Measurement of strong-phase difference between D^0 and $\bar{D}^0 \to K^0_{\mathrm{S/L}}\pi^+\pi^-$ and the role of model-dependent inputs at BESIII

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Belle II Physics Week

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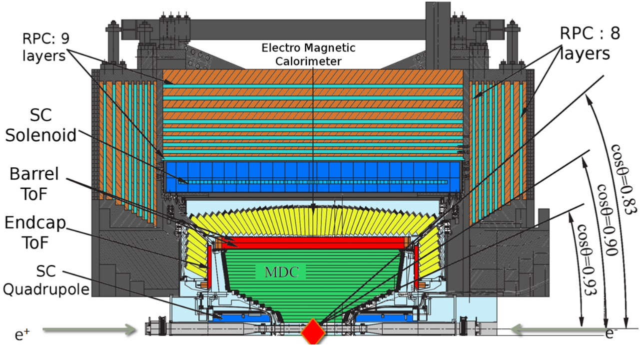
Outline

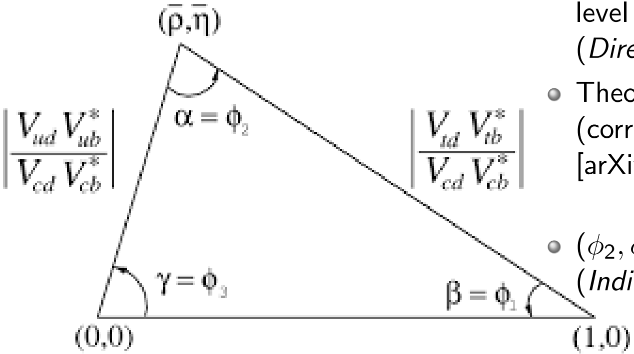
- Introduction – CPV in SM, CKM angle ϕ_3
- $D^0 \to K^0_S \pi^+ \pi^-$ strong-phase measurement
  - statistical advantage
  - $D^0 \to K^0_L \pi^+ \pi^-$ inclusion
- $D^0 \to K^0_{S,L} \pi^+ \pi^-$ amplitude models
  - Importance
  - Formulation
  - Preliminary fits



Beijing Spectrometer (BES-III) (NIM A 614, 345 (2010))
Link

$\mathcal{CP}$ violation in Standard Model

- Unitarity triangle:
  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$



- ϕ_3 $\left(= -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is the only angle measurable from "tree" level $B^{\pm} \to DK^{\pm}$ decays. (*Direct* measurement)
- Theoretically clean (corrections $\mathcal{O}(10^{-7})$) [arXiv:1308.5663]
- $(\phi_2, \phi_2) \Rightarrow \phi_3$ (*Indirect* measurement)

• Direct <u>precision</u> measurement of ϕ_3 is crucial for testing SM description of *CP* violation and for NP.

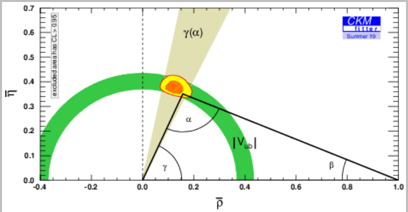
$\mathcal{CP}$ violation in Standard Model

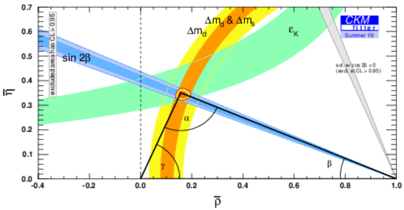
- Unitarity triangle:
  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

- $\phi_3 \left(= -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is the only angle measurable from "tree"

ϕ_3 – Current status (latest LHCb results not included)

[P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)]





Direct: $\phi_3 = (72.1^{+4.1}_{-4.5})^\circ$

Indirect: $\phi_3 = (65.7^{-1.0}_{-2.7})^\circ$

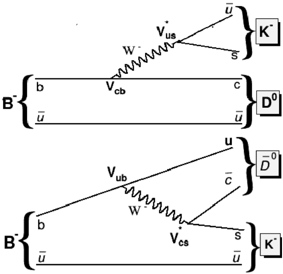
Precision needs to be improved!

- Direct precision measurement of ϕ_3 is crucial for testing SM description of *CP* violation and for NP.

The golden channel — ϕ_3 measurement

- $B^- \to DK^-$, $D \to K^0_S\pi^+\pi^-$ (BPGGSZ [1])
- B^- decay amplitude:

$$\mathcal{A}_{B^-}(m_+^2, m_-^2) = \mathcal{A}_D(m_+^2, m_-^2) + r_B e^{i(\delta_B - \phi_3)}\mathcal{A}_{\bar{D}}(m_+^2, m_-^2)$$



$$\boxed{(m_+^2, m_-^2) \equiv (s_{K^0_S\pi^+}, s_{K^0_S\pi^-})}$$

- N(B) $= \mathcal{F}(\phi_3, r_B, \delta_B, \Delta\delta_D(m_+^2, m_-^2)) \Rightarrow$ Binned analysis

Suppressed to favored B decay amplitudes

Strong-phase difference between D^0 and $\bar{D}^0$ decays

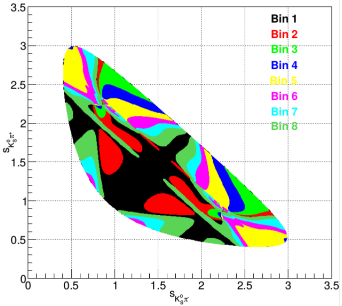
$$N_i^{\pm} = h_B\left[K_{\pm i} + r_B^2 K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_{\pm} c_i \pm y_{\pm} s_i)\right]$$

h_B: Normalization constant

$(x_{\pm}, y_{\pm}) = (r_B\cos(\delta_B \pm \phi_3), r_B\sin(\delta_B \pm \phi_3))$

K_i: Flavor tagged yield in i^{th} bin of Dalitz plot

$c_i(s_i)$: Amplitude averaged cosine(sine) of the strong-phase difference ($\Delta\delta_D$) over the i^{th} bin of DP.



"Equal $\Delta\delta_D$ binning"

The strong-phase ($\Delta\delta_D$) parameters

The parameters of interest:

$$c_i(s_i) = \frac{\int_i |A_D(m_+^2, m_-^2)||A_{\bar{D}}(m_-^2, m_+^2)| \cos(\sin)[\Delta\delta_D(m_+^2, m_-^2)] dm_+^2 dm_-^2}{\sqrt{\int_i |A_D(m_+^2, m_-^2)|^2 dm_+^2 dm_-^2 \int_i |A_{\bar{D}}(m_-^2, m_+^2)|^2 dm_+^2 dm_-^2}}$$

Precision measurement because...

- Crucial for model-independent ϕ_3 measurement in $B^\pm \rightarrow DK^\pm$ *and* other B decays
- High precision strong-phase measurement needed for $D^0 - \bar{D}^0$ mixing and CPV in charm.
  (PRL, **122**, 231802 (2019))

BEPC-II and BES-III

- Symmetric collider
- Quantum-correlated $D^0\bar{D}^0$ pairs produced in e^+e^- collision at $\psi(3770)$ ($\sqrt{s} = 3.773$ GeV) resonance. $(2.93\ \mathrm{fb}^{-1})$
- $2 \times M_{D^0} = 3.73$ GeV $\approx M_\psi$
  $\Rightarrow$ **Negligible background because there are no extra particles**
- $J^{PC}(\psi(3770)) = 1^{--}$

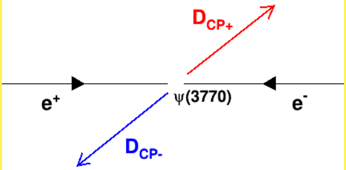
$$|\psi(3770)\rangle = \underbrace{\frac{1}{\sqrt{2}}\left(|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle\right)} = \frac{1}{\sqrt{2}}\left(|D_{CP-}\rangle|D_{CP+}\rangle - |D_{CP+}\rangle|D_{CP-}\rangle\right)$$

"antisymmetric state"

$\mathcal{C} = -1$

$$D_{CP\pm} = \frac{1}{\sqrt{2}}(|D^0\rangle \pm |\bar{D}^0\rangle)$$

$\Rightarrow$ **Flavor & *CP* tagging!**



Model-independent strong-phase measurement
$D^0 \to K_S^0\pi^+\pi^-$ expected yields

- Number of CP-even or CP-odd tagged $D^0 \to K_S^0\pi^+\pi^-$ events in the i^{th} bin of the DP:

$$M_i^{\pm} = h_{CP\pm}\left(K_i - (2\mathcal{F}_{CP} - 1)2c_i\sqrt{K_i K_{-i}} + K_{-i}\right)$$

$c_i^2 + s_i^2 \leq 1$

- $D^0\bar{D}^0 \to (K_S^0\pi^+\pi^-)^2$ yield in the i^{th} bin of D^0 decay and j^{th} bin of $\bar{D}^0$ decay Dalitz plots:

$$M_{ij} = h_{corr}\left[K_i K_{-j} + K_{-i}K_j - 2\sqrt{K_i K_{-j} K_{-i} K_j}(c_i c_j + s_i s_j)\right]$$

Inclusion of $D^0 \to K_L^0\pi^+\pi^-$ mode

- Provides boost in statistics
  - Combinatorics: $\mathrm{N}(K_S^0\pi^+\pi^-\ \textit{vs.}\ K_L^0\pi^+\pi^-) = 2 \times \mathrm{N}(K_S^0\pi^+\pi^-\ \textit{vs.}\ K_S^0\pi^+\pi^-)$
  - Only $K_S^0 \to \pi^+\pi^-$ ($\approx 70\%\ \mathcal{BF}$) reconstruction.
- Introduces additional constraint in the log-likelihood function along with extra parameters to fit.

Decay mode	Data yield [11]
$K_S^0\pi^+\pi^-$ *vs.* $K_S^0\pi^+\pi^-$	899 ± 31
$K_S^0\pi^+\pi^-$ *vs.* $K_L^0\pi^+\pi^-$	3438 ± 72

$D^0 \to K_L^0\pi^+\pi^-$ expected yields

- CP-tagged yield:

$$M_i'^{\pm} = h'_{CP\pm}\left(K_i' - (2\mathcal{F}_{CP}-1)2c_i'\sqrt{K_i'K_{-i}'} + K_{-i}'\right)$$

- $D^0\bar{D}^0 \to K_S^0\pi^+\pi^-(i)$ *vs.* $K_L^0\pi^+\pi^-(j)$ yield:

$$M_{ij}' = h'_{corr}\left[K_iK_{-j}' + K_{-i}K_j' - 2\sqrt{K_iK_{-j}'K_{-i}K_j'}(c_ic_j' + s_is_j')\right]$$

Better precision expected because:

In comparison to PRD82, 112006(2010) (CLEO-collaboration),

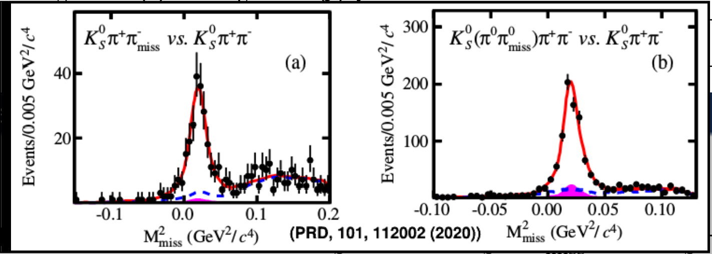
- Larger data set. CLEO: 0.818 fb^{-1}, BESIII: 2.93 fb^{-1}
- Partial reconstruction technique: ($K_S^0\pi^+\pi^-$ *vs.* $K_S^0\pi^+\pi^-_{\mathrm{miss}}$) and ($K_S^0\pi^+\pi^-$ *vs.* $K_S^0(\pi^0\pi^0_{\mathrm{miss}})\pi^+\pi^-$) in addition to fully reconstructed $K_S^0\pi^+\pi^-$ *vs.* $K_S^0\pi^+\pi^-$
- Inclusion of $K_L^0\pi^+\pi^-$ modes: Missing momentum technique
  − loose selection criteria requiring no hadronic showers.
  $\Rightarrow \sim 30\%$ boost in K_L^0 relative reconstruction efficiency
- Extra tag modes included *e.g.* $\pi^+\pi^-\pi^0$ CP tag ($\mathcal{BF} = 1.49\%$) and $K_S^0(\pi^0\pi^0_{\mathrm{miss}})\pi^+\pi^-$

$D^0 \to K^0_{\rm L}\pi^+\pi^-$ expected yields

- CP-tagged yield:

$$M_i'^{\pm} = h'_{CP\pm}\left(K'_i - (2\mathcal{F}_{CP} - 1)2c'_i\sqrt{K'_i K'_{-i}} + K'_{-i}\right)$$

- $D^0\bar{D}^0 \to K^0_{\rm S}\pi^+\pi^-(i)$ *vs.* $K^0_{\rm L}\pi^+\pi^-(j)$ yield:



Better pre

In comparis

- Larger
- Partia $^+\pi^-$ *vs.* $K^0_{\rm S}(\pi^0\pi^0_{\rm miss})\pi^+\pi^-$) in addition to fully reconstructed $K^0_{\rm S}\pi^+\pi^-$ *vs.* $K^0_{\rm S}\pi^+\pi^-$
- Inclusion of $K^0_{\rm L}\pi^+\pi^-$ modes: Missing momentum technique
  – loose selection criteria requiring no hadronic showers.
  $\Rightarrow \sim 30\%$ boost in $K^0_{\rm L}$ relative reconstruction efficiency
- Extra tag modes included *e.g.* $\pi^+\pi^-\pi^0$ CP tag ($\mathcal{BF} = 1.49\%$) and $K^0_{\rm S}(\pi^0\pi^0_{\rm miss})\pi^+\pi^-$

Summary of tags and yields (PRD, **101**, 112002 (2020))

Tag type	Modes
Flavor	$K^+\pi^-$, $K^+\pi^-\pi^0$, $K^+\pi^-\pi^+\pi^-$, $K^+e^-\nu_e$
CP-even	K^+K^-, $\pi^+\pi^-$, $K^0_{\rm S}\pi^0\pi^0$, $\pi^+\pi^-\pi^0$, $K^0_{\rm L}\pi^0$
CP-odd	$K^0_{\rm S}\pi^0$, $K^0_{\rm S}\eta$, $K^0_{\rm S}\eta'$, $K^0_{\rm S}\omega$, $K^0_{\rm L}\pi^0\pi^0$
Mixed *CP*	$K^0_{\rm S}\pi^+\pi^-$

Tag type	DT yield	
	$K^0_{\rm S}\pi^+\pi^-$	$K^0_{\rm L}\pi^+\pi^-$
Flavor	23457±319	40642±423
CP-even	2528±124	5003±178
CP-odd	1725±106	1485±117
$K^0_{\rm S}\pi^+\pi^-$	1833±82	3438±72

Extraction of $c_i^{(\prime)}$ and $s_i^{(\prime)}$

$$
\begin{aligned}
-2log\mathcal{L} = & -2\sum_{i=1}^{8} lnP(N_i^{obs}, \langle N_i^{exp}\rangle)_{CP,K_S^0\pi^+\pi^-} \\
& -2\sum_{i=1}^{8} lnP(N_i^{obs}, \langle N_i^{exp}\rangle)_{CP,K_L^0\pi^+\pi^-} \\
& -2\sum_{n=1}^{72} lnP(N_n^{obs}, \langle N_n^{exp}\rangle)_{K_S^0\pi^+\pi^-,K_S^0\pi^+\pi^-} \\
& -2\sum_{n=1}^{128} lnP(N_n^{obs\prime}, \langle N_n^{exp}\rangle)_{K_L^0\pi^+\pi^-,K_S^0\pi^+\pi^-} \\
& +\chi^2
\end{aligned}
$$

$P(N^{obs}, \langle N^{exp}\rangle)$: Poisson probablity to observe N^{obs} events given the expected number $\langle N^{exp}\rangle$.

$$
\chi^2 = \sum_i \left(\frac{c_i' - c_i - \Delta c_i}{\delta\Delta c_i}\right)^2 + \sum_i \left(\frac{s_i' - s_i - \Delta s_i}{\delta\Delta s_i}\right)^2
$$

$(c_i' - c_i)$ and $(s_i' - s_i)$: Observed difference (model-independent)

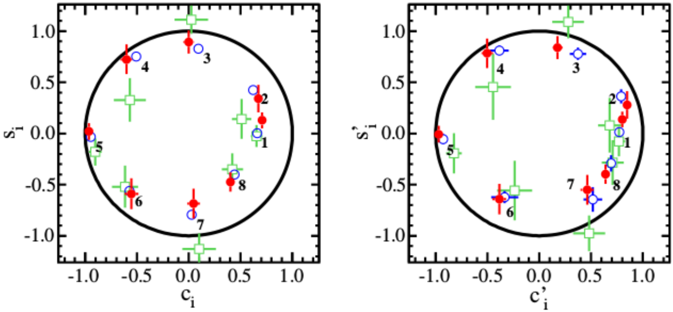
Δc_i and Δs_i: Predicted difference (amplitude model)

$\delta\Delta c_i$ and $\delta\Delta c_i$: Conservative uncertainties on predicted values

Predicted values of $c_i^{(\prime)}$ and $s_i^{(\prime)}$

- c_i, s_i: $D^0 \to K_S^0\pi^+\pi^-$ amplitude model (Phys. Rev. D98, 112012 (2018))
- c_i', s_i': Estimated model that involves assumptions on parameter values. (more later)

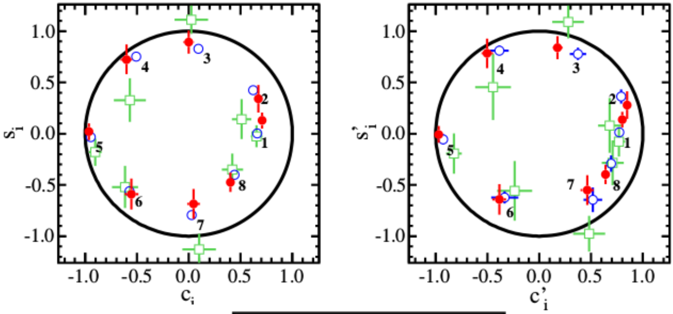
$c_i^{(\prime)}$ and $s_i^{(\prime)}$ results

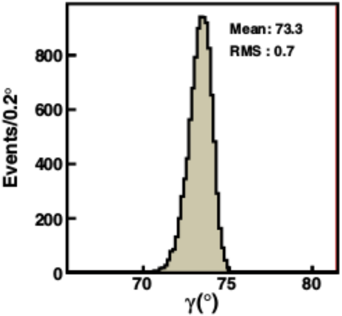


"Equal $\Delta\delta_D$ binning"

- This analysis (Phys. Rev. D, **101**, 112002 (2020))
- Model predicted values (Phys. Rev. D, **98**, 112012 (2018))
- CLEO 2010 results (Phys. Rev. D, **82**, 112006 (2010))

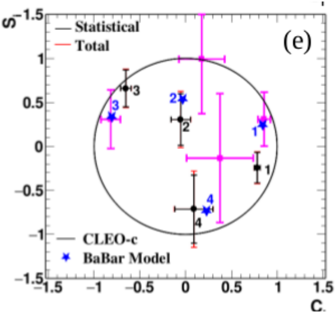
$c_i^{(\prime)}$ and $s_i^{(\prime)}$ results

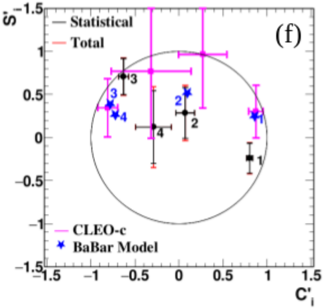




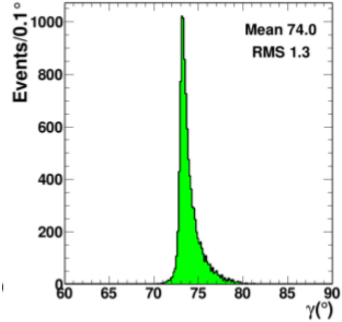
- This analysi 2020))
- Model predi 112012 (2018)
- CLEO 2010)06 (2010))

$c_i^{(\prime)}$, $s_i^{(\prime)}$ results for $D^0 \to K^0_{\mathrm{S/L}} K^+ K^-$ decay mode

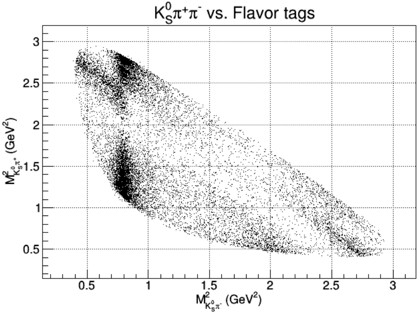




Number of DP bins: 4
(PRD. **102**, 052008 (2020))



Model-dependent parameters (Importance & determination)



$D^0 \to K^0_S\pi^+\pi^-$ – the baseline model (PRD, 98, 112012 (2018))

$$\mathcal{A}\left(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}\right) = \sum_{r\neq(K\pi/\pi\pi)_{L=0}} a_r e^{i\phi_r}\mathcal{A}_r(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) + \mathcal{F}_1(M^2_{\pi^+\pi^-}) + \mathcal{A}_{k\pi(L=0)}(M^2_{K^0_S\pi^-}) + \mathcal{A}_{K\pi(L=0)}(M^2_{K^0_S\pi^+})$$

$D^0 \to K^0_L\pi^+\pi^-$ – building up from $D^0 \to K^0_S\pi^+\pi^-$

1. The DCS resonant components gain a 180° phase shift, *i.e.*

$$A(D^0 \to K^0_S\pi^+\pi^-) = \sum A^{CF}_{\bar{K}^0\pi\pi} + \sum A^{DCS}_{K^0\pi\pi} + \sum A^{CP}_{K^0_S\pi\pi}$$

$$A(D^0 \to K^0_L\pi^+\pi^-) = \sum A^{CF}_{\bar{K}^0\pi\pi} - \sum A^{DCS}_{K^0\pi\pi} + \sum A^{CP}_{K^0_L\pi\pi}$$

Phys. Let. B **349** (1995)

2. *CP* **eigenstate amplitudes,**

$$\frac{A^{DCS}_{K^0\pi\pi}}{A^{CF}_{\bar{K}^0\pi\pi}} = -\tan^2\theta_c\,\hat{\rho}_{(D^0\to K^0_S f^k_{CP})}$$

($\hat{\rho}_k = r_k e^{i\delta_k}$: U-spin breaking)

$$A^{CP}_{K^0_S\pi\pi} = \frac{1}{\sqrt{2}}[A^{CF}_{\bar{K}^0\pi\pi} - A^{DCS}_{K^0\pi\pi}] \text{ and,}$$
$$A^{CP}_{K^0_L\pi\pi} = \frac{1}{\sqrt{2}}[A^{CF}_{\bar{K}^0\pi\pi} + A^{DCS}_{K^0\pi\pi}]$$

$$\boxed{\frac{A^{CP}_{K^0_L\pi\pi}}{A^{CP}_{K^0_S\pi\pi}} = \frac{1 - \tan^2\theta_c\hat{\rho}_k}{1 + \tan^2\theta_c\hat{\rho}_k} \approx \left(1 - 2\times\tan^2\theta_c r_k e^{i\delta_k}\right)}$$

$D^0 \to K^0_S\pi^+\pi^-$ – the baseline model (PRD, 98, 112012 (2018))

$$\mathcal{A}\left(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}\right) = \sum_{r\neq(K\pi/\pi\pi)_{L=0}} a_r e^{i\phi_r}\mathcal{A}_r(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) + \mathcal{F}_1(M^2_{\pi^+\pi^-})$$
$$+ \mathcal{A}_{k\pi(L=0)}(M^2_{K^0_S\pi^-}) + \mathcal{A}_{K\pi(L=0)}(M^2_{K^0_S\pi^+})$$

$D^0 \to K^0_L\pi^+\pi^-$ – building up from $D^0 \to K^0_S\pi^+\pi^-$

2. CP **eigenstate amplitudes,**

1. The DCS...

r_k and δ_k (In model-independent analyses)

- r_k: Gaus(1.0, 0.5) (assumption)
- δ_k: Uniform(0°, 360°) (assumption)

Also, Phys. Rev. Lett. 125, 141802 (2020):

$$BF(D^+ \to K^+\pi^-\pi^+\pi^0) = (1.13 \pm 0.08 \pm 0.03) \times 10^{-3}$$
$$= (6.28 \pm 0.52) \times \tan^4\theta_C \times BF(D^+ \to K^-\pi^+\pi^+\pi^0)$$

$\Rightarrow D^0 \to K^0_L\pi^+\pi^-$ **needs to be modelled.**

Phys. Let. B **349** (1995)

$$\boxed{\frac{A_{K^0_L\pi\pi}}{A^{CP}_{K^0_S\pi\pi}} = \frac{1 - \tan^2\theta_c\hat{\rho}_k}{1 + \tan^2\theta_c\hat{\rho}_k} \approx \left(1 - 2\times\tan^2\theta_c r_k e^{i\delta_k}\right)}$$

$D^0 \to K^0_S\pi^+\pi^-$ amplitude formulation (PRD, **86**, 010001 (2012))

1. P, D-wave resonances: BW barrier factors, Zeemach formalism, Breit-Wigner lineshapes
2. S-wave resonances

   2(a) $\pi\pi$ S-wave ($K^0_S r$): *$\mathcal{K}$-matrix formalism*
   2(b) $K\pi$ S-wave ($\pi^\pm r$): *LASS parametrization*

$$\mathcal{L} = \prod_{i=1}^{N}\left[f_{sig} \times p_{sig}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) + \cancelto{\approx 0}{(1-f_{sig})} \times p_{bkg}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})\right]$$

$$p_{sig}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) = \frac{\epsilon(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|\mathcal{A}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2}{\int_D \epsilon(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|\mathcal{A}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2 dM^2_{K^0_S\pi^+} dM^2_{K^0_S\pi^-}}$$

Event Selection: Similar to model-independent analysis ($K\pi$, $K\pi\pi\pi$, $K\pi\pi^0$ tag modes)

Normalization sample (phase-space signal MC)

Using **MC integration** technique, the normalization factor in the likelihood can be reduced to:

$$\mathcal{N} = \frac{1}{N}\sum_{i=1}^{N}|\mathcal{A}_i(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2$$

$D^0 \to K_S^0\pi^+\pi^-$ amplitude formulation (PRD, **86**, 010001 (2012))

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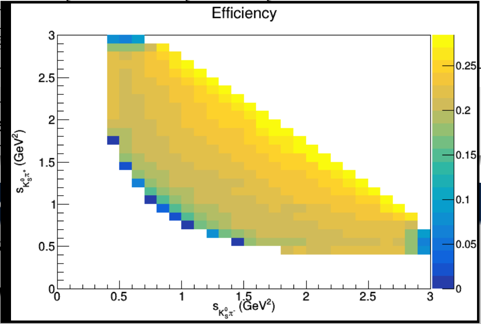
$$p_{sig}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-}) = \frac{\epsilon(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|\mathcal{A}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|^2}{\int_D \epsilon(M^2_{K_S^0\pi^+}, M^2_{K_S^0 \ldots}}$$



Event Selection: Similar to model-independe…

Normalization sample (phase-space signal M…

Using **MC integration** technique, the normaliz… to:

$$\mathcal{N} = \frac{1}{N}\sum_{i=1}^{N}|\mathcal{A}_i(\ldots$$

Corrections to the pure FT model

1. Efficiency correction

Accounts for a constant in -2log$\mathcal{L}$

$$p_{sig}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) = \frac{\epsilon(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|\mathcal{A}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2}{\underbrace{\int_D \epsilon(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|\mathcal{A}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2 dM^2_{K^0_S\pi^+} dM^2_{K^0_S\pi^-}}}$$

$$\mathcal{N} = \frac{1}{N}\sum_{i=1}^{N} |\mathcal{A}_i(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2$$

where, N = Number of fully reconstructed and selected phase-space signal MC events

2. DCS correction

- Data contains DCS contamination i.e. ($D^0 \to K^0_S\pi^+\pi^-$, $\bar{D}^0 \to K^-\pi^+$ *etc.*)
- Model needs to be modified to accomodate such events

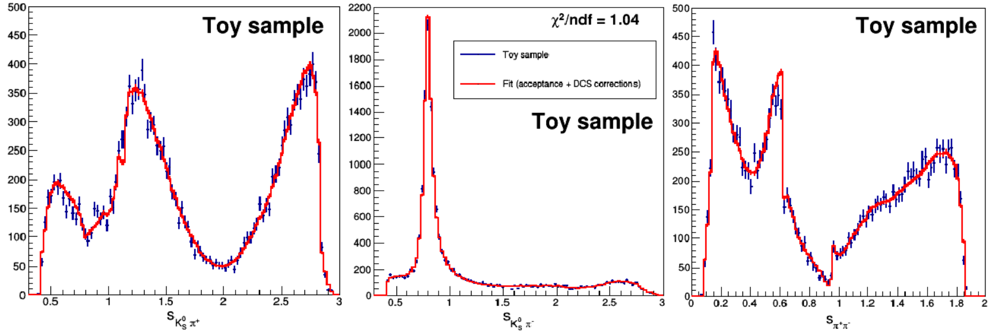
DCS to CF ratios and Coherence factors [11]

f	$r^f_D(\%)$	$\delta^f_D(^\circ)$	R_f
$K\pi$	5.86 ± 0.02	$194.7^{+8.4}_{-17.6}$	1
$K\pi\pi\pi$	5.49 ± 0.06	128^{+28}_{-17}	$0.43^{+0.17}_{-0.13}$
$K\pi\pi^0$	4.47 ± 0.12	198^{+14}_{-15}	$0.81^{+0.06}_{-0.06}$

$$|\mathcal{A}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2 \longrightarrow |\mathcal{A}_{\mathrm{PFT}}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-}) + \mathcal{A}_{\mathrm{DCS}}(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2$$

$$= |\mathcal{A}_{\mathrm{PFT}}|^2 + (r^f_D)^2|A_{\mathrm{DCS}}|^2 - 2r^f_D R_f \mathcal{R}e(e^{i\delta^f_D}\mathcal{A}_{\mathrm{PFT}}A^*_{\mathrm{DCS}})$$

Amplitude fit - On toy sample (I/O check)



Resonance	Starting values (a_r, ϕ_r)	Fit values (a_r, ϕ_r) ($< 3\sigma$ components fixed)
$\rho(770)$	(1.0, 0.0)	(1.0, 0.0)
$\omega(782)$	(0.0388, 120.7)	(0.0342 ± 0.0028, 111.7 ± 4.9)
$f_2(1270)$	(1.43, -36.3)	(1.34 ± 0.11, -58.3 ± 4.4)
$\rho(1450)$	(2.85, 102.1)	(4.28 ± 0.32, 88.0 ± 3.4)
$K^*(892)^-$	(1.72, 136.8)	(1.78 ± 0.02, 136.3 ± 1.2)
$K_2^*(1430)^-$	(1.27, -44.1)	(1.60 ± 0.08, -30.1 ± 3.3)
$K^*(1680)^-$	(3.310, -118.2)	(3.310, -118.2)
$K^*(1410)^-$	(0.29, 99.4)	(0.83 ± 0.10, 173.4 ± 7.5)

..remaining isobar and $\mathcal{K}-$matrix parameter values in backup

Ongoing work

- Simultaneous fits to $D^0 \to K^0_{\mathrm{S}}\pi^+\pi^-$ and $D^0 \to K^0_{\mathrm{L}}\pi^+\pi^-$ to extract values of r_k and δ_k and hence Δc_i and Δs_i
- BESIII is going to collect nearly 20 fB^{-1} data at $\psi(3770)$ by 2023
  $\Rightarrow$ Uncertainties coming from model-dependent parameters will be significant

Summary

- With the largest dataset available at BES-III, the most precise $c_i^{(\prime)}, s_i^{(\prime)}$ measurement till date using $D^0 \to K^0_{\mathrm{S/L}}\pi^+\pi^-$ signal modes have been carried out.
- More and more precise values of these strong-phase parameters would be required with the ever increasing statistical precision on ϕ_3 with LHCb and Belle-II datasets.
- $D^0 \to K^0_{\mathrm{L}}\pi^+\pi^-$ amplitude model inputs are required to better constraint and propagate smaller uncertainties to the strong phase parameters, $c_i(')$ and $s_i(')$ and hence to ϕ_3.
- With $D^0 \to K^0_{\mathrm{S}}\pi^+\pi^-$ as a baseline model, $D^0 \to K^0_{\mathrm{L}}\pi^+\pi^-$ model is constructed with required modifications in DCS and CP amplitudes.

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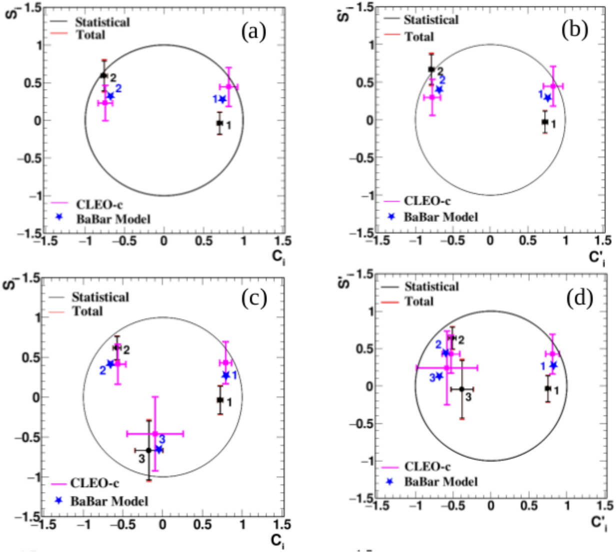
Thank you!

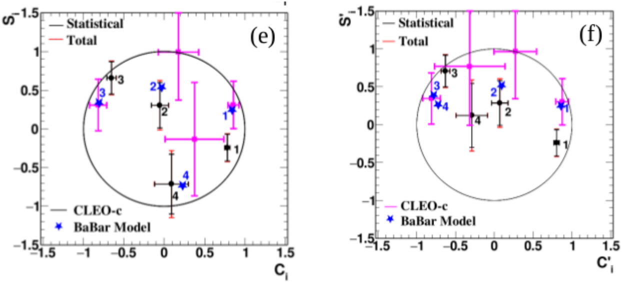
$c_i^{(\prime)}$, $s_i^{(\prime)}$ results for $D^0 \to K^0_{\mathrm{S/L}} K^+ K^-$ decay mode

(Phys. Rev D. **102**, 052008 (2020))
Number of DP bins: 2 and 3



$c_i^{(\prime)}$, $s_i^{(\prime)}$ results for $D^0 \to K^0_{\mathrm{S/L}} K^+ K^-$ decay mode

(Phys. Rev D. **102**, 052008 (2020))
Number of DP bins: 4



$D^0 \to K_S^0 \pi^+ \pi^-$ amplitude formulation (PRD, 98, 112012 (2018))

1. *P*- and *D*-wave resonances (*Isobar ansatz*)
   $a_r e^{i\phi_r} \mathcal{A}_r \left(M^2_{K_S^0 \pi^+}, M^2_{K_S^0 \pi^-} \right) = a_r e^{i\phi_r} \left(\mathcal{F}_D^{(L)}(q, q_0) \times \mathcal{F}_r^{(L)}(p, p_0) \times \mathcal{Z}_L(\Omega) \times \mathcal{T}_r(m) \right)$
   - $\mathcal{F}_D^{(L)}(\mathcal{F}_r^{(L)})$: **Blatt-Weisskopf** form factors for $D \to r h_3 (r \to h_1 h_2)$
   - $\mathcal{Z}_L(\Omega)$: Spin formalism that describes the angular distributions for the decay process
   - $\mathcal{T}_r$: Dynamical function parametrized by relativistic **Breit-Wigner line-shapes**.

1(a) Form factors ($\mathcal{F}_D^{(L)}(q, q_0)$, $\mathcal{F}_r^{(L)}(p, p_0)$) [5][6][7]

Table: Blatt-Weisskopf barrier penetration factors

	L=0	L=1	L=2
$B_L(q)$	1	$\sqrt{\frac{1+z_0}{1+z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$

where, $z = (|q|d)^2$ and $z_0 = (|q_0|d)^2$

- $q(p)$: Momentum of the bachelor particle h_3 (one of the resonances's daughter particles h_1 or h_2) evaluated in the resonance, r rest frame.
- $q_0(p_0)$: Value of $q(p)$ when the invariant mass equals the pole mass of the resonance.
- In present analysis, $d_D = 5\hbar c/\mathrm{GeV} \approx 1$ fm, $d_r = 1.5\hbar c/\mathrm{GeV} \approx 0.3$ fm.

1(b) Spin dependence (Zeemach formalism) Link Link [8]

$\mathcal{Z}'_0(\Omega) = 1$

$\mathcal{Z}'_1(\Omega) = M^2_{h_2 h_3} - M^2_{h_1 h_3} - \frac{(M^2_D - M^2_{h_3})(M^2_{h_2} - M^2_{h_1})}{M^2_{h_1 h_2}}$

etc.

where, Ω: angle between h_3 momentum ($\mathbf{p}$) and break-up three momentum ($\mathbf{q}$), in resonance rest frame.

$\mathbf{q} = \mathbf{p}_{h1} - \mathbf{p}_{h_2}$

1(c) Propagator (dynamical term) – **Breit Wigner line-shapes** [3][5]

$$\mathcal{T}_r(m) = \frac{1}{m_0^2 - m^2 - i m_0 \Gamma(m)}$$

m_0: pole mass of the resonance

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{(2L+1)} \left(\frac{m_0}{m}\right) \mathcal{F}_r^{(L)2}$$

- 11 isobars, 20 free parameters

$D^0 \to K^0_S \pi^+ \pi^-$ amplitude formulation (PRD, 98, 112012 (2018))

2. S-wave resonances
   2(a) $\pi\pi$ S-wave ($K^0_S r$): *$\mathcal{K}$-matrix formalism*
   2(b) $K\pi$ S-wave ($\pi^\pm r$): *LASS parametrization*

2(a). $\pi\pi$ S-wave ($K^0_S r$): *$\mathcal{K}$-matrix formalism*

$$\mathcal{F}_1(s) = [I - iK(s)\rho(s)]^{-1}_{1j}\, \mathcal{P}_j(s)$$

– The index j denotes the particular channels ($\pi\pi$, $K\bar{K}$, $\pi\pi\pi\pi$, $\eta\eta$, $\eta\eta'$) contributing to the scattering process.
– However, in this analysis, only the $\pi^+\pi^-$ final states are considered and $r \to K\bar{K}$ *etc*. contribute to final parametrization of $\mathcal{F}_1(s)$ due to interference processes in the scattering theory.
– The production vector $\mathcal{P}$ parametrizes the initial production of states into open channels.
– The 5x5 $\mathcal{K}$-matrix describes the scattering process and $\rho(s)$ is a diagonal matrix with phase space factors.

– $[I - iK(s)\rho(s)]^{-1}$ {
- Can be viewed as a propagator, carrying on resonances produced by $\mathcal{P}$ to a final state.
- All parameters in this term are fixed at each point on DP.

– The production vector $\mathcal{P}$ is defined as:

$$\mathcal{P}_j(s) = f(\beta_\alpha, f^{prod}_{1j})$$

where, β_α and f^{prod}_{1j} are the complex production couplings and production parameters respectively, to be determined from the fits.
– 16 free parameters

$D^0 \to K_S^0\pi^+\pi^-$ amplitude formulation (PRD, 98, 112012 (2018))

3. $K\pi$ S-wave ($\pi^\pm r$): LASS parametrization

$$\mathcal{A}_{K\pi(L=0)}(s) = A_{prod}\, e^{i\phi_{prod}} \left[R\sin\delta_R e^{i\delta_r} e^{i2\delta_F} + F\sin\delta_F e^{i\delta_F} \right]$$

– Parametrizes the CF $K_0^*(1430)^-$ and the DCS $K_0^*(1430)^+$ contributions.

– 4 free parameters

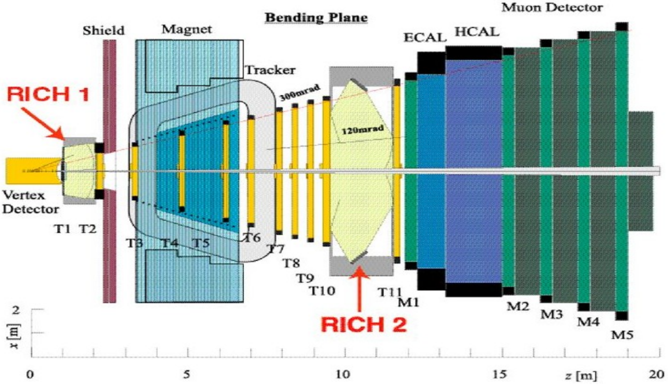
Likelihood function to minimize:

$$\mathcal{L} = \prod_{i=1}^{N} \left[f_{sig} \times p_{sig}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-}) + \overset{\approx 0}{\cancel{(1-f_{sig})}} \times p_{bkg}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-}) \right]$$

$$p_{sig}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-}) = \frac{\epsilon(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|\mathcal{A}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|^2}{\int_D \epsilon(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|\mathcal{A}(M^2_{K_S^0\pi^+}, M^2_{K_S^0\pi^-})|^2 dM^2_{K_S^0\pi^+} dM^2_{K_S^0\pi^-}}$$

The problem of K_S^0 regeneration – an aside

- A pure beam of K_L^0: $$K_L^0 = \frac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right)$$
- K^0: $\bar{s}d$; $\bar{K}^0$: $s\bar{d}$
- While passing through matter (of detector) made of *uud*, K^0 interaction will be more than $\bar{K}^0$ interaction.
- After passing through matter:
  Amplitude of $K^0 = f$
  Amplitude of $\bar{K}^0 = \bar{f}$
  With, $f < \bar{f}$
- Total amplitude after passage through detector:
$$\frac{1}{\sqrt{2}}\left(fK^0 + \bar{f}\bar{K}^0\right) = \frac{1}{2}(f + \bar{f})K_L^0 + \frac{1}{2}(f - \bar{f})K_S^0$$
- $\therefore$ Initial K_L^0 are mis-identified as signal K_S^0 events!



LHCb detector

- A significant number of K_S^0 decay outside VELO and before the magnet.
- Average charged track traverses 60% of a radiation length.
- $\therefore$ Regeneration is a systematic problem to understand.
- A measurement of the $K_L^0\pi\pi$ Dalitz plot will allow a data driven constraint of the systematics involved with $B \to D(K_L^0\pi\pi)K$ being reconstructed as $B \to D(K_S^0\pi\pi)K$ because of regeneration.

Summary of tags and yields (PRD, **101**, 112002 (2020))

	ST			DT			
Mode	ΔE (GeV)	N_{ST}	ϵ_{ST} (%)	$N_{\mathrm{DT}}^{K_S^0\pi^+\pi^-}$	$\epsilon_{\mathrm{DT}}^{K_S^0\pi^+\pi^-}$ (%)	$N_{\mathrm{DT}}^{K_L^0\pi^+\pi^-}$	$\epsilon_{\mathrm{DT}}^{K_L^0\pi^+\pi^-}$ (%)
$K^+\pi^-$	[−0.025, 0.028]	549373 ± 756	67.28 ± 0.03	4740 ± 71	27.28 ± 0.07	9511 ± 115	35.48 ± 0.05
$K^+\pi^-\pi^0$	[−0.044, 0.066]	1076436 ± 1406	35.12 ± 0.02	5695 ± 78	14.45 ± 0.05	11906 ± 132	18.21 ± 0.04
$K^+\pi^-\pi^-\pi^+$	[−0.020, 0.023]	712034 ± 1705	39.20 ± 0.02	8899 ± 95	13.75 ± 0.05	19225 ± 176	18.40 ± 0.04
$K^+e^-\nu_e$		458989 ± 5724	61.35 ± 0.02	4123 ± 75	26.11 ± 0.07		
CP-even tags							
K^+K^-	[−0.020, 0.021]	57050 ± 231	63.90 ± 0.05	443 ± 22	25.97 ± 0.07	1289 ± 41	33.60 ± 0.07
$\pi^+\pi^-$	[−0.027, 0.030]	20498 ± 263	68.44 ± 0.08	184 ± 14	27.27 ± 0.07	531 ± 28	35.60 ± 0.08
$K_S^0\pi^0\pi^0$	[−0.044, 0.066]	22865 ± 438	15.81 ± 0.04	198 ± 16	6.47 ± 0.03	612 ± 35	8.57 ± 0.03
$\pi^+\pi^-\pi^0$	[−0.051, 0.063]	107293 ± 716	37.26 ± 0.04	790 ± 31	14.28 ± 0.06	2571 ± 74	20.29 ± 0.06
$K_L^0\pi^0$		103787 ± 7337	48.97 ± 0.11	913 ± 41	20.84 ± 0.04		
CP-odd tags							
$K_S^0\pi^0$	[−0.040, 0.070]	66116 ± 324	35.98 ± 0.04	643 ± 26	14.84 ± 0.05	861 ± 46	18.76 ± 0.06
$K_S^0\eta_{\gamma\gamma}$	[−0.035, 0.038]	9260 ± 119	30.70 ± 0.11	89 ± 10	12.86 ± 0.05	105 ± 15	16.78 ± 0.06
$K_S^0\eta_{\pi^+\pi^-\pi^0}$	[−0.027, 0.032]	2878 ± 81	16.61 ± 0.13	23 ± 5	6.98 ± 0.03	40 ± 9	8.88 ± 0.03
$K_S^0\omega$	[−0.030, 0.039]	24978 ± 448	16.79 ± 0.05	245 ± 17	6.30 ± 0.03	321 ± 25	8.14 ± 0.03
$K_S^0\eta'_{\pi^+\pi^-\eta}$	[−0.028, 0.031]	3208 ± 88	13.17 ± 0.09	24 ± 6	5.06 ± 0.02	38 ± 8	6.86 ± 0.03
$K_S^0\eta'_{\gamma\pi^+\pi^-}$	[−0.026, 0.034]	9301 ± 139	23.80 ± 0.10	81 ± 10	9.87 ± 0.03	120 ± 14	12.43 ± 0.04
$K_L^0\pi^0\pi^0$		50531 ± 6128	26.20 ± 0.07	620 ± 32	11.15 ± 0.03		
Mixed-CP tags							
$K_S^0\pi^+\pi^-$	[−0.022, 0.024]	188912 ± 756	42.56 ± 0.03	899 ± 31	18.53 ± 0.06	3438 ± 72	21.61 ± 0.05
$K_S^0\pi^+\pi^-_{\mathrm{miss}}$				224 ± 17	5.03 ± 0.02		
$K_S^0(\pi^0\pi^0_{\mathrm{miss}})\pi^+\pi^-$				710 ± 34	18.30 ± 0.04		

Results (Phys. Rev. D, **101**, 112002 (2020))

	Equal $\Delta\delta_D$ binning			
	c_i	s_i	c'_i	s'_i
1	$0.708 \pm 0.020 \pm 0.009$	$0.128 \pm 0.076 \pm 0.017$	$0.801 \pm 0.020 \pm 0.013$	$0.137 \pm 0.078 \pm 0.017$
2	$0.671 \pm 0.035 \pm 0.016$	$0.341 \pm 0.134 \pm 0.015$	$0.848 \pm 0.036 \pm 0.016$	$0.279 \pm 0.137 \pm 0.016$
3	$0.001 \pm 0.047 \pm 0.019$	$0.893 \pm 0.112 \pm 0.020$	$0.174 \pm 0.047 \pm 0.016$	$0.840 \pm 0.118 \pm 0.021$
4	$-0.602 \pm 0.053 \pm 0.017$	$0.723 \pm 0.143 \pm 0.022$	$-0.504 \pm 0.055 \pm 0.019$	$0.784 \pm 0.147 \pm 0.022$
5	$-0.965 \pm 0.019 \pm 0.013$	$0.020 \pm 0.081 \pm 0.009$	$-0.972 \pm 0.021 \pm 0.017$	$-0.008 \pm 0.089 \pm 0.009$
6	$-0.554 \pm 0.062 \pm 0.024$	$-0.589 \pm 0.147 \pm 0.031$	$-0.387 \pm 0.069 \pm 0.025$	$-0.642 \pm 0.152 \pm 0.034$
7	$0.046 \pm 0.057 \pm 0.023$	$-0.686 \pm 0.143 \pm 0.028$	$0.462 \pm 0.056 \pm 0.019$	$-0.550 \pm 0.159 \pm 0.030$
8	$0.403 \pm 0.036 \pm 0.017$	$-0.474 \pm 0.091 \pm 0.027$	$0.640 \pm 0.036 \pm 0.015$	$-0.399 \pm 0.099 \pm 0.026$
	Optimal binning			
	c_i	s_i	c'_i	s'_i
1	$-0.034 \pm 0.052 \pm 0.017$	$-0.899 \pm 0.094 \pm 0.030$	$0.240 \pm 0.054 \pm 0.014$	$-0.854 \pm 0.106 \pm 0.032$
2	$0.839 \pm 0.062 \pm 0.037$	$-0.272 \pm 0.166 \pm 0.031$	$0.927 \pm 0.054 \pm 0.036$	$-0.298 \pm 0.162 \pm 0.029$
3	$0.140 \pm 0.064 \pm 0.028$	$-0.674 \pm 0.172 \pm 0.038$	$0.742 \pm 0.060 \pm 0.030$	$-0.350 \pm 0.180 \pm 0.039$
4	$-0.904 \pm 0.021 \pm 0.009$	$-0.065 \pm 0.062 \pm 0.006$	$-0.930 \pm 0.023 \pm 0.019$	$-0.075 \pm 0.075 \pm 0.007$
5	$-0.300 \pm 0.042 \pm 0.013$	$1.047 \pm 0.055 \pm 0.019$	$-0.173 \pm 0.043 \pm 0.010$	$1.053 \pm 0.062 \pm 0.018$
6	$0.303 \pm 0.088 \pm 0.027$	$0.884 \pm 0.191 \pm 0.043$	$0.554 \pm 0.073 \pm 0.032$	$0.605 \pm 0.184 \pm 0.043$
7	$0.927 \pm 0.016 \pm 0.008$	$0.228 \pm 0.066 \pm 0.015$	$0.975 \pm 0.017 \pm 0.008$	$0.198 \pm 0.071 \pm 0.014$
8	$0.771 \pm 0.032 \pm 0.015$	$-0.316 \pm 0.123 \pm 0.021$	$0.798 \pm 0.035 \pm 0.017$	$-0.253 \pm 0.141 \pm 0.019$
	Modified optimal binning			
	c_i	s_i	c'_i	s'_i
1	$-0.270 \pm 0.061 \pm 0.019$	$-0.140 \pm 0.168 \pm 0.028$	$-0.198 \pm 0.067 \pm 0.025$	$-0.209 \pm 0.181 \pm 0.028$
2	$0.829 \pm 0.027 \pm 0.018$	$-0.014 \pm 0.100 \pm 0.018$	$0.945 \pm 0.026 \pm 0.018$	$-0.019 \pm 0.100 \pm 0.017$
3	$0.038 \pm 0.044 \pm 0.021$	$-0.796 \pm 0.095 \pm 0.020$	$0.477 \pm 0.040 \pm 0.019$	$-0.709 \pm 0.119 \pm 0.028$
4	$-0.963 \pm 0.020 \pm 0.009$	$-0.202 \pm 0.080 \pm 0.014$	$-0.948 \pm 0.021 \pm 0.013$	$-0.235 \pm 0.086 \pm 0.014$
5	$-0.460 \pm 0.044 \pm 0.012$	$0.899 \pm 0.078 \pm 0.021$	$-0.359 \pm 0.046 \pm 0.011$	$0.943 \pm 0.084 \pm 0.022$
6	$0.130 \pm 0.055 \pm 0.017$	$0.832 \pm 0.131 \pm 0.031$	$0.333 \pm 0.051 \pm 0.019$	$0.701 \pm 0.137 \pm 0.029$
7	$0.762 \pm 0.025 \pm 0.012$	$0.178 \pm 0.094 \pm 0.016$	$0.878 \pm 0.026 \pm 0.015$	$0.188 \pm 0.098 \pm 0.016$
8	$0.699 \pm 0.035 \pm 0.012$	$-0.085 \pm 0.141 \pm 0.018$	$0.740 \pm 0.037 \pm 0.014$	$-0.025 \pm 0.149 \pm 0.019$

	Equal $\Delta\delta_D$ binning			
	c_i	s_i	c'_i	s'_i
1	$0.708 \pm 0.020 \pm 0.009$	$0.128 \pm 0.076 \pm 0.017$	$0.801 \pm 0.020 \pm 0.013$	$0.137 \pm 0.078 \pm 0.017$
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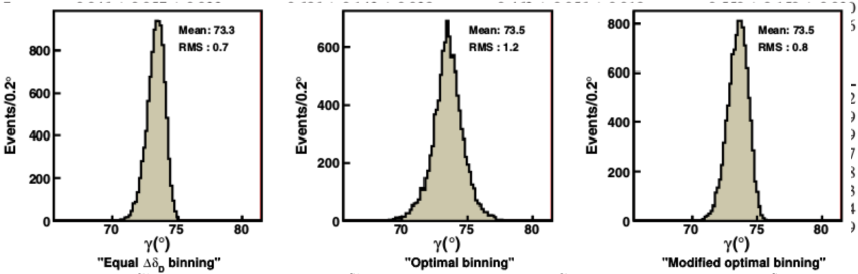
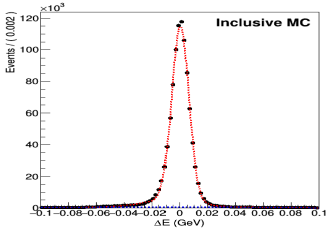


	c_i	s_i	c_i	s_i
1	$-0.270 \pm 0.061 \pm 0.019$	$-0.140 \pm 0.168 \pm 0.028$	$-0.198 \pm 0.067 \pm 0.025$	$-0.209 \pm 0.181 \pm 0.028$
2	$0.829 \pm 0.027 \pm 0.018$	$-0.014 \pm 0.100 \pm 0.018$	$0.945 \pm 0.026 \pm 0.018$	$-0.019 \pm 0.100 \pm 0.017$
3	$0.038 \pm 0.044 \pm 0.021$	$-0.796 \pm 0.095 \pm 0.020$	$0.477 \pm 0.040 \pm 0.019$	$-0.709 \pm 0.119 \pm 0.028$
4	$-0.963 \pm 0.020 \pm 0.009$	$-0.202 \pm 0.080 \pm 0.014$	$-0.948 \pm 0.021 \pm 0.013$	$-0.235 \pm 0.086 \pm 0.014$
5	$-0.460 \pm 0.044 \pm 0.012$	$0.899 \pm 0.078 \pm 0.021$	$-0.359 \pm 0.046 \pm 0.011$	$0.943 \pm 0.084 \pm 0.022$
6	$0.130 \pm 0.055 \pm 0.017$	$0.832 \pm 0.131 \pm 0.031$	$0.333 \pm 0.051 \pm 0.019$	$0.701 \pm 0.137 \pm 0.029$
7	$0.762 \pm 0.025 \pm 0.012$	$0.178 \pm 0.094 \pm 0.016$	$0.878 \pm 0.026 \pm 0.015$	$0.188 \pm 0.098 \pm 0.016$
8	$0.699 \pm 0.035 \pm 0.012$	$-0.085 \pm 0.141 \pm 0.018$	$0.740 \pm 0.037 \pm 0.014$	$-0.025 \pm 0.149 \pm 0.019$

Event selection (BOSS 6.6.4.p02)

- $N_{\text{charged tracks}} \geq 4$
- PID: $\mathcal{L}_\pi > \mathcal{L}_K$ for $\pi^+\pi^-$ from D^0
- For $K^0_S \to \pi^+\pi^-$:
  - Primary and secondary vertex fits
  - $0.485 \leq M_{\pi^+\pi^-} \leq 0.510$ GeV
  - $L/\sigma_L > 2.0$ (L: K^0_S decay length)
  - Exactly one pair of $\pi^+\pi^-$ satisfying these conditions $\Rightarrow$ No multiple combinations allowed
- 6C kinematic fit



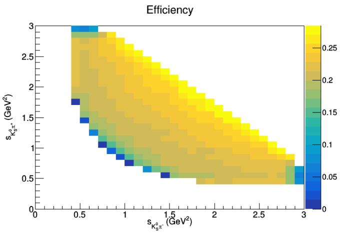
Event selection

- ΔE_{tag} cuts for resolution
- $-0.027 \leq \Delta E_{\text{sig}} \leq 0.030$ GeV

Normalization sample (phase-space signal MC)

- 15M events generated
- Using **MC integration** technique, the normalization factor in the likelihood can be reduced to:

$$\mathcal{N} = \frac{1}{N}\sum_{i=1}^{N} |\mathcal{A}_i(M^2_{K^0_S\pi^+}, M^2_{K^0_S\pi^-})|^2$$



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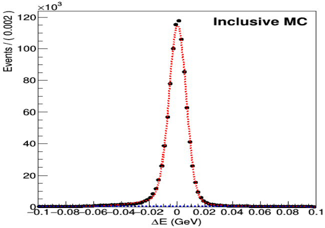

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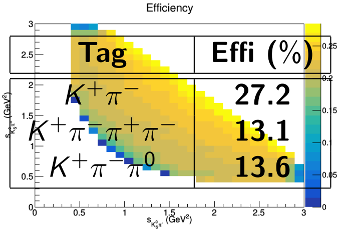


Tag	Effi (%)
$K^+\pi^-$	27.2
$K^+\pi^-\pi^+\pi^-$	13.1
$K^+\pi^-\pi^0$	13.6

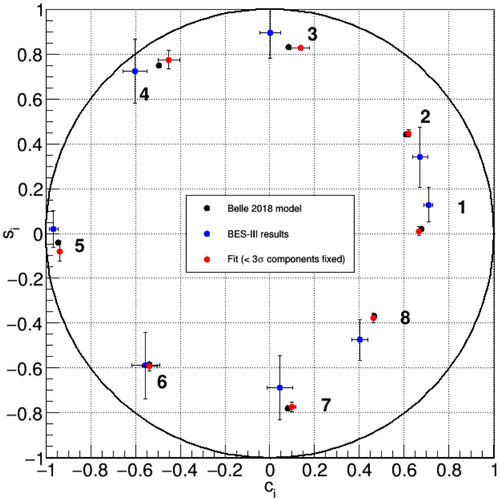
Amplitude fit - On toy sample (I/O check)

Resonance	Starting values (a_r, ϕ_r)	Fit values (a_r, ϕ_r) ($< 3\sigma$ components fixed)
$K^*(892)^+$	(0.164, -42.2)	(0.164, -42.2)
$K_2^*(1430)^+$	(0.10, -89.6)	(0.10, -89.6)
$K^*(1410)^+$	(0.21, 150.2)	(0.21, 150.2)
$K\pi$ S-wave LASS		
$K_0^*(1430)^-$	(2.36, 99.4)	(2.56$\pm$0.08, 105.2$\pm$1.9)
$K_0^*(1430)^+$	(0.11, 162.3)	(0.11, 162.3)
$\pi\pi$ S-wave parameters		
β_1	(8.5, 68.5)	(10.7$\pm$0.4, 98.1$\pm$2.2)
β_2	(12.2, 24.0)	(10.4$\pm$0.4, 27.9$\pm$2.7)
β_3	(29.2, -0.1)	(58.2$\pm$3.8, -15.1$\pm$3.3)
β_4	(10.8, -51.9)	(0.8$\pm$0.6, -59.8$\pm$45.2)
f_1^{prod}	(8.0, -126.0)	(7.9$\pm$0.3, -104.2$\pm$2.9)
f_2^{prod}	(26.3, -152.3)	(24.9$\pm$1.9, -132.0$\pm$4.3)
f_3^{prod}	(33.0, -93.2)	(44.4$\pm$3.3, -92.5$\pm$3.8)
f_4^{prod}	(26.2, -121.4)	(27.5$\pm$1.0, -97.2$\pm$2.6)

Fit fractions (Toy fit)

Resonance	FF (%) Belle	FF (%) Fitted
$K_S^0\rho(770)^0$	20.4	19.9
$K_S^0\omega(782)$	0.5	0.3
$K_S^0 f_2(1270)$	0.8	0.6
$K_S^0\rho(1450)^0$	0.6	1.2
$K^*(892)^-\pi^+$	59.9	60.4
$K_2^*(1430)^-\pi^+$	1.3	1.7
$K^*(1680)^-\pi^+$	0.5	0.5
$K^*(1410)^-\pi^+$	0.1	0.6
$K^*(892)^+\pi^-$	0.6	0.5
$K_2^*(1430)^+\pi^-$	< 0.1	0.01
$K^*(1410)^+\pi^-$	< 0.1	0.04
$\pi^+\pi^-$ S-wave	10.0	8.6
$K\pi$ S-wave		
$K_0^*(1430)^-\pi^+$	7.0	7.7
$K_0^*(1430)^+\pi^-$	< 0.1	0.01
Total	101.6	102.2

Amplitude fit - On toy sample (I/O check) – $\boxed{c_i, s_i}$



Ongoing work

- Simultaneous fits to $D^0 \to K^0_{\rm S}\pi^+\pi^-$ and $D^0 \to K^0_{\rm L}\pi^+\pi^-$ to extract values of r_k and δ_k and hence Δc_i and Δs_i

Summary of tags and yields (PRD, **101**, 112002 (2020))

Mode	ST ΔE (GeV)	ST N_{ST}	ST ϵ_{ST} (%)	DT $N_{DT}^{K_S^0\pi^+\pi^-}$	DT $\epsilon_{DT}^{K_S^0\pi^+\pi^-}$ (%)	DT $N_{DT}^{K_L^0\pi^+\pi^-}$	DT $\epsilon_{DT}^{K_L^0\pi^+\pi^-}$ (%)
$K^+\pi^-$	[−0.025, 0.028]	549373 ± 756	67.28 ± 0.03	4740 ± 71	27.28 ± 0.07	9511 ± 115	35.48 ± 0.05
$K^+\pi^-\pi^0$	[−0.044, 0.066]	1076436 ± 1406	35.12 ± 0.02	5695 ± 78	14.45 ± 0.05	11906 ± 132	18.21 ± 0.04
$K^+\pi^-\pi^-\pi^+$	[−0.020, 0.023]	712034 ± 1705	39.20 ± 0.02	8899 ± 95	13.75 ± 0.05	19225 ± 176	18.40 ± 0.04
$K^+e^-\nu_e$		458989 ± 5724	61.35 ± 0.02	4123 ± 75	26.11 ± 0.07		
CP-even tags							
K^+K^-	[−0.020, 0.021]	57050 ± 231	63.90 ± 0.05	443 ± 22	25.97 ± 0.07	1289 ± 41	33.60 ± 0.07
$\pi^+\pi^-$	[−0.027, 0.030]	20498 ± 263	68.44 ± 0.08	184 ± 14	27.27 ± 0.07	531 ± 28	35.60 ± 0.08
$K_S^0\pi^0\pi^0$	[−0.044, 0.066]	22865 ± 438	15.81 ± 0.04	198 ± 16	6.47 ± 0.03	612 ± 35	8.57 ± 0.03
$\pi^+\pi^-\pi^0$	[−0.051, 0.063]	107293 ± 716	37.26 ± 0.04	790 ± 31	14.28 ± 0.06	2571 ± 74	20.29 ± 0.06
$K_L^0\pi^0$		103787 ± 7337	48.97 ± 0.11	913 ± 41	20.84 ± 0.04		
CP-odd tags							
$K_S^0\pi^0$	[−0.040, 0.070]	66116 ± 324	35.98 ± 0.04	643 ± 26	14.84 ± 0.05	861 ± 46	18.76 ± 0.06
$K_S^0\eta_{\gamma\gamma}$	[−0.035, 0.038]	9260 ± 119	30.70 ± 0.11	89 ± 10	12.86 ± 0.05	105 ± 15	16.78 ± 0.06
$K_S^0\eta_{\pi^+\pi^-\pi^0}$	[−0.027, 0.032]	2878 ± 81	16.61 ± 0.13	23 ± 5	6.98 ± 0.03	40 ± 9	8.88 ± 0.03
$K_S^0\omega$	[−0.030, 0.039]	24978 ± 448	16.79 ± 0.05	245 ± 17	6.30 ± 0.03	321 ± 25	8.14 ± 0.03
$K_S^0\eta'_{\pi^+\pi^-\eta}$	[−0.028, 0.031]	3208 ± 88	13.17 ± 0.09	24 ± 6	5.06 ± 0.02	38 ± 8	6.86 ± 0.03
$K_S^0\eta'_{\gamma\pi^+\pi^-}$	[−0.026, 0.034]	9301 ± 139	23.80 ± 0.10	81 ± 10	9.87 ± 0.03	120 ± 14	12.43 ± 0.04
$K_L^0\pi^0\pi^0$		50531 ± 6128	26.20 ± 0.07	620 ± 32	11.15 ± 0.03		
Mixed-CP tags							
$K_S^0\pi^+\pi^-$	[−0.022, 0.024]	188912 ± 756	42.56 ± 0.03	899 ± 31	18.53 ± 0.06	3438 ± 72	21.61 ± 0.05
$K_S^0\pi^+\pi^-_{\text{miss}}$				224 ± 17	5.03 ± 0.02		
$K_S^0(\pi^0\pi^0_{\text{miss}})\pi^+\pi^-$				710 ± 34	18.30 ± 0.04		