Charm decays with missing energy

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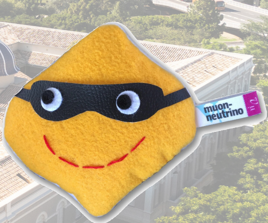
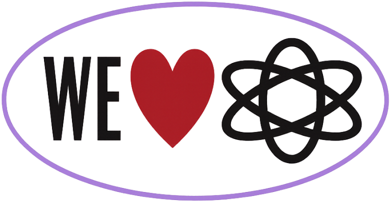


Belle II Physics week, October 6–10, 2025, KEK (Japan)

Universidad Europea VALENCIA

Why are we here?

We love particle physics!

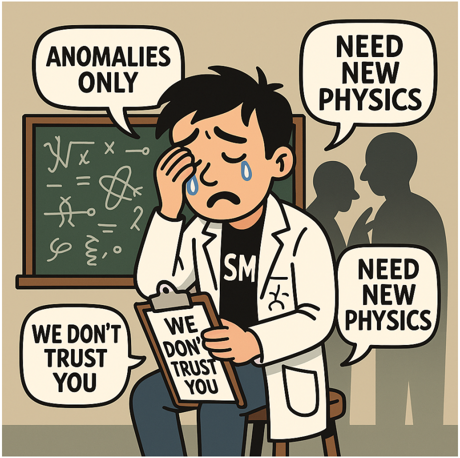


None of us thinks the Standard Model is complete!

Why?

Because there are many whys!

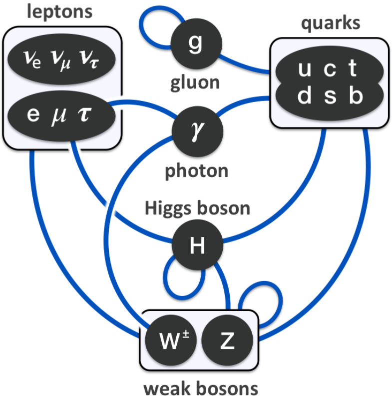
- **Although most experimental data are well explained by the SM.**
- **New physics beyond the SM is needed!**
  e.g., matter–antimatter asymmetry, Dirac/Majorana nature of neutrinos, three families, . . .
- **We are not satisfied!**



Where do we look for new physics beyond the SM?

Wait, what is the SM?

SM = Symmetries + Particle content + SSB pattern



How many fermion combinations?

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$n = 12$ fermions, $k = 2$ (fermions come in pairs)

$$\Downarrow$$

$$\binom{12}{2} = \frac{12!}{2!(12-2)!} = 66$$

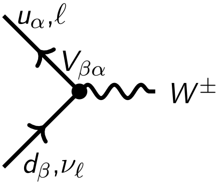
Fermion generations provide a rich environment

Flavour physics is a suitable place to search for physics beyond the SM.

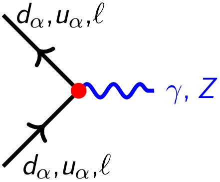
The heart of flavour physics in the SM

Flavour physics describes **interactions** among **fermion generations**.

- **Charged-current interactions**



- **Neutral-current interactions**



with $(u_1, u_2, u_3) = (u, c, t)$ and $(d_1, d_2, d_3) = (d, s, b)$.

What's the game?

- We look for experimental deviations from SM predictions:

$$\Delta\mathcal{O} = (\mathcal{O}_{\text{exp}} - \mathcal{O}_{\text{SM}}) \pm \sqrt{(\delta\mathcal{O}_{\text{SM}})^2 + (\delta\mathcal{O}_{\text{exp}})^2}$$

- If the SM works really well, $\Delta\mathcal{O}$ is consistent with zero. This, of course, depends on the experimental and theoretical uncertainties of the observable.
- We can use the fact that we know how the SM works and search for an observable $\mathcal{O}$ to measure that is zero:

$$\boxed{\text{SM symmetries} \Rightarrow \mathcal{O}_{\rm SM} = 0 \; (\approx 0)}$$

or close to zero due to (soft) symmetries.

Which process (with associated $\mathcal{O}$) do we look for?

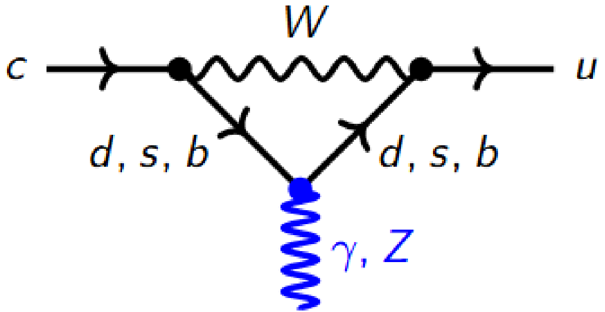
- The SM tells us that, at tree level, we have the following **fermion bilinears**:
  - **Charged:** $u_\alpha d_\beta W$, $\ell \nu_\ell W$, with $\ell = e, \mu, \tau$.
  - **Neutral:** $\psi_\alpha \psi_\alpha \gamma$, $\psi_\alpha \psi_\alpha Z$, with $\psi = u, d, \ell$.
- But what about the following neutral bilinears:

$$\textbf{Down sector: } \boldsymbol{d_\alpha d_\beta X^0} \text{ or } \textbf{Up sector: } \boldsymbol{u_\alpha u_\beta X^0}$$

where X^0 is a neutral particle or a neutral combination of particles.

Example of an SM Feynman diagram in the up sector

They are not present at tree level in the SM, but they can appear at one-loop level; e.g., we can draw this Feynman diagram in the SM:



One can also work out other cases (e.g., $b \to s$, etc.).

How large is this diagram in the SM?

How large is this diagram in the SM?

- The amplitude is the sum of the three diagrams:

$$\mathcal{A} = \sum_{i=d,s,b} \lambda_i \, f_i$$

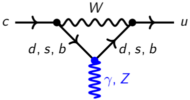
Here, $\lambda_i \equiv V_{ci} V_{ui}^*$ and f_i is a loop function.

- In the SM, the CKM matrix is unitary, i.e., $V^\dagger V = I$, and thus

$$\boxed{\sum_{i=d,s,b} \lambda_i = 0} \rightarrow \lambda_d = -(\lambda_s + \lambda_b).$$

- We can now use this information in our amplitude (eliminating λ_d):

$$\begin{aligned}\mathcal{A} &= \lambda_d f_d + \lambda_s f_s + \lambda_b f_b \\ &= -(\lambda_s + \lambda_b) f_d + \lambda_s f_s + \lambda_b f_b \\ &= \lambda_s \left[(f_s - f_d) + \frac{\lambda_b}{\lambda_s} (f_b - f_d) \right].\end{aligned}$$



$$= \sum_{i=d,s,b} \lambda_i f_i = \lambda_s \left[(f_s - f_d) + \frac{\lambda_b}{\lambda_s} (f_b - f_d) \right]$$

$f_i \sim \frac{m_i^2}{(4\pi)^2 M_W^2}$, $\mathrm{Im}(\lambda_b/\lambda_s) \sim 10^{-3}$

BRs (A_{CP}) are loop-(CKM-) suppressed!

An excellent place to search for BSM physics!

It looks like we are on the right track to find $\mathcal{O}_{SM} \approx 0$!

Naïve estimates of rare decays (attaching $\nu\bar{\nu}$)

Using the branching ratio for a hadron h (with mass m_h and lifetime τ_h) decaying into a final hadronic state F (with mass $m_F \ll m_h$) and two neutrinos (see 2205.07534):

$$\mathcal{B}(h \to F\nu\bar{\nu}) \approx \frac{\tau_h\, G_F^2\, \alpha_e^2\, m_h^3\, \left|\mathcal{C}_L^{\alpha\beta\ell\ell}\right|^2}{16\,(2\pi)^5}$$

we obtain the following estimates for different quark transitions:

$$\mathcal{B}(b \to s\nu\bar{\nu}) \sim 10^{-6}, \qquad \mathcal{B}(b \to d\nu\bar{\nu}) \sim 10^{-7},$$
$$\mathcal{B}(s \to d\nu\bar{\nu}) \sim 10^{-8}, \qquad \mathcal{B}(c \to u\nu\bar{\nu}) \sim 10^{-19}.$$

Experimental comparison: different phenomenology

Using the available experimental information (PDG):

$$\mathcal{B}(b \to s\nu\bar{\nu})_{\text{exp}} \sim 10^{-5}, \qquad \mathcal{B}(b \to d\nu\bar{\nu})_{\text{exp}} \sim 10^{-5},$$
$$\mathcal{B}(s \to d\nu\bar{\nu})_{\text{exp}} \sim 10^{-9}, \qquad \mathcal{B}(c \to u\nu\bar{\nu})_{\text{exp}} \sim 10^{-4}.$$

We obtain the following ratios (remember we are being very naïve - we can be off by an order of magnitude):

$$\frac{\mathcal{B}(b \to s\nu\bar{\nu})_{\text{exp}}}{\mathcal{B}(b \to s\nu\bar{\nu})_{\text{SM}}} \sim 10, \qquad \frac{\mathcal{B}(b \to d\nu\bar{\nu})_{\text{exp}}}{\mathcal{B}(b \to d\nu\bar{\nu})_{\text{SM}}} \sim 100,$$
$$\frac{\mathcal{B}(s \to d\nu\bar{\nu})_{\text{exp}}}{\mathcal{B}(s \to d\nu\bar{\nu})_{\text{SM}}} \sim 10, \qquad \frac{\mathcal{B}(c \to u\nu\bar{\nu})_{\text{exp}}}{\mathcal{B}(c \to u\nu\bar{\nu})_{\text{SM}}} \sim 10^{15}.$$

Any experimental signal in $c \to u\nu\bar{\nu}$ is NP!

What did we find using this naïve estimate?

- Strong GIM suppression of $\nu\bar{\nu}$ modes in the SM!
- Very different phenomenology between the up and down sectors.
- By far, $c \to u$ transitions are the rock stars!
- However, it is important to note that information from all sectors is necessary.

Conclusion: We found $\mathcal{O}_{\text{SM}} \approx 0$!

$$\mathcal{B}(c \to u\nu\bar{\nu}) \sim 10^{-19} \approx 0$$

This is almost zero!

Thanks to the GIM mechanism!

Experimentally, this means that any signal is NP!

An important comment

If NP follows the same GIM-suppression mechanism, NP signals would be tiny $\Rightarrow$ it is more promising to look in other transitions.

But: we don't know what NP looks like - it might not respect GIM!

In addition: the SM involves all sectors - that is, both up and down ($b \to s$, $b \to d$, $s \to d$, $c \to u$) - so one has to explore all possibilities, especially since most experimental data are well explained by the SM.

"What does BSM physics look like?"

Keep an open mind - remember we do not know what BSM looks like!

More nice things about neutrino modes!

- Neutrino flavours are experimentally untagged.
- Any dineutrino observable $\mathcal{O}(\nu_\ell \bar{\nu}_{\ell'})$ requires an incoherent sum over the lepton flavours:

$$\mathcal{O}(\nu\bar{\nu}) = \mathcal{O}(\nu_e \bar{\nu}_e) + \mathcal{O}(\nu_e \bar{\nu}_\mu) + \mathcal{O}(\nu_e \bar{\nu}_\tau) + \mathcal{O}(\nu_\mu \bar{\nu}_\mu) + \cdots$$

- In compact form:

$$\mathcal{O}(\nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{O}(\nu_\ell \bar{\nu}_{\ell'}) = p\, \mathcal{O}_{\max}(\nu_\ell \bar{\nu}_{\ell'}), \qquad p = \frac{\sum_{\ell,\ell'} \mathcal{O}(\nu_\ell \bar{\nu}_{\ell'})}{\mathcal{O}_{\max}(\nu_\ell \bar{\nu}_{\ell'})} \leq 9.$$

Here, $\mathcal{O}_{\max}(\nu_\ell \bar{\nu}_{\ell'})$ denotes the largest contribution among all ℓ, ℓ' configurations.

Inclusiveness leads to an enhancement

1. **LU case:** If LU holds,

$$\mathcal{O}(\nu_\ell\bar{\nu}_{\ell'})\big|_{\mathsf{LU}} = \delta_{\ell\ell'}\, \mathcal{O}^{\mathsf{max}}(\nu_\ell\bar{\nu}_{\ell'}),$$

   then the sensitivity of $\mathcal{O}(\nu\bar{\nu})$ is enhanced by a factor $p = 3$.

2. **General case:** If NP allows nonzero $\mathcal{O}(\nu_\ell\bar{\nu}_{\ell'})$ with $\ell \neq \ell'$, p can be further enhanced, up to $p = 9$.

3. **Beyond neutrinos:** Experimentally, we measure

$$\mathcal{O}(\nu\bar{\nu}) \Rightarrow \mathcal{O}(\text{missing energy}) = \mathcal{O}(\nu\bar{\nu} + \text{particles that are not detected})$$

Notice: These aspects are absent in charged-dilepton observables $\mathcal{O}(\ell^+\ell^-)$, where the lepton flavour is experimentally tagged.

Yeah, very pretty, but are charm decays with missing energy experimentally feasible?

How cool is Belle II?

- **Well suited to e^+e^- colliders** such as **Belle II**.
- **What is the new-physics reach?**

⋆ Fragmentation fractions $f(c \to h_c)$,
1509.01061

⋆ Luminosity $50\,\text{ab}^{-1}$,

⋆ $N(c\bar{c})_{\text{Belle II}} = 65 \times 10^9$, Abada:2019lih

⋆ $N(h_c) = 2\, f(c \to h_c)\, N(c\bar{c})$.

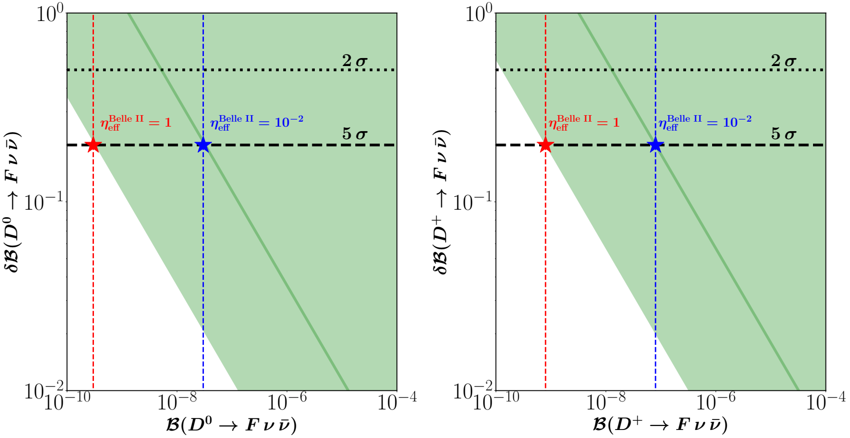
h_c	$f(c \to h_c)$	$N(h_c)_{\text{Belle II}}$
D^0	0.59	8×10^{10}
D^+	0.24	3×10^{10}
D_s^+	0.10	1×10^{10}
Λ_c^+	0.06	8×10^9

$$N(h_c) \sim 10^{10}!$$

Experimental projections: $\delta\mathcal{B}$ versus $\mathcal{B}$

The SM contribution cannot be seen in the plot; it lies well below 10^{-10}.

$$\delta\mathcal{B}(h_c \to F\,\nu\bar{\nu}) = 1/\sqrt{N_F^{\rm exp}} \text{ with } N_F^{\rm exp} = \eta_{\rm eff}\,N(h_c)\,\mathcal{B}(h_c \to F\,\nu\bar{\nu}).$$



Reach of Belle II

If there is no loss of information, Belle II can reach:

BRs $\sim 10^{-10}$!

$\eta_{\text{eff}} = 1$, $\eta_{\text{eff}} = 10^{-2}$, $\delta\mathcal{B} = 1/5$ for $5\,\sigma$

- $\mathcal{B}^{5\,\sigma}_{\text{Belle II}}(D^0 \to F\nu\bar{\nu}) \approx 3 \cdot 10^{-10}$ **and** $3 \cdot 10^{-8}$!
- $\mathcal{B}^{5\,\sigma}_{\text{Belle II}}(D^+ \to F\nu\bar{\nu}) \approx 8 \cdot 10^{-10}$ **and** $8 \cdot 10^{-8}$!

Independent of η_{eff}: red numbers multiplied by $1/\eta_{\text{eff}}$.

Have a look into PDG!

$$D^0 \to \pi^0 \nu \bar{\nu}$$

$D^0 \to \pi^0\nu\bar{\nu}$ **Decay Mode Summary** PDGID:S032.571 JSON (beta) INSPIRE

	Mode (*)	Fraction ($\Gamma_i\,/\,\Gamma$)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_{320}	$D^0 \to \pi^0\nu\bar{\nu}$	$< 2.1 \times 10^{-4}$	CL= 90%	928

Category: ΔC = 1 weak neutral current ($C1$) modes, Lepton Family number (LF) violating modes, Lepton (L) or Baryon (B) number violating modes

The following data is related to the above value:

$\Gamma(D^0 \to \pi^0\nu\bar{\nu})/\Gamma_{\rm total}$ Γ_{320}/Γ

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
$< \mathbf{2.1} \times \mathbf{10^{-4}}$	90	[1] ABLIKIM	2022V	BES3	e^+e^- at 3773 MeV

[1] ABLIKIM 2022V easurement comes from a sample of $10.6 \times 10^6 D^0\overline{D}^0$ pairs .

References

ABLIKIM	2022V	PR D105 L071102	Search for the decay $D^0 \to \pi^0\nu\bar{\nu}$

Have a look into PDG!

$$D^0 \to \nu\bar{\nu}$$

$D^0 \to$ invisibles Decay Mode Summary

PDGID:S032.433 JSON (beta) INSPIRE

	Mode (*)	Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_{18}	$D^0 \to$ invisibles	$< 9.4 \times 10^{-5}$	CL= 90%	

Category: Inclusive modes

The following data is related to the above value:

$\Gamma(D^0 \to \text{invisibles})/\Gamma_{\text{total}}$ Γ_{18}/Γ

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
$< \mathbf{9.4 \times 10^{-5}}$	90	LAI	2017	BELL	e^+e^- at $\Upsilon(nS)$, n=4,5

References

LAI	2017	PR D95 011102	Search for D^0 Decays to Invisible Final States at Belle

Comparison of Belle II with current limits

- Only experimental information so far from $D^0 \to$ invisible (Belle, $\mathcal{C}_{S,P}$) and $D^0 \to \pi^0\nu\bar{\nu}$ (BESIII, $\mathcal{C}_{L,R,S,P}$):

$$\mathcal{B}(D^0 \to \text{invisible}) < 9.4 \times 10^{-5} \quad (90\%\ \text{C.L.}),$$
$$\mathcal{B}(D^0 \to \pi^0\nu\bar{\nu}) < 2.1 \times 10^{-4} \quad (90\%\ \text{C.L.}).$$

- Comparing with the Belle II projections (50 ab^{-1}), in the scenario $\eta_{\text{eff}} = 10^{-2}$ Belle II would be stronger than BESIII by $\sim 10^4$. In the (unrealistic $\eta_{\text{eff}} = 1$) best case, Belle II would be stronger by $\sim 10^6$.

$$\frac{\mathcal{B}(D^0 \to \pi^0\nu\bar{\nu})_{\text{BESIII}}}{\mathcal{B}(D^0 \to \pi^0\nu\bar{\nu})_{\text{Belle II}}} \sim [10^4,\ 10^6]!$$

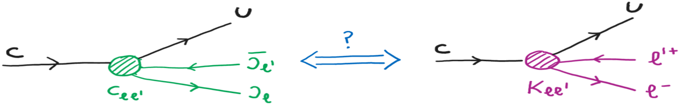
Best limits or NP discover in 20XX: If I had to bet

Shit! We need to wait until our experimental colleagues measure something. . .

Apart from limits on WCs, can we say anything else about rare charm dineutrino modes?

Can we get information from dineutrino modes?

$\boldsymbol{\ell}$ and $\boldsymbol{\nu_\ell}$ (with $\ell = e, \mu, \tau$) belong to the same **SU(2)$_\mathrm{L}$ doublet** in the SM.



$$\begin{pmatrix} c_{ee} & c_{e\mu} & c_{e\tau} \\ c_{\mu e} & c_{\mu\mu} & c_{\mu\tau} \\ c_{\tau e} & c_{\tau\mu} & c_{\tau\tau} \end{pmatrix} \Longleftrightarrow \begin{pmatrix} k_{ee} & k_{e\mu} & k_{e\tau} \\ k_{\mu e} & k_{\mu\mu} & k_{\mu\tau} \\ k_{\tau e} & k_{\tau\mu} & k_{\tau\tau} \end{pmatrix}$$

Neutrino flavour not tagged!

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{B}(c \to u\,\nu_\ell \bar{\nu}_{\ell'})$$

LU, cLFC, or general:

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) \sim \frac{1}{3} \sum_{\ell,\ell'} c_{\ell\ell'}$$

Charged leptons tagged!

LU:

$$R_H \sim \frac{\mathcal{B}(c \to u\,\mu^+\mu^-)}{\mathcal{B}(c \to u\,e^+e^-)} \sim 1 + (k_{\mu\mu} - k_{ee})$$

cLFC or general:

$$\mathcal{B}(c \to u\,\ell'^+\ell^-) \sim k_{\ell\ell'}$$

Is there a link between $c_{\ell\ell'}$ and $k_{\ell\ell'}$?

Low-energy $|\Delta c| = |\Delta u| = 1$ EFT description

$c \to u\, \nu_\ell \bar{\nu}_{\ell'}$ $\overset{?}{\Longleftrightarrow}$ $c \to u\, \ell^- \ell'^+$

$$\mathcal{H}_{\text{eff}}^{\nu_\ell \bar{\nu}_{\ell'}} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{C}_k^{U\ell\ell'}\, Q_k^{U\ell\ell'}$$

$$\mathcal{H}_{\text{eff}}^{\ell^- \ell'^+} = -\frac{4\, G_F}{\sqrt{2}} \frac{\alpha}{4\pi} \sum_k \mathcal{K}_k^{U\ell\ell'}\, O_k^{U\ell\ell'}$$

Only two operators (no RH neutrinos, as in the SM).

$$Q_{\mathrm{L\,(R)}}^{U\ell\ell'} = (\bar{u}_{L\,(R)} \gamma_\mu c_{L\,(R)})(\bar{\nu}_{\ell' L} \gamma^\mu \nu_{\ell L})$$

Additional operators are not connected.

$$O_{\mathrm{L\,(R)}}^{U\ell\ell'} = (\bar{u}_{L\,(R)} \gamma_\mu c_{L\,(R)})(\bar{\ell}'_L \gamma^\mu \ell_L),\ \ldots$$

Dineutrino BR is obtained via an incoherent neutrino flavour sum:

$$\mathcal{B}(c \to u\, \nu\bar{\nu}) = \sum_{\ell,\ell'} \mathcal{B}(c \to u\, \nu_\ell \bar{\nu}_{\ell'}) \sim \sum_{\ell,\ell'} \left| \mathcal{C}_L^{U\ell\ell'} \pm \mathcal{C}_R^{U\ell\ell'} \right|^2$$

$\mathcal{C}^P$ and $\mathcal{K}^P$ are in the mass basis. $P = D$ ($P = U$) $\to$ down-quark sector (up-quark sector).

Correlating neutrinos and charged leptons with $SU(2)_L$

Lowest order $SU(2)_L \times U(1)_Y$–invariant effective theory (1008.4884)

$$\mathcal{L}^{\rm LO}_{\rm SMEFT} \supset \frac{C^{(1)}_{\ell q}}{v^2}\,\bar{Q}\gamma_\mu Q\,\bar{L}\gamma^\mu L + \frac{C^{(3)}_{\ell q}}{v^2}\,\bar{Q}\gamma_\mu \tau^a Q\,\bar{L}\gamma^\mu \tau^a L + \frac{C_{\ell u}}{v^2}\,\bar{U}\gamma_\mu U\,\bar{L}\gamma^\mu L + \frac{C_{\ell d}}{v^2}\,\bar{D}\gamma_\mu D\,\bar{L}\gamma^\mu L\,.$$

1. **Writing in $SU(2)_L$ components:** ($C \to$ dineutrinos and $K \to$ dileptons in the gauge basis)

$$C_L^U = K_L^D = \frac{2\pi}{\alpha}\left(C^{(1)}_{\ell q} + C^{(3)}_{\ell q}\right), \qquad C_R^U = K_R^U = \frac{2\pi}{\alpha}\,C_{\ell u}\,.$$

2. **Mass basis:**

$$\boxed{\mathcal{C}_L^U = W^\dagger\,\mathcal{K}_L^D\,W + \mathcal{O}(\lambda), \qquad \mathcal{C}_R^U = W^\dagger\,\mathcal{K}_R^U\,W}$$

3. **BRs are independent of the PMNS matrix! (field redefinition of ν_L)**

$$\begin{aligned}
\mathcal{B}(c \to u\,\nu\bar{\nu}) \sim \sum_{\ell,\ell'}\left|\mathcal{C}_L^{U\,\ell\ell'} \pm \mathcal{C}_R^{U\,\ell\ell'}\right|^2 &= \mathrm{Tr}\left[(\mathcal{C}_L^U \pm \mathcal{C}_R^U)(\mathcal{C}_L^U \pm \mathcal{C}_R^U)^\dagger\right] \\
&= \mathrm{Tr}\left[W^\dagger(\mathcal{K}_L^D \pm \mathcal{K}_R^U)W\,W^\dagger(\mathcal{K}_L^D \pm \mathcal{K}_R^U)^\dagger W\right] \\
&= \sum_{\ell,\ell'}\left|\mathcal{K}_L^{D\,\ell\ell'} \pm \mathcal{K}_R^{U\,\ell\ell'}\right|^2 + \mathcal{O}(\lambda)\,.
\end{aligned}$$

Predictions for dineutrino rates with different leptonic flavour structures $\mathcal{K}^{\ell\ell'}_{L,R}$ can be probed with lepton-specific measurements!

Possible leptonic flavour structures for $\mathcal{K}_{L,R}^{\ell\ell'}$

$$\mathcal{B}(c \to u\,\nu\bar{\nu}) \sim \sum_{\ell,\ell'} \left|\mathcal{K}_L^{D\ell\ell'} \pm \mathcal{K}_R^{U\ell\ell'}\right|^2$$

i) ***Lepton-universality* (LU).**

$$\begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

ii) ***Charged lepton flavour conservation* (cLFC).**

$$\begin{pmatrix} k_{ee} & 0 & 0 \\ 0 & k_{\mu\mu} & 0 \\ 0 & 0 & k_{\tau\tau} \end{pmatrix}$$

iii) **$\mathcal{K}_{L,R}^{\ell\ell'}$ arbitrary.**

$$\begin{pmatrix} k_{ee} & k_{e\mu} & k_{e\tau} \\ k_{\mu e} & k_{\mu\mu} & k_{\mu\tau} \\ k_{\tau e} & k_{\tau\mu} & k_{\tau\tau} \end{pmatrix}$$

Dineutrino branching ratios

$$\mathcal{B} = A_+ \, x^+ + A_- \, x^-, \qquad x^\pm = \sum_{\ell,\ell'} \left| \mathcal{C}_L^{U\,\ell\ell'} \pm \mathcal{C}_R^{U\,\ell\ell'} \right|^2$$

→ **Long-distance dynamics & kinematics $A_\pm$: LCSR (low q^2) + Lattice (high q^2)**

→ **Short-distance dynamics $x^\pm$: Wilson coefficients (BSM)**

→ **Excellent complementarity of $\mathcal{B}$:**

- **$A_- = 0$ in $D \to P\,\nu\bar{\nu}$ decays.**
- **$A_- > A_+$ in $D \to P_1 P_2\,\nu\bar{\nu}$ decays.**
- **$A_- = A_+$ in inclusive D decays.**

$D \to F$	A_+ $[10^{-8}]$	A_- $[10^{-8}]$
$D^0 \to \pi^0$	0.9	0
$D^+ \to \pi^+$	3.6	0
$D^0 \to \pi^0\pi^0$	0	0.2
$D^0 \to \pi^+\pi^-$(*)	0	0.4
$D^0 \to X$	2.2	2.2
$D^+ \to X$	5.6	5.6

(*) heavy hadron chiral perturbation theory; new results data-driven from $D^+ \to \pi^+\pi^- e^+\nu_e$ (2509.10447): $A_+^{D^0\pi^+\pi^-} = 0.1 \cdot 10^{-8}$ and $A_-^{D^0\pi^+\pi^-} = 0.5 \cdot 10^{-8}$.

Upper limits on dineutrino modes can probe LU!

- **Limits from high-p_T and charged-dilepton D and K decays (†):**[1]

	$\lvert\mathcal{K}_A^{P\ell\ell'}\rvert$	ee	$\mu\mu$	$\tau\tau$	$e\mu$	$e\tau$	$\mu\tau$
$s\,d$	$\lvert\mathcal{K}_L^{D\ell\ell'}\rvert$	$5\times10^{-2\dagger}$	$1.6\times10^{-2\dagger}$	6.7	$6.6\times10^{-4\dagger}$	6.1	6.6
$c\,u$	$\lvert\mathcal{K}_R^{U\ell\ell'}\rvert$	2.9	$0.9^{\dagger}$	5.6	1.6	4.7	5.1

$$x^{\pm} < 2\,x, \qquad x = \sum_{\ell,\ell'}\left(|\mathcal{K}_L^{D\ell\ell'}|^2 + |\mathcal{K}_R^{U\ell\ell'}|^2\right) + \mathcal{O}(\lambda) = \sum_{\ell,\ell'} R^{\ell\ell'} + \mathcal{O}(\lambda)$$

$\boldsymbol{x = 3\,R^{\mu\mu} \lesssim 2.6}$, **(Lepton Universality)** **LU is fixed by muons.**

$\boldsymbol{x = R^{ee} + R^{\mu\mu} + R^{\tau\tau} \lesssim 156}$, **(charged Lepton Flavor Conservation)**

$\boldsymbol{x = R^{ee} + R^{\mu\mu} + R^{\tau\tau} + 2\,(R^{e\mu} + R^{e\tau} + R^{\mu\tau}) \lesssim 655.}$

[1] 2007.05001

Upper limits on dineutrino branching ratios

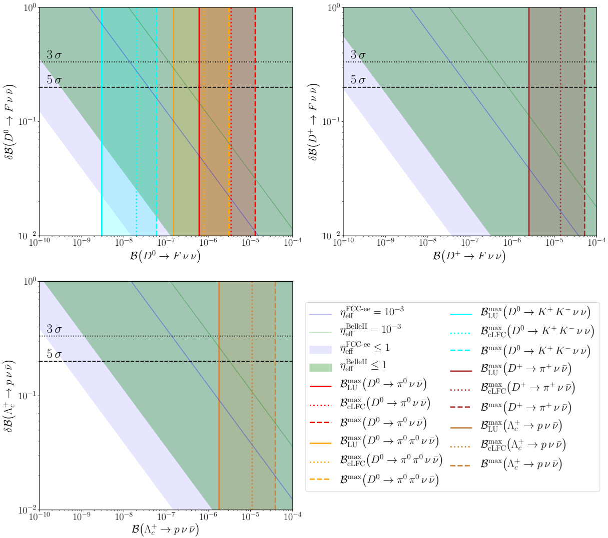
2007.05001, 2010.02225

$h_c \to F$	$\mathcal{B}^{\rm max}_{\rm LU}$ $[10^{-7}]$	$\mathcal{B}^{\rm max}_{\rm cLFC}$ $[10^{-6}]$	$\mathcal{B}^{\rm max}$ $[10^{-6}]$
$\boldsymbol{D^0 \to \pi^0}$	**0.5**	**2.8**	**12**
$D^+ \to \pi^+$	1.9	11	47
$D^0 \to \pi^0\pi^0$	0.1	0.7	2.8
$D^0 \to \pi^+\pi^-$	0.2	1.3	5.4
$\Lambda_c^+ \to p^+$	1.4	8.4	35
$\Xi_c^+ \to \Sigma^+$	2.7	17	70

$\mathcal{B}(D^0 \to \pi^0 \nu\bar{\nu}) < 2.1 \times 10^{-4}$ (BESIII) is about one order of magnitude above our predictions. 2112.14236

$\delta\mathcal{B}$ vs $\mathcal{B}$: exp. projections and theo. predictions

2010.02225



Conclusions

- $c \to u\,\nu\bar{\nu}$ modes are extremely GIM-suppressed.
- Unique phenomenology!
- Well suited to e^+e^- colliders such as Belle II.
- Based on current experimental sensitivities:

  Any signal would be a clear sign of NP!
- $\mathrm{SU}(2)_L$ links between charged leptons and neutrinos!

$$\boxed{c \to u\,\ell\ell} \longrightarrow \boxed{c \to u\,\nu\bar{\nu}} \longleftarrow \boxed{d \to s\,\ell\ell}$$

Allows us to probe lepton flavour in two benchmarks: cLFC and LU!

Thank you for your attention!