Lattice: exclusive semileptonic decays, lifetimes and mixing for heavy mesons

Oliver Witzel

Universität Siegen

TP1 Theoretical Particle Physics

CPPS Center for Particle Physics Siegen

RA2 kick-off meeting

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color meets flavor

Example: $B_s \to D_s^* \ell\nu$

in collaboration with

Matthew Black, Anastasia Boushmelev

[PoS LATTICE2024 (2025) 251]

(RBC-UKQCD collaboration)

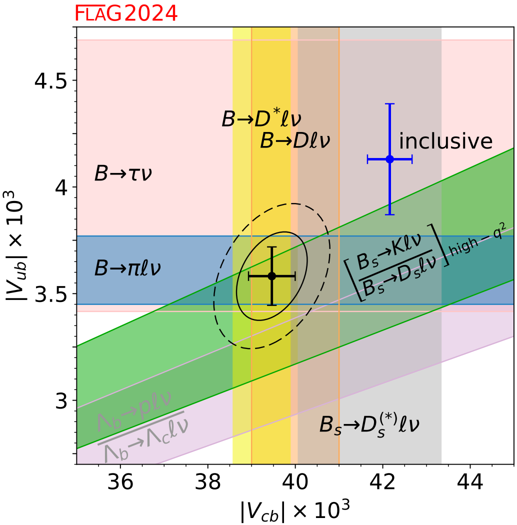
How to determine V_{cb}?

- Leptonic $B_c \to \tau\nu_\tau$ decays
  - $\to$ Experimentally very challenging
- Semileptonic decays with B or B_s initial state
  - $\to$ Inclusive decays $\rightsquigarrow$ Marko Garofalo

    Progress toward first lattice determination
  - $\to$ Exclusive decays
    - hadronic pseudoscalar final state
    - hadronic vector final state

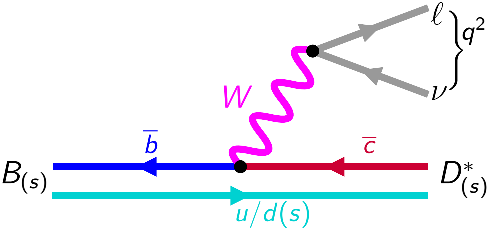
    $B \to D^*\ell\nu$ experimentally preferred
    (BaBar, Belle, Belle II, LHCb)

    $B_s \to D_s^*\ell\nu$ easier for lattice: no chiral extrapolation
    (LHCb)
- Long standing $2-3\sigma$ discrepancy between inclusive and exclusive



[FLAG 2024 arXiv:2411.04268]

Exclusive semi-leptonic decays on the lattice



$$q^2 = M_B^2 + M_{D^*_{(s)}}^2 - 2M_B E_{D^*_{(s)}}$$

- Treat $D^*_{(s)}$ as QCD-stable particle (narrow-width approximation)
- Conventionally parametrized placing the $B_{(s)}$ meson at rest in terms of

$$\frac{d\Gamma(B \to D^*_{(s)}\ell\nu)}{dq^2} = \mathcal{K}_{D^*}(q^2, m_\ell) \cdot |\mathcal{F}(q^2)|^2 \cdot |V_{cb}|^2$$

experiment known theory input (nonperturbative) CKM

Four form factors parametrize $B_{(s)} \to D^*_{(s)}\ell\nu$

- Determine the four form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$ (in relativistic notation) by calculating hadronic matrix elements

$$\langle D^*_{(s)}(k,\varepsilon_\nu)|\mathcal{V}^\mu|B_{(s)}(p)\rangle = V(q^2)\frac{2i\varepsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma}{M_{B_{(s)}}+M_{D^*_{(s)}}}$$

$$\begin{aligned}\langle D^*_{(s)}(k,\varepsilon_\nu)|\mathcal{A}^\mu|B_{(s)}(p)\rangle =& A_0(q^2)\frac{2M_{D^*_{(s)}}\varepsilon^*\cdot q}{q^2}q^\mu \\ &+ A_1(q^2)(M_{B_{(s)}}+M_{D^*_{(s)}})\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu\right] \\ &- A_2(q^2)\frac{\varepsilon^*\cdot q}{M_{B_{(s)}}+M_{D^*_{(s)}}}\left[k^\mu+p^\mu-\frac{M^2_{B_{(s)}}-M^2_{D^*_{(s)}}}{q^2}q^\mu\right]\end{aligned}$$

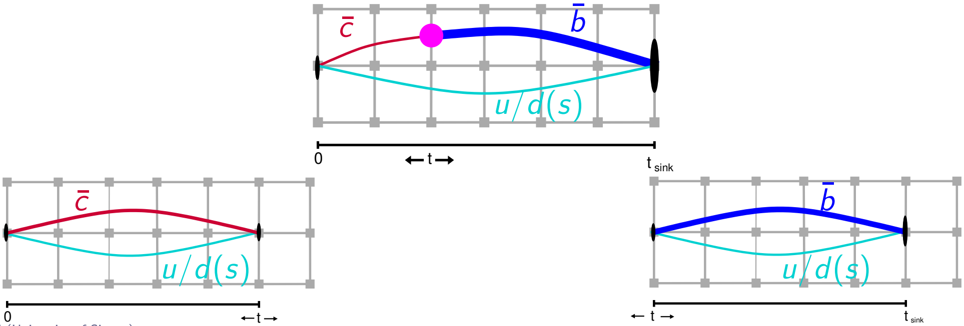
- Alternatively use HQE convention $f(w)$, $g(w)$, $\mathcal{F}_1(w)$, $\mathcal{F}_2(w)$ with $w = v_{D^*}\cdot v_B$

Lattice determination of $B_{(s)} \to D^*_{(s)}\ell\nu$ form factors

- Calculate ratios of 3-point over 2-point functions on the lattice

$$R^{\Gamma,\mu}_{B_{(s)}\to D^*_{(s)}}(t, t_{sink}) = \frac{C^{3pt,\Gamma,\mu}_{B_{(s)}\to D^*_{(s)}}(t, t_{sink}, k)}{\frac{1}{3}\sqrt{\sum_i C^{2pt}_{D^*_{(s)}}(t,k) C^{2pt}_{B_{(s)}}(t_{sink}-t,p)}} \sqrt{\frac{4E_{D^*_{(s)}} M_{B_{(s)}} \sum_j \varepsilon_j(k)\varepsilon^{*j}(k)}{e^{-E_{D^*_{(s)}}} e^{-M_{B_{(s)}}(t_{sink}-t)}}}$$

$$\xrightarrow[t_{sink}-t\to\infty]{t\to\infty} \varepsilon^\mu(k)\langle D^*_{(s)}(k,\varepsilon)|\bar{c}\Gamma b|B_{(s)}(p)\rangle$$



Obtain phenomenological $B_{(s)} \to D^*_{(s)}\ell\nu$ form factors

- Relate ratios to lattice form factor: example $\tilde{A}_0$

$$\tilde{A}_0(q^2) = \frac{1}{2}\frac{M_{D^*_{(s)}}}{E_{D^*_{(s)}} M_{B_{(s)}}}\frac{1}{k^\nu} q_\mu \cdot \varepsilon^\nu(k)\langle D^*_{(s)}(k,\varepsilon)|\overline{c}\gamma_\mu\gamma_5 b|B_{(s)}(p)\rangle$$

- Lattice form factors require renormalization

$$A_0(q^2) = Z_A^{bc} \cdot \tilde{A}_0(q^2)$$

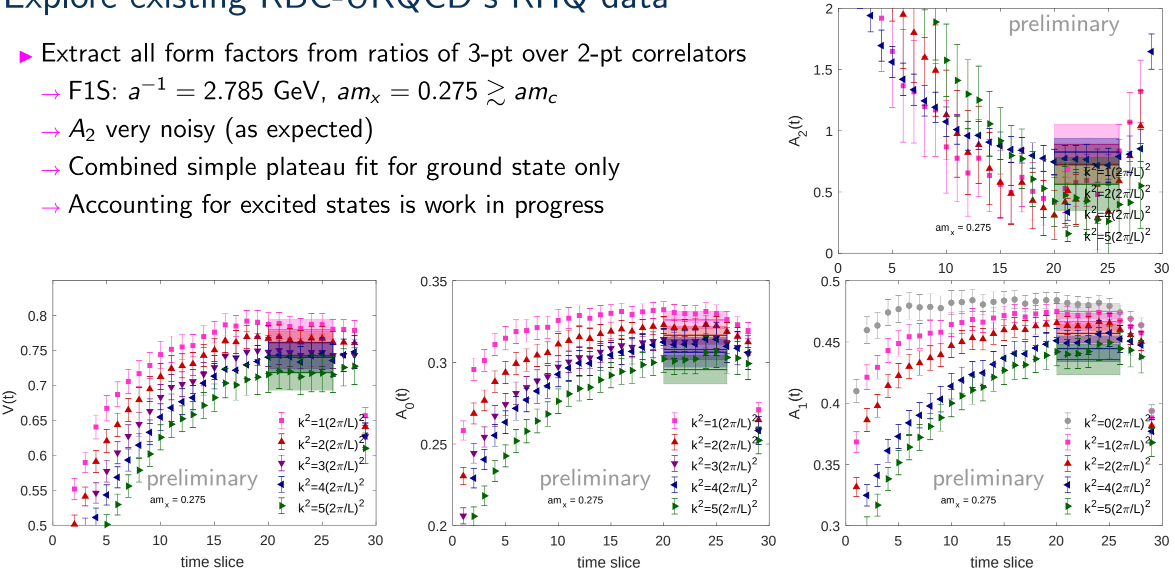
- Obtain Z_A^{bc} and Z_V^{bc} using a mostly nonperturbative setup
[Hashimoto et al. PRD61(1999)014502] [El-Khadra et al. PRD64(2001)014502]

$$Z^{bc} = \varrho^{bc}\sqrt{Z^{bb}Z^{cc}}$$

- Blinded analysis by replacing $\varrho^{bc} \to c \cdot \varrho^{bc}$ and c is hidden to the persons analyzing the data

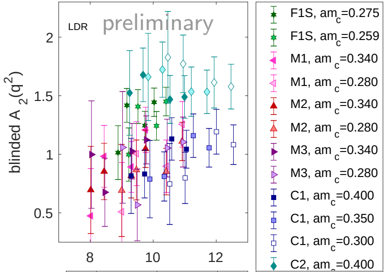
Explore existing RBC-UKQCD's RHQ data

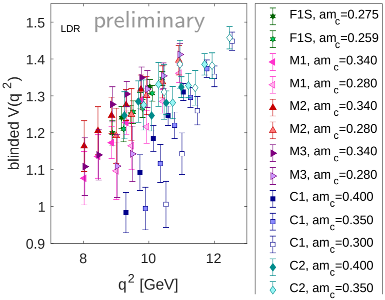
- Extract all form factors from ratios of 3-pt over 2-pt correlators
  - → F1S: $a^{-1} = 2.785$ GeV, $am_x = 0.275 \gtrsim am_c$
  - → A_2 very noisy (as expected)
  - → Combined simple plateau fit for ground state only
  - → Accounting for excited states is work in progress

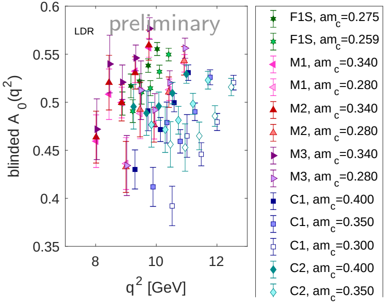


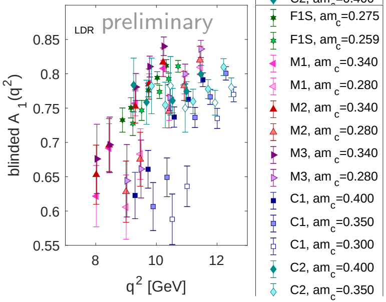
Explore existing RBC-UKQCD's RHQ data

- Extract all form factors from ratios of 3-pt over 2-pt correlators
- Check for improvement using lattice dispersion relation (LDR)
- Manage data from all ensembles and multiple charm masses
  - → Directly covered range $8\,\text{GeV}^2 \lesssim q^2 \lesssim q^2_{\text{max}}$
- Two independent analysis for all steps









Heavy Meson Lifetimes

in collaboration with

Matthew Black, Robert Harlander, Jonas Kohnen, Fabian Lange, Antonio Rago, Andrea Shindler

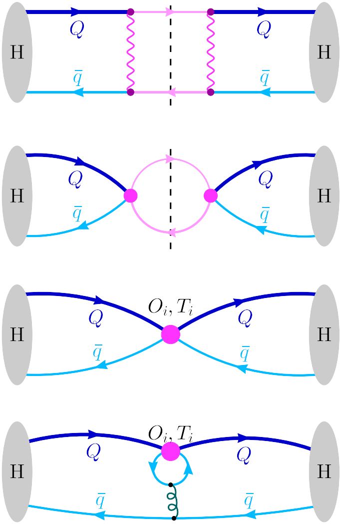
[PoS LATTICE2024 (2025) 243] [PoS LATTICE2023 (2024) 263]

Collaborative Research Center TRR 257

Particle Physics Phenomenology after the Higgs Discovery

Heavy meson lifetimes ($\Delta Q = 0$ operators)

- Heavy quark expansion (HQE): lifetimes of heavy mesons described by 4-quark operators with $\Delta B = 0$
  - → Optical theorem: calculate imaginary part
  - → Integrate out W boson, double insertion $\Delta Q = 1$ eff. weak Hamiltonian
- Operators O_1, O_2, T_1, T_2, contribute
- $\Delta Q = 0$ operators mix under renormalization
  - → To date no complete LQCD determination (only exploratory work 20+ years ago)
  - → Mixing suppressed using GF
- Quark-line disconnected contributions
  - → Notoriously noisy, expensive to calculate on the lattice
  - → In principal we know what to do . . .



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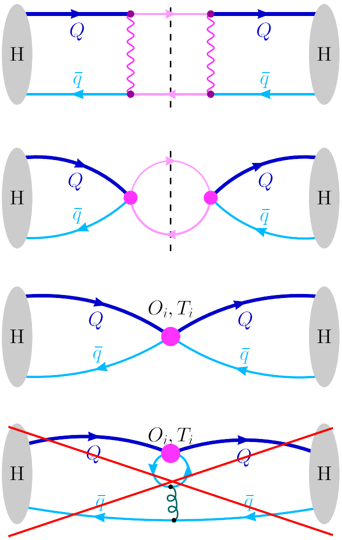
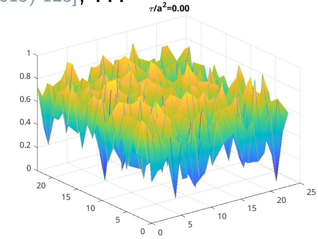


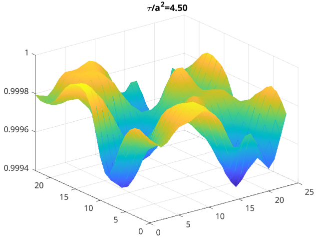
Gradient flow (GF)

- Standard tool for calculating scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
  [Narayanan, Neuberger JHEP 03 (2006) 064] [Lüscher JHEP 08 (2010) 071][JHEP 04 (2013) 123], . . .
- Introduce auxiliary dimension, flow time τ to regularize UV
  - $\rightarrow$ Well-defined smearing of gauge and fermion fields
  - $\rightarrow$ Smoothening UV fluctuations
- First order differential equation

$$\partial_t B_\mu(\tau,x) = \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau,x), \quad B_\mu(0,x) = A_\mu(x)$$
$$\partial_t \chi(\tau,x) = \mathcal{D}^2(\tau)\chi(\tau,x), \quad \chi(0,x) = q(x)$$

- Consider GF as an RG transformation
  [Carosso et al. PRL 121 (2018) 201601] [Hasenfratz et al. PoS Lattice 2021 155]
  [Harlander, Lange, Neumann JHEP 08 (2020) 109]
- Match to $\overline{\text{MS}}$ scheme using short flow-time expansion (SFTX)
  [Lüscher, Weisz JHEP 02 (2011) 051] [Suzuki PTEP 2013 (2013) 083B03]
  [Lüscher PoS Lattice 2013 016] [Makino, Suzuki PTEP (2014) 063B02] . . .





Short flow-time expansion (SFTX)

- Re-express effective Hamiltonian in terms of 'flowed' operators

$$\mathcal{H}_{\text{eff}} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau)$$

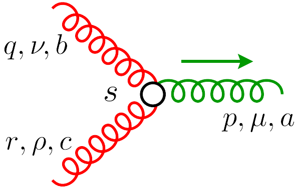
- Relate to regular operators in SFTX

ME of flowed operator (lattice)

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

PT calculated matching matrix

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$



new Feynman diagrams

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in the $\overline{\text{MS}}$ scheme found after taking the $\tau \to 0$
  - → Large systematic effects at very small flow times
  - → Large flow time dominated by operators $\propto O(\tau)$

$\Delta Q = 0$ operators and bag parameters

- The four $\Delta Q = 0$ operators

$$O_1(\tau) = (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma_\mu(1-\gamma_5)Q)(\tau) \qquad T_1(\tau) = (\bar{Q}\gamma_\mu(1-\gamma_5)T^A q)(\bar{q}\gamma_\mu(1-\gamma_5)T^A Q)(\tau)$$

$$O_2(\tau) = (\bar{Q}(1-\gamma_5)q)(\bar{q}(1+\gamma_5)Q)(\tau) \qquad T_2(\tau) = (\bar{Q}(1-\gamma_5)T^A q)(\bar{q}(1+\gamma_5)T^A Q)(\tau)$$

- Determine bag parameters by calculating hadronic matrix elements using GF

$$\langle H|O_1|H\rangle(\tau) = f_H^2\, M_H^2\, B_1^H(\tau) \qquad \langle H|T_1|H\rangle(\tau) = f_H^2\, M_H^2\, \epsilon_1^H(\tau)$$

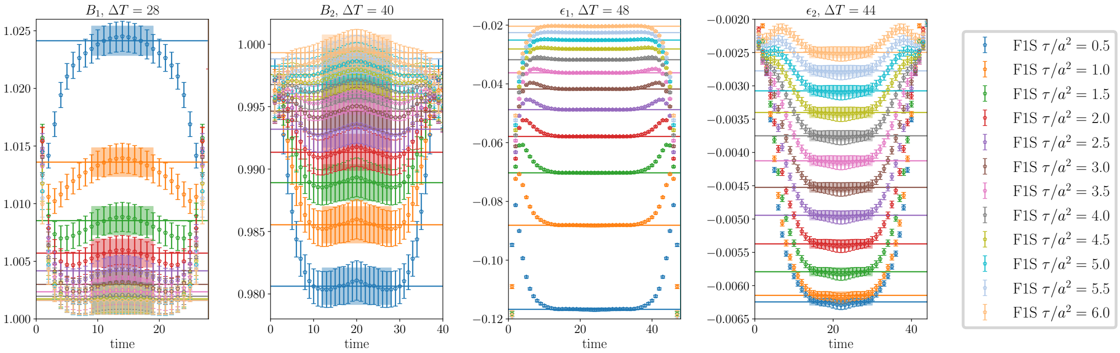
$$\langle H|O_2|H\rangle(\tau) = \frac{M_H^2}{(m_Q+m_q)^2} f_H^2\, M_H^2\, B_2^H(\tau) \qquad \langle H|T_2|H\rangle(\tau) = \frac{M_H^2}{(m_Q+m_q)^2} f_H^2\, M_H^2\, \epsilon_2^H(\tau)$$

- Match to the $\overline{\text{MS}}$ scheme using SFTX

$$\begin{pmatrix} B_i(\tau,\mu) \\ \epsilon_i(\tau,\mu) \end{pmatrix} = \zeta_{B,i}^{-1}(\tau,\mu) \begin{pmatrix} B_i^{\text{GF}}(\tau) \\ \epsilon_i^{\text{GF}}(\tau) \end{pmatrix}$$

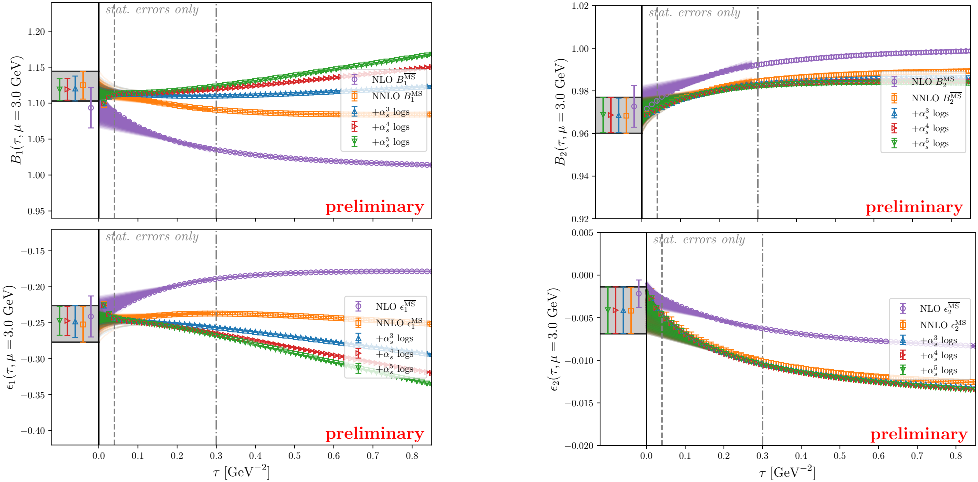
Example: D_s meson lifetimes ($\Delta Q = 0$ operators)

- Extract bag parameters for all ensembles
  - $\rightarrow$ F1S: $a^{-1} = 2.785$ GeV, $M_\pi = 267$ MeV



Example: D_s meson lifetimes ($\Delta Q = 0$ operators)

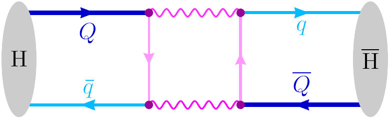
- $\tau \to 0$ limit (work in progress)

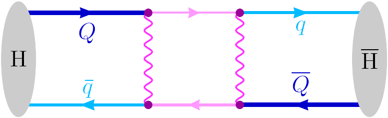


Neutral meson mixing

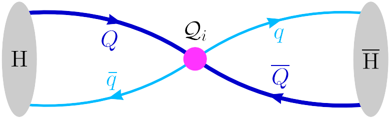
Lattice calculation of neutral $B^0_{(s)}$ meson mixing

- Standard model process described by box diagrams





- Top quark contribution dominates ⇒ short-distance process
  → Describe by point-like 4-quark operators

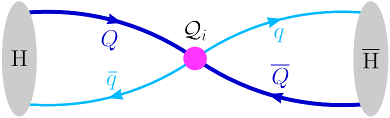


- Parameterize experimentally measured oscillation frequencies Δm_q by

$$\Delta m_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 \hat{B}_{B_q} \left| V_{tq}^* V_{tb} \right|^2, \qquad q = d, s$$

→ Nonperturbative contribution decay constant $f_{B_q}^2$ times bag parameter $\hat{B}_{B_q}$

$\Delta Q = 2$ mixing operators



- Standard model process described by

$$\mathcal{Q}_1^q = \bar{b}^\alpha \gamma^\mu (1-\gamma_5) q^\alpha \; \bar{b}^\beta \gamma_\mu (1-\gamma_5) q^\beta$$

$\rightarrow$ General BSM considerations give rise to four additional dim-6 operators

- Calculate matrix element

$$\langle \mathcal{Q}_1^q \rangle = \langle \overline{B}_q | \mathcal{Q}_1^q | B_q \rangle = \tfrac{8}{3} f_{B_q}^2 M_{B_q} B_{B_q}$$

- Convert "lattice" bag parameter B_{B_q} to RGI bag parameter $\hat{B}_{B_q}$

$\rightarrow$ Renormalization/matching procedures used in the literature

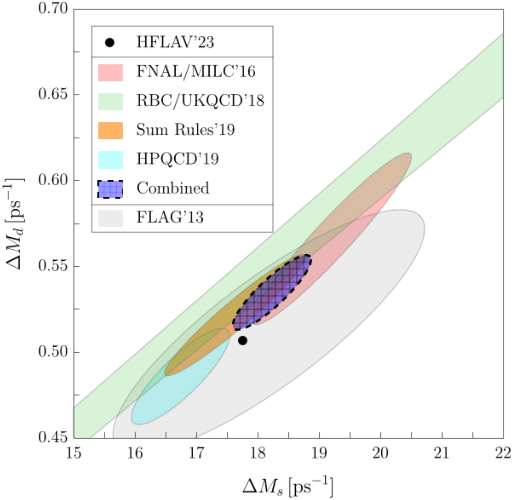
Perturbative scheme: Fermilab/MILC, HPQCD

Nonperturbative scheme: ETMC, RBC-UKQCD

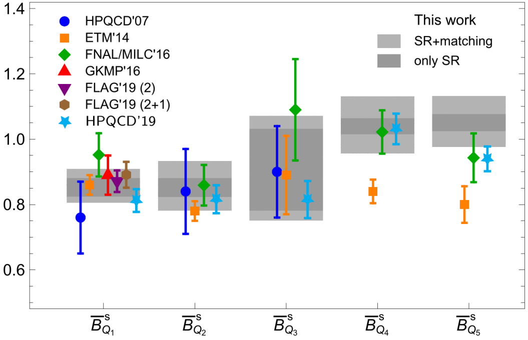
- Operator mixing occurs for non-chiral lattice fermions

$\Delta Q = 2$ mixing operators (literature)

[Albrecht et al. 2402.04224]

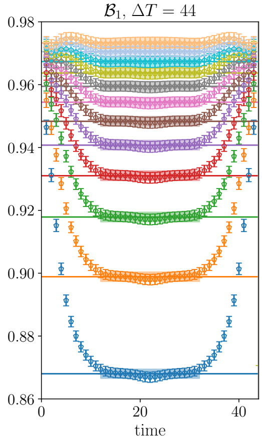


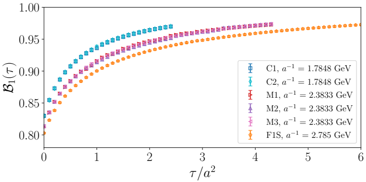
[King Thesis 2022]

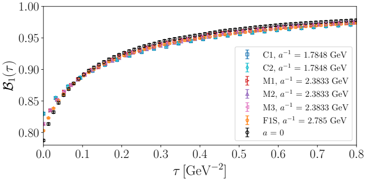


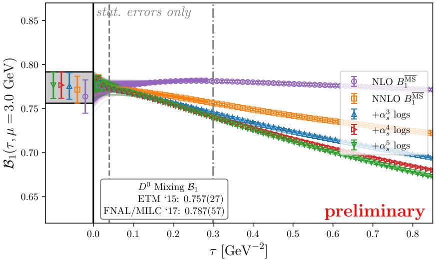
- Ongoing work by RBC-UKQCD+JLQCD
  [Boyle et al. PoS Lattice 2021 224] [Tsang Lattice 2023]
- Dim-7 operators pioneered by HPQCD
  [HPQCD PRL 124 (2020) 082001]

Example: $\Delta Q = 2$ mixing for a "neutral charm-strange meson"









- F1S: $a^{-1} = 2.785$ GeV
  $M_\pi = 267$ MeV
- Consistent with short-distance resultsfor D^0 meson mixing

Numerical setup GF calculations

- RBC/UKQCD's 2+1 flavor DWF + Iwasaki gauge action ensembles, 3 lattice spacings
  [PRD 78 (2008) 114509] [PRD 83 (2011) 074508] [PRD 93 (2016) 074505] [JHEP 12 (2017) 008]
- Setup of the lattice calculation follows [Boyle et al. 1812.08791]
  - → Z2 wall sources for all quark propagators [Boyle et al. JHEP 08 (2008) 086]
  - → Gaussian source smearing for strange quarks [Allton et al. PRD 47 (1993) 5128]
  - → Multiple source separations $\Delta T \in \{10, 30\}$
- Fully-relativistic, chiral action for all quarks
  - → Shamir domain-wall fermions for light and strange quarks
    [Kaplan PLB 288 (1992) 342] [Shamir NPB 406 (1993) 90] [Furman, Shamir NPB 439 (1995) 54]
  - → Stout-smeared Möbius domain-wall fermions for heavy quarks
    [Morningstar, Peardon PRD 69 (2004) 054501] [Brower, Neff, Orginos CPC 220 (2017) 1]
- Simulate "neutral" charm-strange mesons
  - → Easy to tune to physical strange and charm quarks
  - → Avoid more expensive chiral or heavy quark extrapolation

Numerical setup RHQ calculations

- RBC/UKQCD's 2+1 flavor DWF + Iwasaki gauge action ensembles, 3 lattice spacings
  [PRD 78 (2008) 114509] [PRD 83 (2011) 074508] [PRD 93 (2016) 074505] [JHEP 12 (2017) 008]
- Unitary domain-wall up/down quarks; strange quarks at/near the physical value
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90] [Furman, Shamir NPB406(1993)90]
- Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  → Simulate 3 or 2 charm-like masses then extrapolate/interpolate
- Effective relativistic heavy quark (RHQ) action for bottom quarks
  [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
  → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  → Allows to tune the three parameters (m_0a, c_P, ζ) nonperturbatively [PRD 91 (2015) 054502]
  → Smooth continuum limit; heavy quark treated to all orders in $(m_ba)^n$
  → Mostly nonperturbative renormalization: $Z_V^{bl} = \varrho\sqrt{Z_V^{ll} Z_V^{bb}}$
  [Hashimoto et al. PRD61 (1999) 014502][El-Khadra et al. PRD64 (2001) 014502]

RBC-UKQCD's 2+1 flavor SDWF+Iwaski gauge field configurations

	L	a^{-1}(GeV)	am_l	am_s	M_π (MeV)	GF $N_{src} \times N_{conf}$	RHQ $N_{src} \times N_{conf}$	
C1	24	1.785	0.005	0.040	340	32×101	1×1636	[PRD 78 (2008) 114509]
C2	24	1.785	0.010	0.040	433	32×101	1×1419	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	302	32×79	2×628	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	32×89	2×889	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	32×68	2×544	[PRD 83 (2011) 074508]
F1S	48	2.785	0.002144	0.02144	267	24×98	24×98	[JHEP 1712 (2017) 008]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm