

# UNDERSTANDING THE 6-DIM DATA

RESULTS FROM PRINCIPAL COMPONENTS ANALYSIS  
AND SELF-ORGANIZING MAPS

STEPHANIE KÄS



MINI-WORKSHOP  
ANOMALY DETECTION WITH NEURAL NETWORKS  
GIESSEN 21.02.2020

ATTENTION

This event will be  
photographed.



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# OVERVIEW

1. Principal components analysis
2. Antideuteron data set
3. Results of bachelor's thesis



# PART 1

## PRINCIPAL COMPONENTS ANALYSIS



# PRINCIPAL COMPONENTS ANALYSIS

## IDEA

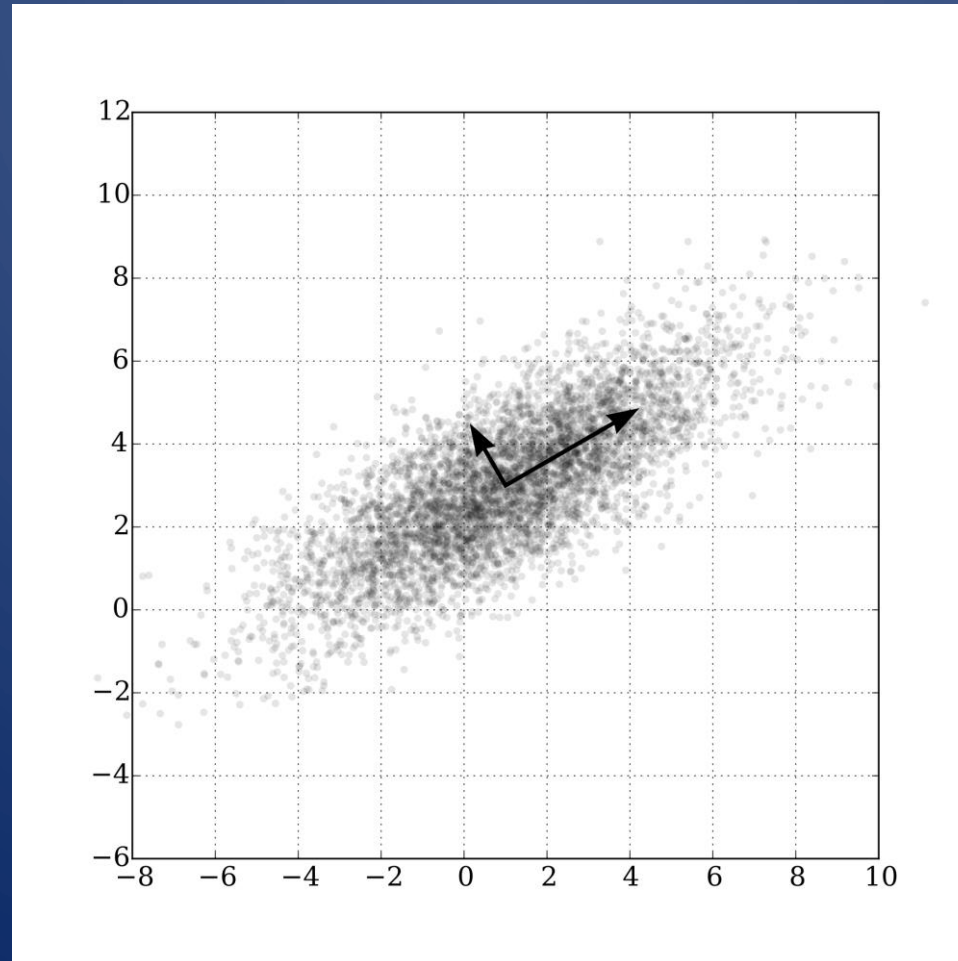


3D duck



2D duck

# PRINCIPAL COMPONENTS ANALYSIS DETAILS



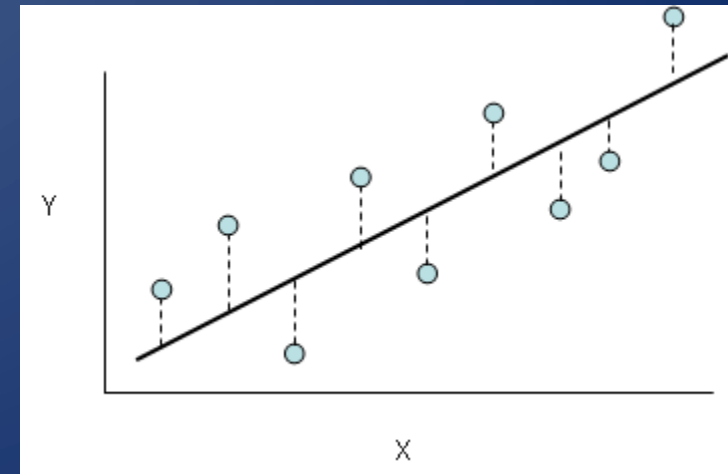
Wikipedia.org

# PRINCIPAL COMPONENTS ANALYSIS DETAILS

*Given: data set of  $n$  attributes*

Find  $n$  new axes by

- Minimization of error squares
- Maximization of variance



# PRINCIPAL COMPONENTS ANALYSIS

## BASIC STATISTICS

### Covariance matrix

$$\Sigma = \begin{pmatrix} \text{var}(\vec{x}) & \text{cov}(\vec{x}, \vec{y}) \\ \text{cov}(\vec{x}, \vec{y}) & \text{var}(\vec{y}) \end{pmatrix}$$

For data set with two attributes  $\vec{x}, \vec{y}$

Correlation matrix = normalized covariance matrix



# PRINCIPAL COMPONENTS ANALYSIS

## DETAILS

Find transformation matrix  $\Gamma$  so that  $\Lambda$  is diagonal.

$$\Lambda = \Gamma^T \Sigma \Gamma = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}$$

Columns of  $\Gamma$  are the axes of the new coordinate system.

# PRINCIPAL COMPONENTS ANALYSIS

## DETAILS

Higher values of  $\lambda_i \leftrightarrow$  higher **information content**

$$\Lambda = \Gamma^T \Sigma \Gamma = \begin{pmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{pmatrix}$$

$$\lambda_1 \geq \lambda_2 \dots \geq \lambda_n$$

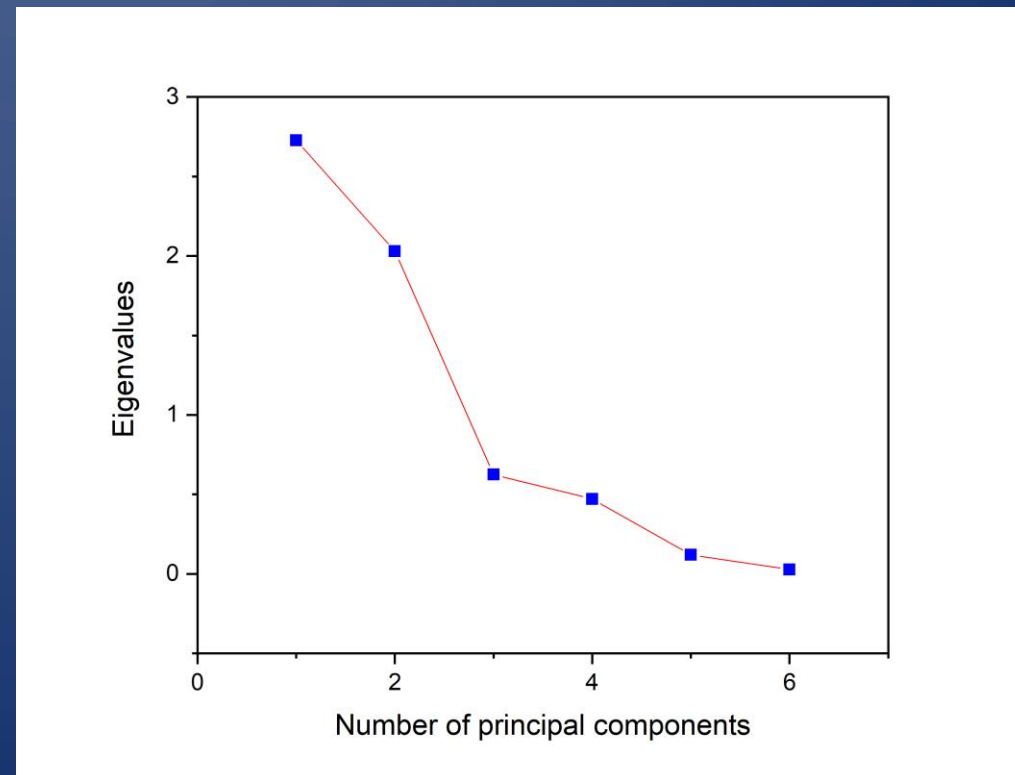
PROJECT 1

# MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

## Eigenvalues

$\lambda_i$	Cum. Sum [%]
2.73	45.46
2.03	79.28
0.62	89.62
0.47	97.55
0.12	99.54
0.03	100

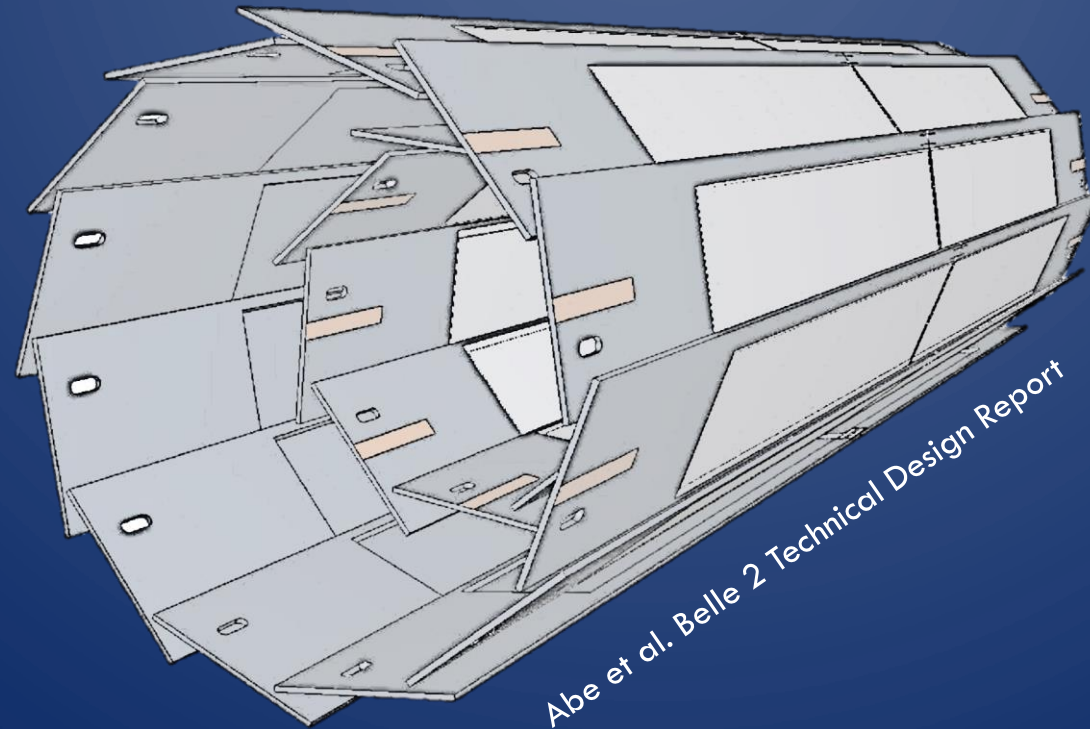
## Scree graph (PXD)



# PART 2

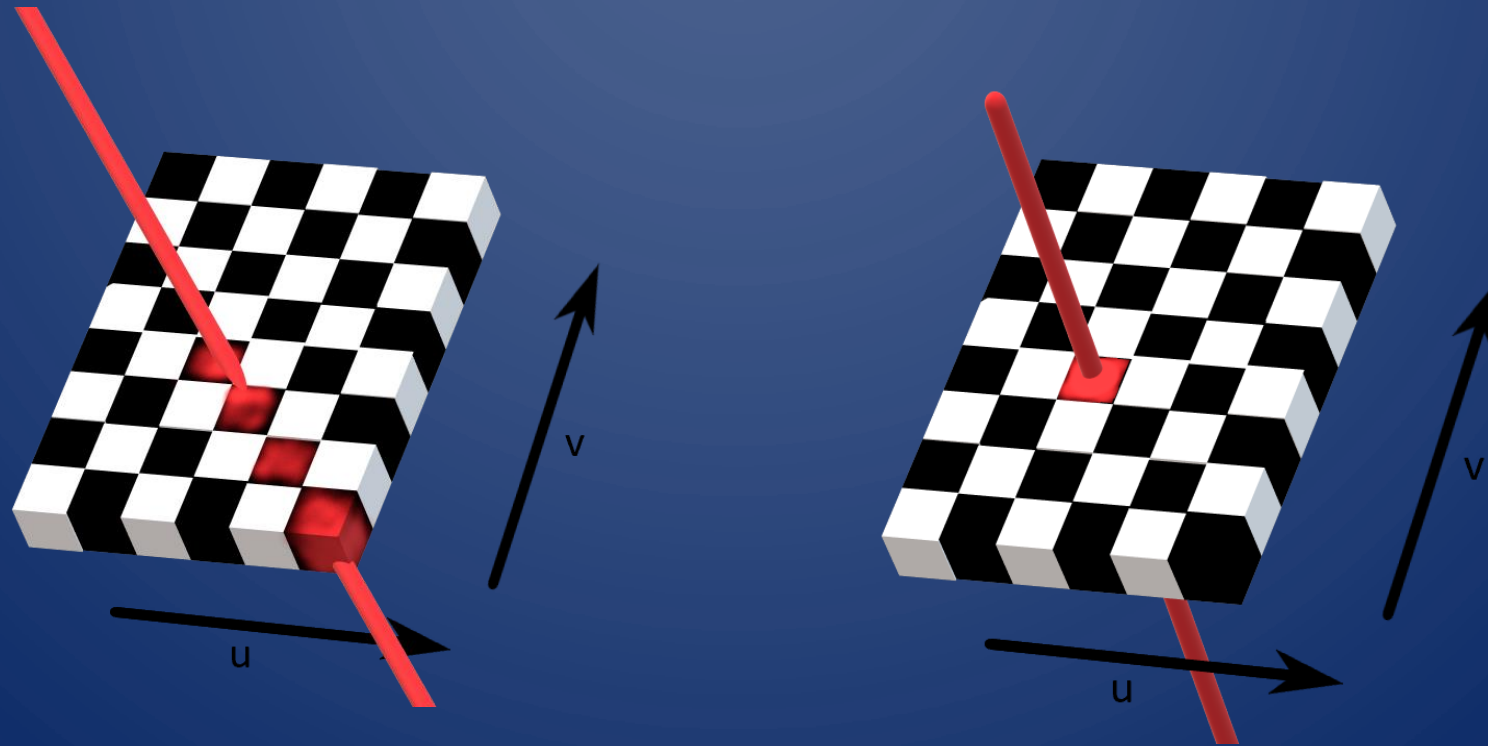
## PIXEL DETECTOR'S ANTIDEUTERON DATA SET

# PIXEL DETECTOR



Abe et al. Belle 2 Technical Design Report

# PIXEL DETECTOR



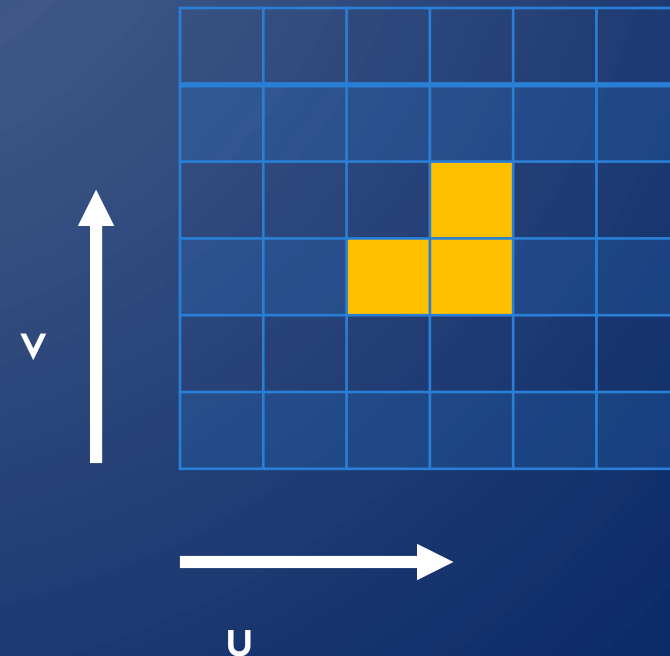
# PIXEL DETECTOR

## THE ANTIDEUTERON DATA SET

Group pixels into clusters

Cluster properties

- Total charge
- Seed charge
- Minimum charge



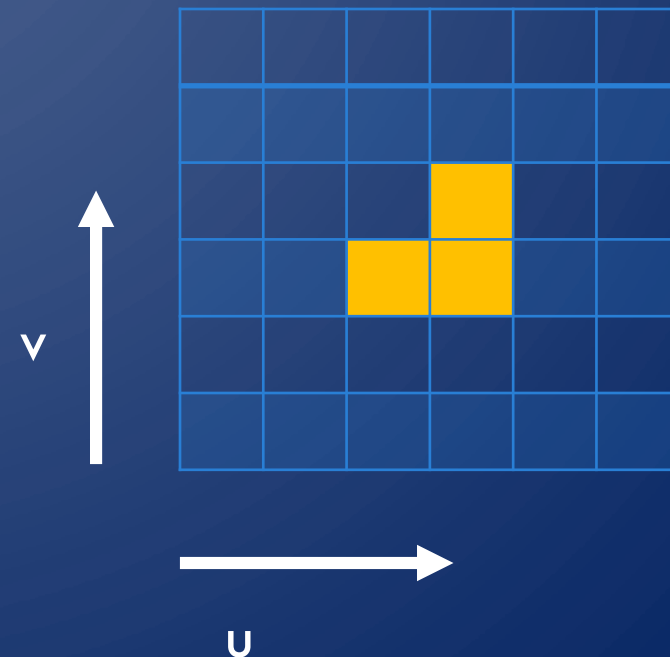
# PIXEL DETECTOR

## THE ANTIDEUTERON DATA SET

Group pixels to clusters

Cluster properties

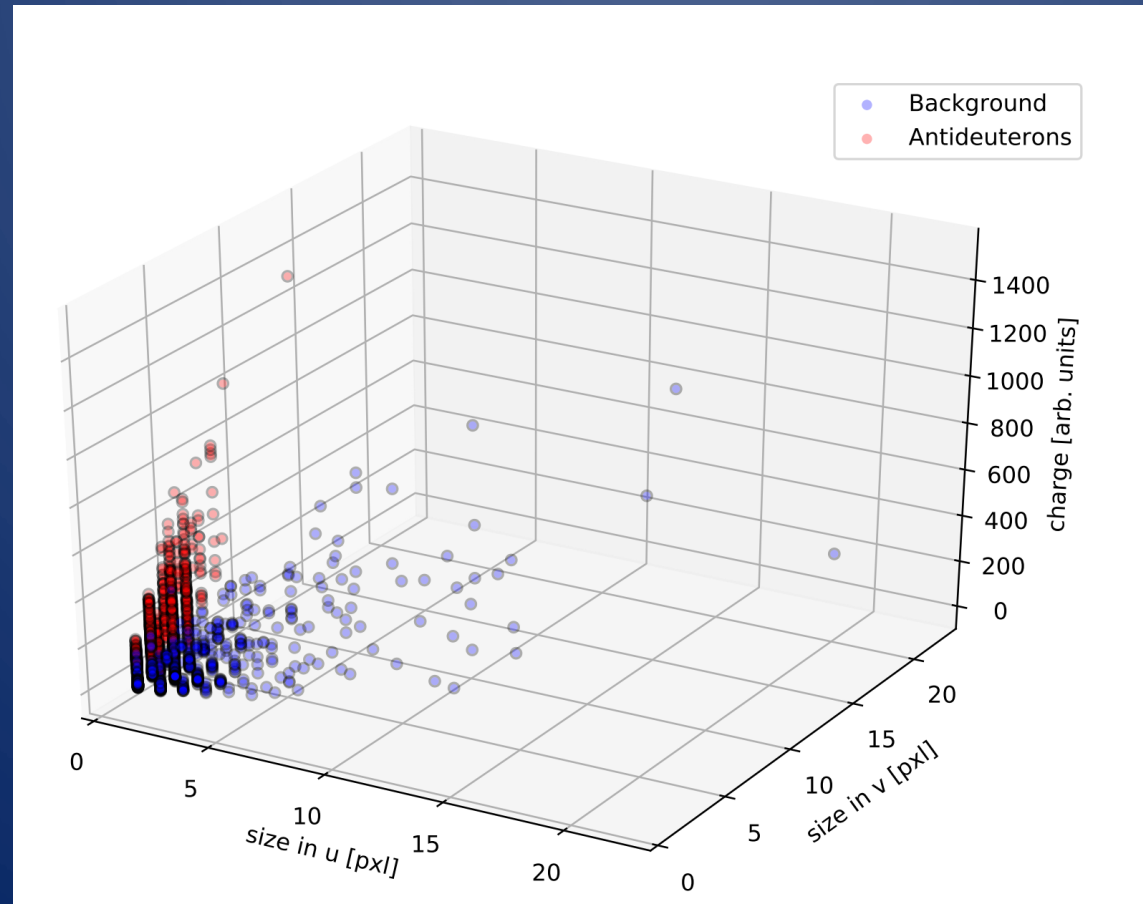
- Total charge
- Seed charge
- Minimum charge
- Total size
- Size in  $u$
- Size in  $v$





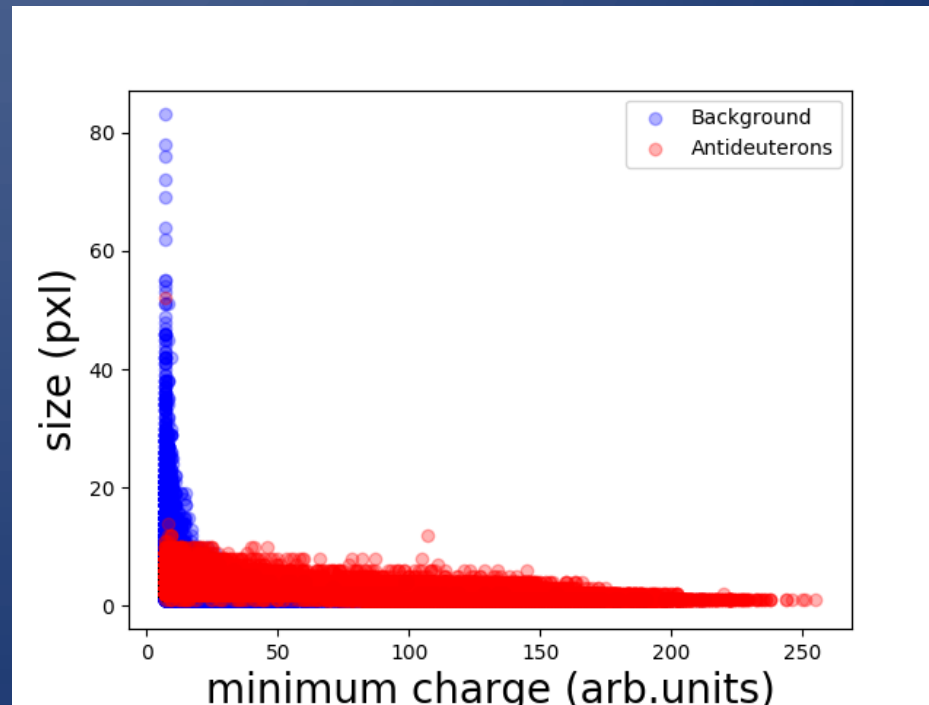
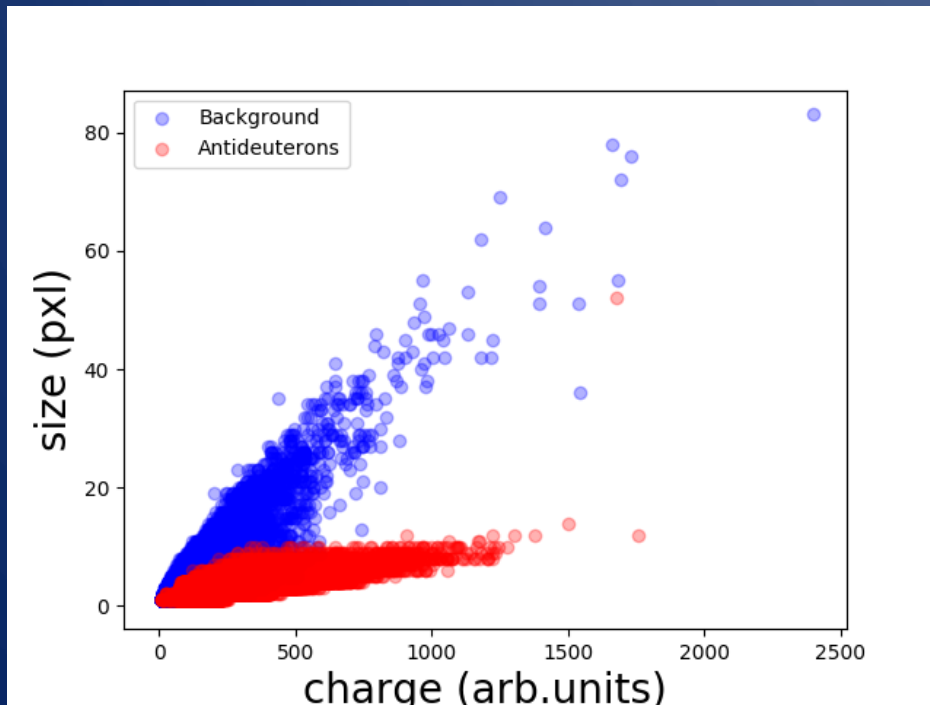
# PIXEL DETECTOR

## THE ANTIDEUTERON DATA SET



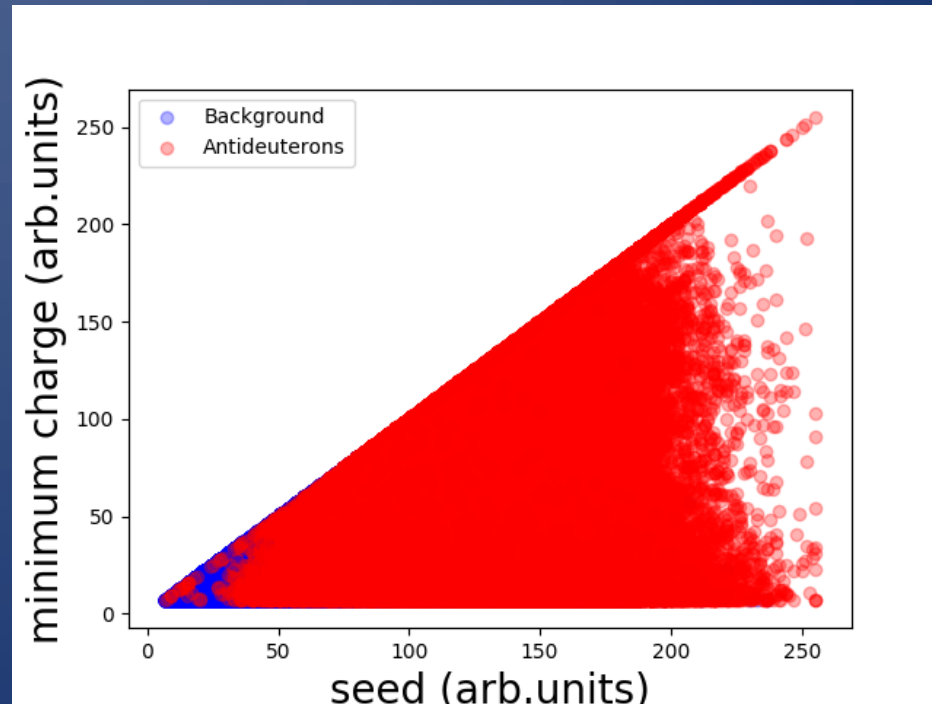
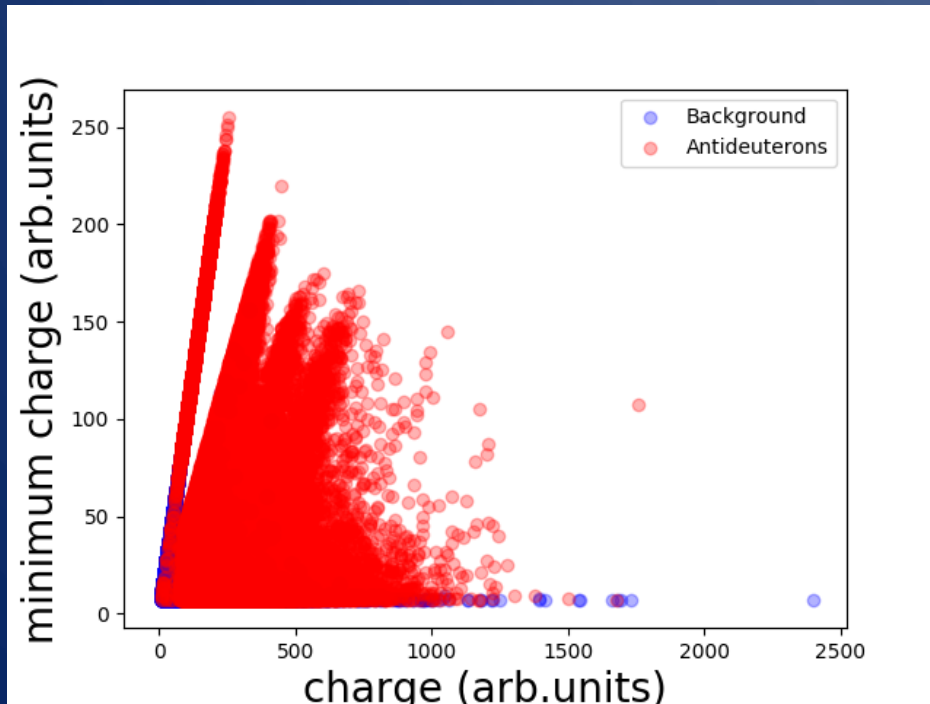
# PIXEL DETECTOR

## THE ANTIDEUTERON DATA SET



# PIXEL DETECTOR

## THE ANTIDEUTERON DATA SET





# PART 3

## BACHELOR'S THESIS



# BACHELOR'S THESIS - GOALS

## 1. Goal: Better understanding of data set

- Find correlations in cluster properties -> PCA
- Cluster shapes

## 2. Goal: Separate particles from background

- Use SOMs: Separate more than 2 particles from background
- Is pre-transformation into PCA-space helpful?

# PROJECT 1: MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

Which correlations between the 6 cluster properties exist ?

Try PCA!

PROJECT 1

# MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

## Correlation matrix (lower half)

Cluster Property	Charge	Min. Charge	Seed	Size	Size in u	Size in v
Charge	1					
Min. Charge	0.2233	1				
Seed	0.7854	0.4771	1			
Size	0.4617	-0.2882	0.0392	1		
Size in u	0.1596	-0.2600	-0.1399	0.8044	1	
Size in v	0.4144	-0.2091	0.0546	0.8414	0.4627	1

Several high correlations!

PROJECT 1

# MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

## Interpretation of principal components

PC1 = Measure for  
size

PC2 = Measure for  
charge

PC3 = ?

79.28 % of total  
information

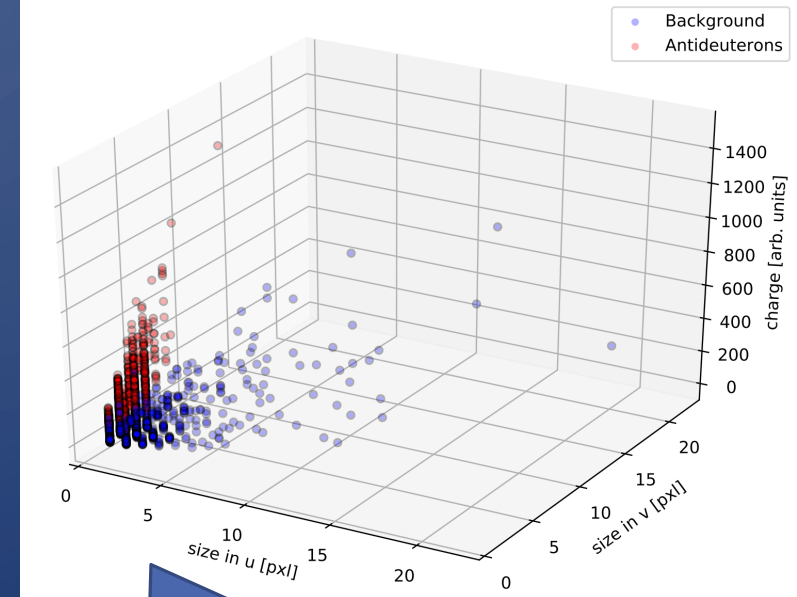
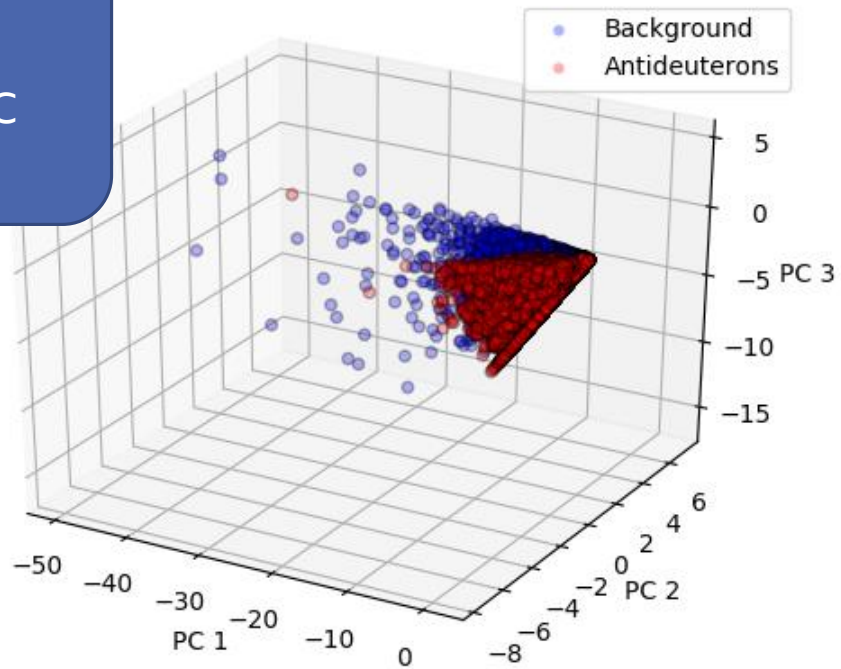
89.69 %



PROJECT 1

# MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

Plot of  
first 3 PC

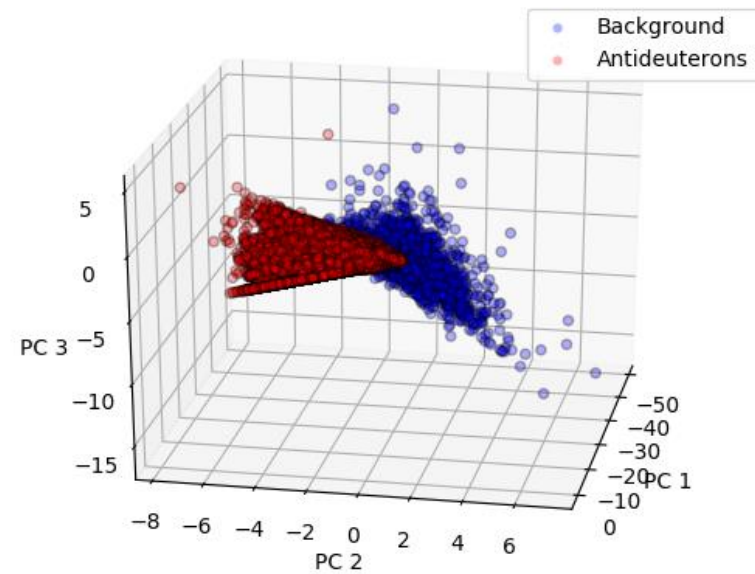
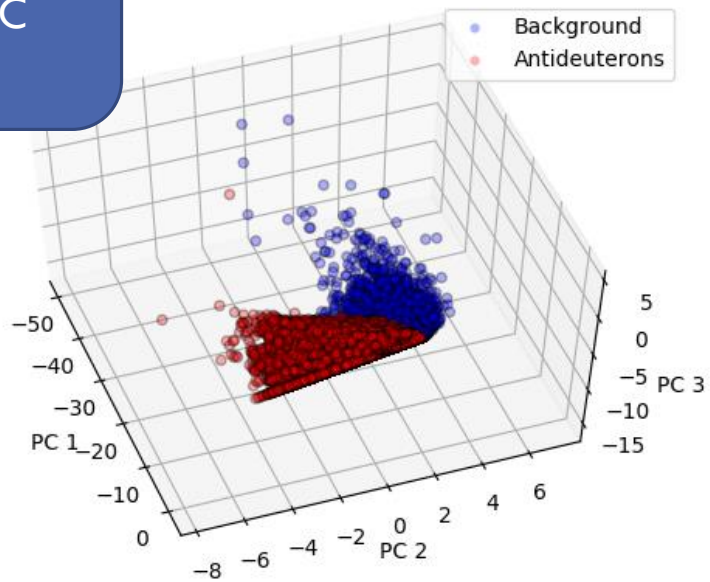


Original data set

PROJECT 1

# MULTIPARAMETER ANALYSIS OF ANTIDEUTERONS

Plot of  
first 3 PC



# BACHELOR'S THESIS - RESULTS

Dimension can be  
reduced to 3D

## PROJECT 2: DATA SEPARATION USING SOMS

Use reduced data set as  
input for SOMs.

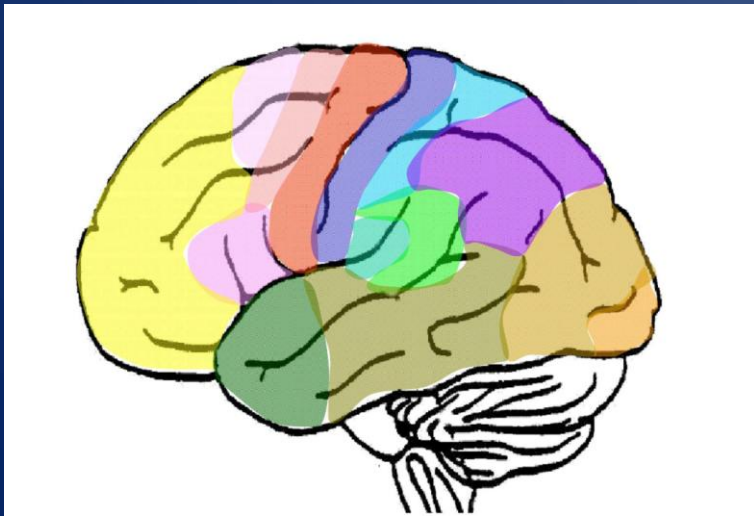
Does PCA-pre-  
transformation enhance  
SOM performance?

Try PCA &  
SOMs!

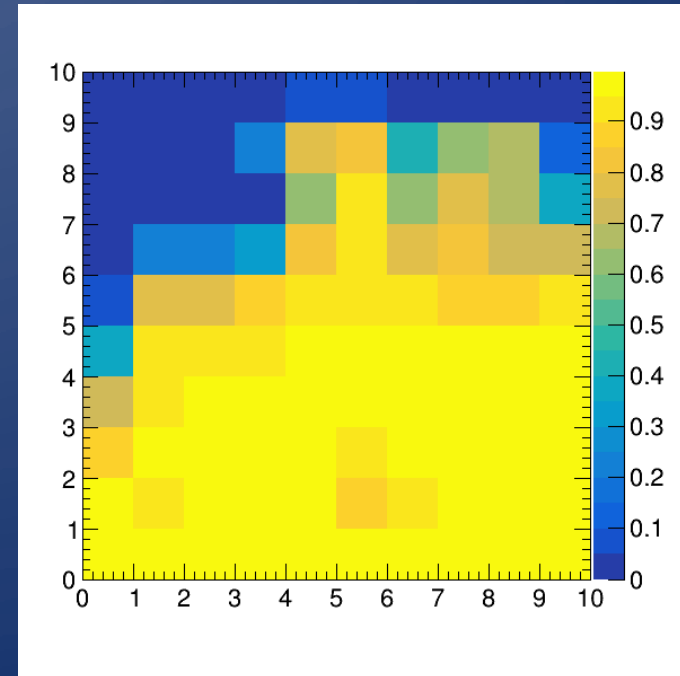
# PROJECT 2: DATA SEPARATION USING SOMS

Idea: Separating particle signals from beam background

Method: Self-organizing maps



R. Westermann,  
VL Neuroanatomie\*



Stephanie Käs

February 20, 2020

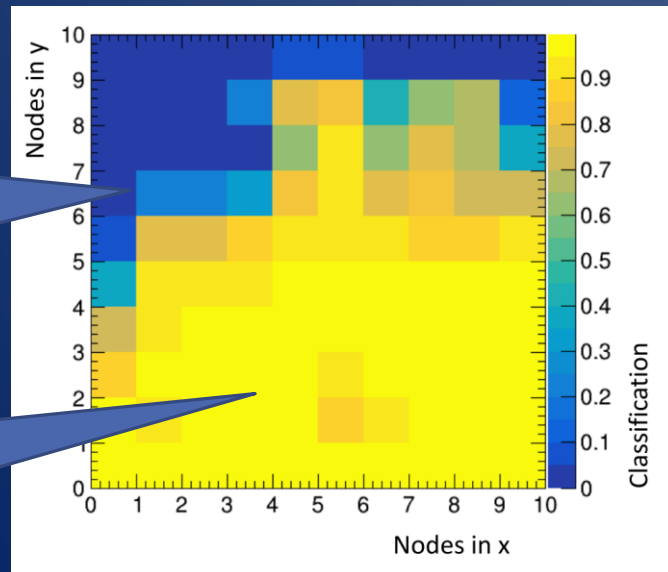
36

PROJECT 2

# DATA SEPARATION USING SOMS

## ANTIDEUTERONS

### ORIGINAL 6-DIM DATA SET



Background-  
like nodes

Antideuteron-  
like nodes

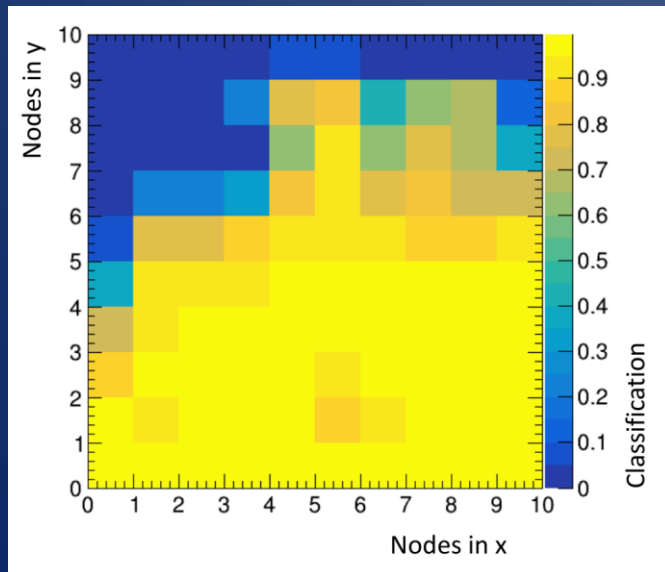
- Separation successful
- Results of Katharina's thesis confirmed

PROJECT 2

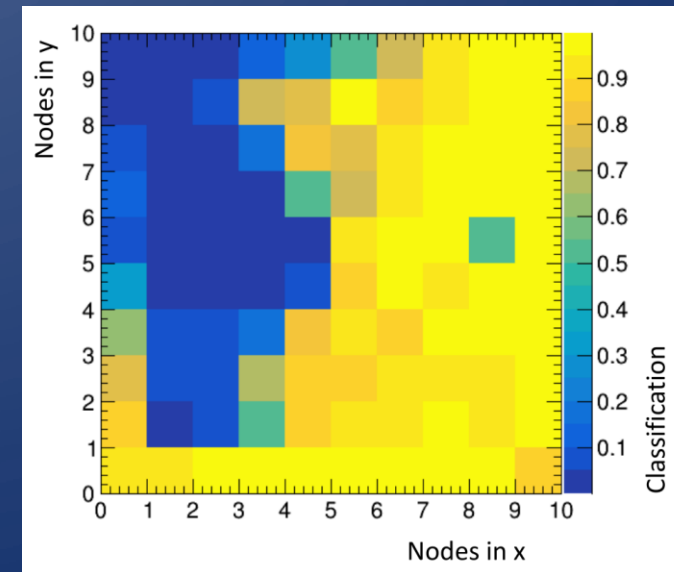
# DATA SEPARATION USING SOMS

## ANTIDEUTERONS

ORIGINAL 6-DIM DATA SET



PCA 6-DIM DATA SET



# BACHELOR'S THESIS - RESULTS

Dimension can be  
reduced to 3D

PCA does not  
enhance  
performance of  
SOMs



PROJECT 2

# DATA SEPARATION USING SOMs

ANTIDEUTERONS, TETRAQUARKS AND PIONS

Can we separate 3 particles from background?

SOMs

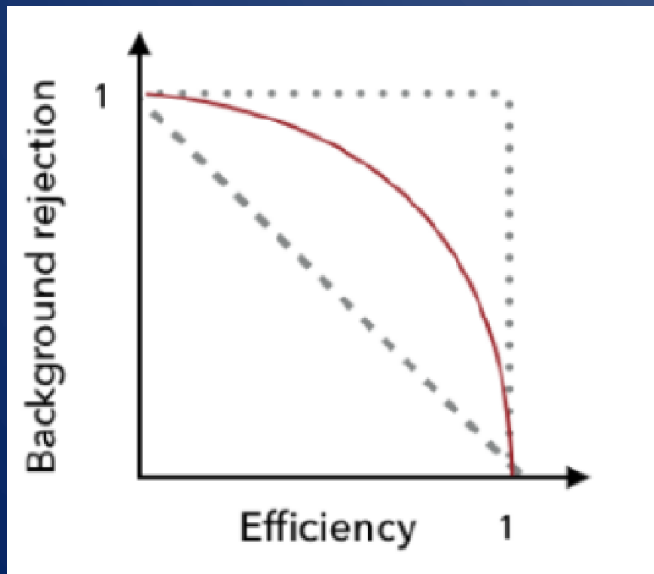
Antideuterons,  
Tetraquarks and  
Pions

PROJECT 2

# DATA SEPARATION USING SOMS

ANTIDEUTERONS, TETRAQUARKS AND PIONS

## ROC-CURVES



“RECEIVER OPERATING CHARACTERISTIC“

Signal efficiency: True positive rate

$$P(\text{classified as signal} \mid \text{signal})$$

Background rejection:

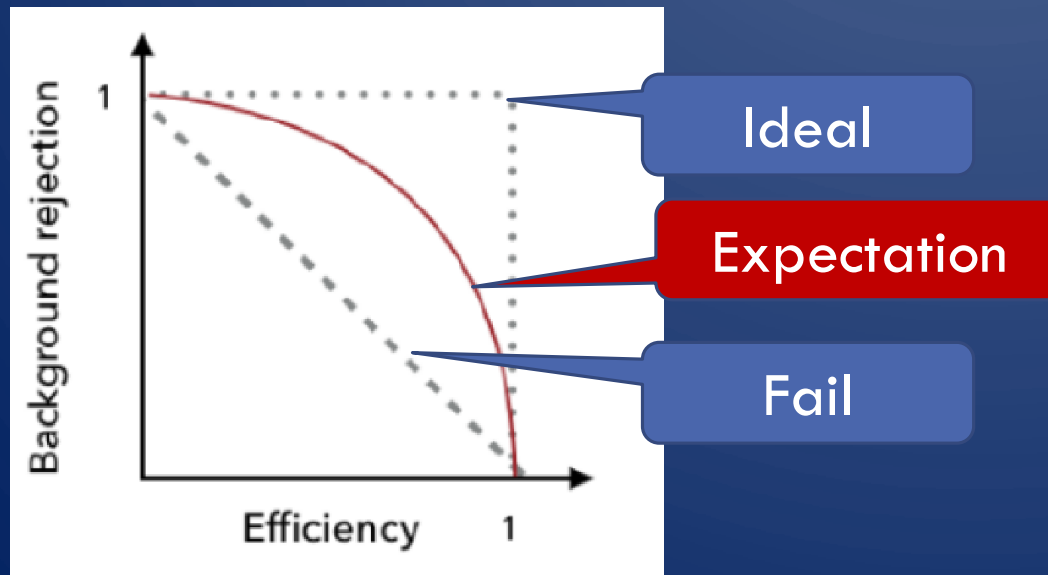
$$1 - P(\text{classified as background} \mid \text{background})$$

## PROJECT 2

# DATA SEPARATION USING SOMS

ANTIDEUTERONS, TETRAQUARKS AND PIONS

## ROC-CURVES

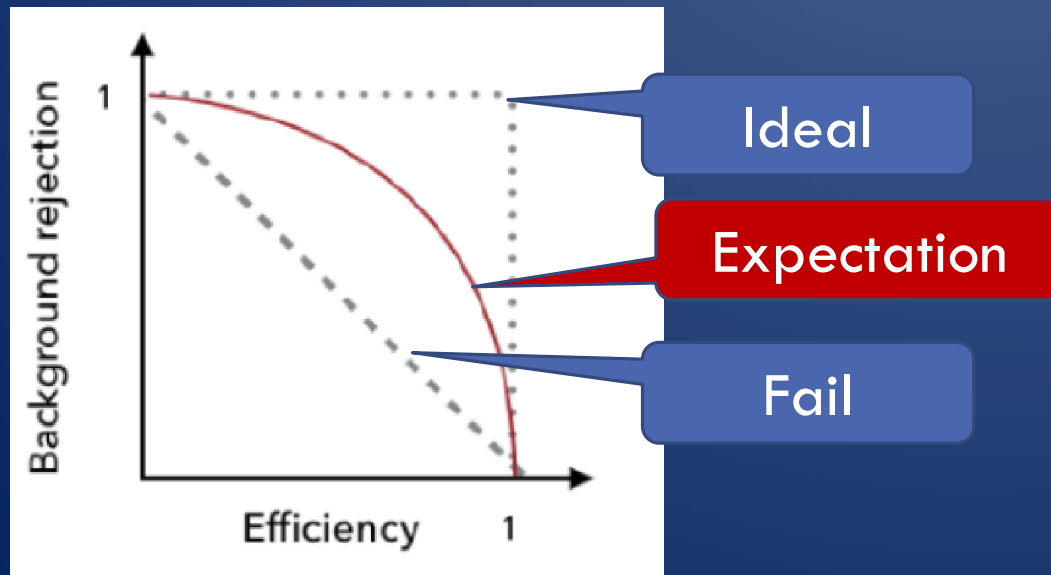


## PROJECT 2

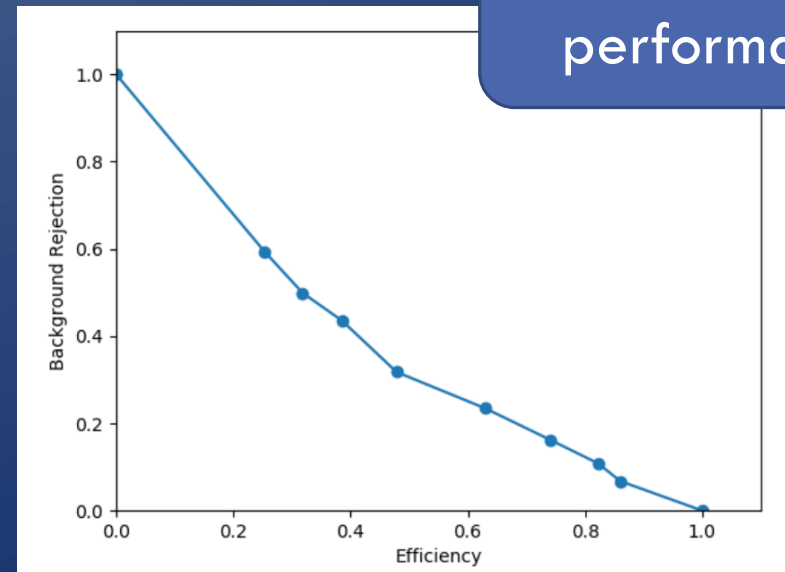
# DATA SEPARATION USING SOMS

ANTIDEUTERONS, TETRAQUARKS AND PIONS

### ROC-CURVES



### RESULT



# BACHELOR'S THESIS - RESULTS

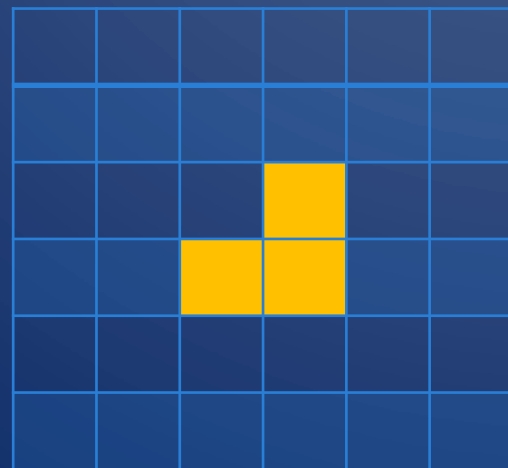
Dimension can be  
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PCA does not  
enhance  
performance of  
SOMs

SOMs cannot  
separate more than  
2 data types

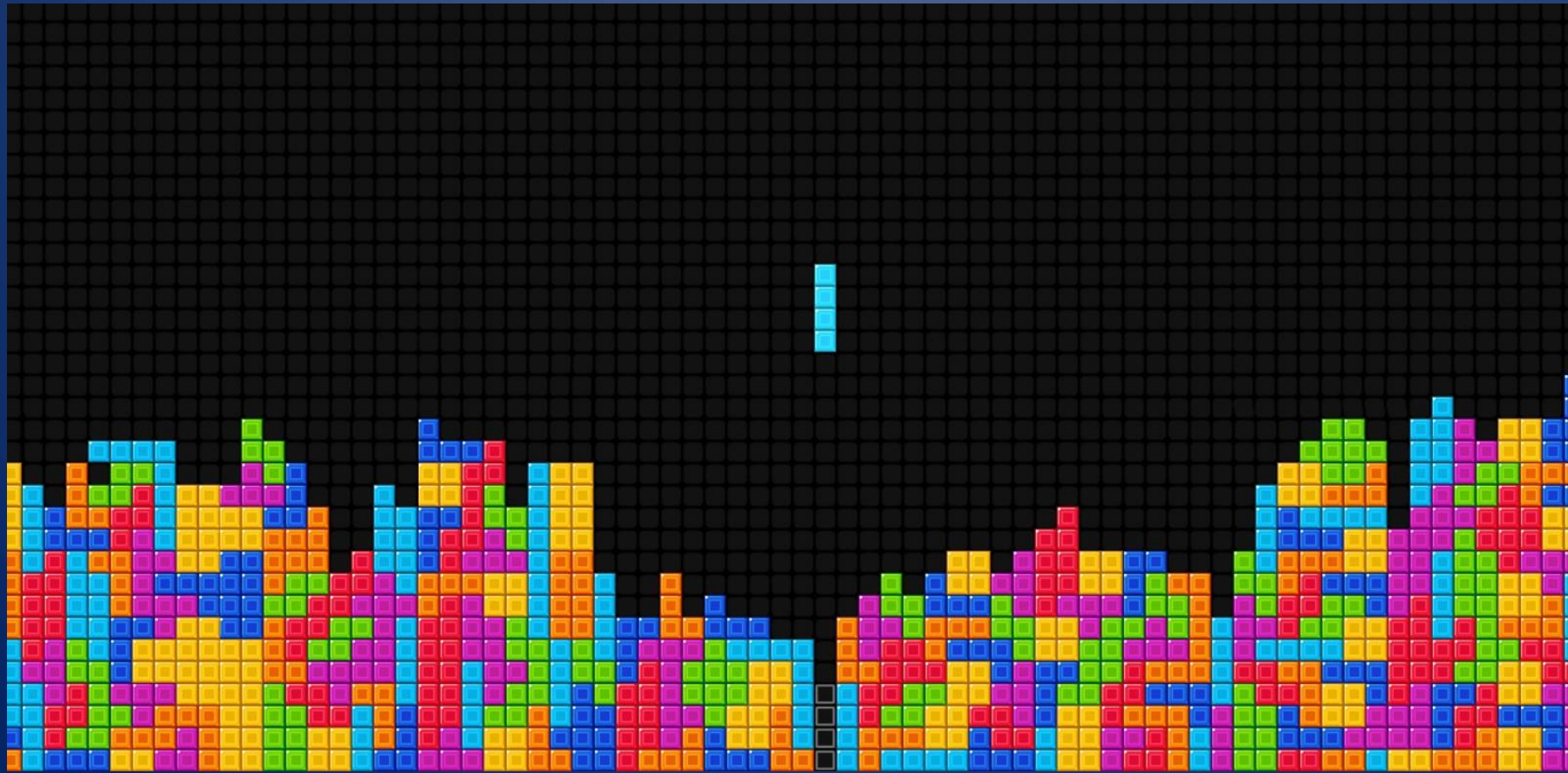
# PROJECT 3: CLUSTER SHAPE ANALYSIS

Which cluster shapes do appear?



PROJECT 3

# CLUSTER SHAPE ANALYSIS



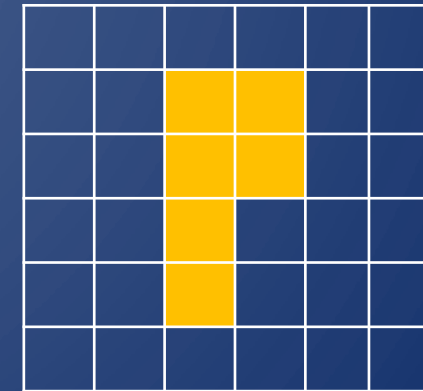
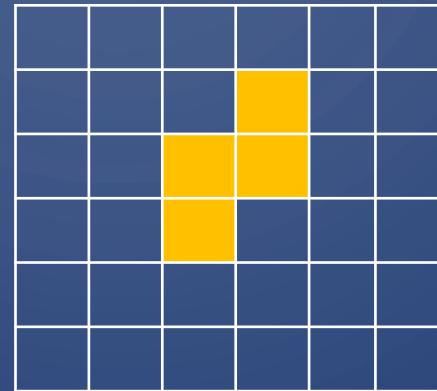
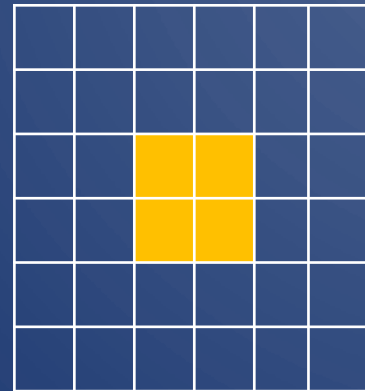
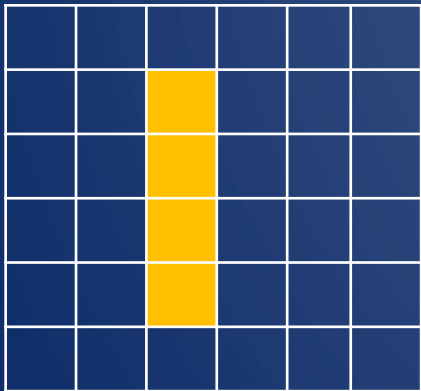
wallpaperaccess.com/retro-game

Stephanie Käs

February 20, 2020

## PROJECT 3

# CLUSTER SHAPE ANALYSIS



Ca. 40%  
narrow  
rectangular

Ca. 17%  
squares

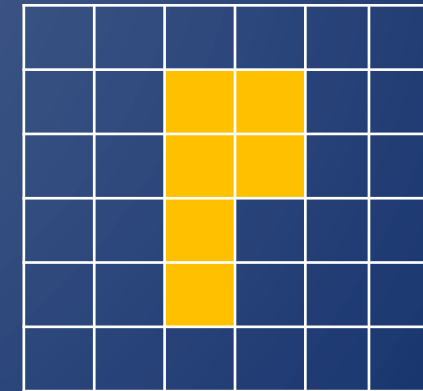
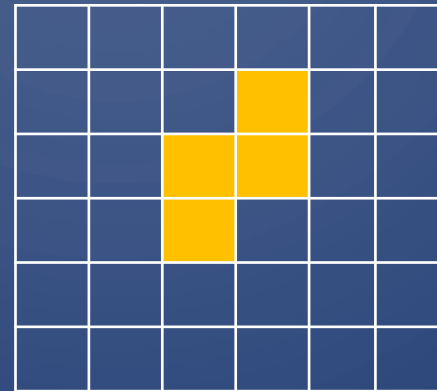
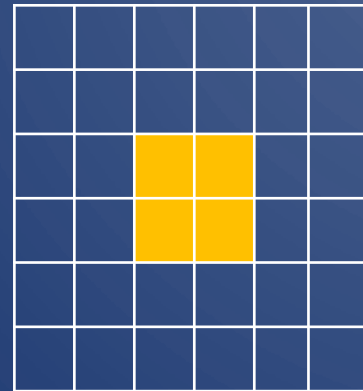
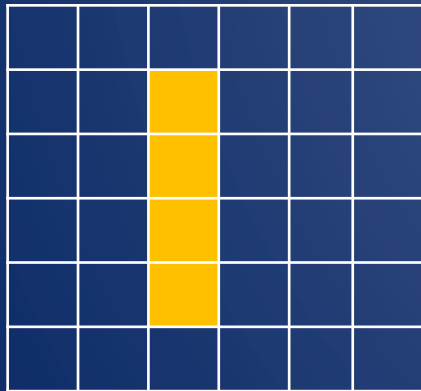
Ca. 1/3 2 pxl  
Ca. 1/7 1 pxl

Ca. 10% > 6  
pxl



PROJECT 3

# CLUSTER SHAPE ANALYSIS



Pattern recognition?

# BACHELOR'S THESIS - RESULTS

Dimension can be  
reduced to 3D

PCA does not  
enhance  
performance of  
SOMs

SOMs cannot  
separate more than  
2 data types

PXD clusters come in many different shapes

# POSSIBLE FUTURE PROJECTS

- Compare different versions of PCA and SOMs
- Use pattern recognition on cluster shapes
  - Try reinforcement learning?
- Try support vector machines

Thank you  
for your attention!



N-te



# PART 4

## DETAILS ON PCA



# PRINCIPAL COMPONENTS ANALYSIS

## BASIC STATISTICS

### Variance

$$\text{var}(\vec{x}) = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{x})^2$$

measure of  
spread of  $x_i$

# PRINCIPAL COMPONENTS ANALYSIS

## BASIC STATISTICS

### Variance

$$\text{var}(\vec{x}) = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{x})^2$$

### Empirical covariance

$$\text{cov}(\vec{x}, \vec{y}) = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{x})(y_i - \hat{y})$$

indication for  
correlations  
between  $\vec{x}$  and  $\vec{y}$

# PRINCIPAL COMPONENTS ANALYSIS

## DETAILS

Higher values of  $\lambda_i \leftrightarrow$  higher information content

$$\lambda_1 \geq \lambda_2 \dots \geq \lambda_n$$

Information percentage of axis  $i$  (cumulative sum):

$$\frac{\lambda_i}{\sum_{i=1}^n \lambda_i} \cdot 100\%$$



# PRINCIPAL COMPONENTS ANALYSIS

## CHOOSING NUMBER OF AXES

Criterion 1:

Choose only  $\lambda_i \geq 1$

Criterion 2:

$$\frac{\lambda_i}{\sum_{i=1}^n \lambda_i} \cdot 100\% \geq 90\%$$

Criterion 3:

Cut at first rapid change of slope in *Scree graph*.

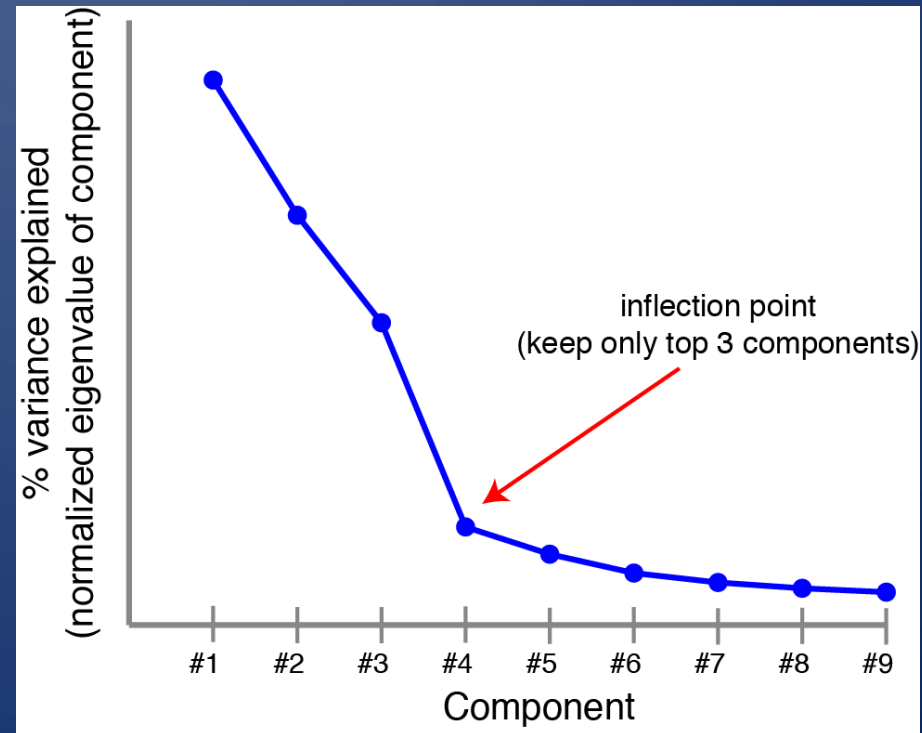
# PRINCIPAL COMPONENTS ANALYSIS

## CHOOSING NUMBER OF AXES

Criterion 3:

Cut at first rapid change of slope in *Scree graph*.

### Scree graph (example)



alexwilliams.info

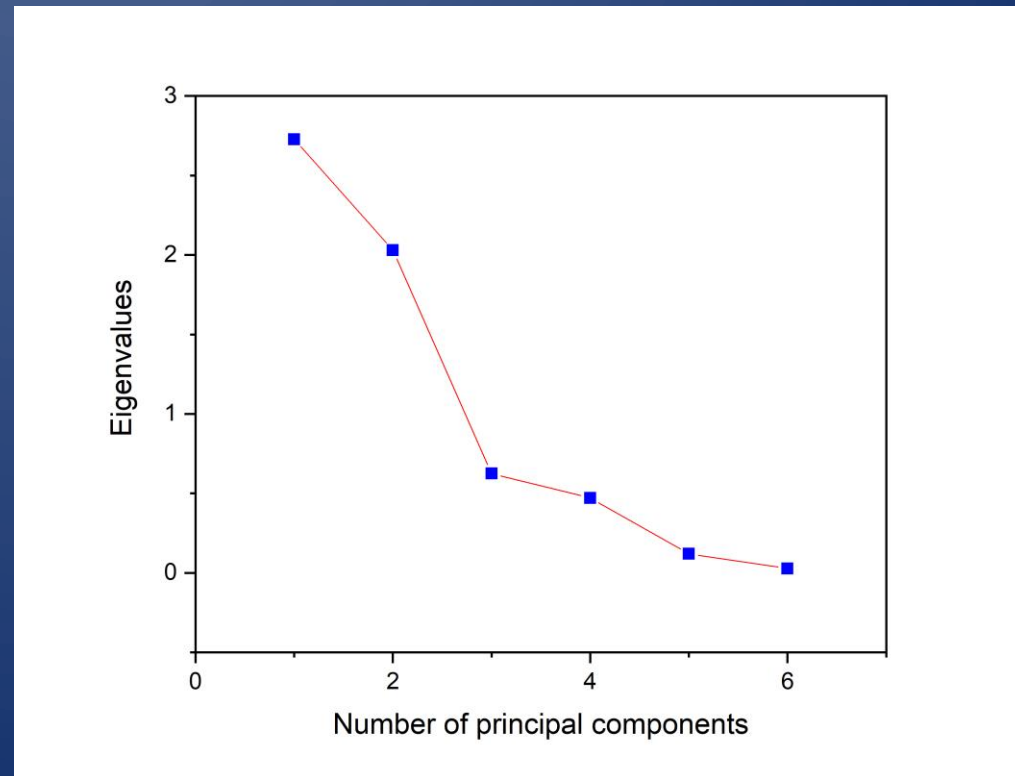
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## Scree graph (PXD)



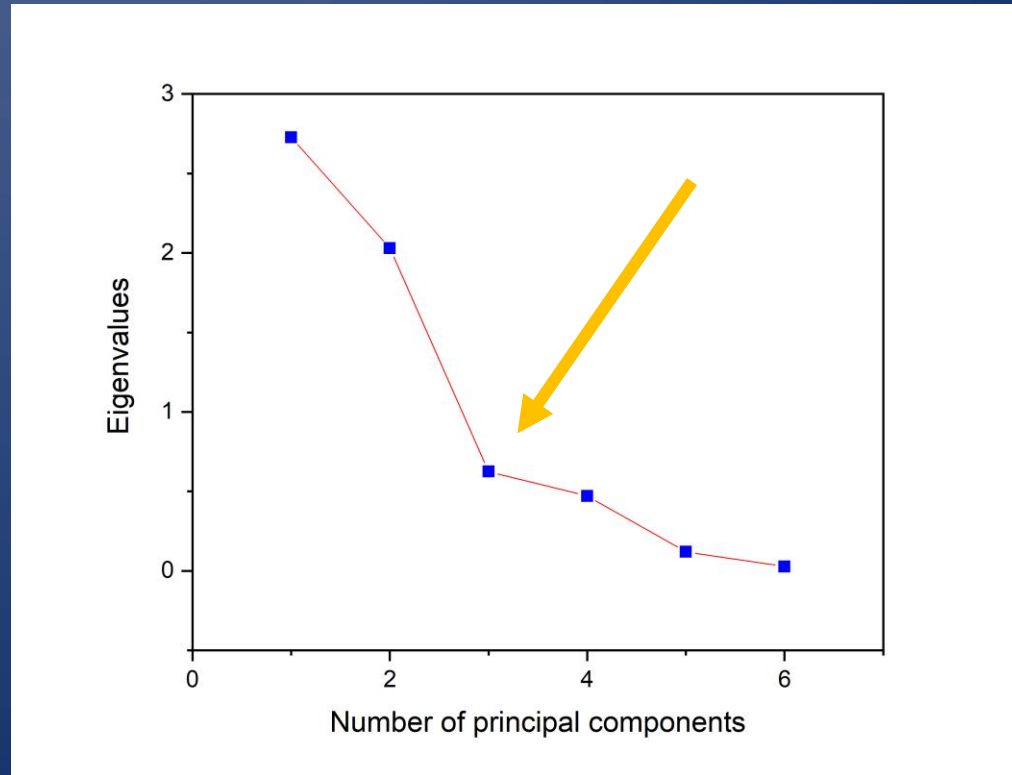
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0.03	100

## Scree graph



# PRINCIPAL COMPONENTS ANALYSIS

## IMPORTANT REMARKS

Pre-processing is a must:

- Normalisation is required
- PCA presumes linear correlations
- if non-linear:  
use *log* or *actan*

PCA is sensitive to outliers!

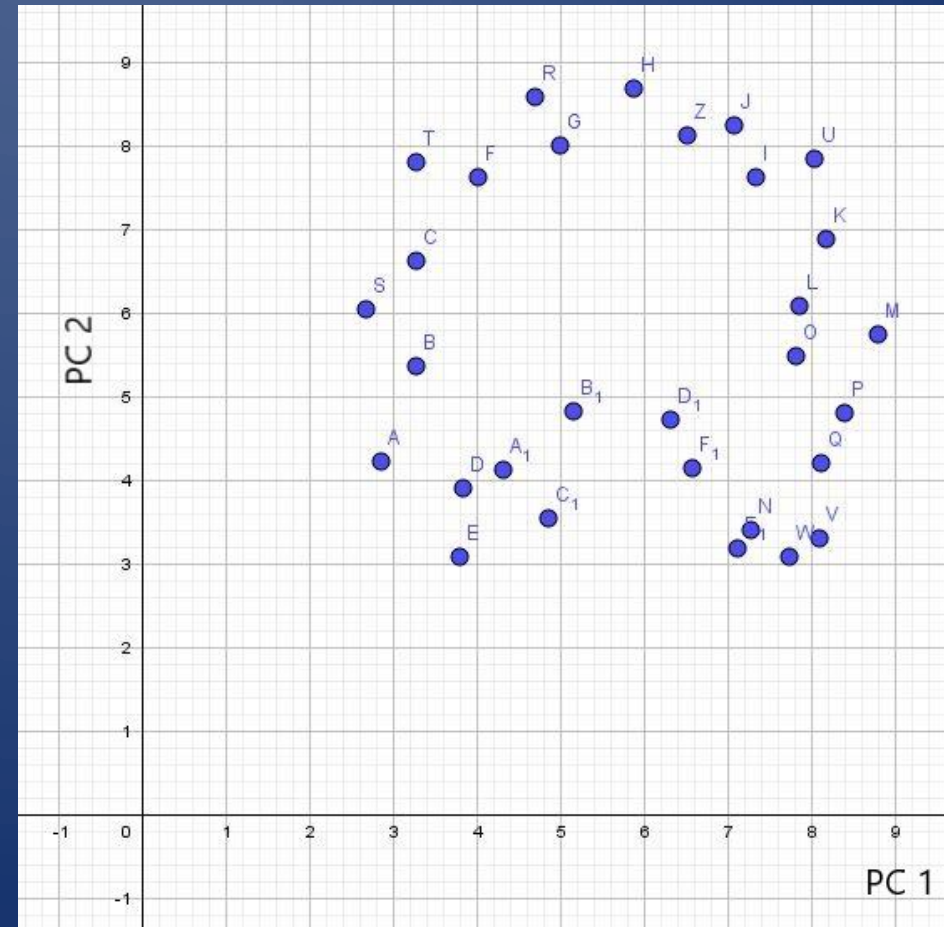
- Plot data set if possible

# PRINCIPAL COMPONENTS ANALYSIS

## IMPORTANT REMARKS

Horseshoe effect

Example of „failure“ of PCA



# PROJECT 3

## CLUSTER SHAPE ANALYSIS

