Pushing towards the ultimate precision in inclusive V_{cb}

Keri Vos

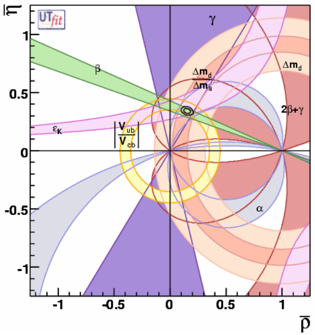
in collaboration with M. Fael, Th. Mannel, K. Olschewsky and M. Rahimi

JHEP 1902 (2019) 177 and work in progress
arXiv:1812.07472

Why V_{cb}?

V_{cb} plays important role in CKM unitarity triangle

- Kaon CP violation via ϵ_K
- In Flavour-Changing-Neutral-Currents (FCNC)



Inclusive versus Exclusive

- $B \to X_c \ell \nu$ versus $B \to D^{(*)} \ell \nu$
- use Heavy Quark Expansion
- Discrepancy between both determinations

Focus on a new method for inclusive $|V_{cb}|$

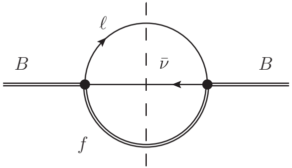
Inclusive B Decays

- Optical Theorem
- Heavy Quark Expansion (HQE)
- Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4$$
$$+ d\Gamma_5 \left[a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_1 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2\right] + \cdots$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$

Operator Product Expansion (OPE)

$2\,\mathrm{Im}$ 

$= \sum_{n,i} \frac{C_i^{(n)}(\mu,\alpha_s)}{m_b^i} \langle B|\mathcal{O}_i^{(n)}|B\rangle_\mu$

- $\mathcal{C}_i(\mu)$: short distance, perturbative coeficients
- $\langle B|\mathcal{O}_i|B\rangle_\mu$: non-perturbative forward matrix elements of local operators
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$$

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD,iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD,iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Inclusive V_{cb} determination

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

$$R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use lepton energy cut

$$R^*(E_{\rm cut}) \quad \langle E^n \rangle_{\rm cut} \quad \langle (M_X^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c)$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} \right]$$

$$\downarrow$$

$$V_{cb} = (42.21 \pm 0.78) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802

State-of-the-art

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)}\right) \right.$$
$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} \Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \cdots \right)$$

- Includes all known α_s and α_s^2 corrections
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063
- Only uses mild external constraints
- Include terms up to $1/m_b^3$
- Assigned 1.4% theo. error due to missing higher orders

Towards the ulitmate precision in inclusive V_{cb}

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}\Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right)\cdots \right)$$

- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov
- Full determination up to $1/m_b^4$ from data
- Reconsider how to deal with backgrounds

Reparametrization Invariance

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel
Mannel, KKV, JHEP 1806 (2018) 115

- Choice of v not unique
- Reparametrization Invariant (RPI) under an infinitesimal change

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\, v_\mu = \delta v_\mu \text{ and } \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

- Reparametrization invariance links different orders in $1/m_b$
  - Gives exact relations between different orders
  - Resums towers of operators
  - Reduces the number of independent parameters
- Up to $1/m_b^4$: 8 parameters versus previous 13

RPI Observables

Mannel, KKV, JHEP 1806 (2018) 115

- Ratio between the rate with and without a cut

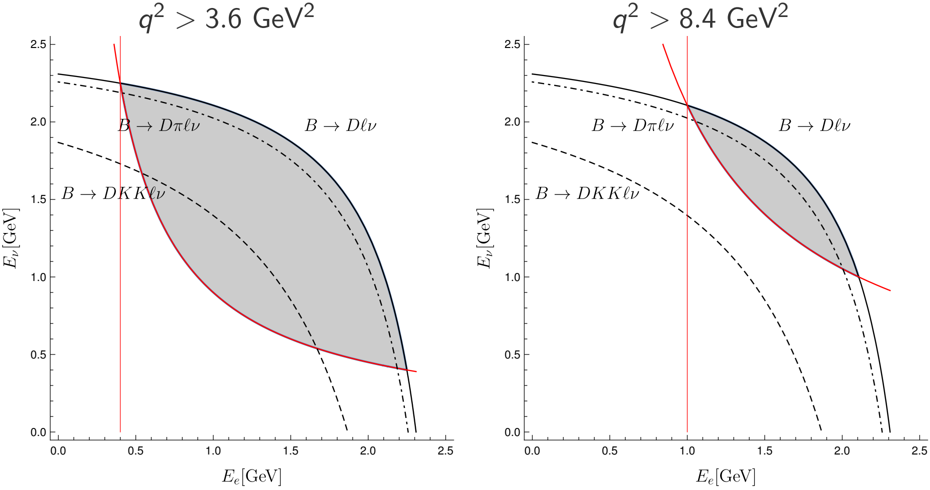
$$R^*(q^2_{\rm cut}) = \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2} \Bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

- q^2 moments

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

- Hadronic mass and lepton energy moments are NOT RPI
- Energy cut is not RPI, but $q^2_{\rm cut}$ is RPI and can be superimposed

q^2 versus energy cut



Alternative V_{cb} determination

Alternative V_{cb} Method

$$R^*(q^2_{\rm cut}) \quad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D}\frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb} = ?$$

Fael, Mannel, KKV, JHEP 02 (2019) 177

Backgrounds in $B \to X_c \ell \nu$

Contamination of the $B \to X_c \ell \nu$ signal

Rahimi, Mannel, KKV, in progress

$b \to u\ell\nu$ contribution

- suppressed by V_{ub}/V_{cb}
- can be calculated precisely in HQE!
- compare used Monte Carlo with theory

$b \to c(\tau \to \mu\nu\bar{\nu})\bar{\nu}$ contribution

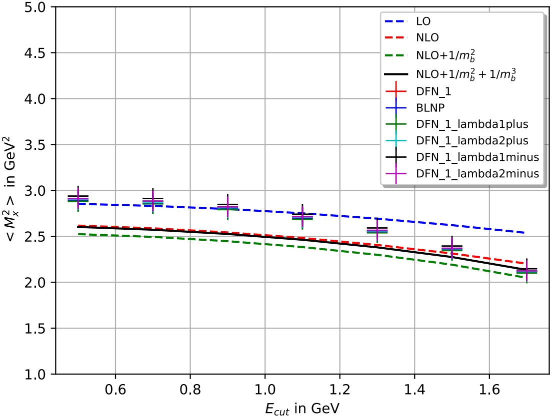
- phase space suppressed
- likewise can be calculated precisely

Analysis of full $B \to X\mu$ data sample?

- including also QED effects

Monte Carlo versus HQE

Rahimi, Mannel, KKV, in progress; MC data by Lu Cao and Florian Bernlochner



Preliminary!

Outlook

Summary & Outlook

New Method:

- RPI reduces number of non-perturbative matrix elements
- Total rate and q^2 moments are RPI: 8 instead of 13 up to $1/m_b^4$
- Extract $|V_{cb}|$ up to $1/m_b^4$, completely data driven
- q^2 moments not (yet) available

In progress:

- Full machinery to obtain V_{cb} from q^2 moments
- Calculation of α_s terms
- Extrapolation of q^2 moments with lepton cut
- $B \to X_u \ell \nu$ and $B \to X_c(\tau \to \mu\nu\bar{\nu})\bar{\nu}$ contamination

Summary & Outlook

New Method:

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In progress:

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Close collaboration between theory and experiment necessary!

Backup

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[\mu_3 - 2\frac{\mu_G^2}{m_b^2} + \left(\frac{34}{3} + 8\log\rho\right)\frac{\tilde{\rho}_D^3}{m_b^2} + \frac{16}{9}(4 + 3\log\rho)\frac{r_G^4}{m_b^4} - \frac{16}{9}(1 + 3\log\rho)\frac{r_E^4}{m_b^4} - \frac{2}{3}\frac{s_B^4}{m_b^4} + \left(\frac{50}{9} + \frac{8}{3}\log\rho\right)\frac{s_E^4}{m_b^4} - \left(\frac{25}{36} + \frac{1}{3}\log\rho\right)\frac{s_{qb}^4}{m_b^4} + O\left(\rho, \frac{1}{m_b^5}\right)\right]$$

with $\rho = m_c^2/m_b^2$

Details of calculation

- Reduction of single γ matrix:

$$\langle \bar{Q}_v \Gamma (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle = \frac{1}{2}\langle \bar{Q}_v \{\Gamma, \not{v}\} \rangle (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle + \frac{1}{2m}\langle \bar{Q}_v \left\{ (i\not{D}) \, , \, (iD_{\mu_1})...(iD_{\mu_n})\Gamma \right\} Q_v \rangle$$

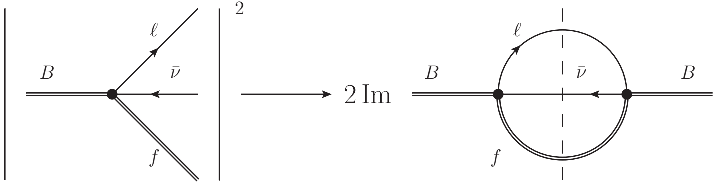
$$\langle \bar{Q}_v \gamma_\beta (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle = v_\beta \langle \bar{Q}_v (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle + \mathcal{O}(1/m_b)$$

- Use E.O.M.

$$\not{v} Q_v = Q_v - \frac{i\not{D}}{m_b} Q_v$$

$$(iv \cdot D) Q_v = -\frac{1}{2m_b}(i\not{D})(i\not{D}) Q_v$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$



Optical Theorem

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v)\rangle|^2$$

$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v)\rangle$$

$$= 2 \, \mathrm{Im} \int d^4x \; e^{-iq\cdot x} \, \langle B(v) | T \left\{ \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) \right\} | B(v)\rangle$$

where $\mathcal{H}_{eff} = J_c^\mu L_\mu, \quad J_c^\mu = \bar{b}\gamma^\mu P_L c$

Inclusive Decays: the OPE

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, · · ·

Heavy Quark Expansion

- B meson: $p_B = m_B v$
- Split the momentum b quark: $p_b = m_b v + k$, expand in $k \sim iD\ Q_v$
- Field-redefinition of the heavy field $Q(x) = exp(-im(v \cdot x))Q_v(x)$

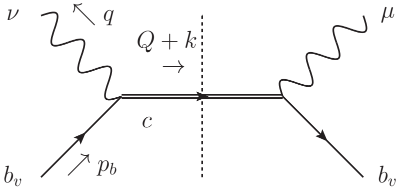
$$\begin{aligned}\Gamma \quad &= 2\ \mathrm{Im} \int d^4x\, e^{-iq\cdot x} \langle B(v)|\, T\left\{\mathcal{H}_{eff}(x)\mathcal{H}^{\dagger}_{eff}(0)\right\}|B(v)\rangle \\ &= 2\ \mathrm{Im} \int d^4x\, e^{i(m_b v - q)\cdot x}\langle B(v)|\, T\left\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^{\dagger}_{eff}(0)\right\}|B(v)\rangle\end{aligned}$$

where $\widetilde{\mathcal{H}}_{eff} = \tilde{J}^{\mu}_c L_\mu, \quad \tilde{J}^{\mu}_c = \bar{b}_v \gamma^\mu P_L c, \quad \Gamma \propto 2\mathrm{Im}\, T^{\mu\nu} L_{\mu\nu}$

$$\Gamma(B \to X_c \ell \nu_\ell) \propto 2 \mathrm{Im}\, T^{\mu\nu} L_{\mu\nu}$$

$$T^{\mu\nu} = i \int dx^4 e^{i(m_b v - q)\cdot x}\, T\left\{\bar{b}_v(x)\gamma^\mu P_L c(x), \bar{c}(0)\gamma^\nu P_L b_v(0)\right\}$$

$$Q = m_b v - q$$



$$= \bar{b}_v \gamma_\mu P_L \left[\frac{i}{\not{Q} + i\not{D} - m_c}\right] \gamma_\nu P_L b_v$$

$$\frac{i}{\not{Q}+i\not{D}-m_c} = \frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \dots$$

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

Theory guidance to include power corrections

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Towards the Ultimate Precision in $|V_{cb}|$

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]; Gambino [in progress]
- Full determination up to $1/m_b^4$ from data possible?

Reparametrization invariance

Total rate at tree level

Mannel, KKV, JHEP 1806 (2018) 115

$$R = \sum_{n=0}^{\infty} C^{(n)}_{\mu_1..\mu_n}(v) \otimes \bar{Q}_v(iD_{\mu_1} \cdots iD_{\mu_n})Q_v$$

$$\delta_{\rm RP} R = 0 \quad = \quad \sum_{n=0}^{\infty} \left[\delta_{\rm RP}\, C^{(n)}_{\mu_1\cdots\mu_n}\right] \bar{Q}_v(iD^{\mu_1} \cdots iD^{\mu_n})Q_v + \sum_{n=0}^{\infty} C^{(n)}_{\mu_1\cdots\mu_n} \left[\delta_{\rm RP}\, \bar{Q}_v(iD^{\mu_1} \cdots iD^{\mu_n})Q_v\right]$$

The RPI relation:

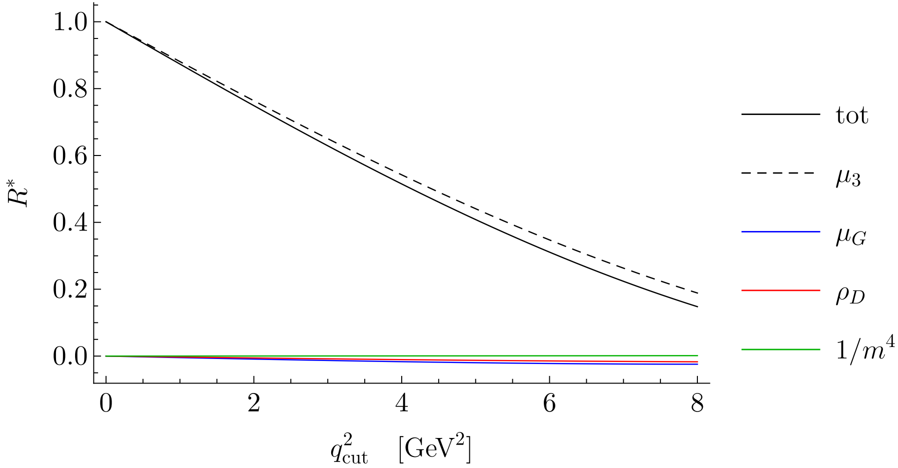
$$\delta_{\rm RP}\, C^{(n)}_{\mu_1\cdots\mu_n} = m_b \delta v^\alpha \left[C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha}\right]$$

Parameter reduction: an example ρ_{LS}

- $1/m_b^2$: $\quad \mu_G^2 \to \underbrace{\eta\,(-i\sigma_{\mu\nu})}_{C^{(2)}_{\mu\nu}} \otimes \bar{Q}_v\,(iD^\mu iD^\nu)\,Q_v$
- $1/m_b^3$: $\quad \rho_{LS}^3 \to \underbrace{\xi\, v_\alpha(-i\sigma_{\mu\nu})}_{C^{(3)}_{\mu\alpha\nu}} \otimes \bar{Q}_v\,(iD^\mu iD^\alpha iD^\nu)\,Q_v$

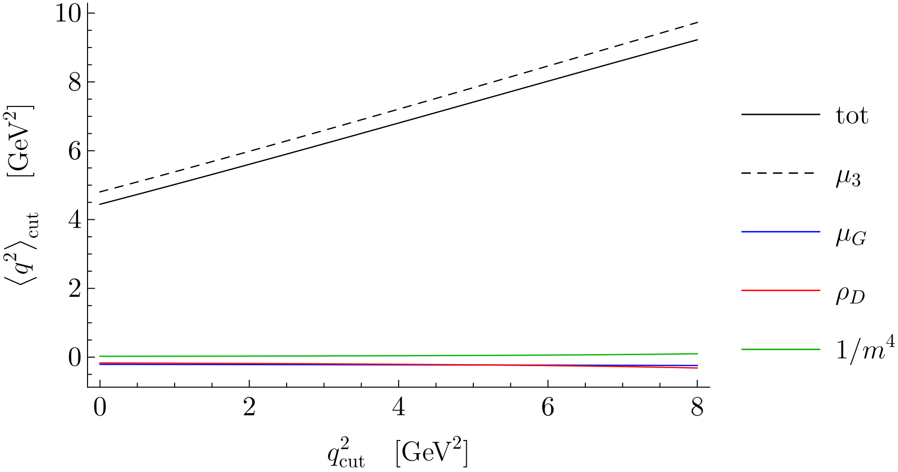
The RPI relation:

$$
\begin{aligned}
\delta_{\mathrm{RP}}\, C^{(2)}_{\mu\nu} &= 0 \\
&= m_b\,\delta v^\alpha \left(C^{(3)}_{\mu\nu\alpha} + C^{(3)}_{\mu\alpha\nu} + C^{(3)}_{\alpha\mu\nu} \right) \\
&= -i m_b\,\xi\,\delta v^\alpha\,(\sigma_{\mu\alpha} v_\nu + \sigma_{\alpha\nu} v_\mu) \\
&\leftrightarrow \xi = 0
\end{aligned}
$$



Fael, Mannel, KKV, JHEP 02 (2019) 177

Benchmark values based on Gambino, Haeley, Turczyk, PLB 763 (2016) 60



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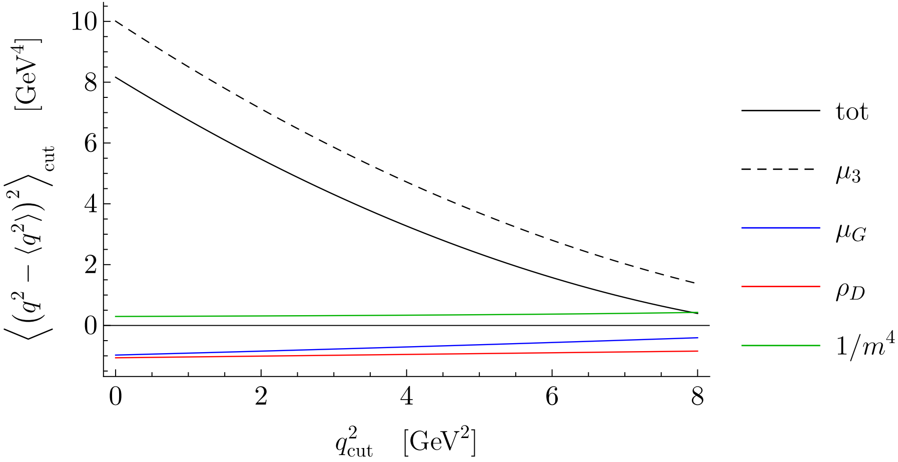

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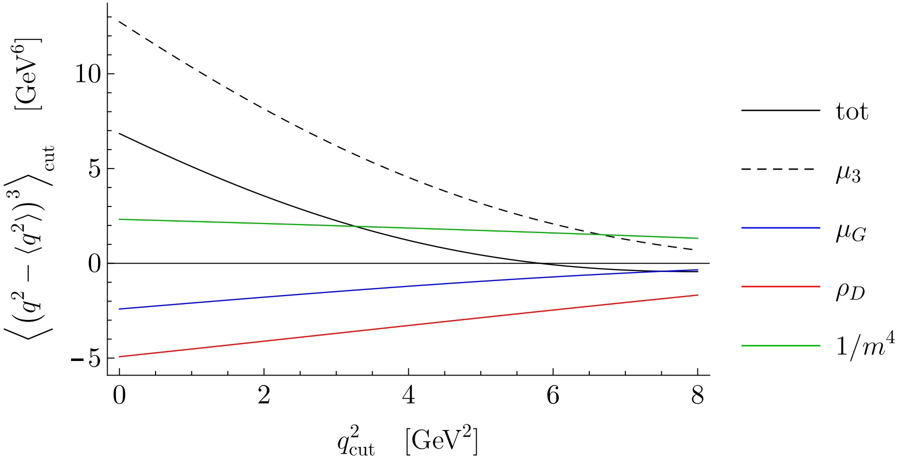

Fael, Mannel, KKV, JHEP 02 (2019) 177

Non-perturbative matrix elements

Mannel, KKV, JHEP 1806 (2018) 115

- 1:
  - $2M_B\mu_3 = \langle B|\bar{b}_v b_v|B\rangle = 2M_B\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b}\right)$
- $1/m_b^2$:
  - $2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$
- $1/m_b^3$:
  - $2M_B\tilde{\rho}_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,\left[\left(ivD + \frac{(iD)^2}{m_b}\right), iD^\mu\right]\right]b_v|B\right\rangle$
- $1/m_b^4$:
  - $2M_B r_G^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, iD_\nu][iD^\mu, iD^\nu]b_v|B\rangle \propto \langle \vec{E}^2 - \vec{B}^2\rangle$
  - $2M_B r_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[ivD, iD_\mu][ivD, iD^\mu]b_v|B\rangle \propto \langle \vec{E}^2\rangle$
  - $2M_B s_B^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu, iD_\alpha][iD^\mu, iD_\beta]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{B}\times\vec{B}\rangle$
  - $2M_B s_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[ivD, iD_\alpha][ivD, iD_\beta]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \vec{\sigma}\cdot\vec{E}\times\vec{E}\rangle$
  - $2M_B s_{qB}^4 \equiv \frac{1}{2}\langle B|\bar{b}_v[iD_\mu,[iD^\mu,[iD_\alpha, iD_\beta]]]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle \Box\,\vec{\sigma}\cdot\vec{B}\rangle$.

Up to $1/m_b^4$: 8 parameters   versus previous 13