Introduction to $b \to d\nu\bar{\nu}$ decays

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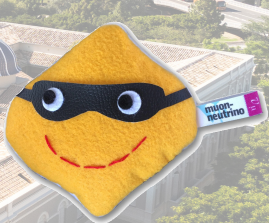

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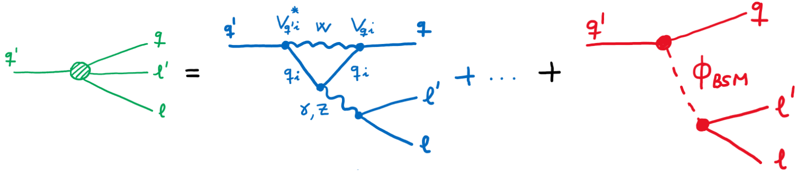
- **FCNCs are loop- and CKM-suppressed in the SM.**



- **BSM contributions could be comparable in size to the SM.**

  Bonus when leptons are attached (rare decays).

- **SM lepton couplings are flavour universal; LU can be tested.**
- **If $\ell \neq \ell'$ (zero in the SM), LFC can be tested as well.**

An excellent place to search for BSM physics!

EFT approach to rare B decays

1. **Symmetries to build all O_i up to desired dimension ($D = 6$):**

$$\mathcal{H}_{\text{eff}} \supset \frac{4\,G_F}{\sqrt{2}} V_{tq}^* V_{tb} \frac{\alpha_e}{4\pi} \sum_i c_i^{(\prime)}\, O_i^{(\prime)}\,, \quad c_i = C_i^{\text{SM}(\prime)} + C_i^{(\prime)}\,,$$

$$O_7^{(\prime)} = \frac{e}{16\pi^2} m_b\, (\bar{q}_{L\,(R)}\, \sigma_{\mu\nu}\, b_{R\,(L)})\, F^{\mu\nu}\,,$$
$$O_8^{(\prime)} = \frac{g_s}{16\pi^2} m_b\, (\bar{q}_{L\,(R)}\, \sigma_{\mu\nu}\, T^a b_{R\,(L)})\, G_a^{\mu\nu}\,,$$
$$O_{9\,(10)}^{(\prime)} = (\bar{q}_{L\,(R)} \gamma_\mu b_{L\,(R)})(\bar{\ell}\,\gamma^\mu (\gamma_5)\,\ell)\,, \ldots$$

2. **Compute $C_i(\mu_{\text{EW}})$ and RGEs to go down $\mu_{\text{low}} \approx m_b$.**

$$C_7^{\text{SM}}(m_b) \approx -0.3\,,\ C_8^{\text{SM}}(m_b) \approx -0.15\,,\ C_9^{\text{SM}}(m_b) \approx 4.1\,,\ C_{10}^{\text{SM}}(m_b) \approx -4.2\,.$$

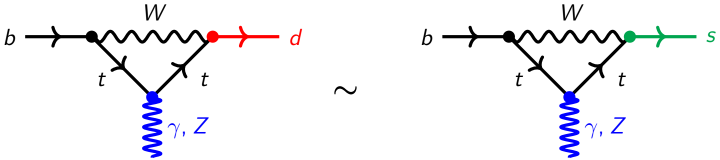
3. **$\langle O_i(\mu_{\text{low}})\rangle$ from non-perturbative techniques (Lattice, LCSR, ...)**

4. **Include resonances.**

Different phenomenology compared with rare charm decays!

Comparisons are odious, but they're necessary here

- **Differences between $b \to d\,\ell\ell$ & $b \to s\,\ell\ell$ in the SM:**



(1) CKM matrix elements: V_{td} vs V_{ts}, **(2)** Light quark masses: m_d vs m_s

$$C_i^{(b\to d)} \approx C_i^{(b\to s)}\ , \text{(CKMs factorized in } \mathcal{H}_{\text{eff}})$$

$$C_i'^{(b\to d)} \approx \left(\frac{m_d}{m_s}\right) C_i'^{(b\to s)}\ , (O_i' \text{ chiral suppression})$$

- **A violation would signal additional BSM sources of quark flavor violation (beyond (1) and (2)); an agreement would indicate similar effects (maybe NP?).**

$b \to s\, ee, \mu\mu$ transitions

- **Over the past decade, tensions with SM predictions have appeared in $b \to s\,\ell\ell$:**
  1. **Branching ratios:** several measurements tend to lie below SM expectations.
  2. **Angular observables:** a persistent tension (notably P_5' at low q^2) in global fits.
  3. **LU ratios:** latest LHCb updates of R_K and R_{K^*} are *consistent with the SM*.

- **Items 1–2 can be explained coherently by NP in a single operator:**

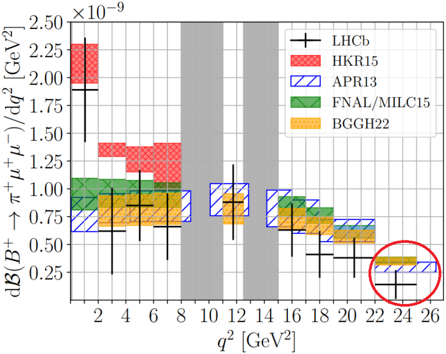
$$C_9^{(bs\mu)} \cdot O_9^{(bs\mu)} \;\approx\; (-1)\cdot(\bar{s}_L\gamma_\mu b_L)\,(\bar{\mu}\gamma^\mu\mu)$$

- **3 suggests discrepancies with the SM in $b \to s\, e^+e^-$, specifically reduced BRs and distorted angular distributions.**

Similar patterns in $B^+ \to \pi^+ \mu^+ \mu^-$

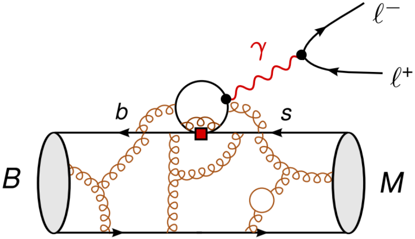
2209.04457

k	$[q^2_{\rm min}, q^2_{\rm max}]$ [GeV^2]	$\mathcal{B}_k^{(B\pi)}$ SM [10^{-9} GeV^{-2}]	experiment [10^{-9} GeV^{-2}]
1	[2, 4]	$0.80 \pm 0.12 \pm 0.05 \pm 0.04$ (FFs, CKMs, scale)	$0.62^{+0.39}_{-0.33} \pm 0.02$ (stat., syst.)
2	[4, 6]	$0.81 \pm 0.12 \pm 0.05 \pm 0.05$	$0.85^{+0.32}_{-0.27} \pm 0.02$
3	[6, 8]	$0.82 \pm 0.11 \pm 0.05 \pm 0.07$	$0.66^{+0.30}_{-0.25} \pm 0.02$
4	[11, 12.5]	$0.82 \pm 0.09 \pm 0.05 \pm 0.09$	$0.88^{+0.34}_{-0.29} \pm 0.03$
5	[15, 17]	$0.73 \pm 0.06 \pm 0.04 \pm 0.06$	$0.63^{+0.24}_{-0.19} \pm 0.02$
6	[17, 19]	$0.67 \pm 0.05 \pm 0.04 \pm 0.05$	$0.41^{+0.21}_{-0.17} \pm 0.01$
7	[19, 22]	$0.57 \pm 0.03 \pm 0.03 \pm 0.04$	$0.38^{+0.18}_{-0.15} \pm 0.01$
8	[22, 25]	$0.35 \pm 0.02 \pm 0.02 \pm 0.02$	$0.14^{+0.13}_{-0.09} \pm 0.01$
9	[15, 22]	$0.64 \pm 0.04 \pm 0.04 \pm 0.05$	$0.47^{+0.12}_{-0.10} \pm 0.01$
10	$[4m_\mu^2, (m_{B^+} - m_{\pi^+})^2]$	$17.9 \pm 1.9 \pm 1.1 \pm 1.5^\dagger$ GeV^2	$18.3 \pm 2.4 \pm 0.5$ GeV^2



- Very good agreement (below 1 σ) except for high-q^2 bins with 1.6 σ.

A lot of discussion on hadronic effects make not possible a consensus about these tensions....



Cleaner place: $b \to s\,\nu\bar{\nu}$ transitions

- **Why interesting?** Clean FCNCs: short–distance Z–penguin and W–box only; no photon pole or charmonium tails. Hadronic input $\Rightarrow$ form factors.

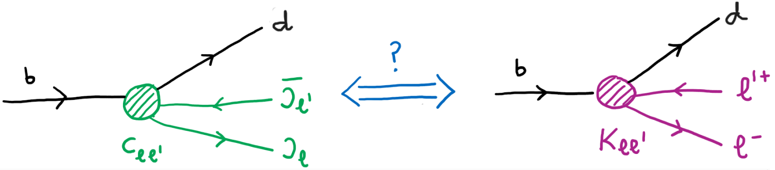
- **Status:** Belle II finds first evidence

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) = \left(2.3^{+0.5}_{-0.4}(\text{stat})^{+0.4}_{-0.3}(\text{syst})\right) \times 10^{-5}\,,$$

  about $\sim 3\sigma$ above SM.

- **While this points to NP, further scrutiny is required before firm conclusions can be drawn.**

$\boldsymbol{\ell}$ and $\boldsymbol{\nu_\ell}$ (with $\ell = e, \mu, \tau$) belong to the same **$\mathbf{SU(2)_L}$ doublet** in the SM.



$$\begin{pmatrix} c_{ee} & c_{e\mu} & c_{e\tau} \\ c_{\mu e} & c_{\mu\mu} & c_{\mu\tau} \\ c_{\tau e} & c_{\tau\mu} & c_{\tau\tau} \end{pmatrix} \overset{?}{\Longleftrightarrow} \begin{pmatrix} k_{ee} & k_{e\mu} & k_{e\tau} \\ k_{\mu e} & k_{\mu\mu} & k_{\mu\tau} \\ k_{\tau e} & k_{\tau\mu} & k_{\tau\tau} \end{pmatrix}$$

Neutrino flavour not tagged!

LU, cLFC, or general:

$$\frac{\mathcal{B}(b \to d\,\nu\bar{\nu})}{\mathcal{B}(b \to d\,\nu\bar{\nu})_{\rm SM}} \sim 1 + \frac{1}{3}\sum_{\ell,\ell'} c_{\ell\ell'}\ .$$

Charged leptons are tagged!

LU:

$$R_H \sim \frac{\mathcal{B}\left(b \to d\,\mu^+\mu^-\right)}{\mathcal{B}\left(b \to d\,e^+e^-\right)} \sim 1 + (k_{\mu\mu} - k_{ee})\ .$$

cLFC or general:

$$\mathcal{B}\left(b \to d\,\ell'^+\ell^-\right) \sim k_{\ell\ell'}\ .$$

Low-energy $|\Delta b| = |\Delta d| = 1$ EFT description

$$\boxed{b \to d\, \nu_\ell \bar{\nu}_{\ell'}} \quad \overset{?}{\Longleftrightarrow} \quad \boxed{b \to d\, \ell^- \ell'^+}$$

$$\mathcal{H}_{\text{eff}}^{\nu_\ell \bar{\nu}_{\ell'}} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{C}_k^{D\ell\ell'}\, Q_k^{D\ell\ell'}$$

$$\mathcal{H}_{\text{eff}}^{\ell^- \ell'^+} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{K}_k^{D\ell\ell'}\, O_k^{D\ell\ell'}$$

Only two operators (no RH neutrinos in the SM).

$$Q_{\mathrm{L\,(R)}}^{D\ell\ell'} = (\bar{s}_{L\,(R)} \gamma_\mu b_{L\,(R)})(\bar{\nu}_{\ell' L} \gamma^\mu \nu_{\ell L})$$

Additional operators are not connected.

$$O_{\mathrm{L\,(R)}}^{D\ell\ell'} = (\bar{s}_{L\,(R)} \gamma_\mu b_{L\,(R)})(\bar{\ell}'_L \gamma^\mu \ell_L),\ \ldots$$

Dineutrino BR is obtained via an incoherent neutrino flavor sum:

$$\mathcal{B}(b \to d\, \nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{B}(b \to d\, \nu_\ell \bar{\nu}_{\ell'}) \sim \sum_{\ell,\ell'} \left| \mathcal{C}_L^{D\ell\ell'} \pm \mathcal{C}_R^{D\ell\ell'} \right|^2$$

$\mathcal{C}^P$ and $\mathcal{K}^P$ are in the mass basis. $P = D$ ($P = U$) $\to$ down-quark sector (up-quark sector).

Correlate neutrinos and charged leptons with $SU(2)_L$

1. **$SU(2)_L \times U(1)_Y$–invariant effective theory:**[1]

$$\mathcal{L}^{\rm LO}_{\rm SMEFT} \supset \frac{C^{(1)}_{\ell q}}{v^2}\,\bar{Q}\gamma_\mu Q\,\bar{L}\gamma^\mu L + \frac{C^{(3)}_{\ell q}}{v^2}\,\bar{Q}\gamma_\mu \tau^a Q\,\bar{L}\gamma^\mu \tau^a L$$
$$+\frac{C_{\ell u}}{v^2}\,\bar{U}\gamma_\mu U\,\bar{L}\gamma^\mu L + \frac{C_{\ell d}}{v^2}\,\bar{D}\gamma_\mu D\,\bar{L}\gamma^\mu L$$

2. **Writing in $SU(2)_L$-components:** ($C \to$ dineutrinos and $K \to$ dileptons in the gauge basis)

$$C_L^D = K_L^U = \frac{2\pi}{\alpha}\left(C^{(1)}_{\ell q} - C^{(3)}_{\ell q}\right), \quad C_R^D = K_R^D = \frac{2\pi}{\alpha}\,C_{\ell d}\,.$$

3. **$C_R^D = K_R^D$ holds in our EFT! But, $C_L^D = K_L^U$!**

4. In terms of mass eigenstates, $Q_\alpha = (u_{L\alpha}, V_{\alpha\beta}\, d_{L\beta})$, $L_i = (\nu_{Li}, W^*_{ki}\,\ell_{Lk})$

$$\boxed{\mathcal{C}_L^D = W^\dagger\,\mathcal{K}_L^U\,W + \mathcal{O}(\lambda), \qquad \mathcal{C}_R^D = W^\dagger\,\mathcal{K}_R^D\,W\ ,}$$

[1] 1008.4884

Connection via "trace identities" in the mass basis

Same story as in $c \to u\nu\bar{\nu}$ transitions (difference with charm is that SM contribution is relevant).

$$\mathcal{B}(b \to d\,\nu\bar{\nu}) \;\sim\; \sum_{\ell,\ell'} \left| \mathcal{C}_L^{D\,\ell\ell'} \pm \mathcal{C}_R^{D\,\ell\ell'} \right|^2 \;=\; \mathrm{Tr}\big[(\mathcal{C}_L^D \pm \mathcal{C}_R^D)(\mathcal{C}_L^D \pm \mathcal{C}_R^D)^\dagger\big]$$

$$=\; \mathrm{Tr}\big[W^\dagger(\mathcal{K}_L^U \pm \mathcal{K}_R^D) W\, W^\dagger (\mathcal{K}_L^U \pm \mathcal{K}_R^D)^\dagger W\big] \;=\; \sum_{\ell,\ell'} \left| \mathcal{K}_L^{U\,\ell\ell'} \pm \mathcal{K}_R^{D\,\ell\ell'} \right|^2 \;+\; \mathcal{O}(\lambda)\,.$$

⋆ **Independent of the PMNS matrix and subleading $\mathcal{O}(\lambda)$ corrections!**

⋆ **Predictions for dineutrino rates from lepton-specific structures $\mathcal{K}_{L,R}^{\ell\ell'}$ can be probed with charged-lepton measurements!**

Dineutrino branching ratios $B \to F\, \nu\bar{\nu}$

$$\mathcal{B}_{BF} = A_{+}^{BF}\, x^{+} + A_{-}^{BF}\, x^{-}, \qquad x^{\pm} = \sum_{\ell,\ell'} \left| \mathcal{C}_L^{D\,\ell\ell'} \pm \mathcal{C}_R^{D\,\ell\ell'} \right|^2$$

→ **Long-distance dynamics $A_{\pm}^{BF}$:** LCSR (low q^2) + lattice (high q^2).

→ **Short-distance dynamics $x^{\pm}$:** Wilson coefficients (SM + BSM).

→ **Excellent complementarity $\mathcal{B}_{BF}$:**

- $A_{-}^{BP} = 0$ **in** $B \to P\, \nu\bar{\nu}$**.**
- $A_{-}^{BV} > A_{+}^{BV}$ **in** $B \to V\, \nu\bar{\nu}$**.**

$B \to F$	A_{+}^{BF} $[10^{-8}]$	A_{-}^{BF} $[10^{-8}]$
$B^0 \to \pi^0$	154 ± 16	0
$B^+ \to \pi^+$	332 ± 34	0
$B^0 \to \rho^0$	59 ± 12	573 ± 233
$B^+ \to \rho^+$	126 ± 26	1236 ± 502

SM predictions, current exp. limits, and future sensitivities

$B \to F$	$\mathcal{B}_{\text{SM}}$ [10^{-8}]	Derived EFT limits [10^{-6}]	Exp. limit (90% C.L.) [10^{-6}]
$B^0 \to \pi^0$	5.4 ± 0.6	6	9
$B^+ \to \pi^+$	12 ± 1	14	14
$B^0 \to \rho^0$	22 ± 8	14	40
$B^+ \to \rho^+$	48 ± 18	30	30

LUV? EFT violation

Any violation implies NP beyond the EFT assumptions.

Belle II: For $50\,\mathrm{ab}^{-1}$, $N(b\bar{b}) \approx 10^{11}$; naively, this implies an order-of-magnitude improvement on the current experimental limits.

1. **SU(2)$_L$ link:** $b_R \to d_R\,\ell^-\ell'^+$ $\rightleftarrows$ $b \to d\,\nu\bar{\nu}$ $\leftrightarrows$ $t_L \to u_L\,\ell^-\ell'^+$

   $\mathcal{K}^{D\ell\ell'}_{R,\mathrm{NP}}$ $\mathcal{B}(B \to F\,\nu\bar{\nu})$ $\mathcal{K}^{U\ell\ell'}_{L,\mathrm{NP}}$

2. **LU limit** $\left(\mathcal{K}^{P\ell\ell'}_{A,\mathrm{NP}} = \mathcal{K}^{P\mu\mu}_{A,\mathrm{NP}}\,\delta_{\ell\ell'}\right)$**:**

   $b_R \to d_R\,\mu^-\mu^+$ $\rightleftarrows$ $b \to d\,\nu\bar{\nu}$ $\leftrightarrows$ $t_L \to u_L\,\mu^-\mu^+$

   Global fits $\mathcal{B}(B \to F\,\nu\bar{\nu})_{\mathrm{LU}}$ Weak bounds

3. **Complementarity among $B \to F\,\nu\bar{\nu}$ modes:**

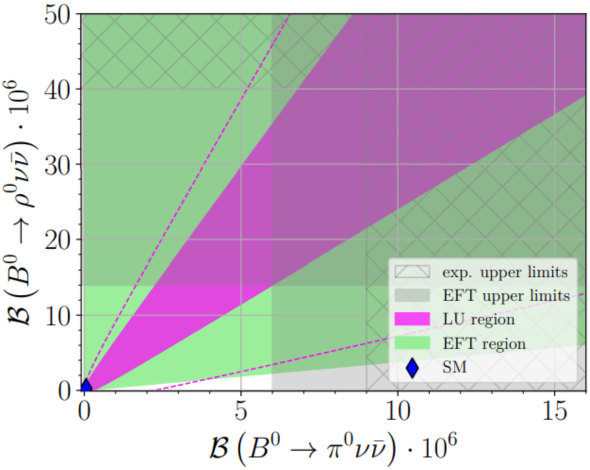
   $B \to \rho\,\nu\bar{\nu}$ $\rightleftarrows$ $b_R \to d_R\,\mu^-\mu^+$ $\leftrightarrows$ $B \to \pi\,\nu\bar{\nu}$

   $\mathcal{B}(B \to \rho\,\nu\bar{\nu})_{\mathrm{LU}}$ Global fits $\mathcal{B}(B \to \pi\,\nu\bar{\nu})_{\mathrm{LU}}$

$$\mathcal{B}(B \to \rho\,\nu\bar{\nu})_{\mathrm{LU}} = \frac{A^{B\rho}_+}{A^{B\pi}_+}\,\mathcal{B}(B \to \pi\,\nu\bar{\nu})_{\mathrm{LU}} + 3\,A^{B\rho}_-\left(\sqrt{\frac{\mathcal{B}(B \to \pi\,\nu\bar{\nu})_{\mathrm{LU}}}{3\,A^{B\pi}_+}} \mp 2\,|\mathcal{K}^{D\mu\mu}_{R,\mathrm{NP}}|\right)^2$$

Testing LU with $b \to d\,\nu\bar{\nu}$ decays

Inputs: FFs 2109.01675,2209.04457 $+$ global fit to $b \to d\,\mu^+\mu^-$ data: $\mathcal{K}_{R,\mathrm{NP}}^{D\mu\mu}$.



$\ell^+\ell'^-$ couplings bounded by $\nu\bar{\nu}$ modes

1. **Derived EFT limits:** $x^+ \lesssim 4.2$, $x^- \lesssim 2.4$

2. ***$SU(2)_L$*–link:** $x^\pm = \sum_{i,j} \left|\mathcal{C}_{L,\,\mathrm{SM}}^{D_{13}ij} + \mathcal{K}_L^{tuij} \pm \mathcal{K}_R^{bdij}\right|^2$, $\mathcal{K}_k^{tuij} \equiv \mathcal{K}_{k,\,\mathrm{NP}}^{U_{13}ij}$, $\mathcal{K}_k^{bdij} \equiv \mathcal{K}_{k,\,\mathrm{NP}}^{D_{13}ij}$,

$$\Longrightarrow \sum_{i,j} \left|X_{\mathrm{SM}}\,\delta_{ij} + \kappa_L^{tuij} + (-)\,\kappa_R^{bsij}\right|^2 \lesssim 5.8\,(3.3)\cdot 10^4 \;,\quad \kappa \equiv \mathcal{K}\cdot\left(V_{tb}V_{td}^*\right)^{-1}$$

Data	$\lvert\kappa_A^{q_1q_2\ell\ell'}\rvert$	ee	$\mu\mu$	$\tau\tau$	$e\mu$	$e\tau$	$\mu\tau$
Rare B decays to Dineutrinos	$\lvert\kappa_R^{bd\ell\ell'}\rvert$	210	210	210	210	210	210
	$\kappa_L^{tu\ell\ell'}$	$[-197, 223]$	$[-197, 223]$	$[-197, 223]$	210	210	210
	$\lvert\kappa_R^{bs\ell\ell'}\rvert$	35	35	35	32	32	32
	$\kappa_L^{tc\ell\ell'}$	$[-22, 47]$	$[-22, 47]$	$[-22, 47]$	32	32	32
Rare B decays to Charged dileptons	$\kappa_R^{bd\ell\ell'}$	~ 10	$[-4, 4]$	~ 2500	~ 20	~ 280	~ 200
	$\kappa_L^{bd\ell\ell'}$	~ 10	$[-8, 2]$	~ 2500	~ 20	~ 280	~ 200
	$\kappa_R^{bs\ell\ell'}$	$\mathcal{O}(1)$	$[0.2, 0.8]$	~ 800	~ 2	~ 50	~ 60
	$\kappa_L^{bs\ell\ell'}$	$\mathcal{O}(1)$	$[-1.6, -1.1]$	~ 800	~ 2	~ 50	~ 60
Drell-Yan	$\lvert\kappa_{L,R}^{bd\ell\ell'}\rvert$	583	314	1122	260	800	866
	$\lvert\kappa_{L,R}^{bs\ell\ell'}\rvert$	331	178	637	142	486	529
$t+\ell$	$\kappa_L^{tt\ell\ell'}$	$[-196, 243]$	$[-196, 243]$	–	–	–	–

Limits on $e\tau$, $\tau\tau$ from dineutrino data
result in factors 1.3 and 12 stronger than charged lepton data!

Improved limits on $b \to d\,\tau^+\tau^-$ decays

Using bounds on $\kappa_A^{bs\tau\tau}$ from dineutrino data, etc, we find the following upper limits:

$$\mathcal{B}(B^0 \to \tau^+\tau^-) \lesssim 6.0 \cdot 10^{-4}\,,$$

well above its SM prediction $\sim \mathcal{O}(10^{-8})$, and a factor 3.5 stronger than its current exp. upper limit (95% C.L., LHCb):

$$\mathcal{B}(B^0 \to \tau^+\tau^-)_{\text{exp}} < 2.1 \times 10^{-3}\ .$$

Belle II with $5\,\text{ab}^{-1}$ ($50\,\text{ab}^{-1}$) is expected to place following (projected) upper limits on the branching ratios 1808.10567

$$\mathcal{B}(B^0 \to \tau^+\tau^-)_{\text{proj}} <\ 3\,(1) \cdot 10^{-4}\ .$$

Any signal between $6.0 \cdot 10^{-4}$ would hint to NP with violation of the assumptions taken.

Similar conclusions for $b \to s\nu\bar{\nu}$ transitions

$$\boxed{b_R \to s_R\, \ell^- \ell'^+} \rightleftarrows \boxed{b \to s\, \nu_\ell \bar{\nu}_{\ell'}} \leftrightarrows \boxed{t_L \to c_L\, \ell^- \ell'^+}$$

Dineutrino modes provide excellent complementarity

$$c_R \to u_R\, \ell^- \ell'^+ \rightleftarrows c \to u\, \nu_\ell \bar{\nu}_{\ell'} \leftrightarrows s_L \to d_L\, \ell^- \ell'^+$$

$$b_R \to d_R\, \ell^- \ell'^+ \rightleftarrows b \to d\, \nu_\ell \bar{\nu}_{\ell'} \leftrightarrows t_L \to u_L\, \ell^- \ell'^+$$

$$b_R \to s_R\, \ell^- \ell'^+ \rightleftarrows b \to s\, \nu_\ell \bar{\nu}_{\ell'} \leftrightarrows t_L \to c_L\, \ell^- \ell'^+$$

$$s_R \to d_R\, \ell^- \ell'^+ \rightleftarrows s \to d\, \nu_\ell \bar{\nu}_{\ell'} \leftrightarrows c_L \to u_L\, \ell^- \ell'^+$$

Conclusions

- Dineutrino modes are powerful probes for discovering BSM physics.
- Within SMEFT, $SU(2)_L$ invariance relates the operators for dineutrinos $q^\alpha q^\beta \nu\bar{\nu}$ and charged dileptons $q^\alpha q^\beta \ell^+ \ell^-$.
- Similar mechanism with applications across charm, beauty, and kaon phenomenology.
- This connection provides complementary tests of LFV and leads to improved limits on the τ couplings.
- Predictions are well suited for present and future e^+e^- facilities like *Belle II*.

Thank you for your attention!