

Λ spin correlation at Belle II

Cynthia Nuñez

Belle II Summer Workshop

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Research supported by:



U.S. DEPARTMENT OF
ENERGY

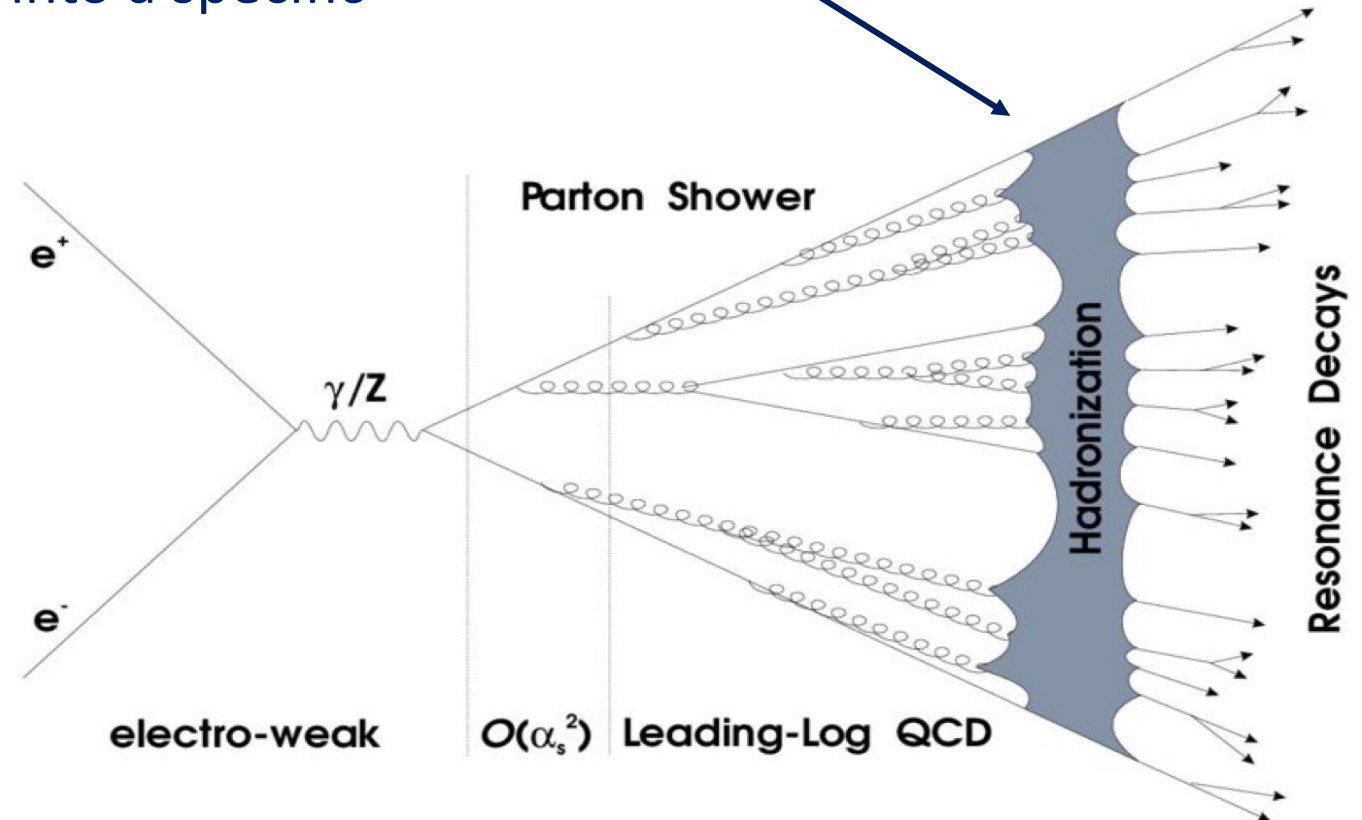
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Hadronization at Belle II

Image source: Buskulic et al. (1995)

- **Hadronization:** how particular hadrons are formed from scattered quarks and gluons (partons)
- **Fragmentation Functions (FF):** probability distribution of a parton fragmenting into a specific hadron

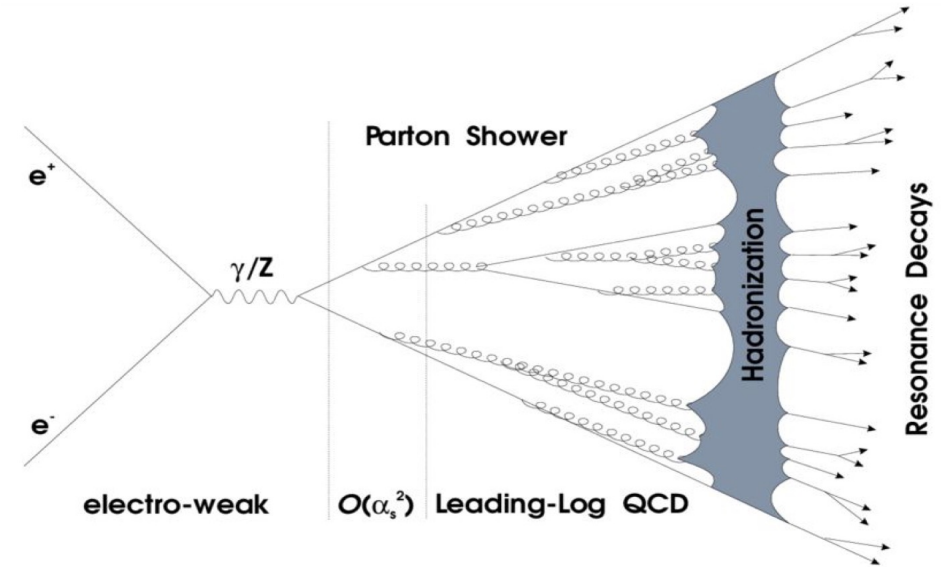
$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes FF$$



Hadronization at Belle II

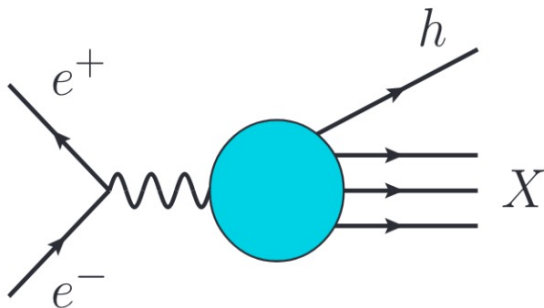
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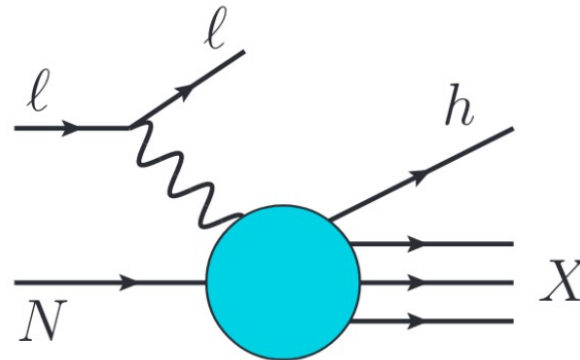
Important processes in studying hadron formation:

$$\sigma^{e^+e^- \rightarrow hX} = \hat{\sigma} \otimes \mathbf{FF}$$



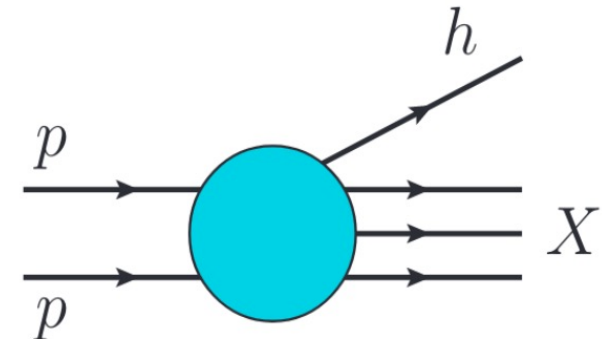
Cynthia Nuñez

$$\sigma^{lN \rightarrow lhX} = PDF \otimes \hat{\sigma} \otimes \mathbf{FF}$$



Duke University

$$\sigma^{pp \rightarrow hX} = PDF \otimes PDF \otimes \hat{\sigma} \otimes \mathbf{FF}$$



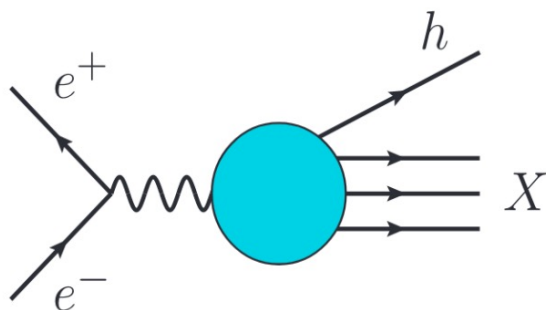
Hadronization at Belle II

Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)

- **Hadronization:** how particular hadrons are formed from scattered quarks and gluons (partons)
- **Fragmentation Functions (FF):** probability distribution of a parton fragmenting into a specific hadron
- **Transverse momentum dependent (TMD):** spin-momentum correlations

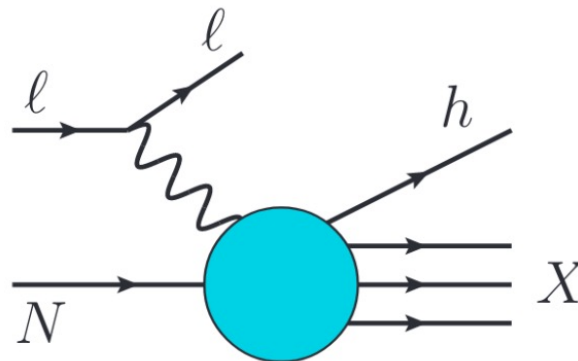
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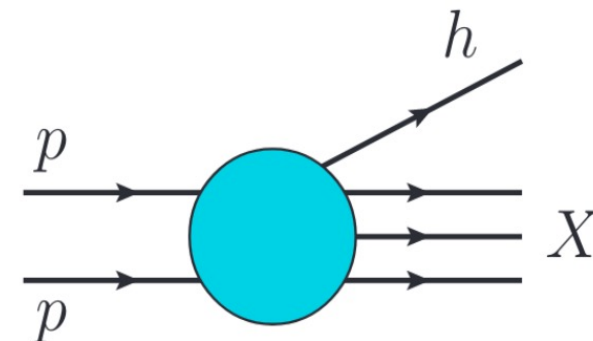
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Duke University

$$\sigma^{pp \rightarrow hX} = PDF \otimes PDF \otimes \hat{\sigma} \otimes \mathbf{FF}$$



Leading Quark TMDFFs

○ → Hadron Spin

● → Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{○} \cdot \text{○}$ Unpolarized		$H_1^\perp = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$ Collins
			$G_1 = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$ Helicity	$H_{1L}^\perp = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$
Polarized Hadrons	L			
	T	$D_{1T}^\perp = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$ Polarizing FF	$G_{1T}^\perp = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$	$H_1 = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$ Transversity $H_{1T}^\perp = \text{○} \cdot \text{○} - \text{○} \cdot \text{○}$

Hadronization at Belle

Belle measurements sensitive to:

- Collins FF
- Di-hadron FF
- Polarizing FF

...

Azimuthal asymmetries in inclusive production of hadron

Phys. Rev. Lett. 96, 232002 (2006) *Phys. Rev. D 78, 032011 (2008)* [*Phys.Rev.D 86, 039905 (2012)*]

Transverse polarization asymmetries of charged pion pairs

Phys. Rev. Lett. 107, 072004 (2011)

Inclusive cross sections for pairs of identified light charged hadrons and for single

Phys. Rev. D 92, 092007 (2015)

Invariant-mass and fractional-energy dependence of inclusive production of di-hadrons

Phys. Rev. D 96, 032005 (2017)

Production cross sections of hyperons and charmed baryons

Phys. Rev. D 97, 072005 (2018)

Transverse $\Lambda/\bar{\Lambda}$ Hyperon

Phys. Rev. Lett. 122, 042001 (2019)

Transverse momentum dependent production cross sections of charged pions, kaons and protons

Phys. Rev. D 99, 112006 (2019)

Inclusive cross sections of single and pairs of identified light charged hadrons

Phys. Rev. D 101, 092004 (2020)

Production cross section of light and charmed mesons

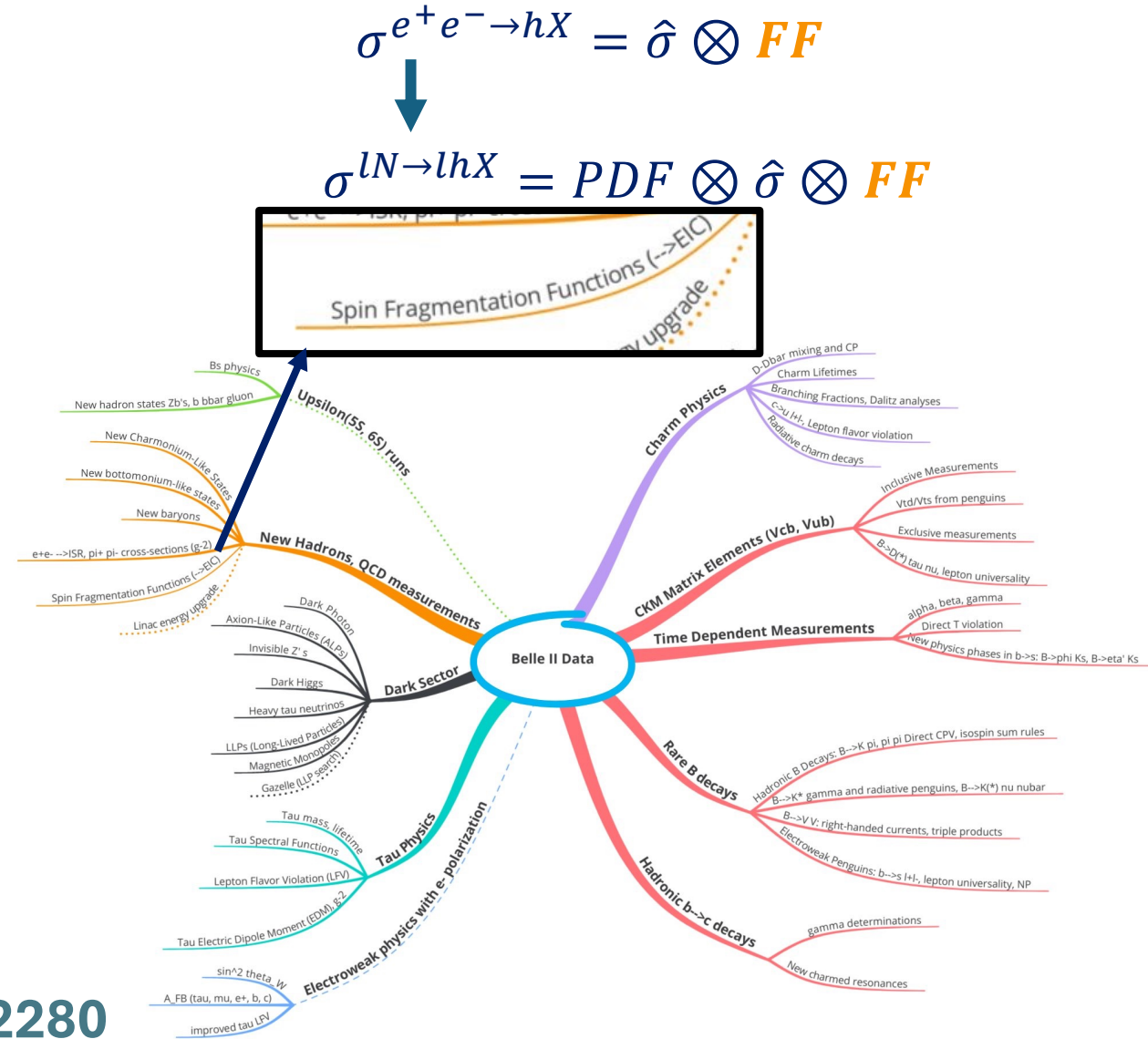
Belle preprint 2024-09, KEK Preprint 2024-30, submitted to PRD

Recent measurement

Hadronization at Belle II and for the EIC

- Belle II can offer high precision, comprehensive measurements essential for the Electron-Ion Collider (EIC)
 - Clean environment for detailed studies of hadronic final states
 - Multi-dimensional analyses of FFs, correlations, heavy flavor, and hadronization effects in jets
 - Essential for understanding transverse momentum of partons in measurements of PDFs and spin-structure of nucleon at the EIC
- + . . .

See Snowmass whitepaper [arXiv:2204.02280](https://arxiv.org/abs/2204.02280)

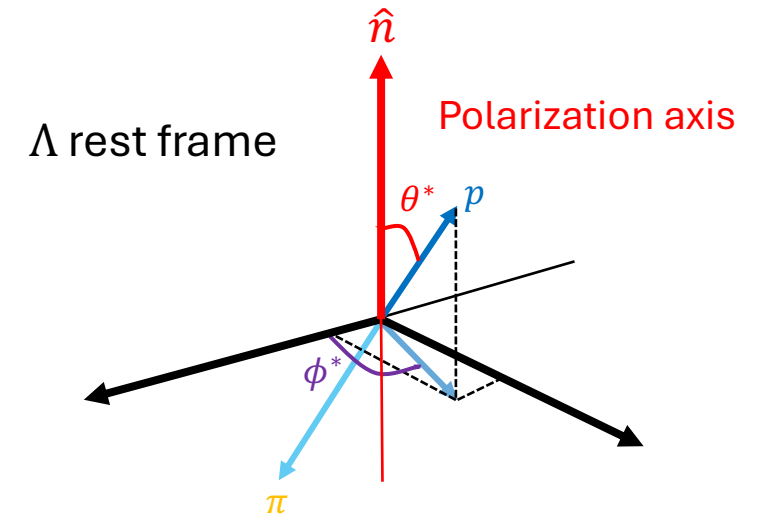


Λ polarization

- $\Lambda \rightarrow p\pi^-$ **self analyzing decay**
- The distribution of θ^* for polarized Λ :

$$\frac{1}{N} \frac{dN}{d\cos\theta^*} = (1 + \alpha_\Lambda P \cos\theta^*)$$

$$\alpha_\Lambda = 0.748 \pm 0.007 \text{ (PDG 2023)}$$



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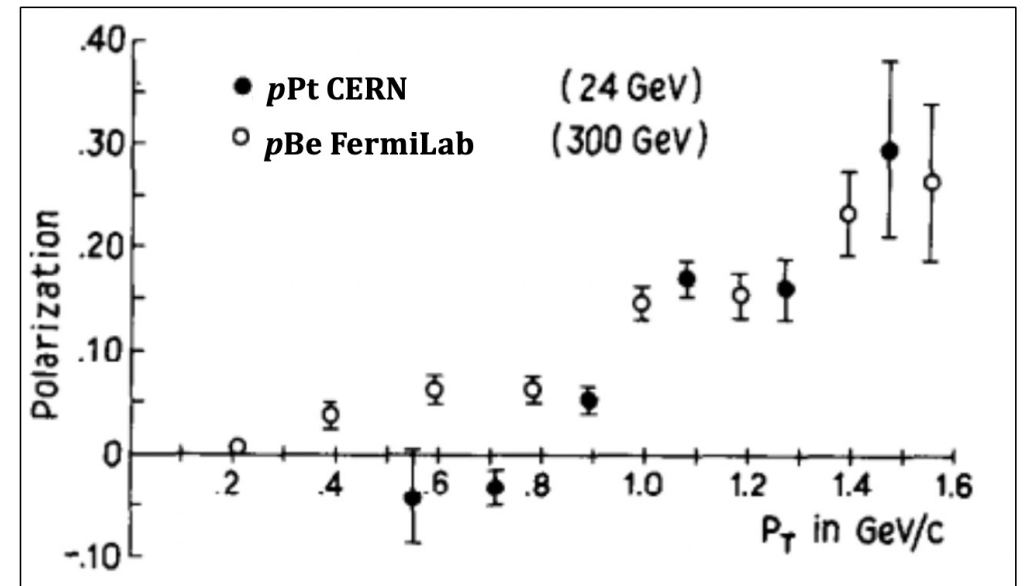
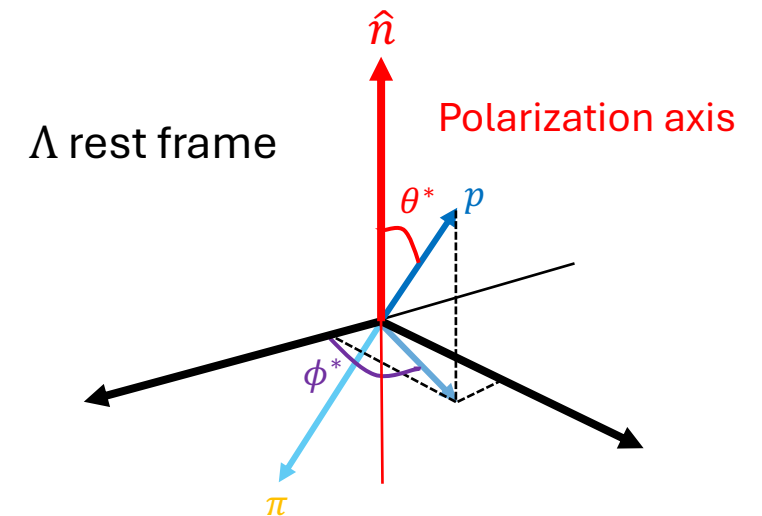
- **Spontaneous transverse Λ Polarization observed in 1976 in unpolarized p Be with polarization values up to 30%**

[PRL36, 1113 \(1976\)](#)

[PLB68, 480 \(1977\)](#)

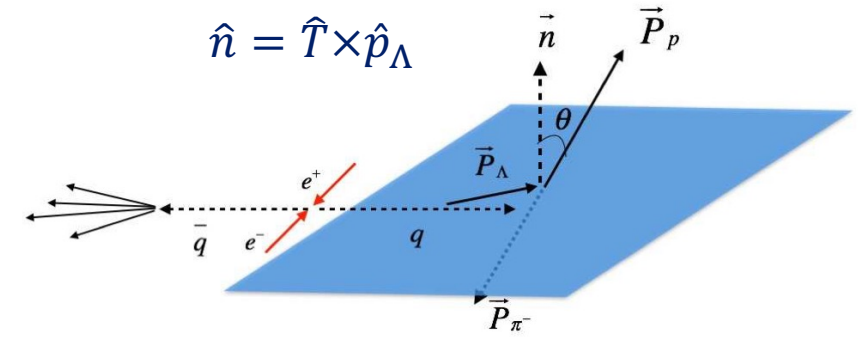
[PRL 41, 1689 \(1978\)](#)

[PRL 122, 042001 \(2019\)](#)



Measurement of Λ polarization Phys. Rev. Lett. 122, 042001 (2019)

- Observed non-zero polarization in $e^+e^- \rightarrow \Lambda X$ at Belle
→ hadronization effect

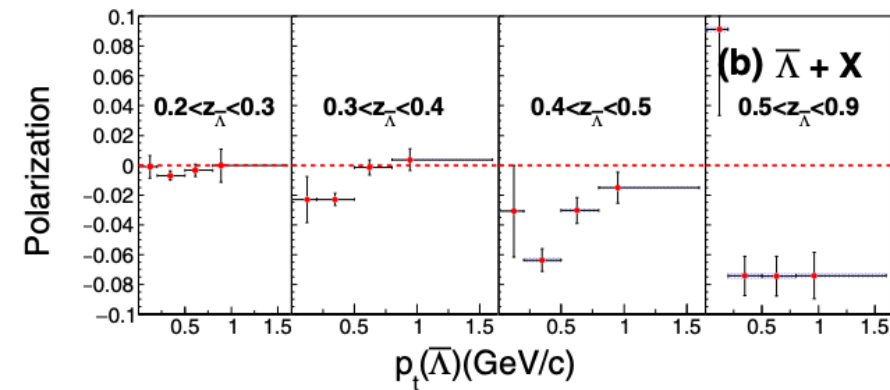
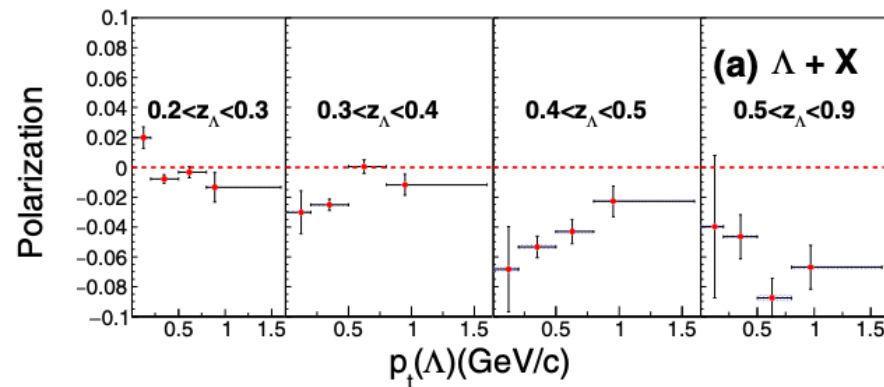
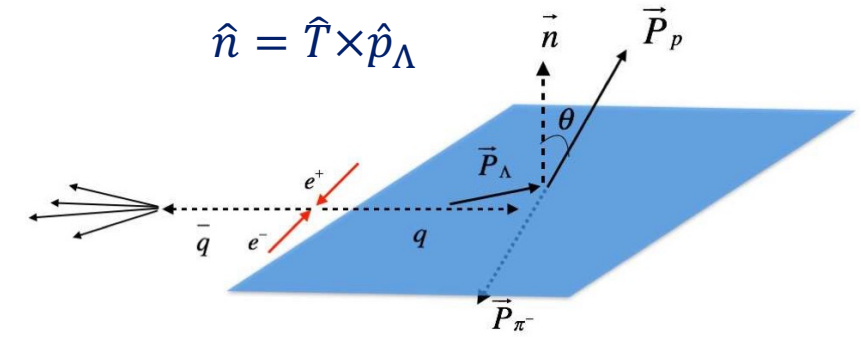


Thrust:

$$T = \max \frac{\sum_h |P_h^{CMS} \cdot \hat{T}|}{\sum_h |P_h^{CMS}|}$$

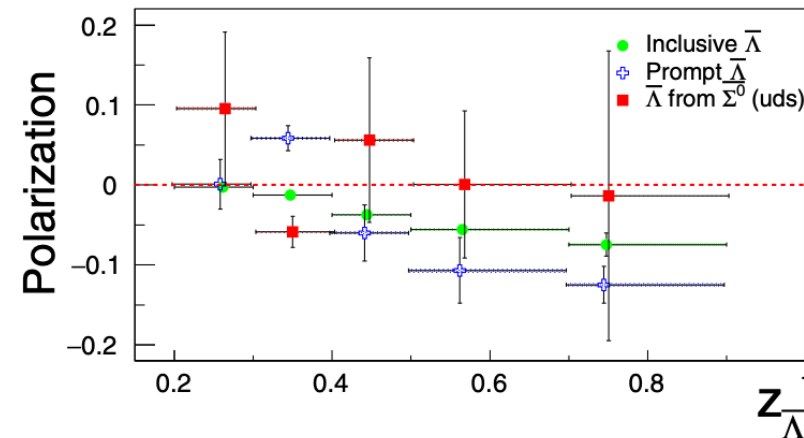
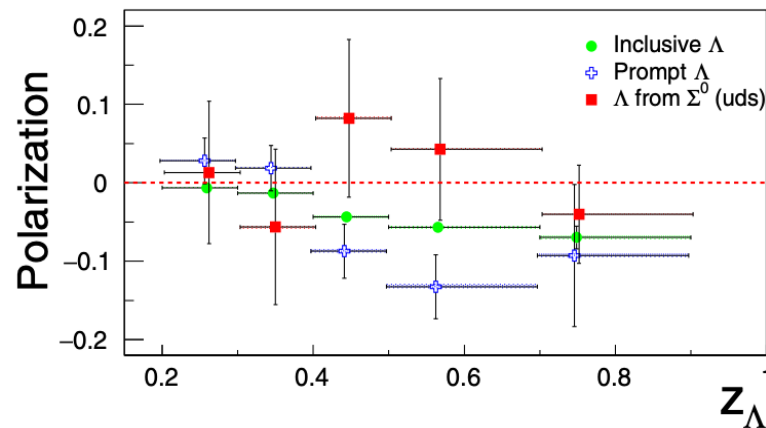
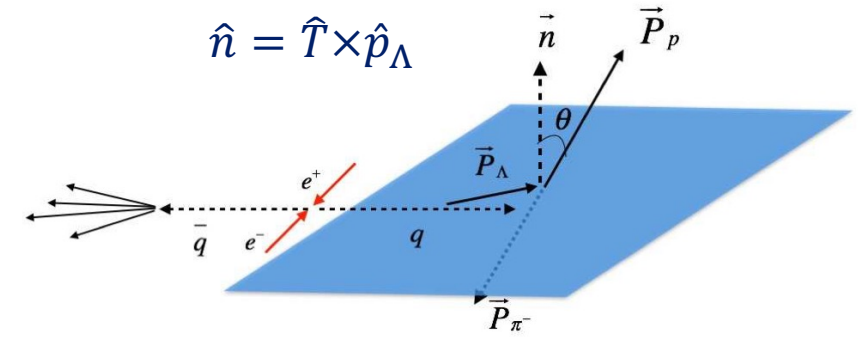
Measurement of Λ polarization $z_h = 2E_h/\sqrt{s}$ Phys. Rev. Lett. 122, 042001 (2019)

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- Nonzero transverse polarization observed for Λ and $\bar{\Lambda}$ as function of z_h and p_T



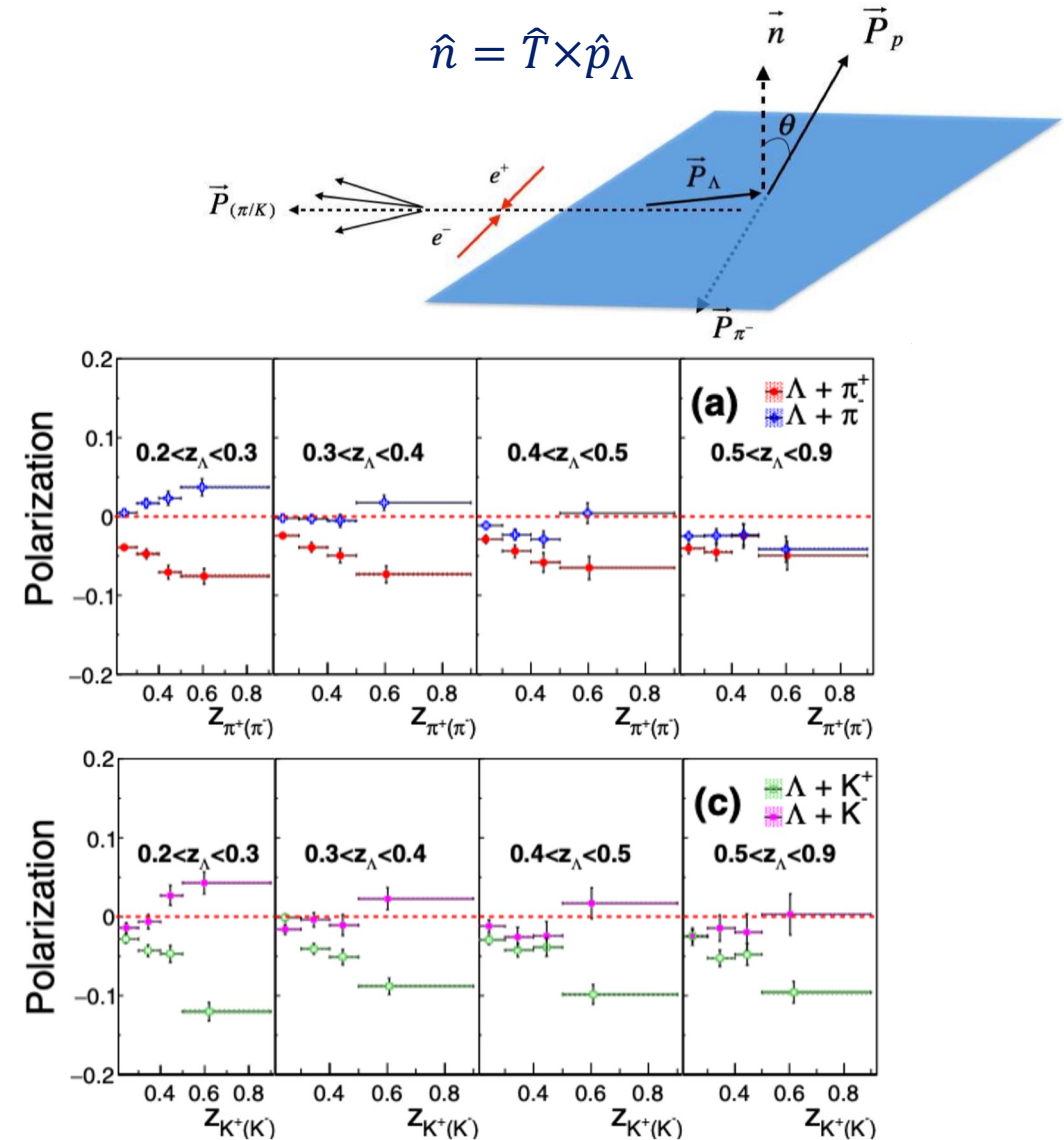
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- Investigate feed-down contributions from Σ^0 and charm decays



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- Polarization measurement also with respect to hadron in opposite hemisphere**



Transverse Λ polarization

- Measurement sensitive to **polarizing transverse-momentum dependent (TMD) fragmentation functions (FF) $D_{1T}^{\perp \Lambda/q}(z, k_{\perp}^2)$**

Leading Quark TMDFFs



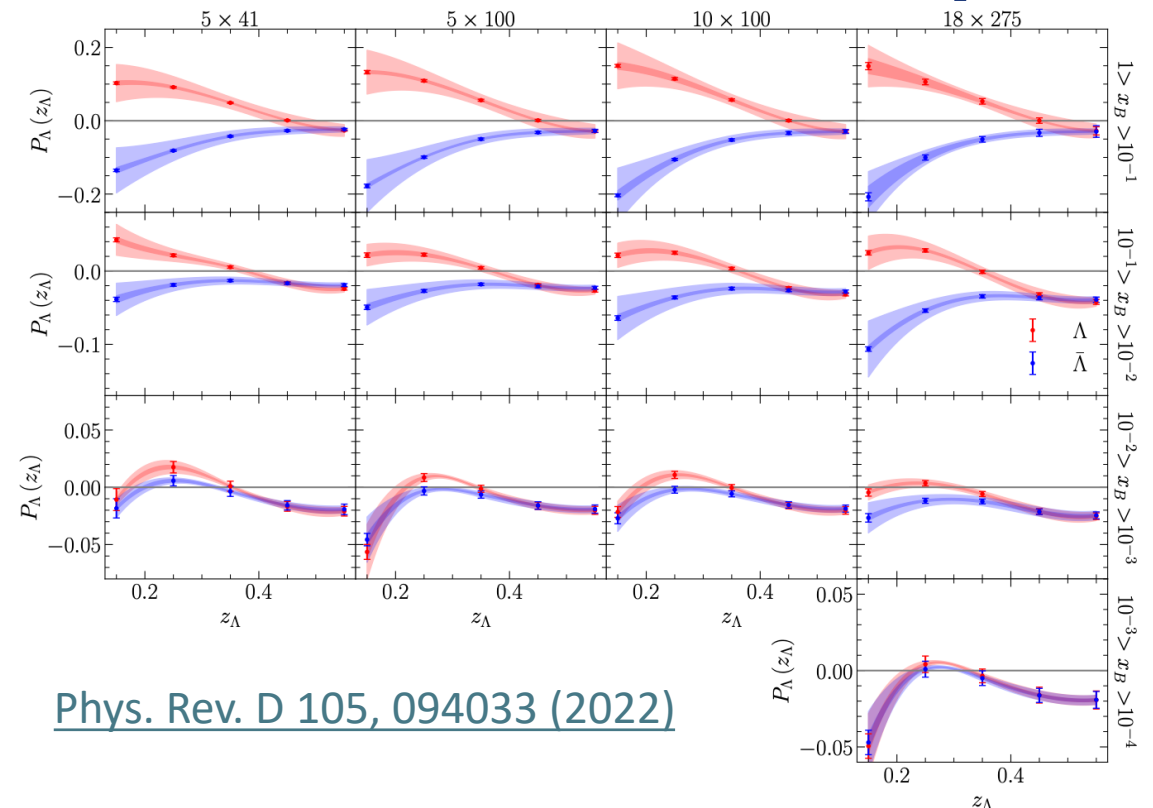
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Polarized Hadrons	Unpolarized (or Spin 0) Hadrons	$D_1 = \text{circle with red dot}$ Unpolarized		$H_1^{\perp} = \text{circle with red dot and arrow up} - \text{circle with red dot and arrow down}$ Collins
	L		$G_1 = \text{circle with red dot and arrow right} - \text{circle with red dot and arrow left}$ Helicity	$H_{1L}^{\perp} = \text{circle with red dot and arrow right and up} - \text{circle with red dot and arrow right and down}$
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Transverse Λ polarization

- Measurement sensitive to **polarizing transverse-momentum dependent (TMD) fragmentation functions (FF) $D_{1T}^{\perp \Lambda/q}(z, k_{\perp}^2)$**
- Belle measurement data accurate enough for **phenomenological studies to extract FF**
 Phys. Rev. D 102, 054001 (2020)
 Phys. Rev. D 102, 096007 (2020)
 Phys. Lett. B 809, 135756 (2020) + ...
- Used for **Λ polarization predictions in $ep \rightarrow \Lambda X$:**

Leading Quark TMDFFs  Hadron Spin  Quark Spin

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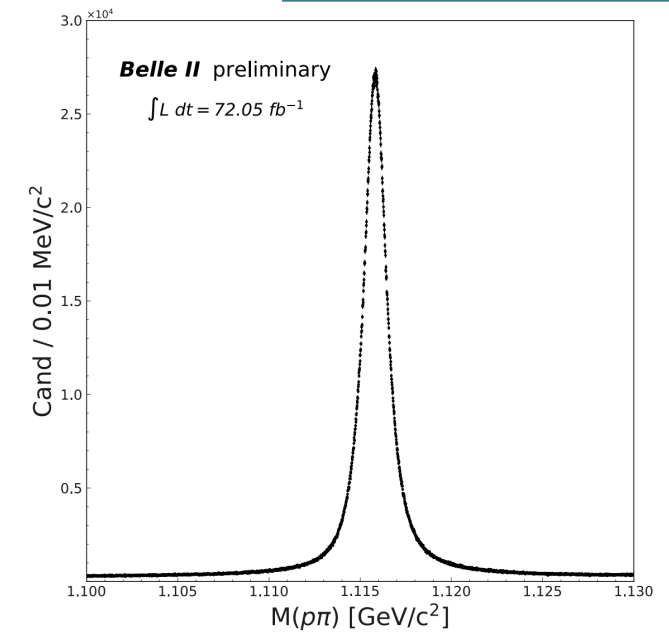


Phys. Rev. D 105, 094033 (2022)

Λ polarization at Belle II

- FFs give insight into spin structure of the Λ
- Λ as polarimeter to explore baryon/hyperon structure
- **Transverse Λ polarization at Belle II**
 - Reduce uncertainties from feed-down and the prompt Λ
 - Λ polarization with respect to the plane spanned by beam axis and Λ momentum

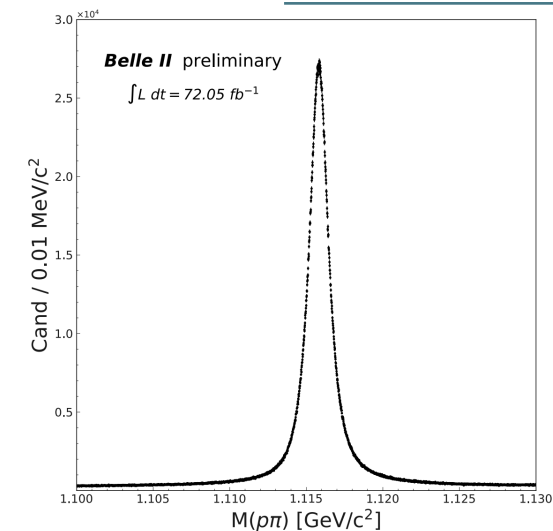
[BELLE2-NOTE-PL-2020-031](#)



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 - Reduce uncertainties from feed-down and the prompt Λ
 - Λ polarization with respect to the plane spanned by beam axis and Λ momentum
- $\Lambda\Lambda$ spin correlations
 - Entanglement as a probe to hadronization
Parton spin correlations and **entanglement** give rise to the Λ polarization ?
 - Entangled $s\bar{s} \rightarrow \Lambda\bar{\Lambda}$
 - Sensitivity to spin transfer FFs $G_{1T}^\perp$ and $H_{1T}^\perp$

BELLE2-NOTE-PL-2020-031



Leading Quark TMDFFs

Hadron Spin
Quark Spin

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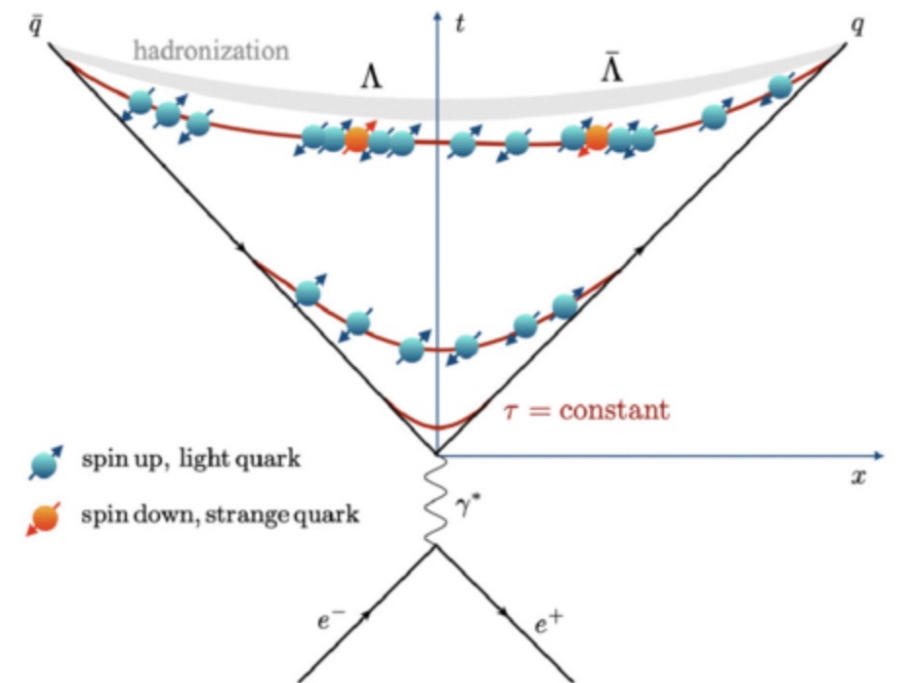
$\Lambda\Lambda$ spin correlation measurement

Entanglement via $\Lambda\bar{\Lambda}$ spin correlations

[Phys. Rev. D 106, L031501 \(2022\)](#)

[Phys. Rev. D 109, 116003 \(2024\)](#)

- **Entanglement as a probe to hadronization**
 - Experimentally track entangled $s\bar{s}$ quark into hadrons
- **Theoretical framework:**
 - Quantum simulations to validate entanglement observable
 - Real time dynamics modeled via 1+1D four-flavor Schwinger model with string-breaking dynamics



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 - Quantum simulations to validate entanglement observable
 - Real time dynamics modeled via 1+1D four-flavor Schwinger model with string-breaking dynamics
- **Experimental:**
 - Spin correlation extracted from the correlation of relative spin projections
$$N \propto 1 + \alpha^2 P_{\Lambda,\Lambda} \cos(n\theta_{ab})$$

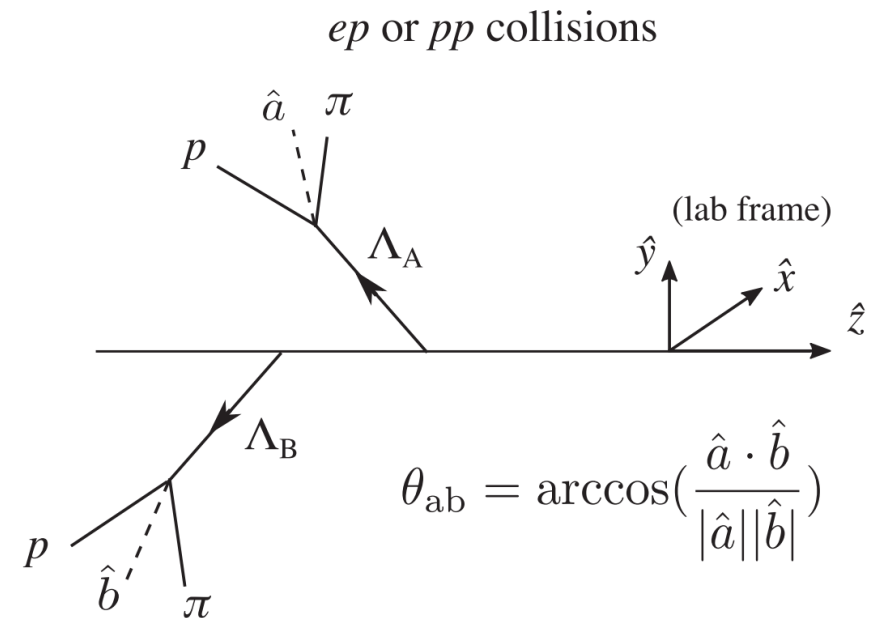


FIG. 3. Illustration of double Λ polarization; here $\hat{a}$ ($\hat{b}$) denotes the momentum direction of Λ_A (Λ_B) daughter particle in the Λ_A (Λ_B) rest frame.

Entanglement via $\Lambda\bar{\Lambda}$ spin correlations

- Past particle correlation measurements have been carried out at a wide variety of collisions
- Limited by low statistics for spin analyses
- Recently, Λ hyperon pair spin-spin correlation in pp collisions measurement at STAR

Λ particle correlation measurement examples:

[DELPHI Collaboration, Phys. Lett. B 318 249-262 \(1993\)](#)

[OPAL Collaboration, Phys. Lett. B 384 377-387 \(1996\)](#)

[ALEPH Collaboration, Phys. Lett. B. 475 395-406 \(1999\)](#)

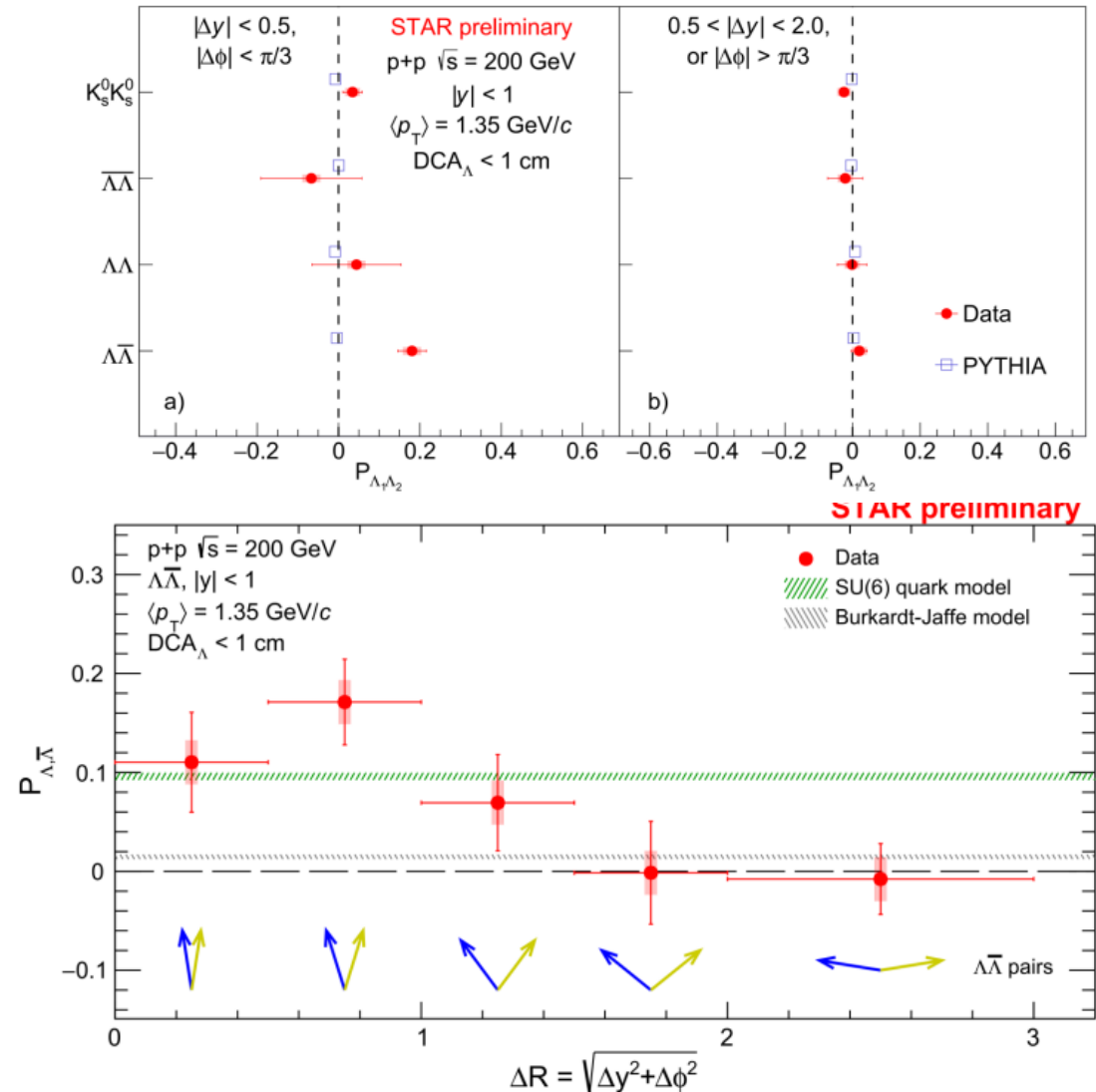
[NA49 Collaboration, Nucl. Phys. A 715 55-64 \(2002\)](#)

[SELEX Experiment, J. Phys.: Conf. Ser. 295 012089 \(2011\)](#)

[STAR Collaboration, Phys. Rev. Lett. 114 022301 \(2015\)](#)

...

Preliminary results from Quark Matter 2025, Jan Vanek



Belle II analysis

$\Lambda + \bar{\Lambda}$ reconstruction

- **The following selection requirements made in inclusive Λ skim**
 - Opposite charged proton and pion candidates combined to common vertex fit
 - Mass range [1.10, 1.13] GeV
 - Proton identification probability using information from all available detectors
$$\mathcal{L}_p / (\mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p + \mathcal{L}_d)$$
 - ProtonID(p) > 0.1
 - Additional selections including on:
 - the cosine angle between momentum and vertex vector (connecting IP and fitted vertex) of the Λ
 - flight distance of Λ
 - and proton and Λ momentum ratio
- **Events with at least one Λ is saved**

$\Lambda + \bar{\Lambda}$ reconstruction

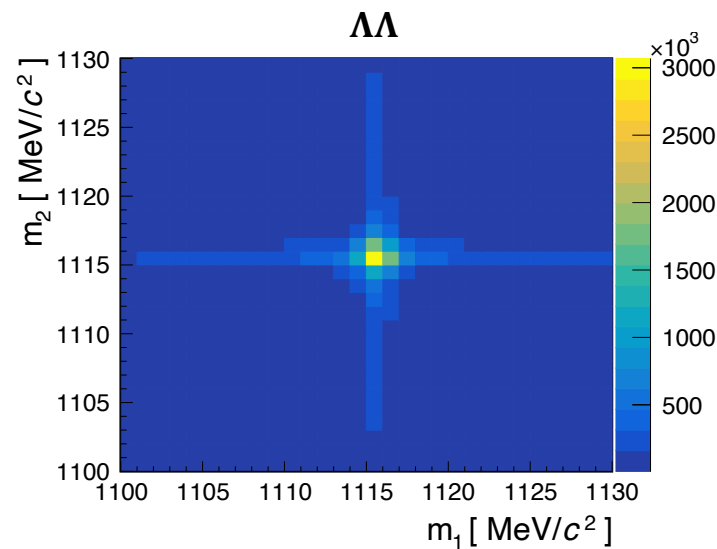
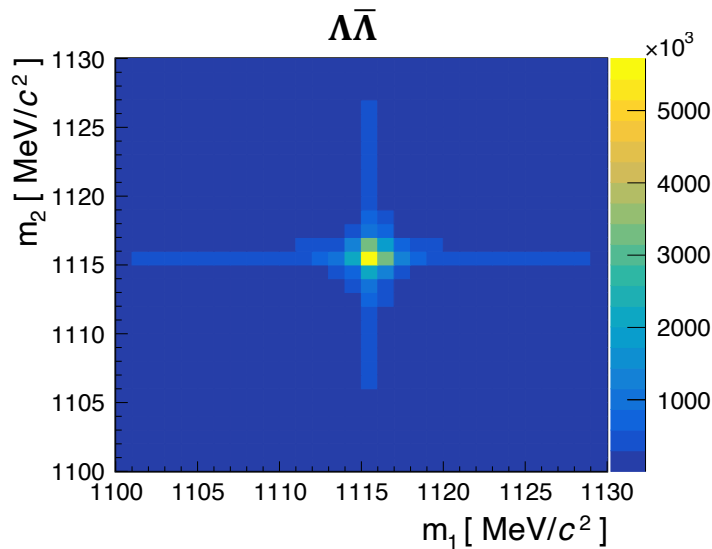
- Only consider pairs in opposite hemispheres.
- Pair sample $\rightarrow$ Events with at least one pair of Λ 's are saved ($\Lambda\bar{\Lambda}$, $\Lambda\Lambda$, or $\bar{\Lambda}\bar{\Lambda}$). All valid pair combinations are considered when multiple candidates present in the same event.

Off-resonance simulation (udsc)

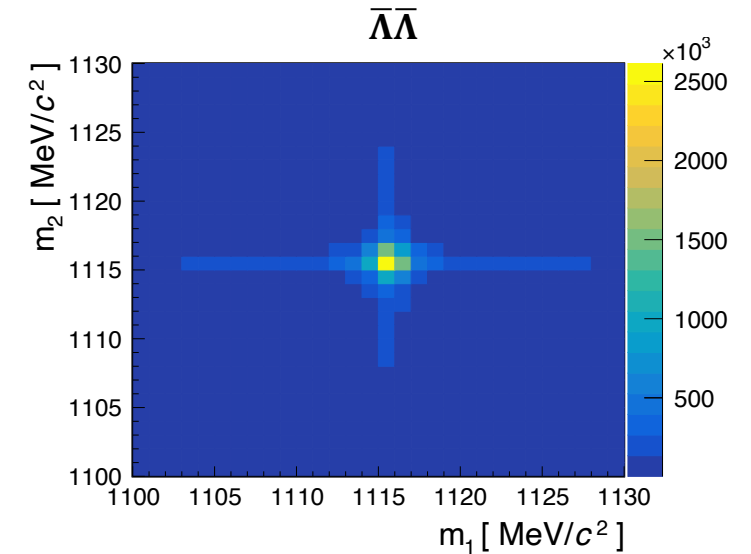
Type of pairs	$\Lambda\bar{\Lambda}$	$\Lambda\Lambda$	$\bar{\Lambda}\bar{\Lambda}$
MC (udsc)	88,523,131	53,483,445	36,281,866
Signal pairs			
MC (udsc)	31,804,757	16,446,138	15,063,164

Signal and background modeling

- Signal will be extracted from 2D invariant mass distributions of pairs
- 2D invariant mass distribution features:
 - Peak: two signal pairs
 - Ridges: signal paired with combinatorial background
 - Continuum: combinatorial background



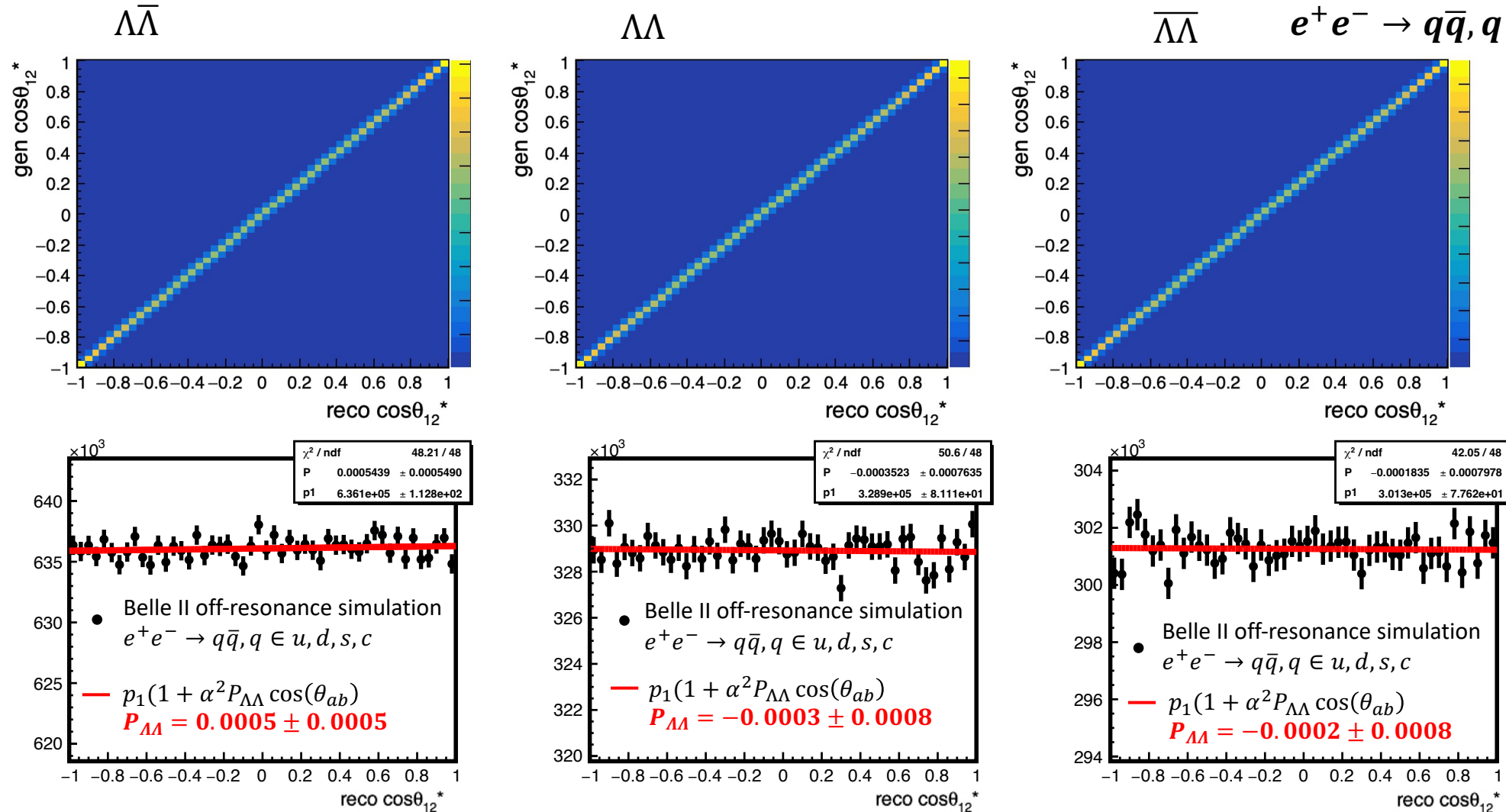
Belle II off-resonance simulation
 $e^+e^- \rightarrow q\bar{q}, q \in u, d, s, c$



Decay angle resolution and correlation fit

- Angular distribution and correlation in simulation (null result)

Belle II off-resonance simulation
 $e^+e^- \rightarrow q\bar{q}, q \in u, d, s, c$



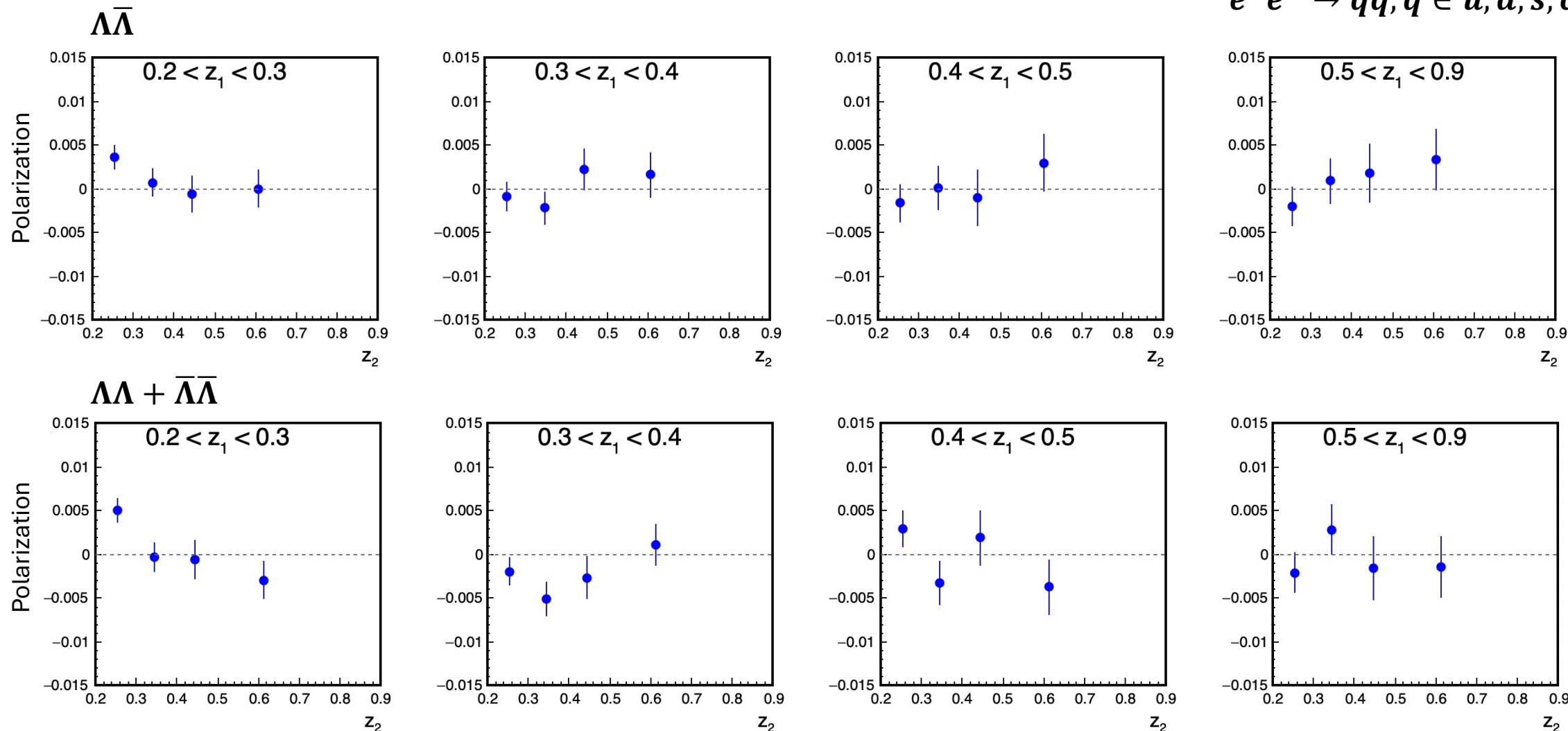
Polarization in simulation

$$z_h = 2E_h/\sqrt{s}$$

- Plan to measure polarization as a function of z_h for each Λ

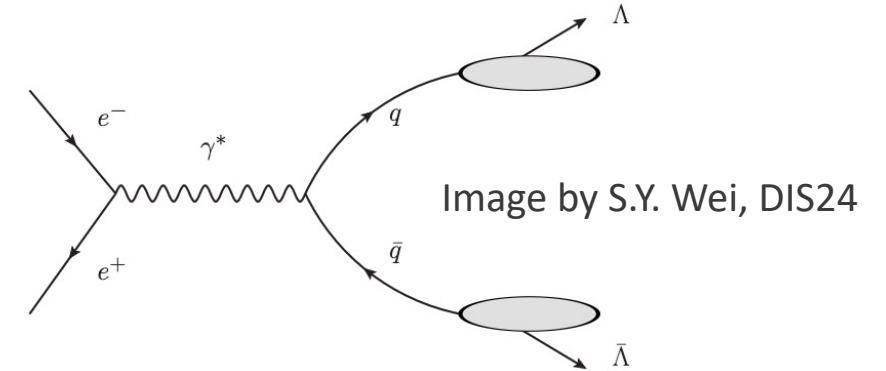
Belle II off-resonance simulation

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Longitudinal spin transfer via dihadron polarization

- Helicity correlation of two produced partons
- Alternative approach to traditional methods using polarized beams and targets



Leading Quark TMDFFs

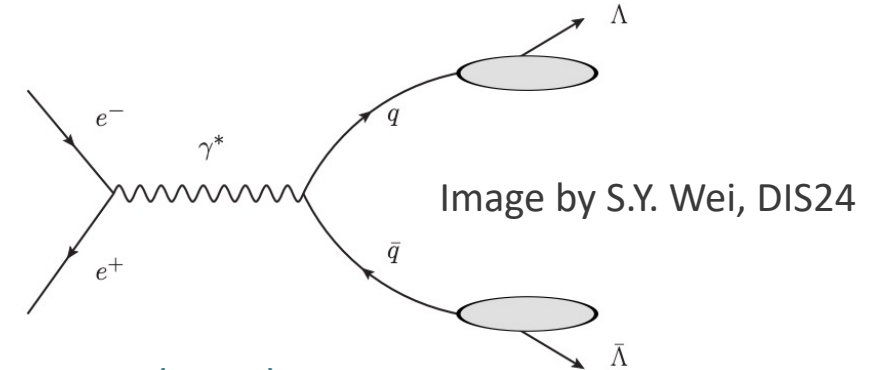


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Polarized Hadrons	L			
	T	$D_{1T}^\perp = \text{Polarizing FF}$	$G_{1T}^\perp$	$H_1 = \text{Transversity}$ $H_{1T}^\perp$

Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)

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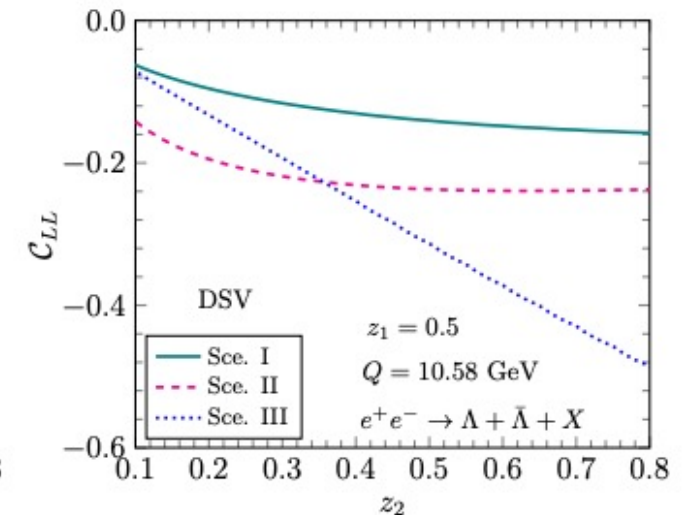
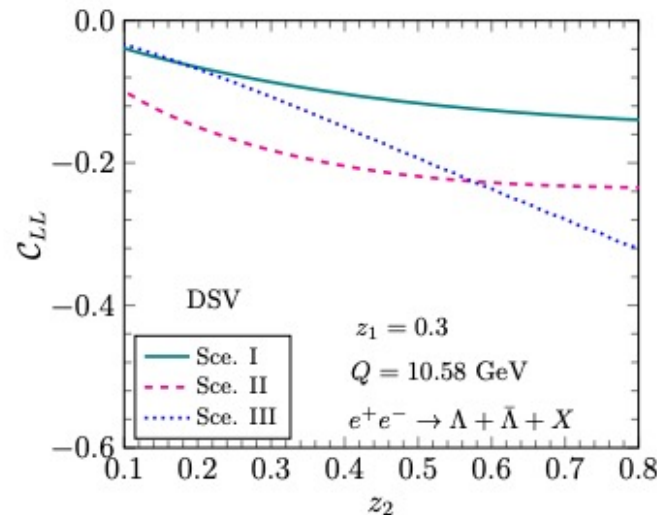
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Image from [arXiv:2304.03302v1](https://arxiv.org/abs/2304.03302v1)

[Phys.Lett.B 839, 137821 \(2023\)](#)



$$\frac{1}{N} \frac{dN}{d \cos \theta_1^* d \cos \theta_2^*} = \frac{1}{4} + P_L^\Lambda \frac{1}{4} \alpha \cos \theta_1^* + P_L^{\bar{\Lambda}} \frac{1}{4} \alpha \cos \theta_2^* + C_{LL} \frac{1}{4} \alpha^2 \cos \theta_1^* \cos \theta_2^*,$$

Longitudinal spin transfer via dihadron polarization

- **Experimental considerations:**
 - Contributions from longitudinal polarization not exactly zero (but expected to be small)
- **Other possible future measurement** [2410.20917](#)
 - Transverse spin correlation of two Λ hyperons sensitive to H_{1T}
 - Measurement of transverse spin correlation of two Λ hyperons normal to the hadron production plane (defined by thrust axis)

Leading Quark TMDFFs

 Hadron Spin
  Quark Spin

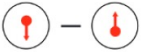
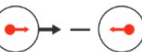
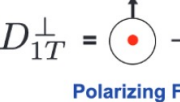
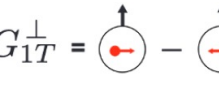
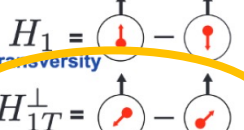
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$ 		$H_1^\perp = \text{Collins}$ 
			$G_1 = \text{Helicity}$ 	$H_{1L}^\perp = \text{Transversity}$ 
Polarized Hadrons	L			
	T	$D_{1T}^\perp = \text{Polarizing FF}$ 	$G_{1T}^\perp = \text{Transversity}$ 	$H_1 = \text{Transversity}$  $H_{1T}^\perp = \text{Transversity}$ 

Image from [arXiv:2304.03302v1](#)

Summary

Summary

Belle II @ Duke

- Belle II plays an important role in understanding hadronization dynamics ([arXiv:2204.02280](https://arxiv.org/abs/2204.02280))
- Λ spin-correlation provides insight on the hadronization process
 - Entanglement as a probe for hadronization process
 - Probe the longitudinal and transverse spin transfer in unpolarized e^+e^-
- Several current ongoing hadronization analyses underway
 - Ongoing analysis studying Λ spin-correlations
 - Ongoing analysis sensitive to the di-hadron fragmentation functions (Katherine Parham)
 - Ongoing analysis sensitive to transverse momentum dependent jet functions (Simon Schneider)
- Future QCD studies with polarized electron beams at SuperKEKB
Chiral Belle Project: [arXiv:2205.12847v3](https://arxiv.org/abs/2205.12847v3)



Anselm Vossen



Frank Meier



Katherine Parham



Simon Schneider



Cynthia Nuñez



Leonel Lin

Back up

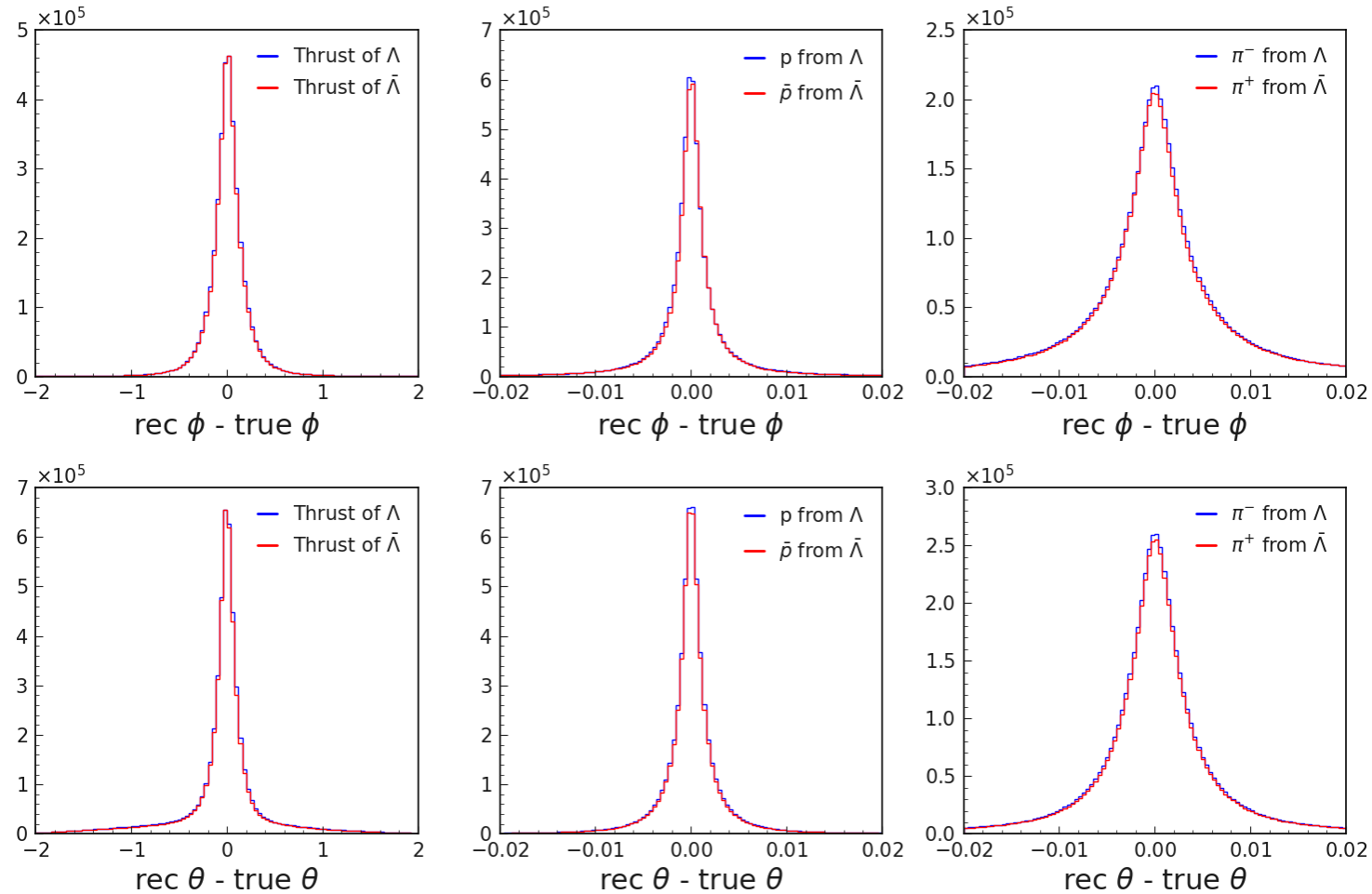
$\Lambda + \bar{\Lambda}$ reconstruction

- InclusiveLambda skim:
- Events with at least one Λ is saved

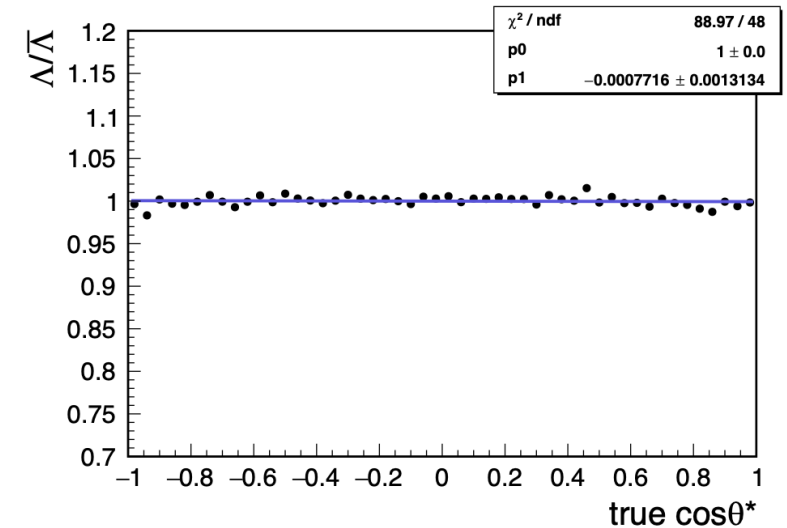
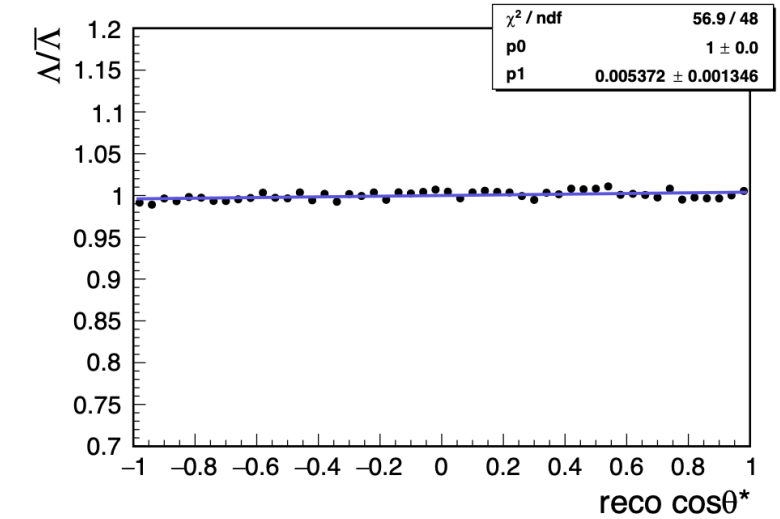
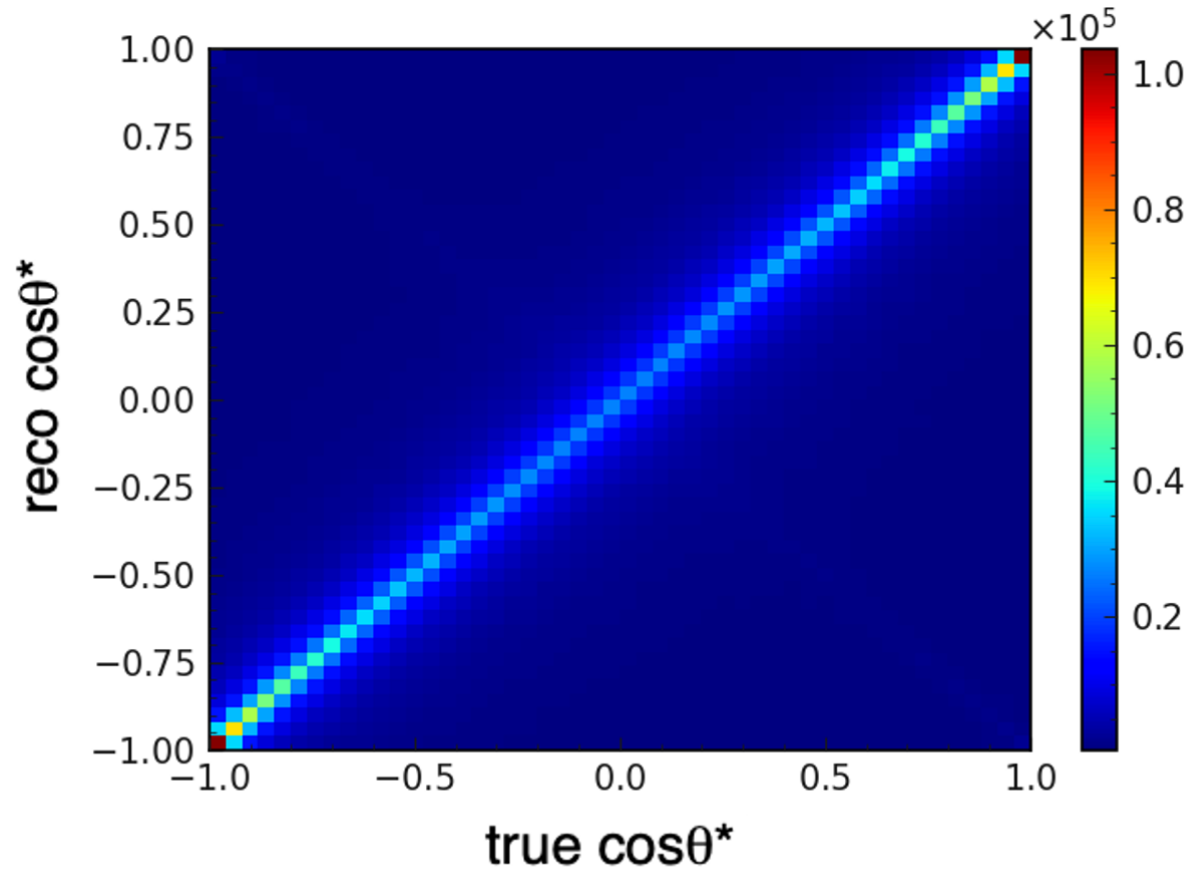
Merged Λ with vertex fit performed	stdLambda Lambda0:merged -> p+:all pi+:all Mass range [1.10, 1.13] GeV Vertex fit with TreeFit
Proton identification probability $\mathcal{L}_p / (\mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p + \mathcal{L}_d)$ Using information from all available detectors	ProtonID (p) > 0.1
Angle between momentum and vertex vector (connecting IP and fitted vertex) of the Λ	cosAngleBetweenMomentumAndVertexVector > 0.75
Flight distance cut on the Λ	flightDistance/flightDistanceErr > 0.
Momentum ratio cut	$0.5 < p_{\text{proton}}/p_{\text{Lambda}} < 1.25 \text{ GeV}/c$

Polarization in simulation

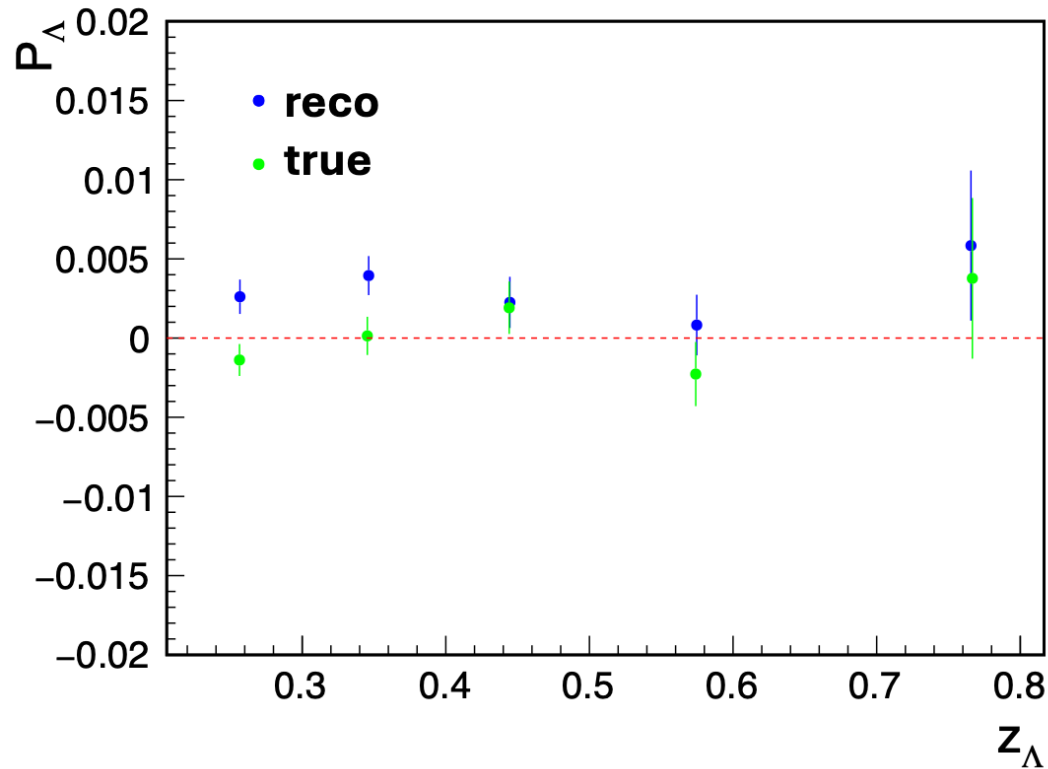
- Simulation has $P_\Lambda = P_{\bar{\Lambda}} = 0$ therefore extracting the polarization from the ratio $\Lambda/\bar{\Lambda}$ should be zero
- No shift observed for Λ and $\bar{\Lambda}$ distributions using updated momentums for proton and pion after vertex fit



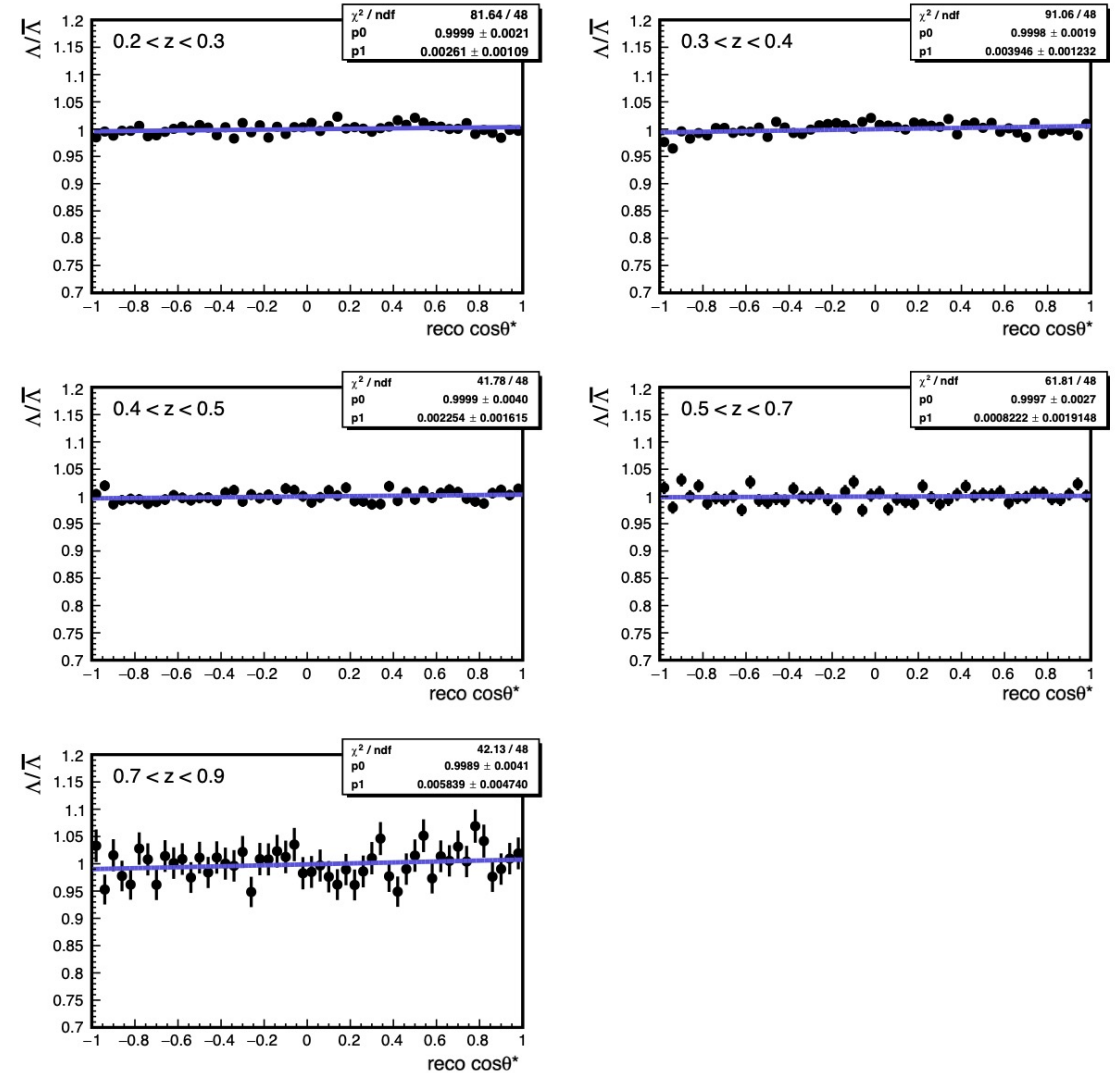
Transverse angular distribution



Transverse polarization



$$z_\Lambda = 2E_\Lambda/\sqrt{s}$$



Decay angle

For a single Λ

$$\frac{dw(n)}{d\Omega_n} = \frac{1}{4\pi} (1 + \alpha_\Lambda \mathbf{P}_\Lambda \cdot \mathbf{n})$$

For double angular distribution

$$\frac{d^2w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}}$$

$$= \frac{1}{16\pi^2} \left(1 + \alpha_{\Lambda_1} \mathbf{P}_{\Lambda_1} \cdot \mathbf{n}_1 + \alpha_{\Lambda_2} \mathbf{P}_{\Lambda_2} \cdot \mathbf{n}_2 + \alpha_{\Lambda_1} \alpha_{\Lambda_2} \sum_{i=1}^3 \sum_{j=1}^3 T_{ij} n_{1i} n_{2j} \right)$$

Integrate over all angles except angle between $\mathbf{n}_1$ and $\mathbf{n}_2$

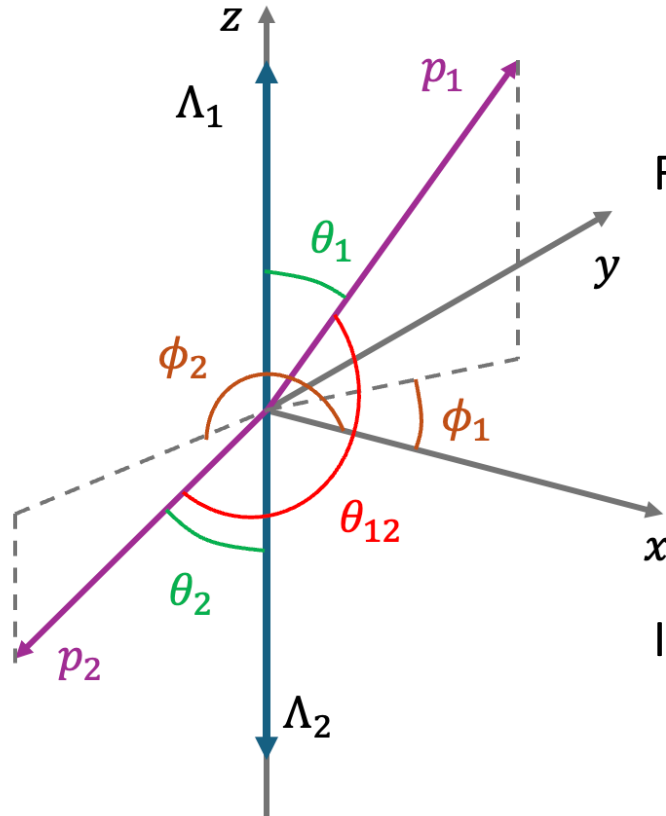
$$\frac{dw(\cos \theta_{12})}{d \cos \theta_{12}} = \frac{1}{2} (1 + \alpha_{\Lambda_1} \alpha_{\Lambda_2} P_{\Lambda_1 \Lambda_2} \cos \theta_{12})$$

Two angle relation:

$$\cos \theta_{12} = \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) - \cos \theta_1 \cos \theta_2$$

$$\langle \phi_1 - \phi_2 \rangle = 0$$

$$\langle \cos \theta_{12} \rangle = \langle \cos \theta_1 \cos \theta_2 \rangle$$



Eur. Phys. J. C. (2012) 72:1877

Nucl. Phys. A820 (2009) 311c–314c

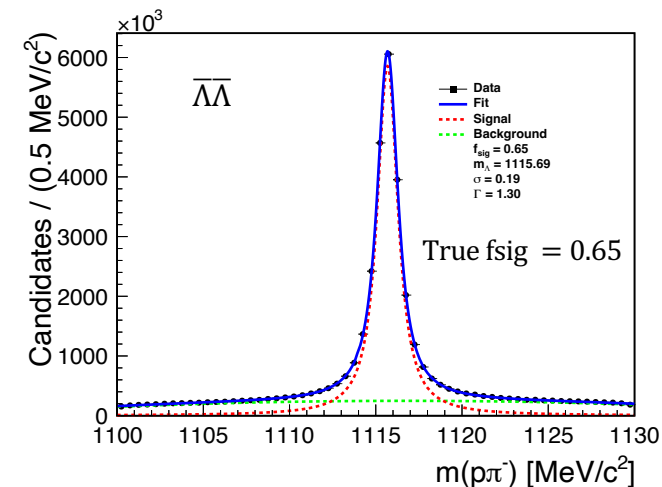
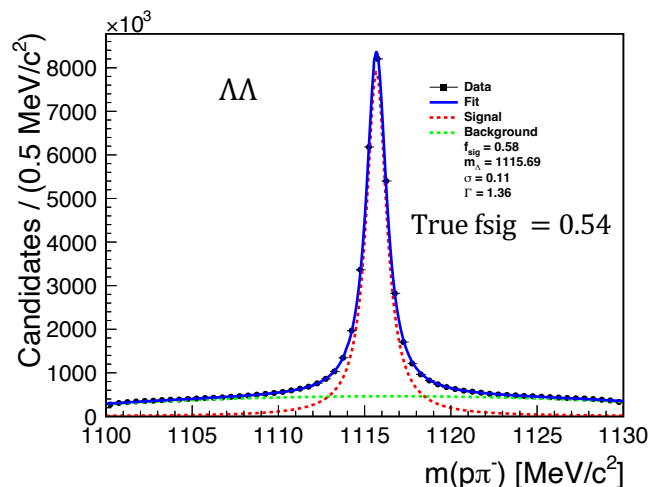
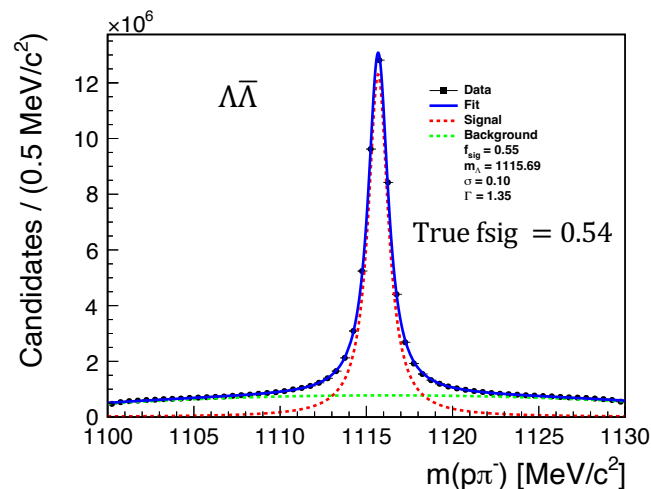
Signal and background modeling

- Fitted with Voigtian and 2nd order Bernstein background

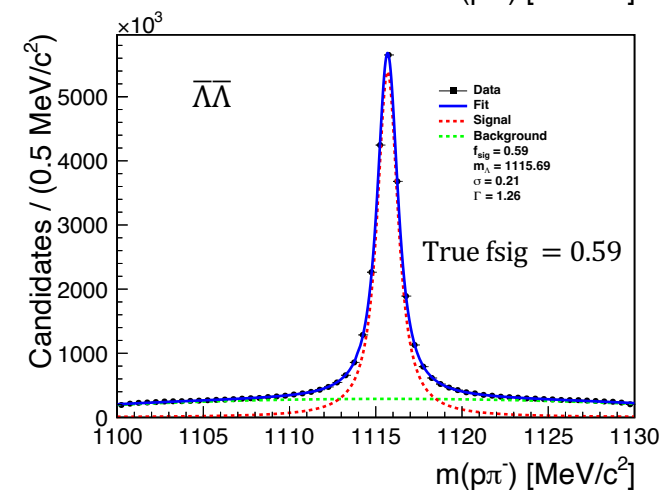
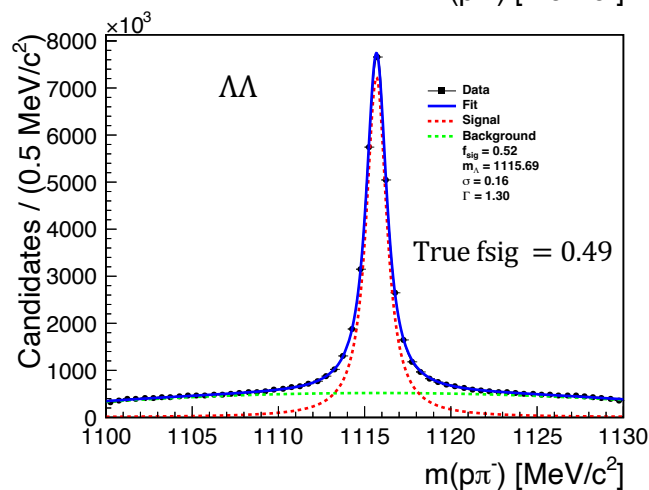
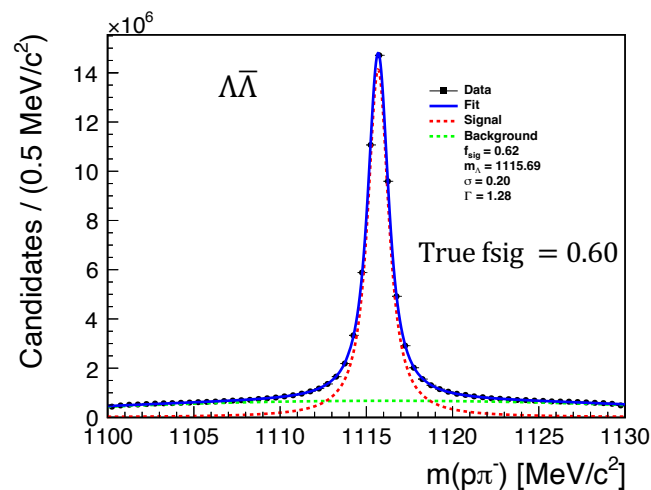
Belle II off-resonance simulation

$$e^+e^- \rightarrow q\bar{q}, q \in u, d, s, c$$

Λ_1



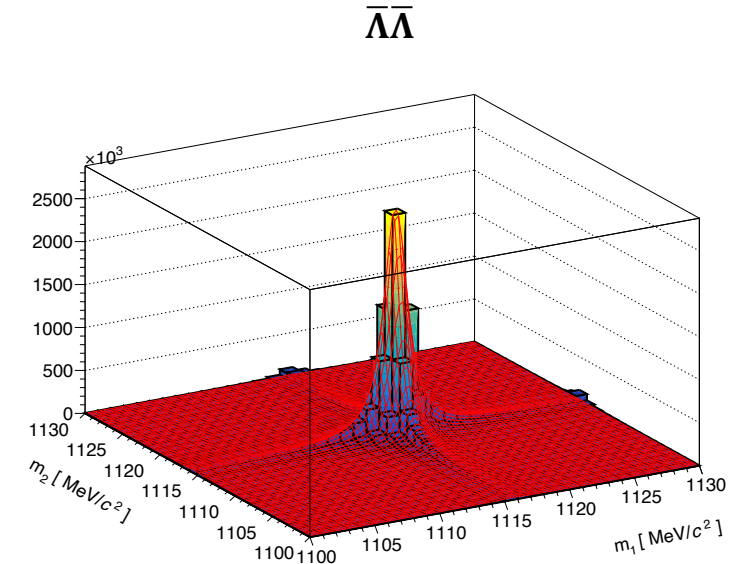
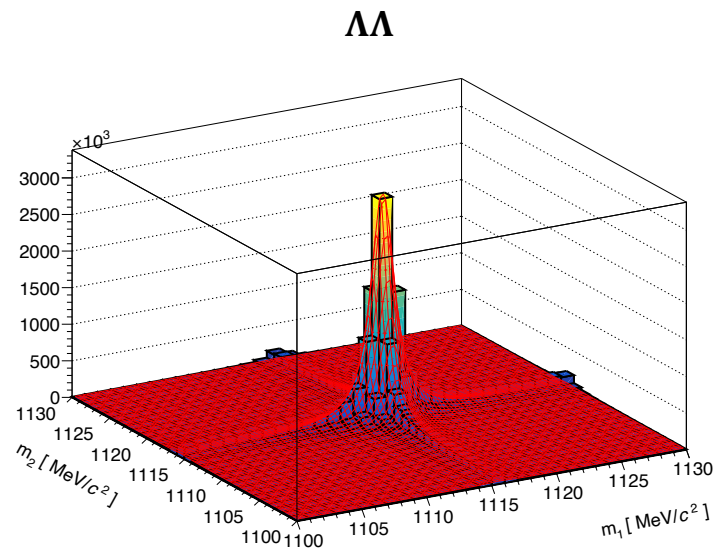
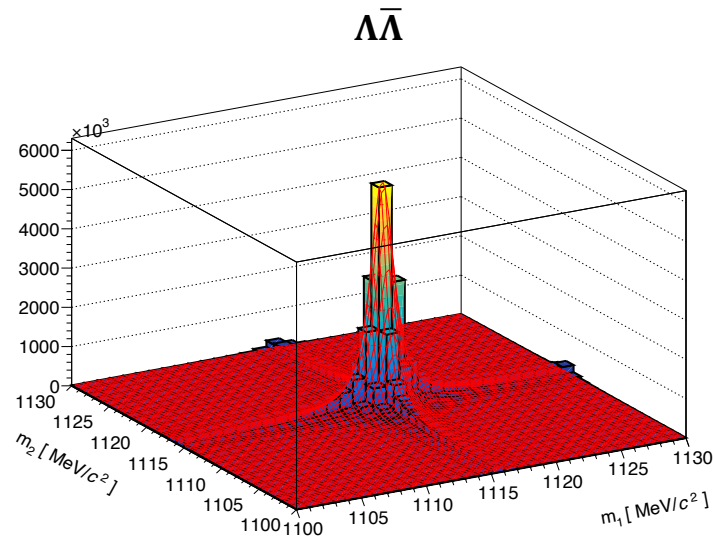
Λ_2



Signal and background modeling

- Signal extracted from 2D invariant mass distributions of pairs
- 2D PDFs is built by multiplying the two 1D PDFs for extended maximum likelihood fit:

$$M(m_1, m_2) = N_{sig} M_{sig}(m_1) M_{sig}(m_2) + N_{bkg} M_{bkg}(m_1) M_{bkg}(m_2)$$



Belle II off-resonance simulation
 $e^+e^- \rightarrow q\bar{q}, q \in u, d, s, c$

In simulation, testing mass fit with 2D Gaussian signal and 2D polynomial background