Global fits:
we are not just a pretty picture…

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Global Fit: tool to test the SM and probe NP

- Introduction and motivations
  - the tool: the Unitarity Triangle (UT) fit
- SM UT analysis:
  - provides the best determination of CKM parameters
  - tests the consistency of the SM (“*direct*” vs “*indirect*” determinations)
  - provides predictions (from data..) for SM observables
- NP UT analysis:
  - model-independent analysis
  - provides limit on the allowed deviations from the SM
  - obtains the NP scale

UTfit collaboration

Talk based on implementation/inputs/plots of the UTfit collaboration:

- Founded in 2003 (2nd CKM workshop in Durham) from the people from the 2000 paper: https://arxiv.org/abs/hep-ph/0012308 with some experimental characters sneaking in..
- Yearly updates: look for us at conferences, the webpage is… often late…🤦
- If you need something specific, let me know!

www.utfit.org

latest paper (2023):
"New UTfit analysis of the unitarity triangle in the Cabibbo–Kobayashi–Maskawa scheme"
10.1007/s12210-023-01137-5

M.Bona, M. Ciuchini, D. Derkach, F. Ferrari, E. Franco, V. Lubicz, G. Martinelli, D. Morgante, M. Pierini, L. Silvestrini, S. Simula, A. Stocchi, C. Tarantino, V. Vagnoni, M. Valli and L. Vittorio

The CKM matrix

The charged current interactions get a flavour structure encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix V:

$$\boxed{\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\left(\bar{\tilde{U}}_L \gamma^\mu W_\mu^+ V \tilde{D}_L + \bar{\tilde{D}}_L \gamma^\mu W_\mu^- V^\dagger \tilde{U}_L\right)}$$

V_{ij} connects left-handed up-type quark of the *i*th generation to left-handed down-type quark of *j*th generation.

Intuitive labelling by flavour:

$$\begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CK @ CKM2006 in Nagoya

Unitarity

The CKM matrix is a 3x3 complex unitary matrix described by 4 (real) parameters:

- 3 can be expressed as (Euler) mixing angles
- the fourth makes the CKM matrix complex (i.e. gives it a phase)
  - weak interaction couplings differ for quarks and antiquarks

- Have you thought about how to obtain this above?
  - You might have… if you have not, let me quickly run through it…

Unitarity

In general, an $n \times n$ unitary matrix has n^2 real and independent parameters:

► a $n \times n$ matrix would have $2n^2$ parameters

► the unitary condition imposes n normalization constraints

► $n(n - 1)$ conditions from the orthogonality between each pair of columns:

thus $2n^2 - n - n(n - 1) = n^2$.

In the CKM matrix, not all of these parameters have a physical meaning:

► with n generations, $2n - 1$ phases are absorbed by the freedom to select the quark field phases

► Each u, c or t phase allows for multiplying a row of the CKM matrix by a phase, while each d, s or b phase allows for multiplying a column by a phase.

thus: $n^2 - (2n - 1) = (n - 1)^2$.

Among the n^2 real independent parameters of a generic unitary matrix:

► $\frac{1}{2} n(n - 1)$ of these parameters can be associated to real rotation angles,

► so the number of independent phases is

$$n^2 - \tfrac{1}{2} n(n - 1) - (2n - 1) = \tfrac{1}{2} (n - 1)(n - 2)$$

n(families)	Total indep. params. $(n-1)^2$	Real rot. angles $\frac{1}{2}n(n-1)$	Complex phase factors $\frac{1}{2}(n-1)(n-2)$
2	1	1	0
3	4	3	1
4	9	6	3

The CKM matrix: rotation decomposition

The CKM matrix can be seen as the product of three rotation matrices and each rotation involves two of the three families:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

which gives the classic exact parameterisation that can be found for example on the PDG:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

with $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, and i,j=1,2,3. δ is the CP violating phase

The CKM matrix: Wolfenstein parameterisation

From measurements, V results hierarchical → $\theta_{13} \ll \theta_{23} \ll \theta_{12}$

We can see this hierarchy via the Wolfenstein parameterisation:

→ the CKM matrix elements are expanded in order of sin θ_{12}

historically called Cabibbo angle θ_C:

→ Wolfenstein parameter $\lambda = \sin\theta_{12} \sim 0.22$

$$V_{CKM} = \boxed{\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}} + \mathcal{O}(\lambda^4)$$

→ Wolfenstein parameters: $\lambda \sim 0.22$, $A \sim 0.83$, $\rho \sim 0.15$, $\eta \sim 0.35$

The CKM matrix: Wolfenstein parameterisation

From the Wolfenstein parameter $\lambda = \sin\boldsymbol{\theta}_{12} \sim 0.22$, we can get an idea on the sizes of the various CKM matrix elements:

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At λ^2 order, the third generation decouples

$\eta \neq 0$ signals CP violation $\rightarrow$ imaginary part of V_{ub} and V_{td} ($1^{st} \rightleftarrows 3^{rd}$ family)

The CKM matrix: Wolfenstein-Buras parameterisation

Usually the Buras correction to the Wolfenstein parameterisation is used:

$$\bar{\rho} = \rho \ (1- \lambda^2/2)$$
$$\bar{\eta} = \eta \ (1-\lambda^2/2)$$

$$V = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Looks indentical to the Wolfenstein one but now the matrix is unitary also in this "approximation" at all λ orders.

Also $\bar{\rho} + i\bar{\eta}$ is phase-convention independent:

$$\boxed{\bar{\rho} + i\bar{\eta} \equiv -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}}$$

The CKM matrix: unitarity relations

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

multiply with its hermitian conjugate (complex conjugate + transpose)
$VV^\dagger = V^\dagger V = \mathbf{1}$

$$\Sigma_i V_{ij} V^*_{ik} = \delta_{jk}$$ column orthogonality

$$\Sigma_j V_{ij} V^*_{kj} = \delta_{ik}$$ row orthogonality

The six vanishing combinations can be represented as triangles in a complex plane

The CKM matrix: unitarity relations

The triangles obtained by taking scalar products of neighboring rows or columns are nearly degenerate. However, the areas of all triangles are the same, half of the Jarlskog invariant J.

$1^{st} \rightleftarrows 2^{nd}$ family

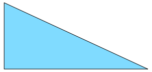
$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* \simeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda) + \mathcal{O}(\lambda^5) = 0$$

column orthogonality

$2^{nd} \rightleftarrows 3^{rd}$ family

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* \simeq \mathcal{O}(\lambda^4) + \mathcal{O}(\lambda^2) + \mathcal{O}(\lambda^2) = 0$$

$1^{st} \rightleftarrows 3^{rd}$ family



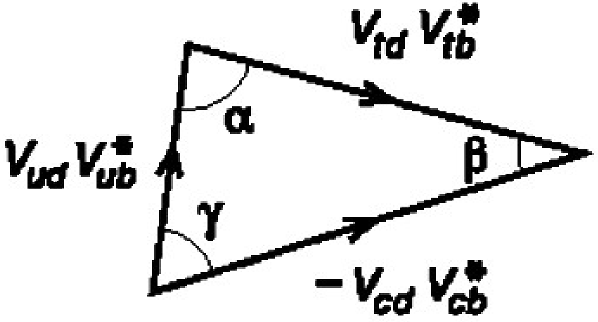
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$

triangles not to scale

The CKM matrix: third unitarity relation

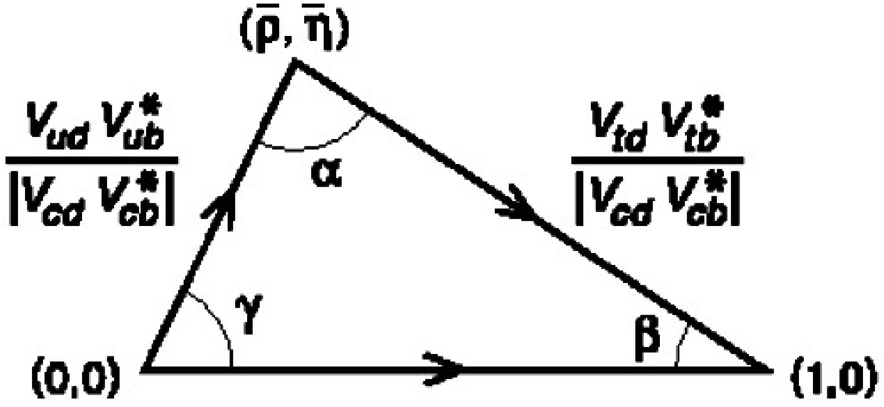
$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*} \simeq \mathcal{O}(\lambda^3)+\mathcal{O}(\lambda^3)+\mathcal{O}(\lambda^3) = 0$$

$V_{id}V^{*}_{ib} = 0$ is the orthogonality condition between the first and the third column: the orientation depends on the phase convention



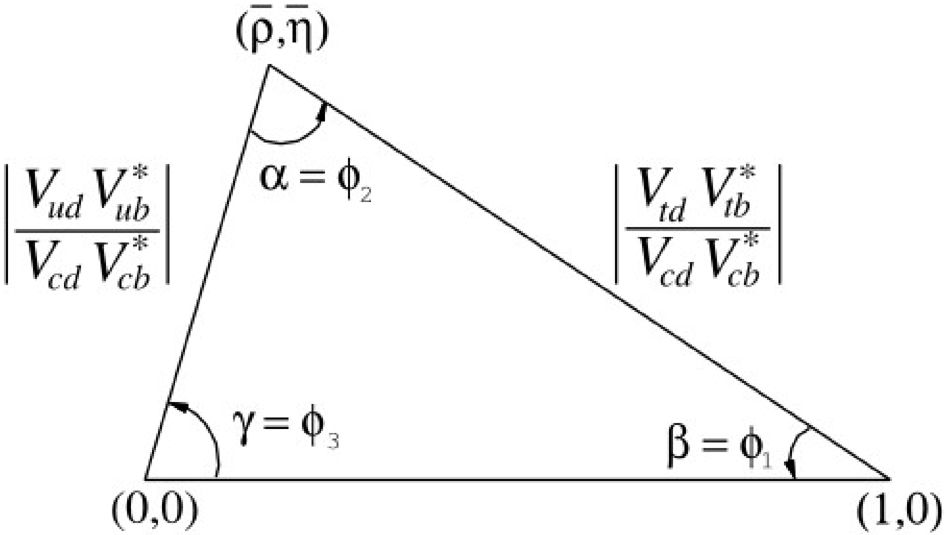
Usually we see re-scaled version where sides have been divided by $|V_{cd}V^{*}_{cb}|$

In the Wolfenstein parameterization, the coordinates are (0, 0), (1, 0) and $(\bar\rho, \bar\eta)$, and the two sides are $(\bar\rho + i\bar\eta)$ and $(1 - \bar\rho - i\bar\eta)$.



The CKM matrix: third unitarity relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^3) = 0$$



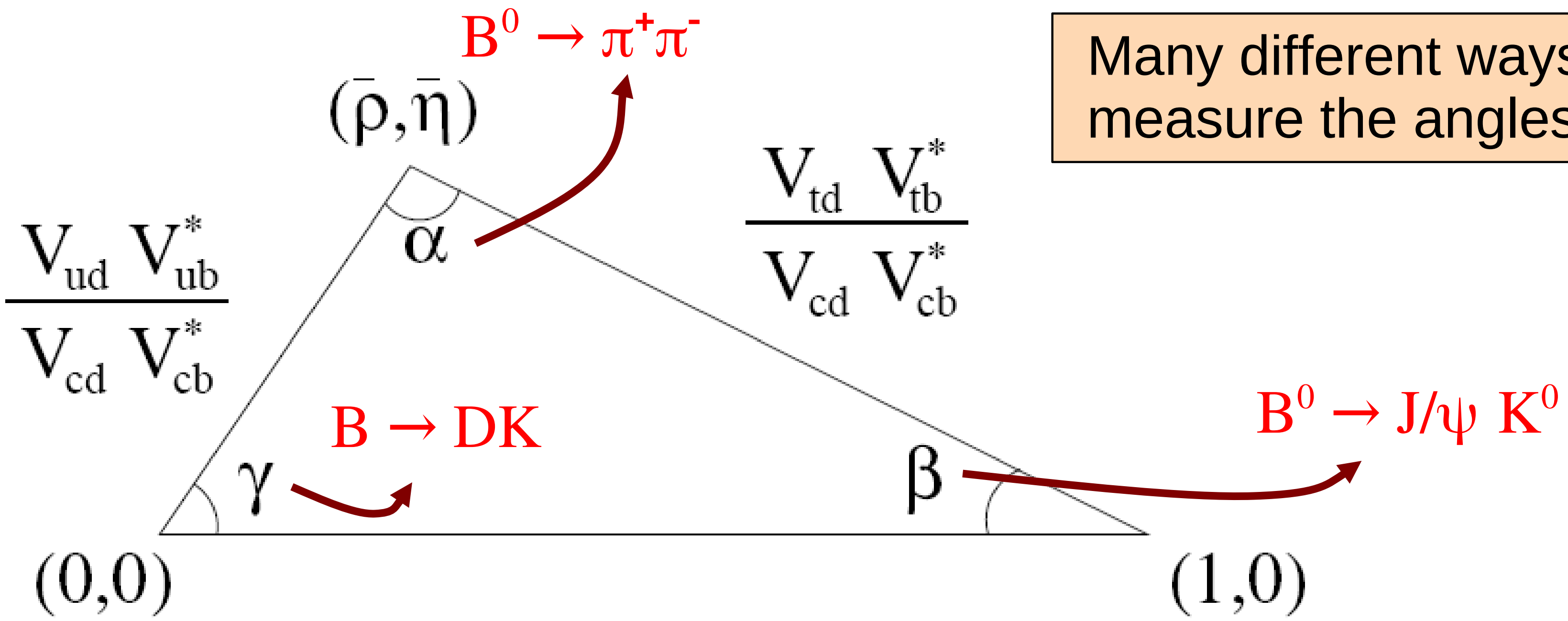
The angles can be written in terms of CKM matrix elements as:

$$\alpha \equiv \arg[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$$
$$\beta \equiv \arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$$
$$\gamma \equiv \arg[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$$

In the Wolfenstein parameterisation:

- the β/ϕ_1 angle corresponds to the phase of V_{td}
- the γ/ϕ_3 angle corresponds to the phase of V_{ub}
- the α/ϕ_2 angle can be obtained with $\pi - \beta - \gamma$ (assumes unitarity)

The Unitarity Triangle



Many different ways to measure the angles and sides.

- We need to measure the angles and sides to over-constrain this triangle, and test that it closes.
- Need to define observables and experiments to measure these quantities

UTfit method and inputs:

$$f(\bar\rho,\bar\eta,X|c_1,...,c_m) \sim \prod_{j=1,m} f_j(\mathcal{C}|\bar\rho,\bar\eta,X) * \prod_{i=1,N} f_i(x_i) f_0(\bar\rho,\bar\eta)$$

Bayes Theorem

$$X \equiv x_1,...,x_n = m_t, B_K, F_B, ...$$

$$\mathcal{C} \equiv c_1,...,c_m = \epsilon, \Delta m_d/\Delta m_s, A_{CP}(J/\psi K_S), ...$$

$(b \to u)/(b \to c)$	$\bar\rho^2 + \bar\eta^2$	$\bar\Lambda, \lambda_1, F(1), \ldots$
ϵ_K	$\bar\eta[(1-\bar\rho)+P]$	B_K
Δm_d	$(1-\bar\rho)^2+\bar\eta^2$	$f_B^2 B_B$
$\Delta m_d/\Delta m_s$	$(1-\bar\rho)^2+\bar\eta^2$	ξ
$A_{CP}(J/\psi K_S)$	$\sin 2\beta$	$-$

, $\mathbf{m_t}$

Standard Model +
OPE/HQET/
Lattice QCD
to go from
quarks to hadrons

Global fit constraints

Before the B factories, the available constraints used were:

- ► Mixing in the K system: ε_K
- ► $\Delta m_{d,s}$ in the $B_{d,s}$ systems
- ► V_{ub}/V_{bc} from semileptonic b to c and b to u

B factories started to improve some of the above
And also adding the measurements of the angles:

- ► β/ϕ_1, γ/ϕ_3 and α/ϕ_2

In any case, we need to reconnect the experimental observables with the fundamental parameters we want to obtain, in our case the CKM matrix elements.

Global fit constraints

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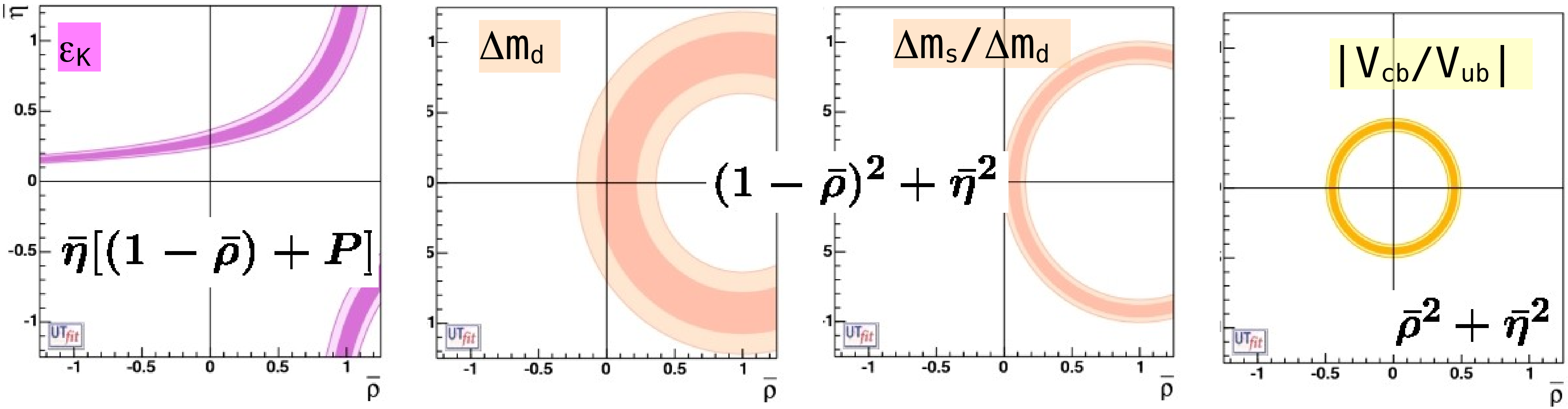


Global fit constraints

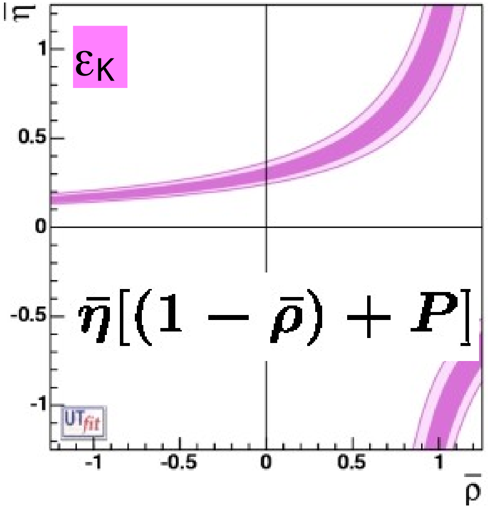
- Mixing in the K system: ε_K

S_0 = Inami-Lim functions for c-c, c-t, e t-t contributions (from perturbative calculations)

$$|\varepsilon_K| = C_\varepsilon B_K A^2\lambda^6\bar{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2)+\eta_3 S_0(x_c,x_t)+\eta_2 S_0(x_t)A^2\lambda^4(1-\bar{\rho})\}$$

from lattice QCD

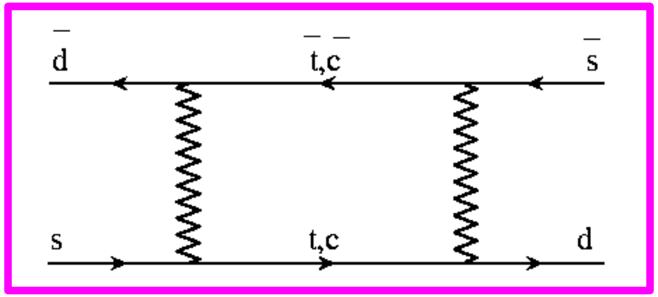
$$B_K = \frac{<K|J_\mu J^\mu|\overline{K}>}{<K|J_\mu|0><0|J^\mu|\overline{K}>}$$



$B_K = 0.7627 \pm 0.0060$

$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3}$

PDG



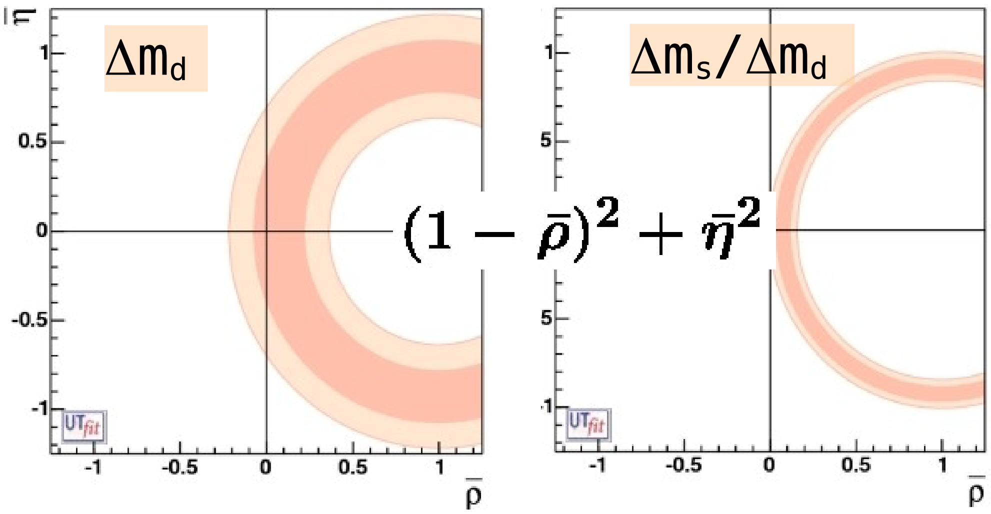
Global fit constraints

S = Inami-Lim function for the t-t contribution (from perturbative calculations)

- $\Delta m_{d,s}$ in the $B_{d,s}$ systems

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{tb}|^2 |V_{td}|^2 = $$
$$= \frac{G_F^2}{6\pi^2} m_W^2 \eta_b S(x_t) m_{B_d} f_{B_d}^2 \hat{B}_{B_d} |V_{cb}|^2 \lambda^2 \left((1-\bar{\rho})^2 + \bar{\eta}^2\right)$$

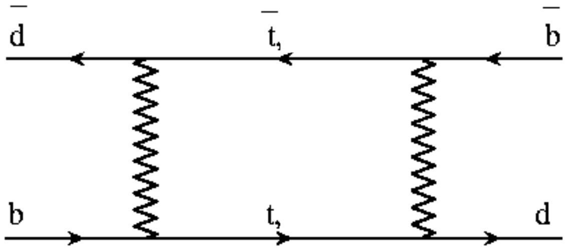
$$\Delta m_d \approx [(1-\rho)^2 + \eta^2] \frac{f_{B_s}^2 B_{B_s}}{\xi^2}$$
$$\Delta m_s \approx f_{B_s}^2 B_{B_s}$$



B_{B_q} and f_{B_q} from lattice QCD

$\Delta m_d = 0.5069 \pm 0.0019\ ps^{-1}$

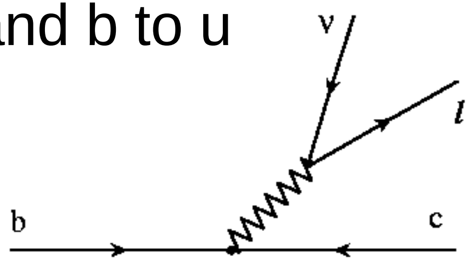
$\Delta m_s = 17.766 \pm 0.006\ ps^{-1}$ HFLAV

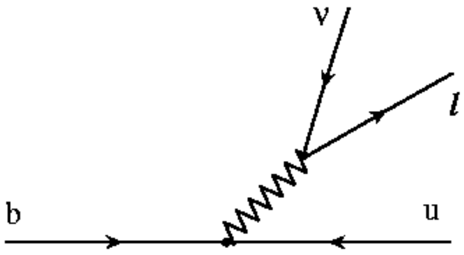


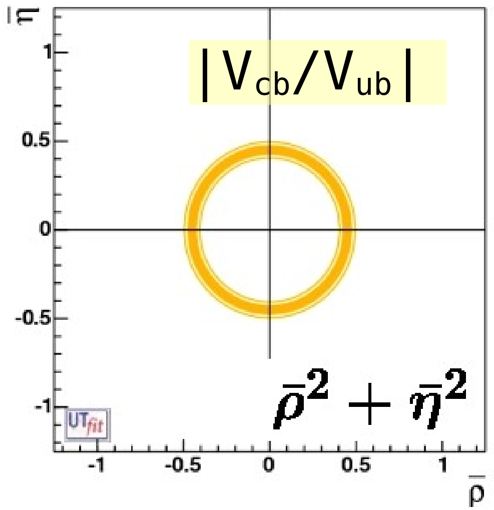
Global fit constraints

► V_{ub}/V_{cb} from semileptonic b to c and b to u

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1-\frac{\lambda^2}{2}}\sqrt{\bar{\rho}^2+\bar{\eta}^2}$$







tree diagrams

- o negligible new physics contributions
- o inclusive and exclusive semileptonic B decay branching ratios

QCD corrections to be included

- o inclusive measurements: OPE
- o exclusive measurements: form factors from lattice QCD

V_{cb} and V_{ub}

average of arXiv:2105.08674, 2204.05925, 2310.03680

$|V_{cb}|$ *(excl)* = $(40.12 \pm 0.55)\ 10^{-3}$

$|V_{cb}|$ *(incl)* = $(41.97 \pm 0.48)\ 10^{-3}$

from arXiv:2310.20324

update of arXiv:2202.10285

$|V_{ub}|$ *(excl)* = $(3.63 \pm 0.26)\ 10^{-3}$

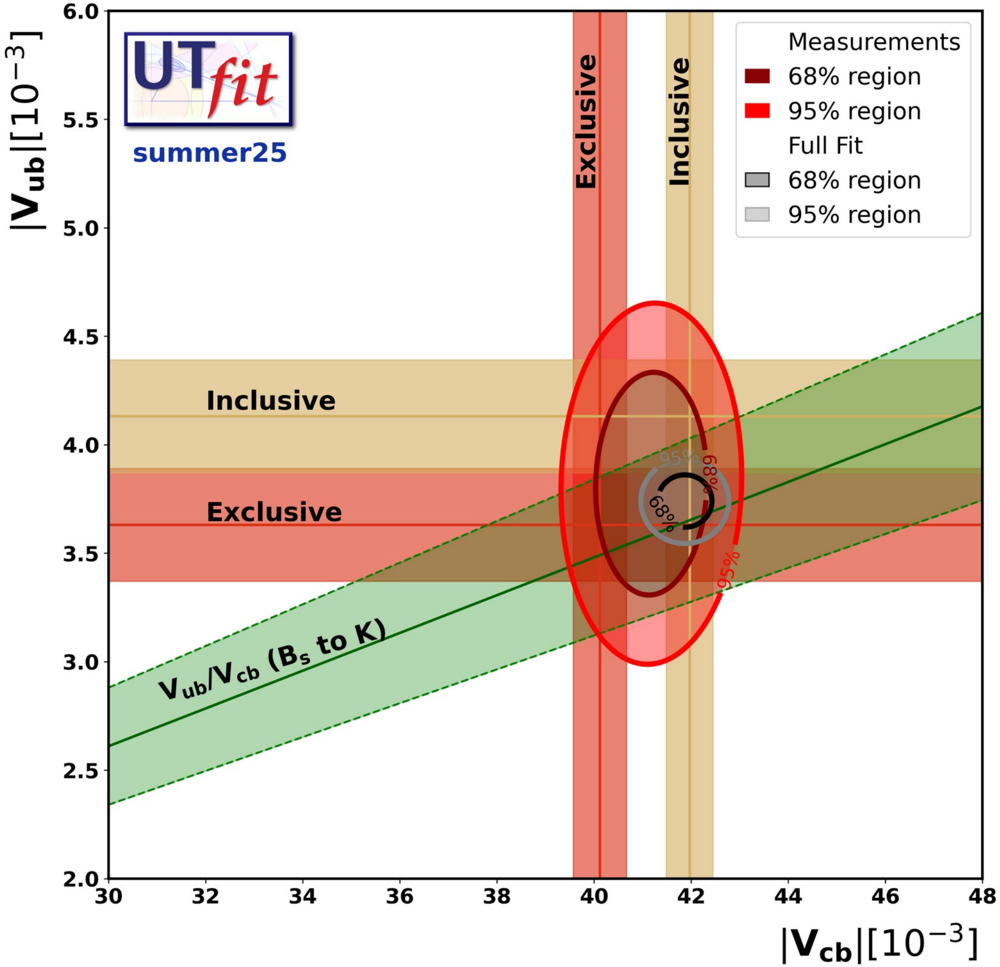
$|V_{ub}|$ *(incl)* = $(4.13 \pm 0.26)\ 10^{-3}$

PDG 2025

from arXiv:2310.03680

$|V_{ub} / V_{cb}| = (8.7 \pm 0.9)\ 10^{-2}$

$|V_{ub} / V_{cb}|$ *(LHCb)* = $(7.9 \pm 0.6)\ 10^{-2}$ Λ_b, excluded following FLAG guidelines



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PDG 2025

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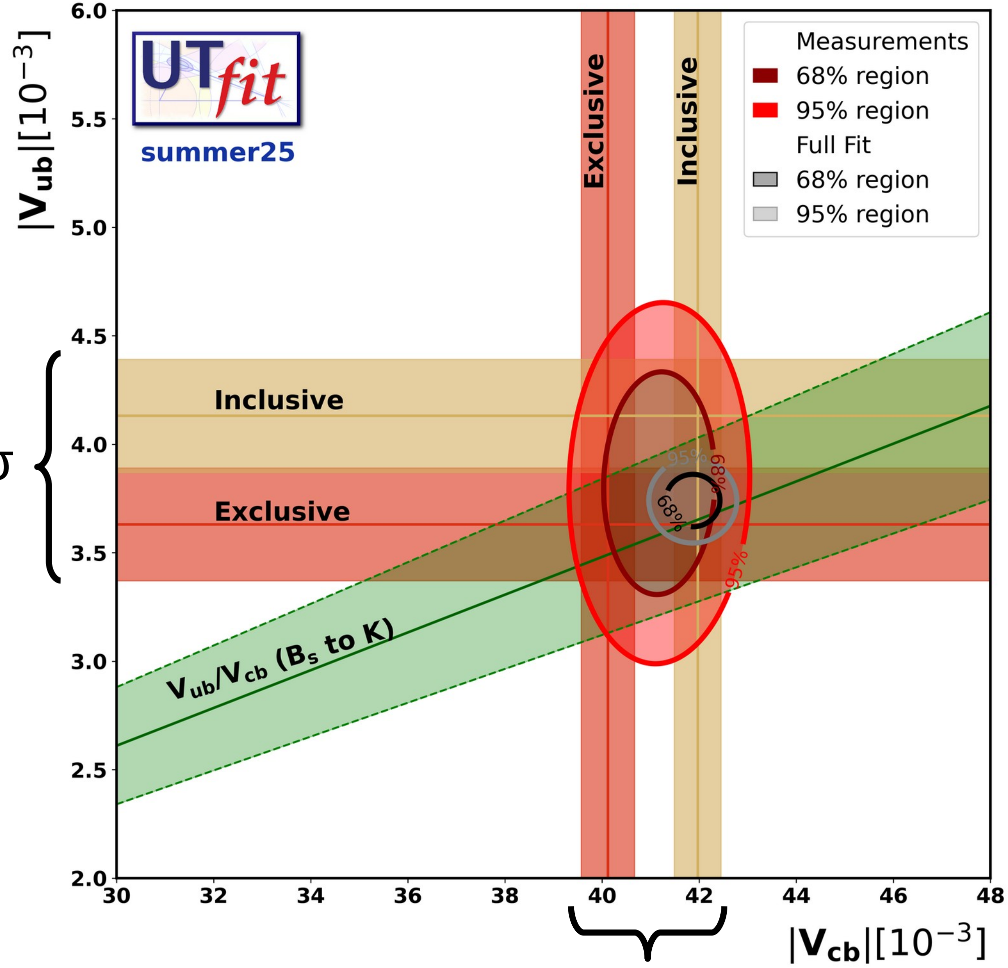
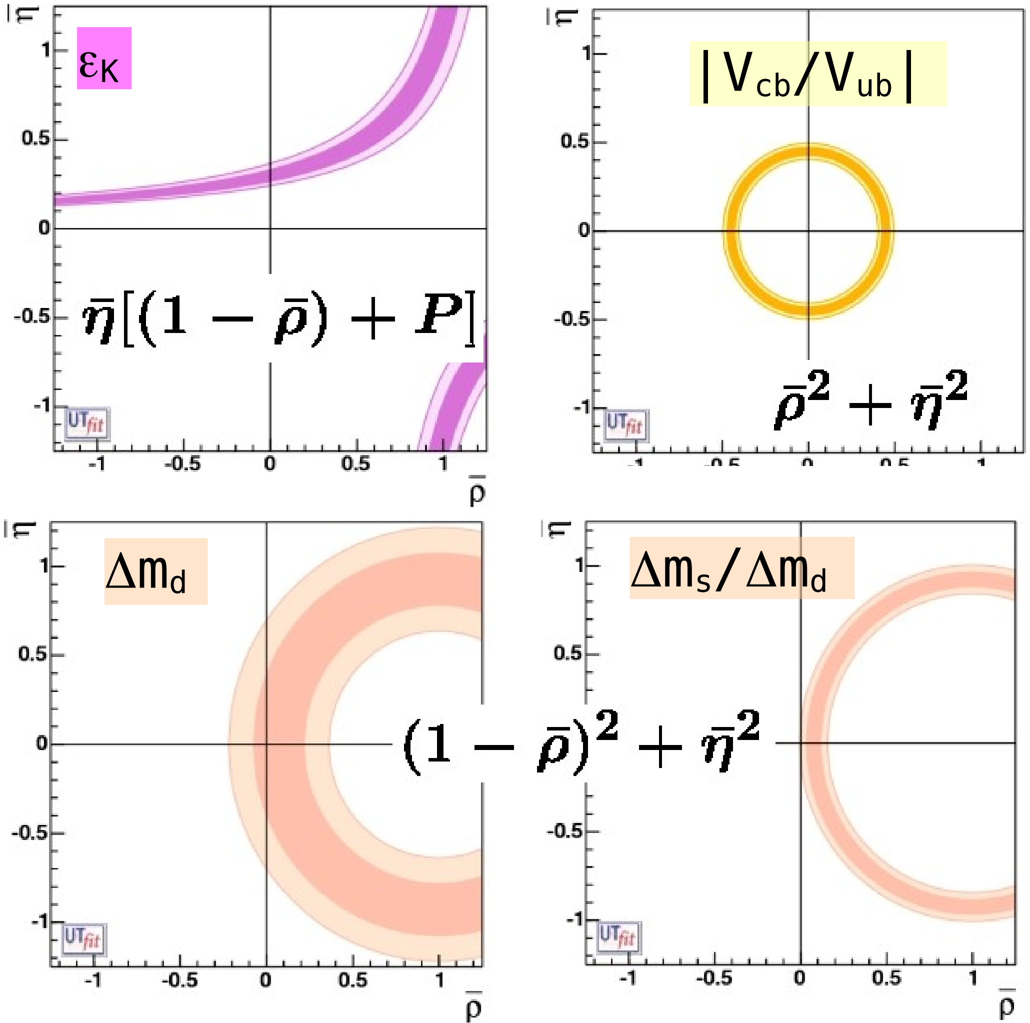
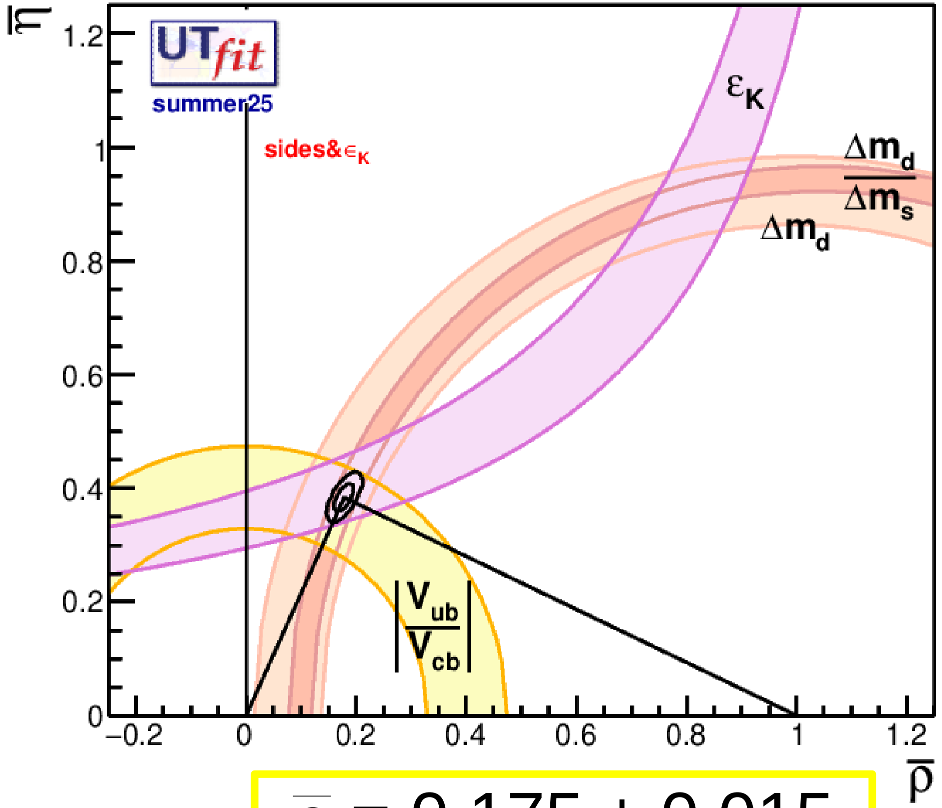
~1σ



~2σ

Global fit with pre-B-factory constraints:



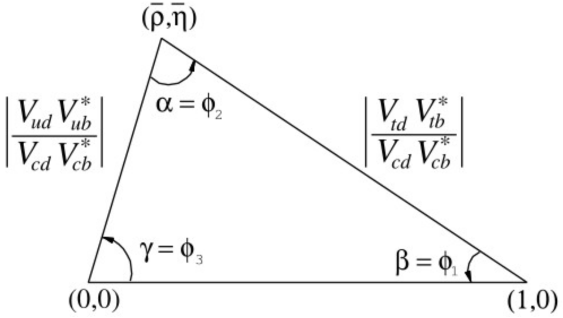


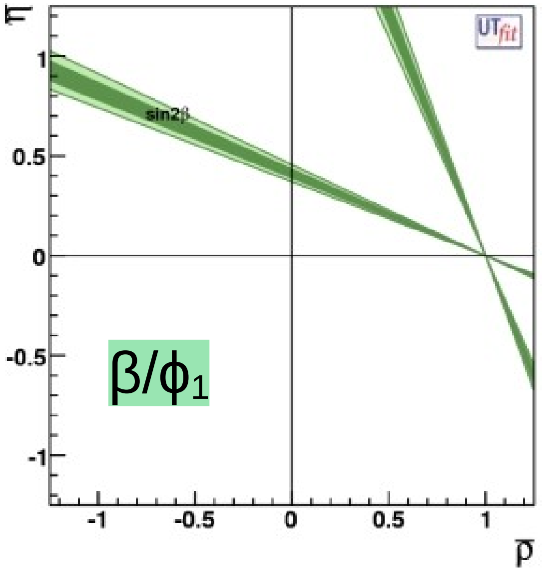
$\bar\rho = 0.175 \pm 0.015$
$\bar\eta = 0.375 \pm 0.022$

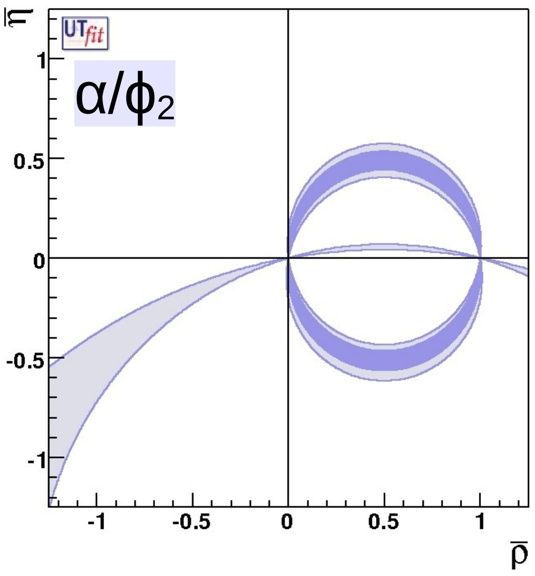
The angles

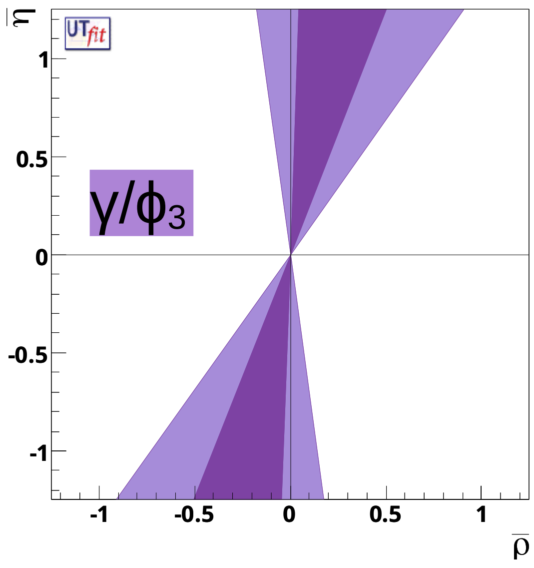
B factories have been groundbreaking in measuring the angles:

- β/ϕ_1, α/ϕ_2 and γ/ϕ_3





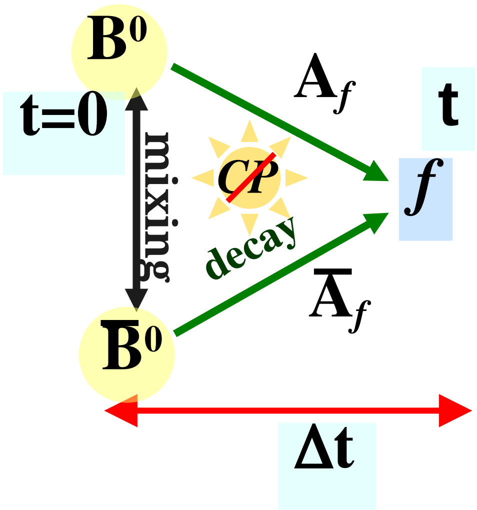


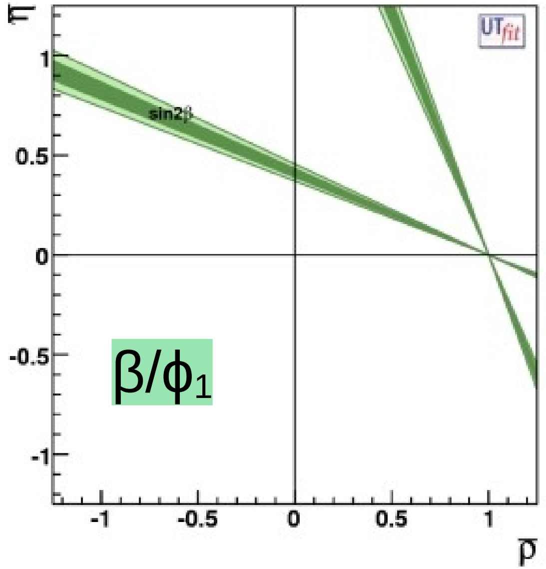


The angles

- β/ϕ_1

sin2β from time-dependent A_{CP} in $B^0 \to J/\psi K^0$



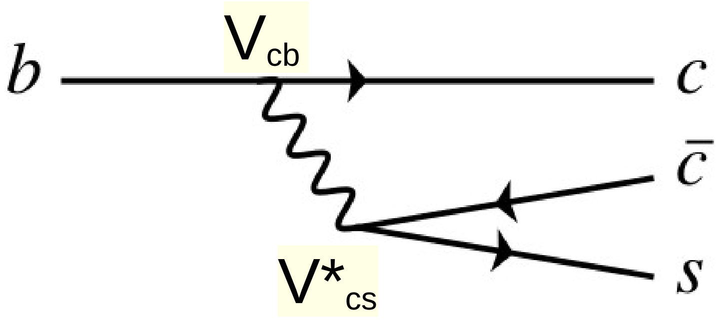


$$a_{f_{CP}}(t) = \frac{\mathrm{Prob}(B^0(t) \to f_{CP}) - \mathrm{Prob}(\overline{B^0}(t) \to f_{CP})}{\mathrm{Prob}(\overline{B^0}(t) \to f_{CP}) + \mathrm{Prob}(B^0(t) \to f_{CP})} = C_f \cos\Delta m_d t \;+\; S_f \sin\Delta m_d t$$

$$\boxed{a_{f_{CP}}(t) = -\eta_{CP} \sin\Delta m_d \Delta t \sin 2\beta}$$

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

leading-order tree decays to $c\bar{c}s$ final states

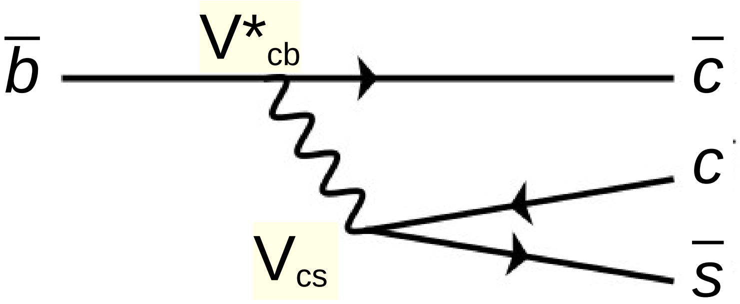


$+\bar{d}$ $\quad \bar{K}^0 \to K_{S,L}$

$\bar{B}^0 \to J/\psi K_{S,L}$

here the CKM elements contributing are $V_{cb}V^*_{cs}$ that in our Wolfenstein CKM parameterisation have no phase.

The CP conjugated case is also leading to (about) the same final state:



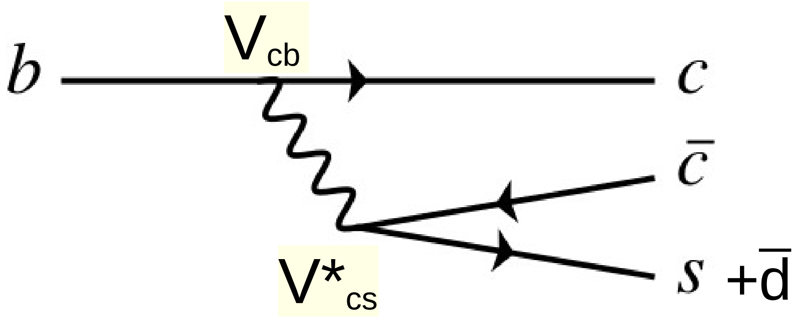
$+d$ $\quad K^0 \to K_{S,L}$

$B^0 \to J/\psi K_{S,L}$

sin2β in golden $b \to c\bar{c}s$ modes

leading-order tree decays to $c\bar{c}s$ final states

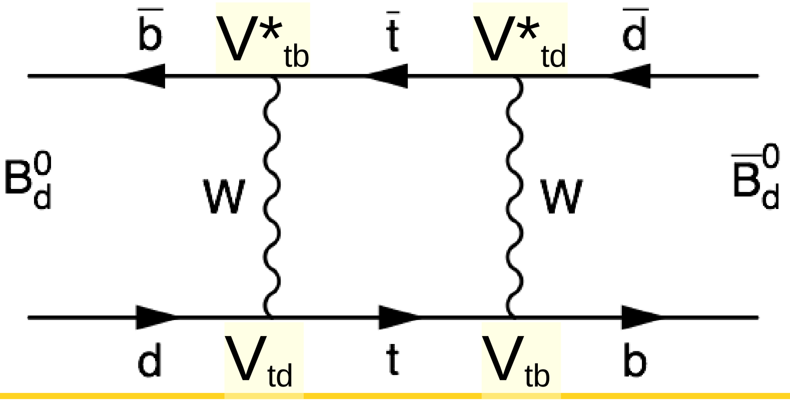
$$\bar{B}^0 \to J/\psi K_{S,L}$$



$$\bar{K}^0 \to K_{S,L}$$

$$\frac{\bar{A}}{A} = \overbrace{\frac{V_{cb}V^*_{cs}}{V^*_{cb}V_{cs}}}^{\text{tree diagram}}\ \underbrace{\frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}}_{\text{K mixing}}$$

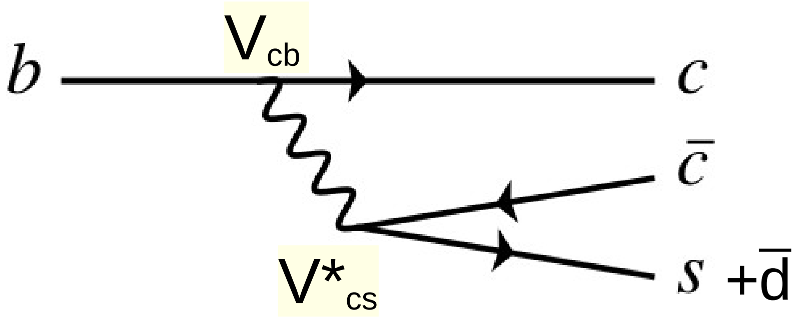
because both B and $\bar{B}$ can decay in this common final state, this can interfere with the oscillation diagram:



$$\lambda = \frac{q}{p}\frac{A(\bar{B} \to f)}{A(B \to f)} = \frac{V^*_{td}V_{tb}}{V_{td}V^*_{tb}}\frac{\bar{A}}{A} \sim e^{-i2\beta}\frac{\bar{A}}{A}$$

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes

leading-order tree decays to $c\bar{c}s$ final states

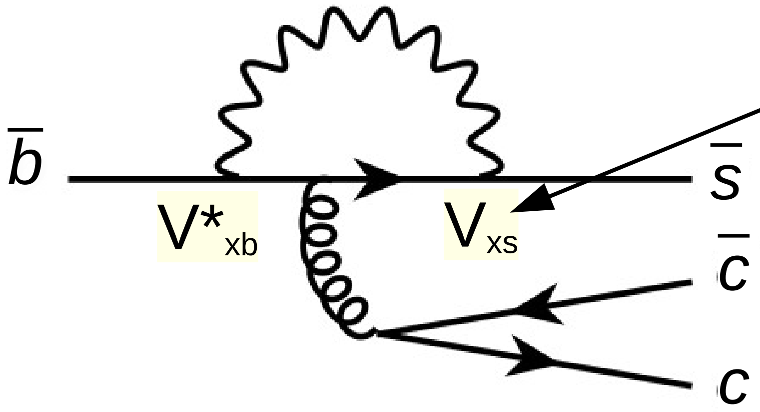


$\bar{B}^0 \to J/\psi K_{S,L}$

$\bar{K}^0 \to K_{S,L}$

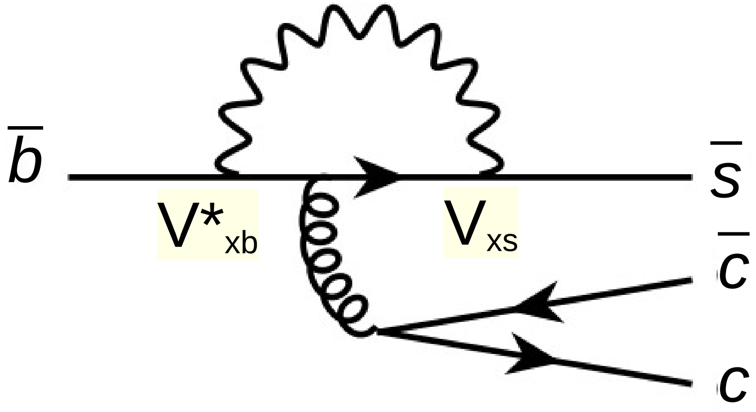
$$\frac{\bar{A}}{A} = \underbrace{\frac{V_{cb}V^*_{cs}}{V^*_{cb}V_{cs}}}^{\text{tree diagram}} \underbrace{\frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}}_{\text{K mixing}}$$

possible penguin contributions:



where x can be any up-type quark hence this counts for three penguin diagrams

$\sin 2\beta$ in golden $b \to c\bar{c}s$ modes



$$\begin{cases} x=u \to P^u \sim V_{ub}V^*_{us} \\ x=c \to P^c \sim V_{cb}V^*_{cs} \\ x=t \to P^t \sim V_{tb}V^*_{ts} \end{cases}$$

using this unitary condition ($2^{nd} \rightleftarrows 3^{rd}$ family), we eliminate $V_{tb}V^*_{ts}$

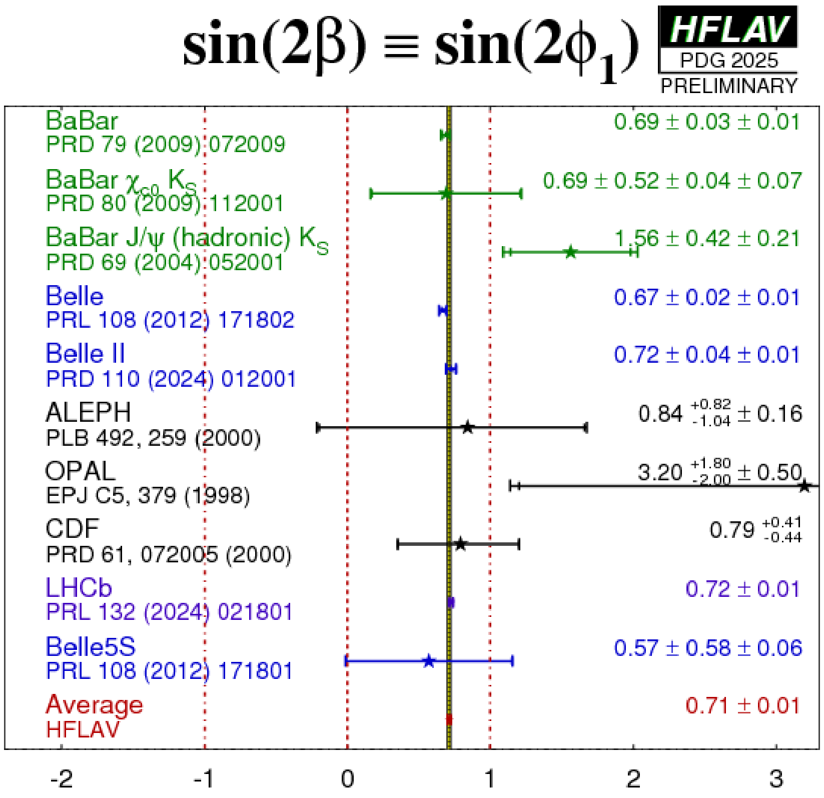
$$V_{ub}V^*_{us} + V_{cb}V^*_{cs} + V_{tb}V^*_{ts} = 0 \quad \to \quad V_{tb}V^*_{ts} = -V_{ub}V^*_{us} - V_{cb}V^*_{cs}$$

thus the amplitude is:

$$A_{ccs} \sim \underbrace{V_{cb}V^*_{cs}}_{\mathcal{O}(\lambda^2)}(T + P^c - P^t) + \underbrace{V_{ub}V^*_{us}}_{\mathcal{O}(\lambda^4)}(P^u - P^t)$$

CKM-suppressed pollution by penguins

sin2β in golden $b \to c\bar{c}s$ modes



Use HFLAV charmonium value

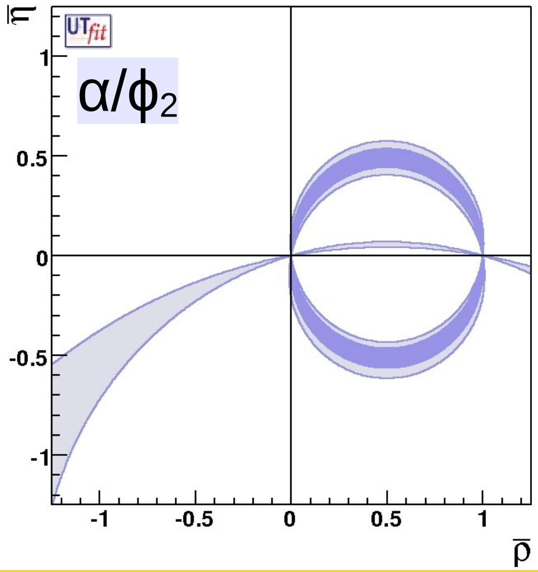
- Include correction due to Cabibbo-suppressed penguin contributions
- Model-independent data-driven estimation from $J/\psi\pi^0$ data
  $\Delta S_{J/\psi K0} = S_{J/\psi K0} - \sin 2\beta = -0.01 \pm 0.01$
- Final corrected number:
  $\sin 2\beta \;=\; 0.700 \pm 0.015$

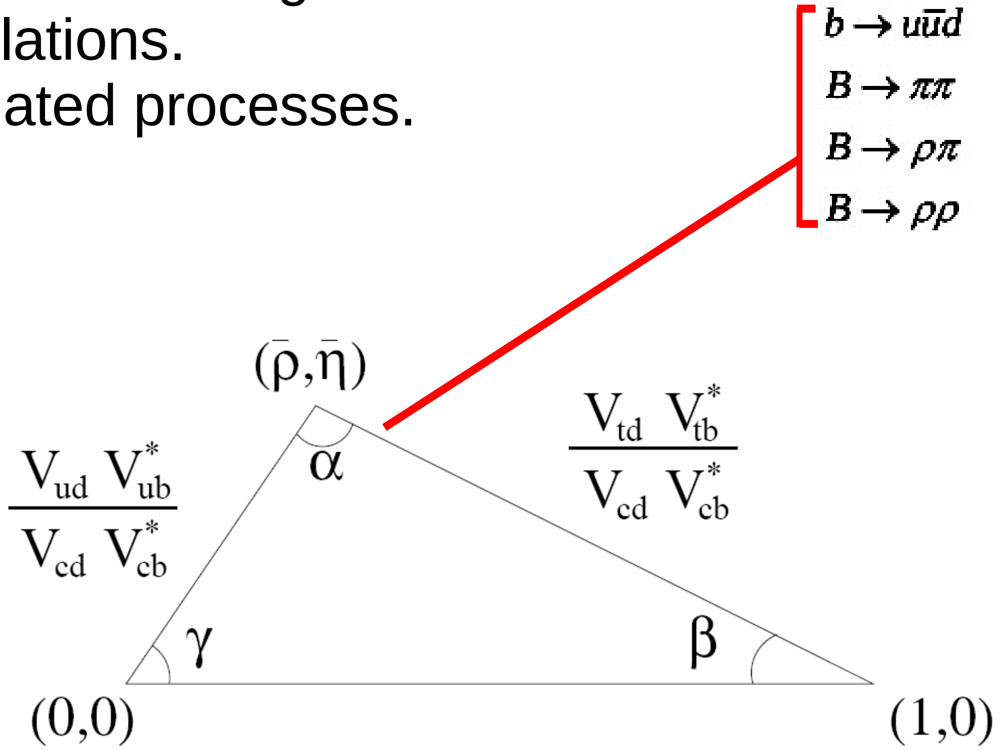
Ciuchini, Pierini, Silvestrini
https://arxiv.org/abs/hep-ph/0507290

The angles

- α/ϕ_2

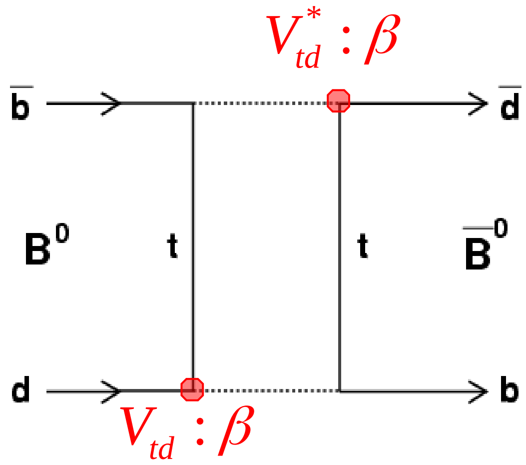
b → u$\bar{u}$d transitions with possible loop contributions. Extract α using
- SU(2) Isospin relations.
- SU(3) flavour related processes.

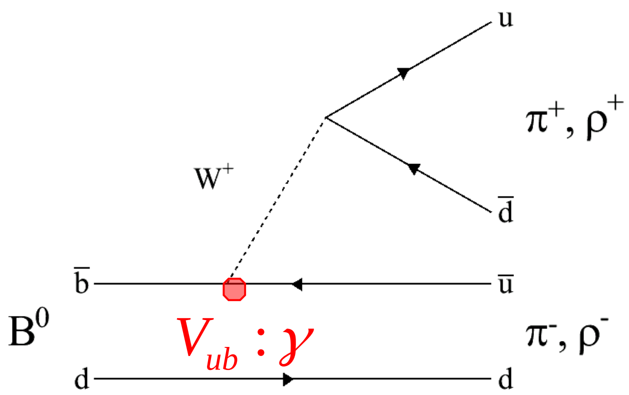




The angle α/ϕ_2

Interference between box and tree results in an asymmetry that is sensitive to α/ϕ_2 in $B \rightarrow hh$ decays: $h = \pi, \rho, \ldots$





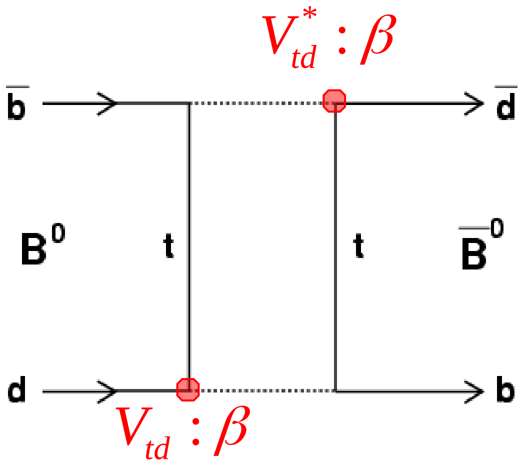
$$C_{hh} = 0$$

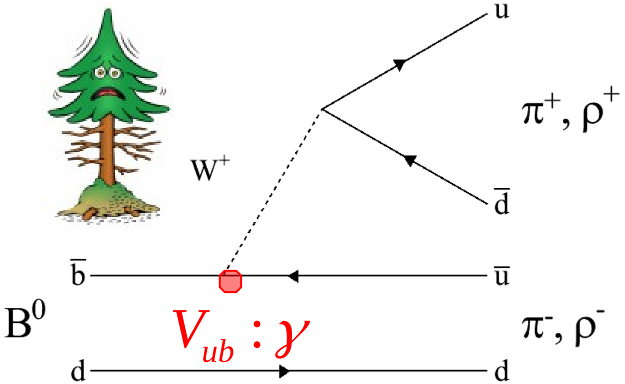
$$S_{hh} = \sin(2\alpha)$$

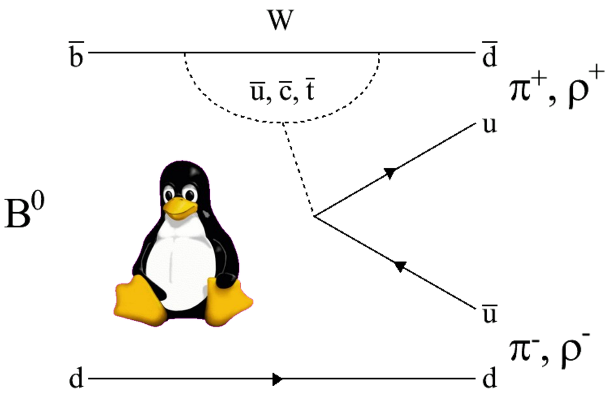
This is again a case of interference between mixing and decay.
This scenario is equivalent to the measurement of sin2β in charmonium decays … but in this case it is more complicated..

The angle α/ϕ_2

Interference between box and tree results in an asymmetry that is sensitive to α/ϕ_2 in $B \rightarrow hh$ decays: $h = \pi, \rho, \ldots$





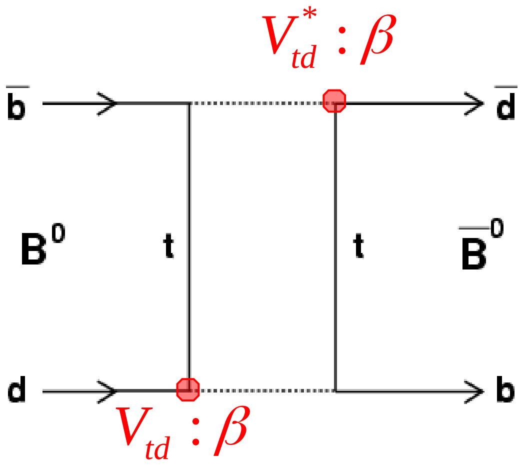


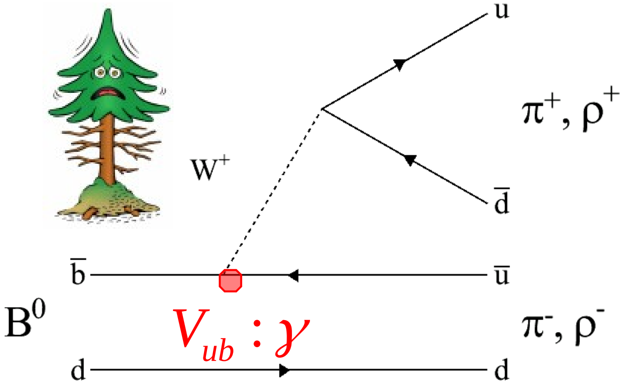
$\mathcal{O}(\lambda^3)$ $\mathcal{O}(\lambda^3)$

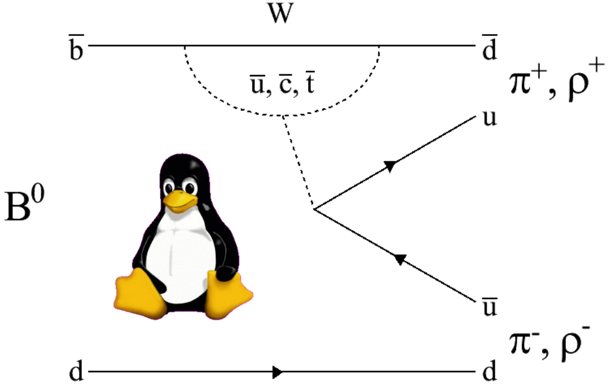
In this case the penguin diagram is not CKM suppressed so it spoils the clean measurement of the CP violation effect

The angle α/ϕ_2

Interference between box and tree results in an asymmetry that is sensitive to α/ϕ_2 in $B \rightarrow hh$ decays: $h = \pi, \rho, \ldots$







$$C_{hh} = 0$$
$$S_{hh} = \sin(2\alpha)$$

$$C_{hh} \propto \sin(\delta)$$
$$S_{hh} = \sqrt{1 - C_{hh}^2}\,\sin(2\alpha_{\mathrm{eff}})$$
$$\delta = \delta_P - \delta_T$$

Measure S $\propto \alpha_{\mathrm{eff}}$
Need to determine $\delta_\alpha = \alpha_{\mathrm{eff}} - \alpha$
[P/T is different for each final state]

The angle α/ϕ_2

Several recipes describe how to bound penguins and measure α/ϕ_2.

These are based on SU(2) [or SU(3)] symmetry.

SU(2)
(Isospin analysis)

M. Gronau and D. London, **65**, 3381 (1990)

$\pi^+\pi^-$ and $\rho^+\rho^-$
Gronau-London
Isospin Triangles

Use charged and neutral B decays to the hh final state to constrain the penguin contribution and measure α.

$\pi^\pm\rho^\pm$
Lipkin (et al.)
Isospin Pentagons

Use charged and neutral B decays to the $\rho\pi$ final state to constrain the penguin contribution and measure α. Remove any overlapping regions in the Dalitz plot.

H. Lipkin *et al.*, Phys. Rev. Lett. D **44**, 1454 (1991)

A. Snyder and H. Quinn, Phys. Rev. Lett. D **48**, 2139 (1993);
H. Quinn and J Silva, Phys. Rev. Lett. D **62**, 054002 (2000).

$\pi^\pm\pi^\pm\pi^0$
Snyder-Quinn (et al.)
Fit Dalitz plot and extract parameters related to α

Regions of the Dalitz plot with intersecting ρ bands are included in this analysis; this helps resolve ambiguities.

The angle α/ϕ_2

from $\alpha_{eff} \rightarrow$ to α: isospin analysis

Channel	Decay Amplitudes
$\pi\pi$	$A(B^+ \to \pi^+\pi^0) = \frac{\sqrt{3}}{2} A_{3/2,2}$ $\frac{1}{\sqrt{2}} A(B^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{12}} A_{3/2,2} - \sqrt{\frac{1}{6}} A_{1/2,0}$ $A(B^0 \to \pi^0\pi^0) = \frac{1}{\sqrt{3}} A_{3/2,2} + \sqrt{\frac{1}{6}} A_{1/2,0}$

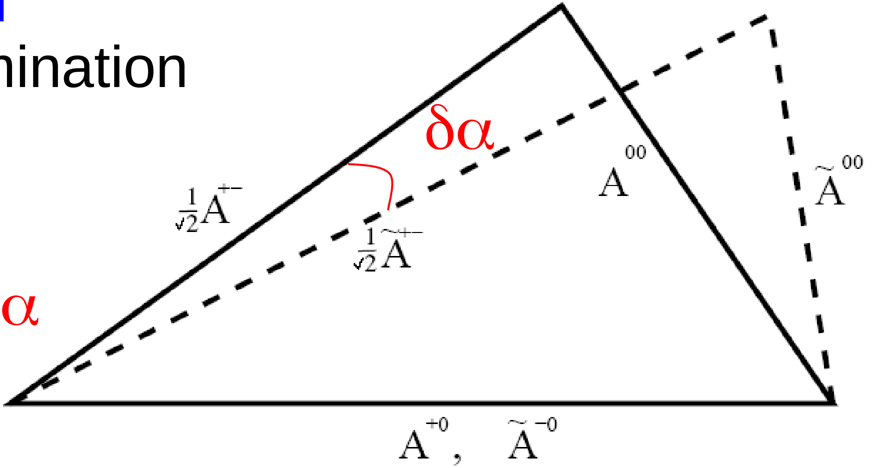
- $B \rightarrow \pi^+\pi^-$, $\pi^+\pi^0$, $\pi^0\pi^0$ connected by isospin relations
- $\pi\pi$ states can have I = 2 or I = 0
  - gluonic penguins contribute only to I = 0 state (ΔI=1/2)
  - $\pi^+\pi^0$ is a pure I = 2 state (ΔI = 3/2) and it gets contribution only from the tree diagram
  - triangular relations allow for the determination of the phase difference induced on α

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}$$

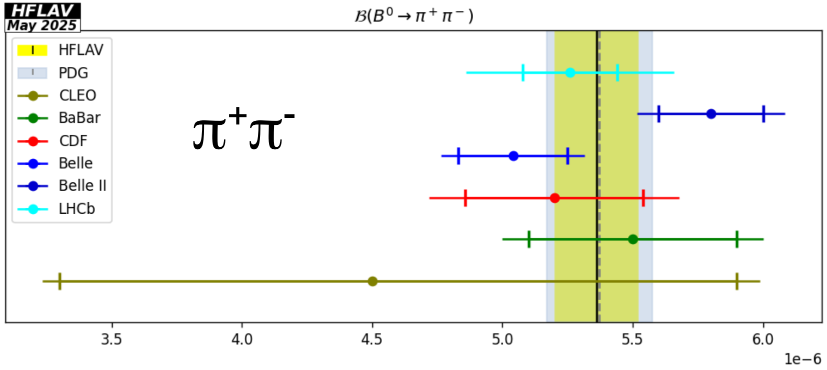
$$\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{+0}$$

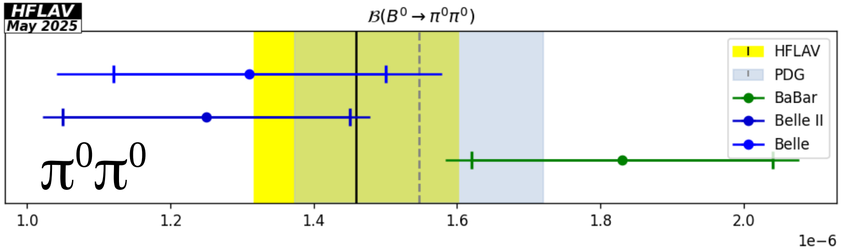
Both BR(B^0) and BR($\bar{B}^0$) have to be measured in all the $\pi\pi$ channels

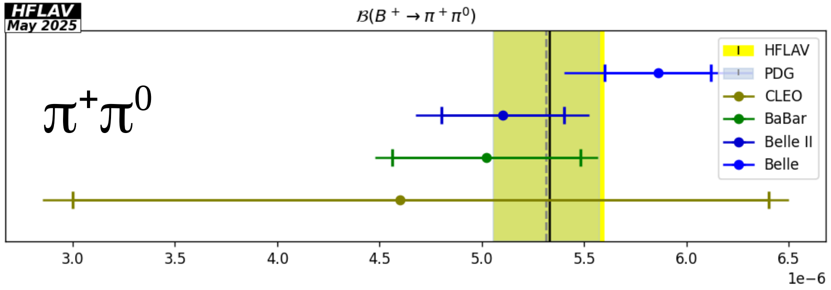
$$\delta\alpha = \alpha_{eff} - \alpha$$



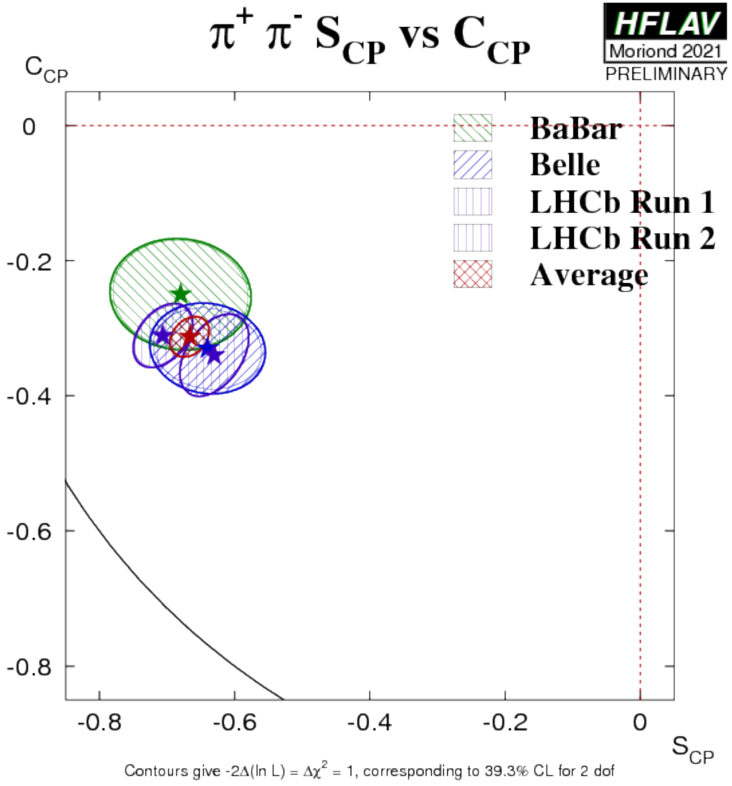
The angle α/ϕ_2



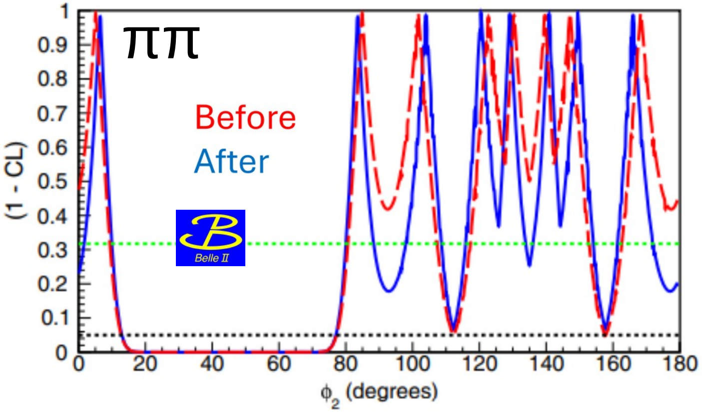


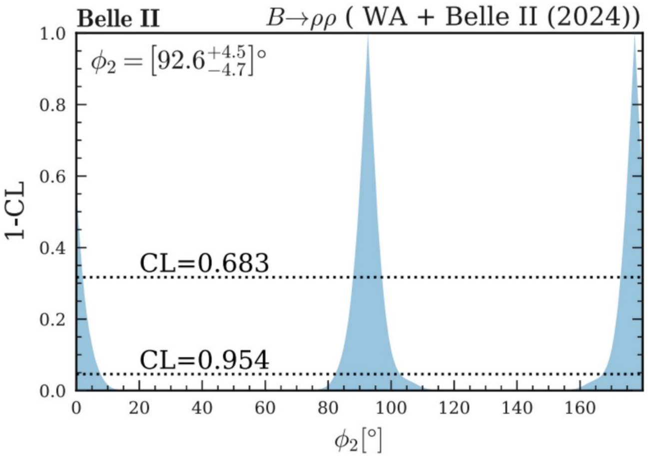


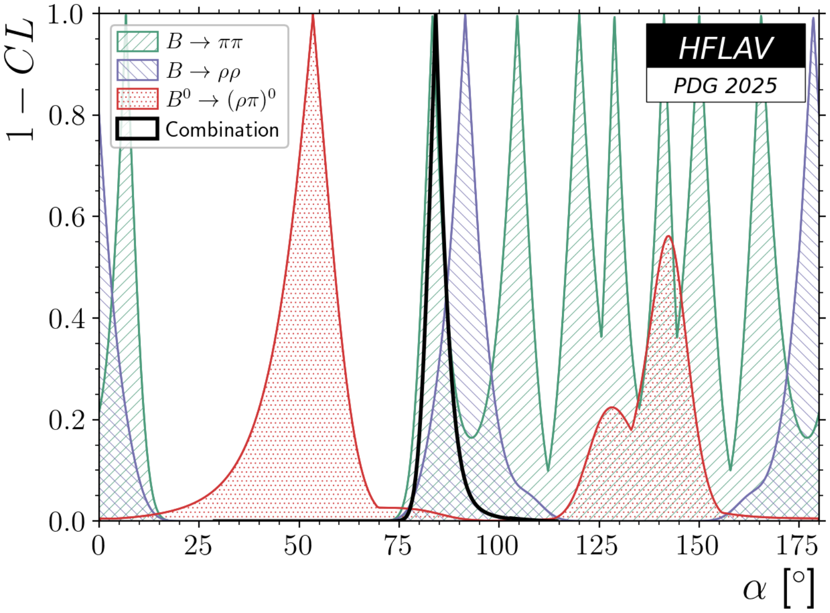
Isospin analysis inputs from HFLAV



The angle α/ϕ_2

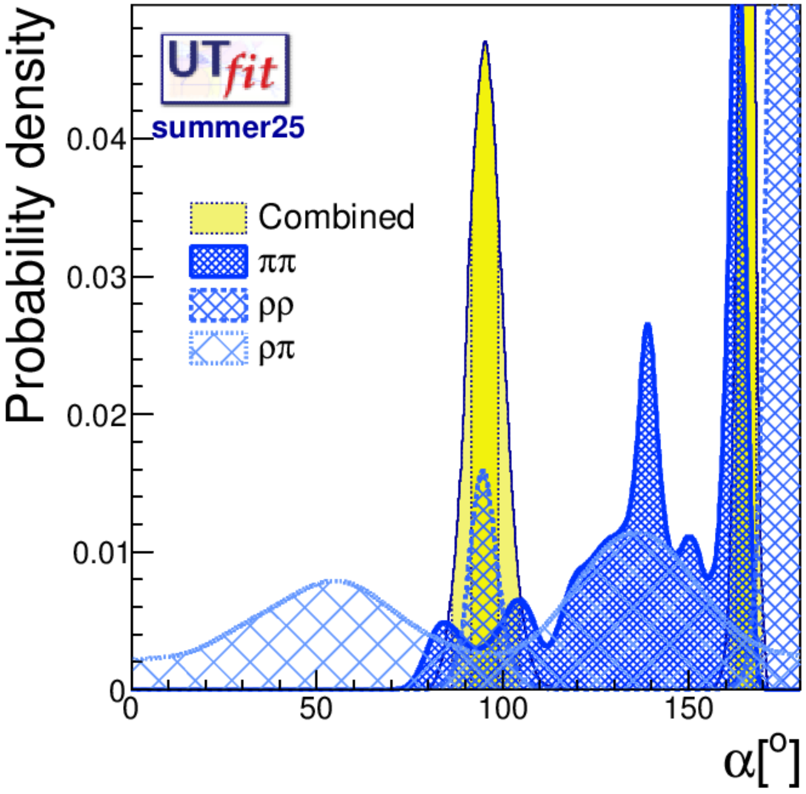


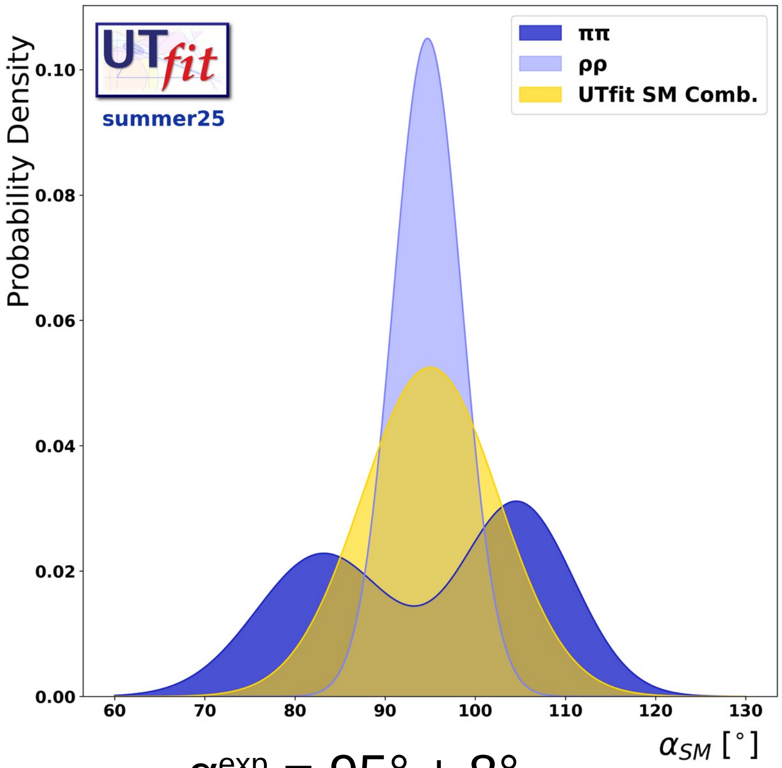




$\alpha^{exp} = 84.5° \pm 3.4°$

The angle α/ϕ_2





$\alpha^{exp} = 95° \pm 8°$

The angles

- γ/ϕ_3

$b \rightarrow c$ interfering with $b \rightarrow u$

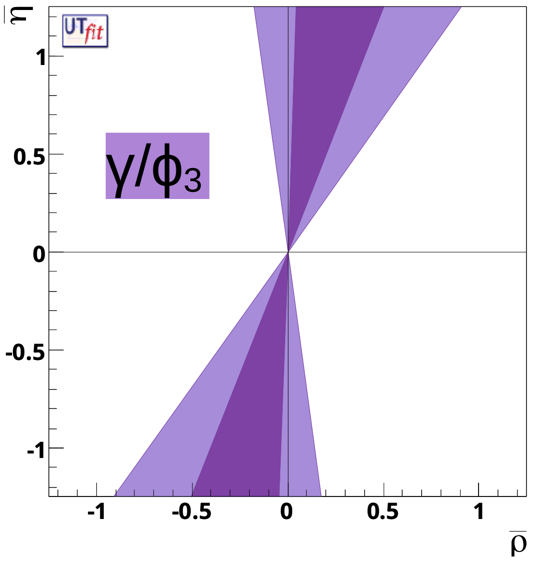
$B \rightarrow D^{(*)}K^{(*)}$

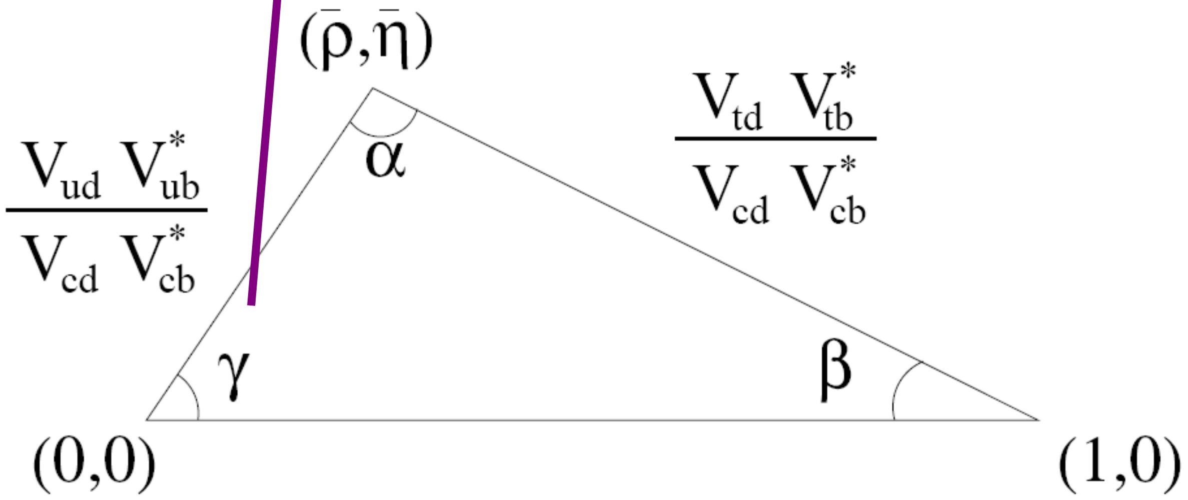
$B^0 \rightarrow D^- K^0 \pi^+$

$B^0 \rightarrow D^{(*)}\pi$

$B^0 \rightarrow D^{(*)}\rho$

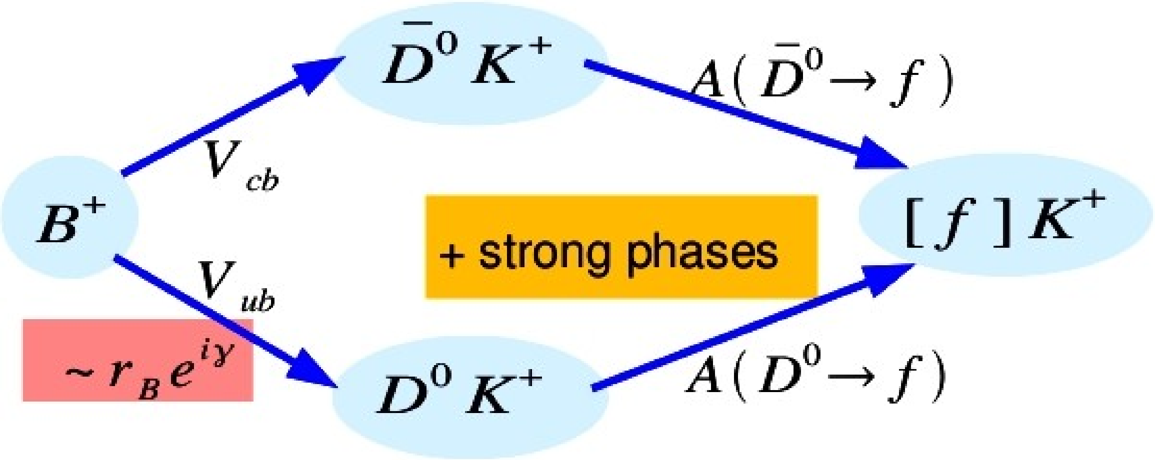
+ *charmless*

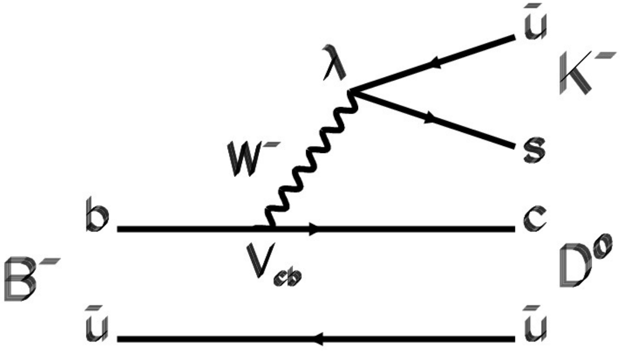




γ/ϕ_3 from B to D decays via tree diagrams

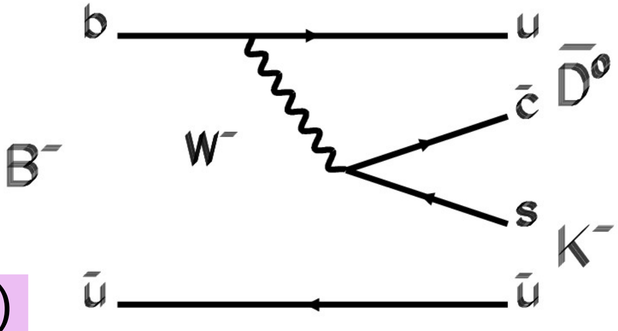
- $D^{(*)}K^{(*)}$ decays: from BRs and BR ratios
  no time-dependent analysis, just rates
- the phase γ is measured exploiting interferences:
  two amplitudes leading to the same final states
- some rates can be really small: ~ 10^{-7}





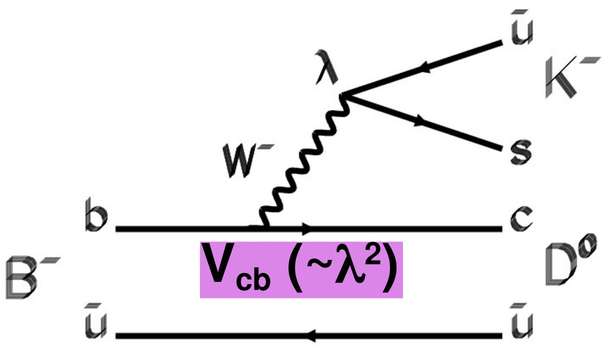
V_{cb} $(\sim\lambda^2)$

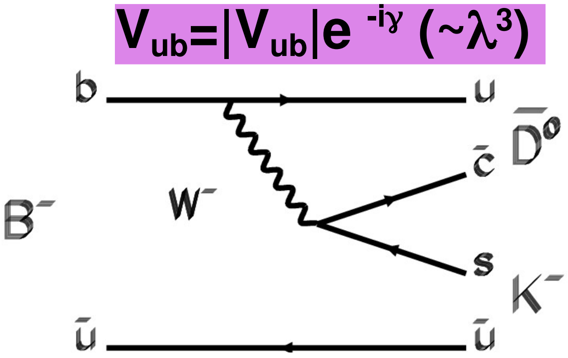
$V_{ub}=|V_{ub}|e^{-i\gamma}$ $(\sim\lambda^3)$



Theoretically clean (no penguins *neglecting the D^0 mixing*)

γ/ϕ_3 from B to D decays via tree diagrams





δ_B = strong phase diff.

$$A(B^- \to D^0 K^-) = A_B$$

$$A(B^+ \to \bar{D}^0 K^+) = A_B$$

$$A(B^- \to \bar{D}^0 K^-) = A_B r_B e^{i(\delta_B - \gamma)}$$

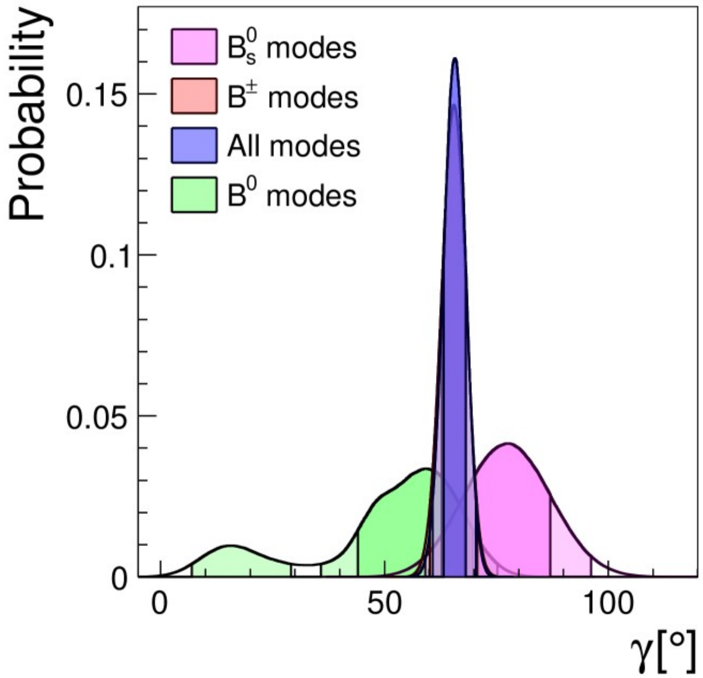
$$A(B^+ \to D^0 K^+) = A_B r_B e^{i(\delta_B + \gamma)}$$

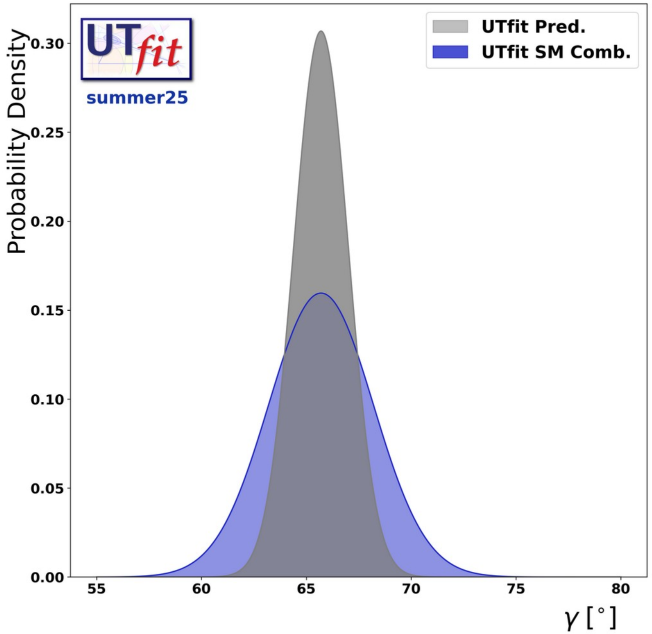
r_B = amplitude ratio

$$r_B = \left| \frac{B^- \to \bar{D}^0 K^-}{B^- \to D^0 K^-} \right| = \sqrt{\bar{\eta}^2 + \bar{\rho}^2} \times F_{CS}$$

hadronic contribution channel-dependent

γ/ϕ_3 from B to D decays via tree diagrams

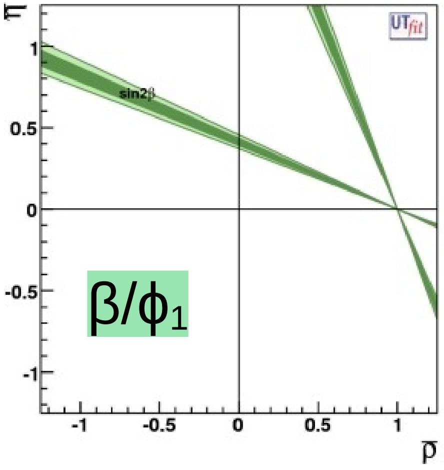


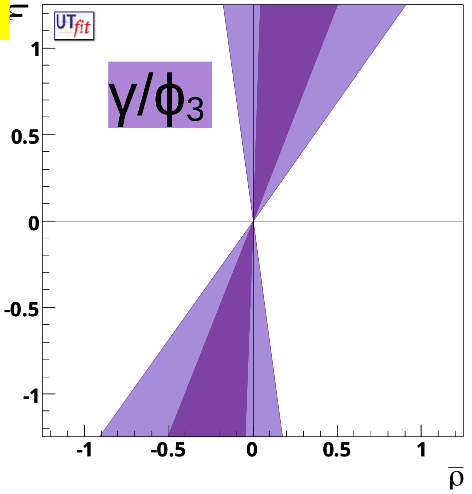


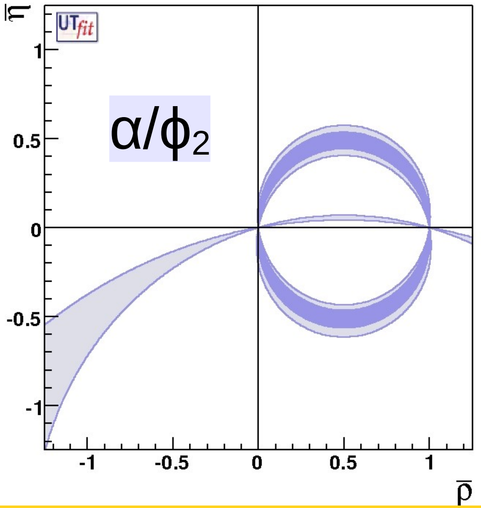
$\gamma = (65.7 \pm 2.5)^o$

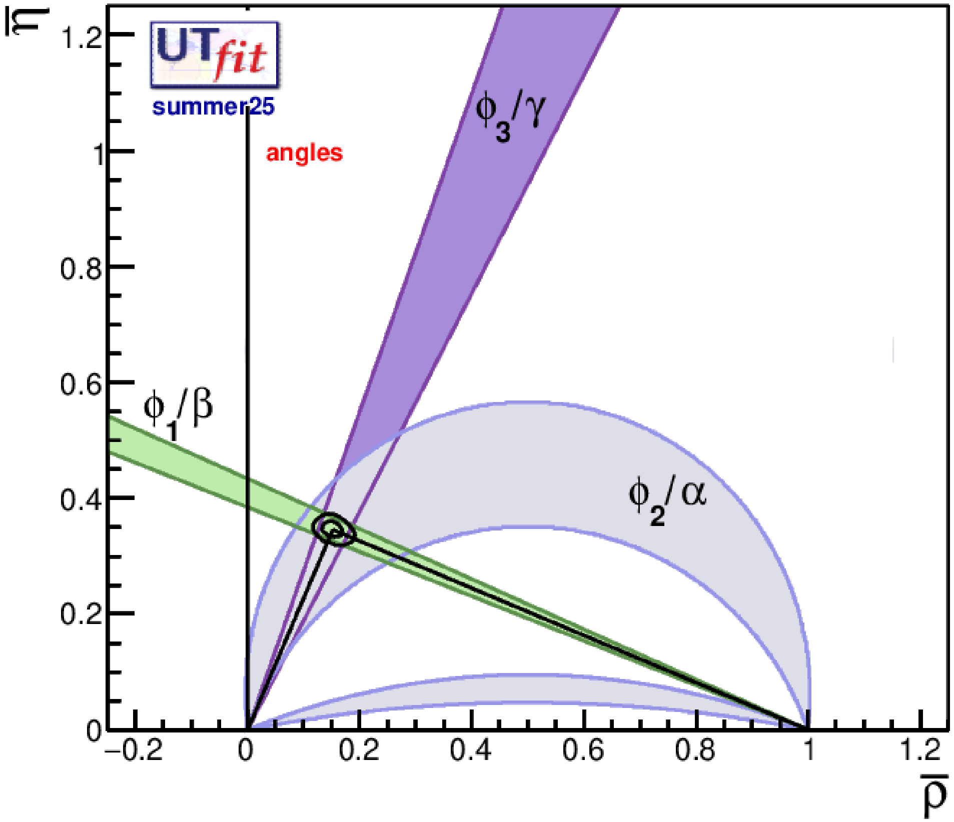
Updated analysis in arXiv:2409.06449
analysis of charm and beauty observables, together with neutral D mixing and CP-violating parameters

The angles



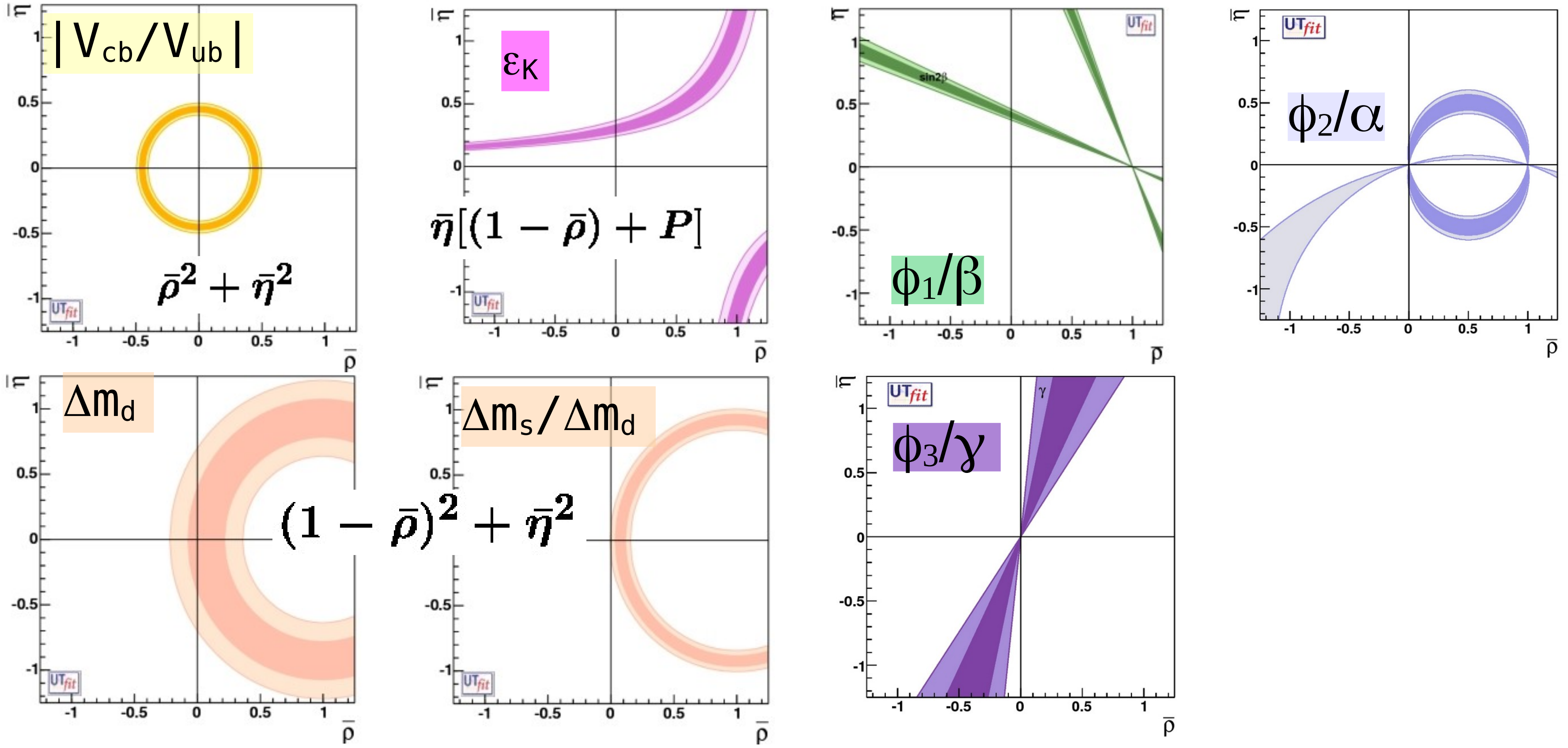




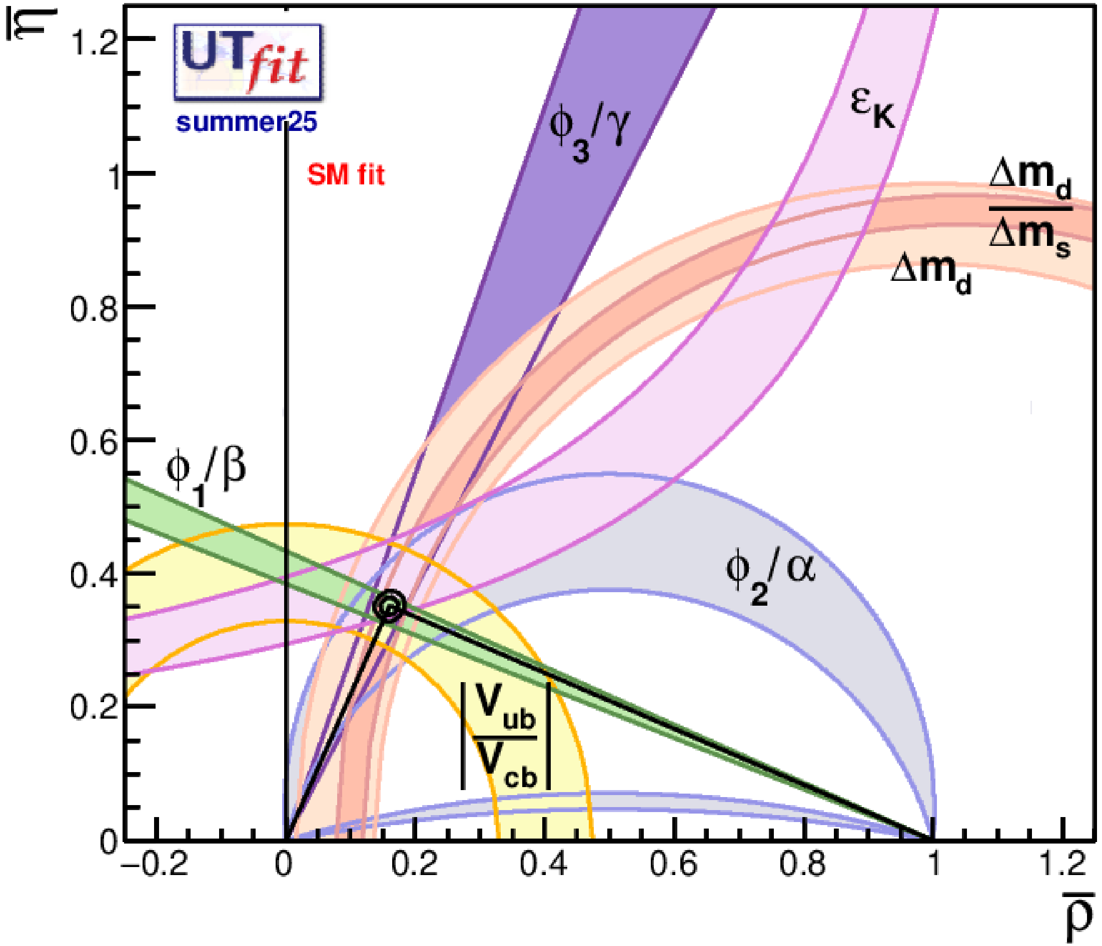


$\bar{\rho} = 0.151 \pm 0.015$
$\bar{\eta} = 0.346 \pm 0.010$

Unitarity Triangle analysis in the SM:



Unitarity Triangle analysis in the SM:



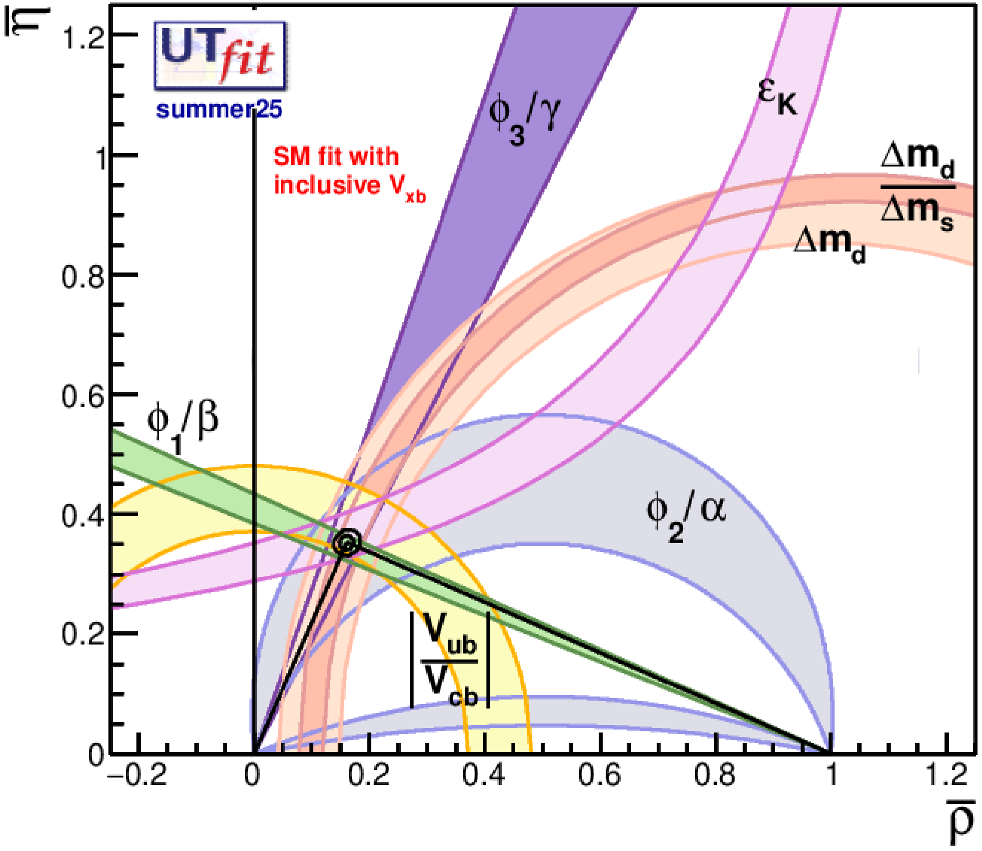
levels @
95% Prob

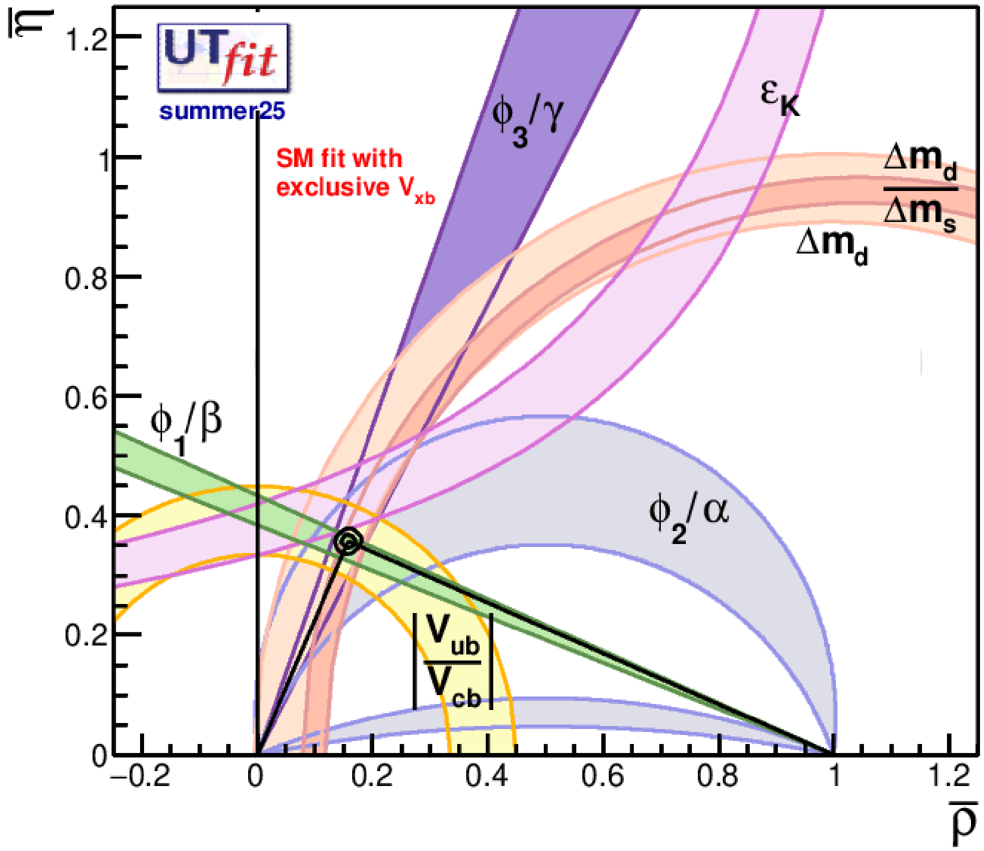
$\bar{\rho} = 0.160 \pm 0.009$
$\bar{\eta} = 0.352 \pm 0.008$

$\lambda = 0.2250 \pm 0.0006$
$A = 0.826 \pm 0.009$

Unitarity Triangle analysis in the SM:

Inclusive V_{xb} inputs vs exclusive V_{xb} inputs





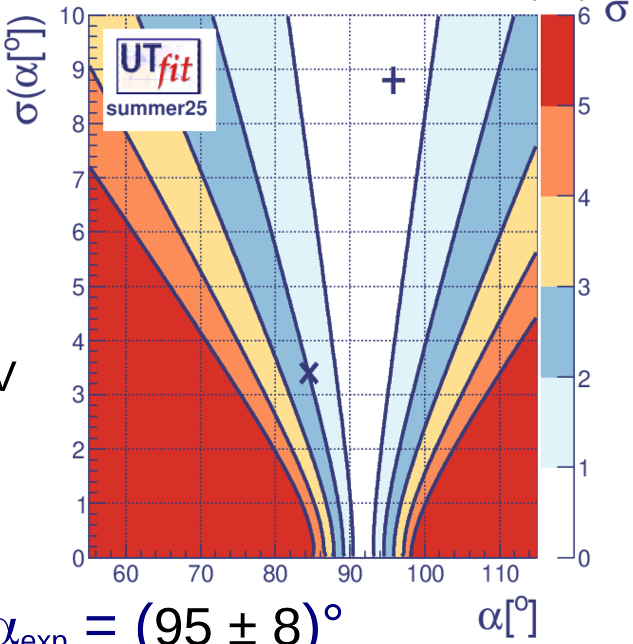
Compatibility plots

A way to "measure" the agreement of a single measurement with the indirect determination from the fit using all the other inputs: test for the SM description of the flavour physics

Colour code: agreement between the predicted values and the measurements at better than 1, 2, ...nσ

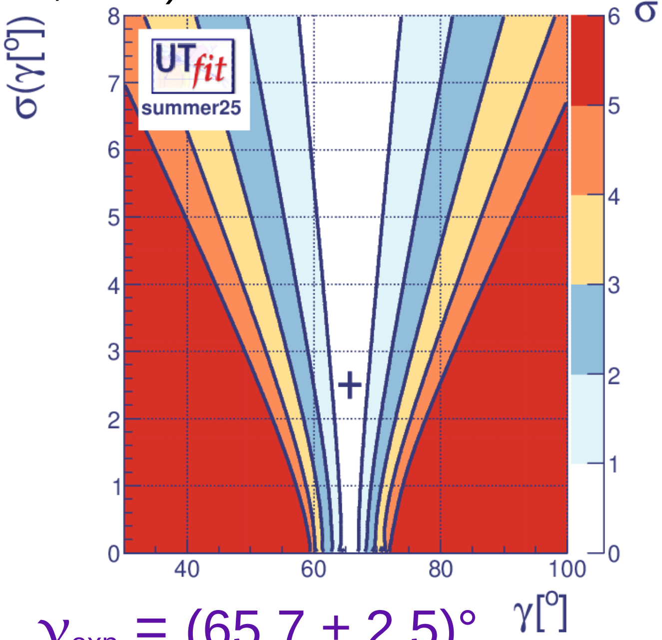
The cross has the coordinates (x,y)=(central value, error) of the direct measurement

X = HFLAV



$\alpha_{exp} = (95 \pm 8)^\circ$

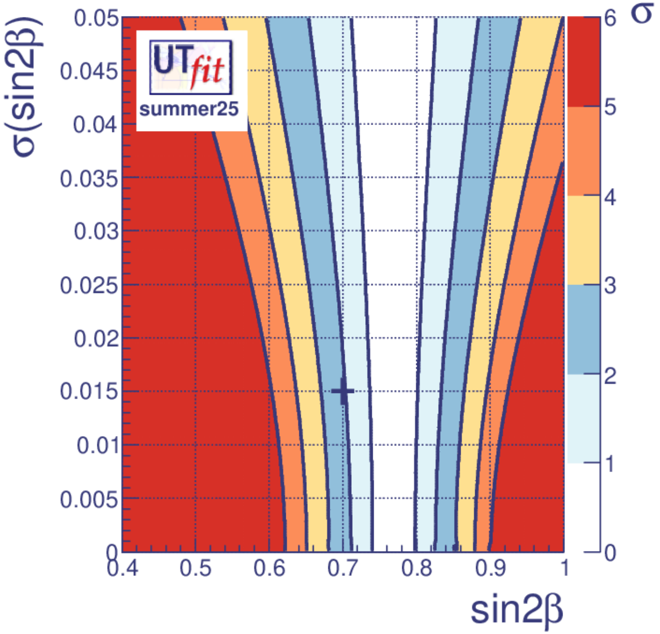
$\alpha_{UTfit} = (91.7 \pm 1.4)^\circ$



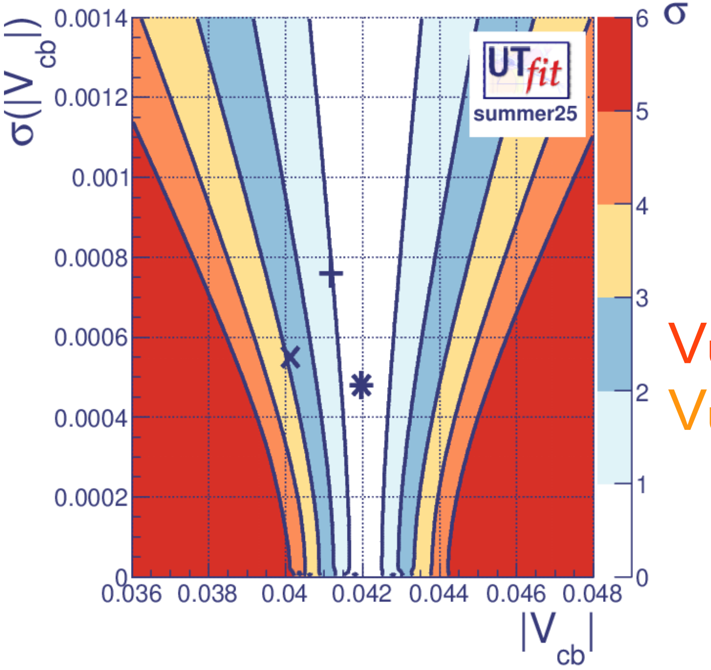
$\gamma_{exp} = (65.7 \pm 2.5)^\circ$

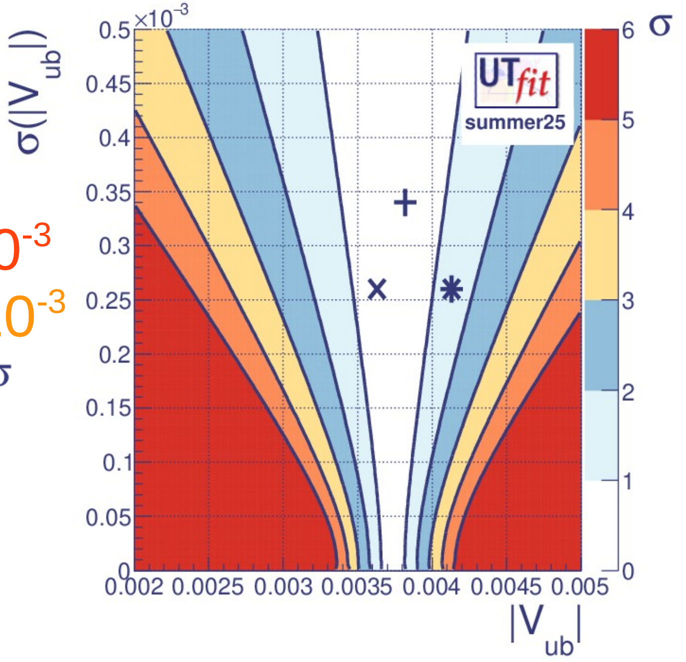
$\gamma_{UTfit} = (65.7 \pm 1.3)^\circ$

Checking the usual *tensions*..



$Vcb_{exp} = (41.18 \pm 0.76) \cdot 10^{-3}$
$Vcb_{UTfit} = (42.07 \pm 0.42) \cdot 10^{-3}$





$Vub_{exp} = (3.82 \pm 0.34) \cdot 10^{-3}$
$Vub_{UTfit} = (3.74 \pm 0.08) \cdot 10^{-3}$

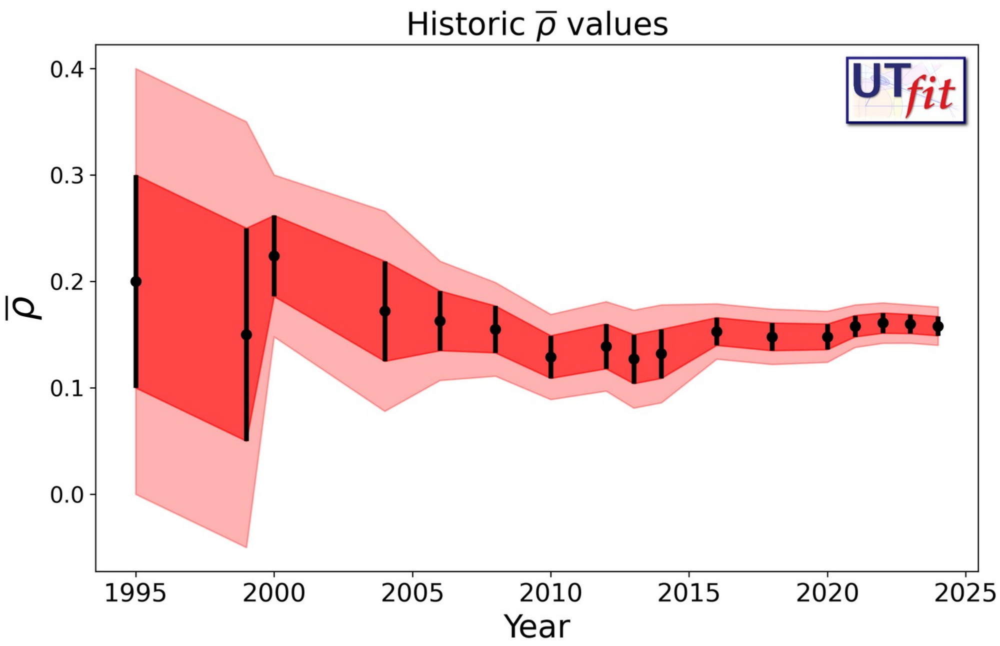
$\sin 2\beta_{exp} = 0.700 \pm 0.015$
$\sin 2\beta_{UTfit} = 0.768 \pm 0.029$

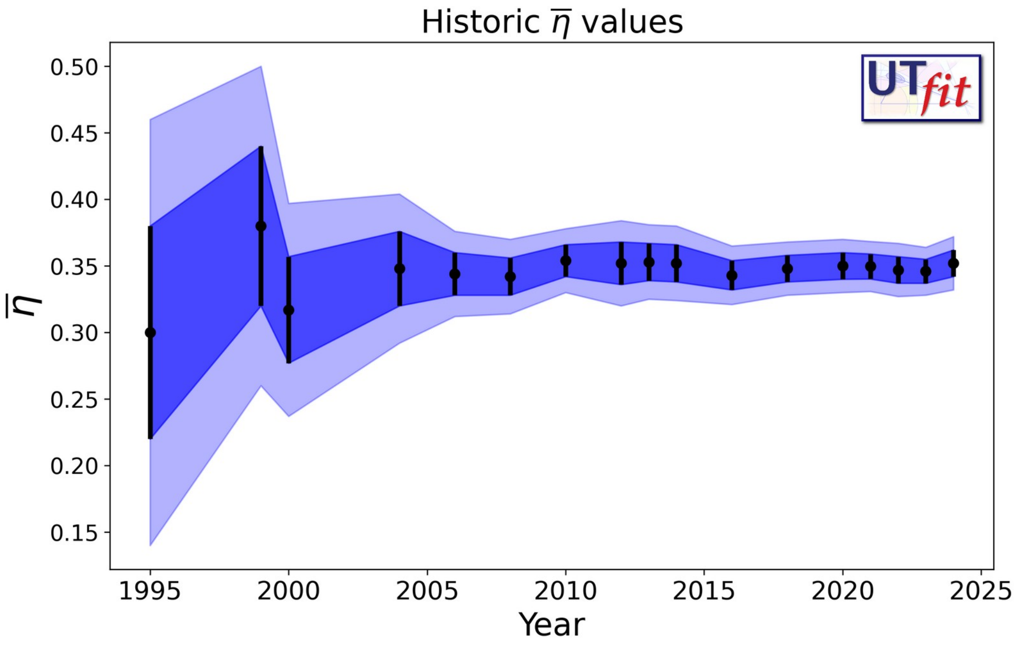
x = exclusive
∗ = inclusive

Result summary

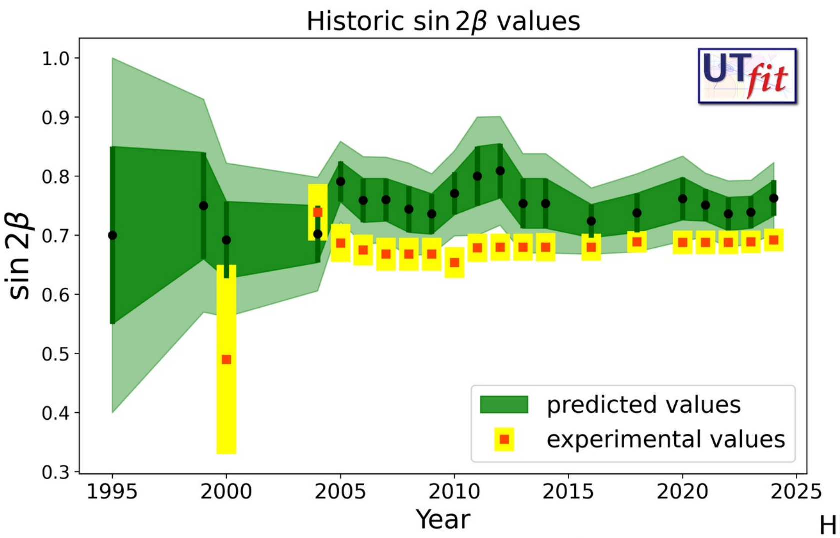
Observables	Measurement	Prediction	Pull (#σ)
$\sin 2\beta$	0.700 ± 0.015	0.768 ± 0.029	~ 2
γ	65.7 ± 2.5	65.7 ± 1.3	< 1
α	95 ± 8	91.7 ± 1.4	< 1
$\lvert V_{cb}\rvert \cdot 10^3$	41.18 ± 0.76	42.07 ± 0.42	~ 1
$\lvert V_{cb}\rvert \cdot 10^3$ (excl)	40.12 ± 0.55		~ 2.8
$\lvert V_{cb}\rvert \cdot 10^3$ (incl)	41.97 ± 0.48		< 1
$\lvert V_{ub}\rvert \cdot 10^3$	3.82 ± 0.34	3.74 ± 0.08	< 1
$\lvert V_{ub}\rvert \cdot 10^3$ (excl)	3.63 ± 0.26	-	< 1
$\lvert V_{ub}\rvert \cdot 10^3$ (incl)	4.13 ± 0.26	-	~ 1.4
$BR(B \to \tau\nu)[10^4]$	1.12 ± 0.21	0.884 ± 0.040	~ 1

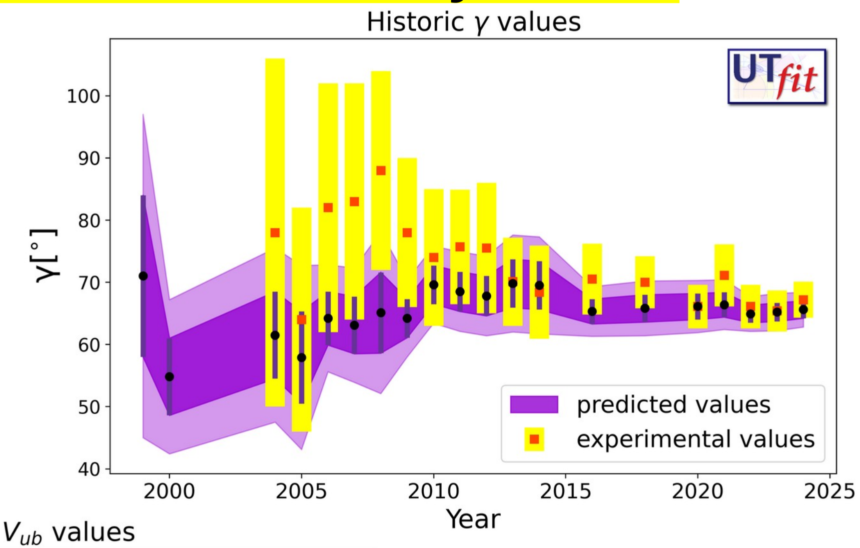
UTfit results across the years:

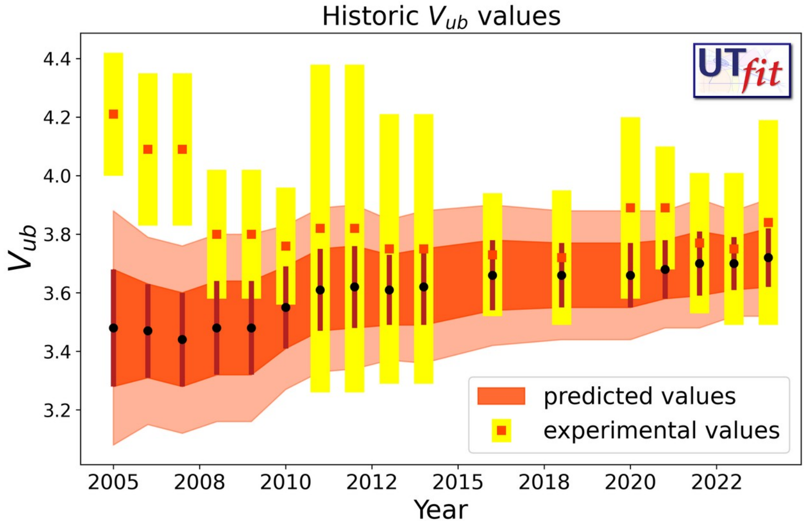




UTfit and experimental results across the years:







UT analysis including new physics

fit simultaneously for the CKM and
the NP parameters (generalized UT fit)

- add most general loop NP to all sectors
- use all available experimental info
- find out NP contributions to ΔF=2 transitions

B_d and B_s mixing amplitudes
(2+2 real parameters):

$$A_q = C_{B_q} e^{2i\phi_{B_q}} A_q^{SM} e^{2i\phi_q^{SM}} = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$

$\Delta m_{q/K} = C_{B_q/\Delta m_K} (\Delta m_{q/K})^{SM}$	$\varepsilon_K = C_\varepsilon \varepsilon_K^{SM}$
$A_{CP}^{B_d \to J/\psi K_s} = \sin 2(\beta + \phi_{B_d})$	$A_{CP}^{B_s \to J/\psi \phi} \sim \sin 2(-\beta_s + \phi_{B_s})$
$A_{SL}^q = \mathrm{Im}(\Gamma_{12}^q / A_q)$	$\Delta\Gamma^q / \Delta m_q = \mathrm{Re}(\Gamma_{12}^q / A_q)$

new-physics-specific constraints

semileptonic asymmetries in B^0 and B_s:
sensitive to NP effects in both size and phase.

$$A^s_{\rm SL} \equiv \frac{\Gamma(\bar{B}_s \to \ell^+ X) - \Gamma(B_s \to \ell^- X)}{\Gamma(\bar{B}_s \to \ell^+ X) + \Gamma(B_s \to \ell^- X)} = {\rm Im}\left(\frac{\Gamma^s_{12}}{A^{\rm full}_s}\right)$$

HFLAV from Cleo, BaBar, Belle, D0 and LHCb

same-side dilepton charge asymmetry:
admixture of B_s and B_d so sensitive to NP effects in both.

D0 arXiv:1106.6308

$$A^{\mu\mu}_{\rm SL} = \frac{f_d \chi_{d0} A^d_{\rm SL} + f_s \chi_{s0} A^s_{\rm SL}}{f_d \chi_{d0} + f_s \chi_{s0}}$$

$$A^{\mu\mu}_{\rm SL} \times 10^3 = -7.9 \pm 2.0$$

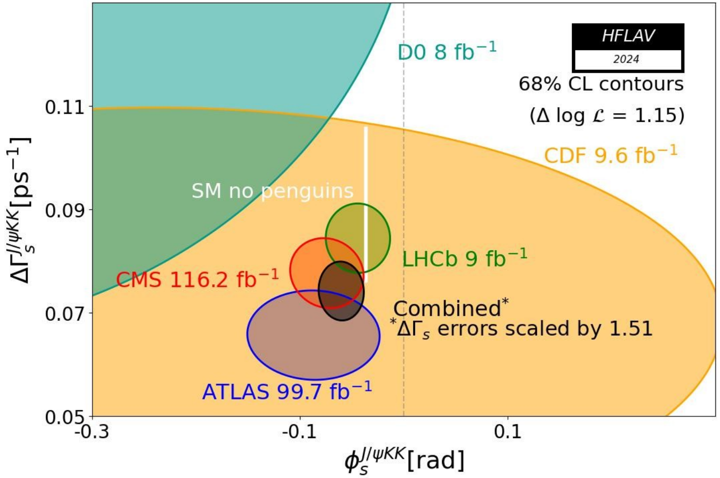
lifetime τ^{FS} in flavour-specific final states:
average lifetime is a function to the width and the width difference

$$\tau^{FS}(B_s) = 1.527 \pm 0.011 \text{ ps}$$

HFLAV

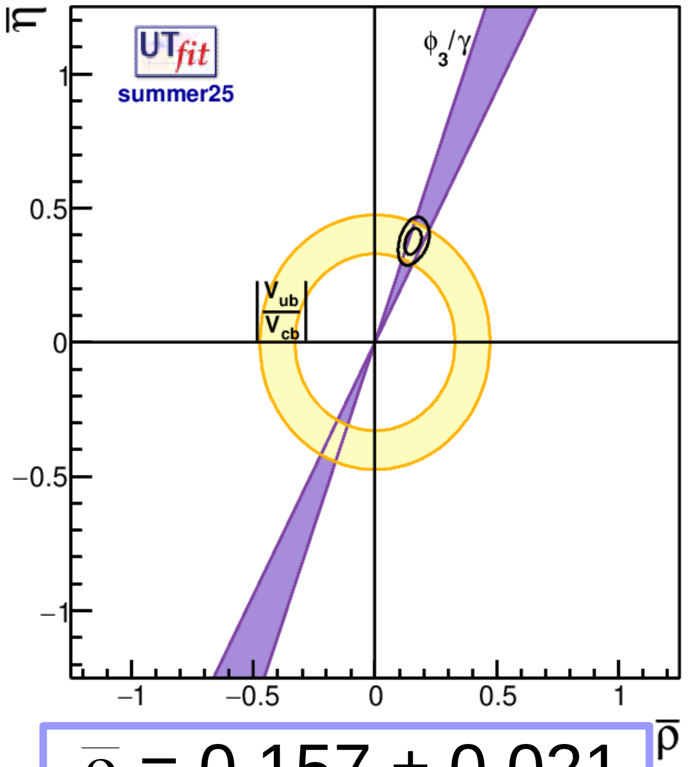
$\phi_s = 2\beta_s$ vs $\Delta\Gamma_s$ from $B_s \to J/\psi\phi$
angular analysis as a function of proper time and b-tagging

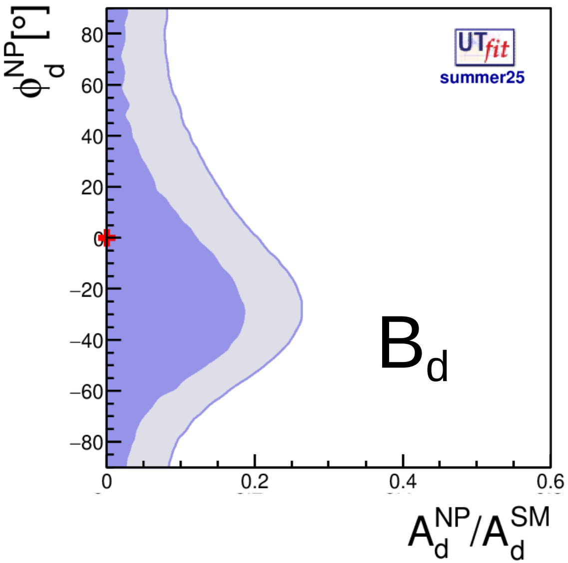
$$\phi_s = -0.039 \pm 0.016 \text{ rad}$$

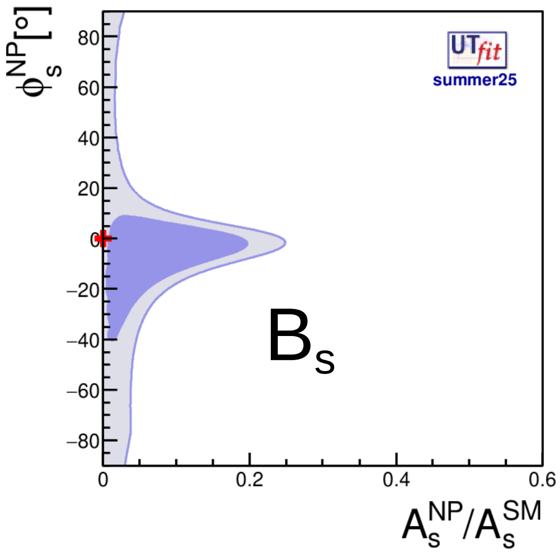


From the NP fit we get

$$A_q = \left(1 + \frac{A_q^{NP}}{A_q^{SM}} e^{2i(\phi_q^{NP} - \phi_q^{SM})}\right) A_q^{SM} e^{2i\phi_q^{SM}}$$







$\bar{\rho} = 0.157 \pm 0.021$
$\bar{\eta} = 0.376 \pm 0.035$

The ratio of NP/SM amplitudes is:
< 18% @68% prob. (25% @95%) in B_d mixing
< 20% @68% prob. (25% @95%) in B_s mixing

Testing the new-physics scale

M. Bona *et al.* (UTfit)
JHEP 0803:049,2008
arXiv:0707.0636

RGE

At the high scale
new physics enters according to its specific features

At the low scale
use OPE to write the most general effective Hamiltonian.
the operators have different chiralities than the SM
NP effects are in the Wilson Coefficients C

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i Q_i^{bq} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{bq}$$

$$\begin{aligned}
Q_1^{q_i q_j} &= \bar{q}_{jL}^{\alpha} \gamma_\mu q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} \gamma^\mu q_{iL}^{\beta} , \\
Q_2^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jR}^{\beta} q_{iL}^{\beta} , \\
Q_3^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jR}^{\beta} q_{iL}^{\alpha} , \\
Q_4^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} q_{iR}^{\beta} , \\
Q_5^{q_i q_j} &= \bar{q}_{jR}^{\alpha} q_{iL}^{\beta} \bar{q}_{jL}^{\beta} q_{iR}^{\alpha} .
\end{aligned}$$

$$C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$$

F_i: function of the NP flavour couplings
L_i: loop factor (in NP models with no tree-level FCNC)
Λ: NP scale (typical mass of new particles mediating ΔF=2 processes)

Testing the new-physics scale

The dependence of C on Λ changes depending on the flavour structure.
We can consider different flavour scenarios:

$$C_i(\Lambda) = F_i \frac{L_i}{\Lambda^2}$$

- Generic: $C(\Lambda) = \alpha/\Lambda^2$ $F_i \sim 1$, arbitrary phase
- NMFV: $C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$ $F_i \sim |F_{SM}|$, arbitrary phase

α (L_i) is the coupling among NP and SM
- $\alpha \sim 1$ for strongly coupled NP
- $\alpha \sim \alpha_W$ (α_S) in case of loop coupling through weak (strong) interactions

If no NP effect is seen lower bound on NP scale Λ

F is the flavour coupling and so
F_{SM} is the combination of CKM factors for the considered process

Results from the Wilson coefficients

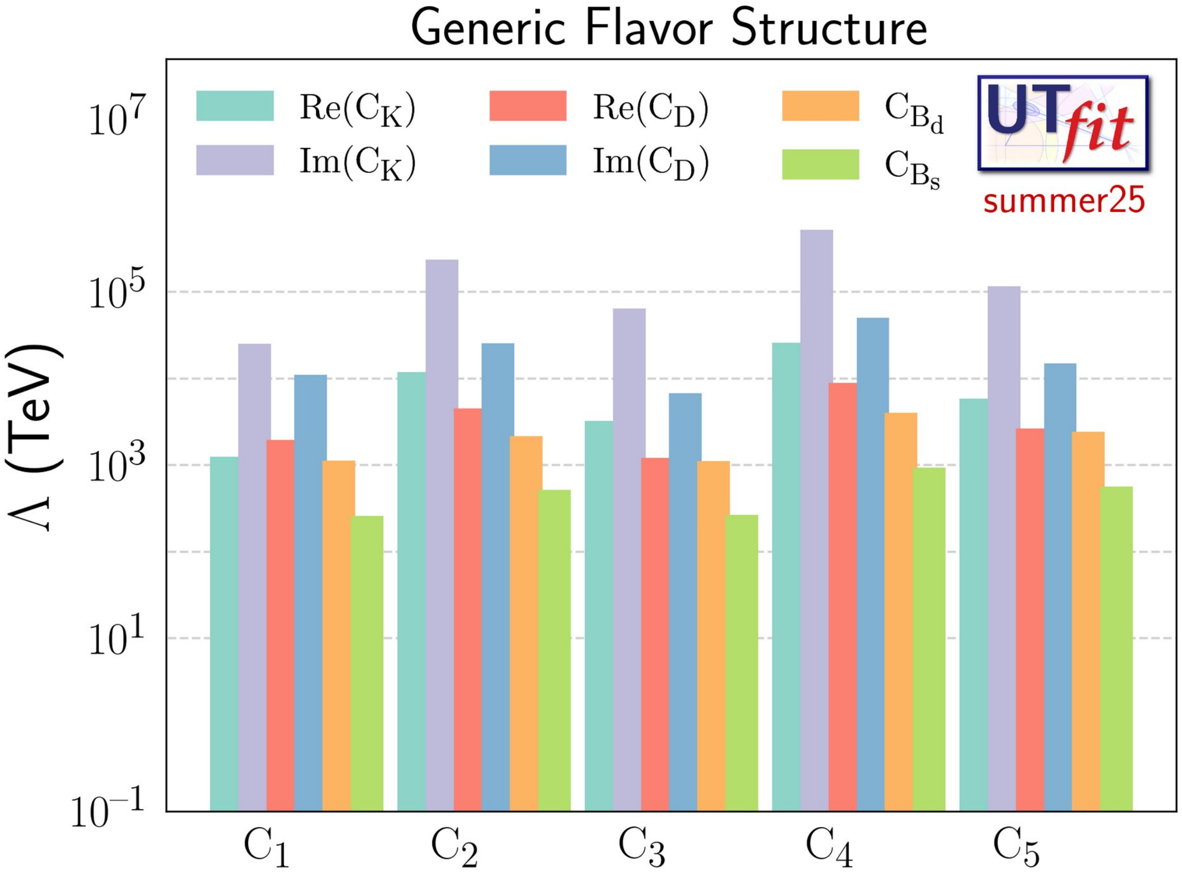
Generic:
$C(\Lambda) = \alpha/\Lambda^2$,
F_i~1, arbitrary phase

α ~ 1 for strongly coupled NP

Λ > 4.9 10^5 TeV

$\alpha \sim \alpha_W$ in case of loop coupling through weak interactions

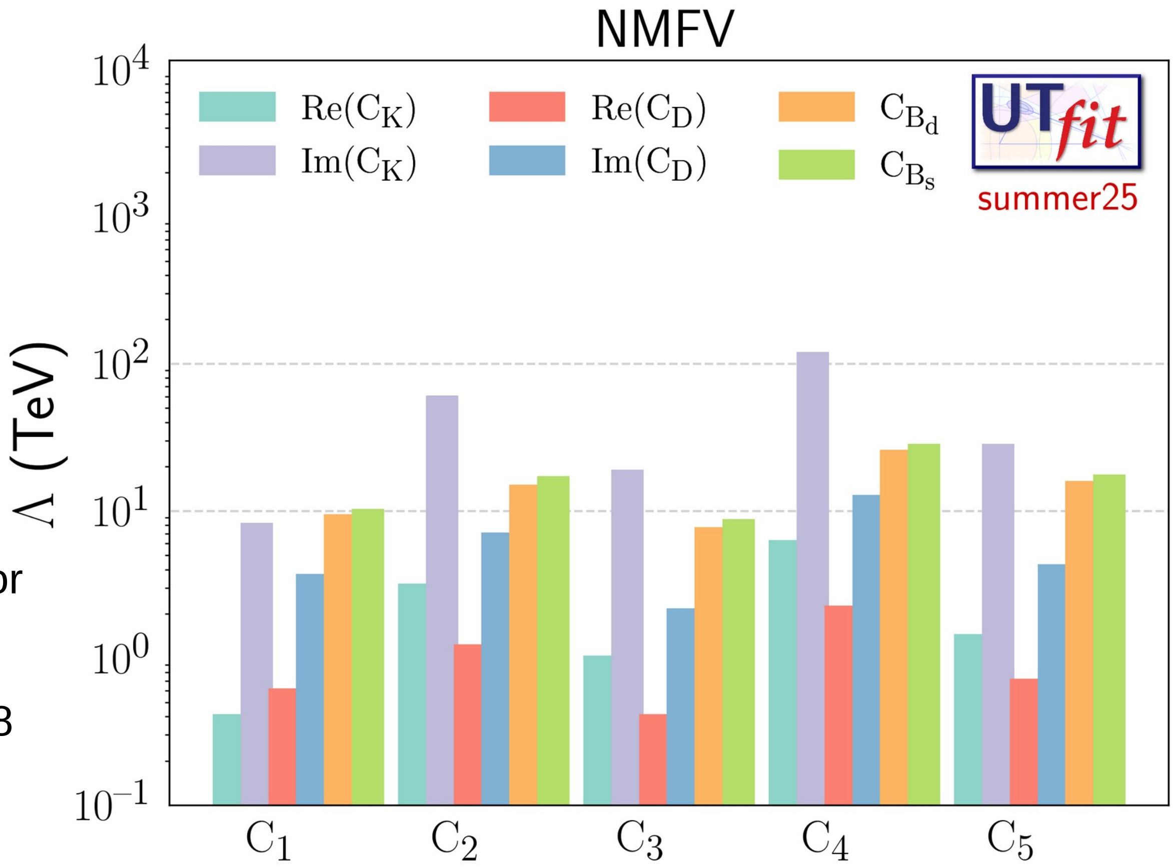
Λ > 1.5 10^4 TeV



the bound has increased of a factor of 2 with respect to the first UTfit NP analysis in JHEP'08 [0707.0636]

for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_W (~ 0.03).

Results from the Wilson coefficients



NMFV:
$C(\Lambda) = \alpha \times |F_{SM}|/\Lambda^2$, $F_i \sim |F_{SM}|$, arbitrary phase

$\alpha \sim 1$ for strongly coupled NP

$\Lambda > 1.5\ 10^2$ TeV

$\alpha \sim \alpha_W$ in case of loop coupling through weak interactions

$\Lambda > 4.5$ TeV

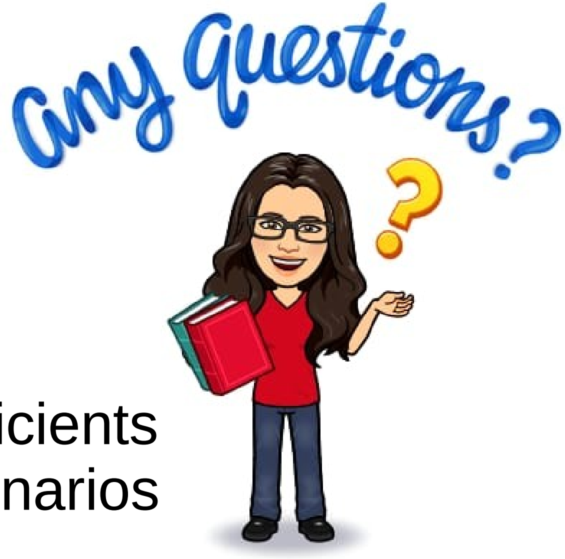
the bound has increased of a factor of 2.5 with respect to the first UTfit NP analysis in JHEP‘08 [0707.0636]

for lower bound for loop-mediated contributions, simply multiply by α_s (~ 0.1) or by α_W (~ 0.03).

some conclusions

- ◉ test of the SM consistency and the CKM mechanism:
  comparison between inputs and indirect determinations
  - ◎ using all the available inputs from experiments and theoretical and lattice QCD calculations
  - ◎ extraction of the most accurate SM predictions

- ◉ model-independent new physics:
  - ◎ overconstraining of the SM fit allows for extraction of generic amplitude and phase for all the systems (K, B_d, B_s)
  - ◎ scale analysis: putting bounds on the Wilson coefficients gives insights into the NP scale in different NP scenarios



Back up slides