Analytic structures in $b \to s\ell\ell$ and CP violation in $B \to K\pi\pi$

Bastian Kubis

HISKP (Theorie), BCTP

Universität Bonn

RA2 Kick-off

Bonn, October 23, 2025

with essential input from Simon Mutke and Leon Heuser

Motivation: non-local form factors in $B \to K^{(*)}\ell\ell$

- **Flavor anomalies**: sizable deviations from SM predictions for decay rates and angular observables (e.g. P_2, P_5') in $b \to s\mu\mu$ decays (e.g. $B^0 \to K^{*0}\mu^+\mu^-$)
- Hadronic matrix element for $B \to K^{(*)}\ell\ell$ in Weak Effective Theory

$$\mathcal{A}(B \to K^{(*)}\ell\ell) \sim \mathcal{N}\left[(C_9 L_V^\mu + C_{10} L_A^\mu)\boldsymbol{\mathcal{F}_\mu} - \frac{L_V^\mu}{q^2}(C_7 \boldsymbol{\mathcal{F}_{T,\mu}} + \boldsymbol{\mathcal{H}_\mu})\right]$$

conventions from Gubernari, van Dyk, Virto 2021

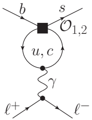
- **Non-local FFs $\mathcal{H}_\mu$**

$\hookrightarrow$ still with large uncertainties

$\hookrightarrow$ calculated with Operator Product Expansion, QCD factorization, ...

$\hookrightarrow$ extrapolated **analytically** from space-like to semileptonic region

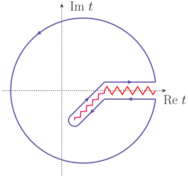
$\hookrightarrow$ need good understanding of the **analytic structure**



Analytic structure of hadronic form factors

- **Singularities in q^2**
  - **Poles**: (infinitely) narrow bound states (J/ψ, $\psi(2S)$)
  - **Thresholds**: $\gamma^* \to \{\pi^+\pi^-, D\bar{D}, \ldots\}$ cuts
  - **Anomalous thresholds**: e.g., triangle diagrams

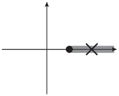
    $\hookrightarrow$ need to modify dispersion relation



- Three cases: (all three can occur Mutke, Hoferichter, BK 2024)

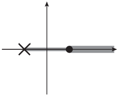
(1) $t_{\rm anom}$ on normal cut

$\hookrightarrow$ analytic continuation of normal discontinuity



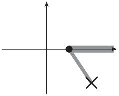
(2) $t_{\rm anom}$ on negative real axis

$\hookrightarrow$ integration deformed along real axis



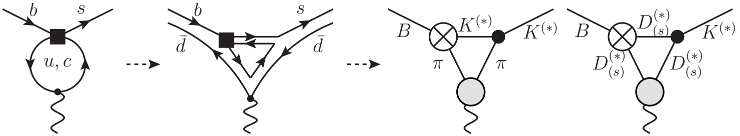
(3) $t_{\rm anom}$ in complex plane

$\hookrightarrow$ integration deformed into complex plane



Triangle loops in non-local $B \to K^{(*)}\gamma^*$ form factors

- **Triangle loop** contributions to non-local FFs:



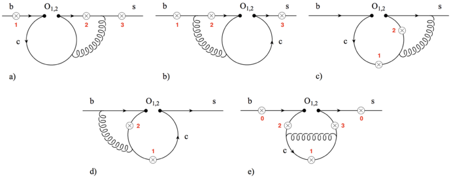
- Started with ***u*-quark loop** and **$\pi\pi$ intermediate states**:
  - CKM-suppressed $\sim \lambda^4$ compared to ***c*-quark loop** $\sim \lambda^2$
  - Input (form factors, branching ratios, polarization fractions ...) well known
  - Sizable energy gap to next state $\pi\omega$

    $\hookrightarrow$ cf. various $D^{(*)}_{(s)}\bar{D}^{(*)}_{(s)}$ for hadronization of charm loop within close proximity
- How important are **anom. contributions**? $\to$ anom. fraction $|\Pi^{\text{anom}}(t)/\Pi^{\text{norm}}(t)|$
- (Almost) all parameters fixed from **data**!
- **Anomalous contributions** can be $\gtrsim$ **10%** away from thresholds, resonances

Mutke, Hoferichter, BK 2024

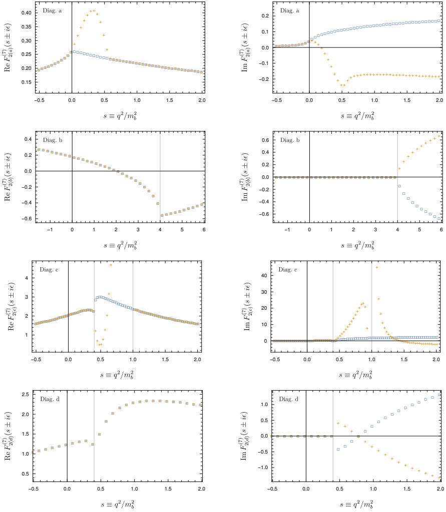
Comparison: analytic structures in $b \to s\ell\ell$ at two-loop

- What about the **quark level**?
- Consider $b \to s\ell\ell$ at two-loop order



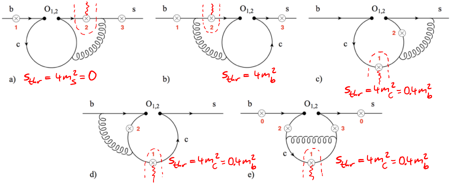
Asatrian, Greub, Virto 2019

- Leads to interesting analytic structures

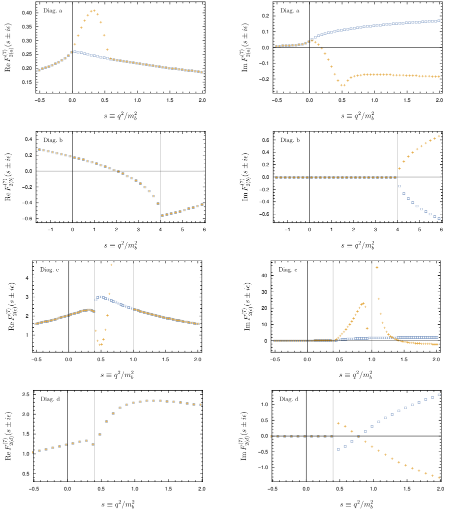


Asatrian, Greub, Virto 2019

- What about the **quark level**?
- Consider $b \to s\ell\ell$ at two-loop order



Asatrian, Greub, Virto 2019



Asatrian, Greub, Virto 2019

- Leads to interesting analytic structures

  ↪ usual **two-particle thresholds**

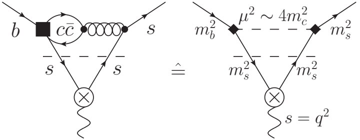
  ↪ what else—anomalous thresholds?

Dispersive representation for $F^{(7)}_{2,(a)}$

- Check **analytic structures** using **dispersion relation**

$$F^{(7)}_{2,(a)}(s) = F^{(7)}_{2,(a)}(s_0) + \frac{s - s_0}{2\pi i} \int_{s_{\text{thr}}=0}^{\infty} \mathrm{d}s' \, \frac{\text{disc}\, F^{(7)}_{2,(a)}(s')}{(s' - s_0)(s' - s)}$$

- Discontinuity determined by two-particle cut



↪ known discontinuity from scalar **triangle diagram** (with $\mu^2 = 4m_c^2$)

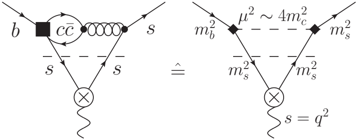
↪ adjust prefactors for correct threshold behavior

Dispersive representation for $F^{(7)}_{2,(a)}$

- Check **analytic structures** using **dispersion relation**

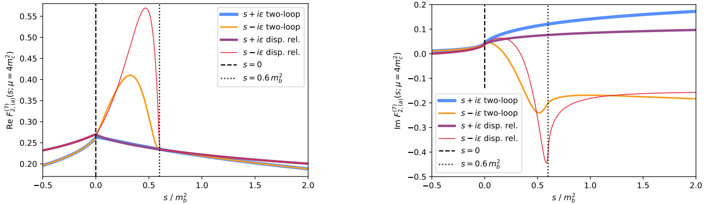
$$F^{(7)}_{2,(a)}(s) = F^{(7)}_{2,(a)}(s_0) + \frac{s-s_0}{2\pi\mathrm{i}} \int_{s_{\rm thr}=0}^{\infty} \mathrm{d}s' \, \frac{\mathrm{disc}\, F^{(7)}_{2,(a)}(s')}{(s'-s_0)(s'-s)}$$

- Discontinuity determined by two-particle cut



$\hookrightarrow$ known discontinuity from scalar **triangle diagram** (with $\mu^2 = 4m_c^2$)

$\hookrightarrow$ adjust prefactors for correct threshold behavior



$\hookrightarrow$ can be improved using **spectral density function**

Mutke et al., in progress

Analytic structures in $b \to s\ell\ell$: upshot, to do

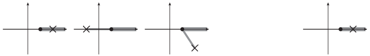
- **Analytic structure** of non-local FFs richer than previously thought

  $\hookrightarrow$ **anomalous thresholds**!

  $\hookrightarrow$ essential for detailed understanding both on **hadronic** and **quark level**

- Subtle, non-trivial mass dependence:

hadronic level	**quark level**



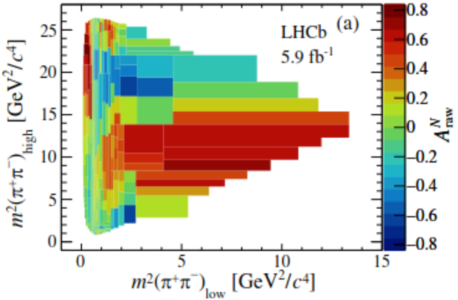
  $\hookrightarrow$ already normal thresholds differ, e.g., $4m_c^2 \leftrightarrow 4M_D^2$

- **Open questions**:
  - Can we quantify anomalous contributions to charm loop?
  - How to match hadronic and quark picture?
  - What are the implications for non-local FF predictions?

discussions and collaboration with Gubernari, Reboud, van Dyk, Virto...

Motivation: CP violation in $B \to K\pi\pi$

- **LHCb**: CP asymmetries in $B^\pm \to h_1^+ h_2^- h_3^\pm$ ($h_i \in \{\pi, K\}$) 5.9 fb^{-1} integrated luminosity
- Huge CPV locally in Dalitz plots
  $\hookrightarrow$ role of **resonances**

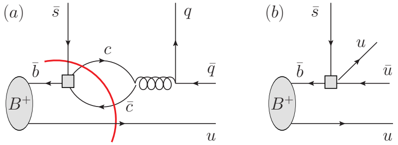


LHCb 2022

- **Goal**: disentangle strong and weak phases
  $\hookrightarrow$ describe final-state interactions **dispersively**
- First step: $B^\pm \to K^\pm\pi^+\pi^-$ at **small $\pi^+\pi^-$ invariant masses** ($\leq 1\ \mathrm{GeV}^2$)

Heuser, Reyes-Torrecilla, Hanhart, BK, Magalhães, Mannel, Peláez 2025

Formalism



- **Eff. weak Hamiltonian** $H_{\text{eff}} = \frac{G_F}{\sqrt{2}}\Big(|V_{cb}^* V_{cs}|(\bar{b}c)(\bar{c}s) + e^{i\gamma}|V_{ub}^* V_{us}|(\bar{b}u)(\bar{u}s)\Big)$

  $\hookrightarrow$ parametrization of short-distance source

$$\bar{\mathcal{A}}_i^{\pm} = \hat{A}_i + e^{\pm i\gamma}\hat{B}_i = a_i + ic_i \pm ib_i$$

- Final-state rescatt.: **Omnès** functions $\Omega_i(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}s'}{s'}\frac{\delta_i(s')}{s'-s}\right\}$

  (**spectator approximation**) $\hookrightarrow$ full amplitude

$$\mathcal{A}^{\pm}(s,t) = \sum_i f_i(s,t)P_i(s)\,\Omega_i(s)\,\bar{\mathcal{A}}_i^{\pm}$$

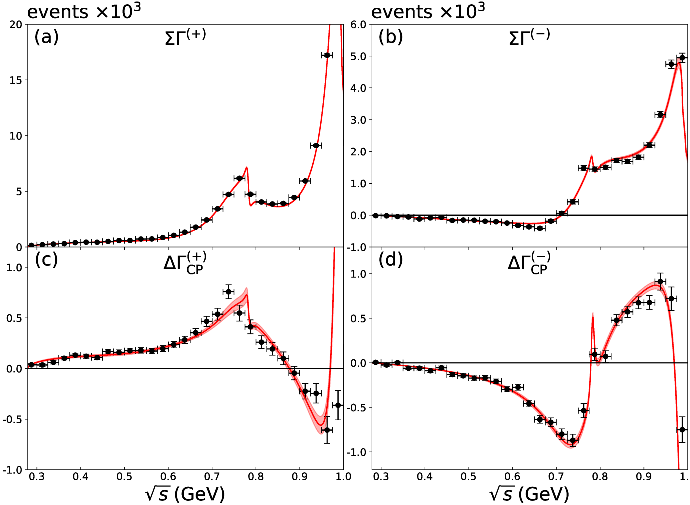
partial waves: $\pi\pi$ $I = 0, 2$ S-waves, $I = 1$ P-wave (+ mixing with ω)

Fits to CP-even/-odd angular projections

angle-symmetric | angle-asymmetric

CP-even

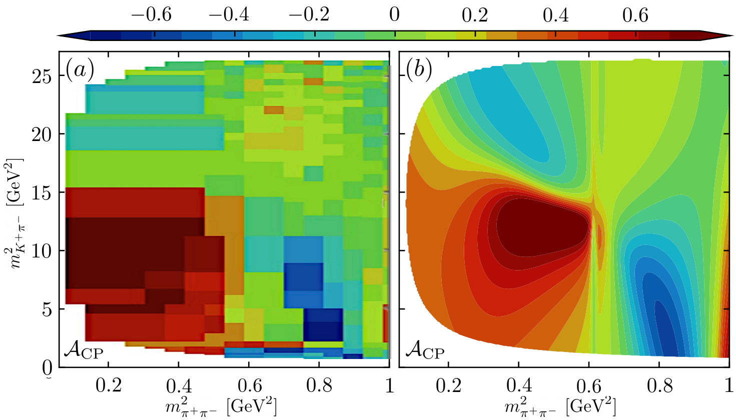
CP-odd



Heuser, Reyes-Torrecilla, Hanhart, BK, Magalhães, Mannel, Peláez 2025

Result: CP asymmetry over Dalitz plot

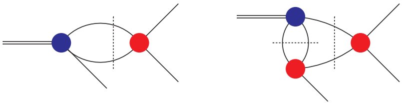
- reconstructed from partial waves (*not* fitted!)



LHCb 2022 vs. Heuser, Reyes-Torrecilla, Hanhart, BK, Magalhães, Mannel, Peláez 2025

CP violation in $B \to K\pi\pi$: upshot; to do

- **CP asymmetries in Dalitz plots** can be understood in terms of $\pi\pi$ rescattering
- Successful **CmF collaboration**! Heuser, Reyes-Torrecilla, Hanhart, BK, Magalhães, Mannel, Peláez 2025
- Generalize
  - to other Dalitz plot corners: small $K\pi$ invariant masses
  - to other final states $B^\pm \to \pi^\pm\pi^+\pi^-$ work in progress
  - to coupled channels $B \to \pi^\pm(\pi^+\pi^- \leftrightarrow \bar{K}K)$
- **Open questions:**
  - Beyond the **spectator approximation**: Khuri–Treiman formalism ($\leftrightarrow$ RA1)



preliminary work by Thanos Kotarelas & Miriam Penners

  - Elucidate role of **charm loops** $\to$ Christoph Hanhart
  - Understand matching to **short-distance operators**