Rare $D, B \to K$ decay form factors from lattice QCD

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Belle II Physics Week, KEK

Belle II

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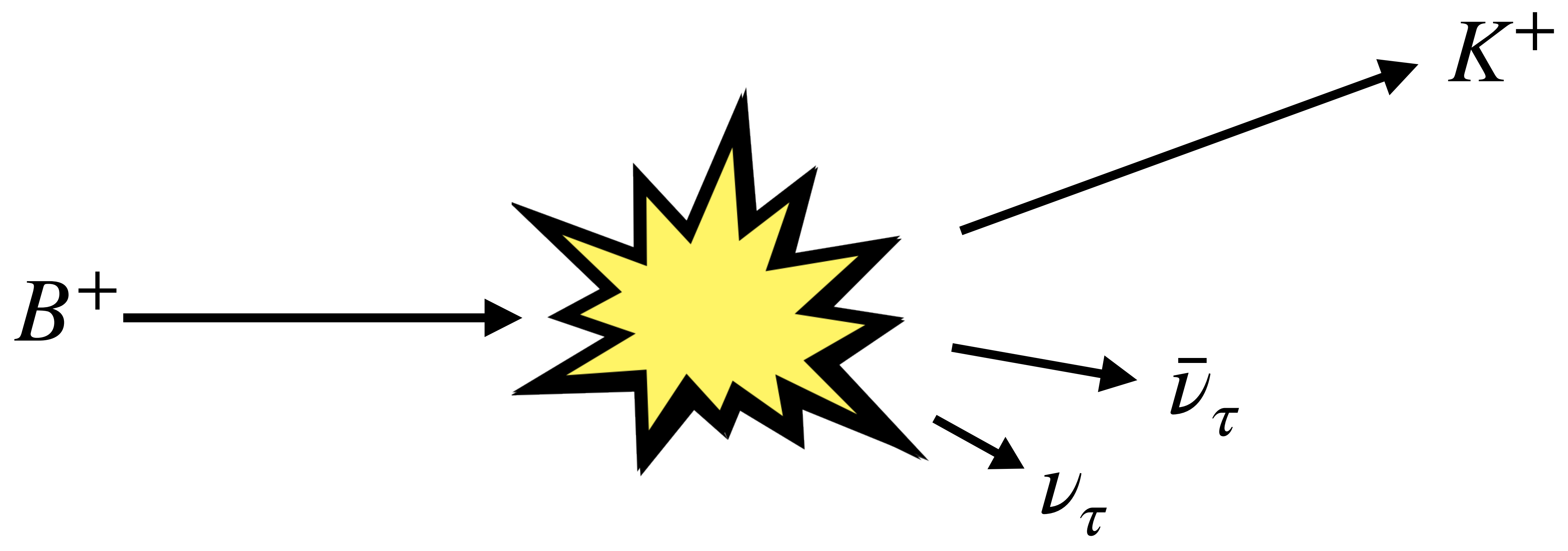
- Motivation
- Role of form factors
- Calculating form factors with lattice QCD
- Conclusion and outlook

Parrott, Bouchard, and Davies, PRD 107 (2023) 014510

Parrott, Bouchard, and Davies, PRD 107 (2023) 119903

Motivation $B \to K\nu\bar{\nu}$: SM contribution small



- rare in the Standard Model (SM) - new physics might play a noticeable role
- in comparison of SM prediction with experiment, precision is key
    - from theory perspective, $\nu\bar{\nu}$ is advantageous as $\ell^+\ell^-$ is contaminated by QCD resonances with non-local interactions

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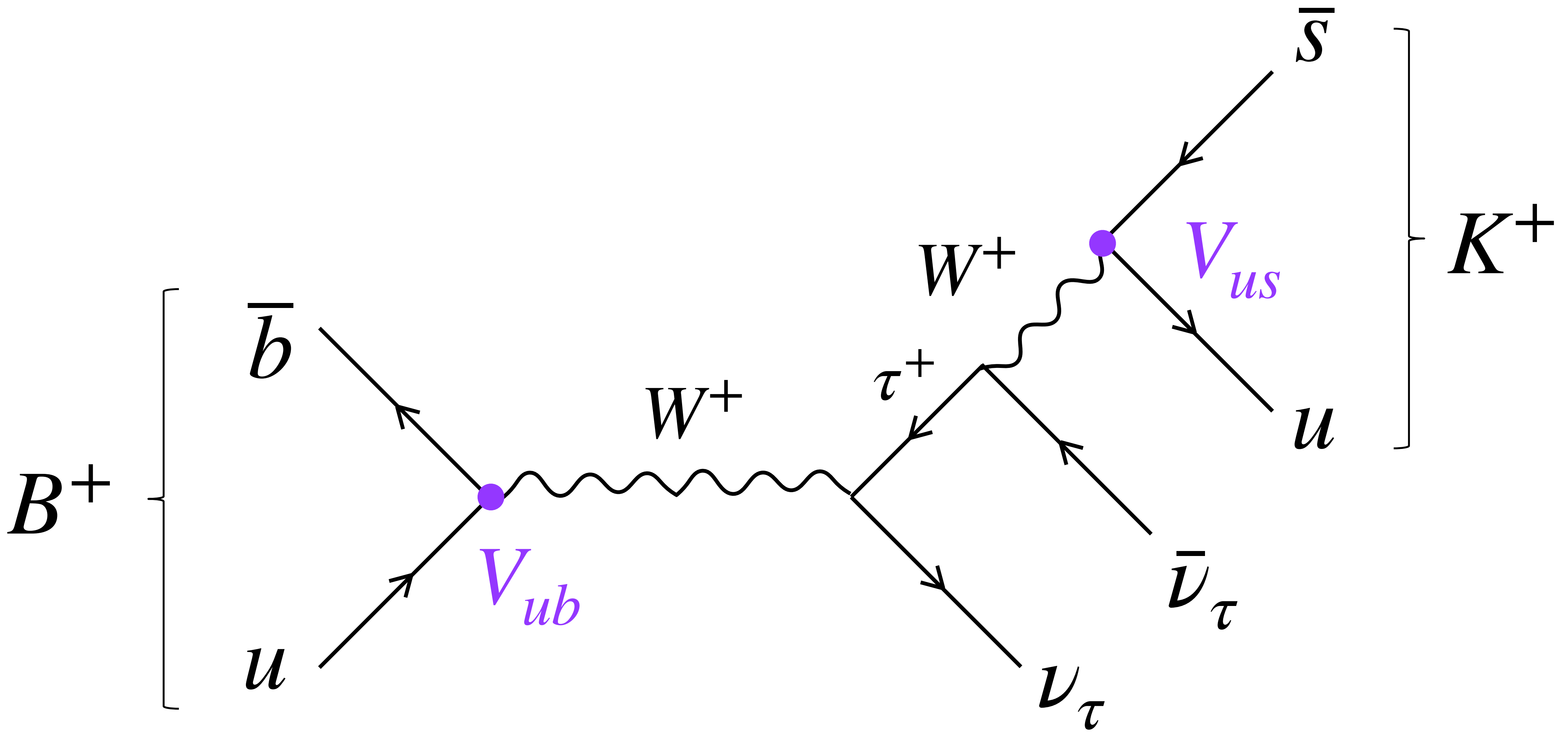

long distance

- weak suppression, branching fraction $\propto G_F^2 \sim (10^{-5}\,\mathrm{GeV}^{-2})^2$
- CKM suppressed, branching fraction $\propto |V_{ub}V_{us}|^2 \sim 7 \times 10^{-7}$

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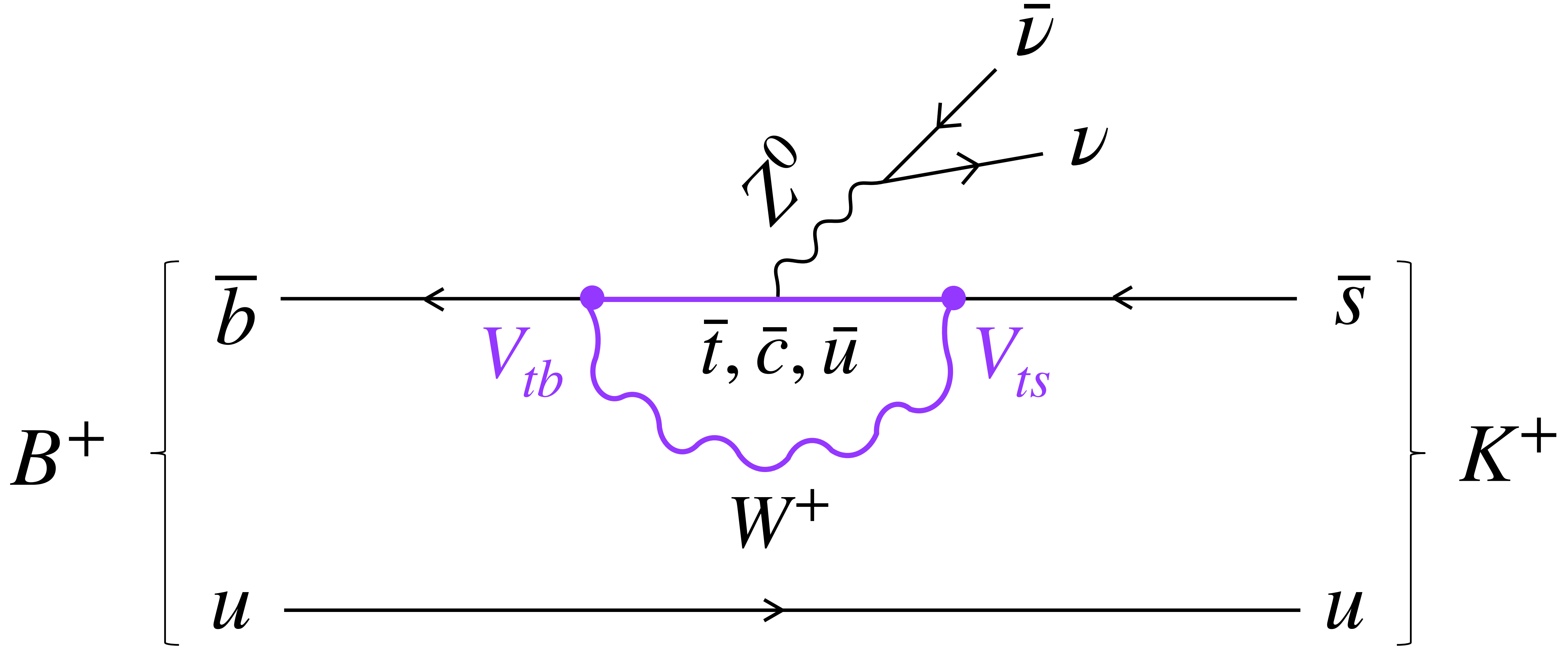

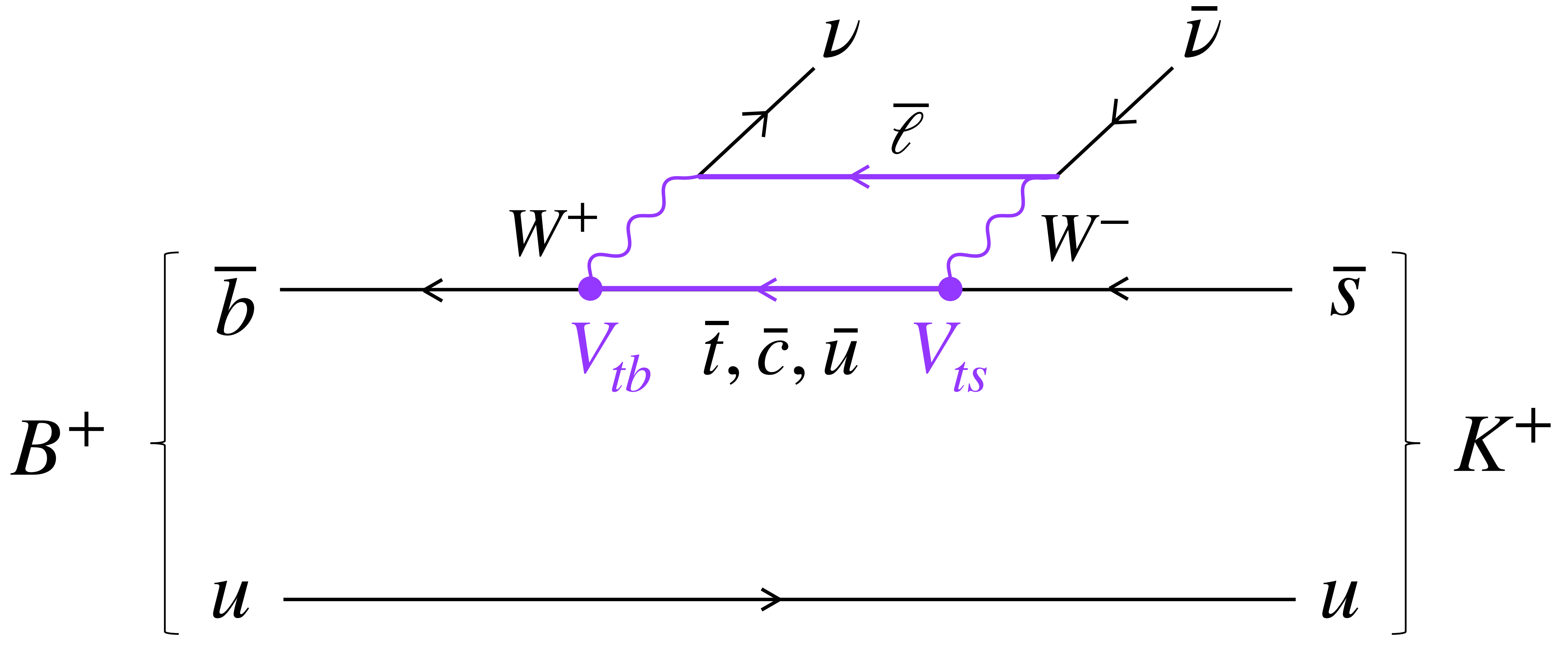

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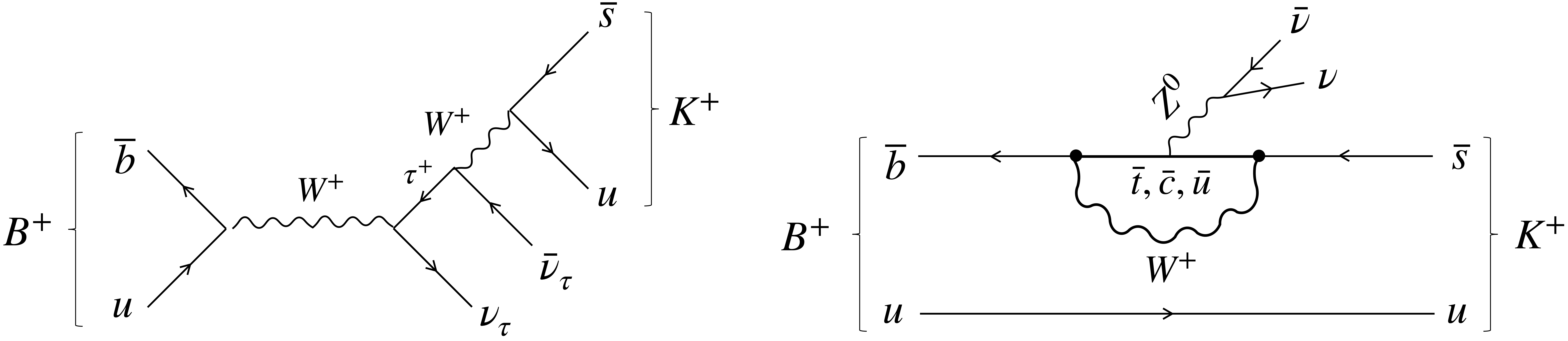
short distance

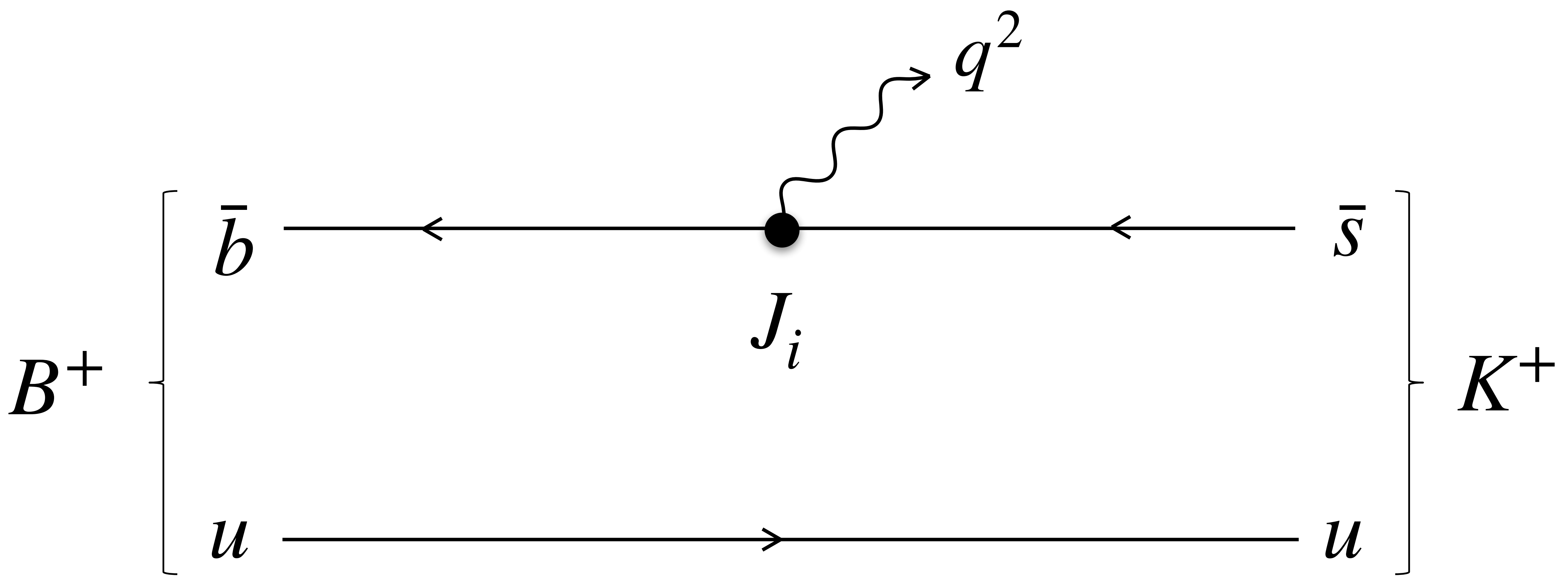
- loop/weak suppression, branching fraction $\propto G_F^2 \sim (10^{-5}\,\mathrm{GeV}^{-2})^2$
- CKM suppression, branching fraction $\propto |V_{tb}V_{ts}|^2 \sim 2 \times 10^{-3}$

Role of form factors



- long distance QCD contributions confined to initial state B^+ and final state K^+
  → decay constants f_{B^+} and f_{K^+}
- short distance includes hadronic contributions conflated with momentum transfer
  → form factors $f_0(q^2)$ and $f_+(q^2)$ for SM; $f_T(q^2)$ for BSM

Role of form factors

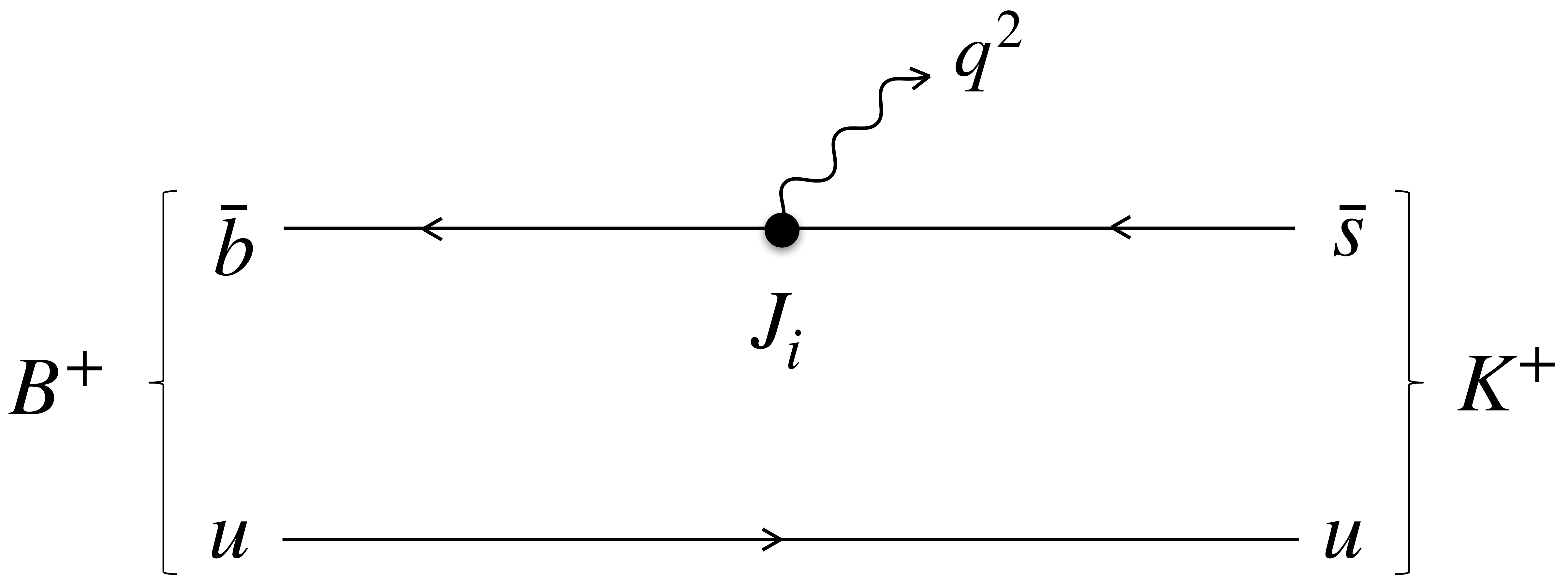


- short distance Standard Model (or BSM) contributions characterised in effective theory by local interaction via currents J_i

- hadronic matrix elements $\langle K | J_i | B \rangle (q^2)$ give the hadronic contributions to the rare decays

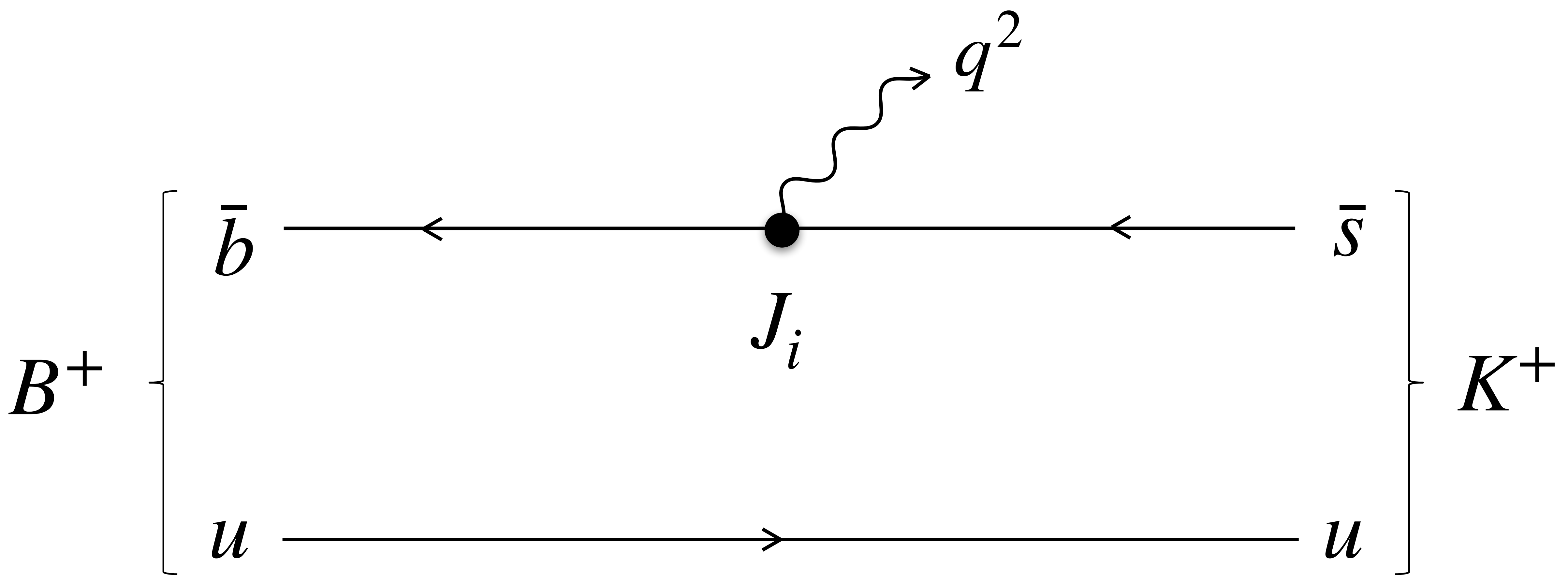
$$f(q^2) \sim \langle K | J | B \rangle (q^2)$$

Calculating form factors: hadronic matrix elements



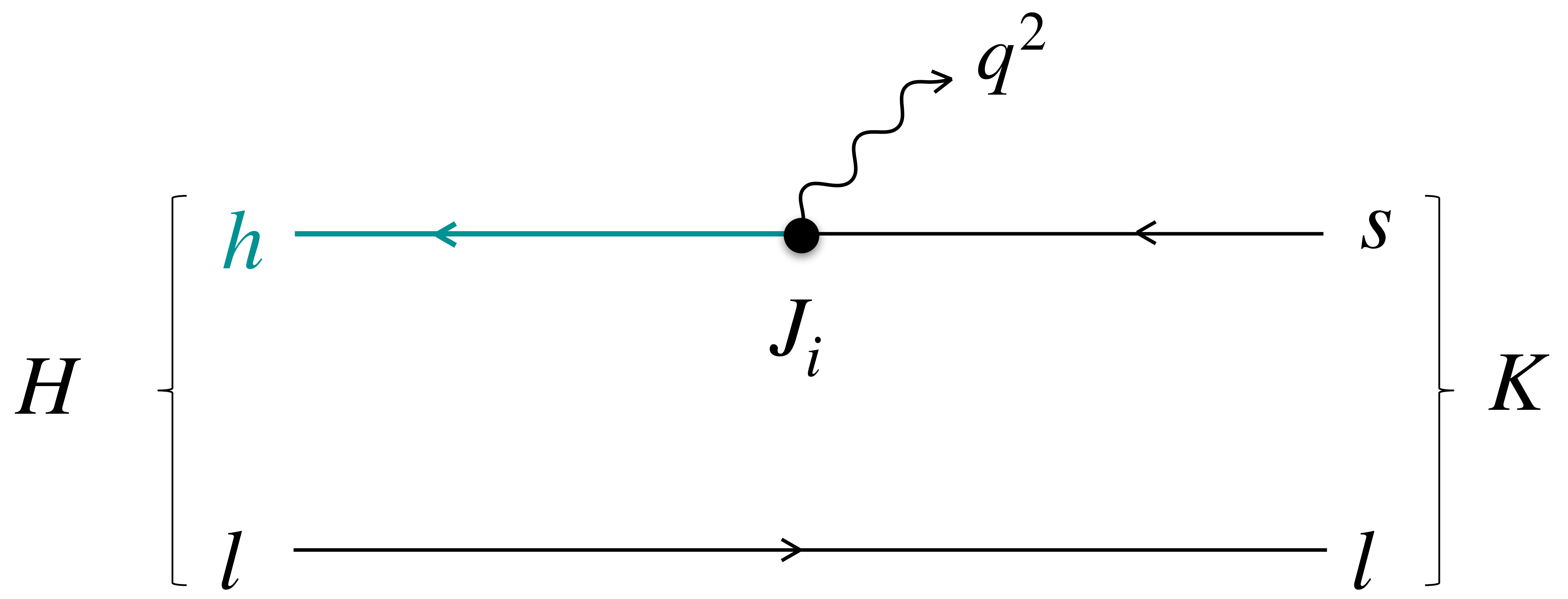
- Lattice QCD discretises spacetime and evaluates correlation functions without need for perturbation theory - necessary for nonperturbative physics.
- Hadronic matrix elements are amplitudes of three point correlators generated on the lattice, $\langle K(T)\ J_i(t)\ B(0)^\dagger \rangle \propto \langle K|J_i|B\rangle\ \mathrm{fn}(t,T)$.
- In what follows, the B is at rest; the K and the J_i recoil off each other.

Calculating form factors: hadronic matrix elements



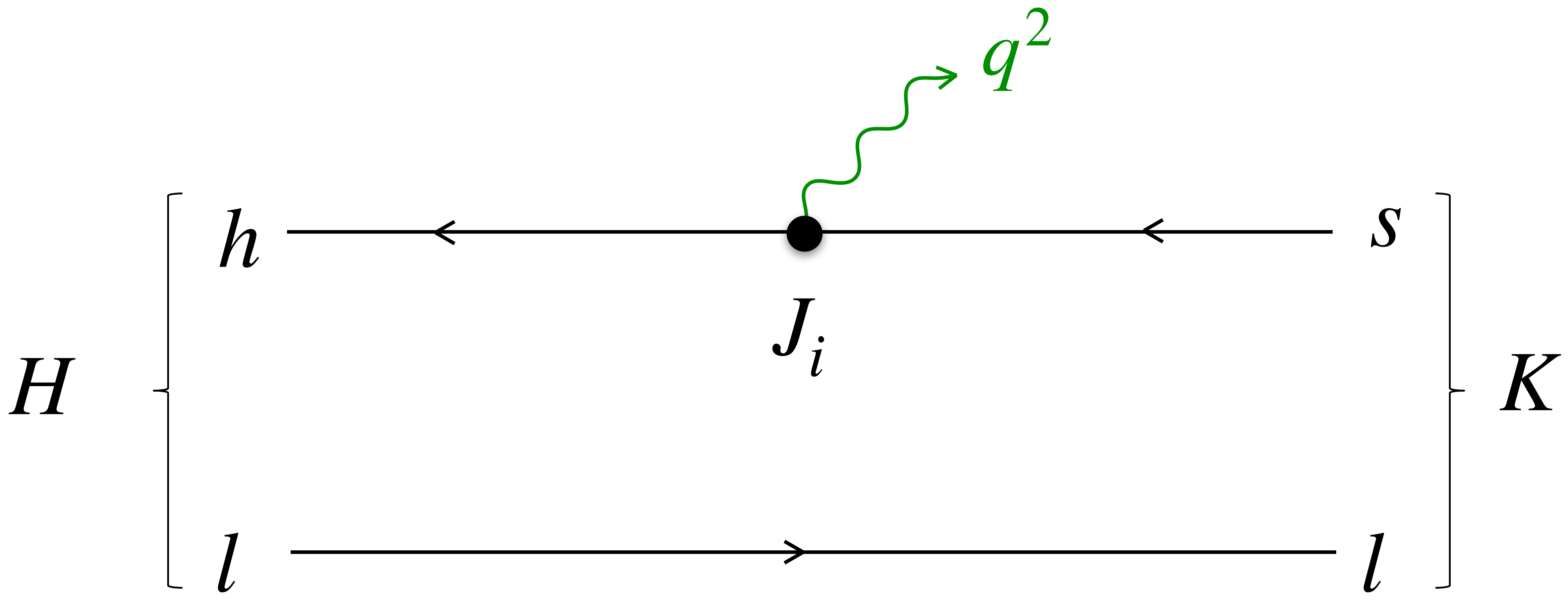
- Most of the choice comes from quarks, ie. discretising the Dirac equation (gluons are relatively simple) - tends to define/divide collaborations.
- HPQCD uses the HISQ (highly improved staggered quark) approach:
  - bad - unphysical time-oscillating states that must be accounted for
  - good - cheaper computationally, so generally good statistics; small discretisation effects, so can be used for heavy quarks $\mathcal{O}(am_h(v_h/c)^4)$

Calculating form factors: hadronic matrix elements



- 'heavy HISQ' h quark with $m_c \leq m_h \lesssim m_b$; HISQ s and l
  - avoids effective theory for h (e.g. NRQCD) and associated matching to QCD
  - simulate over a range of m_h values
  - guided by HQET, extrapolate form factors in m_h from $m_c \to m_b$
  - obtain results for both B and D decays

Calculating form factors: hadronic matrix elements

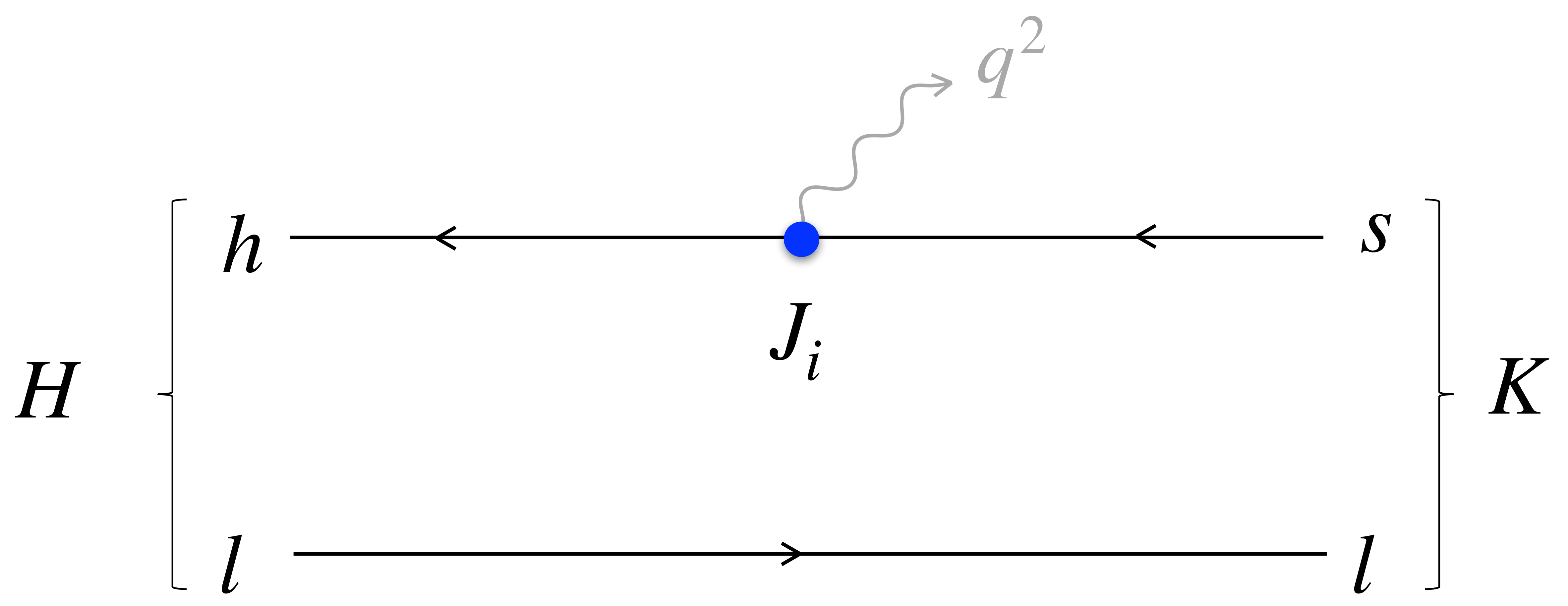


$\langle K | J_i | H \rangle$

hadronic matrix elements have:

- momentum transfer dependence, $0 \leq q^2 \leq q^2_{\max} = (M_H - M_K)^2$

Calculating form factors: hadronic matrix elements



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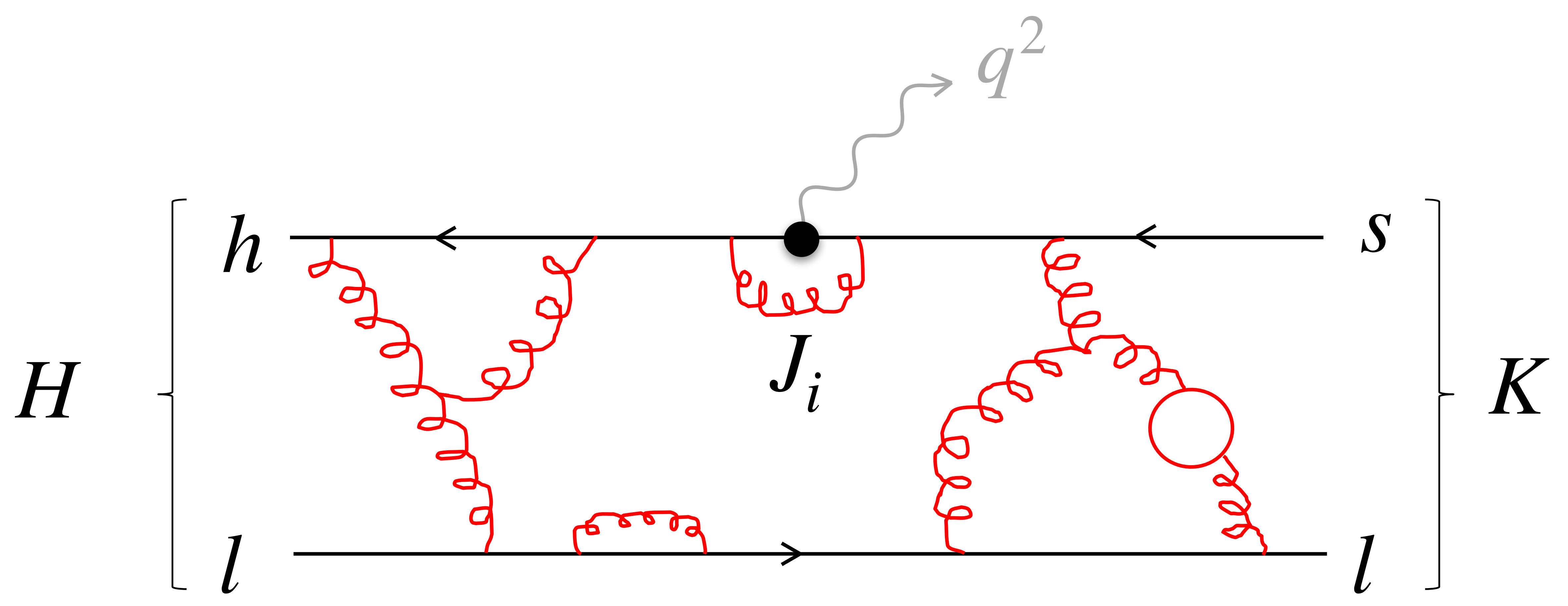
- effects from short distance weak interactions: $M_W \sim \mathcal{O}(100\,\mathrm{GeV})$

Calculating form factors: hadronic matrix elements



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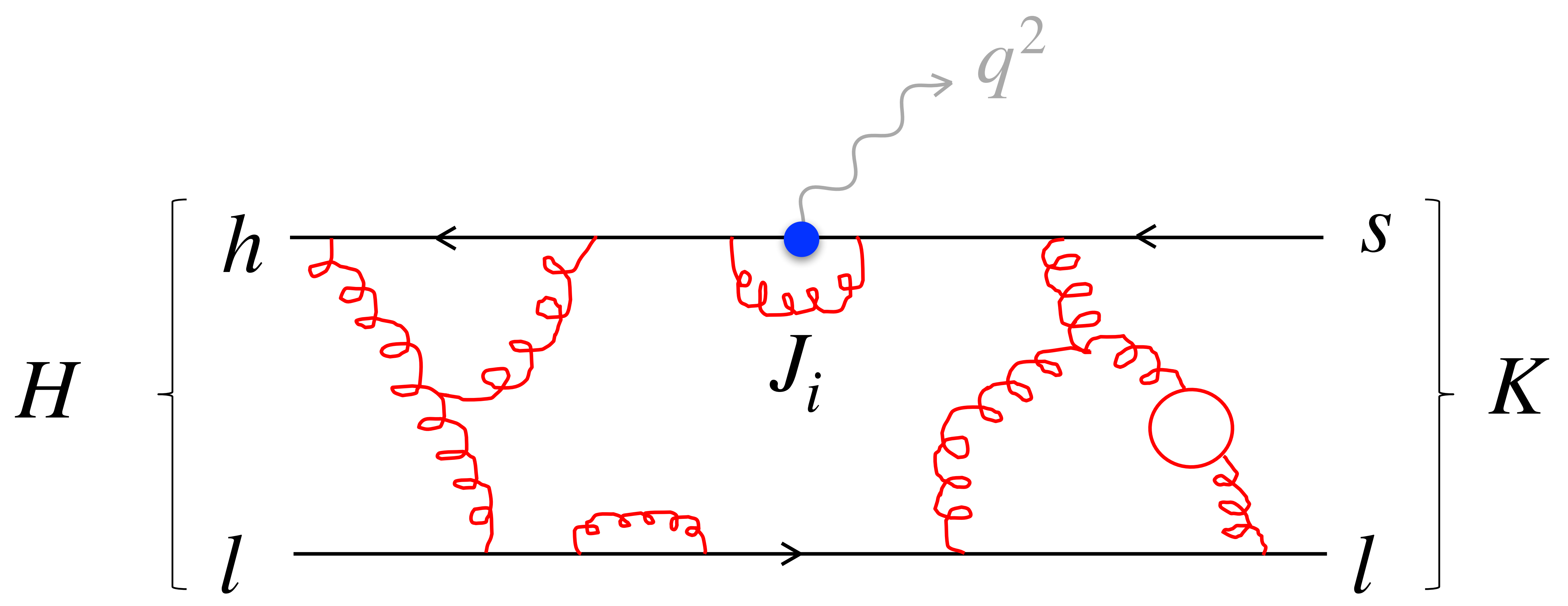
hadronic matrix elements have:

- momentum transfer dependence, $0 \leq q^2 \leq q^2_{\rm max} = (M_H - M_K)^2$
- effects from short distance weak interactions: $M_W \sim \mathcal{O}(100\,\mathrm{GeV})$
- long distance QCD interactions: $\Lambda_{\rm QCD} \sim 0.5\,\mathrm{GeV}$

Calculating form factors: hadronic matrix elements



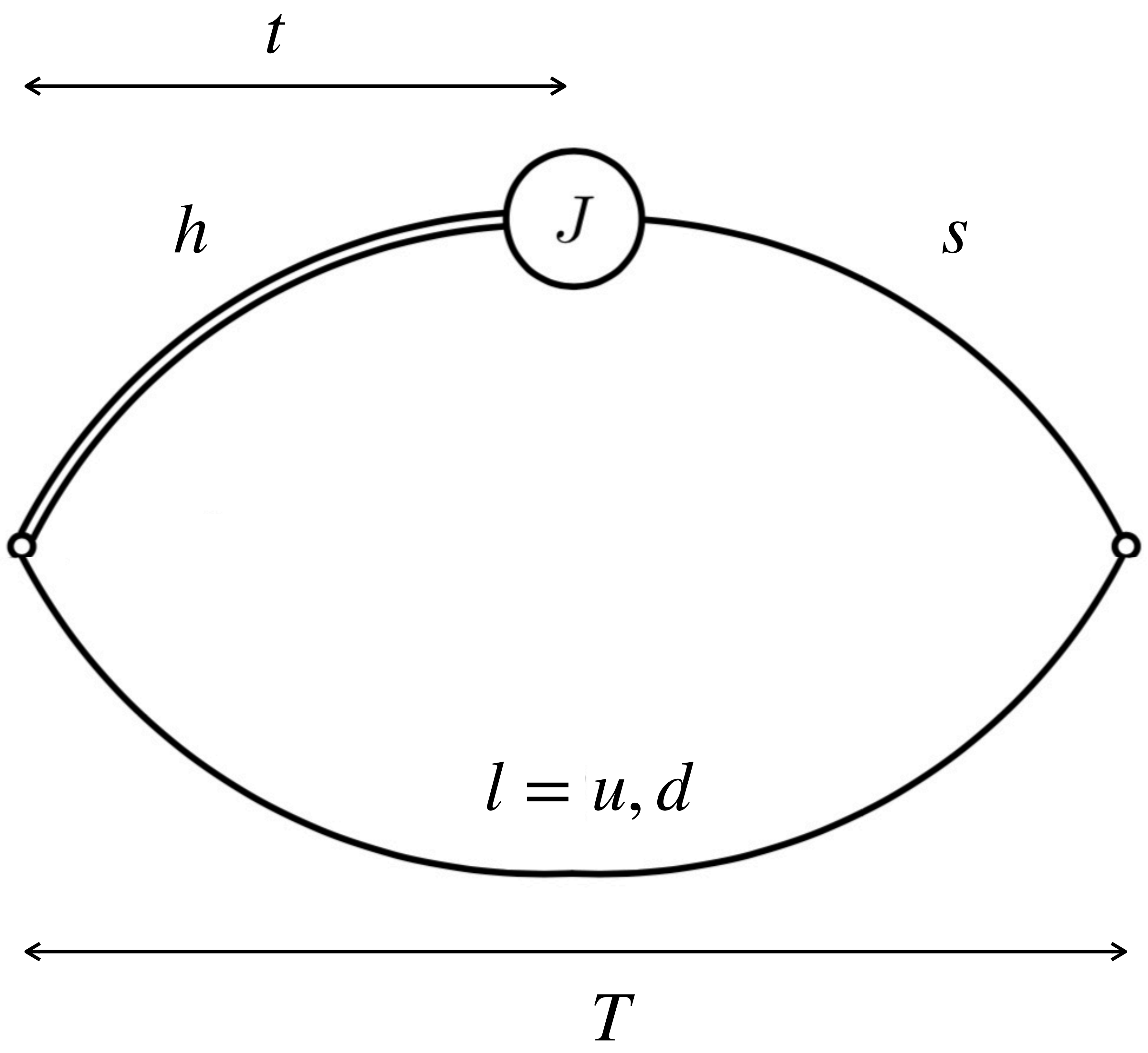
Physics at disparate scales factorizes (up to small corrections)

$$\frac{d\mathscr{B}}{dq^2} = \left| \sum_i C_i \langle K | J_i | H \rangle \right|^2 + \ldots$$

see lectures by Wolfgang and David

- Wilson coefficients: short distance, *perturbative*
- hadronic matrix elements: long distance, *nonperturbative*

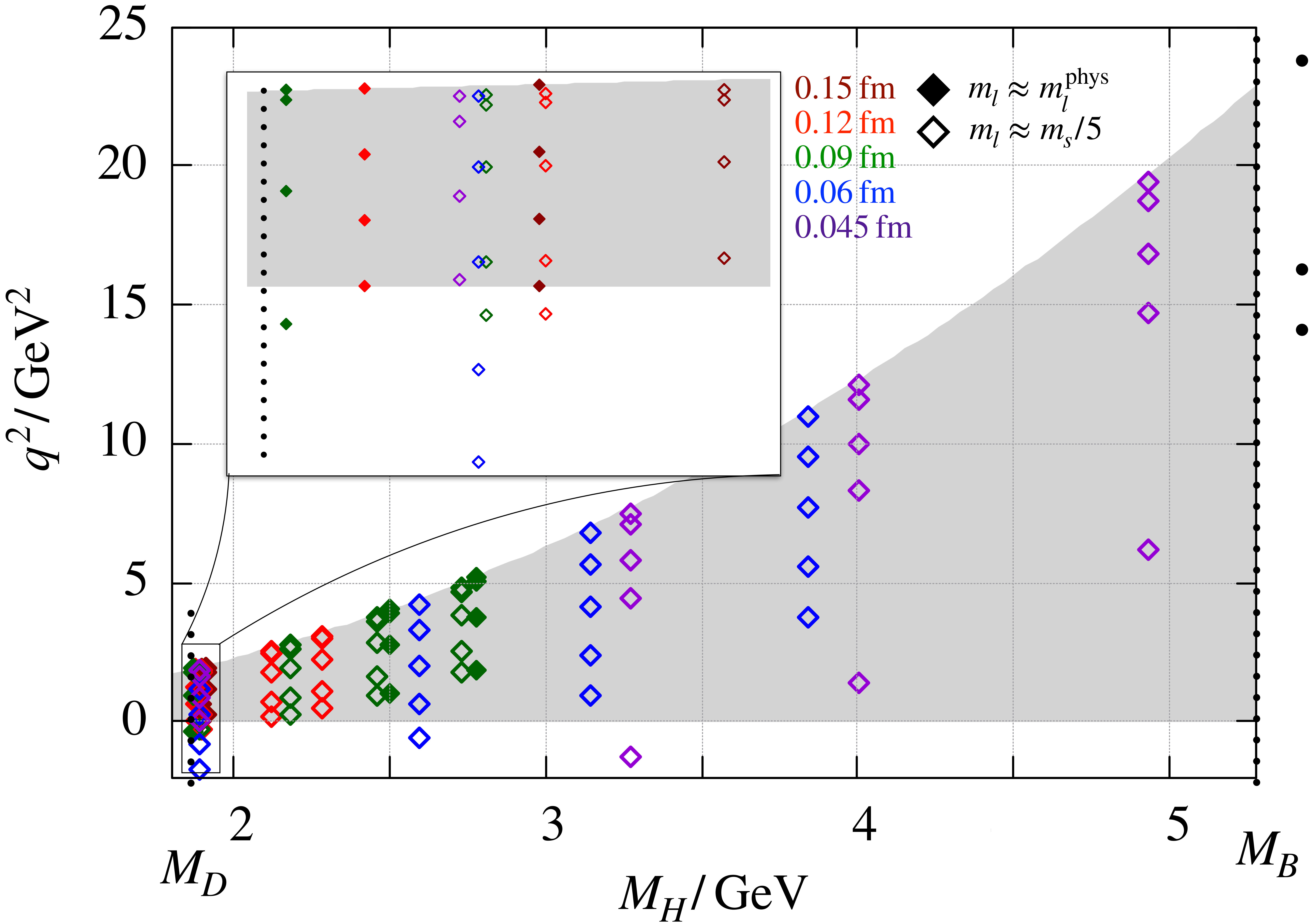
Calculating form factors: 3 point correlators



$\langle K(T)\, J(t)\, H(0)^\dagger \rangle$ data generated with:

- J specifies current quantum numbers
- range of momenta given to daughter s quark
- several (3 or 4) values of T
- all values of t such that $0 \leq t \leq T$
- multiple lattice spacings a of discrete spacetime
- multiple values of m_h with $m_c \leq m_h \lesssim m_b$
- Two values of $m_l = m_l^{\text{phys}},\ m_s/5$

Calculating form factors: kinematic coverage



- MILC HISQ $n_f = 2+1+1$ ensembles
  Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)
- for large range of M_H, cover q^2
- near M_B on finest lattice

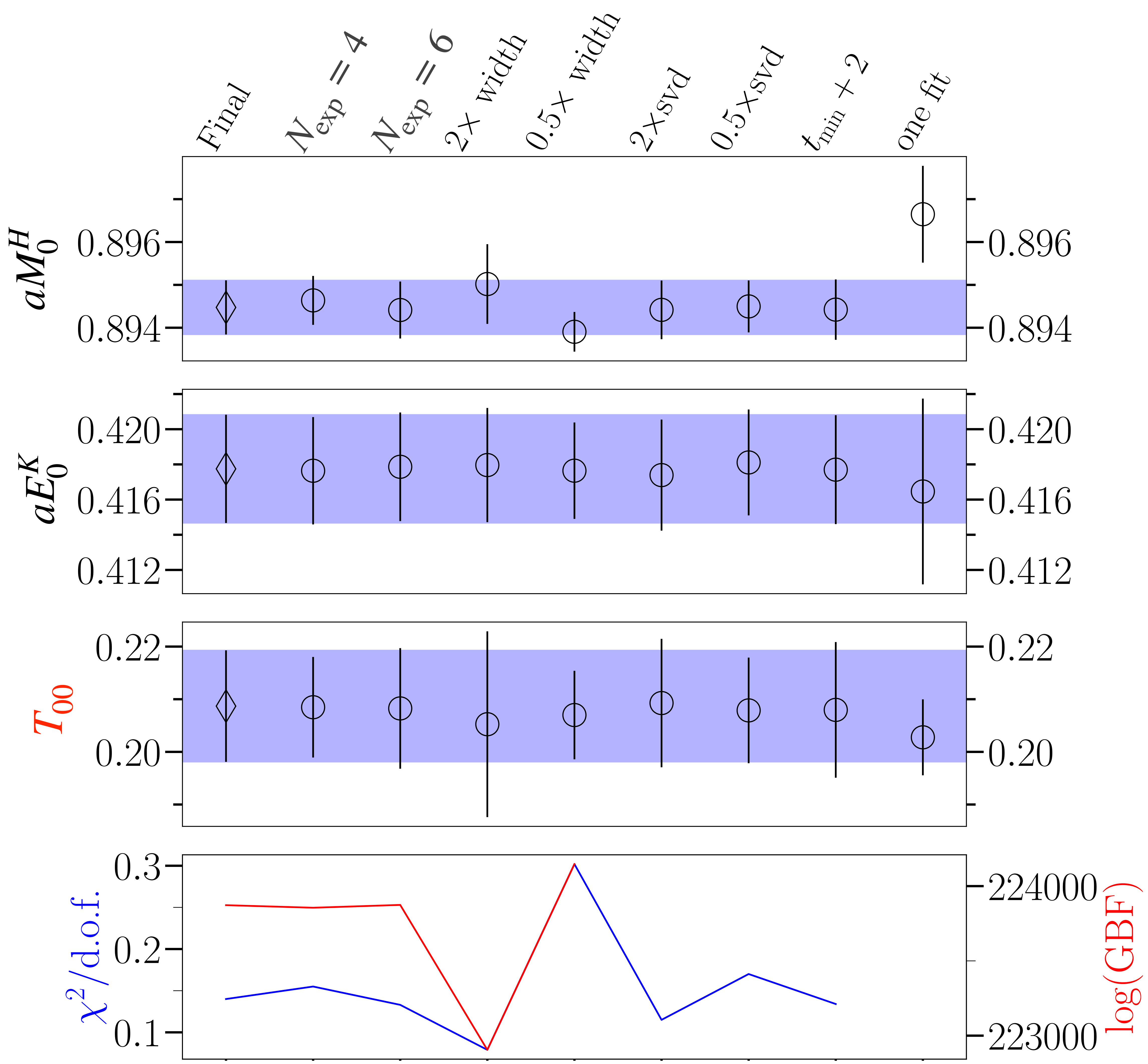
Calculating form factors: extract matrix elements

- matrix element extracted from amplitudes of 3pt correlators
- simultaneous fit 2pt and 3pt correlators, e.g.
  Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)

$$C_2^H(t) = \sum_{i=0}^{N_{\rm exp}} |d_i^H|^2 (e^{-M_i^H t} + e^{-M_i^H (N_t - t)}) + \ldots$$

$$C_2^K(t) = \sum_{i=0}^{N_{\rm exp}} |d_i^K|^2 (e^{-E_i^K t} + e^{-E_i^K (N_t - t)}) + \ldots$$

$$C_3^J(t, T) = \sum_{i,j=0}^{N_{\rm exp}} d_i^H J_{ij} d_j^K e^{-M_i^H t} e^{-E_j^K (T-t)} + \ldots$$



representative fit stability, from $a \approx 0.045$ fm

Calculating form factors: from the matrix elements

- J_{00} from fits are the hadronic matrix elements we need

$$S_{00} = \langle K | S | H \rangle \qquad V_{00} = \langle K | V^\mu | H \rangle \qquad T_{00} = \langle K | T^{0k} | H \rangle$$

- form factors parameterise matrix elements

$$\langle K | S | H \rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2) \qquad Z_T(\overline{\text{MS}}, M_H) \langle K | T^{jo} | H \rangle = \frac{2iM_H p_K^j}{M_H + M_K} f_T(\overline{\text{MS}}, M_H; q^2)$$

$$Z_V \langle K | V^\mu | H \rangle = f_+(q^2)\left(p_H^\mu + p_K^\mu - \frac{M_H^2 - M_K^2}{q^2} q^\mu\right) + f_0(q^2) \frac{M_H^2 - M_K^2}{q^2} q^\mu$$

Calculating form factors: from the matrix elements

- J_{00} from fits are the hadronic matrix elements we need

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$$\langle K|S|H\rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2)$$

$Z_T(\overline{\text{MS}}, M_H)$ Calculated via RI-SMOM at 2 GeV (accounting for nonperturbative contributions)

Hatton, Davies, Lepage, Lytle, PRD 102, 094509 (2020)

Z_V Calculated via PCVC relation, $Z_V = \left.\frac{m_h - m_s\langle K|S|H\rangle}{(M_H - M_K)\langle K|V^0|H\rangle}\right|_{\vec{p}_K=0}$

Na, Davies, Follana, Lepage, PRD 82, 114506 (2010)

- scalar matrix element is absolutely normalised; tensor and vector from lattice must be matched to the continuum via renormalisation factors Z_V, $Z_T(\overline{\text{MS}}, M_H)$

Form Factors: modified z-expansion

- form factors at simulated a, m_{quarks}, V and q^2
- extrapolate to $a \to 0$, $m_{\text{quarks}} \to m_{\text{quarks}}^{\text{phys}}$ and $V \to \infty$ using modified z-expansion

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}} \;; \quad t_+ = (M_H + M_K)^2 \;, \quad \text{we choose} \quad t_0 = 0$$

$$f_{+,T}(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^*}^2} \sum_{n=0}^{N-1} a_n^{+,T}\left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right) \;, \quad f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^0}^2} \sum_{n=0}^{N-1} a_n^0 z^n$$

- $\mathscr{L}(V)$ are hard pion ChPT logs including (small) FV corrections Bijnens, Jemos, NPB 846, 145-166 (2011)
- a_n contains mistuning, heavy quark expansion, discretization, and analytic chiral terms

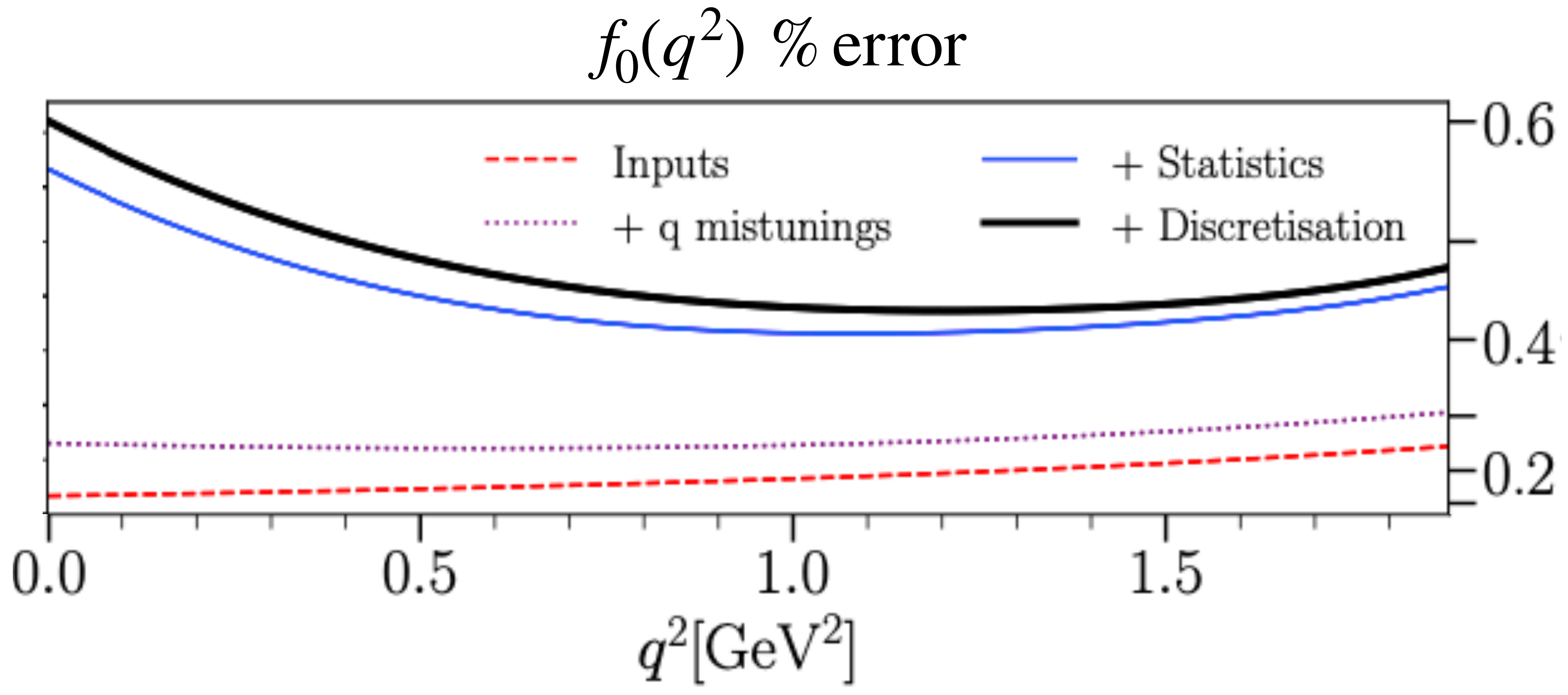
$$a_n^f = \left(1 + \mathcal{N}_n^f\right)\left(\frac{M_D}{M_H}\right)^{\zeta_n}\left(1 + \rho_n^f \log\left(\frac{M_H}{M_D}\right)\right) \sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln}^f \left(\frac{\Lambda}{M_H}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{a\Lambda}{\pi}\right)^{2k} \left(\frac{m_\pi^2 - (m_\pi^{\text{phys}})^2}{(4\pi f_\pi)^2}\right)^l$$

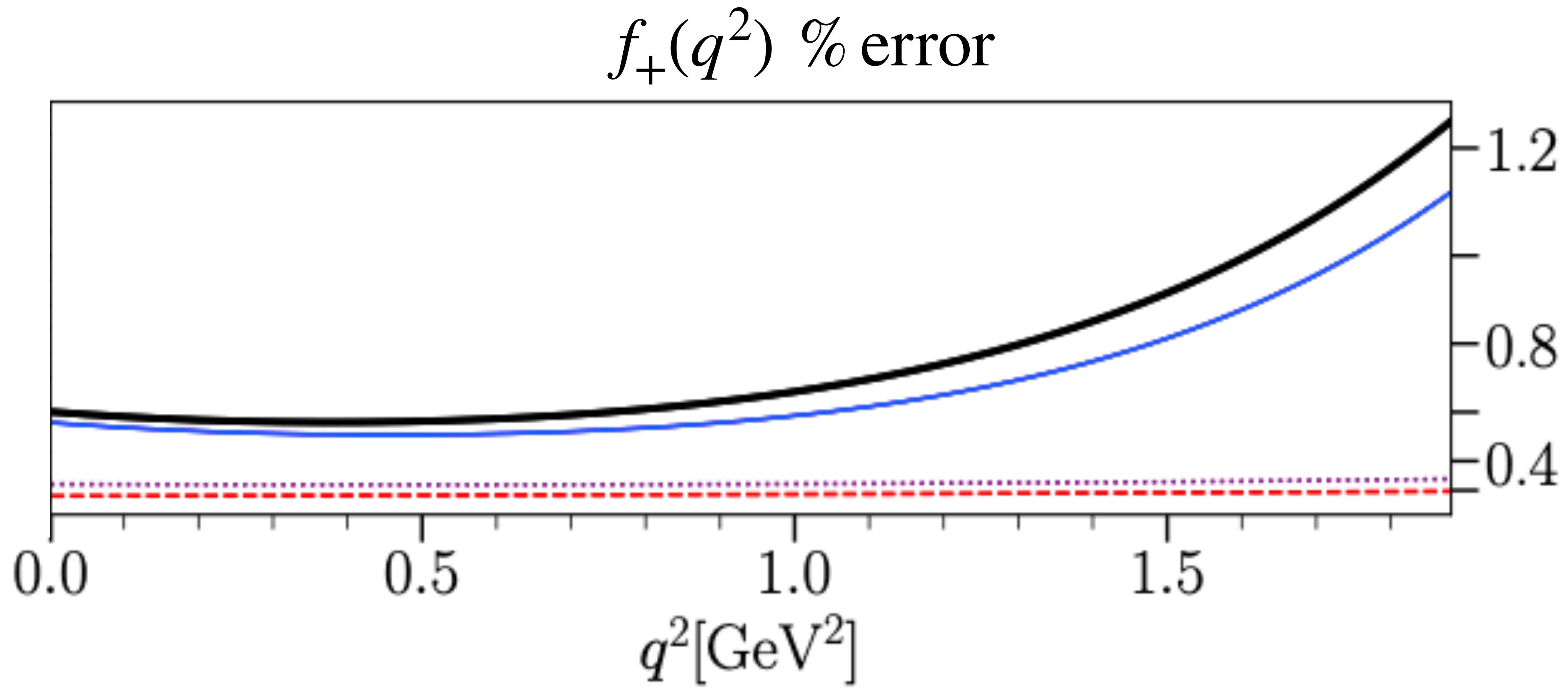
Form Factors: results on the D end

$D \to K$

- bands show continuum, infinite volume, physical quark mass ($m_h = m_c$) form factors
- $f_+(q^2)$ precision 0.6 - 1.2%, depending on q^2
- errors statistics dominated, so improvement straightforward

Form Factors: $D \to K$ stacked variances vs. q^2



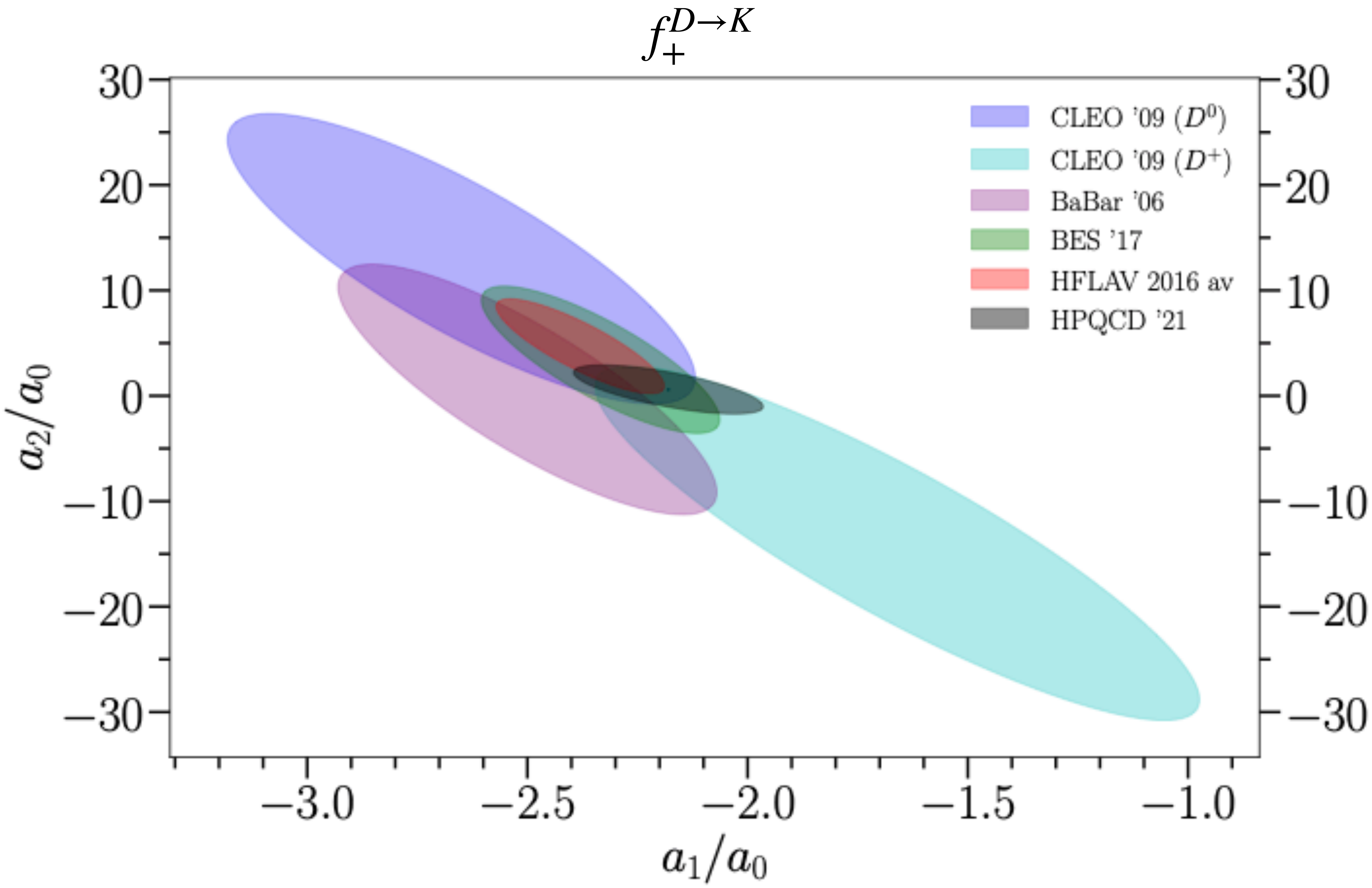


error budget (stacked variances)

- "Inputs" from errors of input quantities, e.g. meson masses
- "q mistunings" mistuned simulation quark masses and chiral effects - ϵ_n and $L(m_l)$ terms
- "Statistics" from finite ensemble size in Monte Carlo evaluation of path integral
- "Discretisation" from uncertainty in extrapolation $a \to 0$

Form Factors: results on the D end

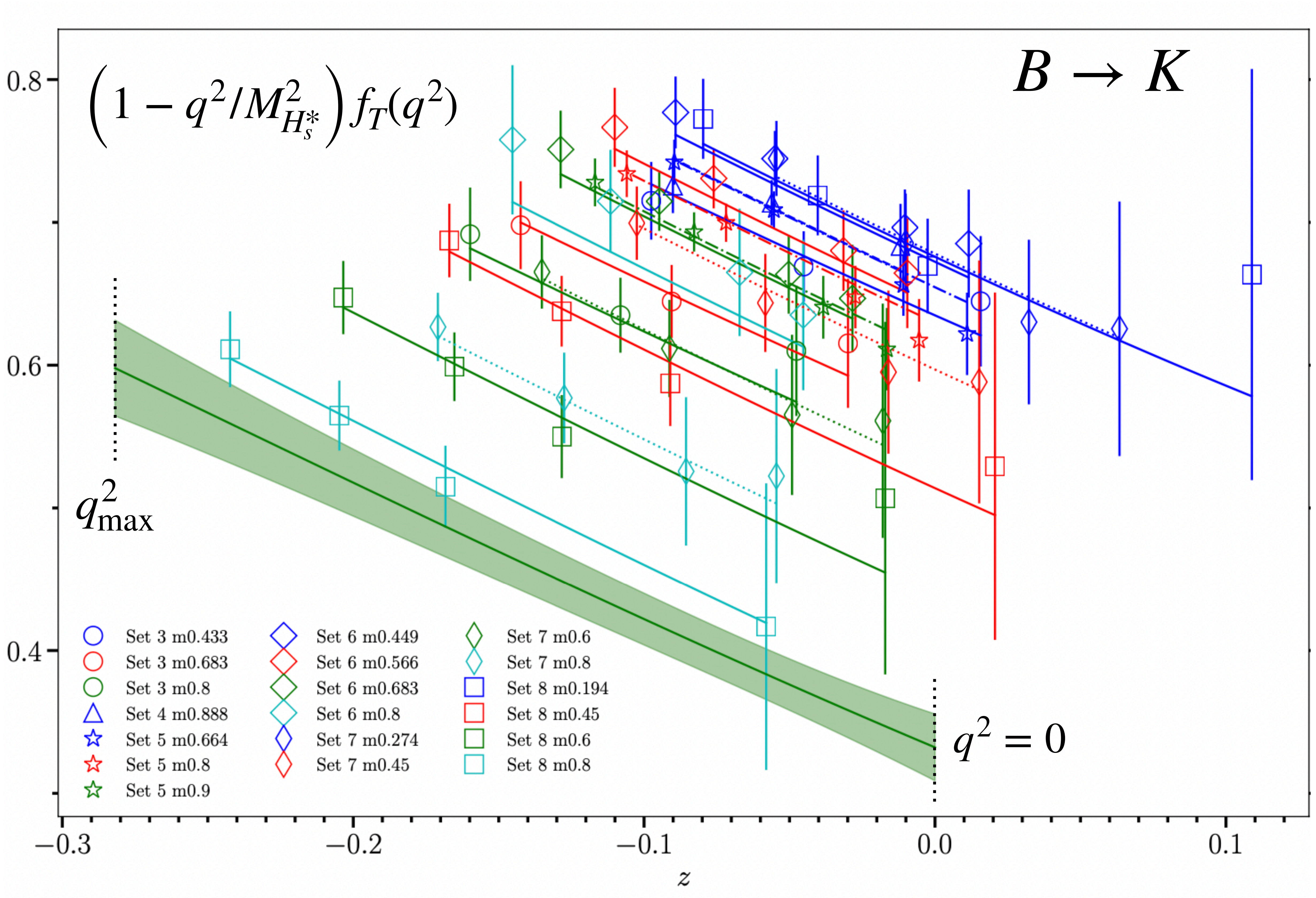
- ratios of z-expansion coefficients give shape information. Favorable comparison with experiment.

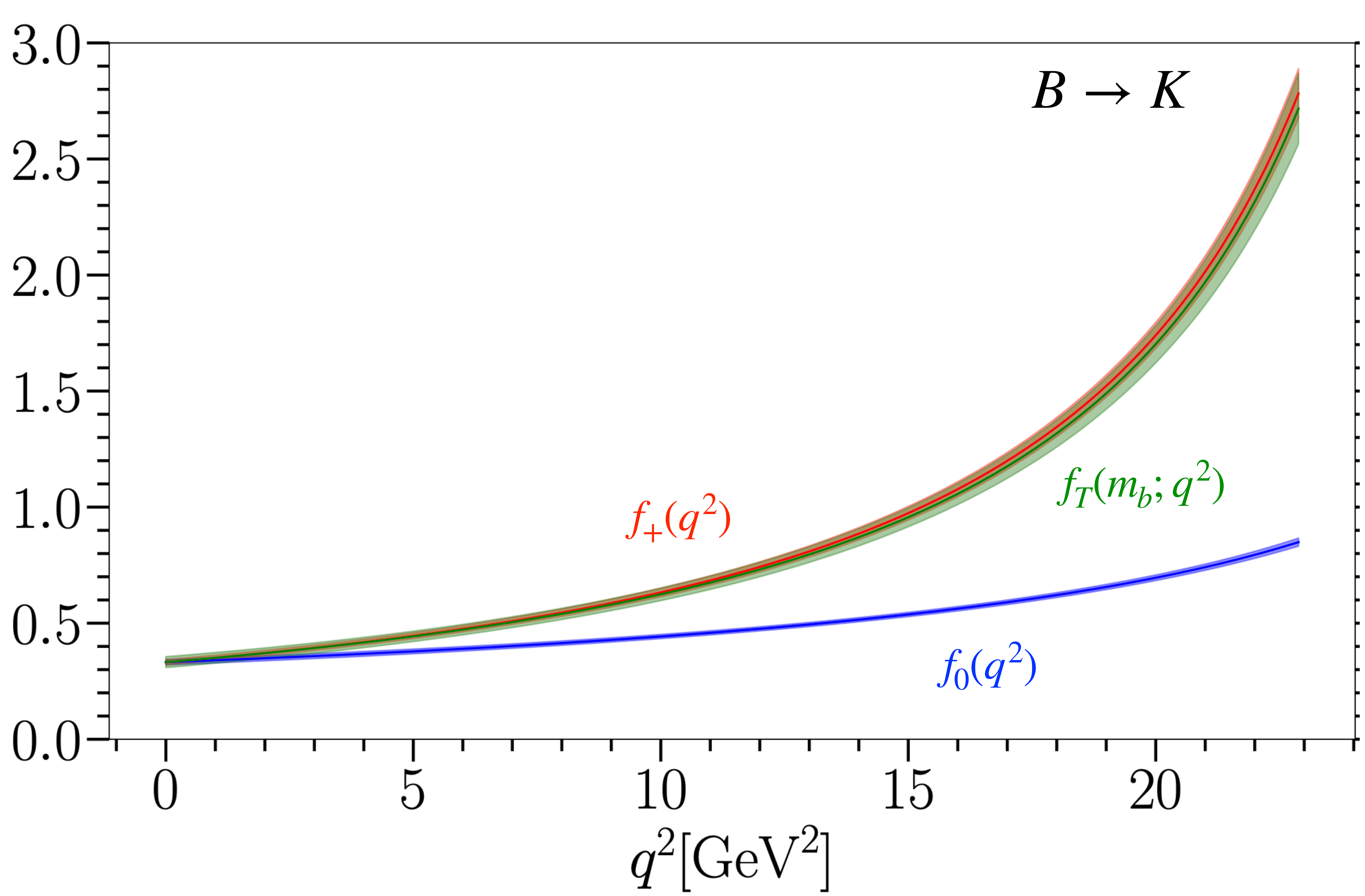


Form Factors: results on the B end

- bands show continuum, infinite volume, physical quark mass form factors
- large f_+ errors at large q^2, when using V^0
  - using spatial component V^k fixes this *

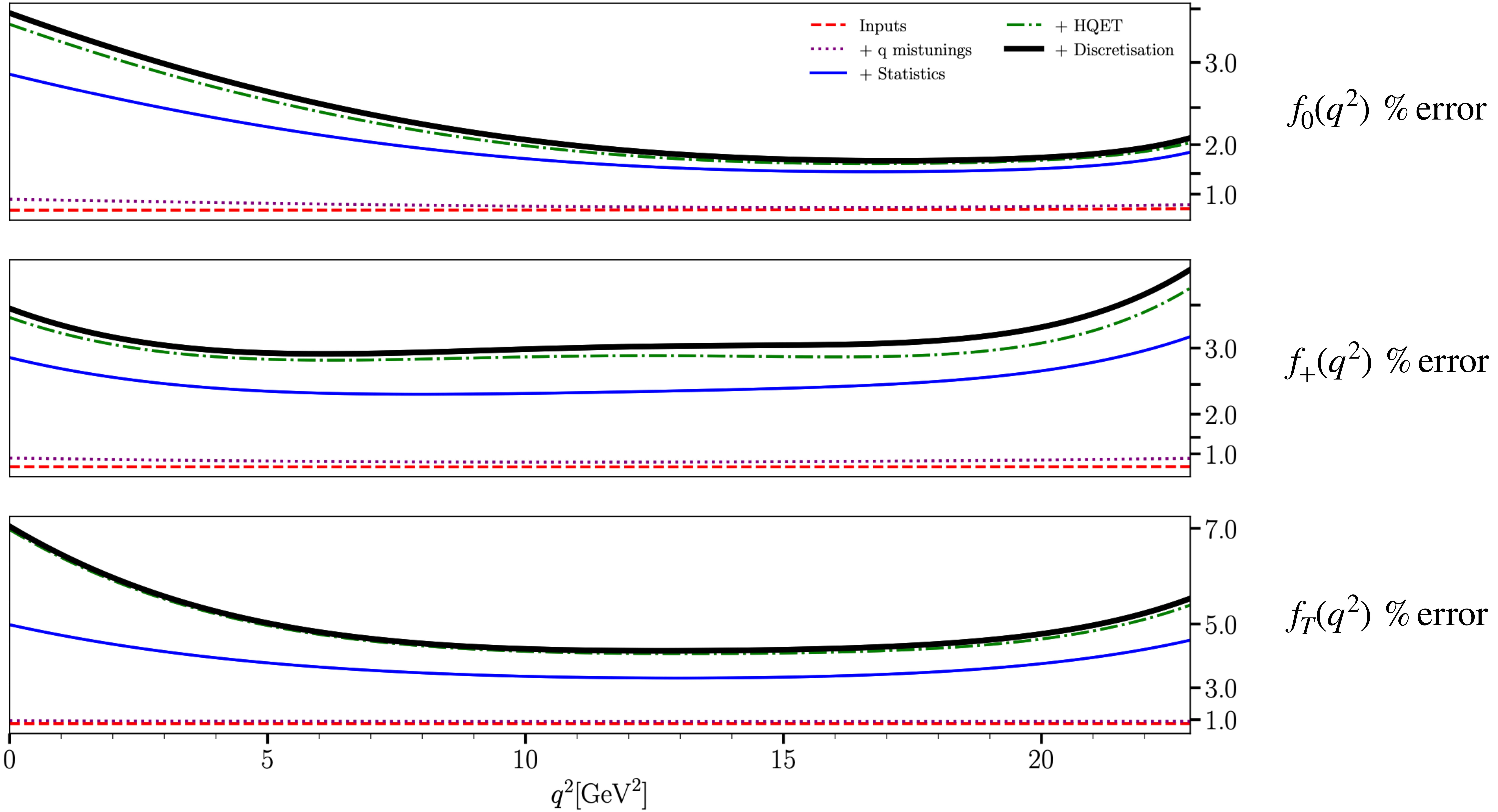
Form Factors: results on the B end





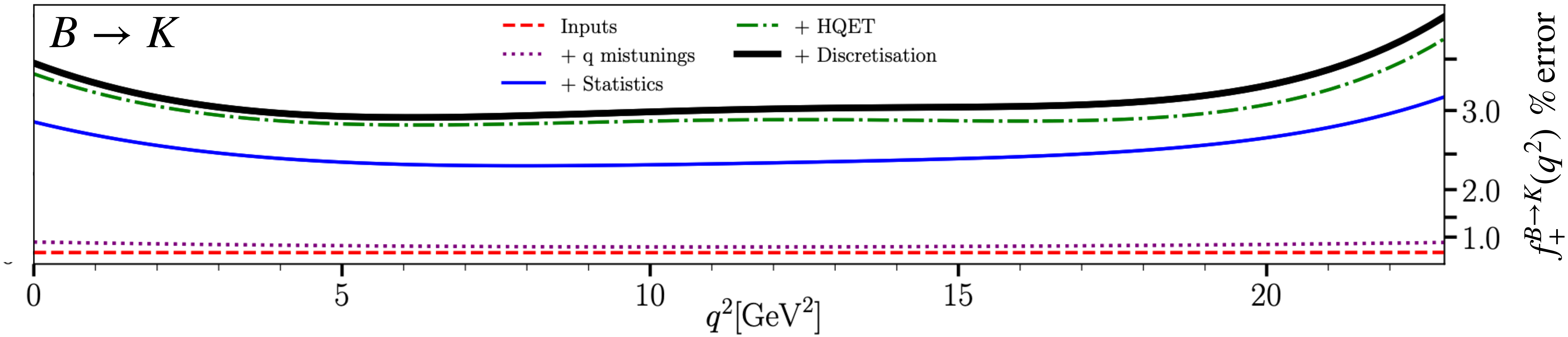
- improved precision, especially at low q^2, where it is needed
- errors statistics dominated, so improvement straightforward

Form Factors: $B \to K$ stacked variances vs. q^2



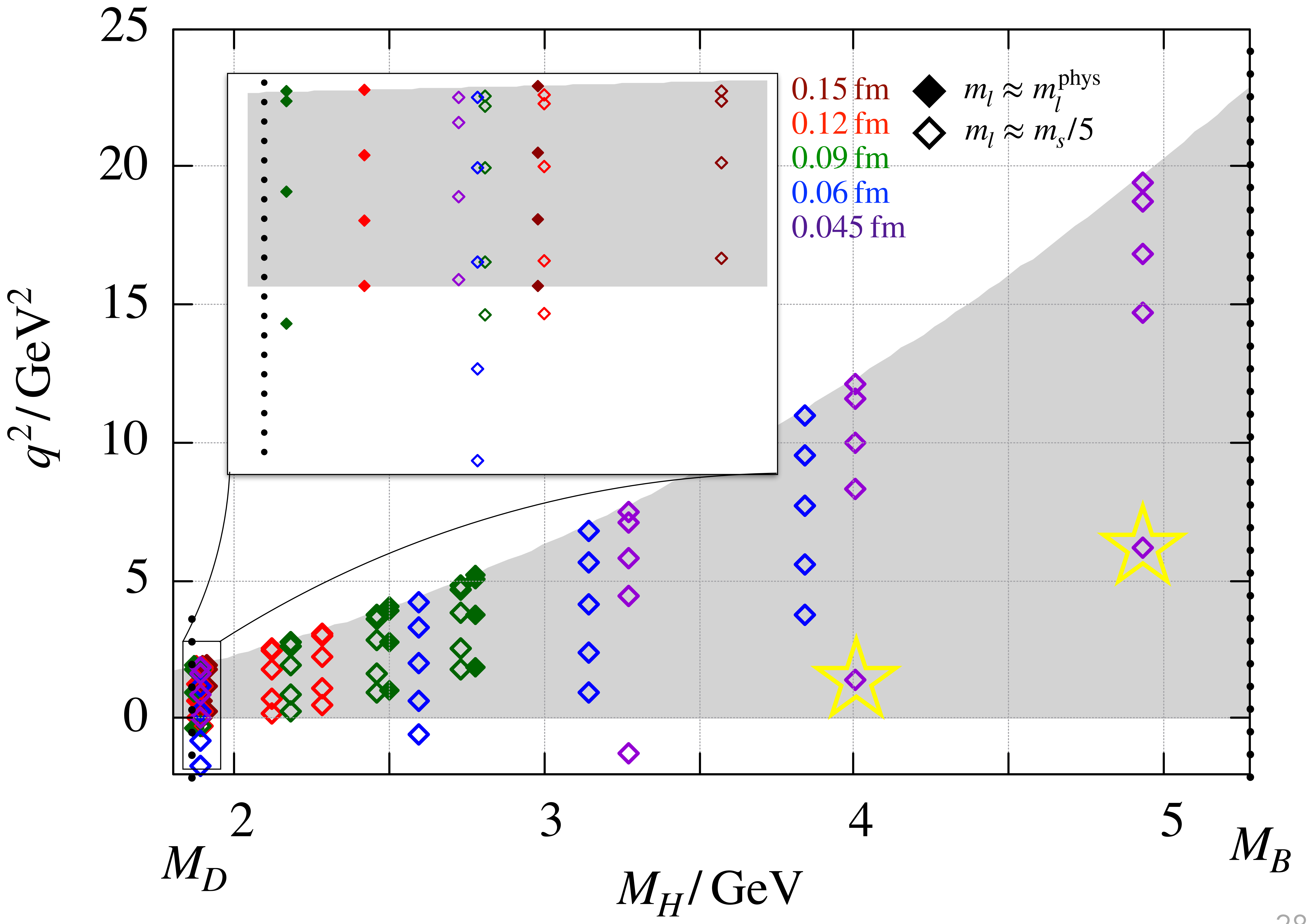
Conclusions and Outlook

- 'heavy HISQ' form factors most precise to date at low q^2
  - statistics limited, improvement straightforward
- FNAL/MILC has heavy HISQ calculations underway with more statistics on finer lattices
  - addresses 'Statistics', 'HQET', and 'Discretization'



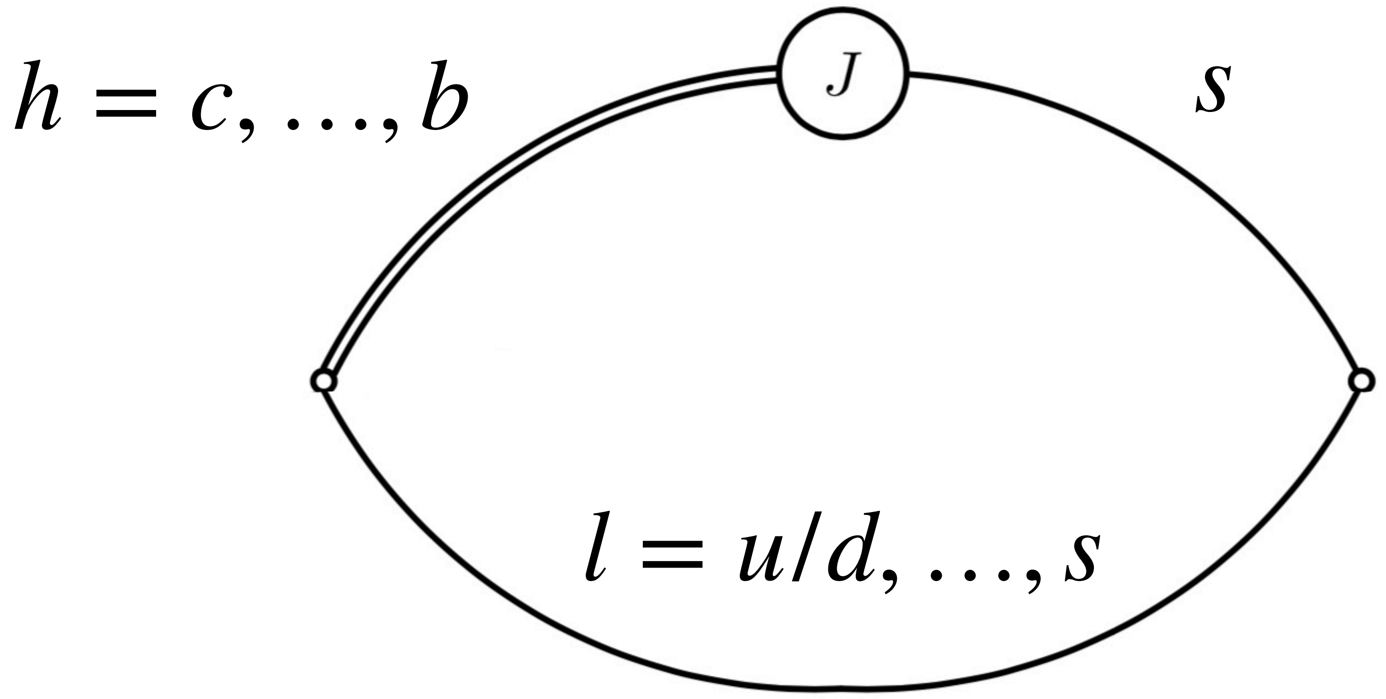
Conclusions and Outlook

- fully relativistic b quark removes EFT matching error and improves q^2 coverage
  - changes the story that LQCD is only applicable at large q^2
- others also using fully relativistic treatments of the b quark, e.g., RBC/UKQCD and JLQCD using DWF

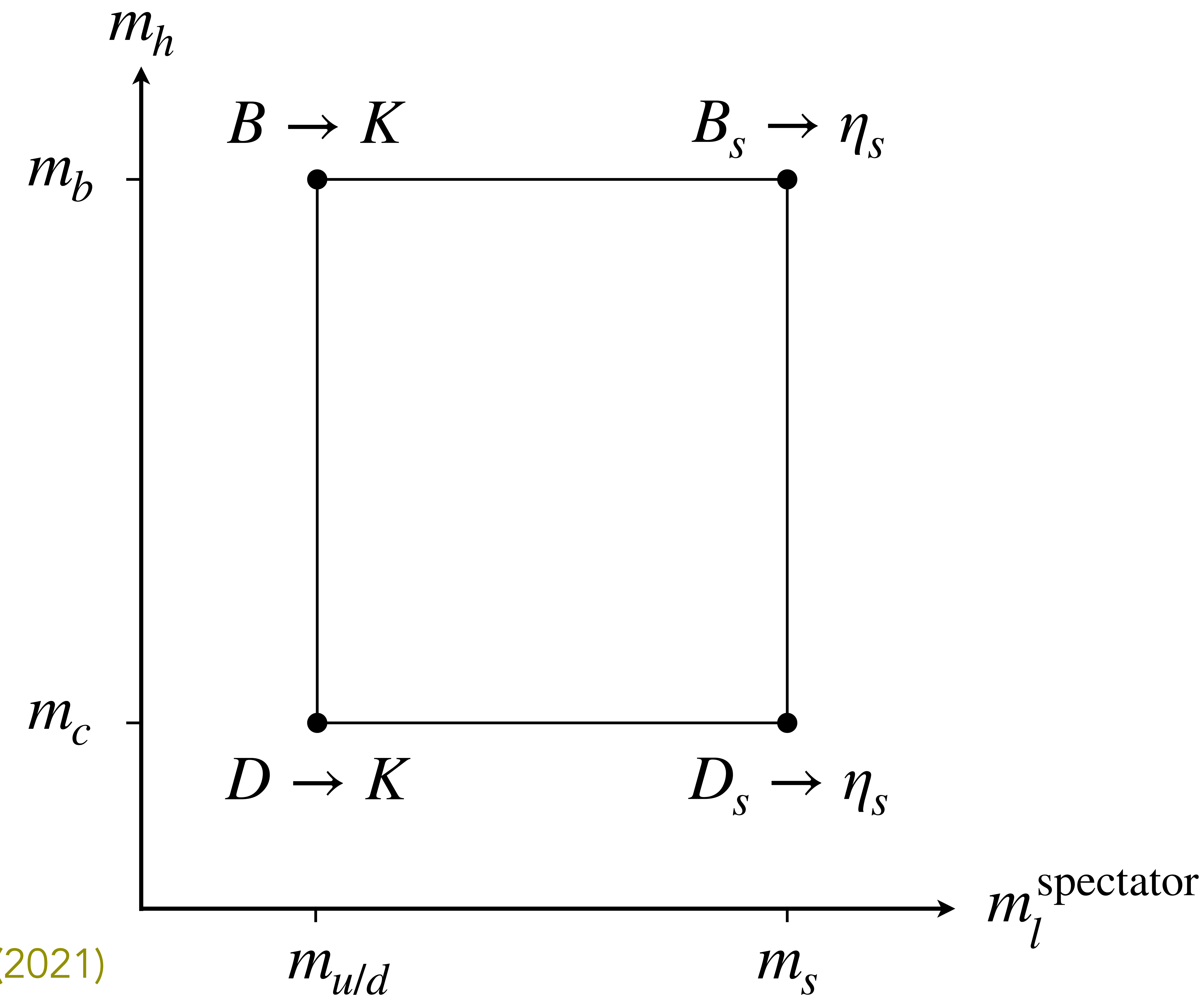


Conclusions and Outlook

- variable mother m_h and spectator m_l

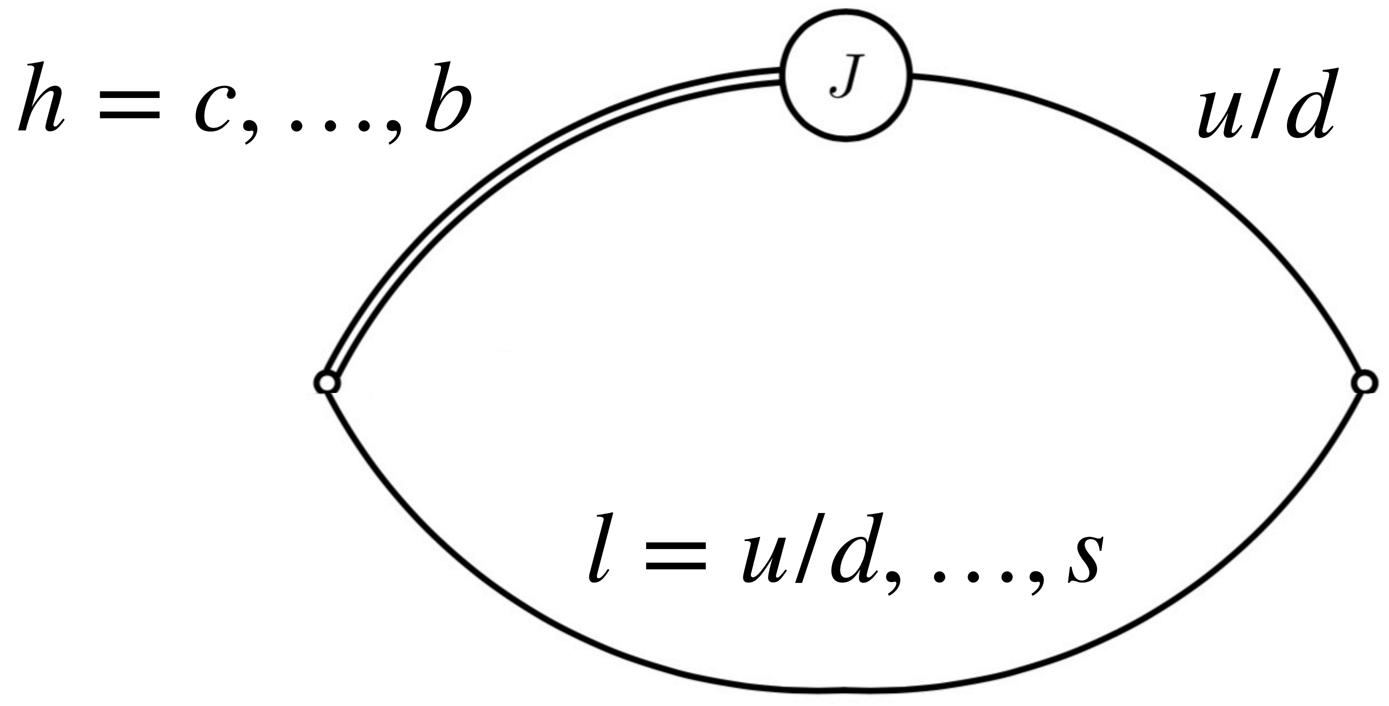


- one calculation gives multiple form factors
- we attacked in piecemeal fashion
  - $H_s \to \eta_s$ Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)
  - $D \to K$ Chakraborty, Parrott, Bouchard, Davies, Koponen, and Lepage, PRD 104 (2021) 034505
  - $B \to K$ Parrott, Bouchard, and Davies, 2207.12468 and 2207.13371



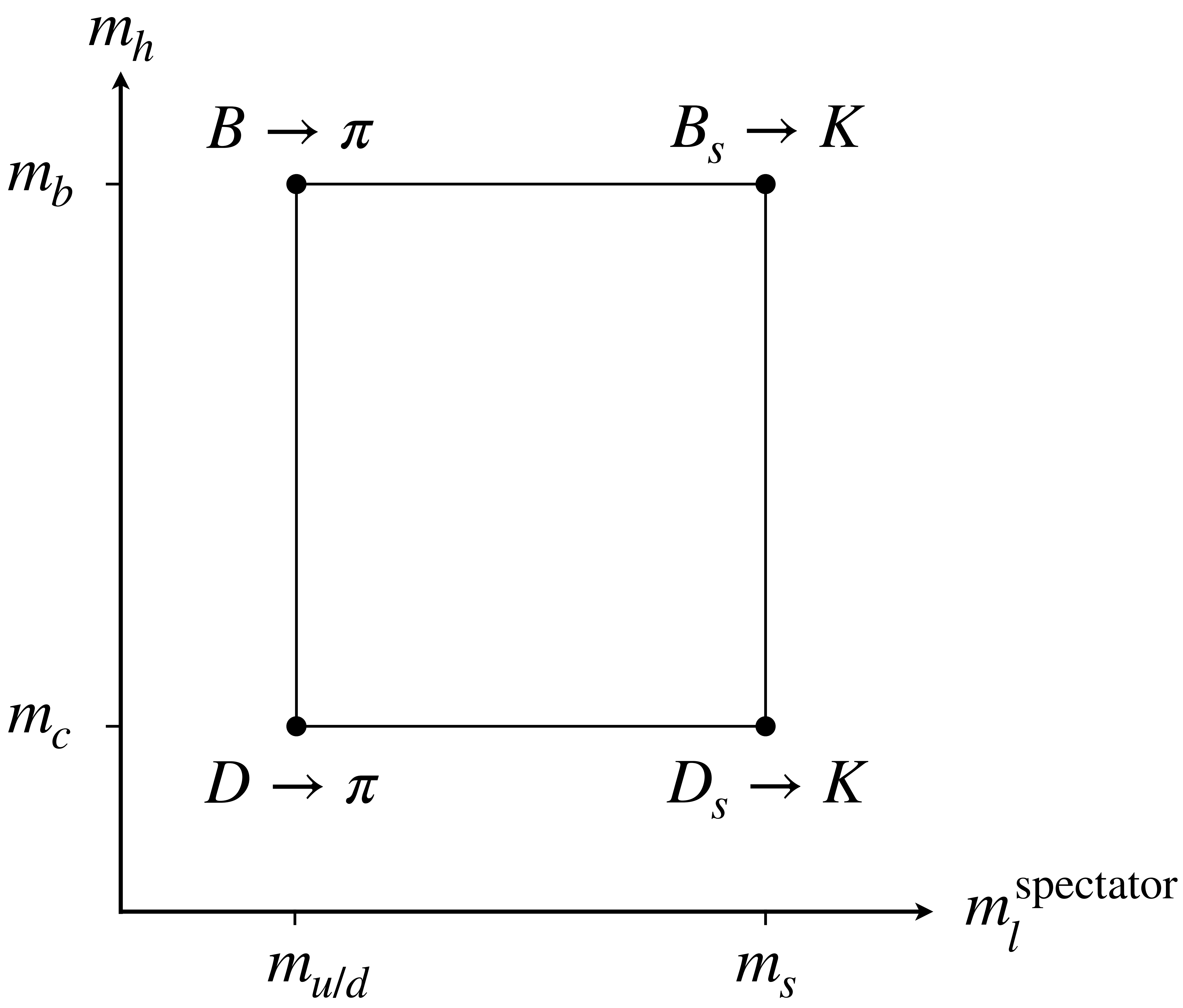
Conclusions and Outlook

- changing daughter quark for u/d



- these form factors are all being obtained from a combined analysis, informing one another

Roberts, Bouchard, Francesconi, Parrott, 2501.18586



Thank you

and thanks to collaborators:

- Bipasha Chakraborty
- Christine Davies
- Jonna Koponen
- Peter Lepage
- Will Parrott

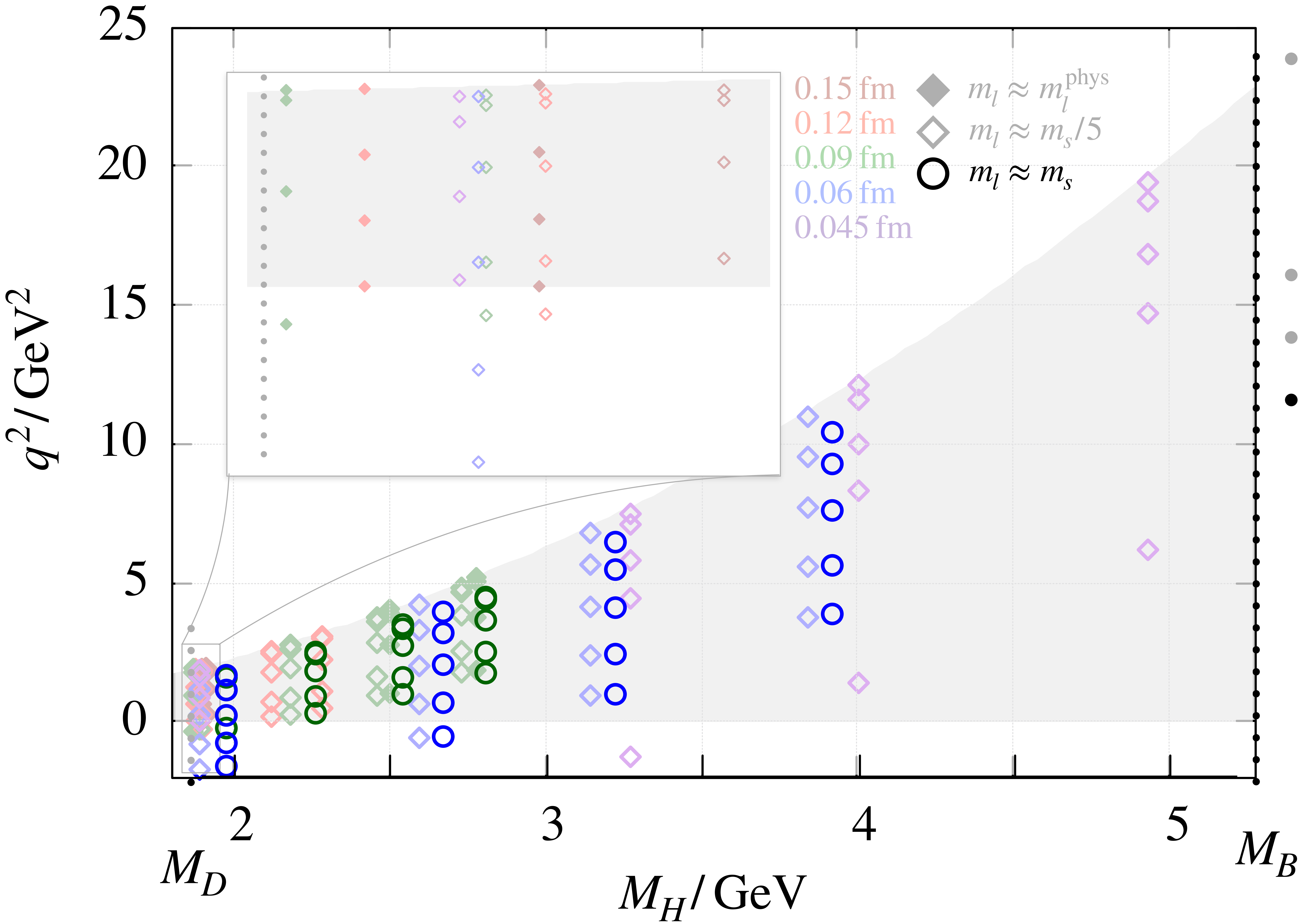
HPQCD

Form Factor calculation: ensembles

MILC $n_f = 2+1+1$ HISQ ensembles Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)

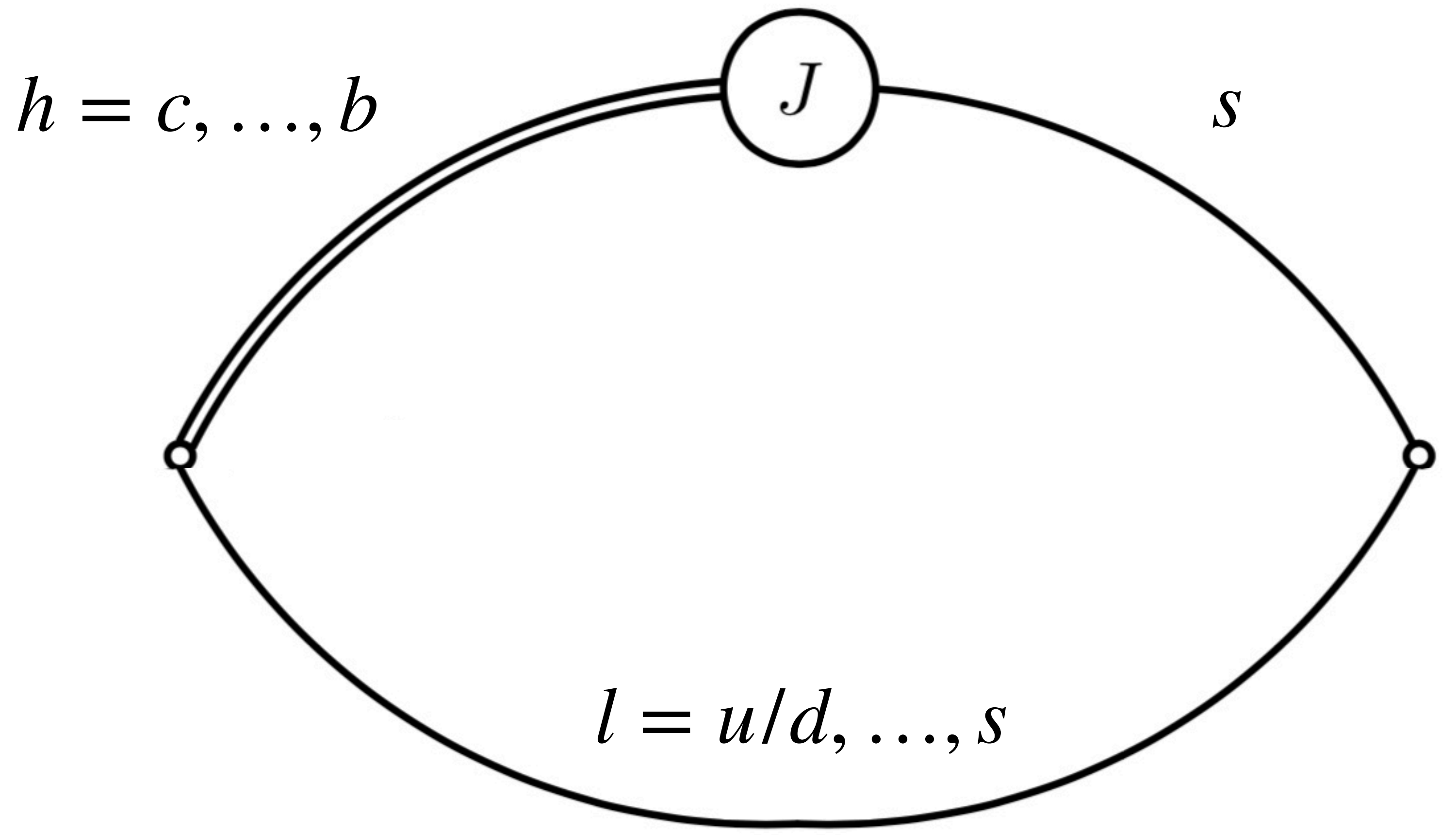
$\approx a/\mathrm{fm}$	$N_s^3 \times N_t$	N_{cfg}	N_{src}	$am_l^{\mathrm{val,\,sea}}$	am_h
0.15	$32^3 \times 48$	998	16	$0.00235 \approx am_l^{\mathrm{phys}}$	0.8605
0.15	$16^3 \times 48$	1020	16	$0.013 \approx am_s/5$	0.888
0.12	$48^3 \times 64$	985	16	$0.00184 \approx am_l^{\mathrm{phys}}$	0.643
0.12	$24^3 \times 64$	1053	16	$0.0102 \approx am_s/5$	0.664, 0.8, 0.9
0.09	$64^3 \times 96$	620	8	$0.0012 \approx am_l^{\mathrm{phys}}$	0.433, 0.683, 0.8
0.09	$32^3 \times 96$	499	16	$0.0074 \approx am_s/5$	0.449, 0.566, 0.683, 0.8
0.06	$48^3 \times 144$	413	8	$0.0048 \approx am_s/5$	0.274, 0.45, 0.6, 0.8
0.045	$64^3 \times 192$	375	4	$0.00316 \approx am_s/5$	0.194, 0.45, 0.6, 0.8

Calculating form factors: kinematic coverage

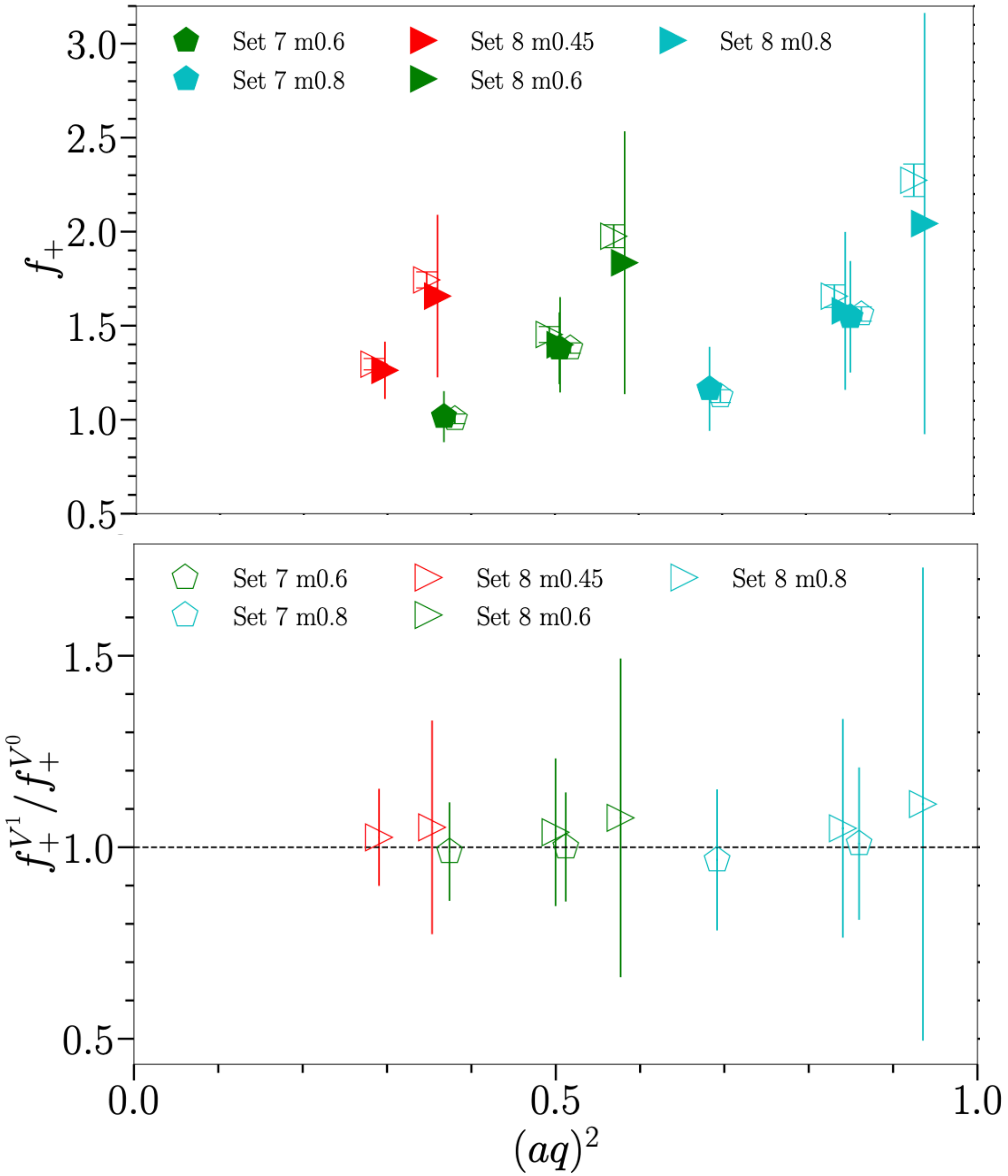


- MILC HISQ $n_f = 2 + 1 + 1$ ensembles
  Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)
- for large range of M_H, cover q^2
- near M_B on finest lattice
- include $H_s \to \eta_s$ data from

  Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)



Form Factor calculation: V^0 vs V^k



- filled symbols: V^0
- open symbols: V^k
- $f_+(q^2_{\max})$ from V^0 relies on a delicate cancelation

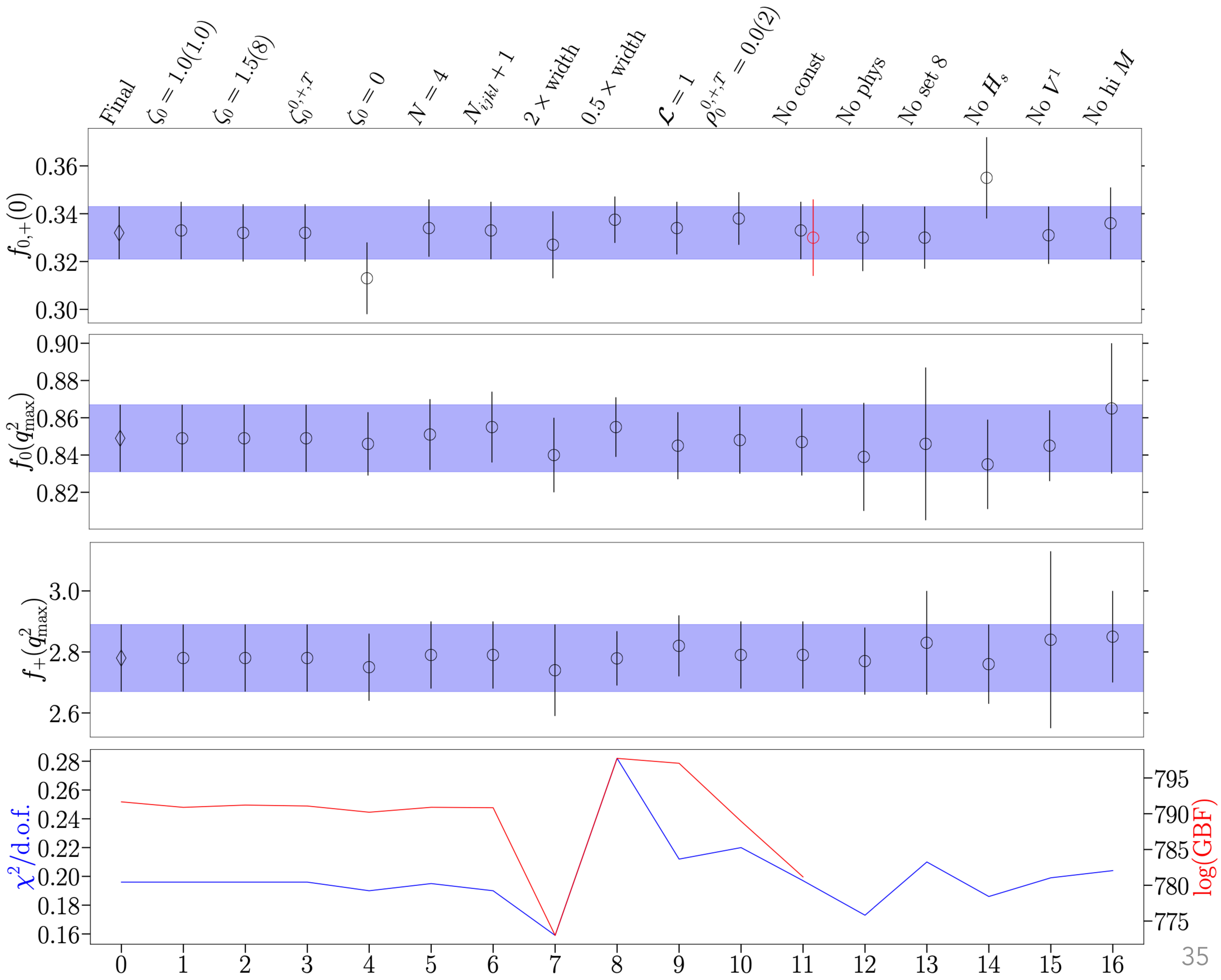
$$f_+(q^2) = \frac{Z_V\langle K|V^\mu|H\rangle - f_0 B^\mu}{p_H^\mu + p_K^\mu - B^\mu}, \quad B^\mu = \frac{M_H^2 - M_K^2}{q^2} q^\mu$$

- no evidence, within errors, of discretisation effects
- accommodate possibility in fit via

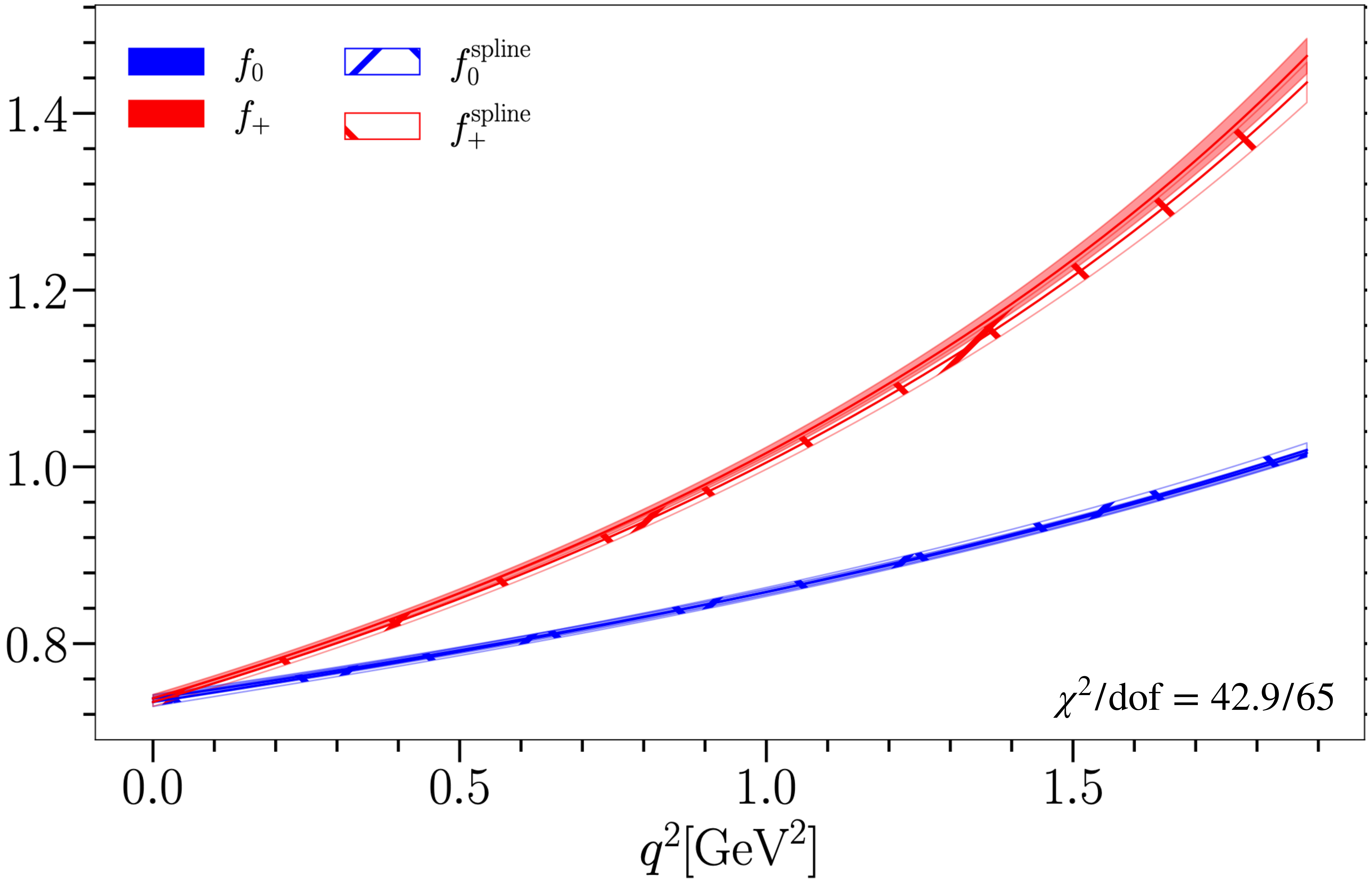
$$f_+^{V^1}(q^2) = \left(1 + \mathscr{C}^{a,m_h}(aq)^2\right) f_+^{V^0}(q^2)$$

$$\text{prior}[\mathscr{C}] = 0.0(1)$$

Form Factors: modified z-expansion stability



Form Factors: test of modified z-expansion



- for $D \to K$, try cubic spline instead of modified z-expansion

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \sum_{n=0}^{N-1} a_n^0 z^n$$

$$\downarrow$$

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \left[\sum_{j=0}^{N} g_j(q^2) \left(\frac{am_c}{\pi} \right)^{2j} + \mathscr{N} \right]$$

- $g_j(q^2)$ are Steffen spline functions
- 4 knots {-3.25, -1.5, 0.25, 2.0} GeV^2

Form Factors: variation with m_h

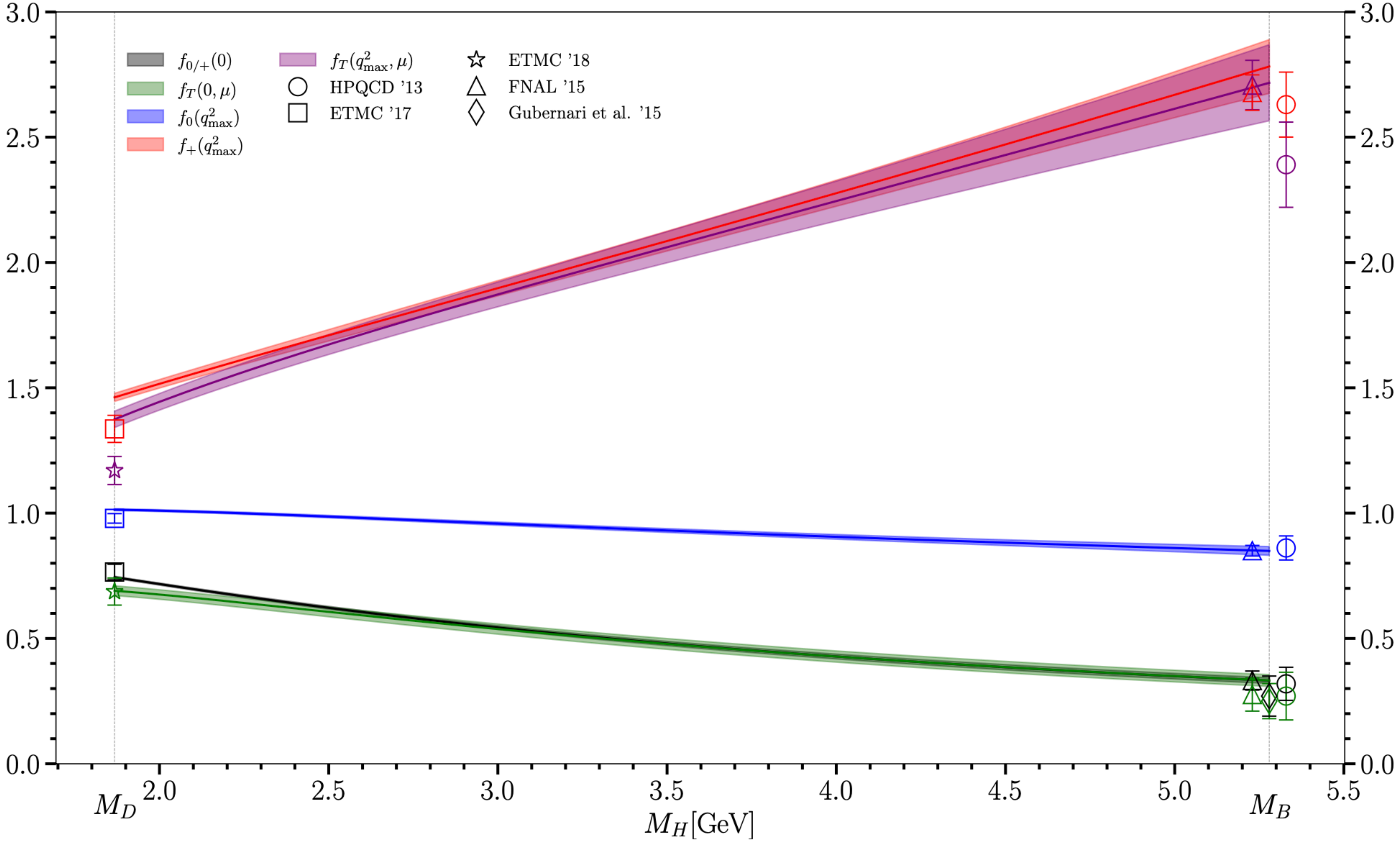
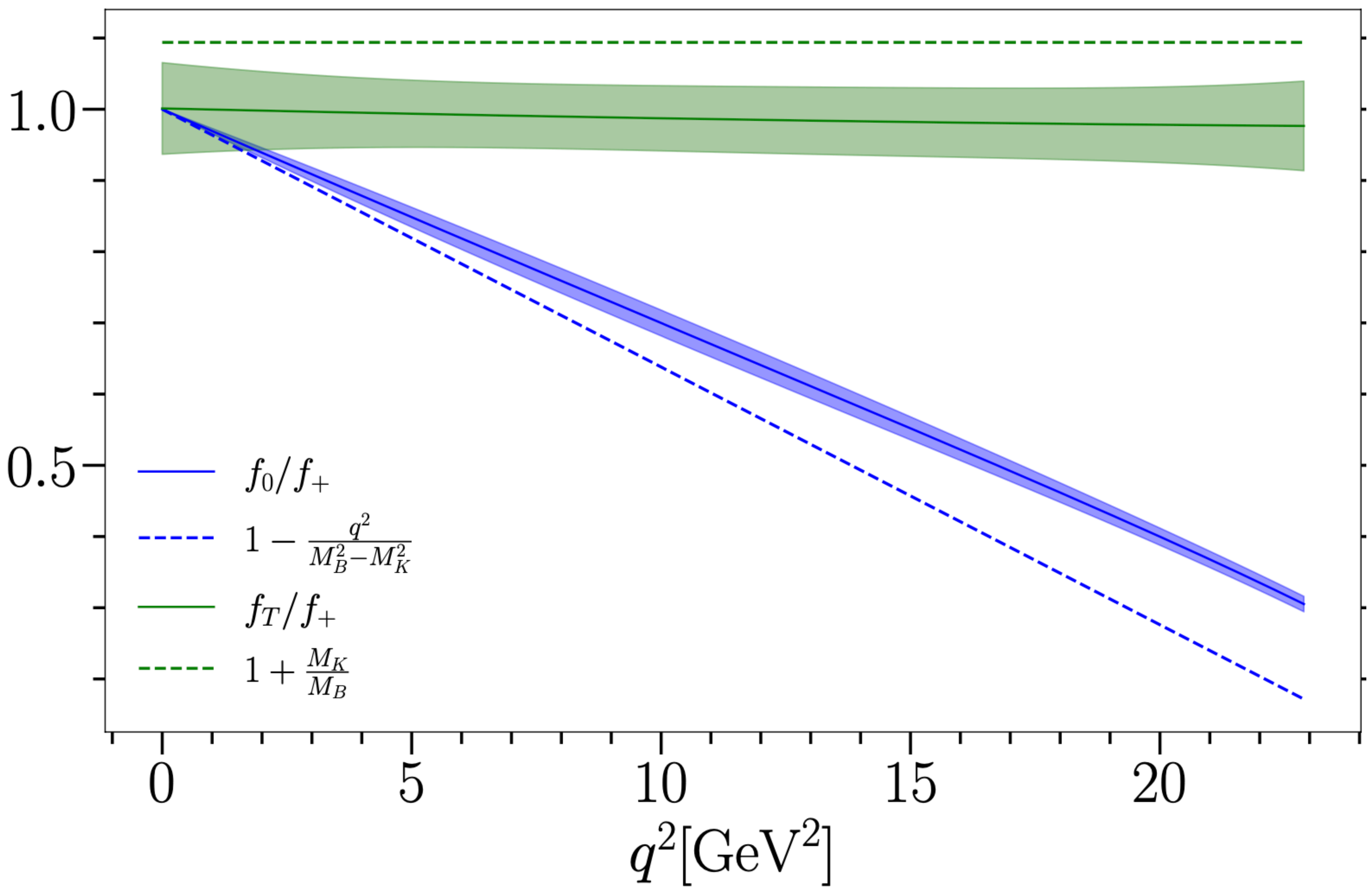


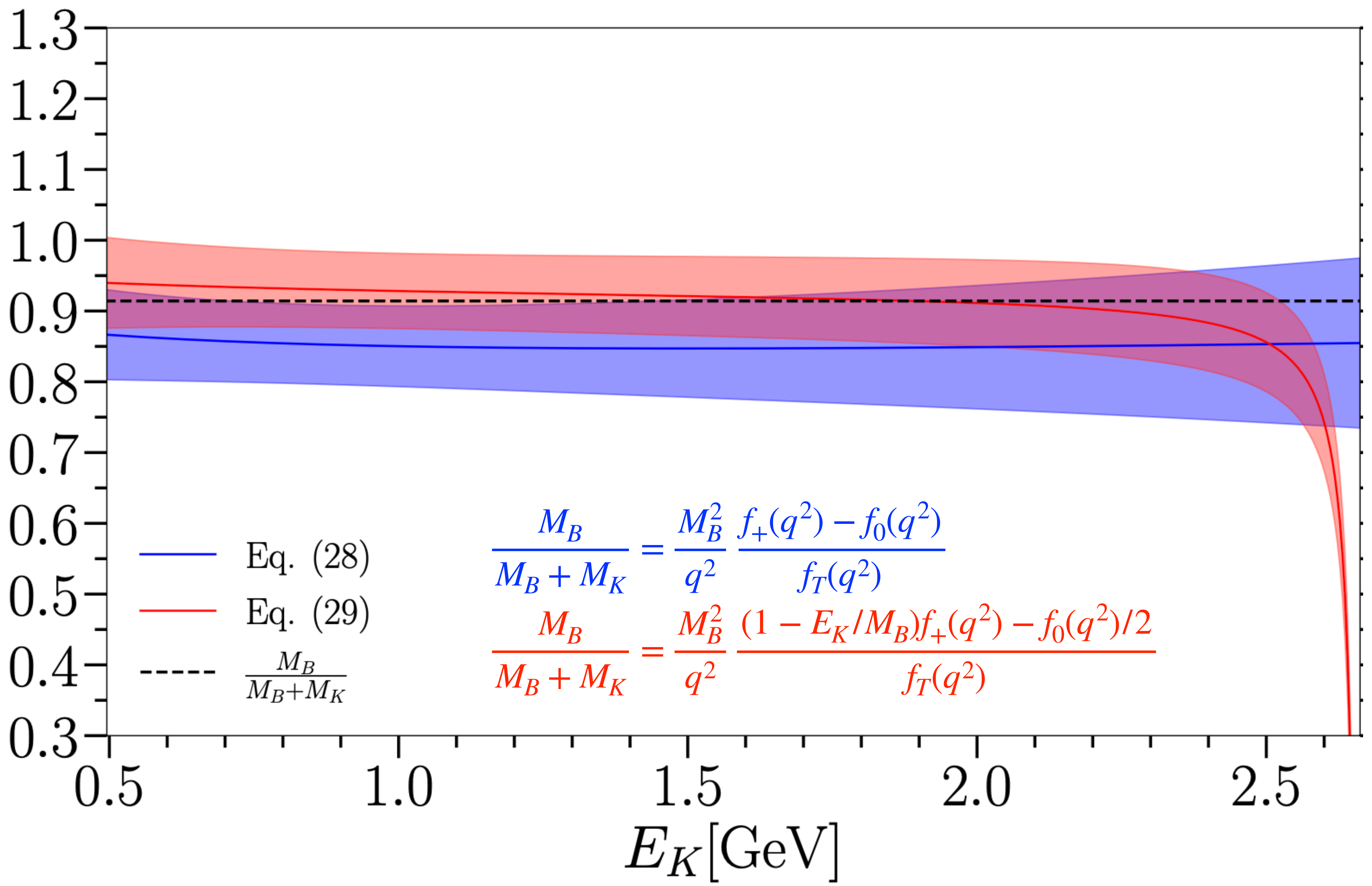
FIG. 10. The form factors at q^2_{max} and $q^2 = 0$ evaluated across the range of physical heavy masses from the D to the B. Other lattice studies [25, 28, 68, 69] of both $D \to K$ and $B \to K$ are shown for comparison. We also include some $B \to K$ results at $q^2 = 0$ from Gubernari et al. [70], a calculation using light cone sum rules. We do not include HPQCD's $D \to K$ results that share data with our calculation here [36]; see text for a discussion of that comparison. At the B end, data points are offset from M_B for clarity. Note that we have run Z_T to scale μ in this plot, where μ is defined linearly between 2 GeV and $m_b = 4.8$ GeV, according to Equation (26). The full running to 2 GeV from m_b results in a factor of 1.0773(17), applied to $f_T^{D\to K}$.

Form Factors: testing EFT expectation



Large Energy Effective Theory expectations

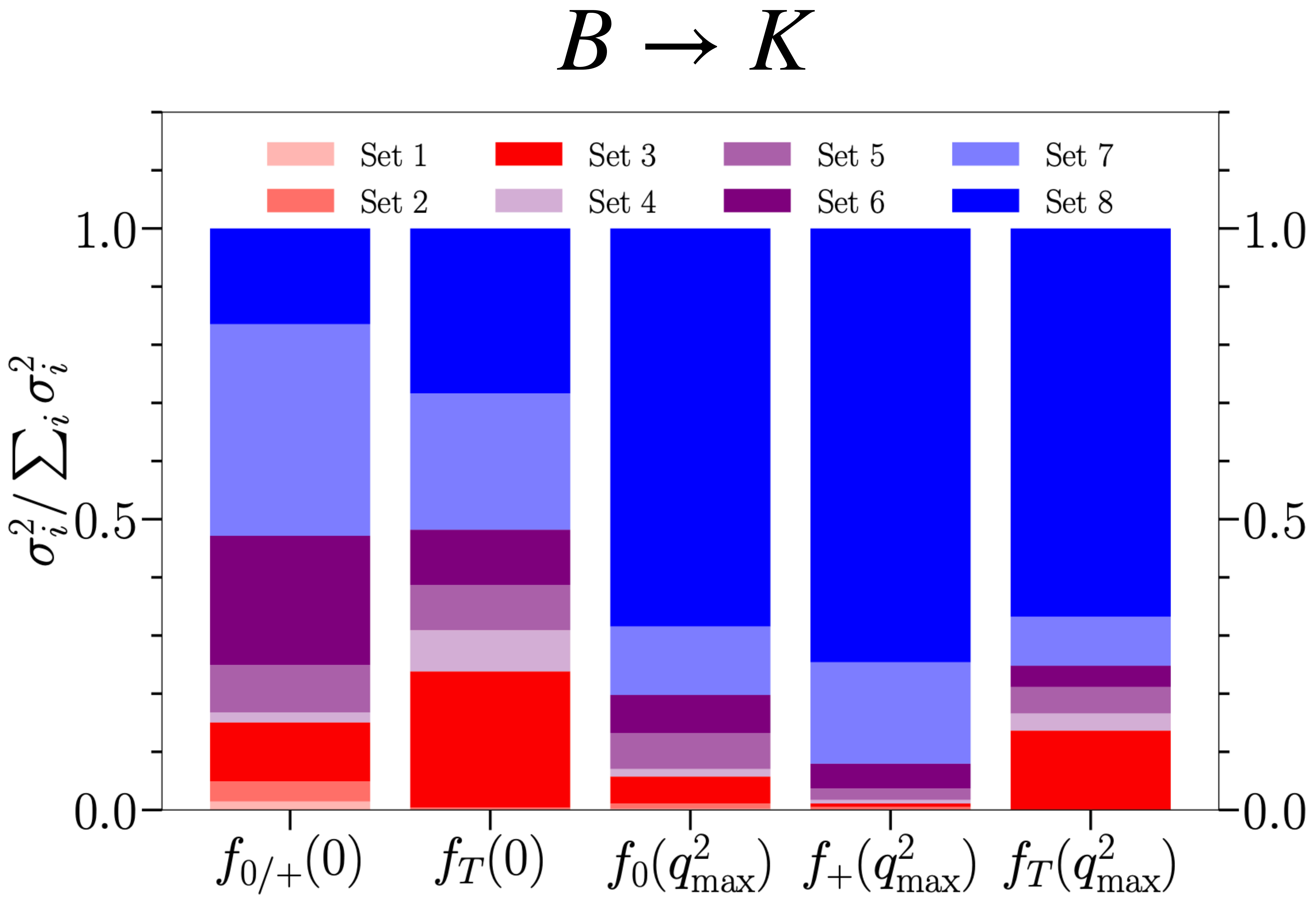
Charles, Le Yaouanc, Oliver, Pene, Raynal, PRD 60, 014001 (1999)

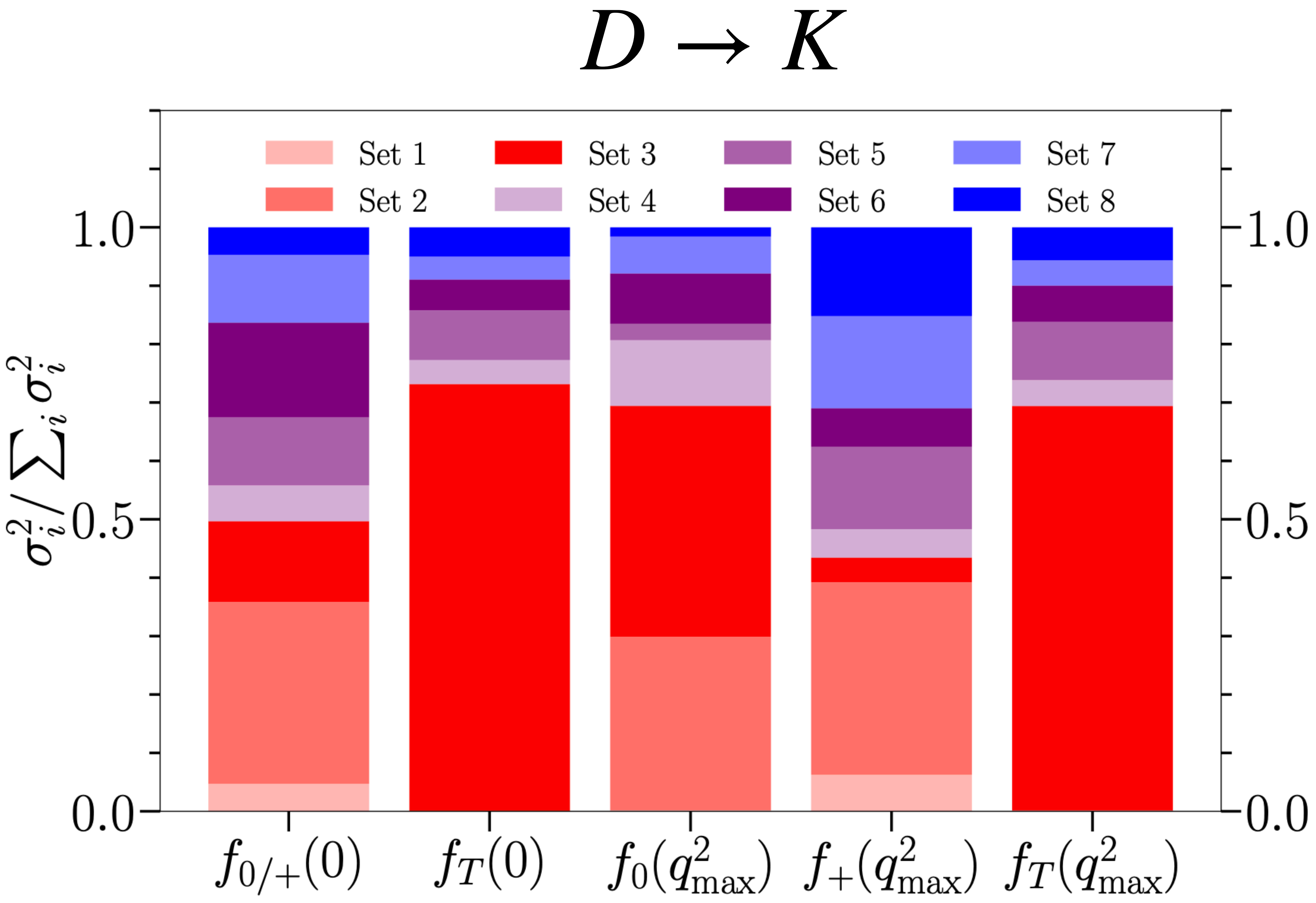


HQET expectations

Hill, PRD 73, 014012 (2006)

Form Factors: error budget by ensemble





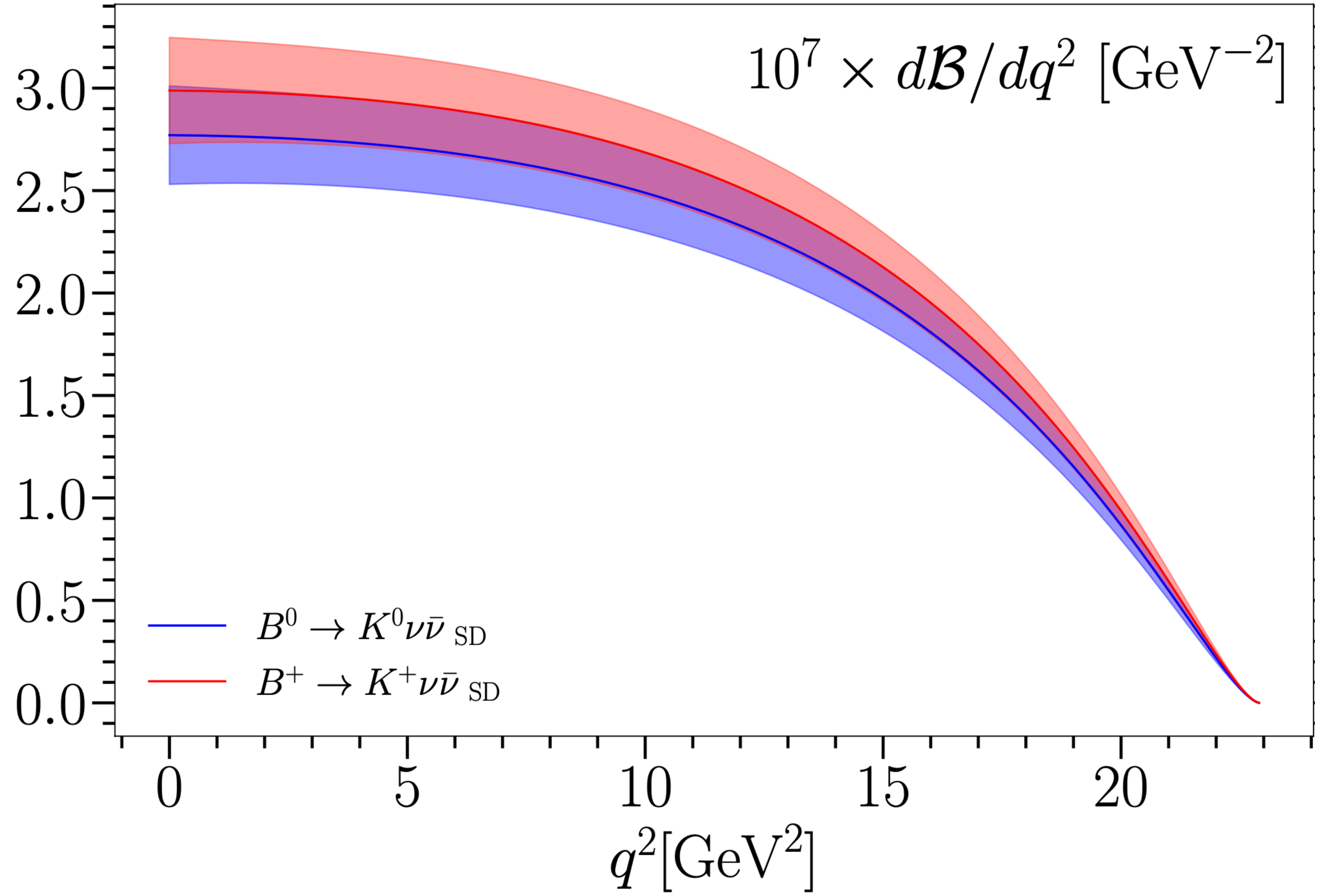
- blue are lattices with finest lattice spacing, needed to reach m_b
- red are lattices with physical light quark mass

Phenomenology: inputs

Parrott, Bouchard, and Davies, PRD 107 (2023) 119903

Parameter	Value	Reference
$\eta_{\mathrm{EW}} G_F$	$1.1745(23) \times 10^{-5}\ \mathrm{GeV}^{-2}$	[45], Eq. (7)
$m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}})$	1.2719(78) GeV	See caption
$m_b^{\overline{\mathrm{MS}}}(\mu_b)$	4.209(21) GeV	[46]
m_c	1.68(20) GeV	-
m_b	4.87(20) GeV	-
f_{K^+}	0.1557(3) GeV	[47–50]
f_{B^+}	0.1894(14) GeV	[51]
τ_{B^0}	1.519(4) ps	[52]
$\tau_{B^\pm}$	1.638(4) ps	[52]
$1/\alpha_{\mathrm{EW}}(M_Z)$	127.952(9)	[45]
$\sin^2\theta_W$	0.23124(4)	[45]
$\lvert V_{tb}V_{ts}^*\rvert$	0.04185(93)	[53]
$C_1(\mu_b)$	-0.294(9)	[54]
$C_2(\mu_b)$	1.017(1)	[54]
$C_3(\mu_b)$	-0.0059(2)	[54]
$C_4(\mu_b)$	-0.087(1)	[54]
$C_5(\mu_b)$	0.0004	[54]
$C_6(\mu_b)$	0.0011(1)	[54]
$C_7^{\mathrm{eff},0}(\mu_b)$	-0.2957(5)	[54]
$C_8^{\mathrm{eff}}(\mu_b)$	-0.1630(6)	[54]
$C_9(\mu_b)$	4.114(14)	[54]
$C_9^{\mathrm{eff},0}(\mu_b)$	$C_9(\mu_b) + Y(q^2)$	-
$C_{10}(\mu_b)$	-4.193(33)	[54]

Phenomenology: $B \to K\nu\bar{\nu}$



Decay	$\mathcal{B} \times 10^6$	Reference	
$B^0 \to K^0_S\nu\bar{\nu}$	< 13 (90% CL) Exp.	[32]	Belle '17
$B^0 \to K^0\nu\bar{\nu}$	< 49 (90% CL) Exp.	[34]	BaBar '13
	4.01(49)	[9]	FNAL '16
	$4.1^{+1.3}_{-1.0}$	[37]	Wang, Xiao '12
	4.67(35)	HPQCD '22	
$B^+ \to K^+\nu\bar{\nu}$	< 16 (90% CL) Exp.	[34]	
	< 19 (90% CL) Exp.	[32]	
	< 41 (90% CL) Exp.	[33]	Belle II '21
	5.10(80)	[75, 78]	Altmanshoffer et al '09; Kamenik, Smith '09
	$4.4^{+1.4}_{-1.1}$	[37]	
	3.98(47)	[76]	Buras et al '14
	4.94(52)	[9]	
	4.53(64)	[83]	Buras, Venturini '21
	4.65(62)	[84]	Buras, Venturini '22
	5.67(38)	HPQCD '22	

- modest improvement in precision
- cleaner theoretically; no resonances or nonfactorizable contributions

- roughly matches expected Belle-II precision at 50 ab^{-1}

Phenomenology: $B \to K\ell^+\ell^-$

- differential decay rate Γ (or branching fraction $\mathscr{B} = \tau_B \Gamma$) is measured

$$\frac{d\Gamma(B \to K\ell^+\ell^-)}{dq^2} = 2a_\ell + \frac{2}{3}c_\ell$$

$$a_\ell = \mathscr{C}\left[q^2|F_P|^2 + \frac{\lambda(q, M_B, M_K)}{4}(|F_A|^2 + |F_V|^2) + 4m_\ell^2 M_B^2 |F_A|^2 + 2m_\ell(M_B^2 - M_K^2 + q^2)\mathrm{Re}(F_P F_A^*)\right]$$

$$c_\ell = -\mathscr{C}\frac{\lambda(q, M_B, M_K)\beta_\ell^2}{4}(|F_A|^2 + |F_V|^2)$$

$$\mathscr{C} = \frac{(\eta_{\mathrm{EW}} G_F)^2 \alpha_{\mathrm{EW}}^2 |V_{tb} V_{ts}|^2}{2^9 \pi^5 M_B^3} \beta_\ell \sqrt{\lambda(q, M_B, M_K)}$$

- prediction depends on $F_{P,A,V}$ - functions of form factors and Wilson coefficients

Phenomenology: $B \to K\ell^+\ell^-$

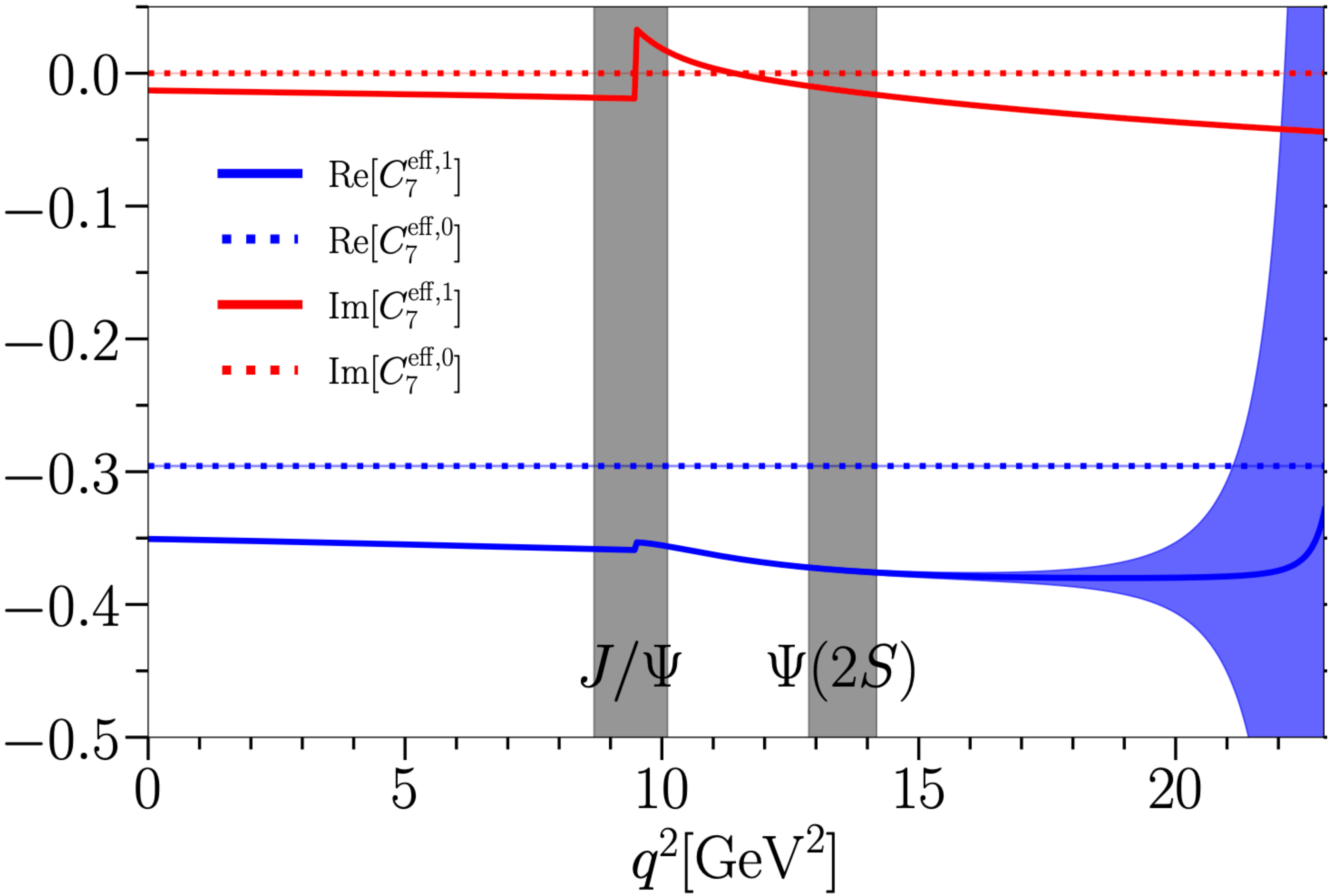
$$F_P = -m_\ell C_{10}\left[f_+ - \frac{M_B^2 - M_K^2}{q^2}(f_0 - f_+)\right]$$

$$F_A = C_{10} f_+$$

$$F_V = C_9^{\mathrm{eff},1} f_+ + \frac{2m_b^{\overline{\mathrm{MS}}}(\mu_b)}{M_B + M_K} C_7^{\mathrm{eff},1} f_T(\mu_b)$$

- $C_9^{\mathrm{eff},1}$ includes nonfactoriazable and $\mathcal{O}(\alpha_s)$ perturbative QCD corrections
- $C_7^{\mathrm{eff},1}$ includes $\mathcal{O}(\alpha_s)$ corrections FNAL/MILC, PRD 93, 034005 (2016)
  - these corrections give $< 1\sigma$ shift, slightly reducing tension with expt
- QED effect from final state radiation: 2% (5%) in $d\mathcal{B}/dq^2$ for $\mu(e)$; 1% in ratio $R(K)$
- other small uncertainties included (e.g. scale dependence of Wilson coefficients, $m_u \neq m_d$)

Phenomenology: $B \to K\ell^+\ell^-$ corrections

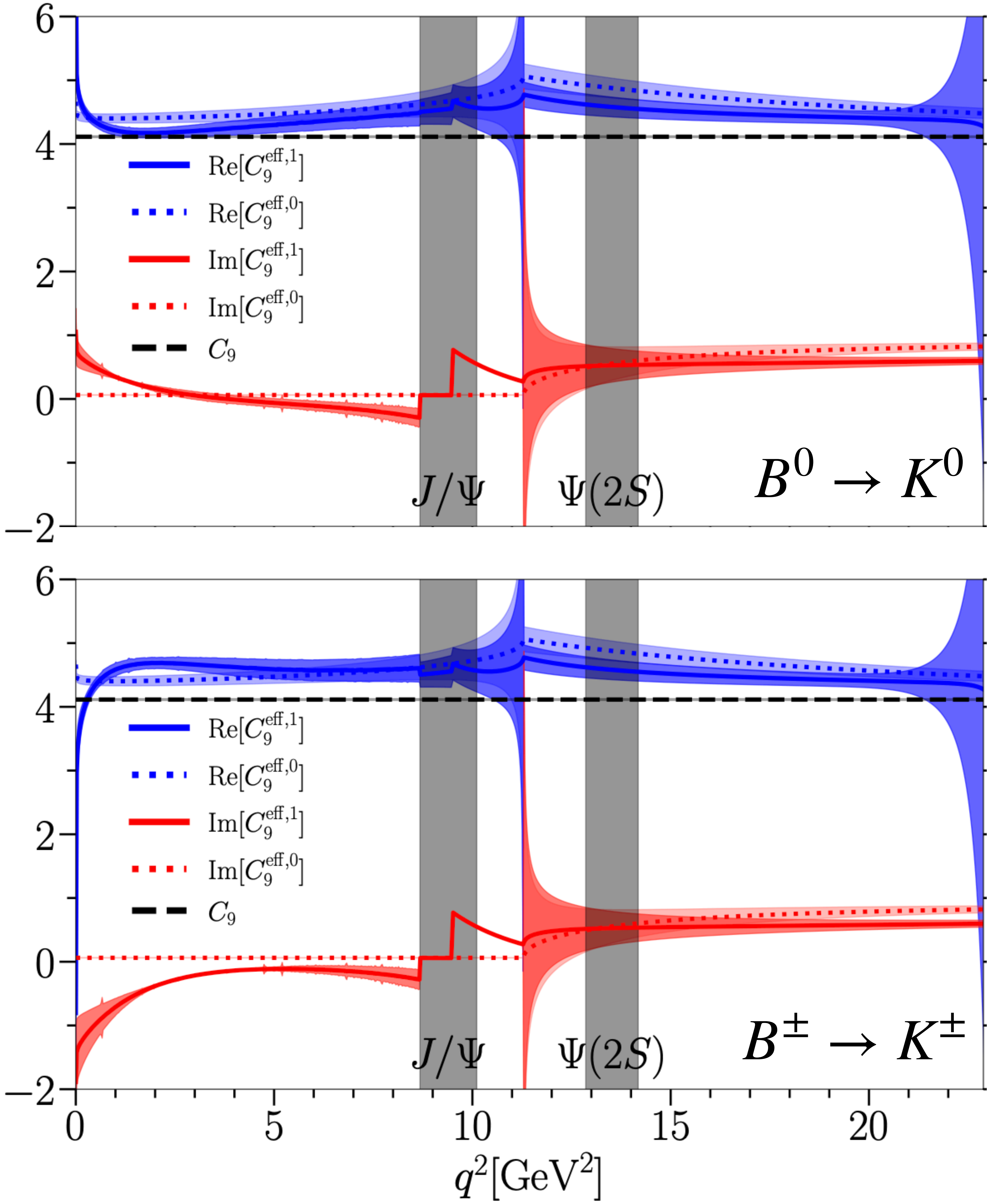


uncorrected C_7^{eff}

corrected C_7^{eff}

Corrections to C_7^{eff} are $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2

Phenomenology: $B \to K\ell^+\ell^-$ corrections



uncorrected C_9^{eff}

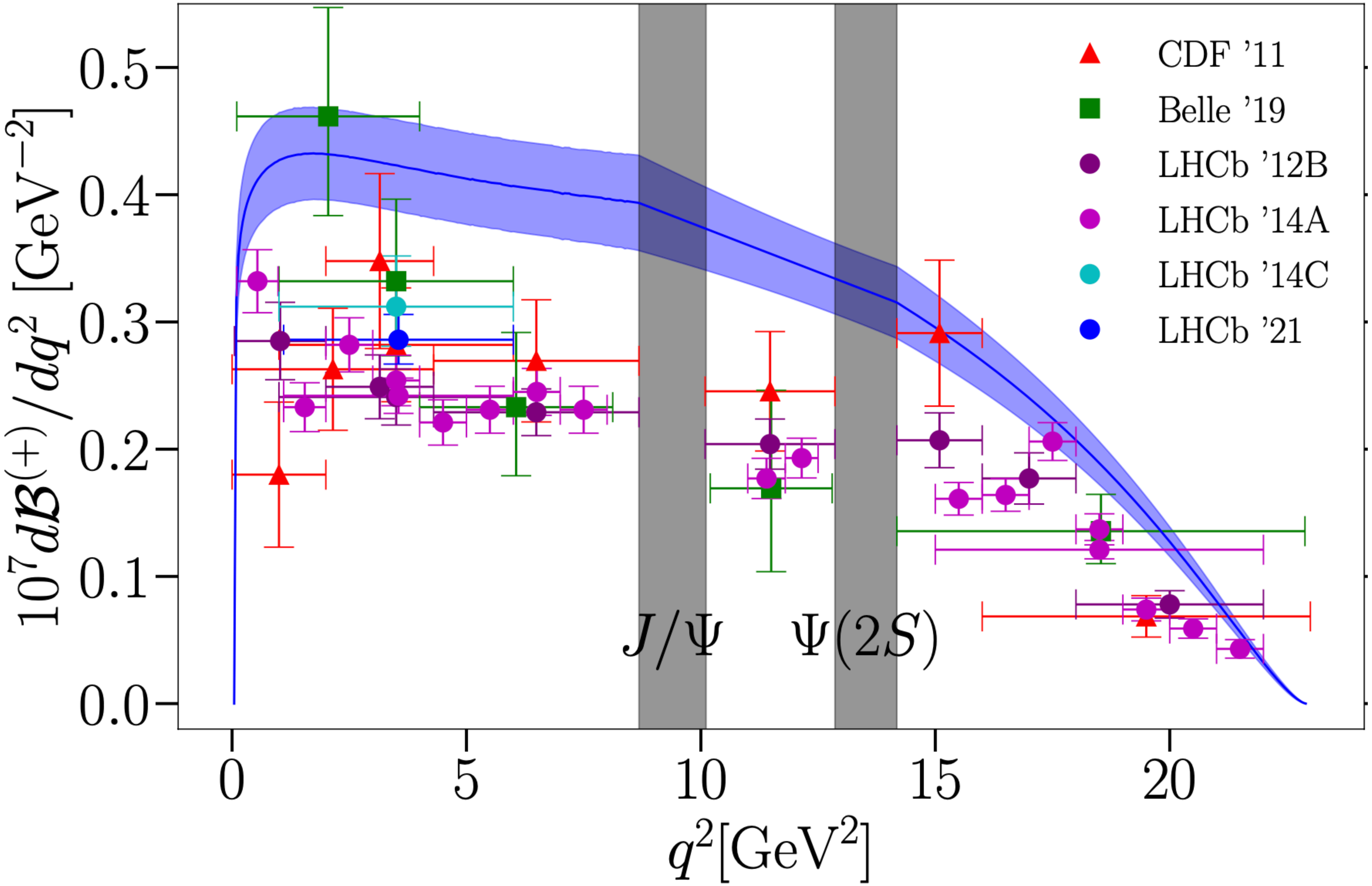
corrected C_9^{eff}

corrections to C_9^{eff} include:

- $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2
- non-factorizable corrections at low q^2

Beneke, Feldmann, Seidel, NPB 612, 25-58 (2001)

Phenomenology: $B \to K\ell^+\ell^-$



Focus on two well-behaved regions:

- $1.1 \le q^2/\mathrm{GeV}^2 \le 6$: below $c\bar{c}$ resonances; improved precision and increased tension
- $15 \le q^2/\mathrm{GeV}^2 \le 22$: above (dominant) $c\bar{c}$ resonances, include 2% uncertainty for broad resonances

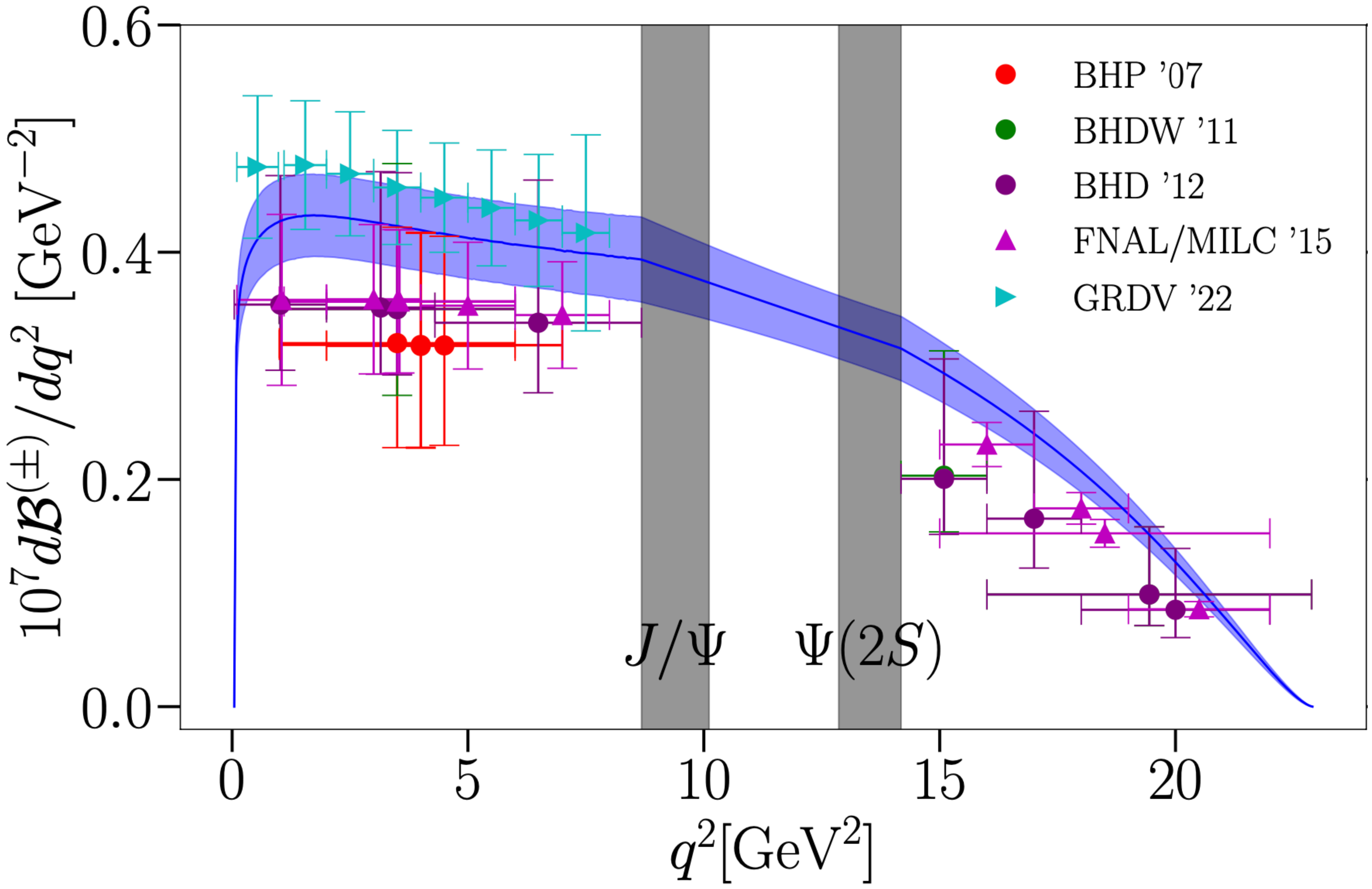
LHCb, Eur. Phys. J. C 77, 161 (2017)

Phenomenology: $B \to K\ell^+\ell^-$

Channel	Result	q^2/GeV^2 range	$\mathcal{B} \times 10^7$	Tension with HPQCD '22
$B^+ \to K^+e^+e^-$	LHCb '21	$(1.1, 6)$	$1.401^{+0.074}_{-0.069} \pm 0.064$	-3.3σ (-3.0σ)
$B^+ \to K^+e^+e^-$	HPQCD '22	$(1.1, 6)$	$2.07 \pm 0.17(\pm 0.10)_{\text{QED}}$	-
$B^+ \to K^+e^+e^-$	Belle '19	$(1, 6)$	$1.66^{+0.32}_{-0.29} \pm 0.04$	-1.2σ (-1.2σ)
$B^+ \to K^+e^+e^-$	HPQCD '22	$(1, 6)$	$2.11 \pm 0.18(\pm 0.11)_{\text{QED}}$	-
$B^0 \to K^0\mu^+\mu^-$	LHCb '14A	$(1.1, 6)$	$0.92^{+0.17}_{-0.15} \pm 0.044$	-3.6σ (-3.5σ)
$B^0 \to K^0\mu^+\mu^-$	HPQCD '22	$(1.1, 6)$	$1.74 \pm 0.15(\pm 0.04)_{\text{QED}}$	-
$B^0 \to K^0\mu^+\mu^-$	LHCb '14A	$(15, 22)$	$0.67^{+0.11}_{-0.11} \pm 0.035$	-3.2σ (-3.1σ)
$B^0 \to K^0\mu^+\mu^-$	HPQCD '22	$(15, 22)$	$1.16 \pm 0.10(\pm 0.02)_{\text{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	Belle '19	$(1, 6)$	$2.30^{+0.41}_{-0.38} \pm 0.05$	$+0.4\sigma$ $(+0.4\sigma)$
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(1, 6)$	$2.11 \pm 0.18(\pm 0.04)_{\text{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	LHCb '14A	$(1.1, 6)$	$1.186 \pm 0.034 \pm 0.059$	-4.7σ (-4.6σ)
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(1.1, 6)$	$2.07 \pm 0.17(\pm 0.04)_{\text{QED}}$	-
$B^+ \to K^+\mu^+\mu^-$	LHCb '14A	$(15, 22)$	$0.847 \pm 0.028 \pm 0.042$	-3.4σ (-3.3σ)
$B^+ \to K^+\mu^+\mu^-$	HPQCD '22	$(15, 22)$	$1.26 \pm 0.11(\pm 0.03)_{\text{QED}}$	-

- consistent tension with LHCb; $(\pm X\sigma)$ includes QED uncertainty
- single experiment (LHCb '14A, $B^+ \to K^+\mu^+\mu^-$, $1.1 \le q^2/\text{GeV}^2 \le 6$) near 5σ

Phenomenology: $B \to K\ell^+\ell^-$ vs other theory



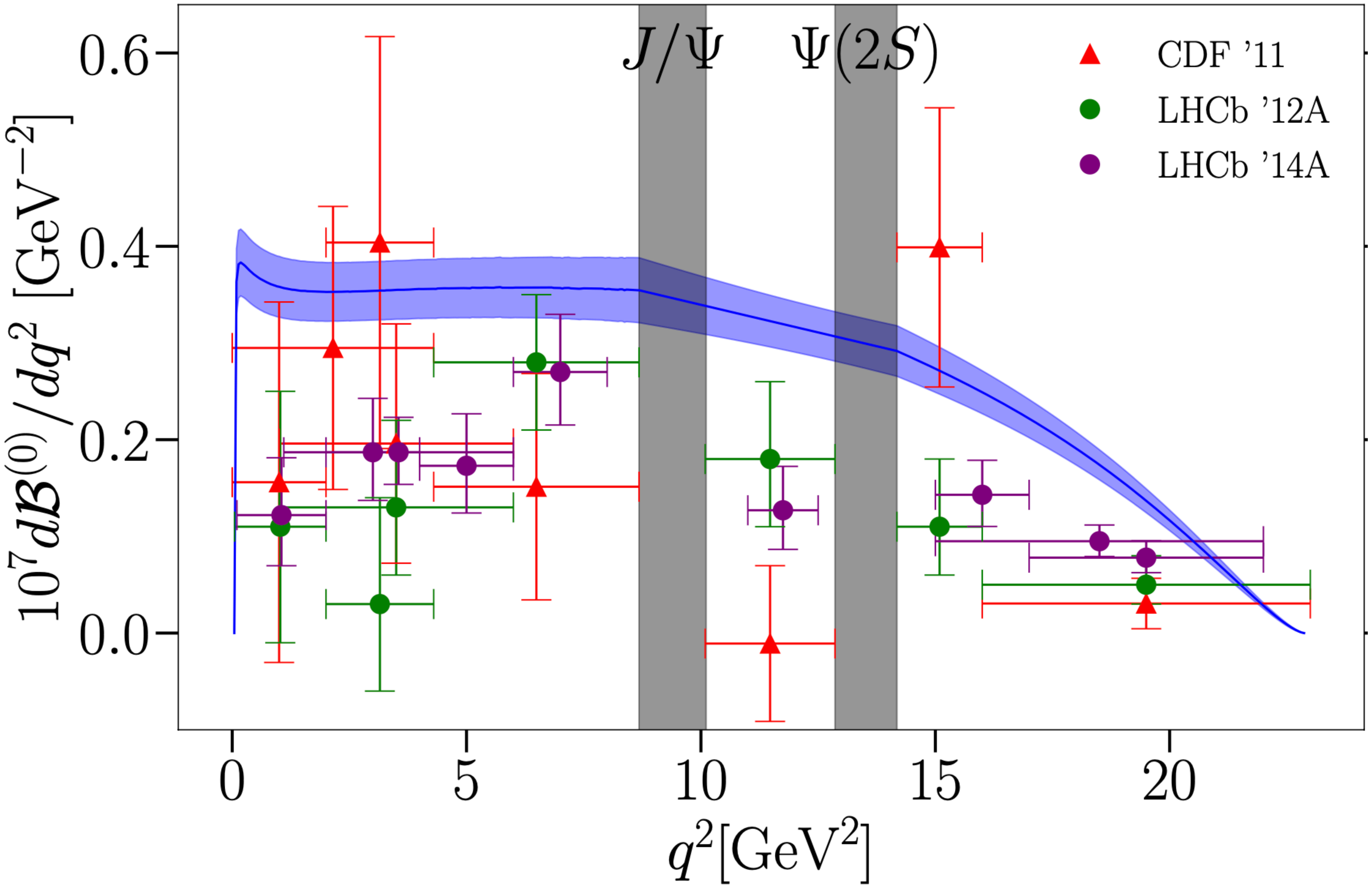
C. Bobeth, G. Hiller, and G. Piranishvili

C. Bobeth, G. Hiller, D. van Dyk, and C. Wacker

C. Bobeth, G. Hiller, and D. van Dyk

N. Gubernari, M. Reboud, D. van Dyk, and J. Virto

Phenomenology: $B \to K\ell^+\ell^-$



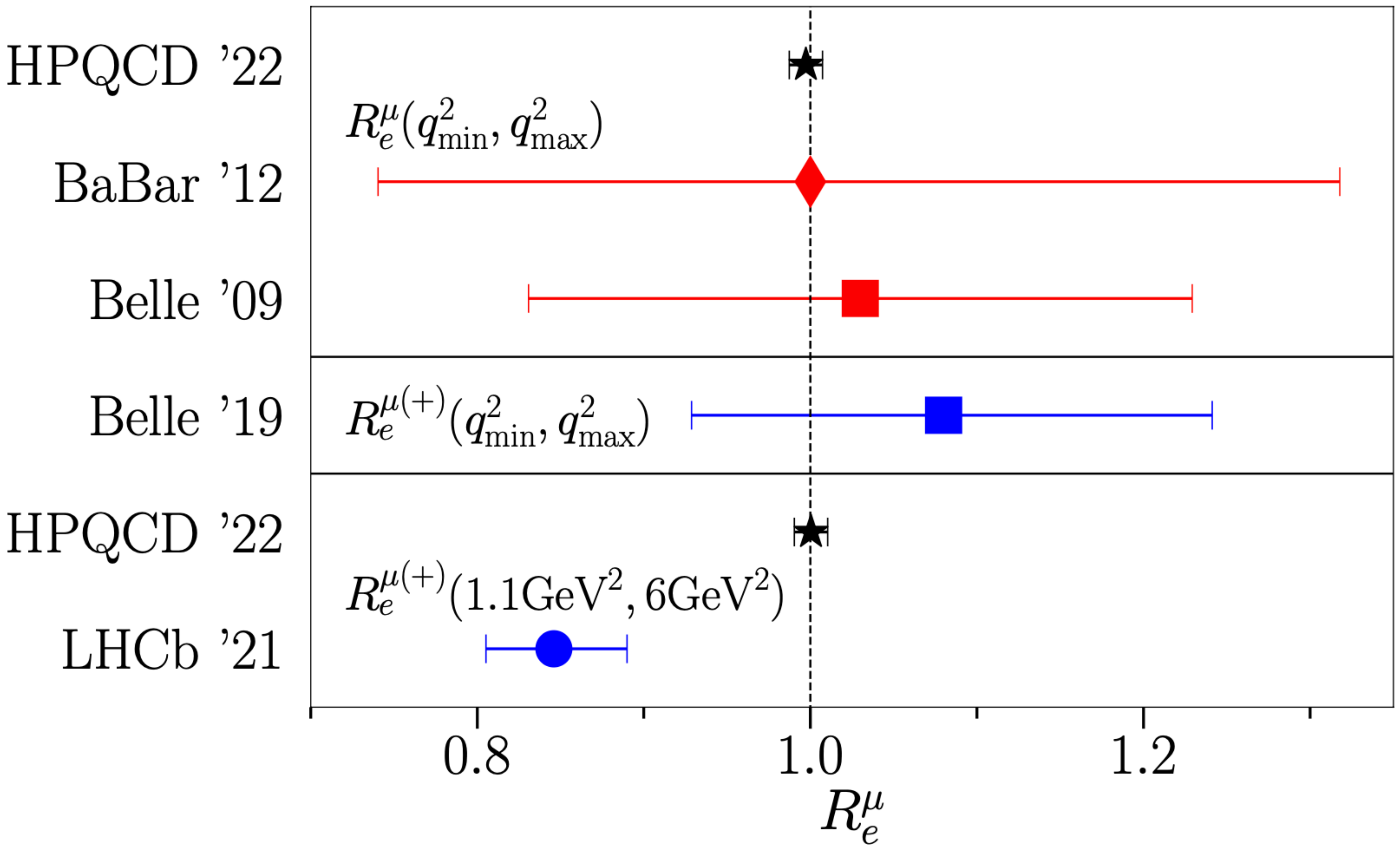
- LQCD calculation omits QED and in isospin limit, with $m_l = (m_u + m_d)/2$
- Differentiate between charged and neutral cases
  - 0.5% for form factor m_l
  - Missing final state radiation in experiment and no QED in form factors: 5% (2%) for e (μ) decay rates; 1% for R_K

Phenomenology: R_K

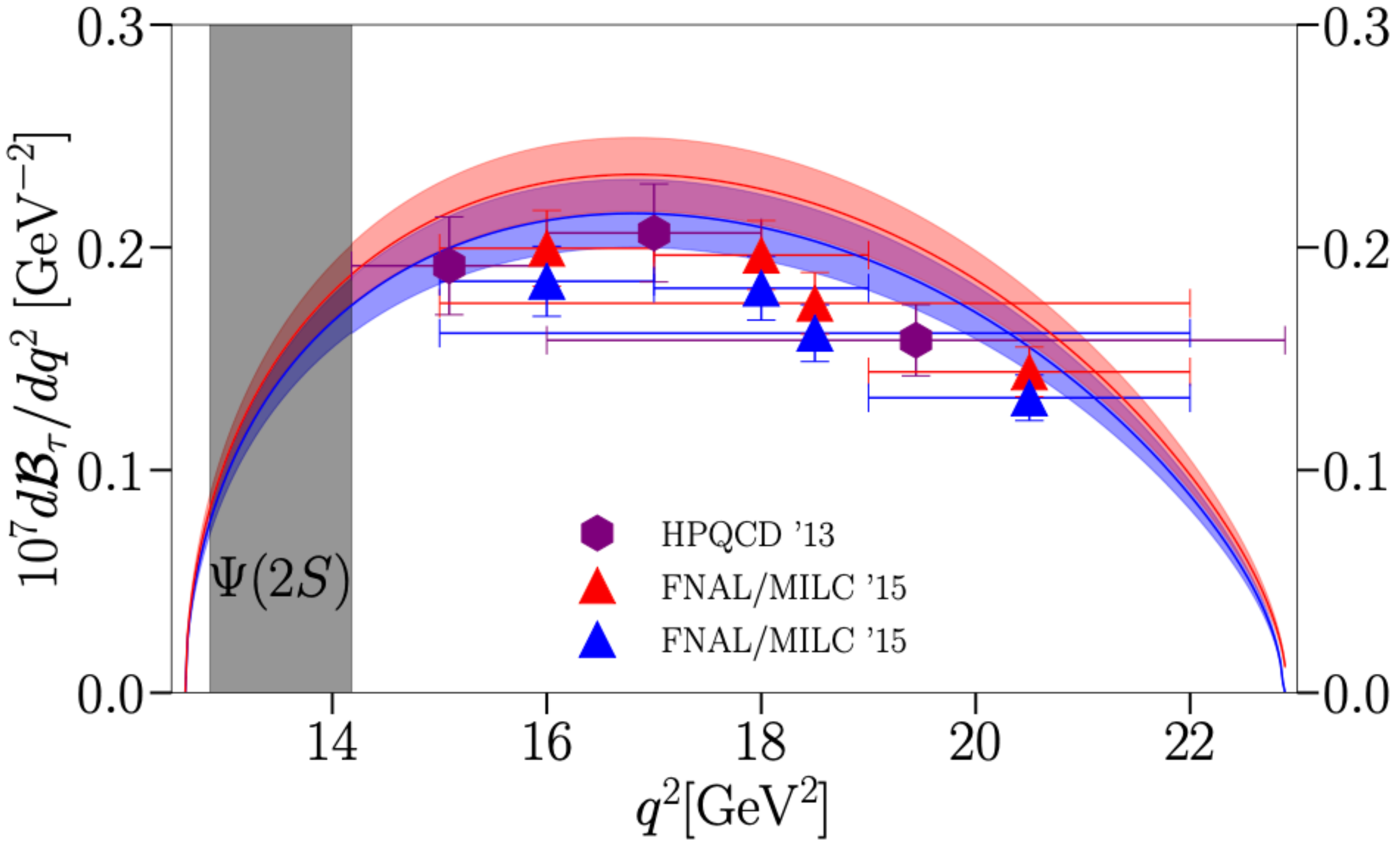
- Ratio of differential branching fractions

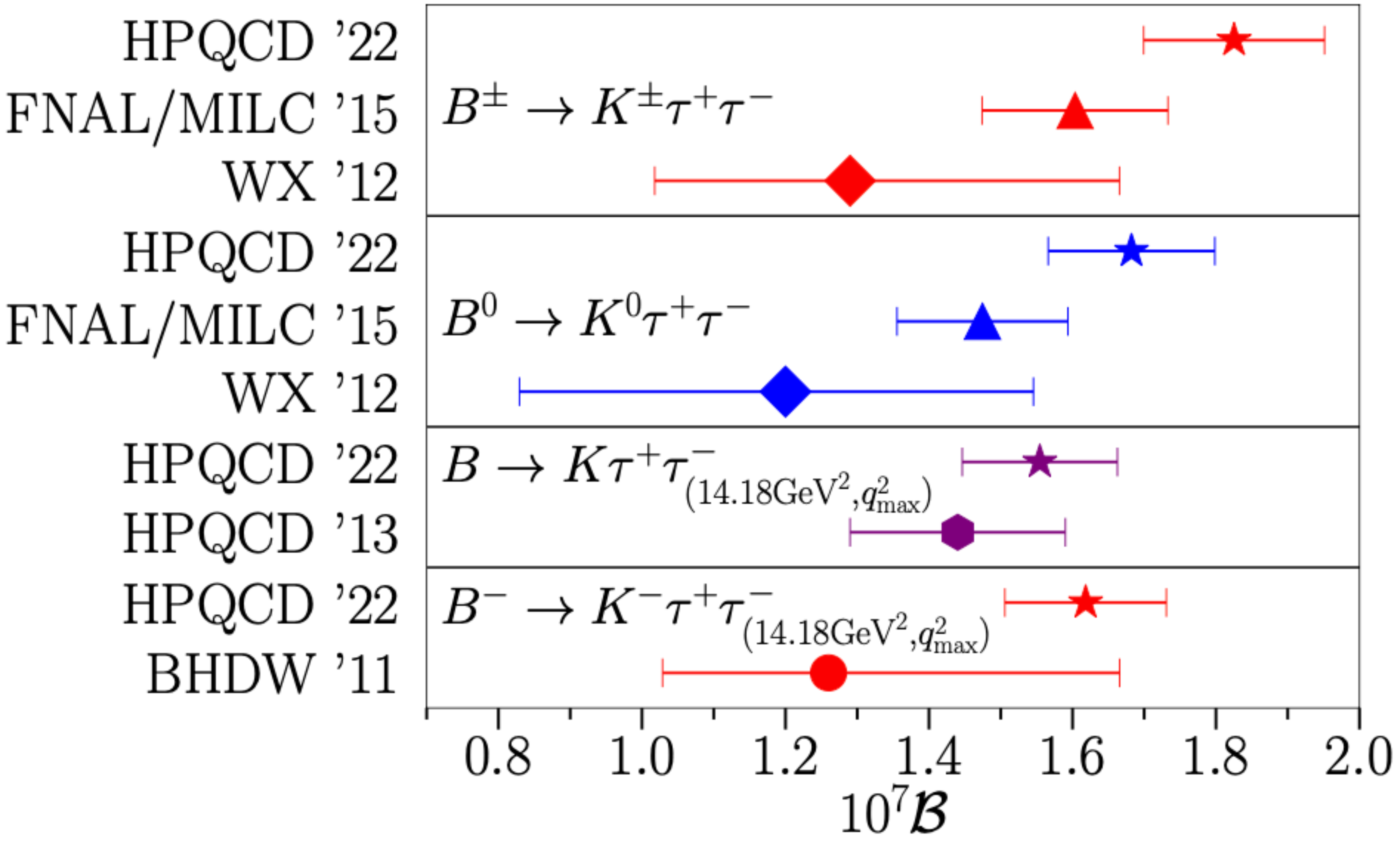
$$R^{\ell_1}_{\ell_2}(q^2_{\rm low}, q^2_{\rm upp}) = \frac{\int_{q^2_{\rm low}}^{q^2_{\rm upp}} \frac{d\mathscr{B}_{\ell_1}}{dq^2} dq^2}{\int_{q^2_{\rm low}}^{q^2_{\rm upp}} \frac{d\mathscr{B}_{\ell_2}}{dq^2} dq^2}$$

- Hadronic uncertainties largely cancel
- LHCb '21 is 3.1σ from SM



Phenomenology: $B \to K\tau^+\tau^-$





- generally, modest improvement over previous predictions