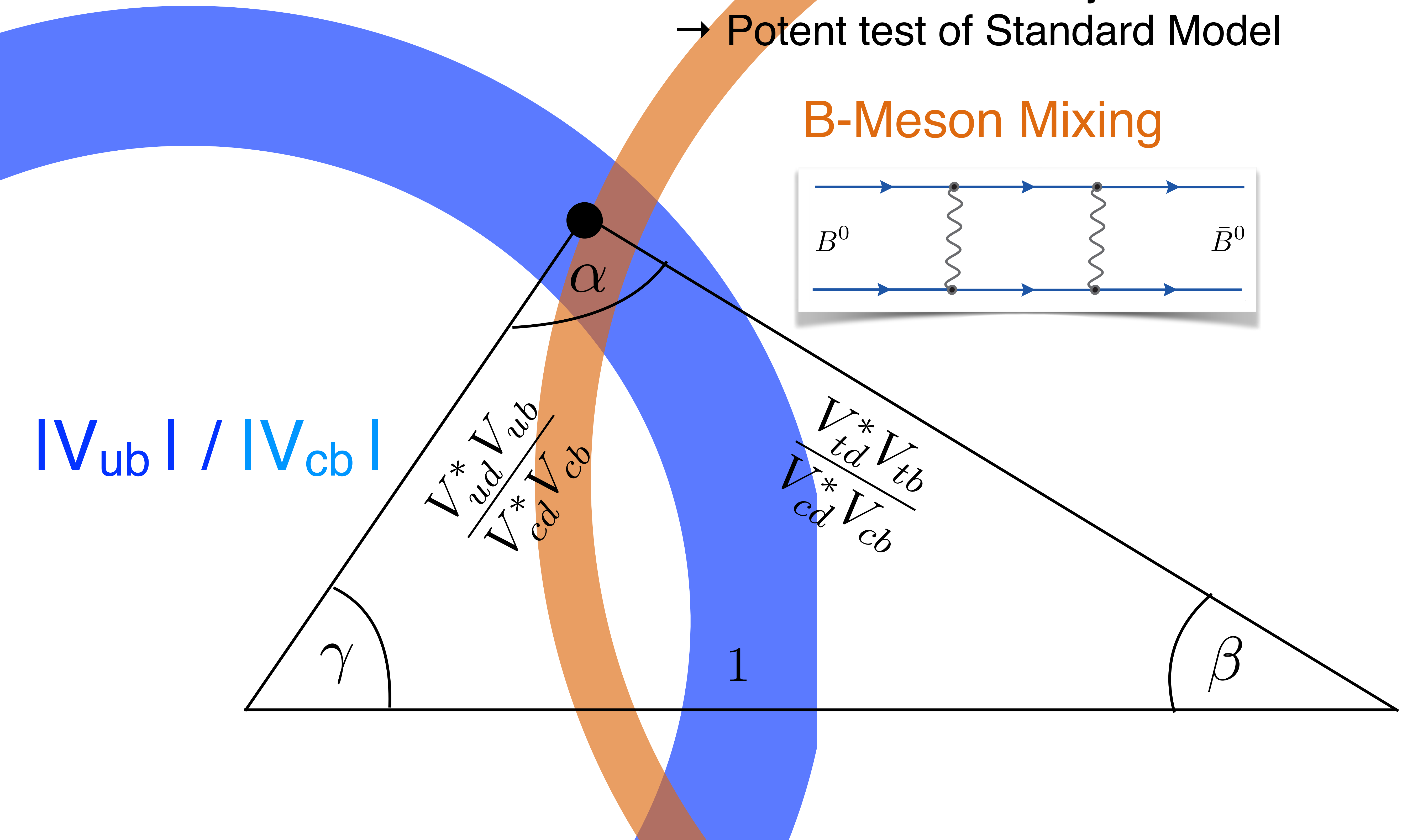
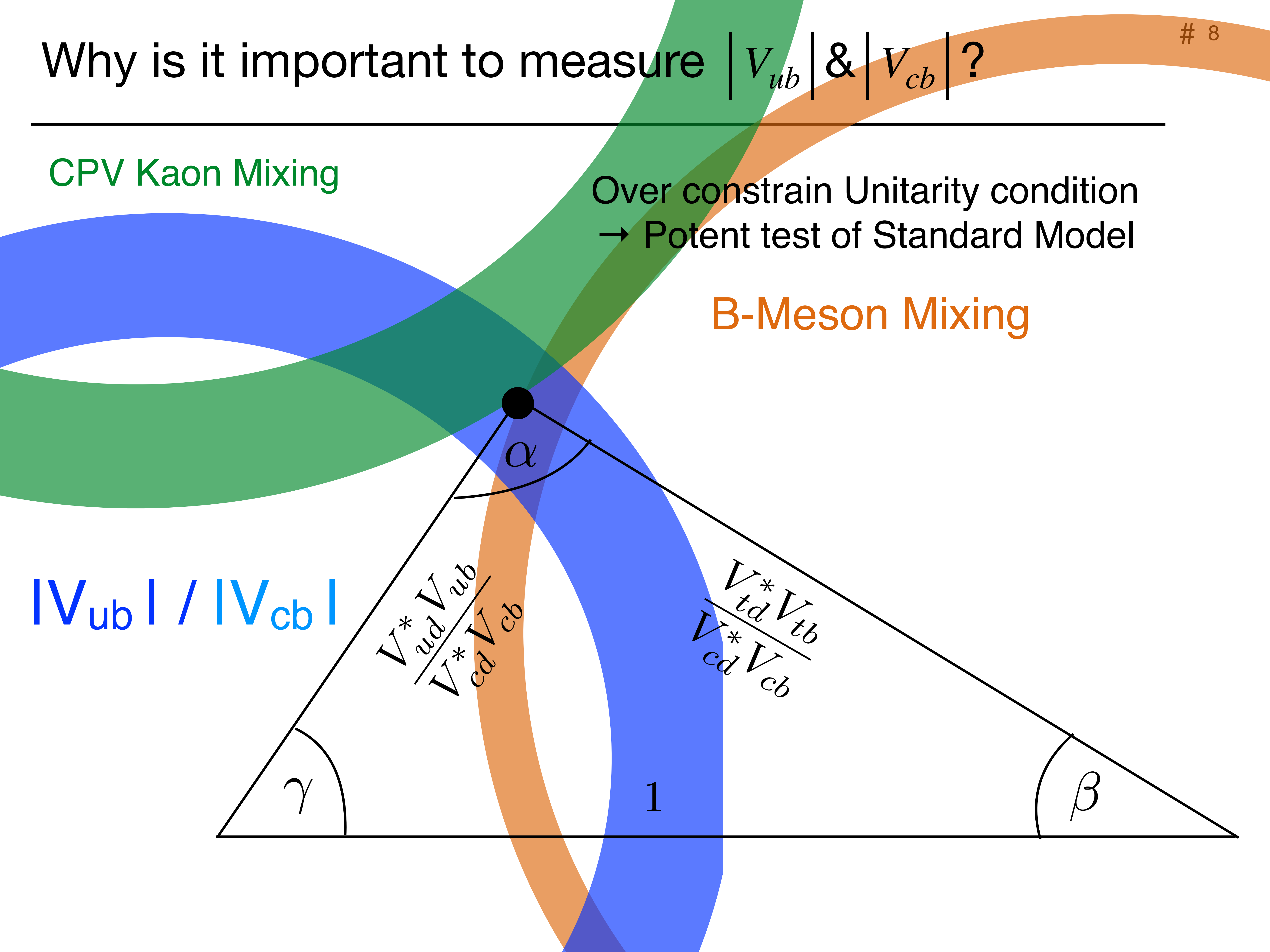


B-Meson Mixing

Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?



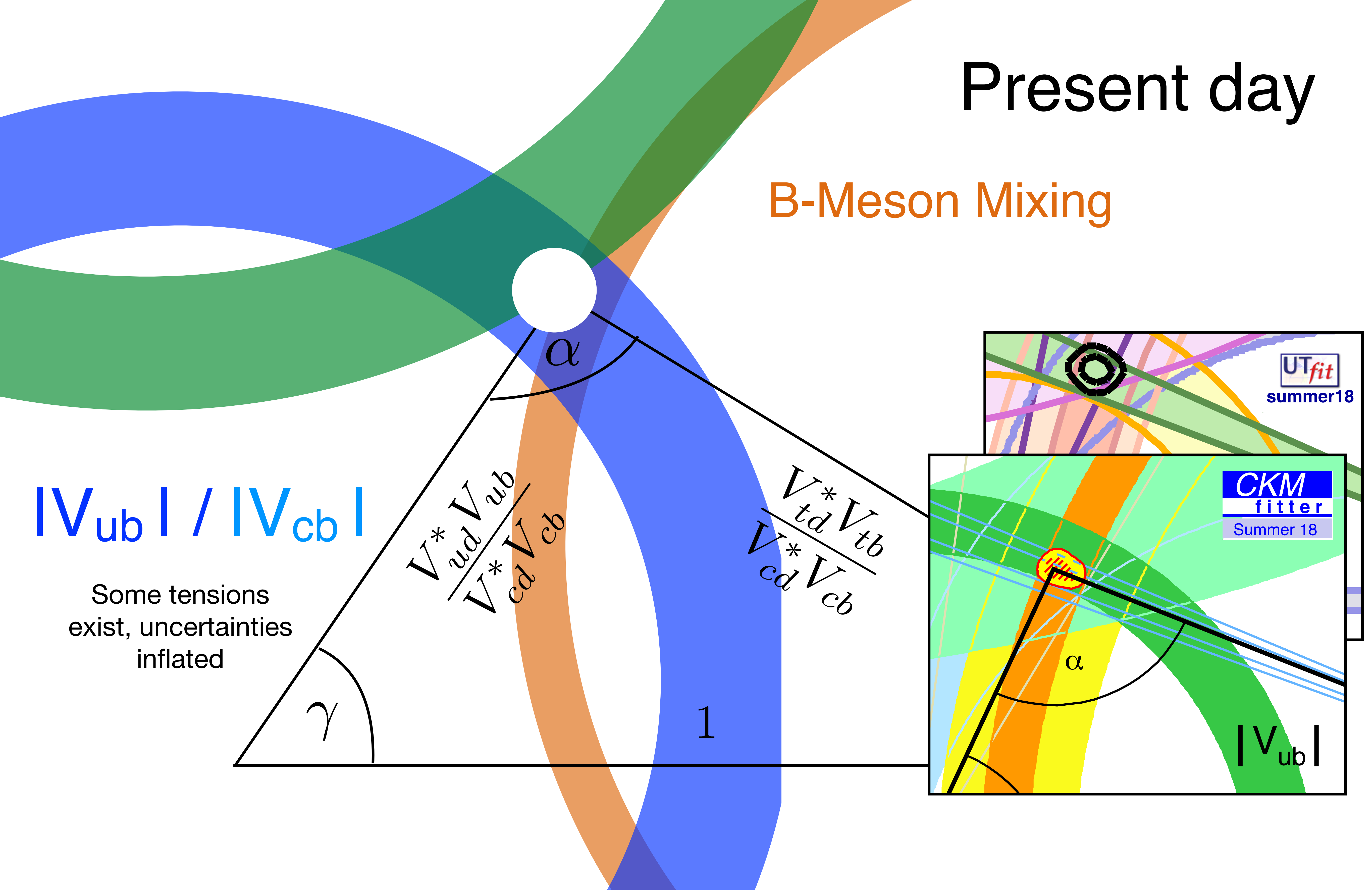
Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

Present day

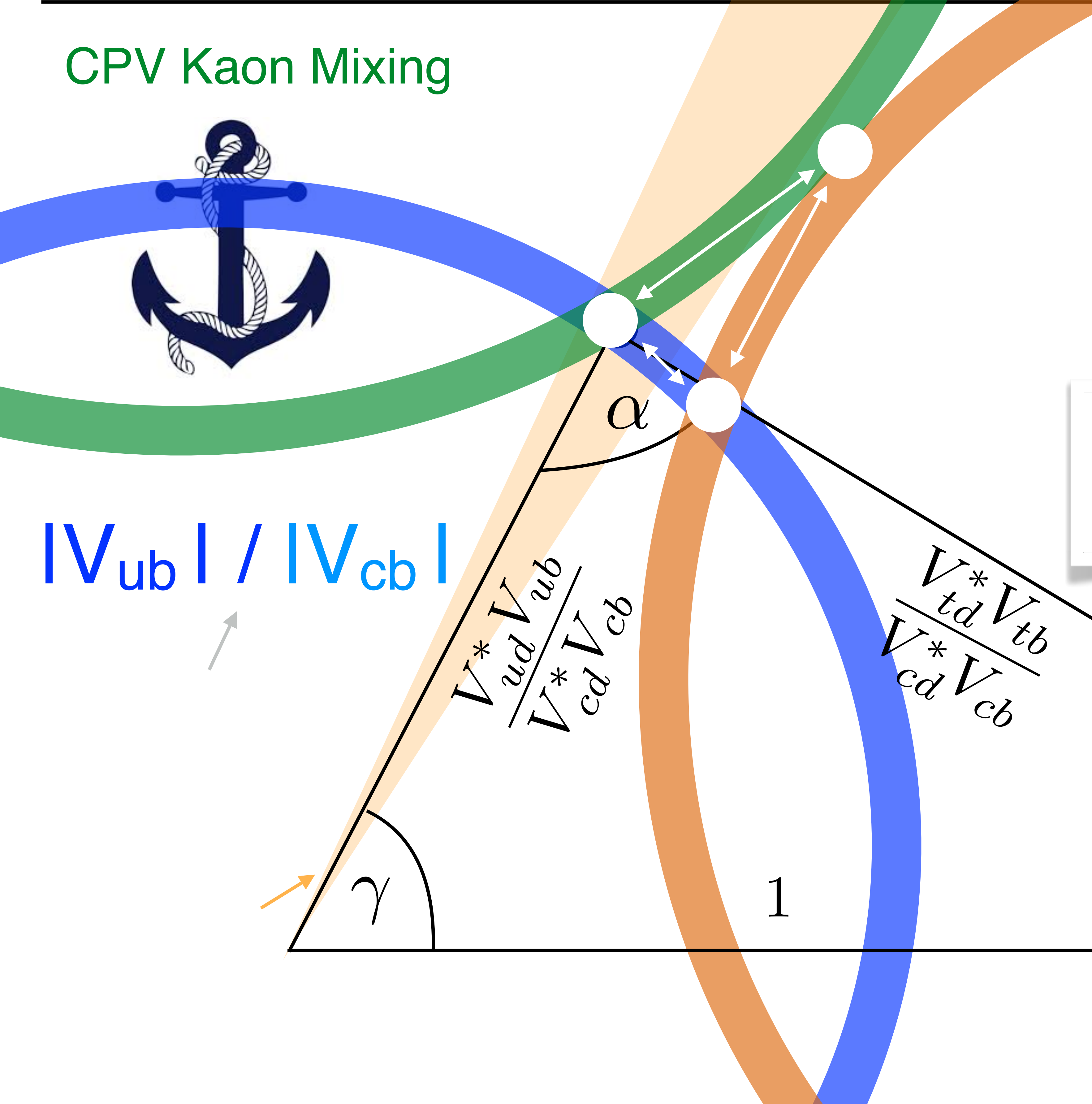
B-Meson Mixing

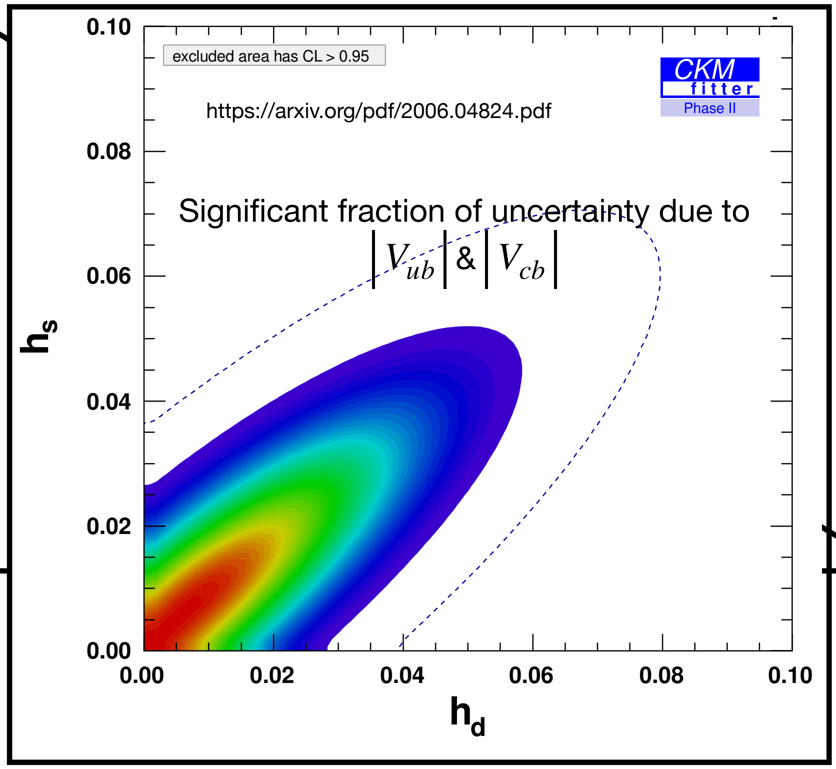
$|V_{ub}| / |V_{cb}|$

Some tensions exist, uncertainties inflated



Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

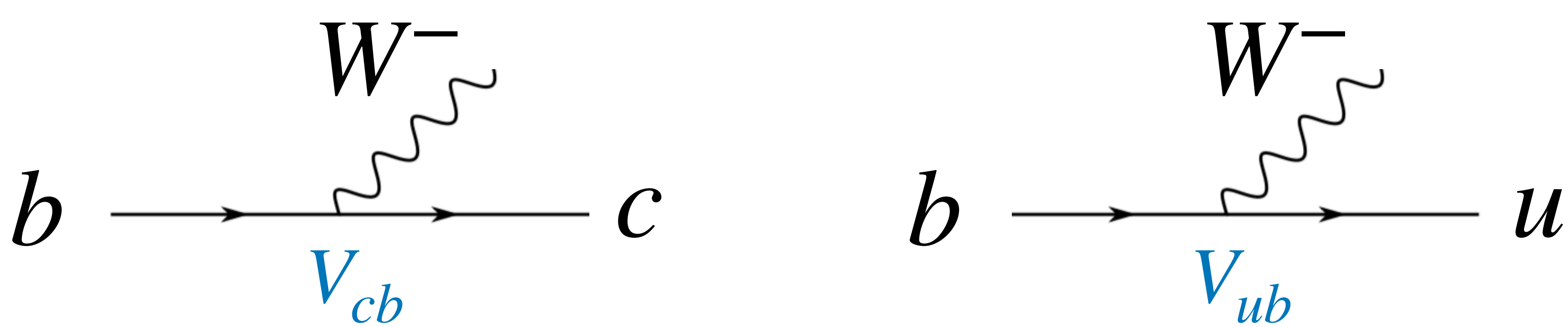




How do we determine $|V_{ub}|$ & $|V_{cb}|$?

At first glance fairly straightforward:

Step 1: Identify a process, in which you have a $b \to cW^-$ or $b \to uW^-$ vertex



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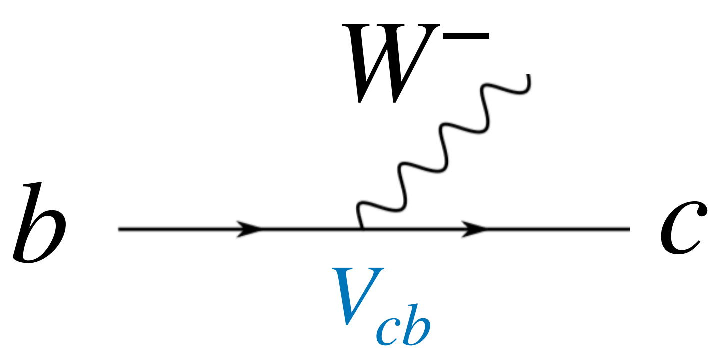
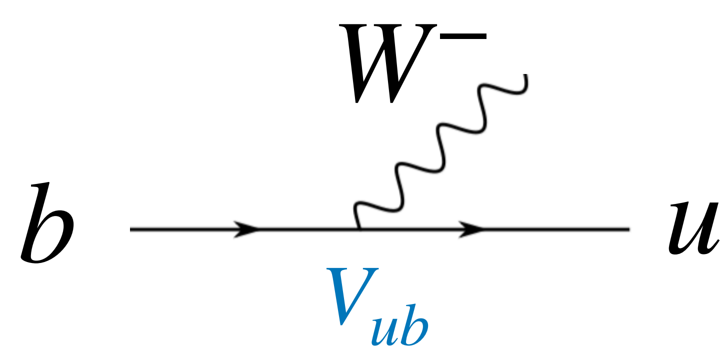




Step 2: Measure how often such a process occurs

$q = c \text{ or } u$

$$\mathcal{B}(b \to qW)$$

and compare this with the expectation from theory w/o CKM factors (or $V_{qb} = 1$)

Mathematically: $$\mathcal{B}(b \to qW) \propto |V_{qb}|^2$$

Predicted partial rate sans CKM factors (or with $V_{qb} = 1$) $$\Gamma(b \to qW)$$

Both quantities are connected as

$$|V_{qb}|^2 \frac{\Gamma(b \to qW)}{\Gamma(b \to \text{Everything})} = \mathcal{B}(b \to qW)$$

so we can solve this using $\tau_b = \hbar/\Gamma(b \to \text{Everything})$

$$|V_{qb}| = \sqrt{\frac{\mathcal{B}(b \to qW)}{\tau_b \Gamma(b \to qW)}}$$

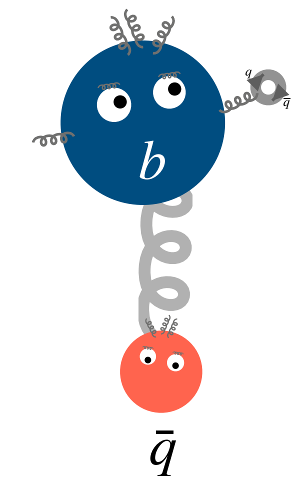
Measured by experiment

Predicted from theory

Great, now we only have to identify suitable processes for this:

1. Complication: Quarks are not free particles

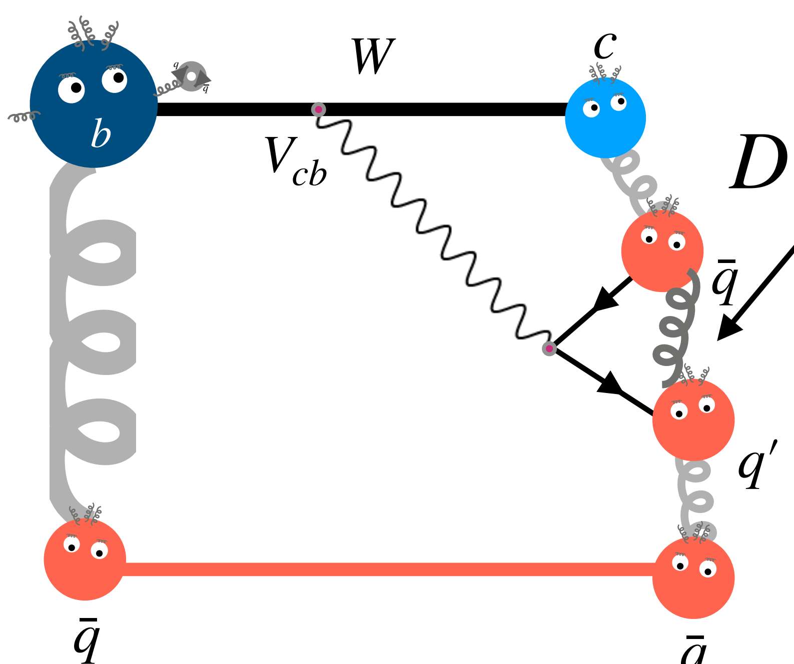
i.e. initial and final state quarks will be bound in hadrons (mesons or baryons)



2. Complication: We need a process, we can describe well from a theory point of view

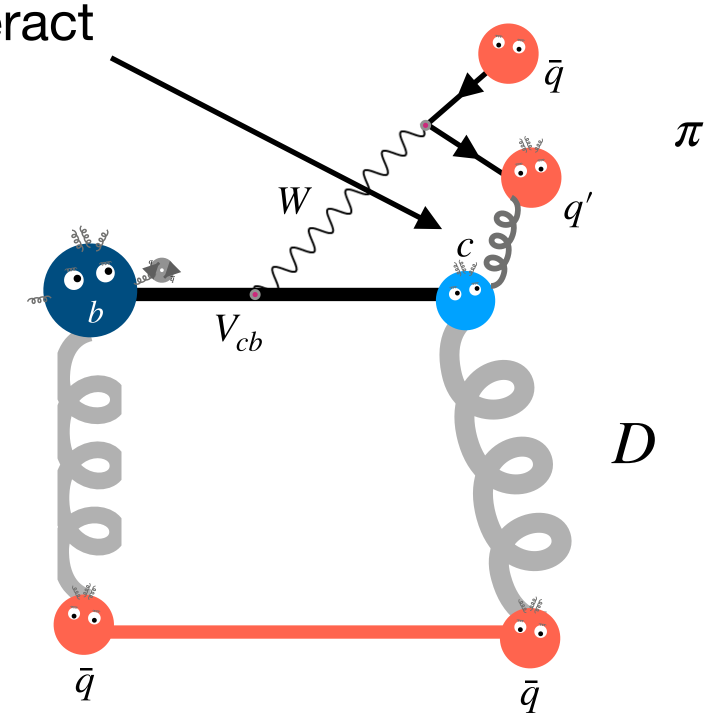
final states involving $W^- \to q\bar{q}'$ introduce additional CKM factors (a priori fine), but also have **color charged constituents**

i.e. final states can strongly interact



e.g. π

+ multiple diagrams



So what are the choices?

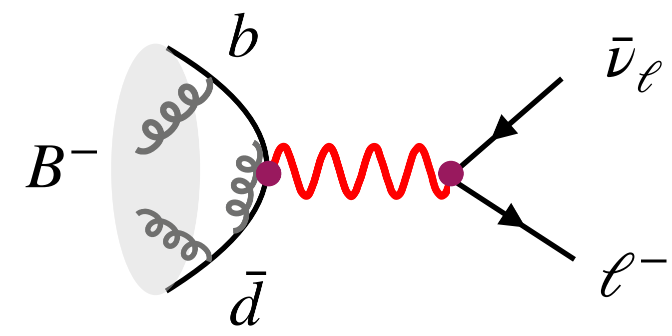
1) Hadronic decays → theory very hard, experimentally "easy"

2) Leptonic decays

→ theory "easy"
experimentally very hard

$$\mathscr{B}(B \to \mu \bar{\nu}_\mu) \sim 10^{-7}$$

$$\mathscr{B}(B \to \tau \bar{\nu}_\tau) \sim 10^{-4}$$



3) Semileptonic decays → theory doable, experimentally doable

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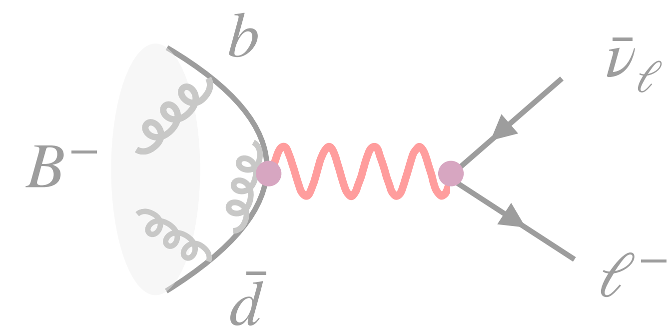

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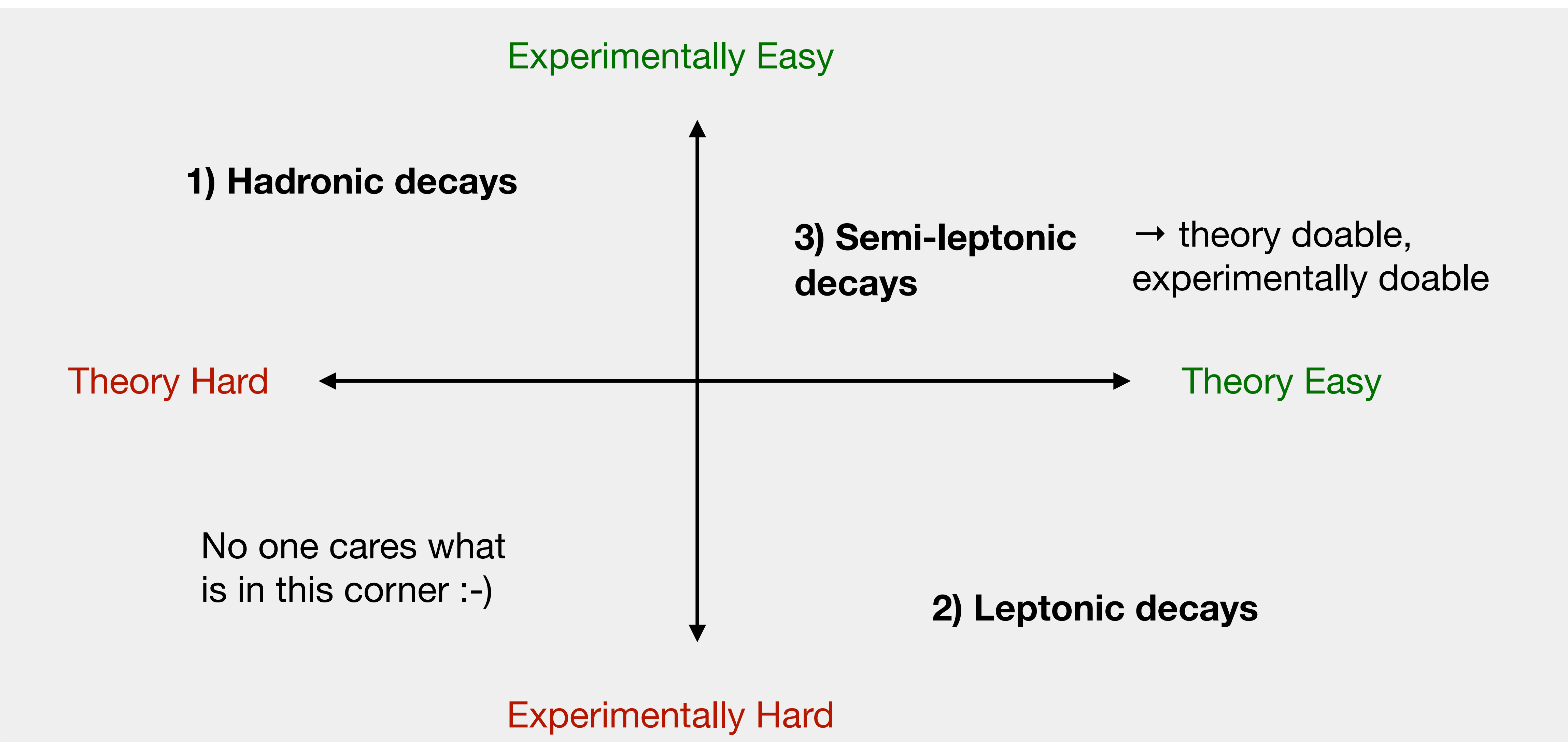
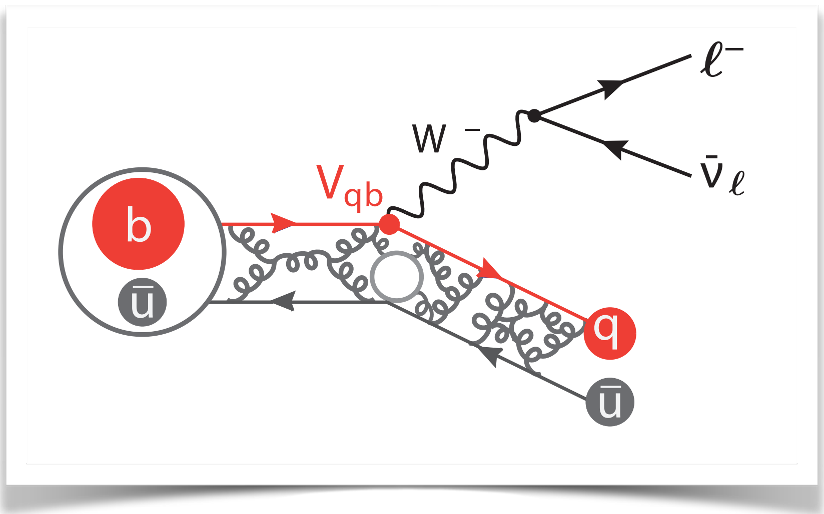


A quick boot-camp: how do we determine $|V_{ub}|$ & $|V_{cb}|$?



Inclusive |Vub|

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

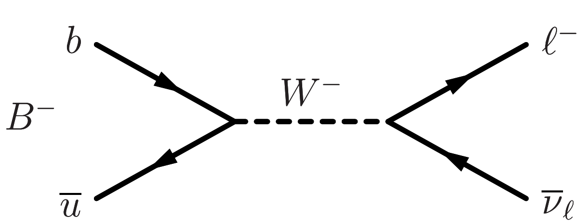
Inclusive |Vcb|

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

'Leptonic' |Vub|



$$\mathcal{B} \propto |V_{ub}|^2 \, f_B^2 \, m_\ell^2$$

B-Meson decay constant

Exclusive |Vub|

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

Exclusive |Vcb|

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

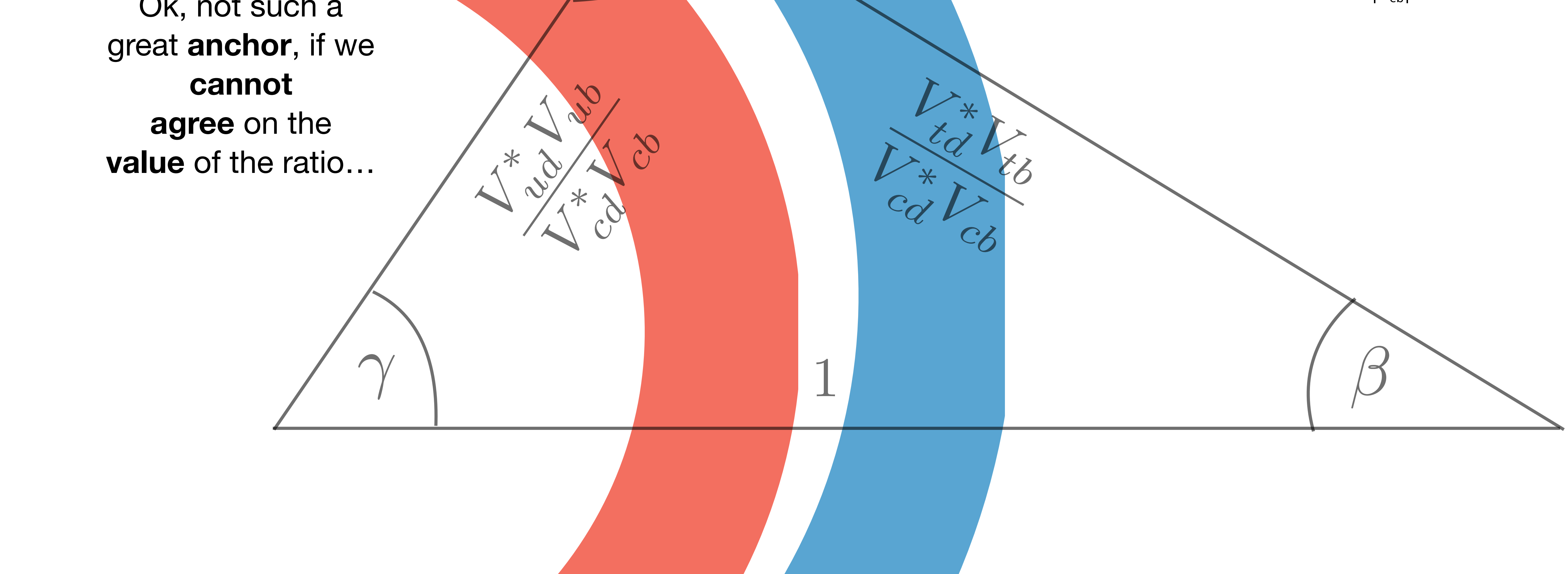
$$\mathcal{B} \propto |V_{cb}|^2 \, f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$

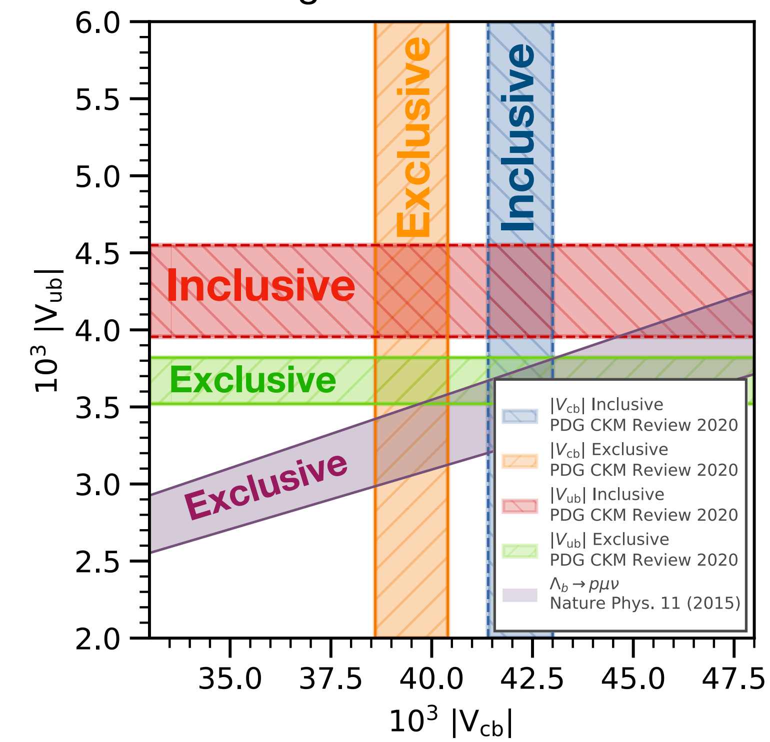
Why is it important to measure $|V_{ub}|$ and $|V_{cb}|$?

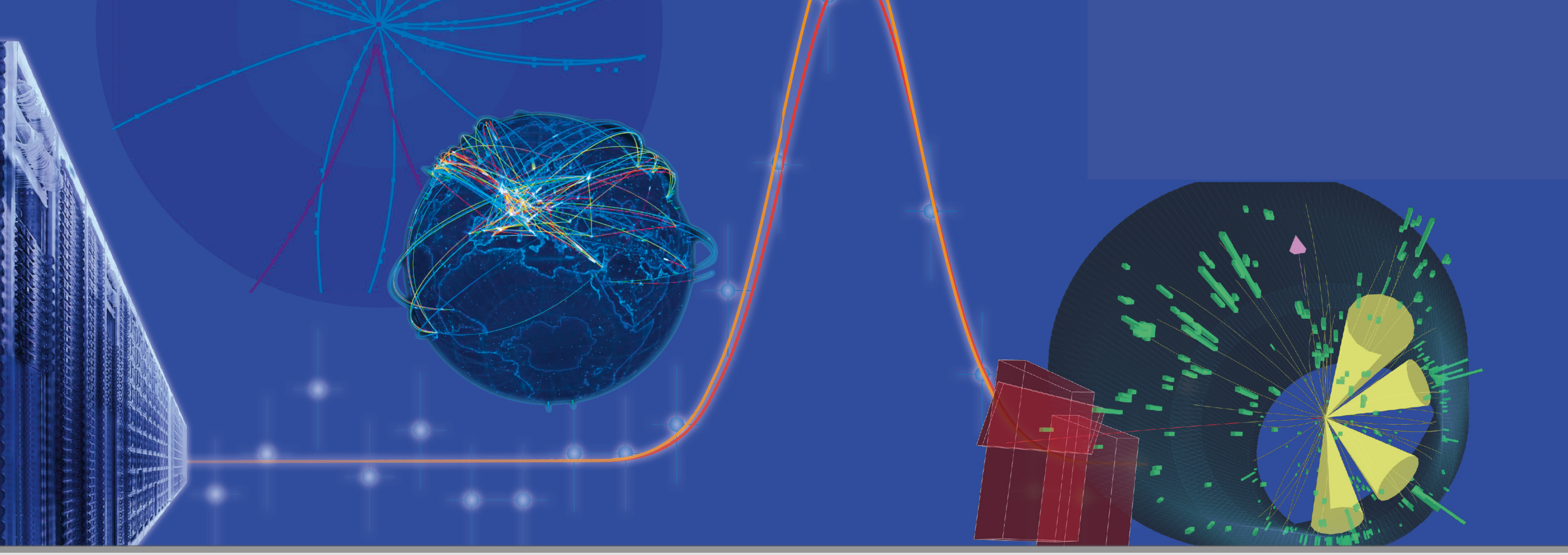
Present day dilemma



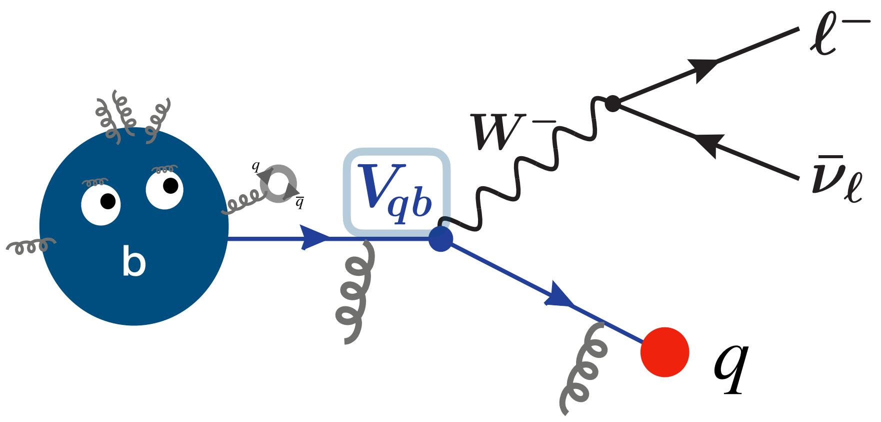
Ok, not such a great **anchor**, if we **cannot agree** on the **value** of the ratio…

Image credit: Markus Prim

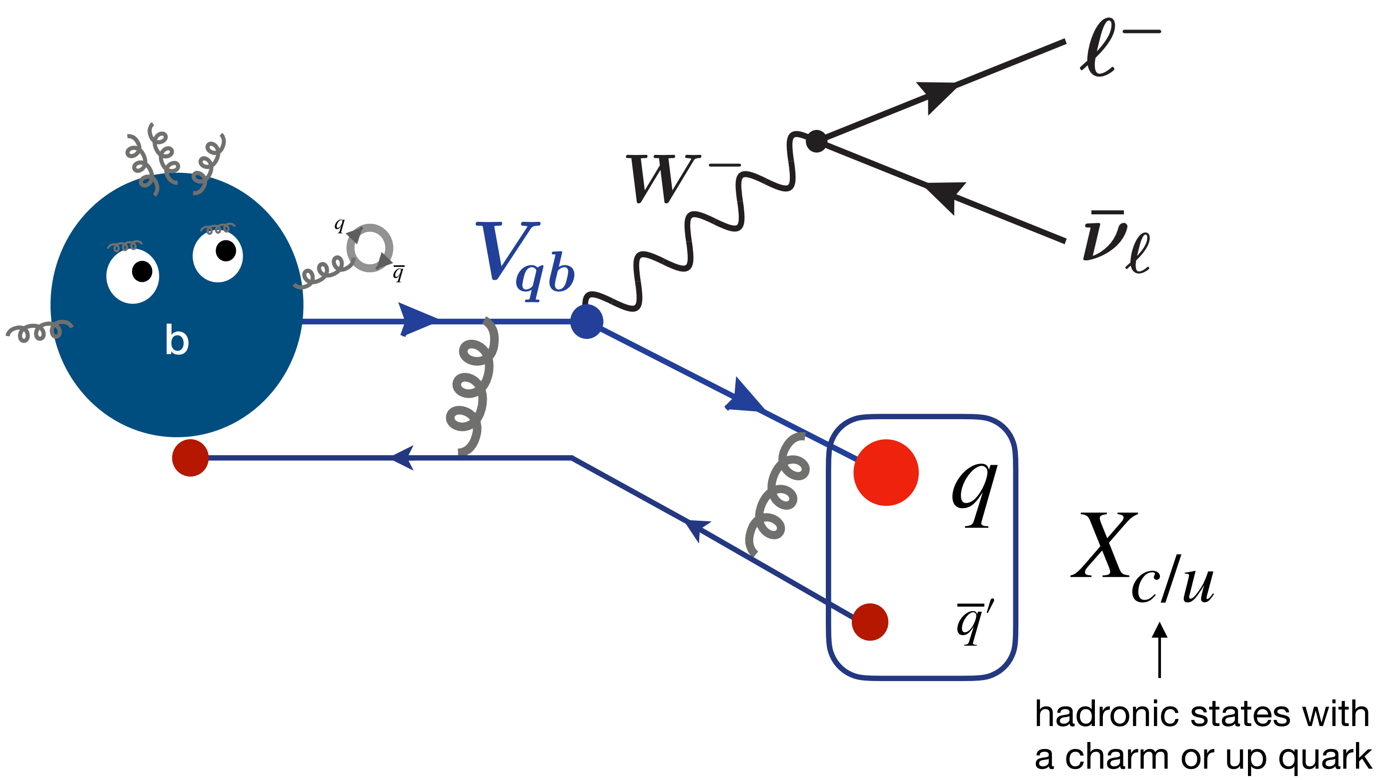




2) Kinematics



Let's first have a look at some of the kinematics



$$p_B = p_X + p_\ell + p_\nu$$

or

$$\begin{pmatrix} E_B \\ \mathbf{p}_B \end{pmatrix} = \begin{pmatrix} E_X \\ \mathbf{p}_X \end{pmatrix} + \begin{pmatrix} E_\ell \\ \mathbf{p}_\ell \end{pmatrix} + \begin{pmatrix} E_\nu \\ \mathbf{p}_\nu \end{pmatrix}$$

$$p_B^2 = m_B^2\,, \quad p_X^2 = m_X^2\,, \quad p_\ell^2 = m_\ell^2\,, \quad p_\nu^2 = 0$$

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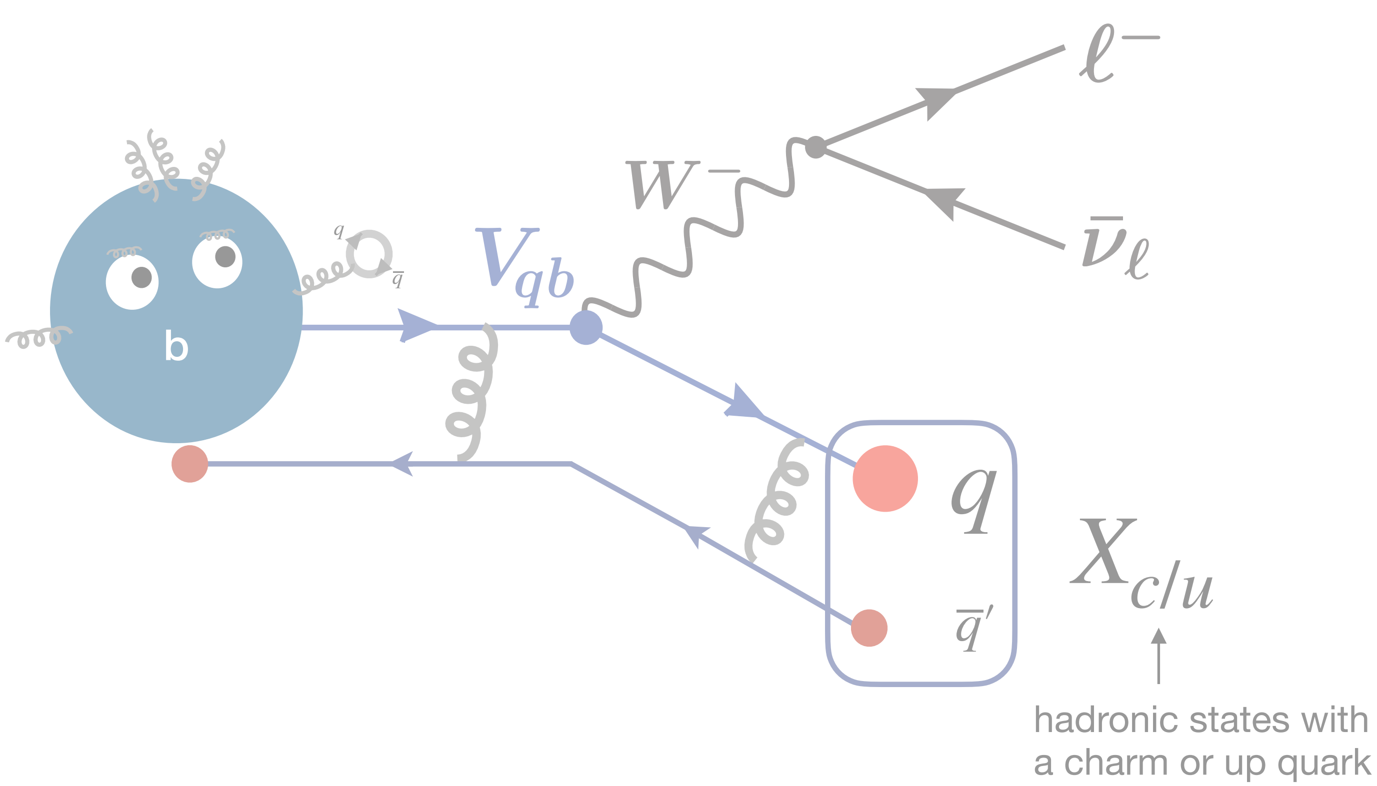

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$$p_B^2 = m_B^2\,, \quad p_X^2 = m_X^2\,, \quad p_\ell^2 = m_\ell^2\,, \quad p_\nu^2 = 0$$

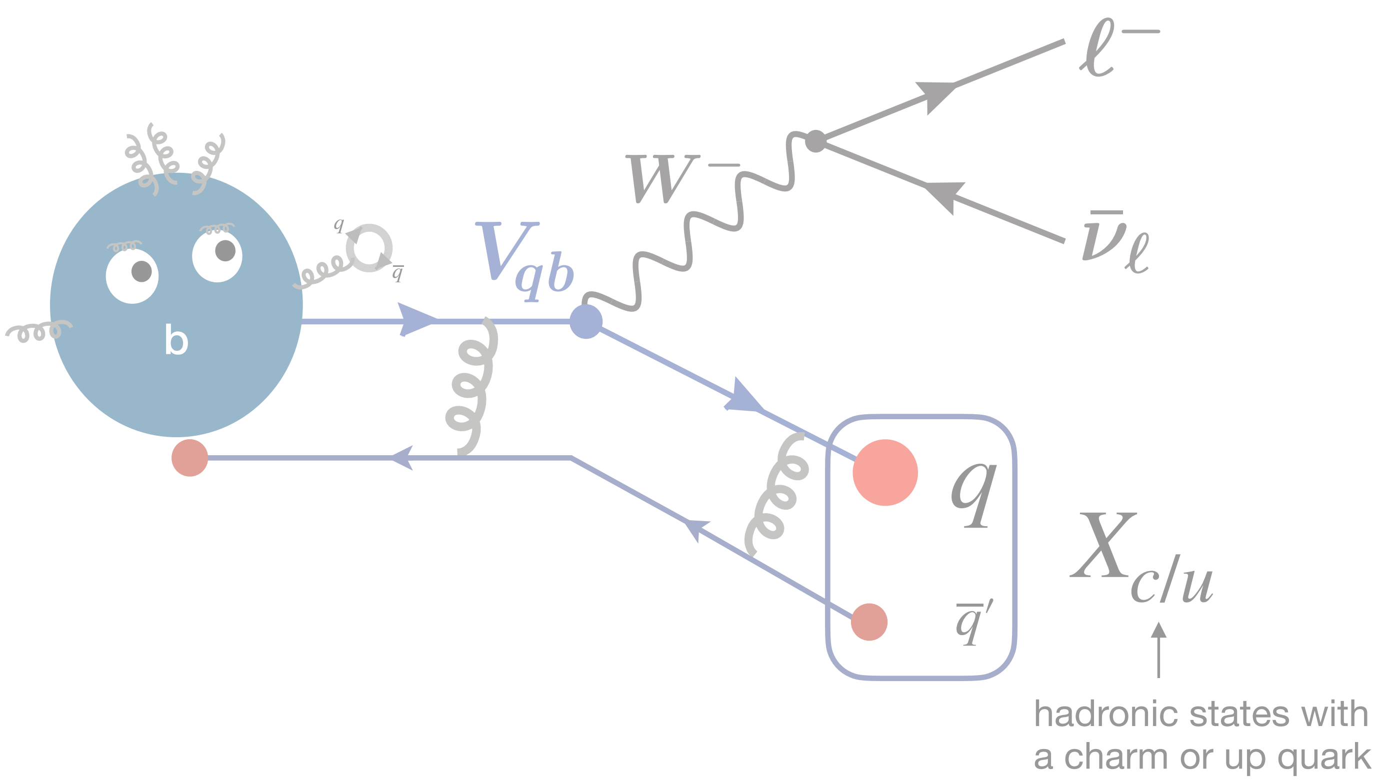
Let's assume we are in the rest frame of the B: $\begin{pmatrix} E_B \\ \mathbf{p}_B \end{pmatrix} = \begin{pmatrix} m_B \\ 0 \end{pmatrix}$

Which variables describe the final state?
Let's for now assume we look at a final state that is a resonance

$$X_c \in \{D, D^*, D^{**}, \dots\}$$

$$X_u \in \{\pi, \rho, f_0, \dots\}$$

Let’s first have a look at some of the kinematics



$$p_B = p_X + p_\ell + p_\nu$$

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$$p_B^2 = m_B^2\,, \quad p_X^2 = m_X^2\,, \quad p_\ell^2 = m_\ell^2\,, \quad p_\nu^2 = 0$$

If we look at final states with a **fixed mass** m_X , we can describe them with **two** kinematic quantities :

$$q^2 = (p_\ell + p_\nu)^2 = (p_B - p_X)^2$$

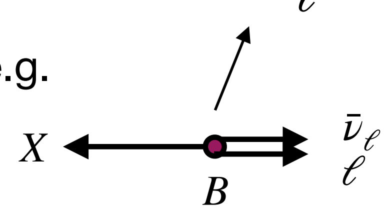
$$E_\ell = \frac{p_B p_\ell}{m_B}$$

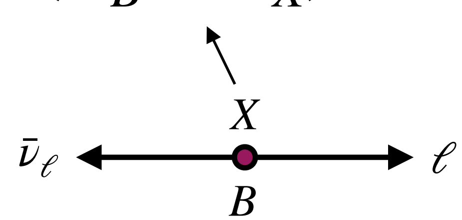
$q^2 : E_\ell$ **not** independent

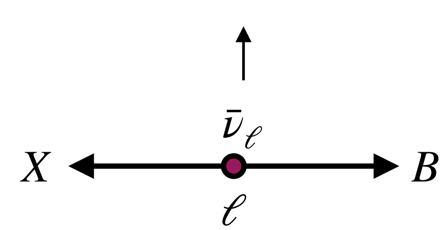
$$m_\ell^2 \leq q^2 \leq (m_B - m_X)^2$$

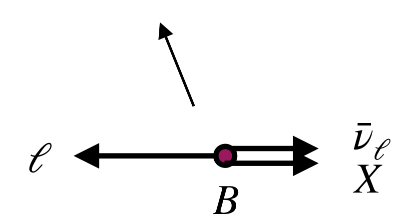
$$m_\ell \leq E_\ell \leq \frac{1}{2m_B}\left(m_B^2 - m_X^2 + m_\ell^2\right)$$

e.g.





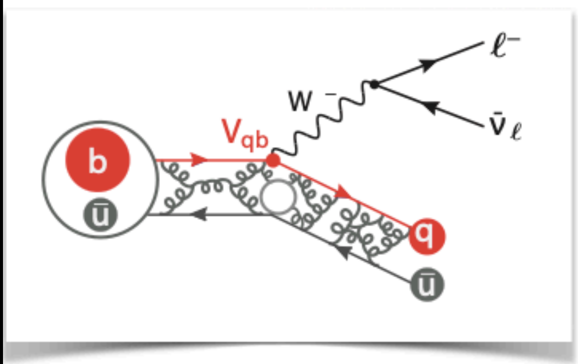




In the context of the **heavy-quark expansion**, it is convenient to introduce

All these quantities are useful, since they **encode** the **non-perturbative decay dynamics**, i.e. you can combine **differential shapes** (or **moments of differential spectra**) with predictions from theory to determine or constrain non-perturbative QCD

A quick boot-camp: how do we determine $|V_{ub}|$ & $|V_{cb}|$



Inclusive $|V_{ub}|$

Inclusive $|V_{cb}|$

Measured Branching Fraction

$$|V_{qb}| = \sqrt{\frac{\mathcal{B}(\bar B \to X_q\,\ell\,\bar\nu_\ell)}{\tau\,\Gamma(\bar B \to X_q\,\ell\,\bar\nu_\ell)}}$$

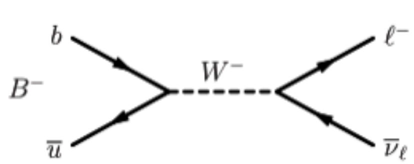
Prediction from **Theory but often also** constrained from **measured differential distributions**

Theory from non-perturbative Methods:

* Lattice QCD (high q²)
* QCD Sum rules (low q²)

$$q^2 = (p - p')^2$$

'Leptonic' $|V_{ub}|$

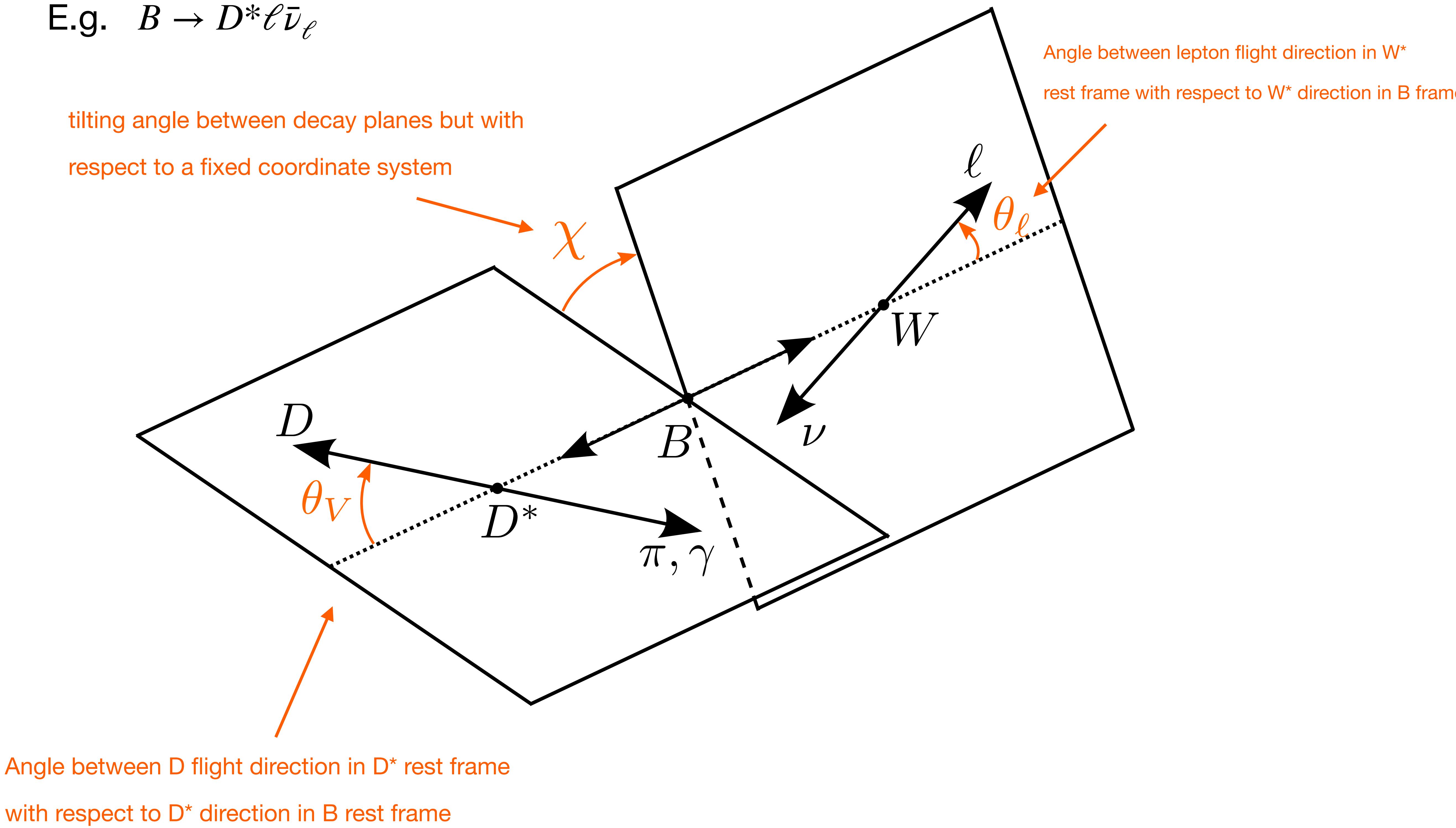


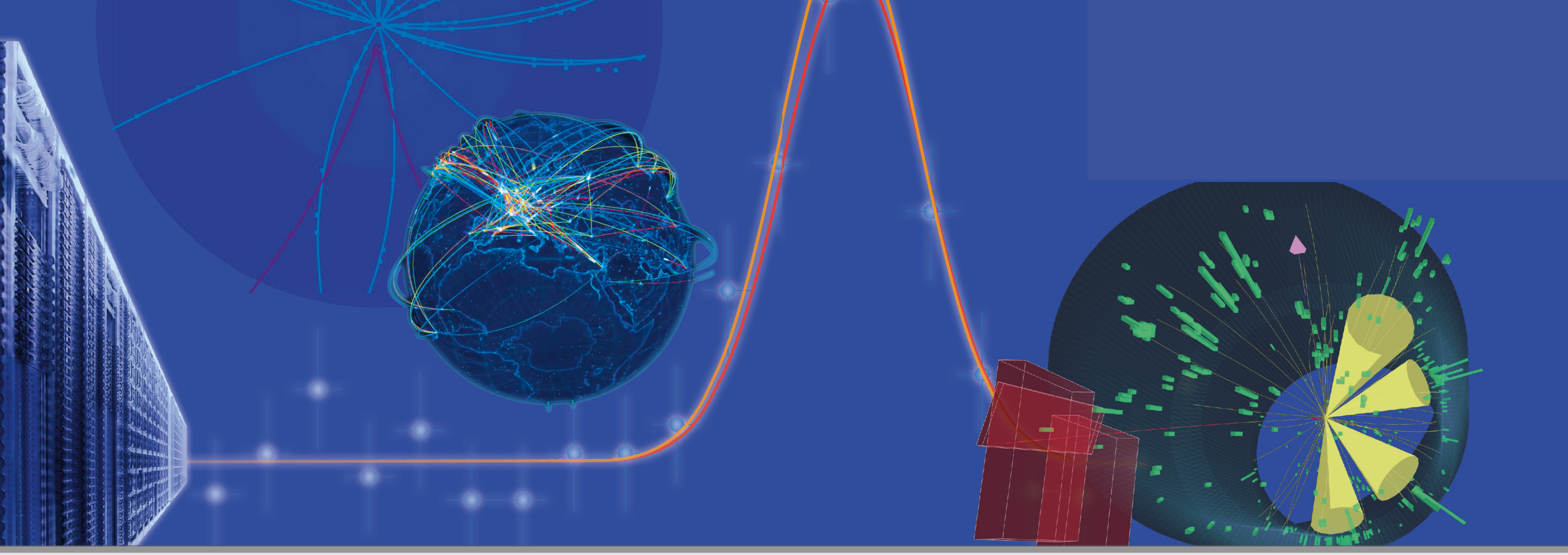
$$\mathcal{B} \propto |V_{ub}|^2\, f_B^2\, m_\ell^2$$

B-Meson decay constant

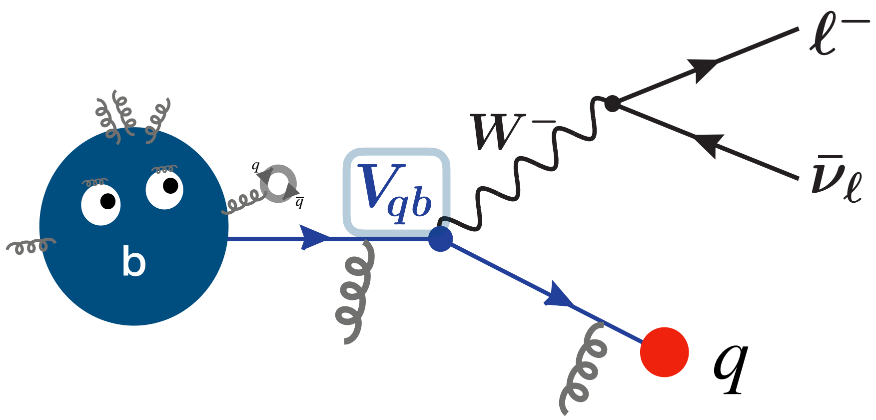
If the final state meson carries spin, **information** is also encoded into the **decay angles**

E.g. $B \to D^* \ell \bar{\nu}_\ell$

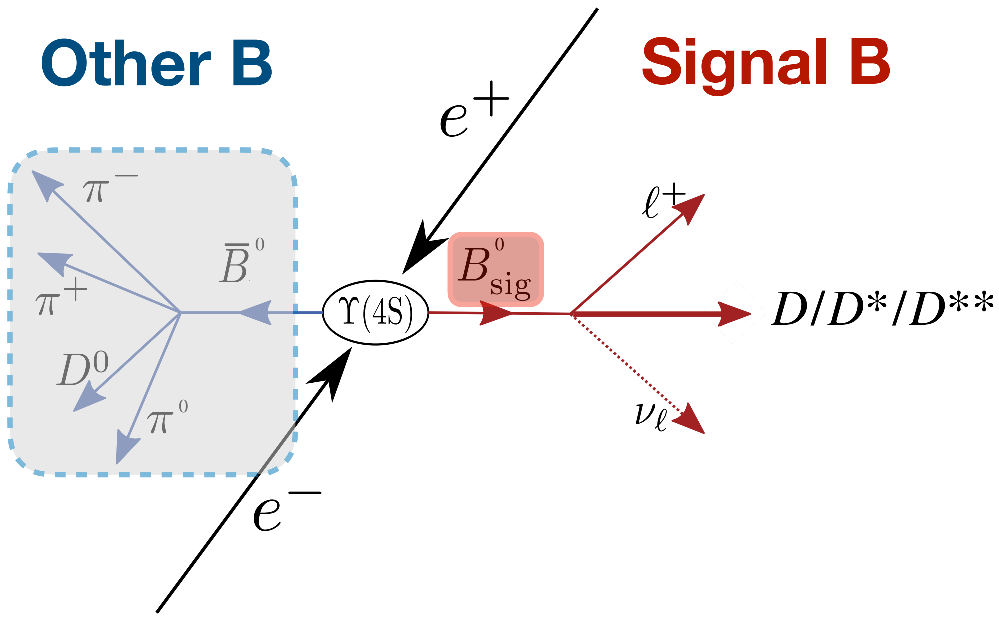




3) Example Measurements

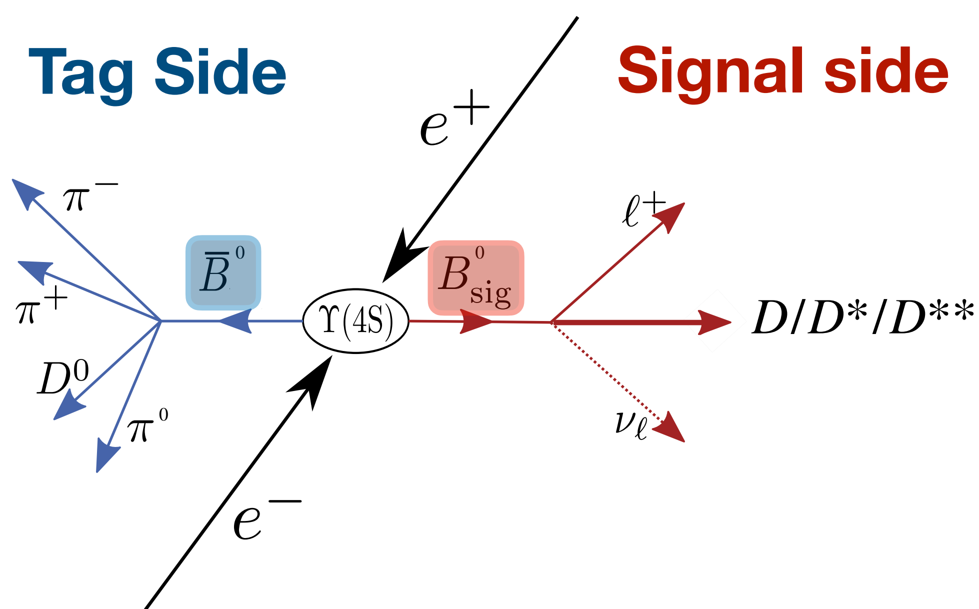


Let's strategize



+ Very high efficiency
+ Measurement of absolute branching fractions straightforward
  (depends on total # of $N_{B\bar{B}}$, understanding efficiencies)

- Less experimental control, e.g. more background from $e^+e^+ \to q\bar{q}$
- Cannot directly access signal B rest frame, need tricks



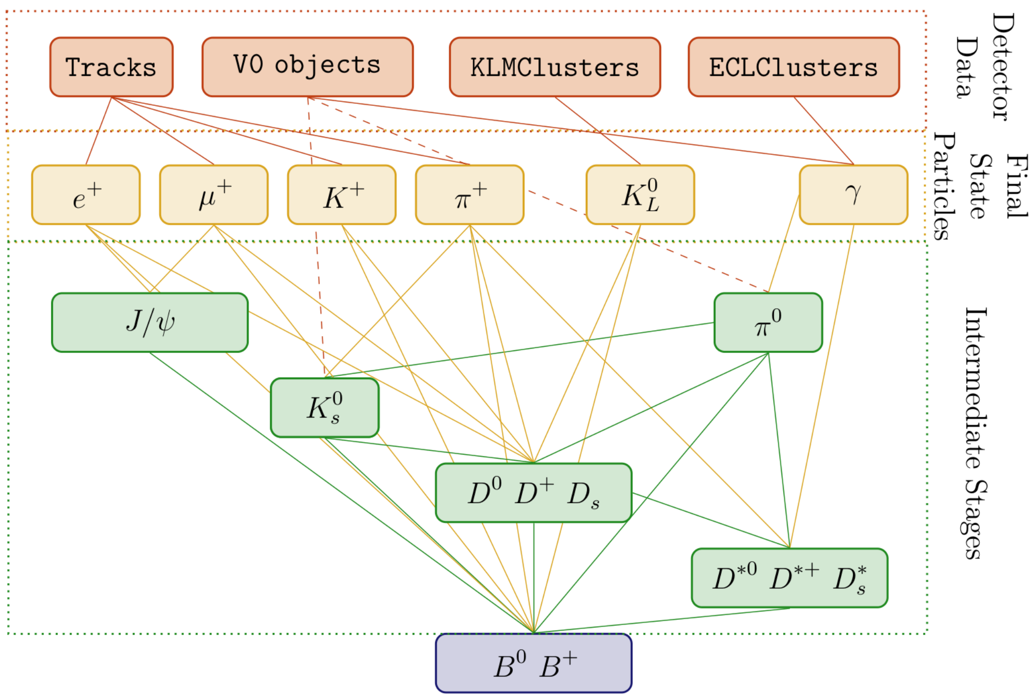
$$p_{sig} = p_{e^+e^-} - p_{tag}$$

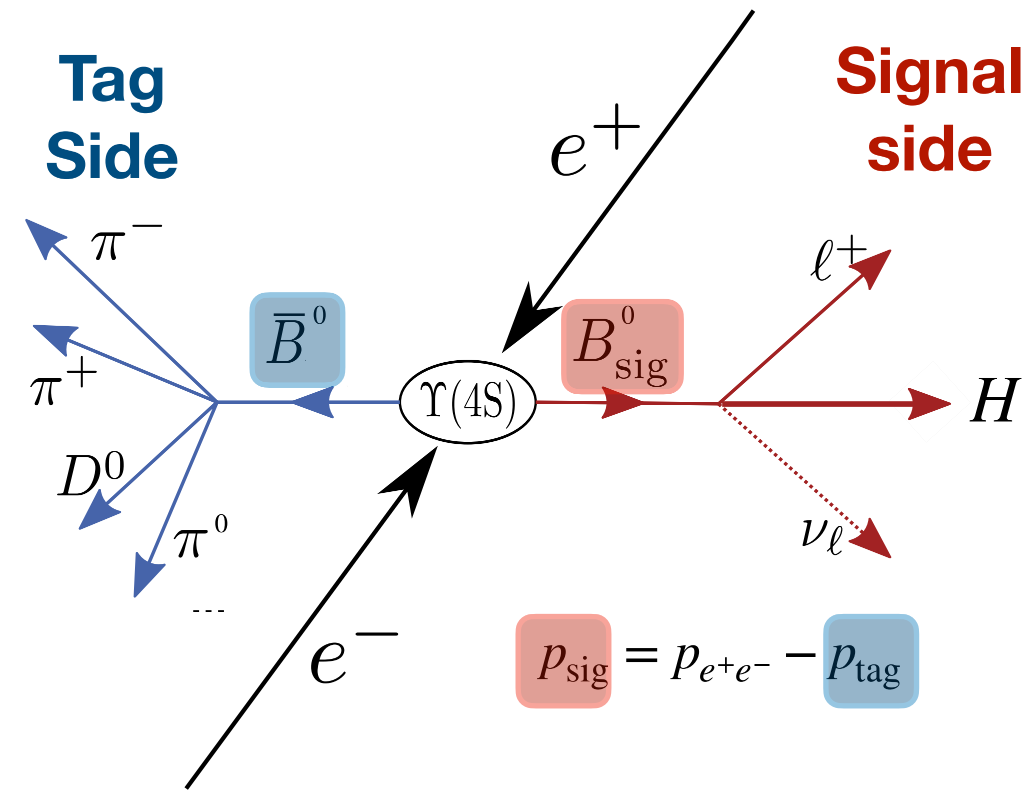
+ High degree of experimental control,
  e.g. can identify all final state particles with either the signal or the tag side
+ If hadronic modes for tagging are used, can reconstruct B rest frame

- Understanding efficiencies is difficult
- Low efficiency reduces the effective statistical power

Tagging in a nutshell

https://arxiv.org/abs/1807.08680





Candidates reconstructed with **hierarchical** approach via e.g. **neural networks (FR) or boosted decision trees (FEI)**

Over 10'000 decay cascades with an **efficiency** of **0.28% / 0.18%** for $B^{\pm}$ and $B^0/\bar{B}^0$

E.g. train a classifier to identify correctly reconstructed electron candidates:

Input variables: all four momenta & particle identification scores

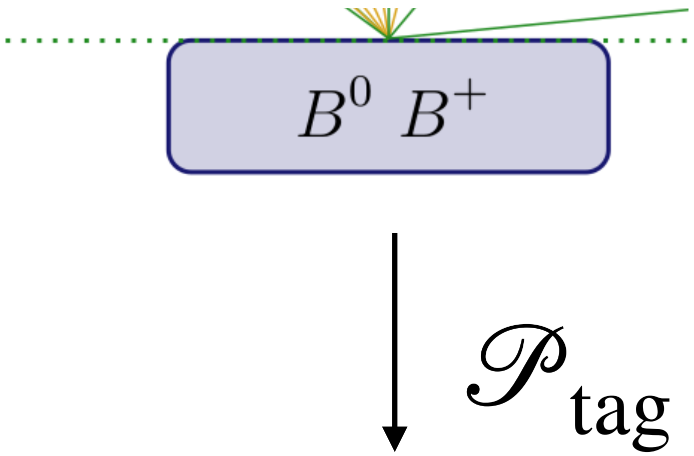
Output: Score $\mathcal{O}_e$

Apply mild selection on $\mathcal{O}_e$ to reduce # of candidate particles

Then train a classifier to identify correctly reconstructed J/ψ candidates

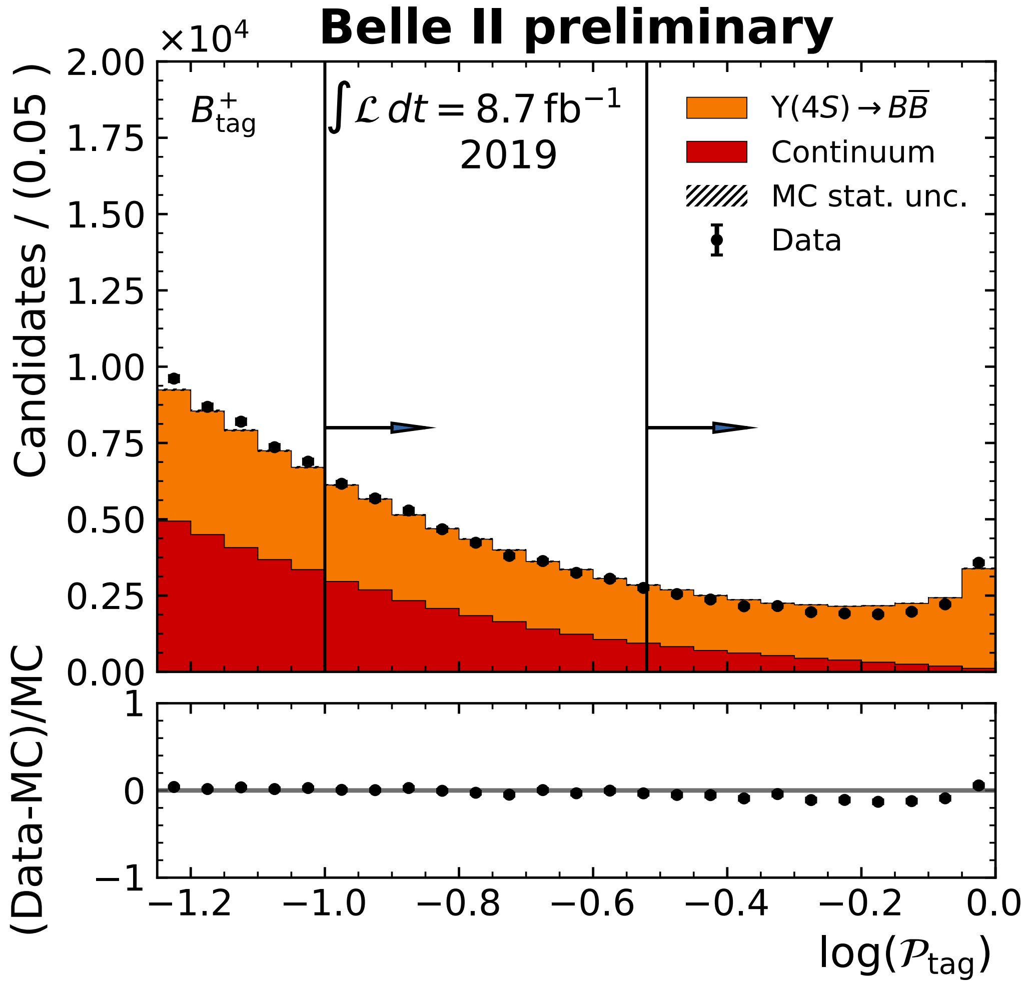
Input variables: all four momenta and output scores of previous layer

Output variable: $\mathcal{O}_{J/\psi}$ […]



$\mathscr{P}_{\text{tag}}$ =

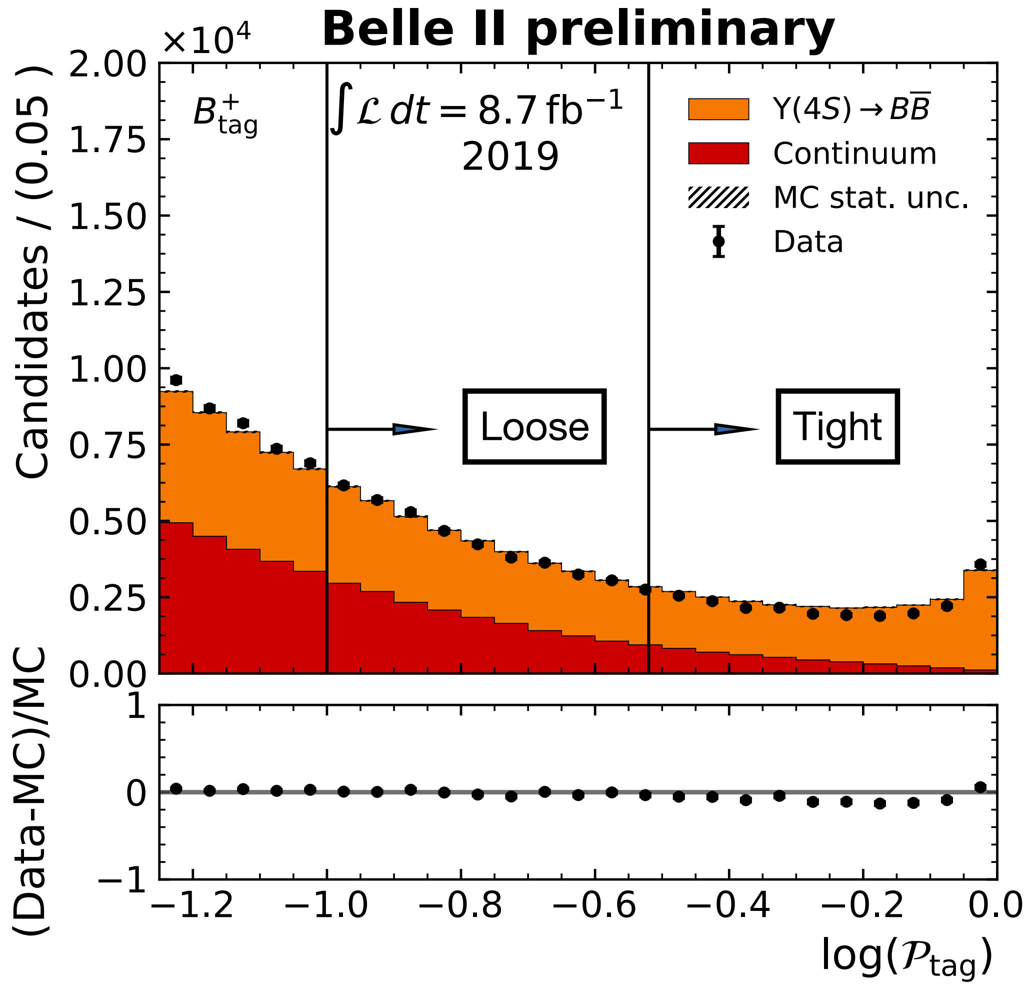
Output classifier = Measure of how well we reconstructed the B-Meson decay



beam constrained mass

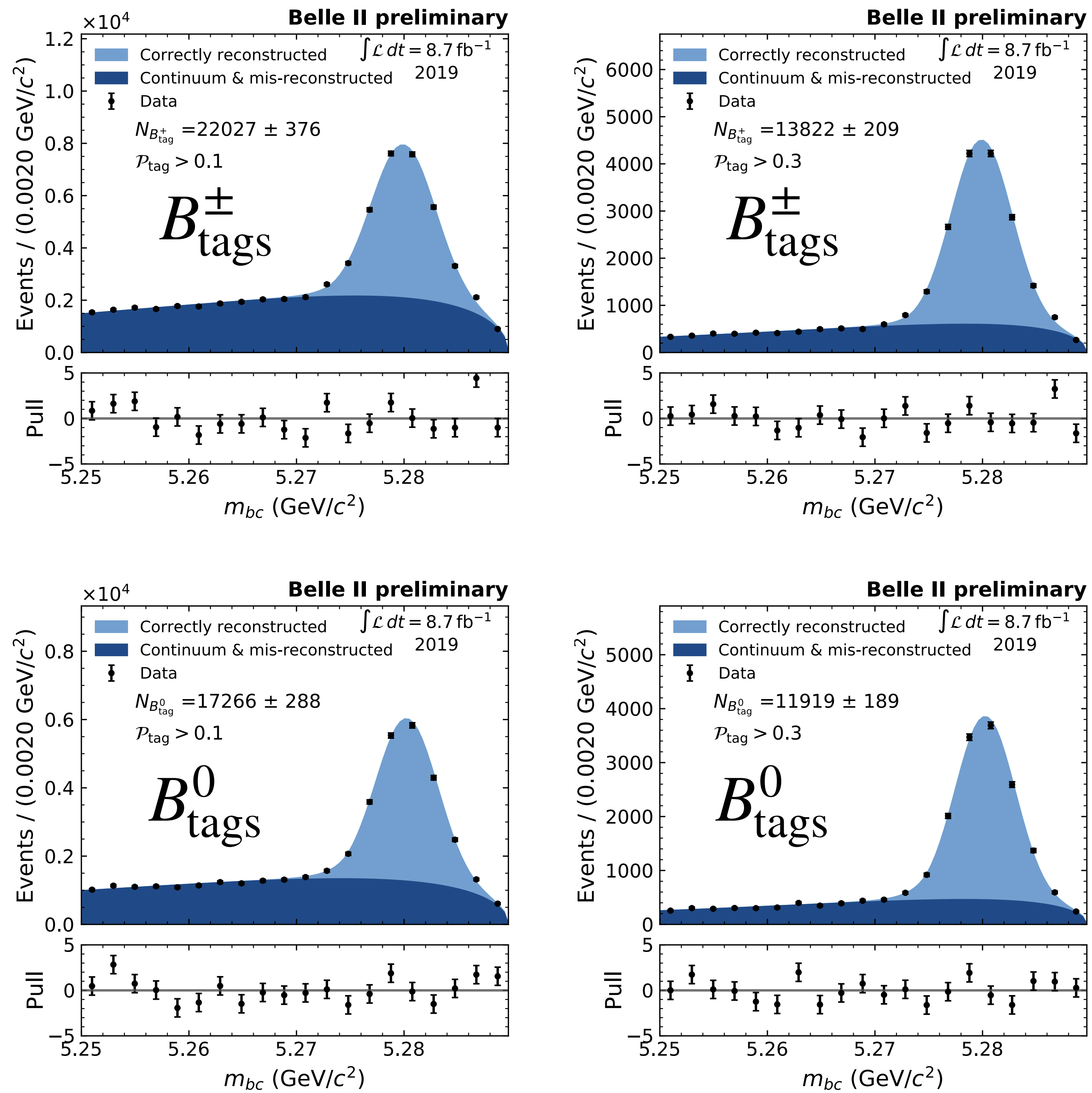
ca. 5.279 GeV

$$m_{bc} = \sqrt{E_{\text{beam}}^2/4 - |\vec{p}_{B_{\text{tag}}}|^2} \simeq m_B$$

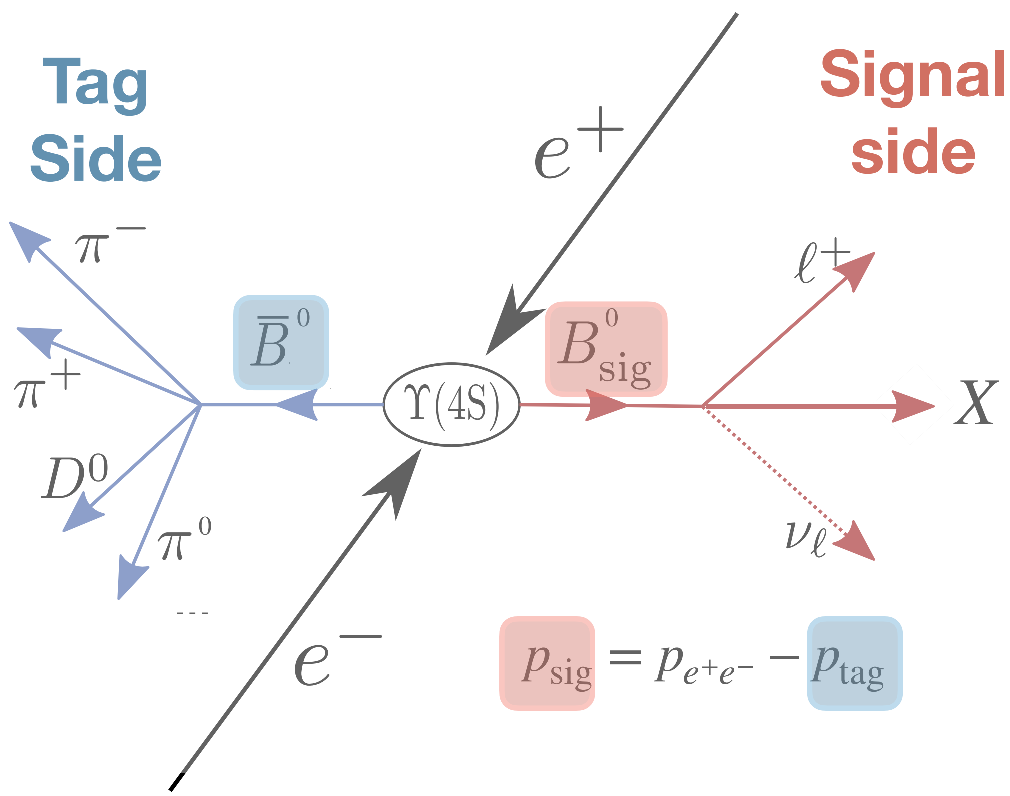


Loose Selection

Tight Selection



Efficiency can be calibrated,
but this has caveats



Why is the efficiency different? Use 10'000 different decays, use uncalibrated detector information, line-shapes differ in simulation
→ all aggregated in $\mathscr{P}_{\rm tag}$

Strategy: use a well measured process, add it to your MC with its measured BF and compare

$$\frac{N^{\rm Data}_{X\ell\bar{\nu}_\ell}}{N^{\rm MC}_{X\ell\bar{\nu}_\ell}}$$

Belle II Collaboration, BELLE2-CONF-PH-2020-005, [arXiv:2008.06096]

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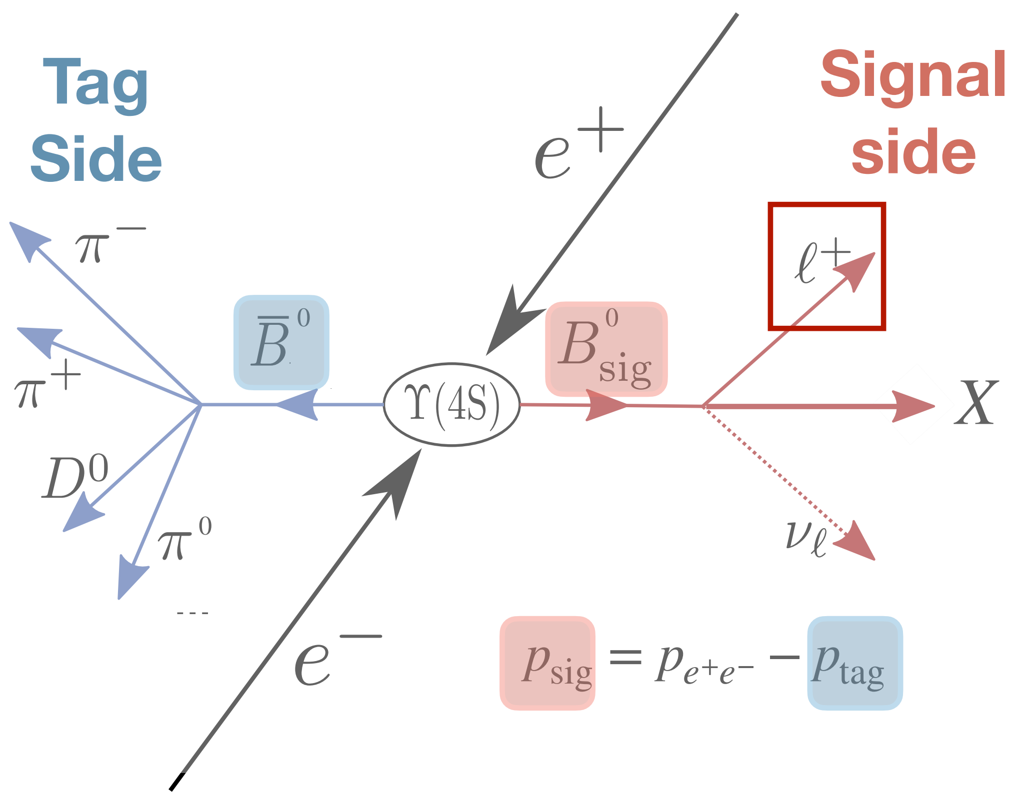
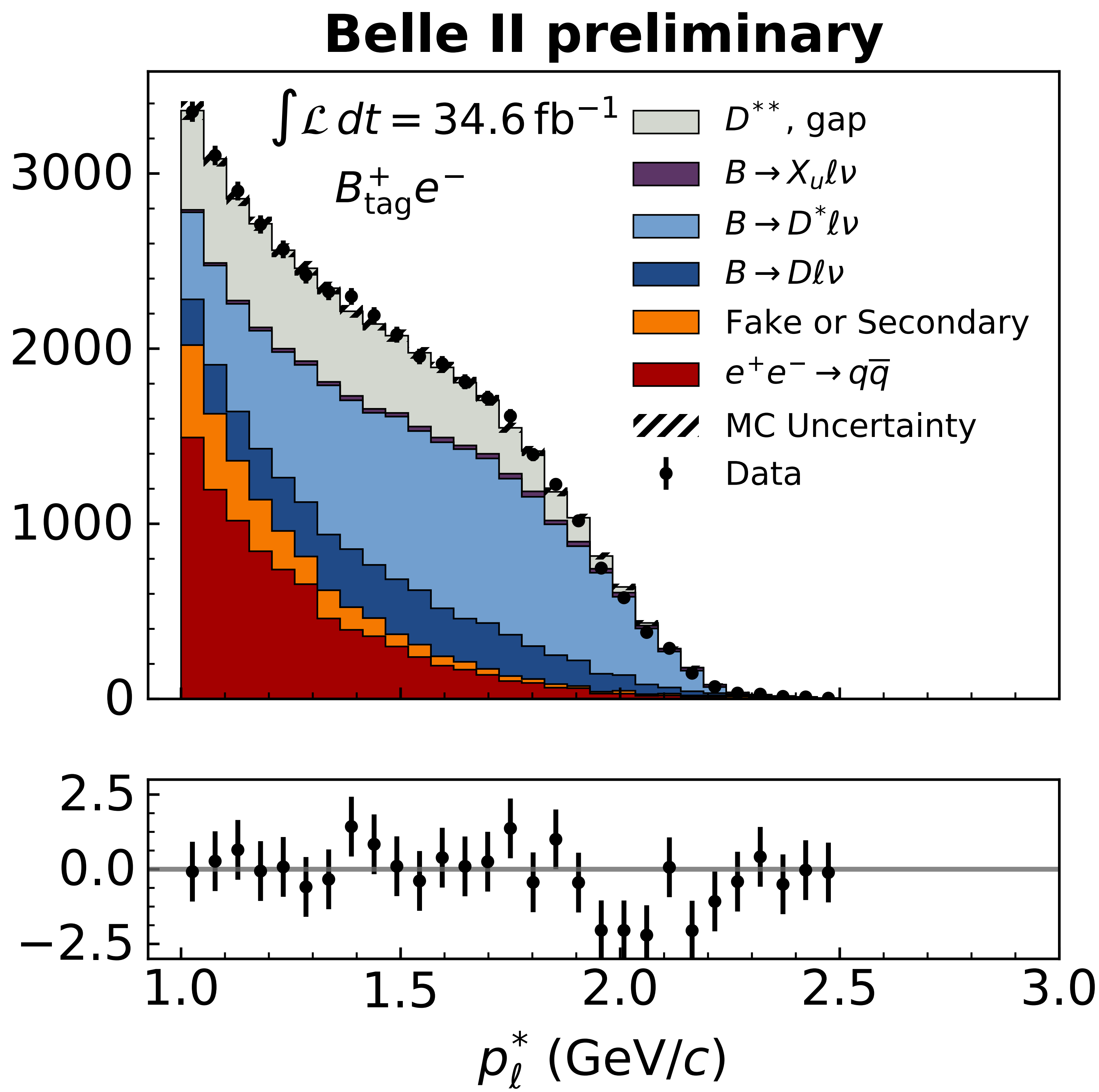


e.g.



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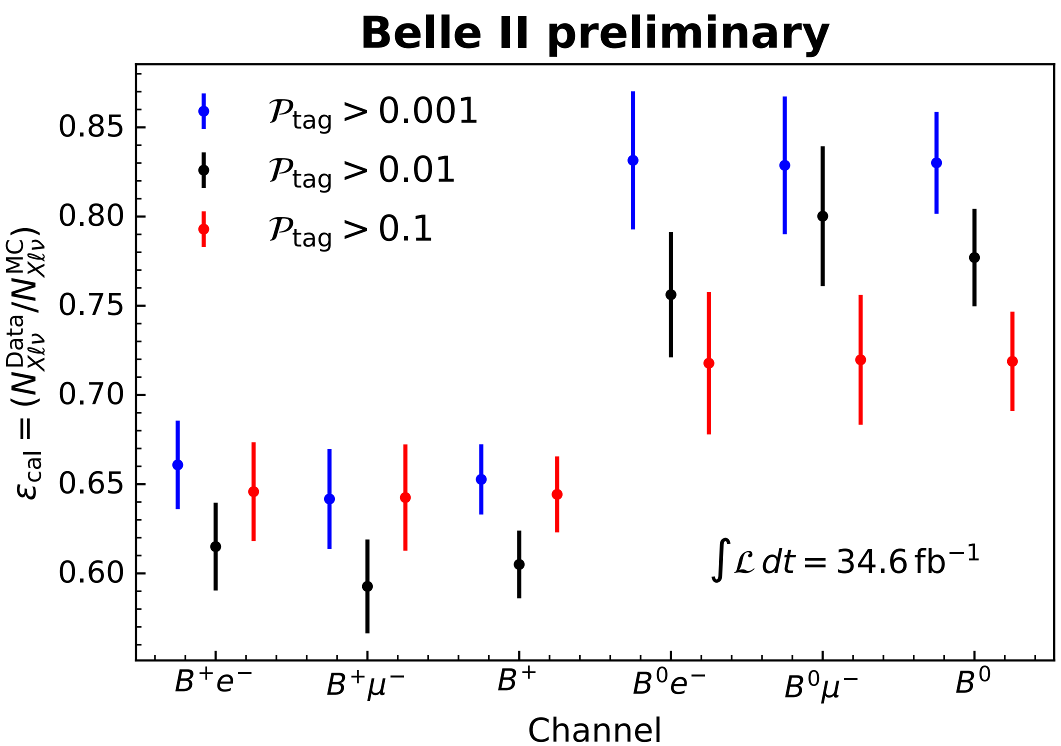
Belle II Collaboration, BELLE2-CONF-PH-2020-005, [arXiv:2008.06096]

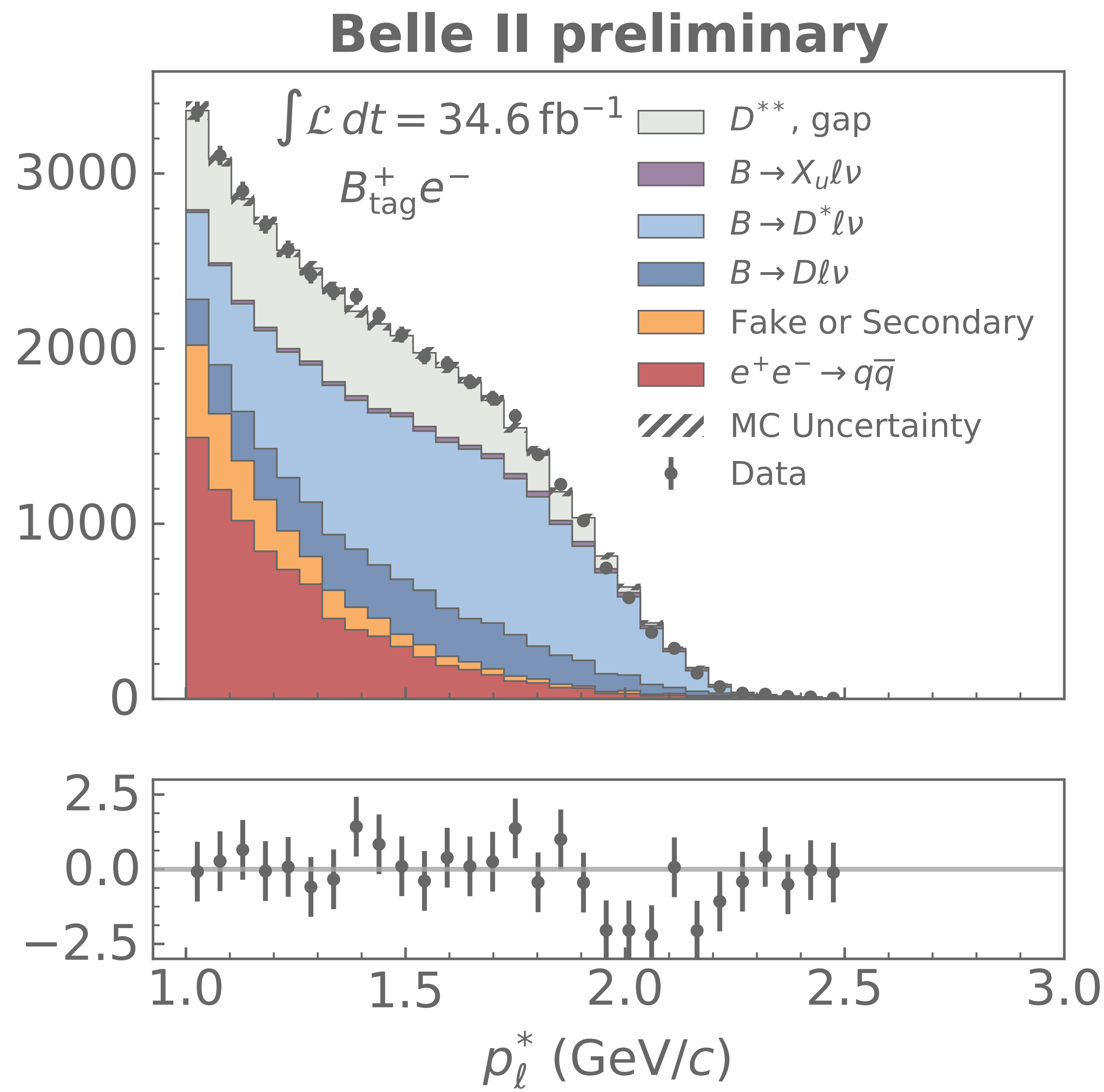
Efficiency Calibration

$$\epsilon_{\text{cal}} = \frac{N^{\text{Data}}_{X\ell\bar{\nu}_\ell}}{N^{\text{MC}}_{X\ell\bar{\nu}_\ell}}$$



B^+		
$\mathcal{P}_{\text{tag}} >$	ϵ	uncertainty [%]
0.001	0.65 ± 0.02	3.0
0.01	0.61 ± 0.02	3.1
0.1	0.64 ± 0.02	3.3

B^0		
$\mathcal{P}_{\text{tag}} >$	ϵ	uncertainty [%]
0.001	0.83 ± 0.03	3.4
0.01	0.78 ± 0.03	3.5
0.1	0.72 ± 0.03	3.9

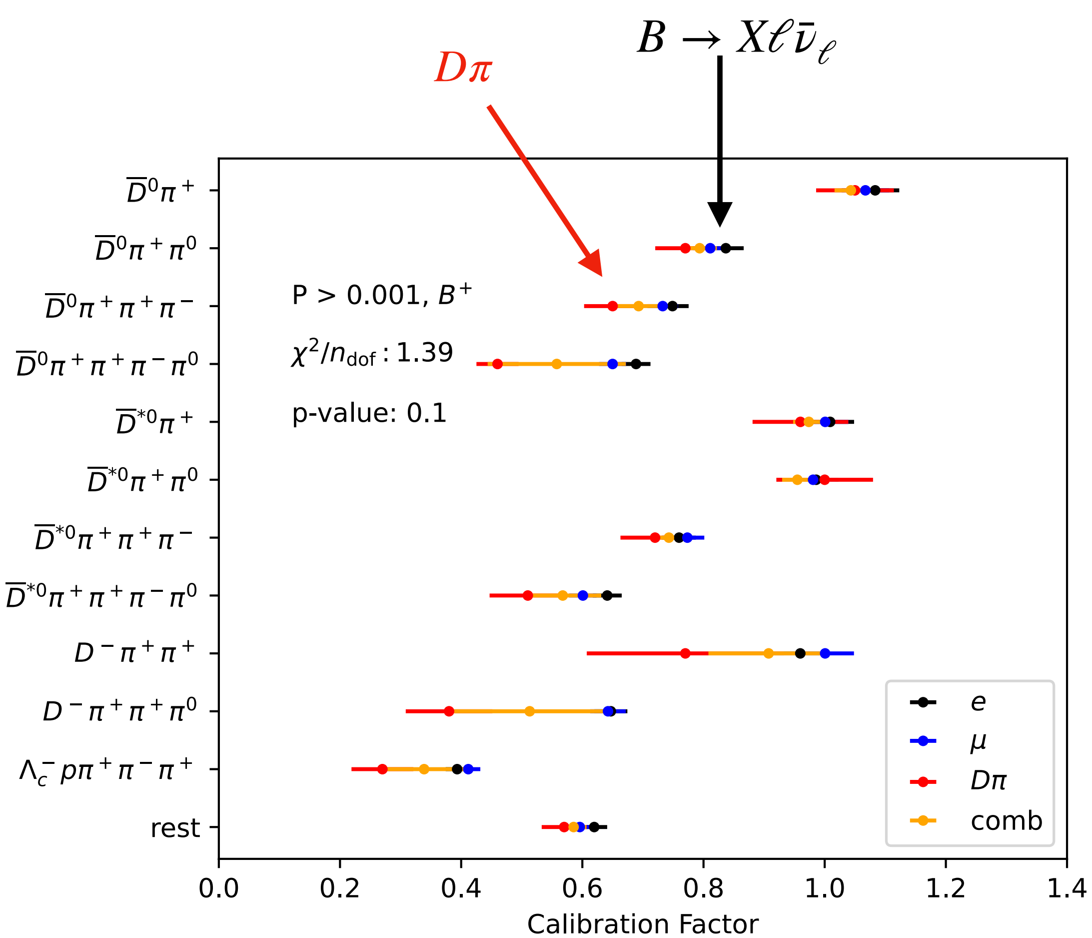


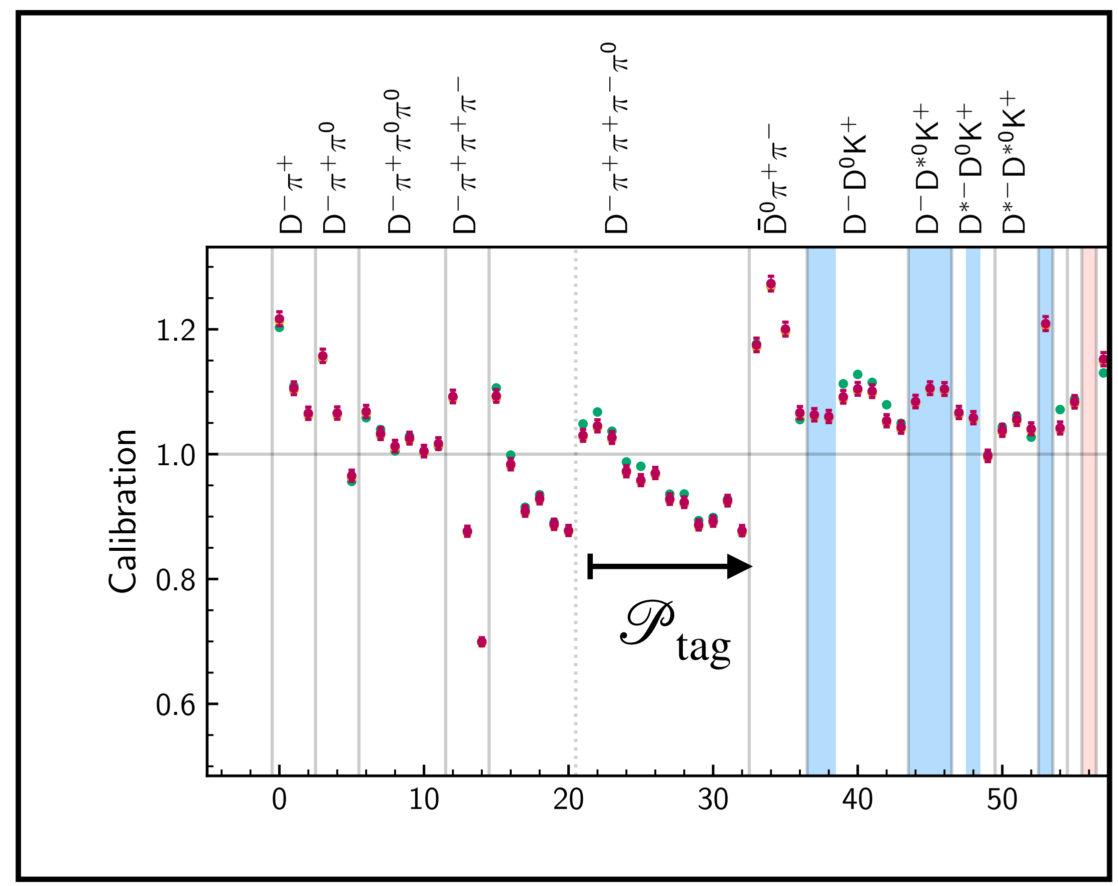
→ Also look at BELLE2-NOTE-PH 2023-004)
and BELLE2-NOTE-PH-2023-008

Unbiased calibration very challenging :

Calibration shows **signal side dependence**

Calibration also dependent on **composition** of **tag-side candidates** and fraction of **good** versus **bad tags**





One needs to carefully check these issues; best to carry out **self calibration** whenever possible

See e.g. PhD thesis of Kilian Lieret: https://edoc.ub.uni-muenchen.de/30193/1/Lieret_Kilian.pdf

Tagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

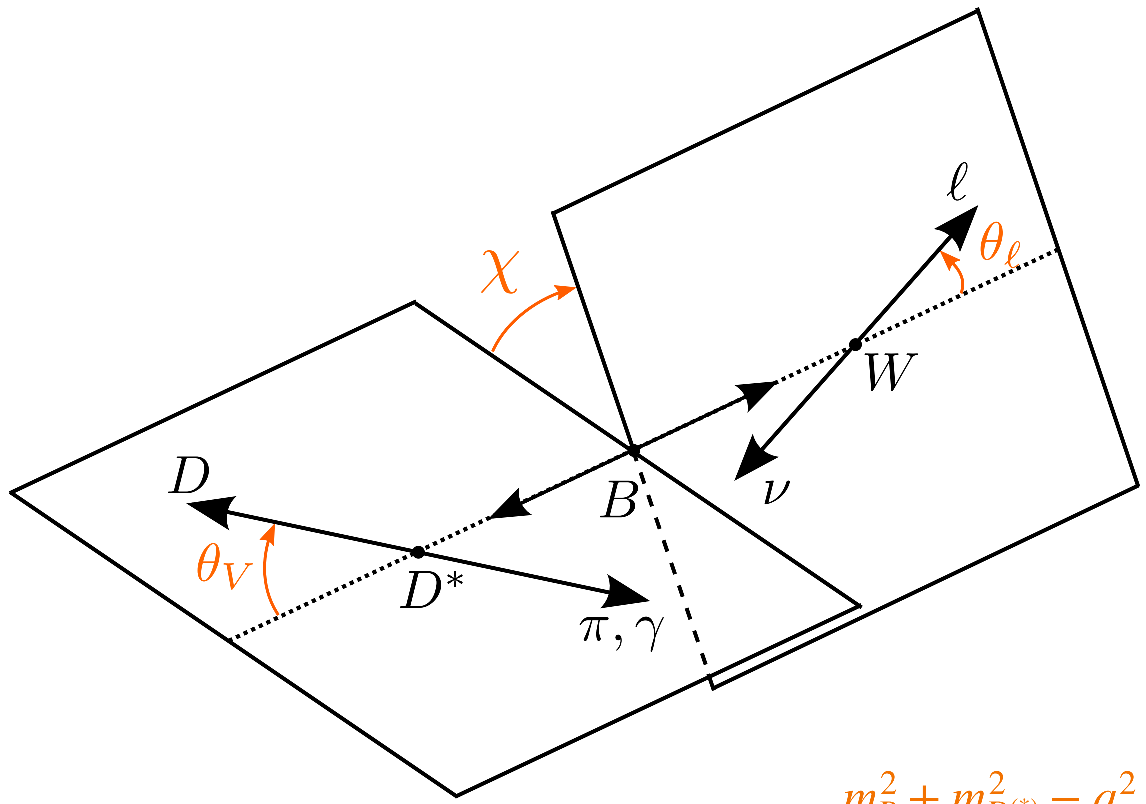
Target B^0 and B^+ and reconstruct D in many modes :

$D^+ \to K^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+\pi^0$, $D^+ \to K^-\pi^+\pi^+\pi^+\pi^-$, $D^+ \to K^0_S\pi^+$, $D^+ \to K^0_S\pi^+\pi^0$, $D^+ \to K^0_S\pi^+\pi^+\pi^-$, $D^+ \to K^0_S K^+$, $D^+ \to K^+K^-\pi^+$, $D^0 \to K^-\pi^+$, $D^0 \to K^-\pi^+\pi^0$, $D^0 \to K^-\pi^+\pi^+\pi^-$, $D^0 \to K^-\pi^+\pi^+\pi^-\pi^0$, $D^0 \to K^0_S\pi^0$, $D^0 \to K^0_S\pi^+\pi^-$, $D^0 \to K^0_S\pi^+\pi^-\pi^0$, and $D^0 \to K^-K^+$.

Reconstruct $D^{*+} \to D^0\pi^+, D^{*+} \to D^+\pi^0, D^{*0} \to D^0\pi^0$

In principle also can do $D^{*0} \to D^0\gamma$ but has different Lorentz structure & angular distributions

Tagged measurement can directly reconstruct **B rest frame** & access $\{w, \cos\theta_\ell, \cos\theta_V, \chi\}$



$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_{D^{(*)}}}$$

Tagged measurements of $B \to D^{(*)} \ell \bar{\nu}_\ell$

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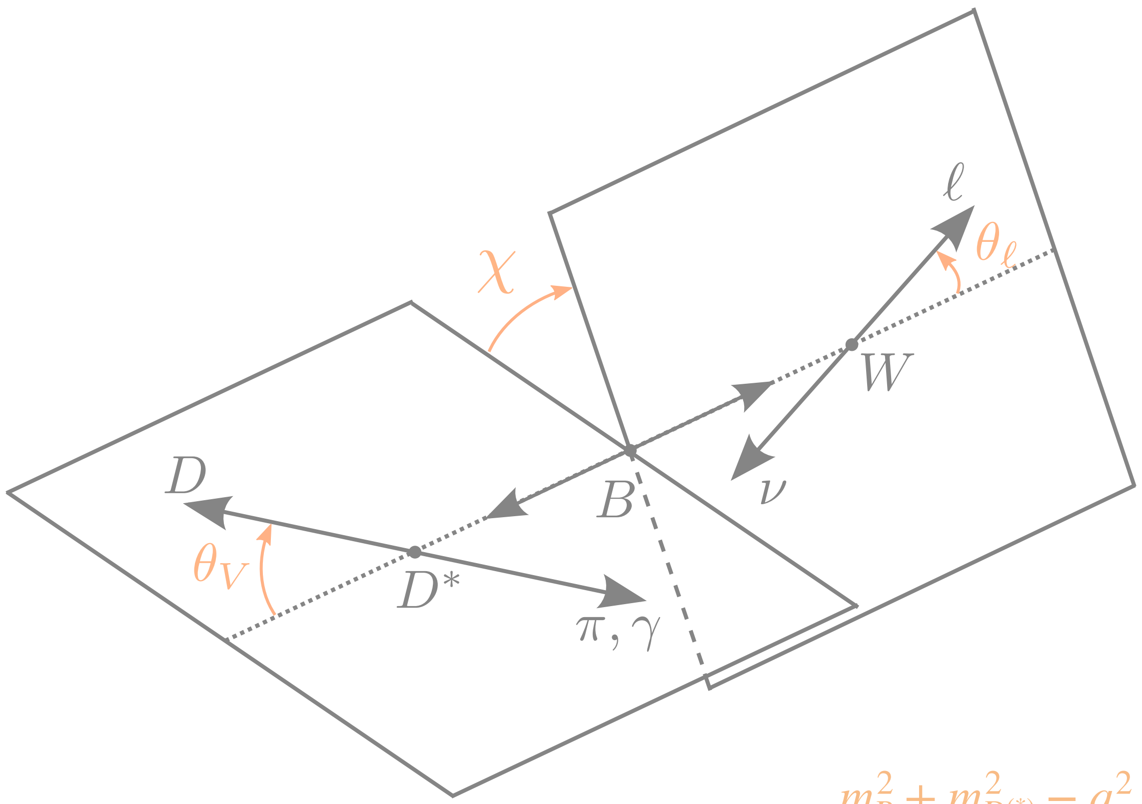

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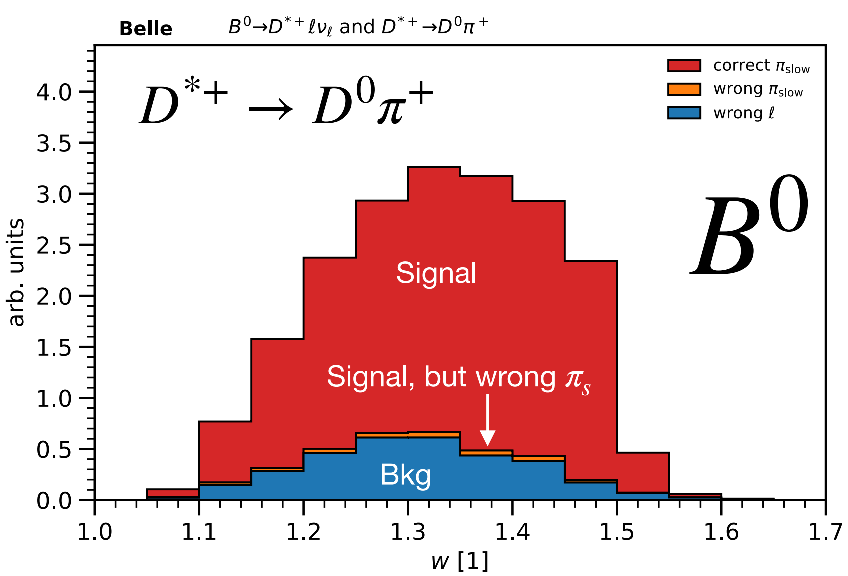

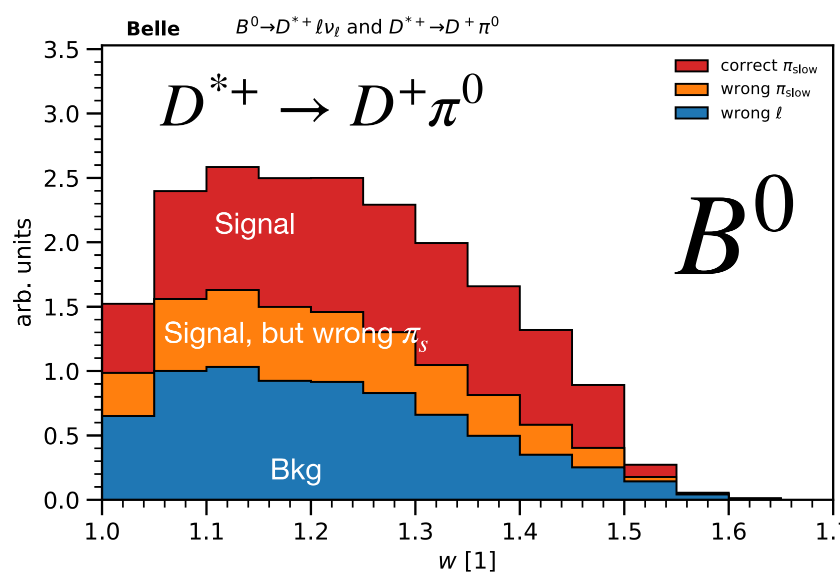

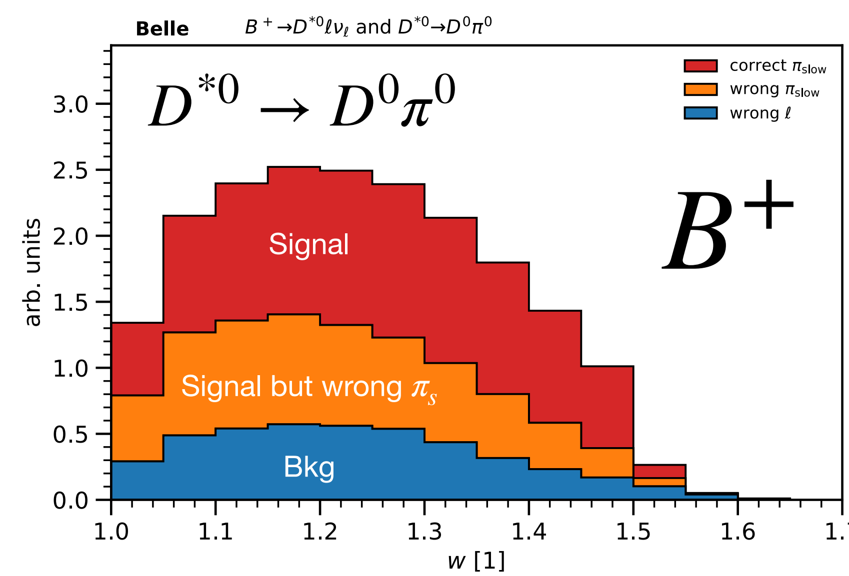

Background subtraction:

Need to subtract residual **background** contributions:

- From other SL decays ($B \to D^{**}\ell\bar{\nu}_\ell$ or $B \to D\ell\bar{\nu}_\ell$)
- From other B decays (with fake or real leptons)
- From Continuum ($e^+e^- \to q\bar{q}$)

Key idea :

$$p_{B_{\rm sig}} = p_{e^+e^-} - p_{B_{\rm tag}}$$

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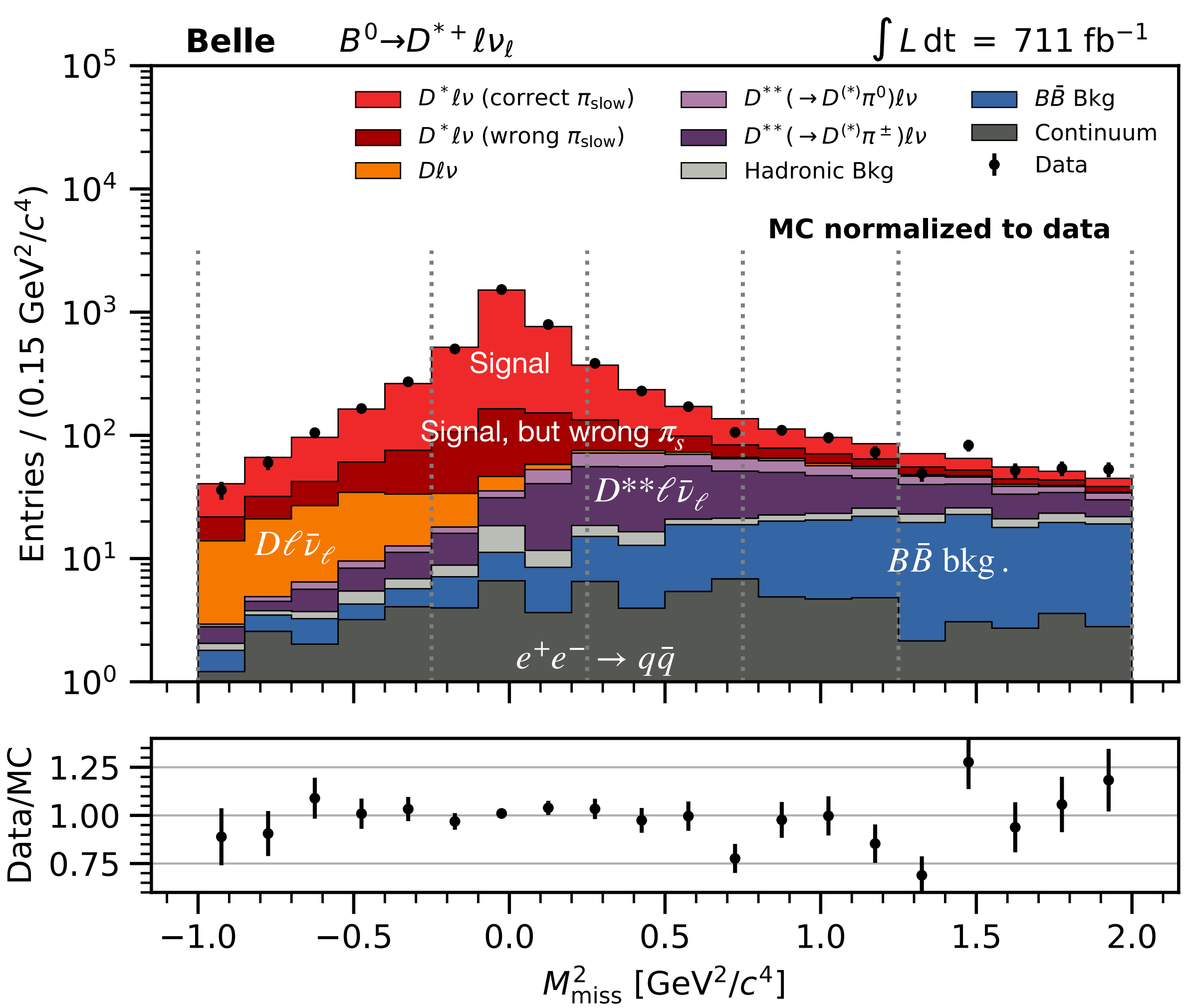
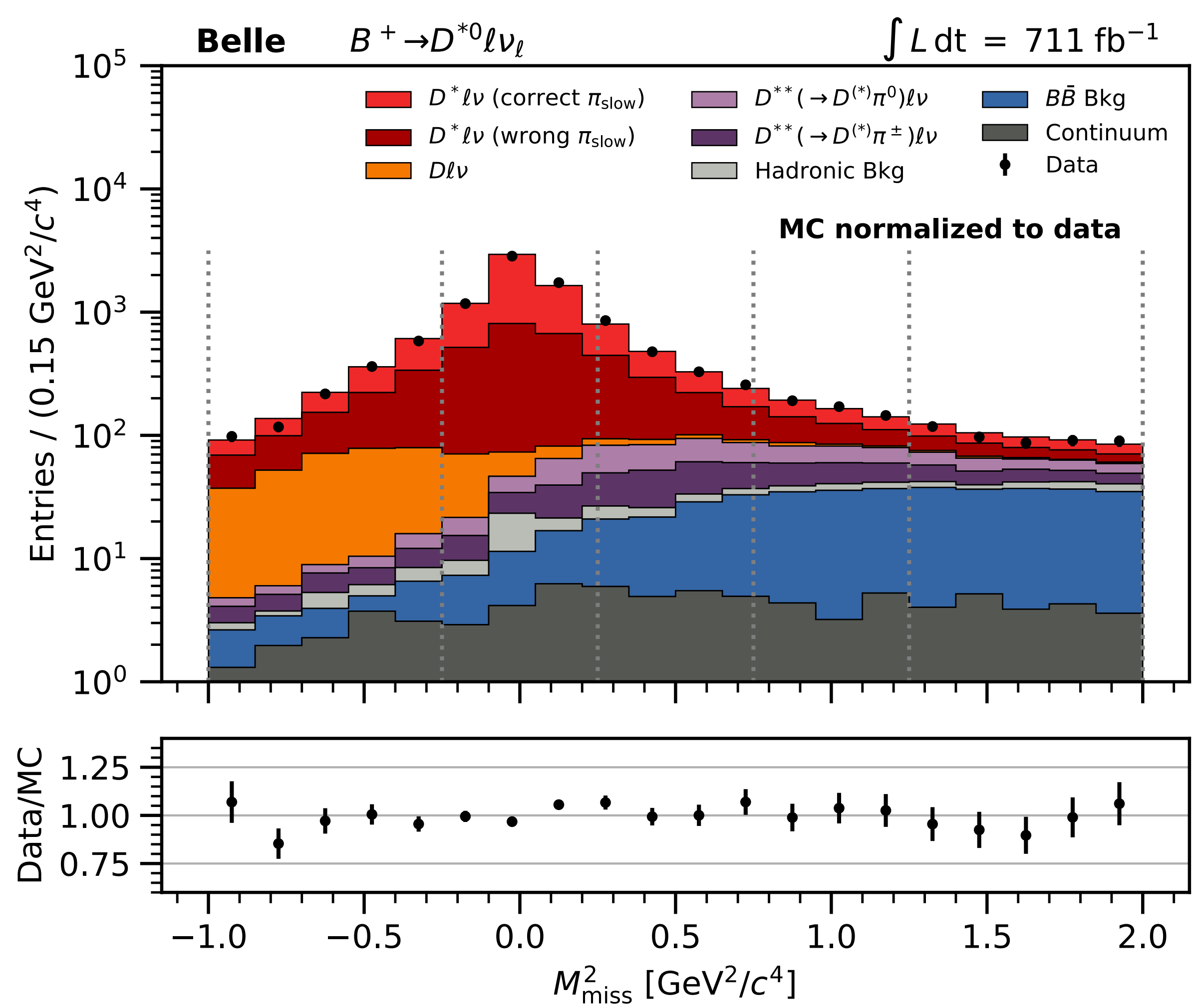
Key idea :

$$p_{B_{\rm sig}} = p_{e^+e^-} - p_{B_{\rm tag}}$$

Use: $0 = m_\nu^2 \simeq M_{\rm miss}^2 = (E_{\rm miss}, \mathbf{p}_{\rm miss})^2 = (p_B - p_{D^*} - p_\ell)^2$ or $U = E_{\rm miss} - |\mathbf{p}_{\rm miss}|$



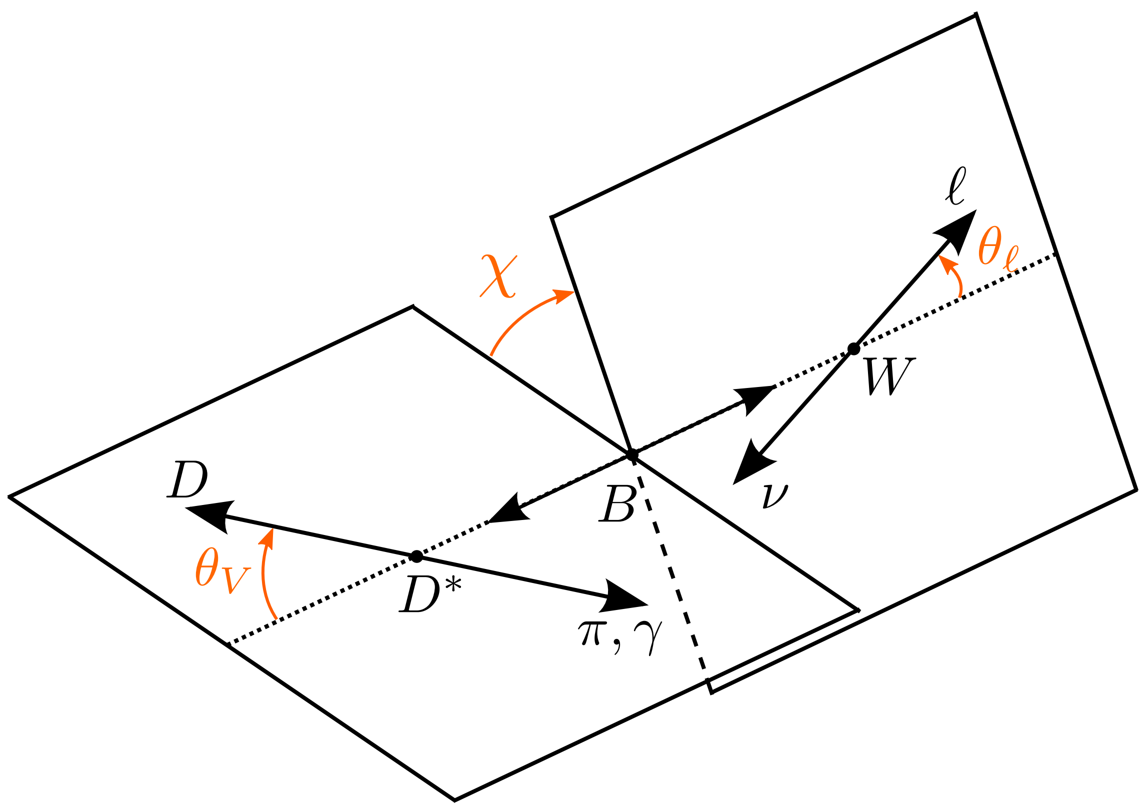


Fit in Bins of $\{w, \cos\theta_\ell, \cos\theta_V, \chi\}$

E.g. Can use **binned likelihood** fit to **1D distributions**
(good to use coarse binning to reduce modelling dependence (Bkg shape, resolution))

4D fit also possible; but binned approach suffers from course of dimensionality

→ **better unbinned (but then need to worry about efficiency & migrations)**



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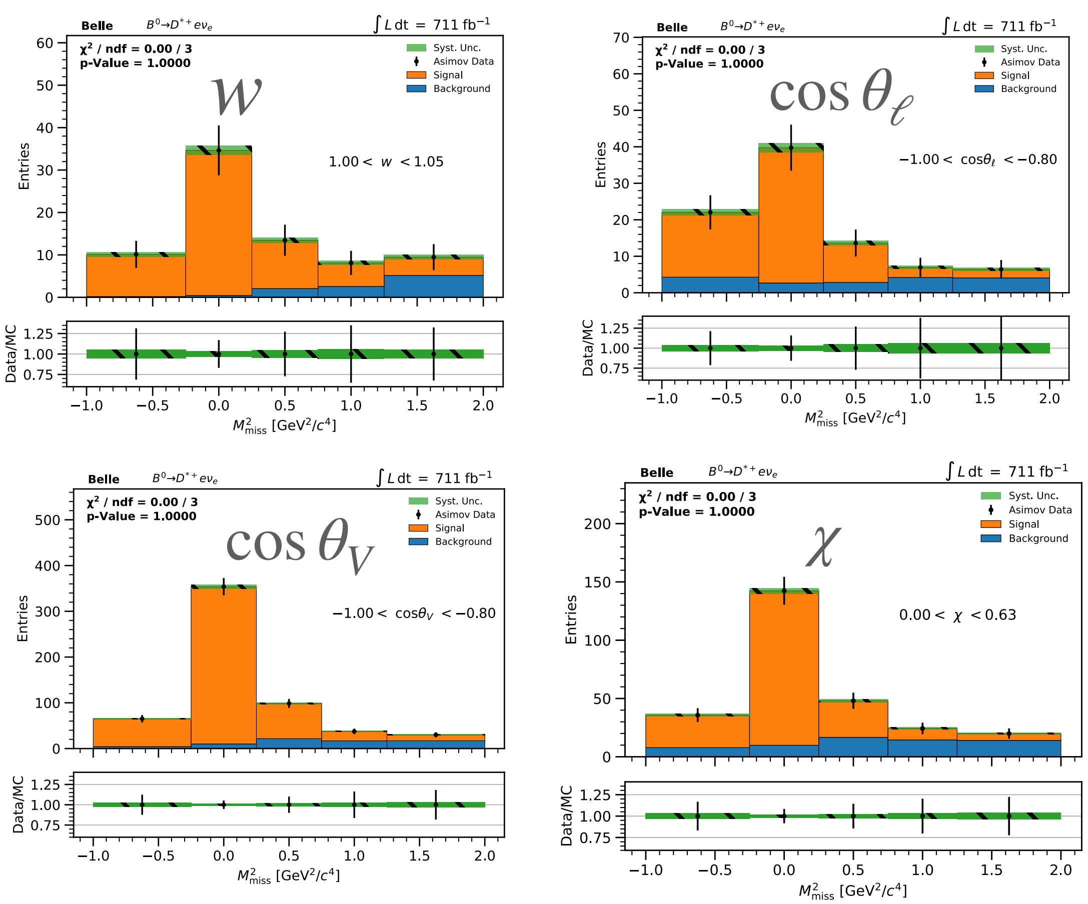
Example **1D fits** to MC (Asimov fits)



Best approach: use folding to extract relevant information

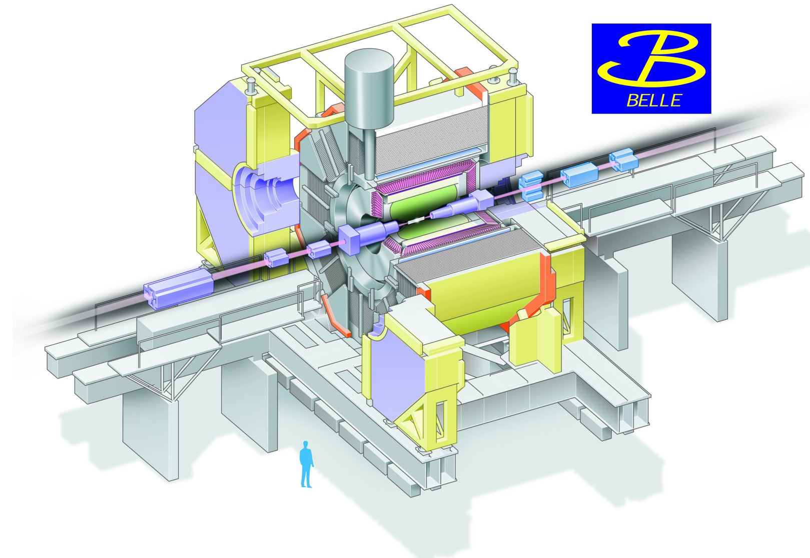
$$\frac{d^4\Gamma}{dq^2\, d\cos\theta^*\, d\cos\theta_\ell\, d\chi} = \frac{9}{32\pi}\Big[\left(I_1^s \sin^2\theta^* + I_1^c \cos^2\theta^*\right) + \left(I_2^s \sin^2\theta^* + I_2^c \cos^2\theta^*\right)\cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta^* \sin^2\theta_\ell \cos 2\chi + I_4 \sin 2\theta^* \sin 2\theta_\ell \cos\chi + I_5 \sin 2\theta^* \sin\theta_\ell \cos\chi$$
$$+ \left(I_6^c \cos^2\theta^* + I_6^s \sin^2\theta^*\right)\cos\theta_\ell + I_7 \sin 2\theta^* \sin\theta_\ell \sin\chi$$
$$+ I_8 \sin 2\theta^* \sin 2\theta_\ell \sin\chi + I_9 \sin^2\theta^* \sin^2\theta_\ell \sin 2\chi\Big],$$

I.e. by building smart asymmetries, can project out the relevant 12 terms (integrated over a certain q^2 range)

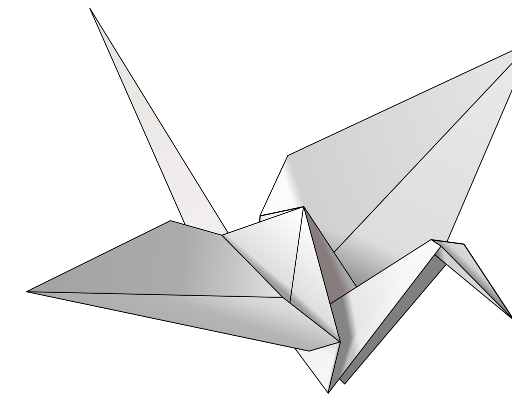
⟶ See e.g. Markus Prim's Belle Analysis (in preparation)

Detector migrations

X: True distribution



M: Detector response



Y: Measured distribution

An event reconstructed in a given *bin i,* might not have had a "true" value corresponding to a *bin j*

Can be parametrized as a **migration matrix:**

$$\mathscr{M}_{ij} = \mathscr{P}(\text{reco. in bin } i \,|\, \text{true value in bin } j)$$

parametrize detector migrations
as **conditional probability**

Detector migrations

An event reconstructed in a given *bin i,* might not have had a “true” value corresponding to a *bin j*

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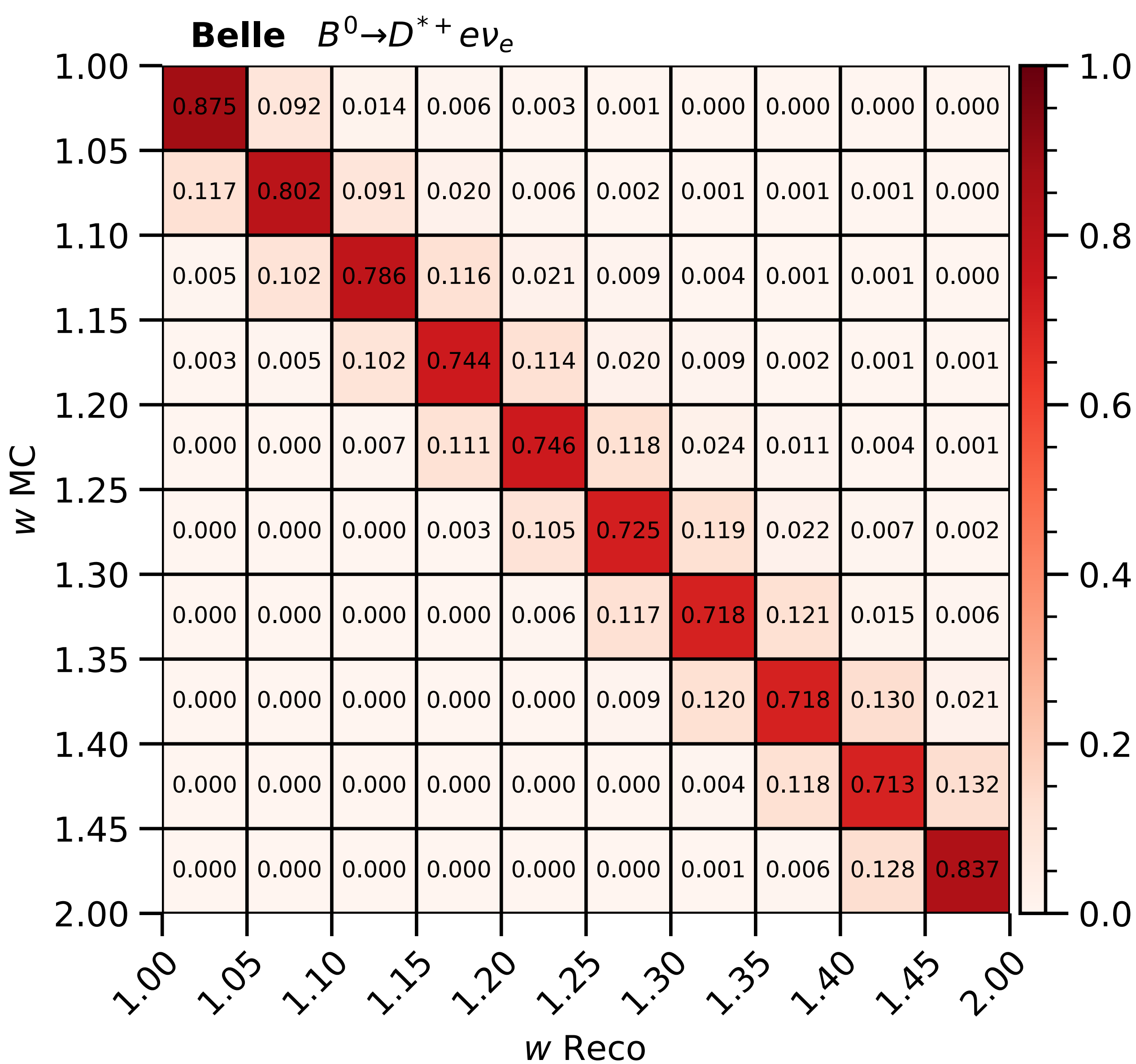


Can recover estimates for true values via “unfolding” determined yields, mapping reco → true

Simplest version: migration matrix inversion

$$\mathbf{x}_{\text{true}} = \mathscr{M}_{ij}^{-1}\,\mathbf{x}_{\text{reco}}$$

Many approaches to dampen impact of increase in variance

(mostly a problem with large migrations → true bin is then the sum of many reco bins with high weights)

or to reduce impact of MC prior

(here less an issue; but Bayesian unfolding can propagate the observed shape to MC to minimize model dependencies)

Acceptance × Efficiency

After migration effects are corrected, need to correct also for selection effects (Acceptance x Efficiency)

$$\boldsymbol{\Delta}\mathscr{B}/\boldsymbol{\Delta}\mathbf{x} = \left(\epsilon_{\text{reco}} \times \epsilon_{\text{tag}}\right)^{-1} \times \mathscr{M}^{-1}\,\mathbf{x}_{\text{reco}} \times \frac{1}{4\,N_{B\overline{B}}}$$

Actually a matrix

$$\left(\epsilon_{\text{reco}} \times \epsilon_{\text{tag}}\right) = \text{diag}\ \left(\ \mathscr{A}(\text{true bin } i)\ \right)$$

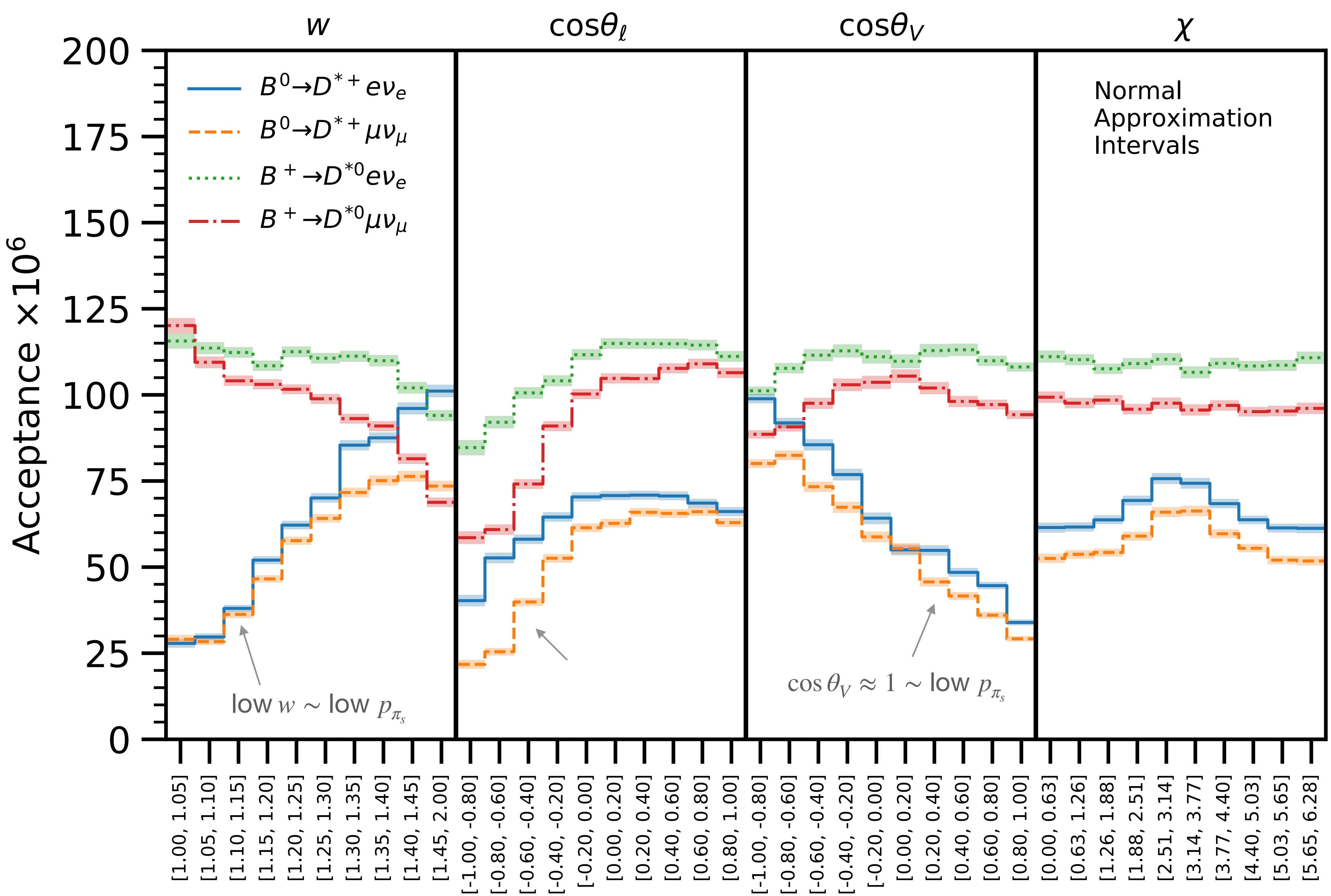
$$2\,N_{B\overline{B}} = \left(1 + f_{+0}\right) N_{B^0} = \left(1 + f_{+0}^{-1}\right) N_{B^+}$$

$$f_{+0} = \frac{\mathscr{B}(\Upsilon(4S) \to B^+B^-)}{\mathscr{B}(\Upsilon(4S) \to B^0\overline{B}^0)}$$

Although it's acceptance × efficiency, we just call this acceptance in the figure on the next slide

Acceptance × Efficiency

After migration effects are corrected, need to correct also for selection effects
(Acceptance x Efficiency)



A word on Efficiencies

Efficiencies can be are a large source of uncertainties

Two examples very relevant for semileptonic decays:

- Lepton Identification Uncertainty

Often based on a global likelihood (or a multivariate classifier) using individual likelihoods (or input features) to calculate a score how likely the identified particle is an electron or a muon

Symbolically:

$$\mathscr{L} = \mathscr{L}_{\mathrm{CDC}} \times \mathscr{L}_{\mathrm{ECL}} \times \mathscr{L}_{\mathrm{TOP}} \times \mathscr{L}_{\mathrm{KLM}}$$

Ionization energy loss

$E/|\vec{p}\,|$

Information from Cherenkov light angles

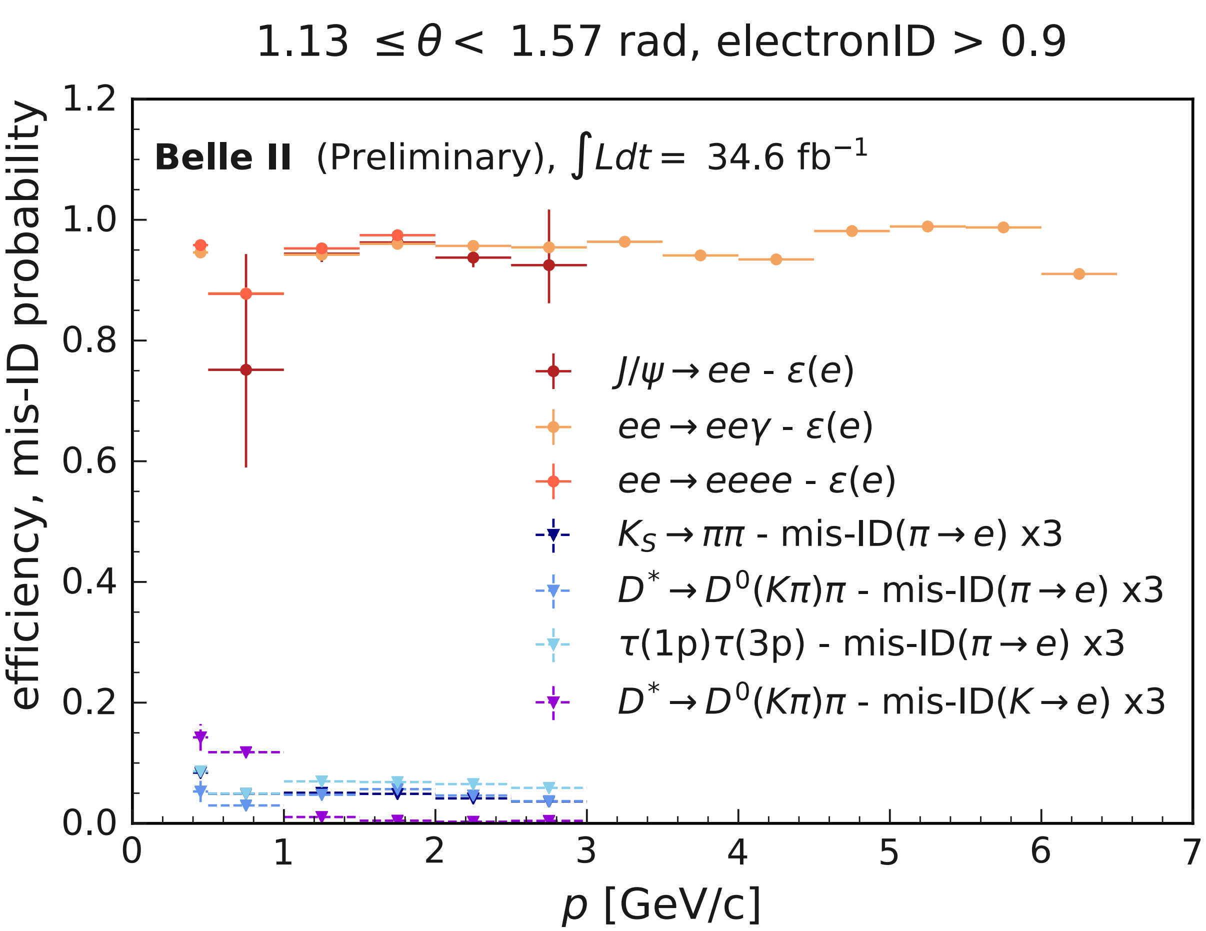
Matched KLM cluster hit?

Use clean physics sample to correct MC efficiencies and fake rates

E.g. $e^+e^- \to \mu\mu\gamma, e^+e^- \to e^+e^-\gamma, J/\psi \to \ell\ell, \ldots$

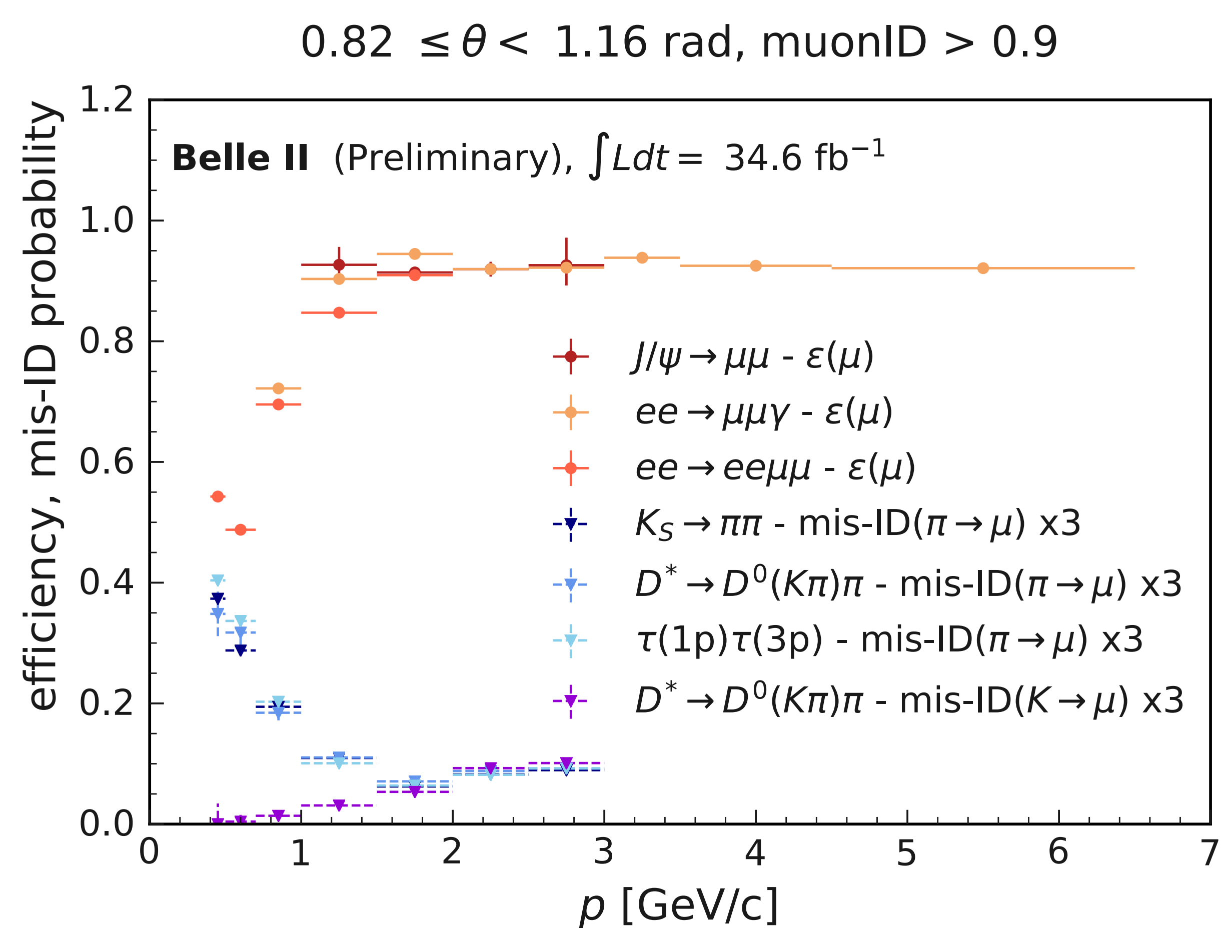
Construct likelihood ratio for Lepton ID: $\ell\mathrm{ID} = \mathscr{L}_e / [\mathscr{L}_e + \mathscr{L}_\mu + \mathscr{L}_\pi + \mathscr{L}_K + \mathscr{L}_p]$

Electrons



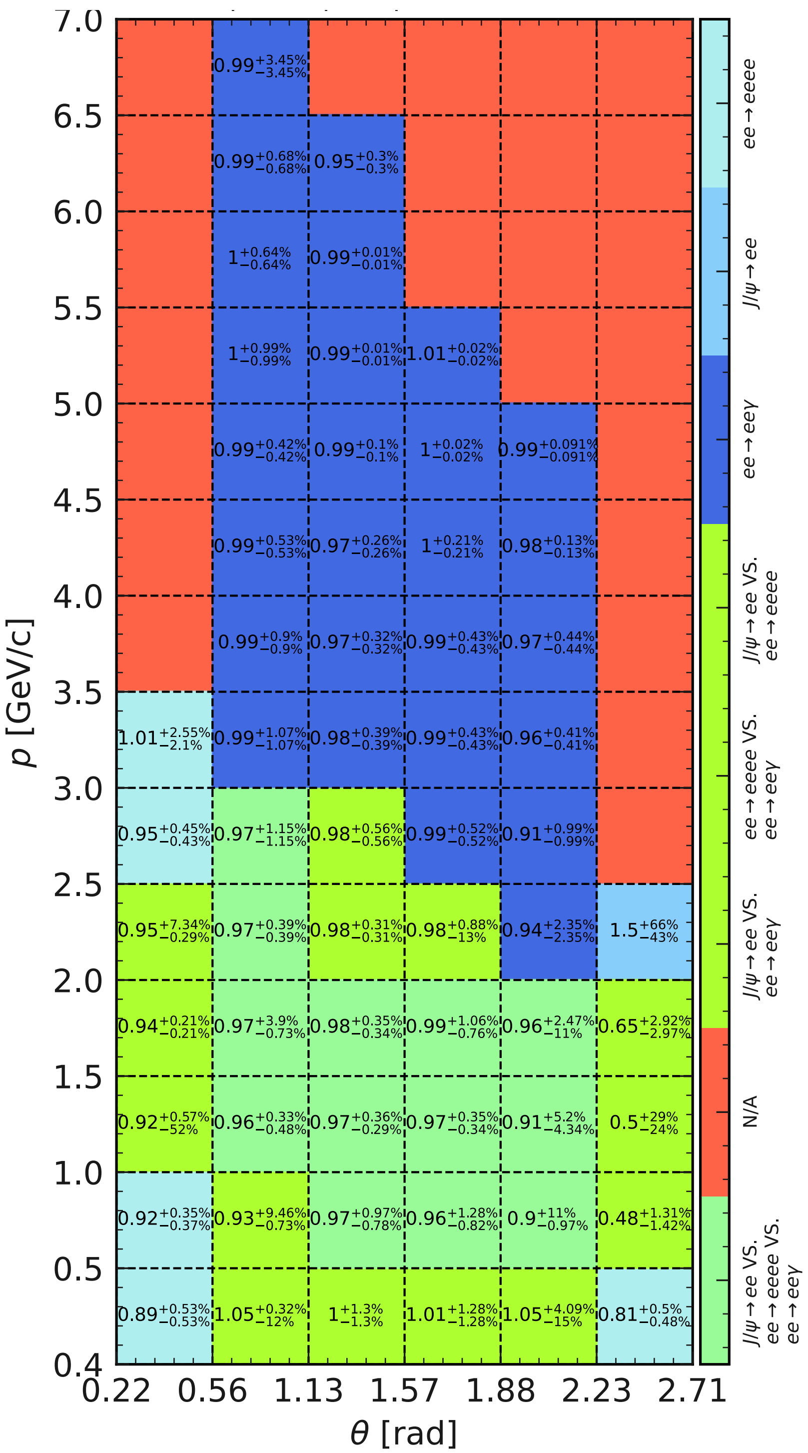
Momentum in lab frame

Muons



Construct correction tables of efficiency ratios $\dfrac{\epsilon_{\text{Data}}}{\epsilon_{\text{MC}}}$

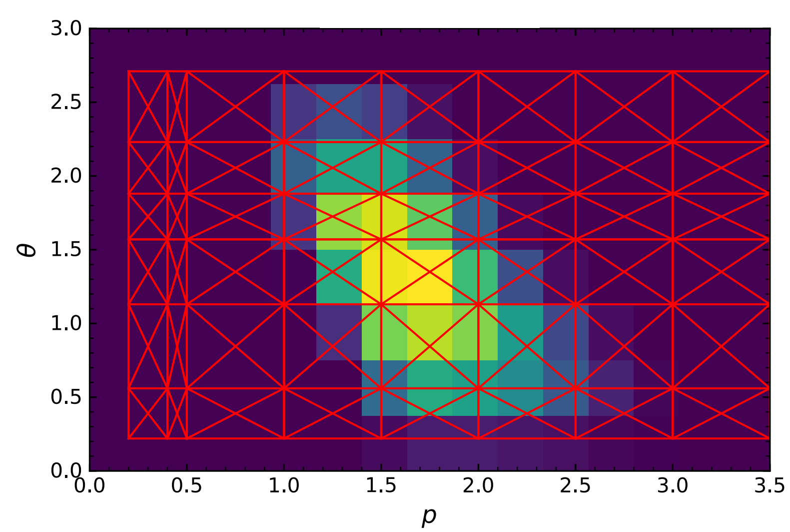
as a function of lab momentum and detector position (polar angle) to correct MC efficiencies



Precision limited by available control channel statistics (i.e. goes down by Lumi)

Non-closure between channels is added as extra uncertainty (limiting factor at very high luminosity)

Coverage of control channels and signal are different, i.e. not all control channels have same relevance)

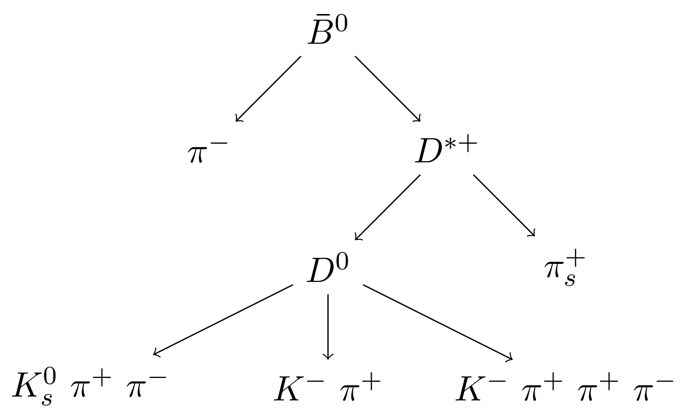


→ Correlation model matters!

Second example:

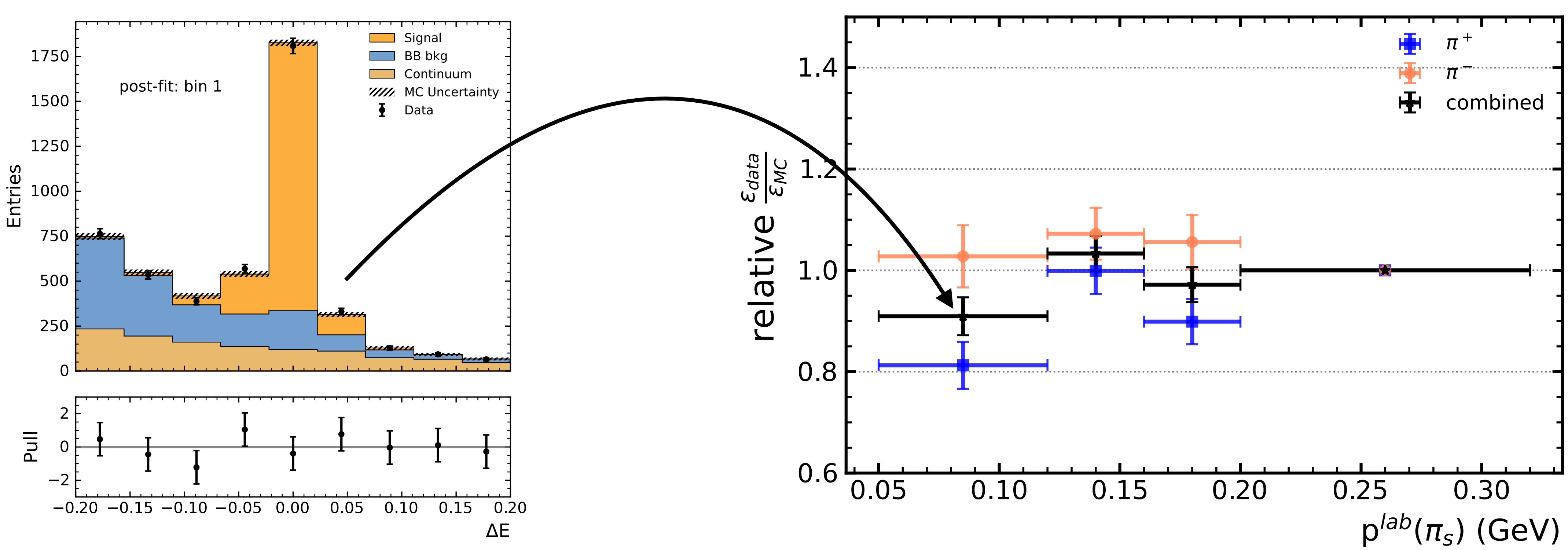
- Slow pion reconstruction efficiency

Also needs to be measured in data, e.g. via $B^0 \rightarrow D^{*+}\pi^-$ decays

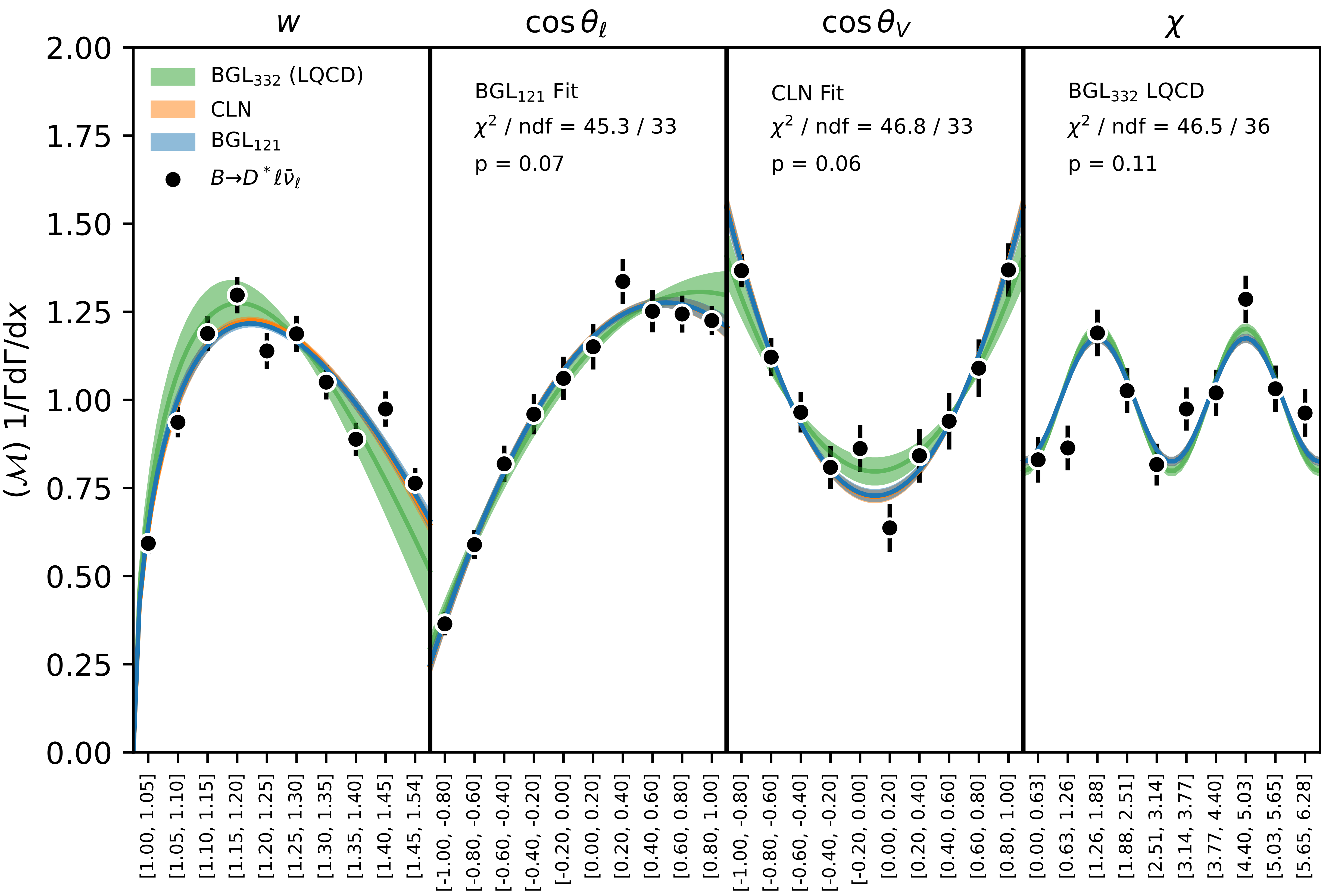


Extract signal in a fit to $\Delta E = \sqrt{s}/2 - E_B$ in bins of $p_{\pi_s}^{\text{lab}}$

Measure ratio efficiency ratio **relative** to high-momentum region of $p_{\pi_s}^{\text{lab}} > 200\,\text{MeV}$



Final result :



Time flies when you are having fun :-)

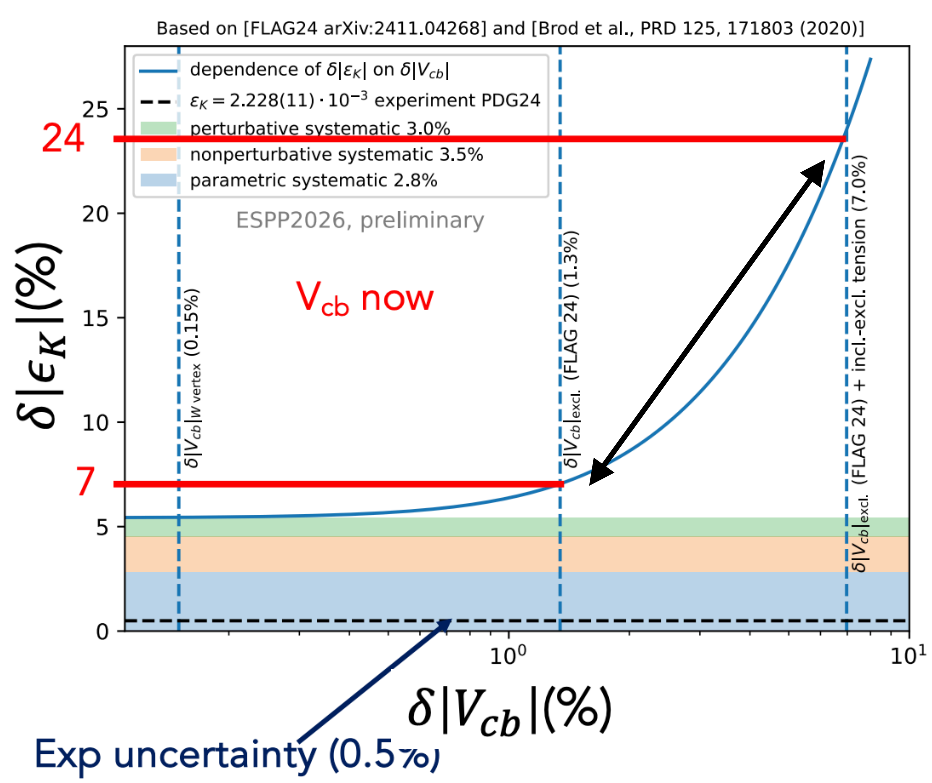
Thank you for your attention!

- More Analyses walkthroughs in the backup for several major topics

Semileptonic measurements, $|V_{cb}|$ and anomalies seen as important topic by the field

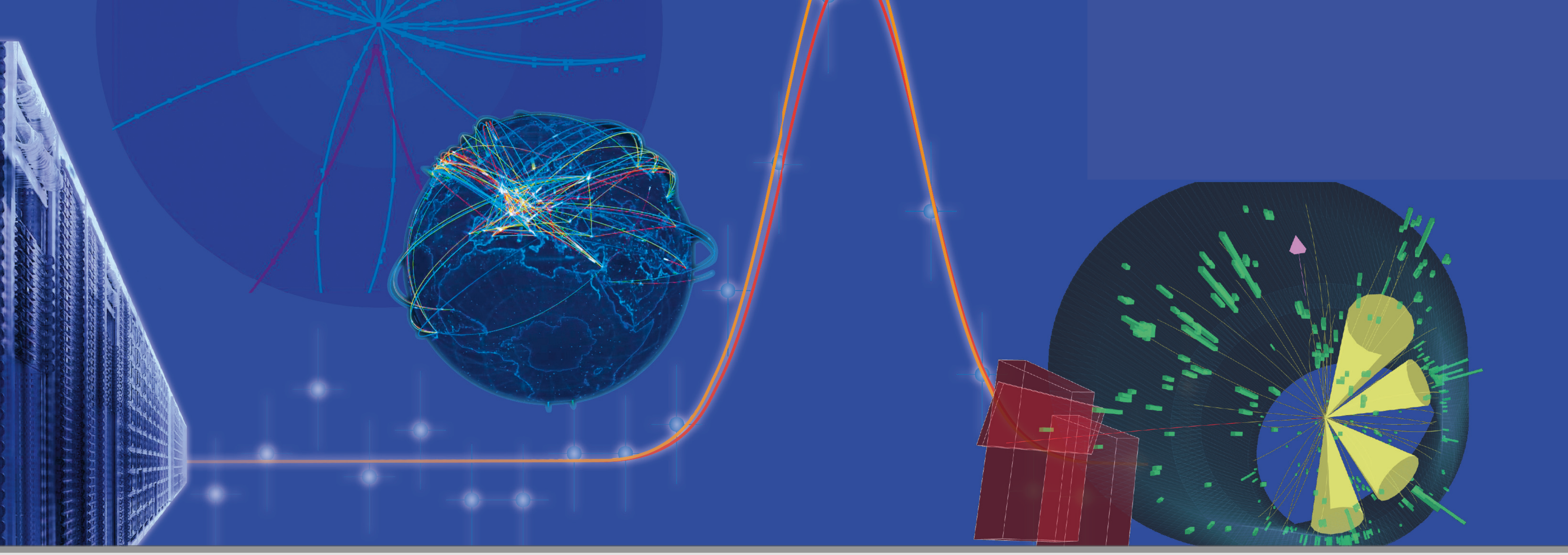
E.g. at **ESPP**

$|V_{cb}|$ enters everywhere in SM predictions (ϵ_K, $BR(B_s^0 \to \mu^+\mu^-)$, …)

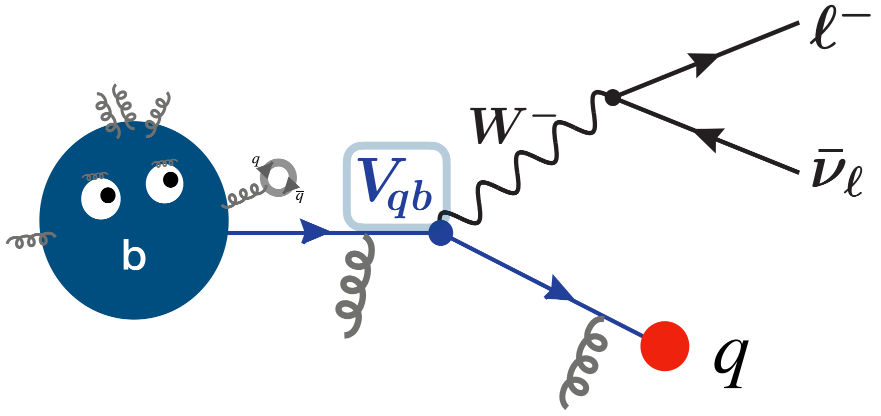


Incl. / excl. Tension

https://agenda.infn.it/event/44943/contributions/263368/attachments/137557/206863/ESPPU206_Venice_MHS.pdf

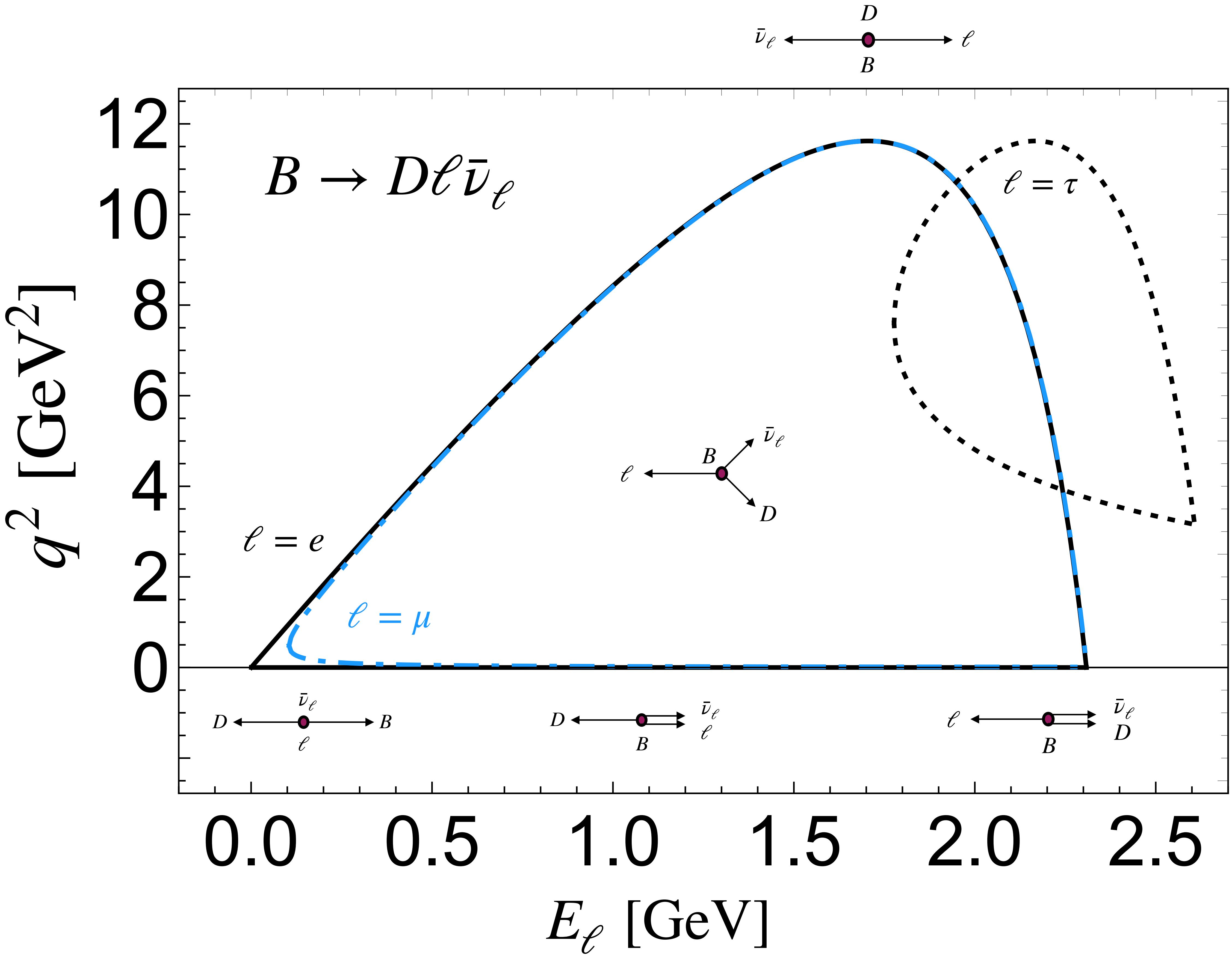


Backup I / More Information on $\ell = e, \mu$ Measurements

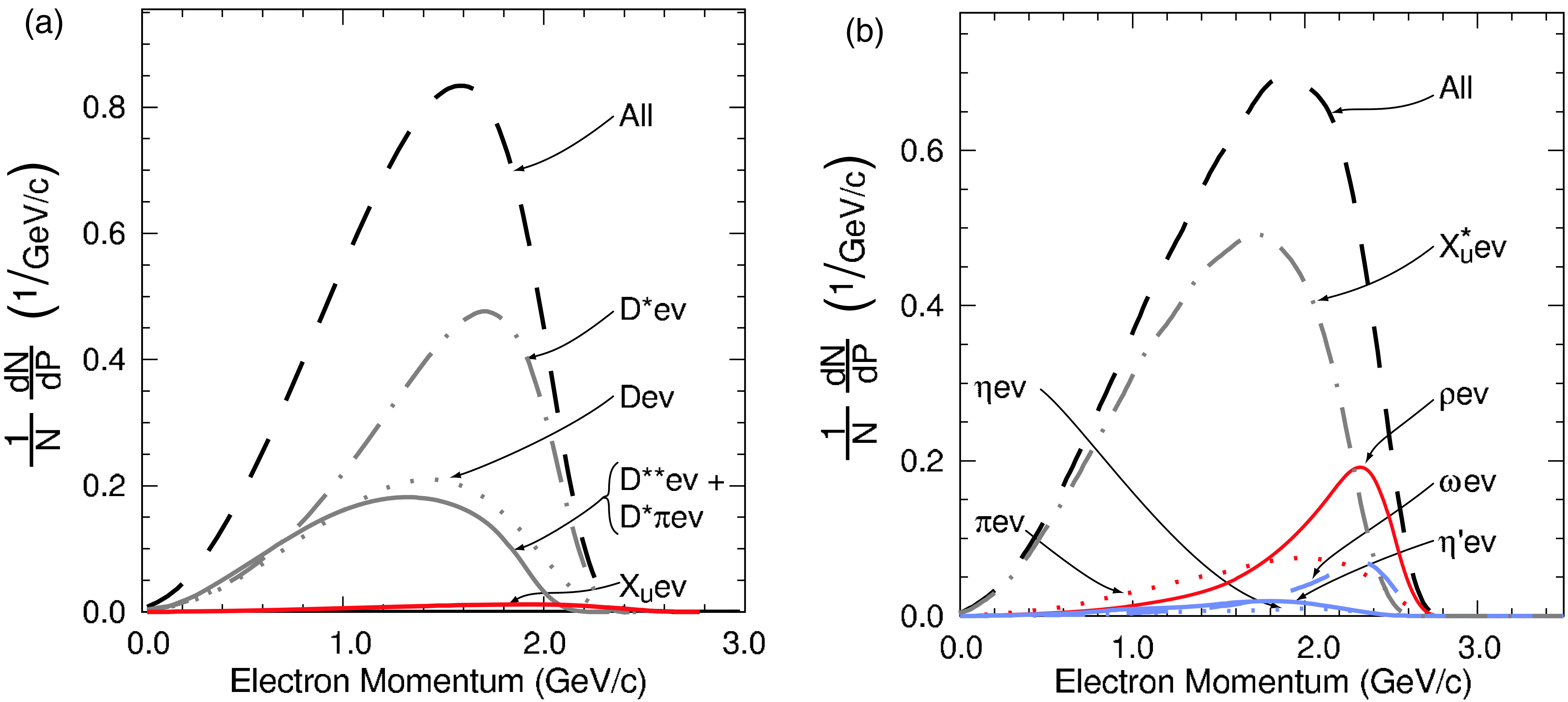


Some more slides on kinematics etc.

But $q^2 : E_\ell$ **not** independent :



The various semileptonic modes have spectra with **different endpoints**, e.g. for $B \to X_c \ell \bar{\nu}_\ell$ and $B \to X_u \ell \bar{\nu}_\ell$:



These already can give you some **experimental intuition**: e.g. if you want to measure $B \to X_u \ell \bar{\nu}_\ell$ its much easier beyond the endpoint of $B \to X_c \ell \bar{\nu}_\ell$

In the context of the **heavy-quark expansion**, it is convenient to introduce **velocities** instead of momenta.

E.g. for the case of heavy mesons like B and D^* one defines

$$v_B = \frac{p_B}{m_B}, \qquad v_{D^{(*)}} = \frac{p_{D^{(*)}}}{m_{D^{(*)}}},$$

$$w = v_B \, v_{D^{(*)}}$$

Here w is the scalar product of the two velocities and used instead of q^2

They are related via $q^2 = m_B^2 + m_{D^{(*)}}^2 - 2 m_B m_{D^{(*)}} w$

Note that :

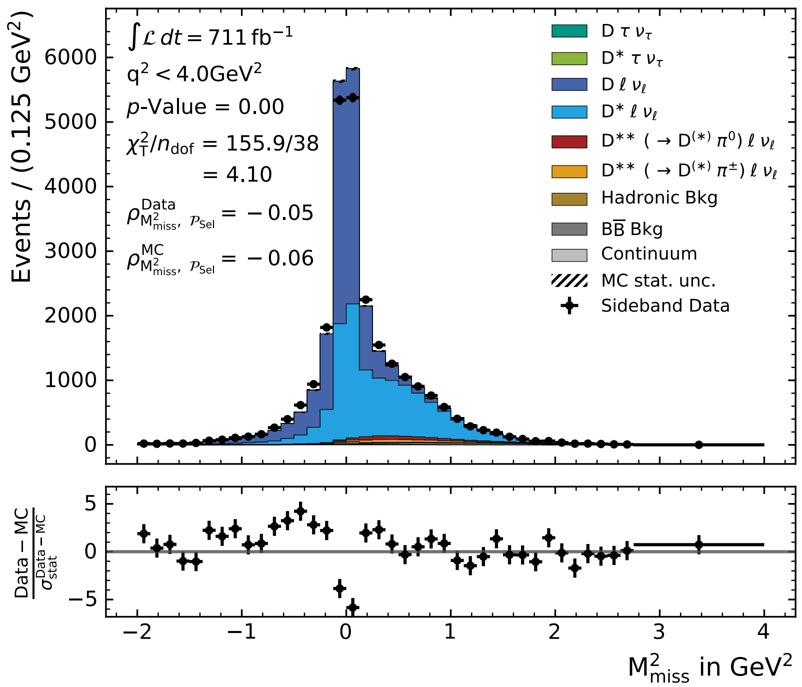
$$w = 1 \quad \longleftrightarrow \quad q^2_{\max} = \left(m_B - m_{D^{(*)}}\right)^2$$

While $q^2 = m_\ell^2 \approx 0$ for light leptons results in the maximal value of w

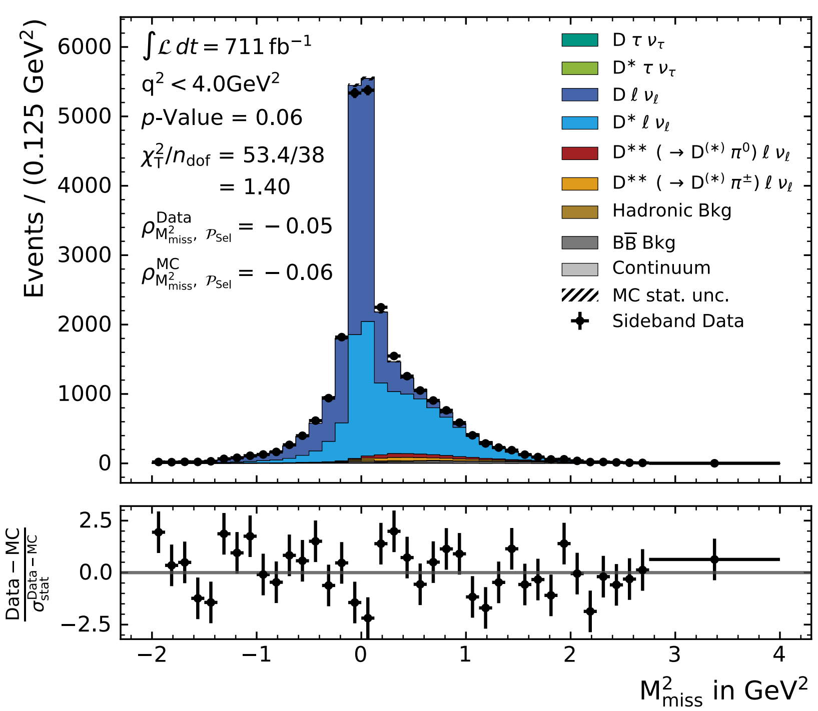
$$\rightarrow 1 \leq w \leq \frac{m_B^2 + m_{D^{(*)}}^2 - m_\ell^2}{2 m_B m_{D^{(*)}}}$$

MC modelling of M^2_{miss} challenging

Need to apply additional corrections to match actual resolution



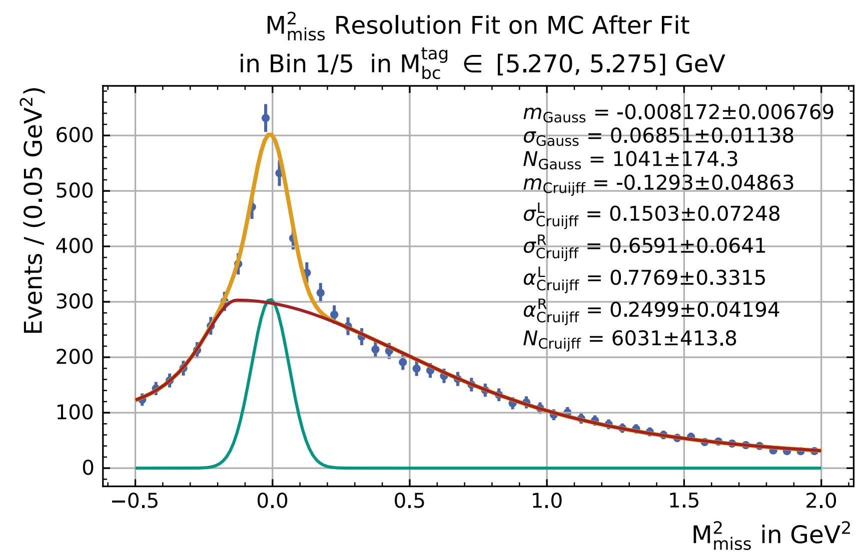
⇓



E.g. use an appropriate smearing function

(e.g. asymmetric Laplace distribution and as a function of m_{bc})

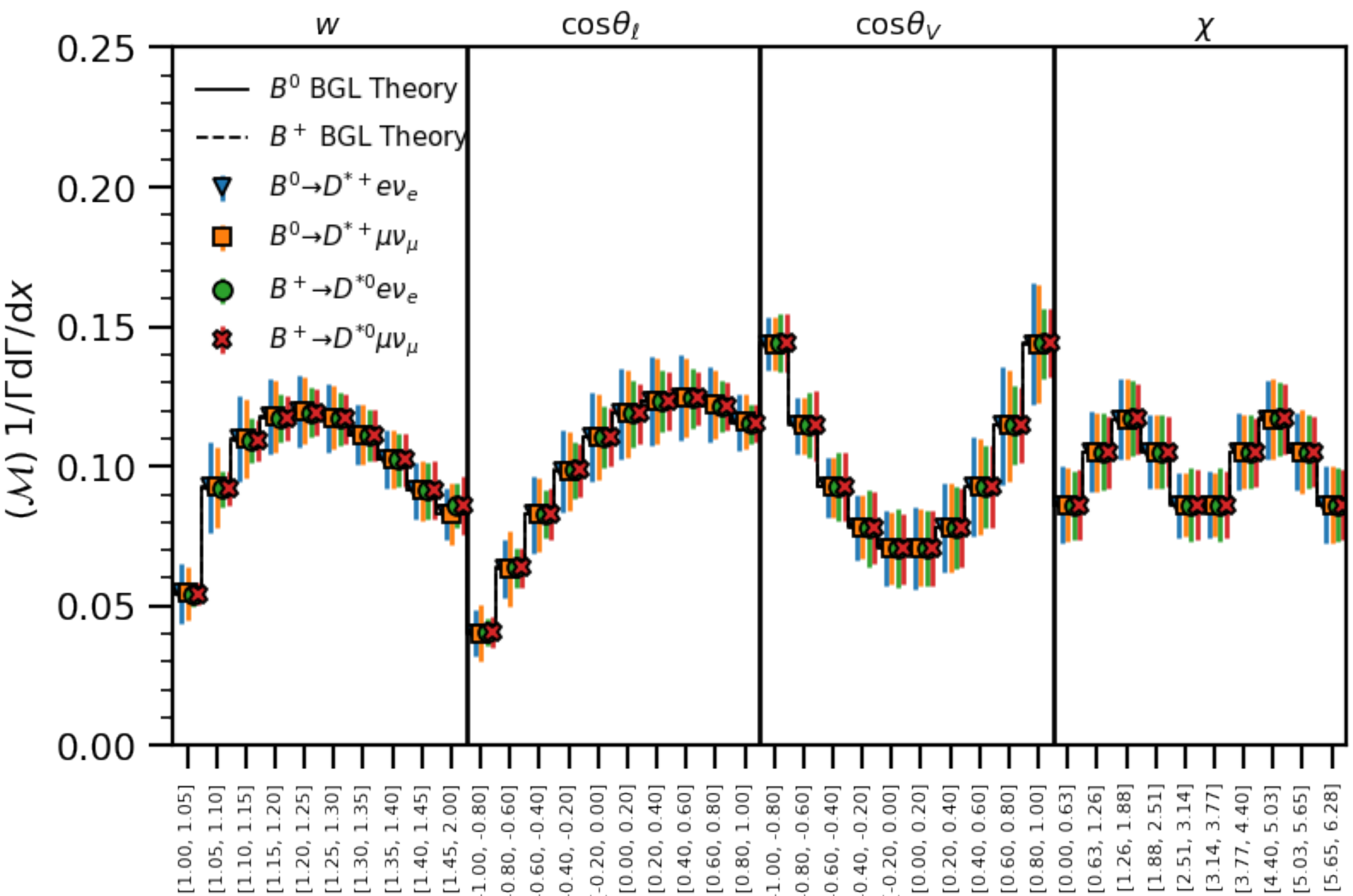
$$f_{\text{AL}}(x; m, \lambda, \kappa) = \frac{\lambda}{\kappa + 1/\kappa} \begin{cases} \exp\left((\lambda/\kappa)(x-m)\right) & \text{if } x < m, \\ \exp(-\lambda\kappa(x-m)) & \text{if } x \geq m, \end{cases}$$



Also other issues which cannot be necessarily solved by smearing alone, e.g. in inclusive analyses the modeling of e.g. D mesons is extremely important

see e.g. Belle II R(X) measurement in preparation

The final result (MC)



Note how the different channels are complementary in different regions of phase-space

(e.g. B^+ has much better precision at low w than B^0, but both have equal precision at high w)

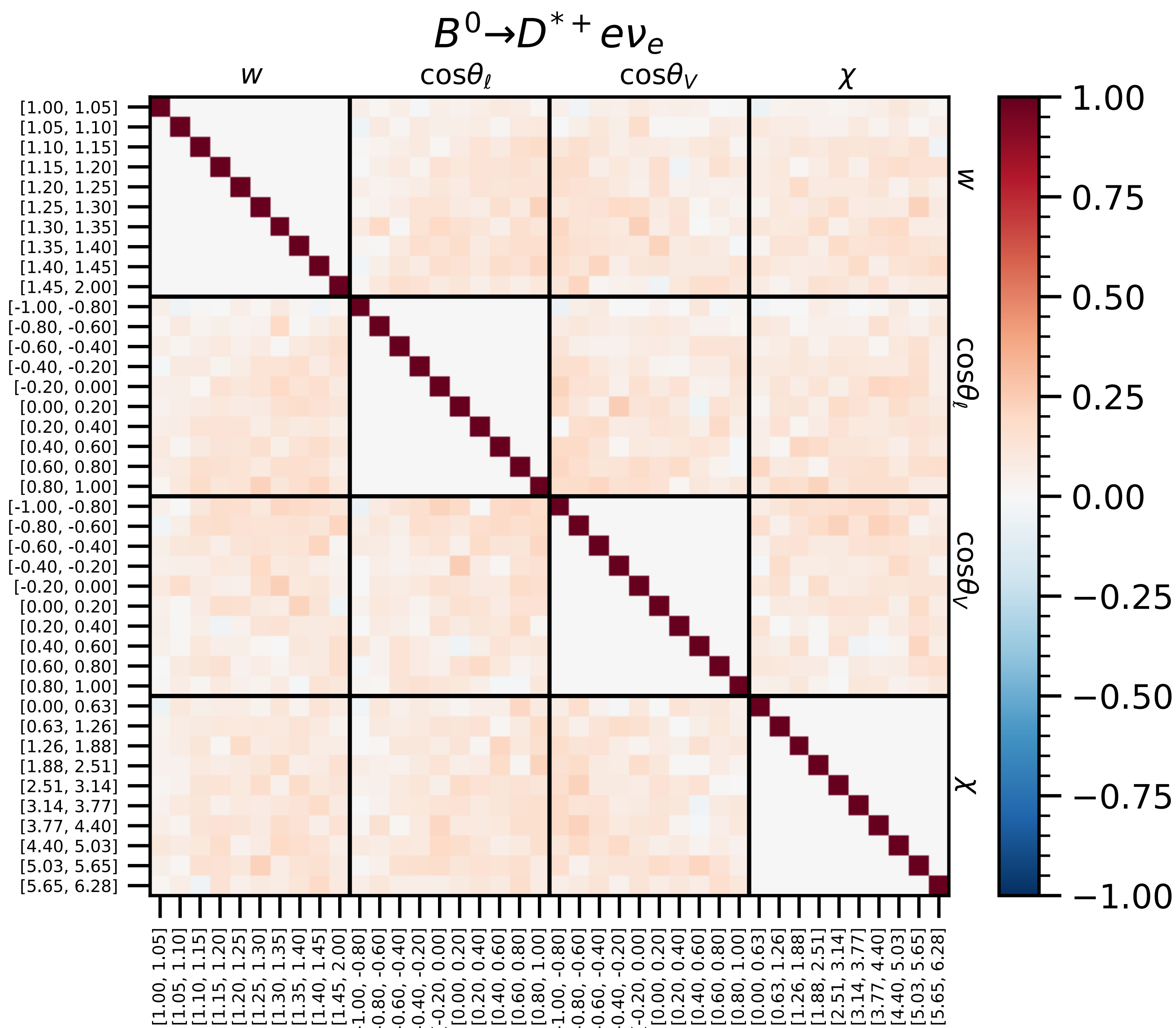
For a simultaneous analysis, need to determine correlations between different 1D projections → can be done using **boostrapping**

Very simple: create a replica of your data set by **sampling with replacement**

Repeat full analysis chain of 4 x1D measurement for **each replica**

Pearson correlator of replica sample provides estimator for statistical correlation between bins:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



But since we measured projections of the same data, the effective **degrees of freedom** are not 40, but 37 (Jung, Van Dyk)

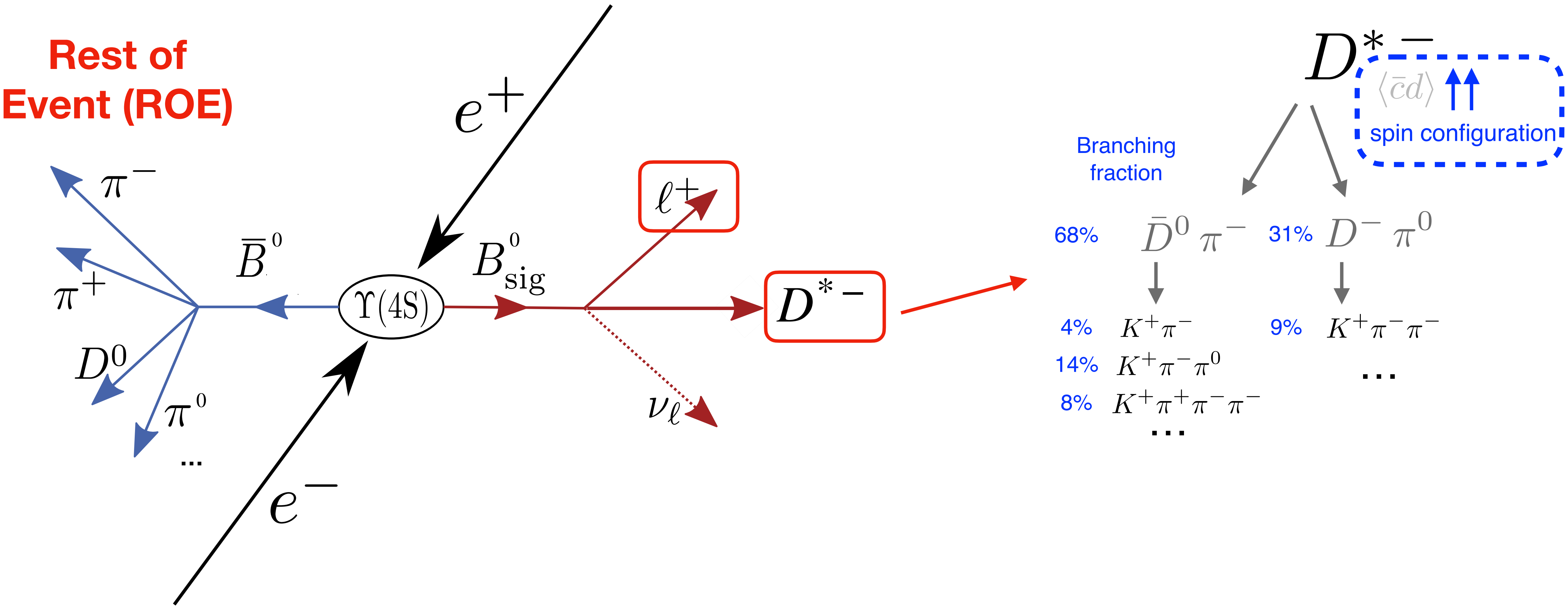
Best use of tagged data:

Fit normalized shapes (and if available total rate)

36 dof from shapes (4*9) and 1 from normalization

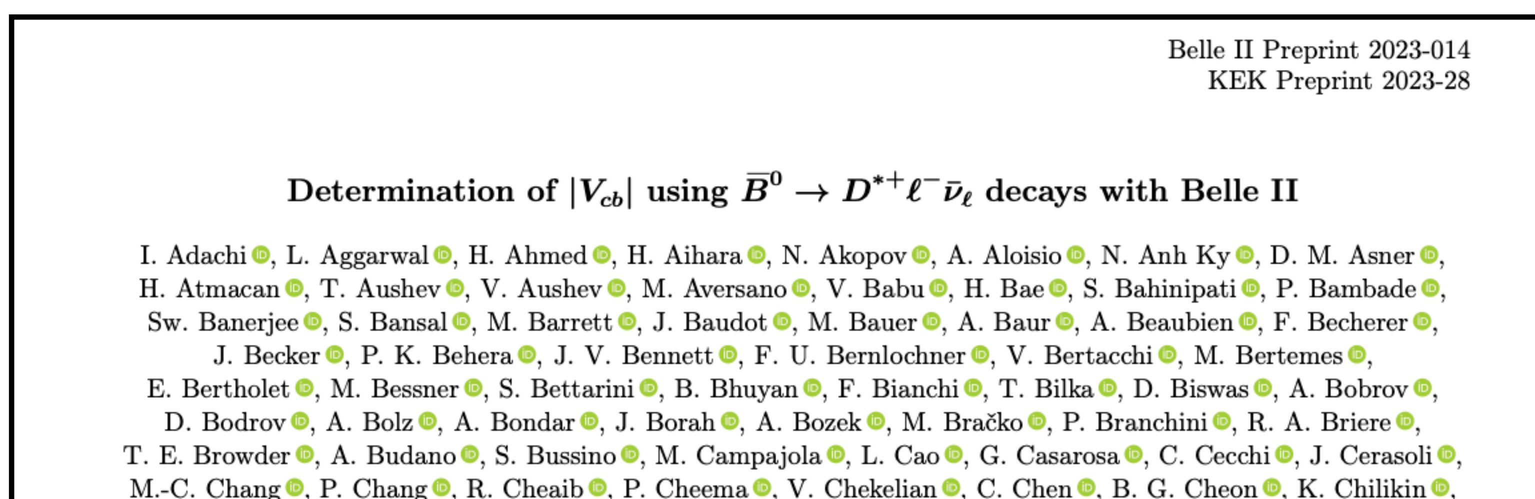
Other Analyses / Approaches / Measurements

Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$



Recent Belle II result:

https://arxiv.org/abs/2310.01170
(accepted by PRD)

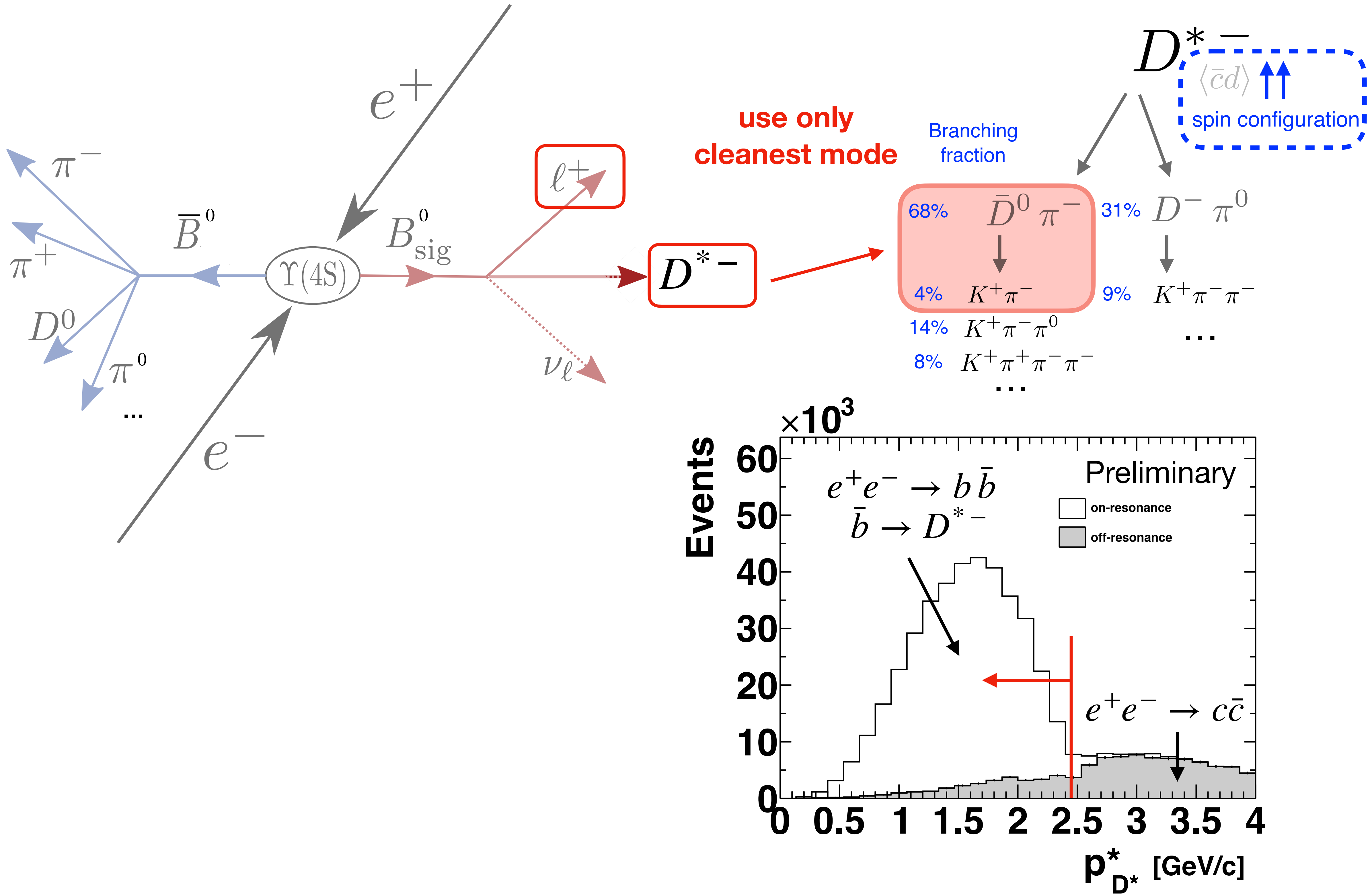


Belle II Preprint 2023-014
KEK Preprint 2023-28

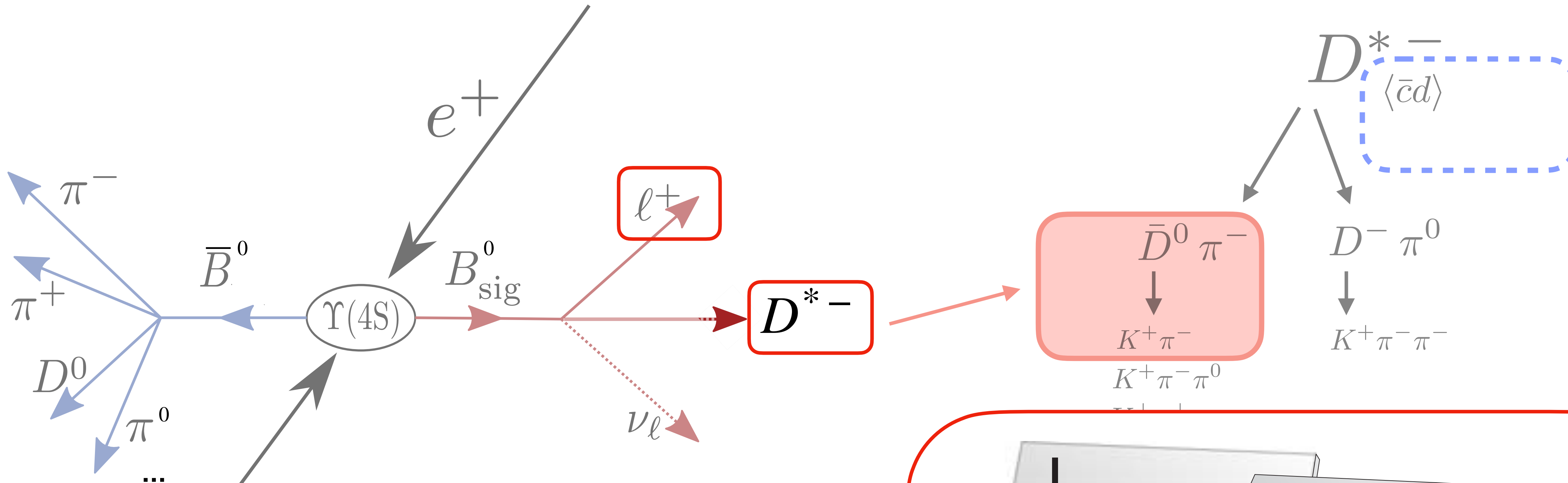
Determination of $|V_{cb}|$ using $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ decays with Belle II

I. Adachi, L. Aggarwal, H. Ahmed, H. Aihara, N. Akopov, A. Aloisio, N. Anh Ky, D. M. Asner, H. Atmacan, T. Aushev, V. Aushev, M. Aversano, V. Babu, H. Bae, S. Bahinipati, P. Bambade, Sw. Banerjee, S. Bansal, M. Barrett, J. Baudot, M. Bauer, A. Baur, A. Beaubien, F. Becherer, J. Becker, P. K. Behera, J. V. Bennett, F. U. Bernlochner, V. Bertacchi, M. Bertemes, E. Bertholet, M. Bessner, S. Bettarini, B. Bhuyan, F. Bianchi, T. Bilka, D. Biswas, A. Bobrov, D. Bodrov, A. Bolz, A. Bondar, J. Borah, A. Bozek, M. Bračko, P. Branchini, R. A. Briere, T. E. Browder, A. Budano, S. Bussino, M. Campajola, L. Cao, G. Casarosa, C. Cecchi, J. Cerasoli, M.-C. Chang, P. Chang, R. Cheaib, P. Cheema, V. Chekelian, C. Chen, B. G. Cheon, K. Chilikin,

Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$



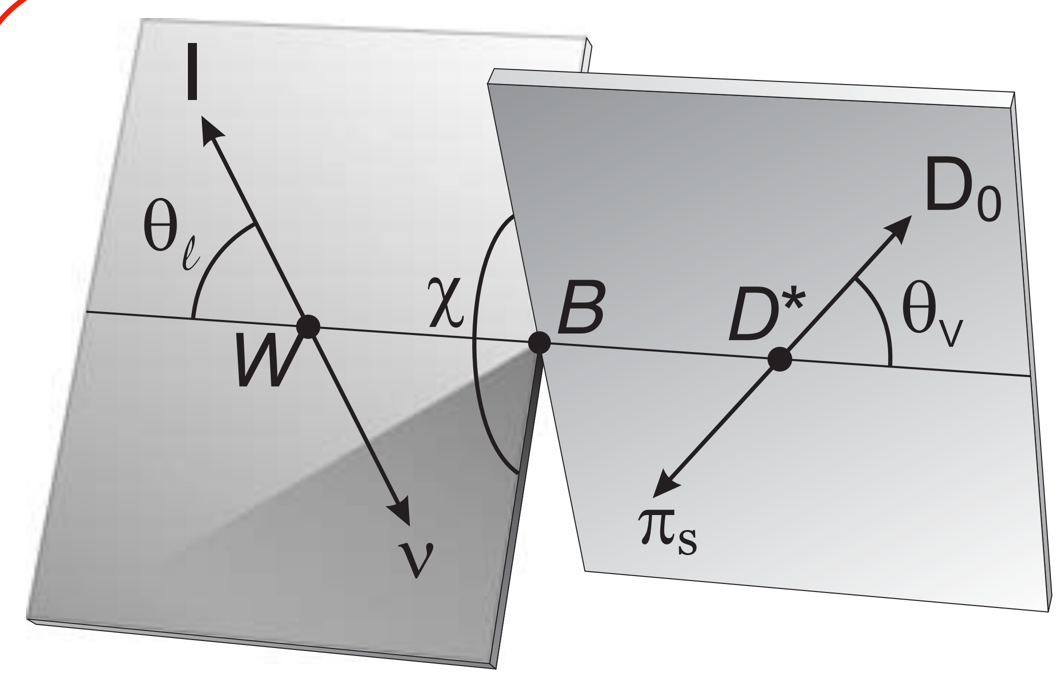
Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$



$$\vec{p}_{\text{incl.}} = \sum_i \vec{p}_i \ ,$$

Reconstruct ROE to estimate B_{sig}^0 momentum

$$\vec{p}_{B_{\text{sig}}} = -\vec{p}_{\text{incl}}$$

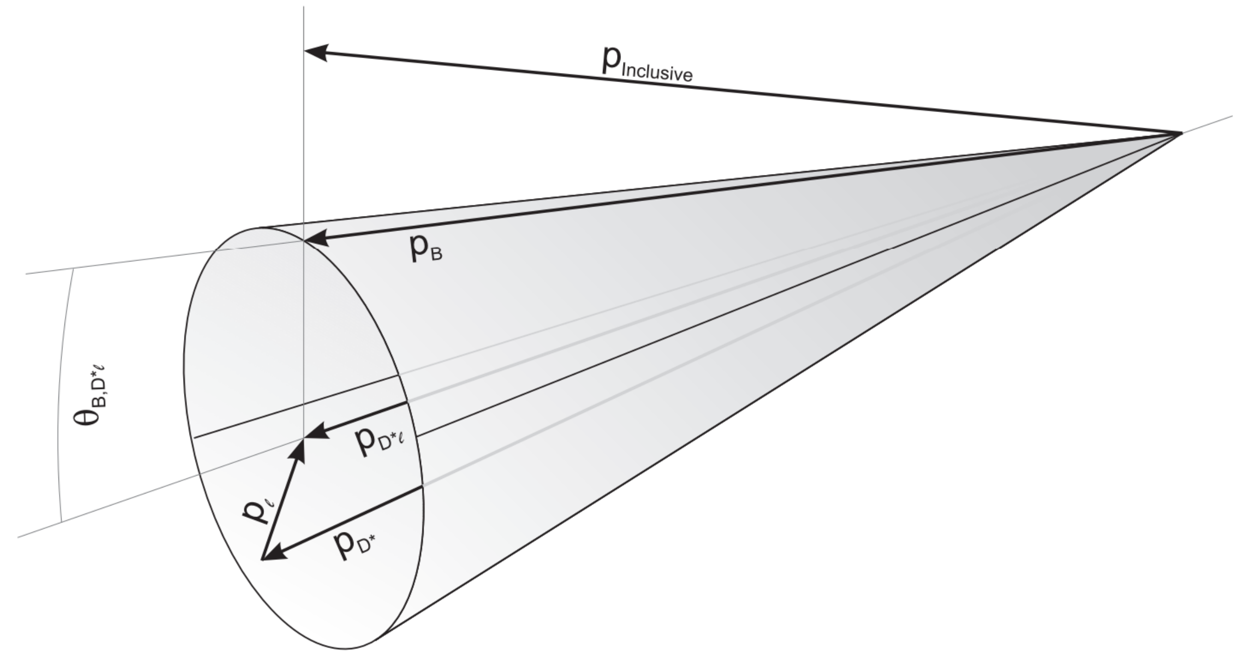


$$w \sim q^2 = (p_B - p_{D^*})^2$$

Improved Reconstruction Methods

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:

$$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}$$



Derivation :

$$0 = p_\nu^2 = \big(p_B - \underbrace{p_{D^*\ell}}_{p_{D^*} + p_\ell}\big)^2 \quad = p_B^2 + p_{D^*\ell}^2 - 2 p_B p_{D^*\ell} = m_B^2 + m_{D^*\ell}^2 - 2E_B E_{D^*\ell} + 2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|\,\cos\theta_{B-D^*\ell}$$

$$\rightarrow \quad \cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}$$

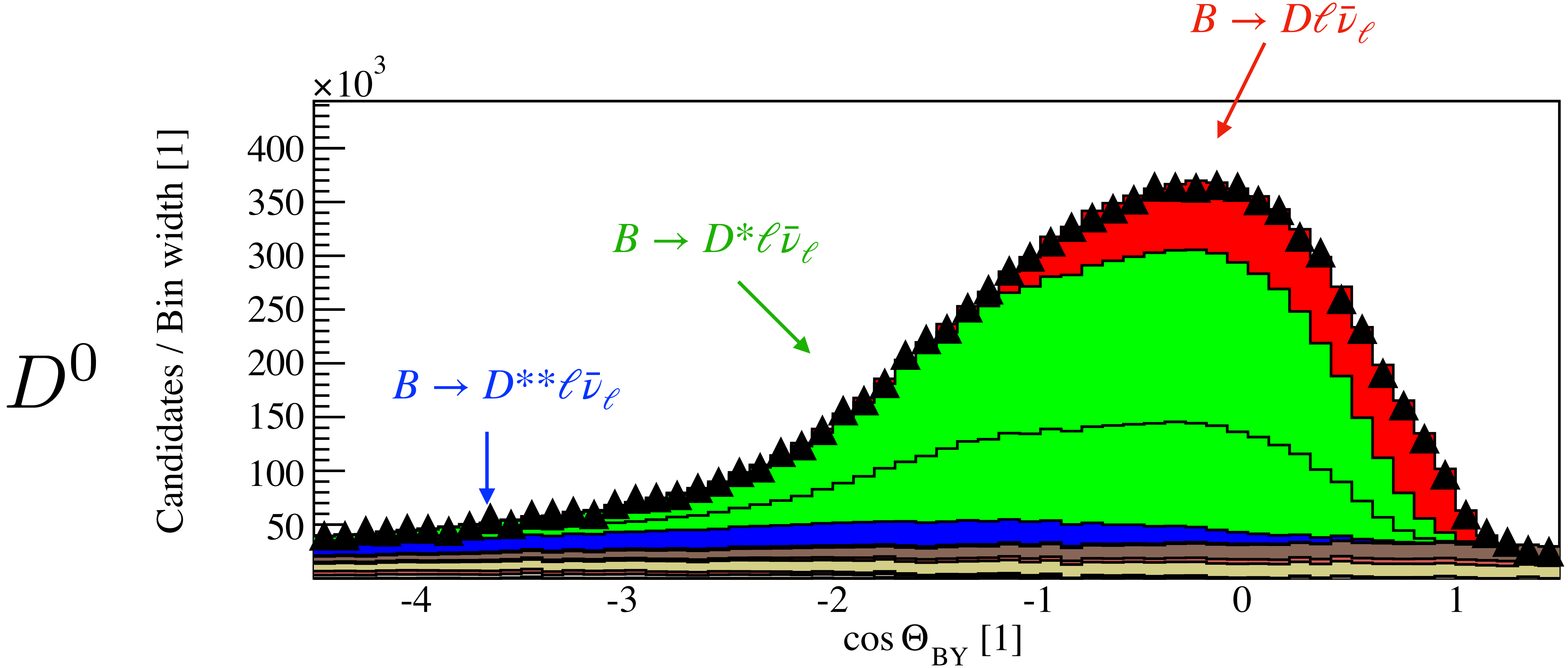
Missing particles :

$$(p_\nu + p_{\text{miss}})^2 = m_B^2 + m_{D^*\ell}^2 - 2E_B E_{D^*\ell} + 2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|\,\cos\theta_{B-D^*\ell} \quad \rightarrow \quad \cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|} + \frac{(p_\nu + p_{\text{miss}})^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}$$

→ shifts $\cos\theta_{B,D^*\ell}$ to **negative** values if not included

Example: reconstruct $B \to D\ell\bar{\nu}_\ell$ (and allow for missing particles, i.e. untagged)

$$B \to DX\ell\bar{\nu}_\ell$$

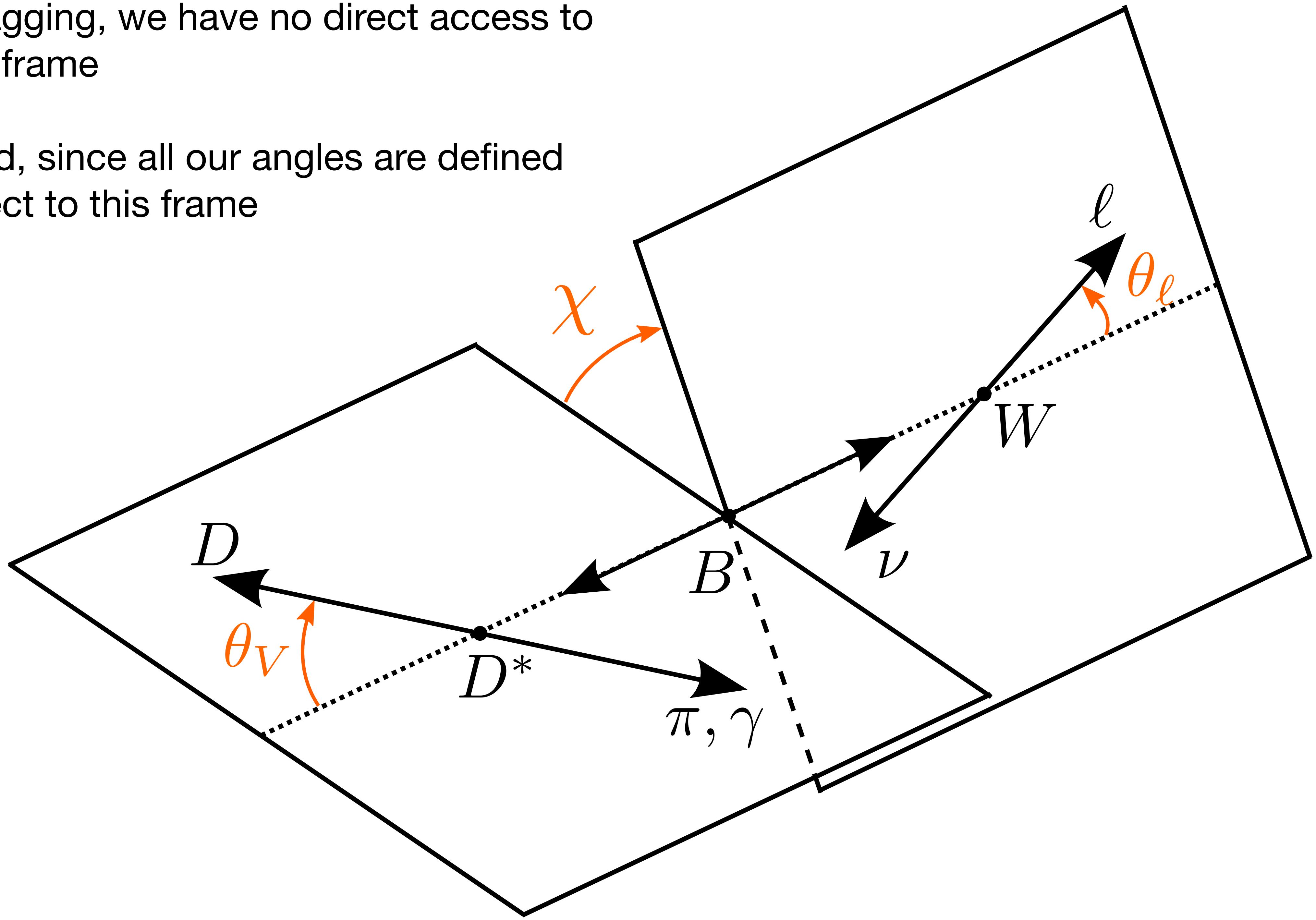


Good discriminating variable, so we will get back to using it.

Estimating the B Frame

Without tagging, we have no direct access to the B rest frame

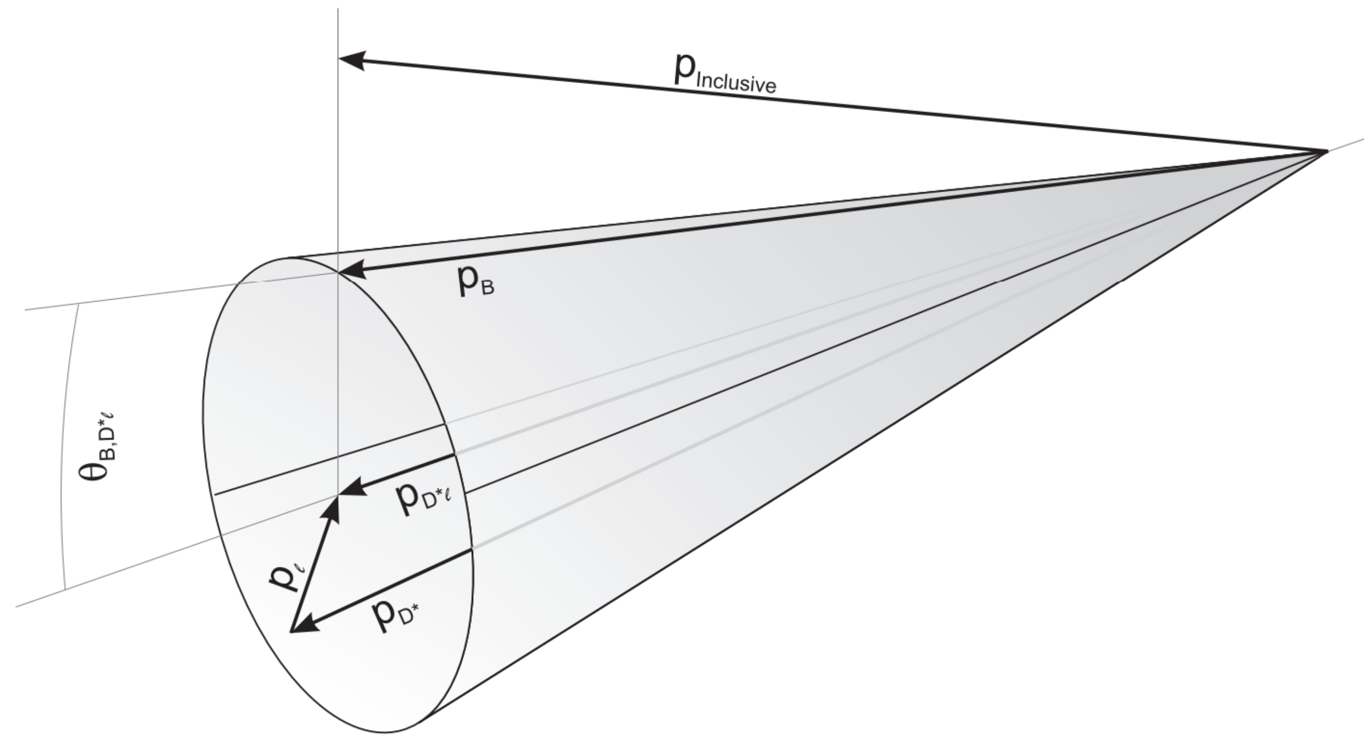
That is bad, since all our angles are defined with respect to this frame



Estimating the B Frame

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:

$$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2\,|\mathbf{p}_B|\,|\mathbf{p}_{D^*\ell}|}$$



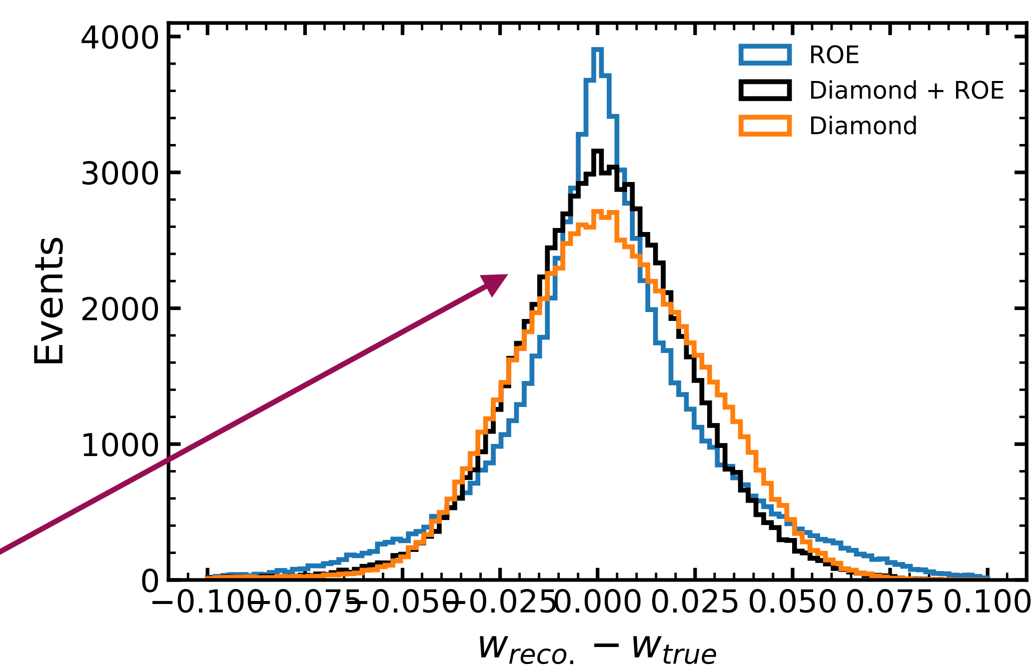
Can use this to estimate B meson direction building a weighted average on the cone

$$(E^B, p_B^x, p_B^y, p_B^z) = (\sqrt{s}/2, |\mathbf{p}_B| \sin\theta_{BY}\cos\phi, |\mathbf{p}_B|\sin\theta_{BY}\sin\phi, |\mathbf{p}_B|\cos\theta_{BY})$$

with weights according to $w_i = \sin^2\theta_i$ with θ denoting the polar angle

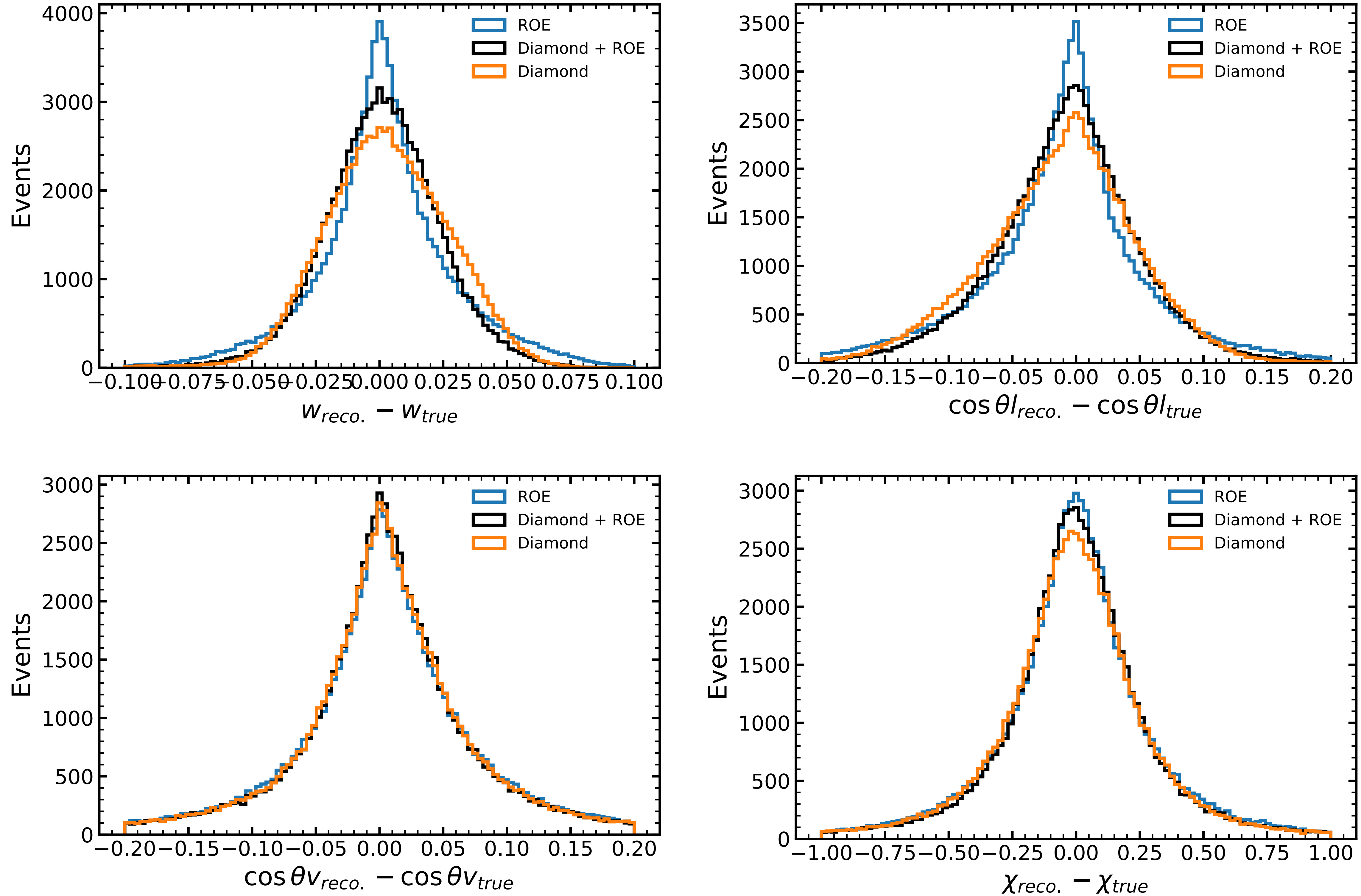
(following the angular distribution of $\Upsilon(4S) \to B\bar{B}$)

One can also **combine** both estimates



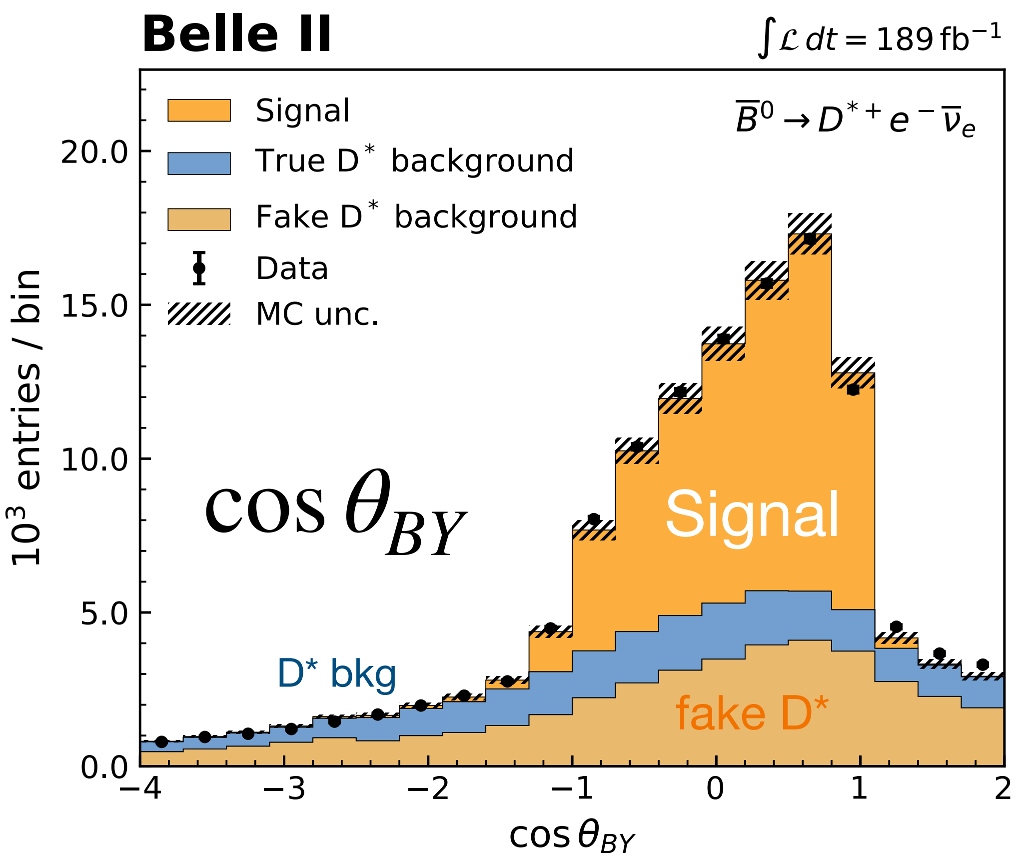
$$\tilde{w}_i = (1 - \hat{p}_{\mathrm{ROE}} \cdot \hat{p}_{B_i}) \sin^2\theta_{B_i}$$

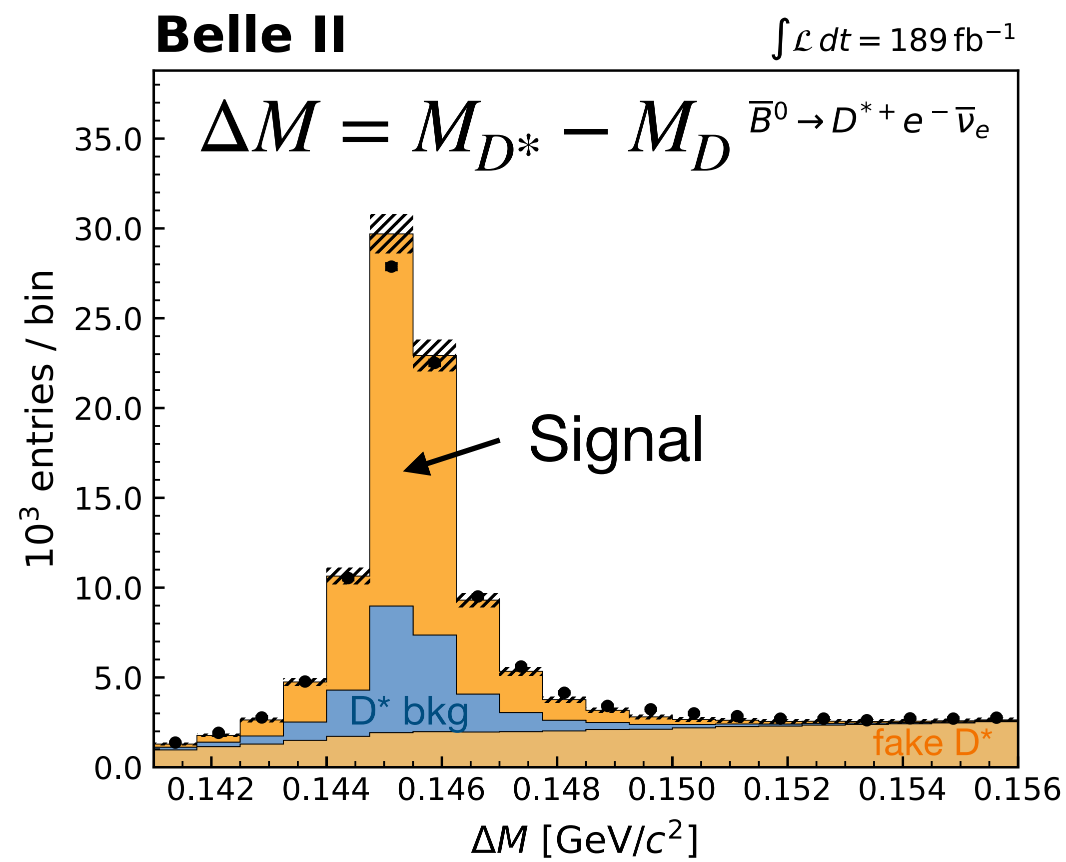
Estimating the B Frame



Background Subtraction

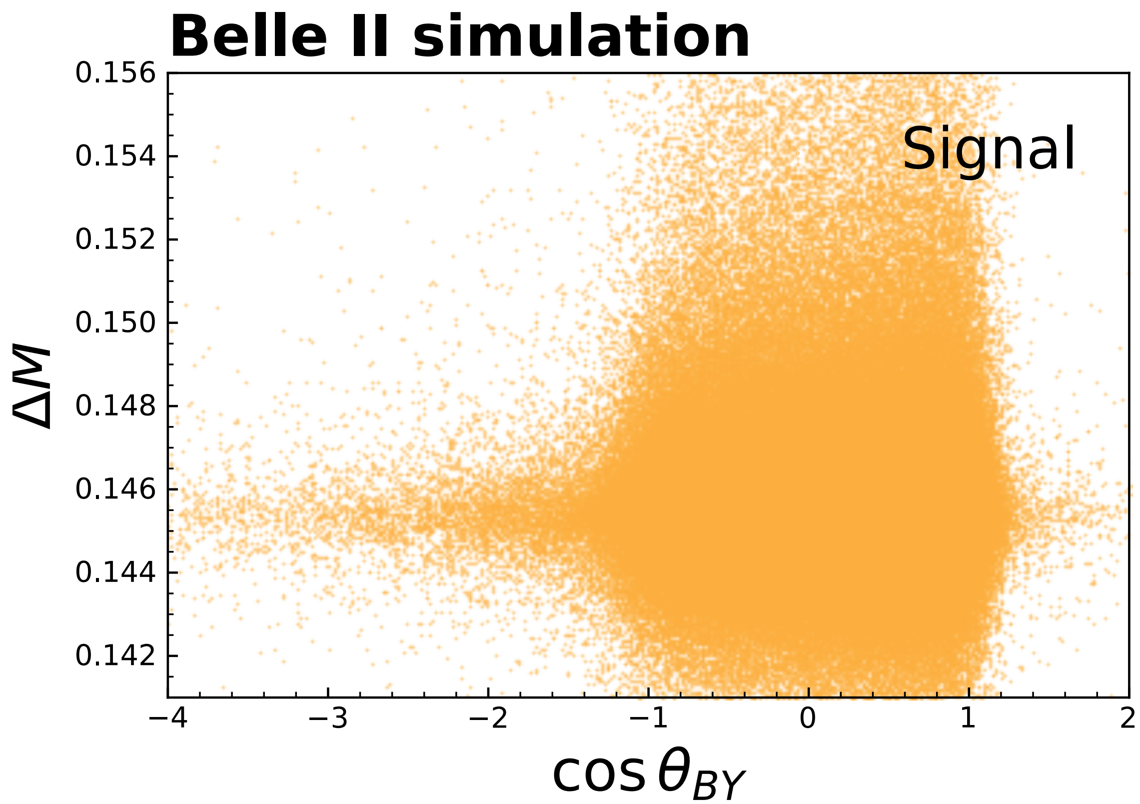
2D Fit of $\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\vec{p}_B||\vec{p}_{D^*\ell}|}$ $\Delta M = m_{D^*} - m_D$

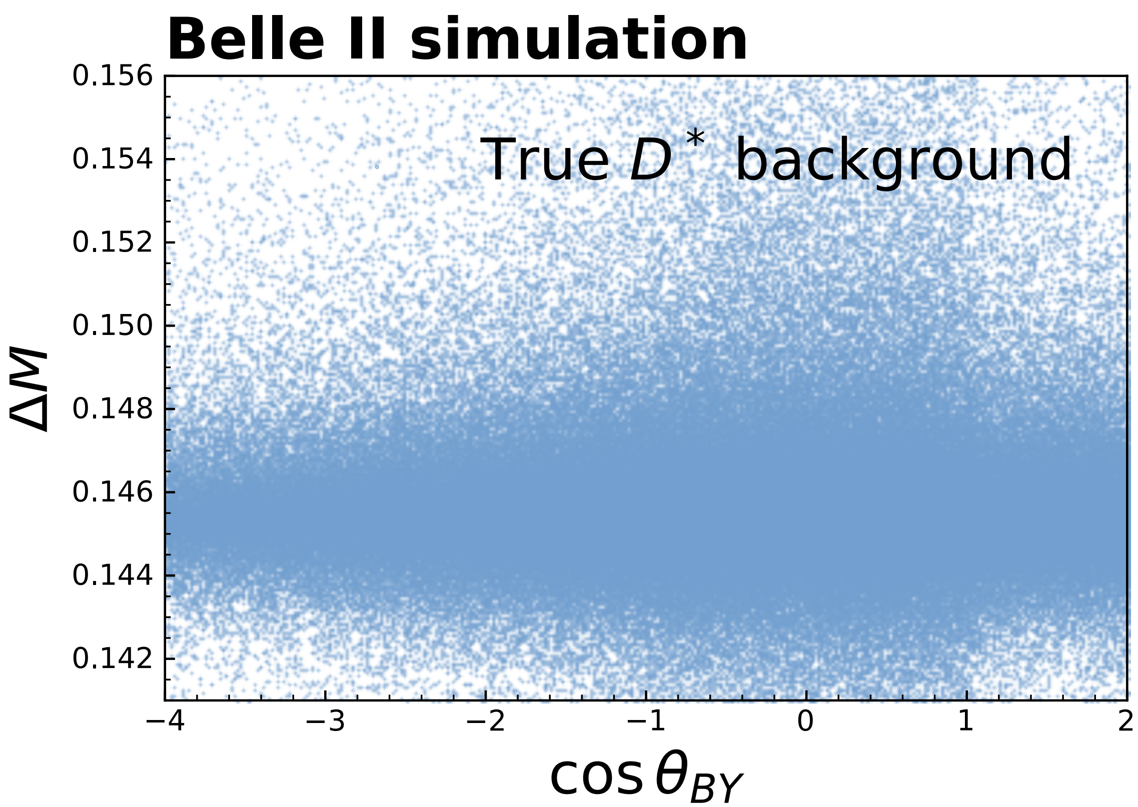


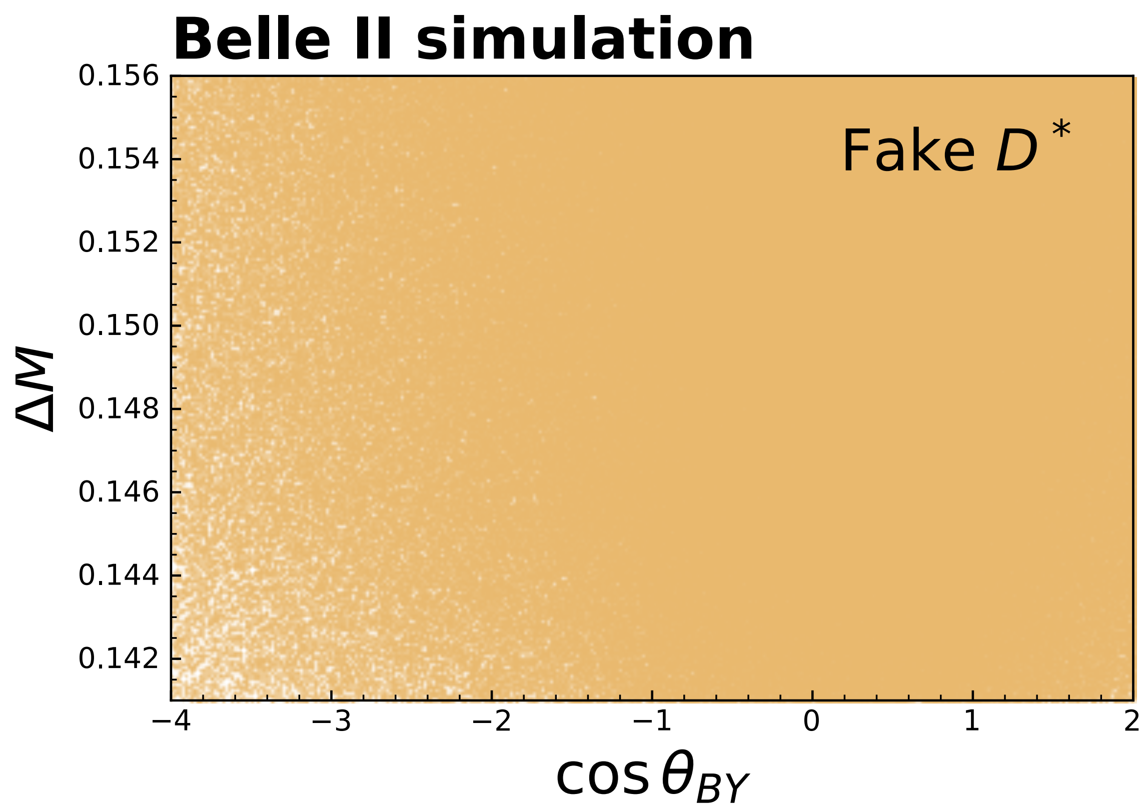


Fit **each bin** of the **kinematic variable**, unfold and correct for selection eff.

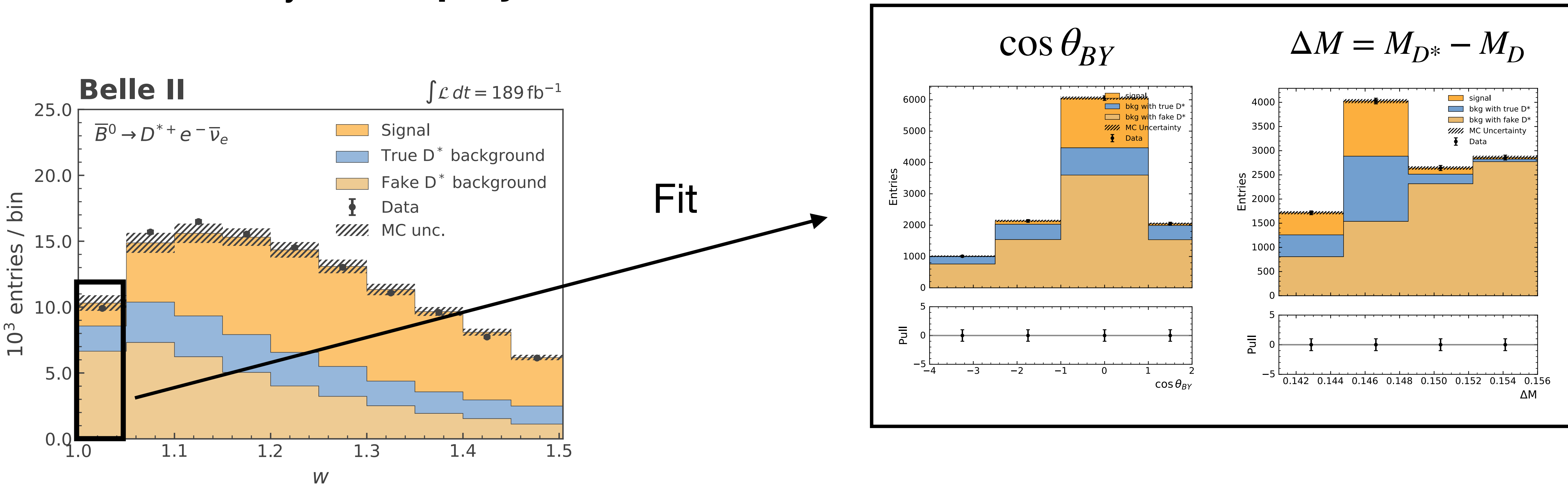
2D separation power :



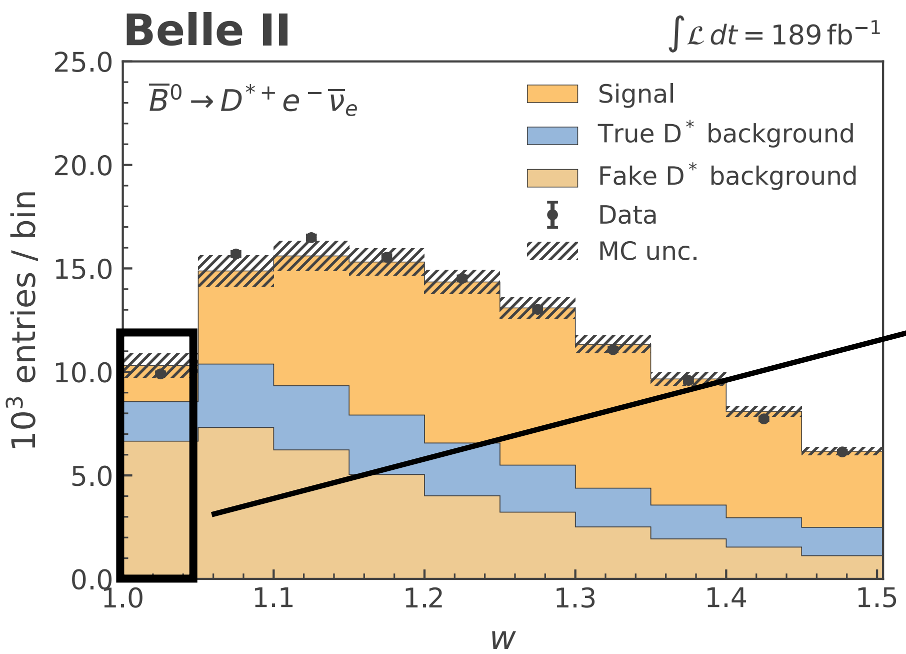




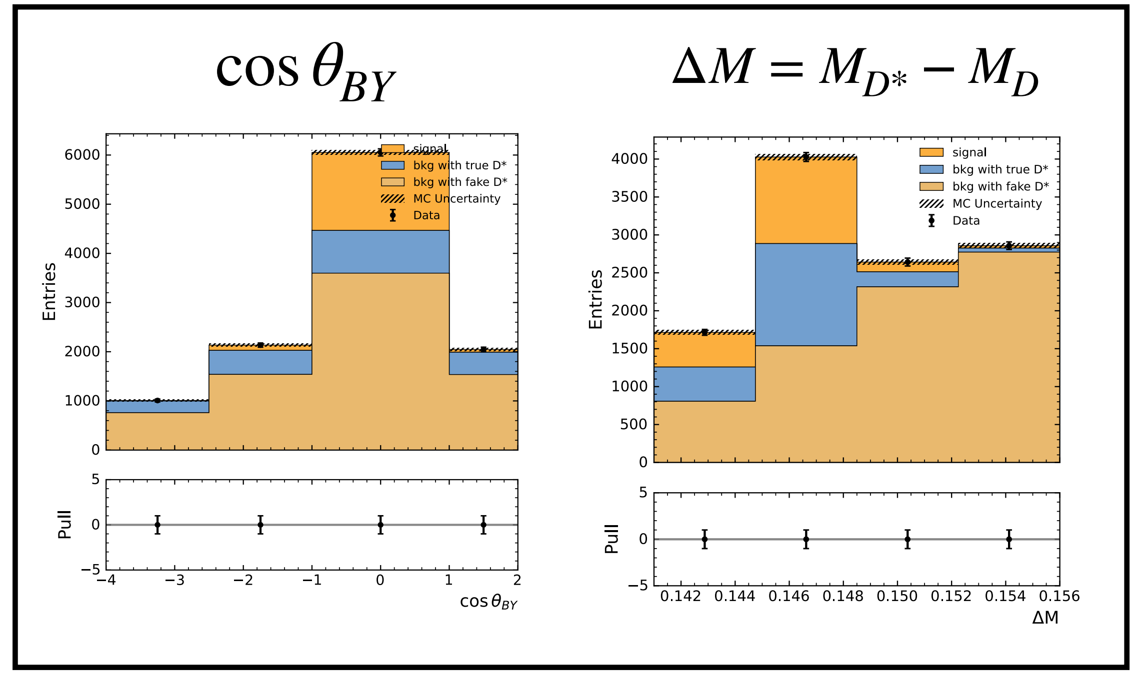
Also focus initially on **1D projections**:



Also focus initially on **1D projections**:

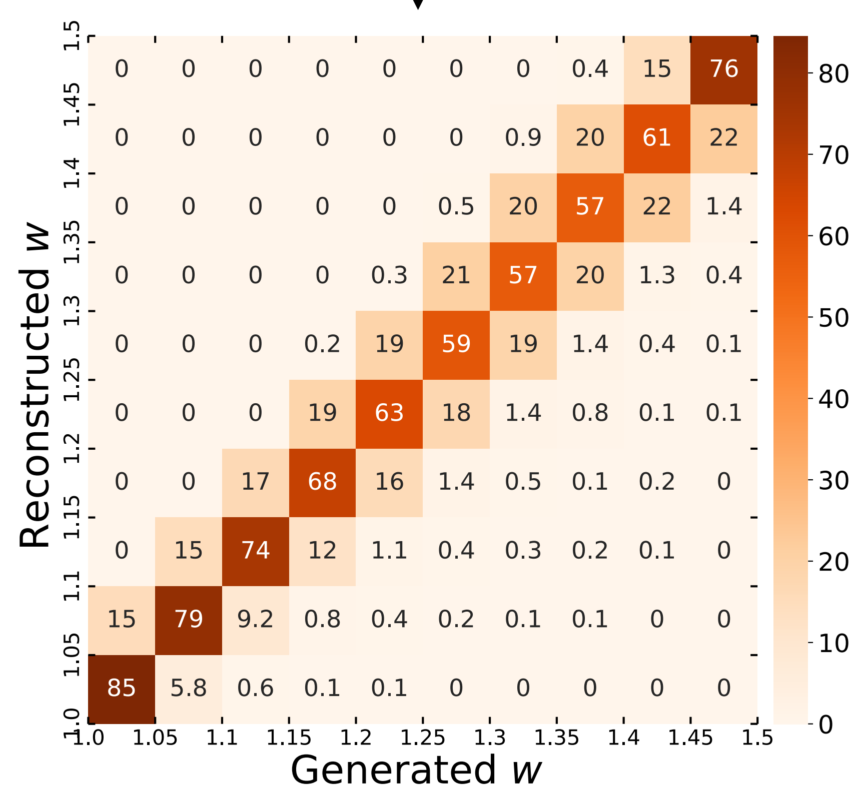


Fit



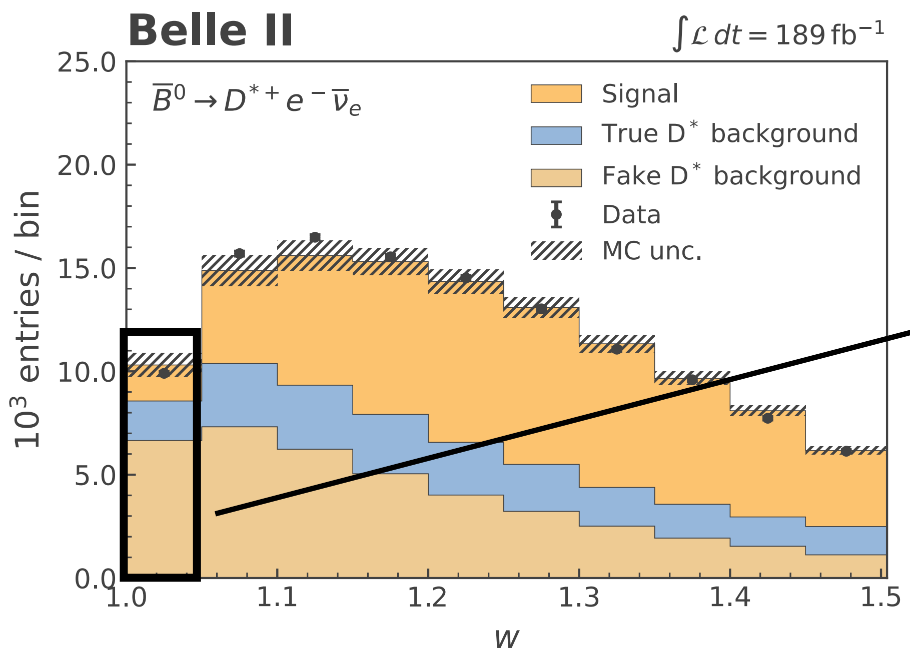
Correct for migration effects:

“Reco”

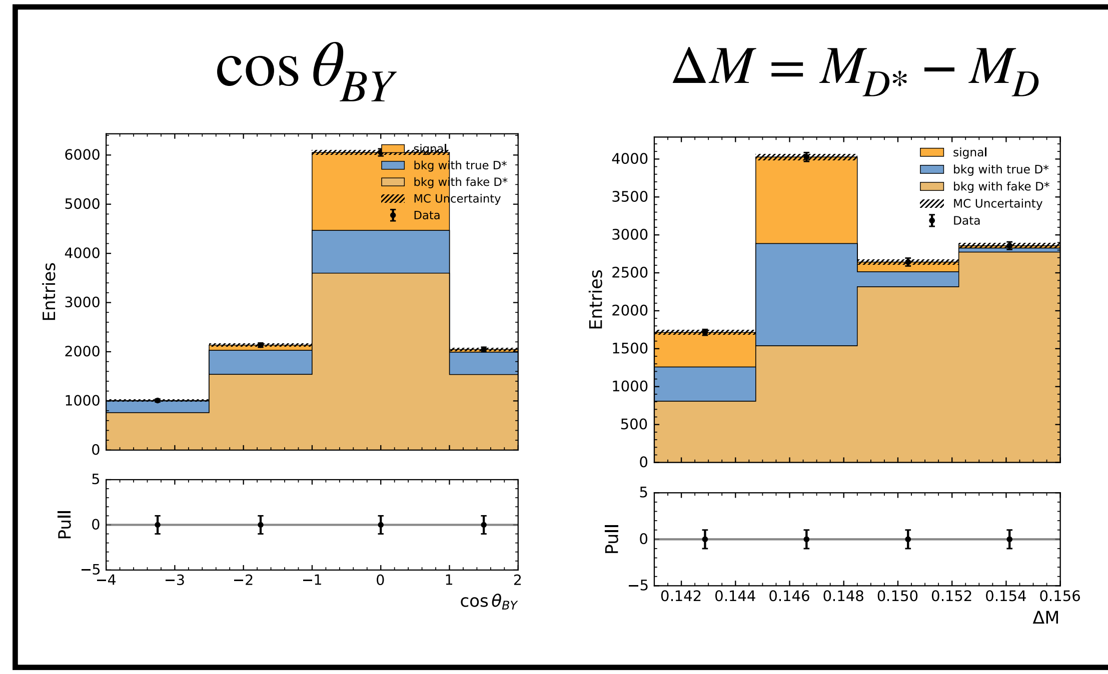


“True”

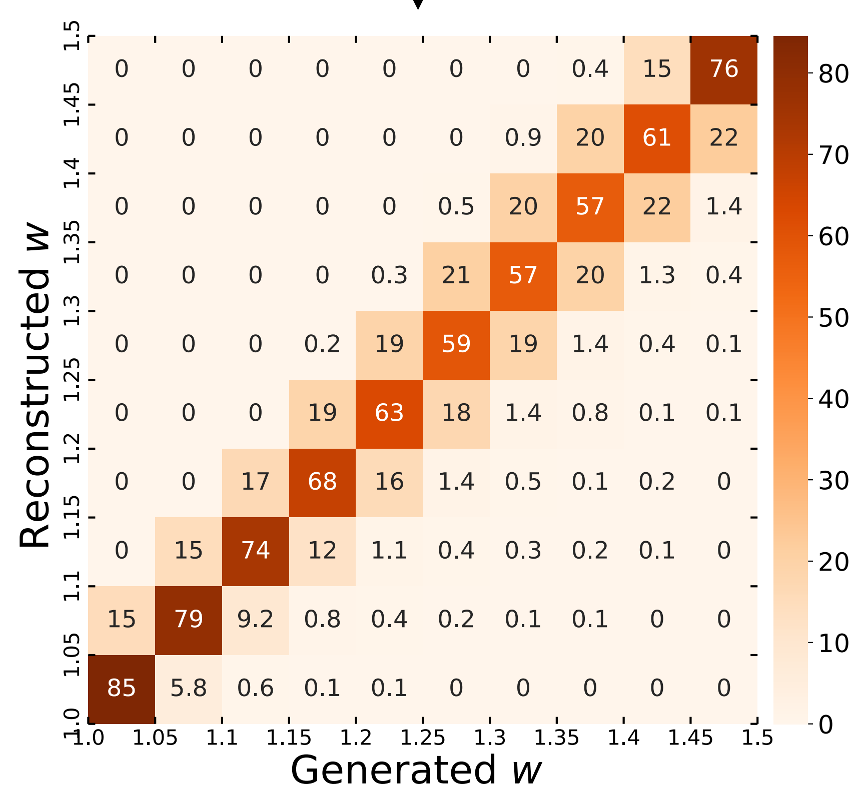
Also focus initially on **1D projections**:



Fit



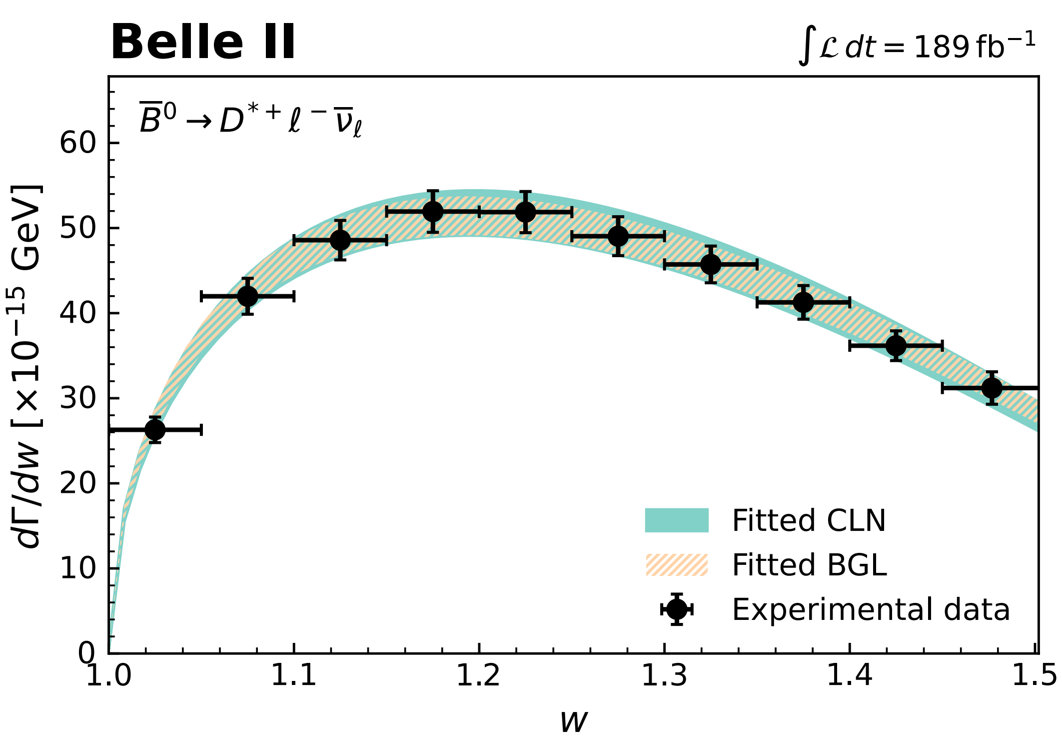
Correct for migration effects:

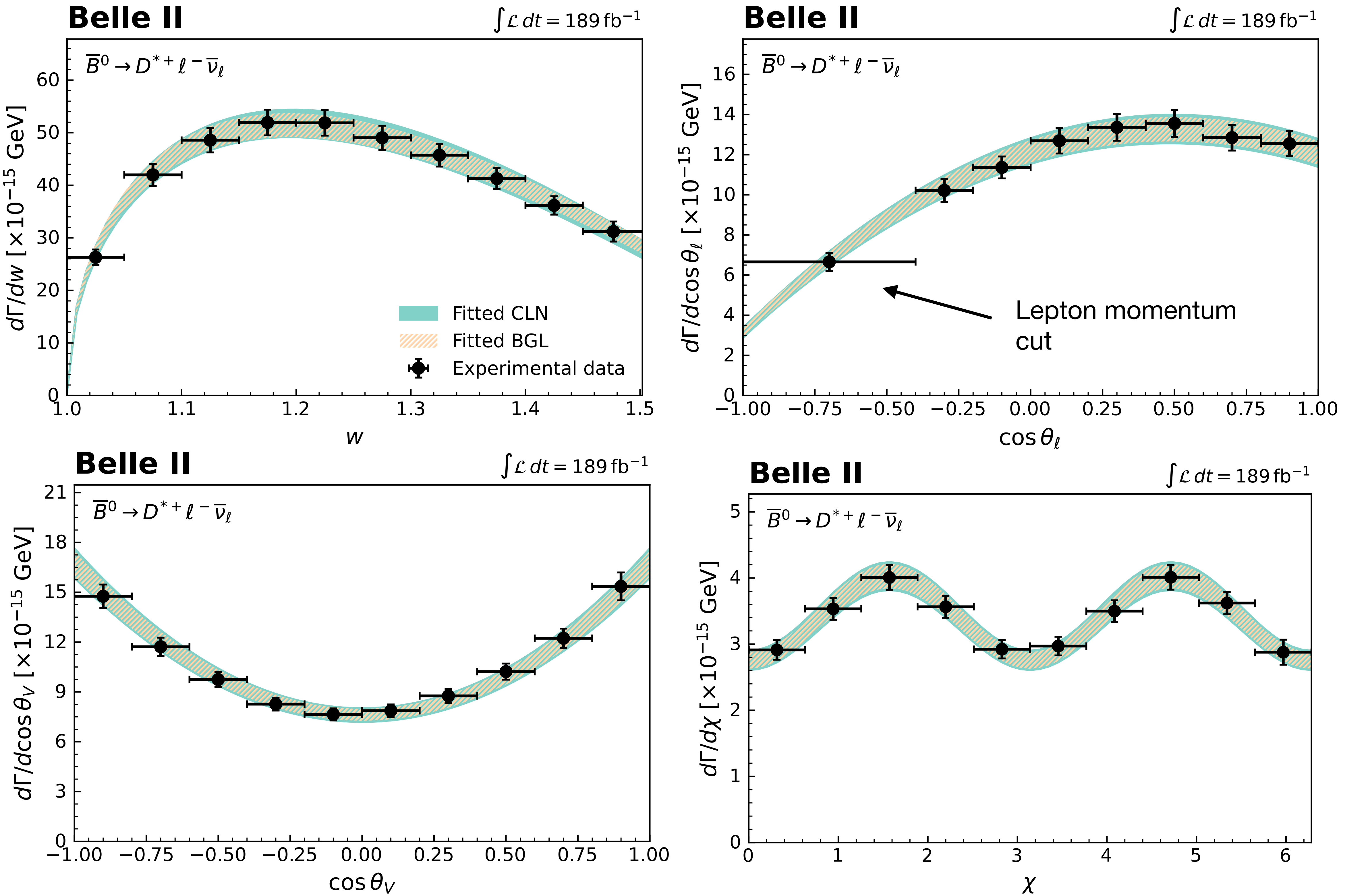


"Reco"

"True"

Correct for acceptance & efficiency





$$|V_{cb}|_{\text{CLN}} = (40.2 \pm 0.3 \pm 0.9 \pm 0.6) \times 10^{-3},$$

$$|V_{cb}|_{\text{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}.$$

← **BGL truncation** order determined using **Nested Hypothesis Test**

(n_a, n_b, n_c)	$\lvert V_{cb}\rvert \times 10^3$	$\rho_{\max}$	χ^2	Ndf	p-value
(1, 1, 2)	40.2 ± 1.1	0.28	40.5	32	14%
(2, 1, 2)	40.1 ± 1.1	0.97	38.6	31	16%
(1, 2, 2)	**40.6 ± 1.2**	**0.57**	**39.1**	**31**	**15%**
(1, 1, 3)	40.1 ± 1.1	0.97	40	31	13%
(2, 2, 2)	40.2 ± 1.3	0.99	38.6	30	13%
(1, 3, 2)	39.8 ± 1.3	0.98	37.6	30	16%
(1, 2, 3)	40.5 ± 1.2	0.97	39	30	13%

Inclusive |Vub |

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive |Vcb |

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \Big[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \Big]$$

Exclusive |Vub |

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

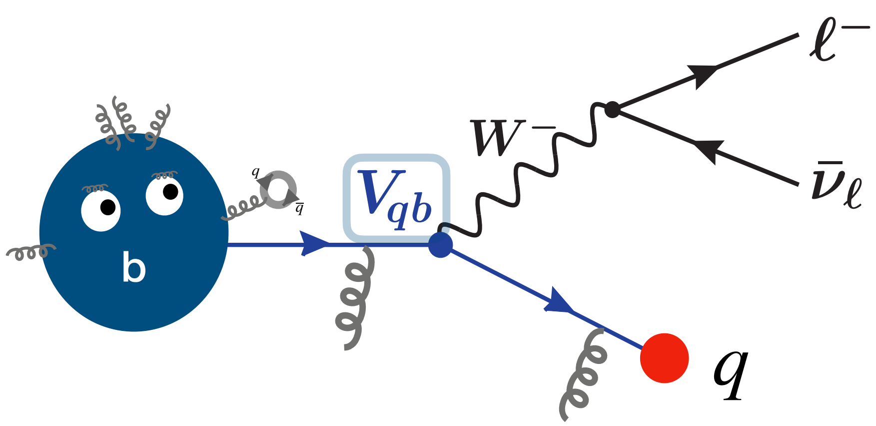
Exclusive |Vcb |

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

$$\mathcal{B} \propto |V_{cb}|^2 \, f^2$$

Form Factors

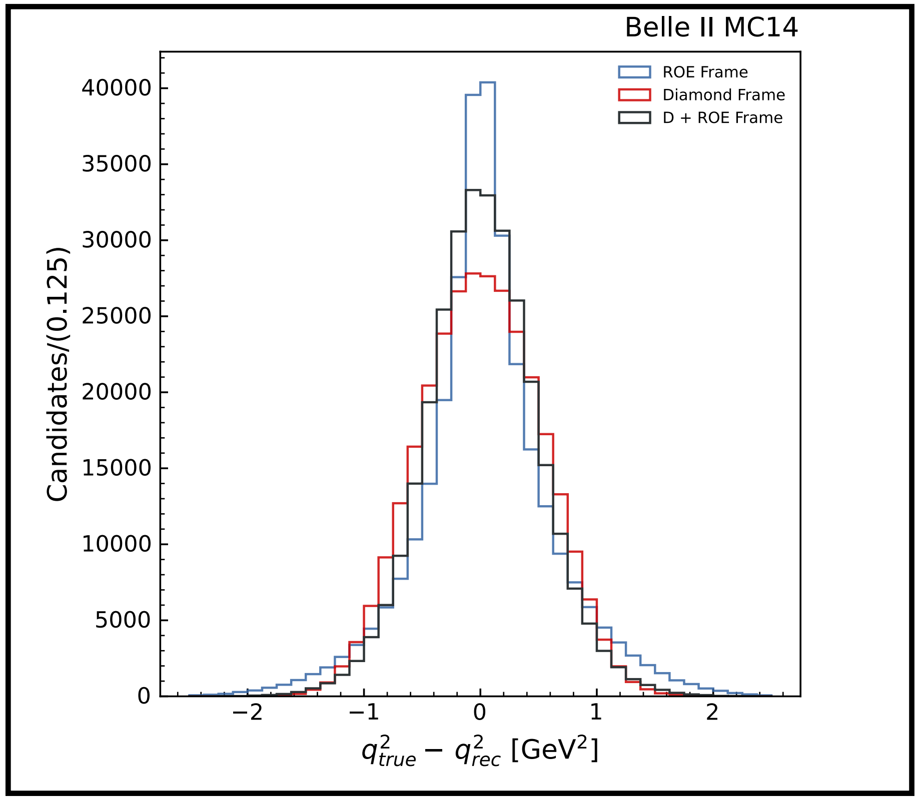
$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$



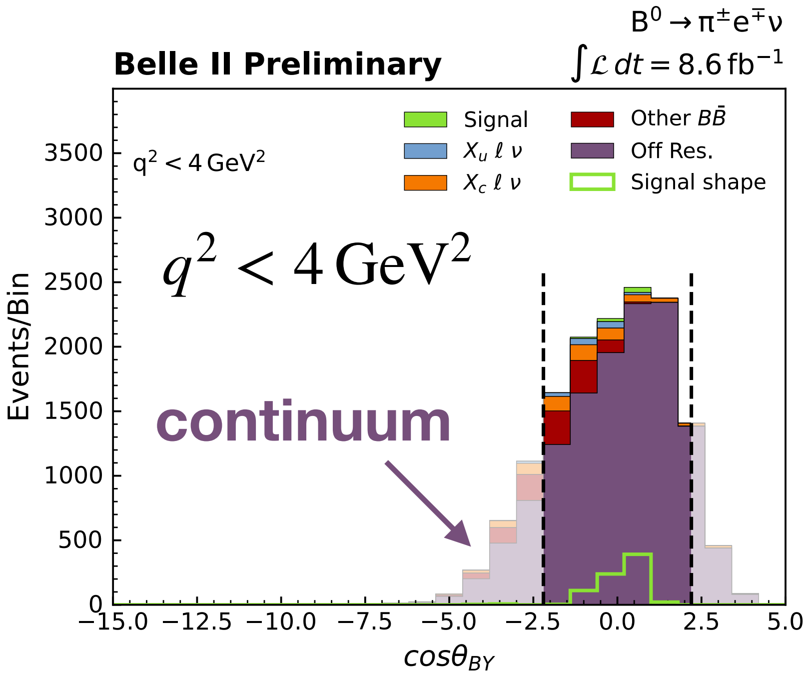
Exclusive measurements of $b \to u \ell \bar{\nu}_\ell$

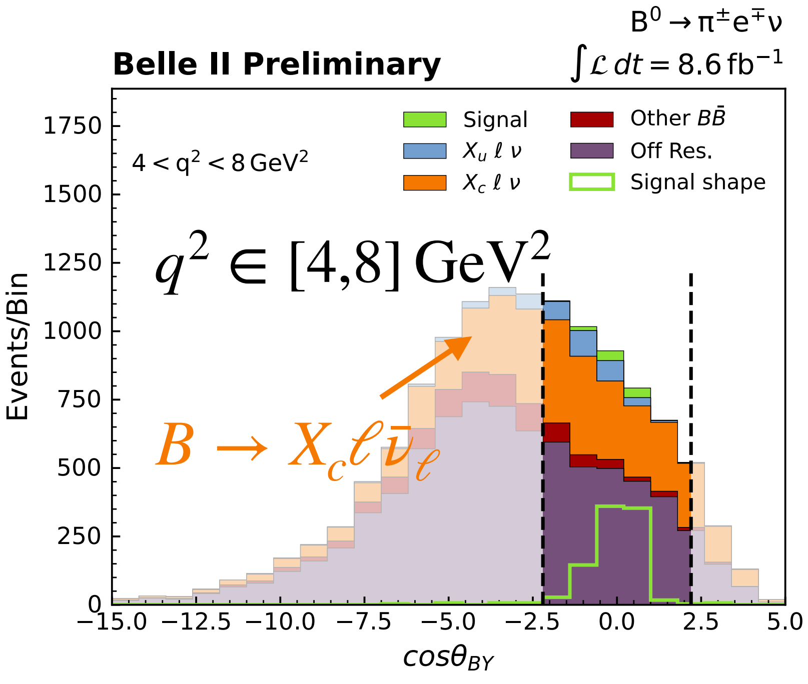
Tagged strategy very similar, but **cross feed** from different modes (e.g. $B \to \rho \ell \bar{\nu}_\ell$) and **large** backgrounds from $B \to D^{(*)} \ell \bar{\nu}_\ell$ (+ other B decays) and **continuum**

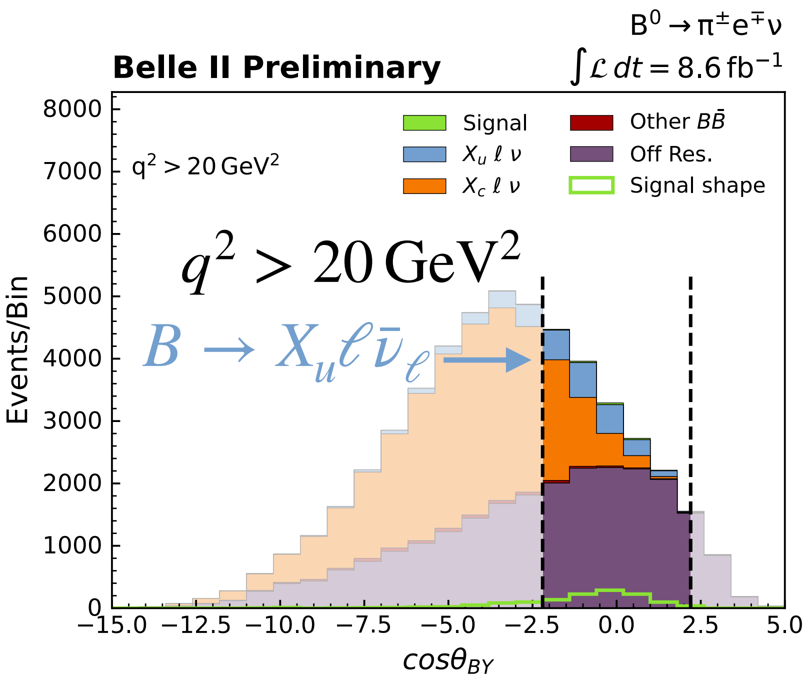
Can reconstruct q^2 with the **same method** as for $B \to D^* \ell \bar{\nu}_\ell$ →



Amount of **background strongly changes** as a function of q^2







Exclusive measurements of $b \to u\ell\bar{\nu}_\ell$

Need **strong multivariate** suppression to carry out analysis :

Due to different S/B and shapes, train separate one for each BDT bin

Continuum

	q1	q2	q3	q4	q5	q6
$\cos\theta_{\mathrm{BY}}$	88	83	61	59	63	59
$\cos\theta_{\mathrm{Thrust}}$	69	53	44	42	47	54
cc1	13	10	11	4	5	13
$\theta^{\mathrm{lab}}_{\mathrm{miss}}$	9	17	24	25	32	38
$\mathrm{Thrust}_{\mathrm{ROE}}$	8	5	5	5	5	5
cc2	5	8	13	4	5	5
cc3	5	5	8	4	2	3
$\mathrm{Thrust}_{\mathrm{sig}}$	3	20	34	56	41	23

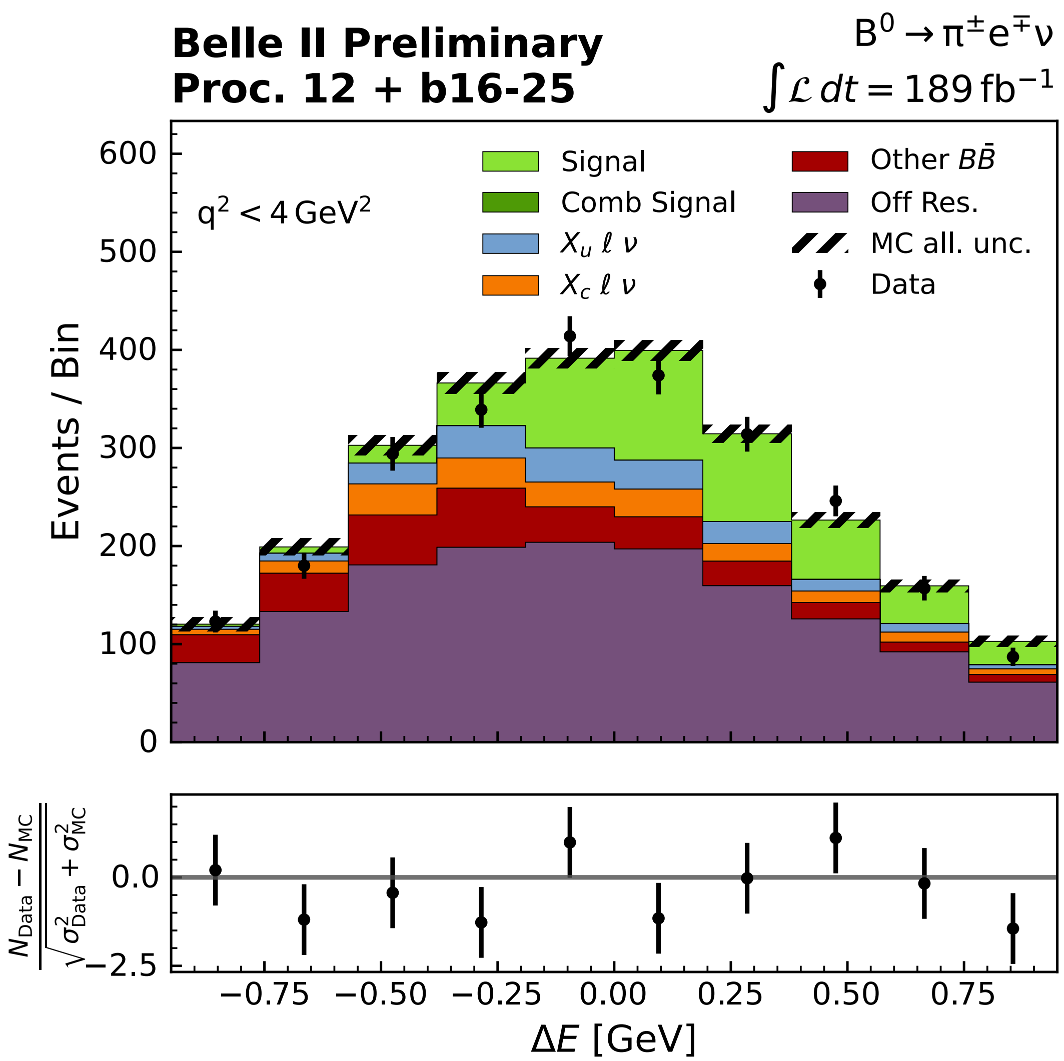
high / low

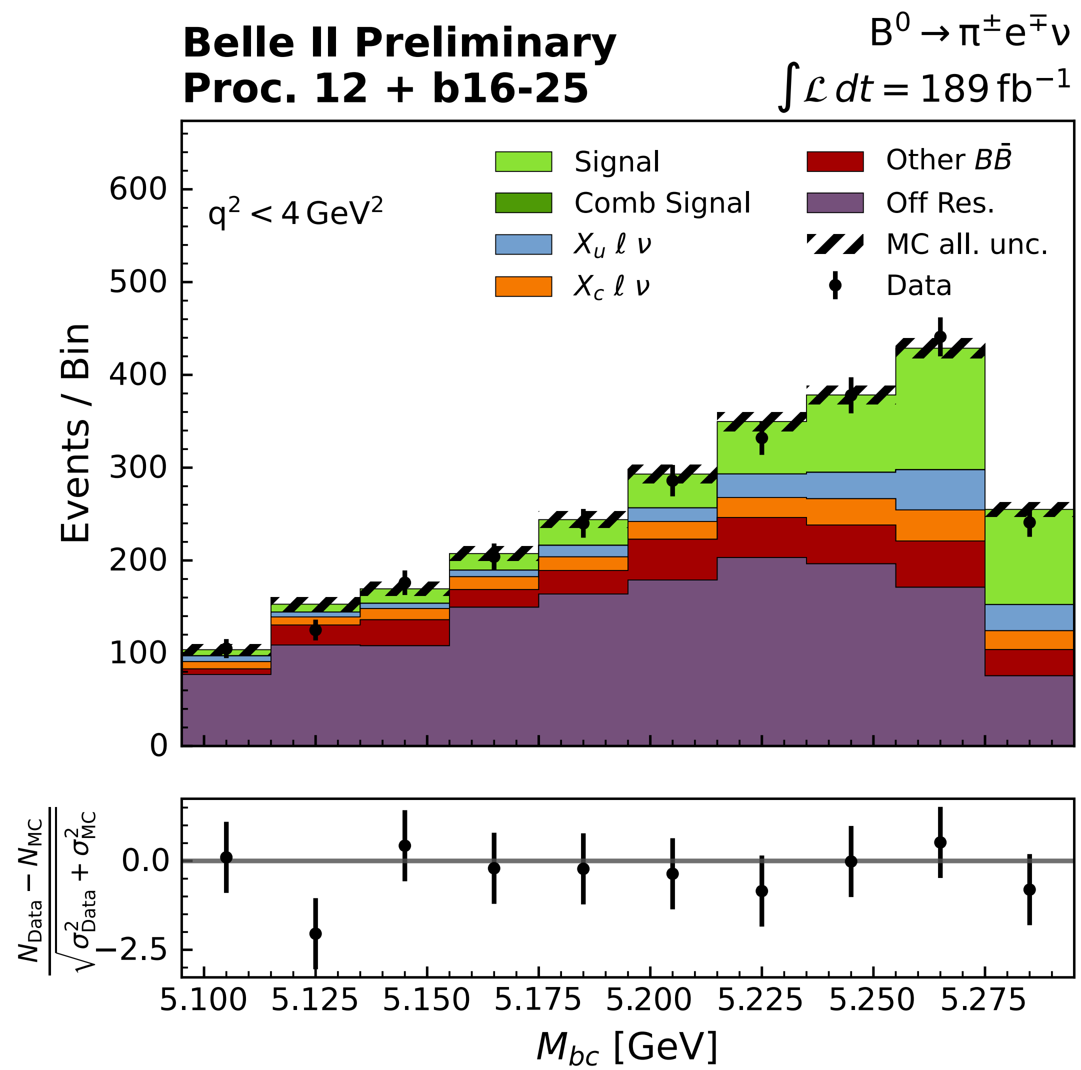
BB

	q1	q2	q3	q4	q5	q6
$\cos\theta_{\mathrm{BY}}$	96	43	34	31	30	44
χ^2_{Fit}	51	82	88	86	80	74
$\cos\theta_{\pi\ell}$	20	22	27	37	49	32
p_{ROE}	14	22	24	24	21	23
$\cos\theta^{\mathrm{XY}}_{\mathrm{p,ip}}$	7	12	9	7	5	5
$\theta^{\mathrm{lab}}_{\mathrm{miss}}$	7	11	11	9	7	8
$\cos\theta_{\mathrm{p,ip}}$	5	6	5	5	5	5
nCleanedTracks	2	2	1	2	3	9

high / low

After BDT selection :





$$\Delta E = E_B^* - E_{\text{beam}}^* = E_B^* - \sqrt{s}/2$$

$$M_{\text{bc}} = \sqrt{E_{\text{beam}}^{*\,2} - \mathbf{p}_\mathbf{B}^*} = \sqrt{s/4 - \mathbf{p}_\mathbf{B}^*}$$

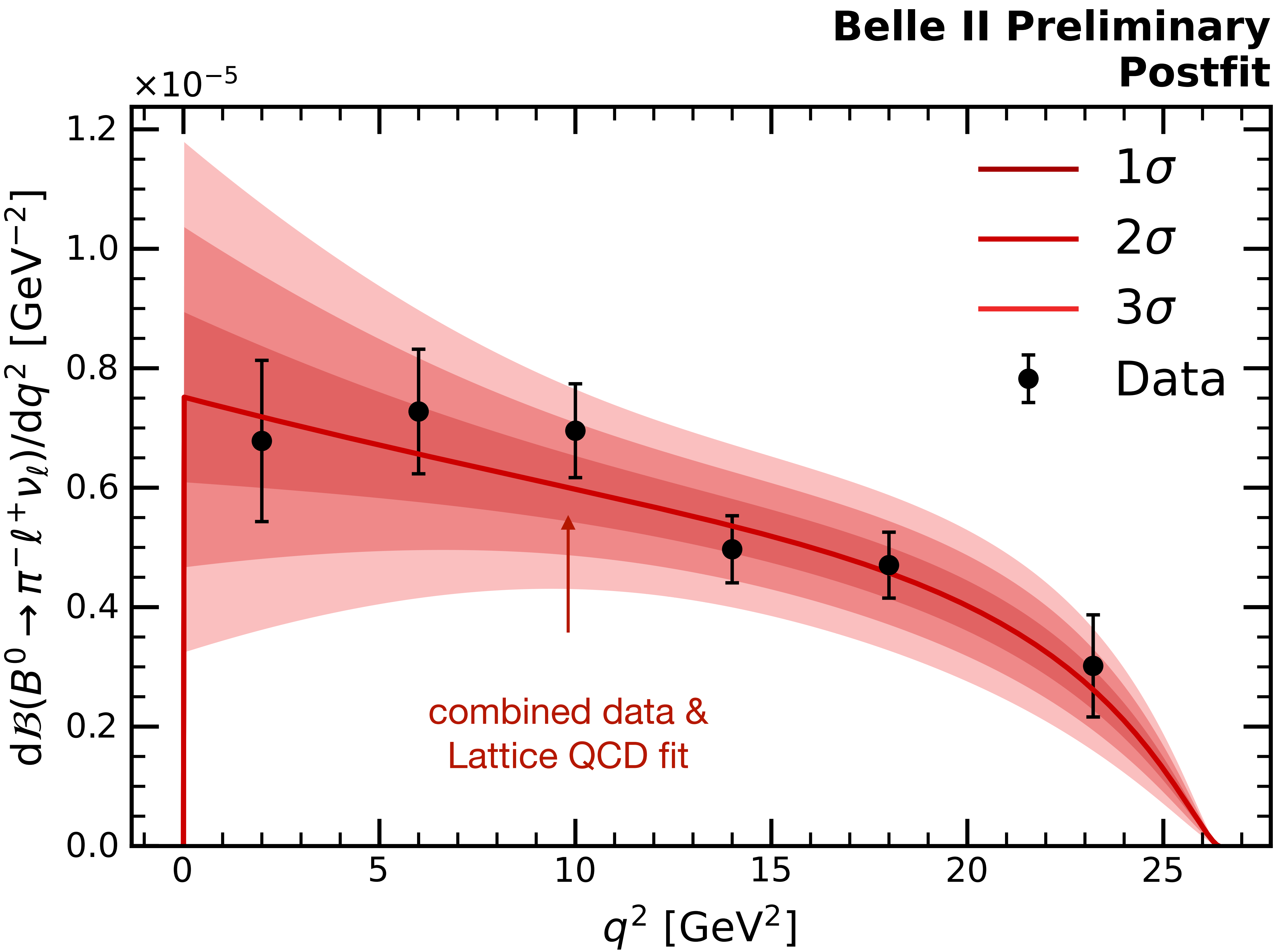
$$E_B^* = E_\pi + E_\ell + E_{\text{miss}}$$

$$\mathbf{p}_\mathbf{B}^* = \mathbf{p}_\pi + \mathbf{p}_\ell + \mathbf{p}_{\text{miss}}$$

Built from clusters & tracks of ROE

$$p_{\text{miss}} = (E_{\text{miss}}, \mathbf{p}_{\text{miss}}) = -\,p_{\text{ROE}}$$

Final Spectrum :



Inclusive |V_{ub}|

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive |V_{cb}|

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \Big[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \Big]$$

Exclusive |V_{ub}|

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

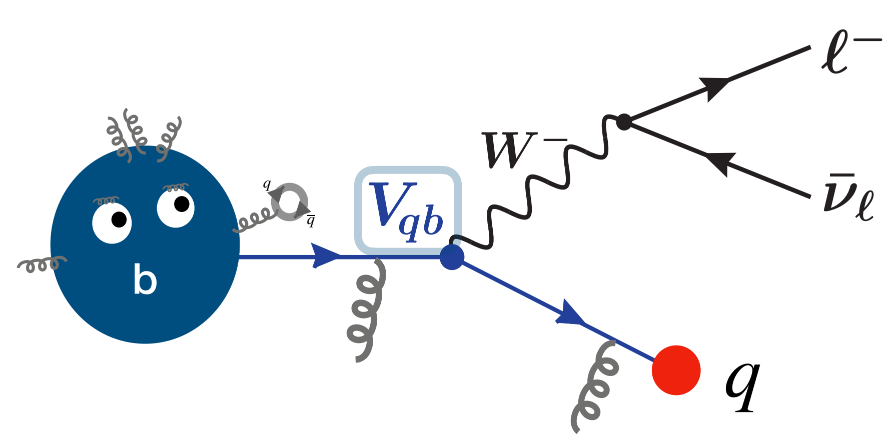
Exclusive |V_{cb}|

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

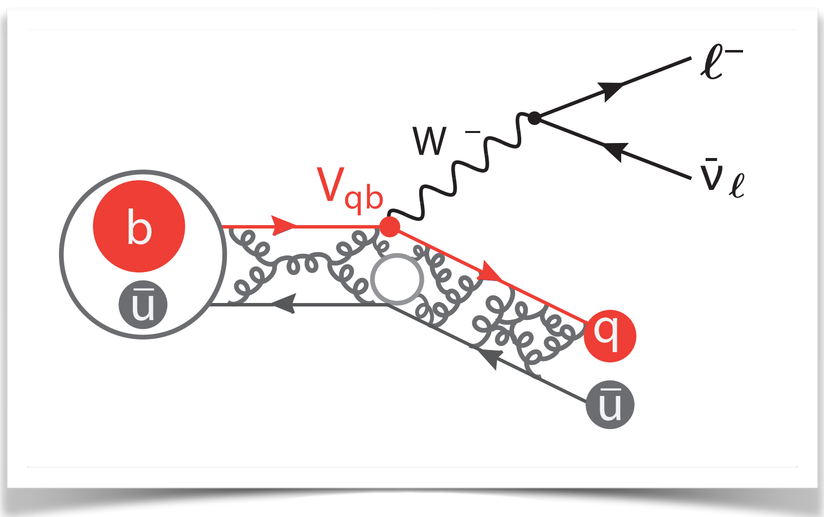
$$\mathcal{B} \propto |V_{cb}|^2 \, f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$



Overview $B \to X_c \ell \bar{\nu}_\ell$



Inclusive $|V_{cb}|$

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Established approach: Use **spectral moments** (hadronic mass moments, lepton energy moments etc.) to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

$d\Gamma$ are calculated perturbatively

Available at $\mathcal{O}(\alpha_s^3)$
Fael, Schönwald, Steinhauser
Phys. Rev. D 104, 016003 (2021)

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ encapsulate non-perturbative dynamics

HQE parameters must be extracted from data

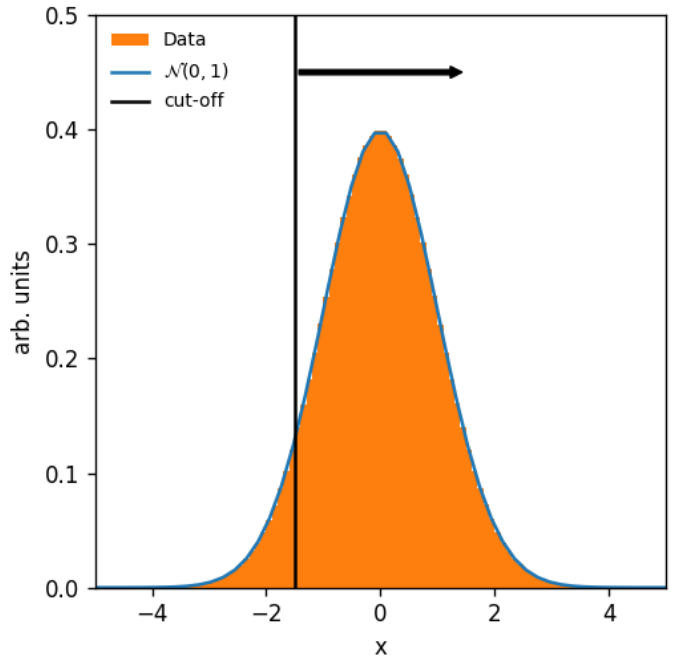
requires the spectral moments of $B \to X_c \ell \nu$

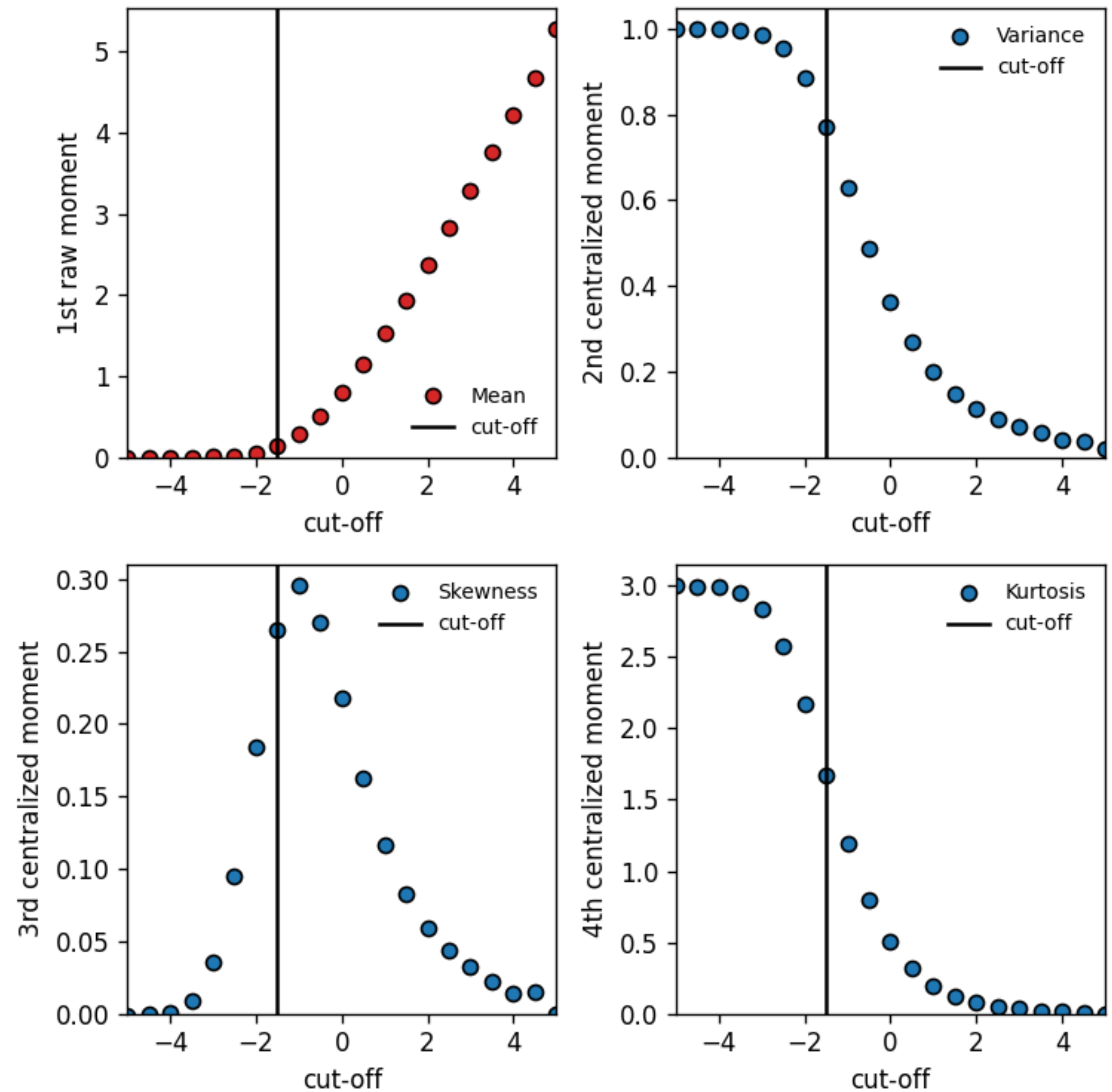
Challenge: Proliferation of HQE parameters at higher order

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Let’s take a moment or two

Illustrations by Markus Prim





$$\mu_n = \int_{-\infty}^{-\infty} (x - c)^n f(x) dx$$

Raw moment: $c = 0$

Central moment: $c = \text{Mean}$

First raw moment: Mean

Measures the location

Second central moment: Variance

Measures the spread

Third central moment: Skewness

Measures asymmetry

Fourth central moment: Kurtosis

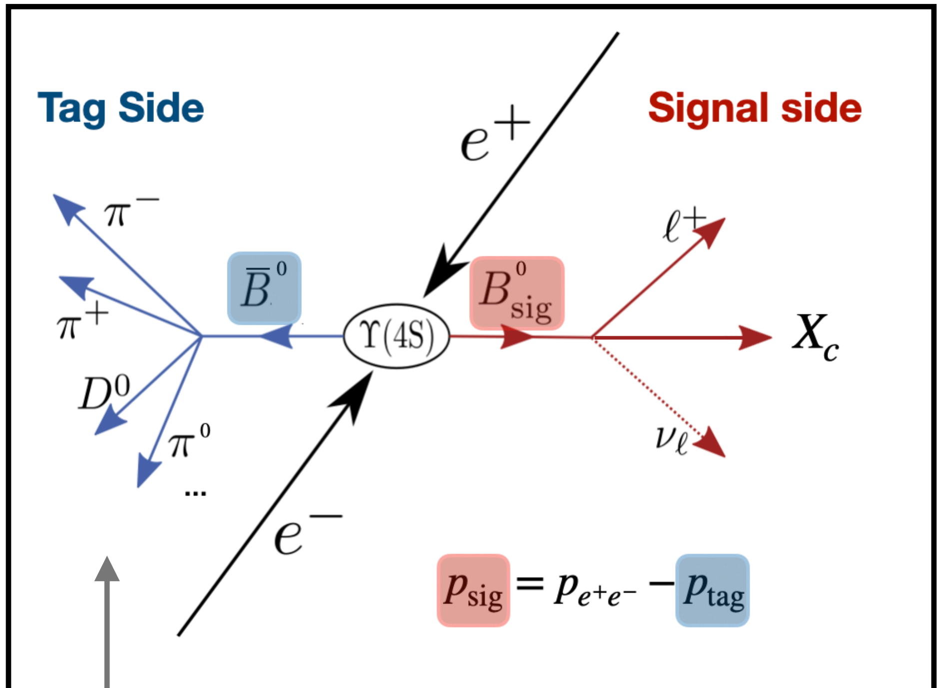
Measures “tailedness”

Moments are measured with progressive cuts in the distribution

→ **highly correlated measurements**

How to measure spectral moments

Key-technique: hadronic tagging



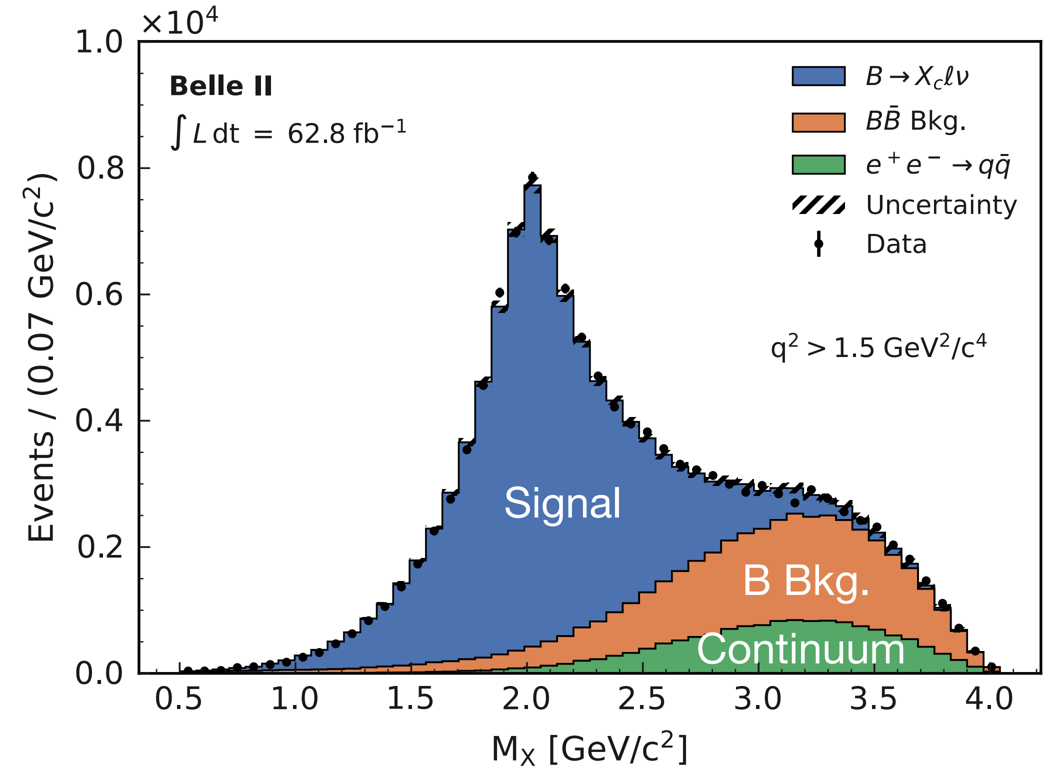
Hadronic Tagging
with **Belle II** algorithm (FEI)

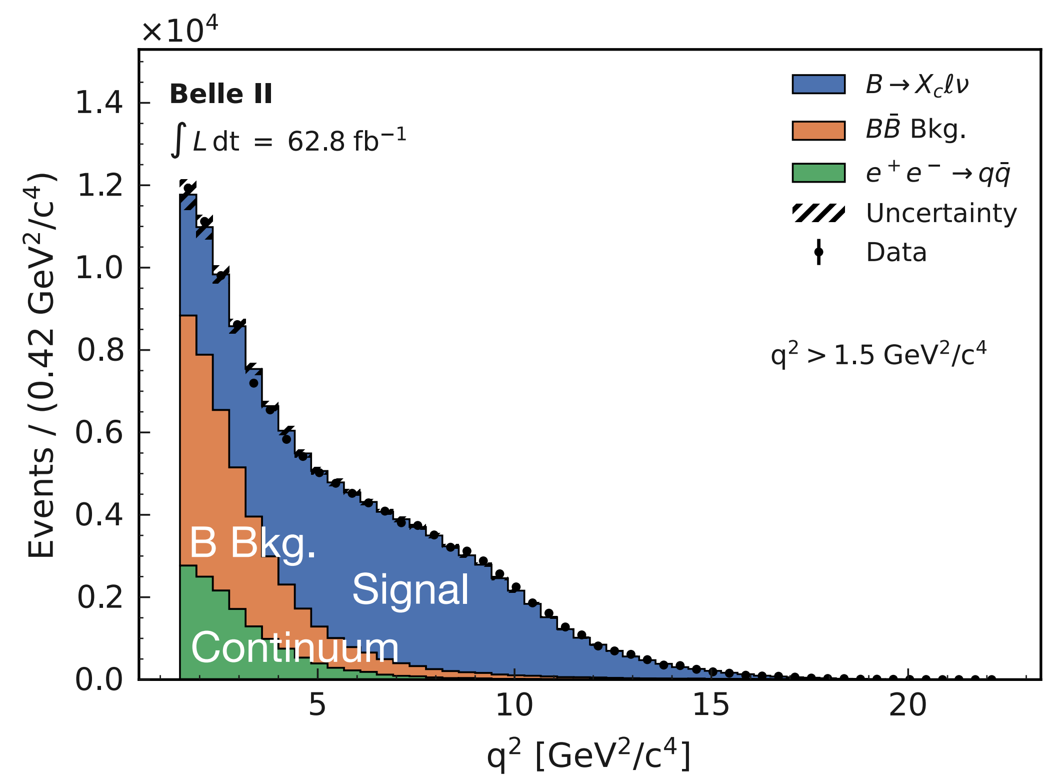
[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

Can identify X_c constituents

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

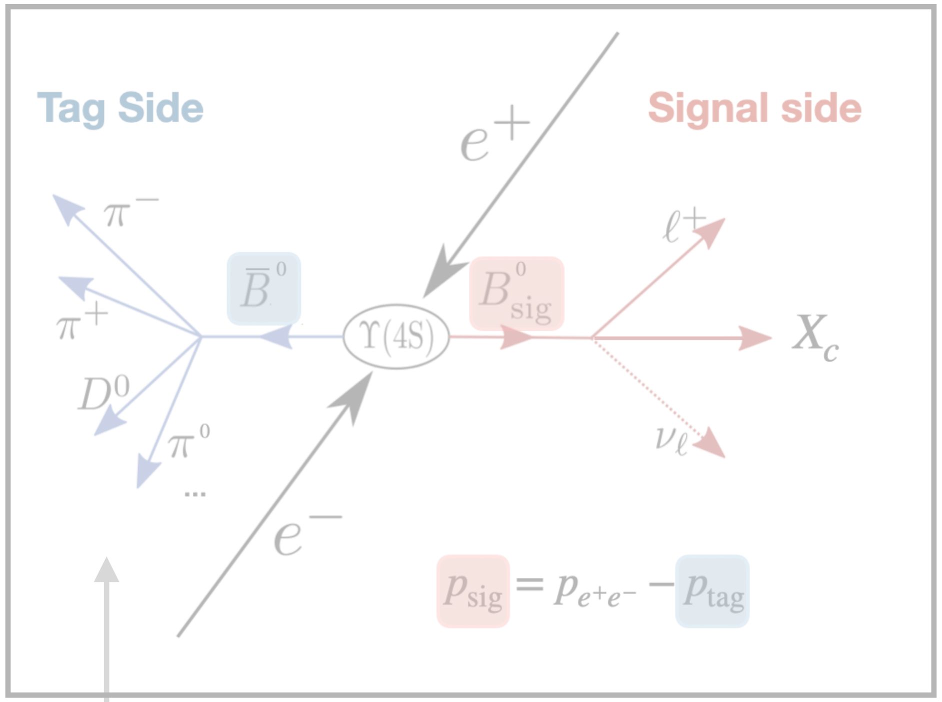
$$q^2 = \left(p_{\rm sig} - p_{X_c}\right)^2$$





How to measure spectral moments

Key-technique: hadronic tagging

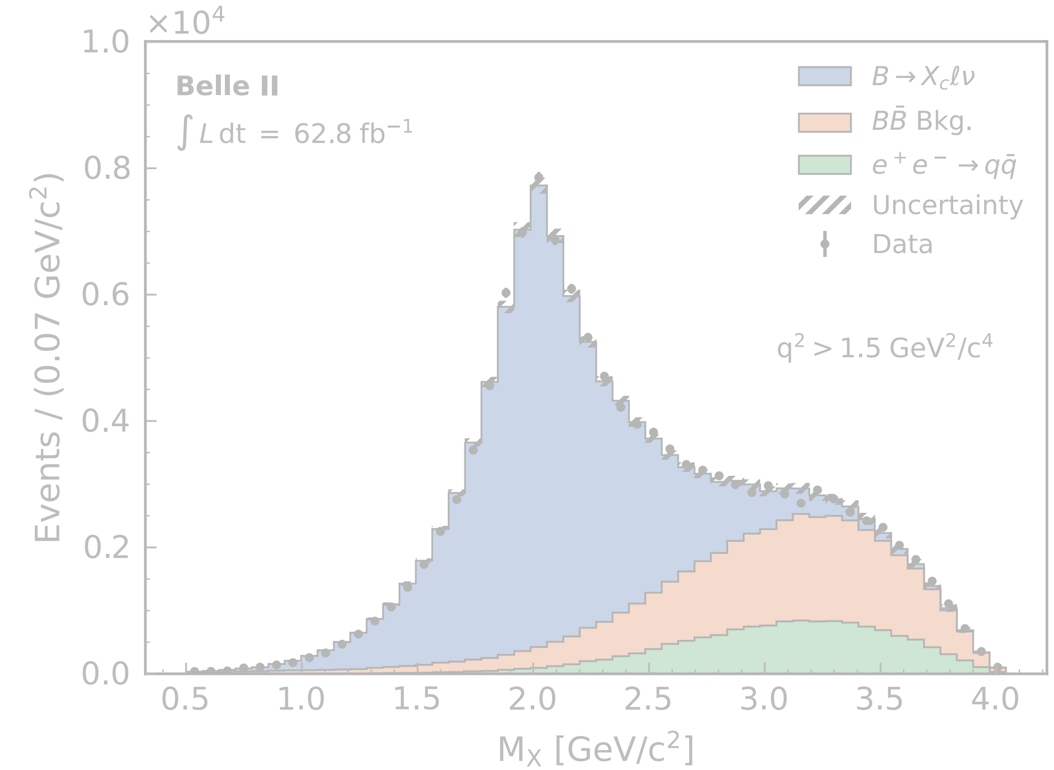


Improved Hadronic Tagging using **Belle II** algorithm (ca. 2 times more efficient)

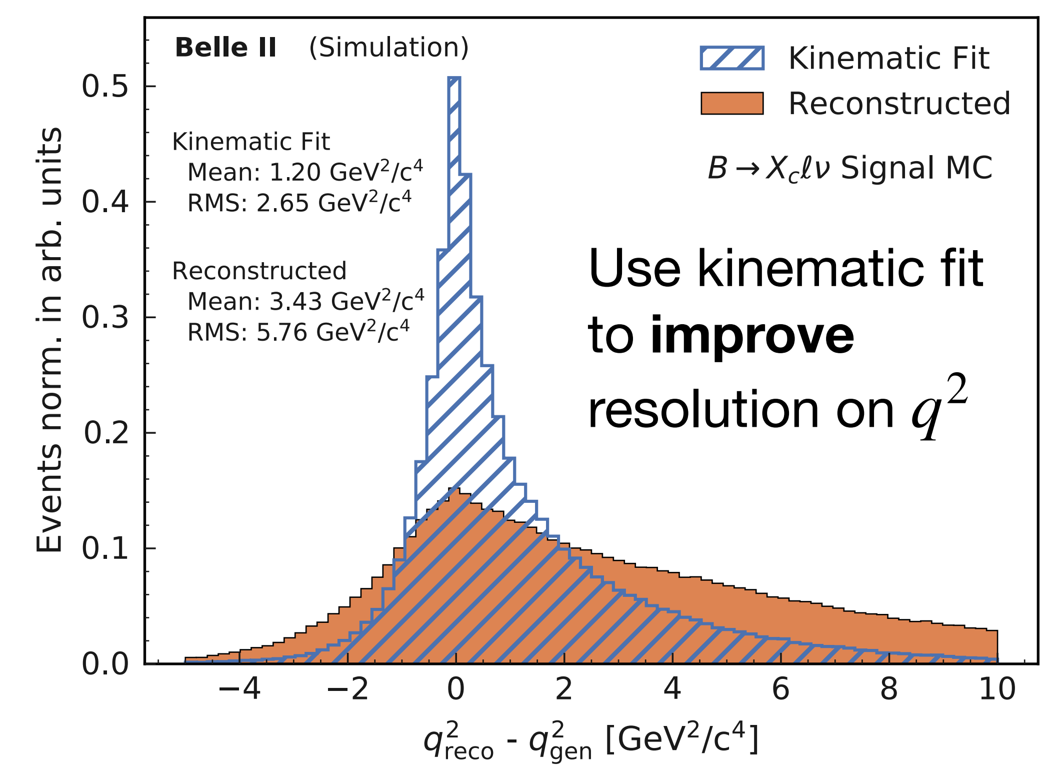
[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

Can identify X_c constituents

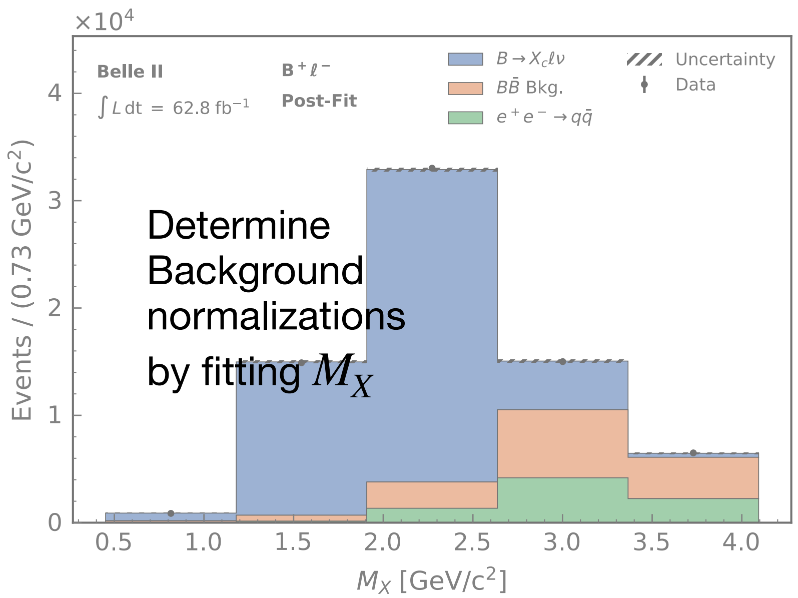
$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$



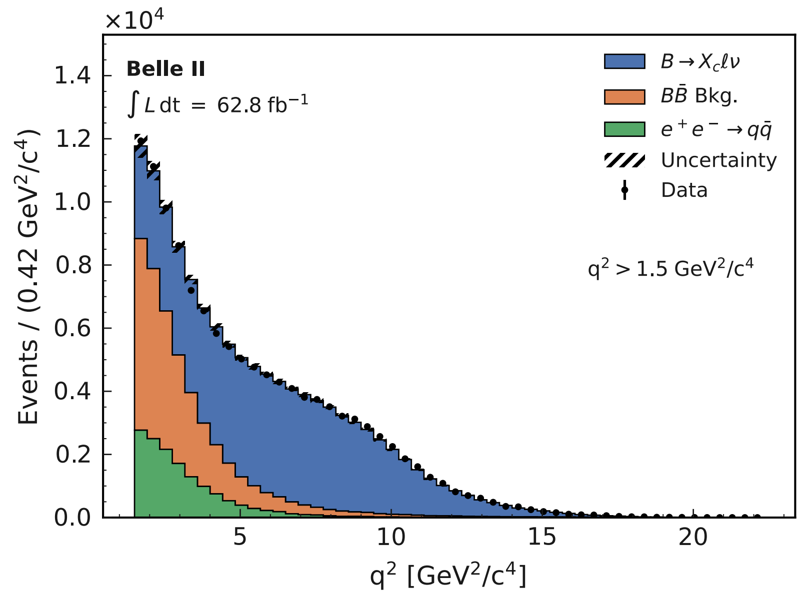
$$q^2 = \left(p_{sig} - p_{X_c}\right)^2$$

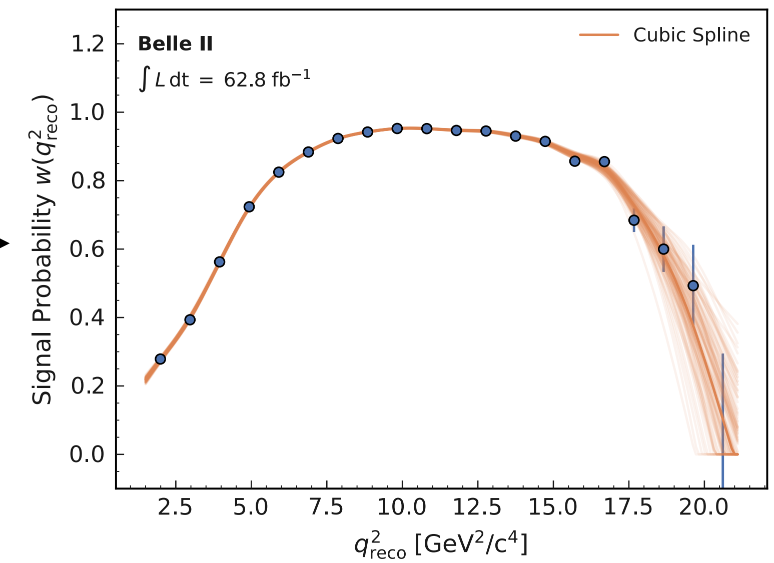


Measurement in a nutshell



Determine Background normalizations by fitting M_X





Step #1: Subtract Background

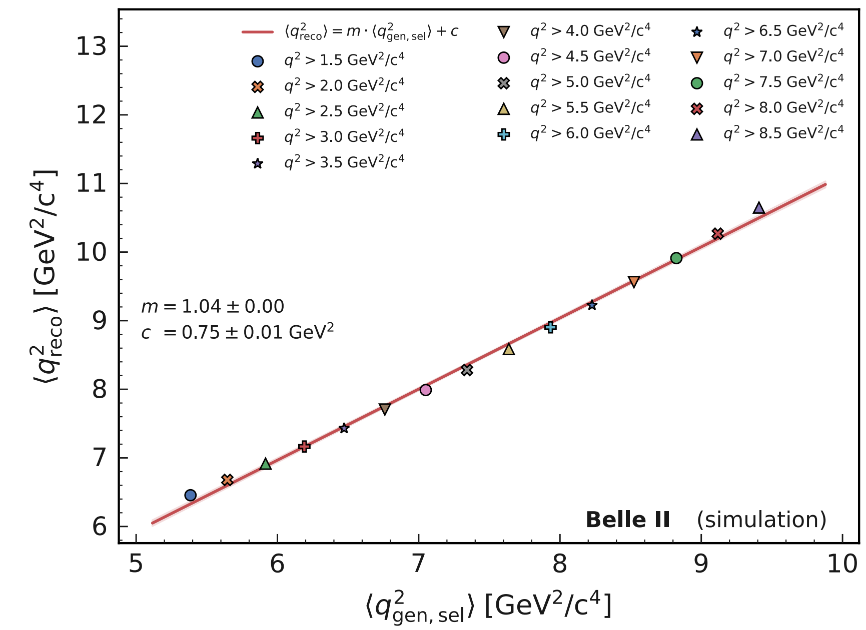
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_\mathrm{data}} w(q^2_\mathrm{reco,i}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_\mathrm{data}} w(q^2_\mathrm{reco,j})} \times \mathcal{C}_\mathrm{calib} \times \mathcal{C}_\mathrm{gen} \,,$$

Measurement in a nutshell

Exploit linear dependence between rec. & true moments

$$q^{2m}_{\mathrm{cal}\,i} = \left(q^{2m}_{\mathrm{reco}\,i} - c\right)/m$$



Step #1: Subtract Background

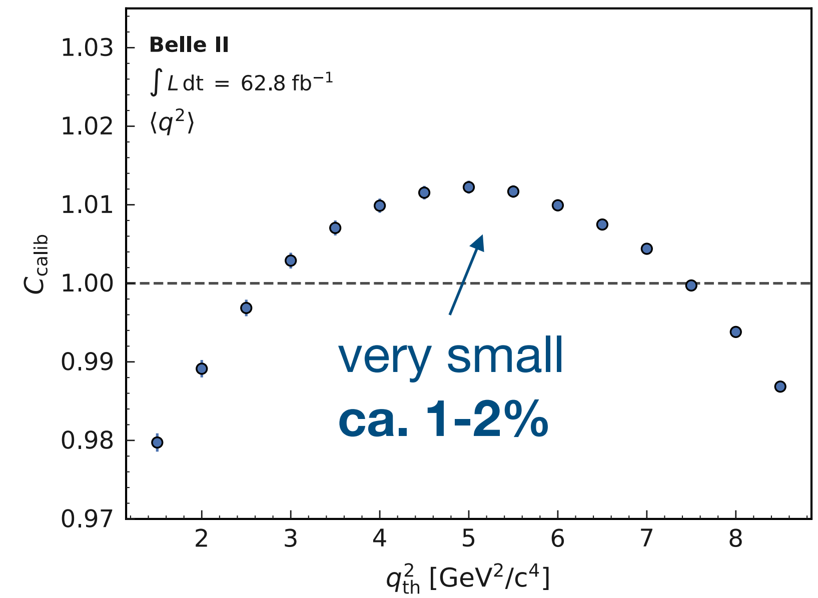
Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,i}}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,j}})} \times \mathcal{C}_{\mathrm{calib}} \times \mathcal{C}_{\mathrm{gen}}\,,$$

Measurement in a nutshell

Very small deviation from linear behavior between reconstruct and true q^2



Step #1: Subtract Background

Step #2: Calibrate moment

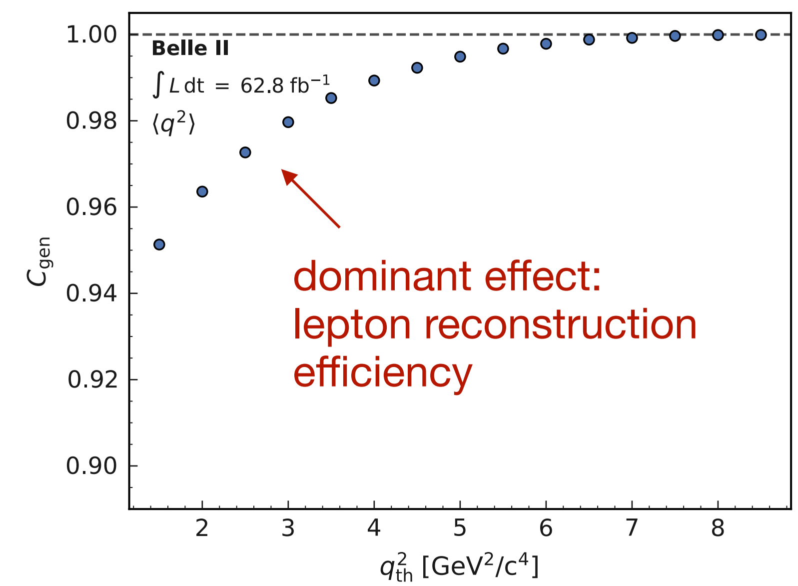
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q^2_{\text{reco,i}}) \times q^{2n}_{\text{calib},i}}{\sum_j^{N_{\text{data}}} w(q^2_{\text{reco,j}})} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}} \,,$$

Step #3: If you fail, try again

Measurement in a nutshell

Account for efficiency & acceptance effects



Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

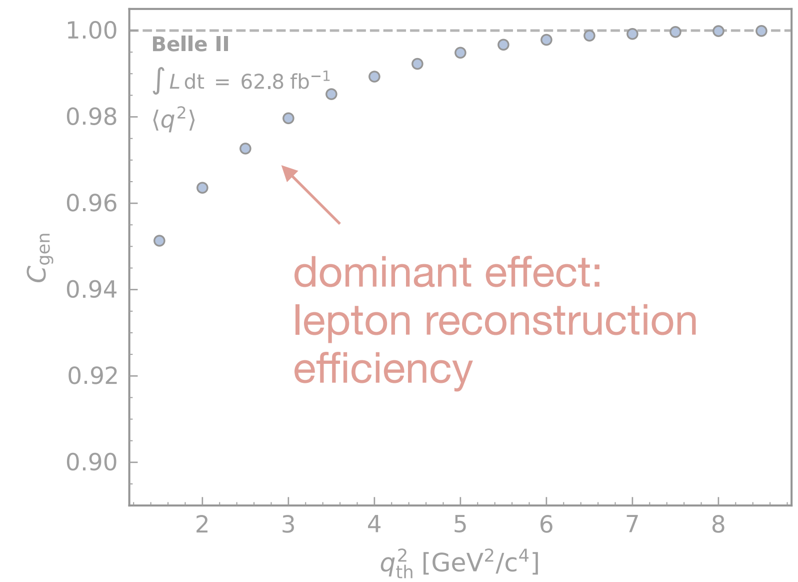
$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,i}}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,j}})} \times \mathcal{C}_{\mathrm{calib}} \times \mathcal{C}_{\mathrm{gen}},$$

Step #3: If you fail, try again

Step #4: Correct for selection effects

Measurement in a nutshell

Account for efficiency & acceptance effects



Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

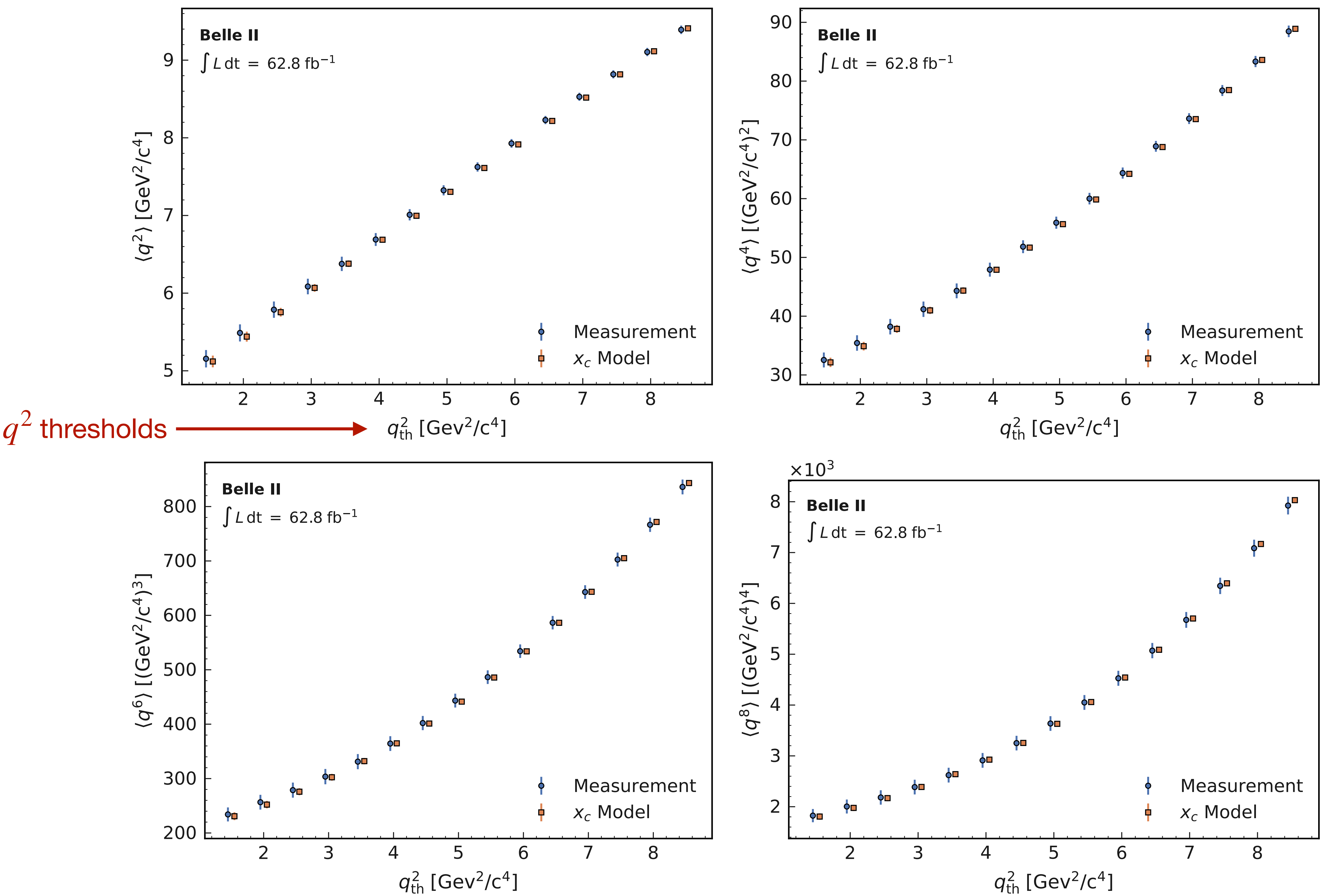
$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\rm data}} w(q^2_{\rm reco,i}) \times q^{2n}_{{\rm calib},i}}{\sum_j^{N_{\rm data}} w(q^2_{\rm reco,j})} \times \mathcal{C}_{\rm calib} \times \mathcal{C}_{\rm gen} \,,$$

Step #3: If you fail, try again

Step #4: Correct for selection effects

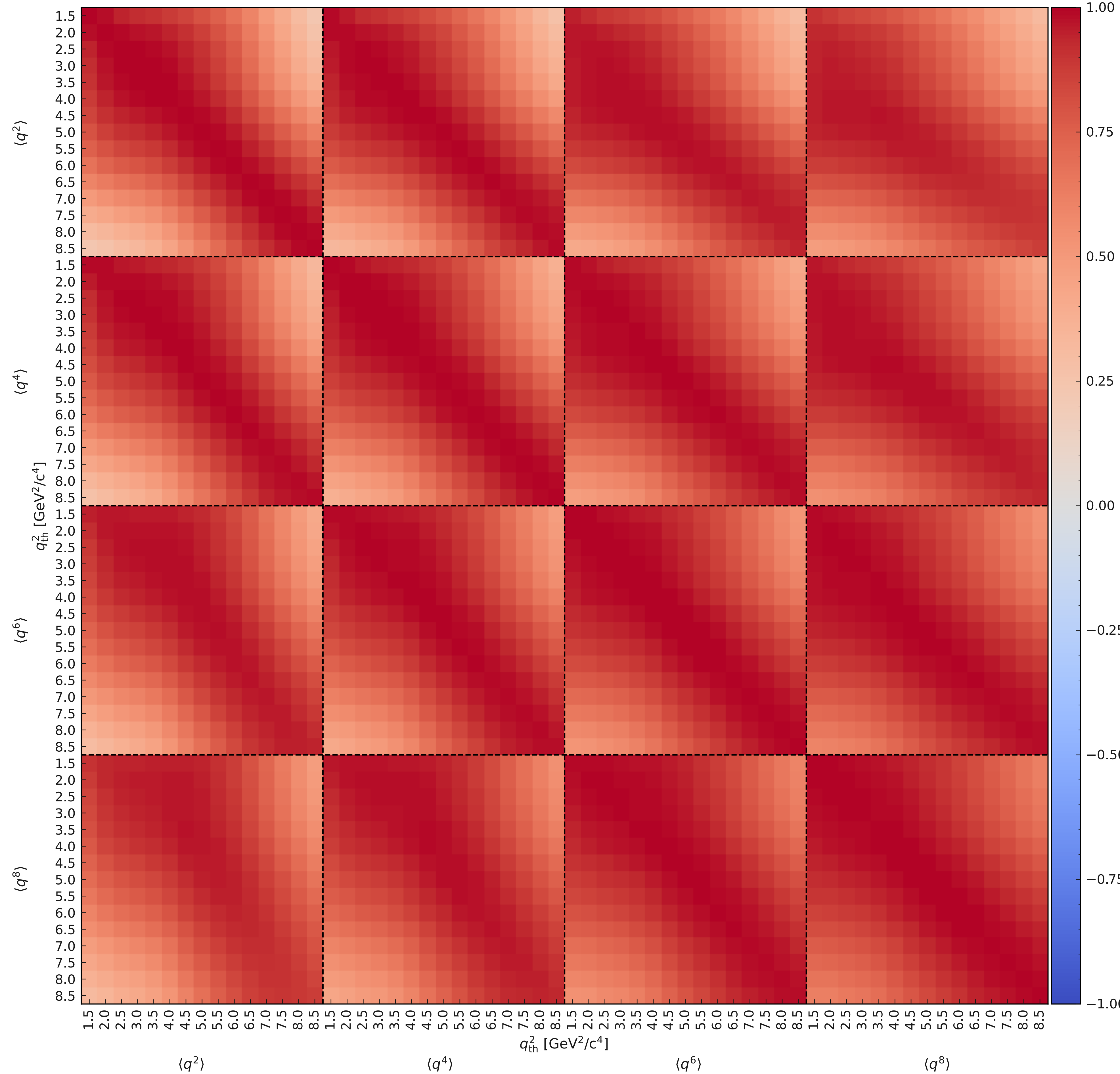
→ Repeat this for many different thresholds cuts $q^2_{\rm th}$

Belle II q^2 spectral moments



Statistical plus **systematic** correlations

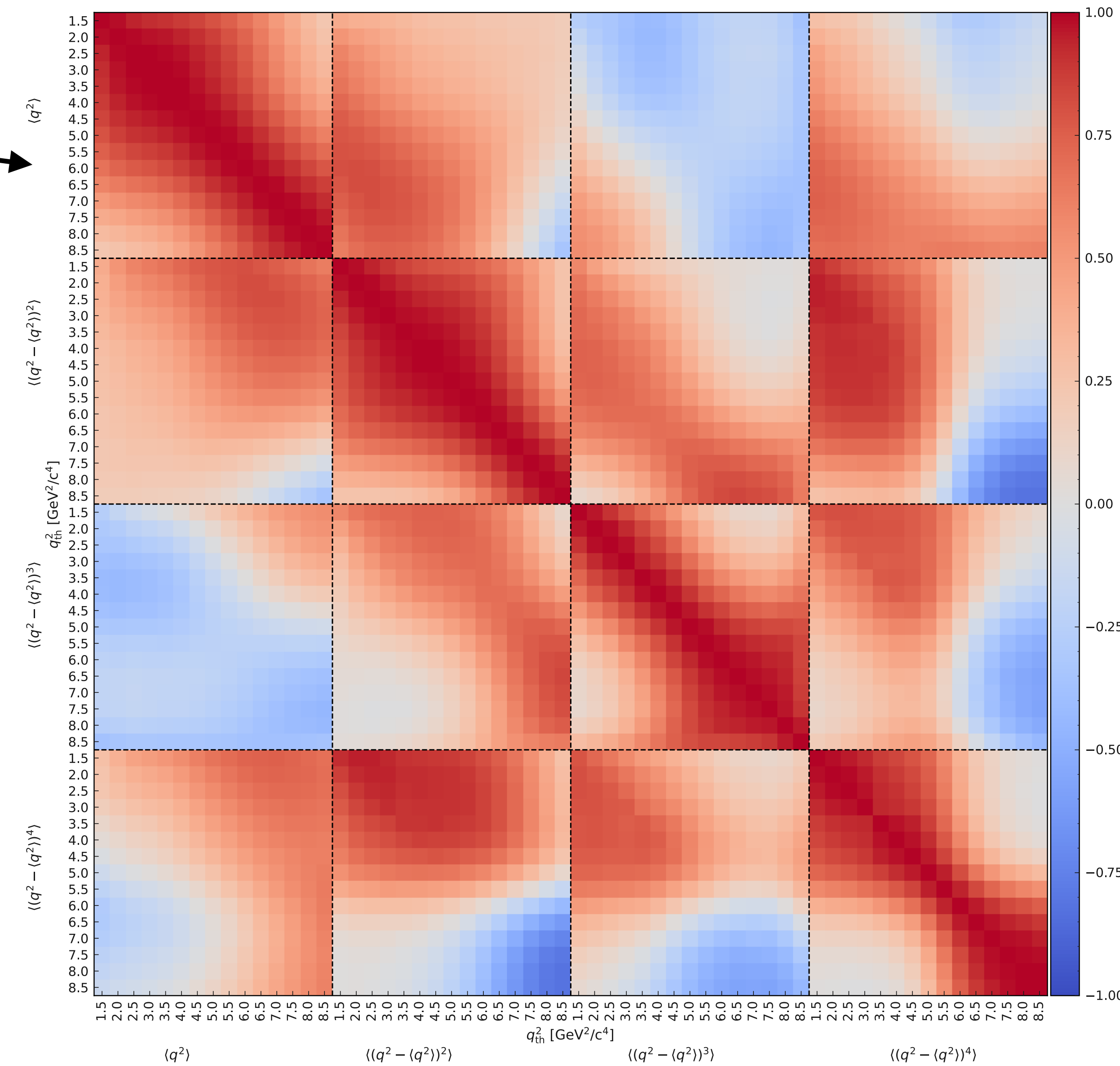
strong correlations!

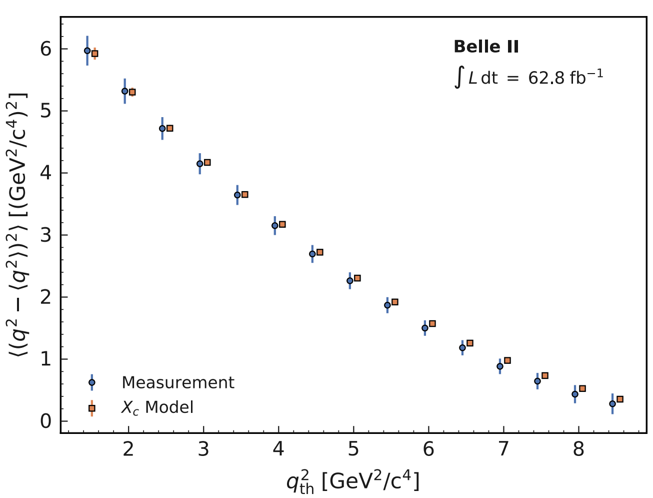


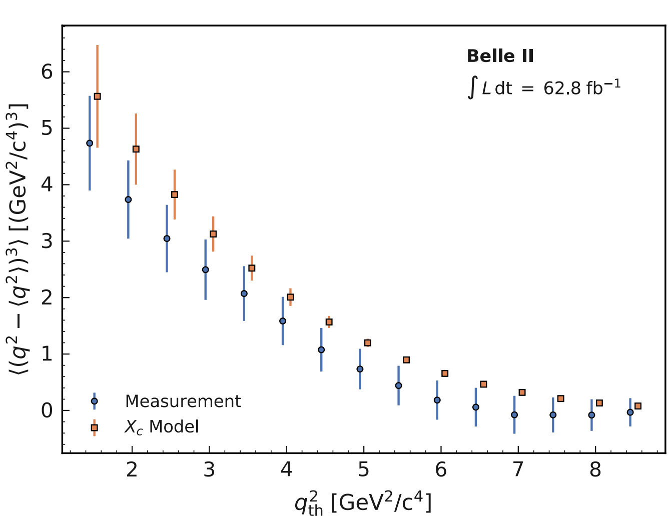
From moments to *central moments*

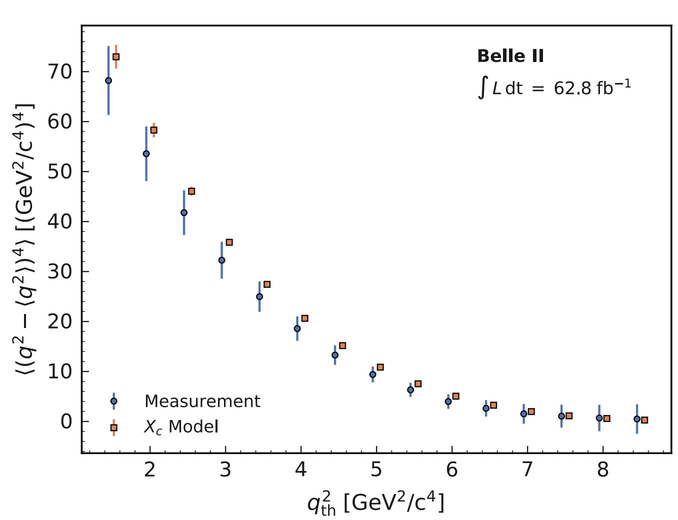
Central moments are **less** strongly correlated

$$\begin{pmatrix} \langle q^2 \rangle \\ \langle q^4 \rangle \\ \langle q^6 \rangle \\ \langle q^8 \rangle \end{pmatrix} \rightarrow \begin{pmatrix} \langle q^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^3 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^4 \rangle \end{pmatrix}$$









Inclusive $|V_{ub}|$

$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$

+ Fermi Motion / Shape Function

Inclusive $|V_{cb}|$

$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive $|V_{ub}|$

$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$

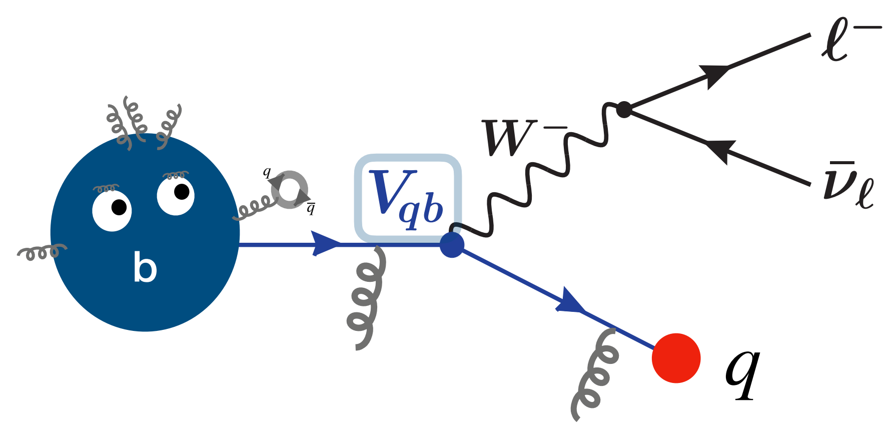
Exclusive $|V_{cb}|$

$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$

$$\mathcal{B} \propto |V_{cb}|^2 \, f^2$$

Form Factors

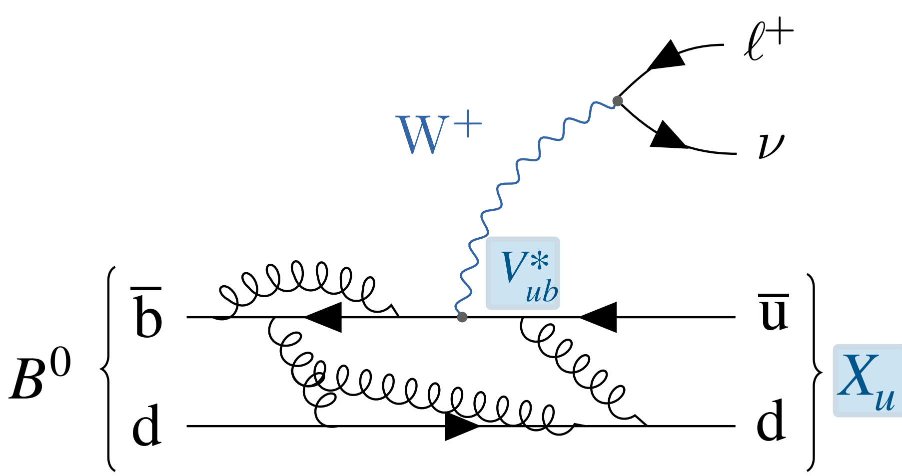
$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$

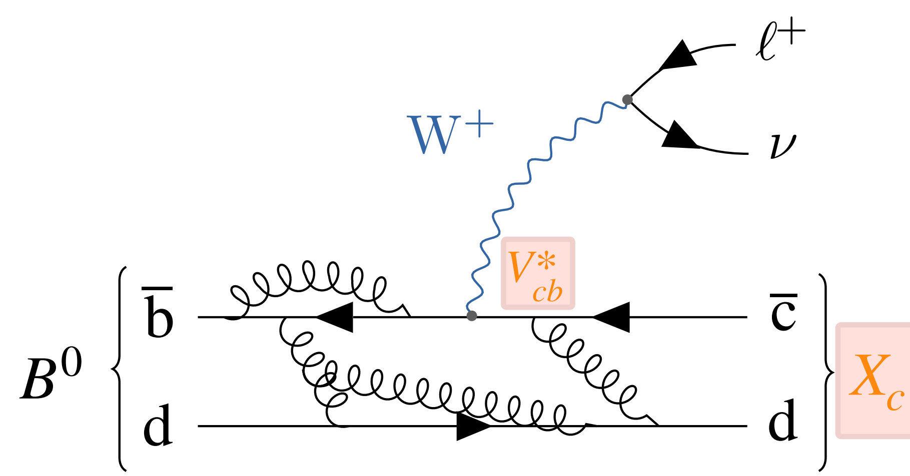


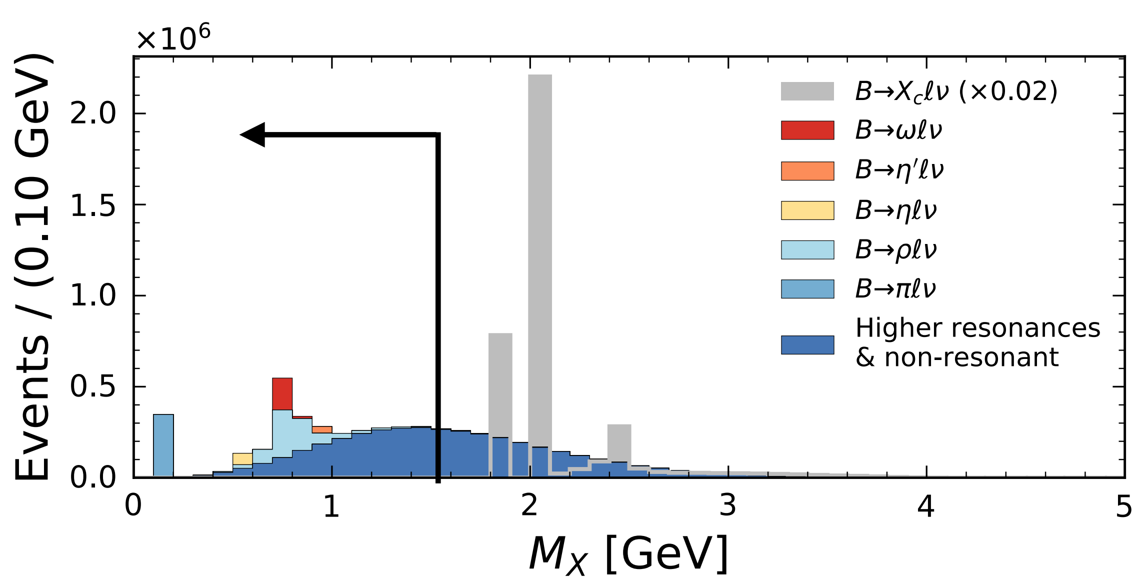
Overview $B \to X_u \ell \bar{\nu}_\ell$

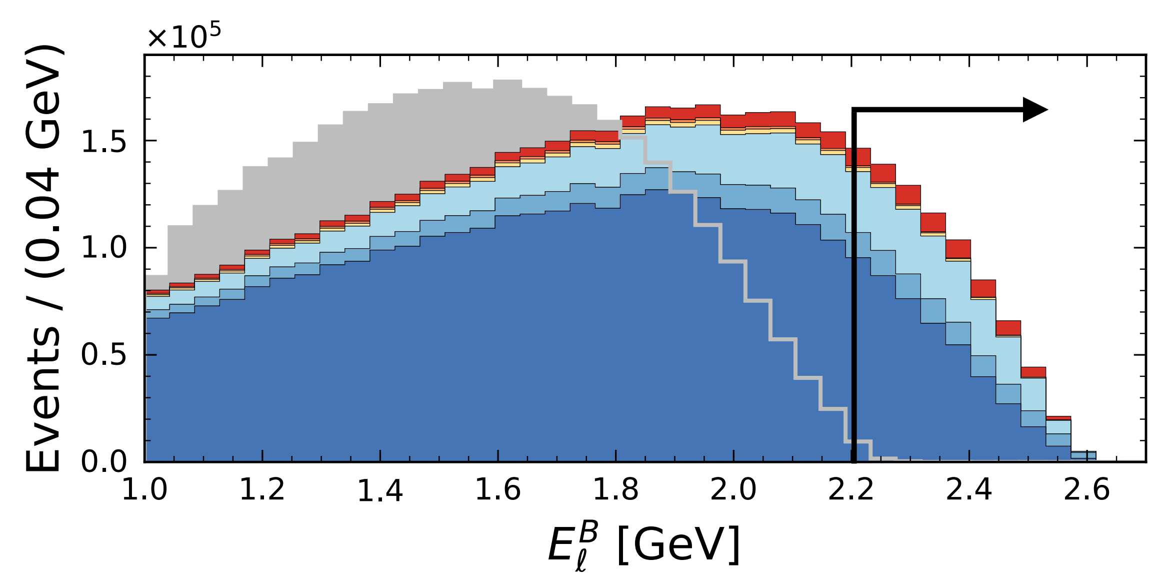
Measuring $|V_{ub}|$ is **hard** due to $B \to X_c \ell \bar{\nu}_\ell$

- x $\mathcal{O}(100)$ more abundant
- Very similar **signature**:
  - high momentum lepton, hadronic system
- Clear separation only in corners of phase space
  - high E_ℓ, low M_X









Going Hybrid : MC for $B \to X_u \ell \bar{\nu}_\ell$

Exclusive make-up of $B \to X_u \ell \bar{\nu}_\ell$:

$\mathcal{B}$	Value B^+	Value B^0
$B \to \pi \, \ell^+ \, \nu_\ell$ [a,e]	$(7.8 \pm 0.3) \times 10^{-5}$	$(1.5 \pm 0.06) \times 10^{-4}$
$B \to \eta \, \ell^+ \, \nu_\ell$ [b,e]	$(3.9 \pm 0.5) \times 10^{-5}$	-
$B \to \eta' \, \ell^+ \, \nu_\ell$ [b,e]	$(2.3 \pm 0.8) \times 10^{-5}$	-
$B \to \omega \, \ell^+ \, \nu_\ell$ [c,e]	$(1.2 \pm 0.1) \times 10^{-4}$	-
$B \to \rho \, \ell^+ \, \nu_\ell$ [c,e]	$(1.6 \pm 0.1) \times 10^{-4}$	$(2.9 \pm 0.2) \times 10^{-4}$
$B \to X_u \, \ell^+ \, \nu_\ell$ [d,e]	$(2.2 \pm 0.3) \times 10^{-3}$	$(2.0 \pm 0.3) \times 10^{-3}$

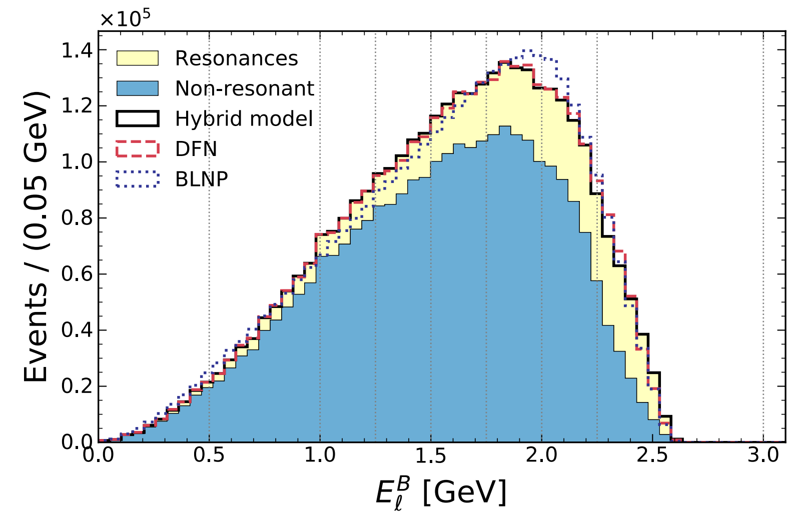
Hybrid = Combining exclusive & inclusive predictions

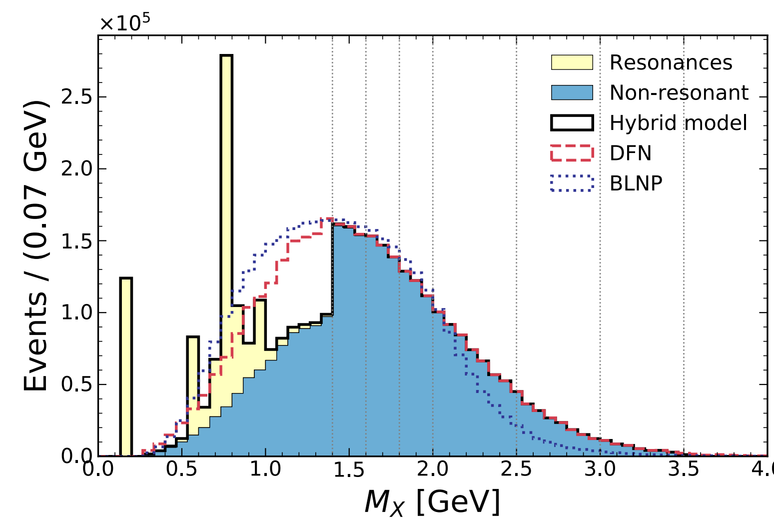
$$\Delta \mathcal{B}^{\text{incl}}_{ijk} = \Delta \mathcal{B}^{\text{excl}}_{ijk} + w_{ijk} \times \Delta \mathcal{B}^{\text{incl}}_{ijk} \, ,$$

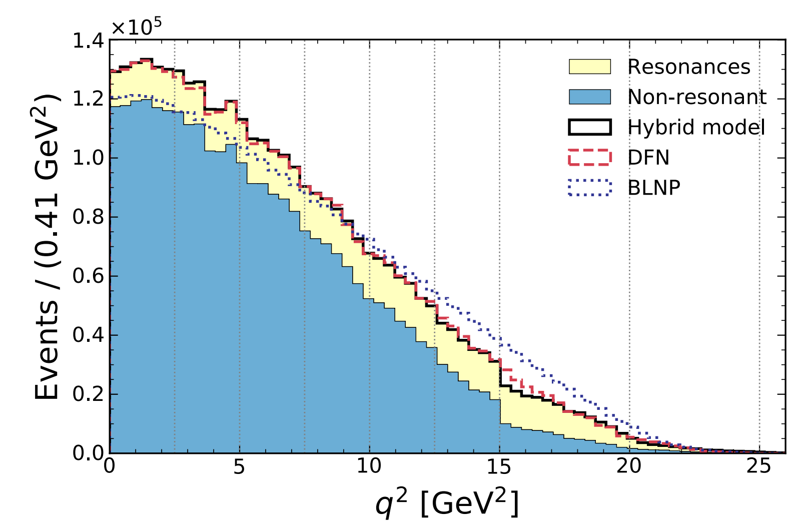
$$q^2 = [0, 2.5, 5, 7.5, 10, 12.5, 15, 20, 25] \, \text{GeV}^2 \, ,$$

$$E_\ell^B = [0, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 3] \, \text{GeV} \, ,$$

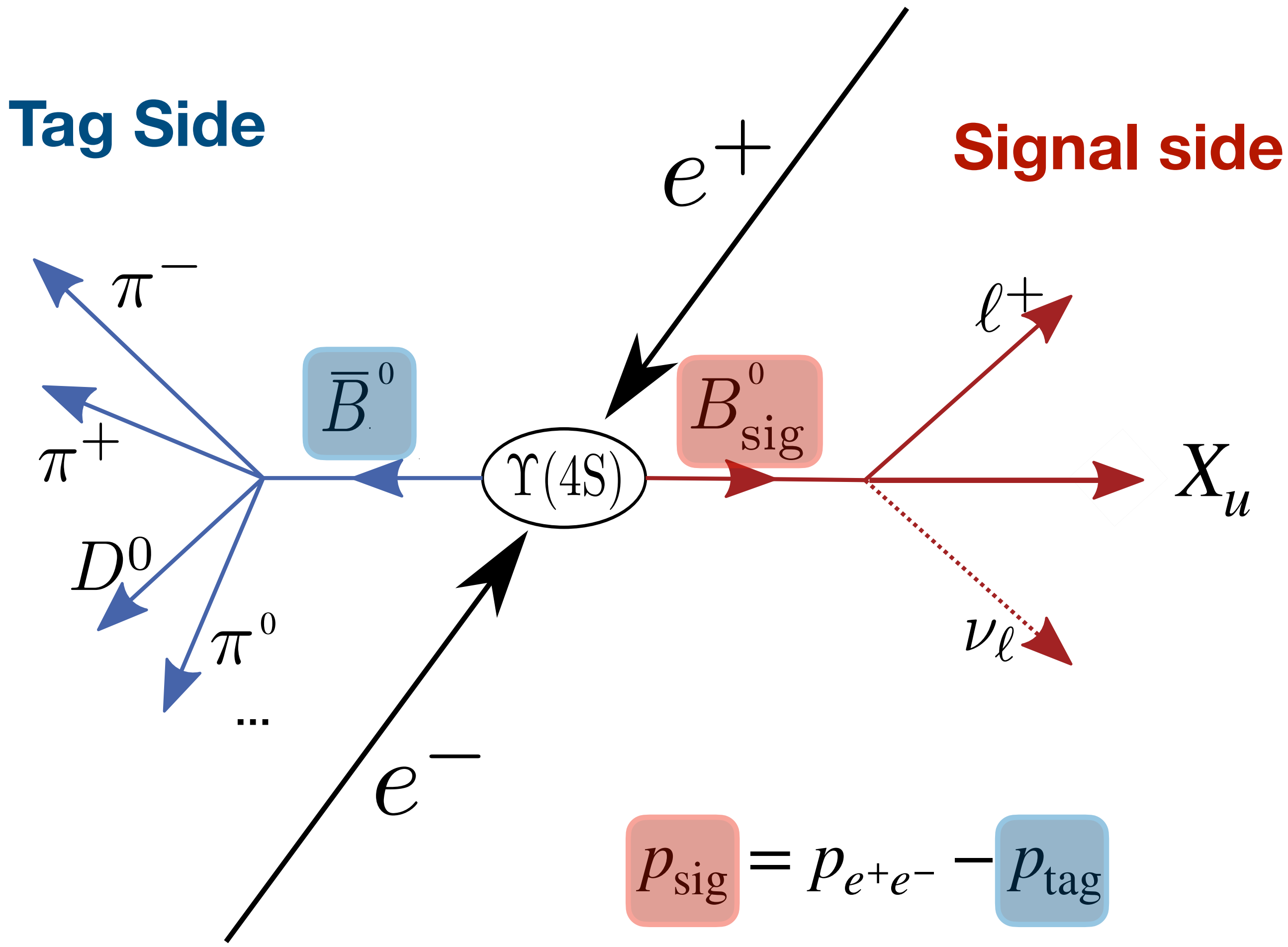
$$M_X = [0, 1.4, 1.6, 1.8, 2, 2.5, 3, 3.5] \, \text{GeV} \, .$$



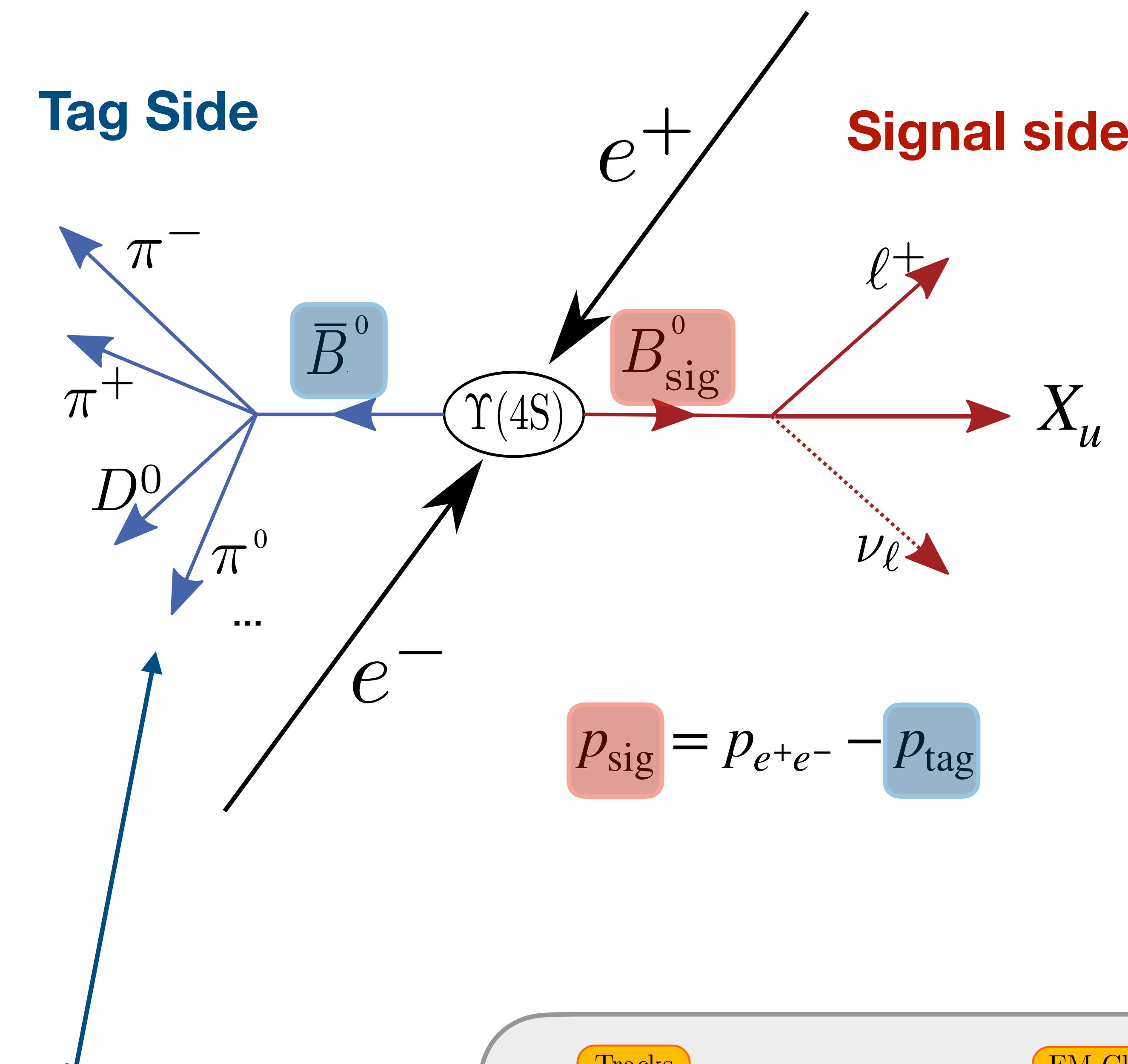




Analysis Strategy with hadronic Tagging

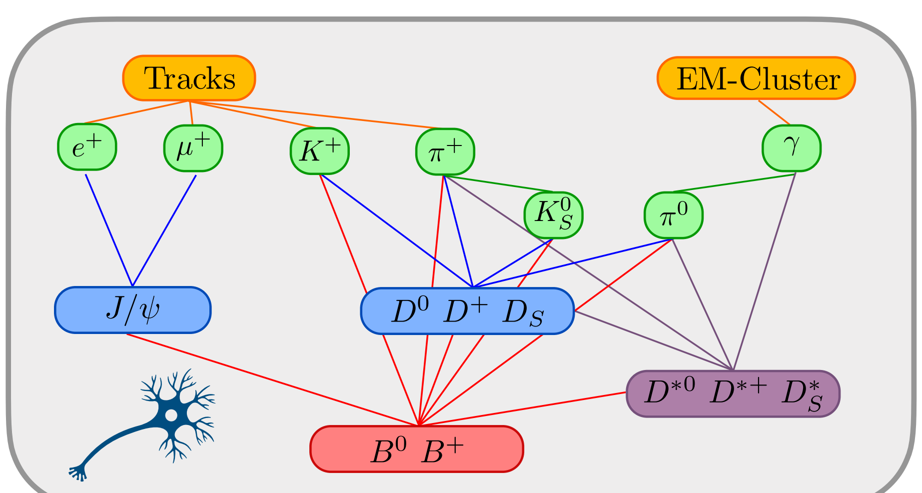


Analysis Strategy with hadronic Tagging



Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

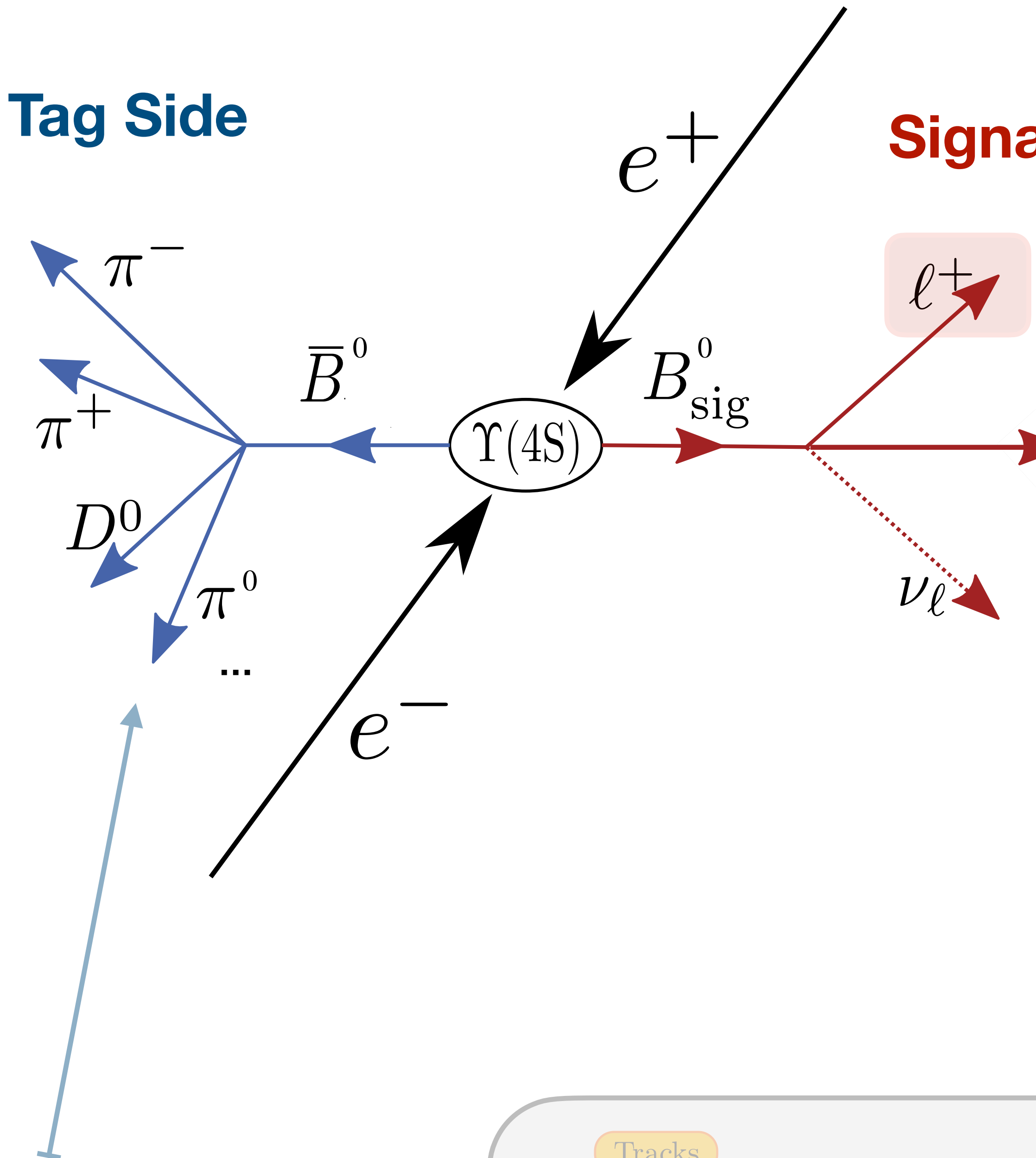
1104 decay cascades used with an **efficiency** of **0.28% / 0.18%** for $B^{\pm}$ and $B^0/\bar{B}^0$



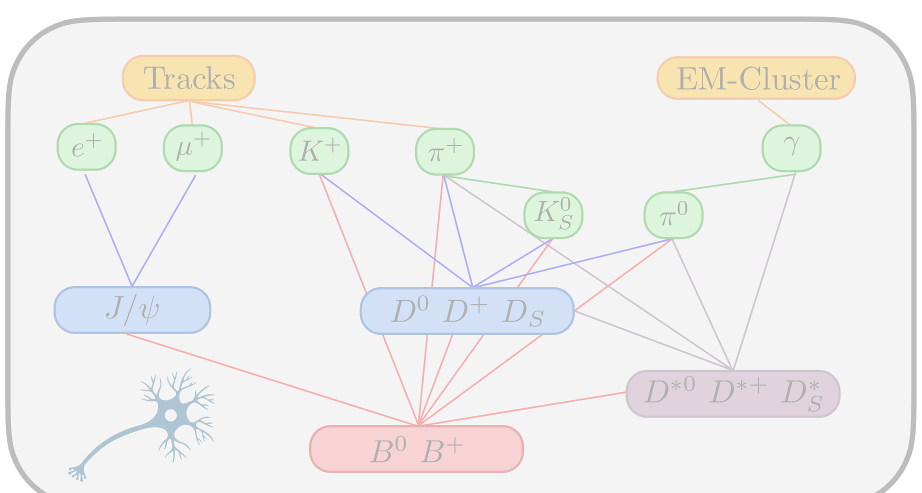
Analysis Strategy with hadronic Tagging

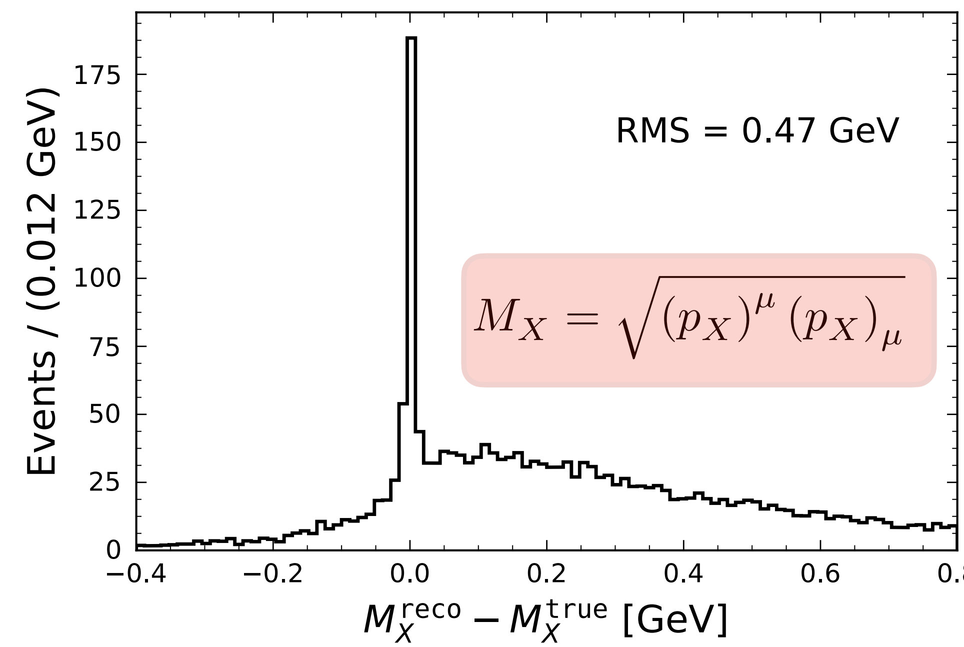
Charged Tracks — Neutral Clusters

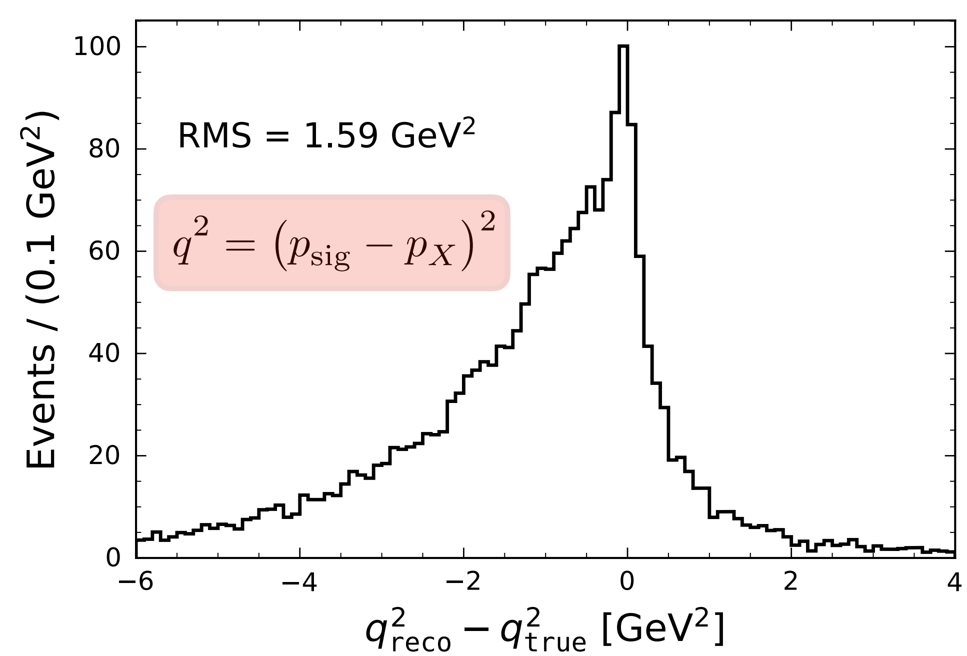
$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j \left(E_j, \mathbf{k}_j \right)$$



Had. Tagging allows to assign particles to X_u system



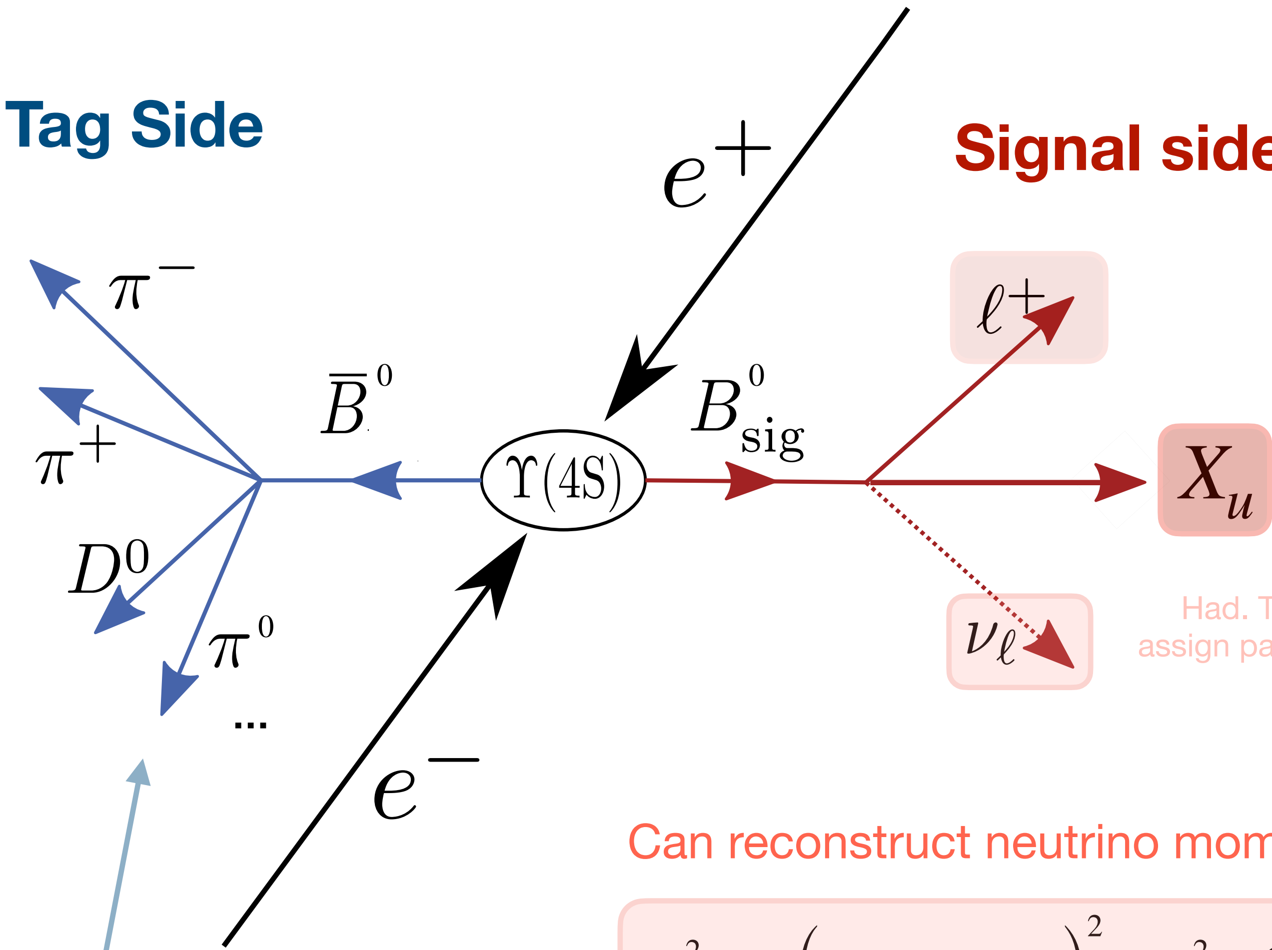




Analysis Strategy with hadronic Tagging

Tag Side

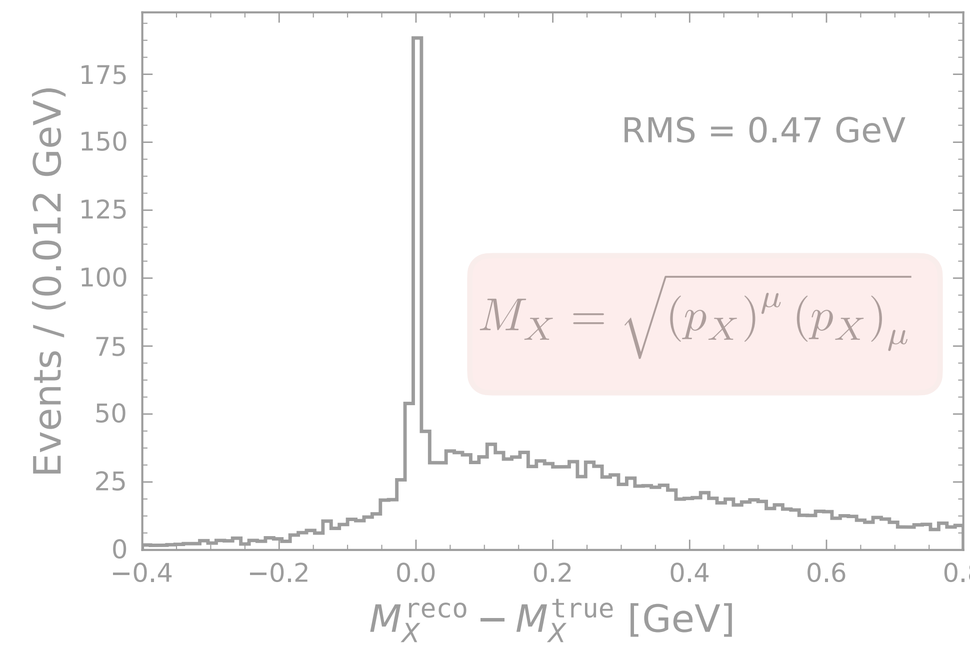
Signal side

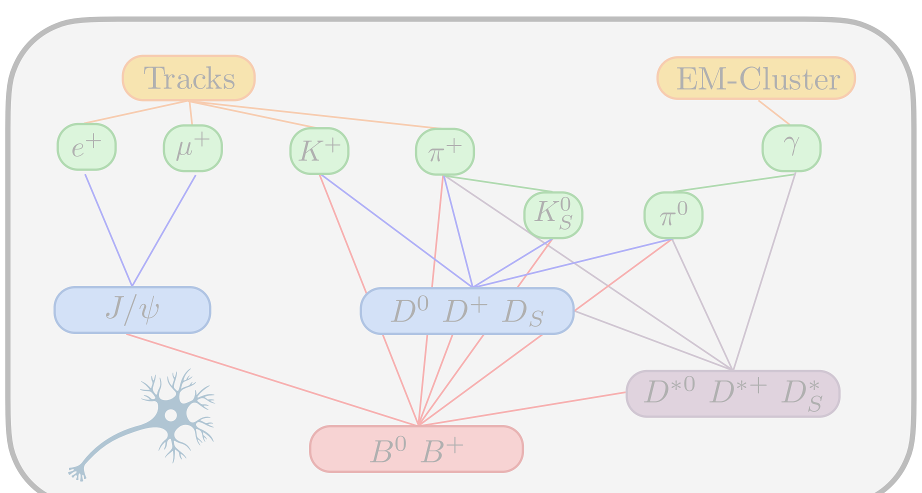


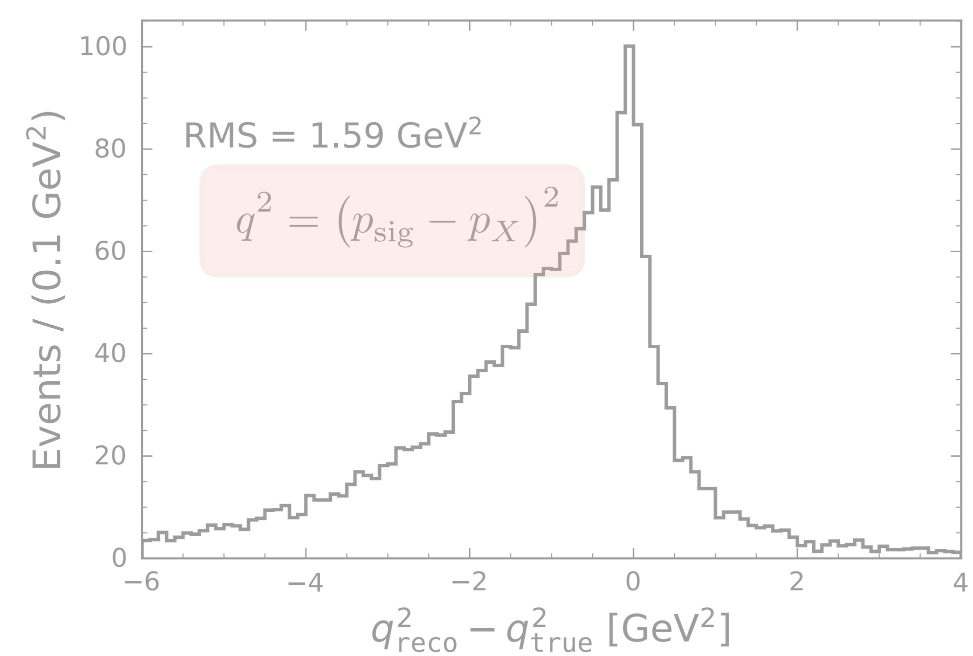
$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i \right) + \sum_j (E_j, \mathbf{k}_j)$$

Can reconstruct neutrino momentum:

$$m^2_{\rm miss} = \left(p_{\rm sig} - p_X - p_\ell\right)^2 \approx m_\nu^2 = 0\,{\rm GeV}^2$$

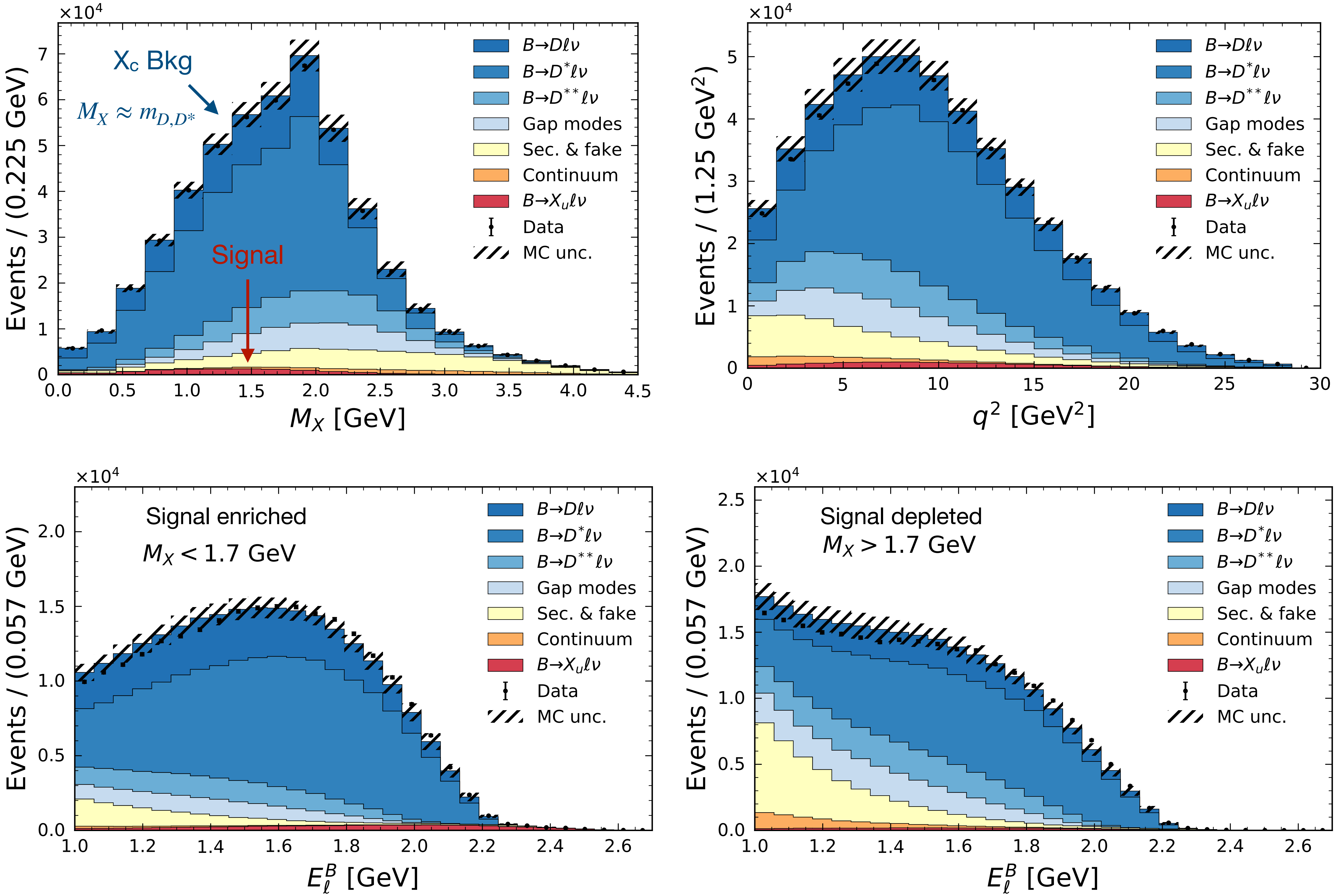






Hadronic Mass $M_X = \sqrt{p_X^2}$

Four-momentum transfer squared $q^2 = (p_B - p_X)^2$



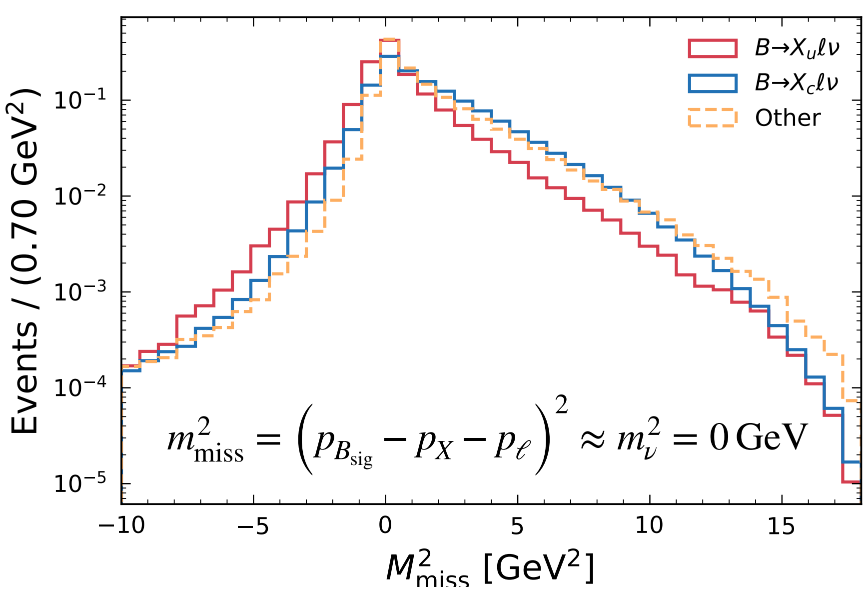
Lepton Energy in signal B rest frame E_ℓ^B

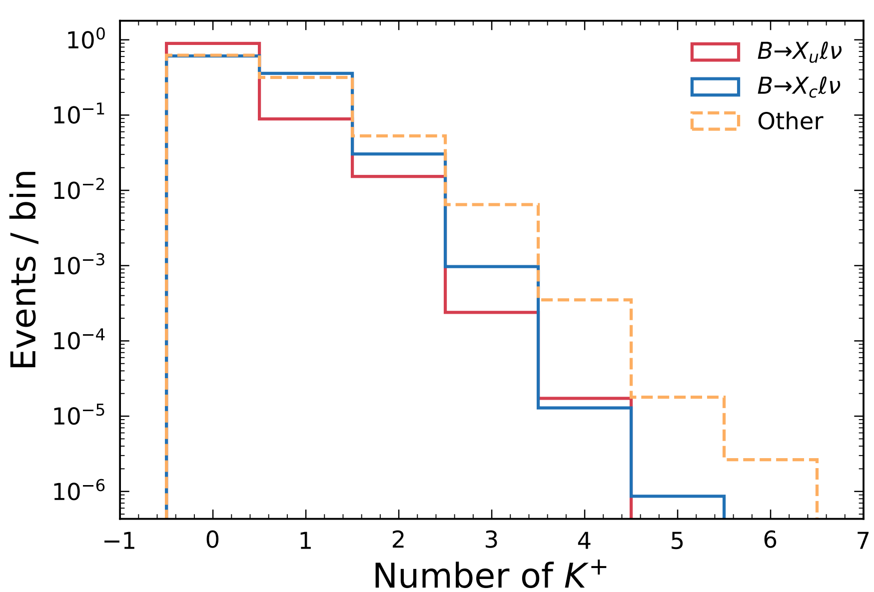
Multivariate Sledgehammer

Can exploit that there are differences:

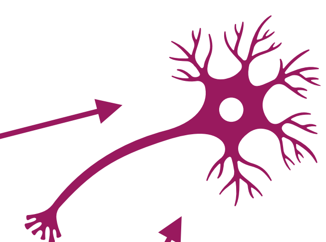
X_u ⟷ X_c

Direct cuts on m_X, E_ℓ **problematic**
(i.e. direct theory / shape-function dependence)



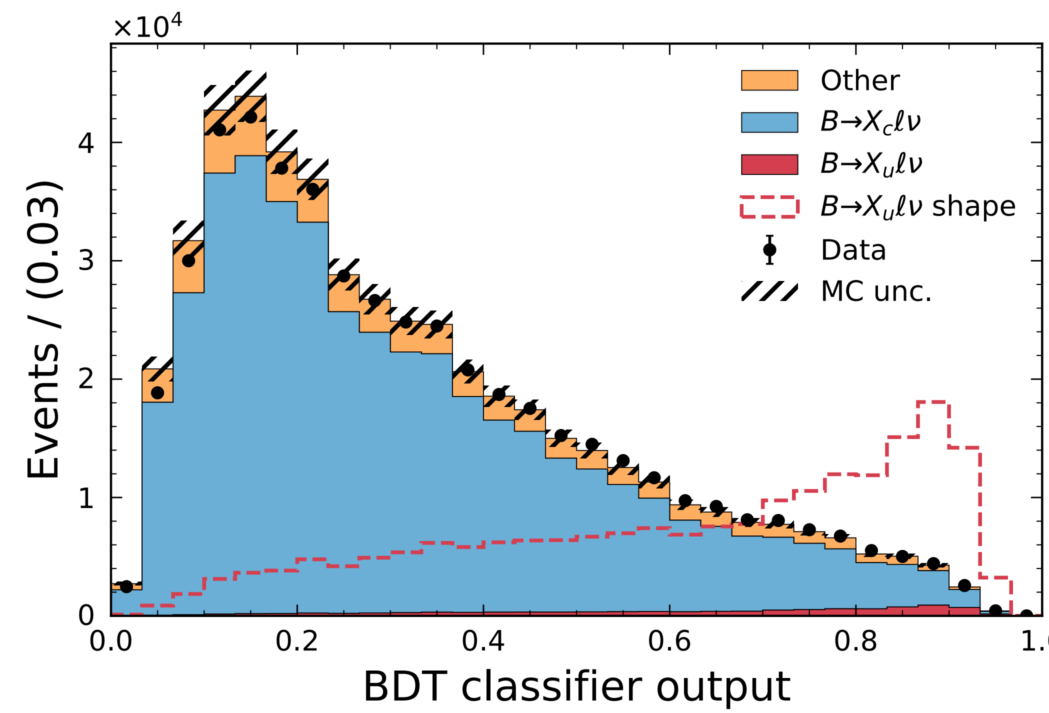


Higher multiplicity
Often come with charged and neutral **Kaons**
D* decays (**slow pions**)
(**Slightly lower E_ℓ**)



+ 9 other variables

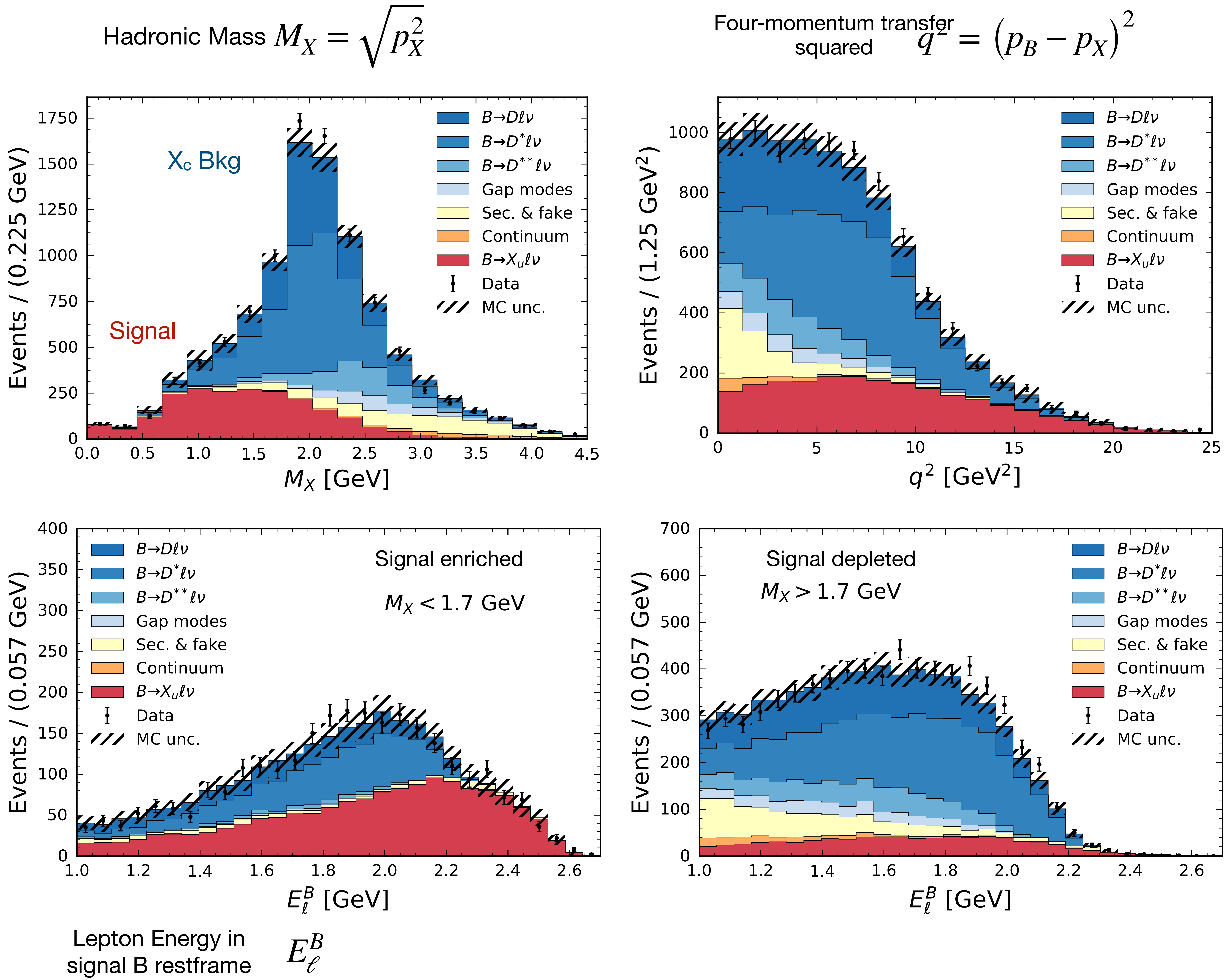
=



Can reject **98.7%** of X_c

Selection	$B \to X_u\, \ell^+\, \nu_\ell$	$B \to X_c\, \ell^+\, \nu_\ell$	Data
$M_{\rm bc} > 5.27\,{\rm GeV}$	84.8%	83.8%	80.2%
$\mathcal{O}_{\rm BDT} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\rm BDT} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\rm BDT} > 0.87$	14.5%	0.9%	1.1%

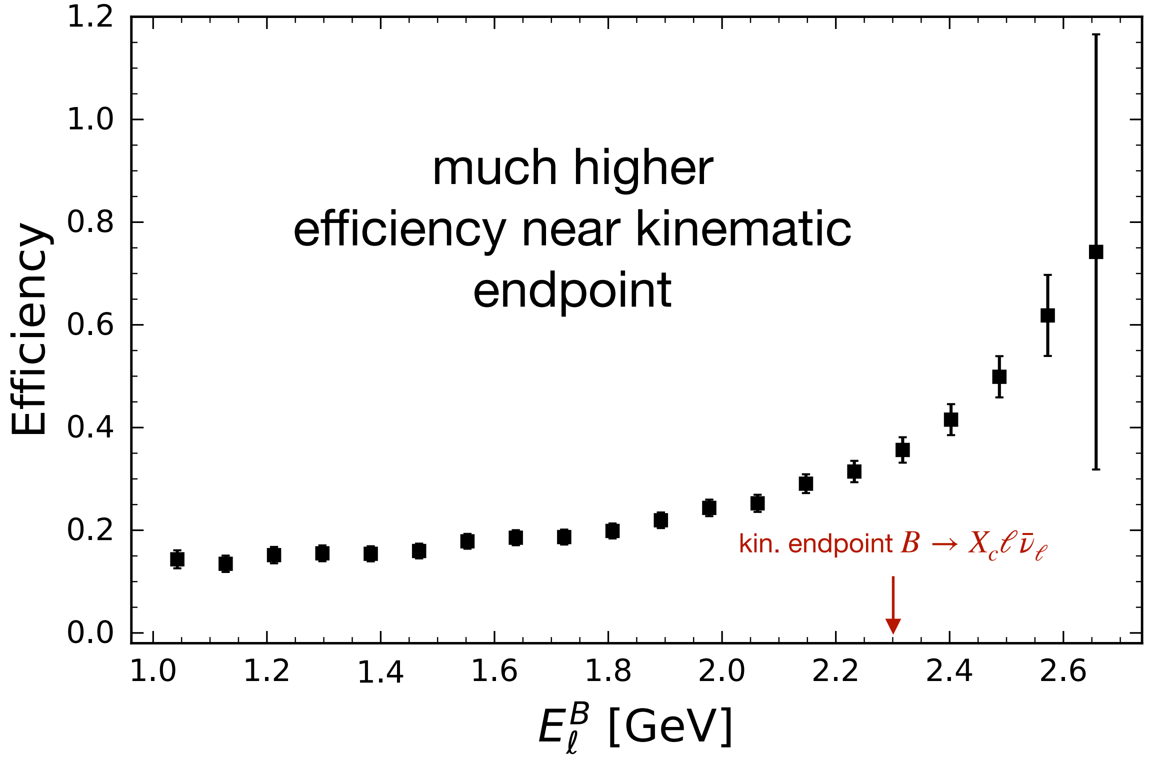
... and retain **18.5%** of X_u

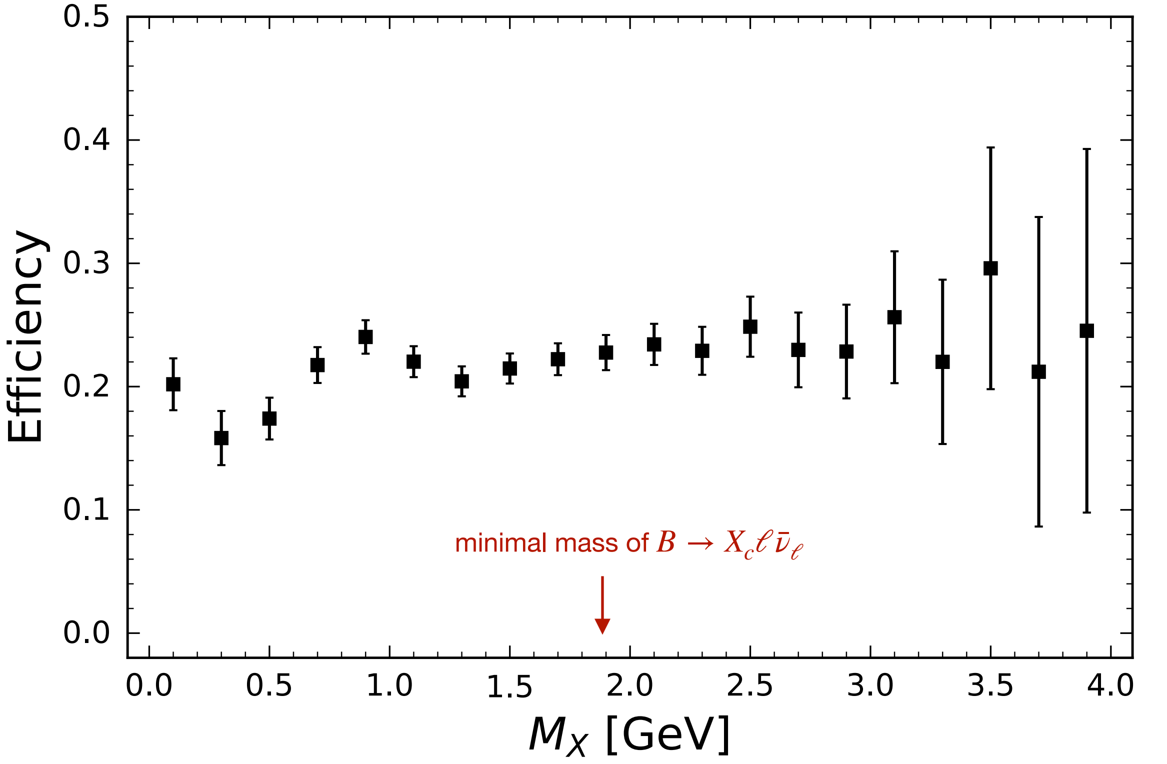


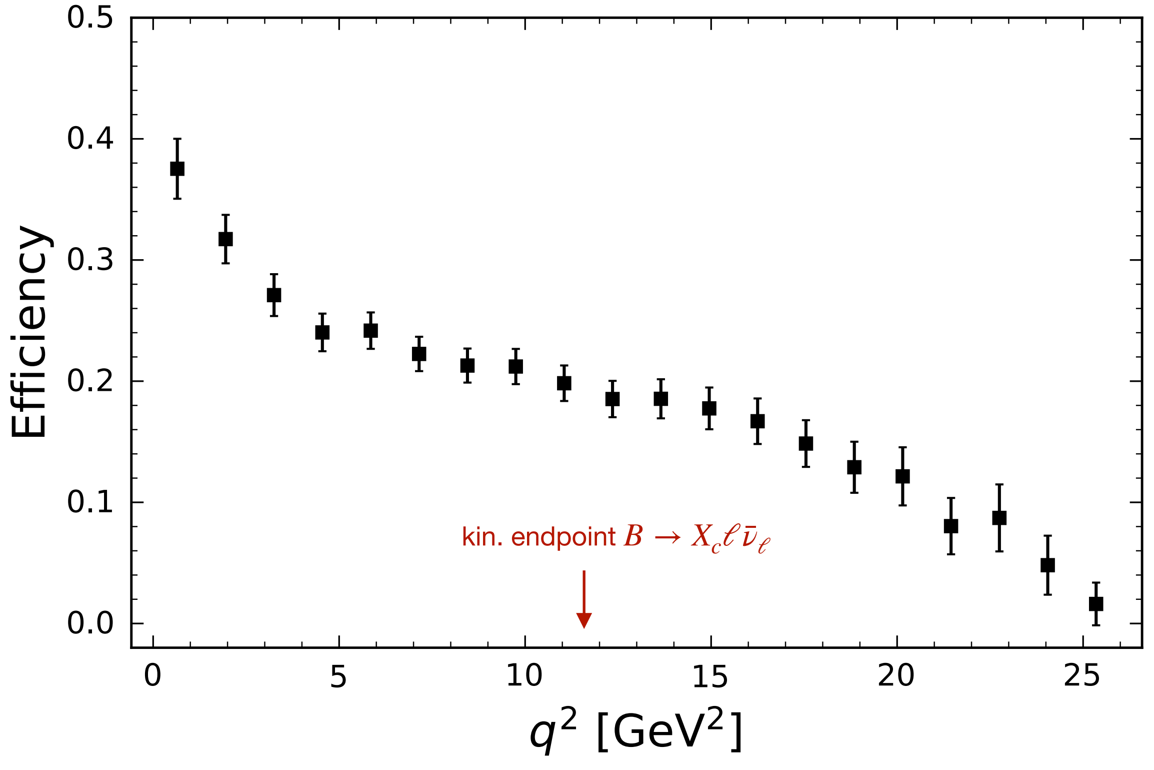
Hadronic Mass $M_X = \sqrt{p_X^2}$
Four-momentum transfer squared $q^2 = (p_B - p_X)^2$
Xc Bkg
Signal
Events / (0.225 GeV)
M_X [GeV]
Events / (1.25 GeV^2)
q^2 [GeV^2]
$B \to D \ell \nu$
$B \to D^* \ell \nu$
$B \to D^{**} \ell \nu$
Gap modes
Sec. & fake
Continuum
$B \to X_u \ell \nu$
Data
MC unc.
Signal enriched
$M_X < 1.7$ GeV
Signal depleted
$M_X > 1.7$ GeV
Events / (0.057 GeV)
E_ℓ^B [GeV]
Lepton Energy in signal B restframe E_ℓ^B

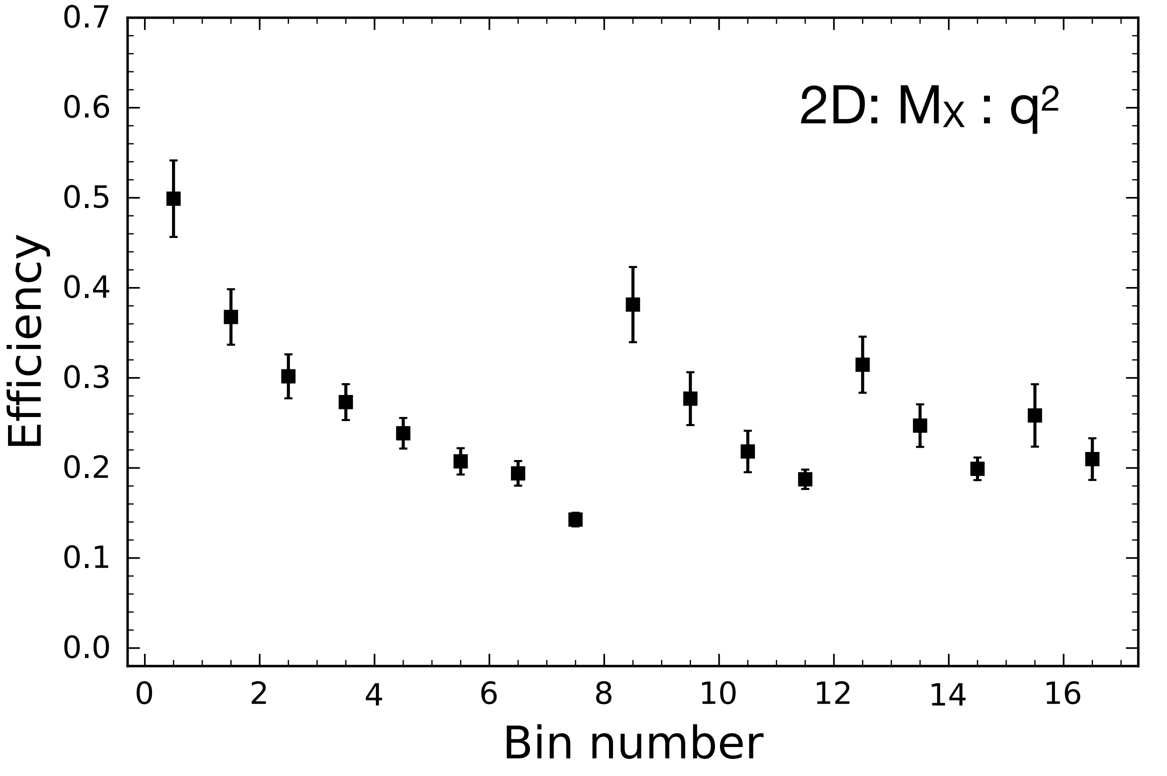
The Cost

Shaping in efficiencies, so some model dependence









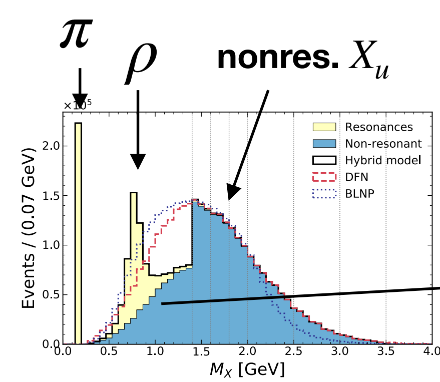
Fit for partial BFs

Subtraction of bkg in fit with coarse binning to minimize X_u modelling dependence

(low m_X, high q^2)

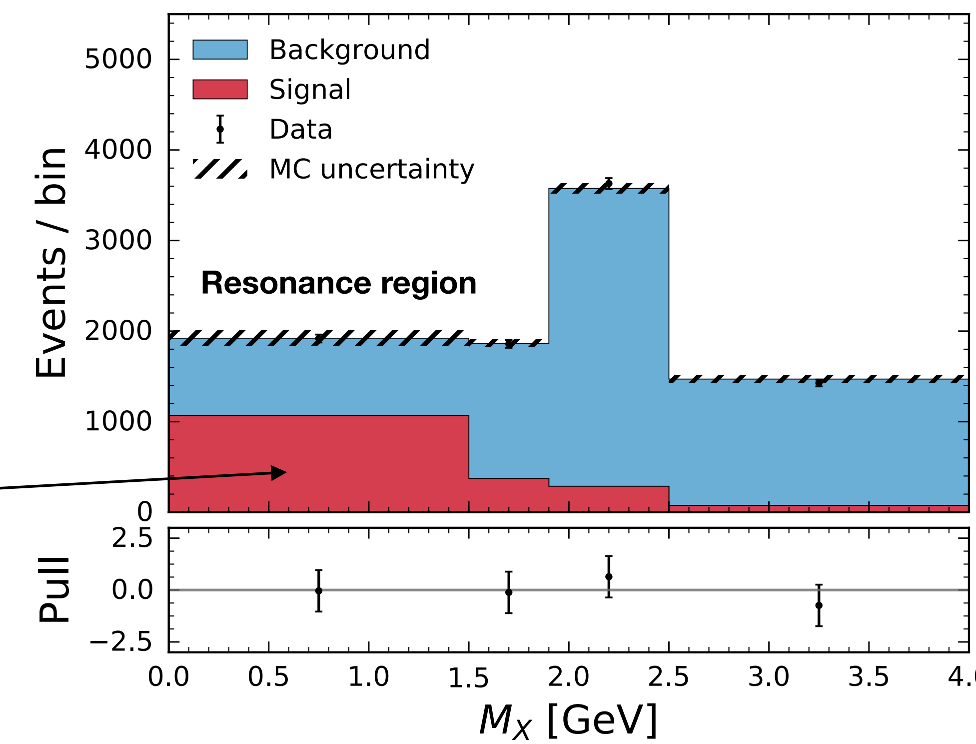
$$\mathcal{L} = \prod_i^{\text{bins}} \mathcal{P}\left(n_i; \nu_i\right) \times \prod_k \mathcal{G}_k \,,$$

Signal and Bkg shape errors included in Fit via NPs



W/o detector smearing

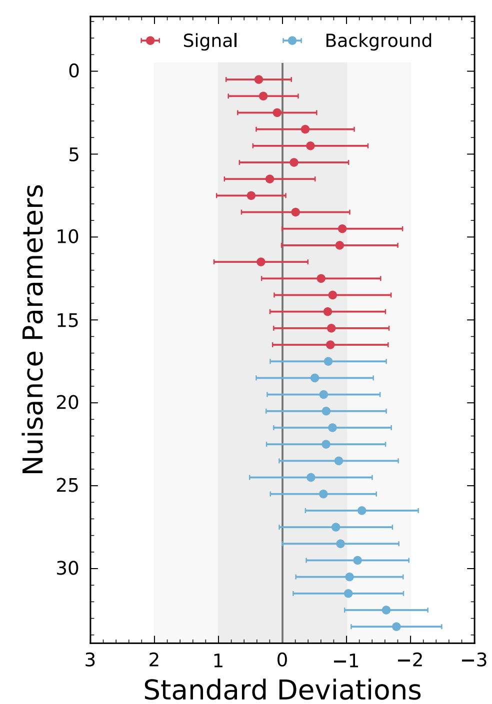
Projections of 2D fit in m_X : q^2

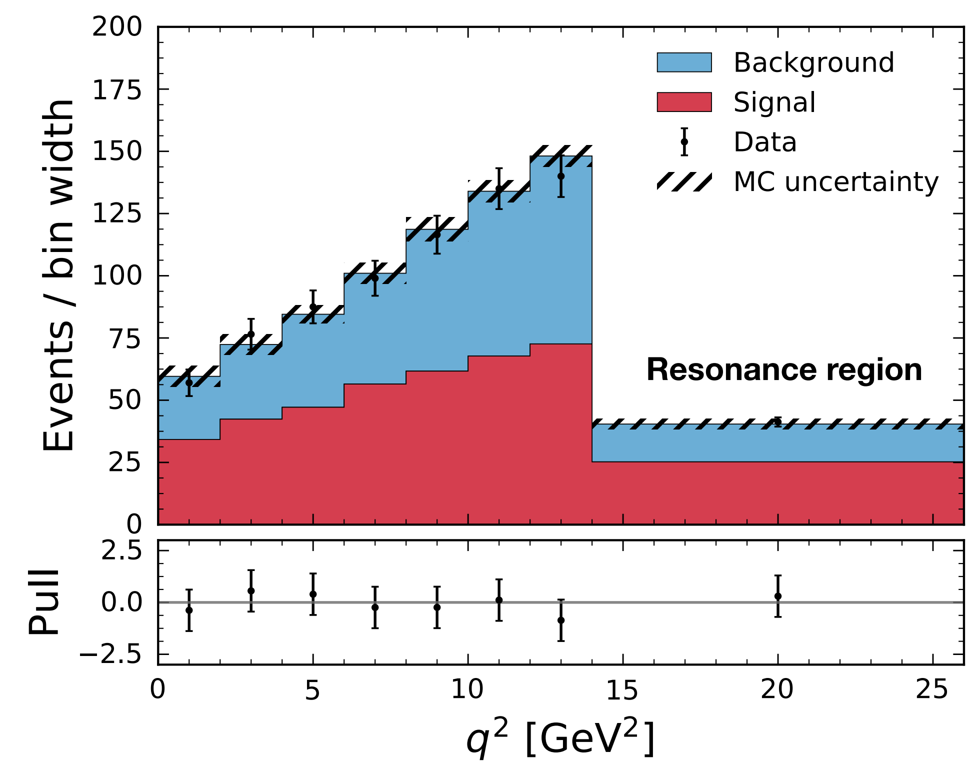


Unfold measured yields to **3 phase-space regions**:

Phase-space region
$M_X < 1.7\,\text{GeV}$
$M_X < 1.7\,\text{GeV},\ q^2 > 8\,\text{GeV}^2$
$E_\ell^B > 1\,\text{GeV}$

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\,\ell^+\,\nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\,\ell^+\,\nu_\ell)}}$$

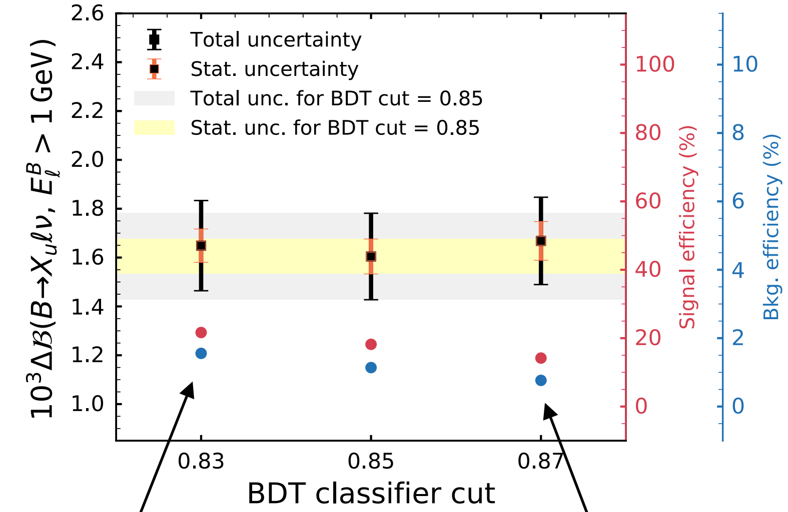




$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell^+ \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u \ell^+ \nu_\ell)}}$$

Fit kinematic distributions and measure partial BF

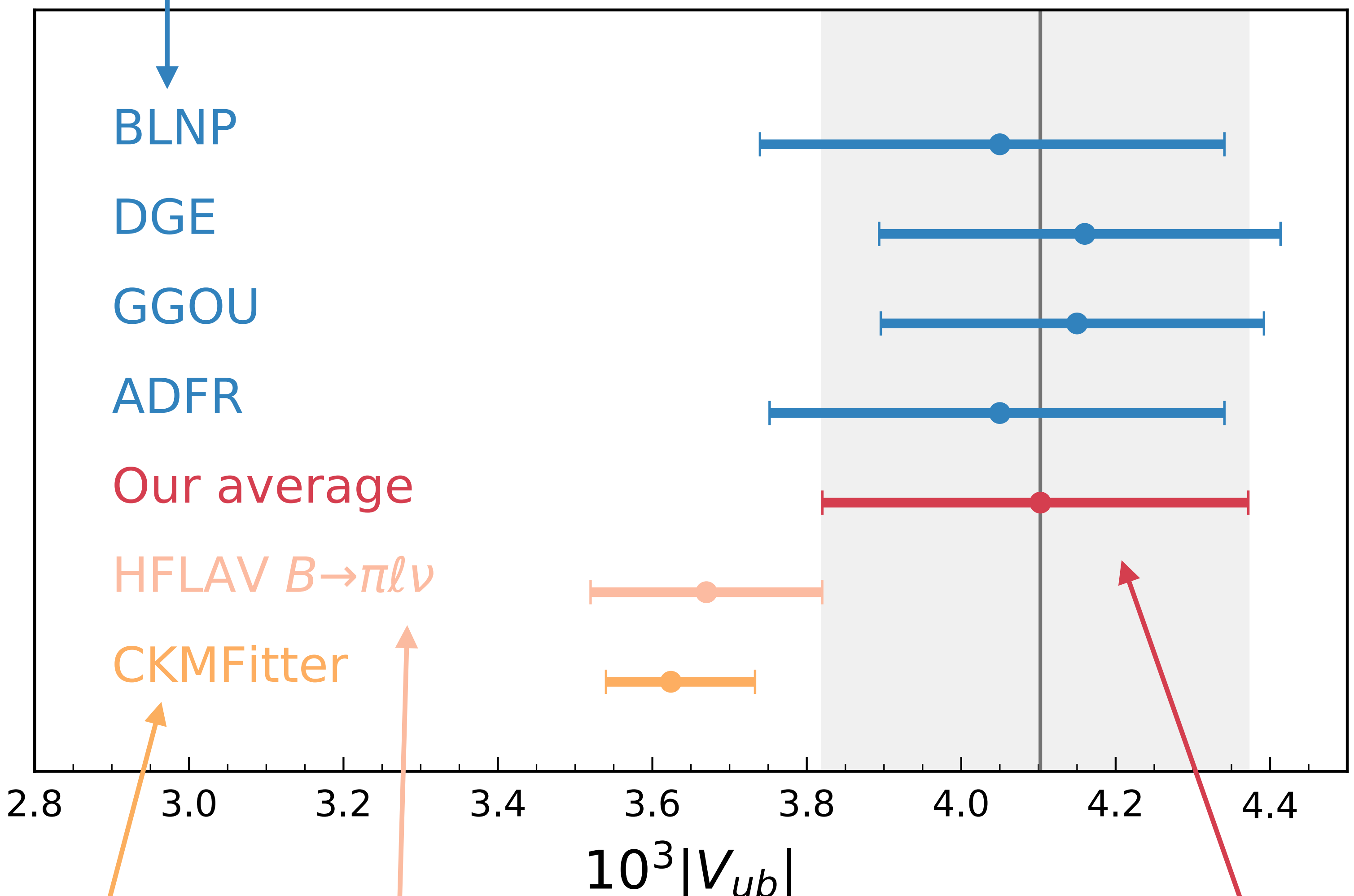
Stability as a function of BDT cut:



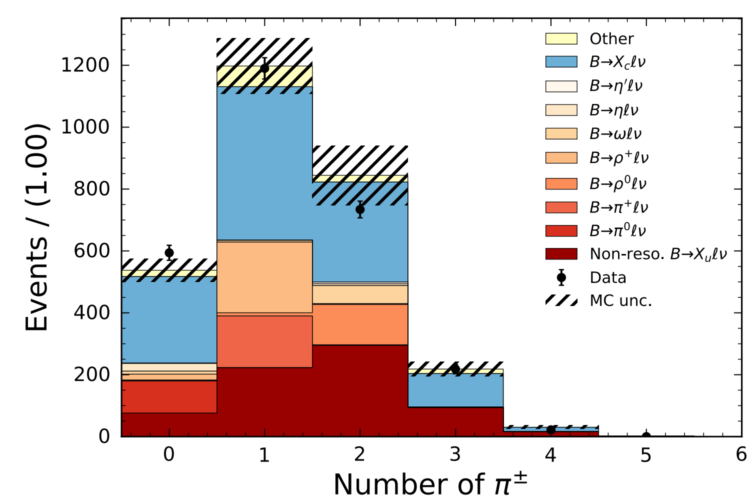
33% more Bkg

37% less Bkg

4 predictions of the partial rate



Post-fit N_{π^+} distribution:

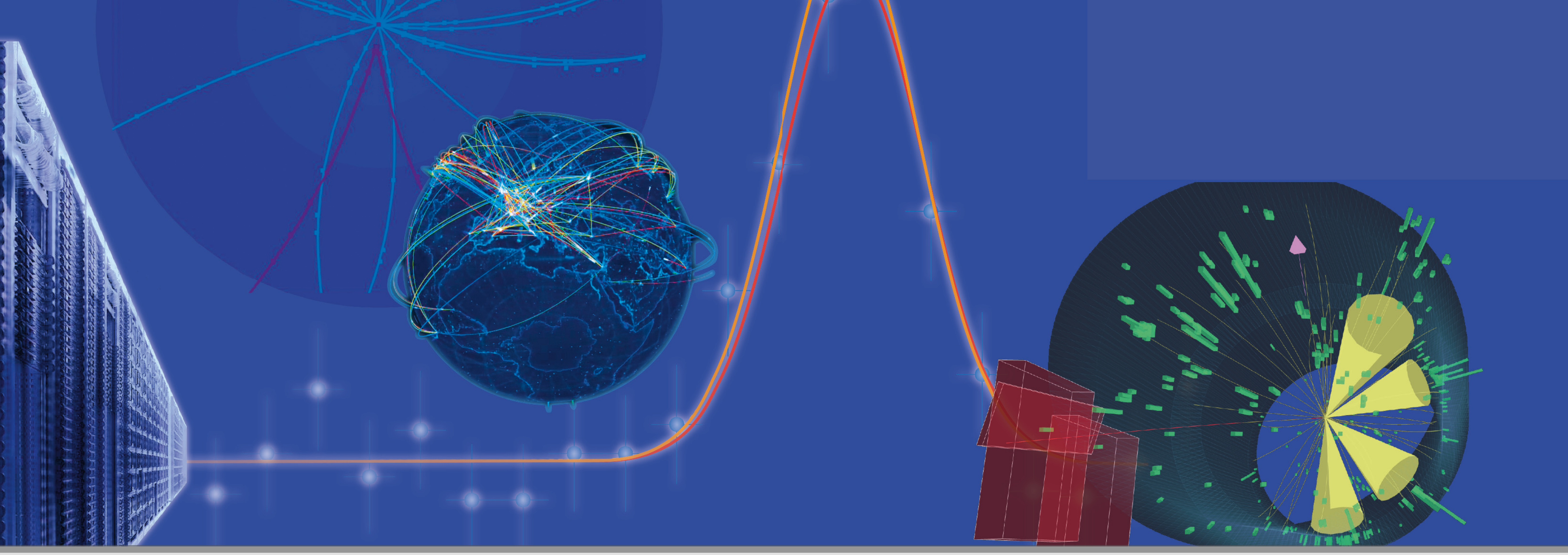


Exclusive Average for $B \to \pi \ell \bar{\nu}_\ell$:
$|V_{ub}| = \left(3.67 \pm 0.09 \pm 0.12\right) \times 10^{-3}$

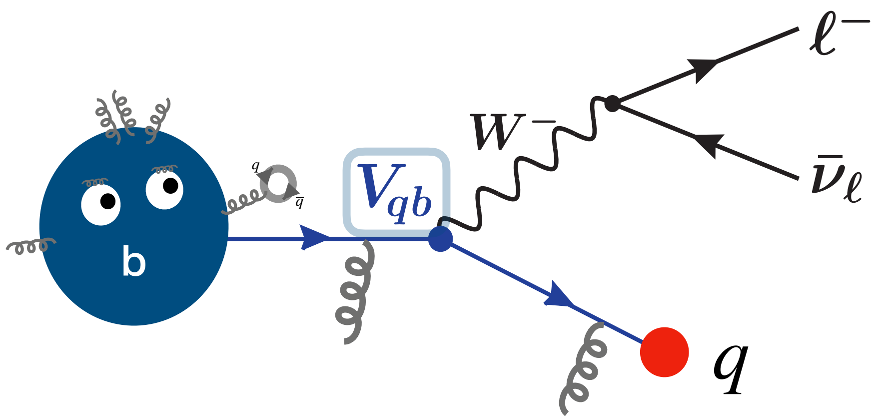
Arithmetic average:
$|V_{ub}| = \left(4.10 \pm 0.09 \pm 0.22 \pm 0.15\right) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = \left(3.62^{+0.11}_{-0.08}\right) \times 10^{-3}$

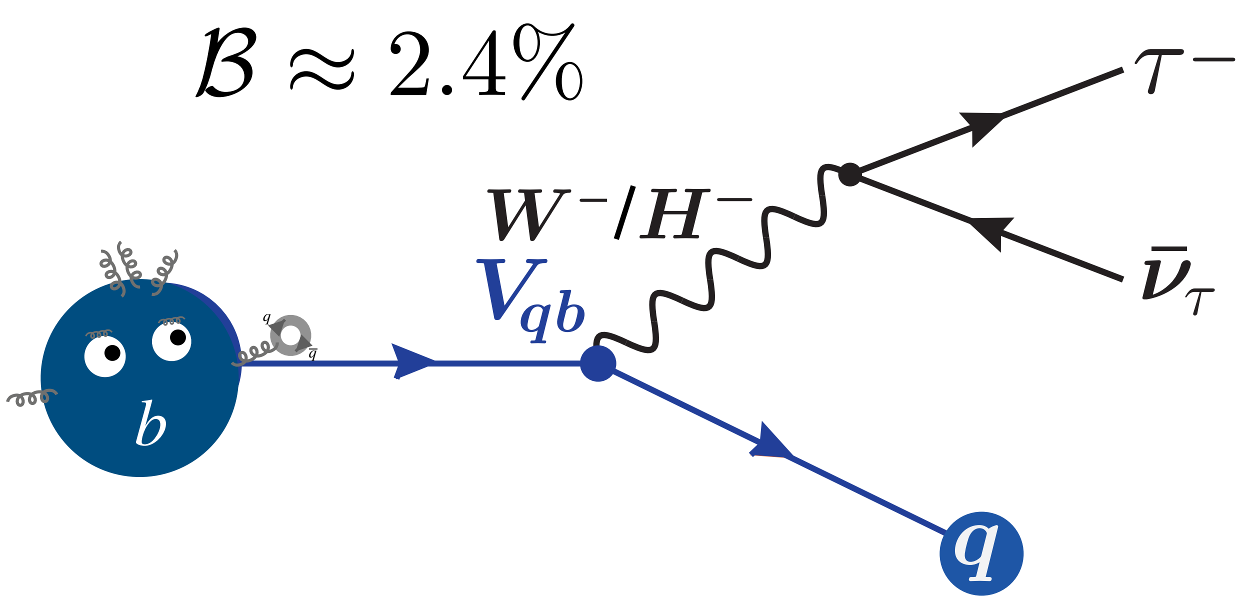
Dr. Lu Cao

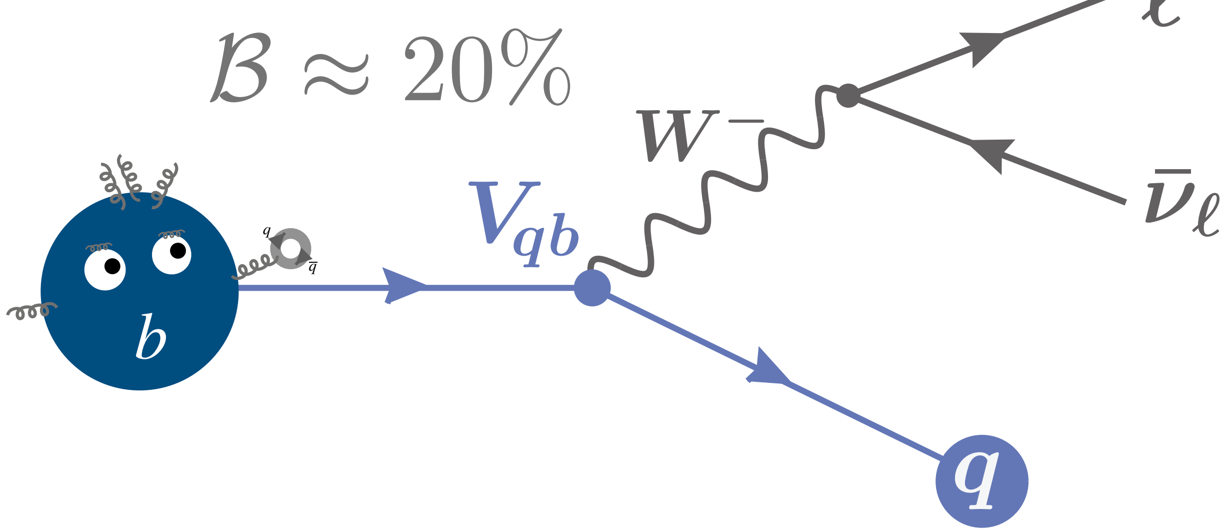


Backup II / More Information on $\ell = \tau$ Measurements



Semileptonic decays with τ





Observable of choice:

$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

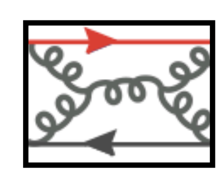
$\ell = e, \mu$

$$\downarrow$$

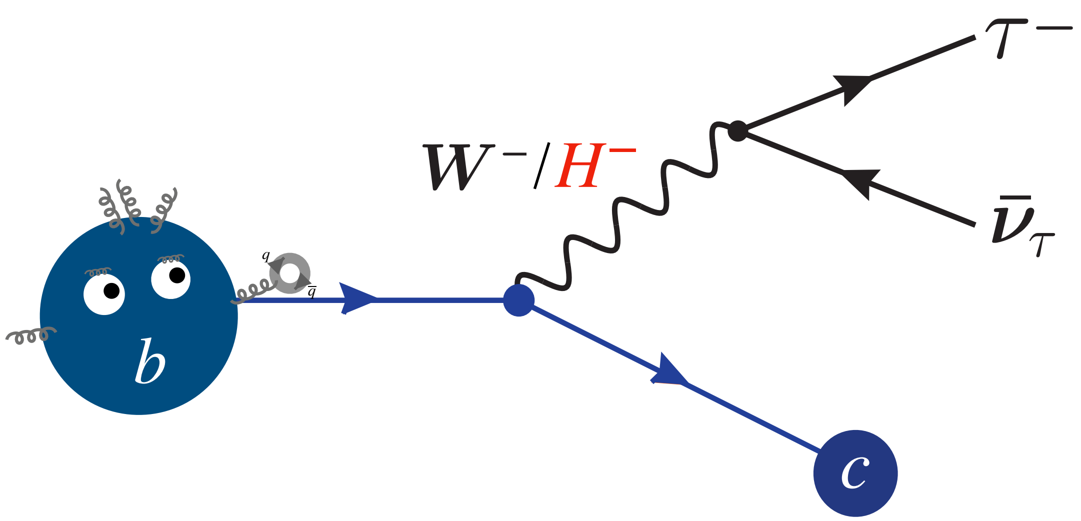
$$R(D^{(*)}, \pi, J/\psi)$$

Benefits:

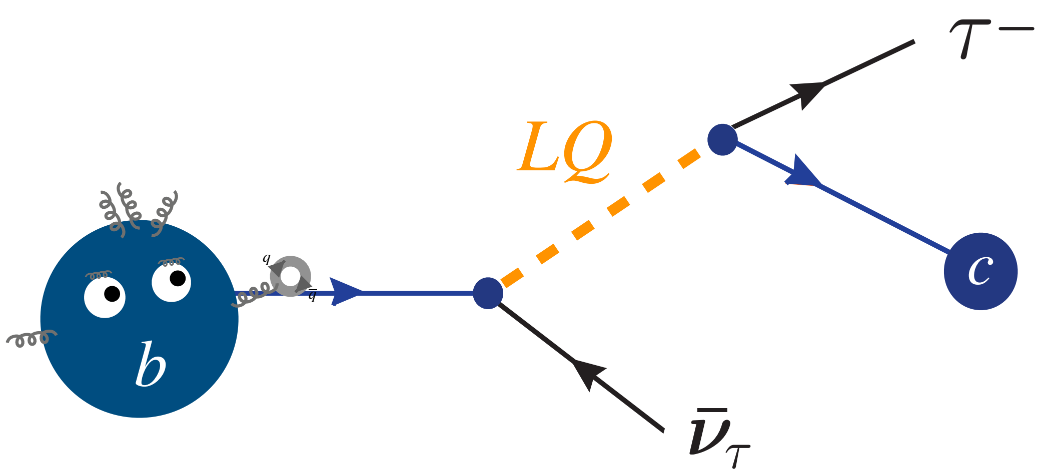
- Experimental systematics **cancel in ratio**
- Theory uncertainties **cancel in ratio**

QCD: 

charged Higgs bosons



Leptoquarks



→ Not the focus of this talk; but I added you some introduction material nonetheless in case you are interested in these!

Measurement Strategies

Signal

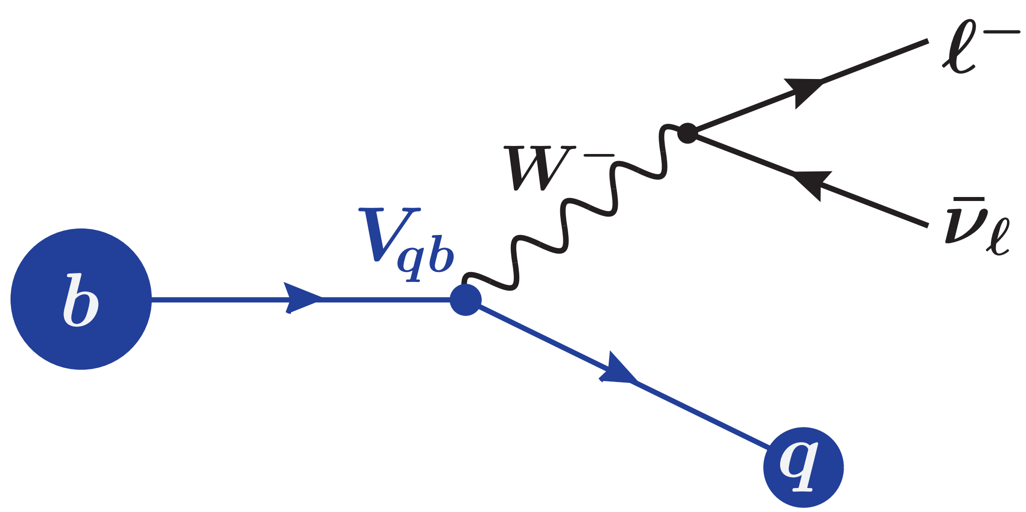
$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

Normalization

$\ell = e, \mu$

1. Leptonic or Hadronic τ decays?

Some properties (e.g. τ **polarization**) readily **accessible** in **hadronic decays.**



2. Albeit not necessarily a rare decay of O(%) in BF, TRICKY to separate from normalisation and backgrounds

LHCb: **Isolation** criteria, **displacement** of τ, **kinematics**

B-Factories: Full reconstruction of event (**Tagging**), **matching topology, kinematics**

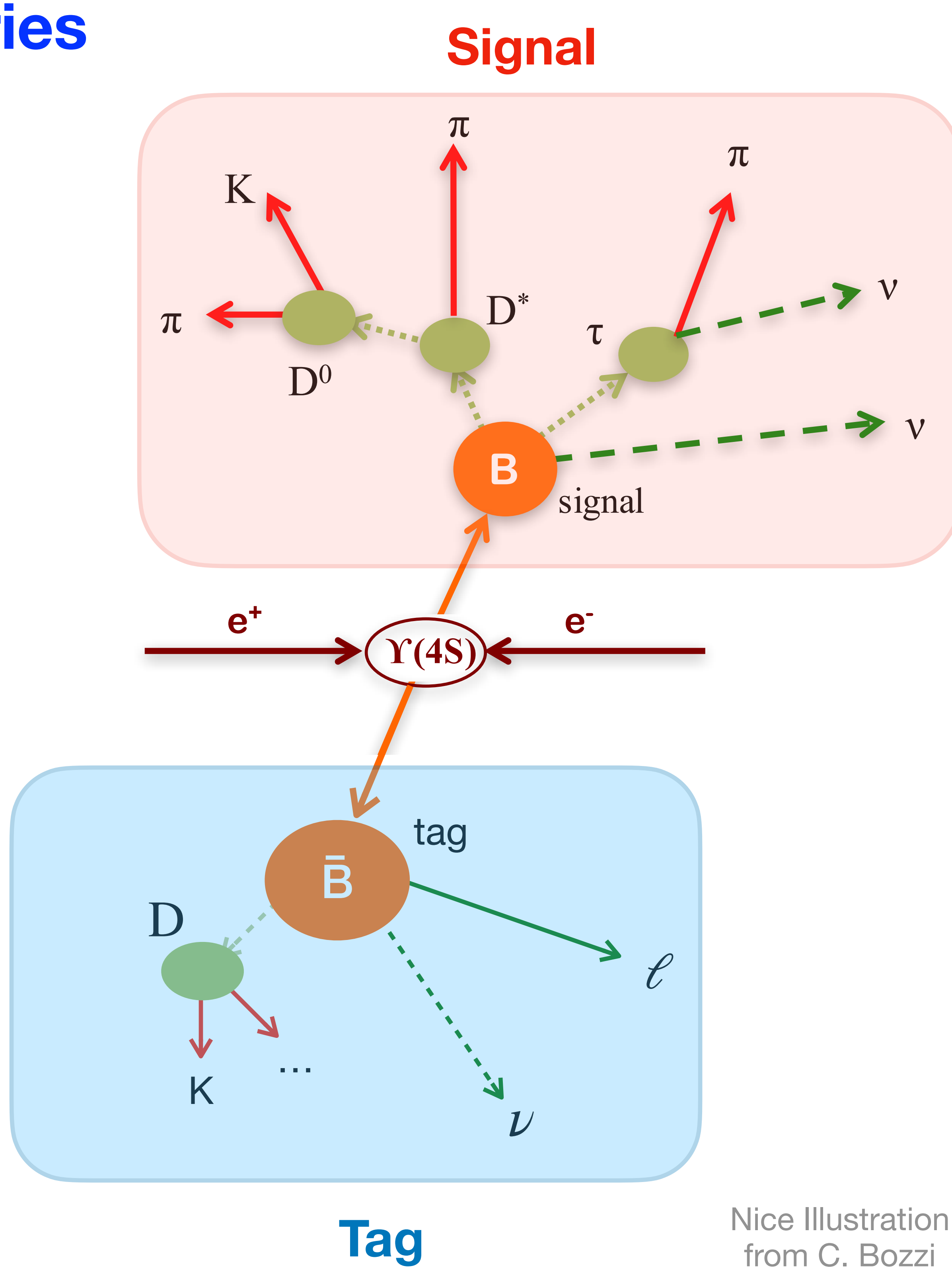
Measurement Strategies

3. Semileptonic decays at B-Factories

- e^+/e^- collision produces ***Y(4S)*** → ***$B\overline{B}$***
- Fully reconstruct one of the two B-mesons ('tag') → **possible** to **assign all particles** to either signal or tag B
- **Missing four-momentum** (**neutrinos**) can be reconstructed with high precision

$$p_{\rm miss} = (p_{\rm beam} - p_{B{\rm tag}} - p_{D^{(*)}} - p_\ell)$$

✓ **Small efficiency (~0.2-0.4%) compensated by large integrated luminosity**



Nice Illustration from C. Bozzi

Measurement Strategies

4. Semileptonic decays at LHCb

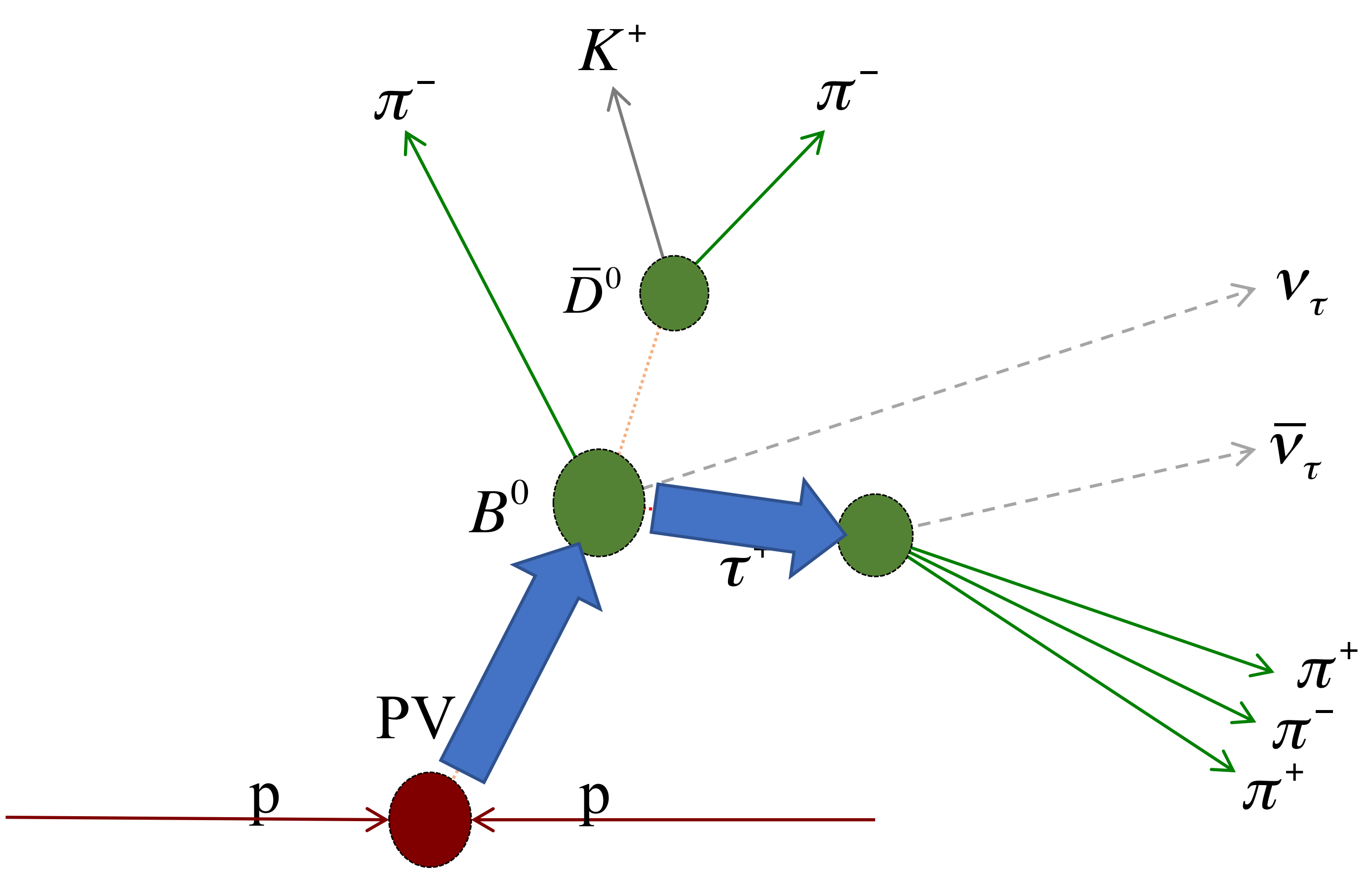
- No constraint from beam energy at a hadron machine, **but..**
- **Large Lorentz boost** with decay lengths in the range of **mm**

  ✓ **Well-separated decay vertices**

  ✓ **Momentum direction of decaying particle is well known**

- With known masses and other decay products can even **reconstruct four-momentum transfer squared q²** up to a two-fold ambiguity

$$q^2 = \left(p_{X_b} - p_{X_q}\right)^2$$



Nice Illustration from C. Bozzi

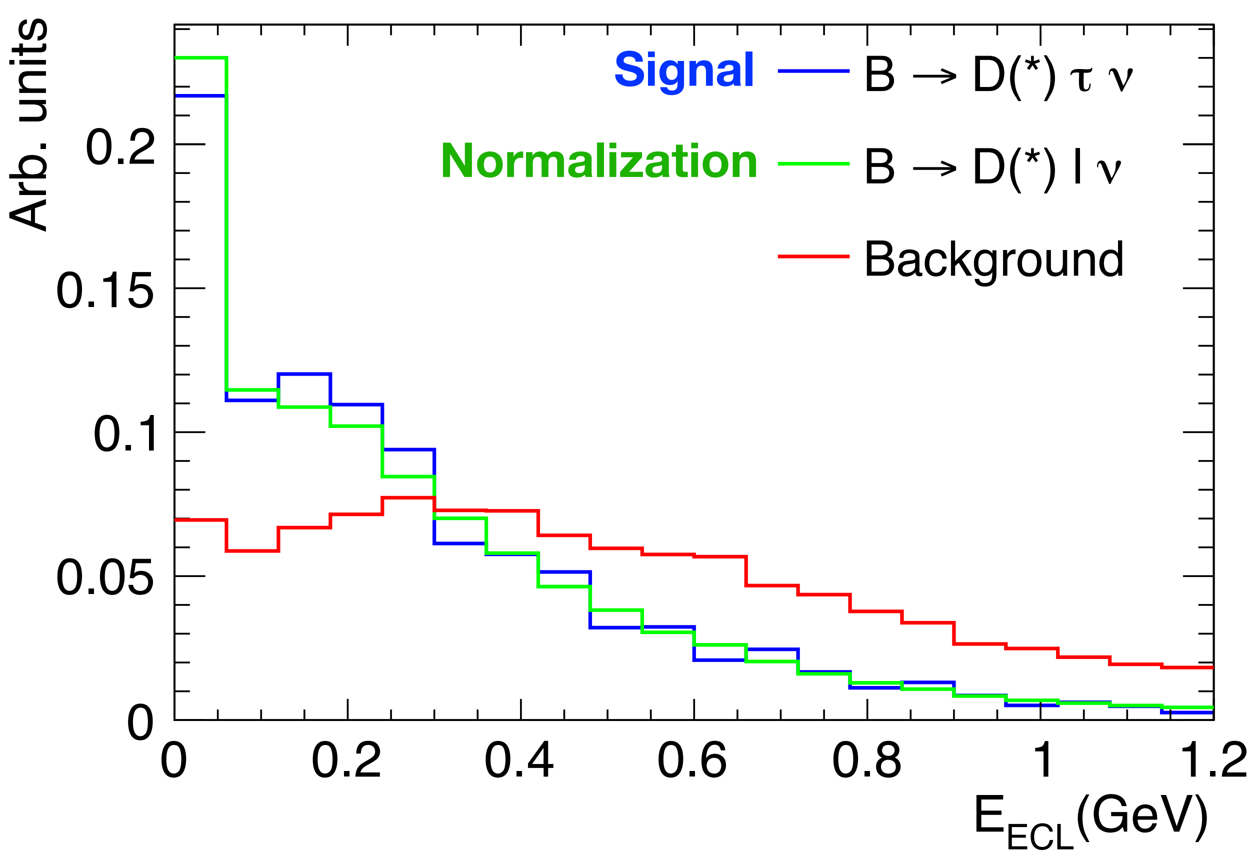
Even bit more complicated for leptonic tau decays

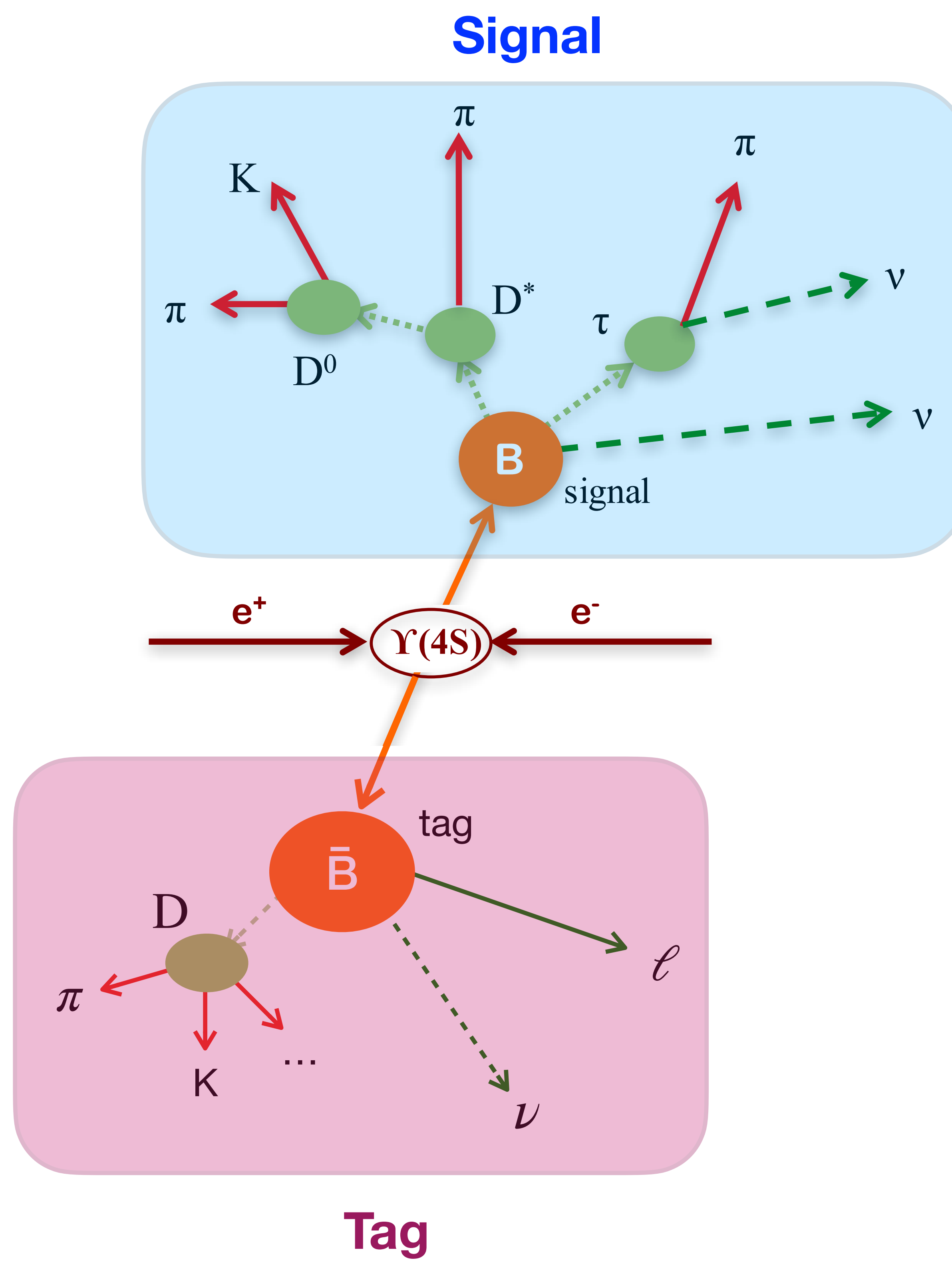
$R(D^{(*)})$ from Belle

G. Caria et al (Belle),
Phys. Rev. Lett. 124, 161803, April 2020
[arXiv:1904.08794]

- Reconstruct one of the two B-mesons ('tag') in **semileptonic modes → possible to assign all particles in detector** to tag- & signal-side
- Demand Matching topology + **unassigned** energy in the calorimeter E_{ECL} to discriminate background from signal

$$E_{\mathrm{extra}} = E_{\mathrm{ECL}} = \sum_i E_i^{\gamma}$$

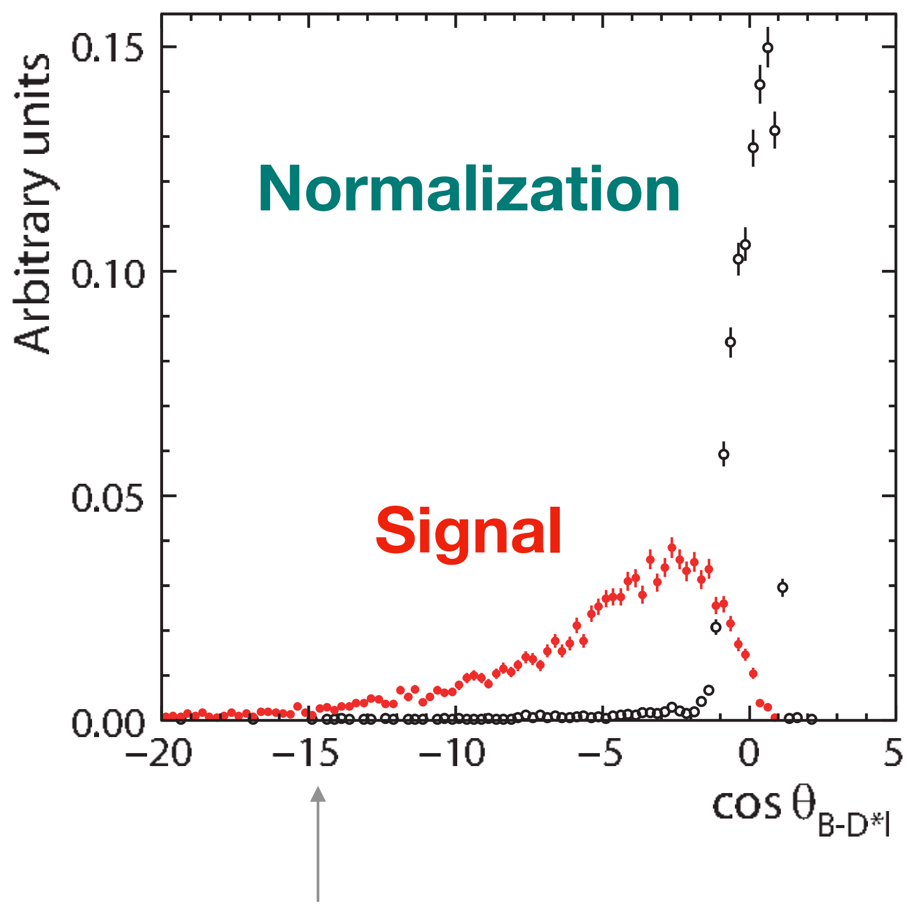


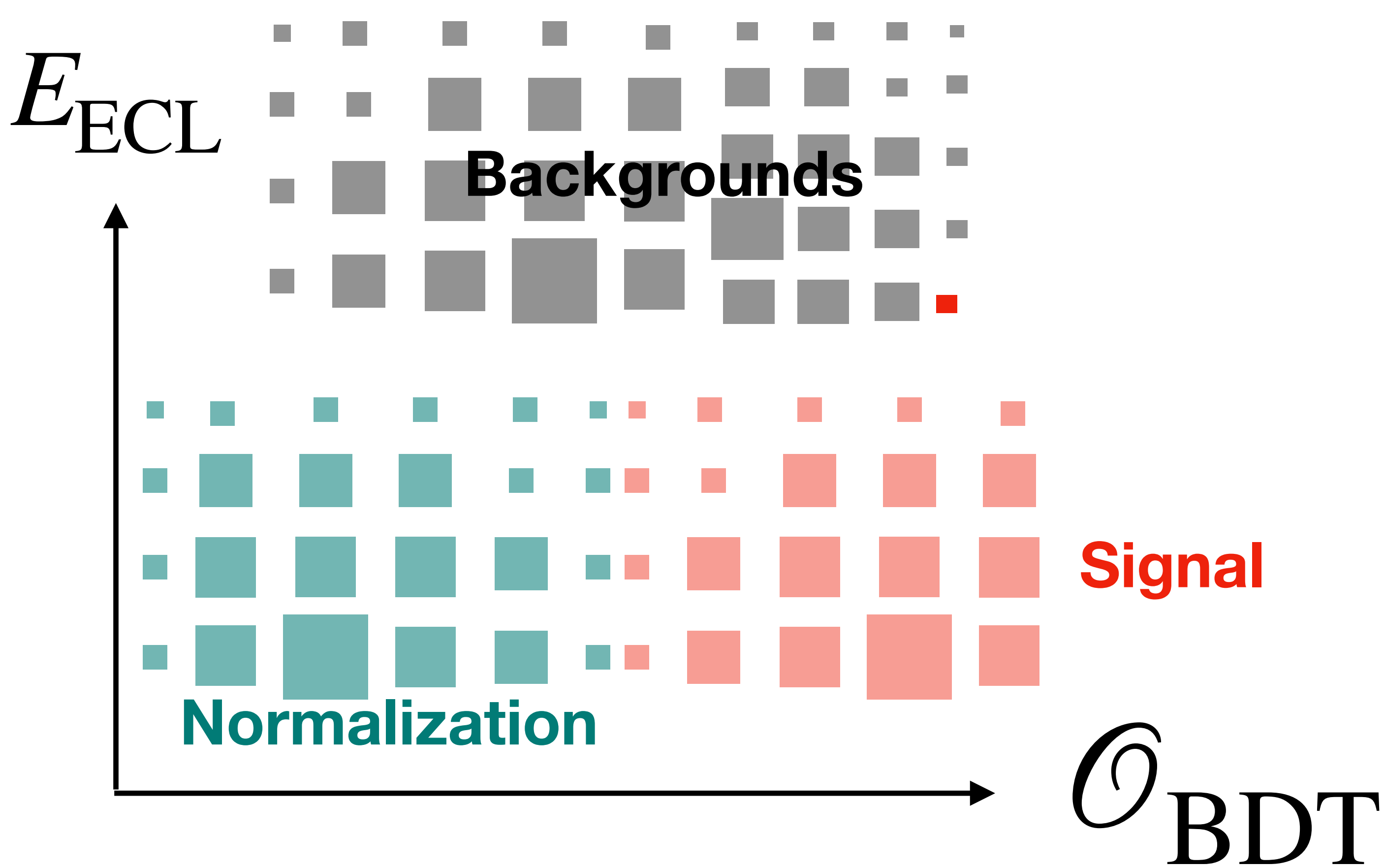


Separation of signal & normalization

- Use kinematic properties to separate $B \to D^{(*)}\tau\nu$ signal from $B \to D^{(*)}\ell\nu$ normalization

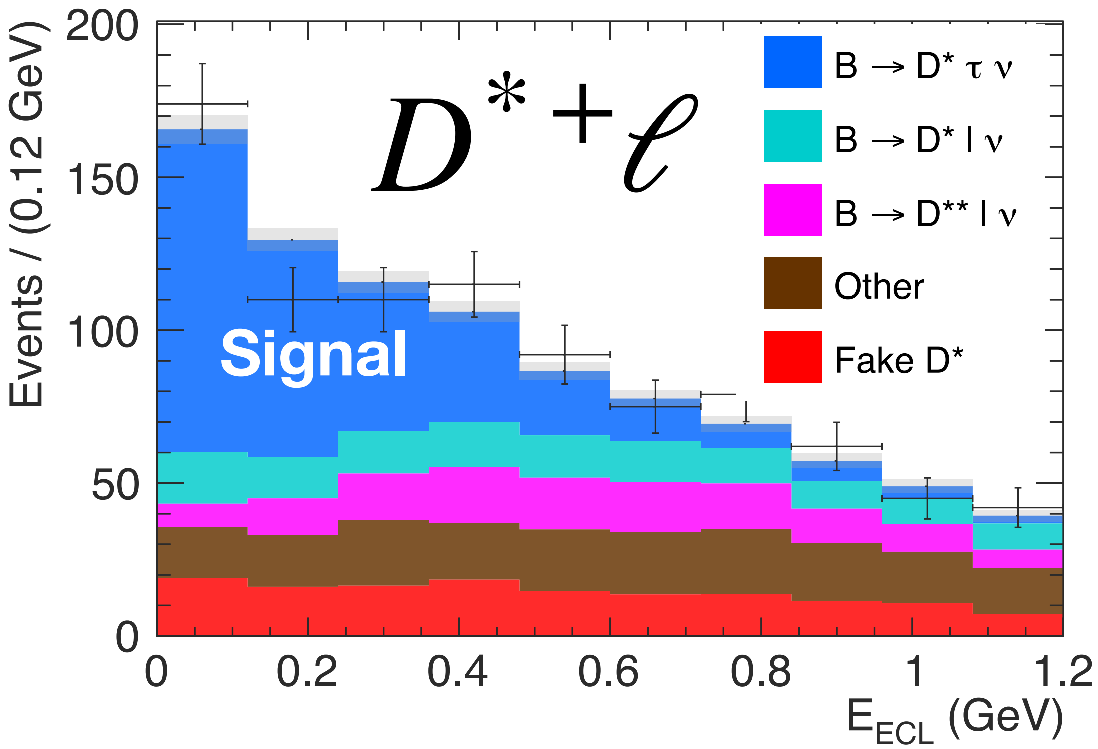
- Construct BDT with 3 variables: $\cos\theta_{B-D^{(*)}\ell}$, E_{vis}, $m^2_{\text{miss}} = p^2_{\text{miss}}$



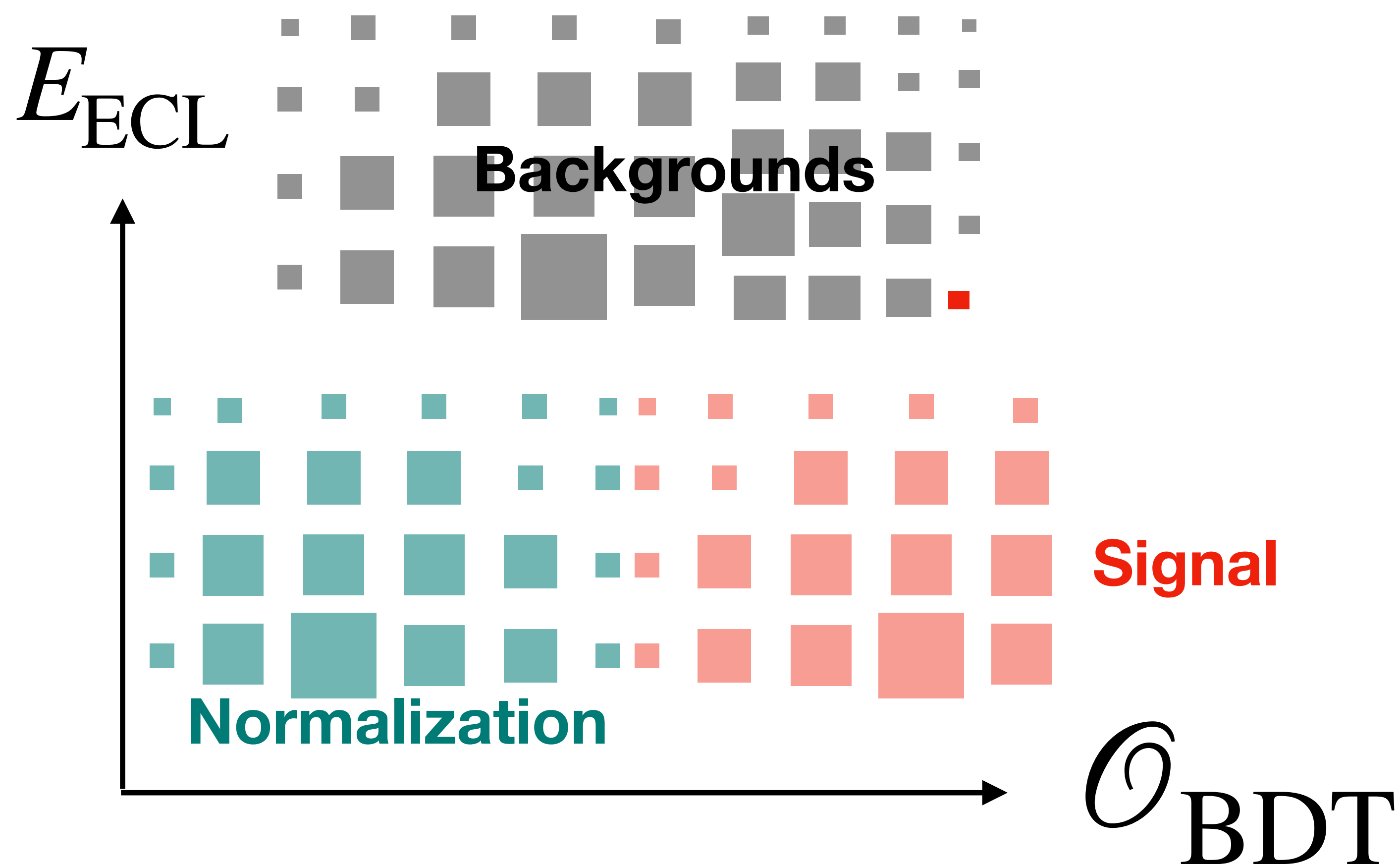


In case you are wondering how a cosine can be outside [-1,1]: it's because the reconstruction uses measured energies and the definition assumes only a single missing neutrino

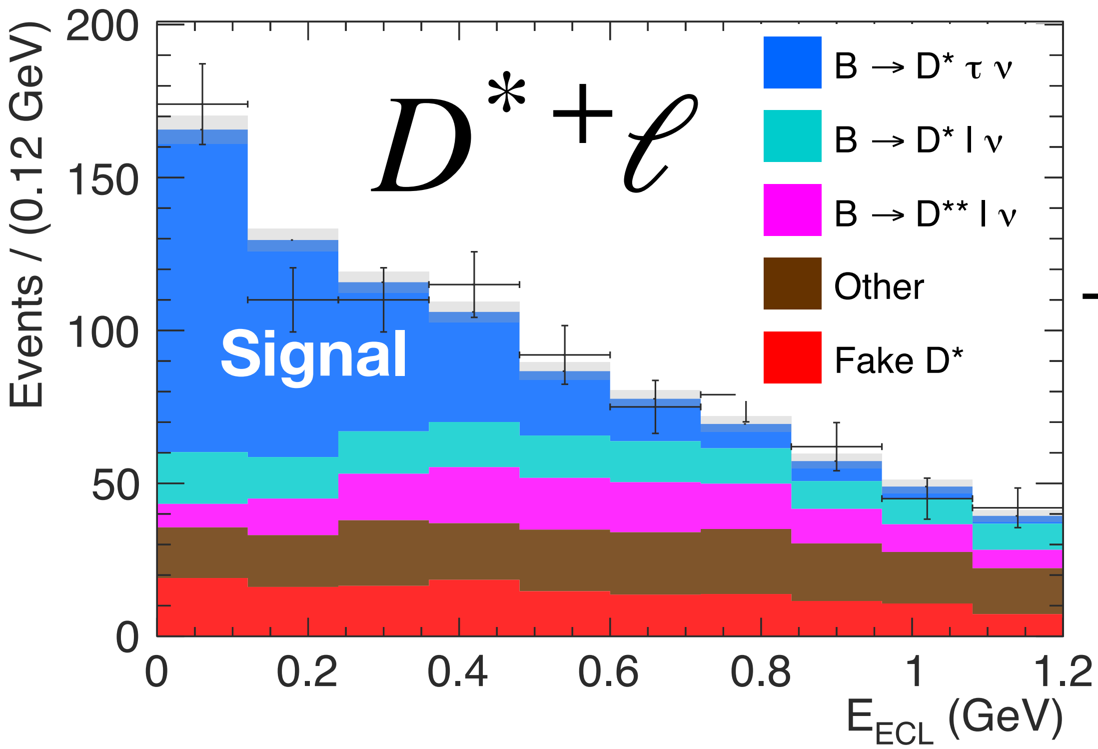
Separation of signal & normalization



Signal-enriched selection with cut on $\mathcal{O}_{\rm BDT}$



Separation of signal & normalization



$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

Most precise measurement to date

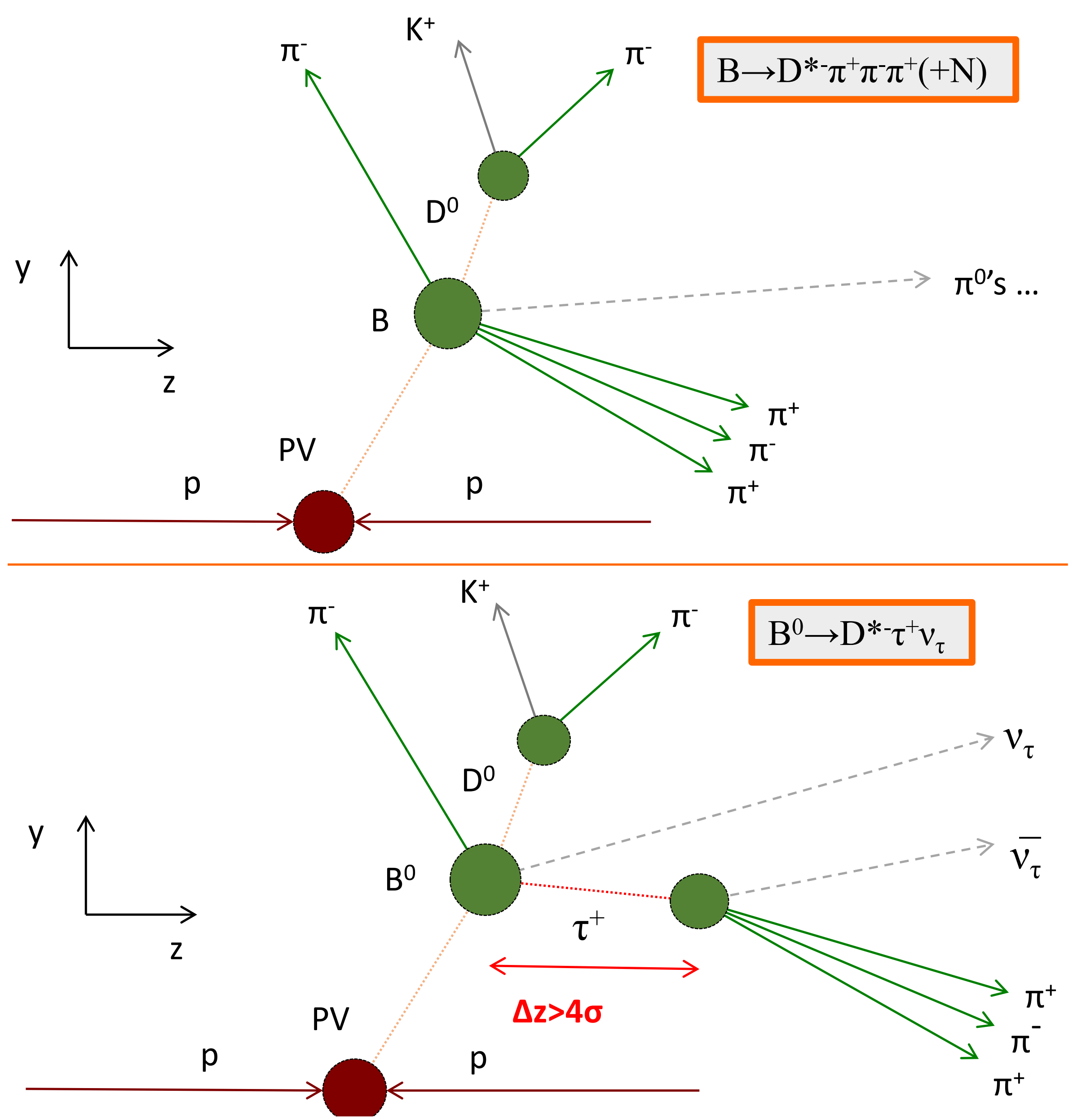
Signal enriched selection with cut on $\mathcal{O}_{\mathrm{BDT}}$

LHCb Measurement of $R(D^*)$

R. Aaij et al (LHCb),
Phys.Rev.Lett.120,171802 (2018) [arXiv:1708.08856]
Phys.Rev.D 97, 072013 (2018) [arXiv:1709.02505]

- Tau reconstructed via ***$\tau \to \pi^+\pi^+\pi^-(\pi^0)\nu$,*** only two neutrinos missing

  Although a semileptonic decay is studied, nearly no background from $B \to D^* X\mu\ \nu$



- Main background: prompt **$X_b \to D^*\pi\pi\pi$ + neutrals**

  BF ~ 100 times larger than signal, all pions are promptly produced

- Suppressed by requiring minimum distance between ***X_b & τ vertices (> 4 $\sigma_{\Delta z}$)***

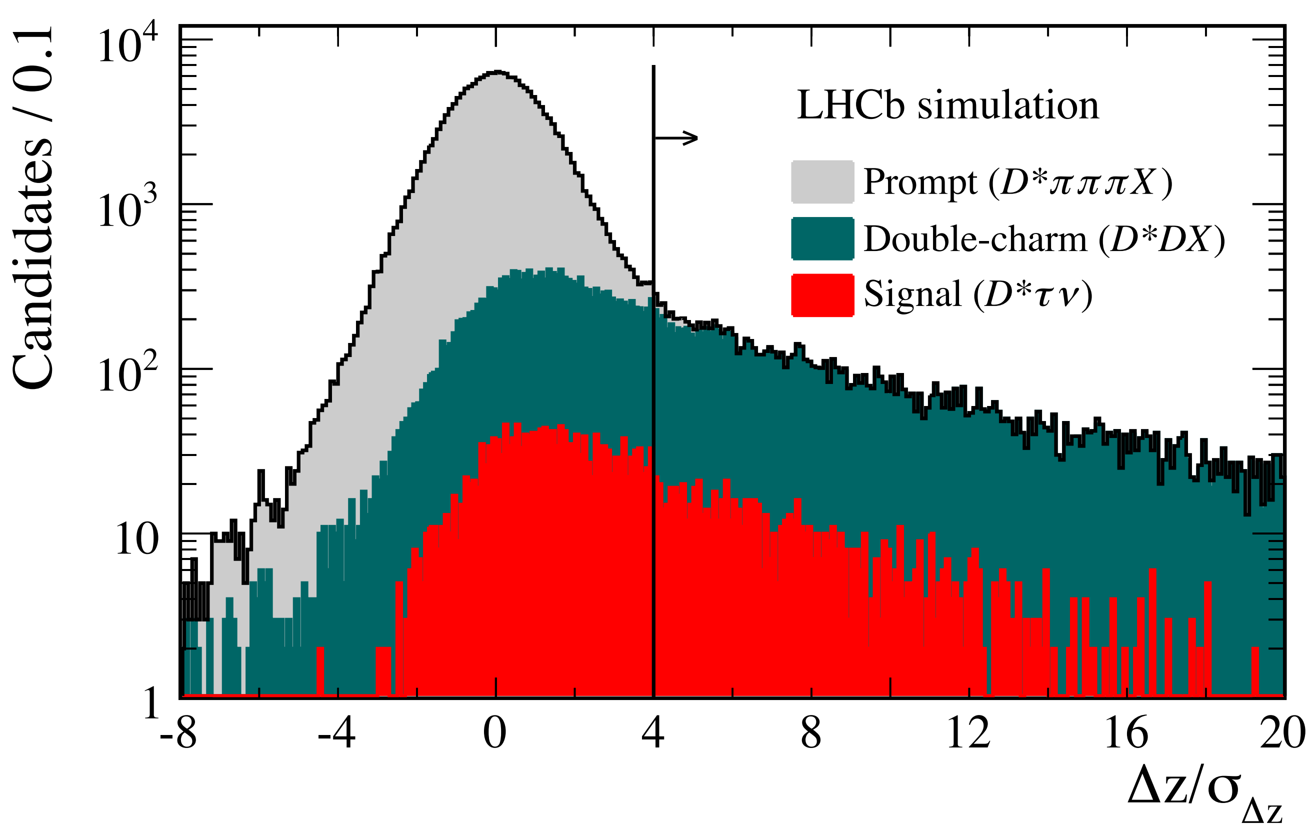
  $\sigma_{\Delta z}$: resolution of vertices separation

- Reduces this background by three orders of magnitude

LHCb Measurement of $R(D^*)$

- Tau reconstructed via $\tau \to \pi^+\pi^+\pi^-(\pi^0)\nu$, only two neutrinos missing

  Although a semileptonic decay is studied, nearly no background from $B \to D^* X\mu\nu$



- Remaining double charm bkgs:

  $X_b \to D^{*-}D_s^+X$ ~ 10 x Signal

  $X_b \to D^{*-}D^+X$ ~ 1 x Signal

  $X_b \to D^{*-}D_{s0}^+X$~ 0.2 x Signal

LHCb Measurement of $R(D^*)$

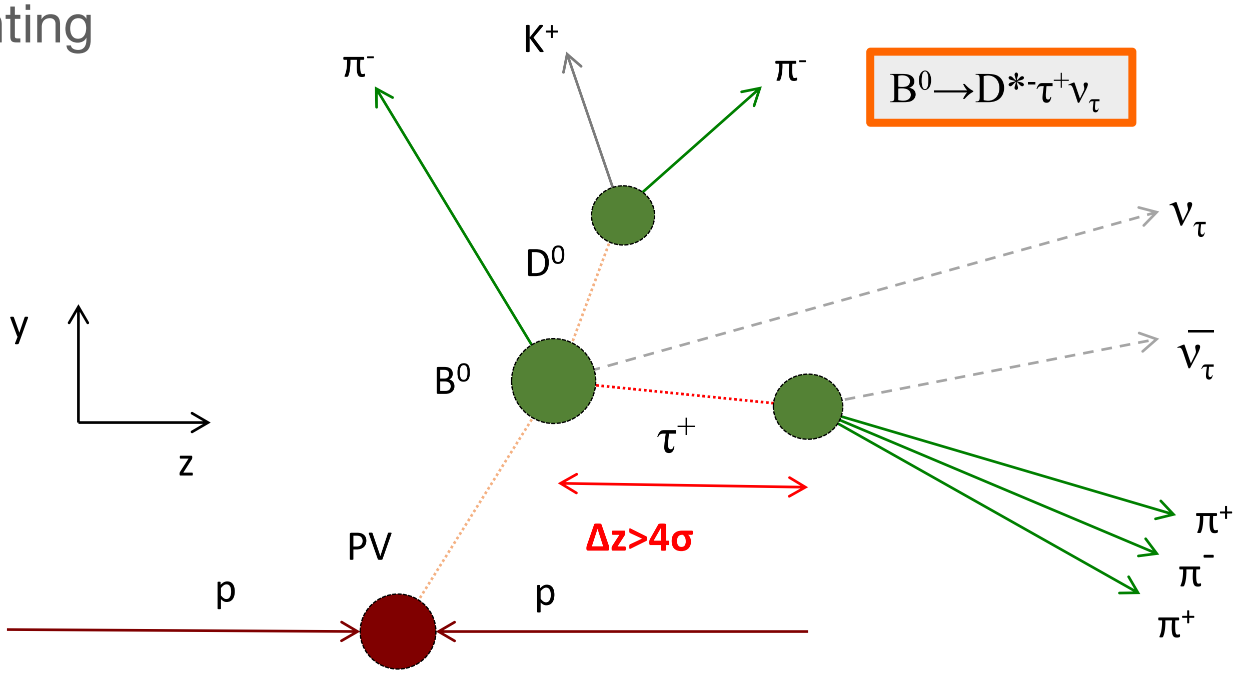
- Remaining backgrounds reduced via isolation & MVA

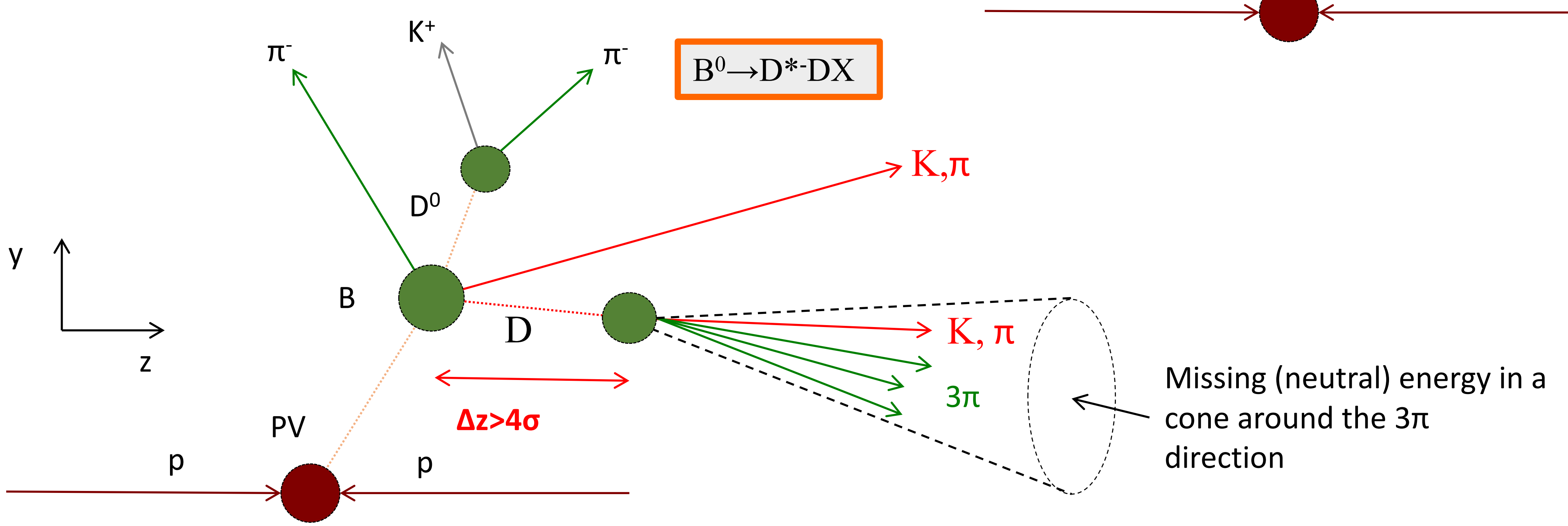
Require signal candidates to be well isolated

i.e. reject events with extra charged particles pointing to the B and/or τ

Events with additional neutral energy are suppressed with a MVA

More information about that in backup





LHCb Measurement of $R(D^*)$

- Extraction in **3D fit** to
  MVA : q² : τ decay time

Invariant masses of 3π system
Invariant mass of $D^*3\pi$ system
Neutral isolation variables

q² reconstructed with some tricks (more in backup)

4 Bins 8 Bins 8 Bins

- Components:

1 Signal component for $\tau \to \pi^+\pi^+\pi^-(\pi^0)\nu$

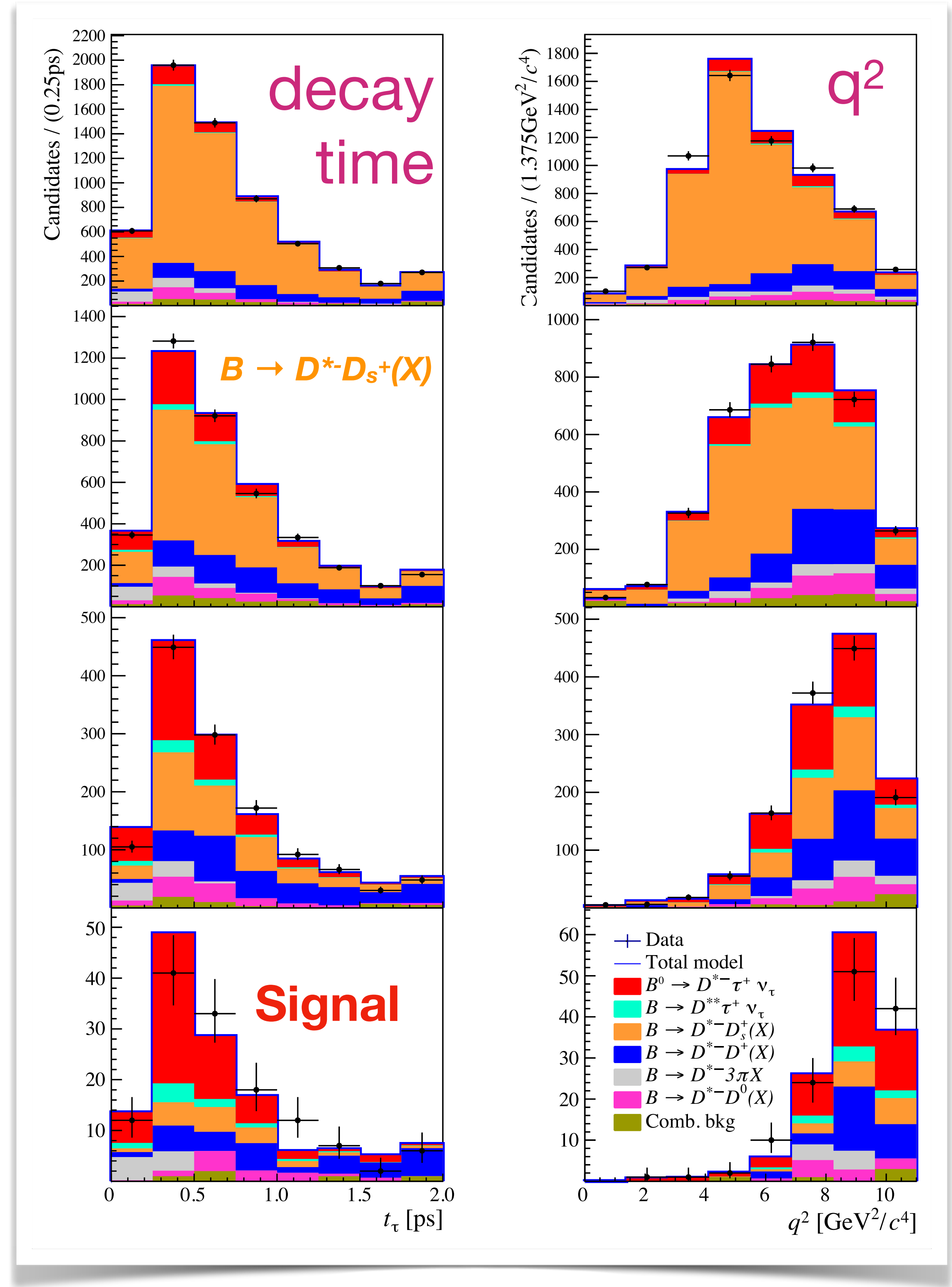
11 Background components

- ~ 1296 ± 86 Signal events
- Using normalization mode and light lepton BFs:

More information about normalization in backup

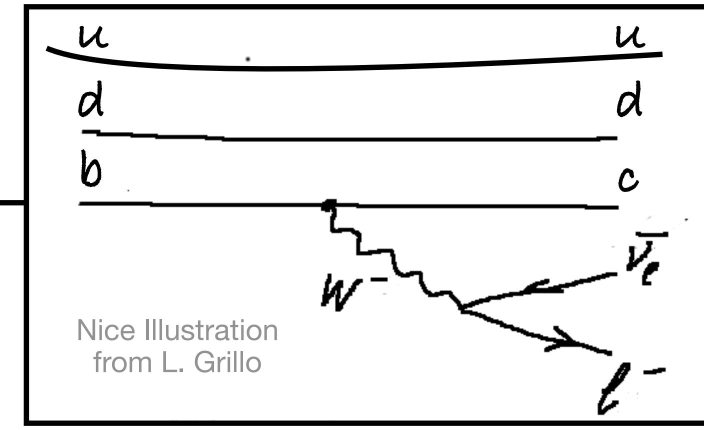
R(D*) = 0.286 ± 0.019 (stat) ± 0.025 (syst) ± 0.021 (norm)

Purer MVA Selection



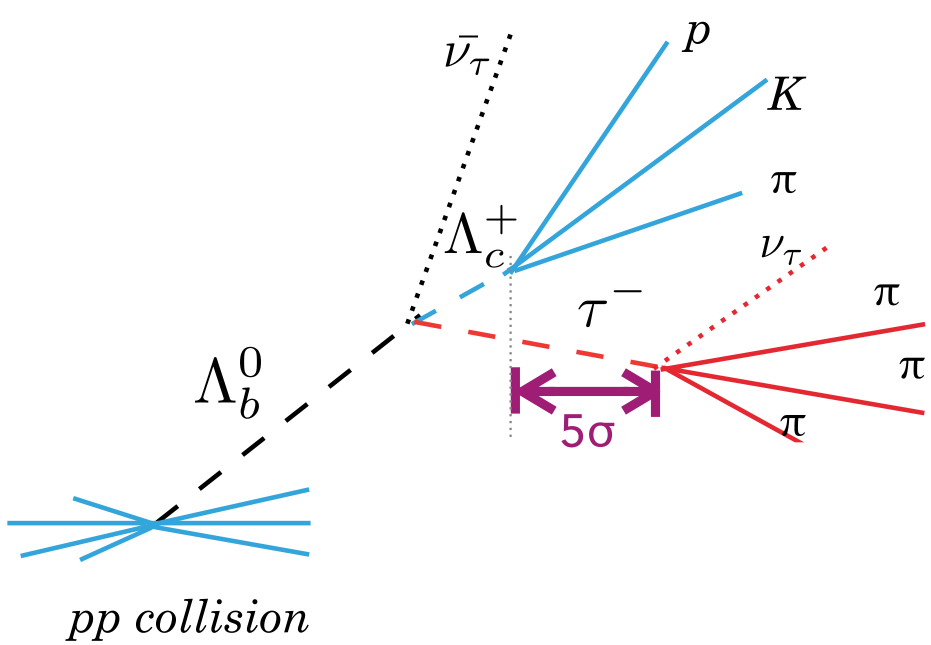
LHCb $R(\Lambda_c)$ Measurement

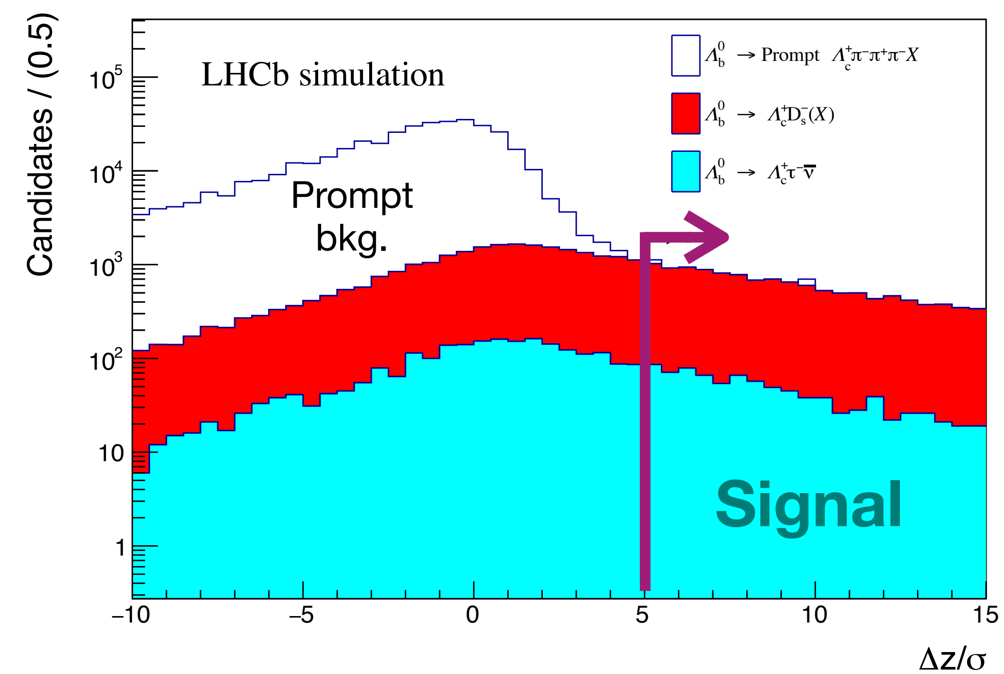
Λ_b



Λ_c

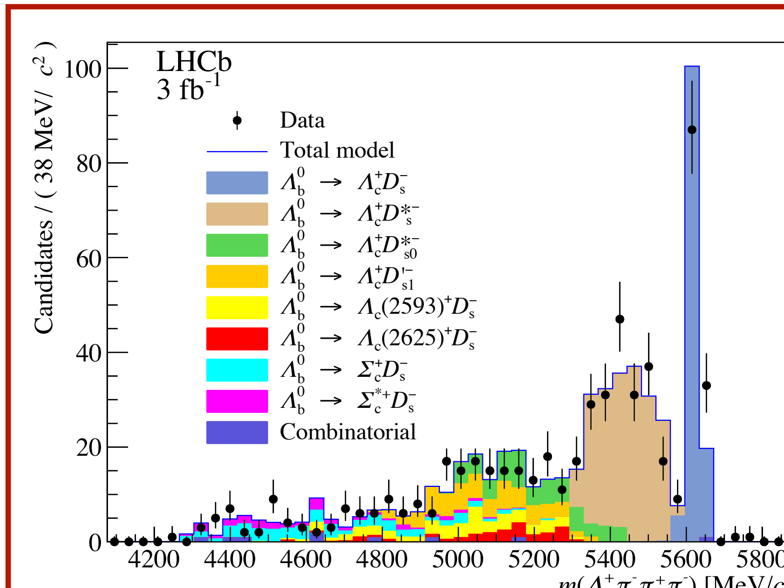
Same experimental Method: exploit vertex **separation**





Long-lived bkg. (D_s)

$m_{3\pi} \in [m_{D_s} - 45\ \text{MeV}, m_{D_s} + 45\ \text{MeV}]$



Bkg. composition constrained by fit to $m_{3\pi}$

Target **ratio**:

$$\mathcal{K}(\Lambda_c^+) = \frac{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ 3\pi)} = \frac{N_{sig}}{N_{norm}} \times \frac{\epsilon_{norm}}{\epsilon_{sig}} \times \frac{1}{\mathcal{B}(\tau^- \to 3\pi(\pi^0)\nu_\tau)}$$

- Extraction in **3D fit** to
  MVA : q^2 : τ decay time

Kinematic and angular information of 3π system, neutral energy in cone around 3π direction

$$N(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau) = 349 \pm 40$$
$$N(\Lambda_b^0 \to \Lambda_c^+ D_s^-(X)) = 2757 \pm 80$$

External input:

$$\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ 3\pi) = (6.14 \pm 0.94) \times 10^{-3}$$

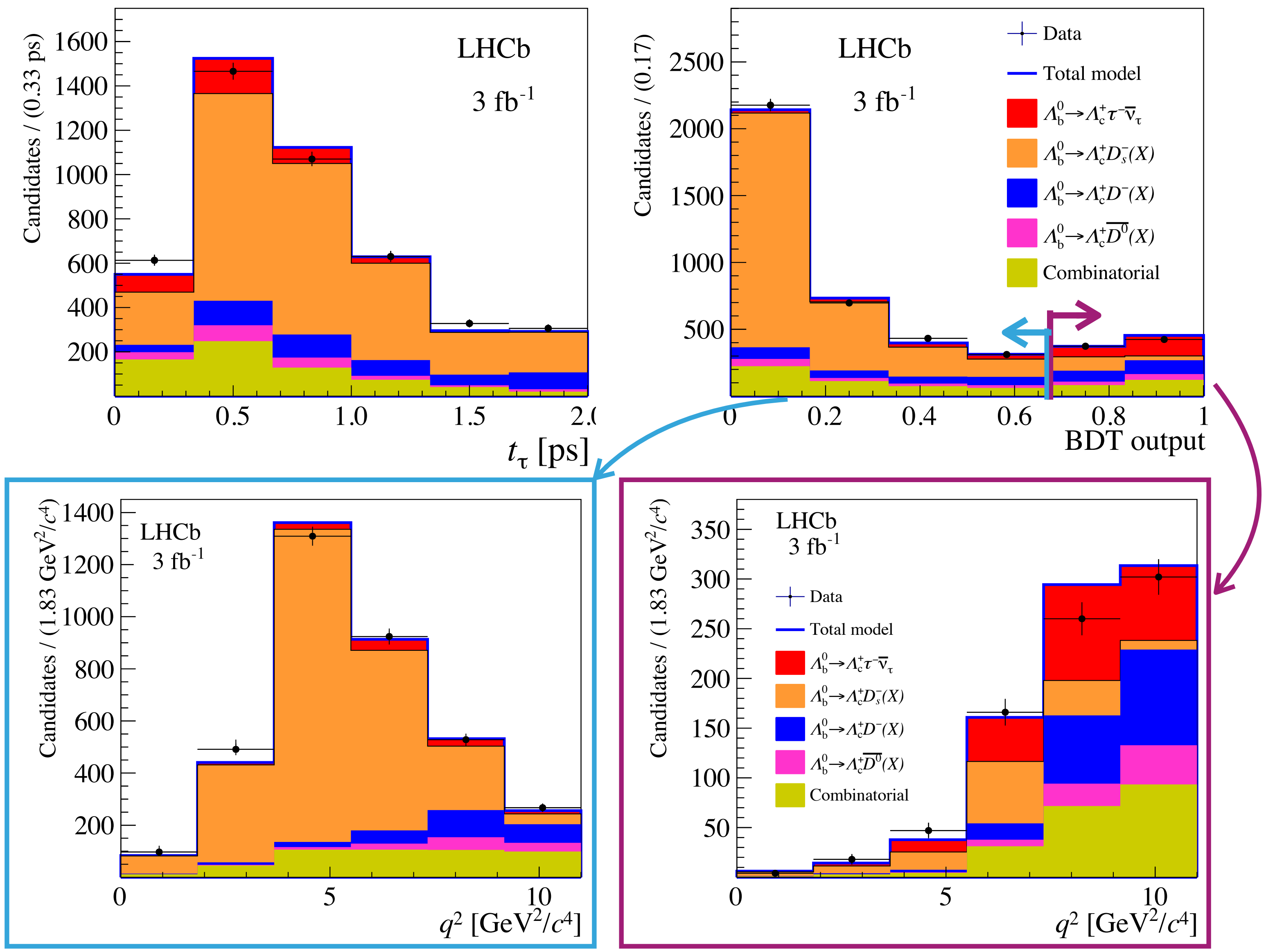
$$\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \tau \bar{\nu}_\tau) = (1.50 \pm 0.16 \pm 0.25 \pm 0.23)\,\%$$

First observation with 6.1 σ !

More external input:

$$\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \mu \bar{\nu}_\mu) = (6.2 \pm 1.4)\,\%$$

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\text{stat}} \pm 0.040_{\text{syst}} \pm 0.059_{\text{ext}}$$



Compatible with SM

$$R(\Lambda_c^+)_{\text{SM}} = 0.340 \pm 0.004$$

F. Bernlochner, Zoltan Ligeti, Dean J. Robinson, William L. Sutcliffe, [arXiv:1808.09464], [arXiv:1812.07593]

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\text{stat}} \pm 0.040_{\text{syst}} \pm 0.059_{\text{ext}}$$

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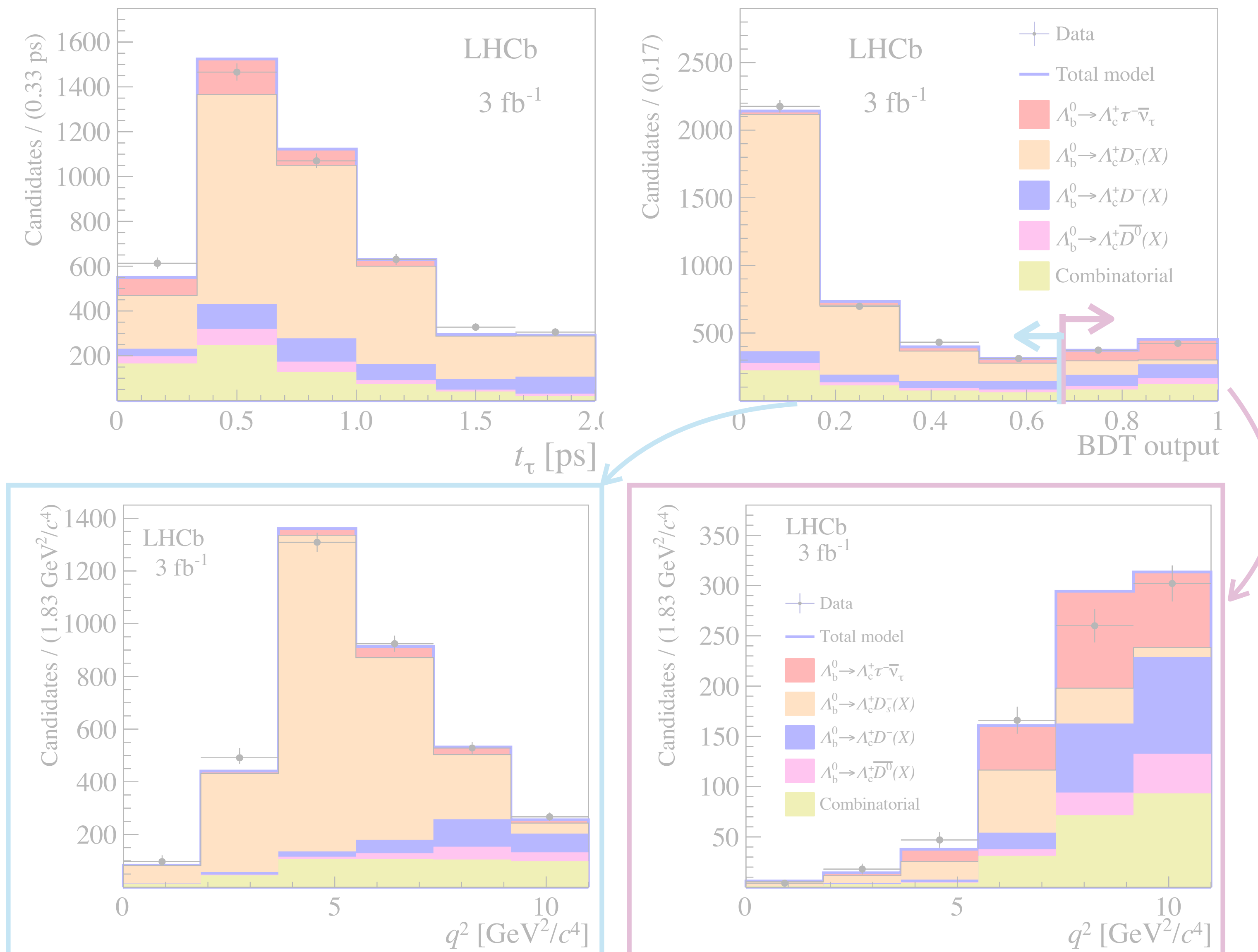
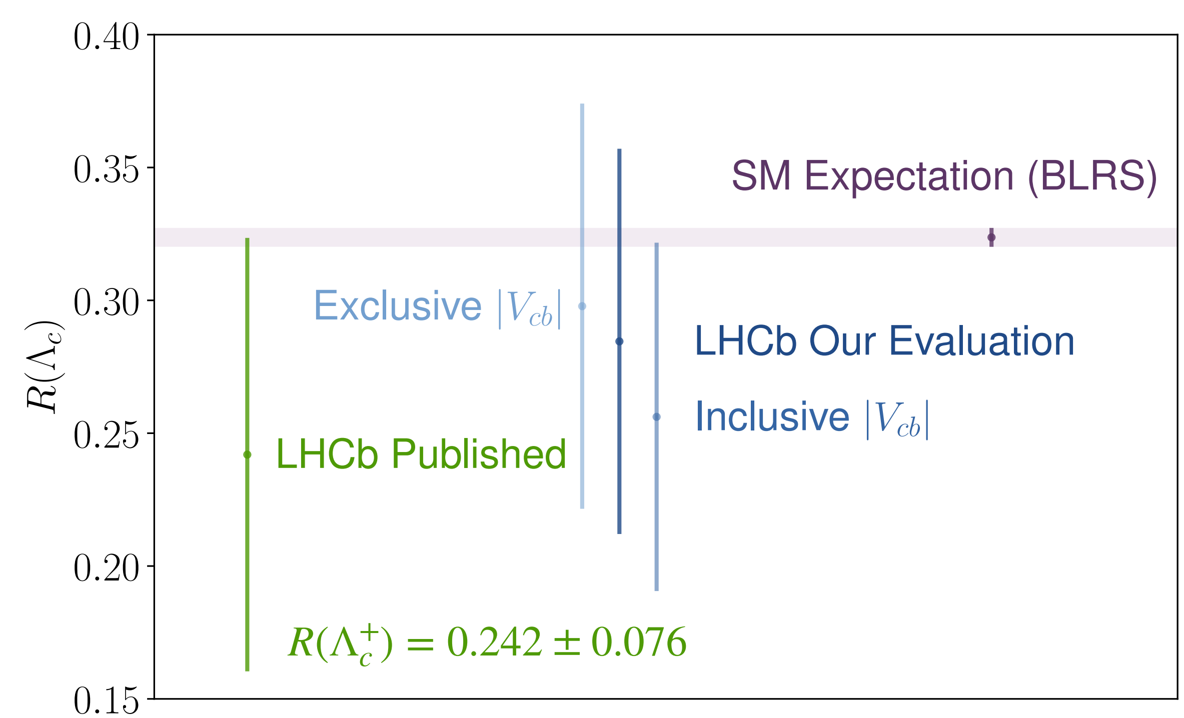
$$\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \tau \bar{\nu}_\tau) = (1.50 \pm 0.16 \pm 0.25 \pm 0.23)\,\%$$



Can also use SM prediction for $\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \mu \bar{\nu}_\mu)$ instead of LEP measurement

FB, Zoltan Ligeti, Michele Papucci, Dean Robinson, [arXiv:2206.11282 [hep-ph]]

$$R(\Lambda_c^+) = 0.285 \pm 0.073$$

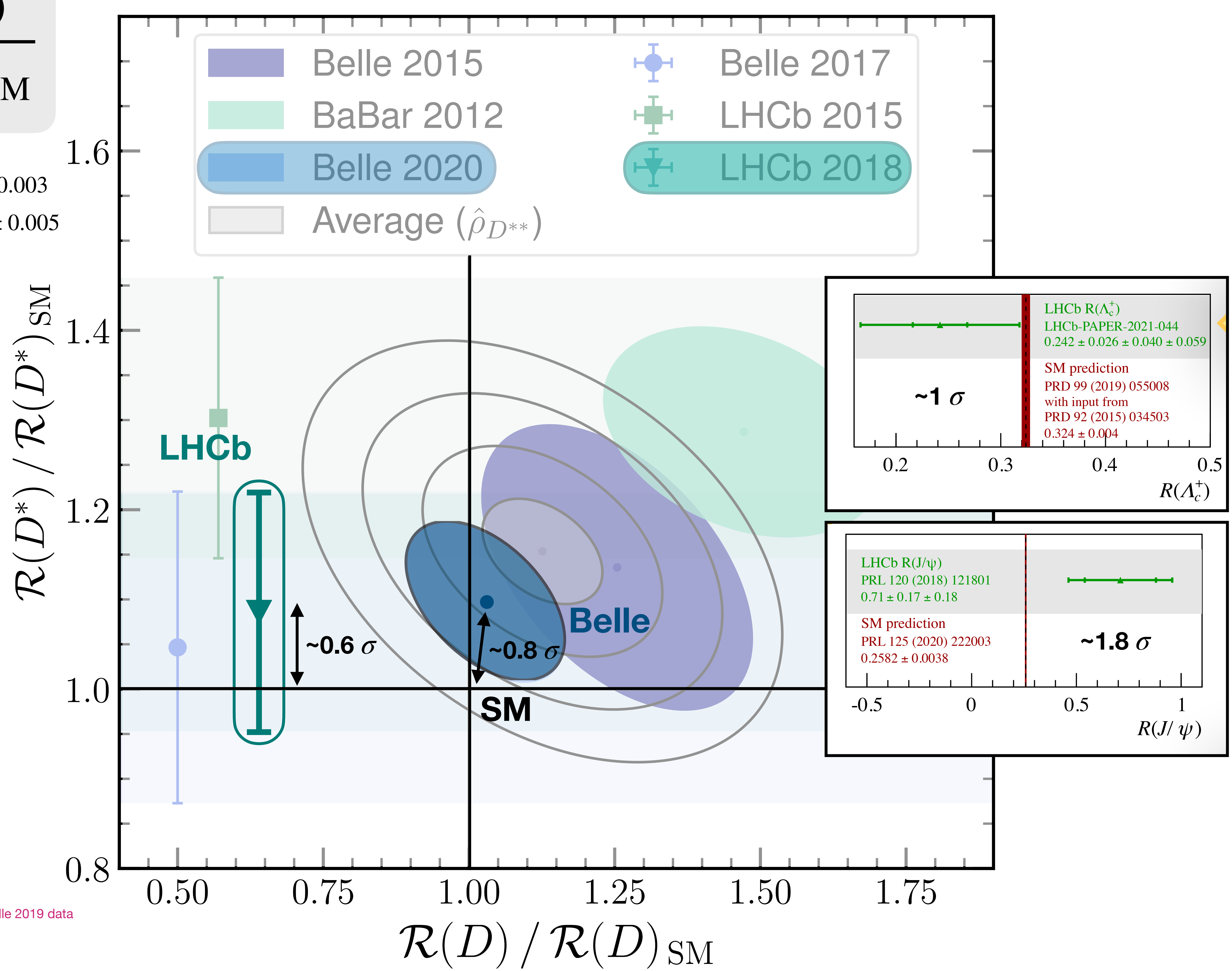


$$\frac{\mathscr{R}(D^{(*)})}{\mathscr{R}(D^{(*)})_{\mathrm{SM}}}$$

$\mathscr{R}(D)_{\mathrm{SM}} = 0.299 \pm 0.003$

$\mathscr{R}(D^*)_{\mathrm{SM}} = 0.258 \pm 0.005$

HFLAV arithmetic average of SM Calculations



More Recent SM Calculations:

BaBar B->D*
https://arxiv.org/abs/1903.10002
- R(D*)=0.253+-0.005

Gambino, Jung, Schacht using Belle 2019 data
https://arxiv.org/abs/1905.08209
- R(D*)=0.254 +0.007 -0.006

Bordone, Jung, van Dyk using Belle 2019 data
https://arxiv.org/abs/1908.09398
- RD=297+-0.003, RD*=0.250+-0.003

See also: https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html