Amplitude Analyses

BELLE II Physics Week, 30 Nov 2020
(originally for B Workshop, Neckarzimmern, 2015)
Jonas Rademacker

Special thanks to Antimo Palano, Marco Pagapallo, and Patricia Magalhães,
from whose excellent talks I lifted a particularly large number of plots.

Why Amplitude Analyses?

- QM is intrinsically complex:

  Wave functions/transition amplitudes etc: $\psi = a\, e^{i\alpha}$. Observable: $|\psi|^2$.

  Only half the information. How do I get the rest?

- Note that the rest is very interesting - CP violation in the SM comes from phases!

- Answer: Interference effects:

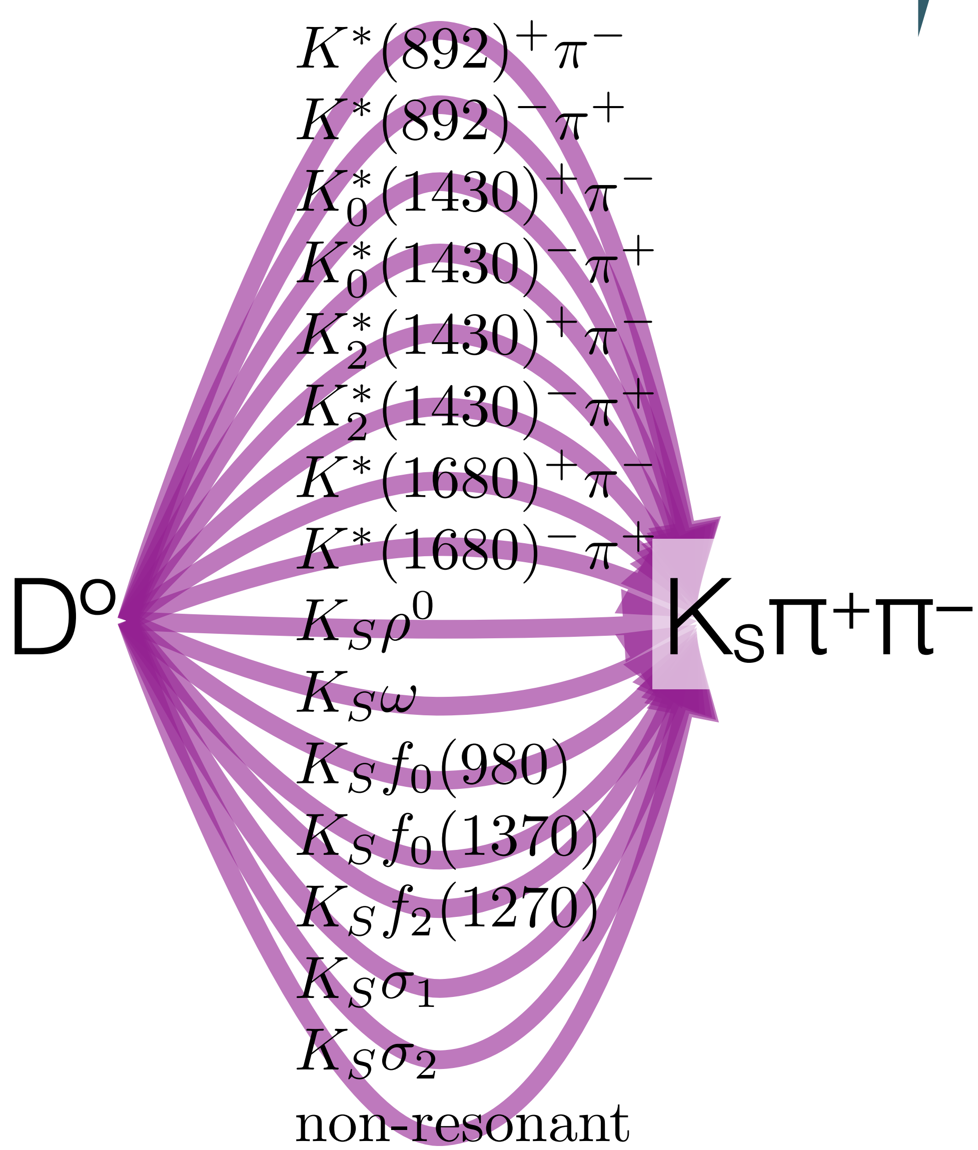
  $\psi_{total} \;\; = \; a\, e^{i\alpha} + b\, e^{i\beta} + \ldots$
  $|\psi_{total}|^2 = \; |a\, e^{i\alpha} + b\, e^{i\beta} + \ldots| = a^2 + b^2 + 2ab\cos(\alpha - \beta) + \ldots$

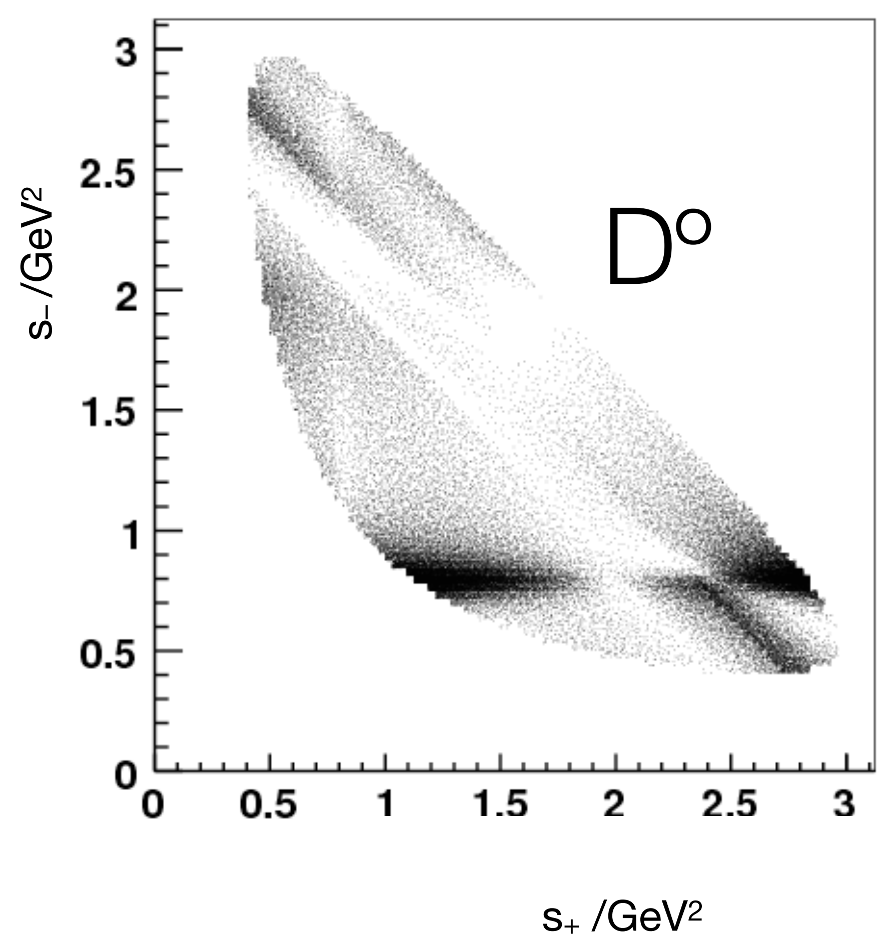
Dalitz plot analyses - lots of interfering amplitudes!

Many interfering decay paths contribute to the same final state

Described by a sum of complex amplitudes

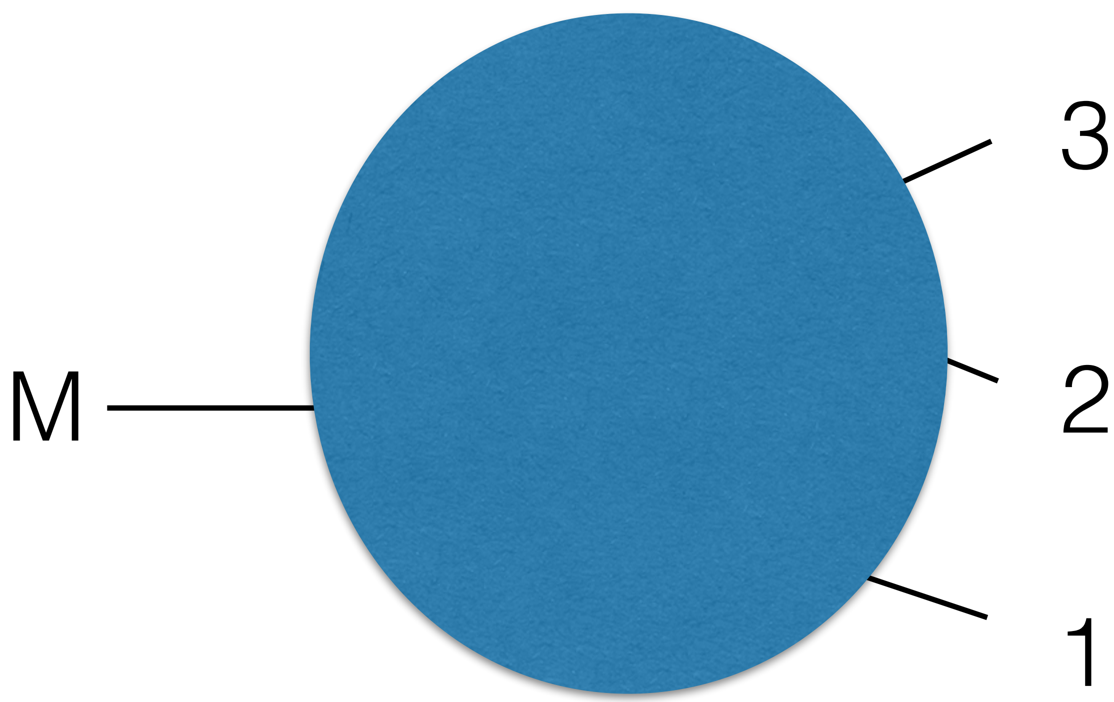
$$A(s_+, s_-) = \sum_k a_k(s_+, s_-)\ e^{i\phi_k(s_+, s_-)}$$





$|A(s_+, s_-)|^2$ represented in a Dalitz plot

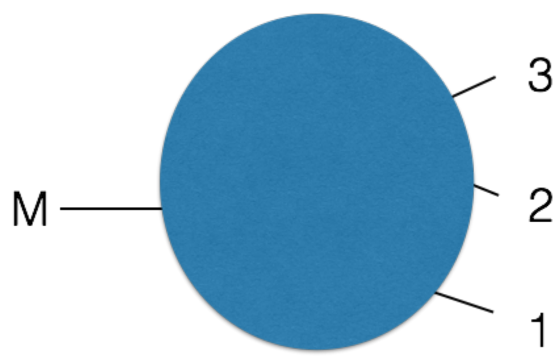
3 body decays

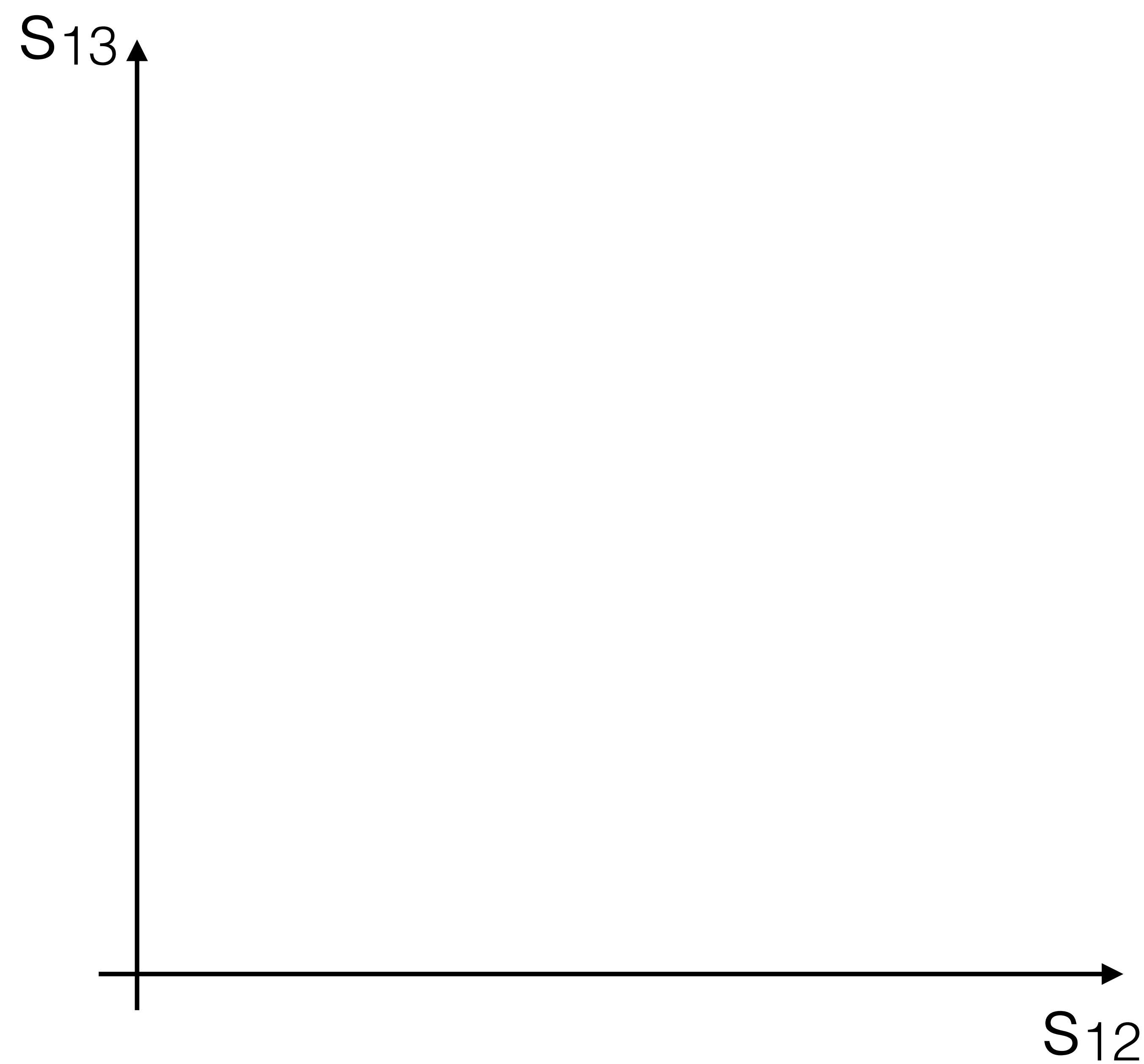


$$
\begin{aligned}
d\Gamma &= |\mathcal{M}_{fi}|^2 d\Phi \\
&= |\mathcal{M}_{fi}|^2 \left| \frac{\partial \Phi}{\partial(s_{12}, s_{13})} \right| ds_{12} ds_{13} \\
&= \frac{1}{(2\pi)^2\, 32 M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}
\end{aligned}
$$

$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

3-body phase space





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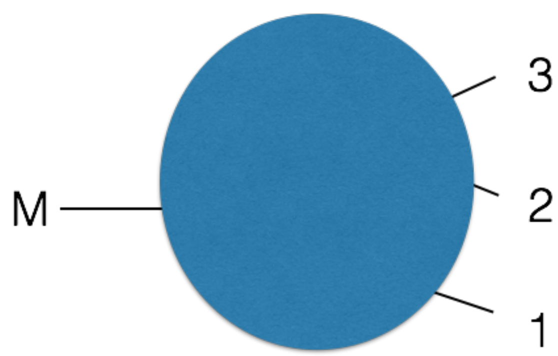

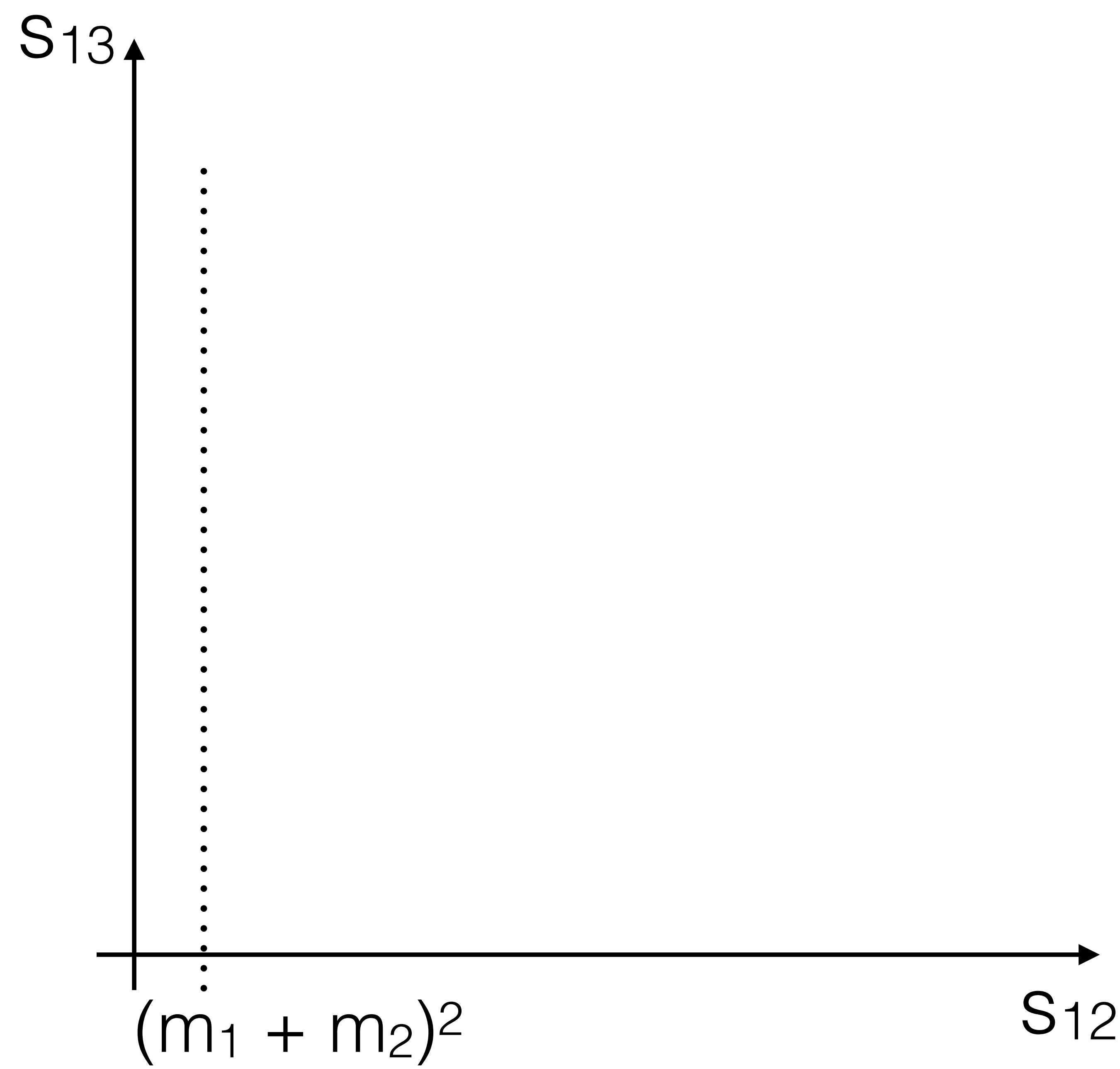

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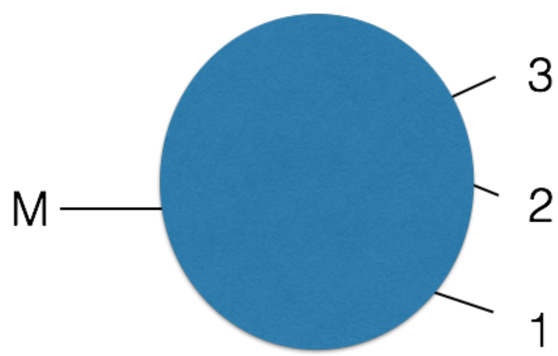

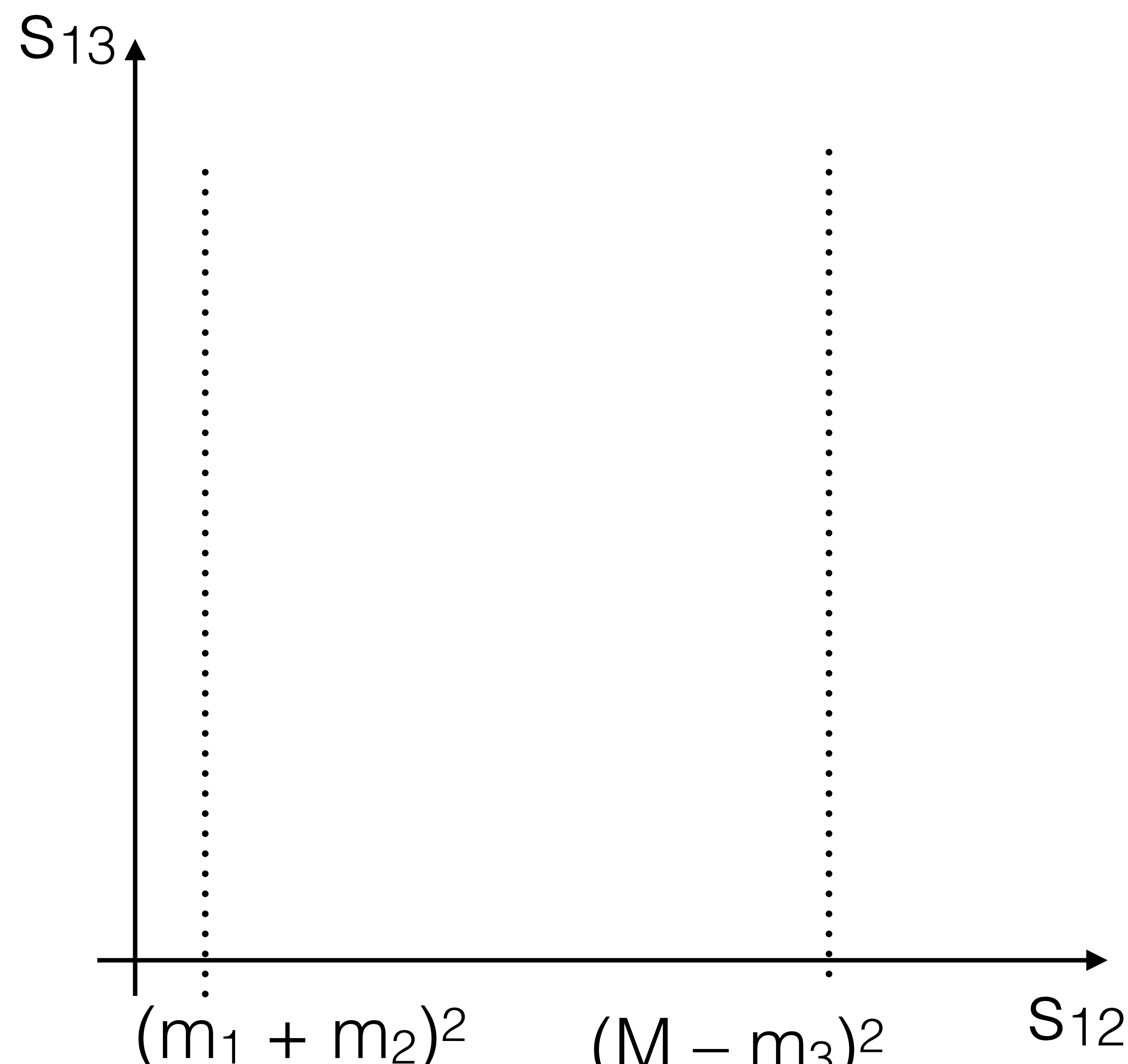

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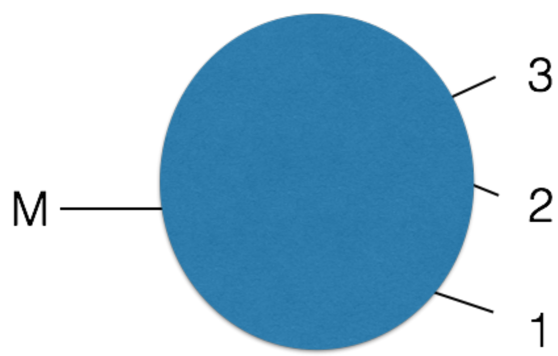

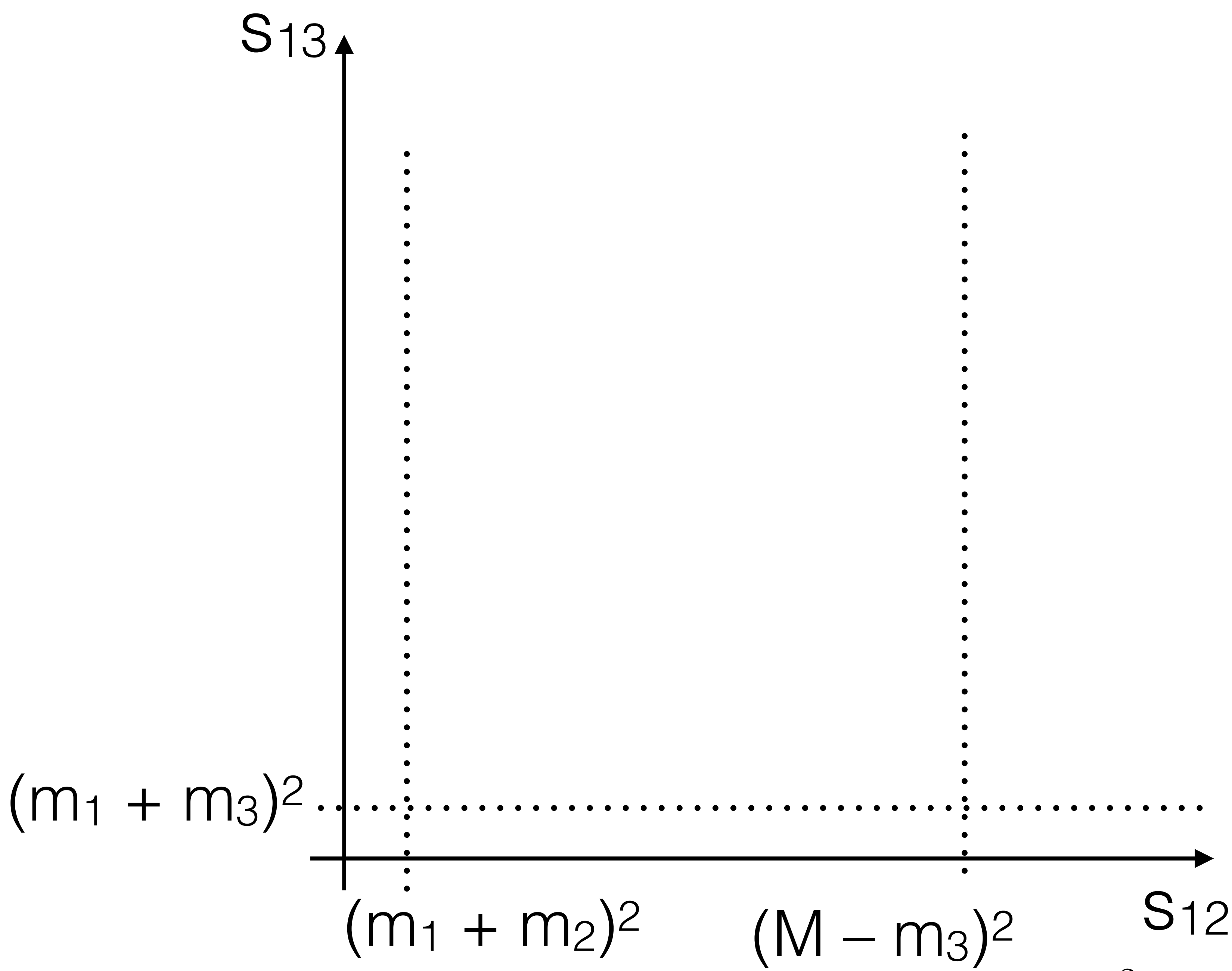

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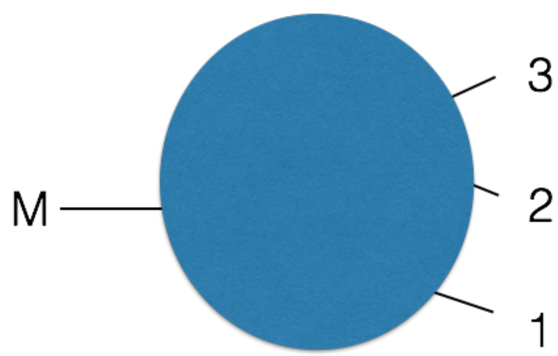

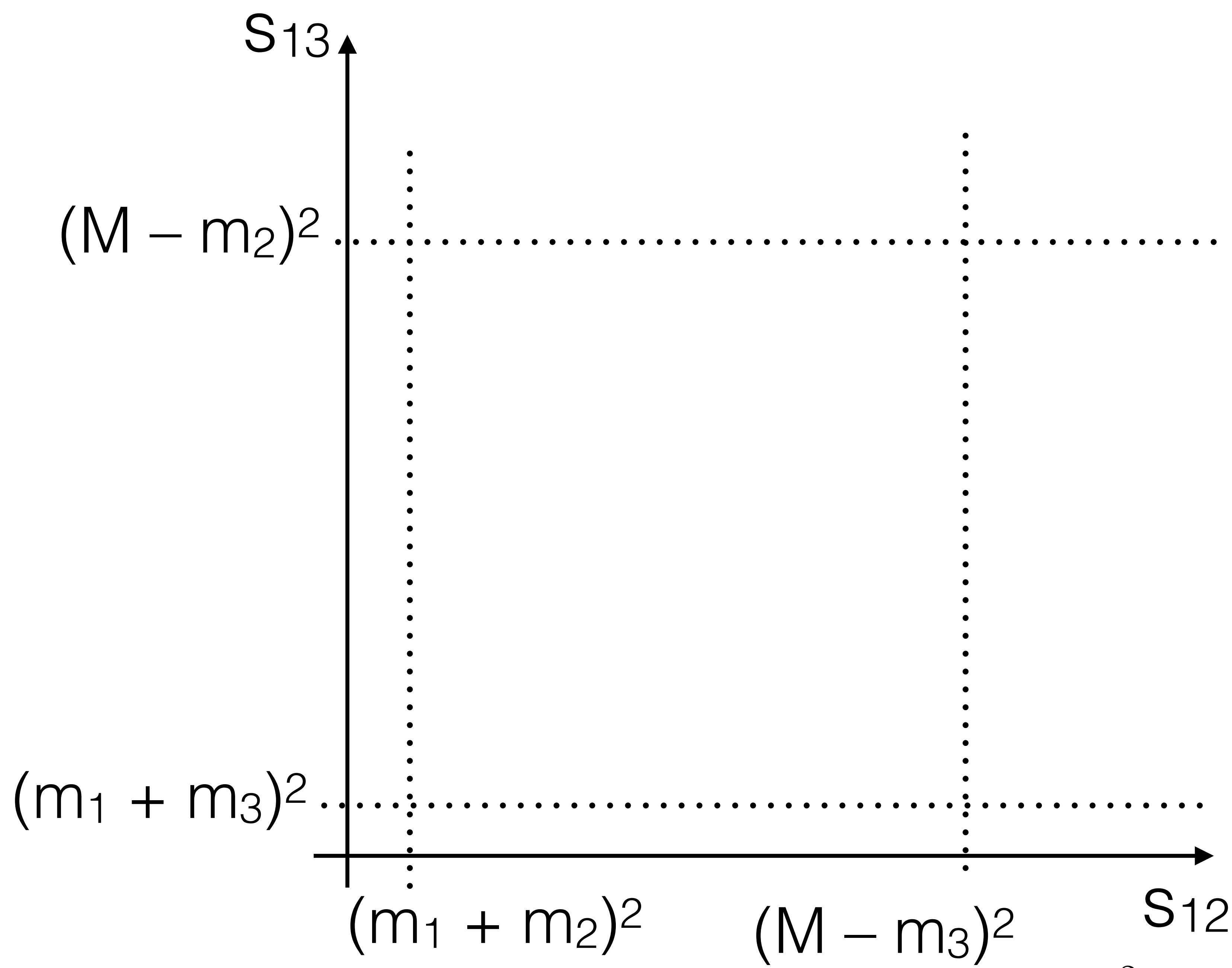

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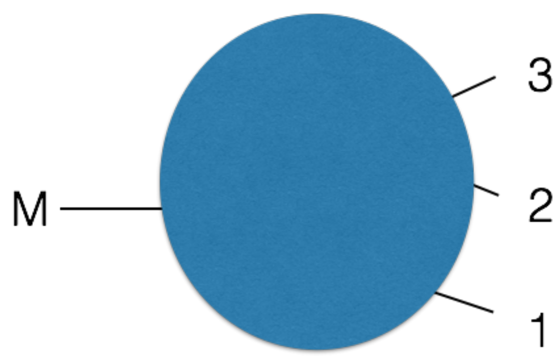

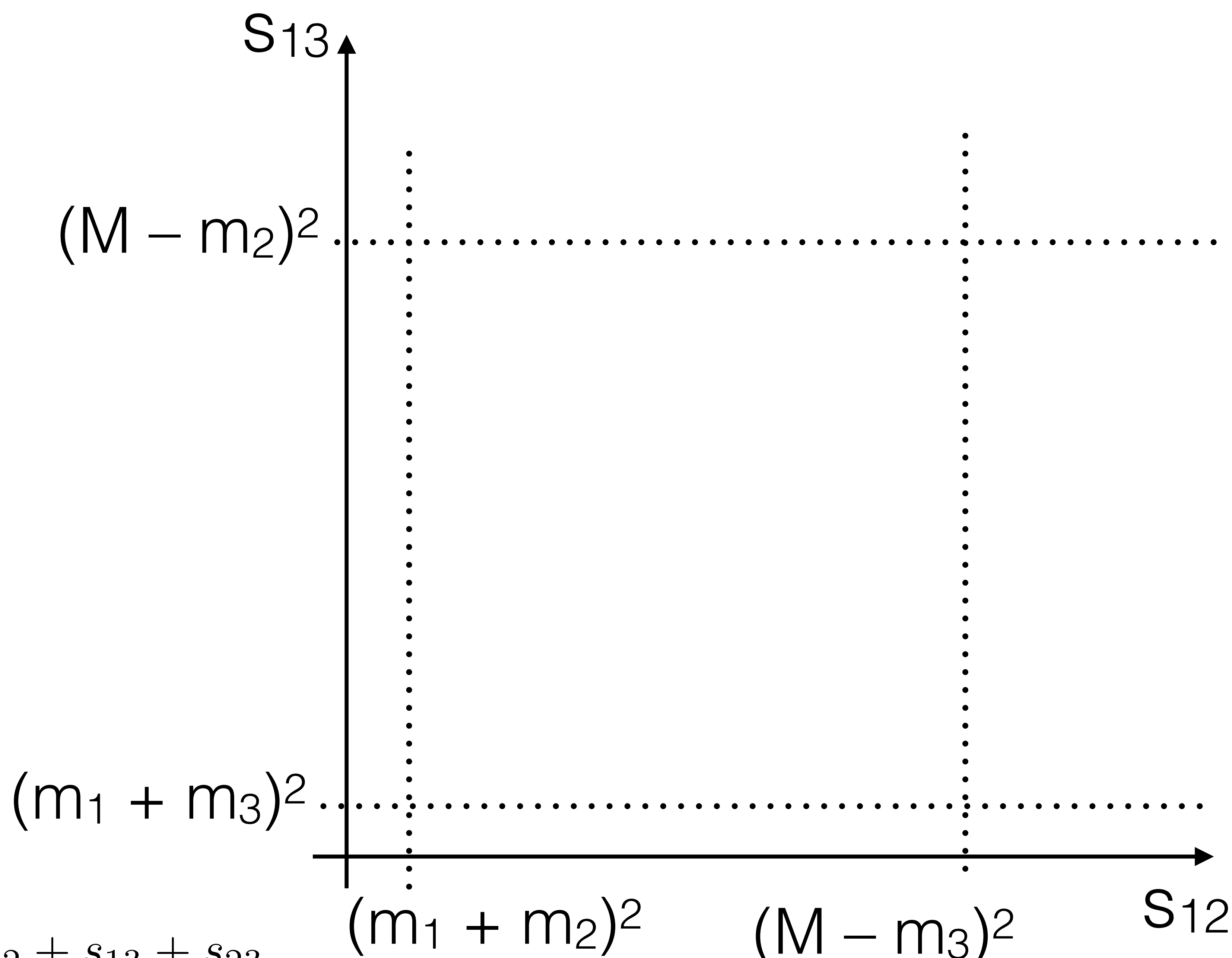

$$M^2 + m_1^2 + m_2^2 + m_3^2 = s_{12} + s_{13} + s_{23}$$

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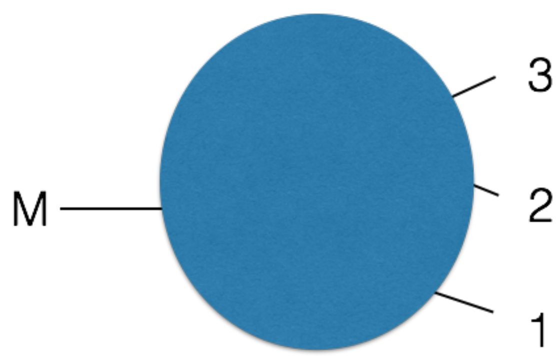

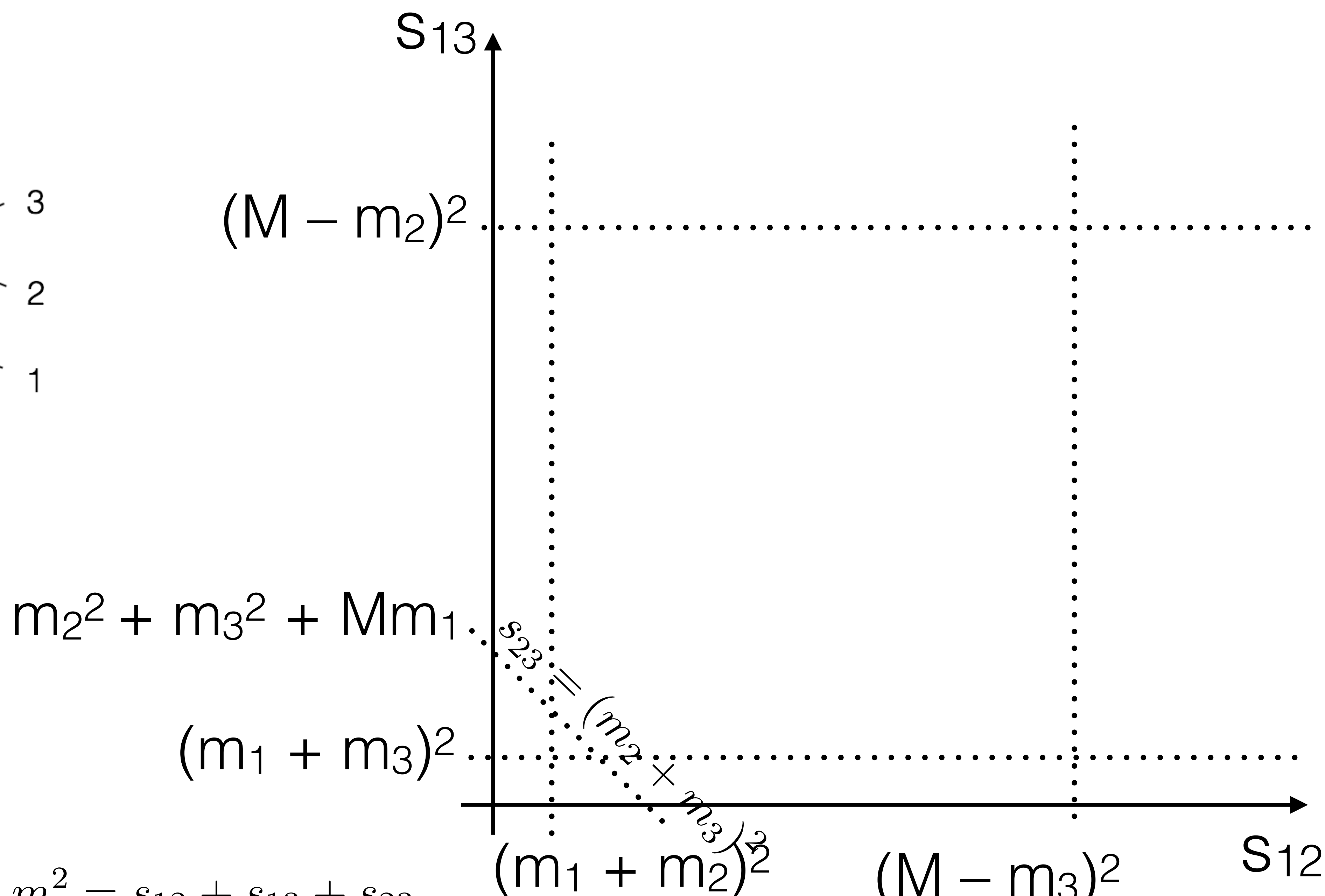

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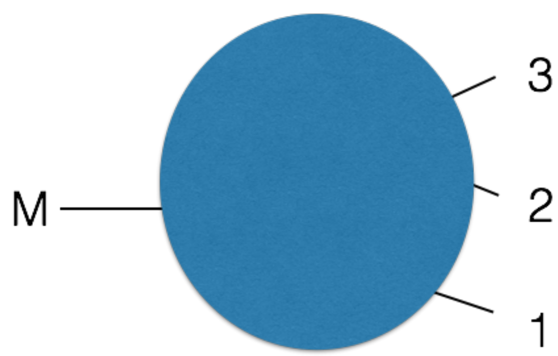

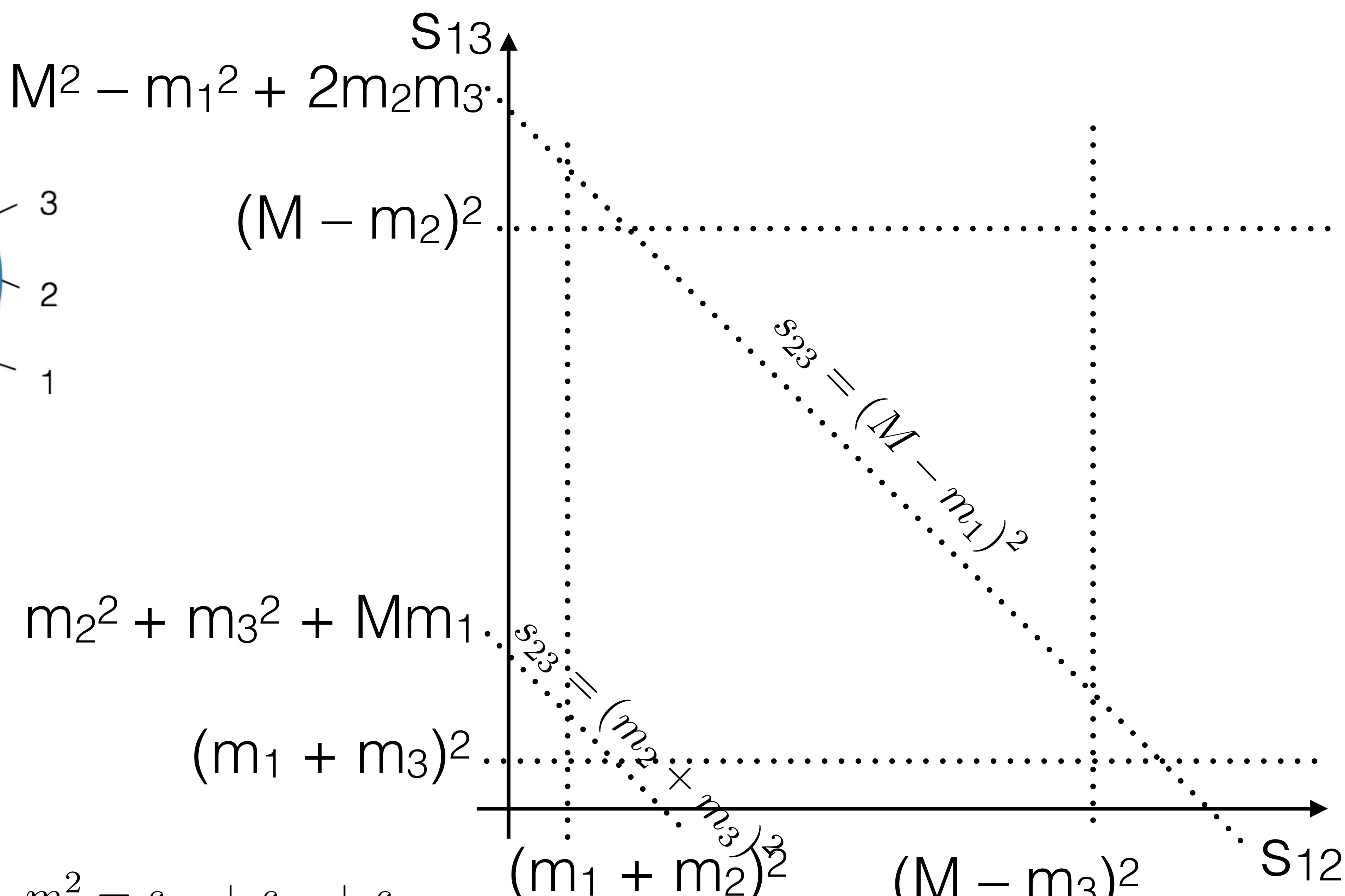

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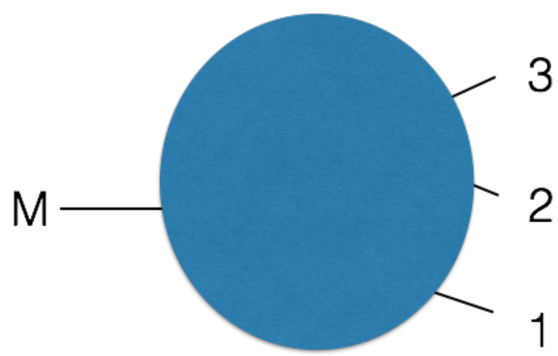

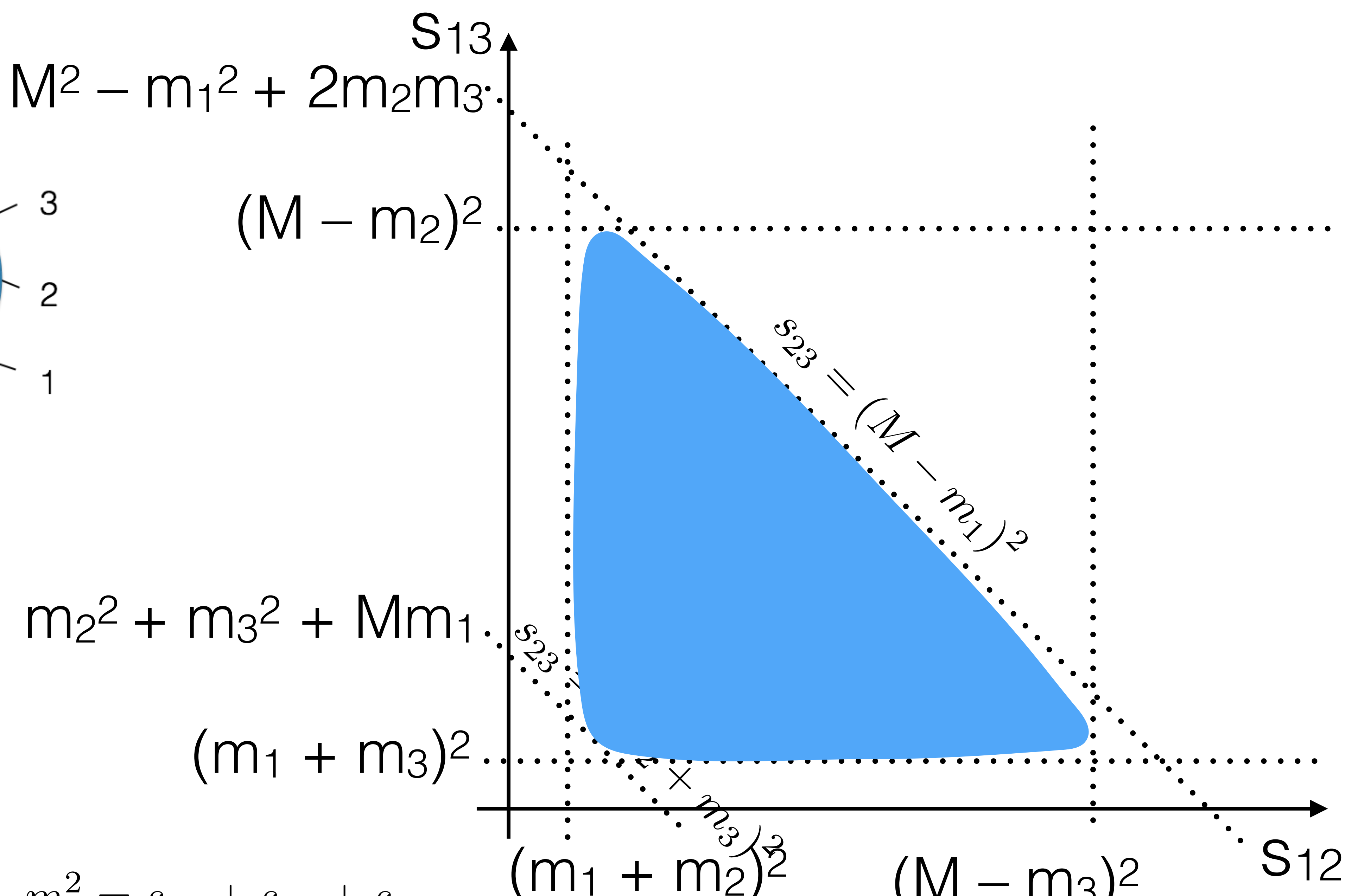

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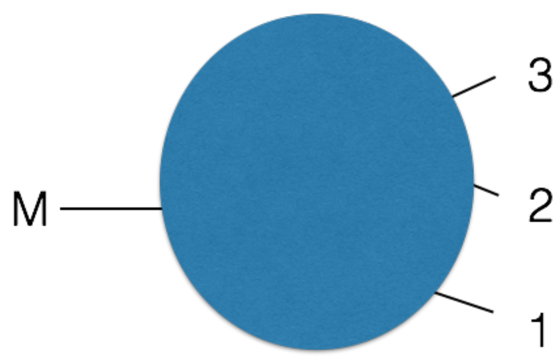
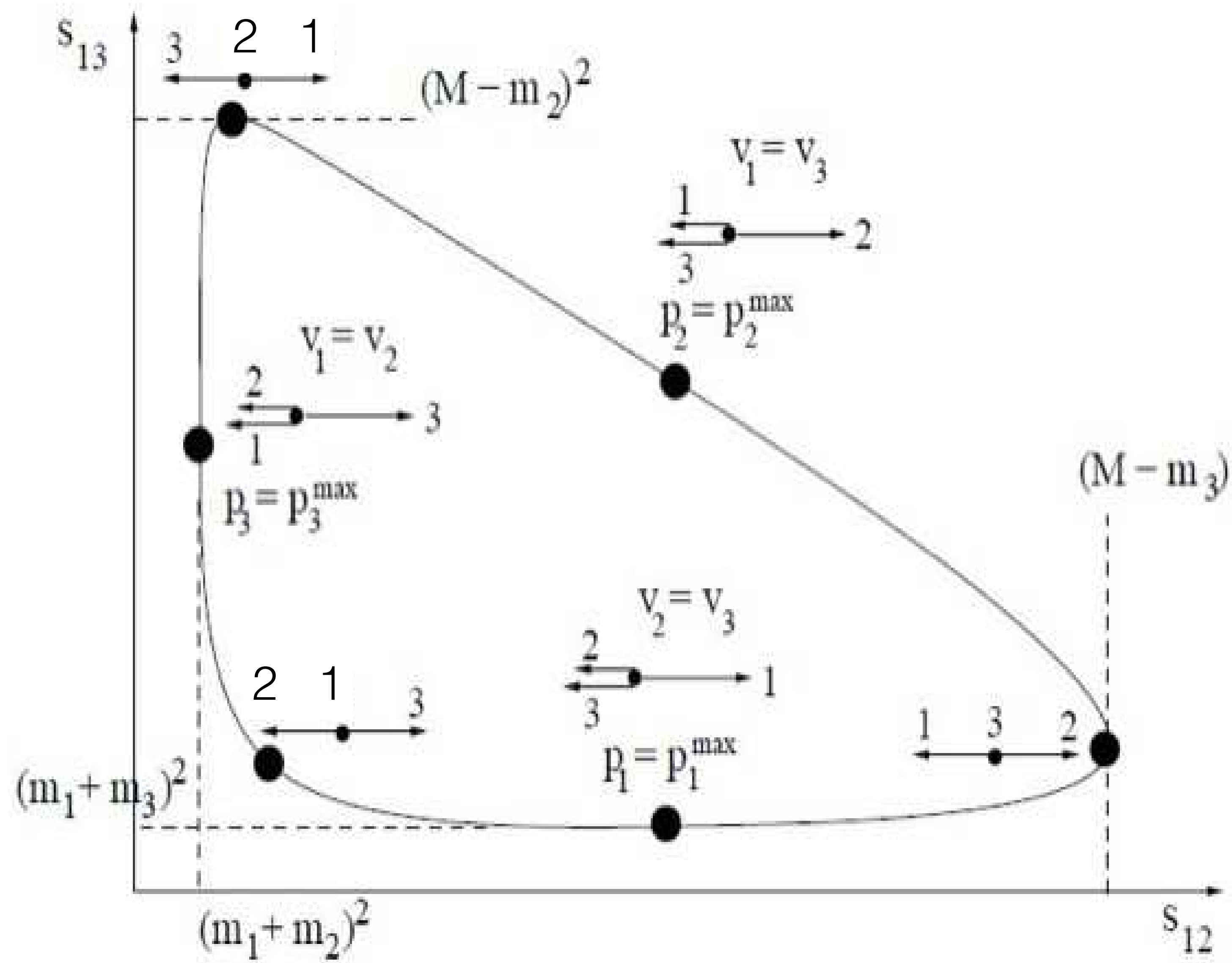
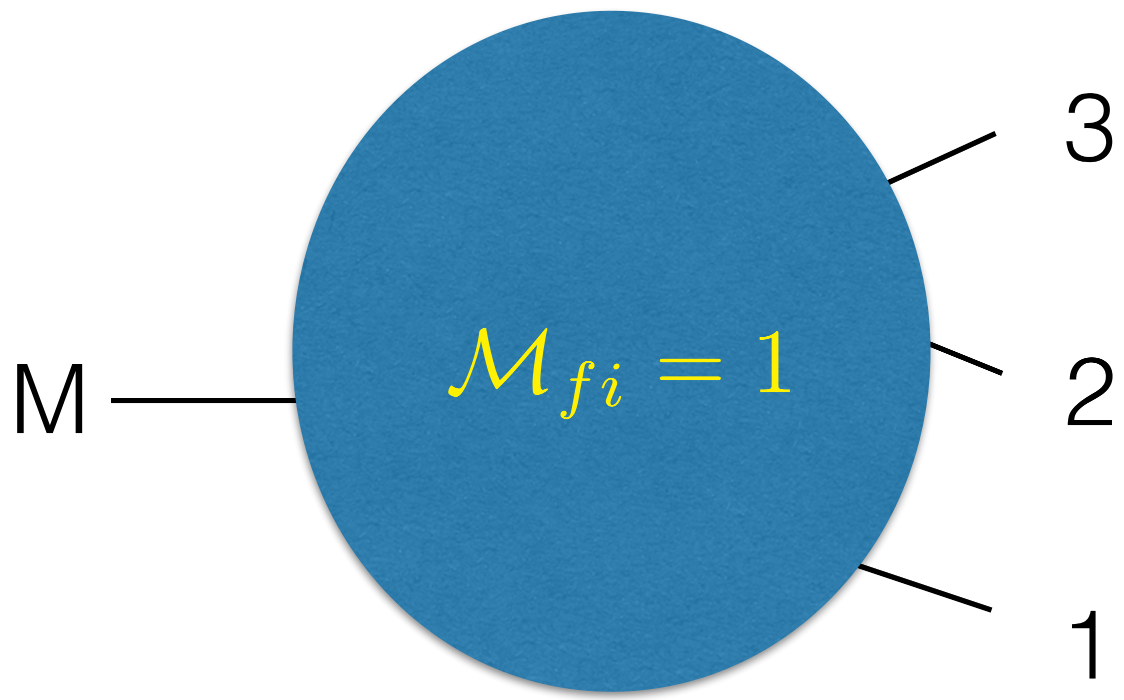
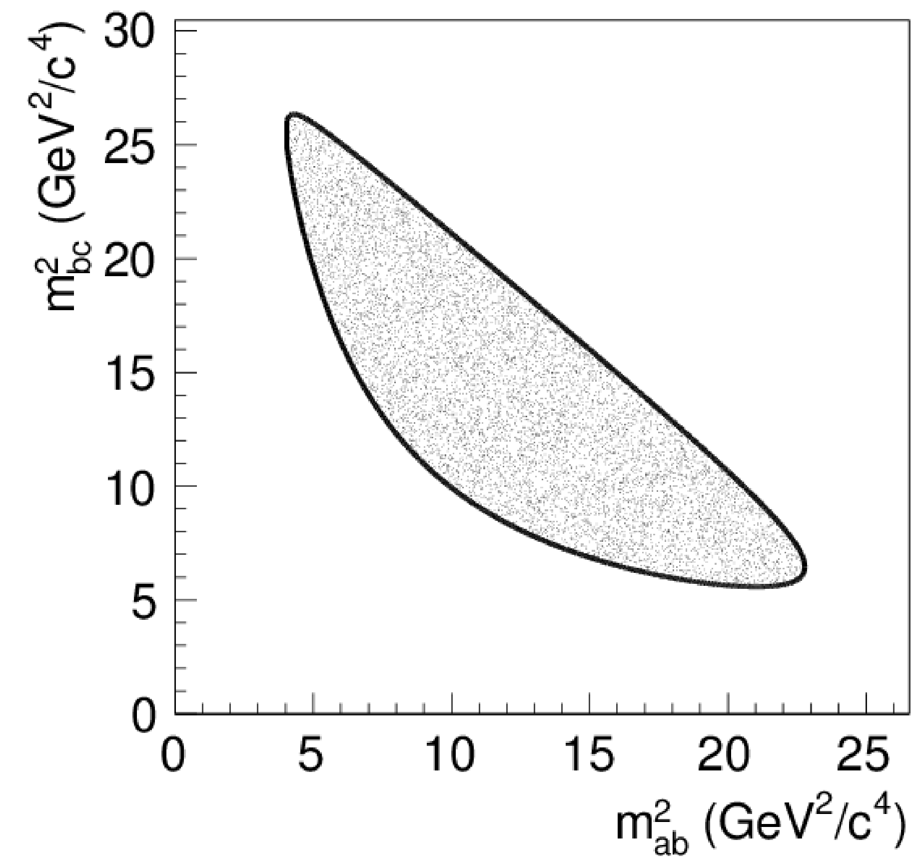




$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

What happens if nothing happens

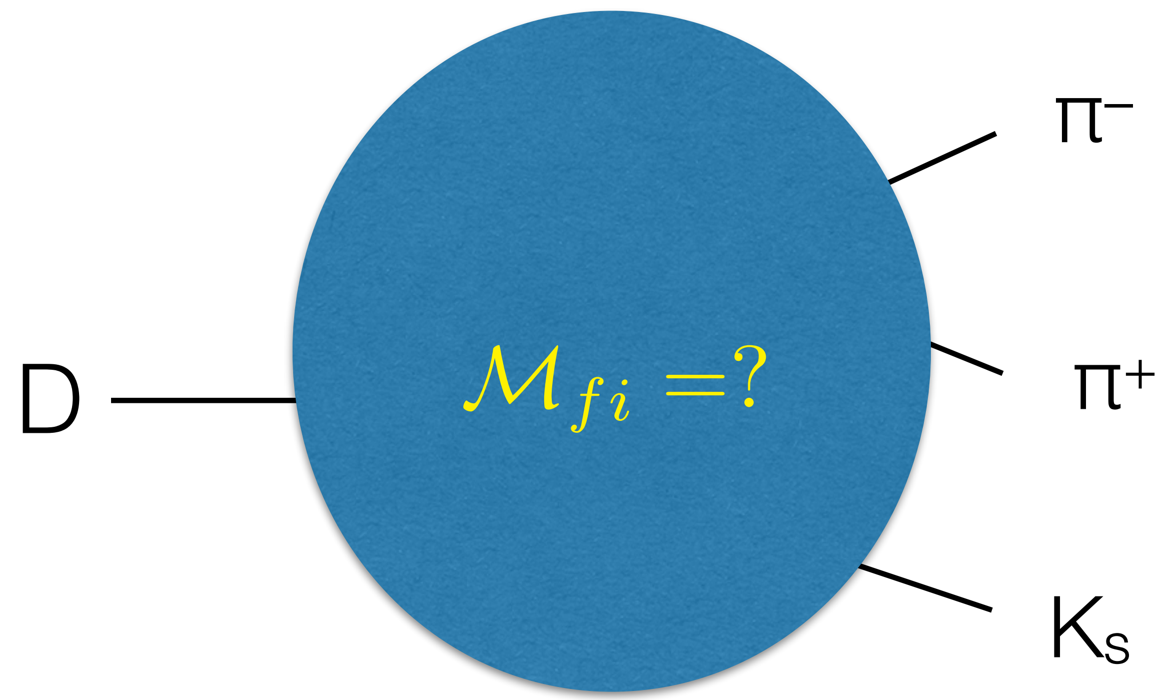


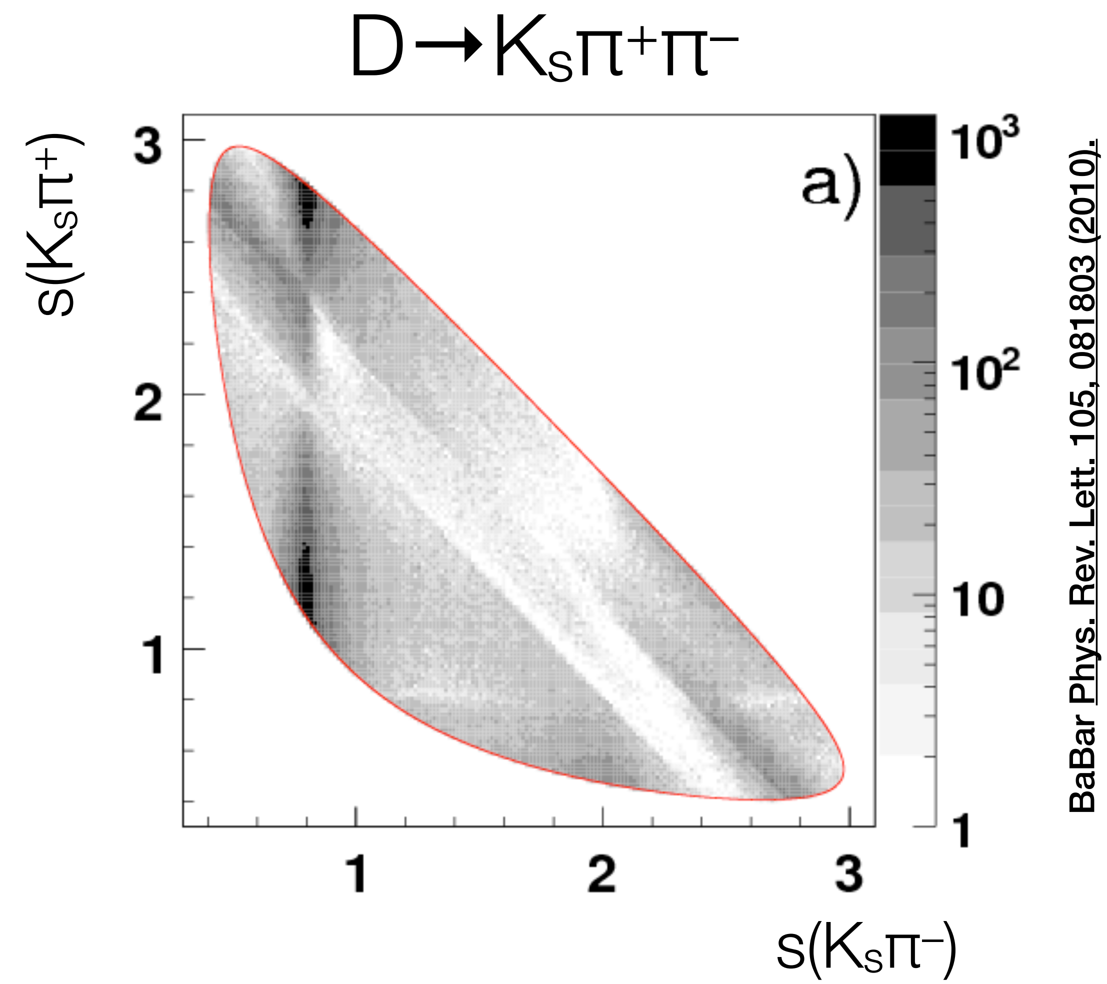


$$s_{ij} \equiv (p_i + p_j)^2 \equiv m^2_{ij}$$

$$d\Gamma = \frac{1}{(2\pi)^2\, 32 M^3} |\cancel{\mathcal{M}}_{fi}|^2 ds_{12} ds_{13}$$

What really happens





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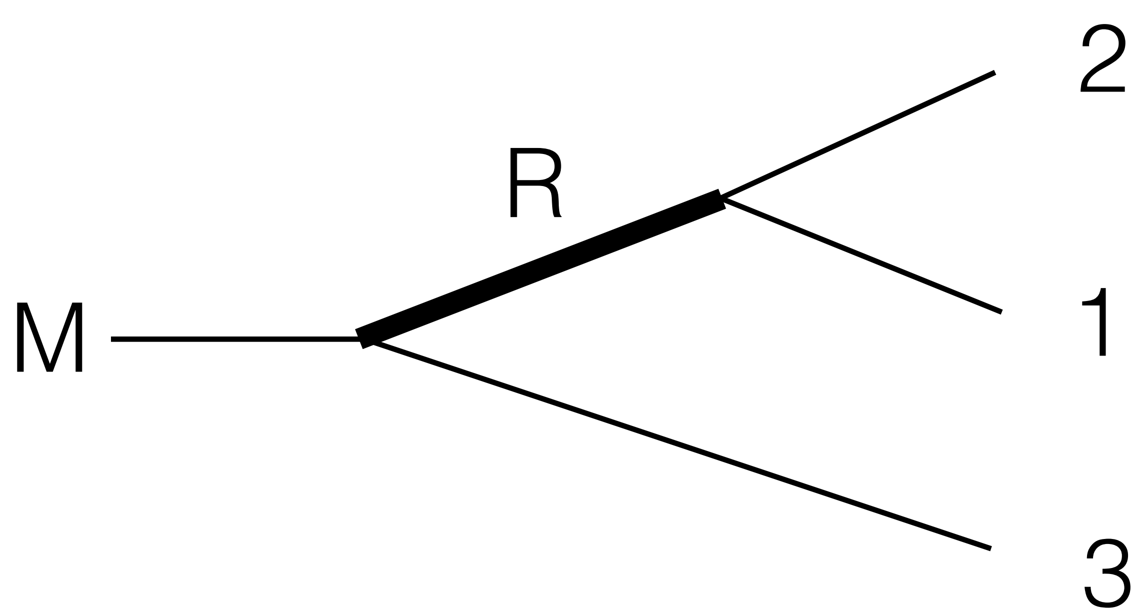

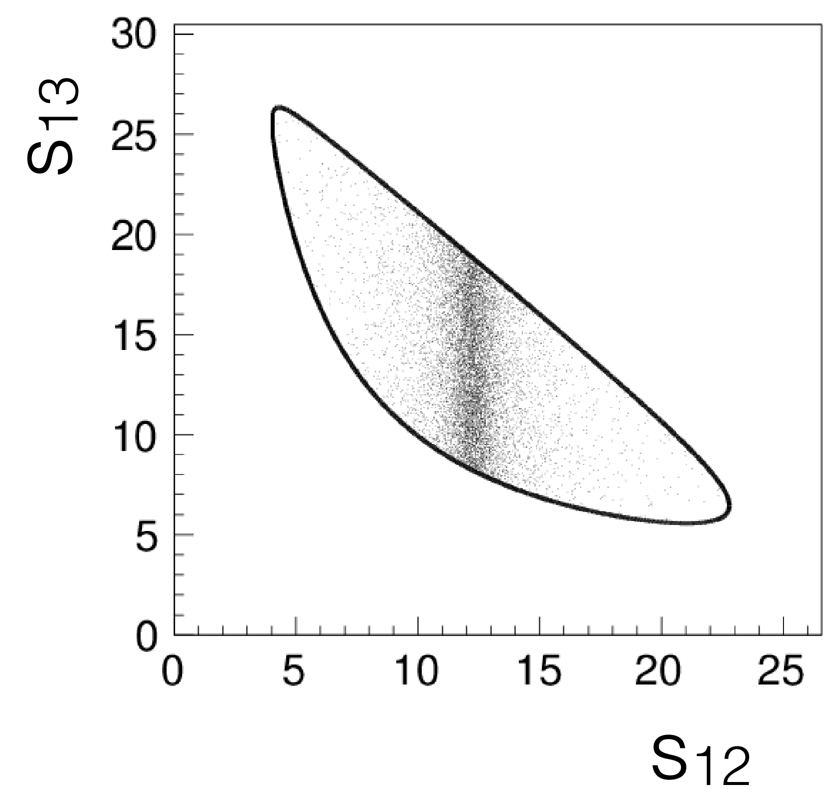

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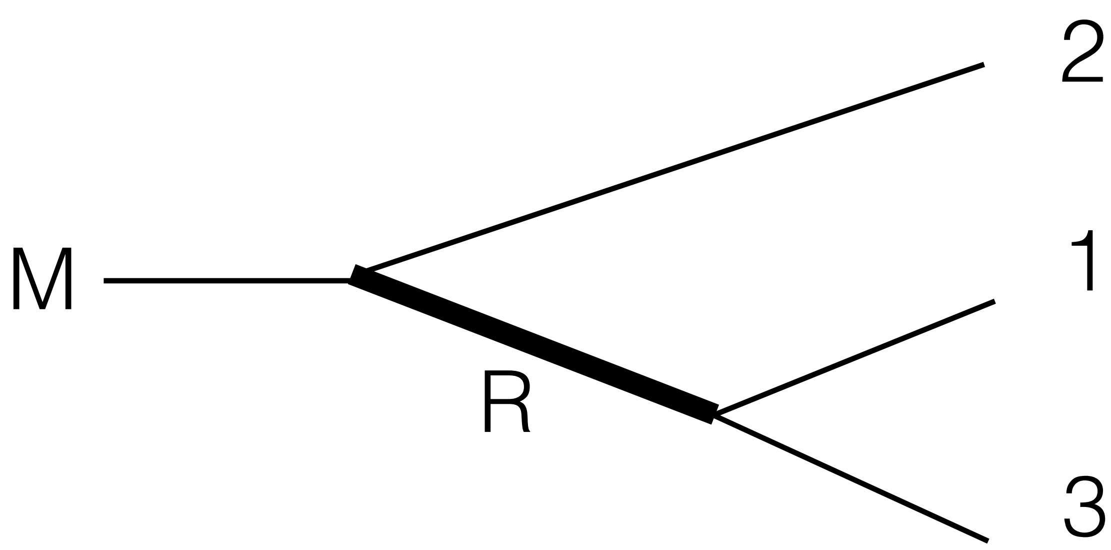

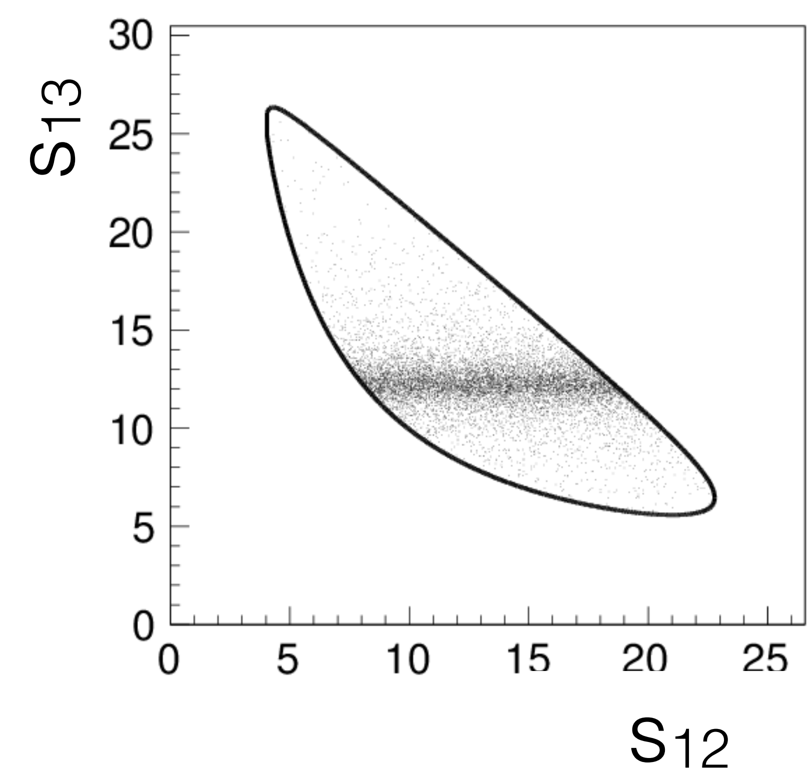

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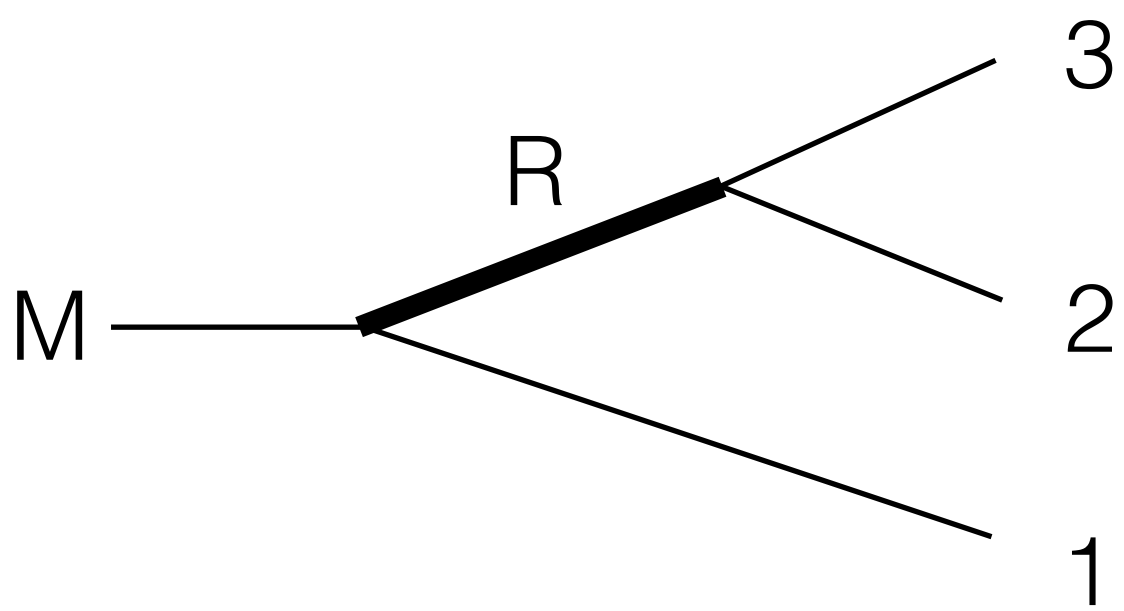

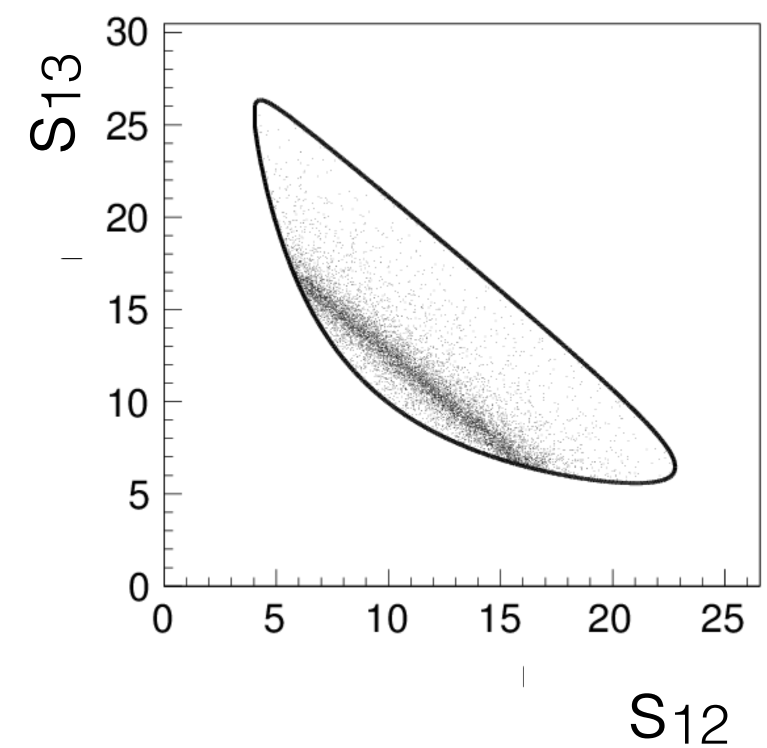

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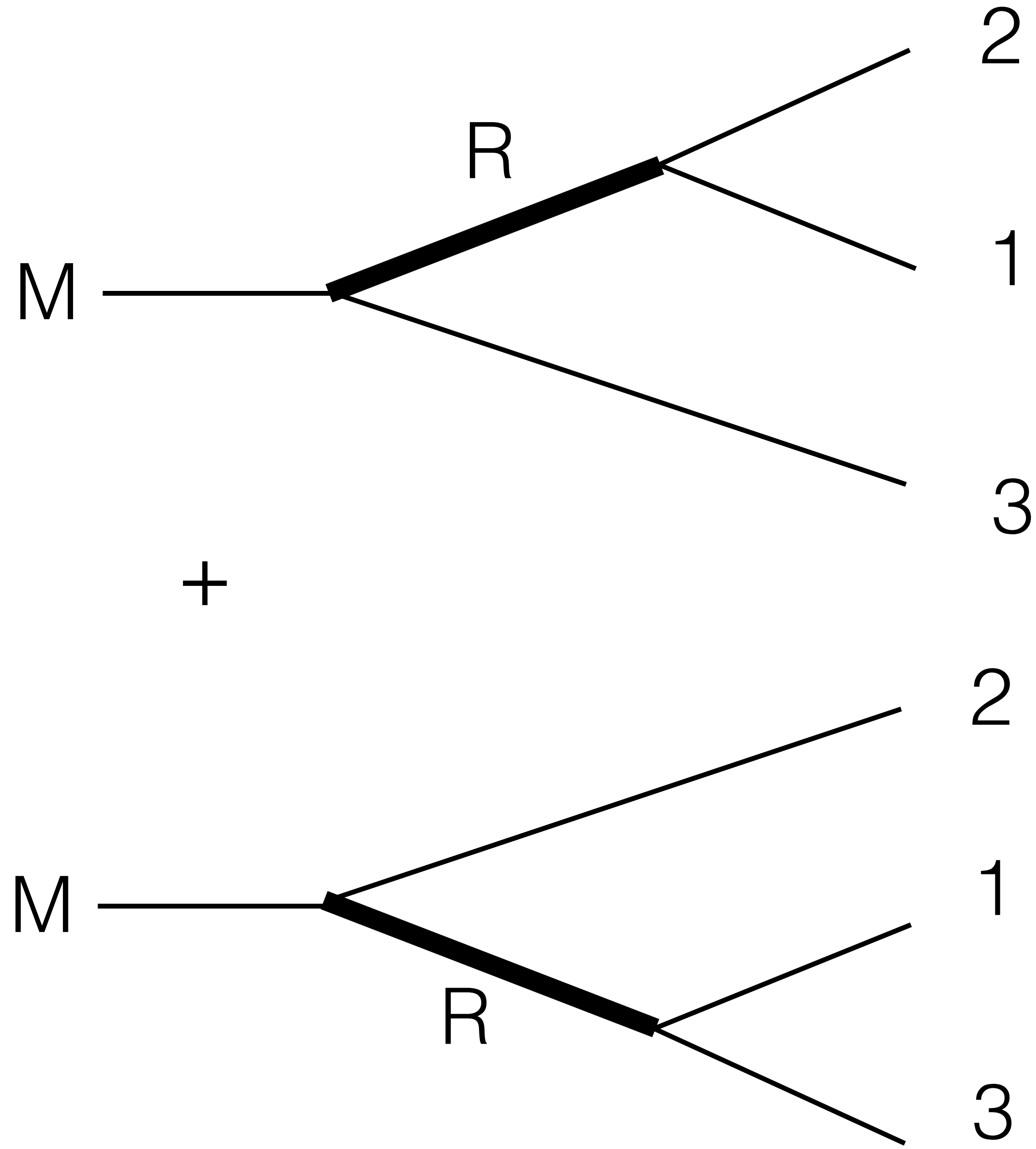
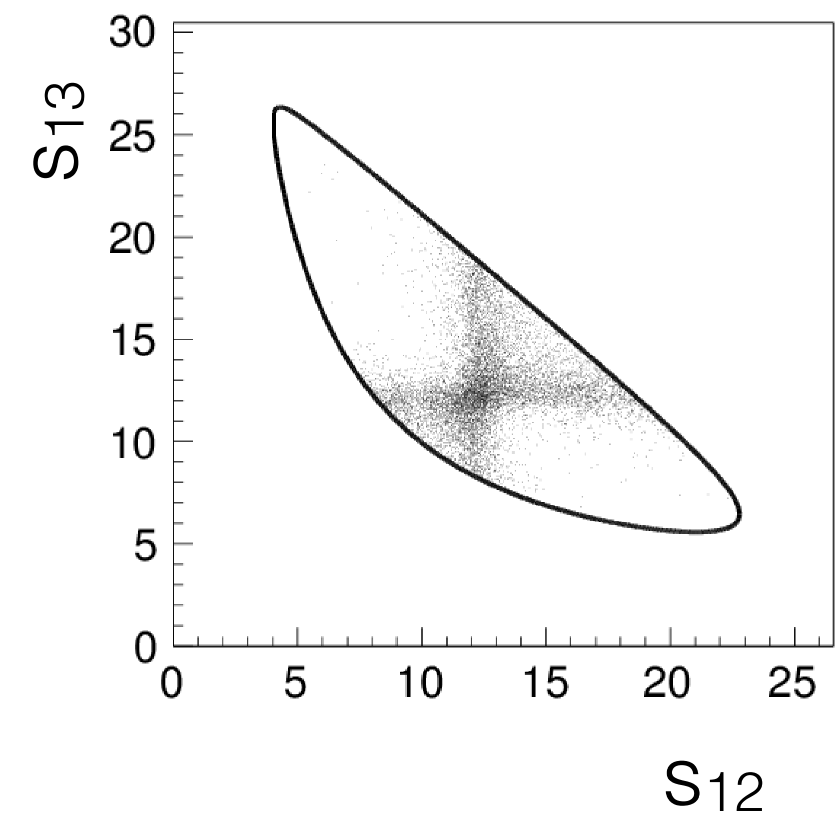
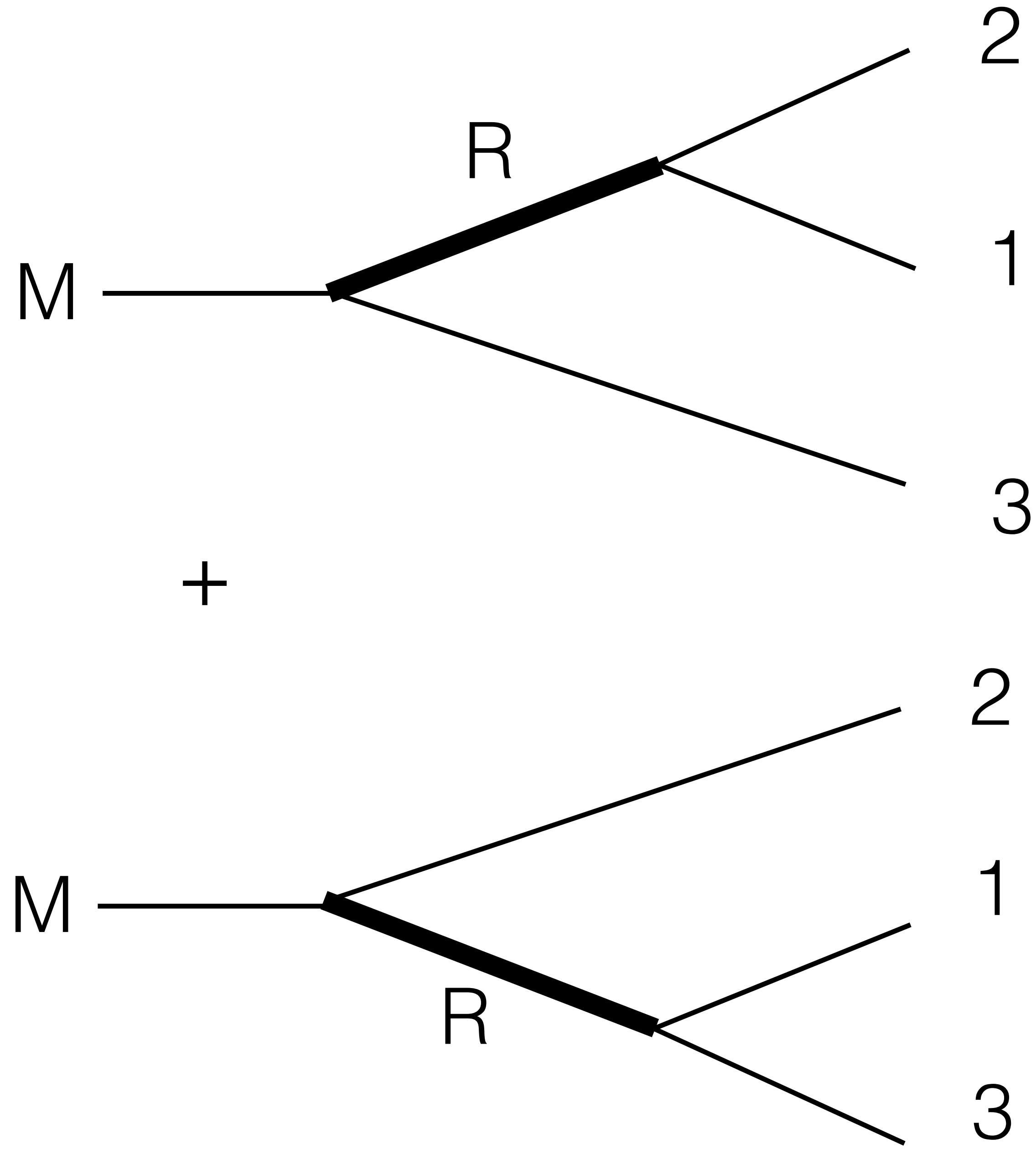
What happens if two things happens

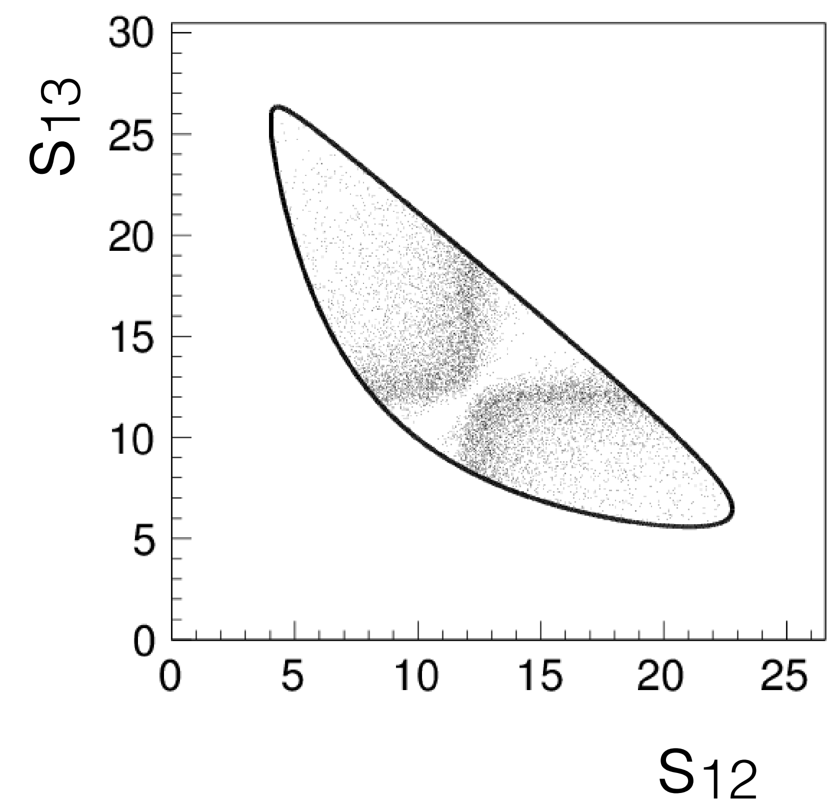




$$d\Gamma = \frac{1}{(2\pi)^2\, 32 M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

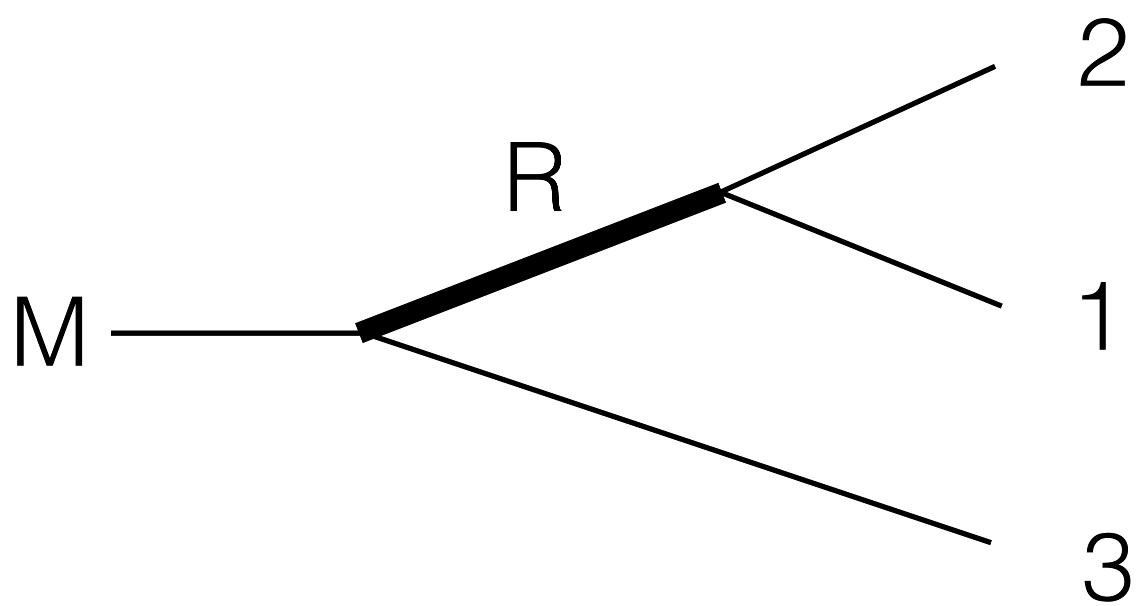
What happens if two things happens

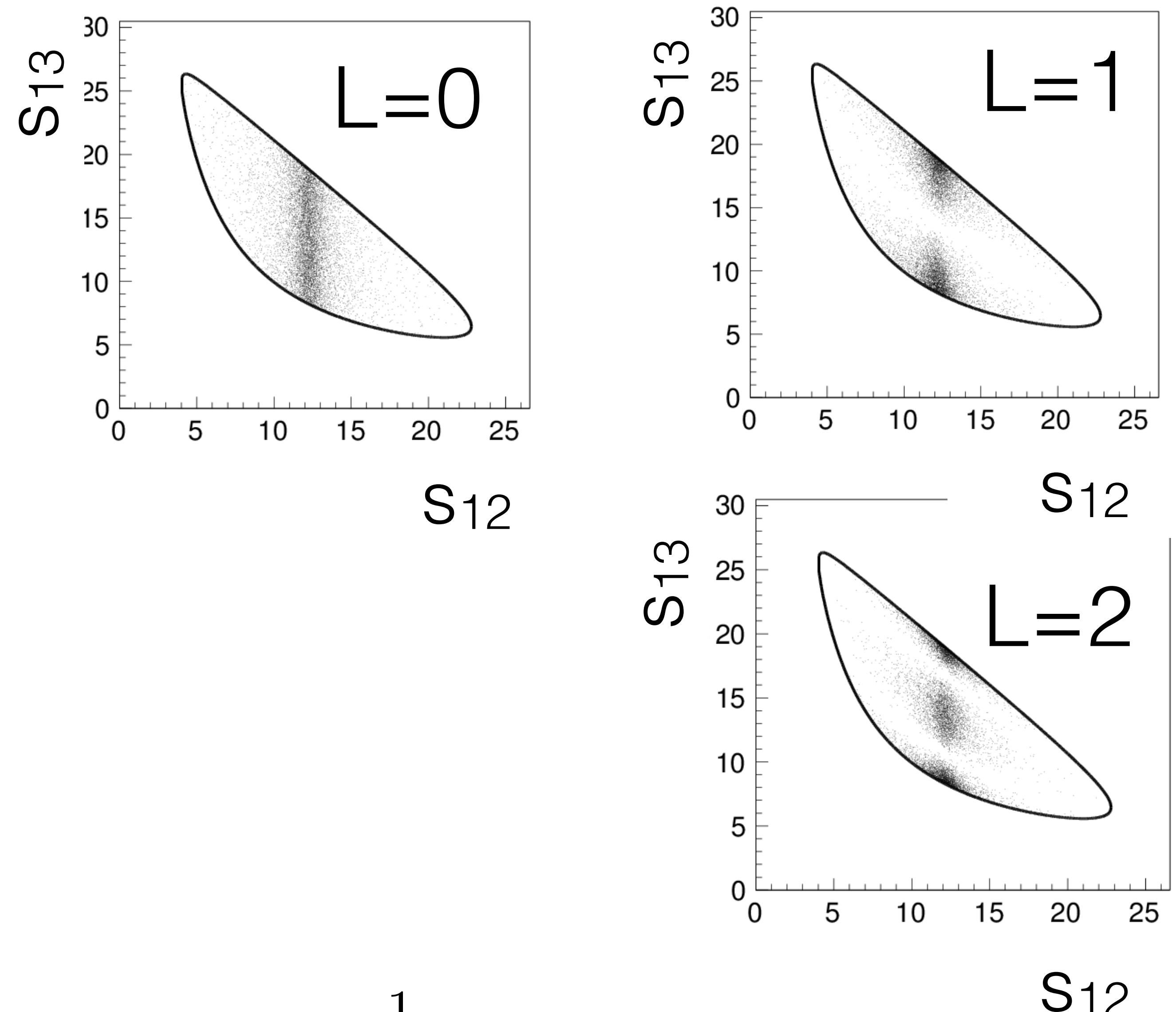




$$d\Gamma = \frac{1}{(2\pi)^2\, 32 M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

What happens if something with spin happens





$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

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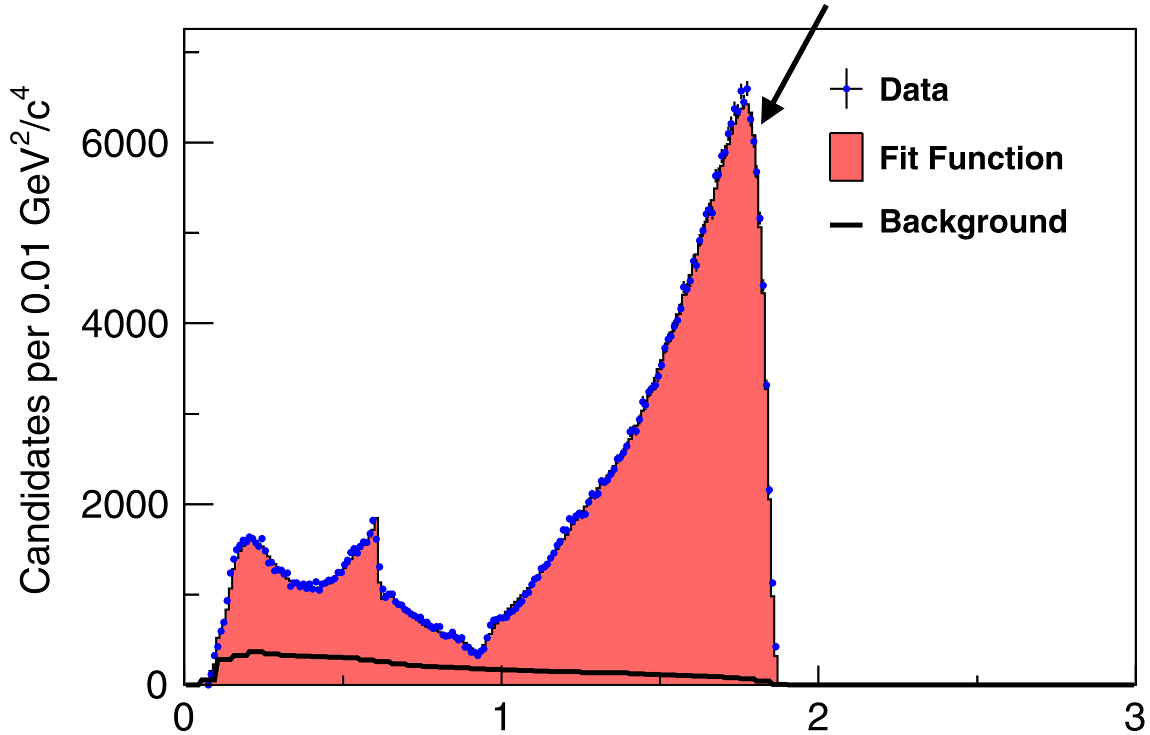
Why you must do an amplitude analyses if you want to find *real* new resonances in multi body decays.

$D^o \rightarrow K_S\pi\pi$

$\pi\pi$ resonance near $m^2 = 2GeV^2$?



$m^2(\pi\pi)/GeV^2$

CDF PHYSICAL REVIEW D 86, 032007 (2012) (no claim of any bogus resonance is made in this paper, it's a completely sound paper about CPV in charm).

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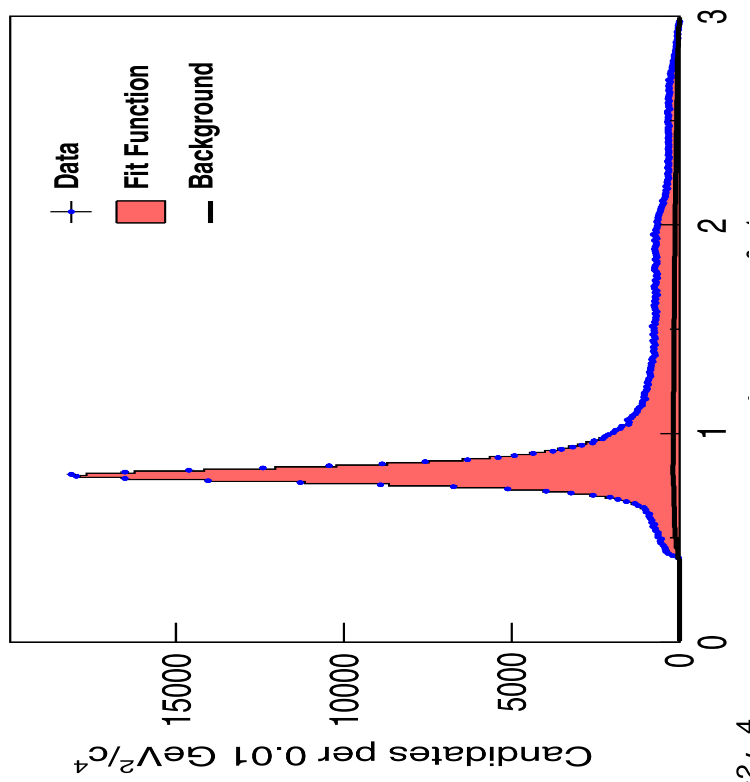

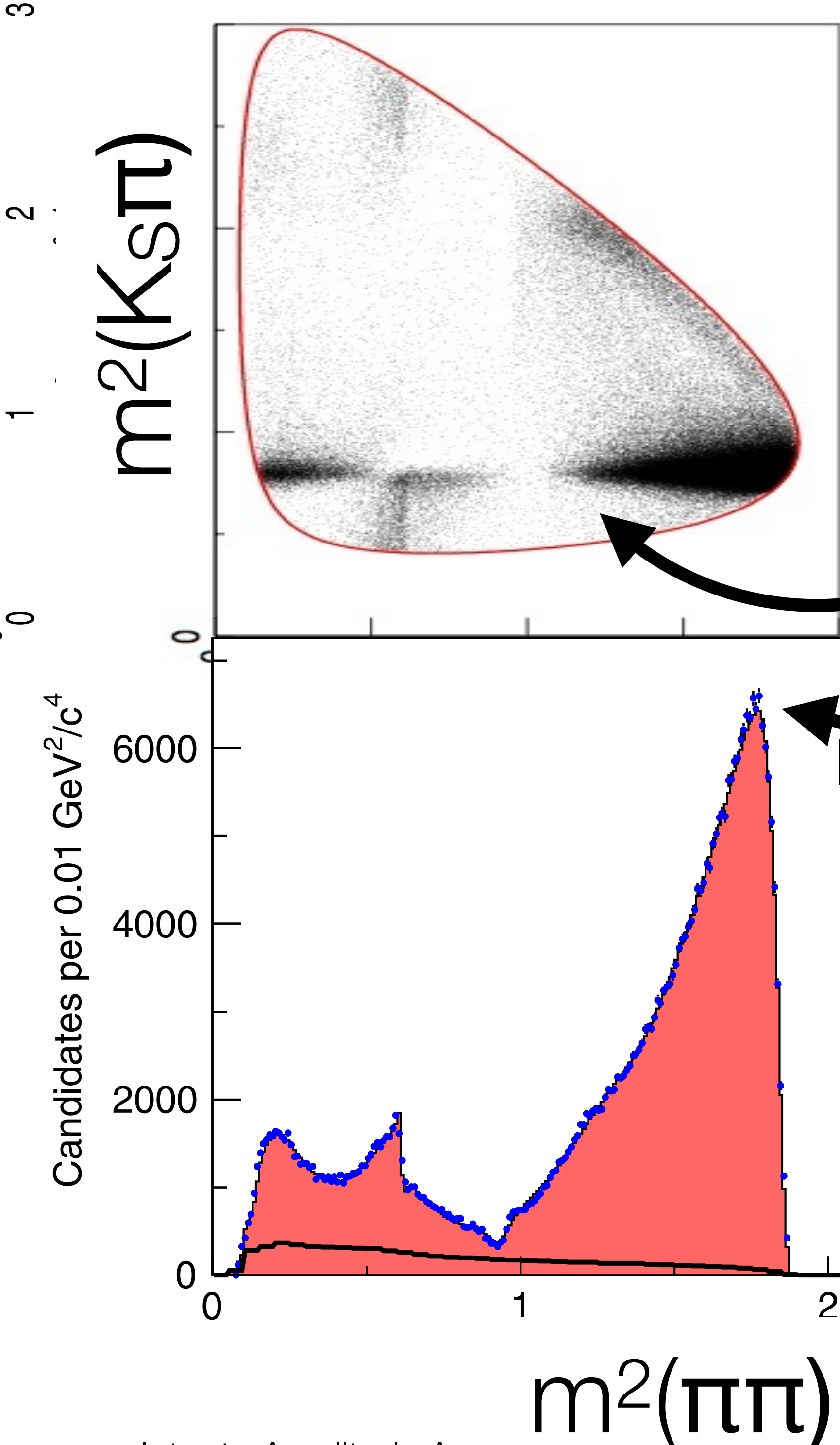

Structure due to angular distribution in D→K*(K$_S$π)π

Not a (new or old) ππ resonance

D^0→K$_S$ππ

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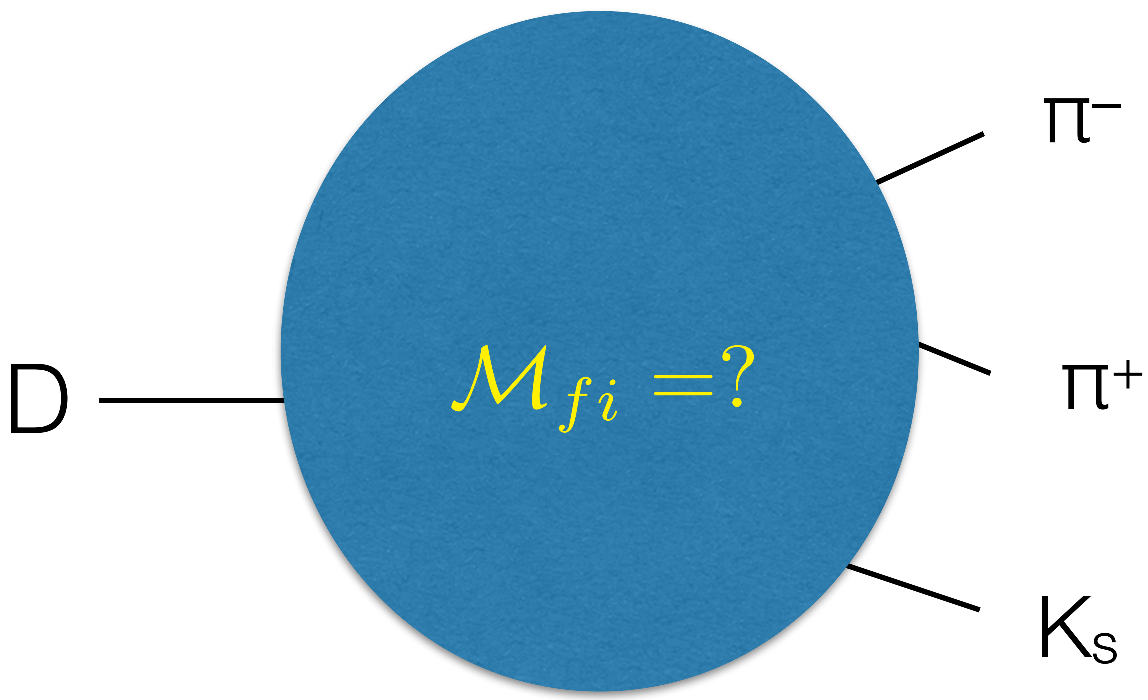
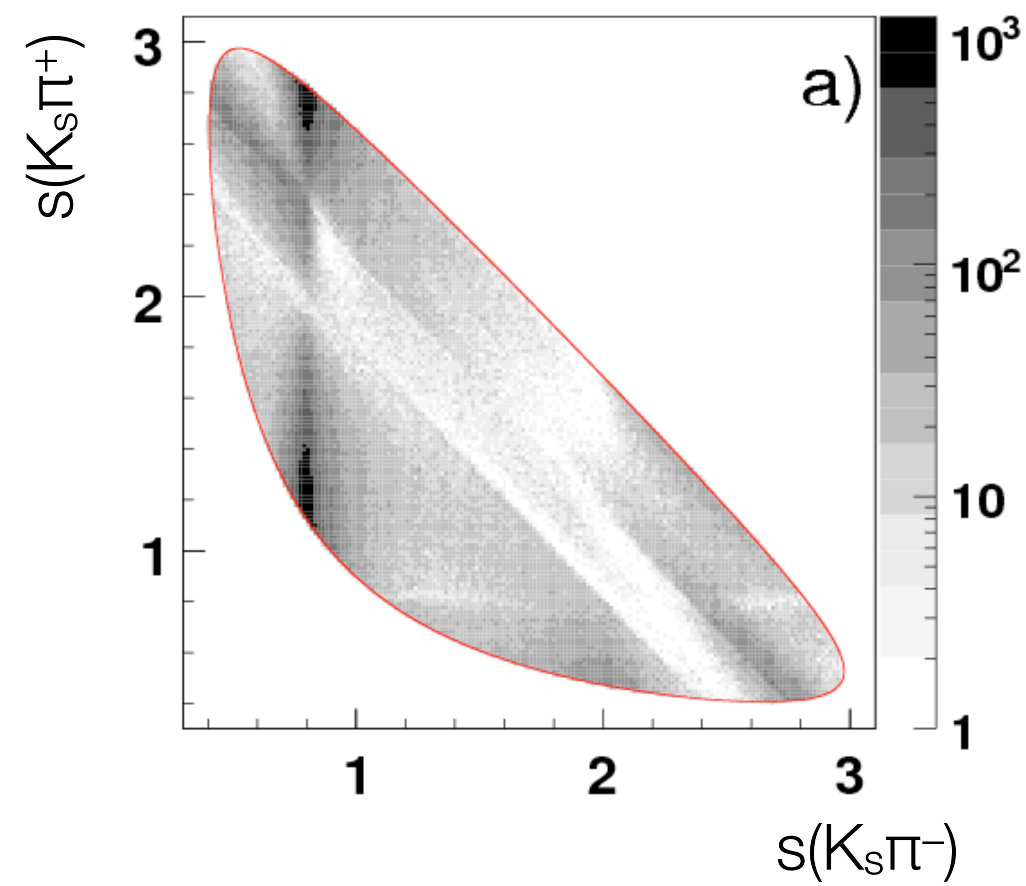
Real Dalitz plots



D→K$_S$π$^+$π$^-$



BaBar Phys. Rev. Lett. 105, 081803 (2010).

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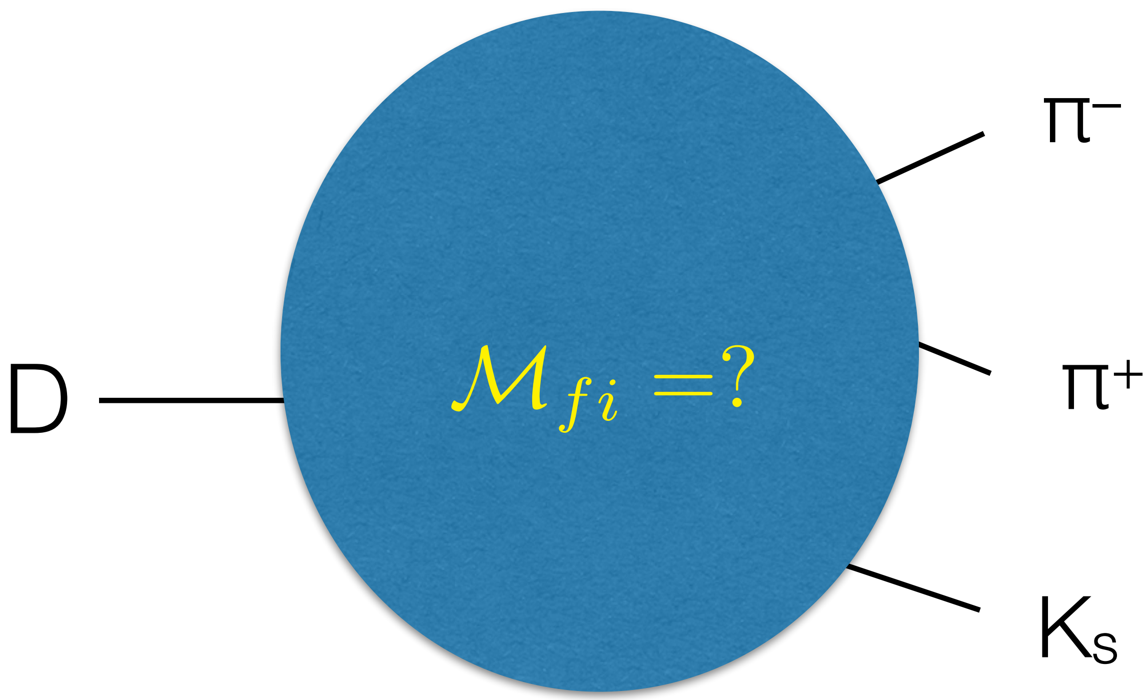
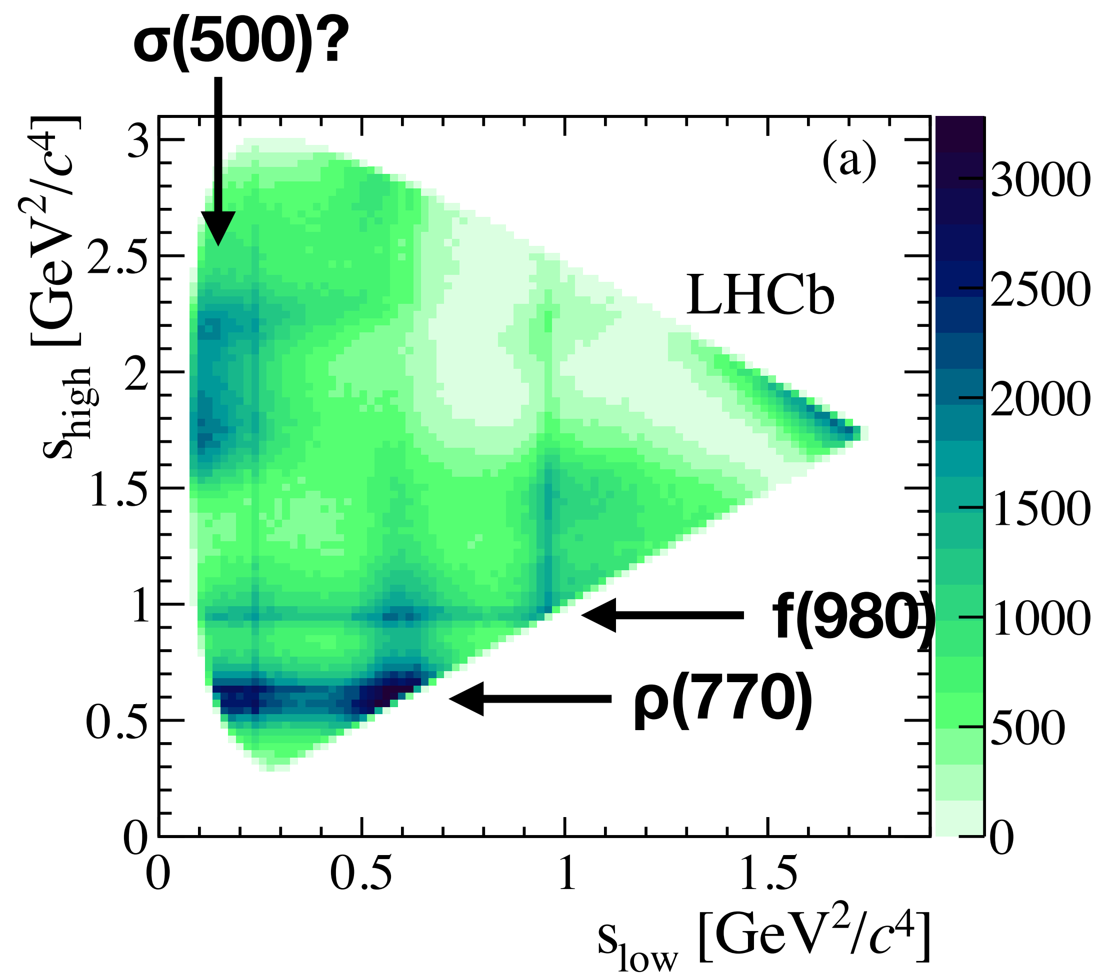
Real Dalitz pots



2.4M D± → π±π∓π± decays (LHCb)



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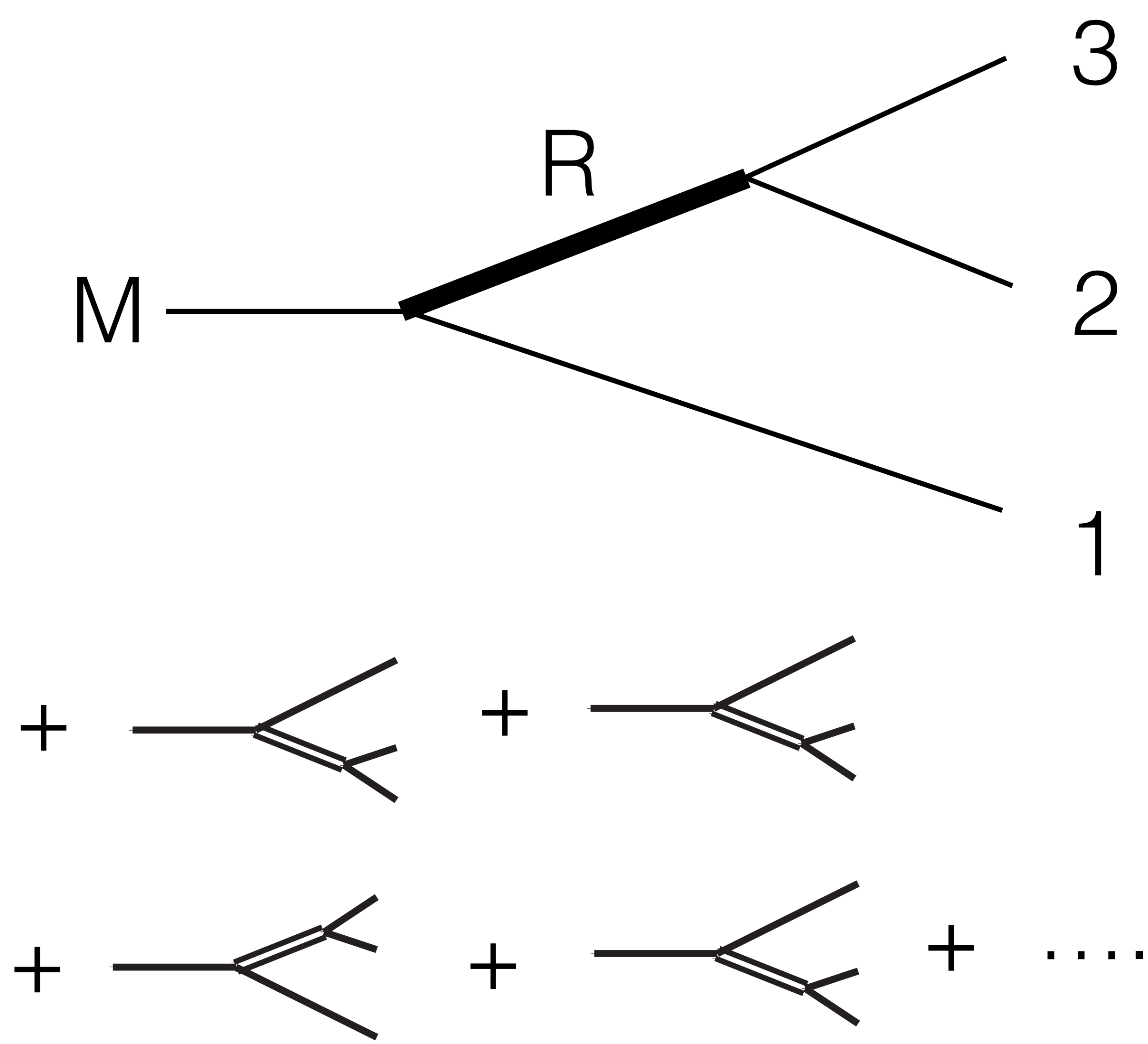
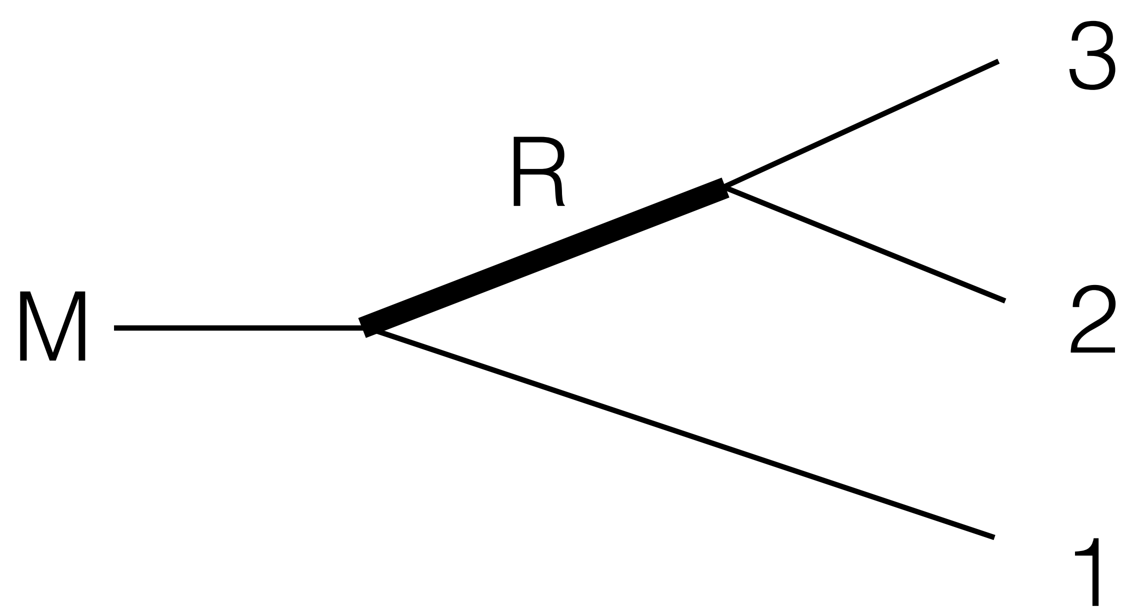
Calculating amplitudes



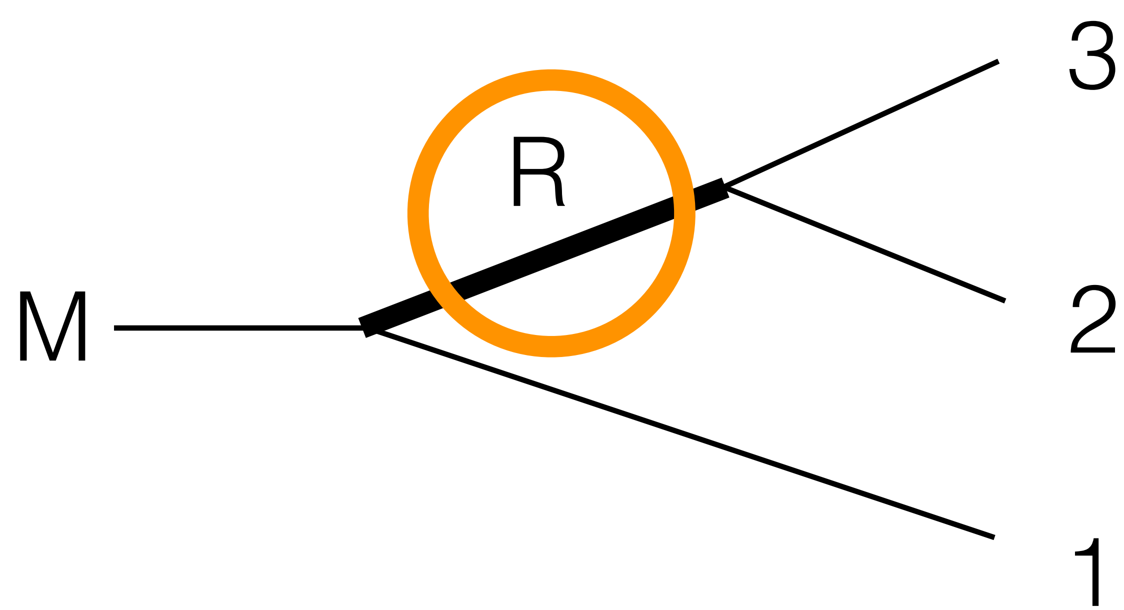
- Let us assume(!) that the full amplitude can be calculated as the sum of essentially independent two body processes.
- Doing this results in the so-called “isobar” model.

Calculating amplitudes

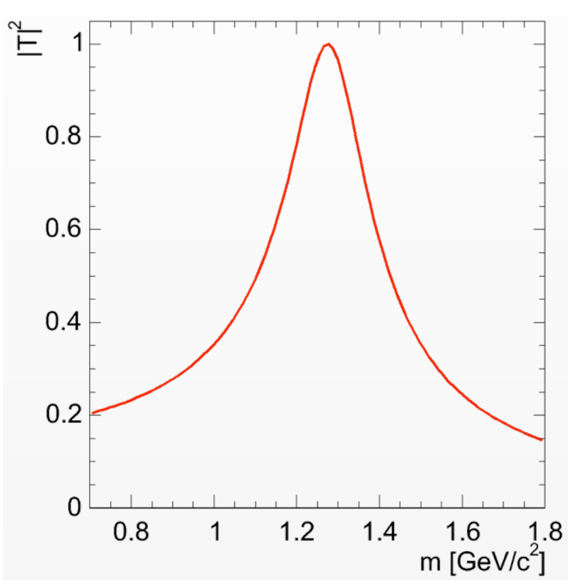


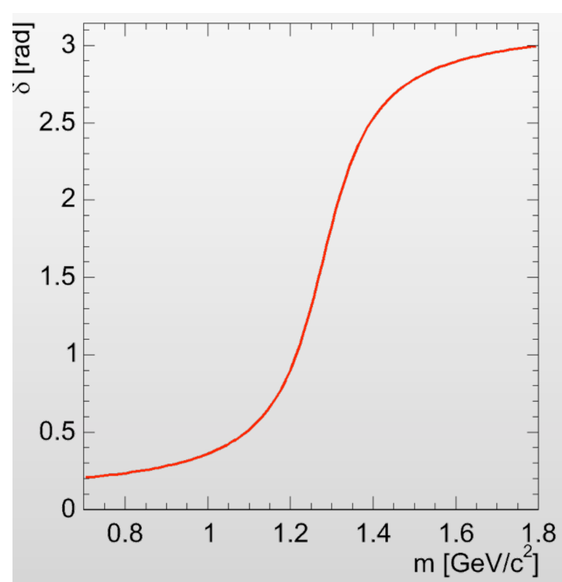
- We don't know anything about the strong interaction dynamics.
- As a first approximation, we treat each particle as point particle.
- We want a Lorentz-invariant matrix element…

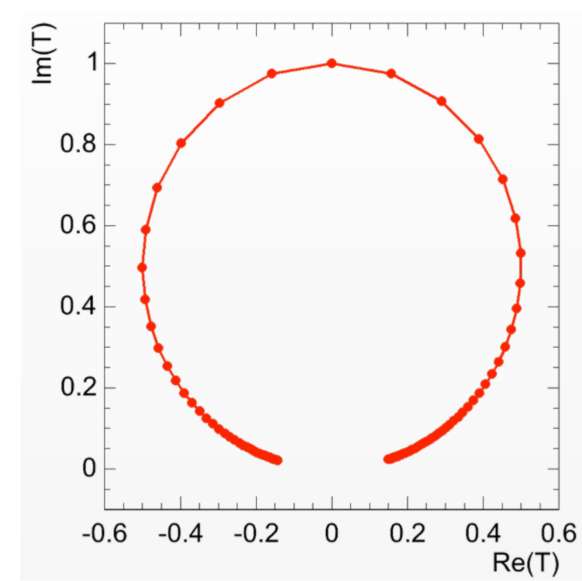
Calculating amplitudes



$$\frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$





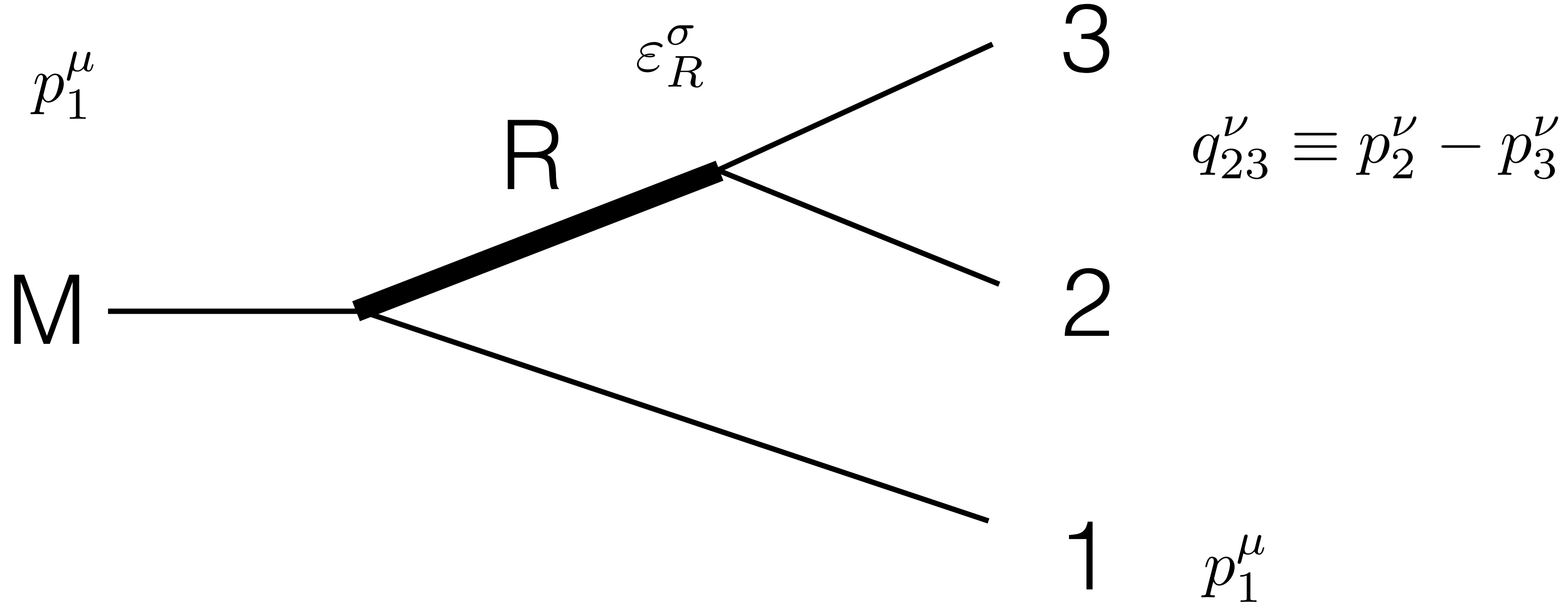


Calculating the amplitudes

ε_R^σ say R has spin 1 (e.g. K*(892), ρ(770) etc)

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



Calculating the amplitudes

ε_R^σ

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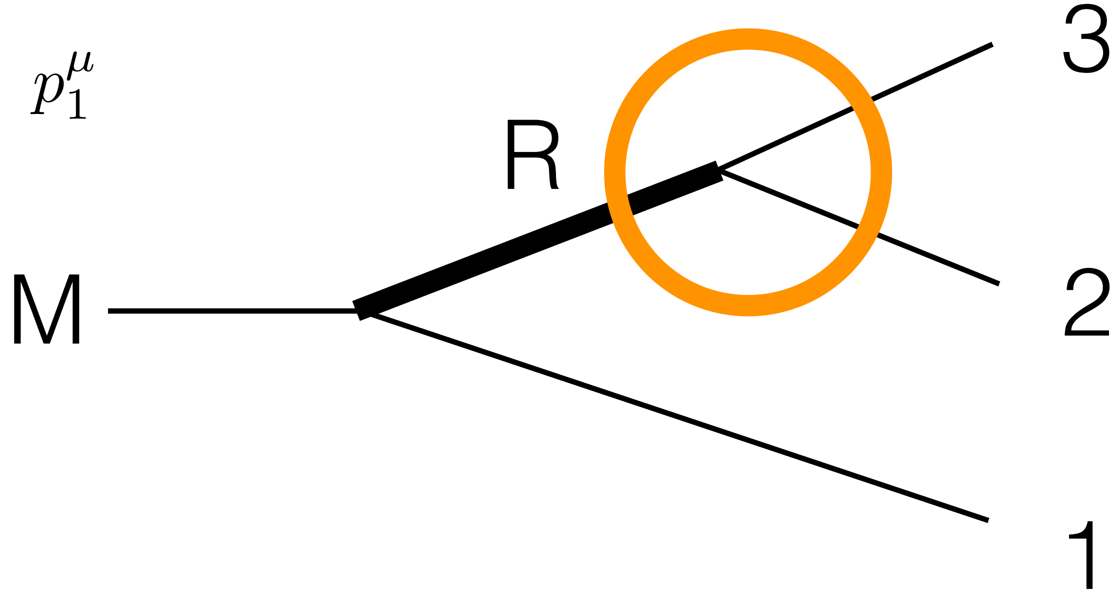
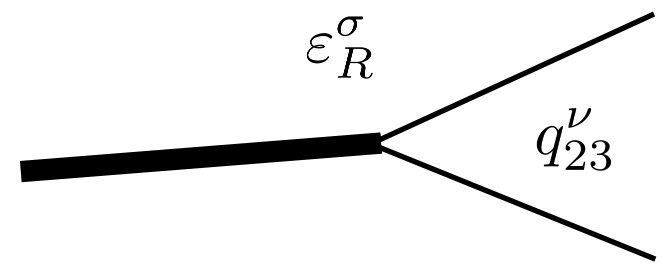




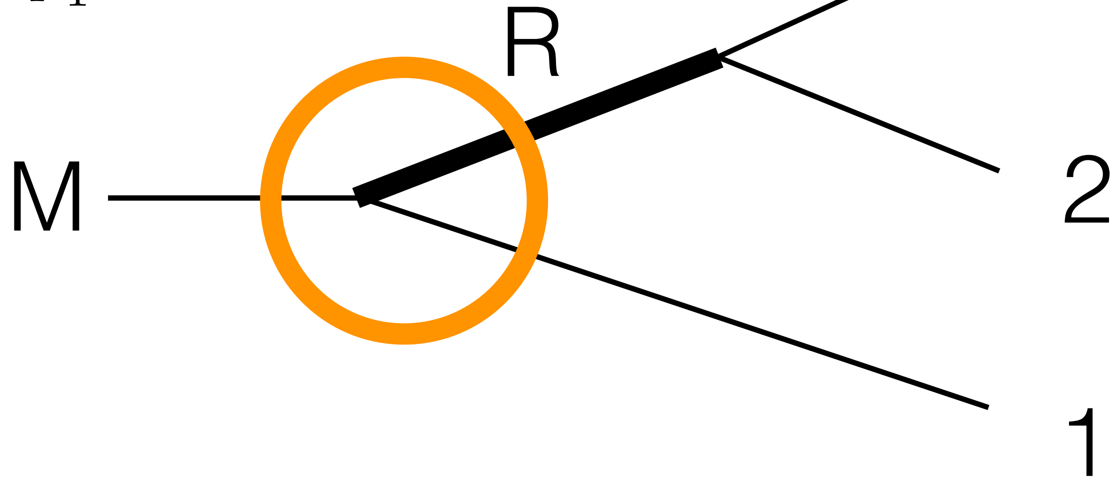
$$\frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

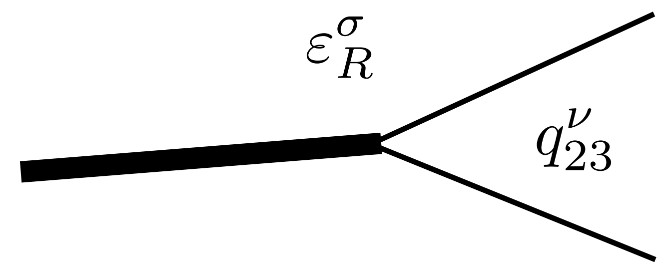
Calculating the amplitudes

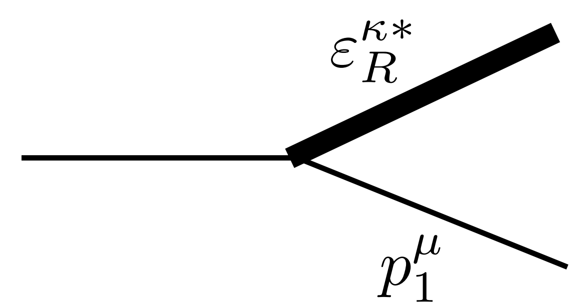
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







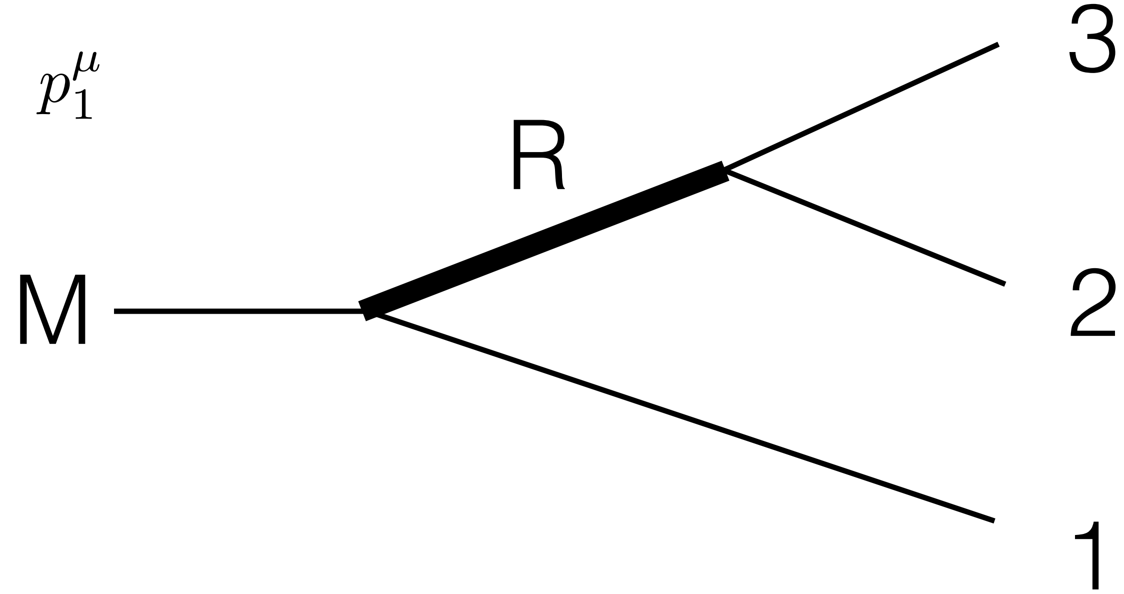
$$p_{1\,\mu} \quad \varepsilon_R^{\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\nu} \quad q_{23\;\nu}$$

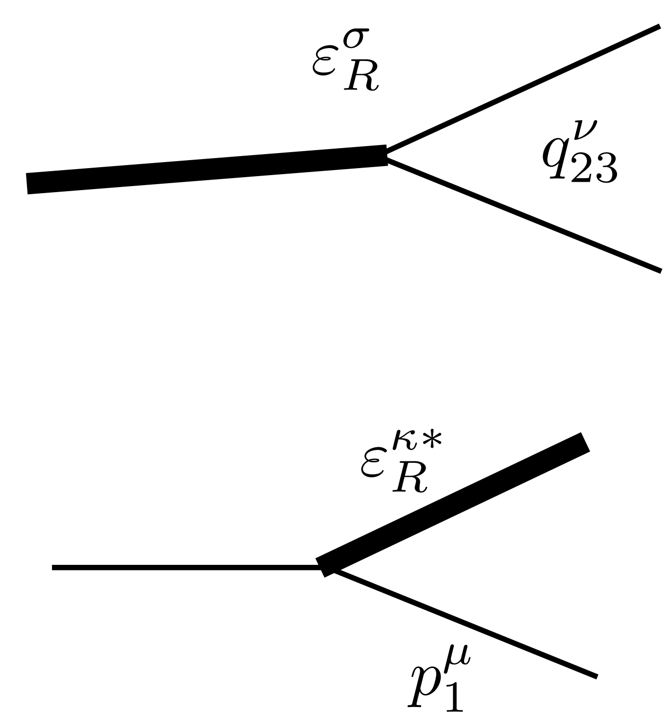
Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}





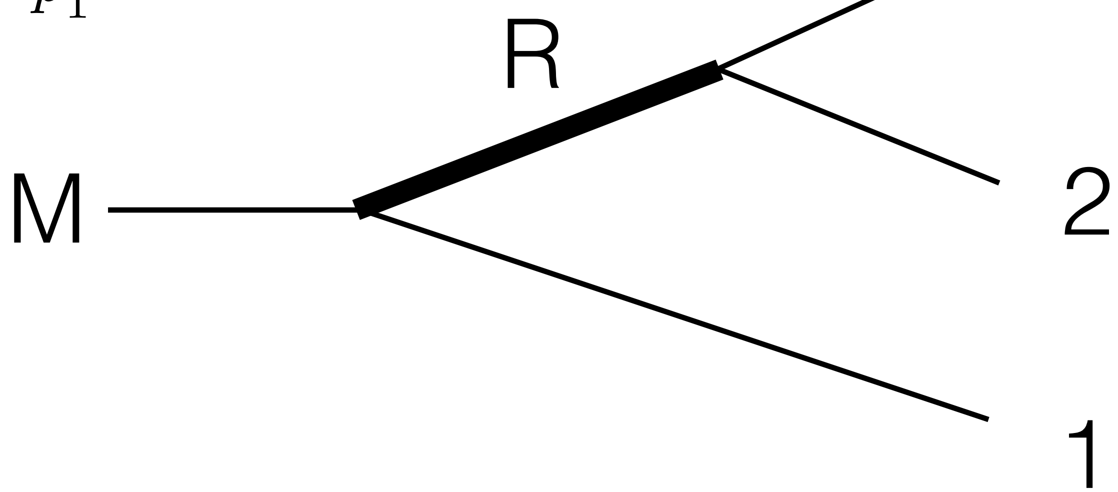
$$\sum_{\text{all } \lambda} \quad p_{1\,\mu} \;\; \varepsilon_R^{\lambda\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\lambda\nu} \;\; q_{23\;\nu}$$

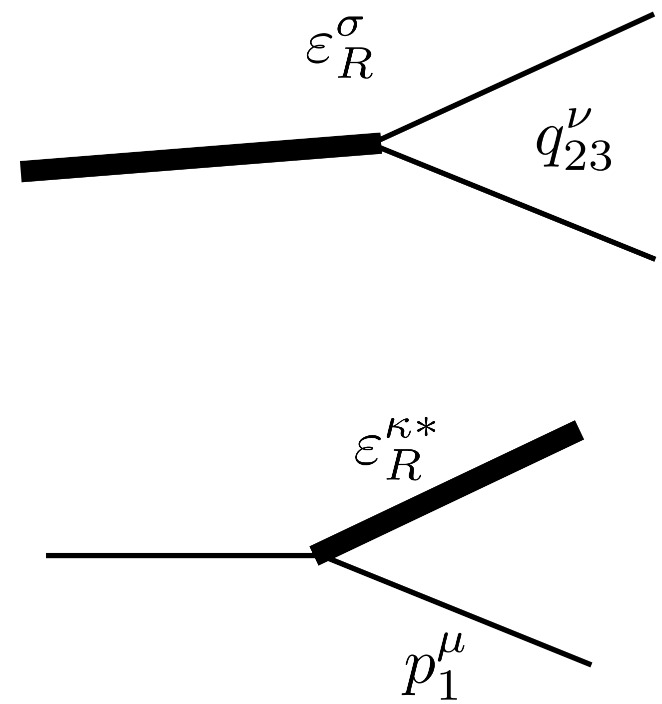
Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}





$$\sum_{\text{all } \lambda} \varepsilon_R^{\lambda\mu *} \varepsilon_R^{\lambda\nu} = -g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}$$

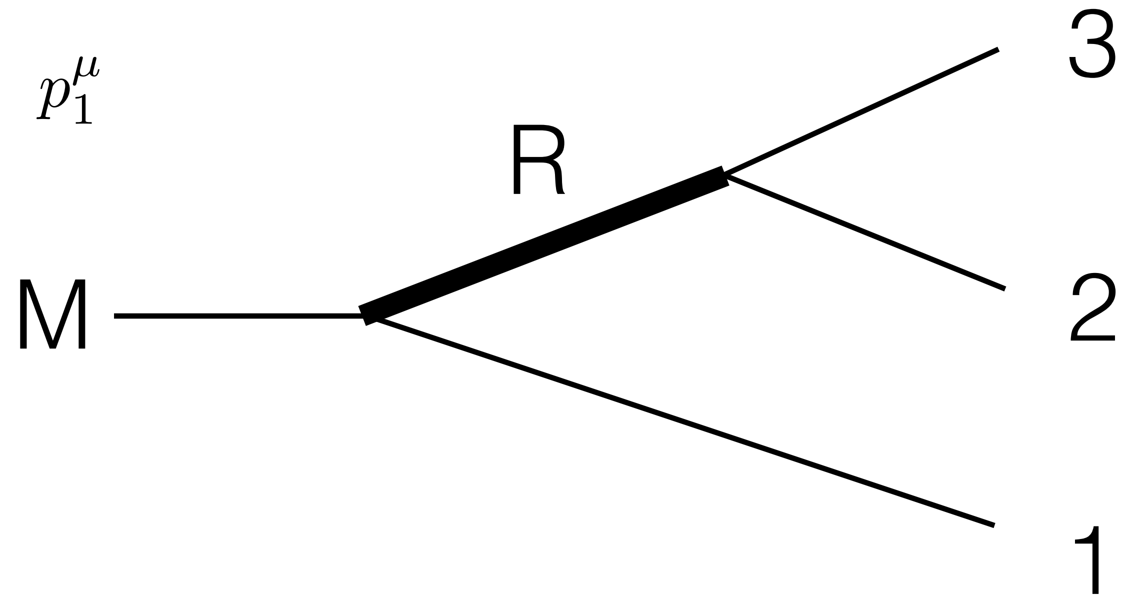
$$\sum_{\text{all } \lambda} \quad p_{1\,\mu} \;\; \varepsilon_R^{\lambda\mu *} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma} \varepsilon_R^{\lambda\nu} \;\; q_{23\,\nu}$$

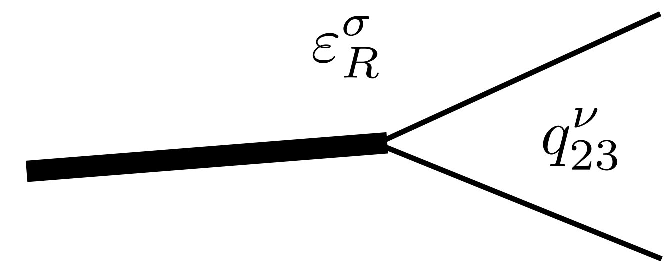
Calculating the amplitudes

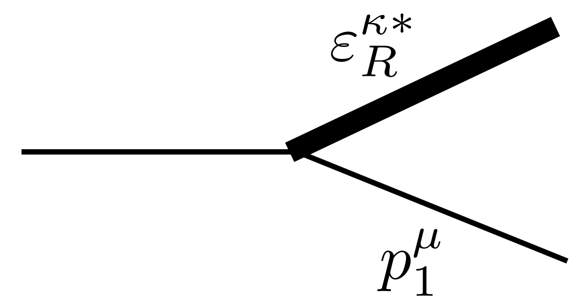
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







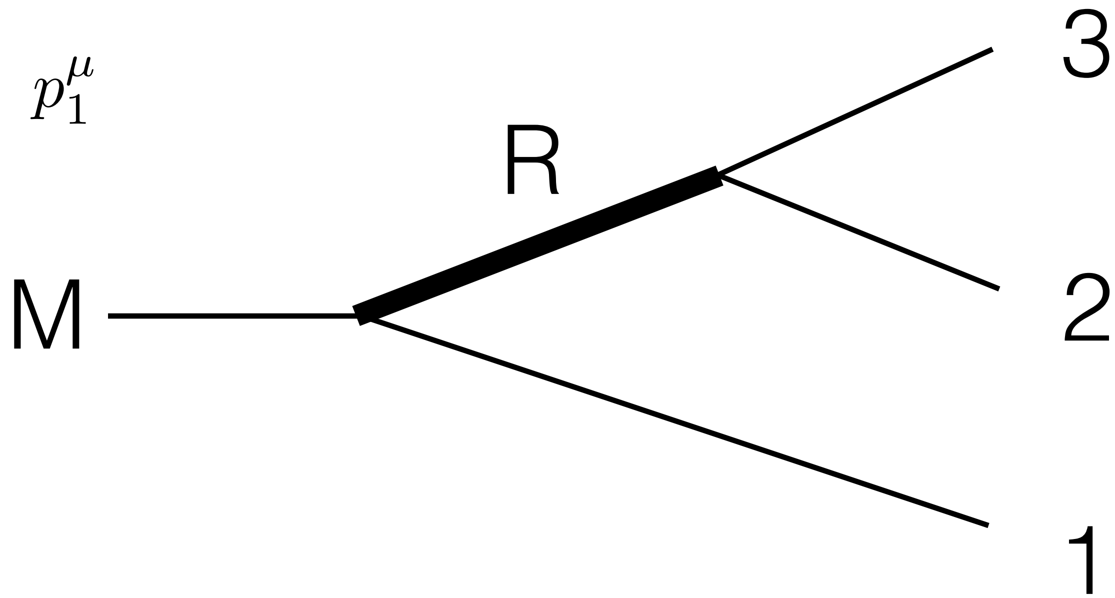
$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\,\nu}$$

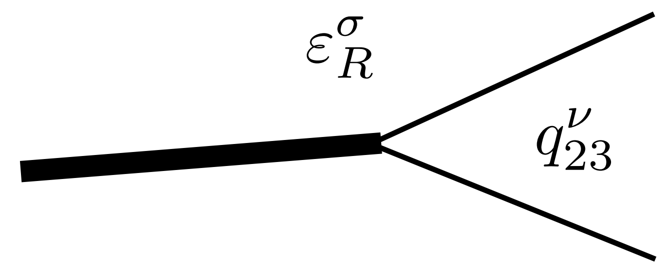
Calculating the amplitudes

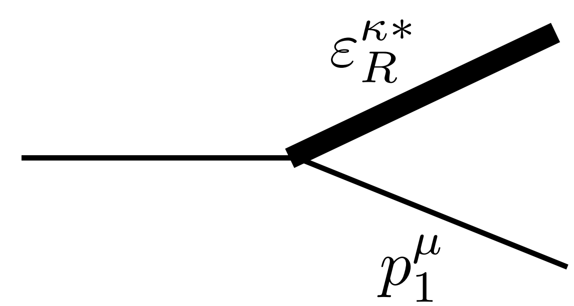
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







$$p_{1\,\mu} \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} q_{23\,\nu}$$

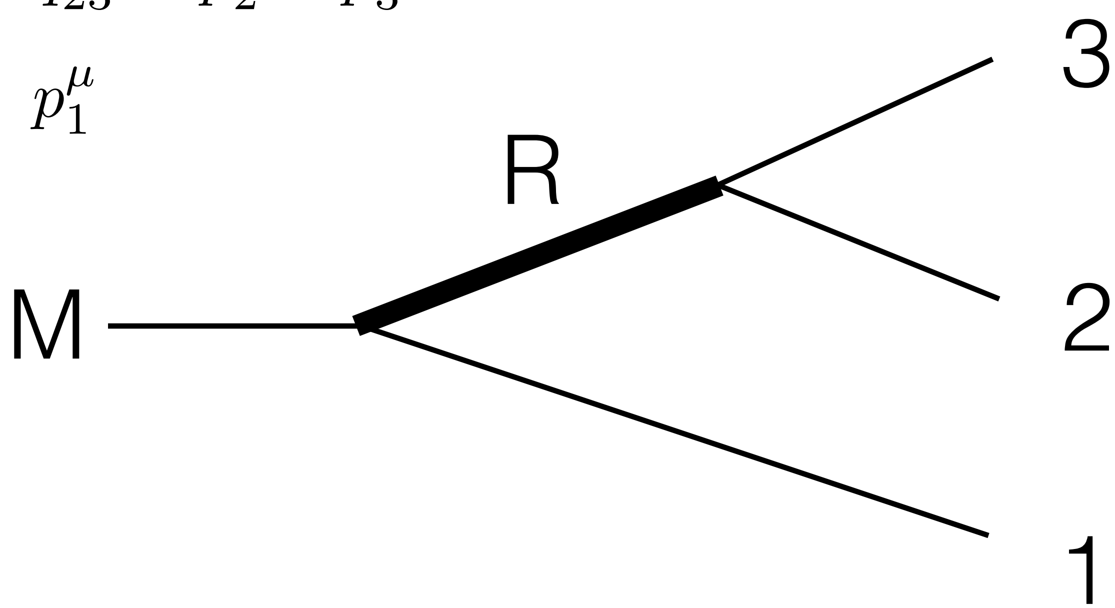
spin factor

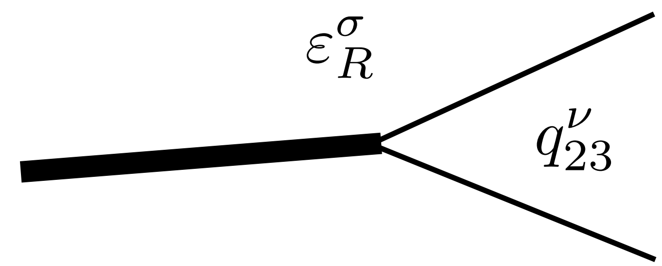
Calculating the amplitudes

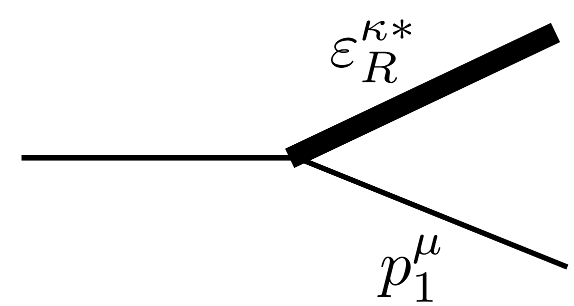
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







$$\boxed{\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2}\right)} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

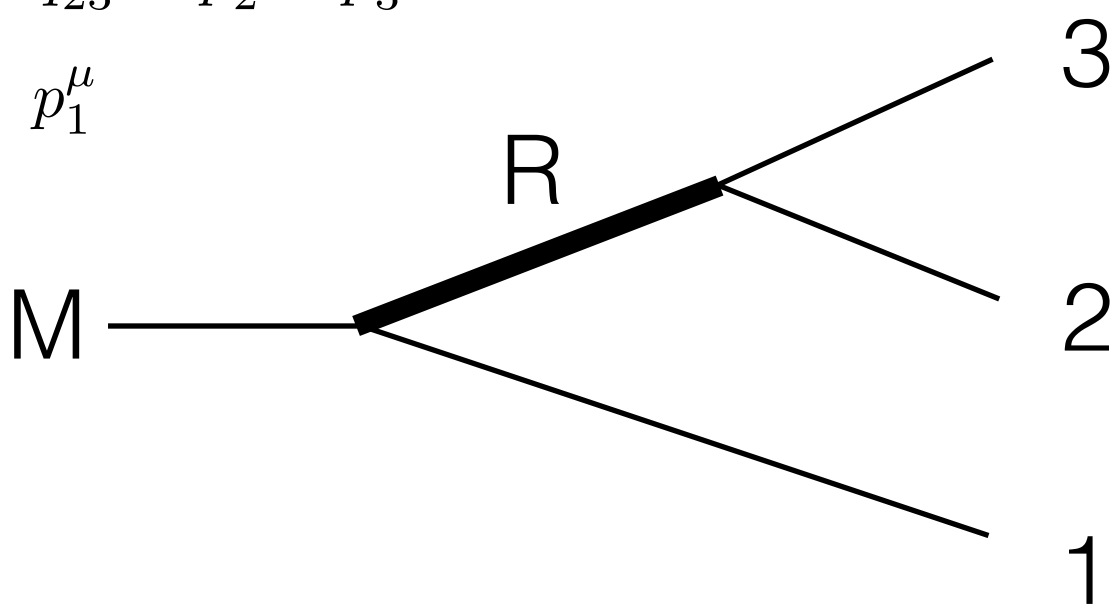
spin factor (here for L=1)

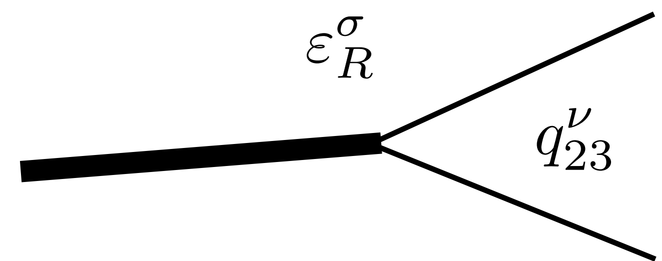
Calculating the amplitudes

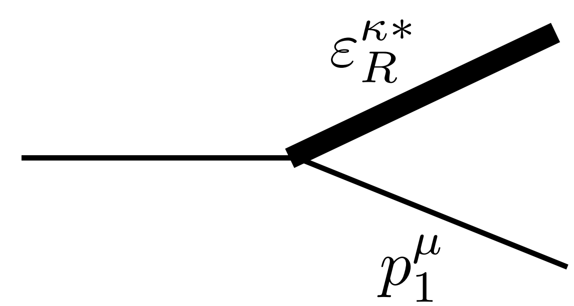
ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ







Express in terms of s_{ij} if you wish, using $p_i \cdot p_j = s_{ij} - m_i^2 - m_2^2$

$$\boxed{\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2}\right)} \frac{1}{s_{23} - m_R^2 - i m_R \Gamma}$$

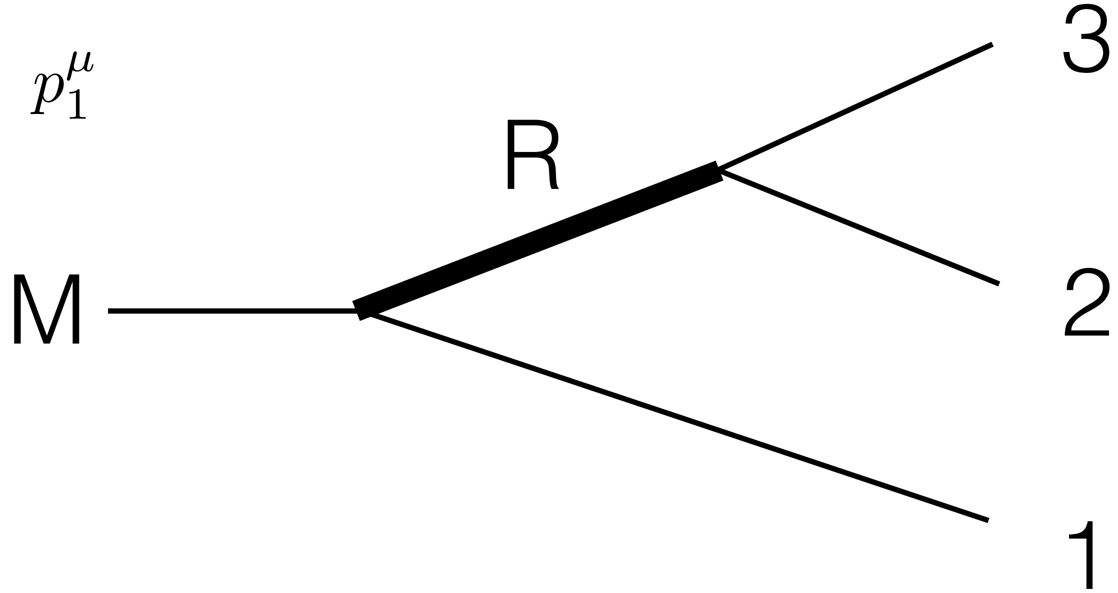
spin factor (here for L=1)

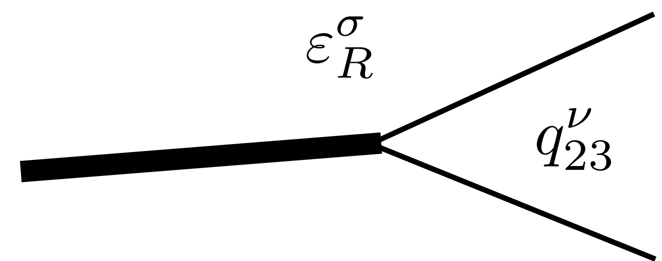
Calculating the amplitudes

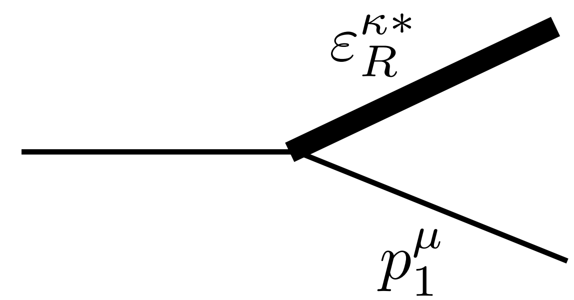
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}







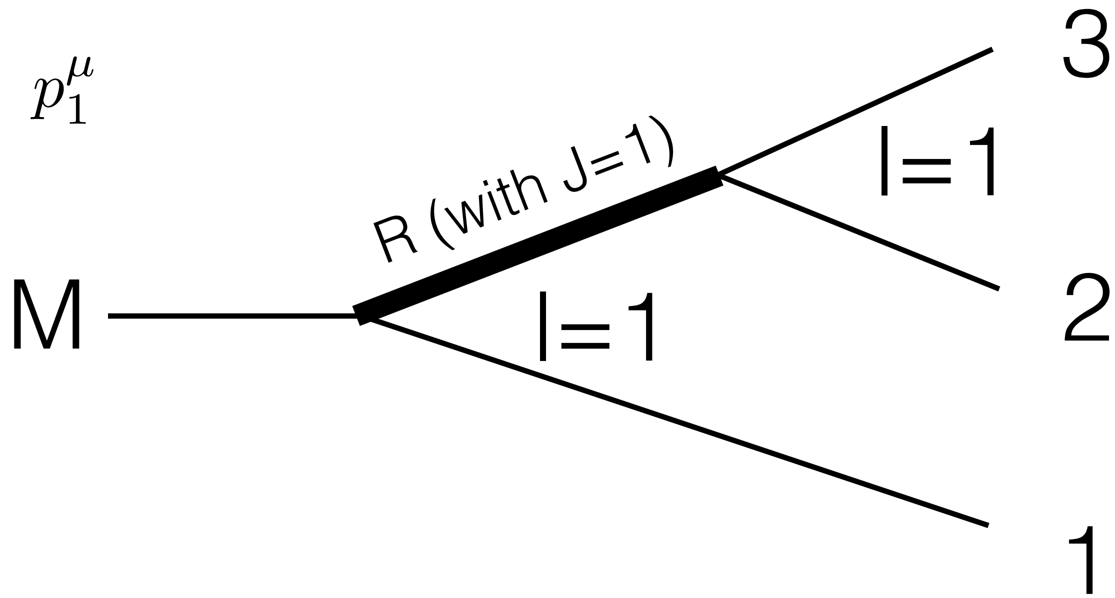
$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\;\nu}$$

Calculating the amplitudes

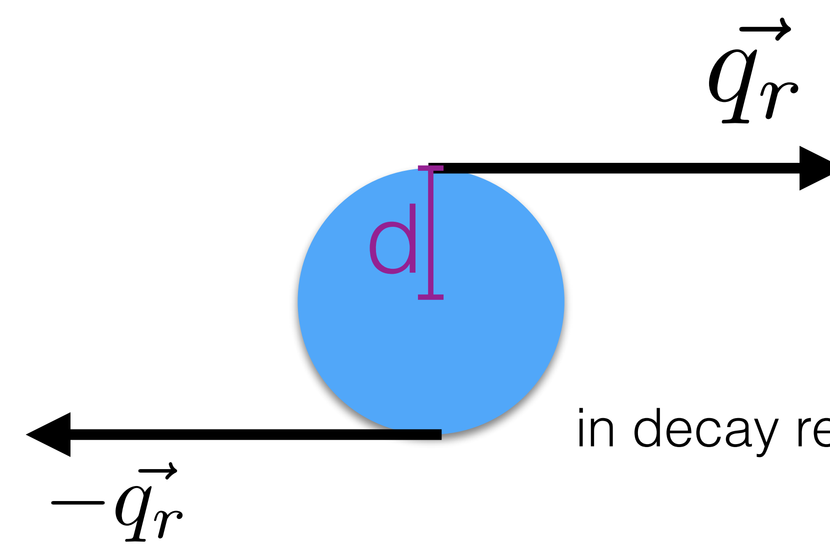
ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}



Angular Momenta require momenta



in decay rest frame

$\vec{L} = 2\,\vec{d} \times \vec{q_r}$ classical mechanics

$L = \sqrt{l(l+1)}$ QM

$$p_{1\,\mu} \quad \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma} \quad q_{23\,\nu}$$

Blatt Weisskopf Penetration Factors

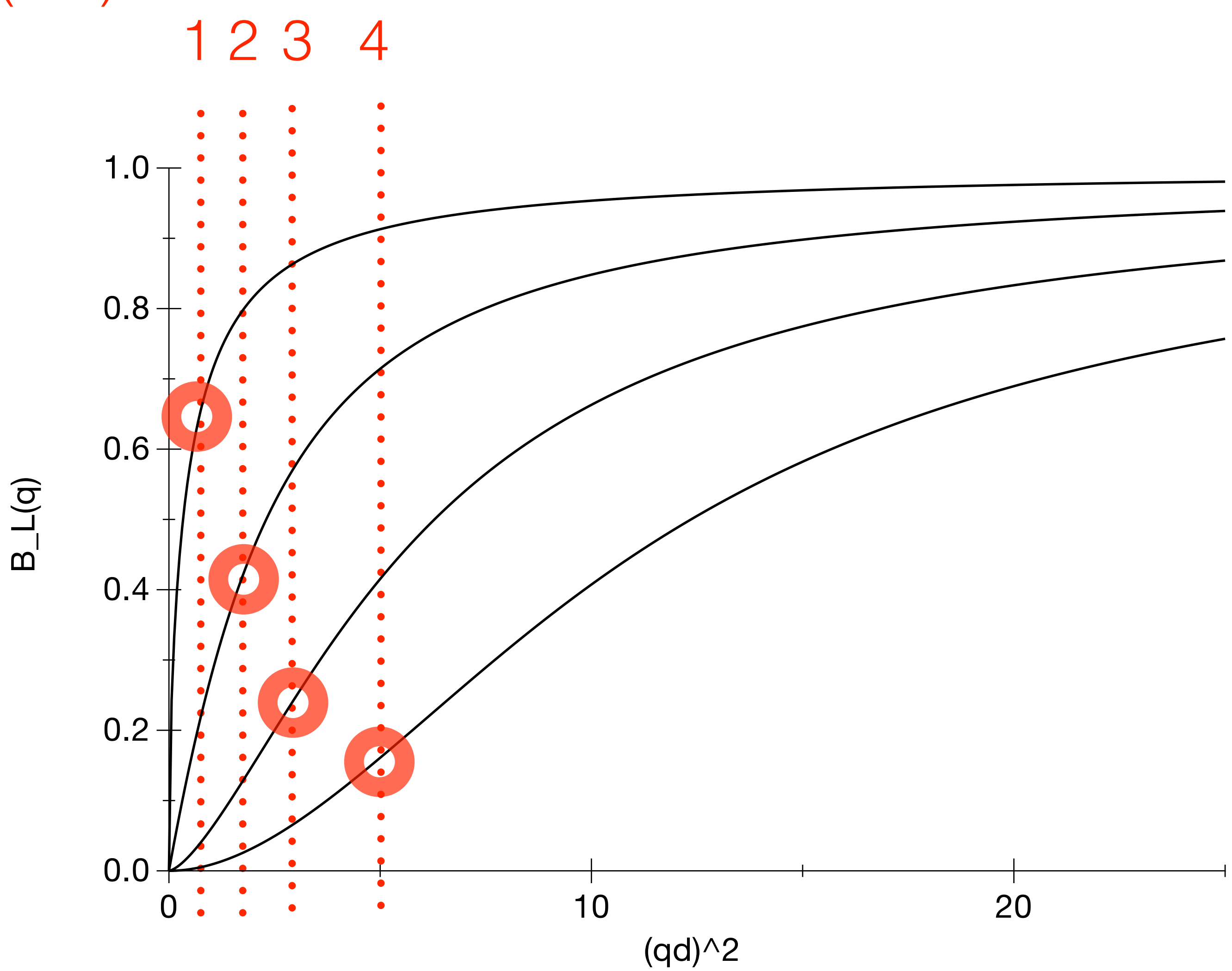
L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$
	where $z = (\lvert q\rvert\, d)^2$ and $z_0 = (\lvert q_0\rvert\, d)^2$	

classical mechanics:
L = 2 qd

QM:
$L^2 = l(l+1)$

Blatt Weisskopf Penetration Factors

1/4 L^2 = 1/4 l(l+1) for l = …



classical mechanics:
L = 2 qd

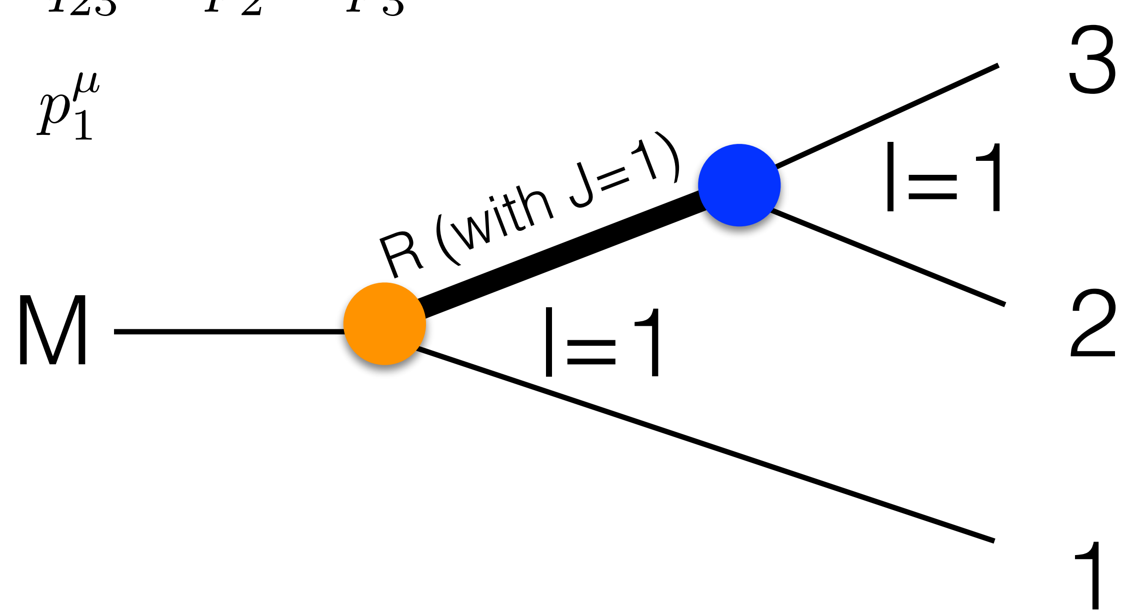
QM:
L^2 = l(l+1)

Calculating the amplitudes

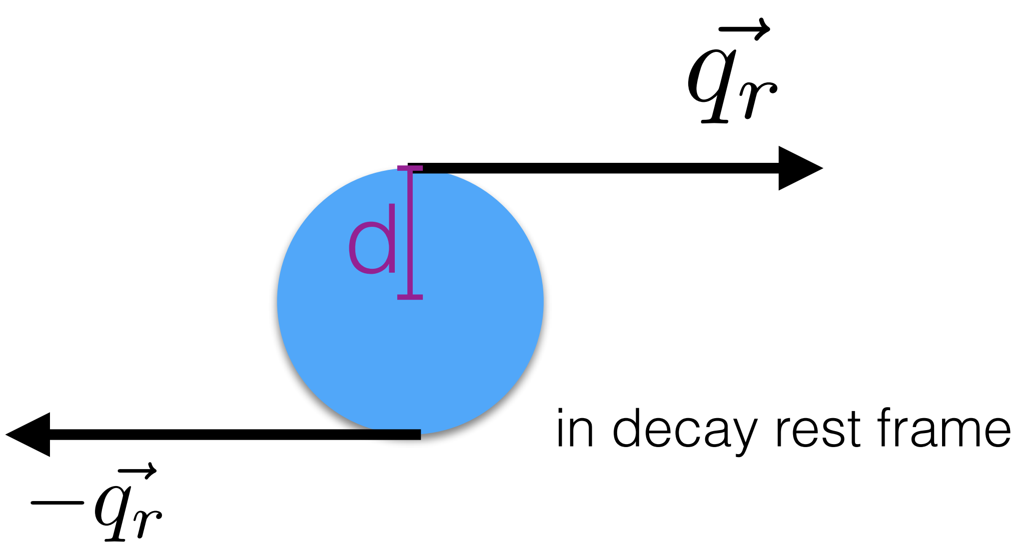
ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



Angular Momenta require momenta



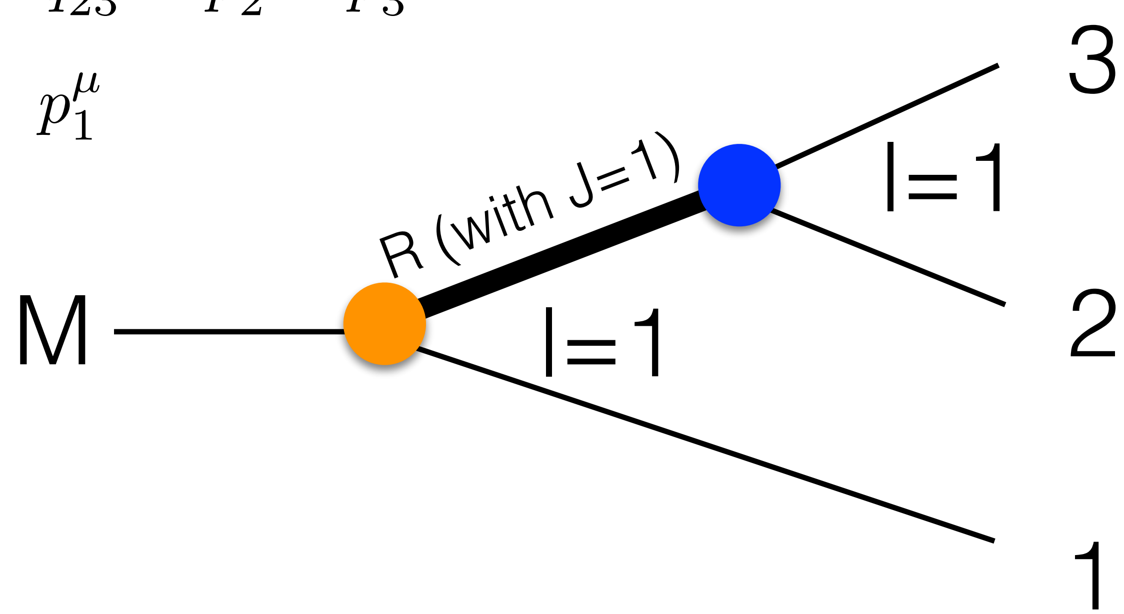
$$p_{1\,\mu}\; B_L'(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma}\; B_L'(q_{rR}, d_R)\; q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.
  break-up momentum
- Rate also depends on B_L.

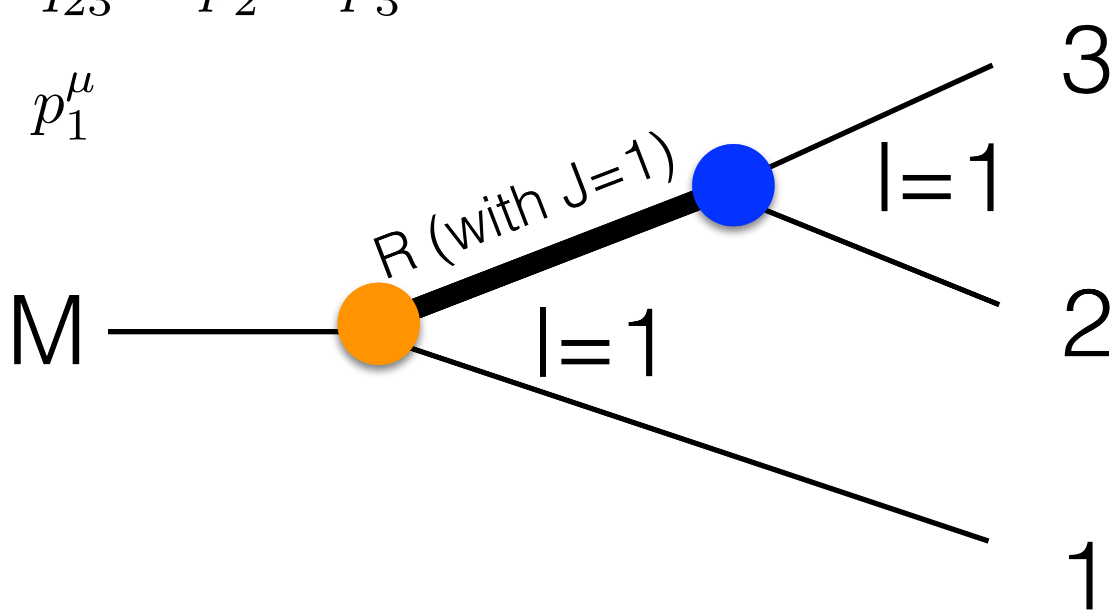
$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma}\; B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

  break-up momentum

- Rate also depends on B_L.

$$p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

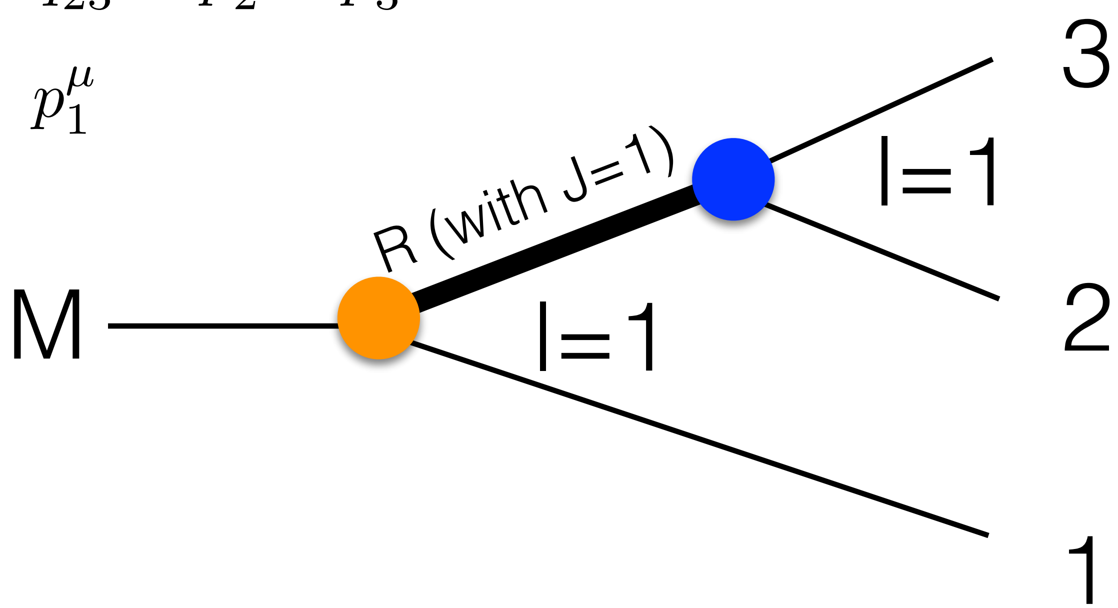
J=1

Calculating the amplitudes

ε_R^{σ}

$q_{23}^{\nu} \equiv p_2^{\nu} - p_3^{\nu}$

p_1^{μ}



- Width Γ = rate, depends on phase space = 2q/m.

  break-up momentum

- Rate also depends on B_L.

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\, B_L(q_{23})}{(q_0/m_R)\;\; B_L(q_0)}$$

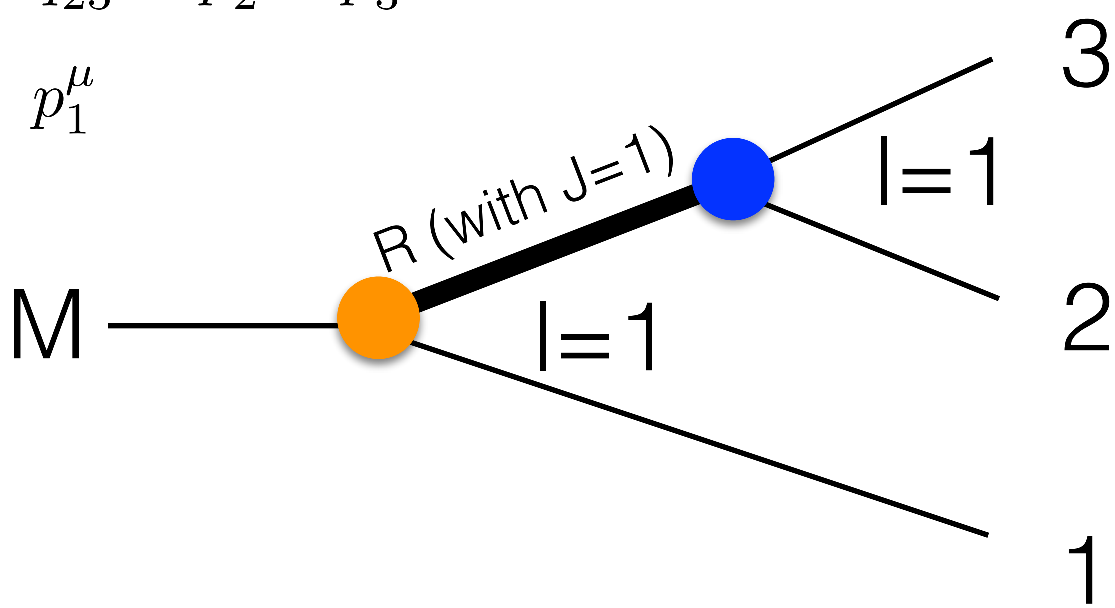
$$p_{1\,\mu}\; B_L(q_{rM}, d_M)\; \frac{-g^{\mu\nu} + \frac{p_R^{\mu} p_R^{\nu}}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\; B_L(q_{rR}, d_R)\; q_{23\;\nu} \qquad \text{J=1}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

break-up momentum in restframe of decaying resonance

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\, B_L(q_{23})}{(q_0/m_R)\;\; B_L(q_0)}$$

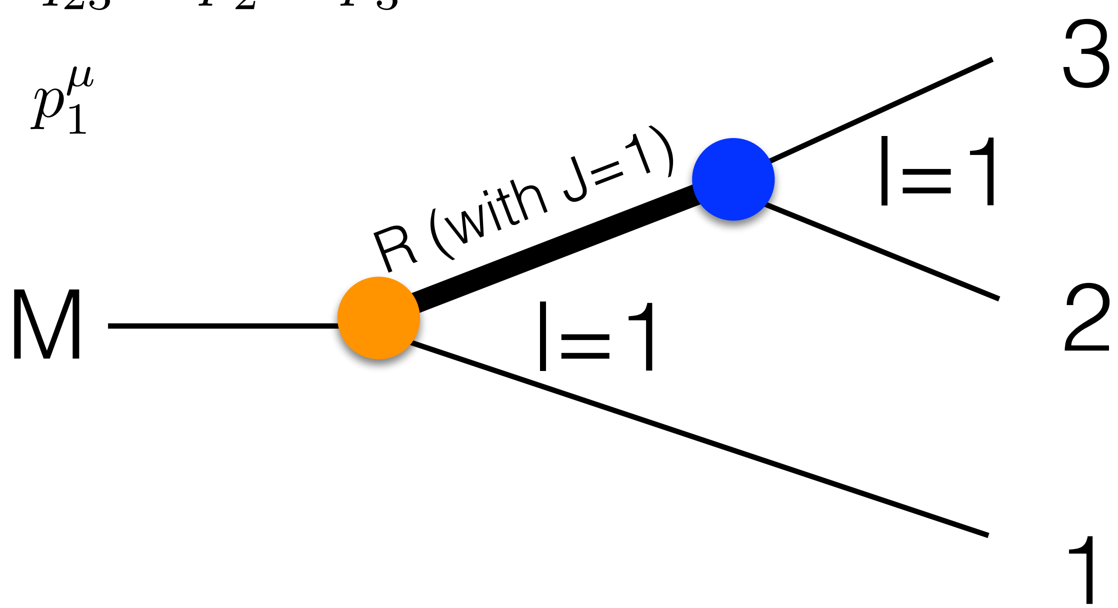
$$p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\; q_{23\,\nu} \qquad \text{J=1}$$

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

centrifugal barrier factor

break-up momentum in restframe of decaying resonance

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\ B_L(q_{23})}{(q_0/m_R)\ \ B_L(q_0)}$$

$$p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\ q_{23\,\nu}$$

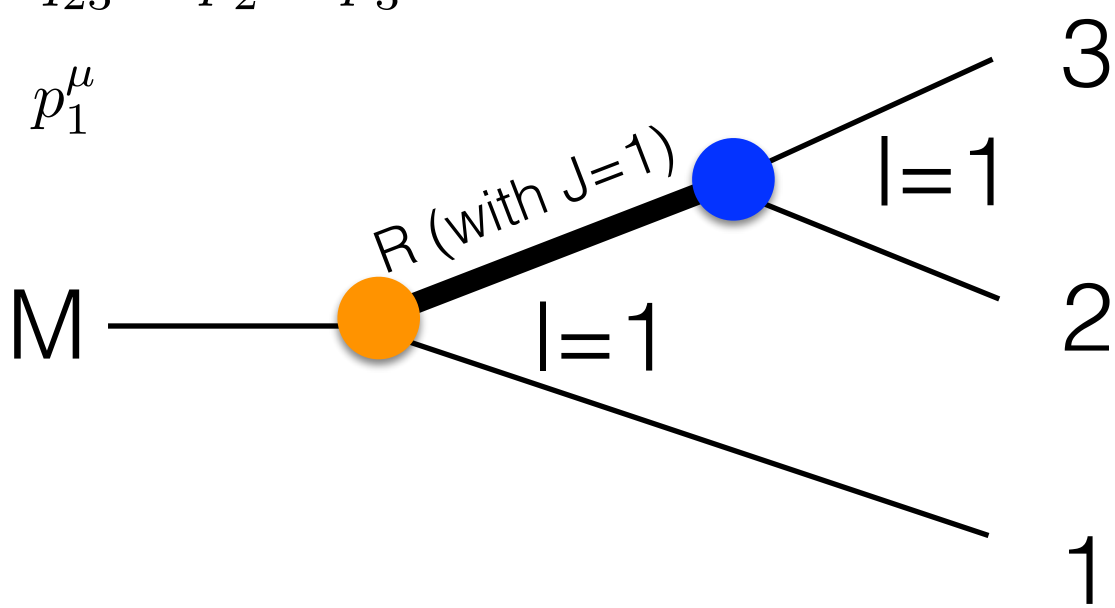
J=1

Calculating the amplitudes

ε_R^σ

$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$

p_1^μ



- Width Γ = rate, depends on phase space = 2q/m.

break-up momentum

- Rate also depends on B_L.

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

centrifugal barrier factor

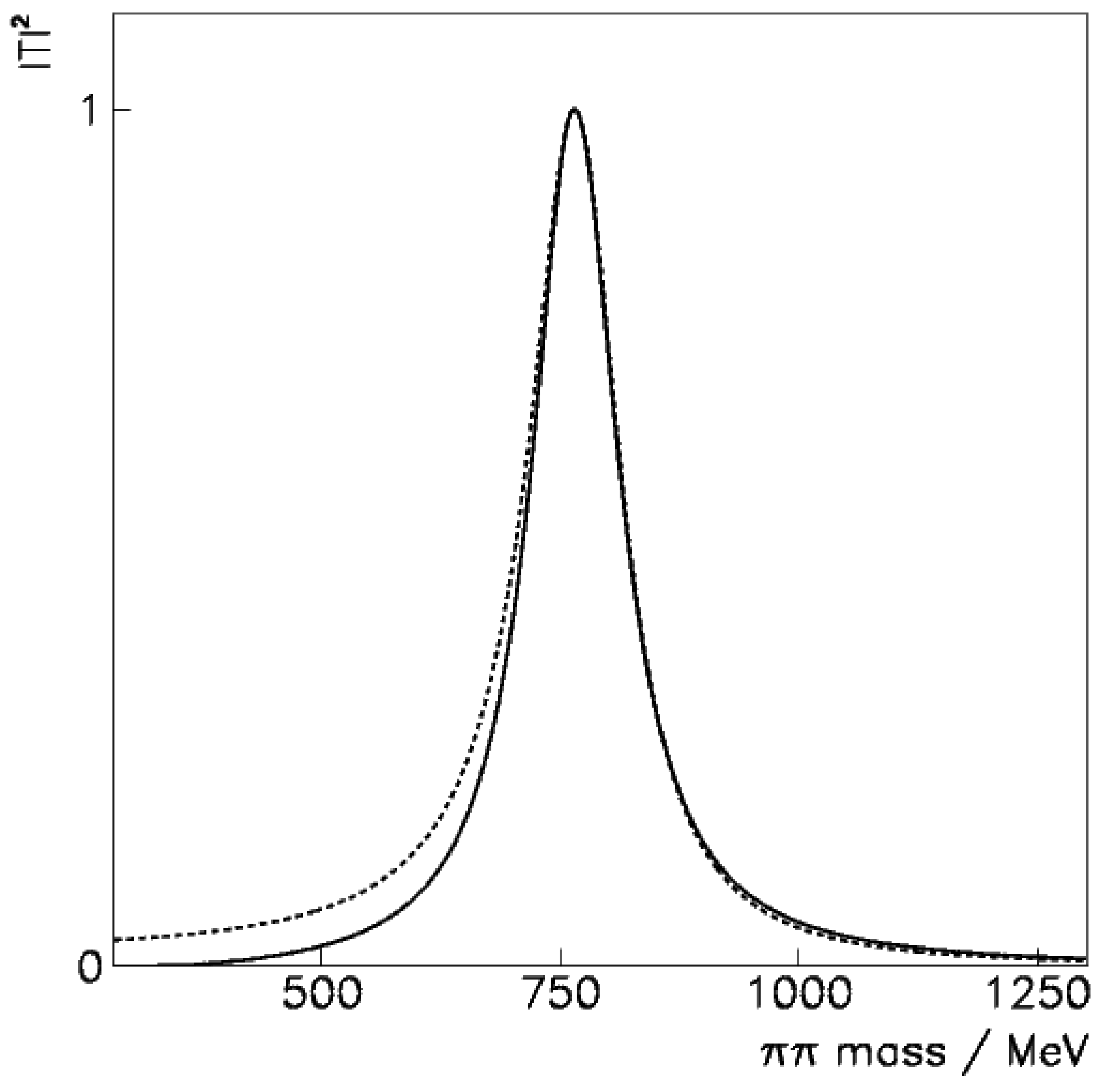
break-up momentum in restframe of decaying resonance

the same as numerator, but calculated for “nominal” (peak) resonance mass.

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})\; B_L(q_{23})}{(q_0/m_R)\;\; B_L(q_0)}$$

$$p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\; q_{23\,\nu}$$ J=1

Mass dependent width (ignoring ang. mom)

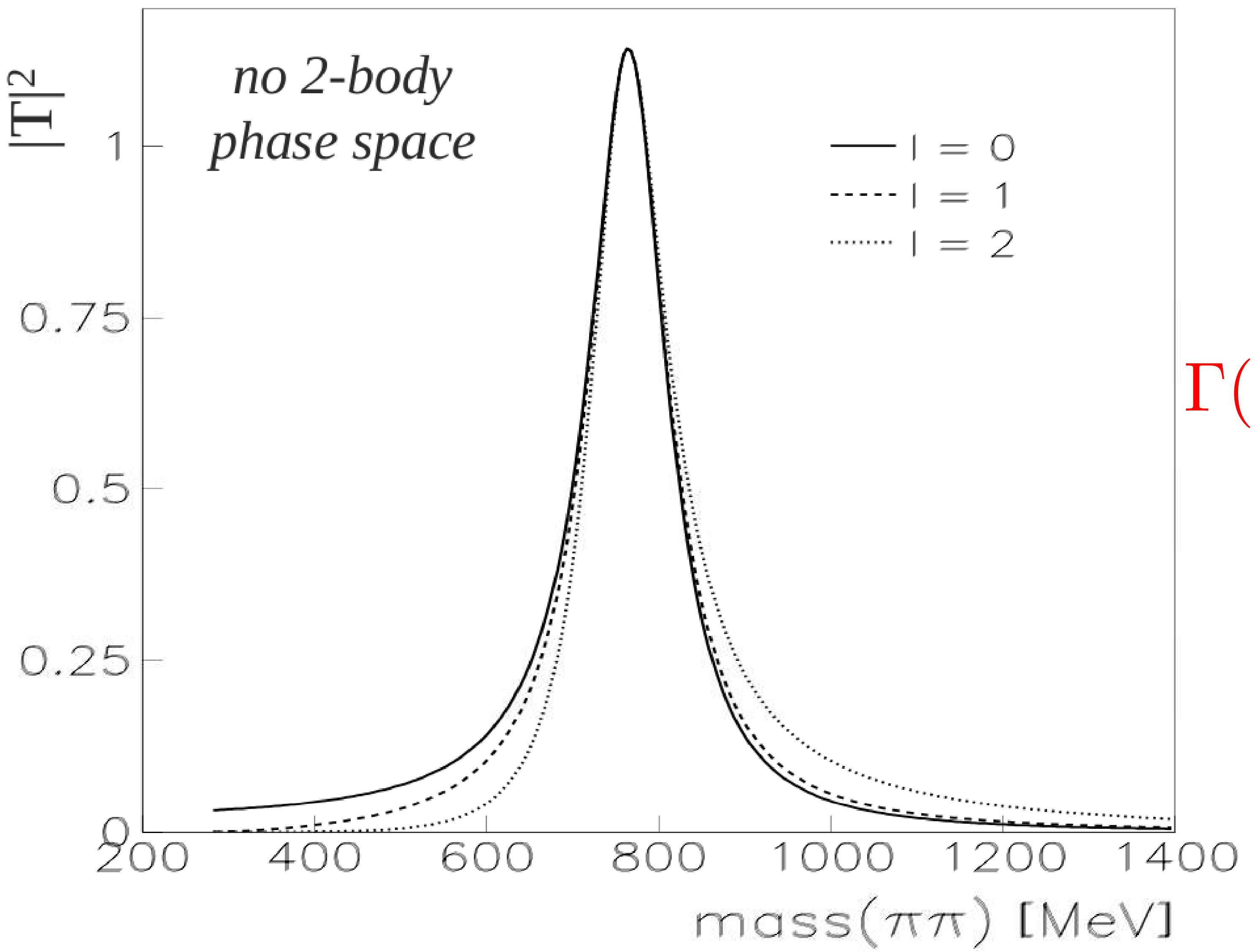


dashed: fixed width

solid: mass dependent width

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})}{(q_0/m_R)} \frac{B_L(q_{23})}{B_L(q_0)}$$

Breit Wigner with angular momentum effects (only)



$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23})}{(q_0/m_R)} \frac{B_L(q_{23})}{B_L(q_0)}$$

Amplitude Model

$$A_R = \; p_{1\,\mu}\, B_L(q_{rM}, d_M)\, \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - i m_R \Gamma(m_{23})}\, B_L(q_{rR}, d_R)\; q_{23\,\nu}$$

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

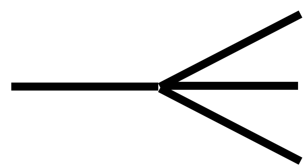
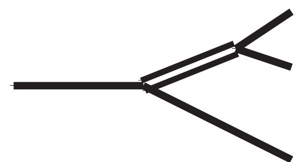
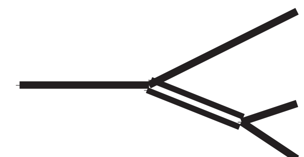
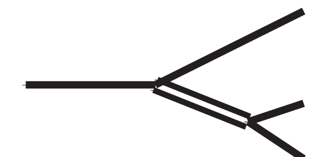
sensitivity to phases is one of the key reasons amplitude analyses are so interesting.

$$P(s_{12}, s_{23}) = \frac{|\mathcal{M}_{fi}|^2 \left|\frac{d\Phi}{ds_{12}\, ds_{23}}\right|}{\int |\mathcal{M}_{fi}|^2 \left|\frac{d\Phi}{ds_{12}\, ds_{23}}\right| ds_{12}\, ds_{23}}$$

$$= \frac{|\mathcal{M}_{fi}|^2}{\int_{\text{within kin boundary}} |\mathcal{M}_{fi}|^2\, ds_{12}\, ds_{23}}$$

Fitters frequently used at LHCb:
MINT (esp for >3 body)
AmpGen (descendent of MINT)
Laura++
z-fit/tensor flow and
GooFit-based fitters

Isobar Model + non-resonant

M=  +  +  +  + ...

$$c_0 \quad + \quad c_1 \quad \frac{1}{s_{23} - m_1^2 - i m_1 \Gamma_1(s_{23})} \quad + \quad \dots.$$

- “Classic” experimentalists’ model: Isobar model with resonances described by Breit Wigner lineshapes, plus “non-resonant” term (also called “contact term”, and, amongst theorists, “background”).
- Despised by theorists - especially the non-resonant term (but in practice it’s often needed to describe the data).

Sum of Breit Wigners

Sum of Breit Wigners with non-resonant term

Last Judgement (Detail) by Fra Angelico

Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

example: **CDF: PHYSICAL REVIEW D 86, 032007 (2012)**

Resonance	a	δ [°]	Fit fractions [%]
$K^*(892)^\pm$	1.911 ± 0.012	132.1 ± 0.7	61.80 ± 0.31
$K_0^*(1430)^\pm$	2.093 ± 0.065	54.2 ± 1.9	6.25 ± 0.25
$K_2^*(1430)^\pm$	0.986 ± 0.034	308.6 ± 2.1	1.28 ± 0.08
$K^*(1410)^\pm$	1.092 ± 0.069	155.9 ± 2.8	1.07 ± 0.10
$\rho(770)$	1	0	18.85 ± 0.18
$\omega(782)$	0.038 ± 0.002	107.9 ± 2.3	0.46 ± 0.05
$f_0(980)$	0.476 ± 0.016	182.8 ± 1.3	4.91 ± 0.19
$f_2(1270)$	1.713 ± 0.048	329.9 ± 1.6	1.95 ± 0.10
$f_0(1370)$	0.342 ± 0.021	109.3 ± 3.1	0.57 ± 0.05
$\rho(1450)$	0.709 ± 0.043	8.7 ± 2.7	0.41 ± 0.04
$f_0(600)$	1.134 ± 0.041	201.0 ± 2.9	7.02 ± 0.30
σ_2	0.282 ± 0.023	16.2 ± 9.0	0.33 ± 0.04
$K^*(892)^\pm$(DCS)	0.137 ± 0.007	317.6 ± 2.8	0.32 ± 0.03
$K_0^*(1430)^\pm$(DCS)	0.439 ± 0.035	156.1 ± 4.9	0.28 ± 0.04
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Nonresonant	1.797 ± 0.147	94.0 ± 5.3	1.64 ± 0.27
Sum			107.25 ± 0.65

Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

example: **CDF: PHYSICAL REVIEW D 86, 032007 (2012)**

Resonance	a	δ [°]	Fit fractions [%]
$K^*(892)^\pm$	1.911 ± 0.012	132.1 ± 0.7	61.80 ± 0.31
$K_0^*(1430)^\pm$	2.093 ± 0.065	54.2 ± 1.9	6.25 ± 0.25
$K_2^*(1430)^\pm$	0.986 ± 0.034	308.6 ± 2.1	1.28 ± 0.08
$K^*(1410)^\pm$	1.092 ± 0.069	155.9 ± 2.8	1.07 ± 0.10
$\rho(770)$	1	0	18.85 ± 0.18
$\omega(782)$	0.038 ± 0.002	107.9 ± 2.3	0.46 ± 0.05
$f_0(980)$	0.476 ± 0.016	182.8 ± 1.3	4.91 ± 0.19
$f_2(1270)$	1.713 ± 0.048	329.9 ± 1.6	1.95 ± 0.10
$f_0(1370)$	0.342 ± 0.021	109.3 ± 3.1	0.57 ± 0.05
$\rho(1450)$	0.709 ± 0.043	8.7 ± 2.7	0.41 ± 0.04
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$$FF_R = \frac{\int \left| c_R e^{i\theta_R} A_R(s_{12}, s_{23}) \right|^2 ds_{12} ds_{23}}{\int \left| \sum_j c_j e^{i\theta_j} A_j(s_{12}, s_{23}) \right|^2 ds_{12} ds_{23}}$$

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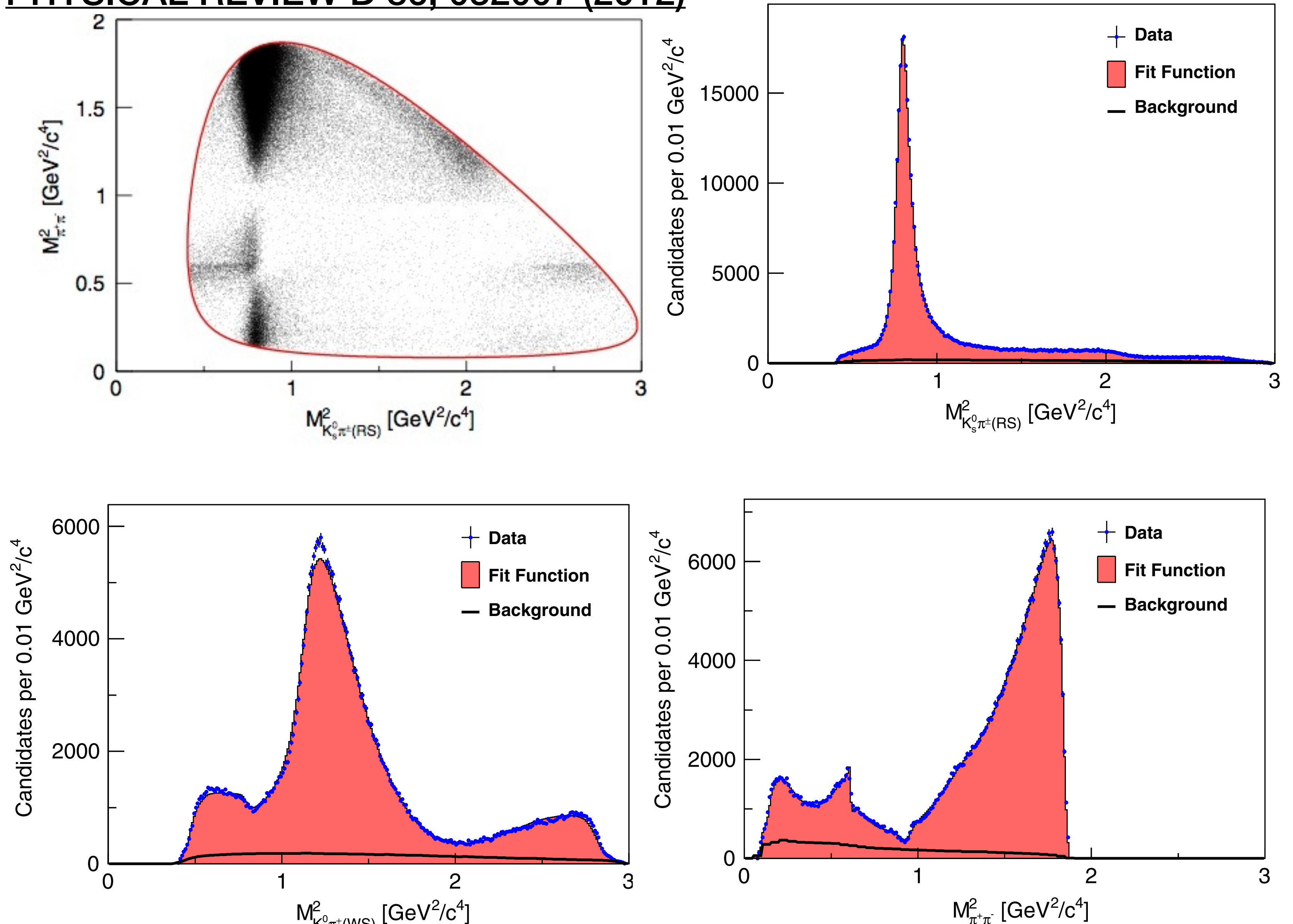

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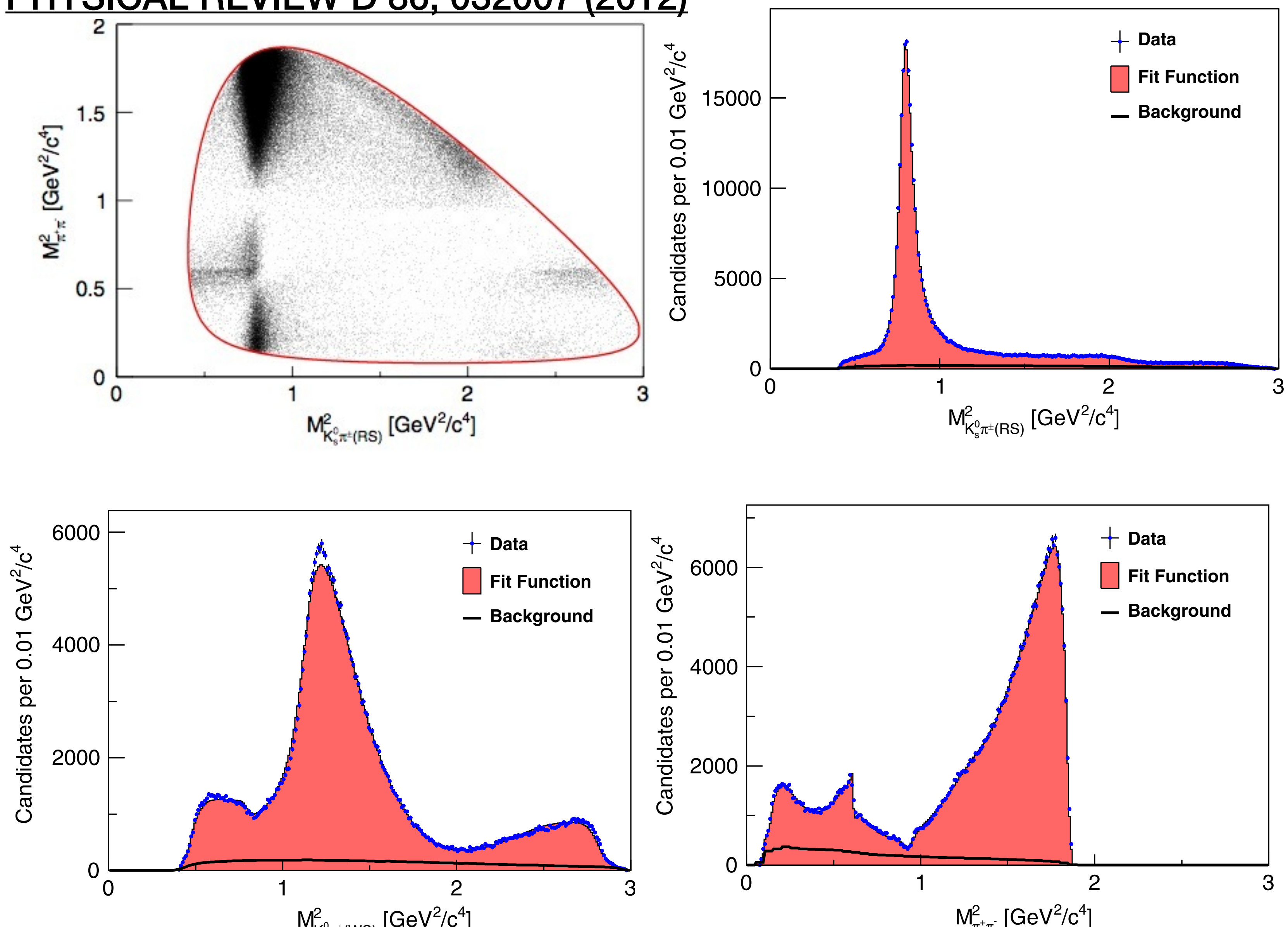
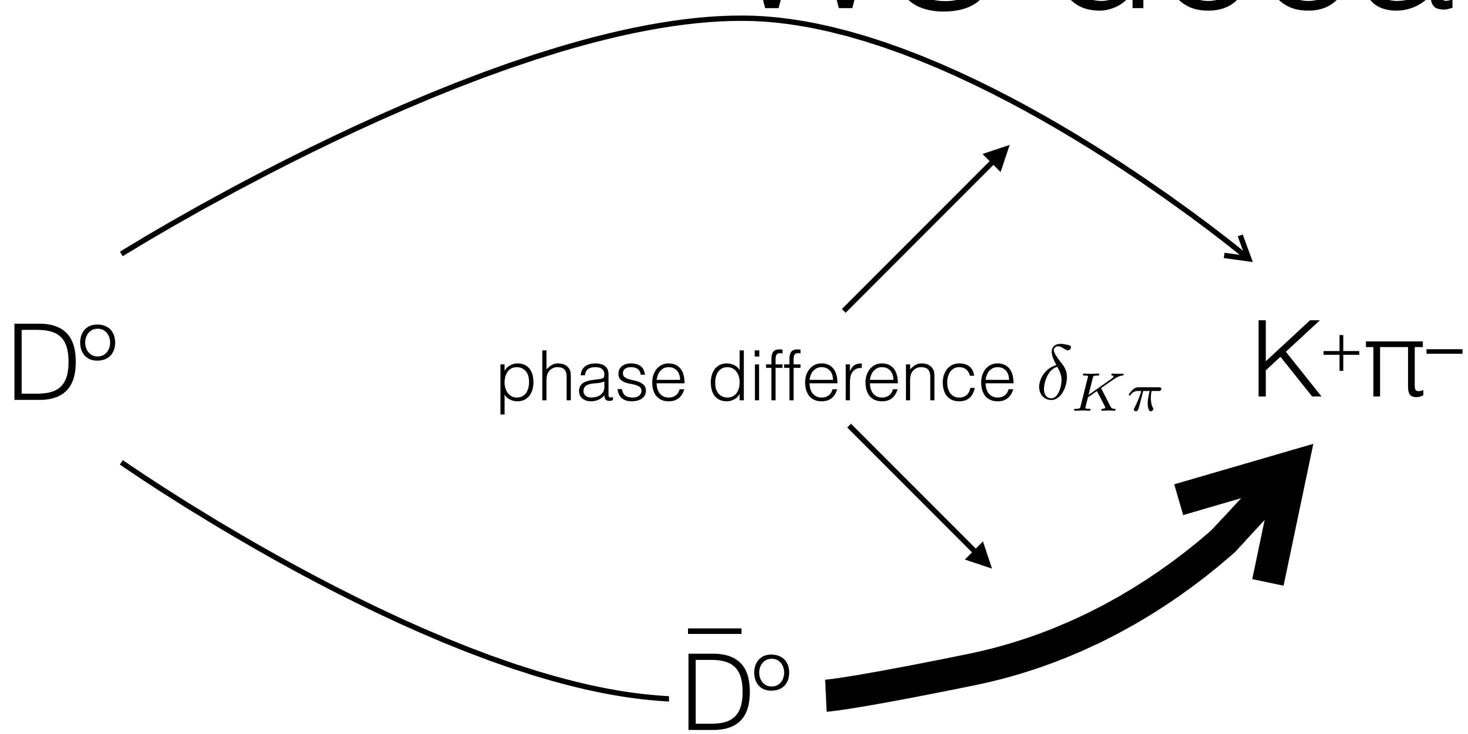
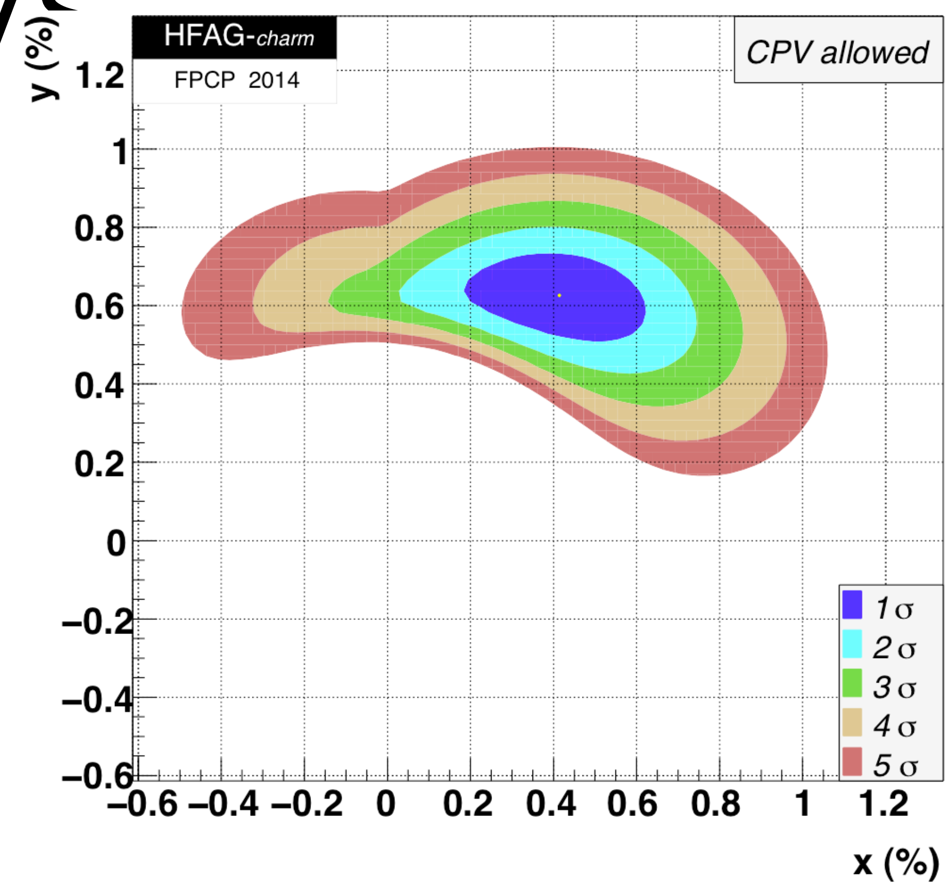


Mixing formalism for 2-body WS decays



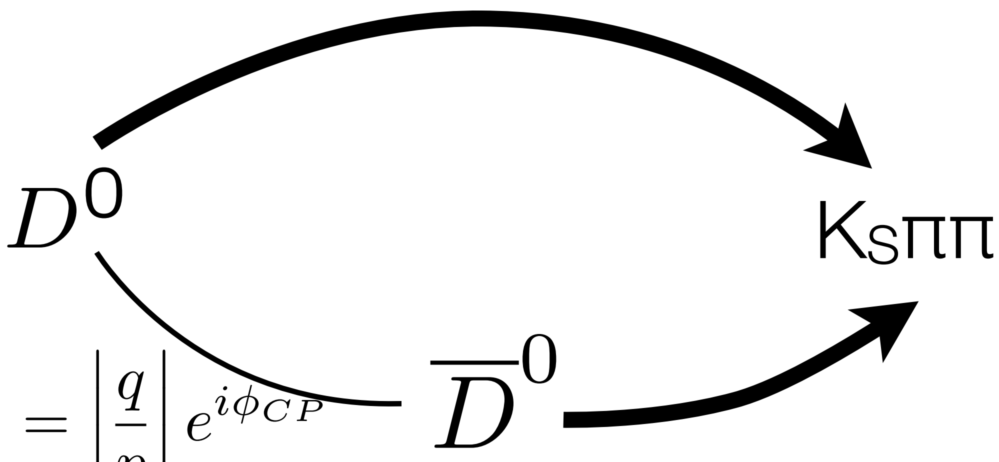


$$\frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)}(t) \approx \left(r_D^{K\pi}\right)^2 + r_D^{K\pi} y'_{K\pi}\Gamma t + \frac{x'^2_{K\pi} + y'^2_{K\pi}}{4}(\Gamma t)^2$$

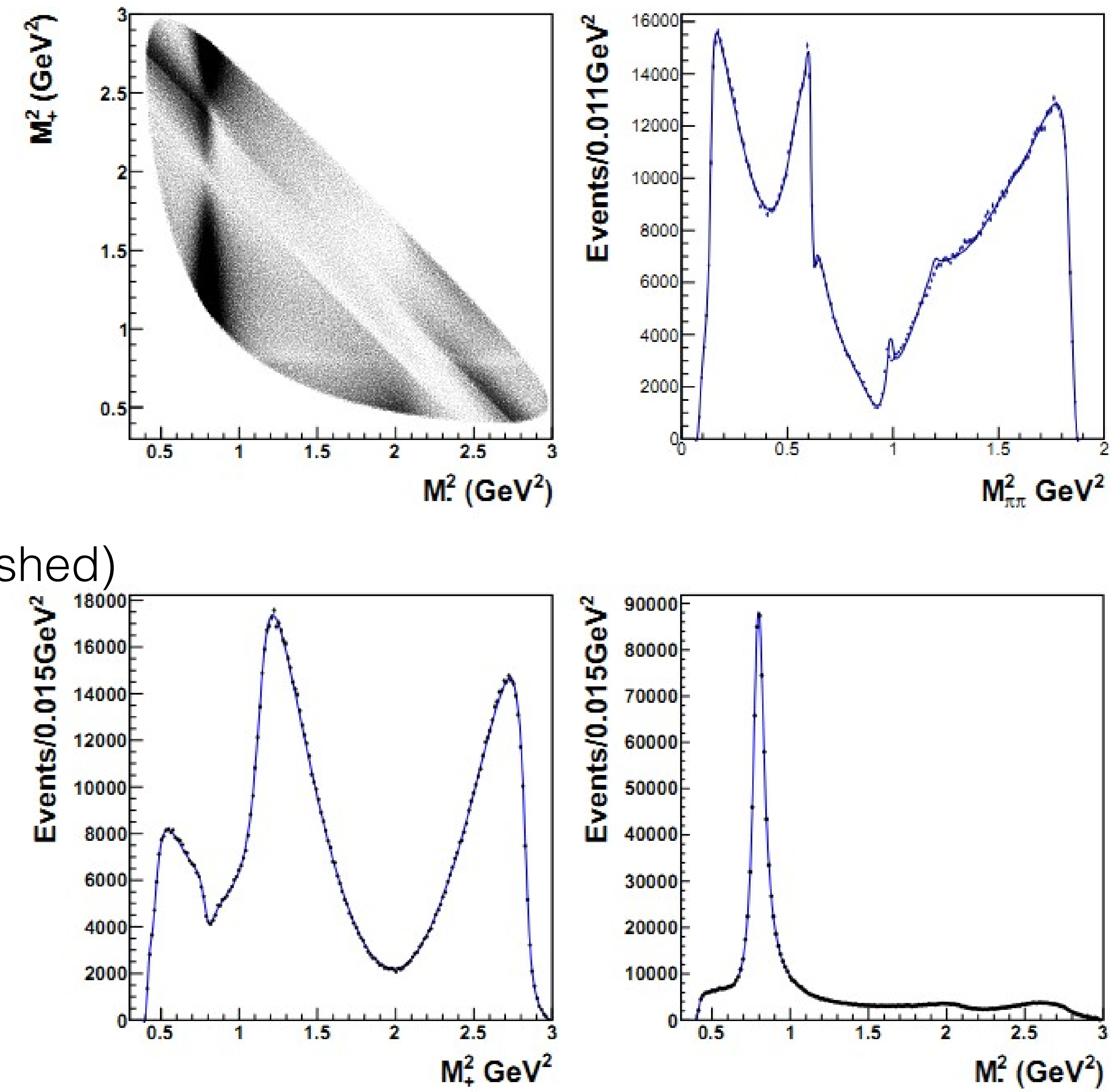
where $\begin{pmatrix} x'_{K\pi} \\ y'_{K\pi} \end{pmatrix} = \begin{pmatrix} \cos\delta_{K\pi} & \sin\delta_{K\pi} \\ \cos\delta_{K\pi} & -\sin\delta_{K\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

Time-dependent CPV $D^0 \to K_S \pi\pi$

see also previous result: Phys. Rev. Lett. 99, 131803 (2007).



$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi_{CP}}$$



(Belle preliminary) (by now published)

Fit case	Parameter	Fit new result
No CPV	$x(\%)$	$0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09}$
	$y(\%)$	$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06}$
No dCPV	$\lvert q/p \rvert$	$0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$
	$\arg q/p(^o)$	$-6 \pm 11^{+3+3}_{-3-4}$

see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

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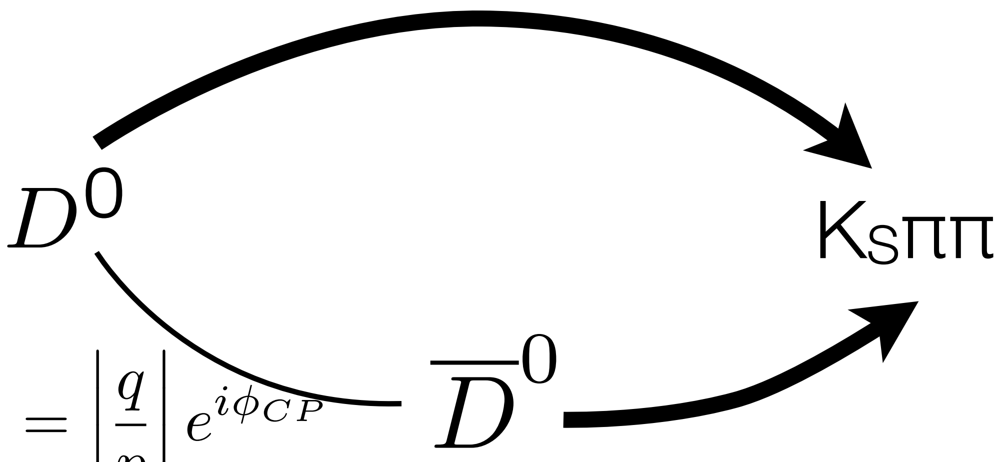

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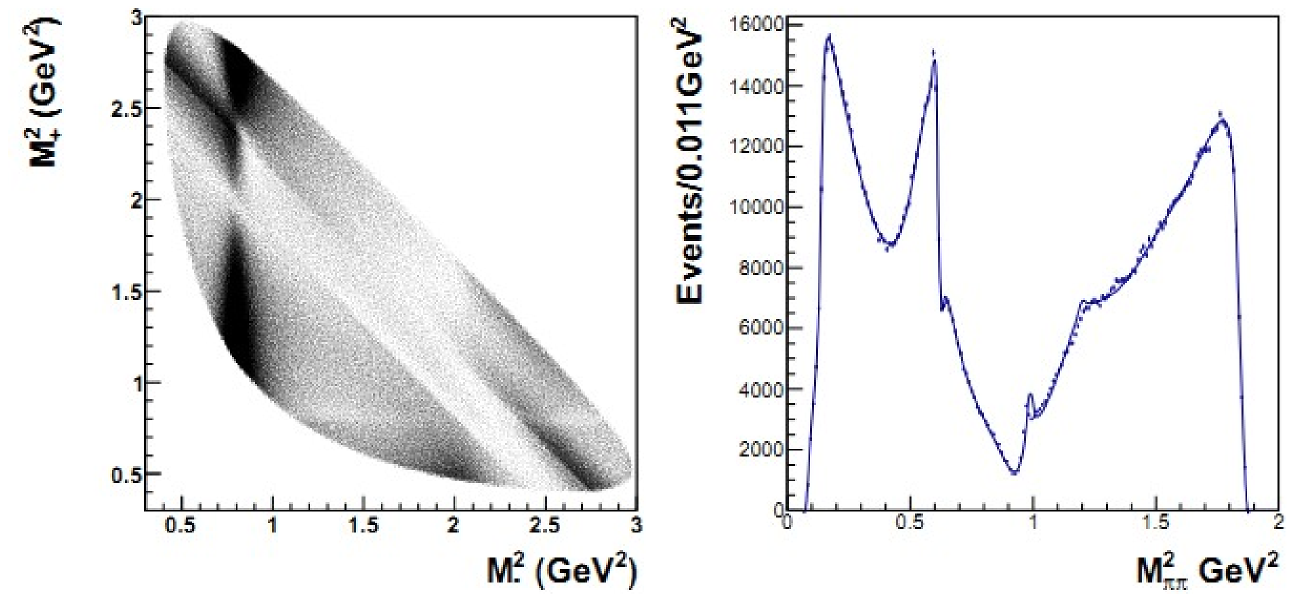

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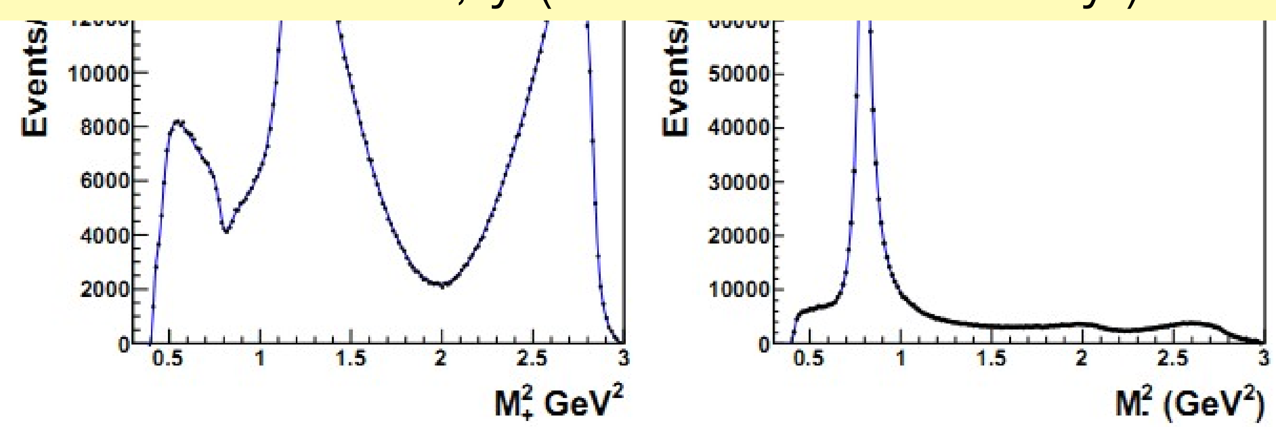
Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x’^{2} and y’)



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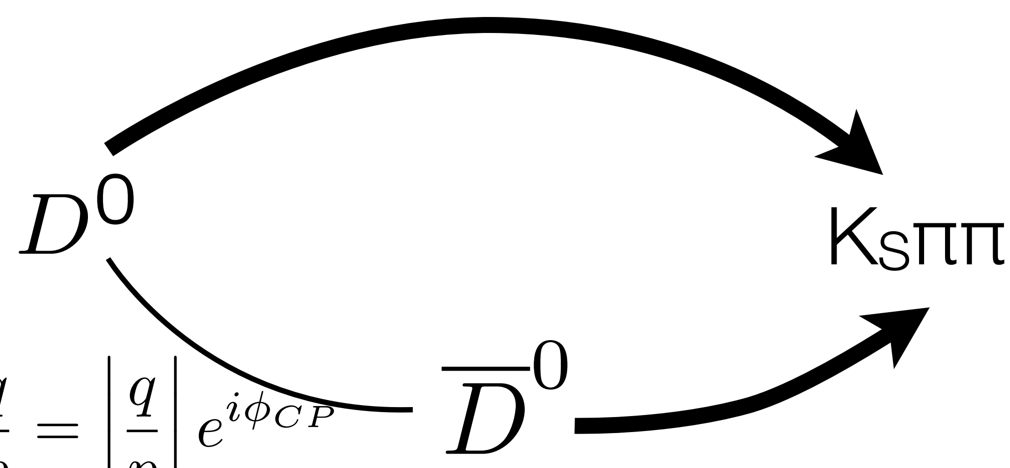

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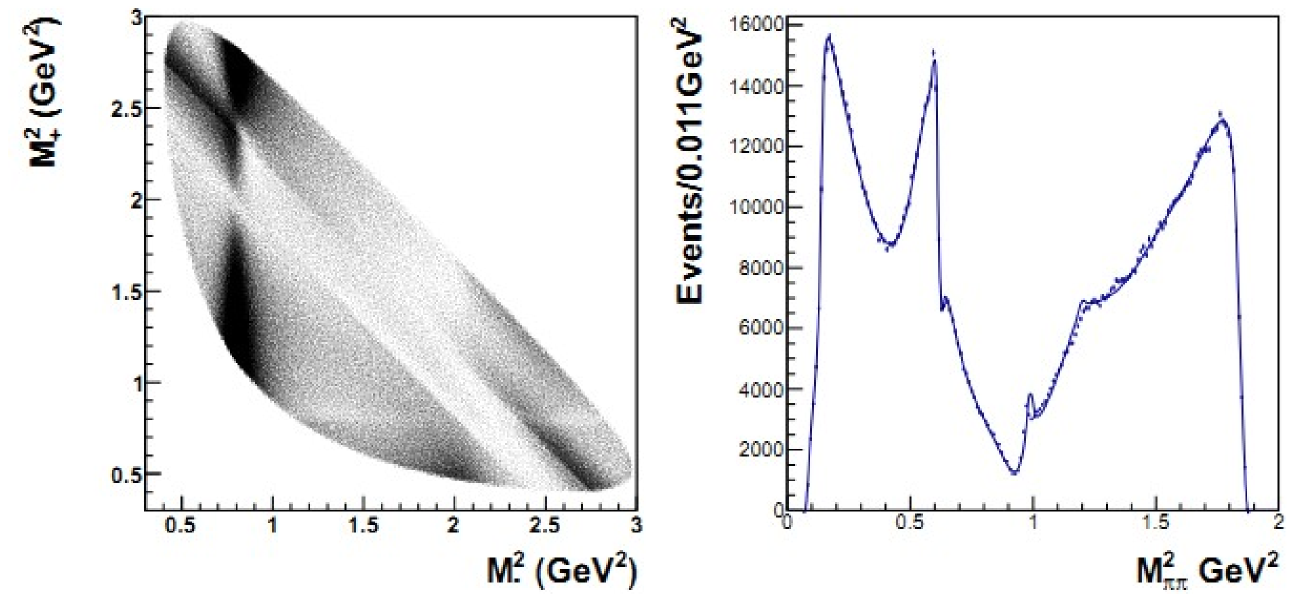

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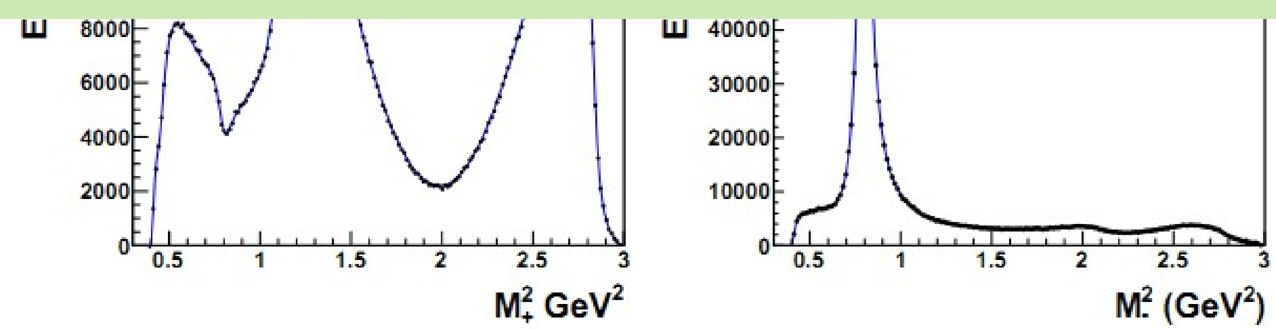
No evidence of CP violation



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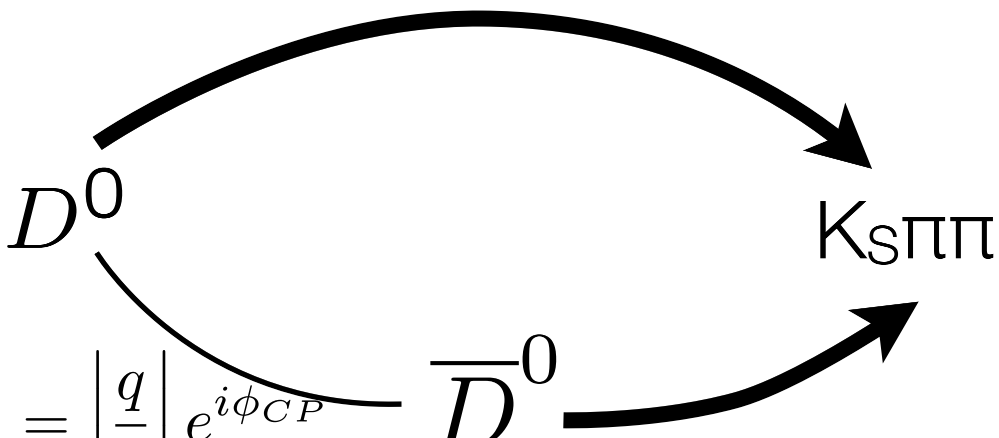

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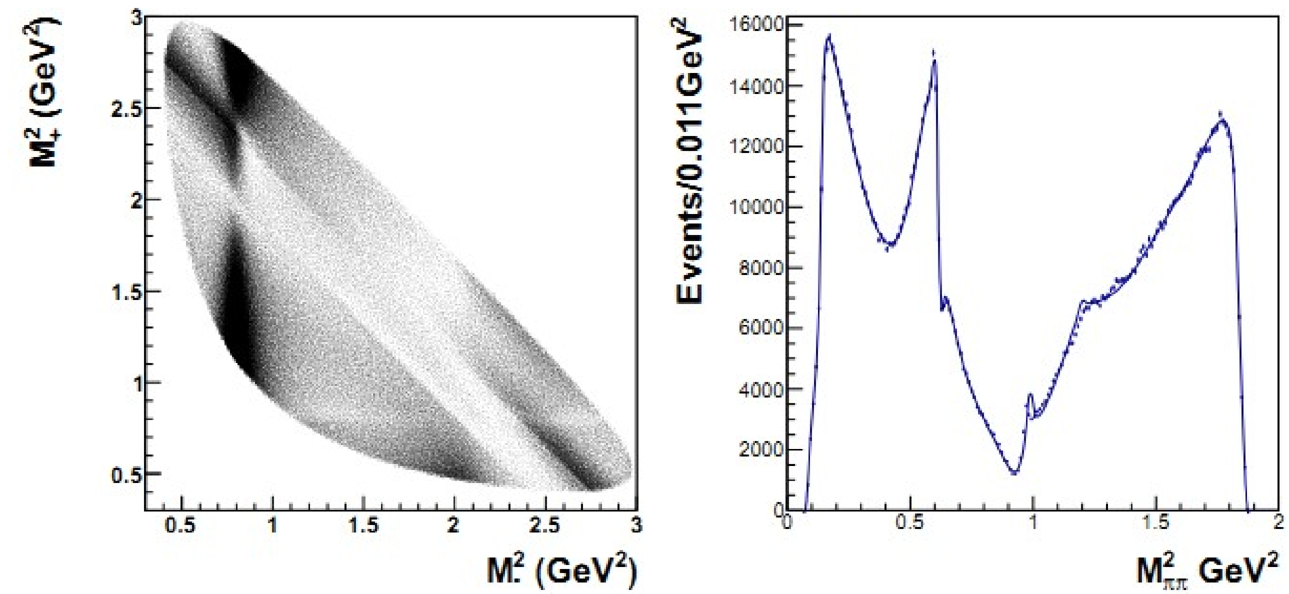

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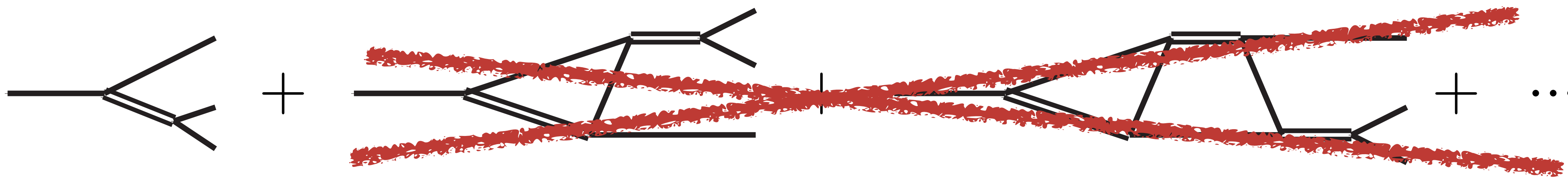
Significant systematic uncertainty from amplitude model dependence. (Could be limiting with future LHCb/upgrade statistics.)

M_+^2 GeV^2 M_-^2 (GeV^2)

see also BaBar Phys. Rev. Lett. 105, 081803 (2010) and CLEO-c Phys. Rev. D 72, 012001 (2005).

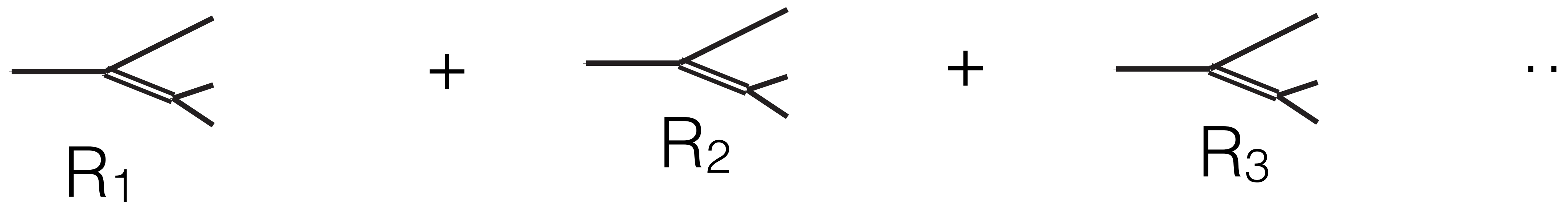
“Isobar” Model

- “Isobar”: Describe decay as series of 2-body processes.



- Usually: each resonance described by Breit Wigner lineshape (or similar) times factors accounting for spin.
- Popular amongst experimentalists, less so amongst theorists: violates unitarity. But not much as long as resonances are reasonably narrow, don’t overlap too much.
- General consensus: Isobar OK-ish for P, D wave, but problematic for S-wave.Alternatives exist, e.g. K-matrix formalism, which respects unitarity.

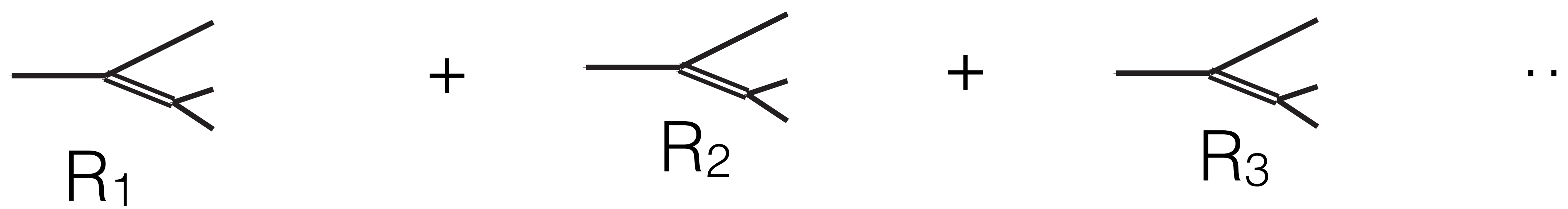
Isobar Model with sum of Breit Wigners



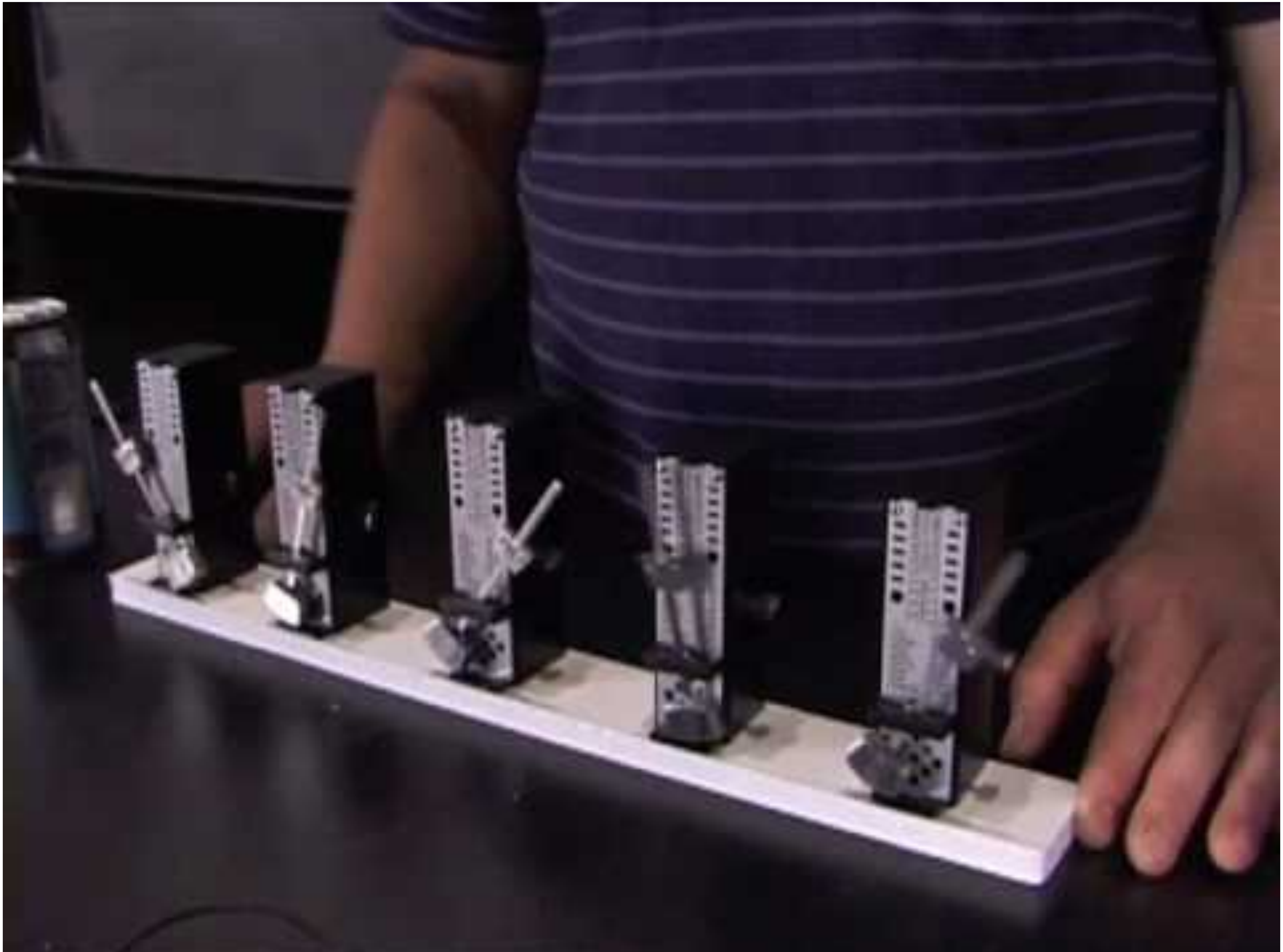
$$\frac{1}{s_{12}-m_1^2-im_1\Gamma_1(s_{12})} + \frac{1}{s_{12}-m_2^2-im_2\Gamma_2(s_{12})} + \frac{1}{s_{12}-m_3^2-im_3\Gamma_3(s_{12})} \cdots$$

- Single narrow resonance well described by Breit Wigner
- Overlapping broad resonances less so. Theoretically problematic: violates unitarity. From a practical point of view problematic as you might get the wrong phase motion.

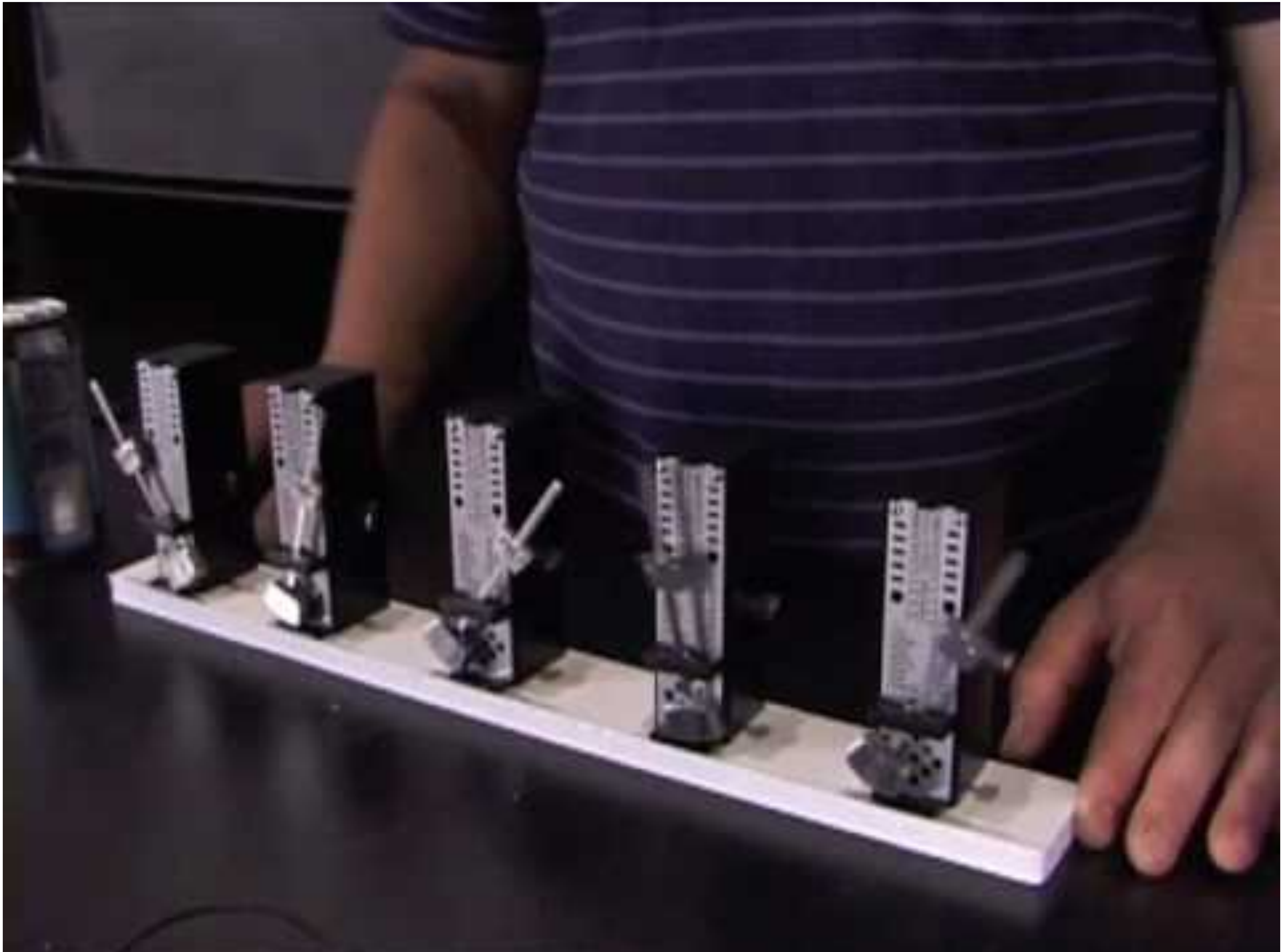
Isobar Model with sum of Breit Wigners



$$\frac{1}{s_{12}-m_1^2-im_1\Gamma_1(s_{12})} + \frac{1}{s_{12}-m_2^2-im_2\Gamma_2(s_{12})} + \frac{1}{s_{12}-m_3^2-im_3\Gamma_3(s_{12})} \cdots$$



link to video



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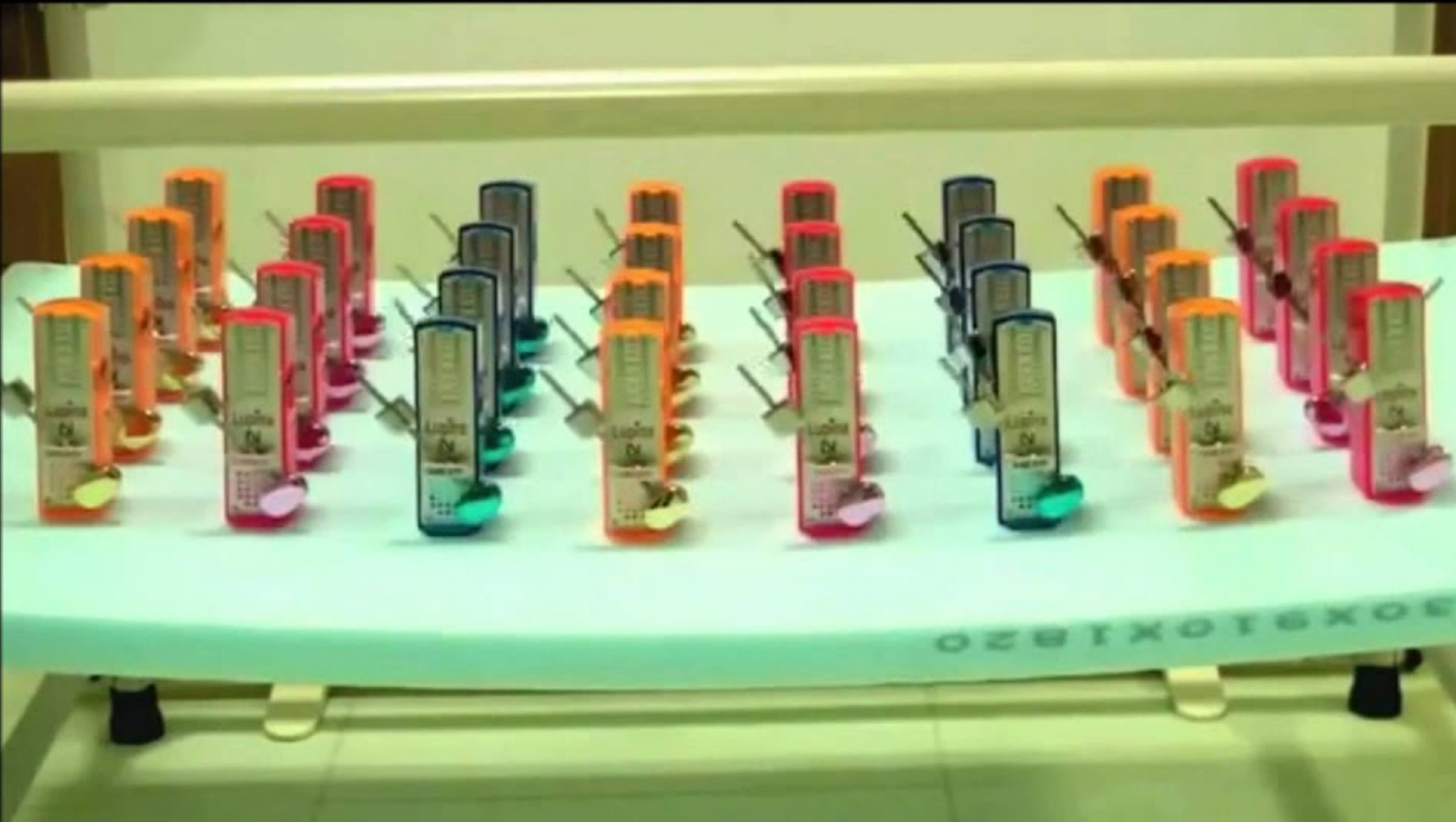

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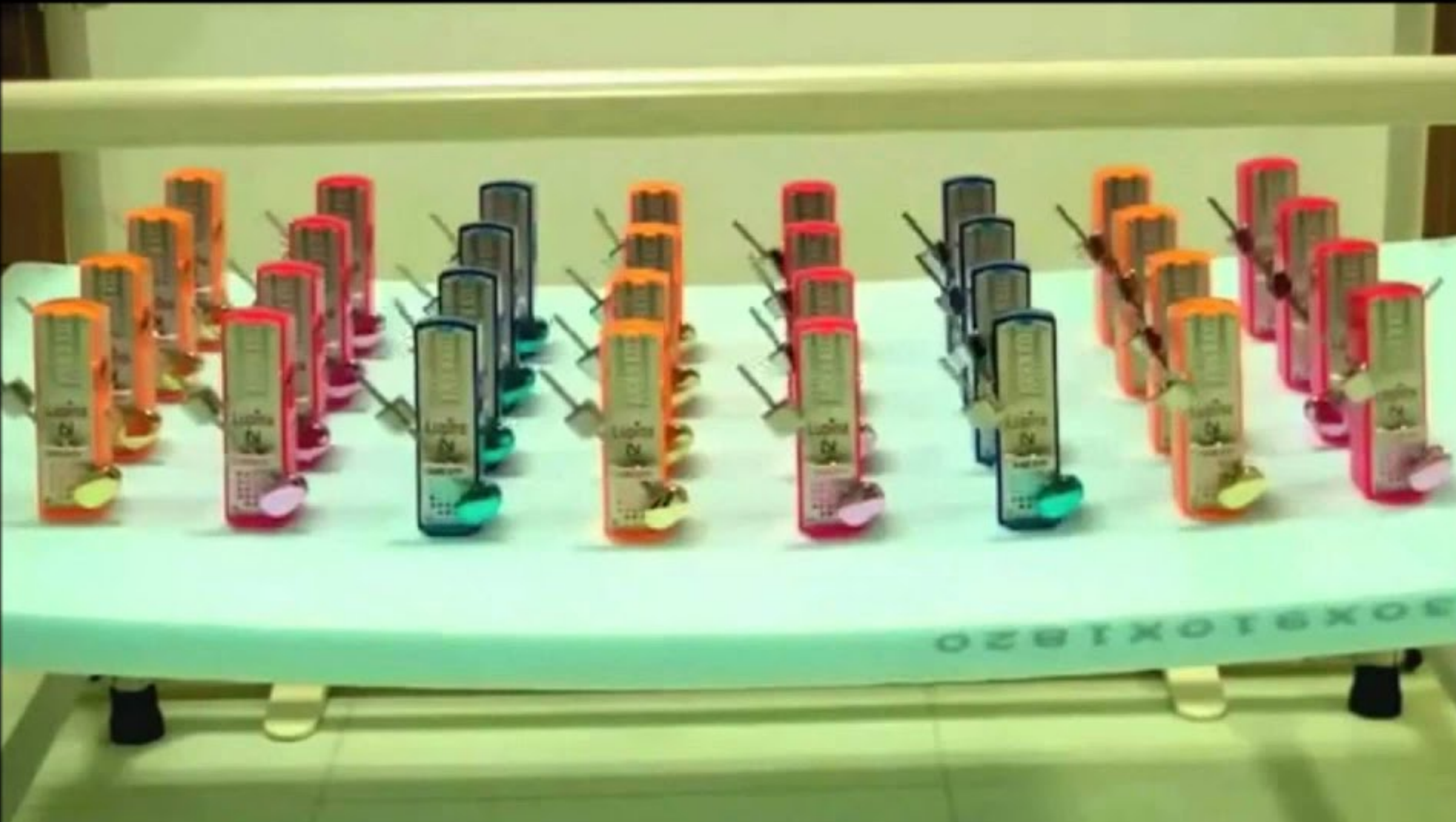

[link to video]

Flatté Formula

- Consider $f_0(980)$ (width $\Gamma \approx$ 40-100 MeV). Decays to $\pi\,\pi$ and KK. To KK only above ~987.4 MeV.
- The availability of the KK final state above 987.4 MeV increases the phase space and thus the width above this threshold.
- Need to take this into account even if I only look at $f_0(980)\rightarrow\pi\pi$.

$$\Gamma_{f_0}(s) = \Gamma_\pi(s) + \Gamma_K(s)$$

$$\Gamma_\pi(s) = g_\pi\sqrt{s/4 - m_\pi^2},$$

$$\Gamma_K(s) = \frac{g_K}{2}\left(\sqrt{s/4 - m_{K^+}^2} + \sqrt{s/4 - m_{K^0}^2}\right)$$

K-matrix

$$S_{fi} = \langle f|S|i\rangle = I + 2iT$$
$$T = K(I - iK)^{-1}$$
$$K_{ij} = \sum_{\alpha} \frac{\sqrt{m_\alpha \Gamma_{\alpha i}}\sqrt{m_\alpha \Gamma_{\alpha j}}}{m_\alpha^2 - m^2}$$

- For single channel: Reproduces Breit Wigner
- For single resonance that can decay to different final state: Reproduces Flatté.

K-matrix

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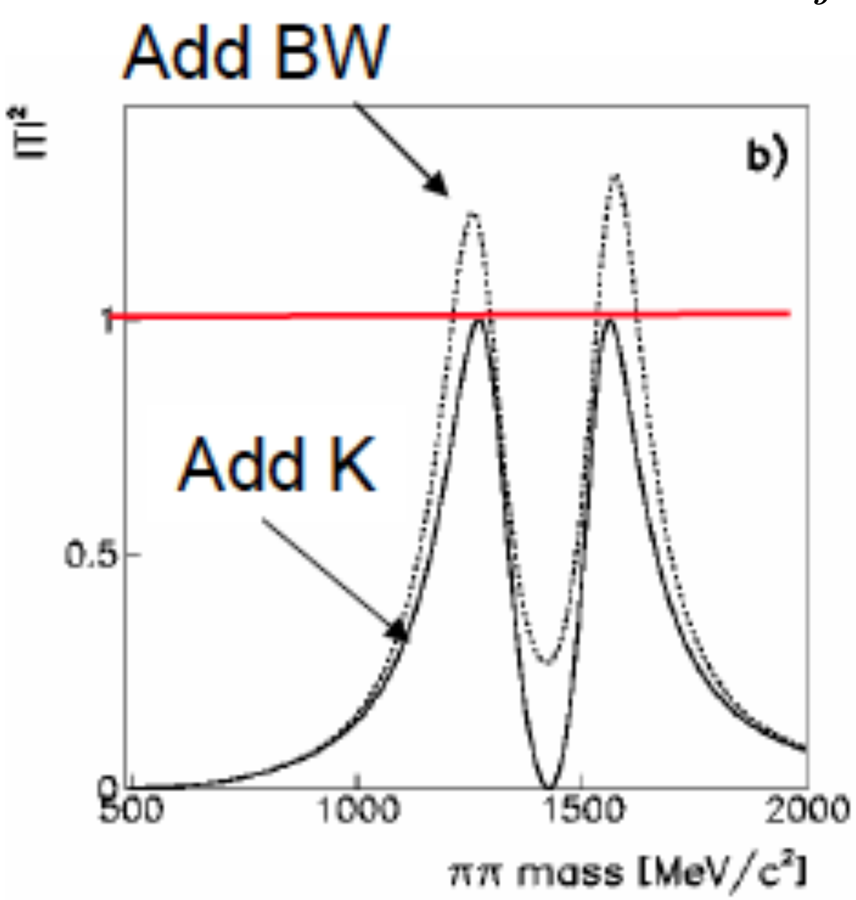
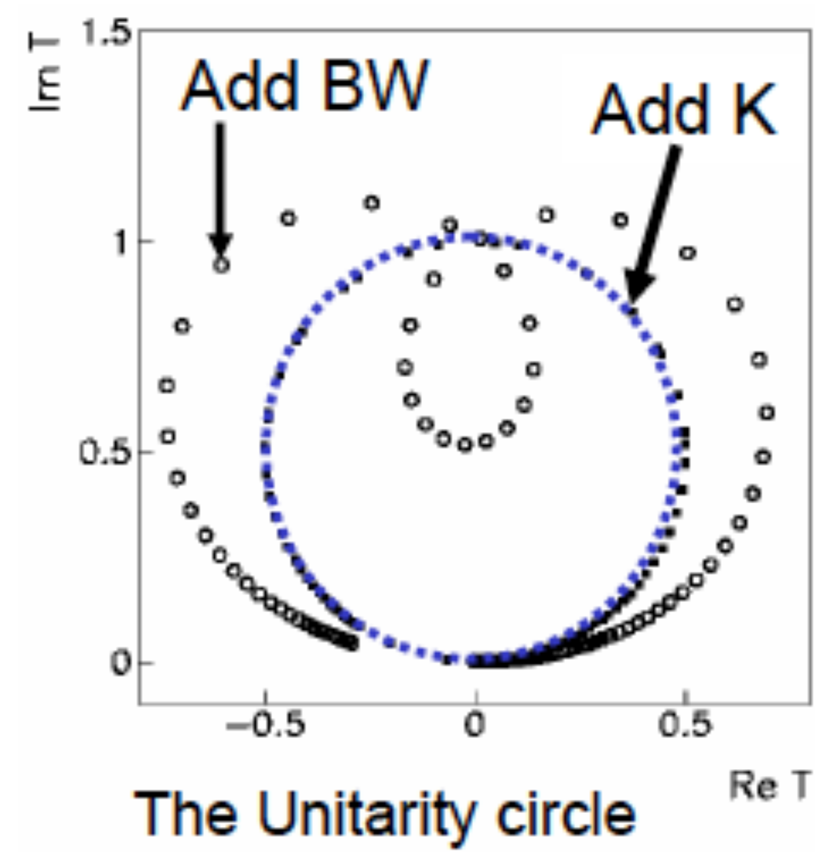
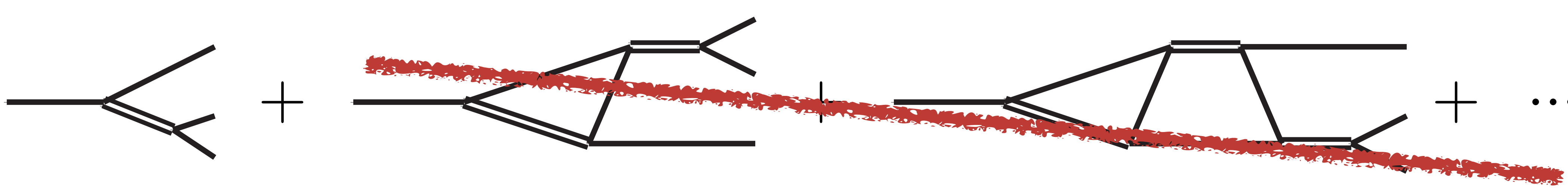




K-matrix

- Note that the K-matrix approach is still an approximation.
- While it ensures unitarity (by construction), it is not completely theoretically sound/motivated (and violates analyticity).
- And it does not in any way address this:



What theorists think of all this

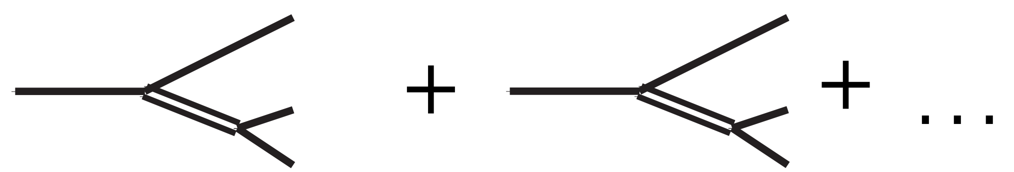
Sum of Breit Wigners

Sum of Breit Wigners with non-resonant term

Last Judgement (Detail) by Fra Angelico

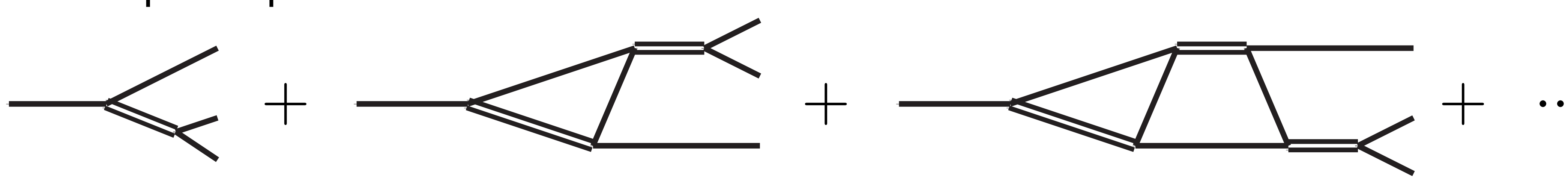
Amplitude Models and their issues

- Let's look at this (effectively 2-body scattering) problem, first:



Describing this with sums of Breit Wigner line shapes violates some fundamental principles, in particular unitarity (which then breaks the relation between magnitude and phase of the amplitude). OK-ish for narrow resonances that do not overlap too much.

- We'll postpone the discussion this for a few slides:



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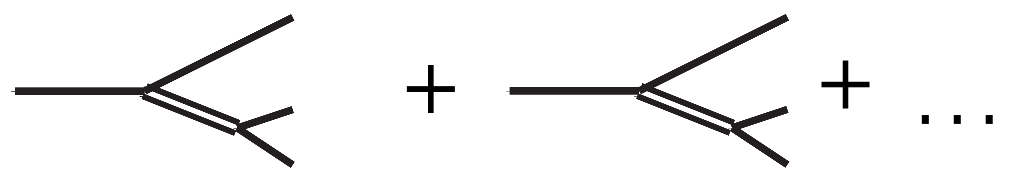

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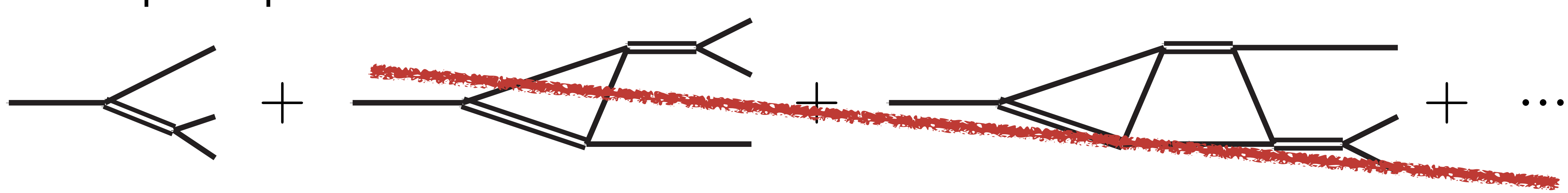


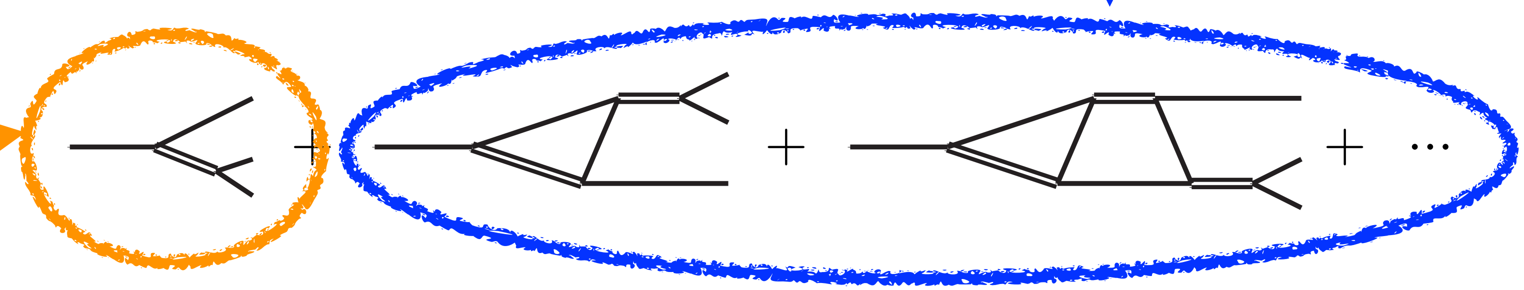
Bastian Kubis

3-body Dalitz plot (theory)

takes into account this

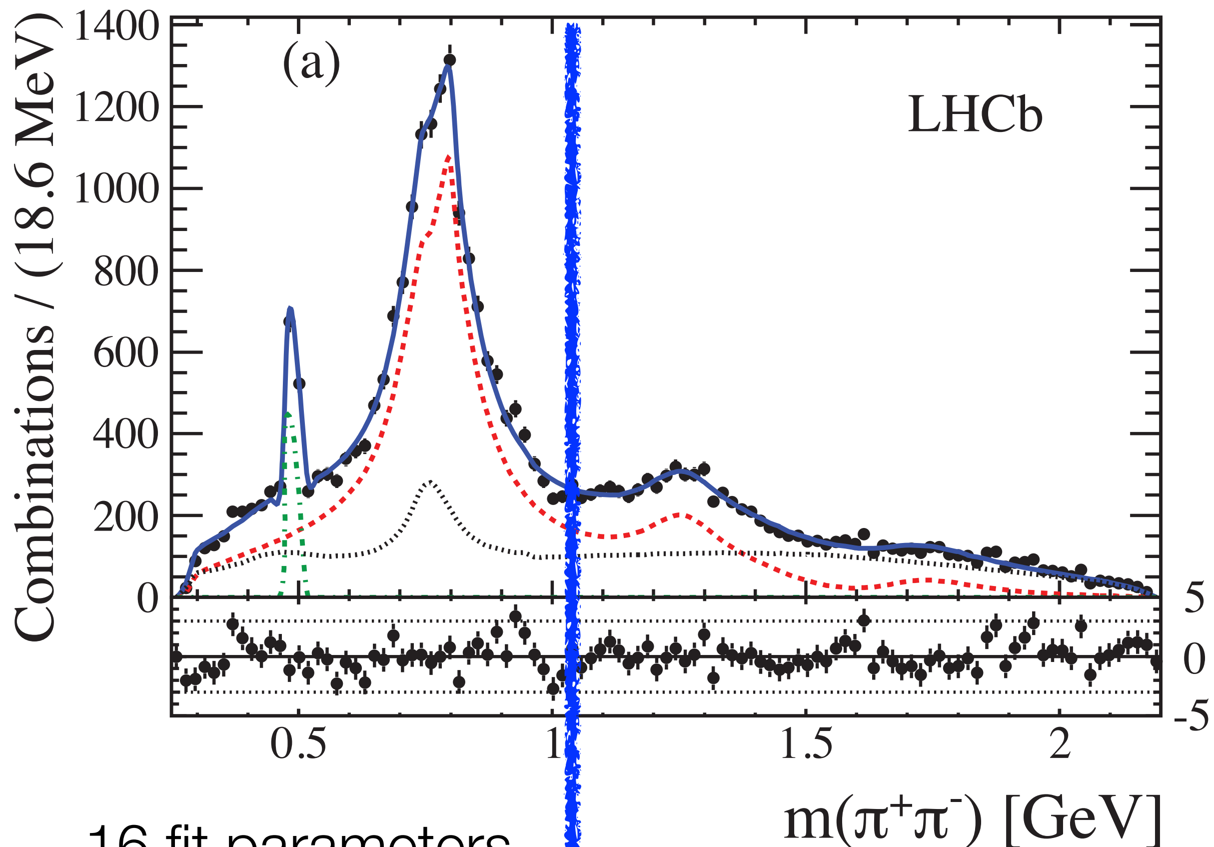
$$\mathcal{F}(s) = a\,\Omega(s)\left\{1+\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\sin\delta_1^1(s')\,\hat{\mathcal{F}}(s')}{|\Omega(s')|(s'-s)}\right\}$$

Omnès takes into account just this



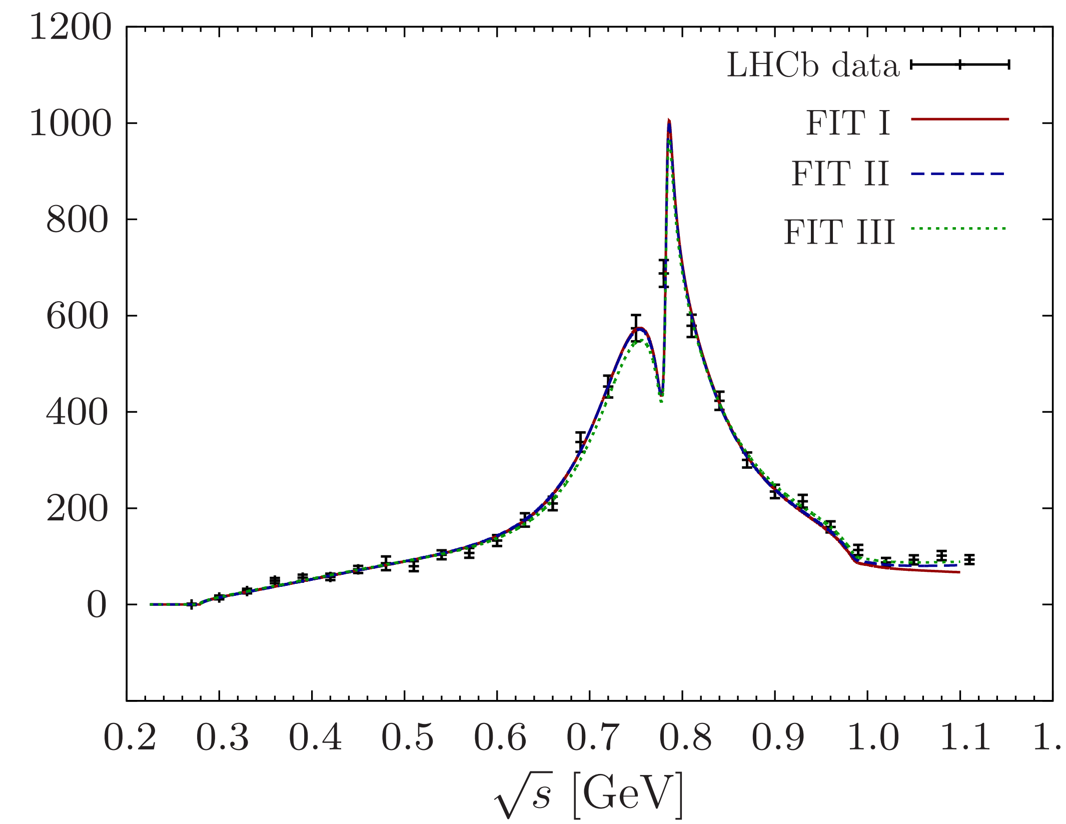
$B_d \to J/\psi\pi\pi$, dispersion relation-based description of $\pi\pi$ S-wave

LHCb fit
(projection on m(ππ))



16 fit parameters for Breit Wigner description of signal in the region < 1.02 GeV

Daub, Hanhart, Kubis' fit to background subtracted, efficiency corrected LHCb data with m(ππ) < 1.02 GeV



3 fit parameters (model I and III)
4 fit parameters for model II

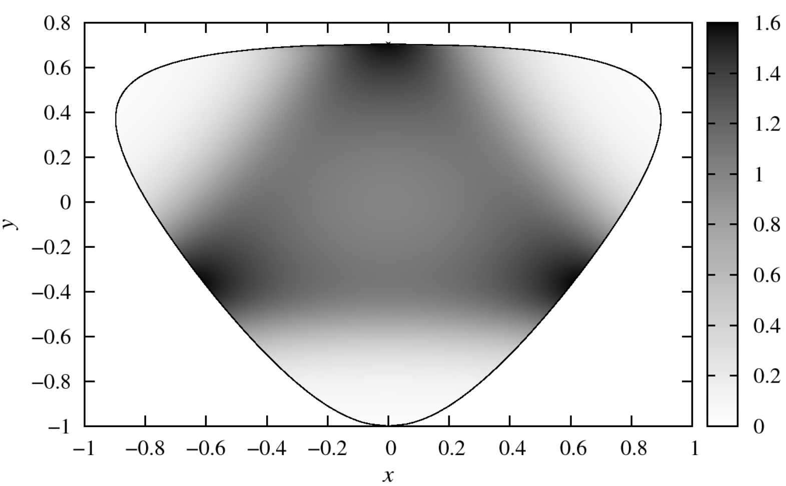
Daub, Hanhart, Kubis: JHEP 1602 (2016) 009

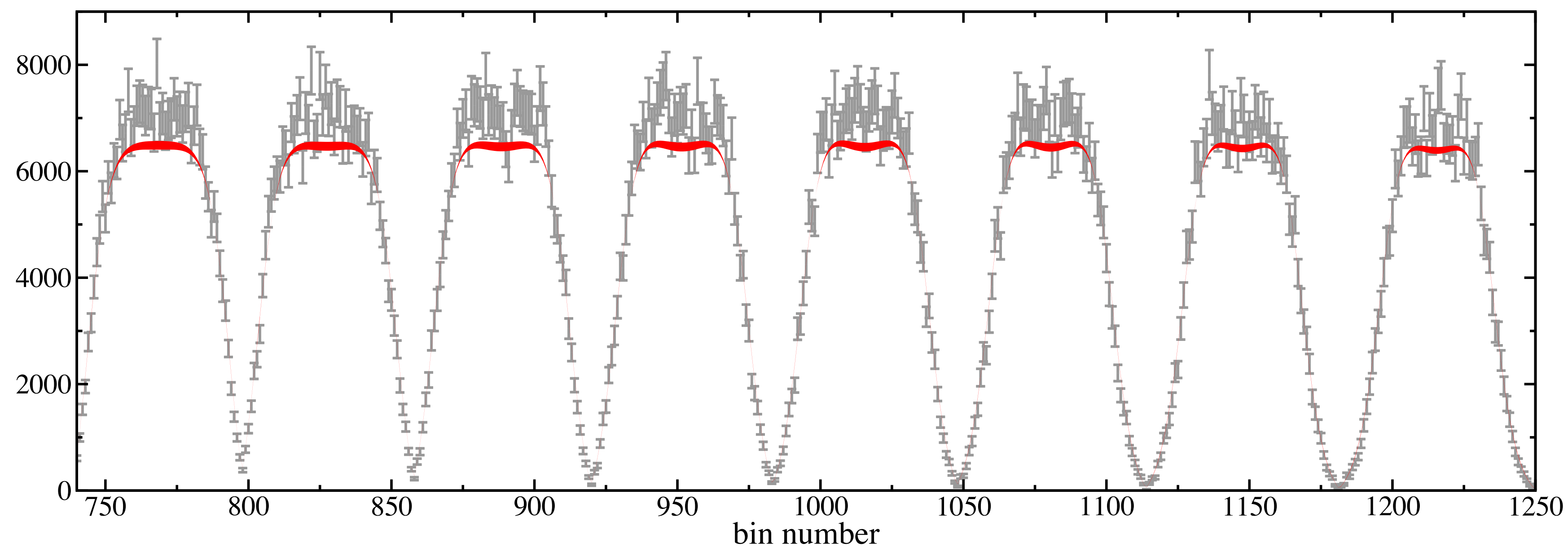
Formalism applied to ϕ→πππ^0

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7\ldots2.1$

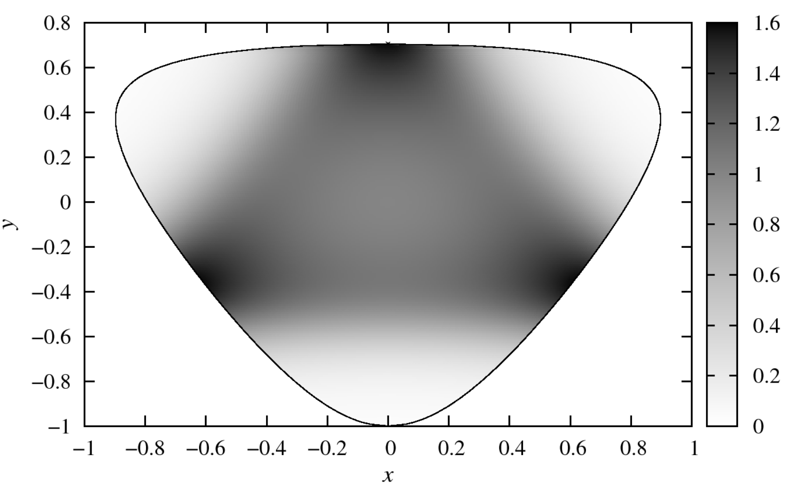
$\longrightarrow$ pairwise interaction only (with correct $\pi\pi$ scattering phase)

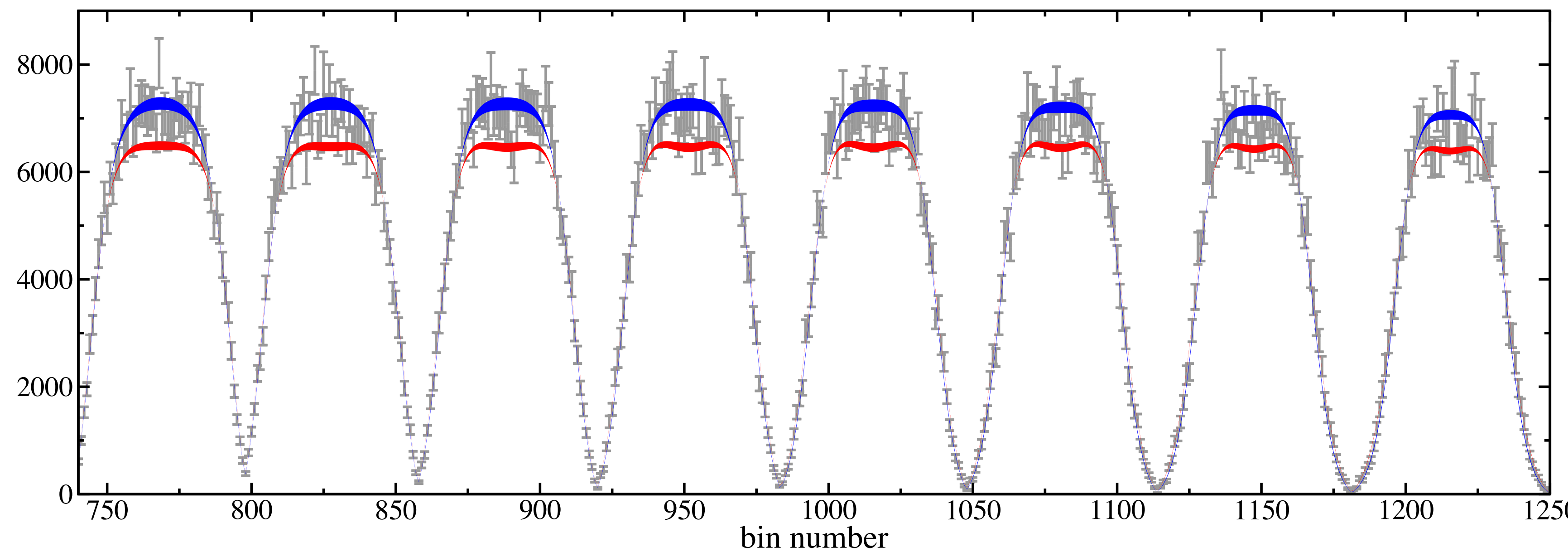
Formalism applied to $\phi \to \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$

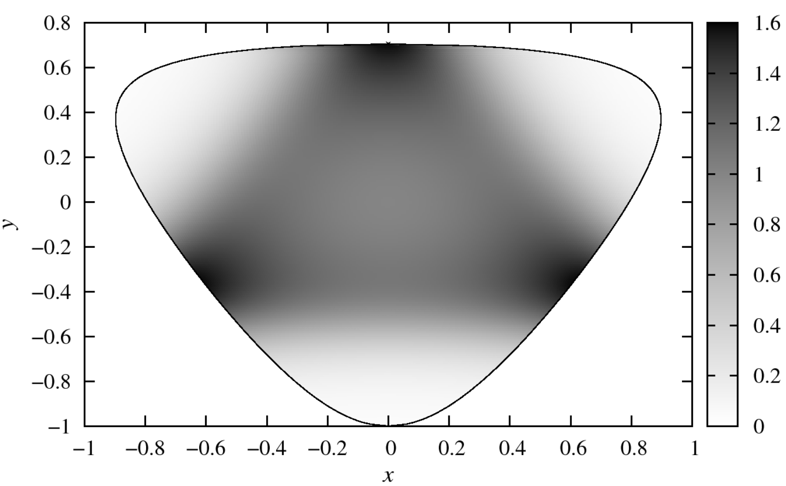
$\longrightarrow$ full 3-particle rescattering, only overall normalization adjustable

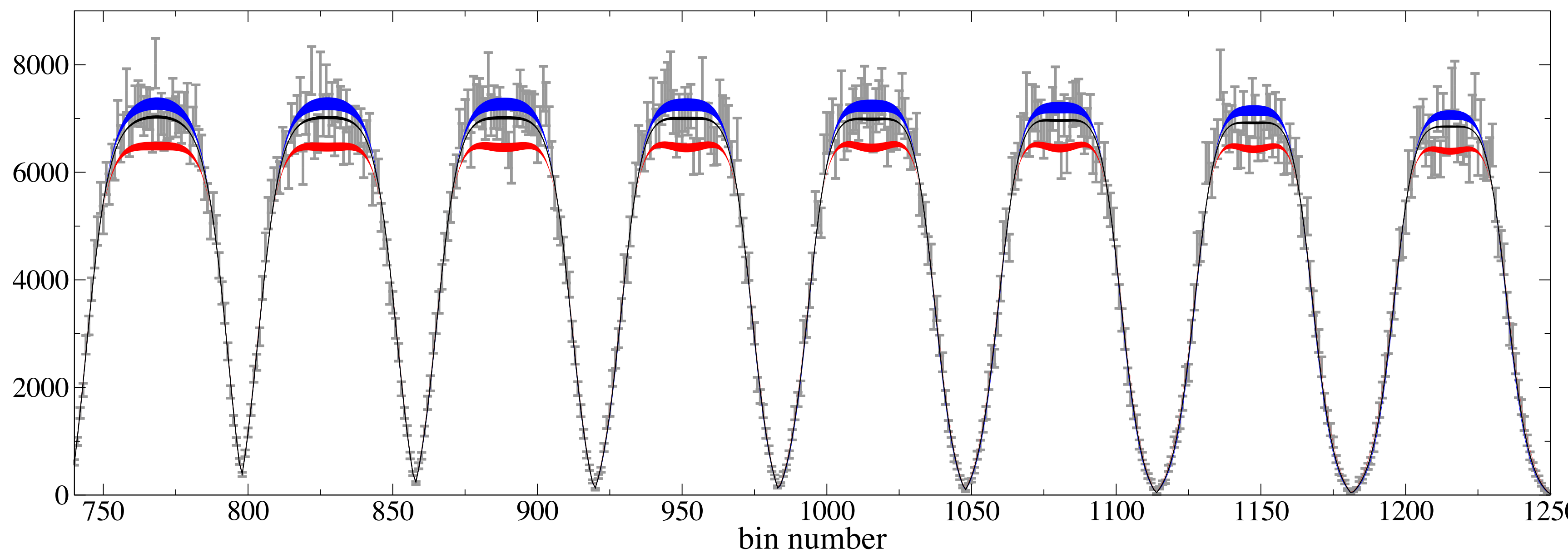
Formalism applied to $\phi \to \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012





χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$	1.0

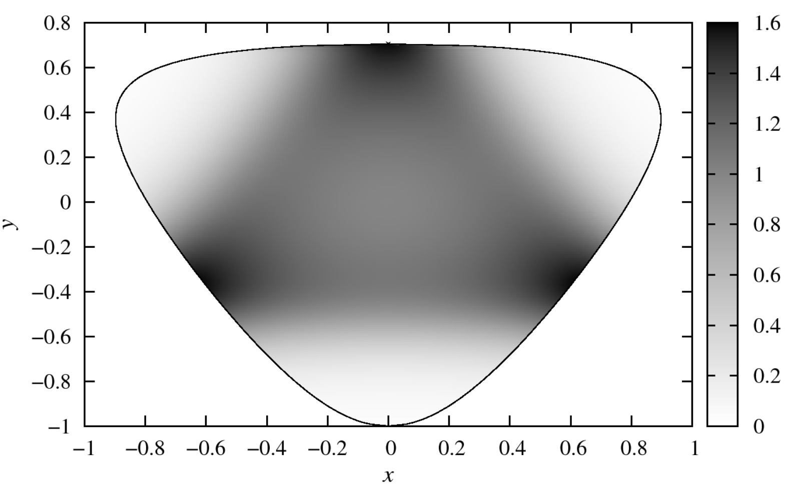
$\longrightarrow$ full 3-particle rescattering, 2 adjustable parameters
(additional "subtraction constant" to suppress inelastic effects)

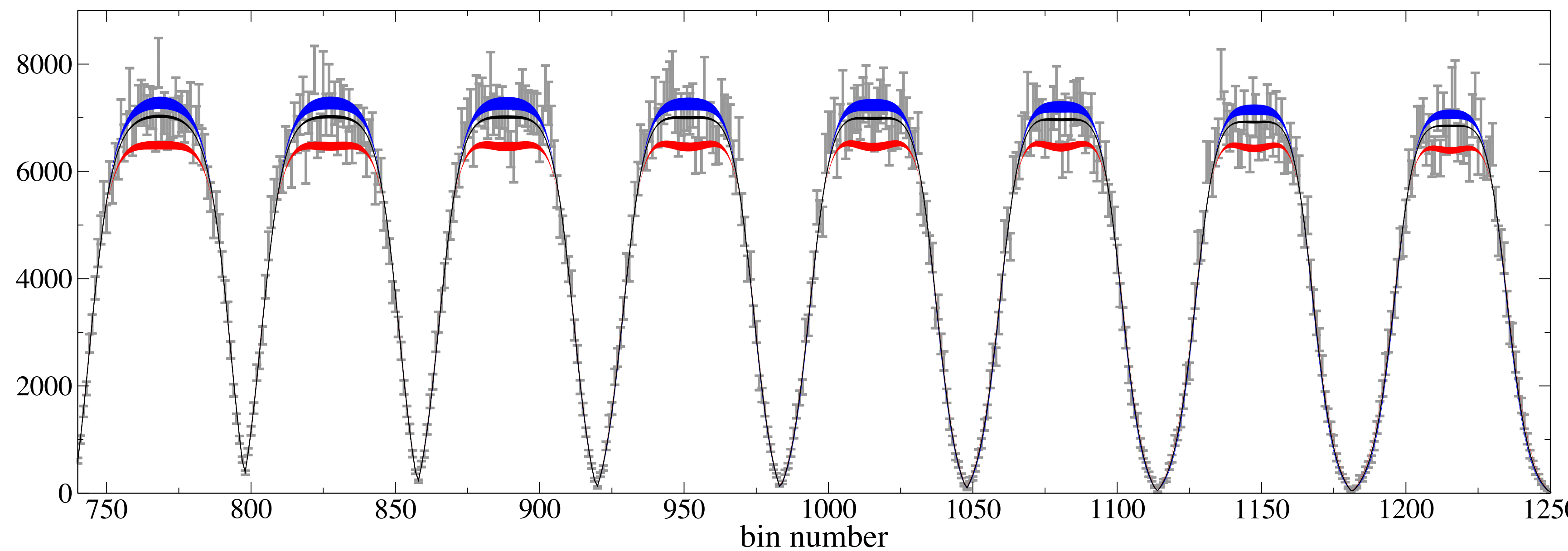
Formalism applied to $\phi \to \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \to 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



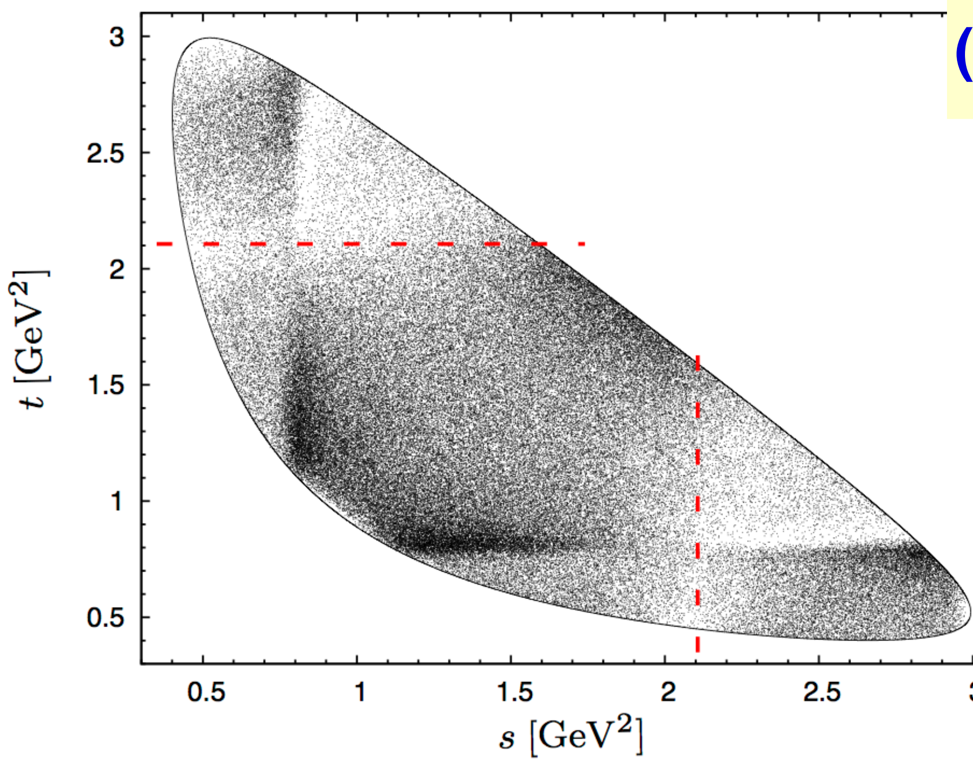


χ^2/ndof	$1.7\ldots2.1$	$1.2\ldots1.5$	1.0

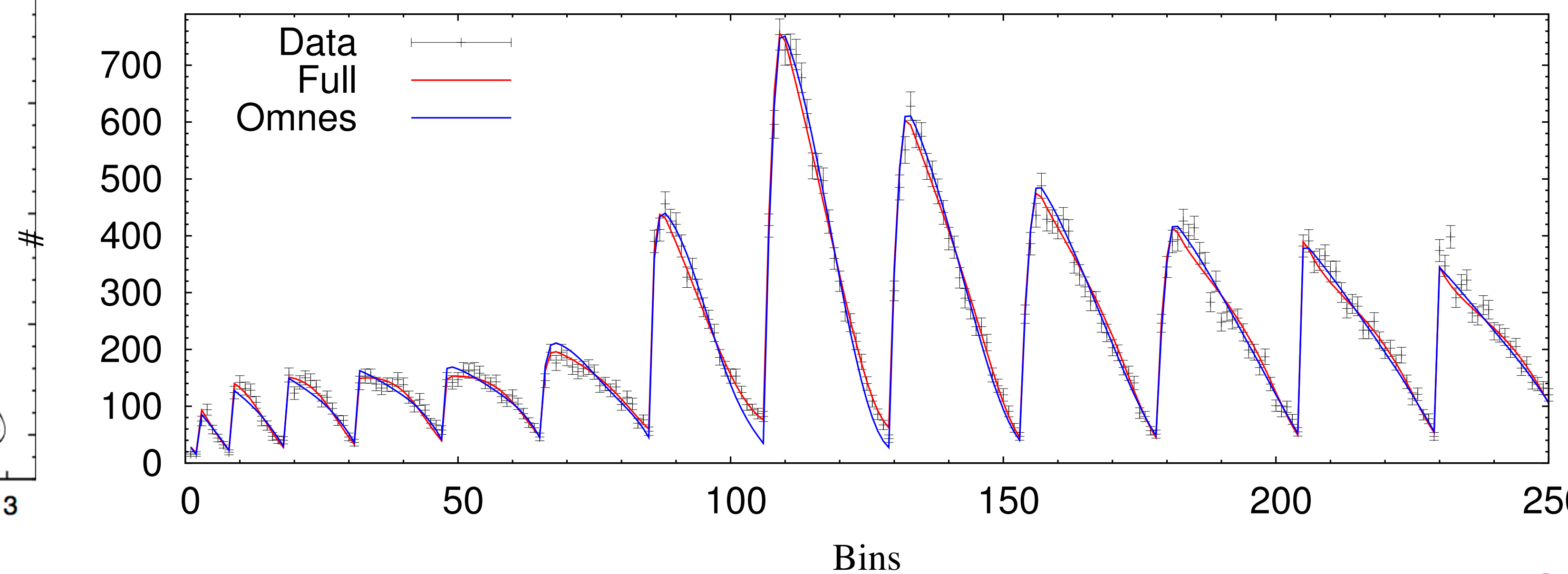
- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"

Formalism applied to D→ππK

Bastian Kubis



(Slices through) Dalitz plot $D^+ \to \pi^+\pi^+K^-$



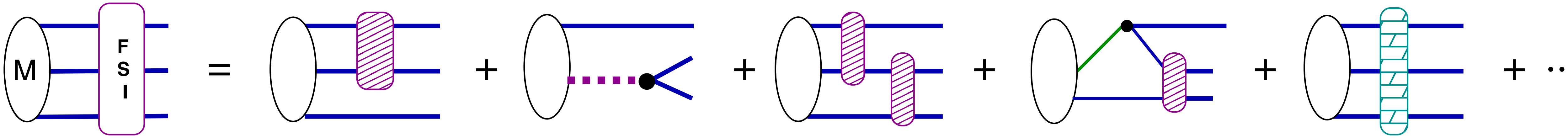
Fit limited to
M(Kπ) < M(η’) + M(K) ≈ 1.45GeV
elastic approximation breaks down beyond.

- Omnès fit: $\chi^2/\text{ndof} \approx 1.42$
  ("isobar model" + non-resonant background waves)
- full dispersive solution: $\chi^2/\text{ndof} \approx 1.11$
  $\longrightarrow$ visible improvement similar to $\phi \to 3\pi$
- full fit in terms of 7 complex subtraction constants
  (–1 phase, –1 overall normalisation)

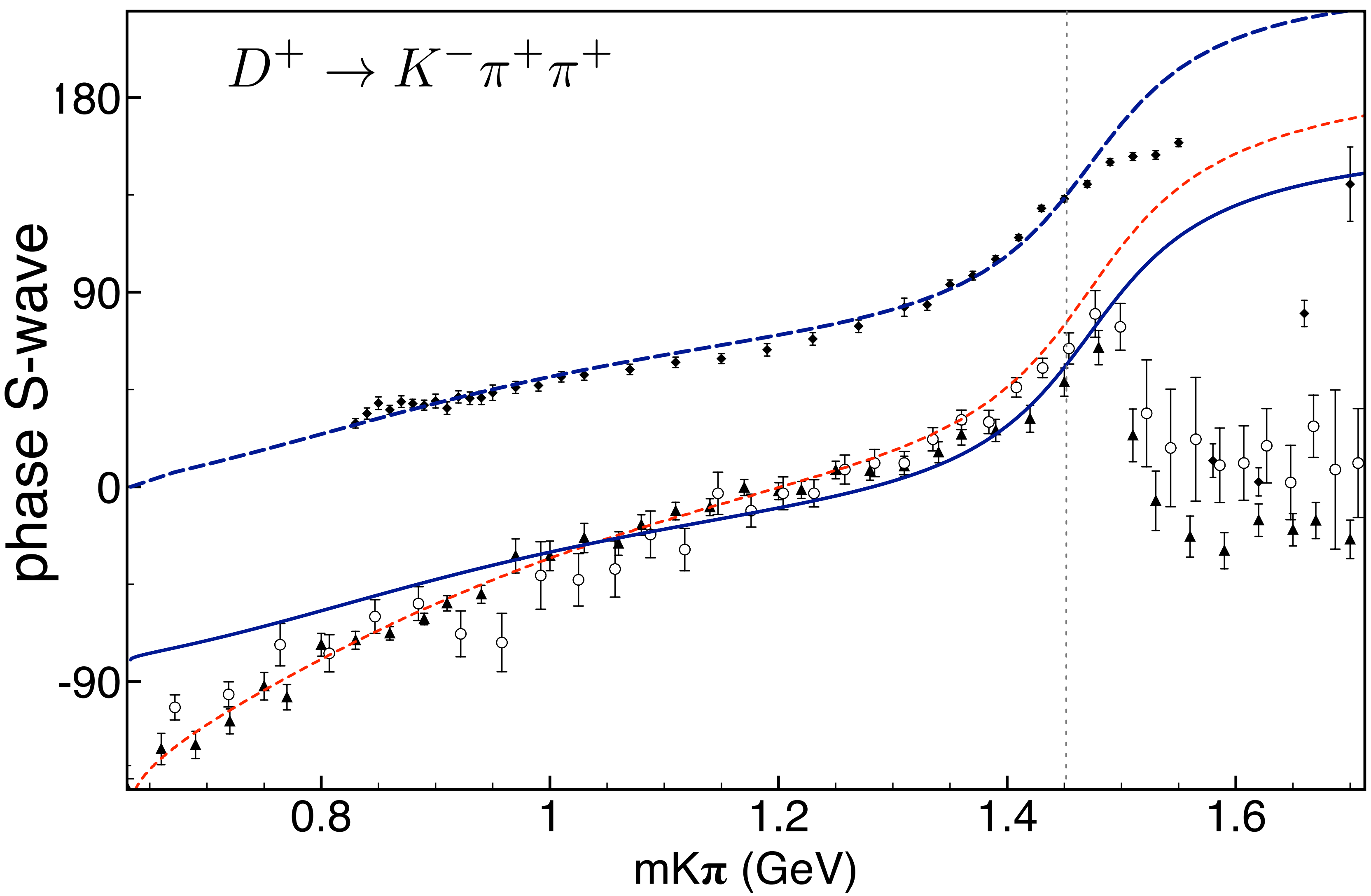
7 fit parameters

Niecknig, BK *in progress*

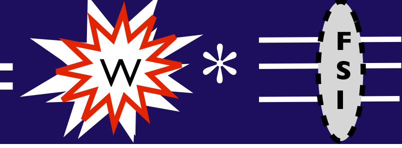
- Three-body FSI (beyond 2+1)



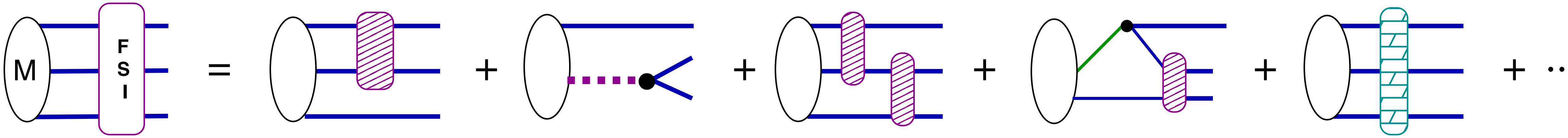
- shown to be relevant on charm sector



Models available

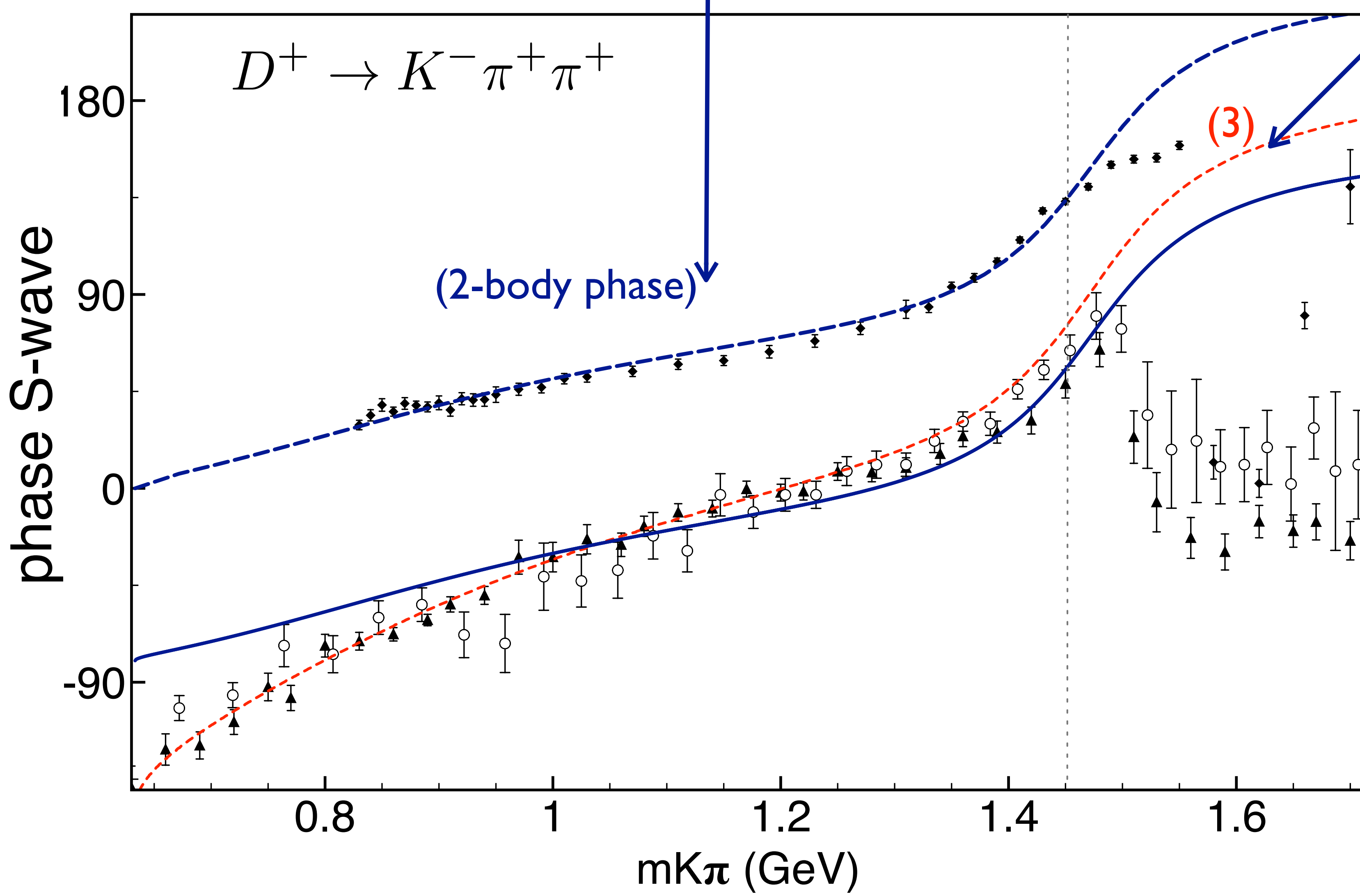
A = 

- Three-body FSI (beyond 2+1)

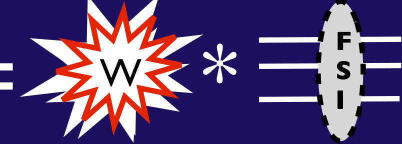


PRD92 094005 (2015)

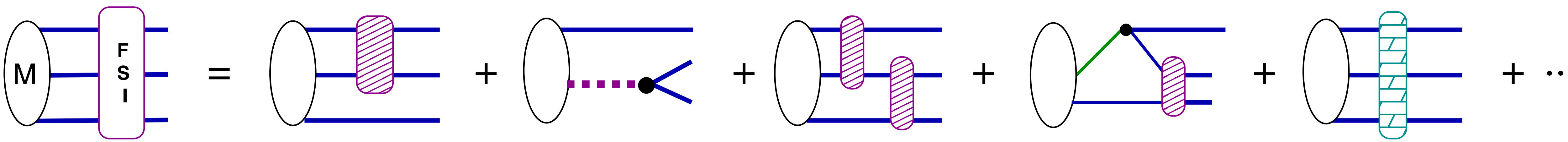
- shown to be relevant on charm sector



Models available

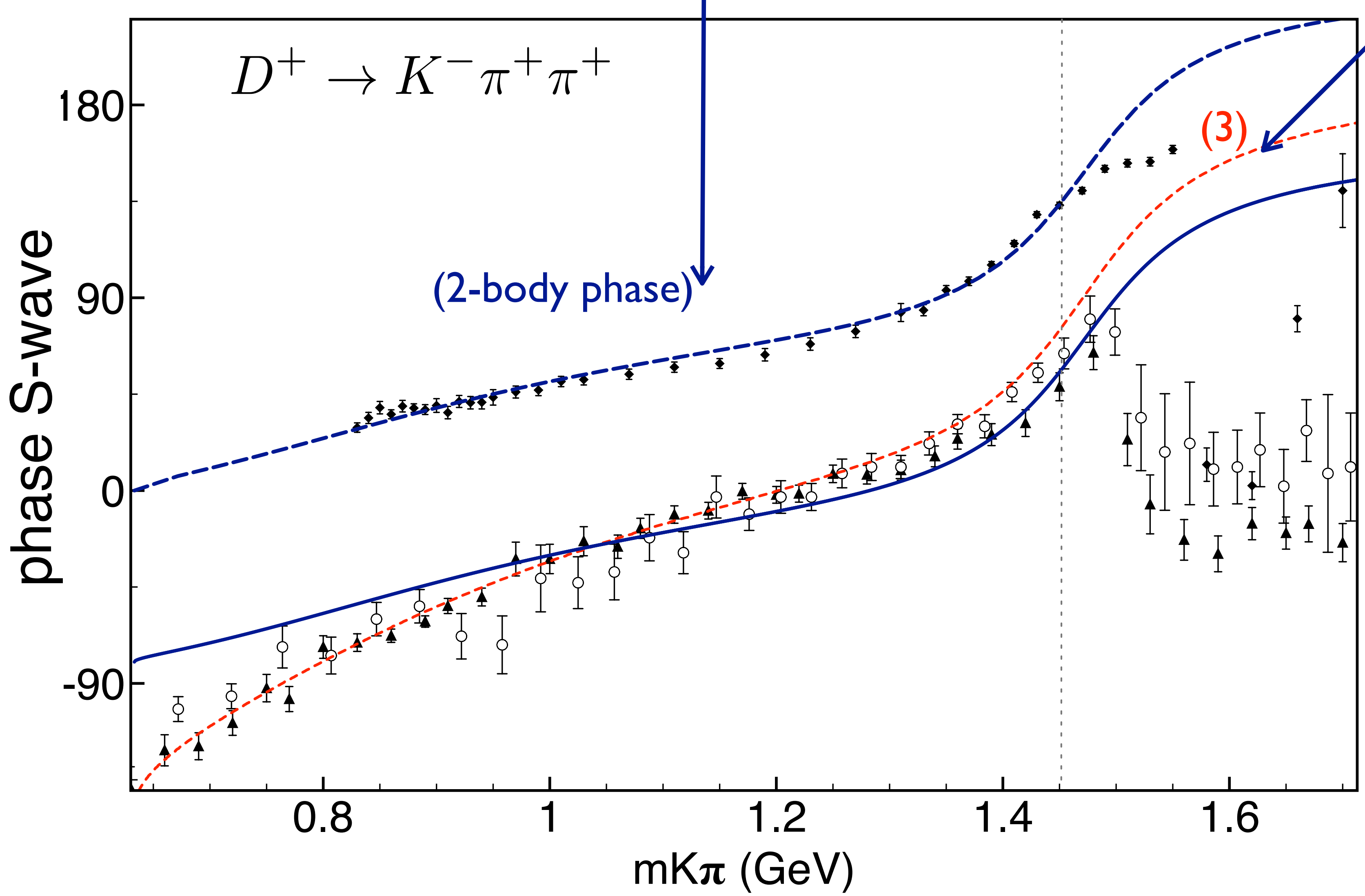
A = 

- Three-body FSI (beyond 2+1)



- shown to be relevant on charm sector

PRD92 094005 (2015)



- 3-body approaches

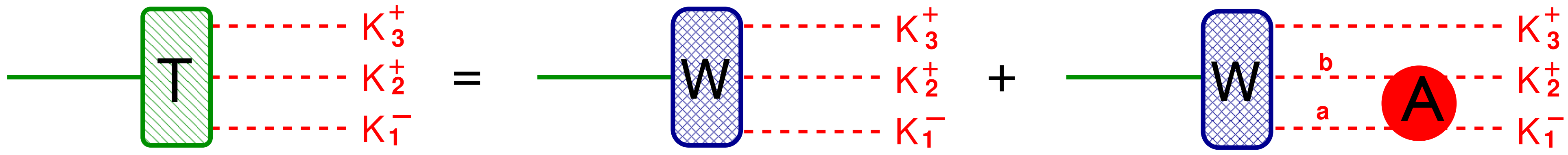
PCM et.al: PRD84 094001 (2011),
S.Nakamura PRD93 014005 (2016)
Niecknig, Kubis, JHEP10 142 (2015)

↳ 3-body FSI play a role

↳ data analysis…

multi meson model - $D^+ \to K^- K^+ K^+$

Triple-M



- alternative to isobar model

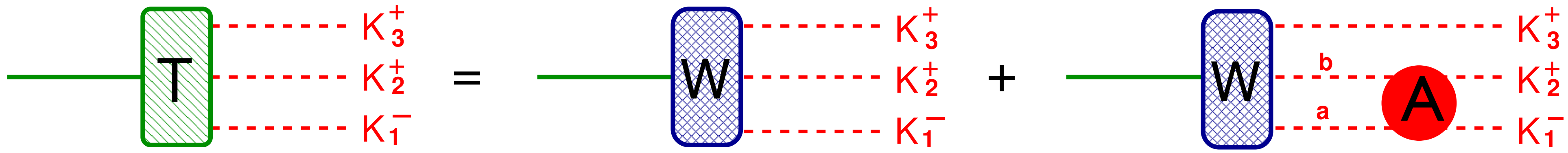
backup

- starts from Chiral Theory

  - A_{ab}^{JI} unitary scattering amplitude for $ab \to K^+ K^-$

    full FSI: coupled channel

Triple-M



- alternative to isobar model

backup

- starts from Chiral Theory

  - A_{ab}^{JI} → unitary scattering amplitude for $ab \to K^+ K^-$
    → full FSI: coupled channel

→ parameters have physical meaning: resonance masses and coupling constants

→ relative phase between partial waves is NOT fitted

Theoretical model

PHYSICAL REVIEW D **98,** 056021 (2018)

arXiv:1805.11764 [hep-ph]

Multimeson model for the $D^+ \to K^+K^-K^+$ decay amplitude

R. T. Aoude,[1,2] P. C. Magalhães,[1,3,*] A. C. dos Reis,[1] and M. R. Robilotta[4]

fitted to LHCb data

JHEP 1904 (2019) 063

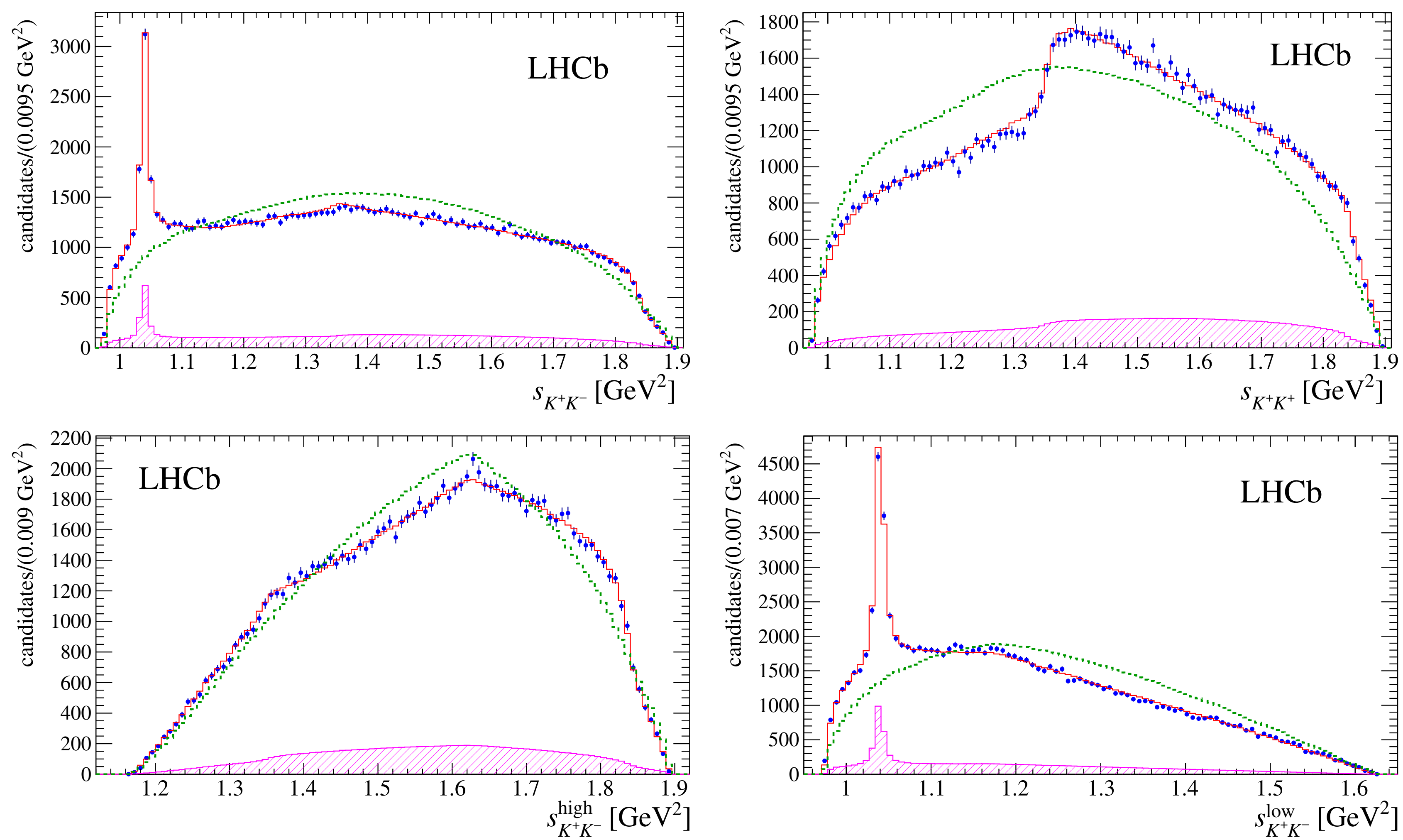
$$T^S = T^S_{NR} + T^{00} + T^{01}$$

$$T^P = T^P_{NR} + T^{11} + T^{10}$$

$\mathrm{FF_{NR}}$	FF^{00}	FF^{01}	FF^{10}	FF^{11}	$\mathrm{FF_{S-wave}}$
14 ± 1	29 ± 1	131 ± 2	7.1 ± 0.9	0.26 ± 0.01	94 ± 1

- free parameters

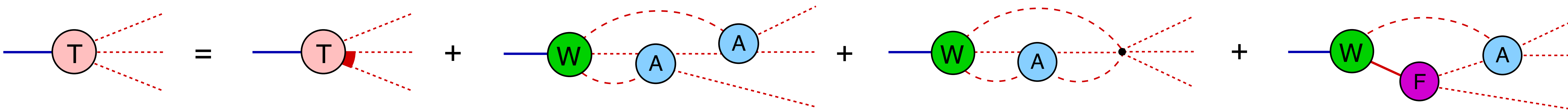
parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5\,\mathrm{MeV}$
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6\,\mathrm{MeV}$
m_{S_o}	$992.0^{+8.5}_{-7.5} \pm 8.6\,\mathrm{MeV}$
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1\,\mathrm{MeV}$
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51\,\mathrm{MeV}$
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9\,\mathrm{MeV}$
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3\,\mathrm{MeV}$
$\tilde{c}_d$	$-6.15^{+0.55}_{-0.54} \pm 0.19\,\mathrm{MeV}$
$\tilde{c}_m$	$-10.8^{+2.0}_{-1.5} \pm 0.4\,\mathrm{MeV}$



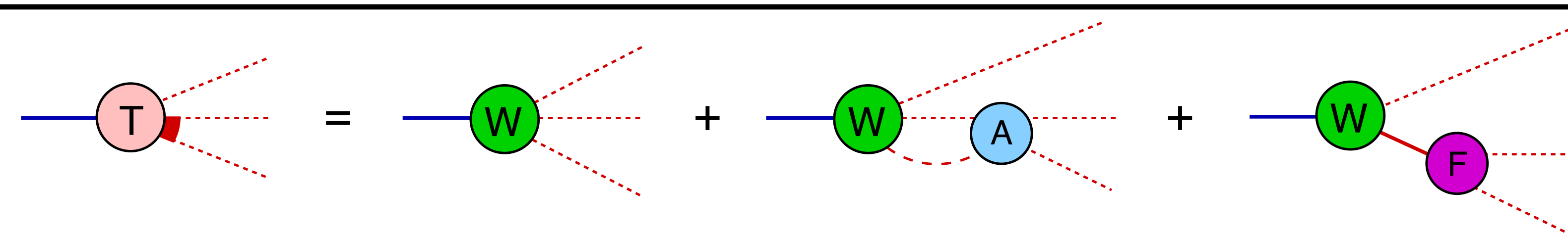
→ good fit with fewer parameters than the isobar

NEW MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

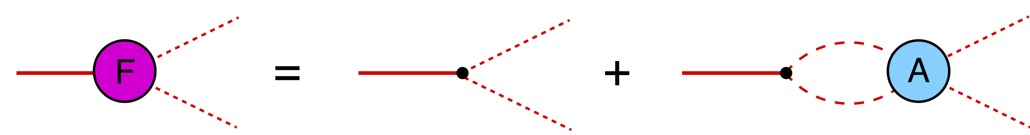
- Any 3-body decay amplitude



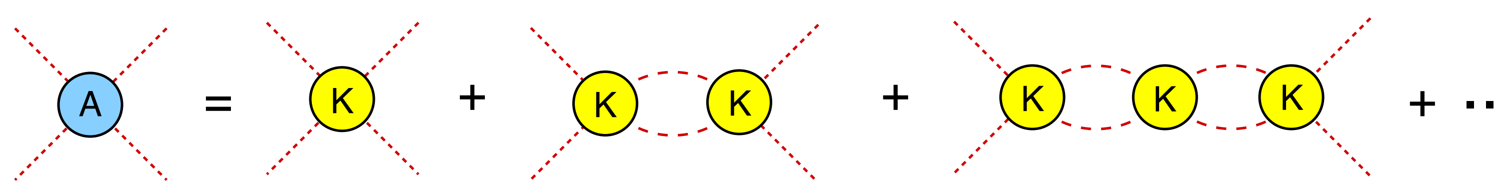
(2+1) approach



Form factor

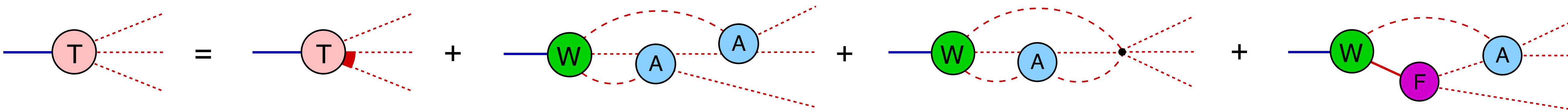


meson-meson

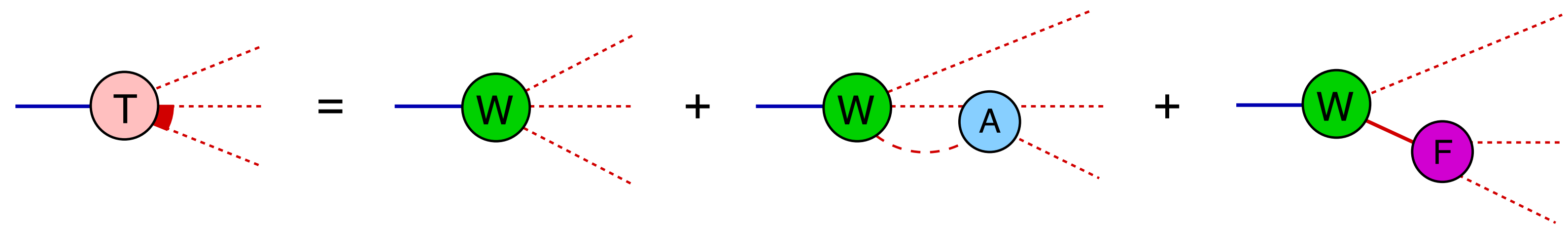


NEW MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

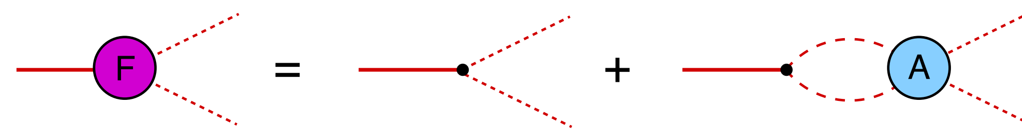
- Any 3-body decay amplitude



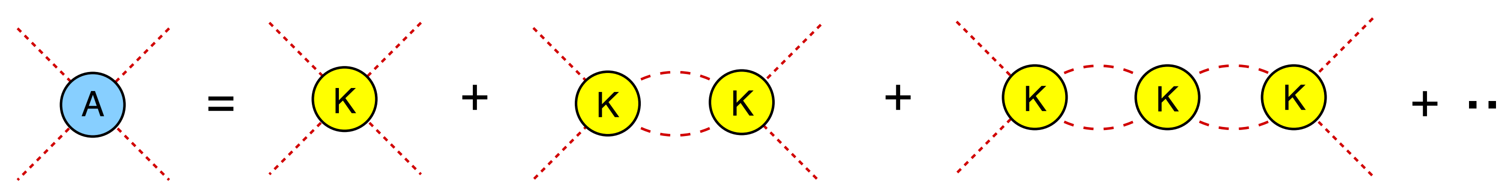
(2+1) approach

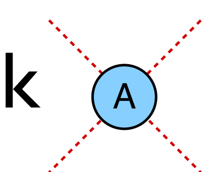


Form factor



meson-meson



↳ provide the building block 

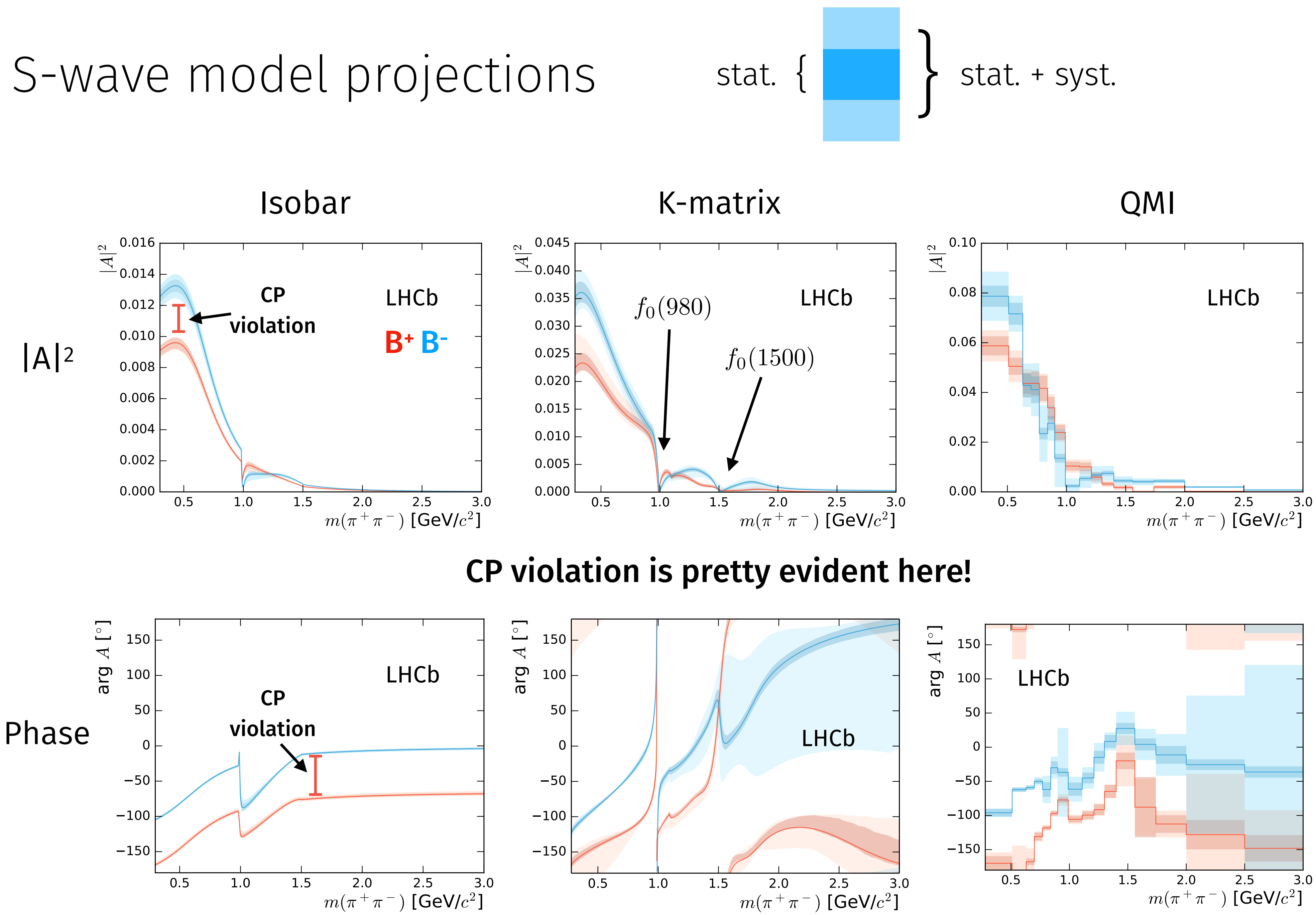
- includes multiple resonances in the same channel (as many as wanted)
- free parameter (massas and couplings) to be fitted to data.

⟶ Available to be implement in data analysis!!

LHCb: Phys. Rev. D101 (2020) 012006

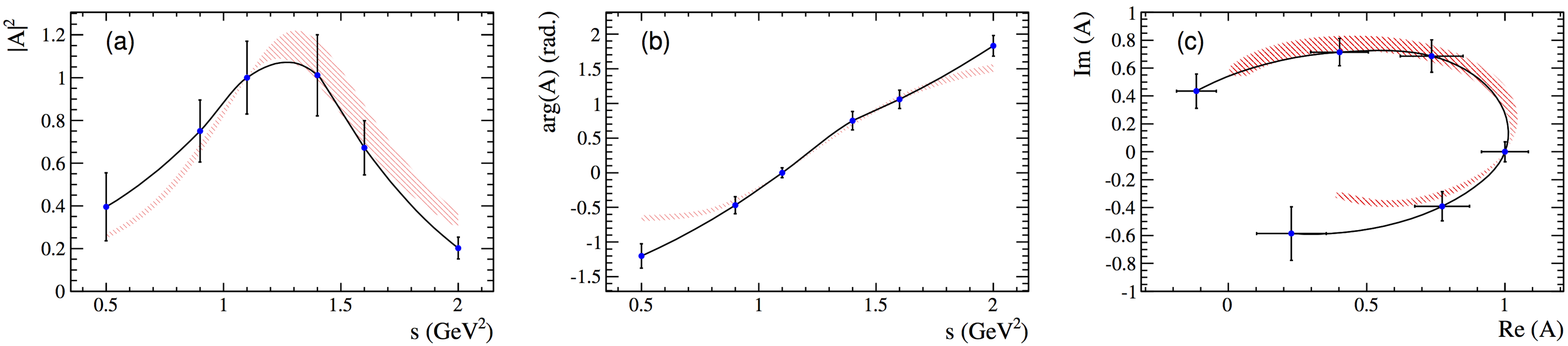
Three $\pi^-\pi^+$ S-wave parametrisation in $B^-\to\pi^-\pi^+\pi^-$

S-wave model projections



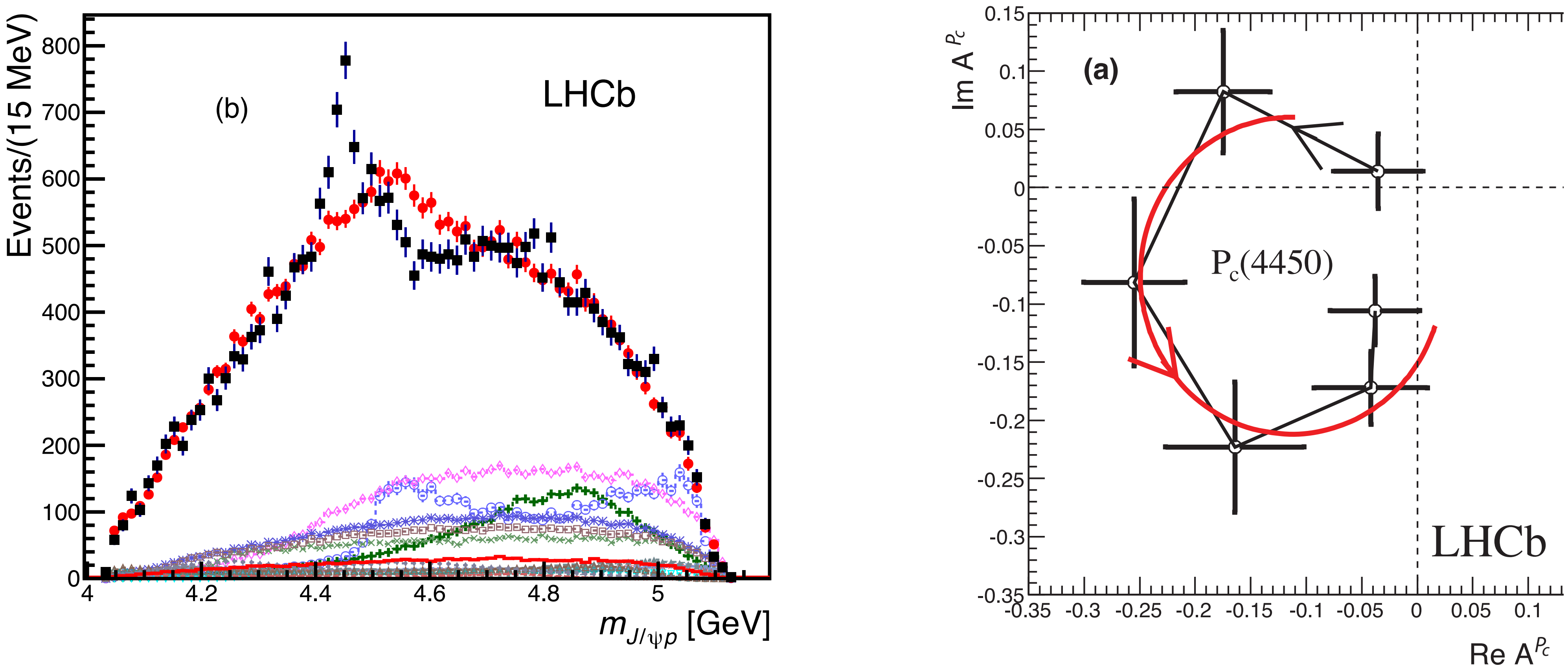
slide by Daniel O'Hanlon

Model-Independent line shapes



a1(1260) line shape in D→$\pi\pi\pi\pi$, CLEO-data

Model-Independent line shapes

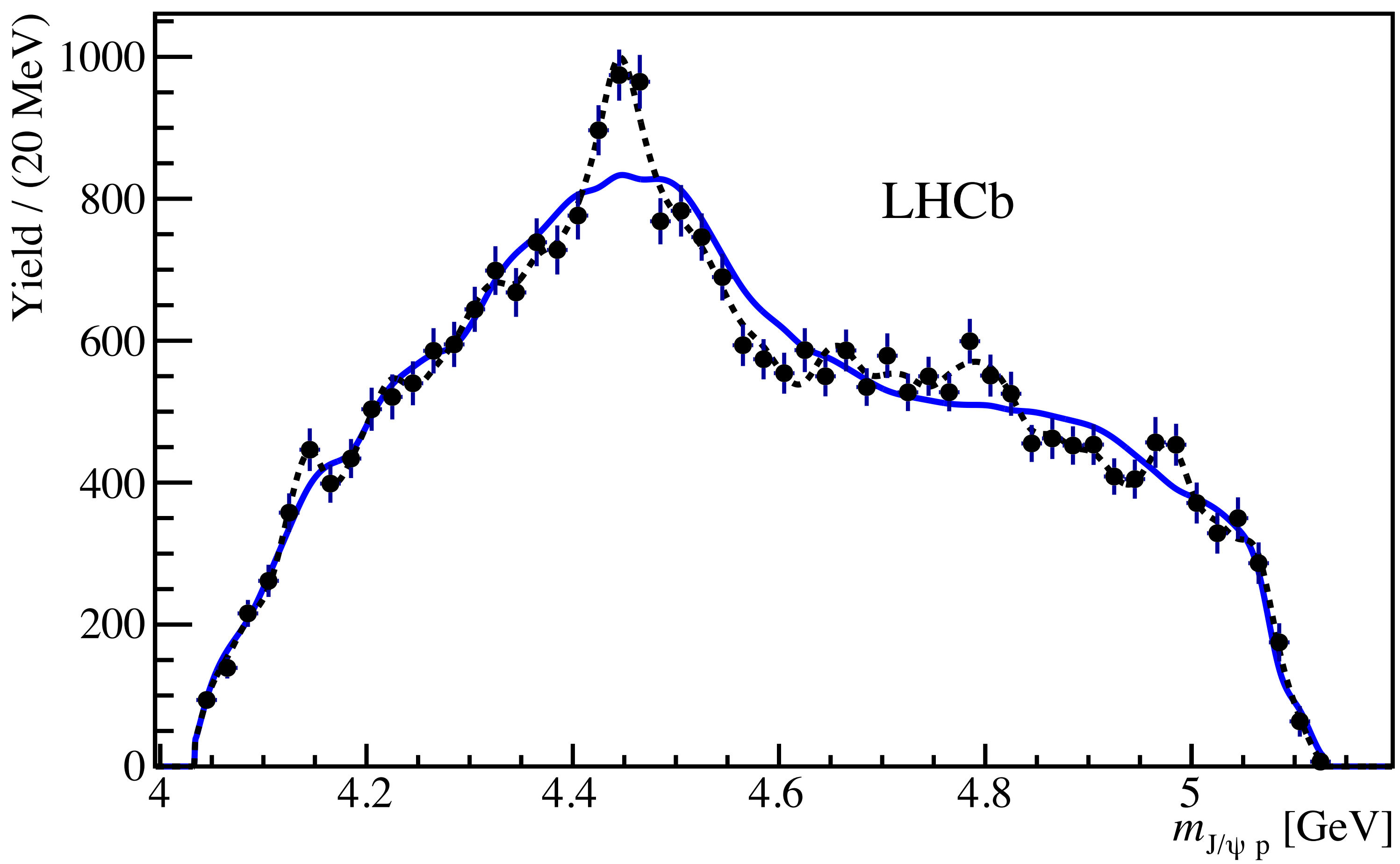


PRL 115 (2015) 072001

Use known resonances as interferometer to obtain model-independent amplitude and phase of resonance.
Example: LHCb Pentaquark discovery in $\Lambda_b^0 \rightarrow J/\psi p K^-$

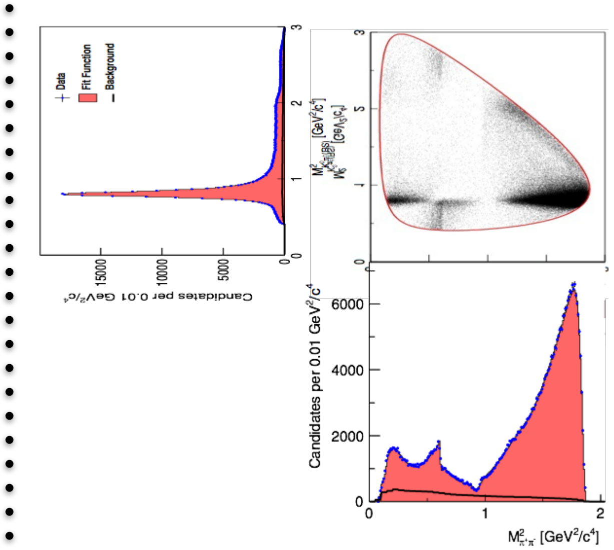
Fully model-independent

Use as input only the well-understood angular distributions of S, P, D, F, … resonances (expanded in terms of Legendre Polynomials) in K^-p, and the measured K^-p mass spectrum, to model $J/\psi p$ mass spectrum (blue). Is the peak just a reflection? No!



PRL 117 (2016) no.8, 082002

This means it's **not** the kind of reflection introduced earlier with the example of $D \to K_S \pi\pi$



See also

- JPAC (Mikhasenko et al): “Dalitz-plot decomposition for three-body decays" Phys.Rev.D 101 (2020) 3, 034033.
  Very useful especially when dealing with complicated spin structures.

- Krinner, Greenwald, Ryabchikov, Grube, Paul, “Ambiguities in model-independet partial-wave analysis” Phys. Rev. D 97, 114008
  Identifies and overcomes difficulties in model-independent amplitude analyses.

Das Model

The Model

- There are, for most cases we care about, no theoretically sound amplitude models…

- However, there are "good enough" models. What's good enough depends on the purpose.

- So what to do? Suggest a mix of….
  - model-independent approaches
  - "good enough" models of various levels of sophistication
  - improve models (there is - and that's fairly new - real, tangible, progress!)