Inclusive semileptonic decays of heavy mesons from lattice QCD

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Extended Twisted Mass Collaboration

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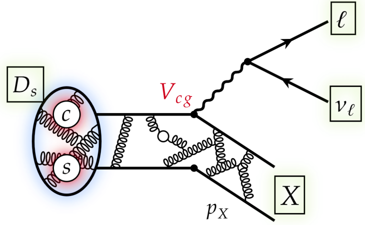
Introduction

Inclusive Semileptonic Decay $H \to X\ell\nu$ from Lattice QCD

- Final state is not necessary
- Flavour-changing process
- non-perturbative; calculations from first principles
- preliminary studies[1]
- Results of the decay $D_s \to X\ell\nu$ were recently accepted for publication by PRL[2] and PRD[3]
- Moving towards the $B_s \to X_{cs}\ell\nu$



[1] Gambino and Hashimoto 2020; Gambino, Hashimoto et al. 2022
[2] De Santis et al. 2025a
[3] De Santis et al. 2025b

- $\Gamma = G_F^2 S_{\text{EW}}(|V_{cs}|^2\Gamma_{cs} + |V_{cd}|^2\Gamma_{cd} + \underbrace{|V_{us}|^2\Gamma_{su}}_{suppressed})$
- $\Gamma_{fg} = \int \frac{\mathrm{d}^3 p_\nu}{(2\pi)^3 2e_\nu} \frac{\mathrm{d}^3 p_\ell}{(2\pi)^3 2e_\ell} \frac{1}{4m_{D_s}^2} L_{\mu\nu}(p_\ell, p_\nu) H_{fg}^{\mu\nu}(p, p - p_\ell - p_\nu)\,,$
- $L_{\mu\nu}$ leptonic tensor from kinematics
- $H_{fg}^{\mu\nu}$ hadronic tensor

$$H_{fg}^{\mu\nu} = \frac{(2\pi)^4}{2m_{D_s}} \langle D_s | J_{fg}^{\mu}(0) \delta^4(\hat{P} - p_X) J_{fg}^{nu\dagger}(0) | D_s \rangle$$

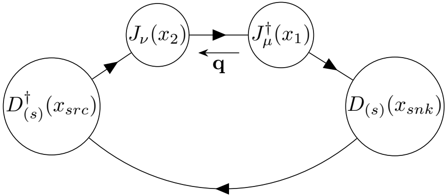
Γ_{fg} from lattice QCD

The hadronic tensor is the **spectral density** of the correlation function:

$$M_{fg}^{\mu\nu}(t, \boldsymbol{\omega}^2) = \int_0^\infty \mathrm{d}\omega_0 \, H_{fg}^{\mu\nu}(\omega_0, \boldsymbol{\omega}^2) e^{-\omega_0 t}$$

On the lattice:

$$M_{fg}^{\mu\nu}(t_2 - t_1, \boldsymbol{\omega}^2) = \lim_{\substack{t_{\rm snk} \mapsto +\infty \\ t_{\rm src} \mapsto -\infty}} \frac{C_{\rm 4pt}^{\mu\nu}(t_{\rm snk}, t_2, t_1, t_{\rm src}; \boldsymbol{\omega}^2)}{C_{\rm 2pt}(t_{\rm snk} - t_2) C_{\rm 2pt}(t_1 - t_{\rm src})}$$



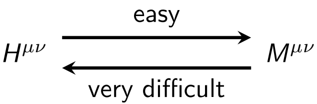
- $t = t_2 - t_1 = a, 2a, \cdots$ **Euclidean time**
- $t_2 - t_{\rm snk}, \; t_{\rm src} - t_1 \gg 0$ checked

Γ_{fg} from lattice QCD

The hadronic tensor is the **spectral density** of the correlation function:

$$M^{\mu\nu}_{fg}(t, \boldsymbol{\omega}^2) = \int_0^\infty \mathrm{d}\omega_0 \, H^{\mu\nu}_{fg}(\omega_0, \boldsymbol{\omega}^2) e^{-\omega_0 t}$$

- Finite number of points for M(t): $30 - 50$
- M has a statistical error
- H is continuous
- inverse Laplace-transform



Γ_{fg} from lattice QCD

$$\frac{1}{\bar{\Gamma}} \frac{\mathrm{d}\Gamma_{fg}}{\mathrm{d}\boldsymbol{\omega}^2} = \sum_{p=0}^{2} |\boldsymbol{\omega}|^{3-p} \int_{\omega_0^{\min}}^{\omega_0^{\max}} \mathrm{d}\omega_0 (\omega_0^{\max} - \omega_0)^p Z^{(p)}$$

- $Z^{(p)}$ are linear combinations of $H_{fg}^{\mu\nu}$
- allowed $\omega_0, \boldsymbol{\omega}^2$ range depends on flavour combination fg

Γ_{fg} from lattice QCD

$$\frac{1}{\bar{\Gamma}}\frac{\mathrm{d}\Gamma_{fg}}{\mathrm{d}\boldsymbol{\omega}^2} = \lim_{\sigma\mapsto 0}\sum_{p=0}^{2}|\boldsymbol{\omega}|^{3-p}\int_{\omega_0^{\min}}^{\infty}\mathrm{d}\omega_0(\omega_0^{\max}-\omega_0)^p\theta_\sigma(\omega_0^{\max}-\omega_0)Z^{(p)}$$

- $Z^{(p)}$ are linear combinations of $H_{fg}^{\mu\nu}$
- allowed $\omega_0, \boldsymbol{\omega}^2$ range depends on flavour combination fg

- Introduce a regularised version of the integral with a θ-function
- σ: smearing parameter
- We can calculate the convolution of the spectral density Z with the kernel θ_σ with HLT-algorithm[4]

[4]Hansen, Lupo and Tantalo 2019

HLT algorithm

- We want to extract $\rho_\sigma = \int d\omega K_\sigma(\omega)\rho(\omega)$ from $C(t) = \int_0^\infty d\omega e^{-\omega t}\rho(\omega)$
- Approximate $K_\sigma(\omega) \approx \sum g_n e^{-\omega t_n}$ minimizing

$$W[\lambda, g] = \frac{A[g]}{A[0]} + \lambda B[g]$$

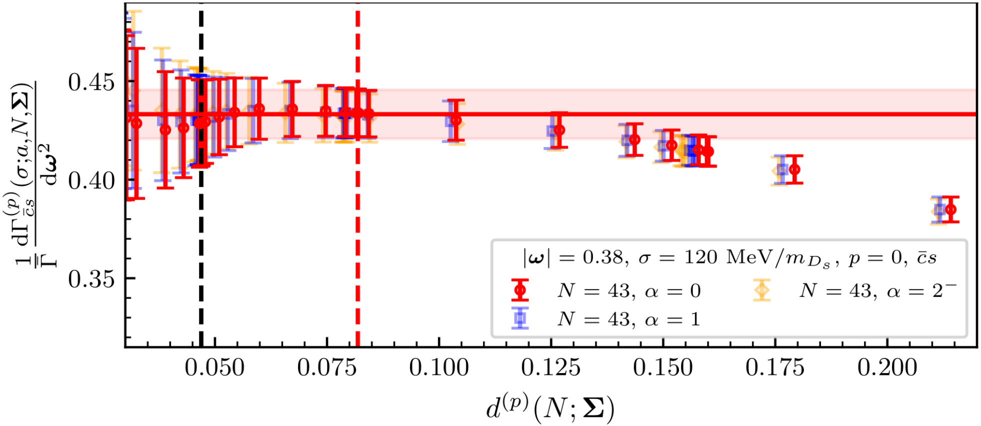
- Accuracy of the approximated kernel

$$A[g] = \int_{E_0}^{\infty} d\omega \left(K_\sigma(\omega) - \sum_{t_n} g_n(\sigma) e^{-\omega t_n} \right)^2$$

- Statistical error

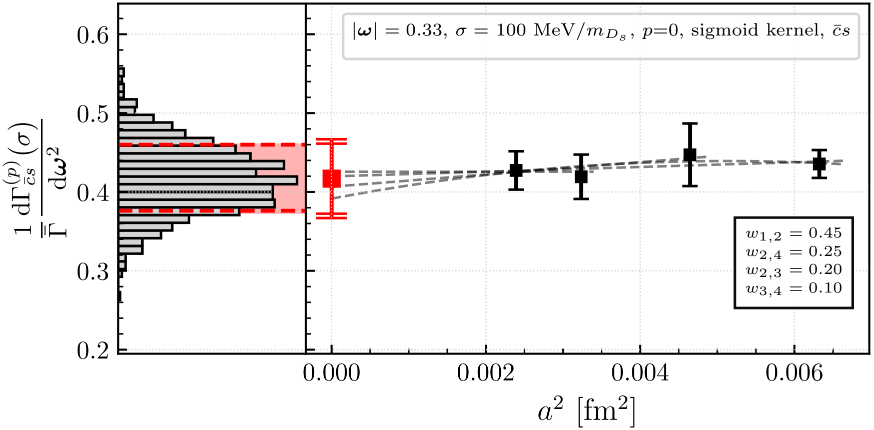
$$B[g] = \boldsymbol{g}^T \mathrm{Cov}[C(t)] \boldsymbol{g}$$

stability analysis



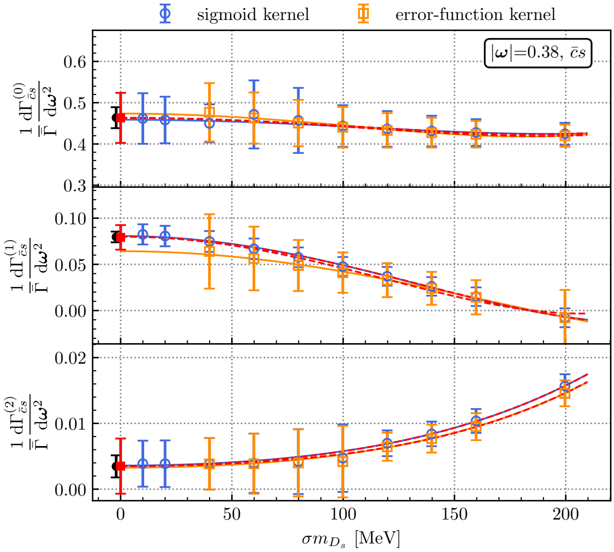
$d^{(p)} = \sqrt{\frac{A[g]}{A[0]}}$ Select plateau of optimal balance between statistical and reconstruction error

Decay rate: Continuum limit



- Combination of several fits with model-averaging criterion.
- Using ETMC ensembles with $N_f = 2 + 1 + 1$ at physical pion mass.

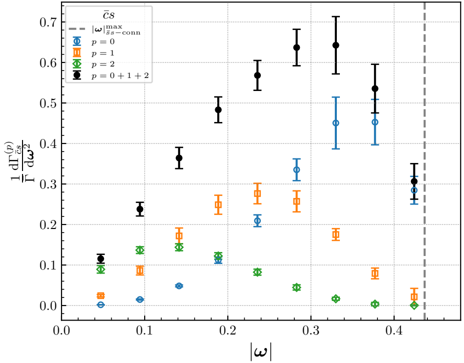
Decay rate: Smearing limit



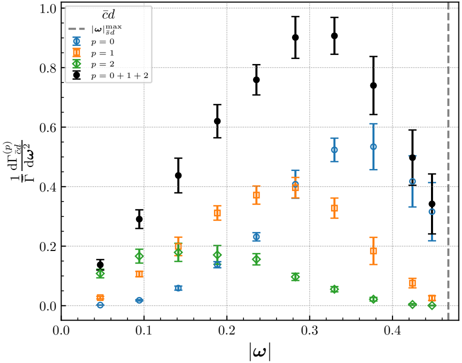
- Combined fit of different reconstruction kernels
- Only even orders of σ contribute

Calculation total decay rates

cs channel



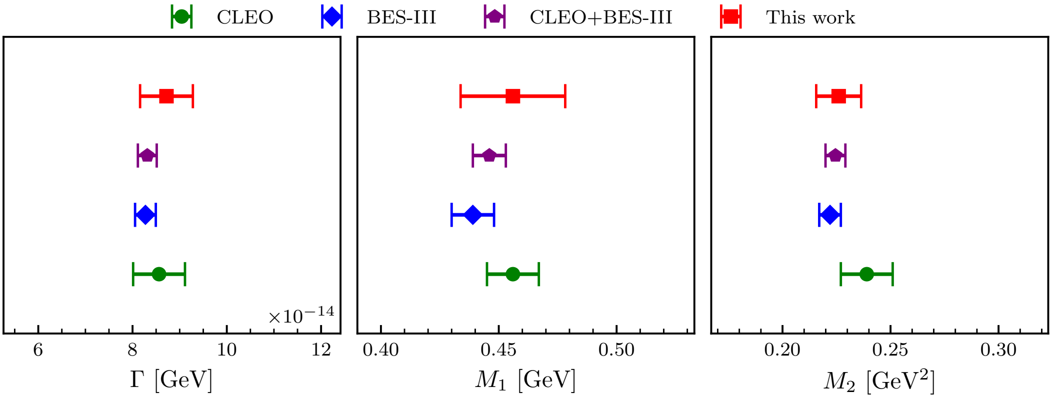
cd channel



- *su* channel negligiable
- Disconneceted diagrams negligiable
- Systematic error estimate

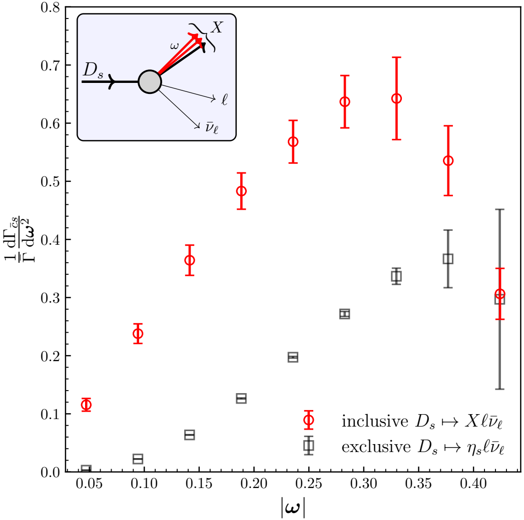
see De Santis et al. 2025a; De Santis et al. 2025b

$D_s \to X\ell\nu$



CLEO
BES-III
CLEO+BES-III
This work
×10^−14
6
8
10
12
Γ [GeV]
0.40
0.45
0.50
M1 [GeV]
0.20
0.25
0.30
M2 [GeV2]

Comparison of the exclusive and inclusive decay



- Inclusive and exclusive results compatible at the end-point of the phase-space

$$\Gamma = |V_{cs}|^2\,\Gamma_{\bar{c}s} + |V_{cd}|^2\,\Gamma_{\bar{c}d} = 8.72(56) \times 10^{-14}\ \mathrm{GeV}$$

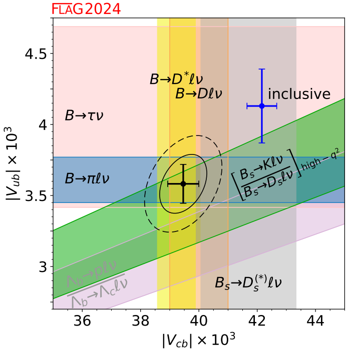
Extract either Γ or $|V_{cs}|$

$$|V_{cs}| = 0.951\,(35)$$
$$|V_{cs}|_{\mathrm{PDG}} = 0.975\,(6)$$

Going to the B_s meson

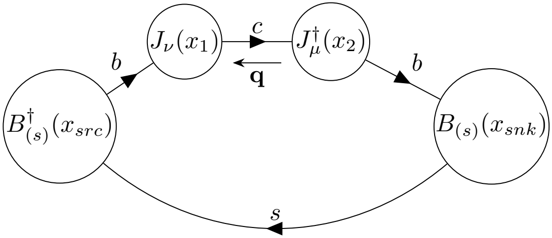
Inclusive Semileptonic Decay $B_s \to X_{cs}\ell\nu$ from Lattice QCD

- Fenomenologically very interesting due to the puzzle between inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$
- b quark not directly simulable on our lattices: $M_{B_s} > 1/a$
- Extrapolate to physical b-mass from lighter unphysical masses with the help of HQEFT

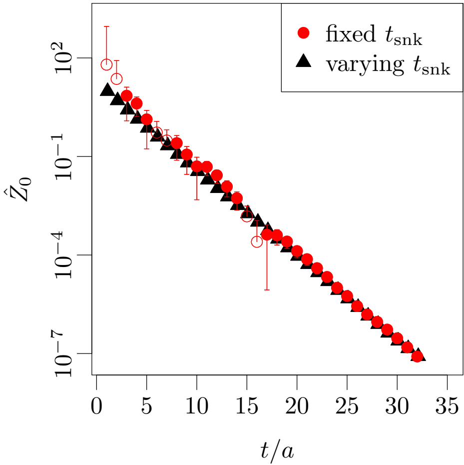


Aoki et al. 2024

Calculation of the 4-Point-Function

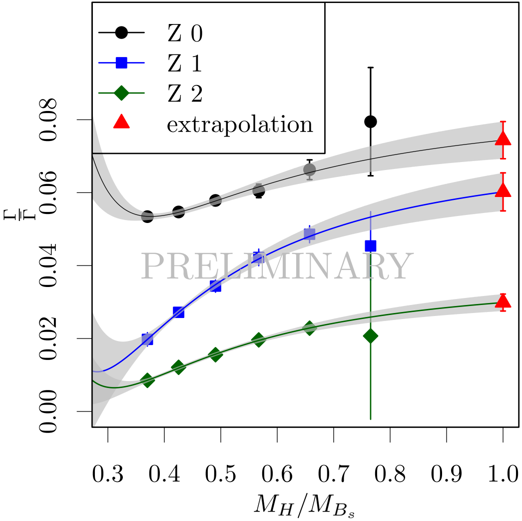


- D_s **case:** Fixed $t_{\rm src}$, $t_{\rm snk}$, and t_2; varied t_1.
- B_s **case:** Noise increases when J_μ is far from the $B_s^\dagger$ source.
- **Strategy:** Keep the $B_s^\dagger$-J_ν and $J_\mu^\dagger$-B_s separation fixed by varying $t_{\rm snk}$.
- Requires extra inversions, but yields a significantly better signal.



$t = t_2 - t_1$

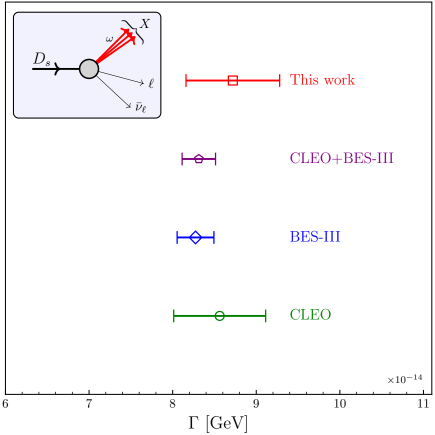
Extrapolation $M_H \to M_{B_s}$



- $\bar{\Gamma} = \dfrac{M_H^5 G_F^2 S_{\mathrm{EW}}}{48\pi^4}$
- $\Gamma(M_H)/\bar{\Gamma} = \left(a_0 + \frac{a_1}{M_H^2} + \frac{a_2}{M_H^3}\right)$
- Only one lattice spacing
- M_H extrapolation better be done after the continuum limit

Summary and Outlook

- first-principles computation of the decay inclusive semileptonic decay rates $D_s \to X\ell\nu$
- Identification and control of the major obstables to extend the calculation to $B_s \to X_{cs}\ell\nu$



Summary and Outlook

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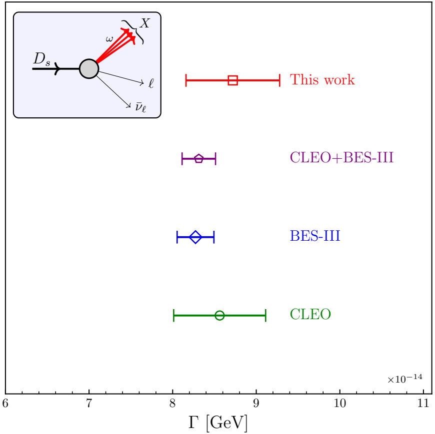

Thank you for your attention!

Configurations

name	L [fm]	a [fm]	M_π [MeV]
B48	3.82	0.080	≈ 135
B64	**5.10**	**0.080**	$\approx$ **135**
B96	7.64	0.080	≈ 135
C80	5.46	0.068	≈ 135
D96	5.46	0.057	≈ 135
E112	5.48	0.049	≈ 135

- ETMC-configurations
- $\mathcal{O}(a)$ and clover improved
- $N_f = 2 + 1 + 1$
- ten momenta per ensemble
- one decay channel
- two smearing kernels (right now: one)
- $\mathcal{O}(10)$ values of σ
- physical pion mass

Bibliography I

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