$\bar{B}_q \to D_q^{(*)}$ Form Factors and Phenomenological Applications

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Introduction

Form Factors (FFs) parametrize fundamental mismatch:

Theory (e.g. SM) for partons (quarks)
vs.
Experiment with hadrons

$$\left\langle D_q^{(*)}(p')|\bar{c}\gamma^\mu b|\bar{B}_q(p)\right\rangle = (p+p')^\mu f_+^q(q^2) + (p-p')^\mu f_-^q(q^2)\,,\; q^2=(p-p')^2$$

Most general matrix element parametrization, given symmetries:
Lorentz symmetry plus P- and T-symmetry of QCD
$f_\pm(q^2)$: scalar functions of one kinematic variable

Issue: how to obtain q^2-dependence?

- Calculable w/ non-perturbative methods (Lattice, LCSR,...)
  Precision?
- Measurable e.g. in semileptonic transitions
  Normalization? Suppressed FFs? NP?

q^2 dependence

- q^2 range can be large, e.g. $q^2 \in [0, 12]\ \mathrm{GeV}^2$ in $B \to D$
- Calculations give usually one or few points
- ➥ Knowledge of functional dependence on q^2 cruical
- This is where discussions start...

Give as much information as possible independent of this choice!

In the following: discuss BGL and HQE ($\to$ CLN) parametrizations

q^2 dependence usually rewritten via conformal transformation:

$$z\left(t = q^2, t_0\right) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$t_+ = (M_{B_q} + M_{D_q^{(*)}})^2$: pair-production threshold

$t_0 < t_+$: free parameter for which $z(t_0, t_0) = 0$

Usually $|z| \ll 1$, e.g. $|z| \leq 0.06$ for semileptonic $B \to D$ decays

➥ Good expansion parameter

The BGL parametrization [Boyd/Grinstein/Lebed, 90's]

FFs are parametrized by a few coefficients the following way:

1. Consider analytical structure, make poles and cuts explicit
2. Without poles or cuts, the rest can be Taylor-expanded in z
3. Apply QCD properties (unitarity, crossing symmetry)
   ➥ dispersion relation
4. Calculate partonic part perturbatively (+condensates)

Result:

$$F(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n [z(t,t_0)]^n \,.$$

- a_n: real coefficients, the only unknowns
- $P(t)$: Blaschke factor(s), information on poles below t_+
- $\phi(t)$: Outer function, chosen such that $\sum_{n=0}^{\infty} a_n^2 \leq 1$

➥ Series in z with bounded coefficients (each $|a_n| \leq 1$)!

➥ Uncertainty related to truncation is calculable!

$V_{cb} + R(D^*)$ w/ data + lattice + unitarity [Gambino/MJ/Schacht'19]

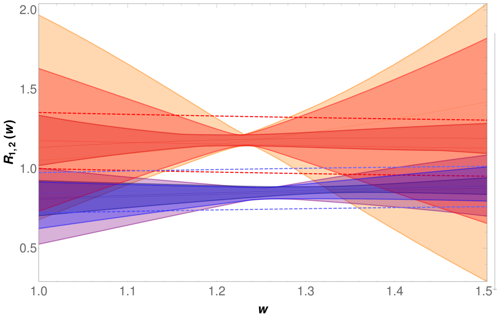
(see also [Fajfer+,Nierste+,Bernlochner+,Bigi+,Grinstein+,Nandi+...])
Recent untagged analysis by Belle with 4 1D distributions [1809.03290]

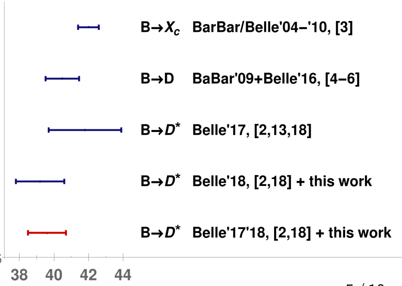
➥ *"Tension with the (V_{cb}) value from the inclusive approach remains"*

Analysis of 2017+2018 Belle data with BGL form factors:

- Datasets roughly compatible
- d'Agostini bias + syst. important
- All FFs to z^2 to include uncertainties
- 2018: no parametrization dependence

$$|V_{cb}^{D^*}| = 39.6^{+1.1}_{-1.0} \times 10^{-3}$$
$$R(D^*) = 0.254^{+0.007}_{-0.006}$$





HQE parametrization

HQE parametrization uses additional information compared to BGL

➥ Heavy-Quark Expansion (HQE)

- $m_{b,c} \to \infty$: all $B \to D^{(*)}$ FFs given by 1 Isgur-Wise function
- Systematic expansion in $1/m_{b,c}$ and α_s
- Higher orders in $1/m_{b,c}$: FFs remain related
  - ➥ Parameter reduction, necessary for NP analyses!

CLN parametrization [Caprini+,'97] :
HQE to order $1/m_{b,c}, \alpha_s$ plus (approx.) constraints from unitarity
[Bernlochner/Ligeti/Papucci/Robinson'17] : identical approach, updated and consistent treatment of correlations

Problem: Contradicts Lattice QCD (both in $B \to D$ and $B \to D^*$)
Dealt with by varying calculable (@$1/m_{b,c}$) parameters, e.g. $h_{A_1}(1)$
➥ Not a systematic expansion in $1/m_{b,c}$ anymore!
➥ Related uncertainty remains $\mathcal{O}[\Lambda^2/(2m_c)^2] \sim 5\%$, insufficient

Solution: Include systematically $1/m_c^2$ corrections
[Bordone/MJ/vDyk'19,Bordone/Gubernari/MJ/vDyk'20] ,using [Falk/Neubert'92]

Theory determination of $b \to c$ Form Factors

SM: BGL fit to data + FF normalization $\to |V_{cb}|$

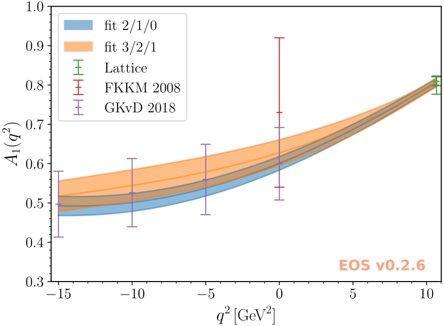
NP: can affect the q^2-dependence, introduces additional FFs

➥ To determine general NP, FF shapes needed from theory

In [MJ/Straub'18,Bordone/MJ/vDyk'19] , we use all available theory input:

- Unitarity bounds (using results from [BGL,Bigi/Gambino(/Schacht)'16'17])
- LQCD for $f_{+,0}(q^2)$ $(B \to D)$, $h_{A_1}(q^2_{\max})$ $(B \to D^*)$ [HPQCD'15,'17,Fermilab/MILC'14,'15]
- LCSR for all FFs (mod f_T) [Gubernari/Kokulu/vDyk'18]
- Consistent HQET expansion [Bernlocher+] to $\mathcal{O}(\alpha_s, 1/m_b, 1/m_c^2)$
  ➥ improved description

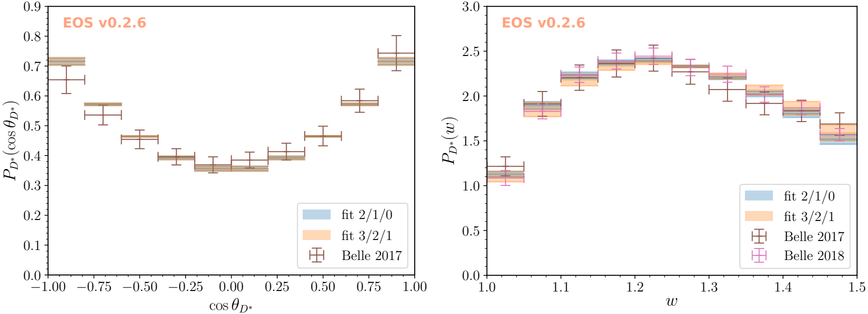
FFs under control; $R(D^*) = 0.247(6)$ [Bordone/MJ/vDyk'19]



Robustness of the HQE expansion up to $1/m_c^2$

[Bordone/MJ/vDyk'19]

Testing FFs by comparing to data and fits in BGL parametrization:



- Fits 3/2/1 and 2/1/0 are theory-only fits(!)
- $k/l/m$ denotes orders in z at $\mathcal{O}(1, 1/m_c, 1/m_c^2)$
- w-distribution yields information on FF shape $\rightarrow V_{cb}$
- Angular distributions more strongly constrained by theory, only

➥ Predicted shapes perfectly confirmed by $B \rightarrow D^{(*)}\ell\nu$ data

➥ V_{cb} from Belle'17 compatible between HQE and BGL!

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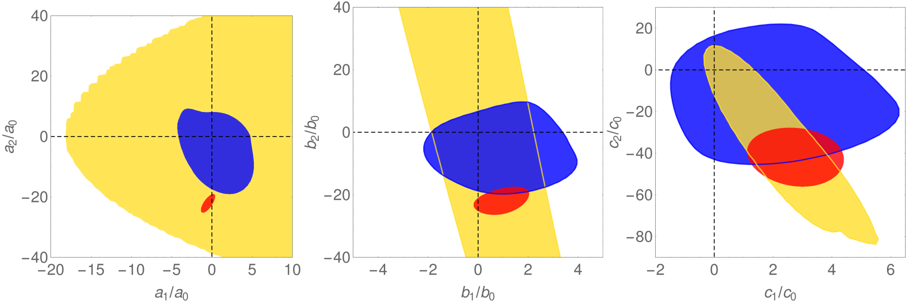


- $B \to D^*$ BGL coefficient ratios from:
  1. Data (Belle'17+'18) + weak unitarity (yellow)
  2. HQE theory fit 2/1/0 (red)
  3. HQE theory fit 3/2/1 (blue)

➥ Again compatibility of theory with data

➥ 2/1/0 underestimates the uncertainties massively

➥ For b_i, c_i ($\to f, \mathcal{F}_1$) data and theory complementary

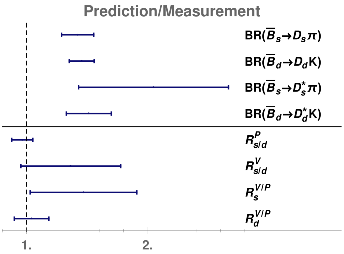
A puzzle in non-leptonic $b \to c$ transitions

[Bordone/Gubernari/Huber/MJ/vDyk'20]

FFs also of central importance in non-leptonic decays:

- Complicated in general, $B \to M_1 M_2$ dynamics
- Simplest cases: $\bar{B}_d \to D_d^{(*)} \bar{K}$ and $\bar{B}_s \to D_s^{(*)} \pi$ (5 diff. quarks)
  - ➥ Colour-allowed tree, $1/m_b^0 @ \mathcal{O}(\alpha_s^2)$ [Huber+'16] , factorizes at $1/m_b$
  - ➥ Amplitudes dominantly $\sim \bar{B}_q \to D_q^{(*)}$ FFs
  - ➥ Used to determine f_s/f_d at hadron colliders [Fleischer+'11]

Updated and extended calculation: tension of 4.4σ w.r.t. exp.!



- Large effect, $\sim -30\%$ for BRs
- Ratios of BRs ok
- QCDf uncertainty $\mathcal{O}(1/m_b^2, \alpha_s^3)$
- Data consistent (too few abs. BRs)
- NP? $\Delta_P \sim \Delta_V \sim -20\%$ possible

➥ We will learn something important!

Conclusions

Form factors essential ingredients in precision-flavour physics!

- q^2 dependence critical
- ➥ Essential to have FF-independent data
- ➥ Inclusion of higher-order (theory) uncertainties important
- BGL: model-independent, truncation uncertainty limited
- ➥ $B \to D^*$: Reduced V_{cb} puzzle, somewhat lower $R(D^*)$ prediction
- Theory determinations for NP required $\to$ HQE to relate FFs
- $\mathcal{O}(1/m_c)$ not good enough for precision analyses
- ➥ First analysis at $1/m_c^2$ provides all $B \to D^{(*)}$ FFs
- ➥ V_{cb} consistent w/ BGL
- 4.4σ tension in non-leptonic decays!
- ➥ Belle II important for “profane” BR measurements

Central lesson: experiment and theory need to work closely together!

Thank you