$B \to K^{(*)}\nu\bar{\nu}$ in the SM and observables beyond rate

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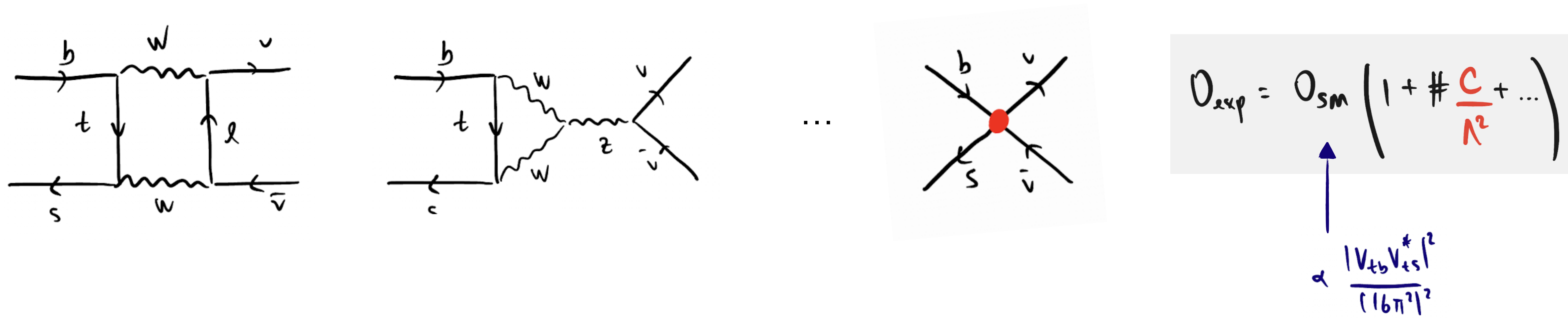
IJCLab (Orsay)

Based on [2301.06990],
in collaboration with D. Becirevic and G. Piazza

Belle-II Physics Week, 8 October, 2025

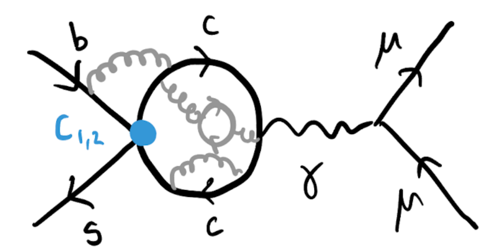
Motivation

- **Flavor Changing Neutral Current (FCNC)** processes are powerful indirect probes of **New Physics (NP)** effects since they are **GIM suppressed** in the SM.



- The **main obstacle** to probing NP at **low energies** is the careful assessment of **hadronic uncertainties**:

  ⇒ Decays based on the $b \to s\nu\bar{\nu}$ transition are **theoretically cleaner** than those based on $b \to s\ell\ell$, since they are **not affected** by **problematic** long-distance contributions from $c\bar{c}$ **loops**.



- **Further motivation** to study these decays:

  ⇒ **First observation** of $B^+ \to K^+\nu\bar{\nu}$ by **Belle-II** and upcoming measurements.

  ⇒ **New determination** of the $B \to K$ **form-factors** on the lattice. [HPQCD, '22]

  ⇒ **Opportunity** to probe **NP** couplings to **3rd gen leptons** — *i.e.*, $L_\tau = (\nu_\tau, \tau_L)^T$.

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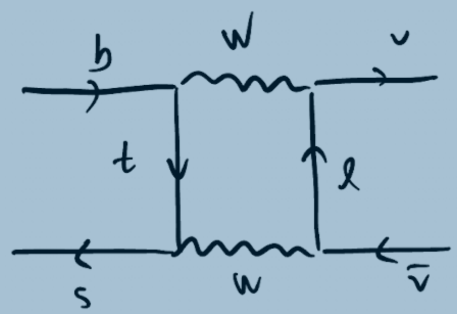

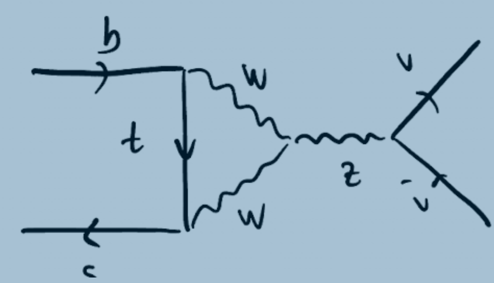

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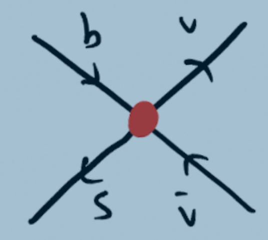
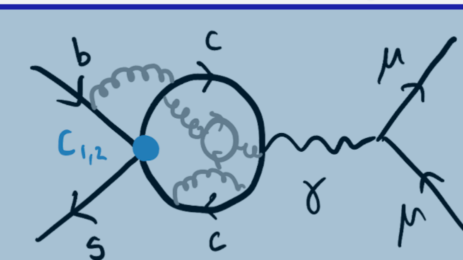


$$O_{exp} = O_{SM}\left(1 + \#\frac{C}{\Lambda^2} + \dots\right)$$

$$\propto \frac{|V_{tb}V_{ts}^*|^2}{(16\pi^2)^2}$$

The $B \to K^{(*)}\nu\nu$ **decays** are among the **most promising** (and **theoretically cleaner!**) **probes of NP** in current flavor experiments — *together with $K^+ \to \pi^+\nu\bar{\nu}$ [NA62] and $K_L \to \pi^0\nu\bar{\nu}$ [KOTO!]*



- **Further motivation** to study these decays:
  - ⇒ First observation of $B^+ \to K^+\nu\bar{\nu}$ by **Belle-II** and upcoming measurements.
  - ⇒ New determination of the $B \to K$ **form-factors** on the lattice. [HPQCD, '22]
  - ⇒ Opportunity to probe **NP** couplings to **3rd gen leptons** — *i.e.*, $L_\tau = (\nu_\tau, \tau_L)^T$.

Motivation

These processes can be used to learn more about the **SM** (in particular, the **CKM parameters** and **non-pertubative QCD**), or possible **New Physics effects** — *provided that the SM predictions are under control*:

Decay rate	=	Kinematics	CKM	Hadronic quantities	+	NP effects?

Exp: Belle-II

See talk by de Marino

Nonperturbative QCD

See theory talks by: Bona; Bouchard, van Dyk; Altmanshofer, Bolton, Cornella, Marzocca, Novoa, Schmidt

This talk:

i. **Critically revisit** the **SM predictions** for $B \to K^{(*)}\nu\bar{\nu}$.

ii. **What can we learn** from $b \to s\nu\bar{\nu}$ **observables** *(in the SM and beyond)*?

Outline

Main focus: quantities beyond the total decay rates — *namely, observables built from kinematical distributions and/or ratios of observables.*

I. $B \to K^{(*)}\nu\bar{\nu}$ in the SM

$B \to K^{(*)}\nu\bar{\nu}$ in the SM

See talk by Altmannshofer

- **Effective Hamiltonian** in the SM:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F \lambda_t}{\sqrt{2}} \frac{\alpha_{\rm em}}{2\pi} \sum_i C_L^{\rm SM} (\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu \nu_{Li}) + \text{h.c.}\,,$$

$\lambda_t = V_{tb}V_{ts}^*$

- **Short-distance** contributions known to **good precision**:

$$C_L^{\rm SM} = -X_t/\sin^2\theta_W$$
$$= -6.32(7)$$

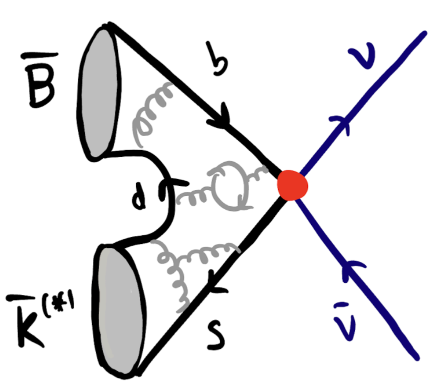
Including NLO QCD and two-loop EW contributions:

$$X_t = 1.462(17)(2)$$

[Buchala et al. '93, '99], [Misiak et al. '99], [Brod et al. '10]

Two main sources of uncertainties:

i) Hadronic matrix-element:



Known Lorentz factors

$$\langle K^{(*)}|\bar{s}_L\gamma^\mu b_L|B\rangle = \sum_a K_a^\mu\, \mathcal{F}_a(q^2)$$

Form-factors (e.g., LQCD)

ii) CKM matrix:

From CKM unitarity:

$$|V_{tb}V_{ts}^*| = |V_{cb}|\,(1 + \mathcal{O}(\lambda^2))$$

Which value to take (incl. vs. excl.)?

I. Form-factors: $B \to K\nu\bar{\nu}$

See talks by Bouchard and van Dyk

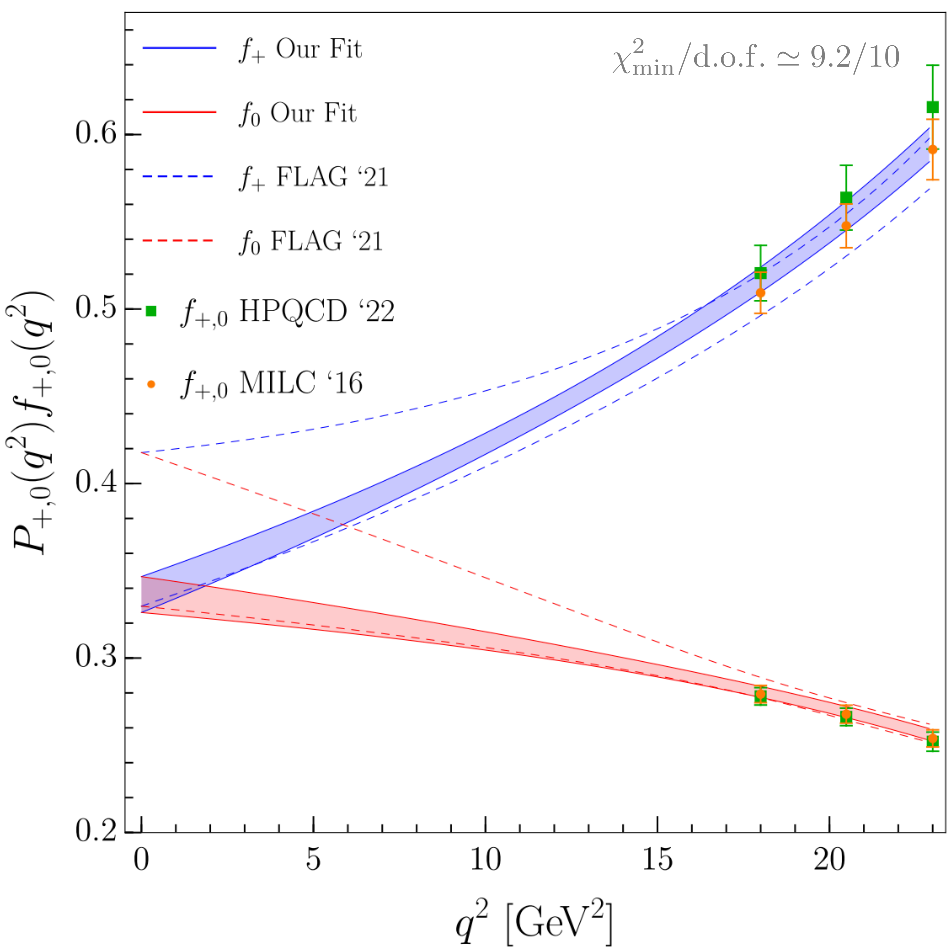
- **Lattice QCD** data available at **nonzero recoil** ($q^2 \neq q^2_{\text{max}}$) for all form-factors:

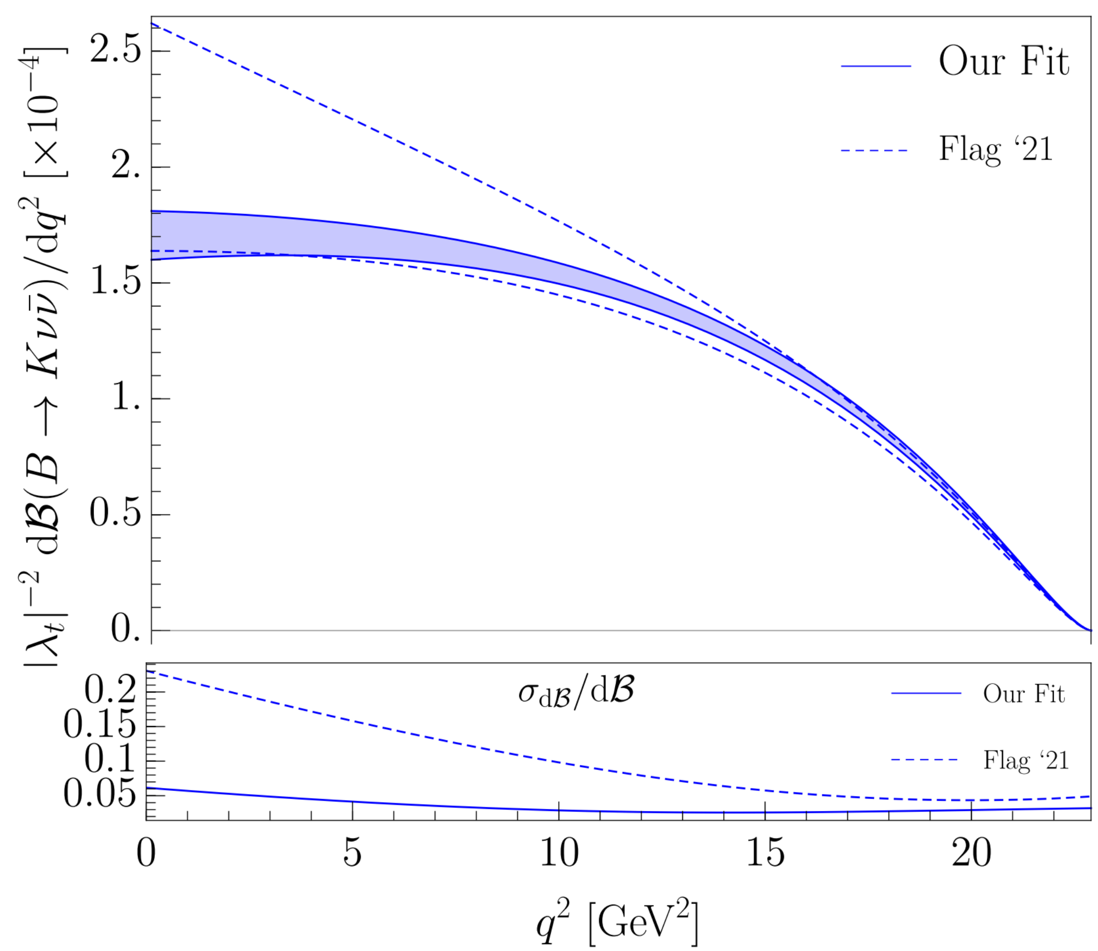
$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_K^2}{q^2}q^\mu\right] f_+(q^2) + q^\mu \frac{m_B^2 - m_K^2}{q^2} f_0(q^2)$$

with $f_+(0) = f_0(0)$.

Only form-factor needed for $B \to K\nu\bar{\nu}$!

- **[NEW]** We update the FLAG average by combining [HPQCD '22] results with [FNAL/MILC '16]:





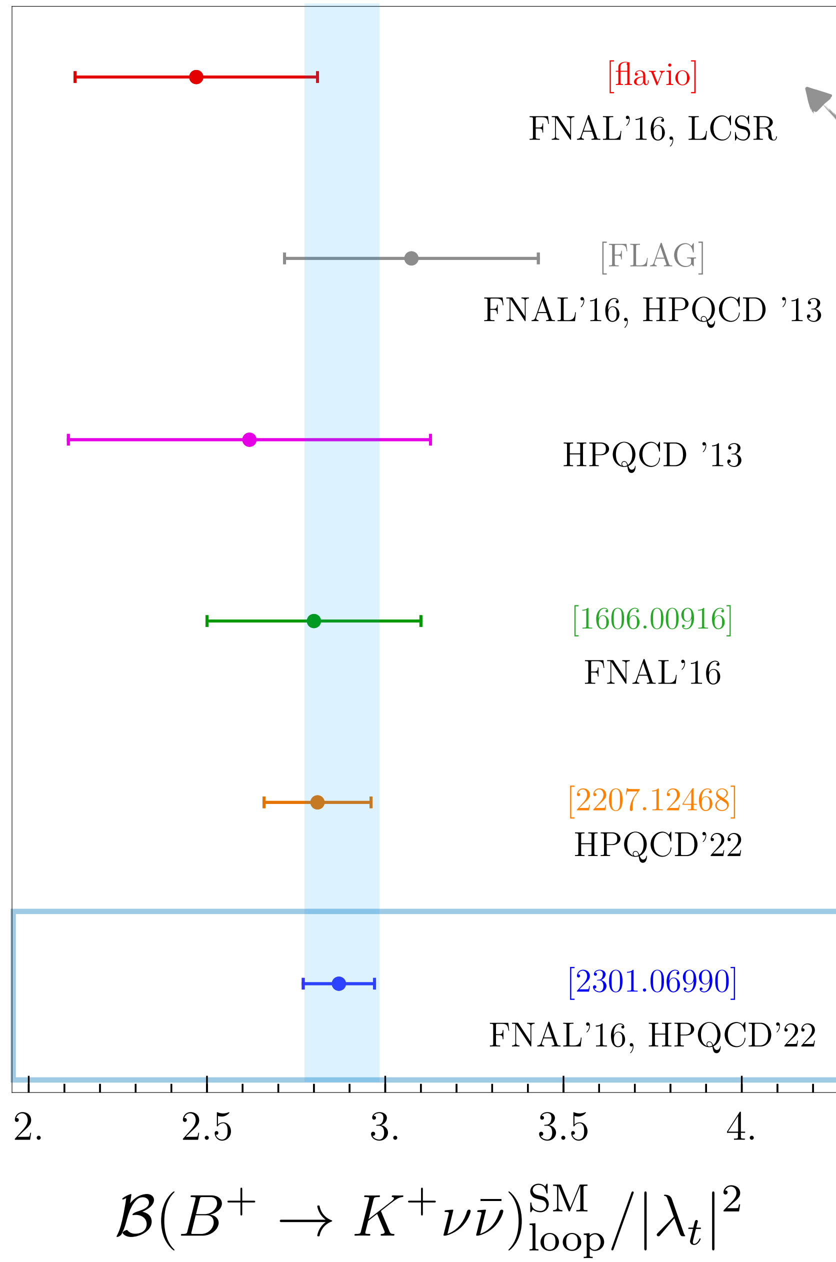
Pole factor:

$P_i(q^2) = 1 - q^2/M_i^2$

[Becirevic, Piazza, **OS**. 2301.06990]

I. Form-factors: $B \to K\nu\bar{\nu}$

**Annihilation contributions not included below (see back-up)*



Form-factors based on Light-Cone Sum Rules (LCSR) lead to smaller branching fractions.

[Bharucha et al. '15, Gubernari et al. '18]

$$\mathcal{B}(B \to K\nu\bar{\nu})^{\mathrm{SM}}/|\lambda_t|^2 = \begin{cases} (1.33 \pm 0.04)_{K_S} \times 10^{-3} \\ (2.87 \pm 0.10)_{K^+} \times 10^{-3} \end{cases}$$

[$\approx 3\,\%$ uncertainty]

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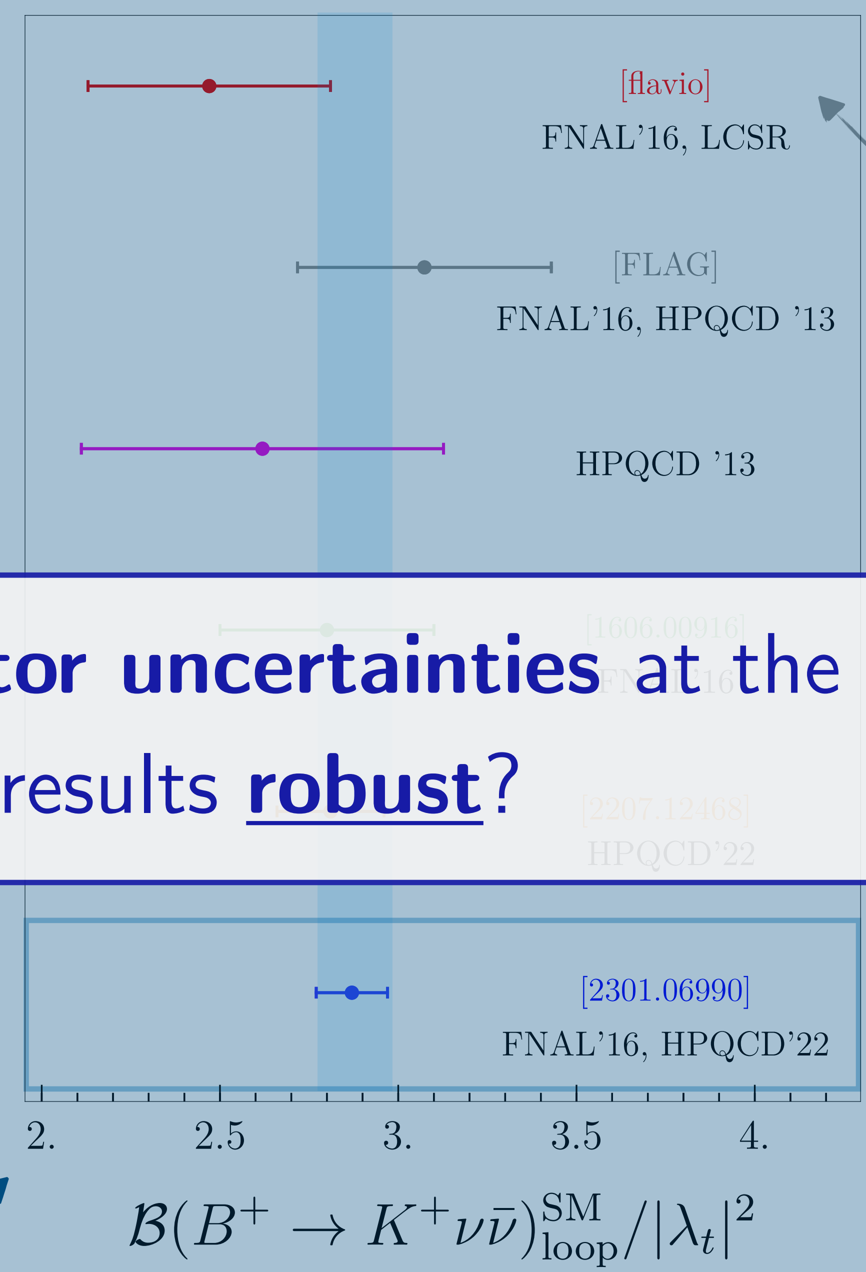

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Form-factor uncertainties at the few % level.

Are these results **robust**?

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[$\approx 3\,\%$ uncertainty]

[Becirevic, Piazza, **OS**. 2301.06990]

[Intermezzo]: Cross-check of $f_+^{B\to K}(q^2)$

- SM predictions depend on the **extrapolation** of the LQCD **form factors to low q^2** values — **parameterisation dependent?**

  ⇒ How can we **test the shape** of the **extrapolated** LQCD **form-factors**?

- We propose to measure:

[Becirevic, Piazza, **OS**. 2301.06990]

$$r_{\text{low/high}} = \frac{\mathcal{B}(B \to K\nu\bar{\nu})_{\text{low}-q^2}}{\mathcal{B}(B \to K\nu\bar{\nu})_{\text{high}-q^2}}$$

⇒ *Independent of λ_t and the form-factor normalisation!*

⇒ *New Physics effects would cancel out in this ratio as well — provided that NP is heavy.*

See next slides!

- For instance, using the bins $(0, q^2_{\text{max}}/2)$ *vs.* $(q^2_{\text{max}}/2, q^2_{\text{max}})$:

$$r_{\text{low/high}} = 1.91 \pm 0.06$$

e.g, using (old) FLAG average:

$$r_{\text{low/high}} = 2.15 \pm 0.26$$

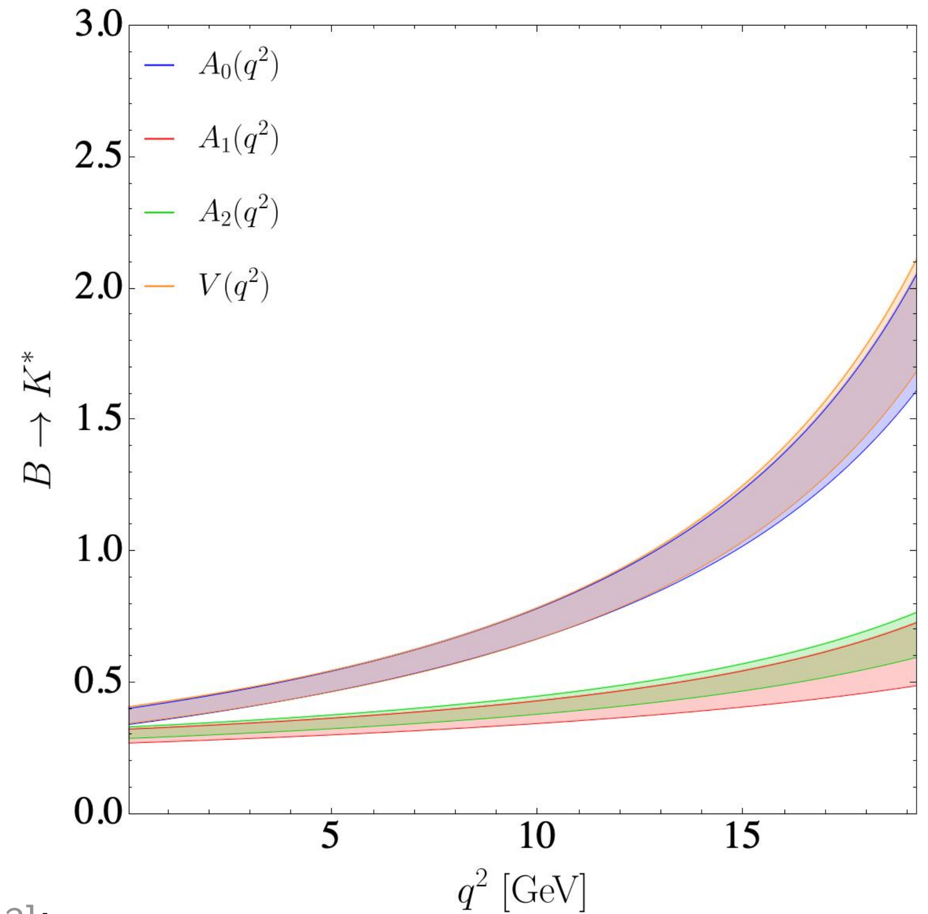
Binned measurements at Belle-II would be a **useful cross-check** of the **consistency** of the q^2**-shape** of **SM predictions**.

I. Form-factors: $B \to K^*\nu\bar{\nu}$

See talk by van Dyk

- $B \to K^*\nu\bar{\nu}$ decays are **more challenging** for several reasons:

$$\langle \bar{K}^*(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = \varepsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}p^\rho k^\sigma \frac{2V(q^2)}{m_B+m_{K^*}} - i\varepsilon^*_\mu(m_B+m_{K^*})A_1(q^2) + i(p+k)_\mu(\varepsilon^*\cdot q)\frac{A_2(q^2)}{m_B+m_{K^*}} + iq_\mu(\varepsilon^*\cdot q)\frac{2m_{K^*}}{q^2}\left[A_3(q^2)-A_0(q^2)\right],$$

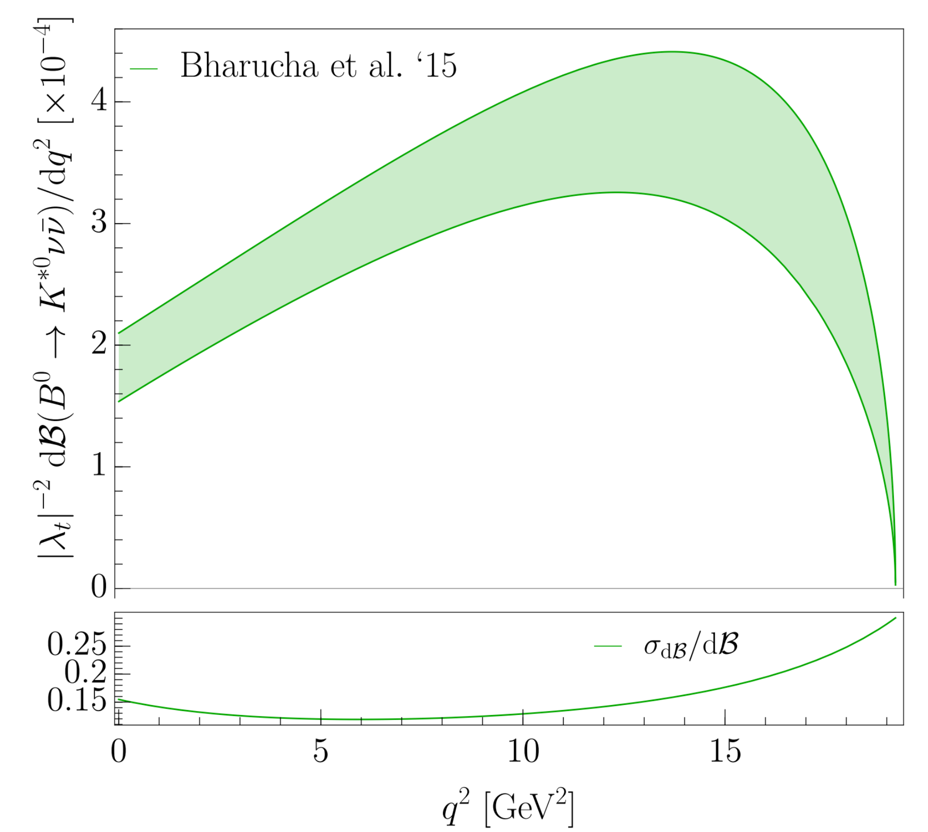


- We use LCSR (+LQCD) results from [Bharucha et al. '15, Horgan et al. '13]:

See also [Gubernari et al. '23]

$$\mathcal{B}(B\to K^*\nu\bar{\nu})^{\rm SM}/|\lambda_t|^2 = \begin{cases}(5.9\pm0.8)_{K^{*0}}\times10^{-3}\\(6.4\pm0.9)_{K^{*+}}\times10^{-3}\end{cases}$$

[$\approx 15\,\%$ uncertainty]



$\Rightarrow$ Relatively small uncertainties, **but are they accurate**?

II. Which CKM value?

See talks by Bona and Dorigo

$$\lambda_t = V_{tb} V_{ts}^*$$

- Using available $b \to c\ell\bar{\nu}$ data:

$$|\lambda_t| \times 10^3 = \begin{cases} 41.4 \pm 0.8\,, & (B \to X_c l\bar{\nu}) \\ 39.3 \pm 1.0\,, & (B \to D l\bar{\nu}) \\ 37.8 \pm 0.7\,, & (B \to D^* l\bar{\nu}) \end{cases}$$

[HFLAV, '22]
[FLAG, '21]
[HFLAV, '22]

... to be compared to CKM global fits:

cf. also [Martinelli et al. '21]

$$|\lambda_t|_{\mathrm{UTfit}} = (41.4 \pm 0.5) \times 10^{-2}$$

$$|\lambda_t|_{\mathrm{CKMfitter}} = (40.5 \pm 0.3) \times 10^{-2}$$

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- Alternative strategy: to use $\Delta m_{B_s} \propto f_{B_s}^2 \hat{B}_{B_s} |\lambda_t|^2$

[Buras, Venturini. '21, '22]

$$|\lambda_t| \times 10^3 = \begin{cases} 41.9 \pm 1.0\,, & (N_f = 2+1+1) \\ 39.2 \pm 1.1\,, & (N_f = 2+1) \end{cases}$$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 256 \pm 6 \text{ MeV} \qquad (N_f = 2+1+1)$$

[HPQCD '19]

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 274 \pm 8 \text{ MeV} \qquad (N_f = 2+1)$$

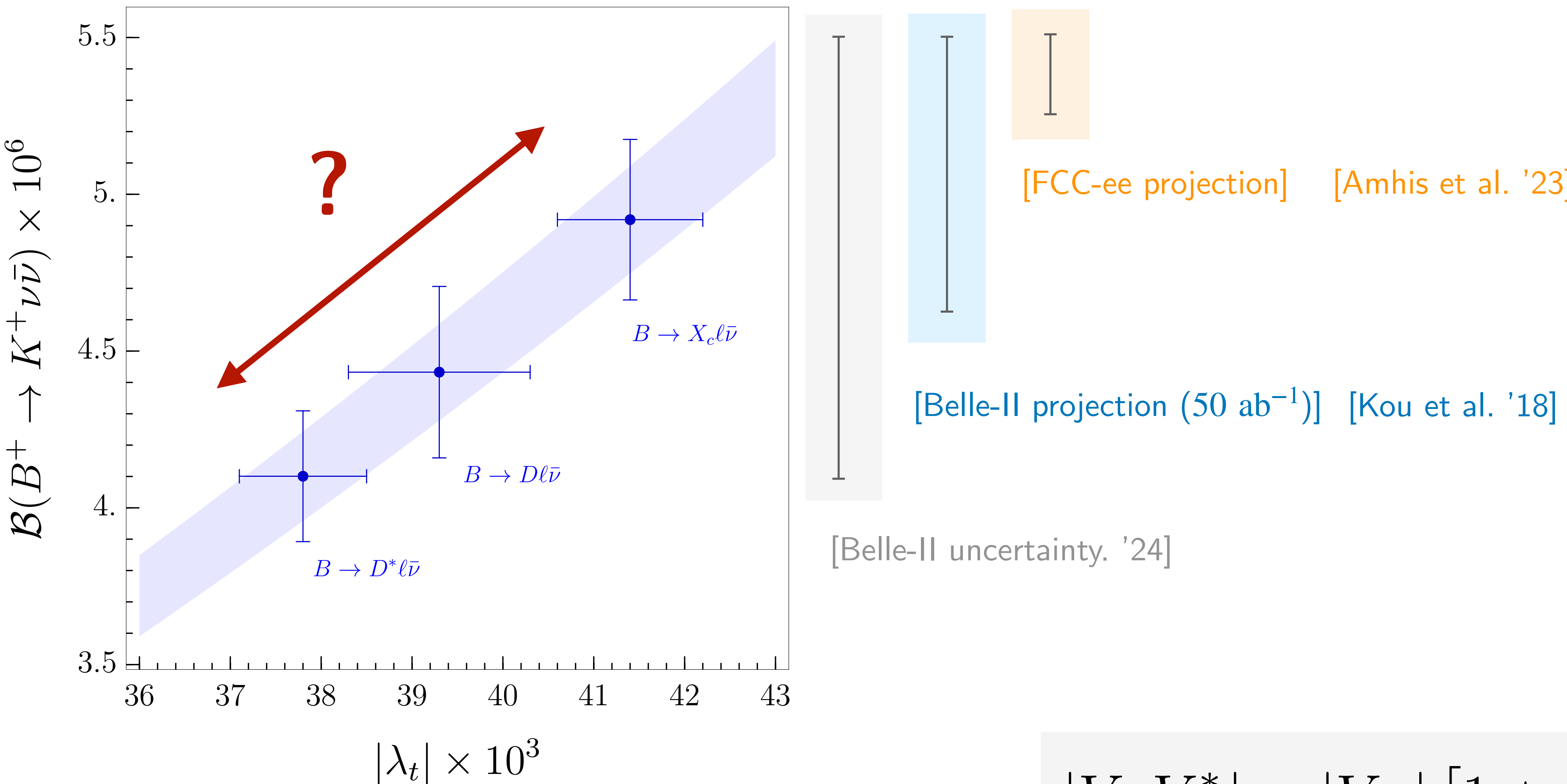
[FLAG '21]

There is **not a clear answer** to this **ambiguity** so far.

CKM and theory uncertainties

[Becirevic, Piazza, **OS**. '23]

see also [Buras et al. '21, '22]



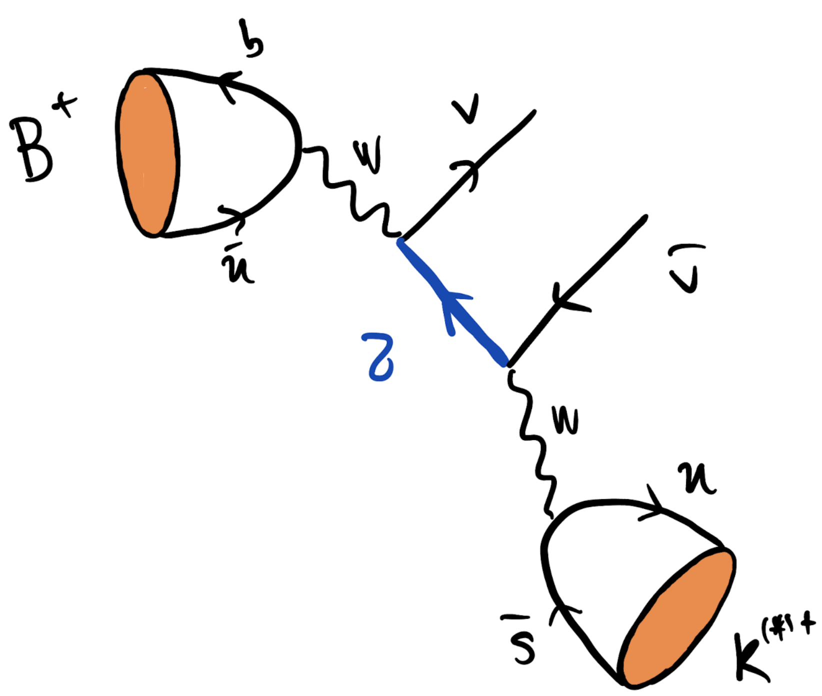
$$|V_{tb}V_{ts}^*| = |V_{cb}| \left[1 + \mathcal{O}(\lambda^2)\right]$$

The **ambiguity** in **determining** V_{cb} can be a **bottleneck** for **SM predictions** of **clean FCNC processes** such as $B \to K\nu\bar{\nu}$ and $B_s \to \mu\mu$ in the long term.

Weak-annihilation contributions

[Kamenik, Smith. '09]

- **<u>To keep in mind</u>**: decay modes with **charged mesons** are affected by **tree-level** weak **annihilation contributions**.



$$m_{K^{(*)+}} \leq m_\tau \leq m_B$$

- Using *narrow-width* approximation:

$$\mathcal{B}(B^+ \to K^{(*)+}\nu\bar{\nu}) \simeq \mathcal{B}(B^+ \to \tau^+\bar{\nu})\,\mathcal{B}(\tau^+ \to K^{(*)+}\nu)$$

- *Non-negligible* contributions:

$$\frac{\mathcal{B}(B^+ \to K^+\nu\bar{\nu})_{\text{tree}}}{\mathcal{B}(B^+ \to K^+\nu\bar{\nu}))_{\text{loop}}} \simeq 14\ \% \qquad \frac{\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu})_{\text{tree}}}{\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu}))_{\text{loop}}} \simeq 11\ \%$$

$\Rightarrow$ They *cannot be removed* by a *simple kinematical cut*...

<u>Belle-II</u>: These contributions are treated as a **background.**

Summary I

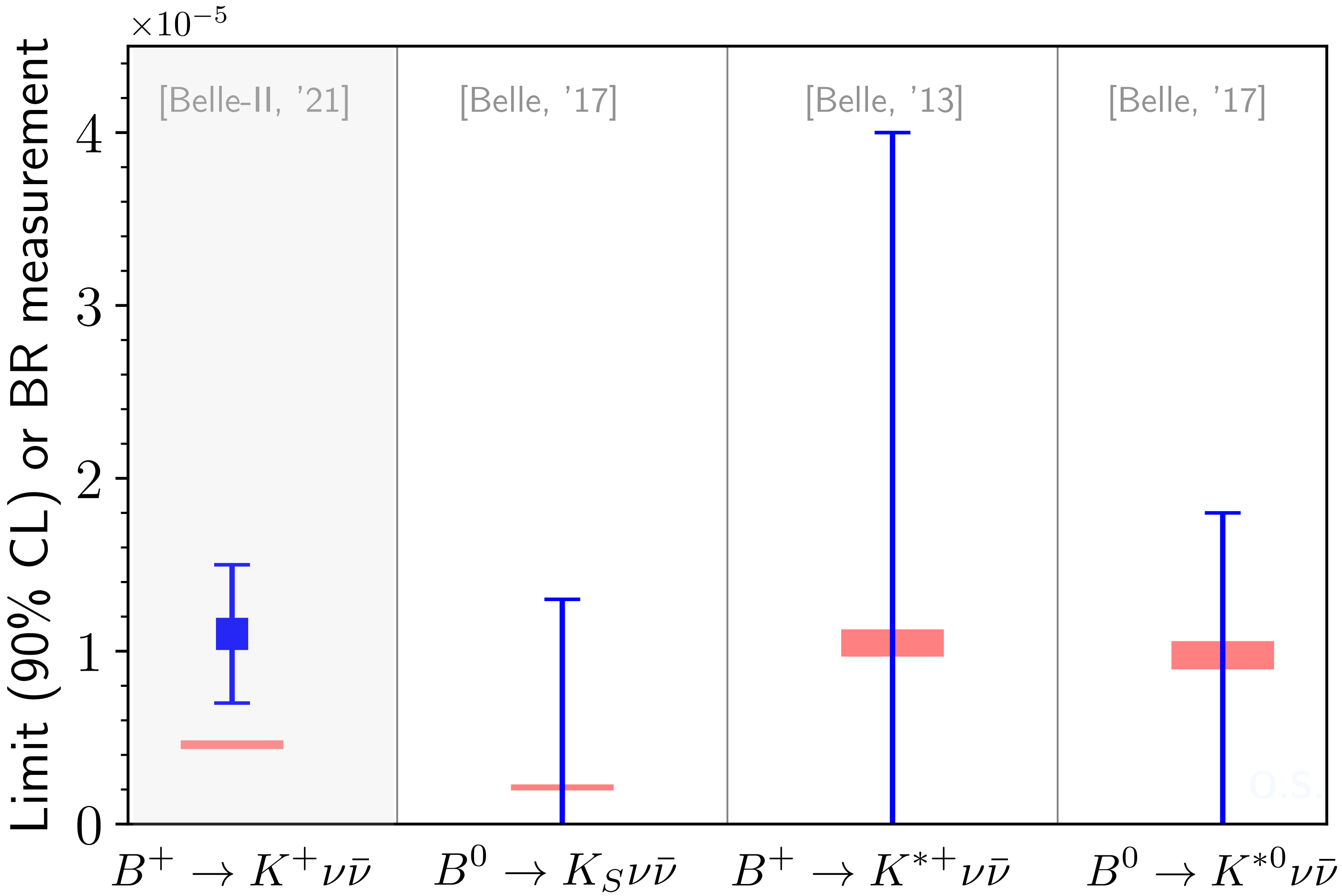
SM predictions

Decay	Branching ratio
$B^+ \to K^+ \nu\bar{\nu}$	$(4.44 \pm 0.14 \pm 0.27) \times 10^{-6}$
$B^0 \to K_S \nu\bar{\nu}$	$(2.05 \pm 0.07 \pm 0.12) \times 10^{-6}$
$B^+ \to K^{*+} \nu\bar{\nu}$	$(9.79 \pm 1.30 \pm 0.60) \times 10^{-6}$
$B^0 \to K^{*0} \nu\bar{\nu}$	$(9.05 \pm 1.25 \pm 0.55) \times 10^{-6}$

FF CKM

*Using V_{cb} $[B \to D\ell\bar{\nu}]$ for illustration — *the central value changes by $-7\,\%$ or $+10\,\%$ if we use $B \to D^*\ell\bar{\nu}$ or $B \to X_c\ell\bar{\nu}$, respectively.*

[Becirevic, Piazza, **OS**. 2301.06990]

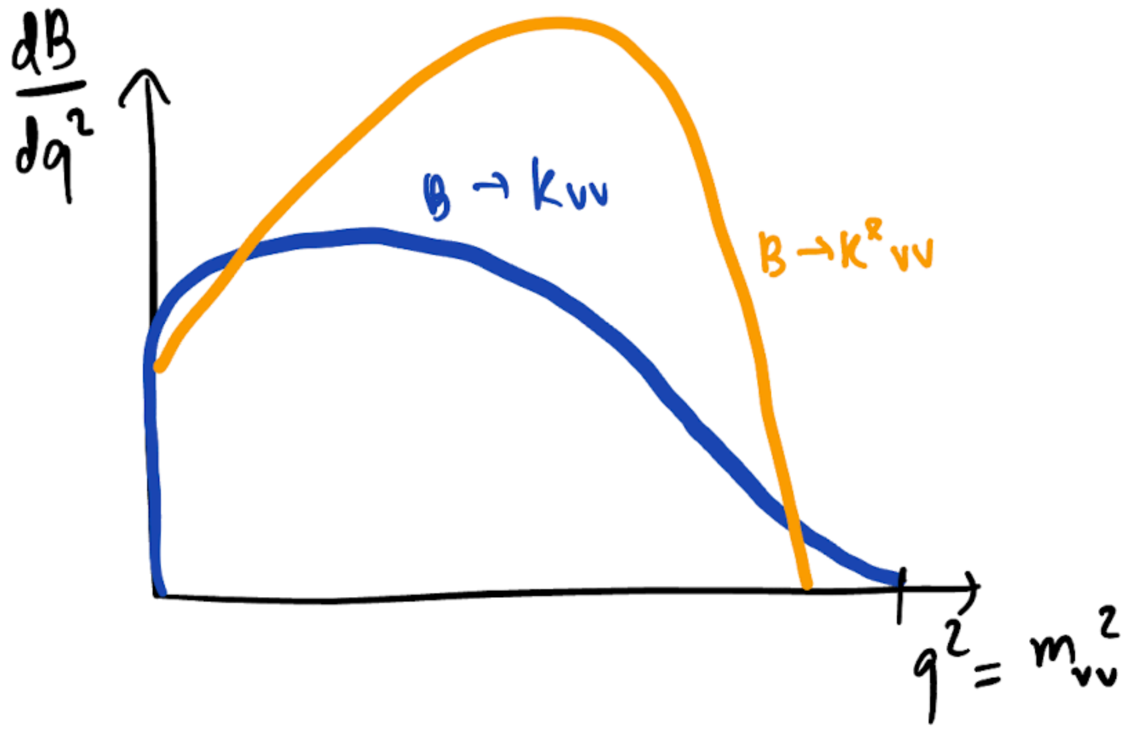


Take-home:

- To remain **cautious** about **hadronic uncertainties** associated with the **form-factors** and the **CKM** values extracted from data — *non-negligible given the projected Belle-II sensitivity.*
- **Binned measurements** at Belle-II would be **valuable** for **testing** the $B \to K^{(*)}$ **form factors**.

See next slides!

II. Differential q^2-distributions



What can we learn from $\mathrm{d}\mathcal{B}(B \to K\nu\bar{\nu})/\mathrm{d}q^2$?

NB. In particular, w/o ν_R

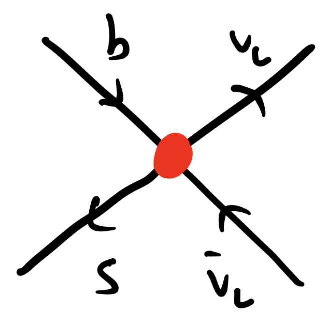
- If **New Physics** is **heavy**, the most general EFT reads:

[Buras et al. '14]

$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\mathrm{em}}}{2\pi}\sum_{ij}\Big[C_L^{\nu_i\nu_j}\,(\bar{s}_L\gamma^\mu b_L)(\bar{\nu}_{Li}\gamma_\mu\nu_{Lj}) + C_R^{\nu_i\nu_j}\,(\bar{s}_R\gamma^\mu b_R)(\bar{\nu}_{Li}\gamma_\mu\nu_{Lj})\Big] + \mathrm{h.c.}$$

$$C_L^{\nu_i\nu_j} \equiv C_L^{\mathrm{SM}}\,\delta^{ij} + \delta C_L^{\nu_i\nu_j}$$

$$C_R^{\nu_i\nu_j} \equiv \delta C_R^{\nu_i\nu_j}$$



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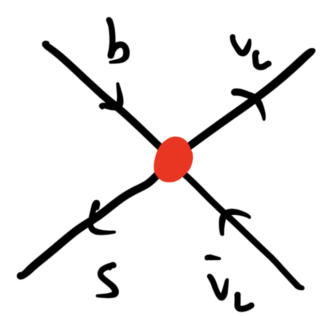
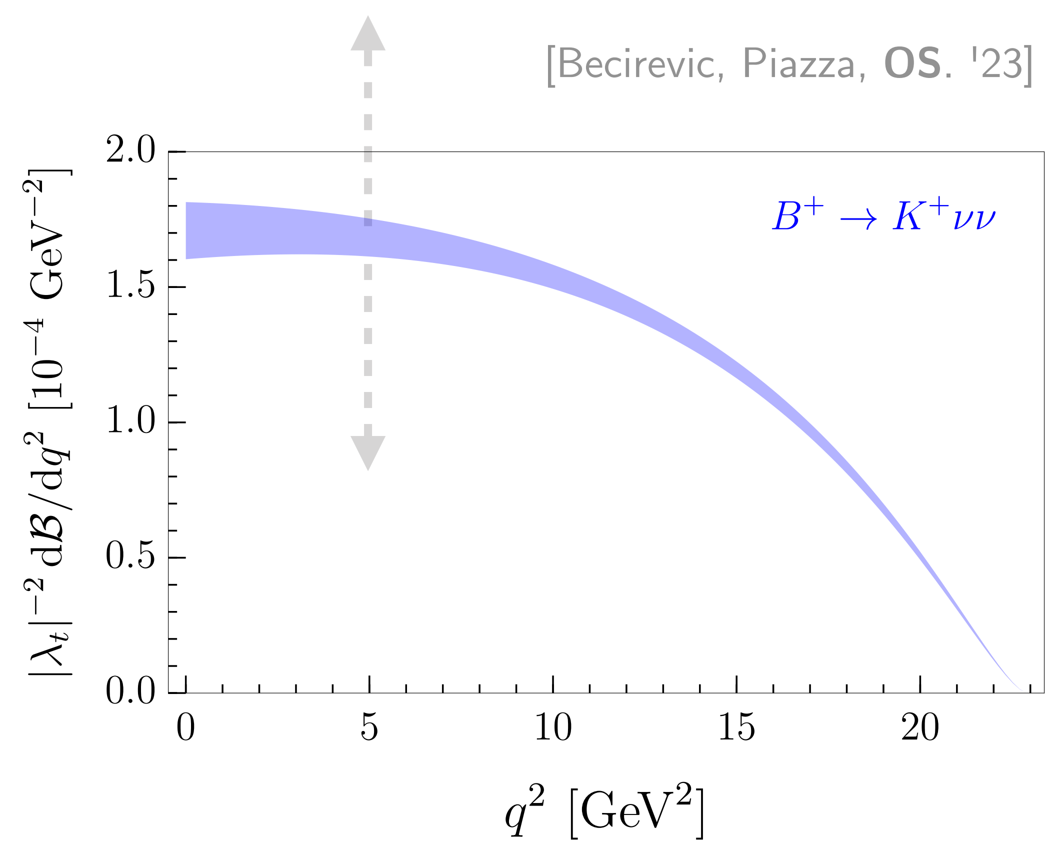


[Becirevic, Piazza, **OS**. '23]



- The **only NP effect** is the **overall rescaling** of $\mathrm{d}\mathcal{B}/\mathrm{d}q^2$:

$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}q^2}(B \to K\nu_i\bar{\nu}_j) \propto |C_L^{\nu_i\nu_j} + C_R^{\nu_i\nu_j}|^2\,\varphi(q^2)\,f_+(q^2)^2$$

$$\langle K(k)|\bar{s}\gamma^\mu b|B(p)\rangle \to f_+(q^2)$$

i.e., with $m_\nu \to 0$

⇒ The q^2**-shape** measured **experimentally** gives **direct access** to the $f_+(q^2)$ **form-factor**!

What can we learn from $\mathrm{d}\mathcal{B}(B \to K\nu\bar{\nu})/\mathrm{d}q^2$?

- The q^2-**shape** of $B \to K\nu\nu$ can **only be modified** in the presence of **light NP**:

$\Rightarrow$ Example: light RH neutrino

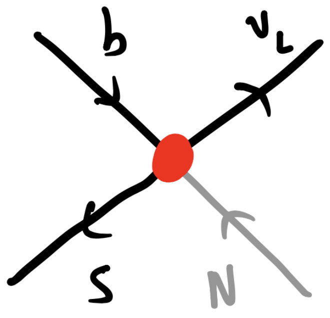
$$\boxed{\mathcal{L}_{\nu\mathrm{EFT}} \supset \sum_i \frac{C_i}{v^2} O_i + \mathrm{h.c.}}$$

Operators made of SM fields and $N \sim (\mathbf{1}, \mathbf{1}, 0)$

New types of operators:

$$O_{S_{LL}} = (\bar{s}_R b_L)(\bar{N}\nu_L)$$

$$O_{T_L} = (\bar{s}_R \sigma^{\mu\nu} b_L)(\bar{N}\sigma_{\mu\nu}\nu_L)$$



... and $(L \leftrightarrow R)$

[Felkl et al., '21], [Rosauro-Alcaraz et al. '24], [Becirevic et al. '24], [Buras et al. '24], [Bolton et al. '25]...

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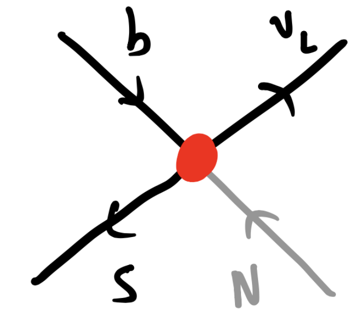
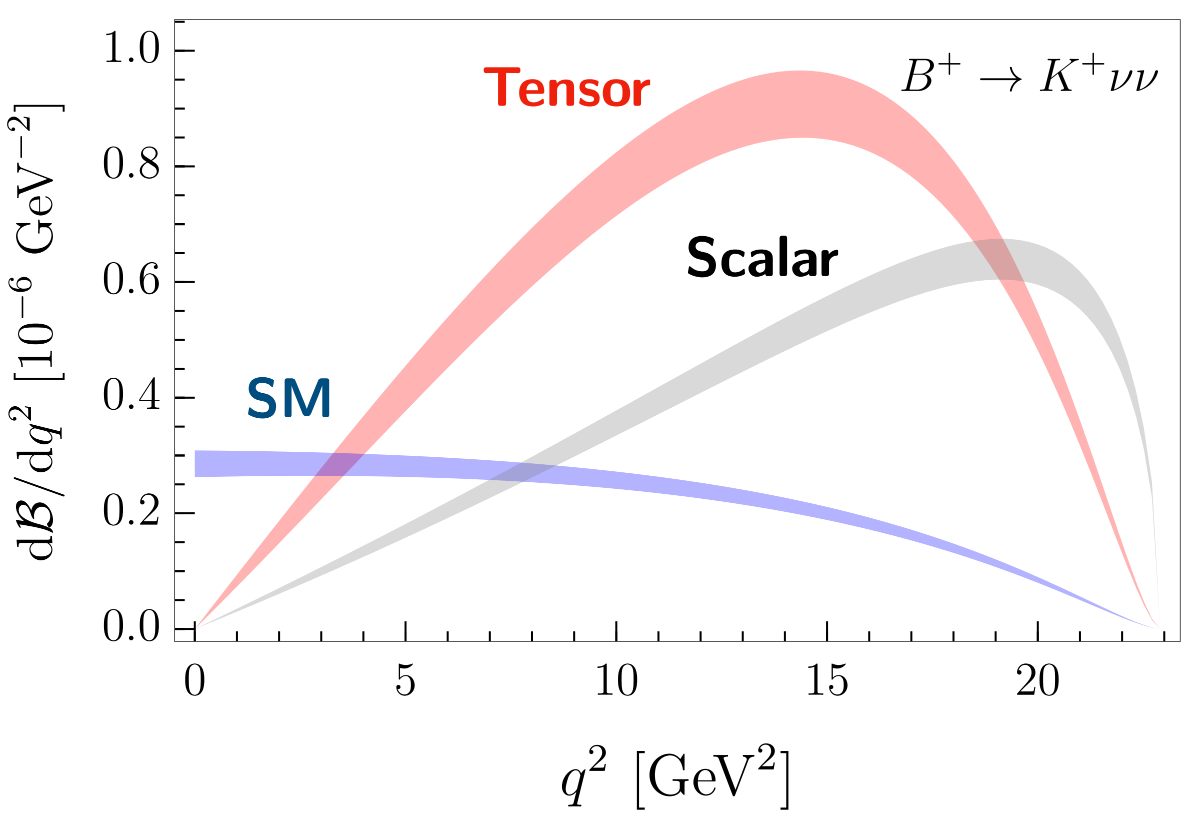


... and $(L \leftrightarrow R)$

[Felkl et al., '21], [Rosauro-Alcaraz et al. '24], [Becirevic et al. '24], [Buras et al. '24], [Bolton et al. '25]...



NB. Separate contributions (to be added to the SM)!

$\Rightarrow$ **Binned q^2-distributions** can probe **EFTs** with **new light d.o.f.** — *provided that $f_+(q^2)$ is well known.*

See talk by Schmidt

$\Rightarrow$ **Reweighting** of the MC is **needed** for consistency.

See talk by Gaertner

What about $\mathrm{d}\mathcal{B}(B \to K^* \nu\bar{\nu})/\mathrm{d}q^2$?

- SM form-factors: $B \to K^*\nu\nu$

$$\langle \bar{K}(k)|\bar{s}\gamma_\mu b|\bar{B}(p)\rangle \to V(q^2)$$

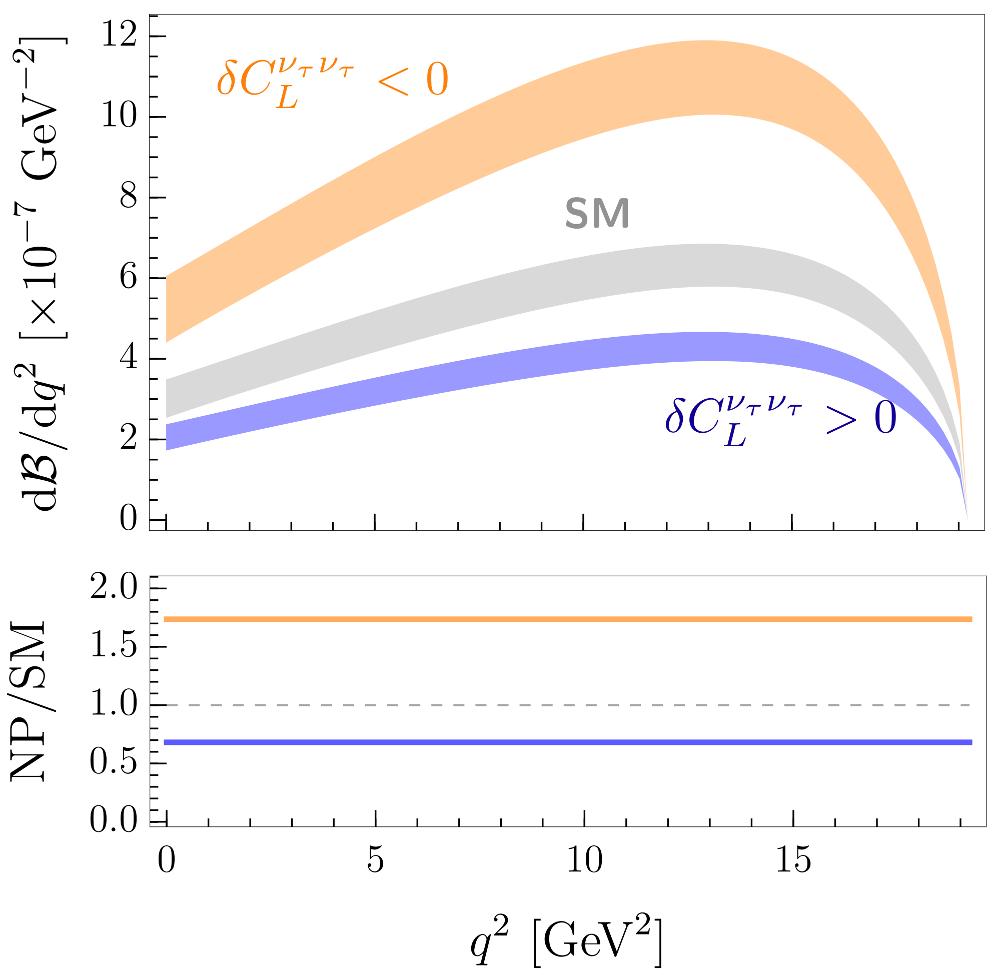
$$\langle \bar{K}(k)|\bar{s}\gamma_\mu\gamma_5 b|\bar{B}(p)\rangle \to A_1(q^2),\ A_{12}(q^2)$$

i.e., with $m_\nu \to 0$

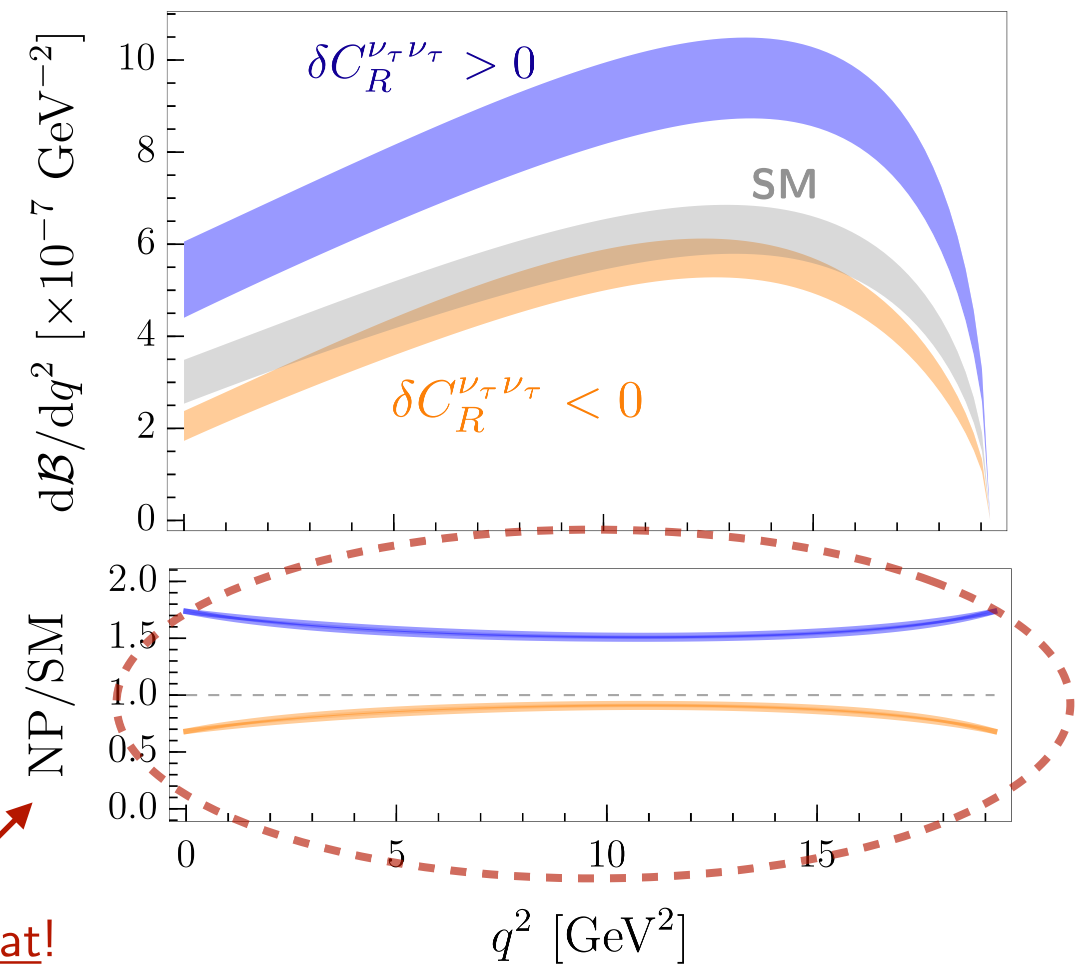
- The q^2-**shapes** of $B \to K^*\nu\nu$ can be **mildly modified** by the **EFT** (with **LH neutrinos):**

LH:

Rescaling of the SM:



RH:



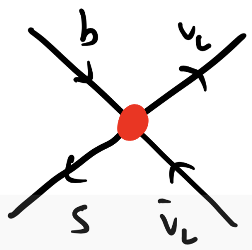
Almost flat!

$\Rightarrow$ The **differential** q^2**-distribution** is **mostly sensitive** to the $B \to K^*$ **form-factors.**

NB. They could be modified by operators with light NP — *cf. back-up.*

Summary II

Differential q^2-distributions



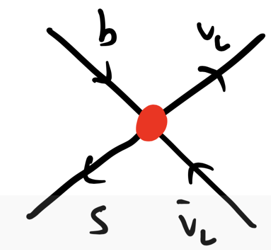
I. If **New Physics** is assumed to be **heavy** [*i.e., SMEFT*]:

- $\mathrm{d}\mathscr{B}(B \to K\nu\nu)/\mathrm{d}q^2$ gives direct access to $f_+(q^2)$ — *i.e., independently of NP effects!*
- $\mathrm{d}\mathscr{B}(B \to K^*\nu\nu)/\mathrm{d}q^2$ is mostly senstitive to the form-factors — *with mild q^2-dependent effects induced by contributions from right-handed currents.*

⇒ **Useful cross-checks of form-factor inputs!**

Summary II

Differential q^2-distributions



I. If **New Physics** is assumed to be **heavy** [*i.e., SMEFT*]:

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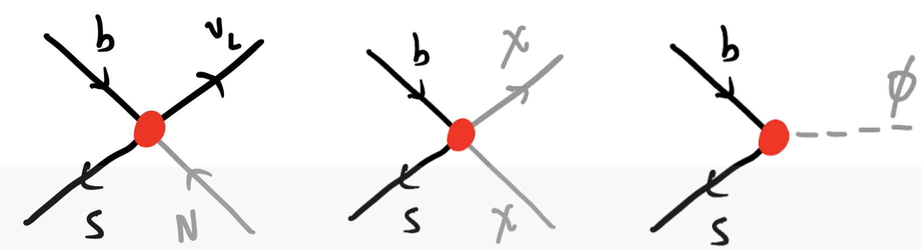
⇒ Useful cross-checks of form-factor inputs!



II. If **New Physics** has **light d.o.f** [*e.g., EFT with light ν_R*]:

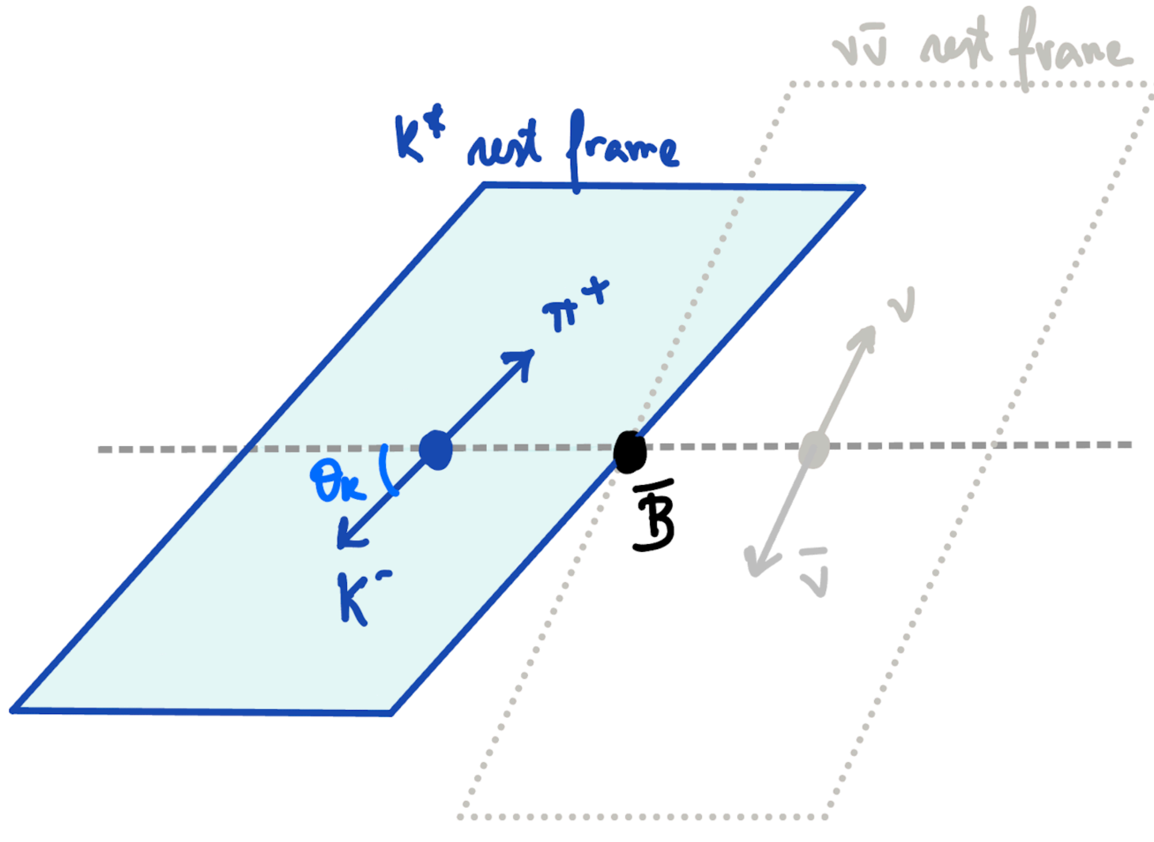
- **Differential distributions** can be used as **additional probes of New Physics** — *assuming that SM predictions are under control.*

See talks by Schmidt and Bolton

Important: Monte Carlo reweighting needed for consistent reinterpretations!

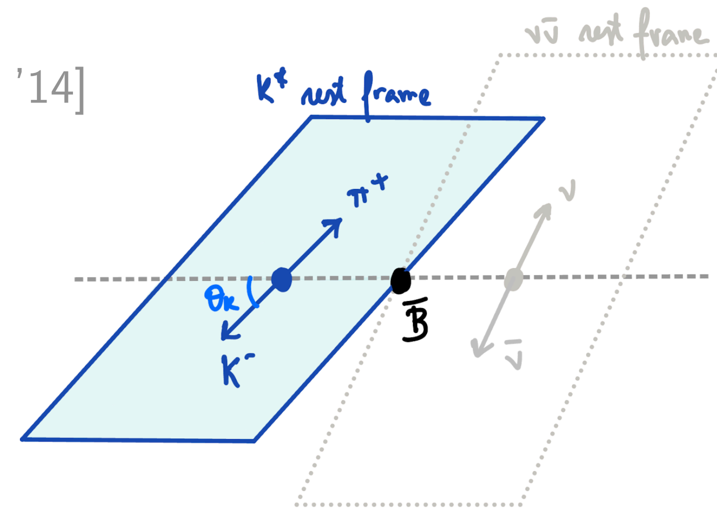
See talk by Gaertner

III. $F_L(B \to K^*\nu\nu)$



$F_L(B \to K^*\nu\nu)$

[Altmannshofer et al. '09, Buras et al. '14]



- The $B \to K^*(\to K\pi)\nu\nu$ distribution reads

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_K} = \frac{3}{2}\frac{d\Gamma_L}{dq^2}\cos^2\theta_K + \frac{3}{4}\frac{d\Gamma_T}{dq^2}\sin^2\theta_K$$

$\Rightarrow$ **Longitudinal** (Γ_L) and **transverse** (Γ_T) polarization fractions can be extracted from data.

- Two independent observables:

$$\frac{d\Gamma}{dq^2} \equiv \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2} \qquad F_L \equiv \frac{d\Gamma_L}{dq^2}\Big/\frac{d\Gamma}{dq^2}$$

$\Rightarrow$ **The longitudinal polarization fraction (F_L)** is a **ratio**, thus independent of the CKM matrix (λ_t) and less sensitive to form-factors uncertainties.

$$\mathcal{B}(B^0 \to K^{*0}\nu\bar{\nu})^{\text{SM}} = (9.1 \pm 1.3 \pm 0.6) \times 10^{-6}$$

FF CKM

$$F_L(B^0 \to K^{*0}\nu\bar{\nu})^{\text{SM}} = 0.49 \pm 0.04$$

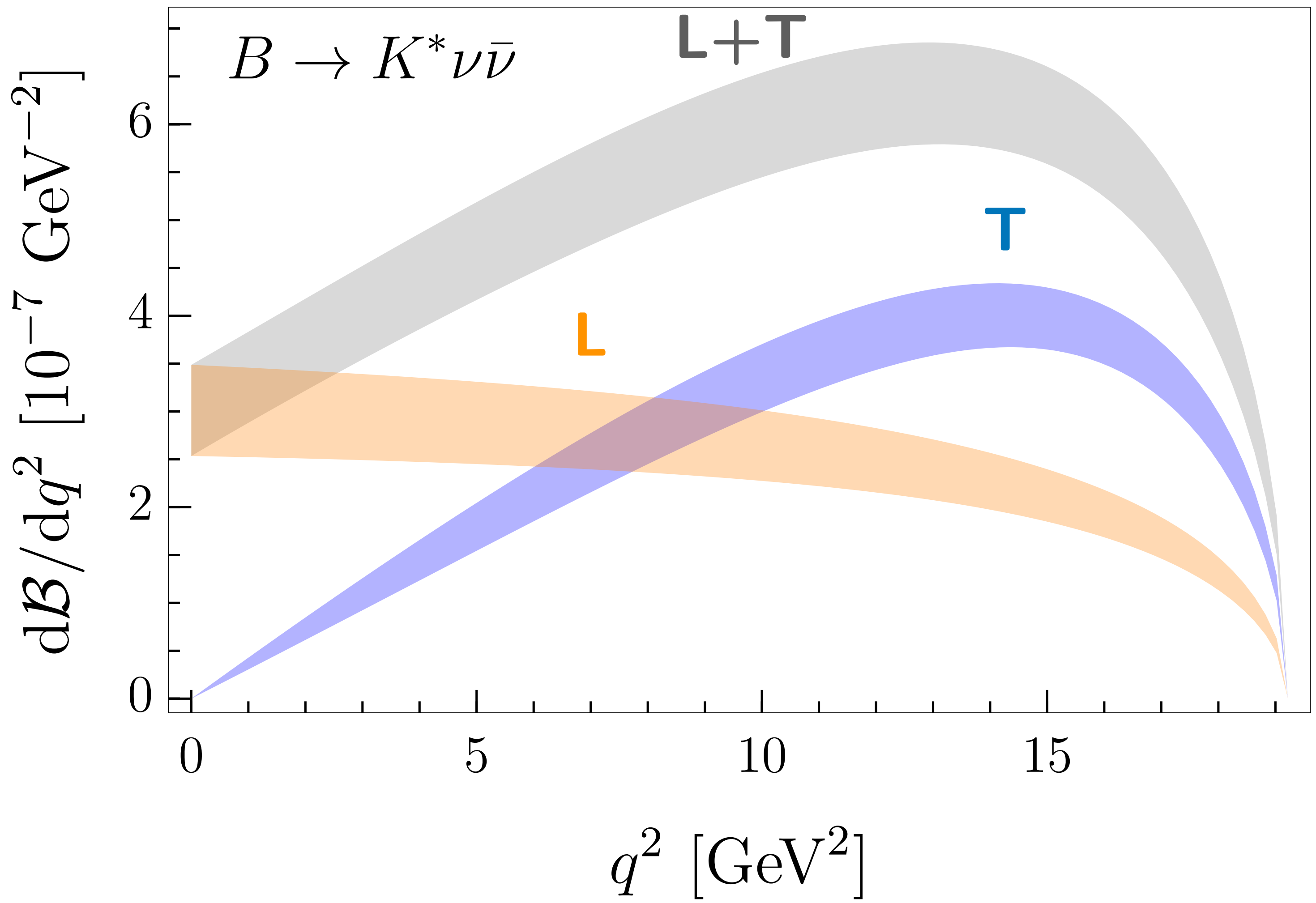
FF

$\Rightarrow$ **Clean probe** of **New Physics** effects!

[Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]

$F_L(B \to K^*\nu\nu)$

$$F_L \equiv \frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} \Big/ \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}$$



$$\frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} \propto |C_L - C_R|^2 A_{12}(q^2)^2$$

$$\frac{\mathrm{d}\Gamma_T}{\mathrm{d}q^2} \propto |C_L - C_R|^2 A_1(q^2)^2 + \# \, |C_L + C_R|^2 V(q^2)^2$$

Contributions from **LH operators** ($\delta C_L \neq 0$) **cancel out** in F_L.

This observable is **sensitive** to **right-handed currents** ($\delta C_R \neq 0$)!

Predictions

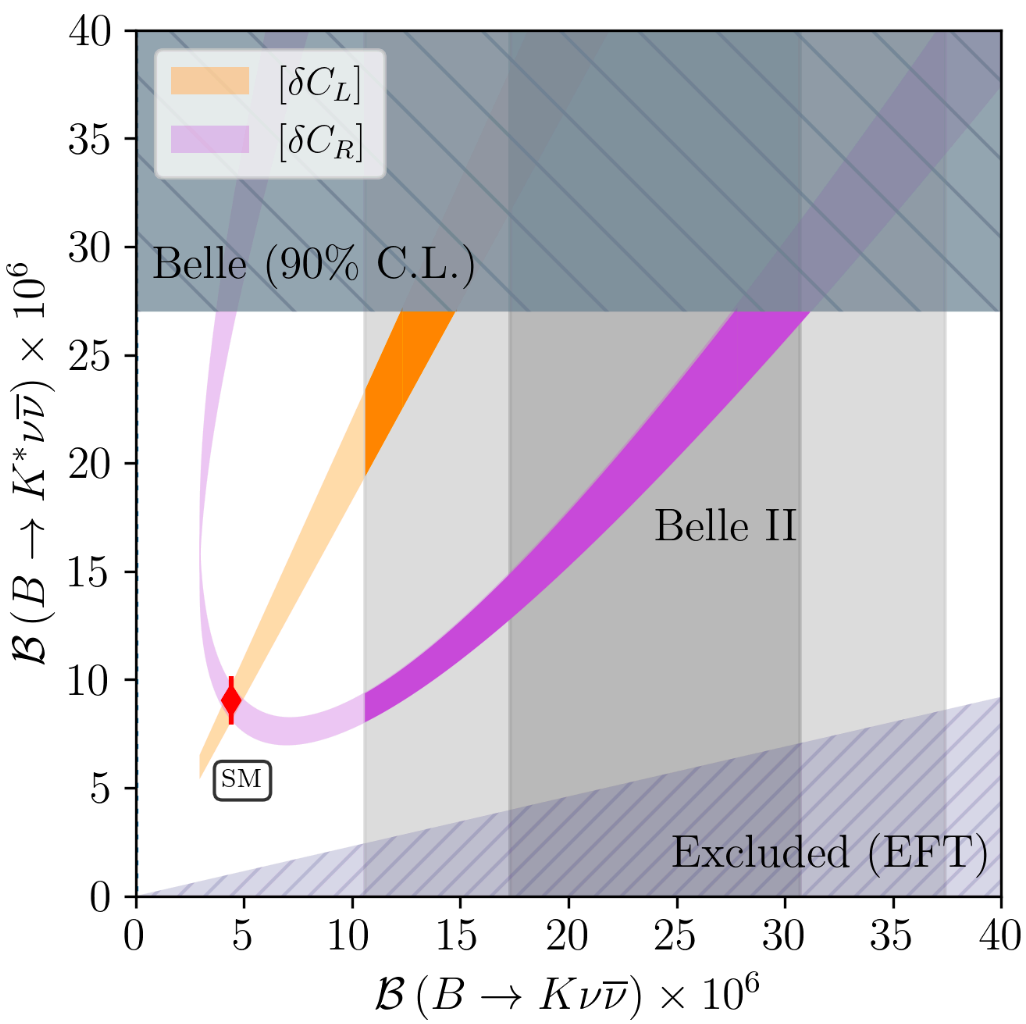
[Altmannshofer et al. '09, Buras et al'. '14, Das et al. '17]

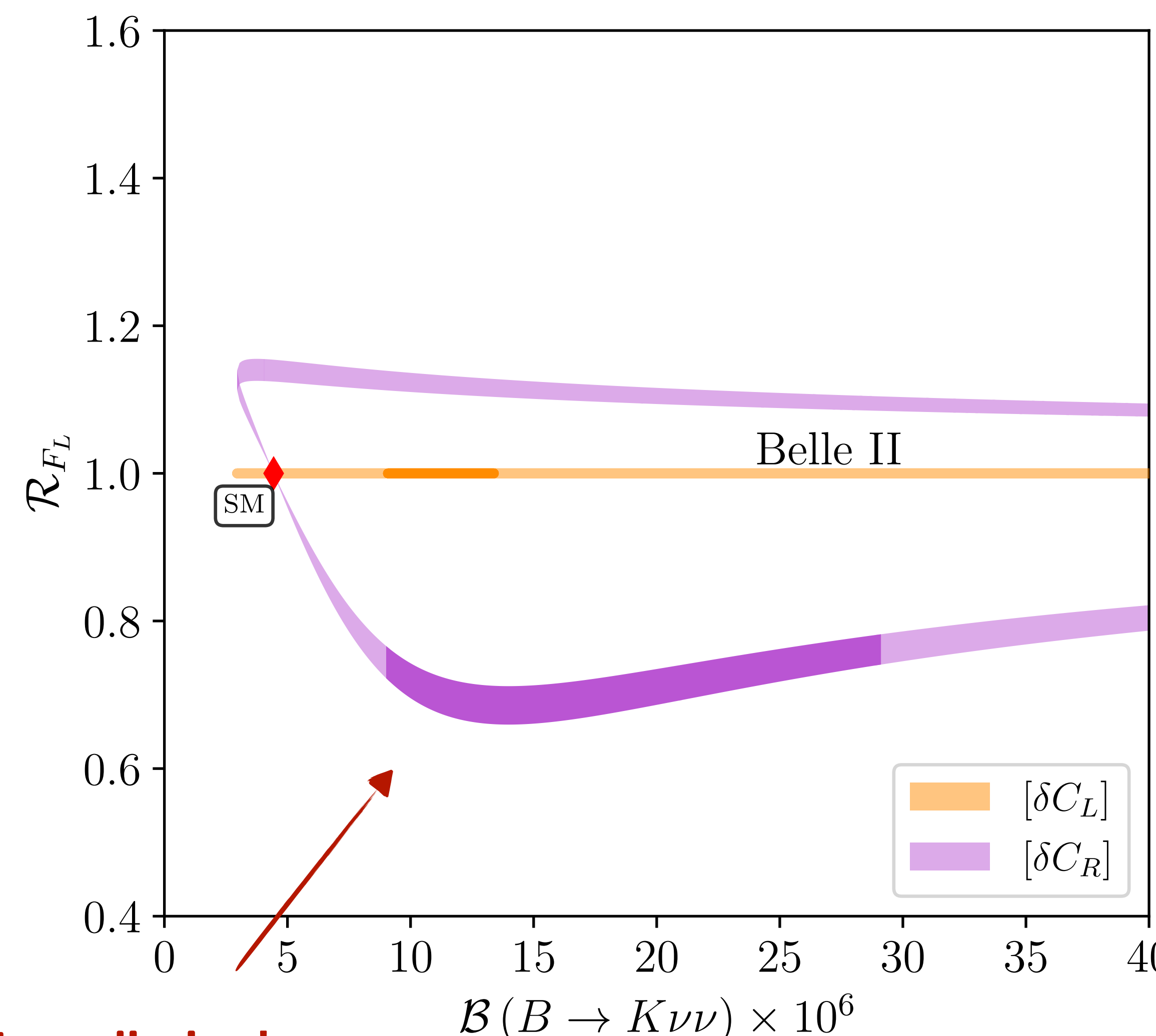
- $F_L(B \to K^*\nu\nu)$ provides an efficient probe of right-handed currents:

$$F_L \equiv \frac{\Gamma_L(B \to K^*\nu\bar{\nu})}{\Gamma(B \to K^*\nu\bar{\nu})}$$

$$F_L(B \to K^*\nu\bar{\nu})^{\mathrm{SM}} = 0.49(4)$$

$$\mathcal{R}_{F_L} \equiv \frac{F_L}{F_L^{\mathrm{SM}}}$$





*Only ν_τ

Depletion of SM prediction!

See also [Bause et al. '23]

[Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]

The measurement of $\mathscr{B}(B \to K^*\nu\bar{\nu})$ and $F_L(B \to K^*\nu\bar{\nu})$ would be **model-independent tests** of the excess in $B^+ \to K^+\nu\nu$ data.

Predictions: LH vs RH neutrinos

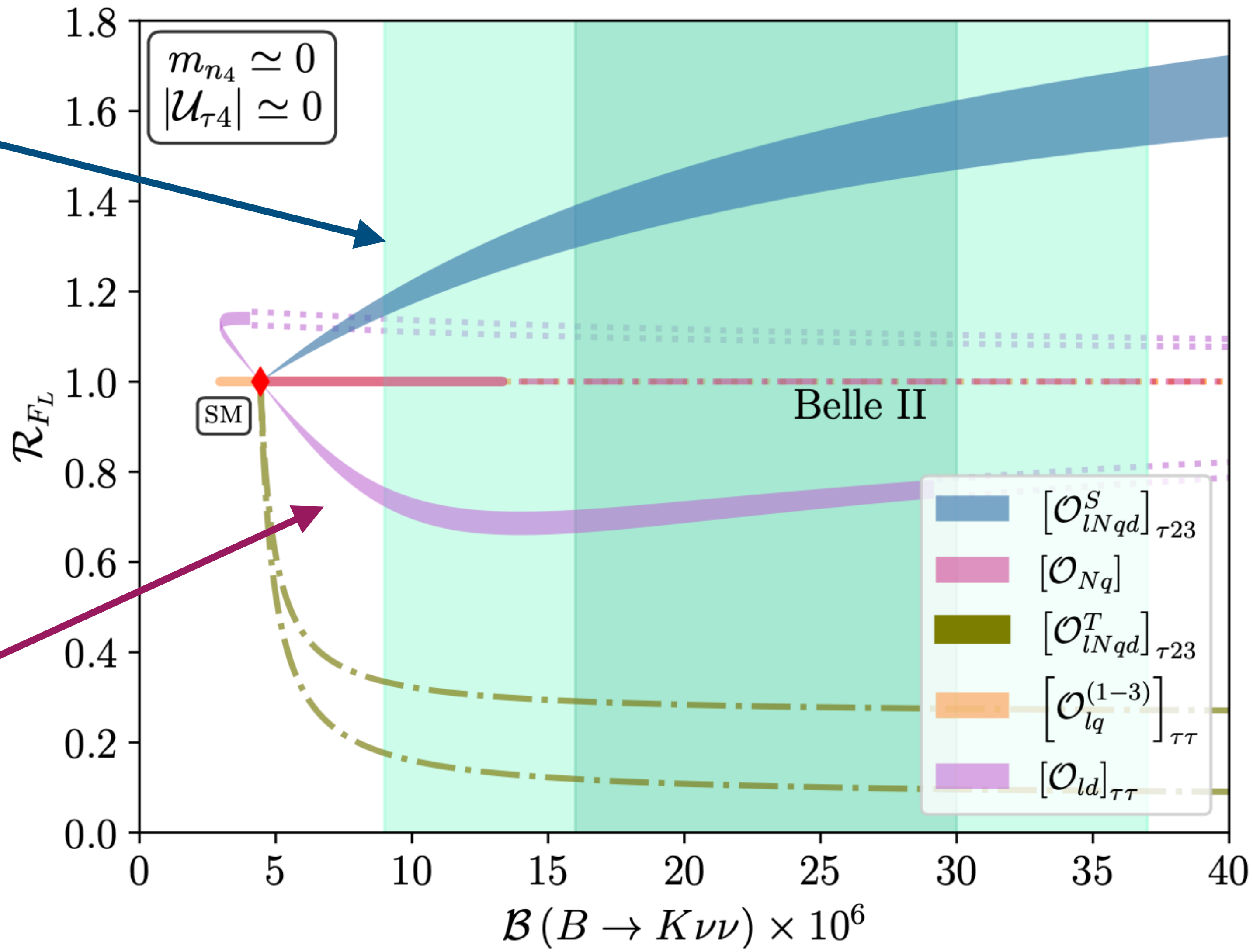
[Rosauro-Alcaraz, Leal. '24]

Scalar operator
(with RH neutrinos)

$$\mathcal{O}^S_{lNqd} = (\bar{l}N)\epsilon(\bar{q}d) \to (\bar{s}_L b_R)(\bar{\nu}_L N) + \ldots$$

Vector operator
(with LH neutrinos)

$$\mathcal{O}_{ld} = (\bar{l}\gamma^\mu l)(\bar{d}\gamma_\mu d) \to (\bar{s}_R\gamma^\mu b_R)(\bar{\nu}_L\gamma_\mu \nu_L) + \ldots$$



$F_L(B \to K^*\nu\nu)$ can **distinguish** between **EFTs** with **left-** and **right-handed neutrinos**

See also [Buras et al. '25]

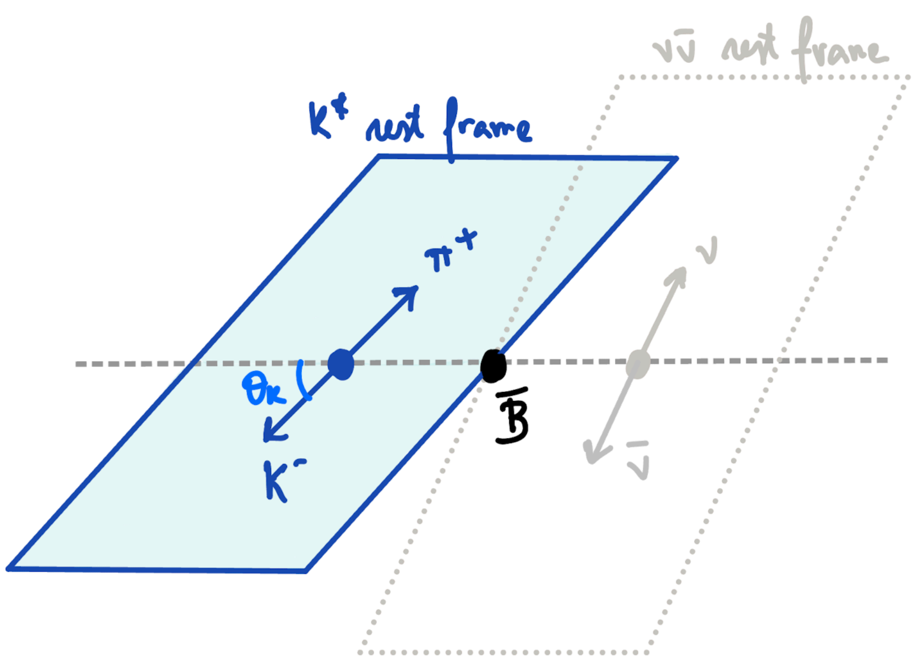
Summary III

$F_L(B \to K^* \nu\nu)$

- The **longitudinal polarization fraction (F_L)** is **theoretically cleaner** than the $B \to K^* \nu\nu$ branching fraction — *i.e., it is independent of the λ_t and form-factors uncertainties are reduced in this ratio.*

- Within the usual **EFT scenarios** (with **SM neutrinos**), F_L can directly probe effective operators with **right-handed quark currents.**

- A peculiar feature is that operators with **LH** and **RH neutrinos** explaining the excess in $B^+ \to K^+ \nu\nu$ data lead to **different predictions** to F_L — *this complementarity is worth exploring!*

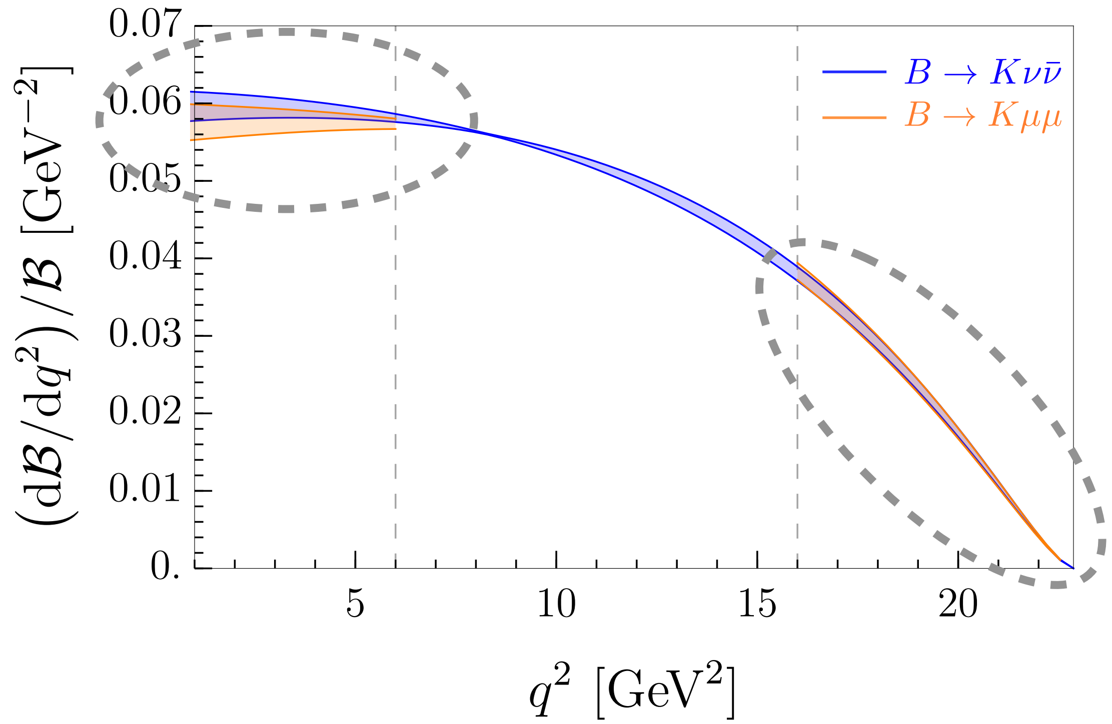


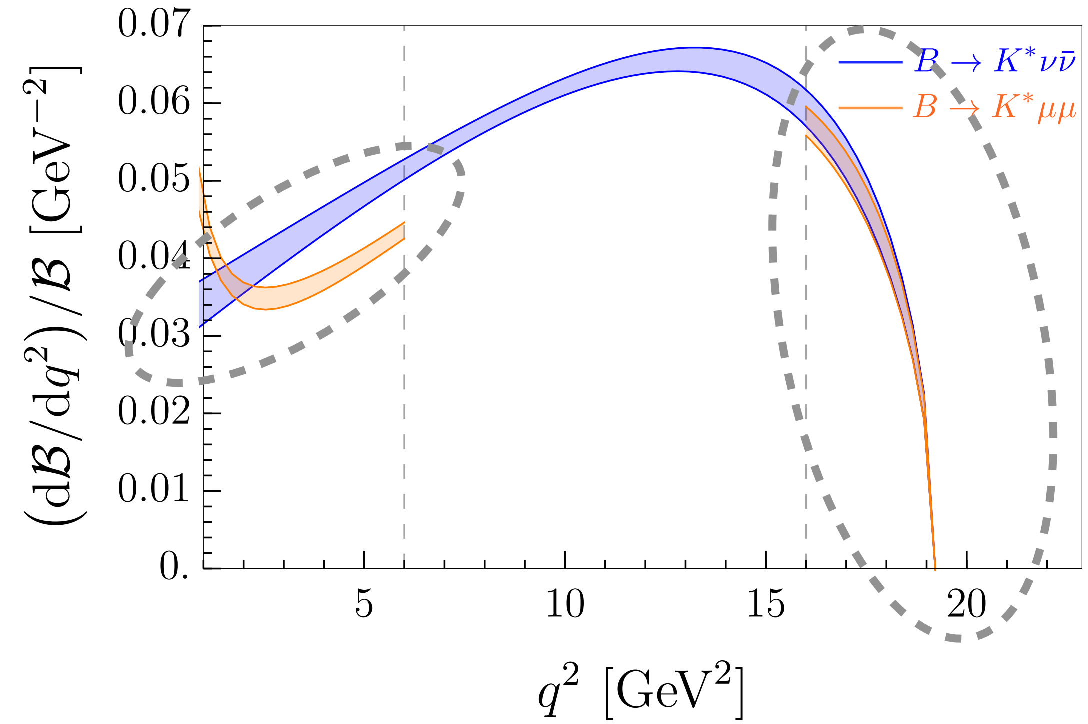
Remarks on ν/μ ratios

Remarks on $B \to K^{(*)}\nu\nu / B \to K^{(*)}\mu\mu$

- $B \to K^{(*)}\nu\nu$ and $B \to K^{(*)}\mu\mu$ have a similar decay spectrum away from the narrow $c\bar{c}$ resonances:

[Becirevic, Piazza, **OS**. 2301.06990] [Bartsch et al. '09]





*using 2-loop results for $c\bar{c}$ loops from [Asatryan et al. '09]

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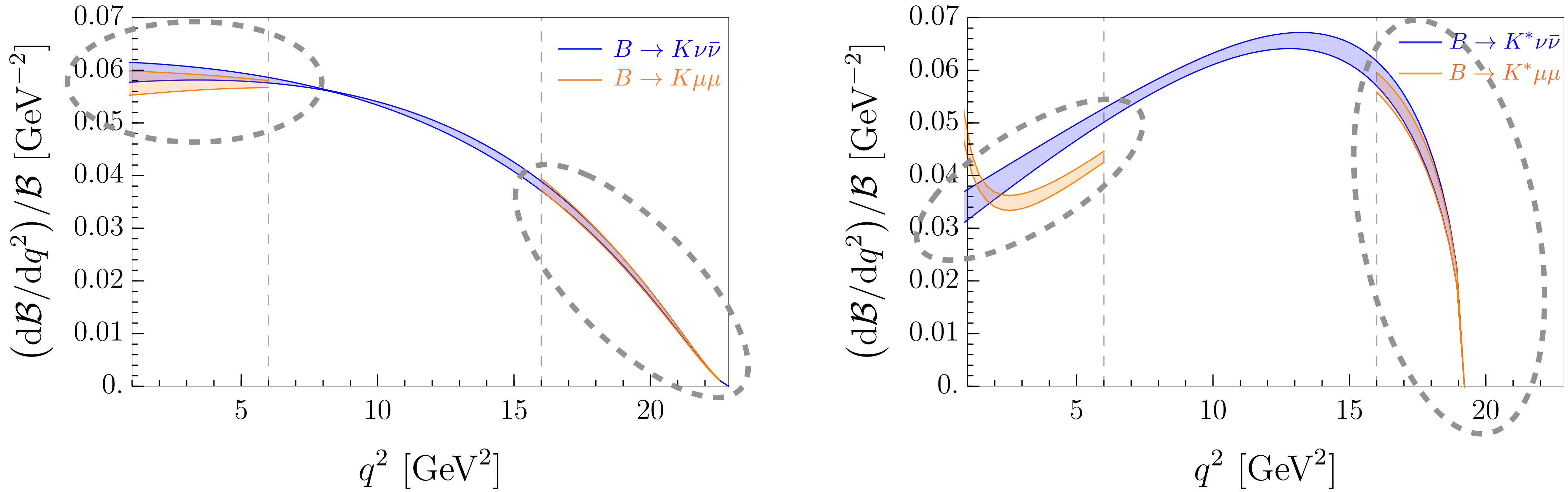
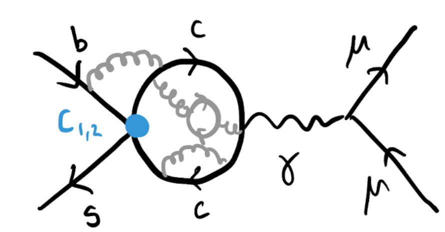


*using 2-loop results for $c\bar{c}$ loops from [Asatryan et al. '09]

- We can defined the **CKM-free ratio**:

$$\mathcal{R}^{(\nu/l)}_{K^{(*)}}[q_0^2, q_1^2] \equiv \left. \frac{\mathcal{B}(B \to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B \to K^{(*)}ll)} \right|_{[q_0^2, q_1^2]}$$

Ratio of partial branching fractions integrated in the same q^2-bin.



$\Rightarrow$ **Form-factor** uncertainties **cancel out** to a good extent for $q^2 \gg m_\ell^2$.

$\Rightarrow$ Neglecting NP contributions, this ratio can be used to **directly probe** $C_9^{\mu\mu}$ — *independently of form-factors!*

Predictions using perturbative calculation of $c\bar{c}$ loops: using [Asatryan et al. '09]

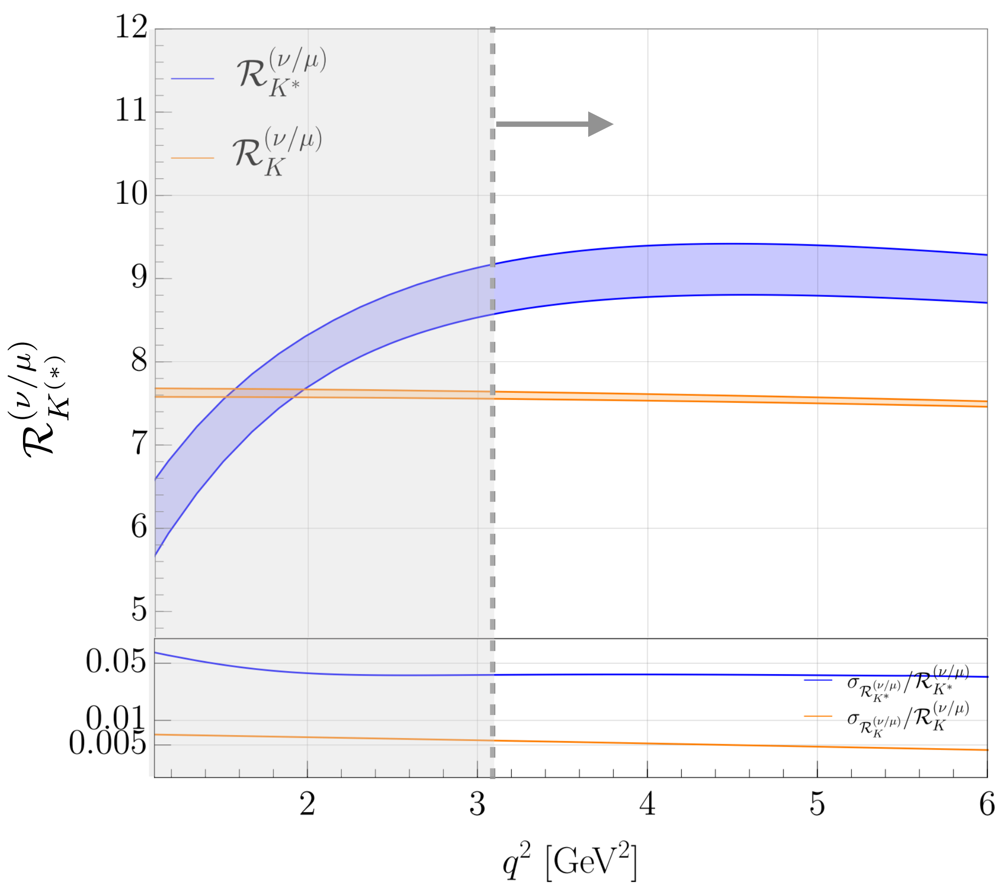
$$\mathcal{R}_K^{(\nu/l)}[1.1,6]\Big|_{\rm SM} = 7.58 \pm 0.04$$

$$\mathcal{R}_{K^*}^{(\nu/l)}[1.1,6]\Big|_{\rm SM} = 8.6 \pm 0.3$$

with the following dependence on $C_9^{\rm eff}$: [Becirevic, Piazza, **OS**. 2301.06990]

$$\frac{1}{\mathcal{R}_K^{(\nu/l)}[1.1,6]}\Bigg|_{\rm SM} \simeq \left[7.15 - 0.45 \cdot C_9^{\rm eff} + 0.42 \cdot \left(C_9^{\rm eff}\right)^2\right] \times 10^{-2}$$

$$\frac{1}{\mathcal{R}_{K^*}^{(\nu/l)}[1.1,6]}\Bigg|_{\rm SM} \simeq \left[9.98 - 1.45 \cdot C_9^{\rm eff} + 0.42 \cdot \left(C_9^{\rm eff}\right)^2\right] \times 10^{-2}$$



- The q^2-**dependence** from **kinematics** and **form-factors** fully **cancels out** in these **ratios**.
- **Binned measurements** could provide us with **additional information** on $C_9^{\rm eff}$ — *e.g., is it q^2-independent?*

Could such measurements **help us to understand** the **various anomalies** in $b \to s\mu\mu$ data?

Summary & Outlook

Summary & Outlook

- **$B \to K^{(*)}\nu\nu$:** rather theoretically clean and very sensitive to NP effects — *in particular, to operators with 3rd generation leptons*!

- **Hadronic uncertainties:** nonperturbative QCD effects remain the main obstacle to using low-energy observables to probe new physics — *caution is (always) advised*!

- **Differential distributions:** the precise measurement of $\mathrm{d}\mathscr{B}/\mathrm{d}q^2$ could provide a helpful cross-check of the relevant form-factors, and a potential probe of EFT scenarios with light NP.

- **V_{cb}:** theory and exp. progress is needed to solve this issue — *needed to fix the parametric uncertainties of rare decays in the SM...* Belle-II data and new LQCD results will be essential.

- **Belle-II:** More data and further cross-checks are needed to understand the first Belle-II results — *e.g., $B^0 \to K_S\nu\bar{\nu}$, $B \to K^*\nu\bar{\nu}$ and $F_L(B \to K^*\nu\bar{\nu})$*.

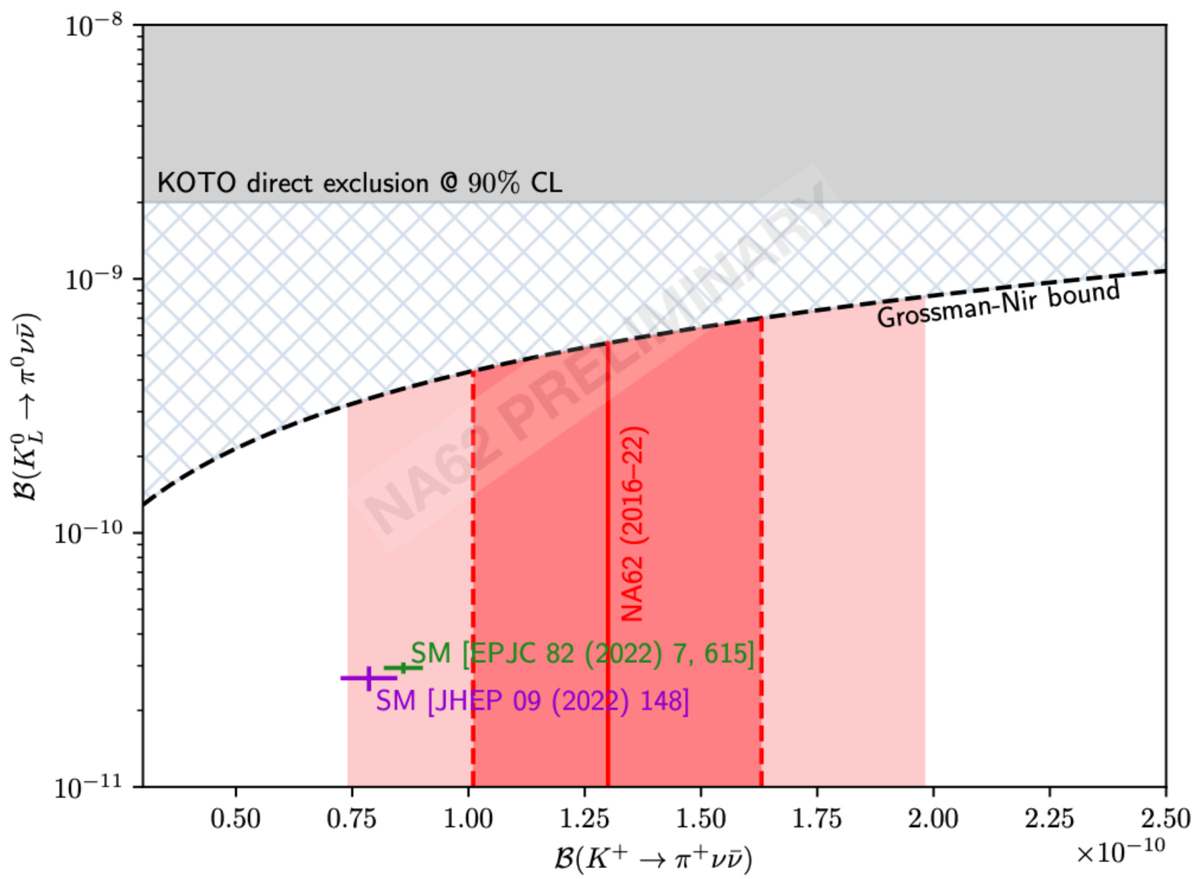
Many opportunities to explore physics (B)SM with Belle-II!

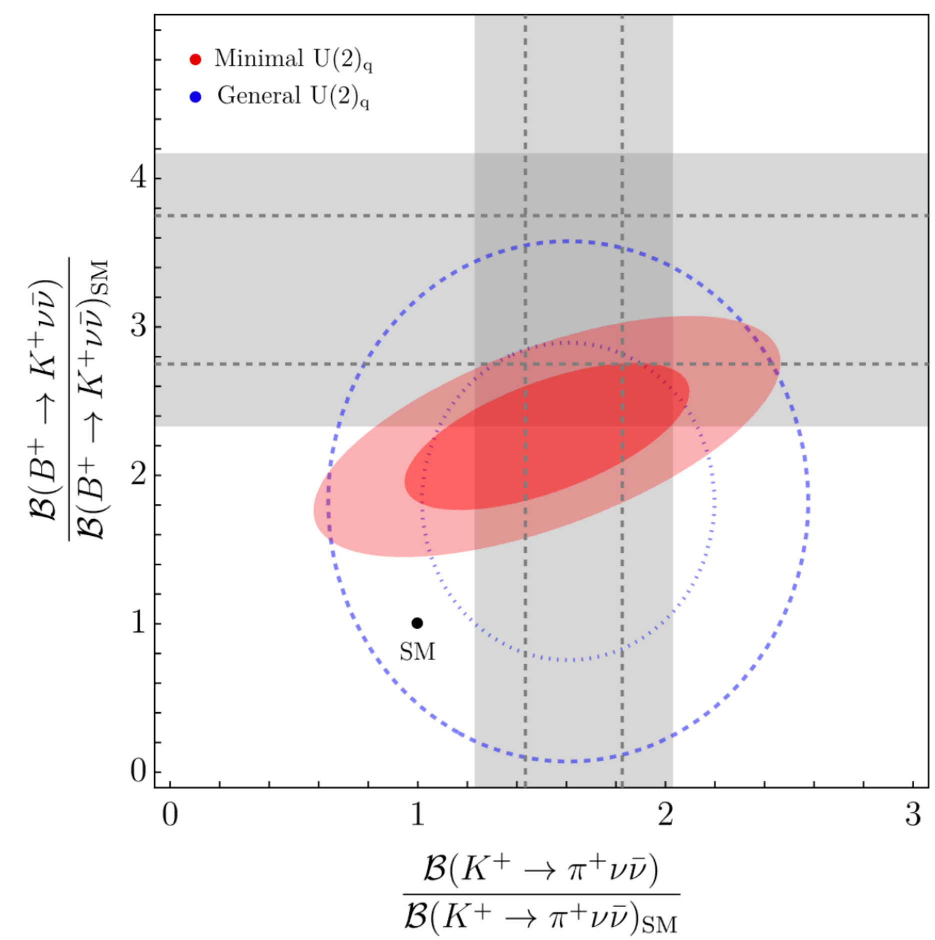
Final words — to not be forgotten:

See talks by Cornella and Nanjo

- $K^+ \to \pi^+ \nu\nu$ and $K_L \to \pi^0 \nu\nu$ are also **very clean probes** of **New Physics!**
- These decays can probe **energy scales** up to ≈ 100 TeV and they can be connected to $B \to K\nu\nu$ in **concrete flavor scenarios**.

[Allwicher et al. '24]

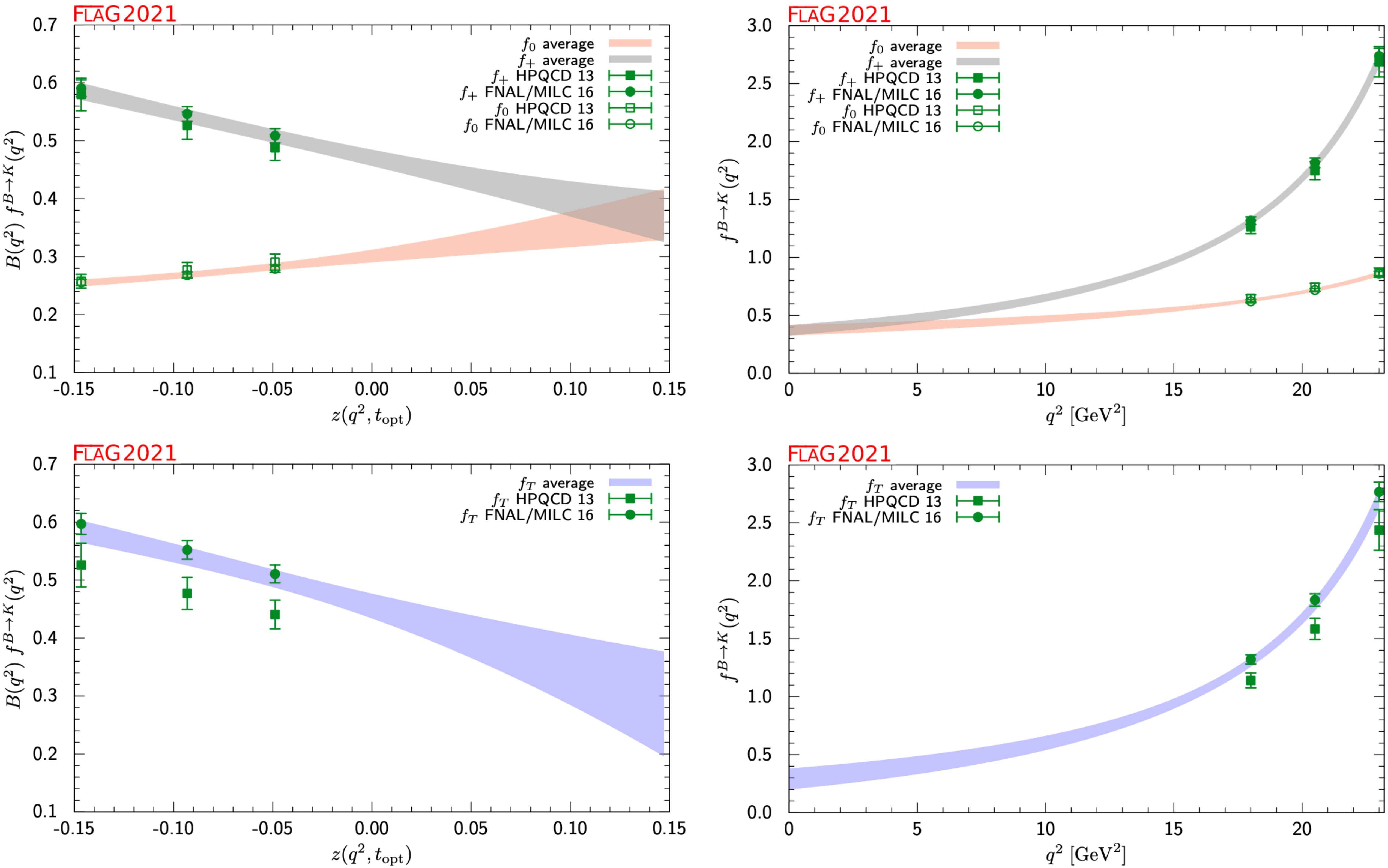




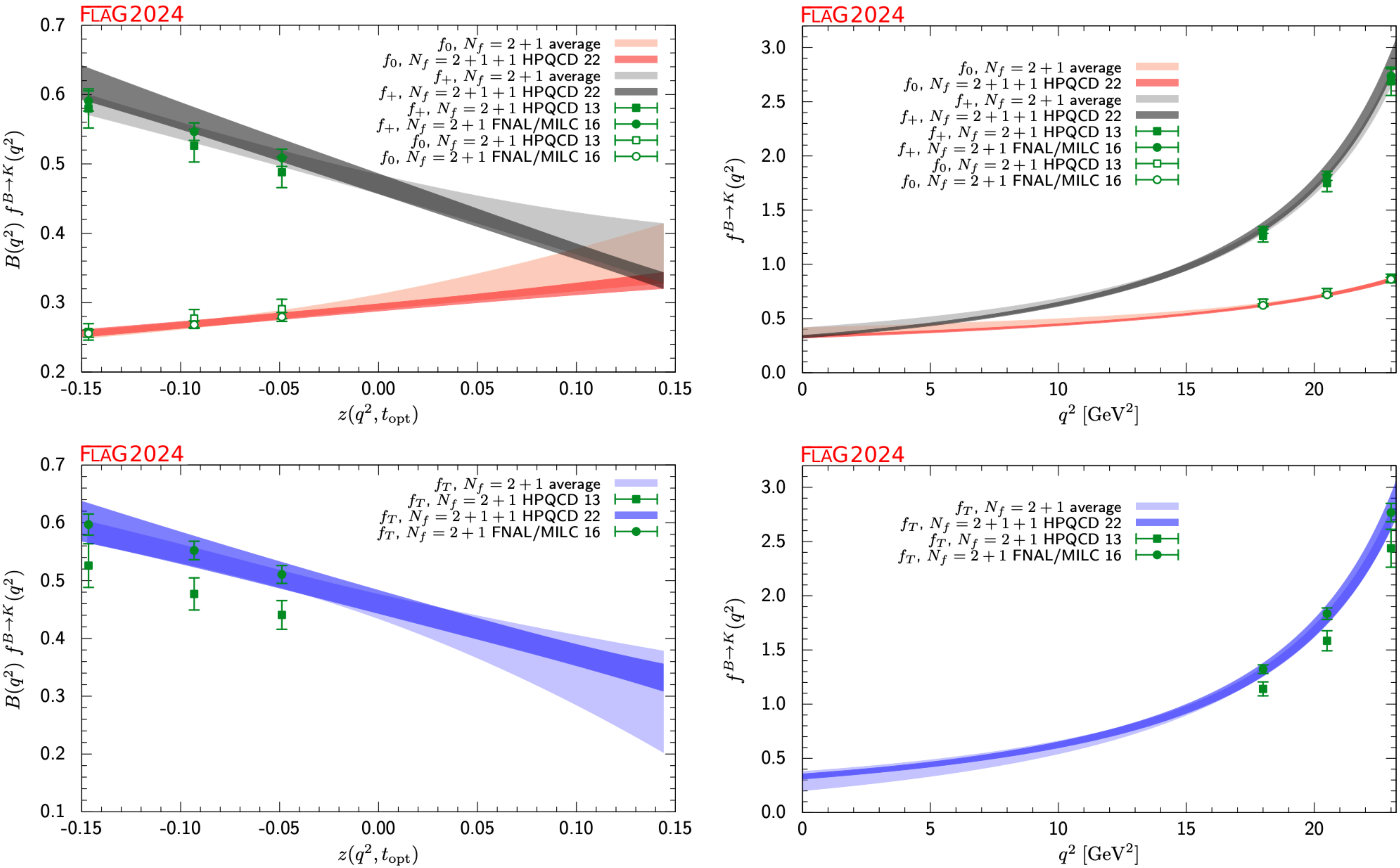
It is **fundamental** to **continue investigating** rare **kaon decays** with **NA62** and, most importantly, **KOTO** and its upgrades!

Thank you!

Back-up



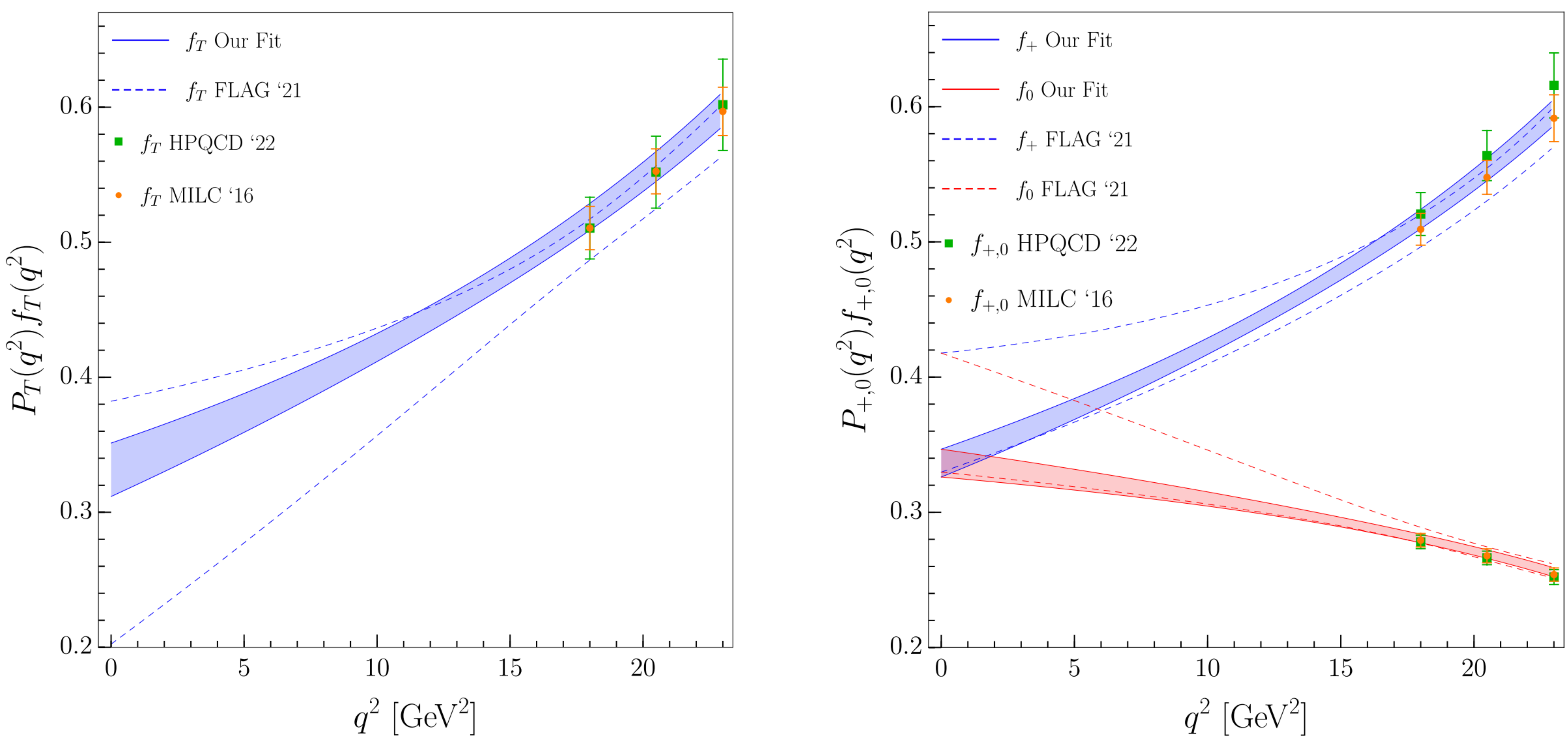
FLAG2021
f_0 average
f_+ average
f_+ HPQCD 13
f_+ FNAL/MILC 16
f_0 HPQCD 13
f_0 FNAL/MILC 16
$B(q^2)\ f^{B\to K}(q^2)$
$z(q^2, t_{\rm opt})$
$f^{B\to K}(q^2)$
q^2 [GeV^2]
f_T average
f_T HPQCD 13
f_T FNAL/MILC 16


FLAG2024
f_0, $N_f = 2+1$ average
f_0, $N_f = 2+1+1$ HPQCD 22
f_+, $N_f = 2+1$ average
f_+, $N_f = 2+1+1$ HPQCD 22
f_+, $N_f = 2+1$ HPQCD 13
f_+, $N_f = 2+1$ FNAL/MILC 16
f_0, $N_f = 2+1$ HPQCD 13
f_0, $N_f = 2+1$ FNAL/MILC 16
$B(q^2)\ f^{B\to K}(q^2)$
$z(q^2, t_{\rm opt})$
$f^{B\to K}(q^2)$
q^2 [GeV^2]
f_T, $N_f = 2+1$ average
f_T, $N_f = 2+1$ HPQCD 13
f_T, $N_f = 2+1+1$ HPQCD 22
f_T, $N_f = 2+1$ FNAL/MILC 16
f_T, $N_f = 2+1+1$ HPQCD 22
f_T, $N_f = 2+1$ HPQCD 13

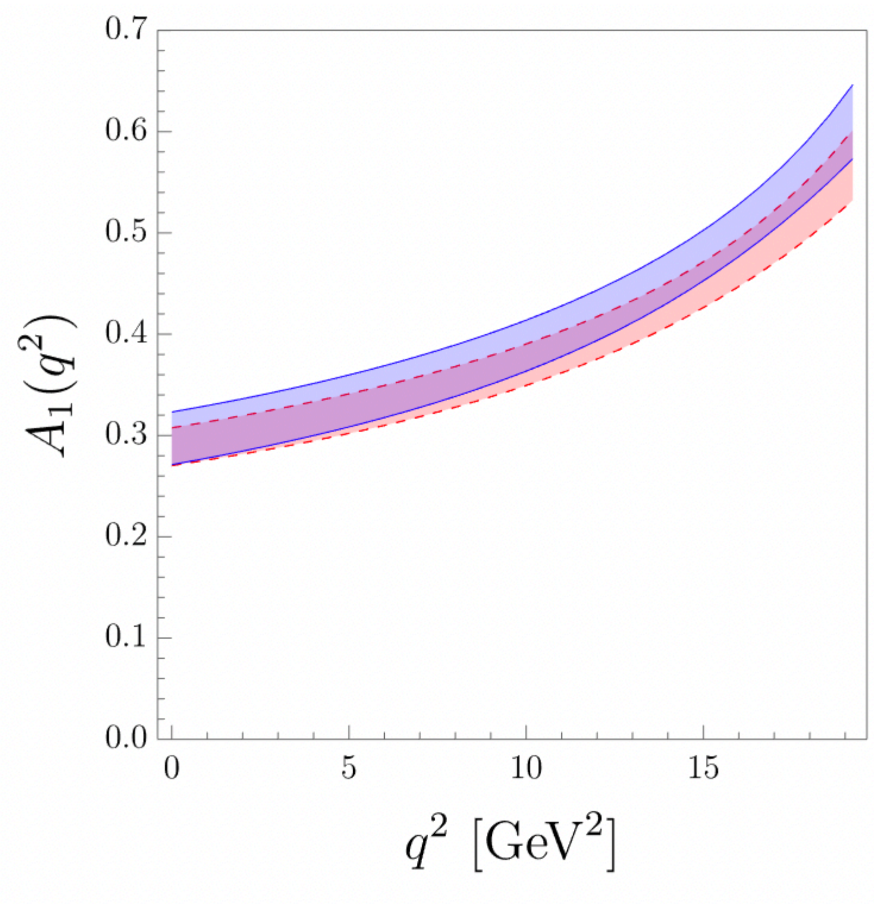
$$f_+(q^2) = \frac{1}{P_+(q^2)} \sum_{n=0}^{N-1} a_n^+ \left[z^n - (-1)^{n-N} \frac{n}{N} z^N\right] \qquad P_i(q^2) = 1 - q^2/M_i^2\,,$$

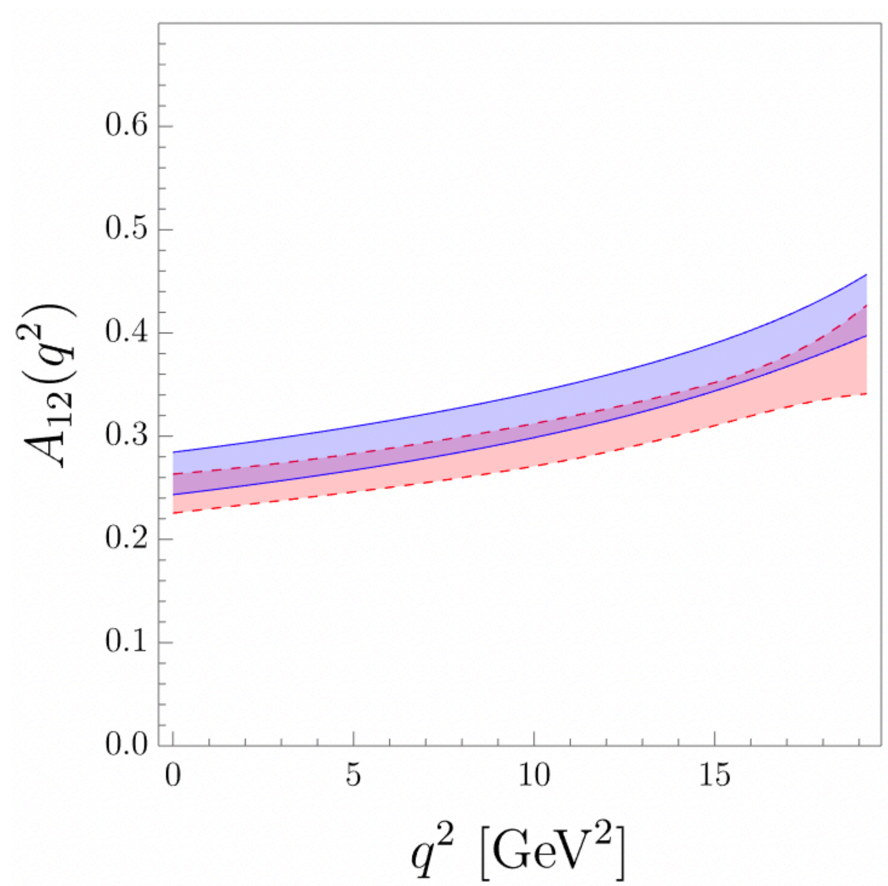
$$f_T(q^2) = \frac{1}{P_T(q^2)} \sum_{n=0}^{N-1} a_n^T \left[z^n - (-1)^{n-N} \frac{n}{N} z^N\right]$$

$$f_0(q^2) = \frac{1}{P_0(q^2)} \sum_{n=0}^{N-1} a_n^0\, z^n\,,$$



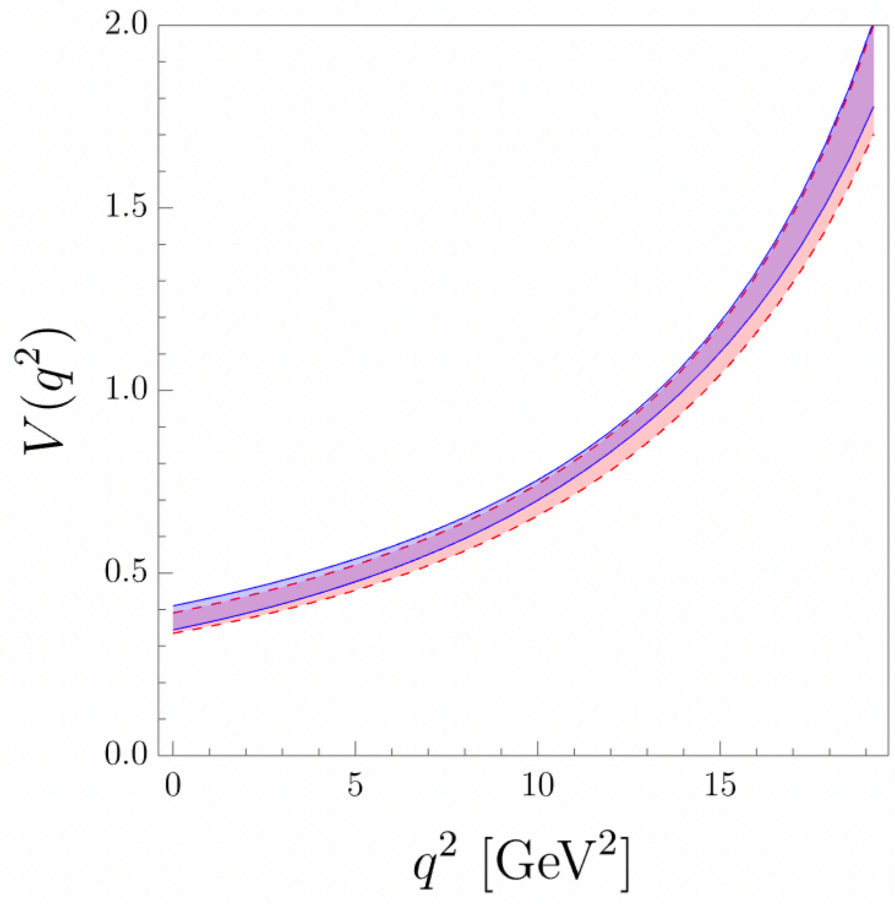
Form-factors: $B \to K^*$





Bharucha et al. '15

Gubernari et al. '23



FLAG - Bag parameters

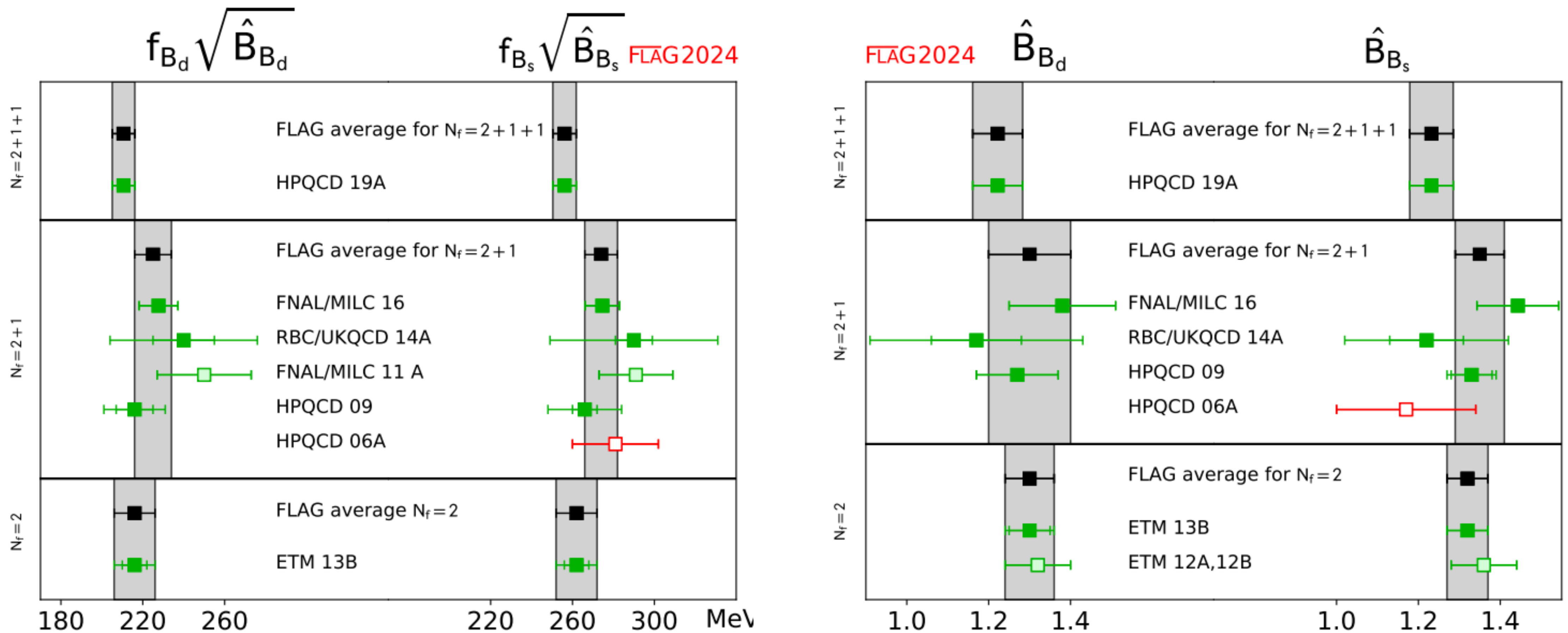
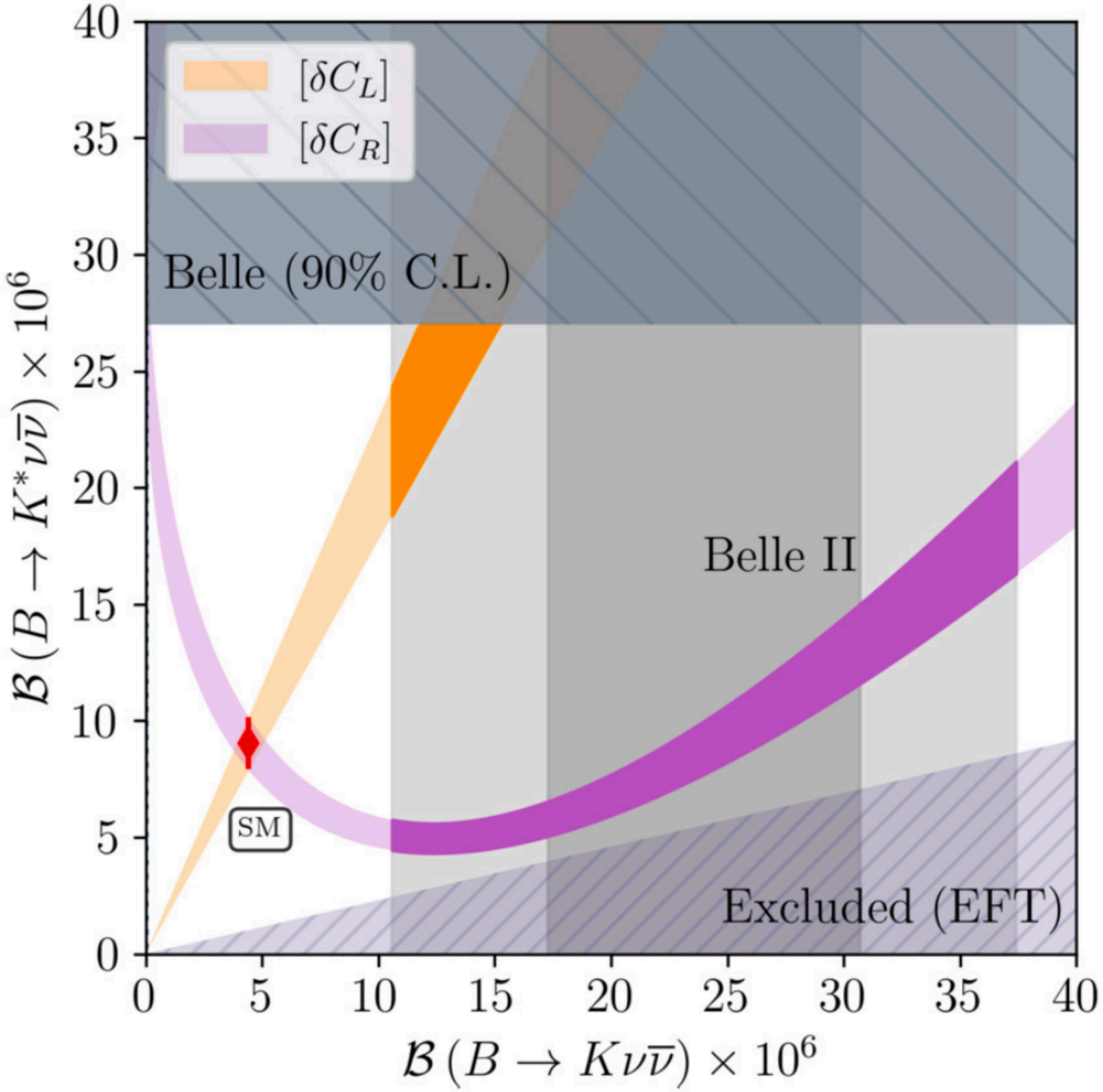
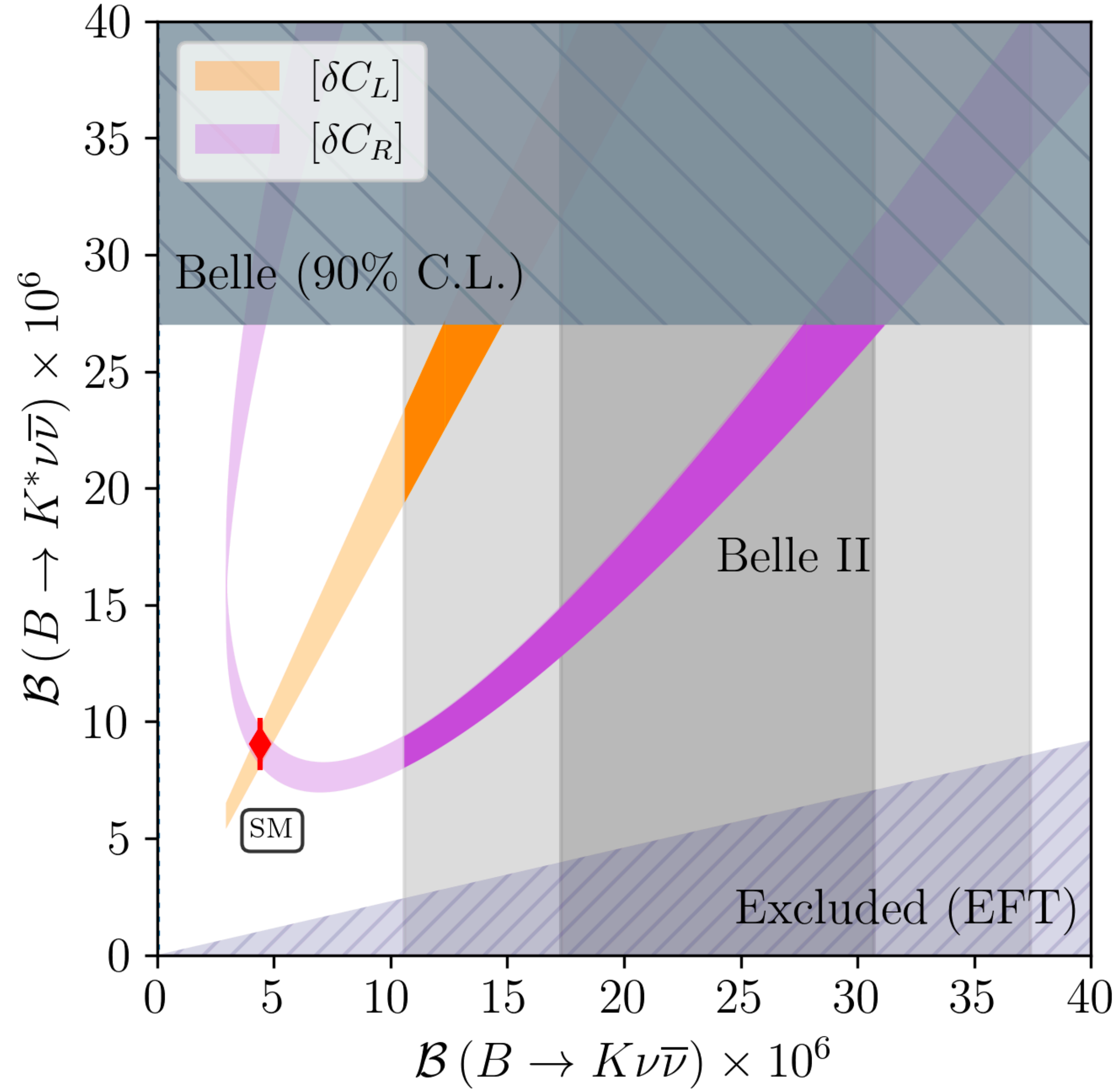


Figure 23: Neutral B- and B_s-meson-mixing matrix elements and bag parameters [values in Tab. 35 and Eqs. (182), (185), (188), (183), (186), (189)].

LFU vs. single-flavor

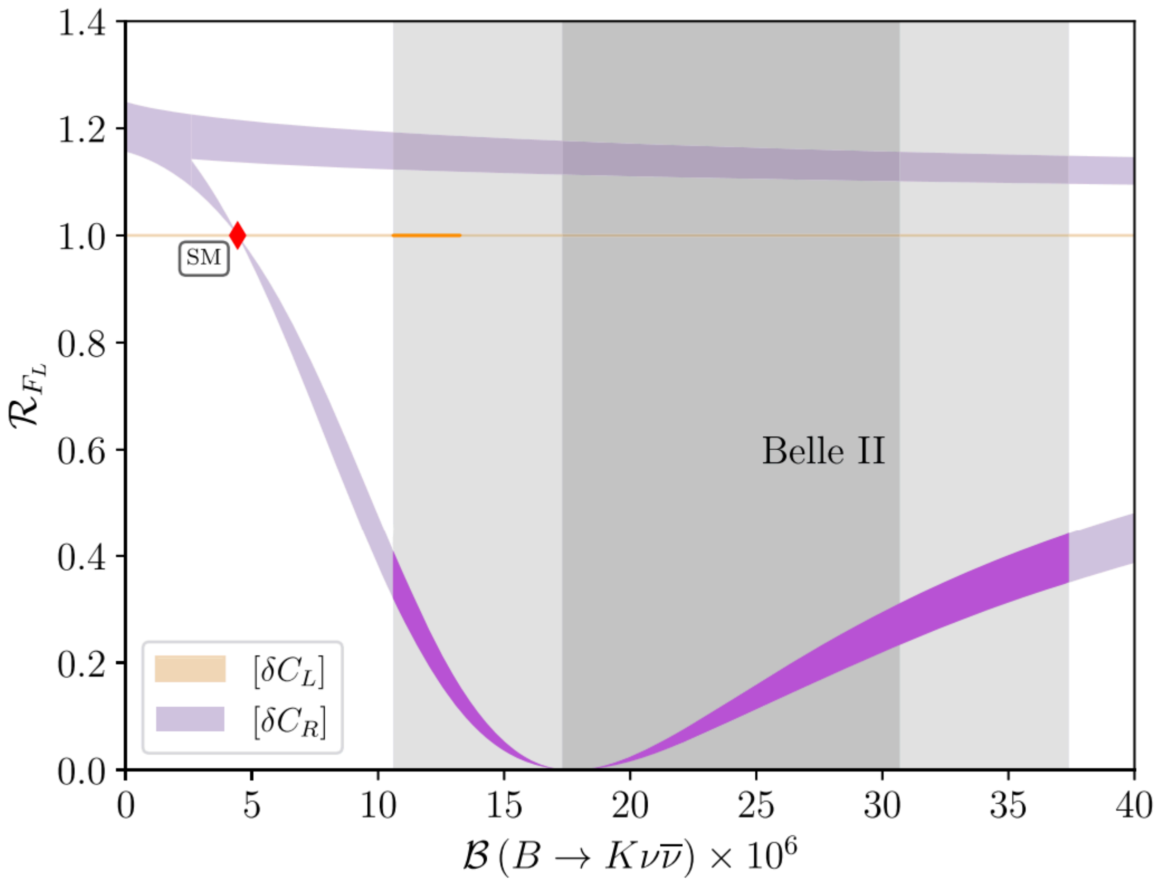
[Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]

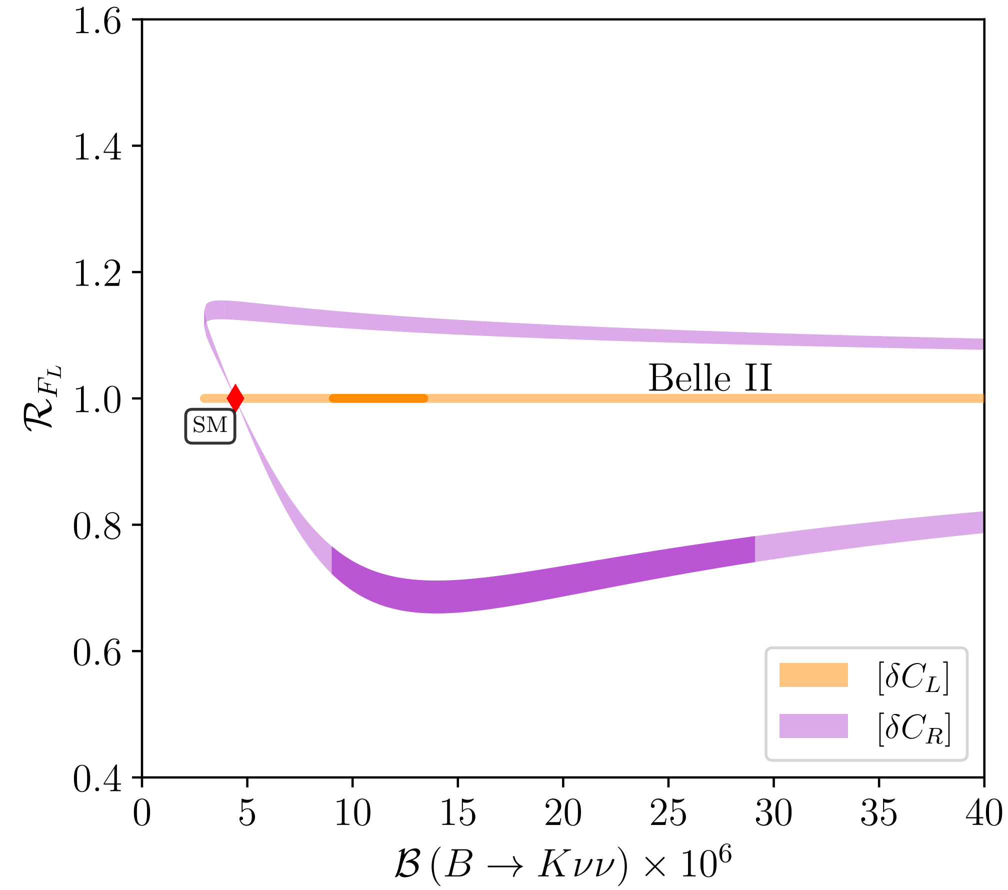




LFU vs. single-flavor

[Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]





EFT for $b \to s\nu\bar{\nu}$

- Low-energy EFT:

see e.g. [Buras et al. '14]

$$\mathcal{L}_{\rm eff}^{b\to s\nu\nu} = \frac{4G_F\lambda_t}{\sqrt{2}}\frac{\alpha_{\rm em}}{2\pi}\sum_{ij}\Big[C_L^{\nu_i\nu_j}\,(\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_{Li}\gamma^\mu\nu_{Lj}) + C_R^{\nu_i\nu_j}\,(\bar{s}_R\gamma_\mu b_R)(\bar{\nu}_{Li}\gamma^\mu\nu_{Lj})\Big] + \text{h.c.}\,,$$

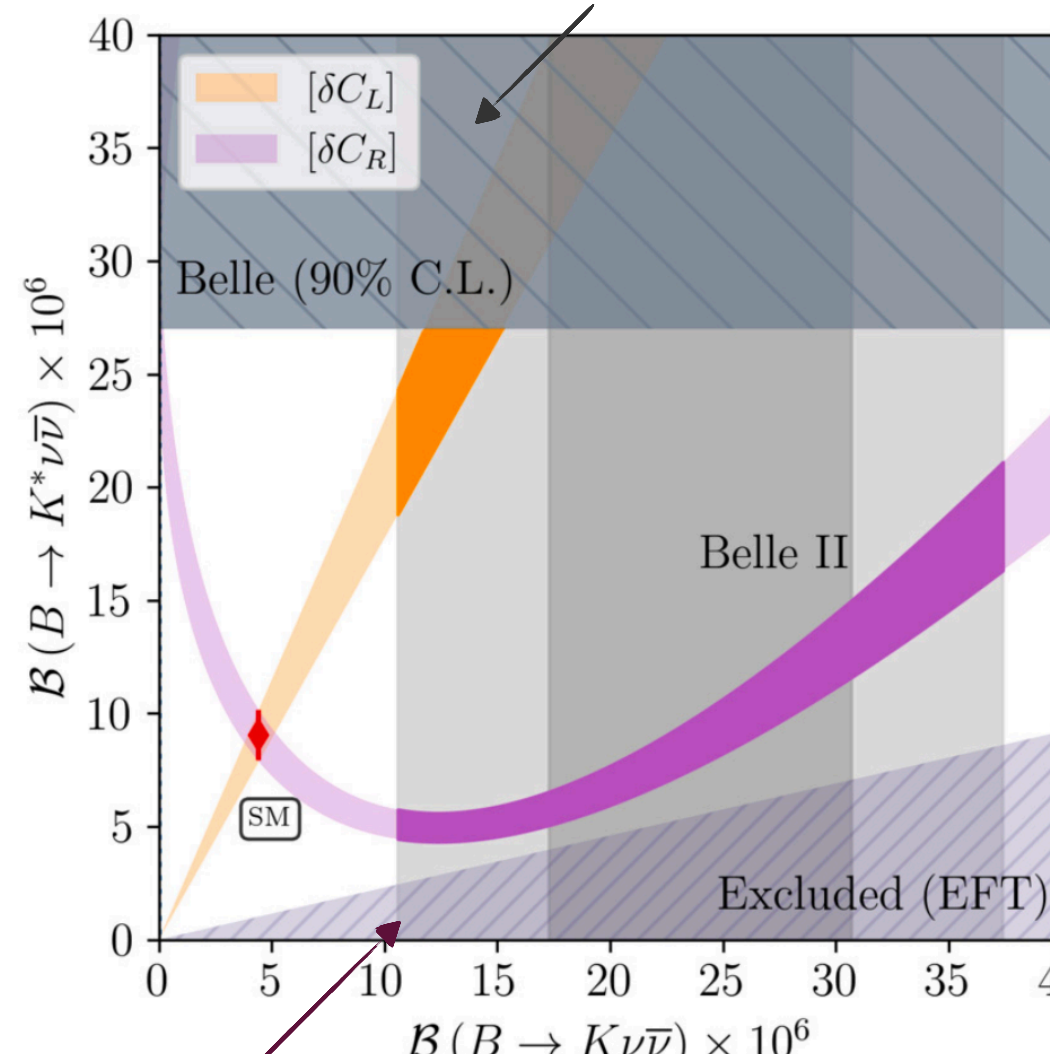
- Complementarity of $B\to K\nu\bar{\nu}$ and $B\to K^*\nu\bar{\nu}$:

$$\frac{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})}{\mathcal{B}(B\to K^{(*)}\nu\bar{\nu})^{\rm SM}} = 1 + \sum_i \frac{2\mathrm{Re}[C_L^{\rm SM}\,(\delta C_L^{\nu_i\nu_i}+\delta C_R^{\nu_i\nu_i})]}{3|C_L^{\rm SM}|^2} + \sum_{i,j}\frac{|\delta C_L^{\nu_i\nu_j}+\delta C_R^{\nu_i\nu_j}|^2}{3|C_L^{\rm SM}|^2} - \eta_{K^{(*)}}\sum_{i,j}\frac{\mathrm{Re}[\delta C_R^{\nu_i\nu_j}(C_L^{\rm SM}\delta_{ij}+\delta C_L^{\nu_i\nu_j})]}{3|C_L^{\rm SM}|^2}\,,$$

$\eta_K = 0$
$\eta_{K^*} = 3.5(1)$

[Becirevic, Piazza, **OS**. '22]

Exclusion from Belle/BaBar



Forbidden region in the EFT approach

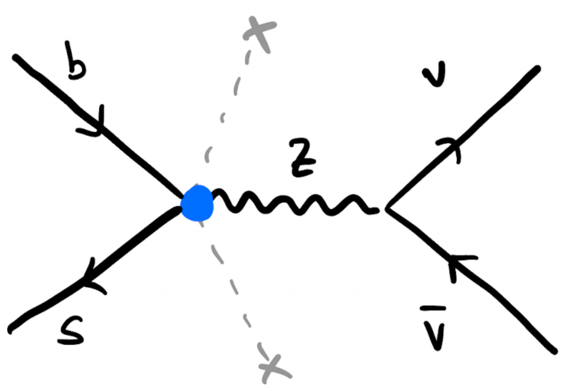
[Bause et al. '23]

[Allwicher et al (**OS**). '23]

SMEFT for $b \to s\nu\nu$ (and $b \to s\ell\ell$)

- **SMEFT** is formulated for $\Lambda \gg v_{\rm ew}$ with $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant operators.
- Gauge invariance **correlates** $b \to s\nu\bar{\nu}$ with $b \to s\ell\ell$ since $L_i = (\nu_{Li}, \ell_{Li})^T$.
- Two types of $d = 6$ **contributions** at tree-level: [Buchmuller & Wyler. '85, Gradkowski et al. '10]

i) $\psi^2 H^2 D$:

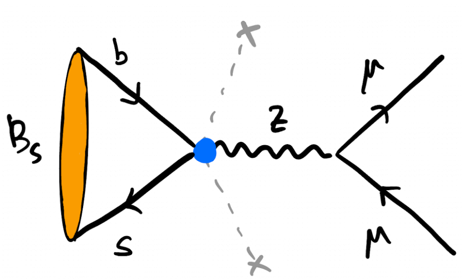


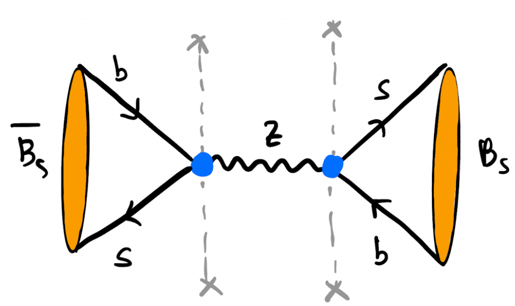
e.g.,

$\mathcal{O}_{Hd} = (H^\dagger \overleftrightarrow{D}_\mu H)(\bar{s}_R \gamma^\mu b_R)$

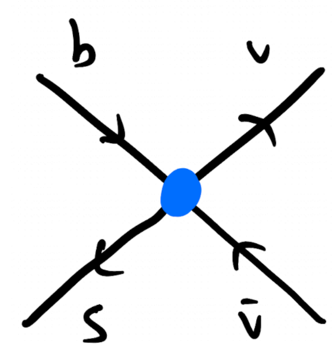
Lepton flavor universal!

$\Rightarrow$ <u>Severely constrained</u> by $\mathcal{B}(B_s \to \mu\mu)$ and Δm_{B_s} :





ii) ψ^4 :



e.g.,

$\mathcal{O}_{ld} = (\bar{L}\gamma^\mu L)(\bar{s}_R \gamma_\mu b_R)$

$\Rightarrow$ Only viable option!

$$\frac{\mathcal{C}}{\Lambda^2} \simeq (5\ \text{TeV})^{-2}$$

[Bause et al. '23] [Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]

SMEFT for $b \to s\nu\nu$ (and $b \to s\ell\ell$)

- ψ^4 operators invariant under $SU(2)_L \times U(1)_Y$:

$b \to s\ell\ell$ $\qquad$ $b \to s\nu\bar{\nu}$

$$[\mathcal{O}_{lq}^{(1)}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{Q}_k\gamma_\mu Q_l)$$
$$[\mathcal{O}_{lq}^{(3)}]_{ijkl} = (\overline{L}_i\gamma^\mu\tau^I L_j)(\overline{Q}_k\tau^I\gamma_\mu Q_l)$$
$$[\mathcal{O}_{ld}]_{ijkl} = (\overline{L}_i\gamma^\mu L_j)(\overline{d}_k\gamma_\mu d_l)$$
$$[\mathcal{O}_{eq}]_{ijkl} = (\overline{e}_i\gamma^\mu e_j)(\overline{Q}_k\gamma_\mu Q_l)$$
$$[\mathcal{O}_{ed}]_{ijkl} = (\overline{e}_i\gamma^\mu e_j)(\overline{d}_k\gamma_\mu d_l)$$

$$\begin{aligned}[\mathcal{O}_{lq}^{(1)}]_{ijkl} &= (\overline{L}_i\gamma^\mu L_j)(\overline{Q}_k\gamma_\mu Q_l)\\ &= (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + \dots\end{aligned}$$

$$\begin{aligned}[\mathcal{O}_{lq}^{(3)}]_{ijkl} &= (\overline{L}_i\gamma^\mu\tau^I L_j)(\overline{Q}_k\gamma_\mu\tau^I Q_l)\\ &= (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) - (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Lk}\gamma_\mu d_{Ll}) + \dots\end{aligned}$$

$$\begin{aligned}[\mathcal{O}_{ld}]_{ijkl} &= (\overline{L}_i\gamma^\mu L_j)(\overline{d}_k\gamma_\mu d_l)\\ &= (\overline{\ell}_{Li}\gamma^\mu\ell_{Lj})(\overline{d}_{Rk}\gamma_\mu d_{Rl}) + (\overline{\nu}_{Li}\gamma^\mu\nu_{Lj})(\overline{d}_{Rk}\gamma_\mu d_{Rl})\end{aligned}$$

Which flavor?

[Bause et al. '23] [Allwicher, Becirevic, Piazza, Rousaro-Alcaraz **OS**. '23]

i. *Couplings to muons* are *tightly constrained* by $\mathcal{B}(B_s \to \mu\mu)$ and $R_{K^{(*)}}$. ✘

ii. The **only viable option** is coupling to τ's *(due to weak exp. limits on $b \to s\tau\tau$).* ✔

⇒ **Predictions:**

$$\frac{\mathcal{B}(B_s \to \tau\tau)}{\mathcal{B}(B_s \to \tau\tau)^{\rm SM}} \simeq \frac{\mathcal{B}(B \to K^{(*)}\tau\tau)}{\mathcal{B}(B \to K^{(*)}\tau\tau)^{\rm SM}} \simeq 10$$

However, **experimentally challenging**…

see e.g. Capdevilla et al. '17

Right-handed neutrinos

Vector

$$\mathcal{O}_{V_{LL}} = (\bar{s}_L \gamma^\mu b_L)(\bar{\nu}_L \gamma_\mu \nu_L)$$

$$\mathcal{O}_{V_{RL}} = (\bar{s}_R \gamma^\mu b_R)(\bar{\nu}_L \gamma_\mu \nu_L)$$

Scalar

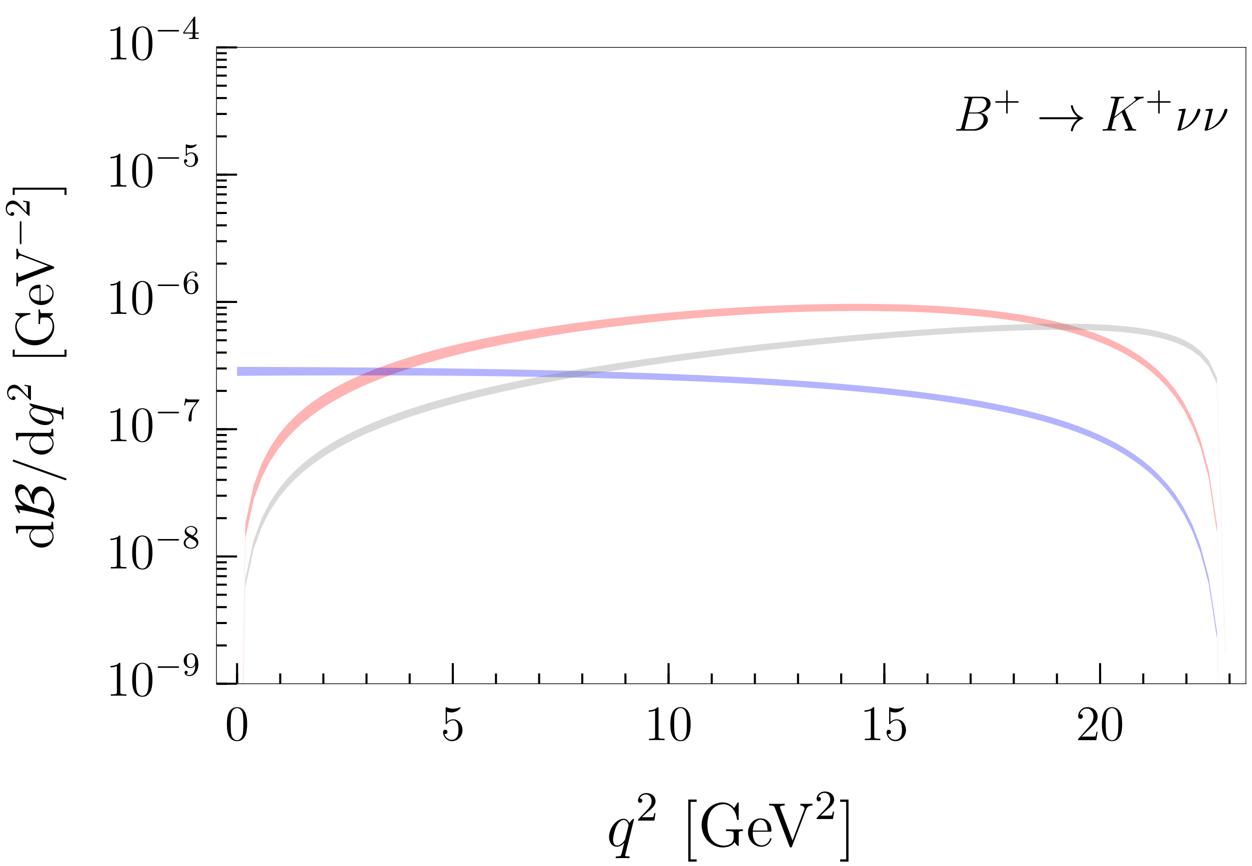
$$\mathcal{O}_{S_{LL}} = (\bar{s}_R b_L)(\bar{\nu}_L N)$$

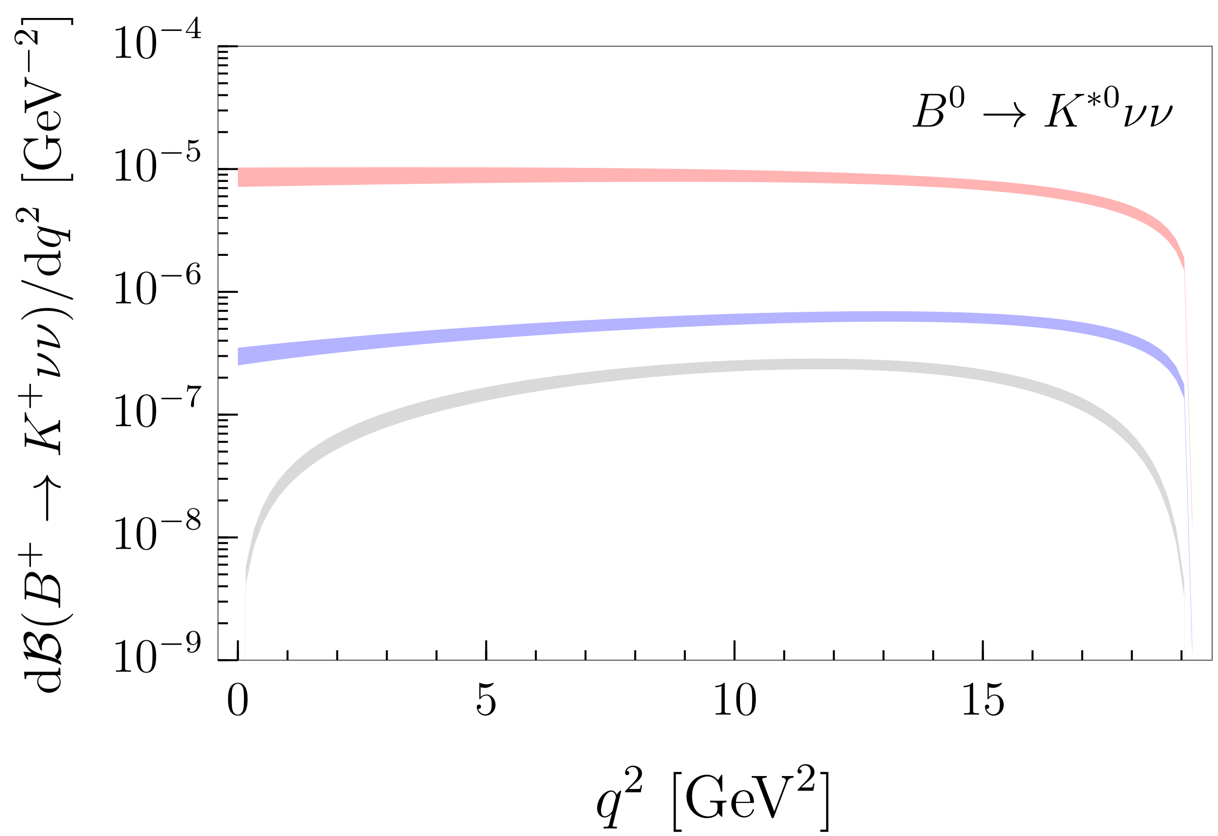
$$\mathcal{O}_{S_{RL}} = (\bar{s}_L b_R)(\bar{\nu}_L N)$$

Tensor

$$\mathcal{O}_{T_R} = (\bar{s}_L \sigma^{\mu\nu} b_R)(\bar{\nu}_L \sigma_{\mu\nu} N)$$

$+\ldots$





[Belle-II, SnowMass - 2207.06307]

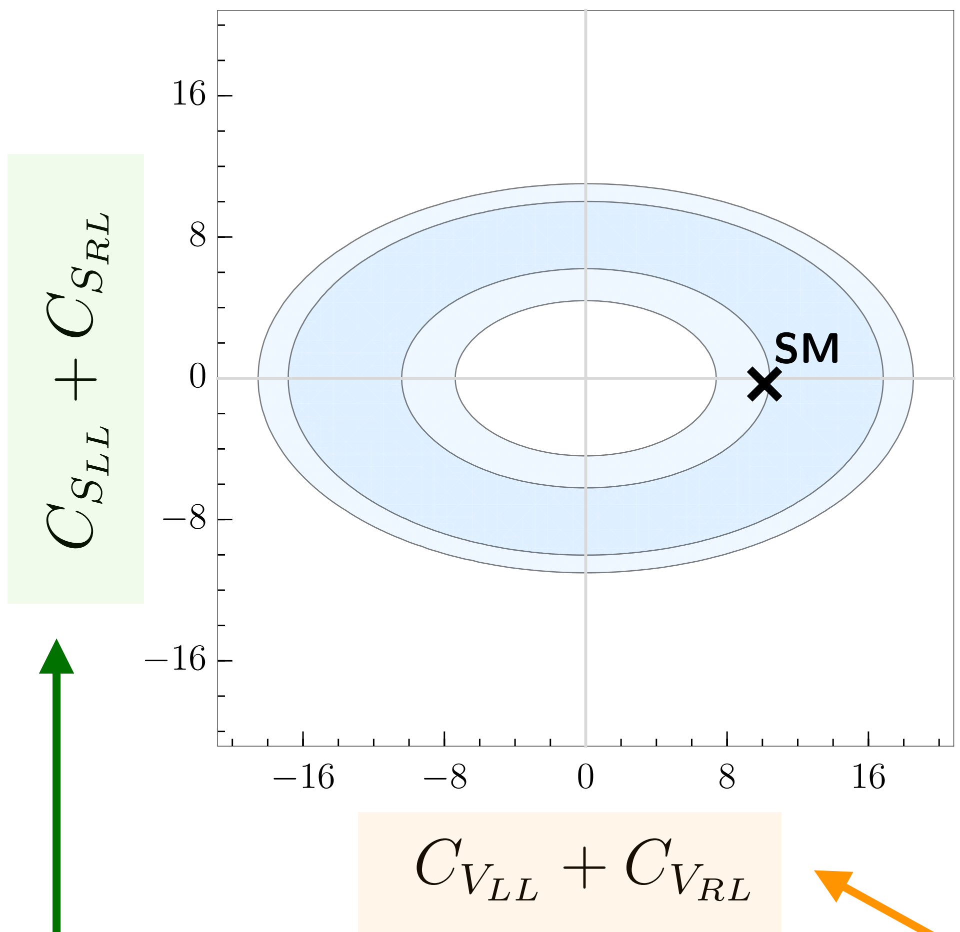
Table 3: Baseline (improved) expectations for the uncertainties on the signal strength μ (relative to the SM strength) for the four decay modes as functions of data set size.

Decay	$1\,\mathrm{ab}^{-1}$	$5\,\mathrm{ab}^{-1}$	$10\,\mathrm{ab}^{-1}$	$50\,\mathrm{ab}^{-1}$
$B^+ \to K^+\nu\bar{\nu}$	0.55 (0.37)	0.28 (0.19)	0.21 (0.14)	0.11 (0.08)
$B^0 \to K^0_{\mathrm{S}}\nu\bar{\nu}$	2.06 (1.37)	1.31 (0.87)	1.05 (0.70)	0.59 (0.40)
$B^+ \to K^{*+}\nu\bar{\nu}$	2.04 (1.45)	1.06 (0.75)	0.83 (0.59)	0.53 (0.38)
$B^0 \to K^{*0}\nu\bar{\nu}$	1.08 (0.72)	0.60 (0.40)	0.49 (0.33)	0.34 (0.23)

[Intermezzo: Using the public likelihood]

See talk by Gaertner

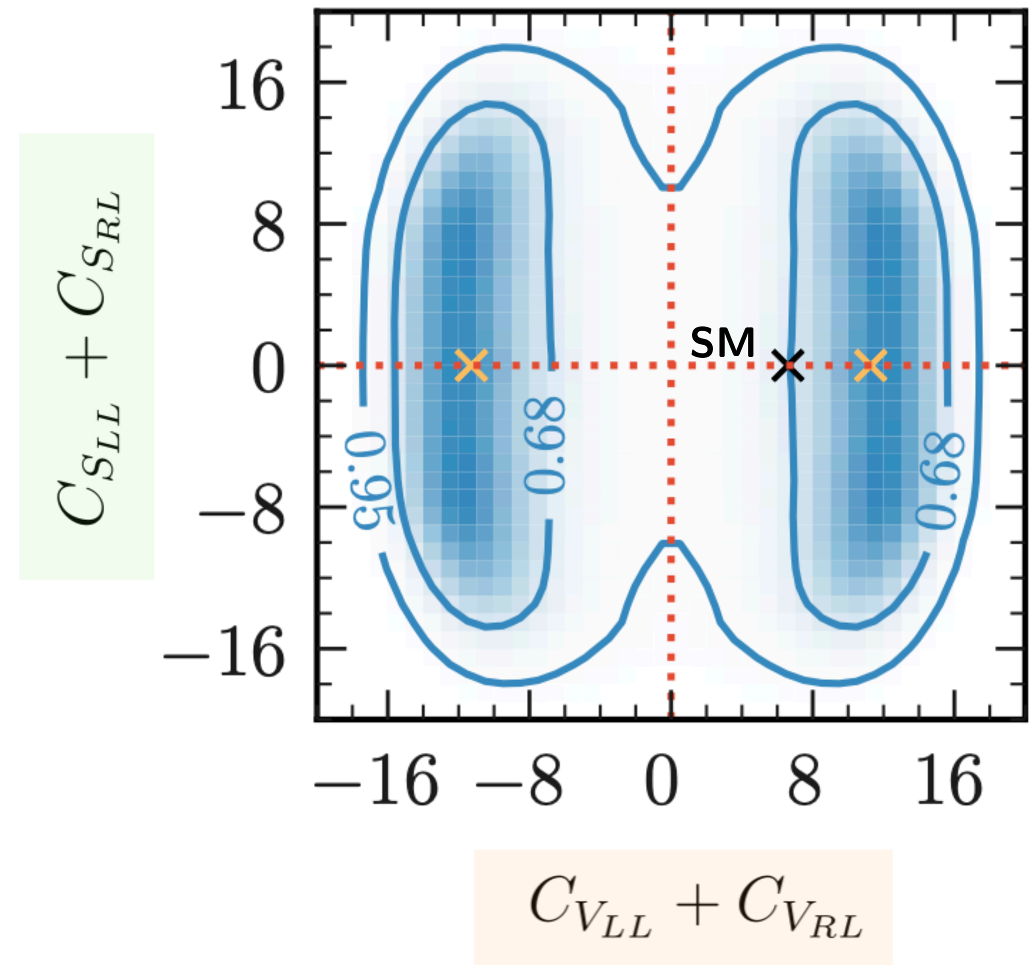
Naive reinterpretation:



vs.

Correct reinterpretation:

[Belle-II, '25], [Gartner et al. '24]



Scalar:

$$O_{S_{LL}} = (\bar{s}_R b_L)(\bar{N}\nu_L)$$
$$O_{S_{RL}} = (\bar{s}_L b_R)(\bar{N}\nu_L)$$

Vector:

$$O_{V_{LL}} = (\bar{s}_L \gamma^\mu b_L)(\bar{\nu}_L \gamma_\mu \nu_L)$$
$$O_{V_{RL}} = (\bar{s}_R \gamma^\mu b_R)(\bar{\nu}_L \gamma_\mu \nu_L)$$

Constraints on **vector operators** are **remain unaffected,** but those on **scalar/tensor ones change significantly!**