Outlook for theoretical calculations

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Points for discussion

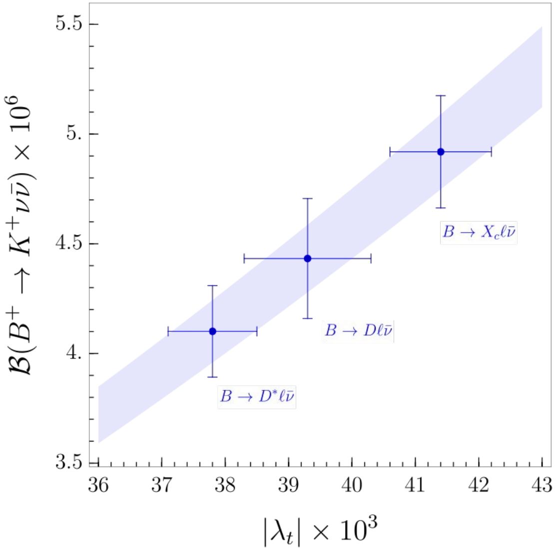
- Improving SM parametric inputs:
  - Exclusive Vcb and B -> D(*) form factors from LQCD
  - Comparing B -> D with Bs -> Ds(*) form-factors
  - Improving kinematic parameterization with unitarity constraints
  - Vcb inclusive: Inclusive decays from the lattice
- Hadronic matrix elements for rare decays:
  - Charm loops: lattice vs dispersive bound vs hadronic models
  - QCD-unstable final states, e.g., B -> K*
- Inputs for NP calculations:
  - Improvements in inclusive calculations
  - Need f_T for B -> D, tensor FFs have been calculated for B -> D*
  - New observables to reduce theoretical uncertainties

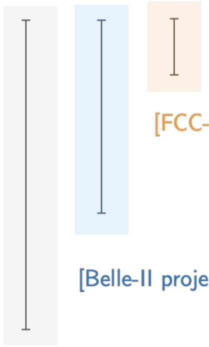
Which CKM value to use?

$$\frac{|V_{ts}^* V_{tb}|}{|V_{cb}|} = 0.965(1)$$

$$|\lambda_t| \times 10^3 = \begin{cases} 41.4 \pm 0.8\,, & (B \to X_c l\bar{\nu}) \\ 39.3 \pm 1.0\,, & (B \to D l\bar{\nu}) \\ 37.8 \pm 0.7\,, & (B \to D^* l\bar{\nu}) \end{cases}$$

[HFLAV, '22]
[FLAG, '21]
[HFLAV, '22]

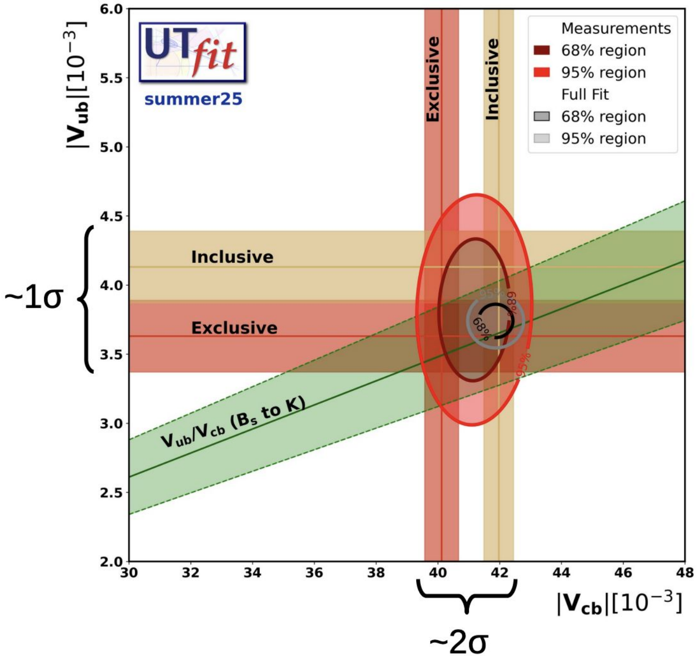




[FCC-ee projection] [Amhis et al. '23]

[Belle-II projection (50 ab^{-1})] [Kou et al. '18]

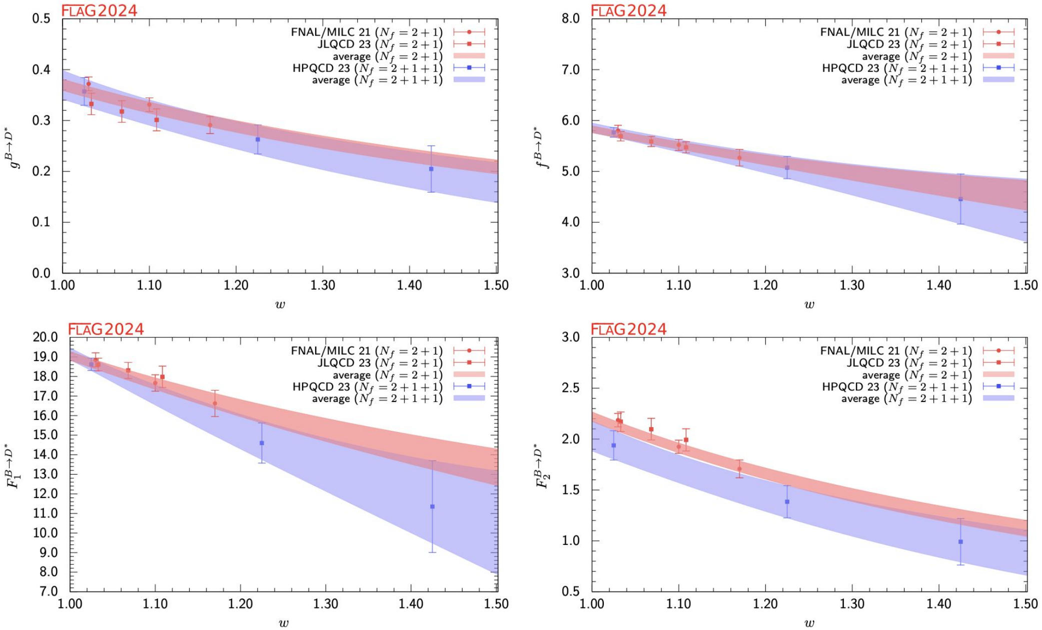
[Belle-II uncertainty. '24]

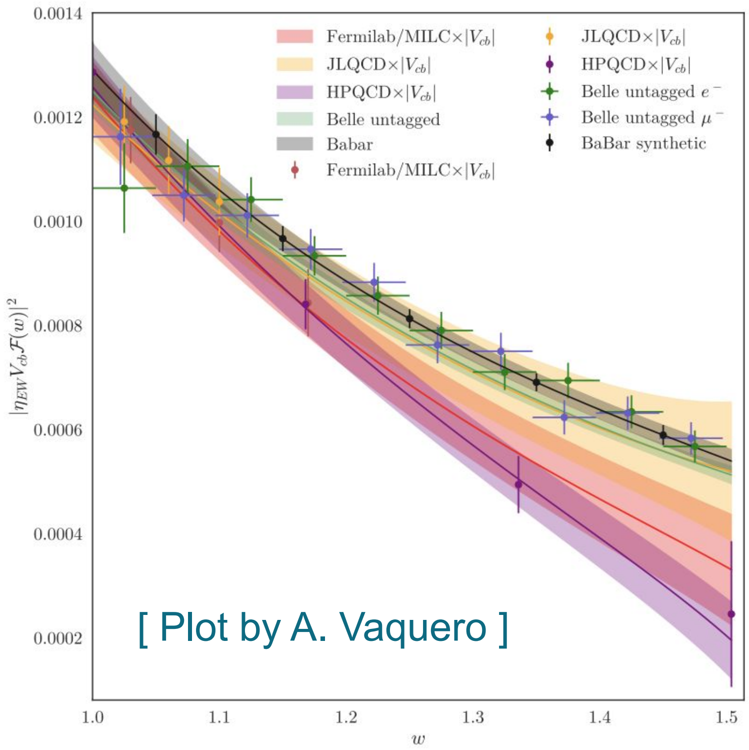


B -> D*: Lattice vs. Exp. data

$$\frac{d\mathcal{B}}{dq^2}(B \to D^*\ell\nu) \propto |V_{cb}|^2\, |\mathcal{F}(w)|^2$$

$$\left[w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \right]$$

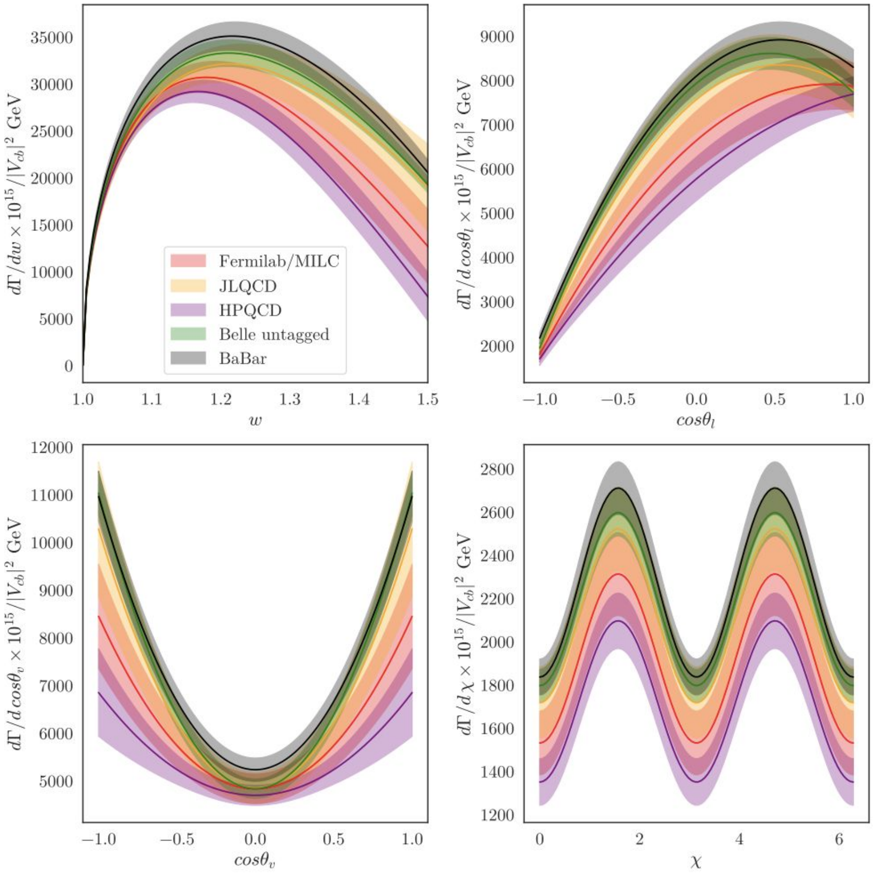




$$|V_{cb}|^{\mathrm{HPQCD}} = 39.31(74) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{FerMILC}} = 38.17(85) \times 10^{-3}$$

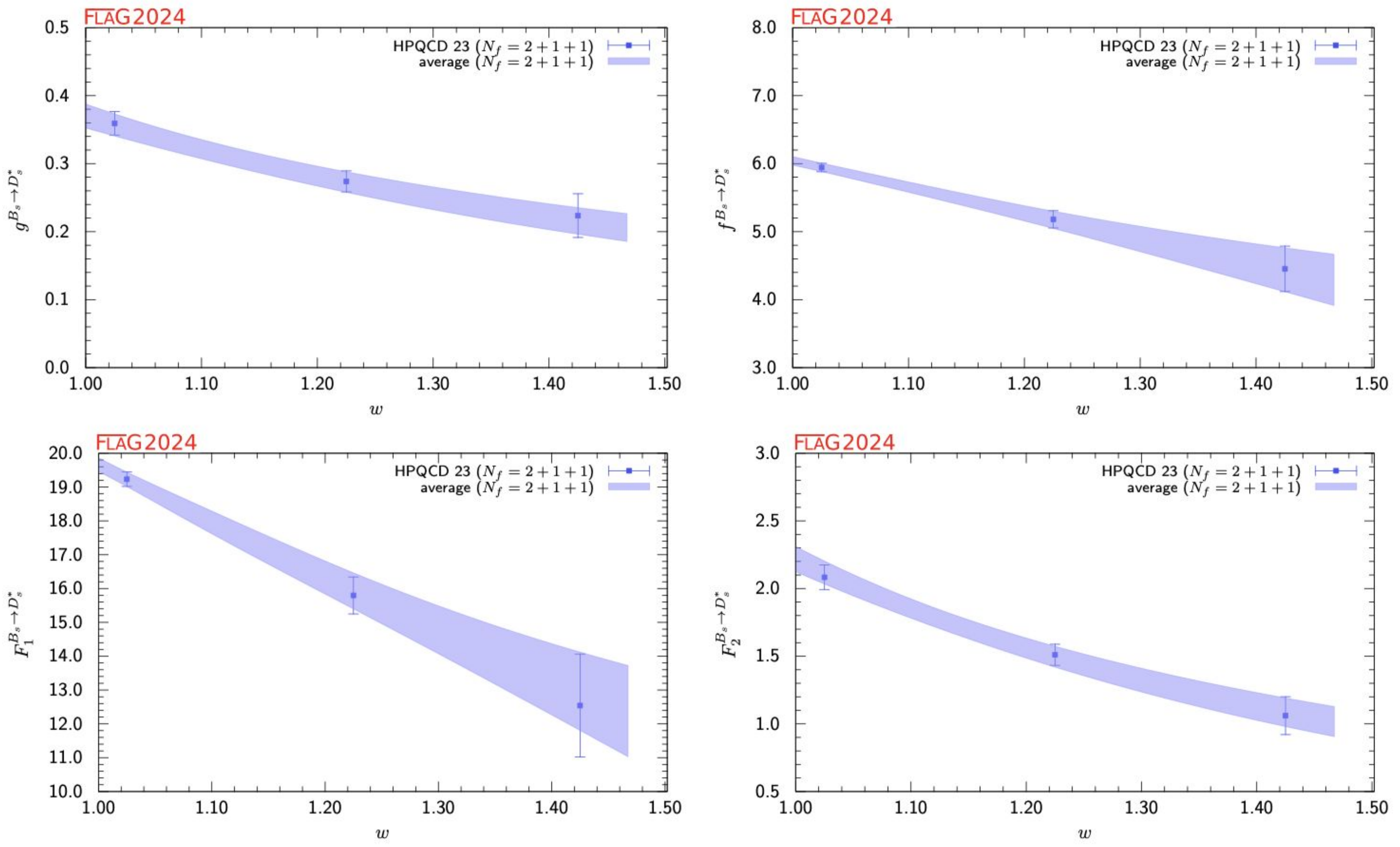
B -> D*: Lattice vs. Exp. data



- Belle-II data and more/improved lattice inputs will be fundamental to solve this discrepancy.
- HPQCD has also computed Bs->Ds*.

[Plots by A. Vaquero]

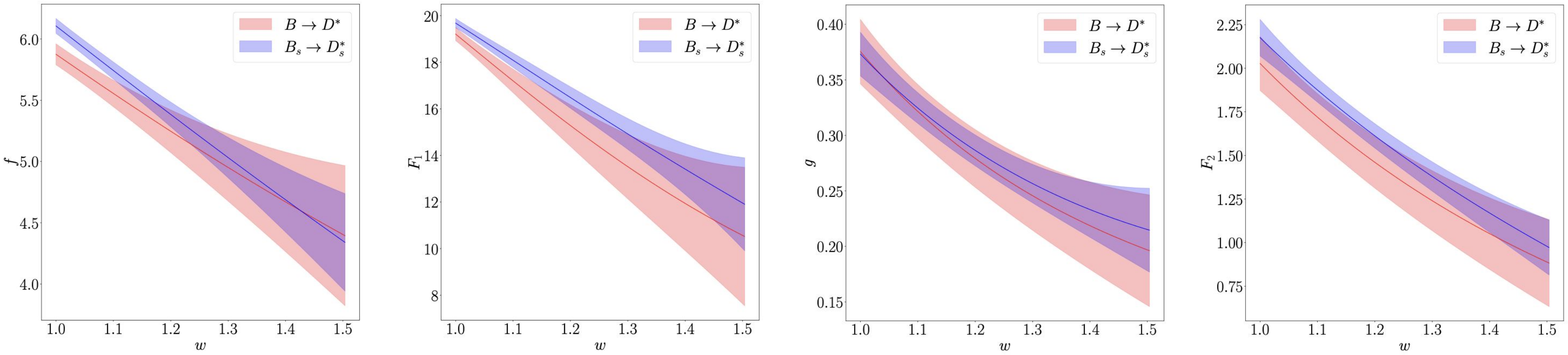
Bs -> Ds*



- Bs->Ds* easier on lattice than B->D*
- Should be similar to B->D* ; offers a sanity check
- Experimentally determined shape would be useful

[FLAG 2024]

Comparing B -> D* with Bs -> Ds*



[(HPQCD) Harrison and Davies, 2304.03137]

Improving kinematic parameterizations

- Dispersive matrix methods - from continuum results at large q2, provides shape for all q2 without explicit parameterization or associated truncation error

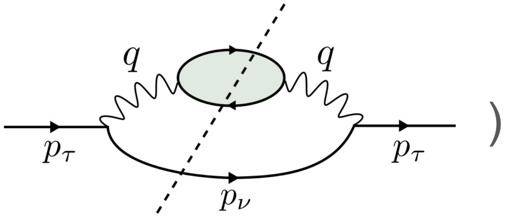
  [Martinelli et al. 2105.08674 , …]

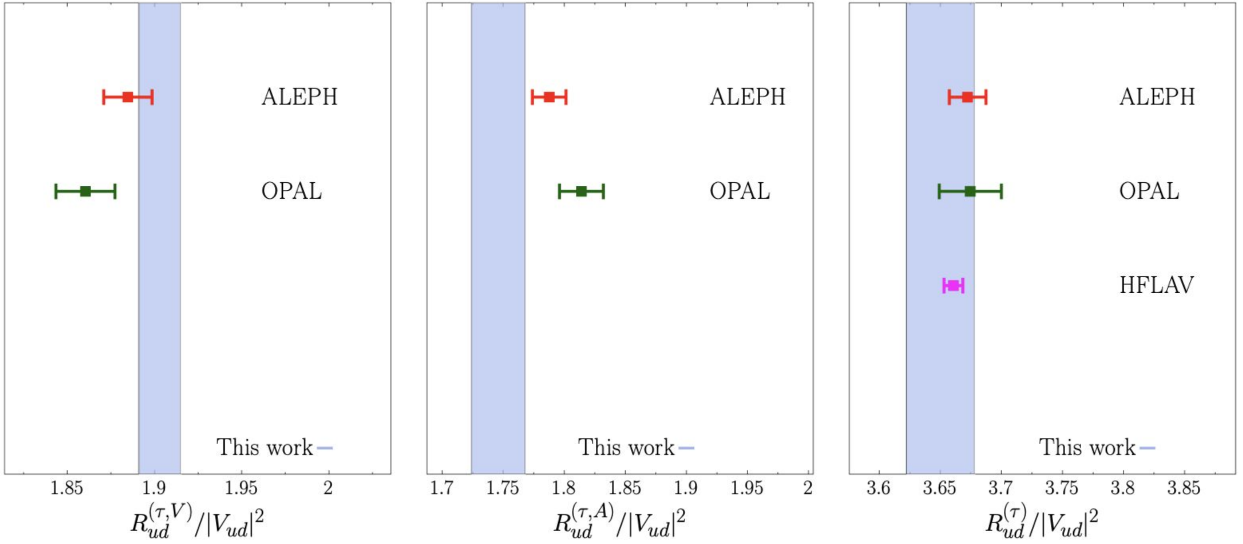
- Improved parameterization - build unitarity constraints into z-parameterization
- Evaluate truncation error

  [(RBC/UKQCD) Flynn et al. 2303.11280]

  [Danny’s talk]

Lattice for inclusive decays: Tau

$\Gamma(\tau \to X\nu_\tau) \propto \mathrm{Im}($  $)$



$$R_{ud}^{(\tau)} = \frac{\Gamma(\tau \to X_{ud}\,\nu_\tau)}{\Gamma(\tau \to e\,\bar{\nu}_e\,\nu_\tau)}$$

$$\Delta_{V-A}^{(\tau)} \equiv \frac{R_{ud}^{(\tau,V)} - R_{ud}^{(\tau,A)}}{R_{ud}^{(\tau)}}$$

$$\Delta_{V-A}^{(\tau)} = 0.042\ (5)$$

vs experiment

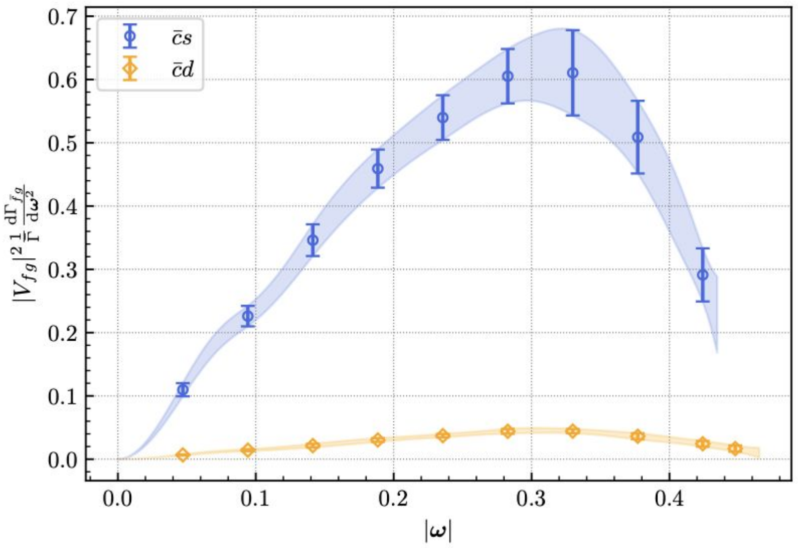
$$\Delta_{V-A}^{(\tau)}(\mathrm{ALEPH}) = 0.026\ (7)$$

$$\Delta_{V-A}^{(\tau)}(\mathrm{OPAL}) = 0.013\ (7)$$

[(ETMC) Evangelista et al. 2308.03125]

Lattice for inclusive decays: Charm

Ds -> X lv



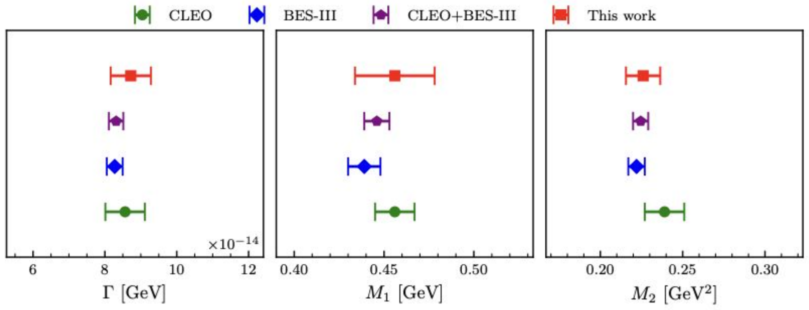


FIG. 4. Comparison between the experimental results from Refs. [44–46, 56] and our theoretical prediction (red points), for the decay rate (left-panel) and for the first (middle-panel) and second (right-panel) lepton moment.

[(ETMC) De Santis et al. 2504.06064]

Lattice for inclusive decays: Bs

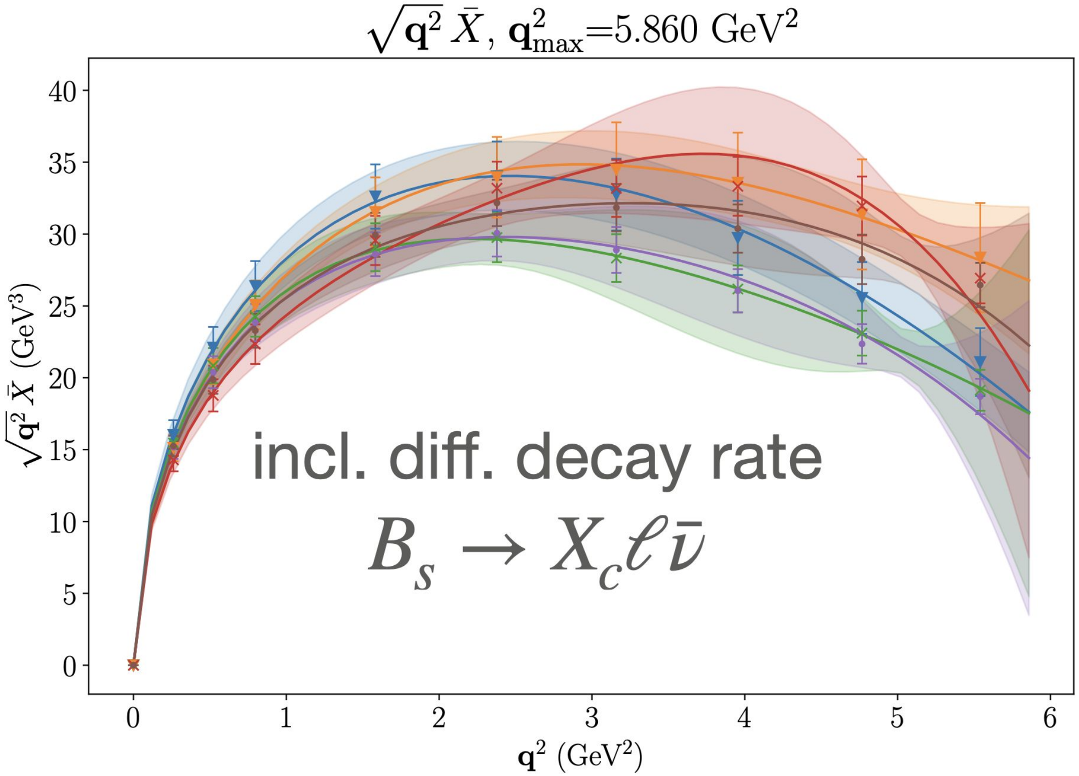
- Lattice inclusive calculations are making rapid progress, working toward the B

- Barone et al. calculated inclusive differential decay rate for Bs -> Xc l nu

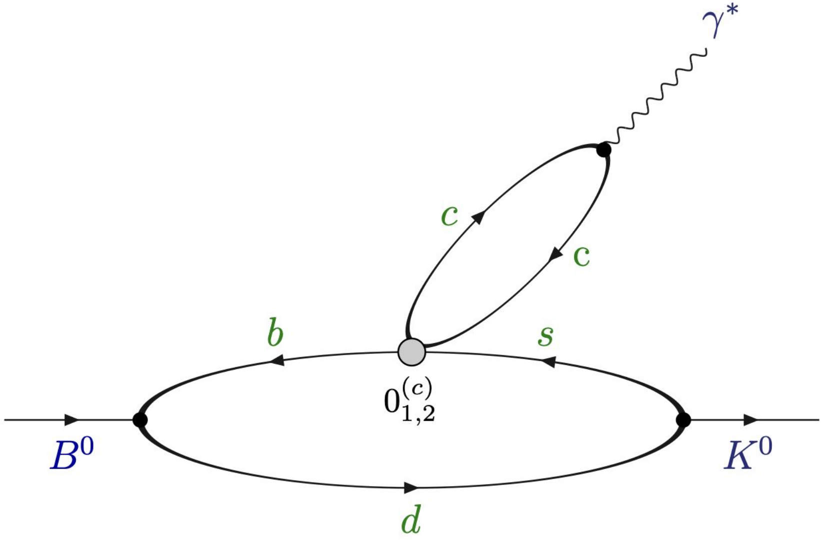
  [Barone et al. 2305.14092]

- Important to understand systematic effects involved

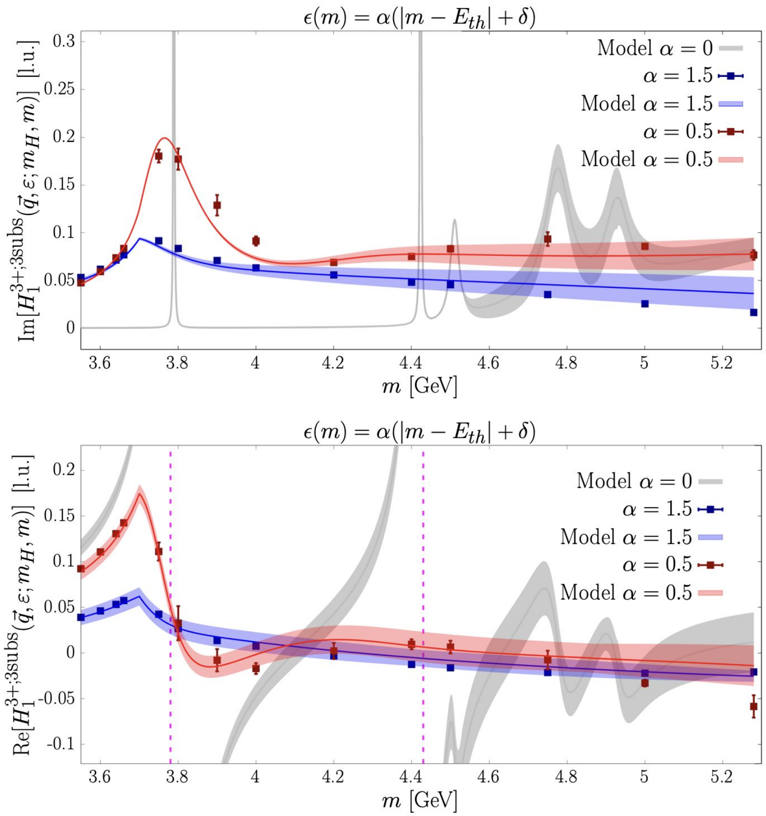
  [Kellermann et al. 2504.03358]



Charm loops



$B_s \to \eta_s \ell^+ \ell^-$



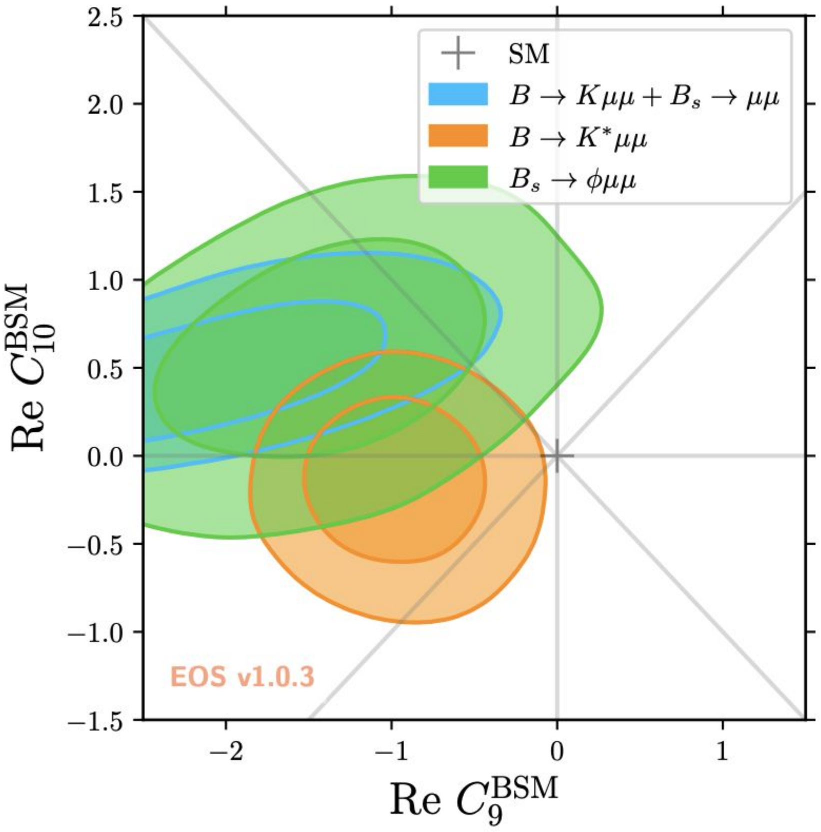
- As with inclusive decays, 4 pt correlators give access to the relevant physics, ie. non-local form factors
- Free from assumptions about factorization

[Frezotti et al. 2508.03655]

Charm loops

[Gubernari et al. 2206.03797]

- Instead of calculating directly, Gubernari et al. use dispersive bounds to constrain the contribution from non-local form factors
- Find local form factors responsible for majority of theory uncertainty
- Tension between SM and experiment remains



New Physics in B -> Xc l nu

[Carvunis et al. 2507.22123]

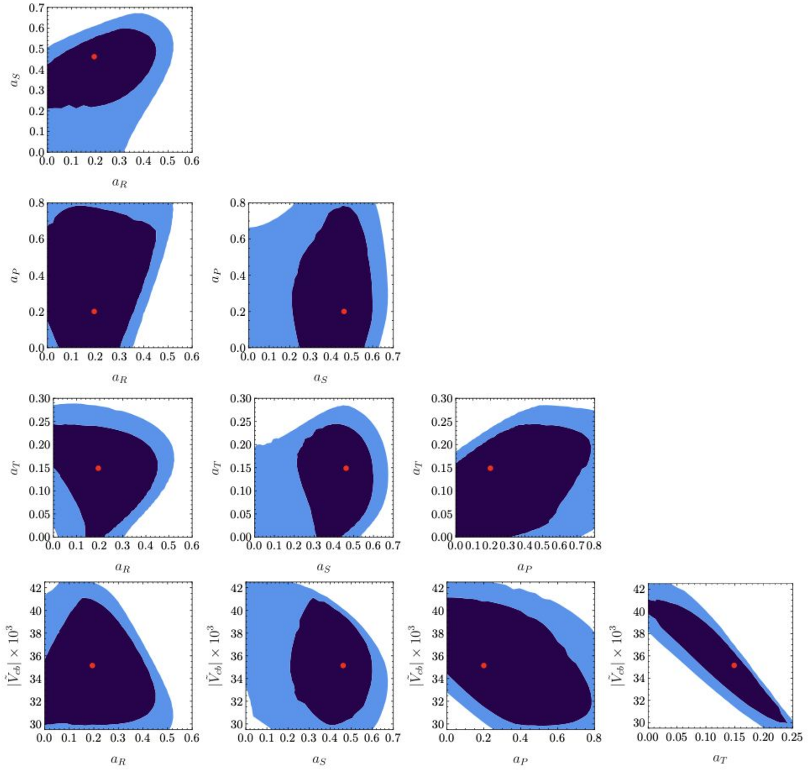


Figure 3. 2D contour plots for pairs of fit parameters in the global fit with $\Delta\chi^2 \leq 2.30$ (dark blue) and $\Delta\chi^2 \leq 5.99$ (light blue) (plots for the phases are not shown for simplicity). The red dot indicates the best fit point.

m_b [GeV]	m_c [GeV]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]	BR$_{c\ell\bar\nu}$ [%]	$10^3 \lvert V_{cb} \rvert$
4.574	1.090	0.435	0.278	0.164	-0.090	10.62	41.64
0.012	0.010	0.040	0.048	0.018	0.089	0.15	0.47
1	0.390	-0.229	0.560	-0.022	-0.181	-0.064	-0.421
	1	0.015	-0.238	-0.028	0.084	0.034	0.071
		1	-0.097	0.535	0.266	0.144	0.346
			1	-0.261	0.004	0.001	-0.271
				1	-0.014	0.025	0.172
					1	-0.010	0.056
						1	0.694
							1

Table 3. Results for the global fit with SM only. Central values on the first line, uncertainties on the second line and correlation matrix below.

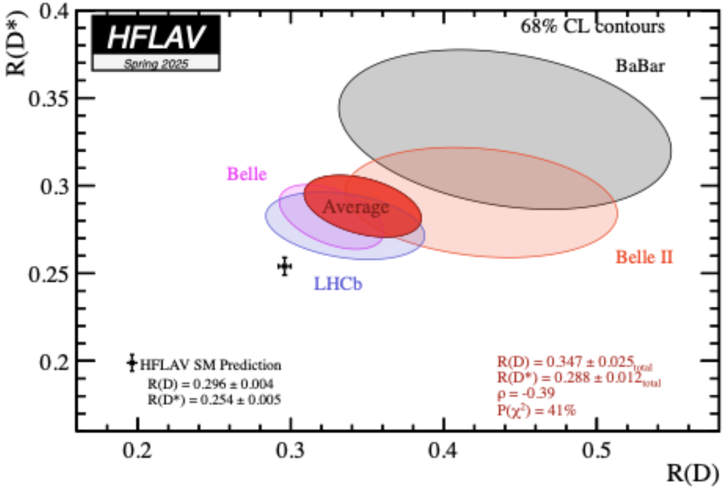
m_b [GeV]	m_c [GeV]	μ_π^2 [GeV^2]	μ_G^2 [GeV^2]	ρ_D^3 [GeV^3]	ρ_{LS}^3 [GeV^3]	BR$_{c\ell\bar\nu}$ [%]
4.569	1.092	0.454	0.342	0.195	-0.126	10.72
$^{+0.017}_{-0.016}$	$^{+0.011}_{-0.011}$	$^{+0.049}_{-0.050}$	$^{+0.065}_{-0.067}$	$^{+0.027}_{-0.029}$	$^{+0.096}_{-0.104}$	$^{+0.16}_{-0.16}$
a_R	a_S	a_P	a_T	$\cos(\delta_R)$	$\cos(\delta_{ST})$	$\cos(\delta_{PT})$
0.19	0.46	0.20	0.15	0.99	-0.94	0.91
$^{+0.21}_{-0.16}$	$^{+0.10}_{-0.14}$	$^{+0.43}_{-0.20}$	$^{+0.07}_{-0.11}$	$^{+0.01}_{-0.81}$	$^{+0.92}_{-0.06}$	$^{+0.09}_{-1.91}$

Table 4. Results for the global fit with full set of NP operators. Central values on the first line, 68.3% confidence level intervals on the second line. The value of $|\tilde{V}_{cb}|$ extracted from the fit parameters is shown in (3.12).

Optimized observables

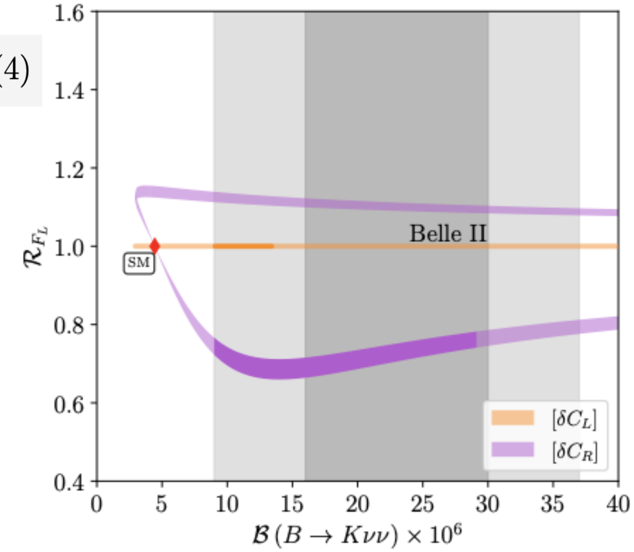
$$R_{H_c} = \frac{\mathcal{B}(B \to H_c \tau \nu)}{\mathcal{B}(B \to H_c \ell \nu)}$$

$$F_L(B \to K^* \nu\nu) = \frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} \Big/ \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}$$



$$F_L(B \to K^* \nu\bar{\nu})^{\mathrm{SM}} = 0.49(4)$$

$$\mathcal{R}_{F_L} \equiv \frac{F_L}{F_L^{\mathrm{SM}}}$$



- Can we devise other observables less sensitive to hadronic uncertainties (while remaining sensitive to New Physics effects)?

Final comments/questions

- Opportunity for progress for parametric (Vcb) and form-factor uncertainties.
- Many exploratory directions on the lattice (e.g., B->K* form factors, inclusive decays, charm loops, ...).
- The public B -> Kvv likelihood is very useful for theory interpretations (both in the SM and beyond). Can this be extended to other channels (e.g., b->c tau nu modes)?
- Interplay between theory and experiment is fundamental to verify precision theory calculations.
- Anything else?