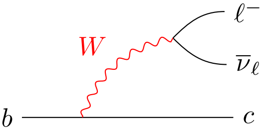
Theory of semileptonic B decays

Keri Vos
Maastricht University

Semileptonic B decays



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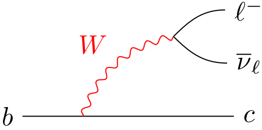
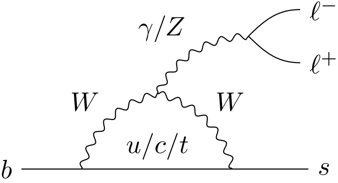
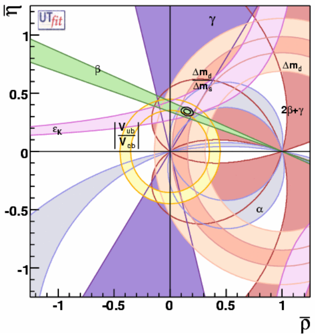




Why (semileptonic) B decays?

- Weak interaction
  - → Test the SM
  - → Understand CP violation (CKM angles)
  - → Search for new physics
- Strong interaction
  - → Insights into QCD
  - → Heavy quark expansion



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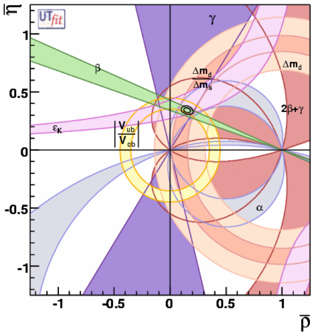
Semileptonic Decays

- $b \to u\ell\nu$ and $b \to c\ell\nu$
- V_{ub} and V_{cb}
- Exclusive versus inclusive
- Theory versus experiment?
- SM or New Physics?



The challenge

- Weak interaction: Transitions between quarks
- Observations: Transitions between hadrons
- Dealing with QCD at large distances/small scales
- Precise predictions required
  - $\rightarrow$ Precise error estimates
- Effective Field Theory methods
  - $\rightarrow$ Heavy quark expansion in QCD

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Outline

- Effective Field Theory methods
  - $\rightarrow$ Heavy quark limit
  - $\rightarrow$ Heavy quark expansion in QCD
  - $\rightarrow$ OPE for inclusive decays
- Phenomenology of semileptonic decays
  - $\rightarrow$ Inclusive and Exclusive V_{ub}
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Effective Field Theories

Effective Field Theories

- Indispensable tool to separate different energy scales
- Semileptonic decays: many different scales involved

  $\rightarrow$ $\Lambda_{\rm QCD}, m_c, m_b, m_t, m_W$ and possible NP scale
- Operator Product Expansion (scales $x \gg 1/\Lambda$)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{(4)} + \frac{1}{\Lambda}\mathcal{L}_{\rm eff}^{(5)} + \frac{1}{\Lambda^2}\mathcal{L}_{\rm eff}^{(6)} + \cdots$$

- At low energies the high mass degrees of freedom irrelevant
- Massive degrees of freedom can be removed (integrated out)

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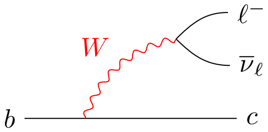
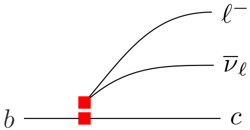


- Start from the Standard Model: Integrate out W, Z, t
- Effective Weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{g^2}{\sqrt{2}m_W^2} V_{q'q} \left[\bar{q}'\gamma_\mu(1-\gamma_5)q\right] \left[\bar{\nu}_\ell\gamma_\mu(1-\gamma_5)\ell\right]$$
$$\to \frac{4G_F}{\sqrt{2}} V_{q'q}\, j_{\mu,\text{had}}\, j^\mu_{\text{lep}}$$

- QED corrections: $\alpha_{\text{em}}/\pi \log(m_W^2/m_b^2)$
- No QCD corrections of that order (unlike in non-leptonic decays)

Effective Weak Hamiltonian



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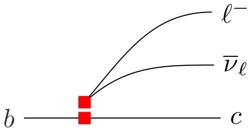

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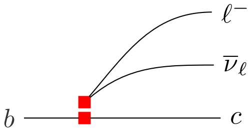

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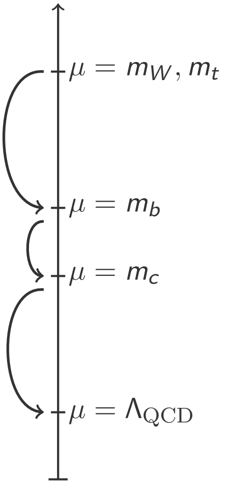

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{q'q} \sum_i C_i \left(\frac{\Lambda}{\mu}\right) \mathcal{O}_i(\mu)$$

- Typical procedure:
  - Matching at the scale $\mu = m_W$
  - RGE running to the scale $\mu = m_b$

$$0 = \mu \frac{d}{d\mu} H_{\rm eff}$$

- Resummations of large logs
  - Matching at the scale $\mu = m_b$
  - RGE running to the scale $\mu = m_c$
- $\cdots$

Heavy Quark Limit

- $1/m_Q$ Expansion: substantial theoretical progress
- Typical momentum transfer between $Q\bar{q} \sim \Lambda_{\rm QCD}$
- Velocity v of heavy quark Q almost unchanged by QCD
- Static limit $m_b, m_c \to \infty$ with fixed velocity ($Q = b, c$)

$$v_Q = \frac{p_Q}{m_Q}$$

- For $m_Q \to \infty$: $m_{hadron} = m_Q$, $p_{hadron} = p_Q \to v_{hadron} = v_Q$
- Heavy quark does not feel recoil from the light quarks and gluons

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Heavy Quark Symmetries

Heavy quark flavor symmetry

- Dynamics unchanged under the exchange of heavy quark flavors
- Interaction of gluons identical for all heavy quarks
- Similarly $m \to 0$ gives Chiral Flavor symmetry (isospin)

Heavy quark spin symmetry

- Spin-dependent interactions proportional to chromomagnetic moment of the quark

$$H_{\rm int} = \frac{g}{2m_Q}\bar{Q}(\vec{\sigma}\cdot\vec{B})Q \overset{m_Q\to\infty}{=} 0$$

- Dynamics unchanged under arbitrary transformations on the spin of the heavy quark
- Heavy Quark Spin Symmetry → Spin Flavour Symmetry Multiplets

Heavy Quark Effective Theory (HQET)

- Effective theory with manifest heavy quark symmetry in the $m_Q \to \infty$ limit
- Valid at scales much smaller than mass m_Q
- Expansion in inverse powers of m_Q
- Define the static field b_v with velocity v

$$b_v(x) = e^{im_b v\cdot x}\frac{1}{2}(1+\not{v})b(x)\ , \qquad p_B = m_b v + k$$

- HQET Lagrangian

$$\mathcal{L} = \bar{b}_v(iv\cdot D)b_v + \frac{1}{2m_b}\bar{b}_v(i\not{D})^2 b_v + \cdots$$

- Derive Feynman rules to do loop calculations/calculate radiative corrections!

Exclusive Decays

Application: Heavy Quark Symmetries

Exclusive Decays

- Hadronic matrix elements parametrized by form factors ($q^2 = (p_B - p_p)^2$)

$$\langle P(p_P)|\bar{q}\gamma^\mu b|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_P^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

$$\langle P(p_P)|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = 0$$

- Heavy quark symmetries limited implications for $B \to \pi$ (next lecture)

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$B \to D^{(*)}$ decays

- Form factors are related by Heavy Quark Symmetry
- Express momenta in velocities with $w = v \cdot v'$

$$\langle H^{(c)}(v')|\bar{c}_v \Gamma b_v|H^{(b)}(v)\rangle = \mathrm{Tr} X \bar{H}^{(c)}\Gamma H^{(b)} \to -\xi(w)\mathrm{Tr}\bar{H}^{(c)}\Gamma H^{(b)} \ ,$$

- Isgur-Wise function ξ form factor
- HQET gives normalization in zero-recoil point $\xi(w=1) = 1$

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$B \to D^{(*)}$ decays

- Express in terms of velocities: $w = v \cdot v'$ Bernlochner, Ligeti, Papucci, Robinson [2017]

$$\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle = \sqrt{m_B m_D}h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu \ ,$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle = i\sqrt{m_B m_D^*}h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle = \sqrt{m_B m_D^*}h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^*\cdot v)v^\mu - h_{A_3}(\epsilon^*\cdot v)v'^\mu$$

- In the Heavy Quark limit only a single Isgur-Wise function ξ form factor:
$h_+ = h_V = h_{A_1} = h_{A_2} = \xi(w)$, $h_- = h_{A_2} = 0$ with $\xi(w=1) = 1$

Application: Heavy Quark Symmetries

- Decay rate for $B \to D\ell\nu$:

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3 (w^2-1)^{3/2}|\eta_{\rm ew}\mathcal{F}(w)|^2$$

with

$$\mathcal{F}(w) = h_+(w) - \frac{m_B-m_D}{m_B+m_D}h_-(w)$$

- Unitarity and Dispersion relations constrain form factors Boyd, Grinstein, Lebed [1995]
- Add HQET conditions and $1/m_Q$ corrections Caprini, Lellouch, Neubert [1997]; Bigi, Gambino [2016]

$$\mathcal{F}(w) \sim \mathcal{F}(1)\left[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3\right]$$

$$z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$$

- Different parametrization for $B \to D^*$
- Zero-recoil point: $w = 1 \to z = 0$ (HQ limit $\mathcal{F}(w=1) = 1$)
- Only one fit parameter: slope parameters ρ
- Devised to predict the form factors!
- Too rigid at current level of precision? $\to$ Next lecture!

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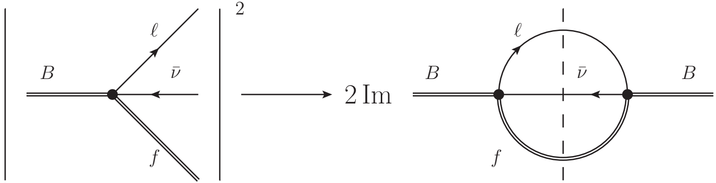
Inclusive B decays

Inclusive B Decays

- Optical Theorem
- Operator Product Expansion (OPE) = Heavy Quark Expansion (HQE)

$$d\Gamma = d\Gamma_0 + d\Gamma_2 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 + d\Gamma_4 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^4$$
$$+ d\Gamma_5 \left[a_0 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^5 + a_1 \left(\frac{\Lambda_{\rm QCD}}{m_b}\right)^3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2\right] + \cdots$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$



Optical Theorem

$$\Gamma \propto \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | B(v)\rangle|^2$$
$$= \int d^4x \, \langle B(v) | \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) | B(v)\rangle$$
$$= 2 \,\mathrm{Im} \int d^4x \, e^{-iq\cdot x} \langle B(v) | T \left\{ \mathcal{H}_{eff}(x) \mathcal{H}^\dagger_{eff}(0) \right\} | B(v)\rangle$$

where $\mathcal{H}_{eff} = J_c^\mu L_\mu, \quad J_c^\mu = \bar{b}\gamma^\mu P_L c$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, ...

Heavy Quark Expansion

- B meson: $p_B = m_B v$
- Split the momentum b quark: $p_b = m_b v + k$, expand in $k \sim iD\ Q_v$
- Field-redefinition of the heavy field $Q(x) = exp(-im(v \cdot x))Q_v(x)$

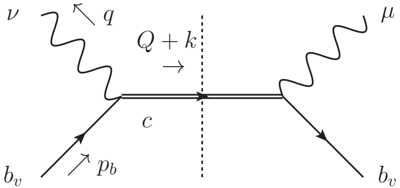
$$\begin{aligned}\Gamma \quad &= 2\ \text{Im} \int d^4x\, e^{-iq\cdot x} \langle B(v)| T\left\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\right\}|B(v)\rangle \\ &= 2\ \text{Im} \int d^4x\, e^{i(m_b v - q)\cdot x} \langle B(v)| T\left\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\right\}|B(v)\rangle\end{aligned}$$

where $\widetilde{\mathcal{H}}_{eff} = \tilde{J}^\mu_c L_\mu$, $\quad \tilde{J}^\mu_c = \bar{b}_v \gamma^\mu P_L c$, $\quad \Gamma \propto 2\text{Im}\, T^{\mu\nu} L_{\mu\nu}$

$$\Gamma(B \to X_c \ell \nu_\ell) \propto 2\text{Im}\, T^{\mu\nu} L_{\mu\nu}$$

$$T^{\mu\nu} = i \int dx^4 e^{i(m_b v - q)\cdot x}\, T\left\{\bar{b}_v(x)\gamma^\mu P_L c(x), \bar{c}(0)\gamma^\nu P_L b_v(0)\right\}$$

$$Q = m_b v - q$$



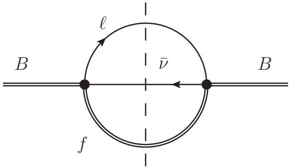
$$= \bar{b}_v \gamma_\mu P_L \left[\frac{i}{\not{Q} + i\not{D} - m_c}\right] \gamma_\nu P_L b_v$$

$$\frac{i}{\not{Q}+i\not{D}-m_c} = \frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c}(-i\not{D})\frac{i}{\not{Q}-m_c} + \dots$$

Chay, Georgi, Bigi, Shifman, Uraltsev, Vainstain, Manohar, Wise, Neubert, Mannel, $\cdots$

Operator Product Expansion (OPE)

$2\,\mathrm{Im}$



$$= \sum_{n,i} \frac{C_i^{(n)}(\mu,\alpha_s)}{m_b^i} \langle B|\mathcal{O}_i^{(n)}|B\rangle_\mu$$

- $\mathcal{C}_i(\mu)$: short distance, perturbative coeficients
- $\langle B|\mathcal{O}_i|B\rangle_\mu$: non-perturbative forward matrix elements of local operators
- operators contain chains of covariant derivatives

$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v (iD_\mu)\ldots(iD_{\mu_n}) b_v|B\rangle$$

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

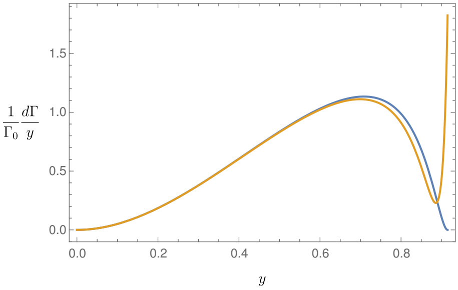
- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\rangle$$
$$2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu,[ivD,iD^\mu]]\,b_v|B\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\langle B|\bar{b}_v\,\{iD_\mu,[ivD,iD_\nu]\}\,(-i\sigma^{\mu\nu})b_v|B\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109



- Reliable calculation in HQE possible for the moments of the spectrum
- End point region: $\rho = m_c^2/m_b^2, y = 2E_\ell/m_b$

$$\frac{d\Gamma}{dy} \sim \theta(1-y-\rho)\left[2 - \frac{\mu_\pi^2}{(m_b(1-y)^2}\left(\frac{\rho}{1-y}\right)^2\left(3-4\frac{\rho}{1-y}\right)\right]$$

$B \to X_u \ell \bar{\nu}$

Modified Heavy Quark Expansion

- Cuts needed to suppress large charm background
- Pushes towards specific corner of the phase space
  - Local OPE as in $b \to c$ cannot work
  - Sensitivity to b-quark wave function properties (Fermi motion)
  - Deal with energetic light degrees of freedom
  - More than two scales involved!
- Expansion parameter $\Lambda_{\rm QCD}/(m_b - 2E_\ell)$
- Use light-cone OPE with light-cone directions n and $\bar{n}$
- Introduce shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n\cdot D))b_v|B(v)\rangle$$

  - Universal
  - Similar to parton distribution in DIS

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Shape functions

Bigi, Shifman, Uraltsev, Luke, Neubert, Mannel, $\cdots$

- Leading order shape functions

$$2m_B f(\omega) = \langle B(v)|\bar{b}_v \delta(\omega + i(n\cdot D))b_v|B(v)\rangle$$

- Charged Lepton Energy Spectrum (at leading order)

$$\frac{d\Gamma}{dy} \sim \int d\omega\theta(m_b(1-y)-\omega)f(\omega)$$

- Moments of the shapefunction are related to HQE ($b \to c$) parameters:

$$f(\omega) = \delta(\omega) + \frac{\mu_\pi^2}{6m_b^2}\delta''(\omega) - \frac{\rho_D^3}{m_b^3}\delta'''(\omega) + \cdots$$

- Shape function is non-perturbative and cannot be computed

Shape functions

Lange, Neubert, Bosch, Paz

- Systematic framework: Soft Collinear Effective Theory (SCET)
- Separates the different scales in the problem

$$d\Gamma = H \otimes J \otimes S$$

$\rightarrow$ H: Hard scattering kernel at $\mathcal{O}(m_b)$
$\rightarrow$ J: universal Jet function at $\mathcal{O}(\sqrt{m_b \Lambda_{\rm QCD}})$
$\rightarrow$ S: Shape function at $\mathcal{O}(\Lambda_{\rm QCD})$

- Framework to include radiative corrections (+ NNLL resummation)
- Introduces 3 subleading shape functions

- Other approach: OPE with hard-cutoff μ Gambino, Giordano, Ossola, Uraltsev
  - Use pert. theory above cutoff and parametrize the infrared
  - Different definition of the shape functions
- Shape functions have to be parametrized and obtained from data $\rightarrow$ next lecture!

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Heavy Quark Expansion for Charm?

Heavy Quark Expansion for Charm

Voloshin [2001]; Bauer, Luke, Falk [1997]; Bigi, Uraltsev, Zwicky [2007]; Bigi, Mannel, Turczyk, Uraltsev, JHEP 1004 (2010)
Breidenbach, Feldmann, Mannel, Turczyk, Phys. Rev. D78 (2008) 014022; Gambino, Kamenik, Nucl. Phys. B840 (2010) 424

- Notoriously difficult $\rightarrow$ no good expansion parameter
- Large D lifetime differences can be qualitatively explained in HQE
- Turn vice into virtue $\rightarrow$ larger dependence on higher-order terms

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- Notoriously difficult $\rightarrow$ no good expansion parameter
- Large D lifetime differences can be qualitatively explained in HQE
- Turn vice into virtue $\rightarrow$ larger dependence on higher-order terms
- $b \rightarrow c$: charm as perturbative scale, expand in fixed $\rho = m_c^2/m_b^2$
- $b \rightarrow u$: massless quarks $\rho \rightarrow 0$ limit exists
- IR sensitivity to light quark gives additional four-quark non-pert. parameters:

$$\langle B|(\bar{b}_v \gamma^\nu P_L q)(\bar{q}\gamma^\mu P_L b_v)|B\rangle = 2M_B \left[T_1(\mu) g^{\mu\nu} + T_2(\mu) v^\mu v^\nu\right]$$

HQE for Charm revisited

- The general structure of the expansion for $D \to X_s \ell \bar{\nu}$:

$$d\Gamma = d\Gamma_0 + d\Gamma_{(2,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^2 + d\Gamma_{(2,2)} \left(\frac{m_s}{m_c}\right)^2 + d\Gamma_3 \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^3 + d\Gamma_{(4,1)} \left(\frac{\Lambda_{\rm QCD}}{m_c}\right)^4 + d\Gamma_{(4,2)} \left(\frac{m_s}{m_c}\right)^4 + \cdots$$

- Expansion parameters:
  - $1/m_c$
  - α_s
  - m_s/m_c

Fael, Mannel, KKV, hep-ph/1910.05234

Fael, Mannel, KKV

- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{4q}|D\rangle$
- Up to $1/m_c^3$ only <u>one</u> extra HQE param:

$$\tau_0 = 128\pi^2\left(T_1(\mu) - T_2(\mu) - 2\frac{T_3(\mu)}{m_c} + \frac{T_4(\mu)}{m_c}\right)$$
$$+\log\left(\frac{\mu^2}{m_c^2}\right)\left[8\tilde{\rho}_D^3 + \frac{1}{m_c}\left(\frac{16}{3}r_G^4 - \frac{16}{3}r_E^4 + \frac{8}{3}s_E^4 - \frac{1}{3}s_{qB}^4 - 12m_s^4\right)\right]$$

- Up to $1/m_c^4$ only <u>two</u> extra HQE params: $\boldsymbol{\tau_m}$ and $\boldsymbol{\tau_\epsilon}$.

HQE for charm revisited

$\rho = m_s^2/m_c^2$ Fael, Mannel, KKV

$$\frac{\Gamma(D \to X_s \ell \nu)}{\Gamma_0} = \left(1 - 8\rho - 10\rho^2\right)\mu_3 + (-2 - 8\rho)\frac{\mu_G^2}{m_c^2} + 6\frac{\tilde{\rho}_D^3}{m_c^3} + \frac{16}{9}\frac{r_G^4}{m_c^4} + \frac{32}{9}\frac{r_E^4}{m_c^4} - \frac{34}{3}\frac{s_B^4}{m_c^4} + \frac{74}{9}\frac{s_E^4}{m_c^4} + \frac{47}{36}\frac{s_{qB}^4}{m_c^4} + \frac{\tau_0}{m_c^3}$$

- Data required to test description
- Extract four-quark operators (weak annihilation) from data
  - Also contribute in $b \to u$
  - Effect is $(m_b/m_c)^3$ enhanced in D compared to B decays
  - Crucial for precise $B \to X_{d,s}\ell\ell$ predicitions Hurth, Huber, Lunghi, Qin, Jenkins, KKV
- Comparison of extracted HQE parameters with B decays

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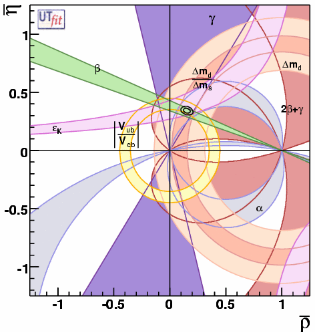
Key question: HQE indeed applicable to inclusive charm decays?

Phenomenology

Motivation

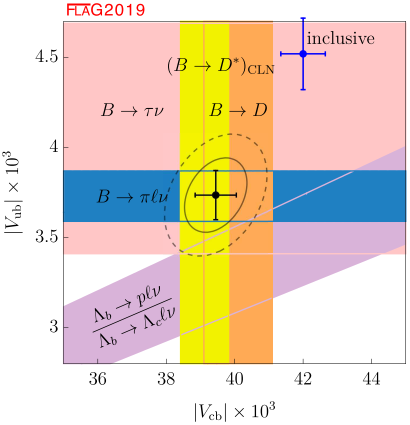
- Semileptonic decays offer Inclusive and Exclusive V_{cb} and V_{ub} determinations
- Crucial to understand the SM CKM mechanism



Inclusive versus Exclusive

- Discrepancy between both determinations
- Next steps . . .

Inclusive versus Exclusive decays



$|V_{cb}|$

- Exclusive $B \to D^{(*)}\ell\bar{\nu}$
- Inclusive $B \to X_c\ell\nu$

$|V_{ub}|$

- Exclusive $B \to \pi\ell\nu (B \to \tau\nu)$
- Inclusive $B \to X_u\ell\nu$

$|V_{ub}|/|V_{cb}|$

- First determination in baryons

$$\frac{\mathcal{B}(\Lambda_b \to p\ell\nu)}{\mathcal{B}(\Lambda_b \to \Lambda_c\ell\nu)} = (1.5 \pm 0.1) \left|\frac{V_{ub}}{V_{cb}}\right|^2$$

LHCb'18; Detmold, Lehner, Meinel'15

Inclusive V_{cb} determination

Inclusive Decays: Recap

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 + \cdots$$

- Operator Product Expansion = Heavy Quark Expansion (HQE)
- Introduces non-perturbative matrix elements at every order in $1/m_b$
- Reliable calculations for moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\left\langle (M_X^2)^n \right\rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

$$R^*(E_{\rm cut}) = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}{\int_0 dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

- Moments up to $n = 3, 4$ and with several energy cuts available
- Experimentally necessary to use lepton energy cut

$$R^*(E_{\rm cut}) \quad \langle E^n \rangle_{\rm cut} \quad \langle (M_X^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3, m_b, (m_c)$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} \right]$$

$$\downarrow$$

$$V_{cb} = (42.21 \pm 0.78) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802

State-of-the-art

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi} \Gamma^{(\pi,1)} \right) + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi} \Gamma^{(G,1)} \right) + \frac{\rho_D^3}{m_b^3} \Gamma^{(D,0)} + \mathcal{O}\left(\frac{1}{m_b^4}\right) \cdots \right)$$

- Includes all known α_s and α_s^2 corrections
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063
- Only uses mild external constraints
- Include terms up to $1/m_b^3$
- Assigned 1.4% theo. error due to missing higher orders
  - Missing power-corrections and α_s-corrections
  - Quark-hadron duality violation expected to be small

QCD Corrections

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

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- QCD corrections calculated in the pole mass
- Very large corrections $\Gamma_0^{(1)}$ and $\Gamma_0^{(2)}$
- Renormalon issues require short-distance mass
- $\overline{\mathrm{MS}}$ for scales μ above heavy quark mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off μ

$$m_H = m_Q(\mu) + \bar{\Lambda}(\mu) + \frac{\mu_\pi^2(\mu)}{2m_Q(\mu)} + \cdots$$

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\Lambda_{\rm QCD} < \mu < m_Q$: expansion parameters μ/m_Q
  - Typical choice $\mu = 1$ GeV: $\mu/m_B \simeq 0.2$
  - Challenging for charm decays!
- Also hadronic matrix elements get shifted!

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Towards higher precision

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290.

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- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
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Next steps:

- Include α_s corrections to for ρ_D^3 Mannel, Pivovarov [in progress]; Gambino [in progress]
- Full determination up to $1/m_b^4$ from data
- Lattice?!

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon = m_P - m_B \sim 0.4 \text{ GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- Assume fully saturated by the lowest state (B, B^* or P-wave excitation)
- Matrix elements with time derivative saturated by P wave states
- LSSA only estimate! Set with 60% Gaussian uncertainty
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60
- -0.25% shift due to power corrections

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

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Lattice determinations of meson masses

Melis, Simular, Gambino [1704.06105], ETMC

Bazavov, TUMQCD, Fermilab-MILC [1802.04248]

$$m_H = m + \bar{\Lambda} + \frac{1}{2m}(\mu_\pi^2 - \mu_G^2)_{m\to\infty} + \cdots$$

- Calculate meson masses for several quark masses
- Extracted in the static B limit (versus physical B states)

$$\mu_\pi^2 = \mu_\pi^2|_\infty - \frac{\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3}{m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

$$\mu_G^2 = \mu_G^2|_\infty + \frac{\rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3}{m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

$\rho_{A,S,\pi\pi,\pi G}$ non-local matrix elements

- Precise determination of $\bar{\Lambda}, \mu_G^2$ and μ_π^2
- Gives estimate of ρ_D^3 in agreement with semileptonic determinations Gambino, Mannel, Uraltsev [1206.2296]

Reparametrization Invariance

Reparametrization invariance

Dugan, Golden, Grinstein, Chen, Luke, Manohar, Hill, Solon, Heinonen, Mannel, KKV

- Choice of v not unique, result independent of v (Lorentz invariance)
- Reparametrization Invariant (RPI) under an infinitesimal change

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\, v_\mu = \delta v_\mu \ \text{ and } \ \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

- Links different orders in $1/m_b$
  - Gives exact relations between different orders
  - Resums towers of operators
  - Reduces the number of independent parameters

Reparametrization invariance

Total rate at tree level

Mannel, KKV, JHEP 1806 (2018) 115

$$R = \sum_{n=0}^{\infty} C^{(n)}_{\mu_1..\mu_n}(v) \otimes \bar{b}_v (iD_{\mu_1} \cdots iD_{\mu_n}) b_v$$

$$\begin{aligned}\delta_{\rm RP} R = 0 \quad = \quad & \sum_{n=0}^{\infty} \left[\delta_{\rm RP}\, C^{(n)}_{\mu_1\cdots\mu_n}\right] \bar{b}_v (iD^{\mu_1} \cdots iD^{\mu_n}) b_v \\ & + \sum_{n=0}^{\infty} C^{(n)}_{\mu_1\cdots\mu_n} \left[\delta_{\rm RP}\, \bar{b}_v (iD^{\mu_1} \cdots iD^{\mu_n}) b_v\right]\end{aligned}$$

The RPI relation:

$$\delta_{\rm RP}\, C^{(n)}_{\mu_1\cdots\mu_n} = m_b \delta v^{\alpha} \left[C^{(n+1)}_{\alpha\mu_1\cdots\mu_n} + C^{(n+1)}_{\mu_1\alpha\mu_2\cdots\mu_n} + \cdots + C^{(n+1)}_{\mu_1\cdots\mu_n\alpha} \right]$$

Parameter reduction: an example ρ_{LS}

- $1/m_b^2: \quad \mu_G^2 \to \underbrace{\eta\,(-i\sigma_{\mu\nu})}_{C^{(2)}_{\mu\nu}} \otimes \bar{b}_v\,(iD^\mu iD^\nu)\,b_v$
- $1/m_b^3: \quad \rho_{LS}^3 \to \underbrace{\xi\, v_\alpha(-i\sigma_{\mu\nu})}_{C^{(3)}_{\mu\alpha\nu}} \otimes \bar{b}_v\,(iD^\mu iD^\alpha iD^\nu)\,b_v$

The RPI relation:

$$\begin{aligned}
\delta_{\rm RP}\, C^{(2)}_{\mu\nu} &= 0 \\
&= m_b\,\delta v^\alpha \left(C^{(3)}_{\mu\nu\alpha} + C^{(3)}_{\mu\alpha\nu} + C^{(3)}_{\alpha\mu\nu} \right) \\
&= -i m_b\,\xi\,\delta v^\alpha \left(\sigma_{\mu\alpha} v_\nu + \sigma_{\alpha\nu} v_\mu\right) \\
&\leftrightarrow \xi = 0
\end{aligned}$$

Non-perturbative matrix elements

- 1: Mannel, KKV, JHEP 1806 (2018) 115
  - $2M_B\mu_3 = \langle B|\bar{b}_v b_v|B\rangle = 2M_B\left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b}\right)$
- $1/m_b^2$:
  - $2M_B\mu_G^2 = \langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\rangle$
- $1/m_b^3$:
  - $2M_B\tilde{\rho}_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,\left[\left(ivD + \frac{(iD)^2}{m_b}\right), iD^\mu\right]\right] b_v|B\right\rangle$
- $1/m_b^4$:
  - $2M_B r_G^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu, iD_\nu]\,[iD^\mu, iD^\nu]\,b_v|B\rangle \propto \langle \vec{E}^2 - \vec{B}^2\rangle$
  - $2M_B r_E^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[ivD, iD_\mu]\,[ivD, iD^\mu]\,b_v|B\rangle \propto \langle \vec{E}^2\rangle$
  - $2M_B s_B^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu, iD_\alpha]\,[iD^\mu, iD_\beta]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle\vec{\sigma}\cdot\vec{B}\times\vec{B}\rangle$
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  - $2M_B s_{qB}^4 \equiv \frac{1}{2}\langle B|\bar{b}_v\,[iD_\mu,[iD^\mu,[iD_\alpha, iD_\beta]]]\,(-i\sigma^{\alpha\beta})b_v|B\rangle \propto \langle\Box\,\vec{\sigma}\cdot\vec{B}\rangle$.

Up to $1/m_b^4$: 8 parameters versus previous 13

Alternative V_{cb} determination

RPI Observables

- RPI reduced set to directly fit the higher order terms
- Requires RPI observables

$$O = \int w(v, p_e, p_\nu) \langle \mathrm{Im}\, T(S) \rangle L(p_e, p_\nu) d\Phi_3$$

- The observable O is RPI if $\delta_{\mathrm{RP}} w(v, p_e, p_\nu) = 0$

O	$w(v, p_e, p_\nu)$	RPI
Total Rate	1	✓
Moments charged lepton energy	$(v \cdot p_e)^n$	✗
Moments hadronic invariant mass	$(M_B v - q)^{2n}$	✗
Moments leptonic invariant mass	$(q^2)^n$	✓

Fael, Mannel, KKV, JHEP 02 (2019) 177

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Fael, Mannel, KKV, JHEP 02 (2019) 177

RPI Observables

- Ratio between the rate with and without a cut

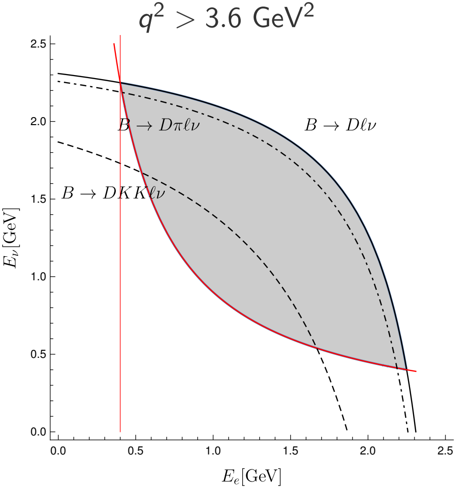
$$R^*(q^2_{\rm cut}) = \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2} \Bigg/ \int_0 dq^2 \, \frac{d\Gamma}{dq^2}$$

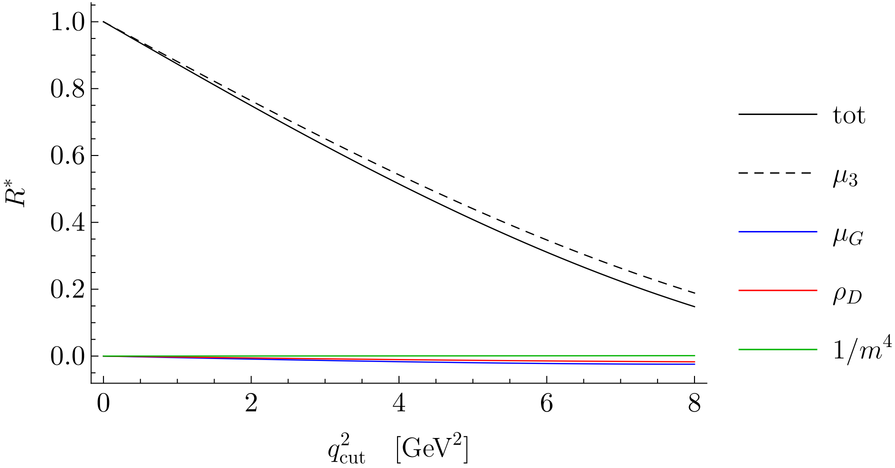
- q^2 moments

$$\langle (q^2)^n \rangle_{\rm cut} = \int_{q^2 > q^2_{\rm cut}} dq^2 \, (q^2)^n \, \frac{d\Gamma}{dq^2} \Bigg/ \int_{q^2 > q^2_{\rm cut}} dq^2 \, \frac{d\Gamma}{dq^2}$$

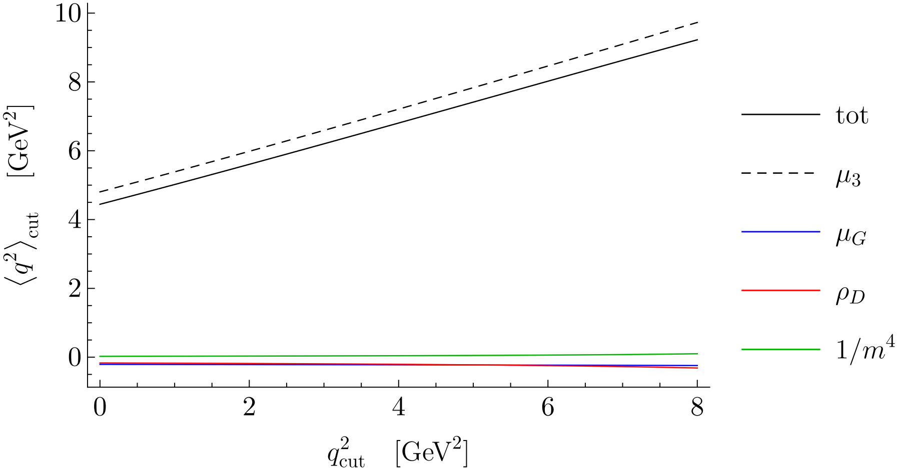
Fael, Mannel, KKV, JHEP 02 (2019) 177

Note; Energy cut is not RPI, but $q^2_{\rm cut}$ is RPI and can be used to remove the low electron energies


$q^2 > 3.6$ GeV^2
$q^2 > 8.4$ GeV^2
$B \to D\pi\ell\nu$
$B \to D\ell\nu$
$B \to DKK\ell\nu$
E_ν[GeV]
E_e[GeV]



Fael, Mannel, KKV, JHEP 02 (2019) 177
Benchmark values based on Gambino, Haeley, Turczyk, PLB 763 (2016) 60



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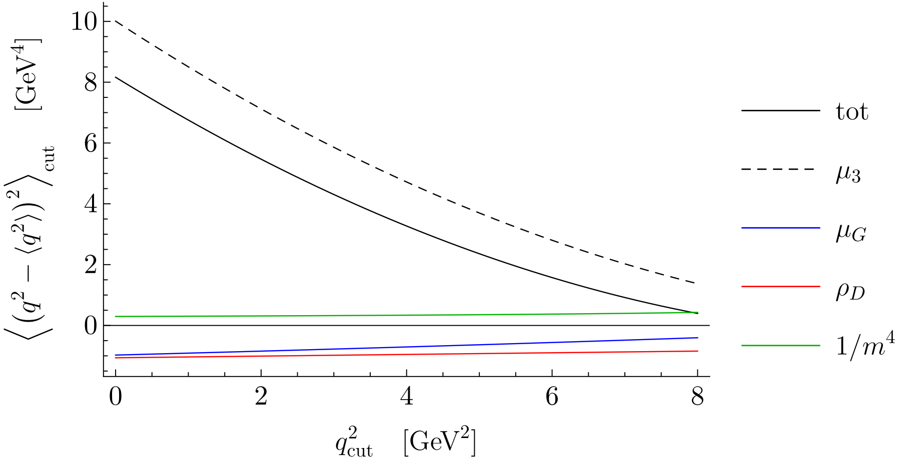
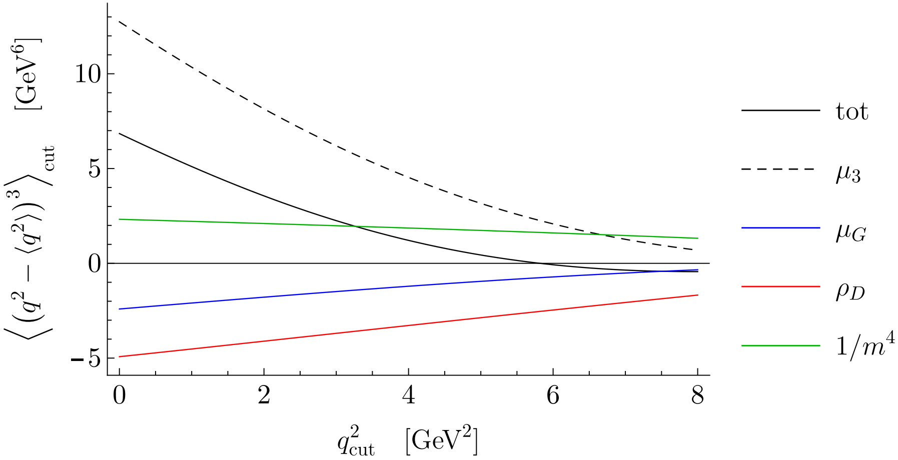


Fael, Mannel, KKV, JHEP 02 (2019) 177



Fael, Mannel, KKV, JHEP 02 (2019) 177

$$R^*(q^2_{\rm cut}) \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3}\mu_3 + \Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D}\frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E}\frac{r_E^4}{m_b^4} + \Gamma_{r_G}\frac{r_G^4}{m_b^4} + \Gamma_{s_B}\frac{s_B^4}{m_b^4} + \Gamma_{s_E}\frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}}\frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb} = ?$$

Fael, Mannel, KKV, JHEP 02 (2019) 177

Rahimi, Mannel, KKV, in progress

$b \to u \ell \nu$ contribution

- suppressed by V_{ub}/V_{cb}
- can be calculated precisely in HQE!
- compare used Monte Carlo with theory

$b \to c(\tau \to \mu \nu \bar{\nu})\bar{\nu}$ contribution

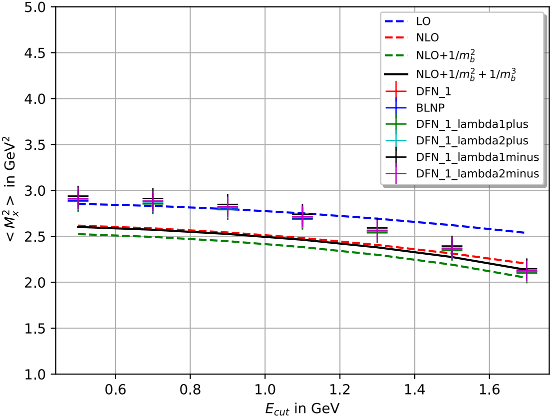
- phase space suppressed
- likewise can be calculated precisely

Analysis of full $B \to X\mu$ data sample?

- including also QED effects

Monte Carlo versus HQE

Rahimi, Mannel, KKV, in progress; MC data by Lu Cao and Florian Bernlochner



Preliminary!

Exclusive V_{cb} determination

Exclusive V_{cb}

$B \to D\ell\nu$

FNAL/MILC,HPQCD

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} |\eta_{\rm ew}\mathcal{F}(w)|^2$$

- $\mathcal{F}(w)$ available from Lattice at different kinematical points!
- Of crucial importance!
- Different parametrization: BGL and CLN
  - BGL: model independent
  - CLN: adds HQET relations: simple, effective parametrization up to certain precision!
- Recently a lot of attention for the V_{cb} puzzle! Apologies for missed citations
- Triggered also by Belle data in format that allows independent reanalyses

BGL or CLN?

BGL Boyd, Grinstein, Lebed [1995]

- Start with z-expansion

$$F(z) = \frac{1}{B_F(z)\phi_F(z)} \sum_{j=0}^{\infty} a_j^F z^j$$

$$z(q^2) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \quad w = \frac{m_B^2 + m_D^2 - q^2}{2 m_B m_D} \quad 0 < z < 0.056$$

- $B_F(z)$ Blaschke factors removes poles below threshold
- $\phi(z)$ outer functions: phase space factors
- Unitarity and Dispersion relations constrain EACH form factor:

$$\sum_{j=0}^{\infty} \left(a_j^F\right)^2 \leq 1$$

- Weak unitarity constraints: saturation by single channel
- For $B \to D^*$ form factors axial form factors f and $\mathcal{F}_1$ related
- Need to truncate the series at some N

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BGL or CLN?

BGL Boyd, Grinstein, Lebed [1995]

- Model independent parameterization
- Remember: truncation at some N Bernlochner, Ligeti, Robinson [2019]
  - Optimal: adding terms until fit is not altered in relevant way Gambino, Jung, Schacht [2019]
- Does not include all available information!
- In general may not give the strongest possible constraints

BGL or CLN?

Strong Unitarity Bound

Boyd, Grinstein, Lebed [1995]Caprini, Lellouch, Neubert [1997]

- Include all four $b \to c$ channels: $B^{(*)} \to D^{(*)}$
- Additional constraint on unitarity sum of channels with same quantum numbers:

$$\sum_{i=1}^{H} \sum_{n=0}^{N} \left(b_n^i \right)^2 \leq 1$$

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- Constraints can be included using HQE relations
- Idea of CLN

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BGL or CLN?

CLN Caprini, Lellouch, Neubert [1997]

- Simple parametrization including strong unitarity constraints
- HQET used to strongly reduce number of parameters

$$\mathcal{F}(w) \sim \mathcal{F}(1)\left[1 - 8\rho^2 z + (51\rho^2 - 10)z^2 - (252\rho^2 - 84)z^3\right]$$

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Different coefficients for $B \to D^*$
- Neglects HQE relations between $B \to D$ and $B \to D^*$ form factors
- Only one fit parameter: slope parameter ρ
- Uncertainties on fixed parameters never included in exp. analyses!
- Lattice data that test and complement HQE relations not included
- Shortcomings addressed in many recent papers Bigi, Schacht, Gambino, Jung, Straub, Bernlochner, Ligetti, Robinson,Bordone, van Dyk, Gubernari
- Too simple for the current level of precision
- Do not use determinations based on CLN! Summary Semileptonic MITP workshop [2006.07287]

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Optimal Determination(?):

- Include strong unitarity
- Include higher-order corrections to HQE
- Adding $1/m_c^2$ crucial Bordone, Jung, van Dyk [2019]
- Include Lattice + QCD sum rule info on form factors
- Requires a global fit!

$B \to D\ell\nu$

FNAL/MILC,HPQCD

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} |\eta_{\rm ew} \mathcal{F}(w)|^2$$

- $\mathcal{F}(w)$ available from Lattice at different kinematical points!
- Latest Belle analysis uses both BGL and CLN Belle [1510.03657]
  - Compatible results and in agreement with inclusive determination
- Global fit including strong unitarity

Bigi, Gambino, Phys. Rev. D94 (2016) 094008

$$|V_{cb}|_{\text{excl, Global Fit},D} = (40.5 \pm 1.0) \times 10^{-3}$$

- In agreement with inclusive determination

Exclusive V_{cb}

$B \to D\ell\nu$

FNAL/MILC,HPQCD

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3(w^2 - 1)^{3/2}|\eta_{\rm ew}\mathcal{F}(w)|^2$$

- $\mathcal{F}(w)$ available from Lattice at different kinematical points!
- Latest Belle analysis uses both BGL and CLN Belle [1510.03657]
  - Compatible results and in agreement with inclusive determination
- Global fit including strong unitarity

Bigi, Gambino, Phys. Rev. D94 (2016) 094008

$$|V_{cb}|_{\text{excl, Global Fit},D} = (40.5 \pm 1.0) \times 10^{-3}$$

- In agreement with inclusive determination

$B \to D^* \ell \nu$

Caprini, Lellouch, Neubert (1998); Boyd, Grinstein, Lebed (1997)

$$\frac{d\Gamma}{dw} = \frac{G_F^2 m_B^5}{48\pi^3} |V_{cb}|^2 (w^2 - 1)^{1/2} P(w) (\eta_{\rm ew} \mathcal{F}(w))^2 \text{ with } w = v_B \cdot v_D$$

- Form factor $\mathcal{F}(w = 1)$ computed on the lattice Fermilab/MILC
- Extrapolation to zero-recoil point necessary
- Far more sensitive to the specific parameterization!
- Too many discussions to review here triggered by Belle data Belle [1702.01521]

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HFLAV'17

$$|V_{cb}|_{\text{excl, CLN}} = (39.2 \pm 0.7) \times 10^{-3}$$

Grinstein, Kobach, PLB 771 359 (2017)
Bigi, Gambino, Schacht, PLB 769 441 (2017)

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- BGL always closer to inclusive decay; larger uncertainty
- Without Lattice data on the slope of the form factor; BGL offers more conservative and reliable choice

Exclusive V_{cb}

$B \to D\ell\nu + B \to D^*\ell\nu$ Bordone, Jung, van Dyk [2019]

- Including lattice + QCD sumrules
- Fully exploiting HQE + $1/m_c^2$ corrections
- Recent analysis: agreement between inclusive and exclusive $|V_{cb}|$

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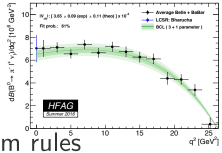
- Theory only BGL fit parameters including errors also available Bordone, Jung, Gubernari, van Dyk [2019]

Exclusive V_{ub} determination

Exclusive $B \to \pi \ell \nu$

Bourrely, Caprini, Lellouch, PRD79, 013008 (2009); Bharucha, JHEP 1205 (2012)

$$\frac{d\mathcal{B}(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2 \tau_B}{24\pi^3} |V_{ub}|^2 p_\pi^3 |f_+^{B\pi}(q^2)|^2$$



- Only one form factor required
- Combined inputs from Lattice QCD (BCL) and QCD sum rules

$$|V_{ub}|_{\text{excl}} = (3.70 \pm 0.16) \times 10^{-3} \quad \text{PDG'18}$$

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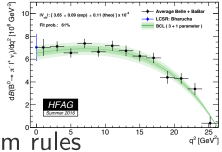

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Other probes:

- $B_s \to K \ell \nu$
  - Form factors available in QCD sum rules Khodjamirian, Rusov, JHEP 08 (2017) 112
  - on the Lattice Fermilab/MILC [1901.02561]
- $B \to \rho \ell \nu$
- Pure leptonic $B \to \tau \nu$

Inclusive V_{ub} determination

Inclusive V_{ub} determination

- Modified HQE for $b \to u$ due to cuts to suppress charm!
- Requires the leading and subleading shapefunctions
  - Comparison with $B \to X_s\gamma$ (challenging)
  - Relation with HQE parameters
  - Modeling
- Different approaches
  - BLNP Bosch, Lange, Neubert, Paz
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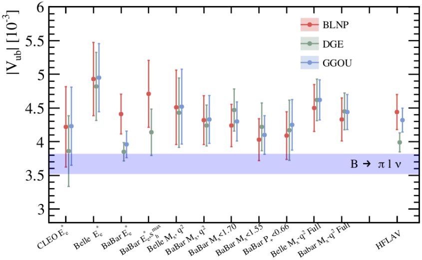

[2006.07287]

- Contrary to V_{cb} puzzle not much recent attention!
- Need to be scrutinized

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$$|V_{ub}|_{\text{incl}} = (4.5 \pm 0.3) \times 10^{-3} \quad \text{PDG'18}$$

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Next steps:

- Extract shape functions from global fit (SIMBA and NNVub)
- Implementing higher-order corrections in BLNP KKV, Mannel, Lange; in progress

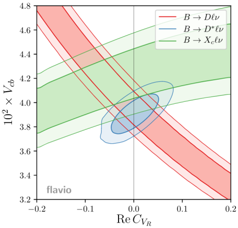
New Physics

New Physics

- Too many to count: exclusive $B \to D^{(*)}$ in combination with

$$R_{D^{(*)}} = \frac{B \to D^{(*)}\tau\nu}{B \to D^{(*)}\mu\nu}$$

- For inclusive $b \to c$ less analyses
  - RH-current, scalar and tensor NP contributions to rate Jung, Straub [2018]
  - RH-current to moments Feger, Mannel, et. al. [2010]
  - NP in semitauonic mode Rusov, Mannel, Shahriaran [2017]
  - NP for moments KKV, Fael, Olschewsky [in progress]



Summary & Outlook

Inclusive $|V_{cb}|$ and exclusive V_{ub} least disputed

Next steps:

- New modes
- Perturbative corrections
- Proliferation of non-perturbative matrix elements: q^2 moments

Inclusive $|V_{ub}|$ and exclusive V_{cb} require more studies

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- Lattice form factors of $B \to D^{(*)}$ at different kinematical points
- Studies of weak annihilation in D and D_s
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Thank you for your attention

Backup

$$\Gamma = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 \left[\mu_3 - 2\frac{\mu_G^2}{m_b^2} + \left(\frac{34}{3} + 8\log\rho\right)\frac{\tilde{\rho}_D^3}{m_b^2} \right.$$

$$+ \frac{16}{9}(4 + 3\log\rho)\frac{r_G^4}{m_b^4} - \frac{16}{9}(1 + 3\log\rho)\frac{r_E^4}{m_b^4} - \frac{2}{3}\frac{s_B^4}{m_b^4}$$

$$\left. + \left(\frac{50}{9} + \frac{8}{3}\log\rho\right)\frac{s_E^4}{m_b^4} - \left(\frac{25}{36} + \frac{1}{3}\log\rho\right)\frac{s_{qb}^4}{m_b^4} + O\left(\rho, \frac{1}{m_b^5}\right)\right]$$

with $\rho = m_c^2/m_b^2$

Details of calculation

- Reduction of single γ matrix:

$$\langle \bar{Q}_v \Gamma (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle = \frac{1}{2} \langle \bar{Q}_v \{\Gamma, \slashed{v}\} \rangle (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle$$
$$+ \frac{1}{2m} \langle \bar{Q}_v \left\{ (i\slashed{D}) \, , \, (iD_{\mu_1})...(iD_{\mu_n}) \Gamma \right\} Q_v \rangle$$

$$\langle \bar{Q}_v \gamma_\beta (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle = v_\beta \langle \bar{Q}_v (iD_{\mu_1})...(iD_{\mu_n}) Q_v \rangle + \mathcal{O}(1/m_b)$$

- Use E.O.M.

$$\slashed{v} Q_v = Q_v - \frac{i\slashed{D}}{m_b} Q_v$$
$$(iv \cdot D) Q_v = -\frac{1}{2m_b} (i\slashed{D})(i\slashed{D}) Q_v$$

Renormalization Group

$$0 = \sum_i \left(\mu \frac{d}{d\mu} C_i(\Lambda/\mu) \right) \mathcal{O}_i(\mu) + C_i(\Lambda/\mu) \left(\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) \right)$$

- Operator mixing:

$$\mu \frac{d}{d\mu} \mathcal{O}_i(\mu) = \sum_j \gamma_{ij}(\mu) \mathcal{O}_j(\mu)$$

- The operators $\mathcal{O}_j$ form a basis:

$$\sum_i \left[\delta_{ij} \mu \frac{d}{d\mu} + \gamma_{ij}^T(\mu) \right] C_j(\Lambda/\mu) = 0$$

- Wilson coefficient C_j also depends on $\alpha_s(\mu)$ introducing the QCD β-function:

$$\mu \frac{d}{d\mu} = \left(\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right)$$

Renormalization Group

- Renormalization Group Equations (RGE):

$$\sum_i \left[\delta_{ij}\left(\mu\frac{\partial}{\partial\mu} + \beta(\alpha_s)\frac{\partial}{\partial\alpha_s}\right) + \gamma_{ij}^T(\alpha_s)\right] C_j(\Lambda/\mu) = 0$$

- Running: Use RGE to relate coefficients at different scales μ
- Resummation of large logarithms. Leading Log Approx. (LLA):

$$C_i(\Lambda/\mu, \alpha_s) \sum_{n=0}^{\infty} b_i^{nn} \left(\frac{\alpha_s}{4\pi}\right)^n \ln^n \frac{\Lambda}{\mu}$$

- At next order include next-to-leading logs etc..