The CKM Matrix

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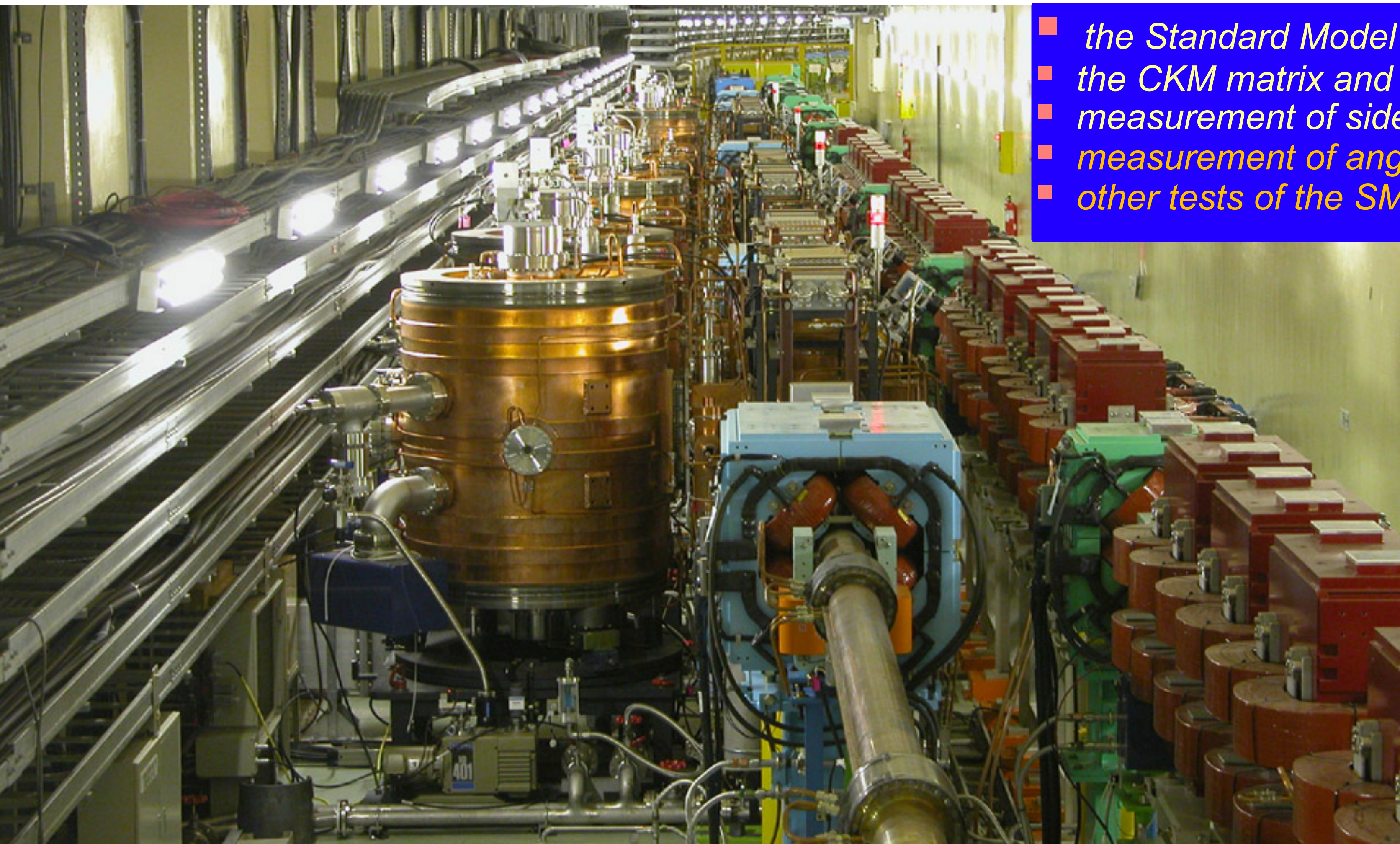
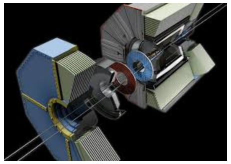


- *the Standard Model (SM)*
- *the CKM matrix and Unitarity triangle*
- *measurement of sides*
- *measurement of angles → Soeren Prell*
- *other tests of the SM → Tom Browder*



Fundamental Particles: Quarks

FERMIONS matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

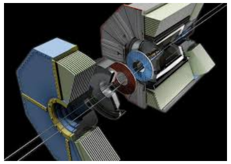
Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

Quark "flavors" are organized into doublets, as that makes it easy to keep track of how they couple to each other via the ***weak interaction:***
(each up-like flavor couples to each down-like flavor)

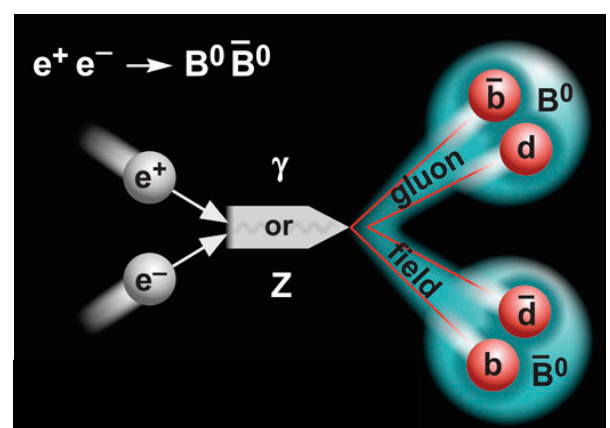
$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

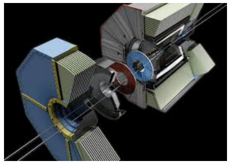


Quarks bind to form mesons and baryons (= hadrons):

Mesons $q\bar{q}$					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.776	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

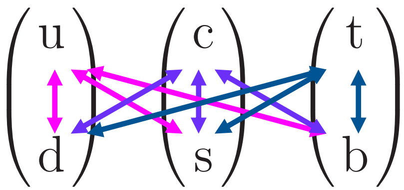
Belle and Belle II produce B^0 and anti-B^0 mesons by colliding electrons and positrons:





The CKM Matrix and Unitarity Triangle

All flavor coupling constants ("coupling strengths") can be arranged in a matrix:

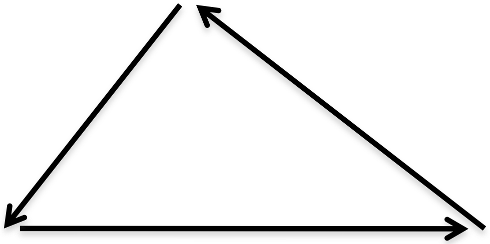


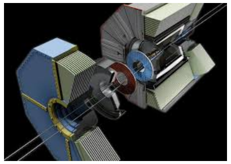
$$U \equiv \begin{array}{c|c} \begin{matrix} \quad d & \quad s & \quad b \end{matrix} & \\ \hline \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} & \begin{matrix} u \\ c \\ t \end{matrix} \end{array}$$

Unitarity ($U^\dagger U=1$) prescribes 6 complex equations:

$$\begin{aligned}
V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} &= 0 \\
V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} &= 0 \\
V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} &= 0 \\
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\
V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0
\end{aligned}$$

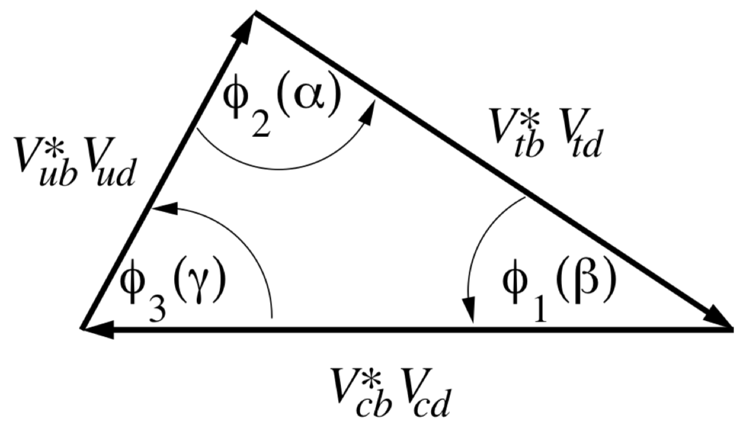
Each equation can be plotted in the complex plane as the sum of three vectors:





The Unitarity Triangle

$$\boxed{V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} \;=\; 0}$$



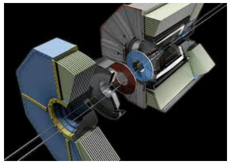
The internal angles of this triangle are phase differences, which can be measured:

$$\phi_1\ (\beta) \;=\; \arg\left(\frac{V_{cb}^* V_{cd}}{-V_{tb}^* V_{td}}\right)$$

$$\phi_2\ (\alpha) \;=\; \arg\left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}}\right)$$

$$\phi_3\ (\gamma) \;=\; \arg\left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}}\right)$$

Convention:
V_{td} and V_{ub} are taken to be complex, others real



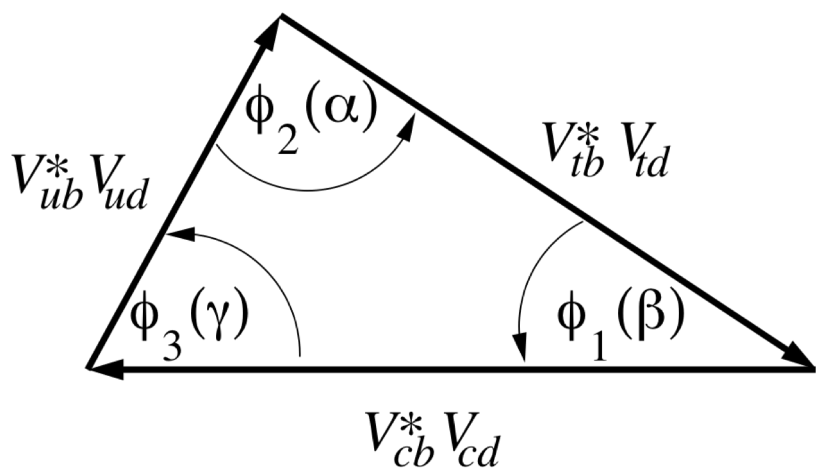
Unitarity triangle – determining the angles

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} \;=\; 0$$

The internal angles of this triangle are phase differences that can be measured via various methods (Belle/BaBar, LHCb):

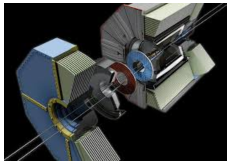
See Soeren Prell's talk

$B \to \pi^+\pi^- / \pi^+\pi^0 / \pi^0\pi^0$
$B \to \rho^+\rho^- / \rho^+\rho^0 / \rho^0\rho^0$
$B^0 \to \rho\,\pi$
$B^0 \to a_1(\rho\pi)^+\,\pi^-$



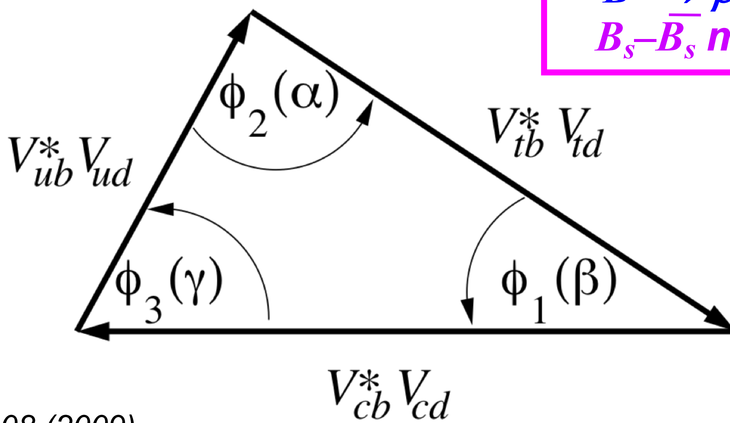
$B^- \to D^{(*)}{}_{CP}\,K^{(*)-}$
$B^0 \to D_{CP}\,K^{*0}$
$B^- \to D^{(*)}(K^+\pi^-)\,K^{(*)-}$
$B^- \to D^{(*)0}\,\pi^-$
$B^- \to D^{(*)}(K_S\,\pi^+\pi^-)\,K^{(*)-}$
$B^- \to D(\pi^0\pi^+\pi^-)\,K^-$
$B^- \to D(K_S\,K^+\pi^-)\,K^-$

$B^0 \to J/\psi\,K_S$
$B^0 \to J/\psi\,K_L$
$B^0 \to \psi'\,K_S$
$B^0 \to \chi_c\,K_S$
$B^0 \to \eta_c\,K_S$
$B^0 \to D^{(*)}{}_{CP}\,h^0$
$B^0 \to (\phi/\eta'/\pi^0/f^0)K^0$
$B^0 \to (K_S K_S/\rho^0/\omega)K_S$



Unitarity triangle – determining the sides

Belle
LHCb



$B^0 \to \pi \ell^+ \nu$
$B^0 \to X_u \ell \nu$
$B^+ \to \tau^+ \nu$
$\Lambda_b \to p \ell^+ \nu$

Bourrely et al., PRD 79, 013008 (2009)
FLAG, arXiv:1607.00299 (2016)
Bharucha, JHEP 05, 092 (2012)
Detmold et al., PRD 92, 034503 (2015)
Faustov and Galkin, PRD 94, 073008 (2016)

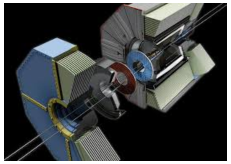
Lange et al. (BLNP), PRD 72, 073006 (2005)
Andersen, Gardi (DGE), JHEP 601, 97 (2006)
Gambino et al. (GGOU), JHEP 10, 058 (2007)
Aglietti et al. (ADFR), EPJ C59 (2009)
Bauer et al. (BLL), PRD 64, 113004 (2001)

$B^0 \to \rho^0 \gamma$
B_s–$\overline{B}_s$ mixing

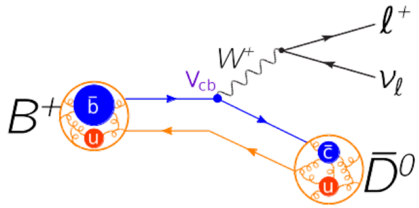
Jubb et al., Nucl. Phys. B 915, 431 (2017)
Artuso et al., RMP 88, 045002 (2016)
Lenz, Nierste, arXiv:1102.4274 (2011)
FNAL/MILC, PRD 93, 113016 (2016)
FLAG, EPJC 77, 112 (2017)

$B^0 \to D^{(*)} \ell \nu$
$B^0 \to X_c \ell \nu$ (ℓ energy, hadron mass moments)
$B^0 \to X_s \gamma$ (γ energy moments)

Caprini et al., Nucl. Phys. B530, 153 (1998)
FNAL/MILC, PRD 89, 114504 (2014)
FNAL/MILC, PRD 92, 034506 (2015)
Benson et al., Nucl. Phys. B665, 367 (2003)
Gambino, Uraltsev, EPJ C34, 181 (2004)
Gambino, JHEP 09, 055 (2011)
Alberti et al., PRL 114, 061802 (2015)
Bauer, Ligeti, et al., PRD 70, 094017 (2004)
Gambino and Schwanda, PRD 89, 014002 (2014)



Semileptonic Decays: some formalism



$$d\Gamma \;\propto\; |\mathcal{A}|^2 \;=\; G_F^2 |V_{cb}^2| \cdot |H^\mu L_\mu|^2$$

$$L_\mu \;=\; \langle P_\ell P_\nu | \bar{\ell}\gamma_\mu (1-\gamma^5)\nu_\ell | 0\rangle \quad \text{(leptonic current)}$$

$$H^\mu \;=\; \langle D | \bar{c}\gamma^\mu b | B\rangle \quad \text{(hadronic current)}$$

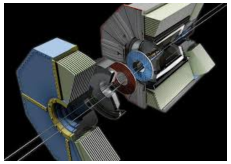
Evaluating the leptonic current gives $\bar{\ell}\gamma_\mu(1-\gamma^5)\nu_\ell$, where ℓ and ν are spinor wavefunctions. We cannot evaluate the hadronic current because we do not know the $|B\rangle$ and $\langle D|$ quantum states. However, the hadronic current must be a four-vector, and, since B and D are spinless, the only four-vectors available are P_B and P_D. Thus:

$$\begin{aligned}
\langle D|\bar{c}\gamma^\mu b|B\rangle \;&=\; A\cdot P_B^\mu + B\cdot P_D^\mu \\
&\rightarrow\; f_+(P_B+P_D)^\mu + f_-(P_B-P_D)^\mu \qquad \text{(form factors)} \\
&=\; f_+(q^2)(P_B+P_D)^\mu + f_-(q^2)q^\mu \qquad \text{where}\;\; q^\mu \equiv (P_B-P_D)^\mu
\end{aligned}$$

Each of these terms gets contracted with the leptonic current $\bar{\ell}\gamma_\mu(1-\gamma^5)\nu$. The second term gives:

$$\begin{aligned}
q^\mu\bar{\ell}\gamma_\mu(1-\gamma^5)\nu \;&=\; (P_B-P_D)^\mu\bar{\ell}\gamma_\mu(1-\gamma^5)\nu \;=\; (P_\ell+P_\nu)^\mu\bar{\ell}\gamma_\mu(1-\gamma^5)\nu \\
&=\; (P_\ell+P_\nu)^\mu\bar{\ell}\gamma_\mu\nu \;-\; (P_\ell+P_\nu)^\mu\bar{\ell}\gamma_\mu\gamma^5\nu \\
&=\; \bar{\ell}(\not{p}_\ell+\not{p}_\nu)\nu \;-\; \bar{\ell}(\not{p}_\ell+\not{p}_\nu)\gamma^5\nu \\
&=\; (-m_\ell+m_\nu)\,\bar{\ell}\nu \;-\; (-m_\ell-m_\nu)\,\bar{\ell}\gamma^5\nu \\
&\qquad\qquad [\text{applying the Dirac equations } (\not{p}-m)\psi = 0 \text{ and } \bar{\psi}(\not{p}+m)=0] \\
&=\; (-m_\ell+m_\nu)\,\bar{\ell}\nu \;+\; (m_\ell+m_\nu)\,\bar{\ell}\gamma^5\nu \\
&\approx\; 0 \qquad [\text{since } \mathrm{m}_\nu \simeq 0 \text{ and } \mathrm{m}_\ell \ll \mathrm{M_B}, \mathrm{M_D}]
\end{aligned}$$

Thus, for $\ell = (e,\mu)$, the contribution of $f_-(q^2)$ is negligible, and the decay rate depends only on $f_+(q^2)$. This is sometimes called **"current conservation."**



$|V_{ub}|$ via exclusive $B \to \pi l \nu$

$$\frac{d\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2}{24\pi^3} p^{*3} |V_{ub}|^2 f_+^2(q^2)$$

$$f_+(q^2) = \frac{1}{(1 - q^2/M_{B^*}^2)} \sum_{k=0}^{3} b_k \left[z^k - (-1)^k \frac{k}{4} z^4 \right]$$

Bourrely, Caprini, Lellouch, PRD 79, 013008 (2009)

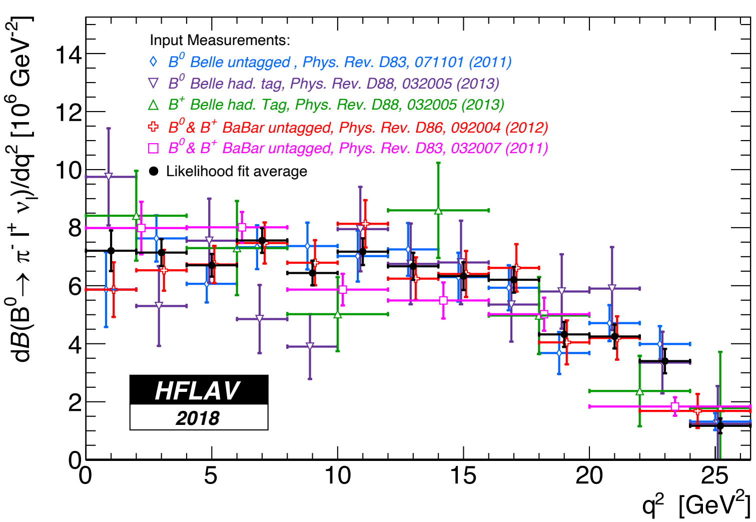
$$\text{where } z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

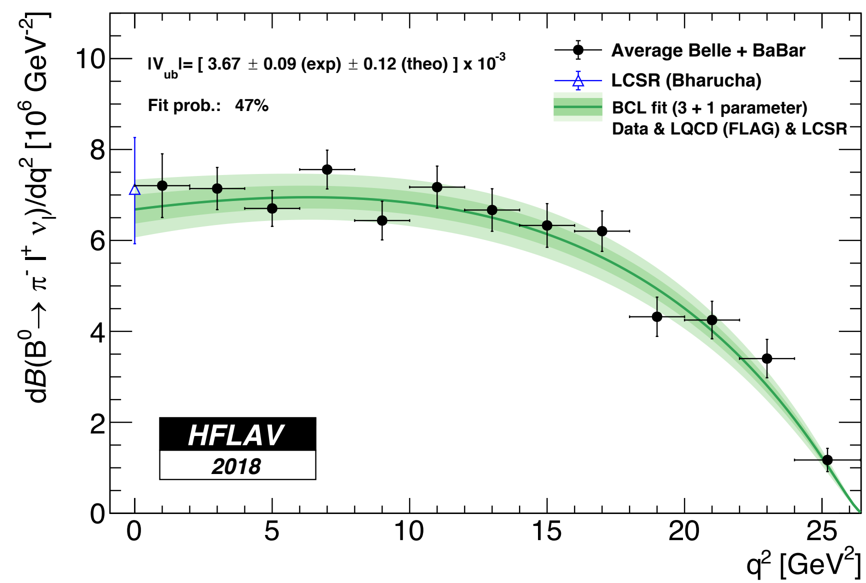
$$t_+ = (M_B + M_\pi)^2 = 29.4 \text{ GeV}^2,$$

$$t_0 = (M_B + M_\pi)\left(\sqrt{M_B} - \sqrt{M_\pi}\right)^2 = 20.1 \text{ GeV}^2$$

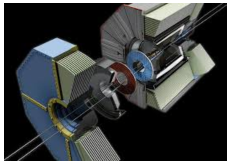
*Fit q^2 spectrum + **LCSR** + **LQCD** for BCL parameters and $|V_{ub}|$:*

***(LQCD:** Aoki et al., EPJC 77, 112, (2017);*
***LCSR:** Bharucha, JHEP 05, 092, (2012))*

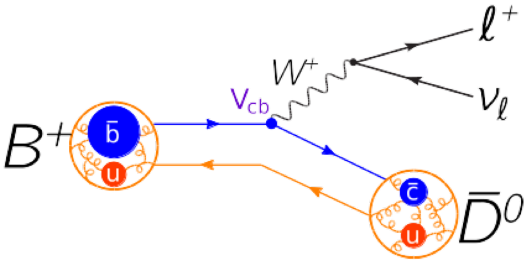




$|V_{ub}| = (3.67 \pm 0.09_{exp} \pm 0.12_{th}) \times 10^{-3}$



$|V_{cb}|$ from $B \to D^{(*)} l \nu$



New kinematic variable w (rather than q^2):

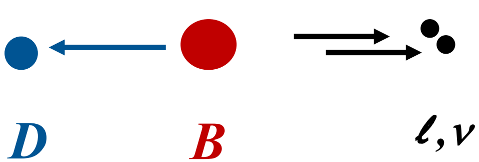
$$w \equiv \frac{P_B \cdot P_{D^*}}{M_B\, M_{D^*}} = \frac{-(P_B - P_{D^*})^2 + P_B^2 + P_{D^*}^2}{2\, M_B\, M_{D^*}} = \frac{M_B^2 + M_{D^*}^2 - q^2}{2\, M_B\, M_{D^*}}$$

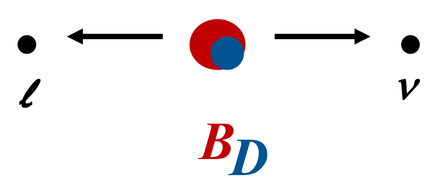
[Recall that $q^2 = (P_B - P_{D^*})^2 = (P_\ell + P_\nu)^2$]

Two extreme situations:

$q^2 \approx 0 \to w = w_{max}$
$= (M_B^2 + M_{D^*}^2)/(2M_B M_{D^*})$
$= 1.6$

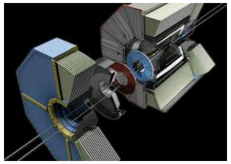
$q^2 = q^2_{max} = (M_B - M_{D^*})^2$
$= 10.69\ (\mathrm{GeV})^2 \to w_{min} = 1$



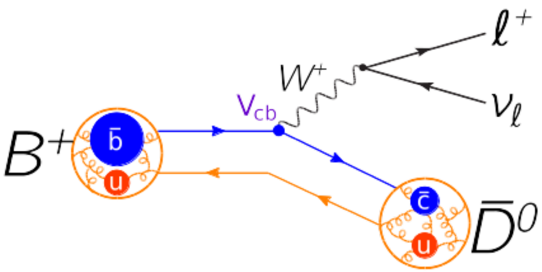


(LCSR reliable, LQCD not)

("zero recoil" : LQCD reliable, LCSR not)



$|V_{cb}|$ from $B \to D^{(*)} l \nu$



$$w \equiv v_B \cdot v_D = \frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}$$

$B \to D l \nu$ decay rate:

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} M_D^3 (M_B + M_D)^2 (w^2 - 1)^{3/2} |V_{cb}|^2 \eta_{EW}^2 \, G^2(w)$$

form factor

Caprini, Lelouch, Neubert:

$$G(w \to z) = G(1) \left[1 - 8\rho^2 z + (51\rho^2 - 10) z^2 - (252\rho^2 - 84) z^3 \right]$$

$$\text{where} \quad z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$$

$B \to D^* l \nu$ decay rate:

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} M_{D^*}^3 (M_B - M_{D^*})^2 \sqrt{w^2 - 1}\,(w+1)^2 |V_{cb}|^2 \eta_{EW}^2 \, F^2(w)$$

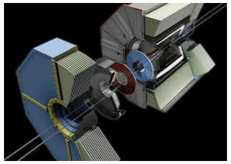
form factor

$$F^2(w) = h_{A_1}^2(w) \left\{ 2 \left[\frac{1 - 2wr + r^2}{(1-r)^2} \right] \left[1 + R_1^2(w)(w-1) \right] + \left[1 + (1 - R_2(w)) \frac{w-1}{1-r} \right]^2 \right\}$$

$$\text{where} \quad r = M_{D^*}/M_B$$

Caprini, Lelouch, Neubert:

$$\begin{aligned} h_{A_1}(z) &= h_{A_1}(1) \left[1 - 8\rho^2 z + (53\rho^2 - 15) z^2 - (231\rho^2 - 91) z^3 \right] \\ R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2 \\ R_2(w) &= R_2(1) - 0.11(w-1) + 0.06(w-1)^2 \end{aligned}$$

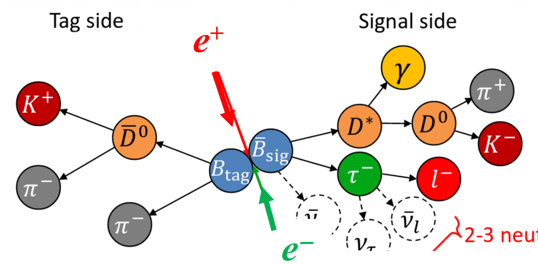


$|V_{cb}|$ from $B \to D l \nu$

711 fb^{-1} Glattauer at al. (Belle), PRD 93, 032006 (2016)

$B \to D\ell\nu$ Reconstruction:

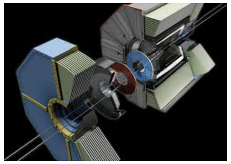
Divide event into 2 hemispheres: "signal" side and "flavor tag" side. Tag side is fully reconstructed (using neural net)



charged tags	neutral tags
$B^- \to D^{*0}\pi^-$	$B^0 \to D^{*+}\pi^-$
$B^- \to D^{*0}\pi^-\pi^0$	$B^0 \to D^{*+}\pi^-\pi^0$
$B^- \to D^{*0}\pi^-\pi^+\pi^-$	$B^0 \to D^{*+}\pi^-\pi^+\pi^-$
$B^- \to D^{*0}\pi^-\pi^+\pi^-\pi^0$	$B^0 \to D^{*+}\pi^-\pi^+\pi^-\pi^0$
$B^- \to D^0\pi^-$	$B^0 \to D^+\pi^-$
$B^- \to D^0\pi^-\pi^0$	$B^0 \to D^+\pi^-\pi^0$
$B^- \to D^0\pi^-\pi^+\pi^-$	$B^0 \to D^+\pi^-\pi^+\pi^-$
$B^- \to D^{*0}D_s^{*-}$	$B^0 \to D^{*+}D_s^{*-}$
$B^- \to D^{*0}D_s^-$	$B^0 \to D^{*+}D_s^-$
$B^- \to D^0 D_s^{*-}$	$B^0 \to D^+ D_s^{*-}$
$B^- \to D^0 D_s^-$	$B^0 \to D^+ D_s^-$
$B^- \to J/\psi\, K^-$	$B^0 \to J/\psi\, K_S$
$B^- \to J/\psi\, K^-\pi^+\pi^-$	$B^0 \to J/\psi\, K^-\pi^+$
$B^- \to J/\psi\, K^-\pi^0$	$B^0 \to J/\psi\, K_S\pi^+\pi^-$
$B^- \to J/\psi\, K_S\pi^-$	
	$B^0 \to D^0\pi^0$
$B^- \to D^0 K^-$	
$B^- \to D^+\pi^-\pi^-$	

charged signals	neutral signals
$D^+ \to K^-\pi^+\pi^+$	$D^0 \to K^-\pi^+$
$D^+ \to K^-\pi^+\pi^+\pi^0$	$D^0 \to K^-\pi^+\pi^0$
$D^+ \to K^-\pi^+\pi^+\pi^+\pi^-$	$D^0 \to K^-\pi^+\pi^+\pi^-$
$D^+ \to K^-K^+\pi^+$	$D^0 \to K^-\pi^+\pi^+\pi^-\pi^0$
$D^+ \to K_S\,\pi^+$	$D^0 \to K_S\,\pi^+\pi^-$
$D^+ \to K_S\,\pi^+\pi^0$	$D^0 \to K_S\,\pi^+\pi^-\pi^0$
$D^+ \to K_S\,\pi^+\pi^+\pi^-$	$D^0 \to K_S\,\pi^0$
$D^+ \to K_S\,K^+$	
	$D^0 \to K^-K^+$
$D^+ \to \pi^+\pi^0$	$D^0 \to \pi^+\pi^-$
$D^+ \to \pi^+\pi^+\pi^-$	$D^0 \to K_S K_S$
	$D^0 \to \pi^0\pi^0$
	$D^0 \to K_S\,\pi^0\pi^0$
	$D^0 \to \pi^+\pi^+\pi^0$

Note: over 1000 decay topologies considered.
[This is straightforward at an e^+e^- machine but very difficult at a hadron machine]



$|V_{cb}|$ from $B \to D l \nu$

711 fb^{-1}

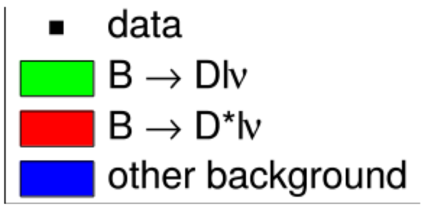
Glattauer at al. (Belle), PRD 93, 032006 (2016)

$B \to D \ell \nu$ Reconstruction:

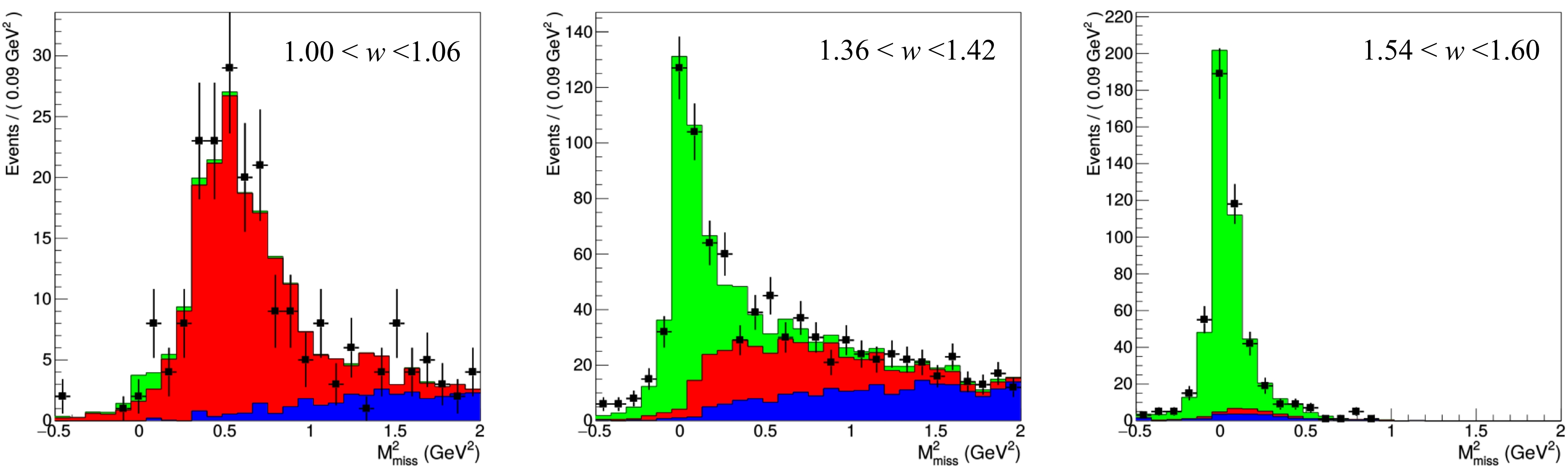
After tag side reconstructed, tracks are "removed" and signal side D reconstructed. After D reconstructed, e or μ is added to decay and missing mass calculated:

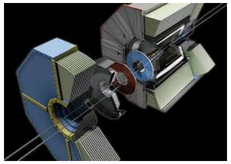
$$M^2_{\rm miss} = \left(P_{\rm beam} - P_D - P_\ell\right)^2$$

Missing mass spectrum (in bins of w) is fit for signal yield; from signal yield one calculates $\Delta\Gamma/\Delta w$.



$B^0 \to D^+ e^- \nu$ (2848 signal events)

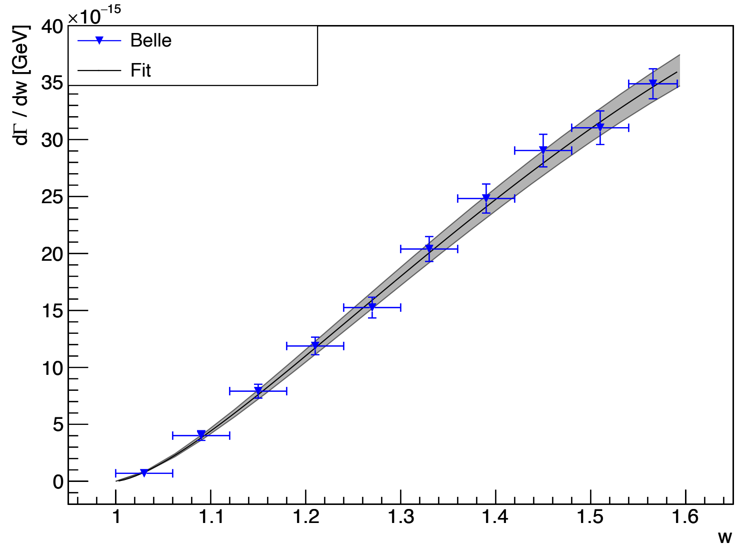


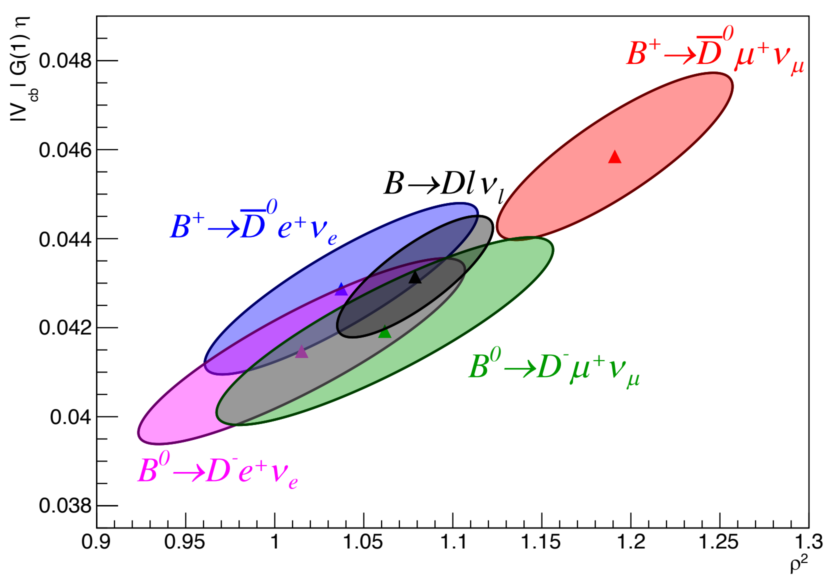


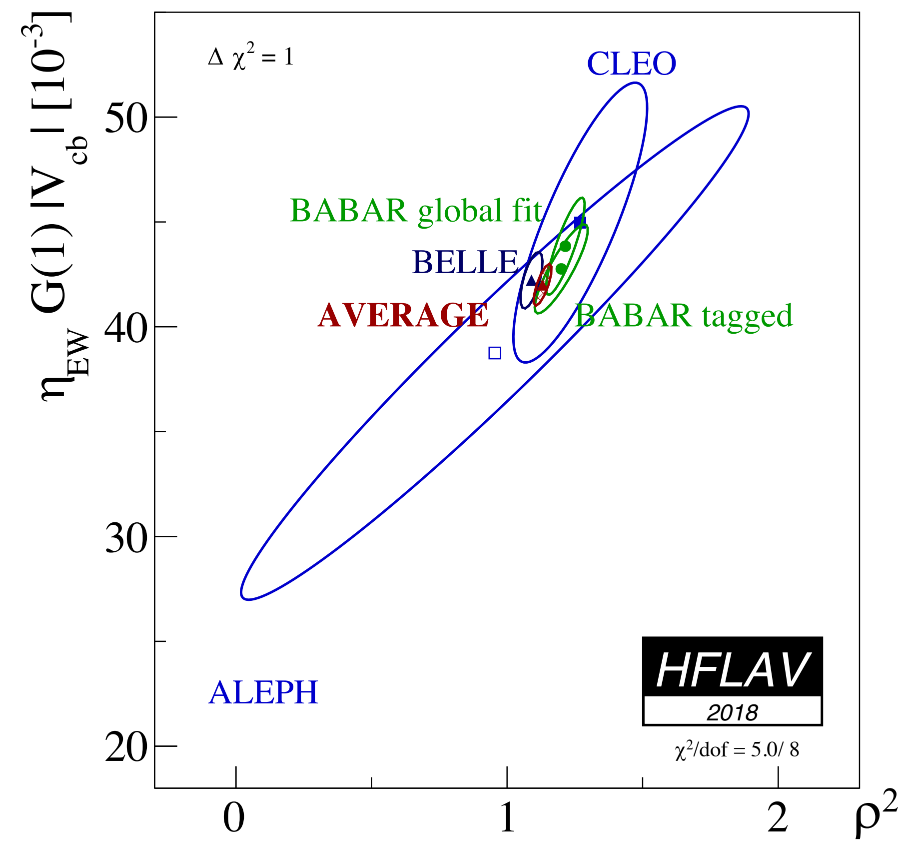
$|V_{cb}|$ from $B \to D l \nu$

711 fb^{-1} Glattauer at al. (Belle), PRD 93, 032006 (2016)

Results: CLN (2 params, heavy quark symmetry)



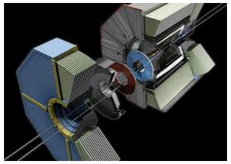




$\eta_{EW}\, G(1)\, |V_{cb}| = (42.00 \pm 1.00) \times 10^{-3}$

Using G(1) = 1.0541 ± 0.0083 [MILC, PRD 92, 034506, (2015)]
η_{EW} = 1.0066 ± 0.0050 [Sirlin, Nucl. Phy. B196, 83 (1982)]

$|V_{cb}| = (39.58 \pm 0.94_{exp} \pm 0.37_{theor}) \times 10^{-3}$



$|V_{cb}|$ from $B \to D^* l \nu$

711 fb^{-1}

Waheed at al. (Belle), PRD 100, 052007 (2019)

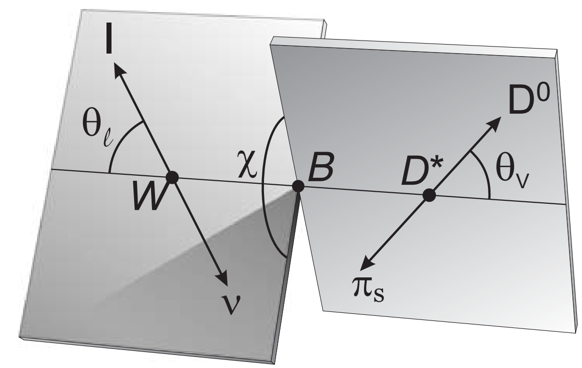
Advantages:

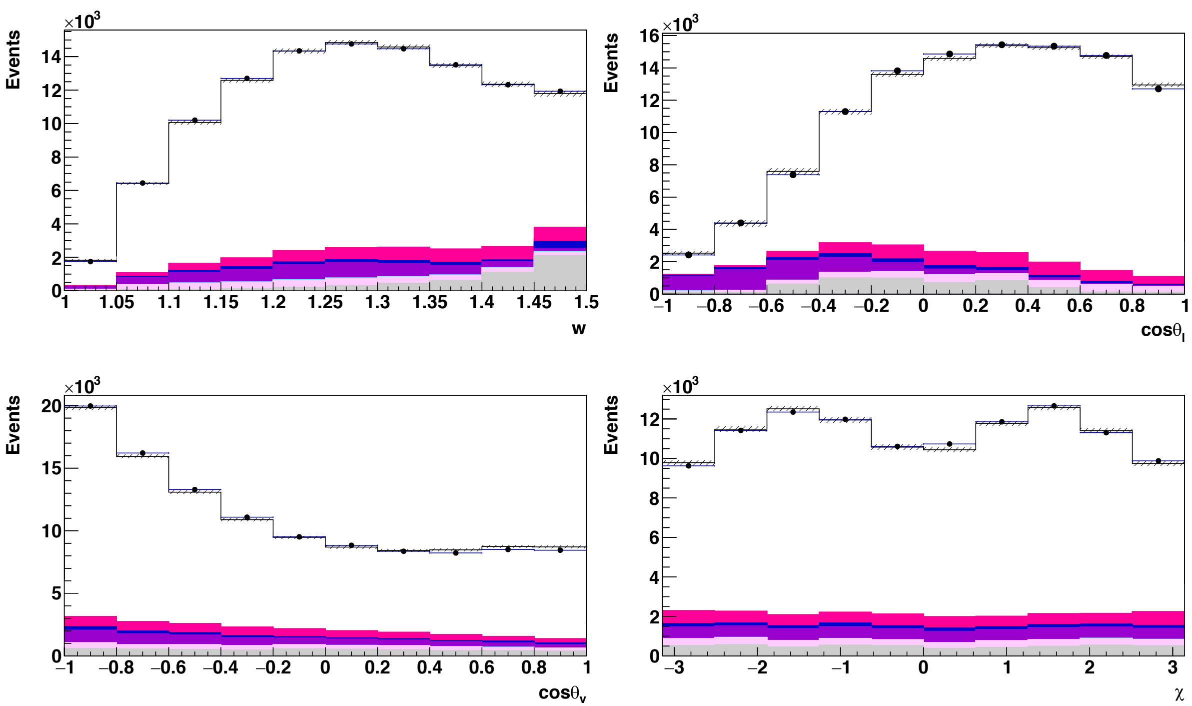
- (2.2–2.4)x larger branching fraction
- hadronic tag reconstruction not needed due to D^*

⇒ much higher statistics (180k signal events, vs. 17k for $B \to D l \nu$)

Statistics are high enough to fit the w, cosθ_ℓ, cosθ_V, χ distributions

to fully differential decay rate $$\frac{d\Gamma(B^0 \to D^{*-}\ell^+\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi}$$





Result:

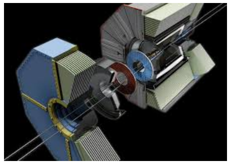
$\eta_{EW}\, F(1)\, |V_{cb}| = (35.06 \pm 0.58) \times 10^{-3}$

Using $F(1) = 0.906 \pm 0.013$ [MILC, PRD 89, 114504, (2014)]
$\eta_{EW} = 1.0066 \pm 0.0050$ [Sirlin, Nucl. Phy. B196, 83 (1982)]

$|V_{cb}| = (38.4 \pm 0.63_{exp} \pm 0.6_{theor}) \times 10^{-3}$

slightly better than $B \to D l \nu$ result:

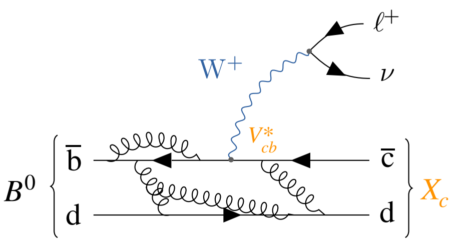
$|V_{cb}| = (39.58 \pm 0.94_{exp} \pm 0.37_{theor}) \times 10^{-3}$



Inclusive $|V_{cb}|$

Gambino and Schwanda, PRD 89, 014022 (2014)

Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)

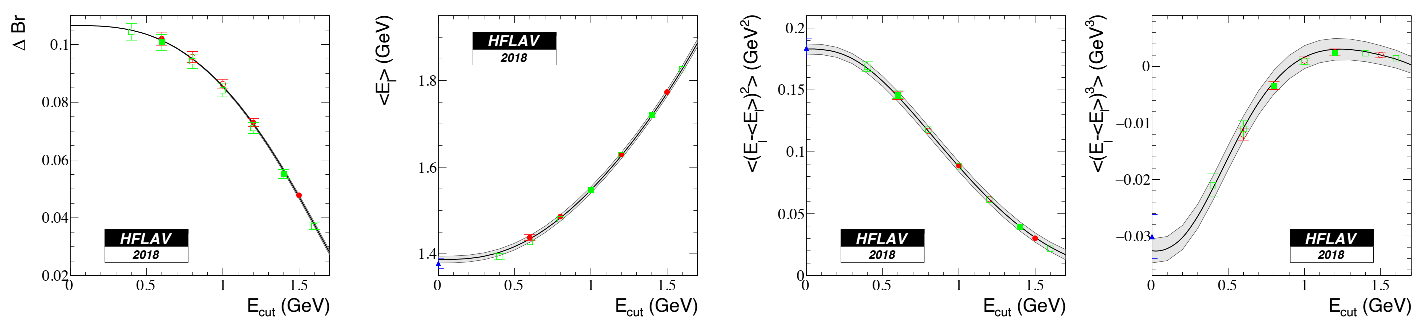


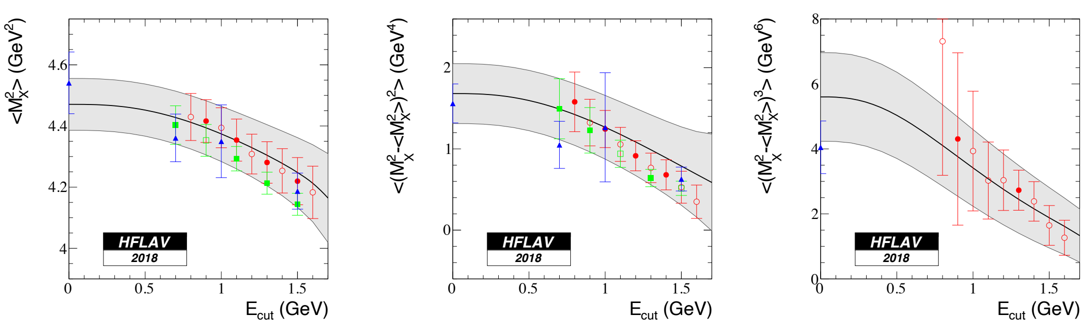
An **"inclusive" search** means $B \to X_c\, l \nu$:, where X_c denotes final state hadrons containing charm.

- *Experimentally, no specific final state is reconstructed. Statistics are high, but backgrounds are high*
- *Theoretically, one calculate a b $\to$ c transition, not a $\langle D^* | \mathcal{H} | B \rangle$ matrix element (parameterized by form factors). Typically this gives less theoretical uncertainty*
- *a decay mode with a specific final state is called an **"exclusive"** decay*

Strategy: *the inclusive b $\to$ clv decay rate can be calculated via the Operator Product Expansion (OPE) [see Takeuchi-san's talk on Tues/Wed]. This is a double expansion in small (perturbative) parameters α_s and (Λ_{QCD}/m_b). The expansion depends on unknown B matrix elements of local operators. However, these matrix elements also determine moments of the lepton energy and recoil hadronic mass in $B \to X\, l \nu$ decays. These moment distributions have been measured (Belle, BaBar, others), and thus one can fit the moment distributions and the measured width for $B \to X\, l \nu$ to extract $|V_{cb}|$*

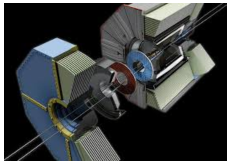
$$\langle E_\ell^n \rangle = \frac{\int_{E_{\rm cut}}^{E_{\rm max}} dE_\ell\, (E_\ell)^n \frac{d\Gamma}{dE_\ell}}{\int_{E_{\rm cut}}^{E_{\rm max}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$





$|V_{cb}| = (42.19 \pm 0.78) \times 10^{-3}$ *(kinetic scheme)*

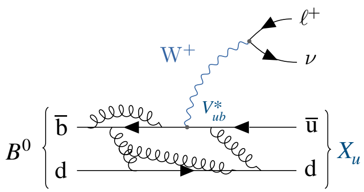
$|V_{cb}| = (41.98 \pm 0.45) \times 10^{-3}$ *(1S scheme)*



Inclusive $|V_{ub}|$

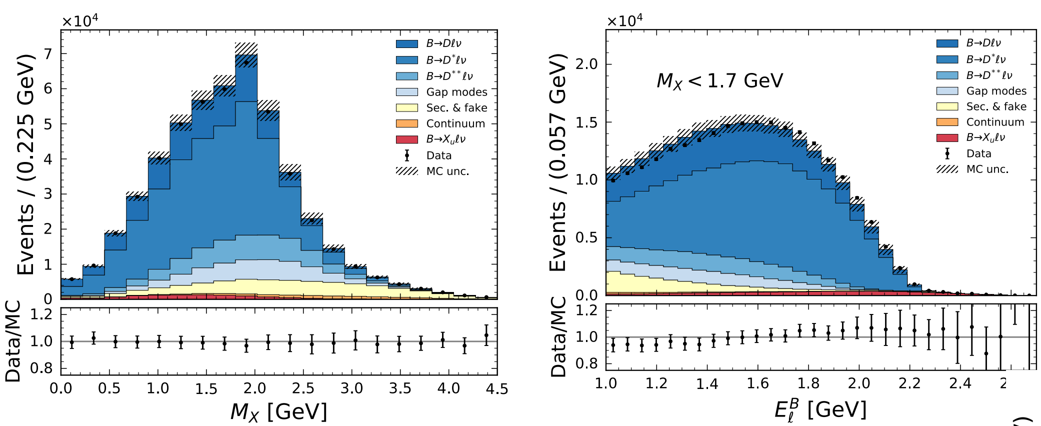
L. Cao et al. (Belle), arXiv:2102.00020 (to appear in PRD)

Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)

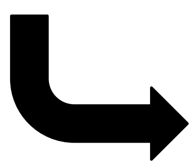


Very challenging to measure $B \to X_u\, l\nu$ (X_u denotes final state hadrons not coming charm), because $B \to X_c\, l\nu$ background is ~50x larger and swamps the signal.

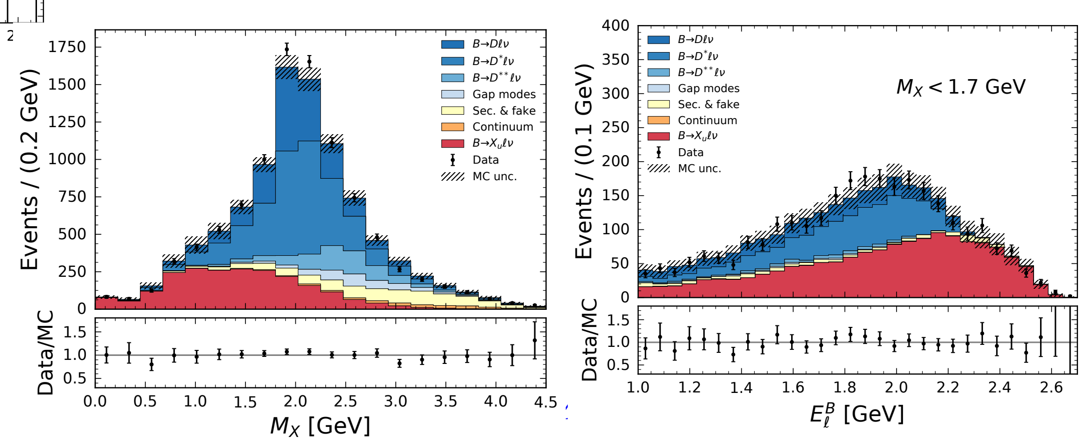
Strategy: *fit data in limited regions of M_X, E_ℓ, and q^2 where $B \to X_c\, l\nu$ background is suppressed, e.g., at lower values of M_X, higher values of E_ℓ, and higher values of q^2. Requiring such limited phase space regions complicates the perturbatve QCD calculations needed to extract $|V_{ub}|$ from the measured rate. Different theoretical models use different parameterizations of the "shape functions" needed to evaluate the unmeasured regions of phase space. Five theory models are commonly used: BLNP, DGE, GGOU, ADFR, and BLL, but no theoretical approach is preferred over the others.*



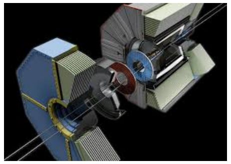
To beat down $B \to X_c l\nu$, Belle uses a sophisticated BDT based on M_{miss}^2, finding a soft π^+ from D^* decay, number of kaons, B_{sig} vertex, and Q_{tot}. Cutting on BDT output rejects 98.7% of $X_c l\nu$, keeping 18% of $X_u l\nu$:



BDT



A. J. Schwartz



Inclusive $|V_{ub}|$

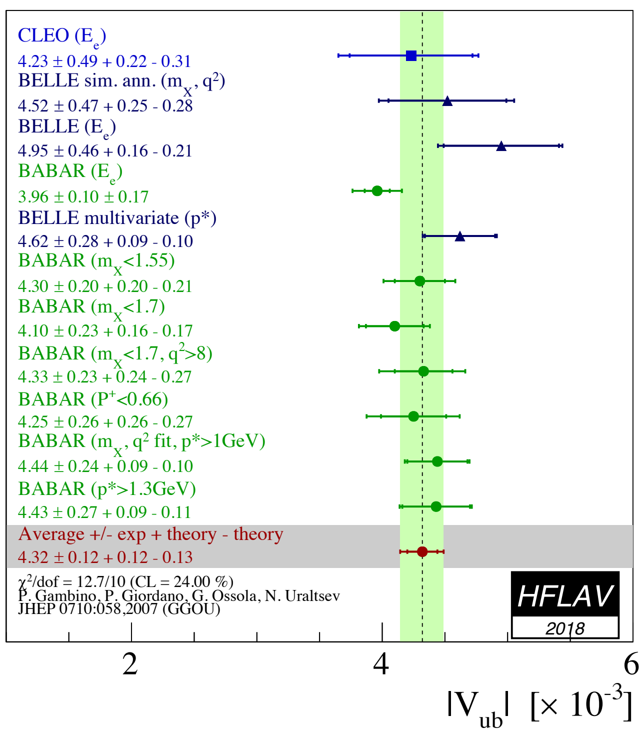
L. Cao et al. (Belle), arXiv:2102.00020 (to appear in PRD)

Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)

Measurement	Accepted region	$\Delta\mathcal{B}[10^{-4}]$	Notes
CLEO [564]	$E_e > 2.1$ GeV	$3.3 \pm 0.2 \pm 0.7$	
BABAR [563]	$E_e > 2.0$ GeV, $s_{\rm h}^{\rm max} < 3.5\,{\rm GeV}^2$	$4.4 \pm 0.4 \pm 0.4$	
BABAR [560]	$E_e > 1.0$ GeV	$1.55 \pm 0.08 \pm 0.09$	Using the GGOU model
Belle [565]	$E_e > 1.9$ GeV	$8.5 \pm 0.4 \pm 1.5$	
BABAR [555]	$M_X < 1.7\,{\rm GeV}/c^2, q^2 > 8\,{\rm GeV}^2/c^4$	$6.9 \pm 0.6 \pm 0.4$	
Belle [566]	$M_X < 1.7\,{\rm GeV}/c^2, q^2 > 8\,{\rm GeV}^2/c^4$	$7.4 \pm 0.9 \pm 1.3$	
Belle [567]	$M_X < 1.7\,{\rm GeV}/c^2, q^2 > 8\,{\rm GeV}^2/c^4$	$8.5 \pm 0.9 \pm 1.0$	Used only in BLL average
BABAR [555]	$P_+ < 0.66$ GeV	$9.9 \pm 0.9 \pm 0.8$	
BABAR [555]	$M_X < 1.7\,{\rm GeV}/c^2$	$11.6 \pm 1.0 \pm 0.8$	
BABAR [555]	$M_X < 1.55\,{\rm GeV}/c^2$	$10.9 \pm 0.8 \pm 0.6$	
Belle [554]	(M_X, q^2) fit, $p_\ell^* > 1$ GeV/c	$19.6 \pm 1.7 \pm 1.6$	
BABAR [555]	(M_X, q^2) fit, $p_\ell^* > 1$ GeV/c	$18.2 \pm 1.3 \pm 1.5$	
BABAR [555]	$p_\ell^* > 1.3$ GeV/c	$15.5 \pm 1.3 \pm 1.4$	
Belle (2021)	$E_\ell > 1.0$ GeV	$15.9 \pm 0.7 \pm 1.6$	

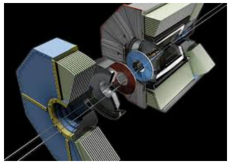
$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B\to X_u\ell^+\nu)}{\tau_B\cdot\Delta\Gamma_{\rm th}(B\to X_u\ell^+\nu)}}$$

Using GGOU for $\Delta\Gamma_{th}$:

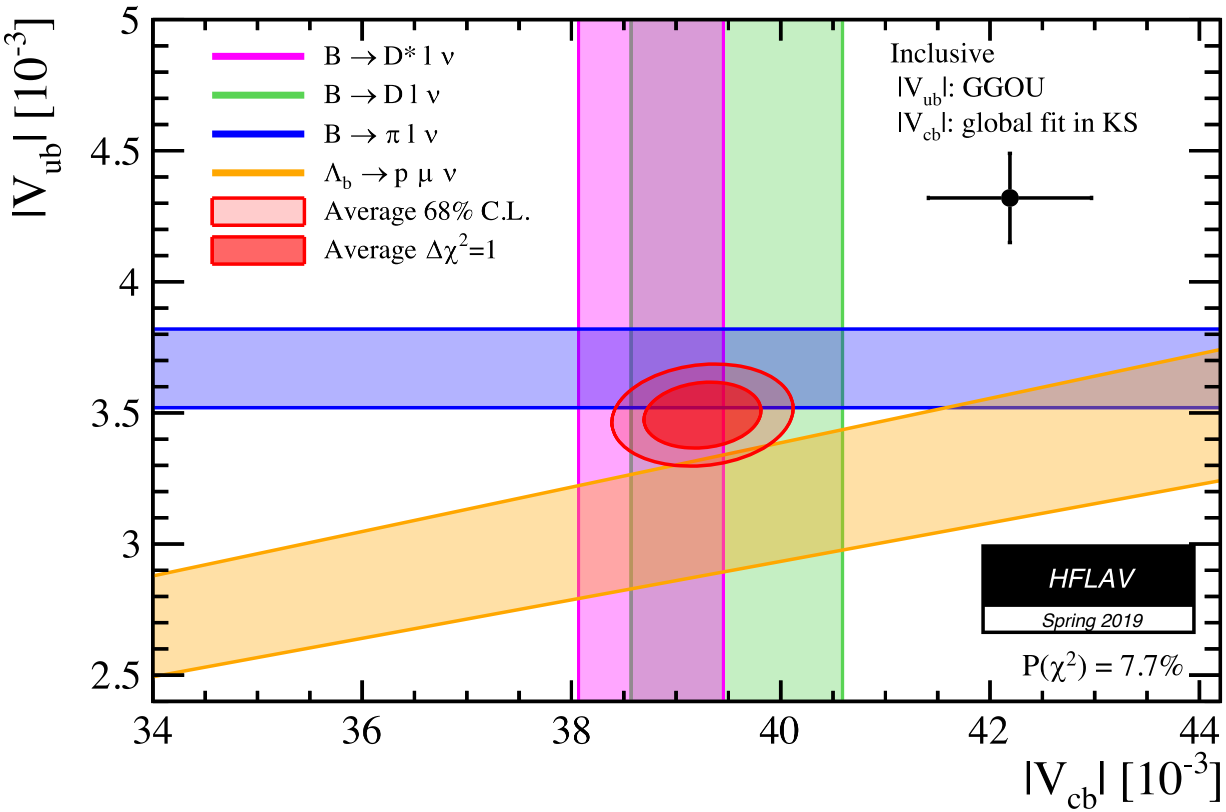


Cao et al. (Belle, 2021):

$$|V_{ub}|\ ({\rm BLNP}) = \left(4.05 \pm 0.09\,^{+0.20}_{-0.21}\,^{+0.18}_{-0.20}\right) \times 10^{-3}$$
$$|V_{ub}|\ ({\rm DGE}) = \left(4.16 \pm 0.09\,^{+0.21}_{-0.22}\,^{+0.11}_{-0.12}\right) \times 10^{-3}$$
$$|V_{ub}|\ ({\rm GGOU}) = \left(4.15 \pm 0.09\,^{+0.21}_{-0.22}\,^{+0.08}_{-0.09}\right) \times 10^{-3}$$
$$|V_{ub}|\ ({\rm ADFR}) = \left(4.05 \pm 0.09\,^{+0.20}_{-0.21} \pm 0.18\right) \times 10^{-3}$$



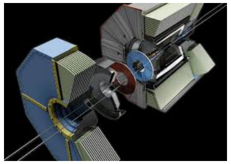
Putting all together: Inclusive vs. Exclusive $|V_{cb}|$, $|V_{ub}|$



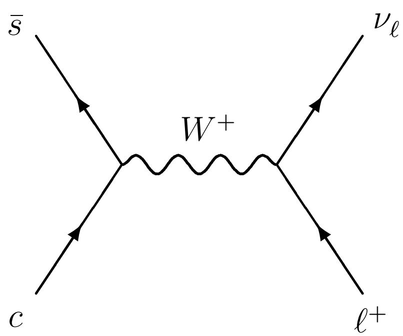
Latest lattice results:

Bailey (MILC), PRD 89, 114504 (2014)
Bailey (FNAL/MILC), PRD 92, 034506 (2015)
Aoki (FLAG), EPJC 77, 112 (2017)
Detmold et.al., PRD 92, 034503 (2015)

	Exclusive (x 10^{-2})	Inclusive (x 10^{-2})	Difference
$\lvert V_{cb}\rvert$	$3.876 \pm 0.042 \pm 0.055$ ($D^*\ell\nu$ CLN) $3.83 \pm 0.07 \pm 0.06$ ($D^*\ell\nu$ BGL [Belle]) $3.958 \pm 0.094 \pm 0.037$ ($D\ell\nu$)	4.219 ± 0.078 (kinetic scheme) 4.198 ± 0.045 (1S scheme)	2.2–3.3 σ
$\lvert V_{ub}\rvert$	$0.367 \pm 0.009 \pm 0.012$ ($\pi\ell\nu$)	$0.432 \pm 0.012 \pm 0.013$ (GGOU) $0.444 \pm 0.014 \pm 0.022$ (BLNP)	2.6–2.8 σ

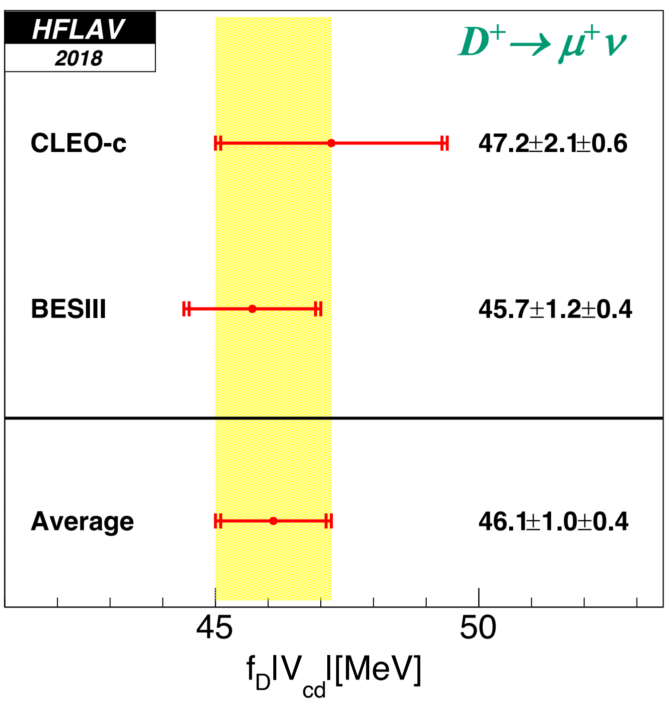


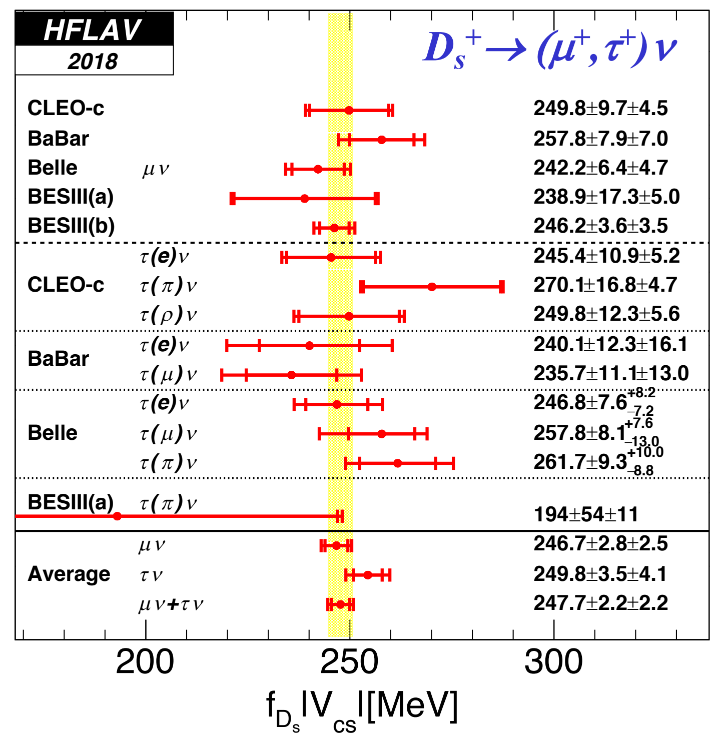
$|V_{cs}|$, $|V_{cd}|$ from leptonic decays $D_{(s)}^+ \to \ell^+ \nu$



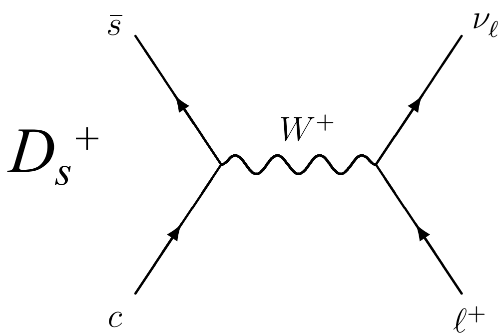
$$\mathcal{B}(D_{(s)}^+ \to \ell^+ \nu) \;=\; \frac{G_F^2}{8\pi} f_{D_{(s)}}^2 |V_{cs,cd}|^2 \, \tau_D \, m_D \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_D^2}\right)^2$$

1) Measure $\mathcal{B}$, calculate f_D on lattice, extract $|V_{cs,cd}|$ (compare to unitarity)
2) Measure $\mathcal{B}$, take $|V_{cs,cd}|$ from other measurements + unitarity, extract f_D (compare to lattice)
3) Note that rate vanishes if $m_\ell = 0$ ⇒ helicity suppression





Leptonic Decays $D_{(s)}{}^{+} \to \ell^{+} \nu$



$$\boxed{\mathcal{B}(D^+_{(s)} \to \ell^+ \nu) \;=\; \frac{G_F^2}{8\pi} f^2_{D_{(s)}} |V_{cs,cd}|^2 \, \tau_D \, m_D \, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_D^2}\right)^2}$$

1) Measure $\mathcal{B}$, calculate f_D on lattice, extract $|V_{cs,cd}|$ (compare to unitarity)
2) Measure $\mathcal{B}$, take $|V_{cs,cd}|$ from other measurements + unitarity, extract f_D (compare to lattice)

Using the latest LQCD results:

f_{Ds} = (249.9 ± 0.5) MeV

f_D = (212.0 ± 0.7) MeV

Aoki et al. (Flavor Lattice Averaging Group), arXiv:1902.08191
FNAL/MILC, PRD 98, 074512 (2018)
ETM, PRD 91, 054507 (2015)

gives:

$|V_{cs}|$ = 0.991 ± 0.013 (exp) ± 0.002 (LQCD)

$|V_{cd}|$ = 0.2173 ± 0.0051 (exp) ± 0.0007 (LQCD)

Using CKM Unitarity: (CKMFitter)

$|V_{cs}| = 0.973394^{+0.000074}_{-0.000096}$

$|V_{cd}| = 0.22529^{+0.00041}_{-0.00032}$

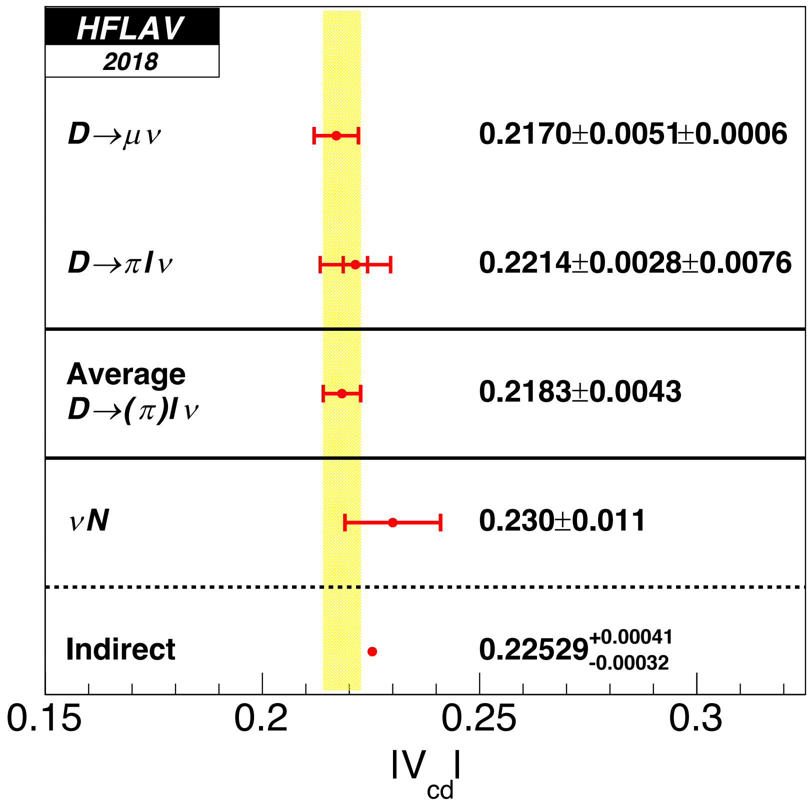
gives:

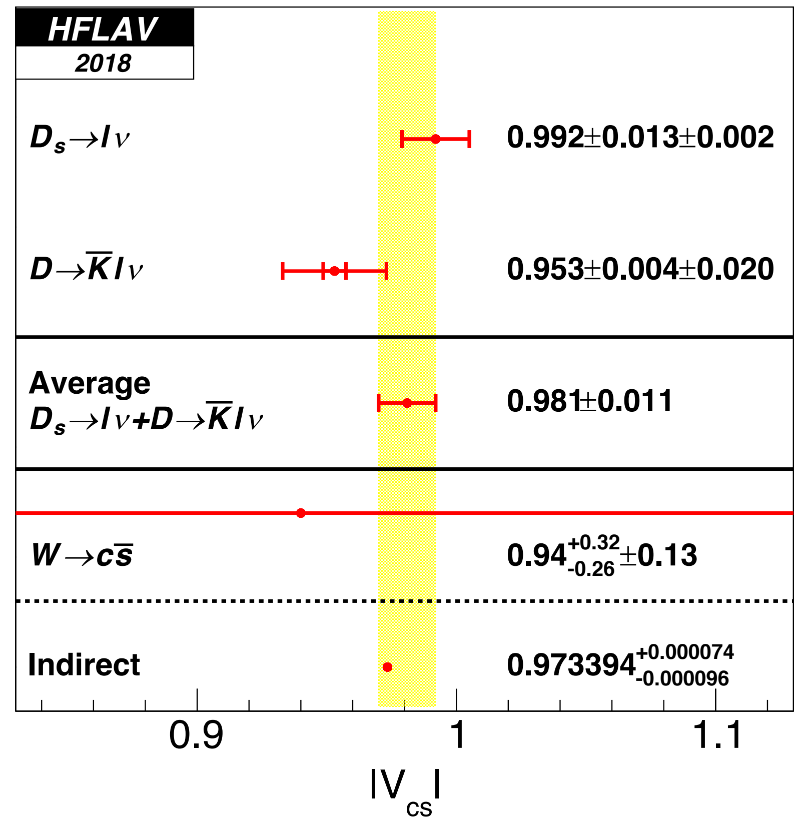
f_{Ds} = (254.5 ± 3.2) MeV

f_D = (205.4 ± 4.8) MeV

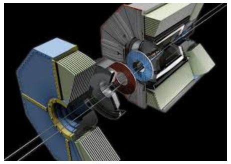
Summary of $|V_{cd}|$, $|V_{cs}|$

Dominated by BESIII [Ablikim et al., PRD 92, 072012 (2015); Ablikim et al., PRD 96, 012002 (2017)]





- *Leptonic decays have larger statistical errors due to smaller samples, but smaller LQCD errors (i.e., decay constants calculations are more accurate than those for form factors)*
- *Combining errors: best knowledge currently comes from leptonic decays*

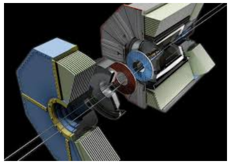


Summary of CKM measurements

- $|V_{cb}|$ is measured via exclusive $B \to D^* \ell \nu$ and $B \to D \ell \nu$ decays. Uncertainty arises from form factors, of which there are two common choices: CLN and BGL
- $|V_{cb}|$ is measured via inclusive $B \to X_c \ell \nu$ decays and using OPE. Uncertainty arises from matrix elements of local operators. These are determined by fitting moment distributions. Two theory schemes available: kinetic scheme and 1S scheme.
- The measurements differ: inclusive $|V_{cb}|$ is higher than exclusive by 2.2–3.3σ

- $|V_{ub}|$ is measured via exclusive $B \to \pi \ell \nu$ decays. Uncertainty arises from form factors, of which there is one common choice: BCL
- $|V_{cb}|$ is measured via inclusive $B \to X_u \ell \nu$ decays. Many cuts are made to reduce huge $B \to X_c \ell \nu$ background, and this makes it challenging to theoretically predict the rate. Five theory schemes available: BLNP, DGE, GGOU, ADFR, and BLL.
- The measurements differ: inclusive $|V_{ub}|$ is higher than exclusive by 2.6–2.8σ

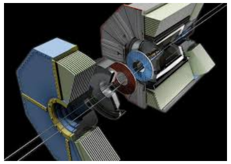
- $|V_{cs}|$ is measured via exclusive $D_s^+ \to \ell^+ \nu$ and $D \to K \ell \nu$ decays. Uncertainty arises from decay constants and form factors, respectively. Results agree. $D \to K \ell \nu$ has much higher statistics, but theory error from form factors is larger, so overall precision is worse.
- $|V_{cd}|$ is measured via exclusive $D^+ \to \ell^+ \nu$ and $D \to \pi \ell \nu$ decays. Uncertainty arises from decay constants and form factors, respectively. Results agree. $D \to \pi \ell \nu$ has much higher statistics, but theory error from form factors is larger, so overall precision is worse.

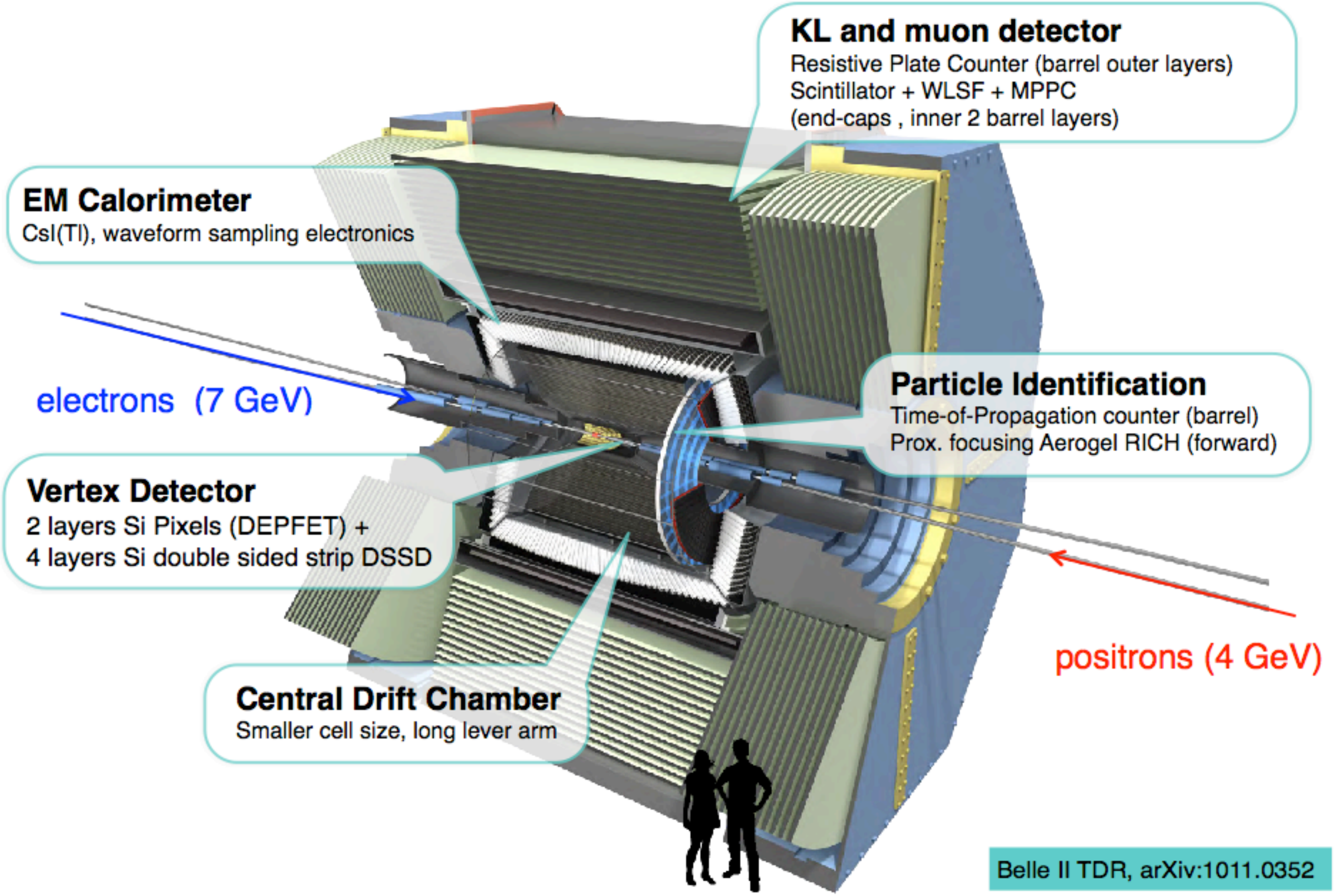


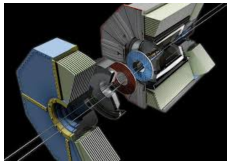
Extra

Extra Slides

The Belle II Detector







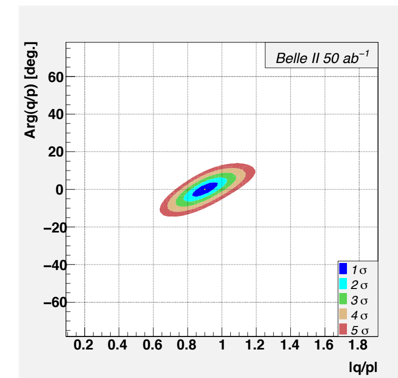
Belle II physics

The Belle II Physics Book
E. Kou et al., PTEP 2019, 123C01 (2019)
[arXiv:1808.10567]

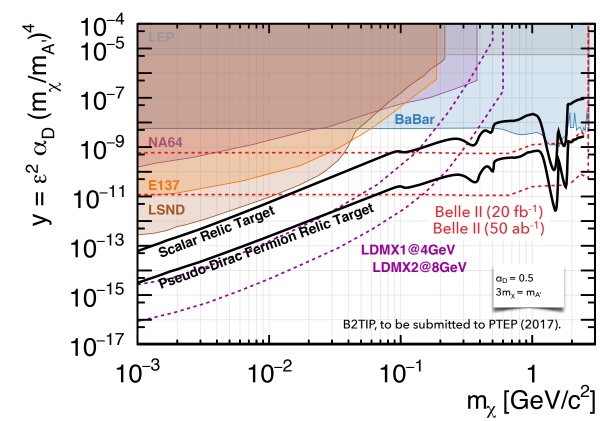
B physics:

Observables	Expected exp. uncertainty	Facility (2025)
UT angles & sides		
ϕ_1 [°]	0.4	Belle II
ϕ_2 [°]	1.0	Belle II
ϕ_3 [°]	1.0	LHCb/Belle II
$\lvert V_{cb}\rvert$ incl.	1%	Belle II
$\lvert V_{cb}\rvert$ excl.	1.5%	Belle II
$\lvert V_{ub}\rvert$ incl.	3%	Belle II
$\lvert V_{ub}\rvert$ excl.	2%	Belle II/LHCb
CPV		
$S(B \to \phi K^0)$	0.02	Belle II
$S(B \to \eta' K^0)$	0.01	Belle II
$\mathcal{A}(B \to K^0\pi^0)[10^{-2}]$	4	Belle II
$\mathcal{A}(B \to K^+\pi^-)$ $[10^{-2}]$	0.20	LHCb/Belle II
(Semi-)leptonic		
$\mathcal{B}(B \to \tau\nu)$ $[10^{-6}]$	3%	Belle II
$\mathcal{B}(B \to \mu\nu)$ $[10^{-6}]$	7%	Belle II
$R(B \to D\tau\nu)$	3%	Belle II
$R(B \to D^*\tau\nu)$	2%	Belle II/LHCb
Radiative & EW Penguins		
$\mathcal{B}(B \to X_s\gamma)$	4%	Belle II
$A_{CP}(B \to X_{s,d}\gamma)$ $[10^{-2}]$	0.005	Belle II
$S(B \to K_S^0\pi^0\gamma)$	0.03	Belle II
$S(B \to \rho\gamma)$	0.07	Belle II
$\mathcal{B}(B_s \to \gamma\gamma)$ $[10^{-6}]$	0.3	Belle II
$\mathcal{B}(B \to K^*\nu\overline{\nu})$ $[10^{-6}]$	15%	Belle II
$\mathcal{B}(B \to K\nu\overline{\nu})$ $[10^{-6}]$	20%	Belle II
$R(B \to K^*\ell\ell)$	0.03	Belle II/LHCb

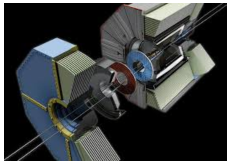
Charm



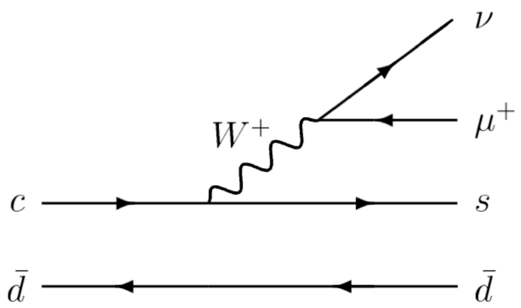
Dark Photon/Sector



Tau physics
Quarkonium-like
B_s physics at Υ(5S)



Semileptonic Decays



$D \rightarrow (K,\pi)\,\ell^+\nu$:

$$\boxed{\frac{d\Gamma}{dq^2} = \frac{G_F^2\,p_h^3}{24\pi^3}\,|V_{cs,cd}|^2\,\left|f_+(q^2)\right|^2}$$

⇒ *Take $f_+(q^2)$ form factor from theory, determine $|V_{cs}|$ or $|V_{cd}|$*

Simple pole:
$$f_+(q^2) = \frac{f_+(0)}{(1-q^2/m^2_{\rm pole})}$$

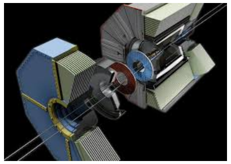
Modified pole model:
$$f_+(q^2) = \frac{f_+(0)}{(1-q^2/m^2_{\rm pole})(1-\alpha_p q^2/m^2_{\rm pole})}$$

z expansion:
$$t_\pm = (m_D \pm m_P)^2 \qquad t_0 = t_+(1-\sqrt{1-t_-/t_+})$$

$$z(q^2,t_0) = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$$

$$f_+(q^2) = \frac{1}{P(q^2)\phi(q^2,t_0)}\sum_{k=0}^{\infty} a_k z^k$$

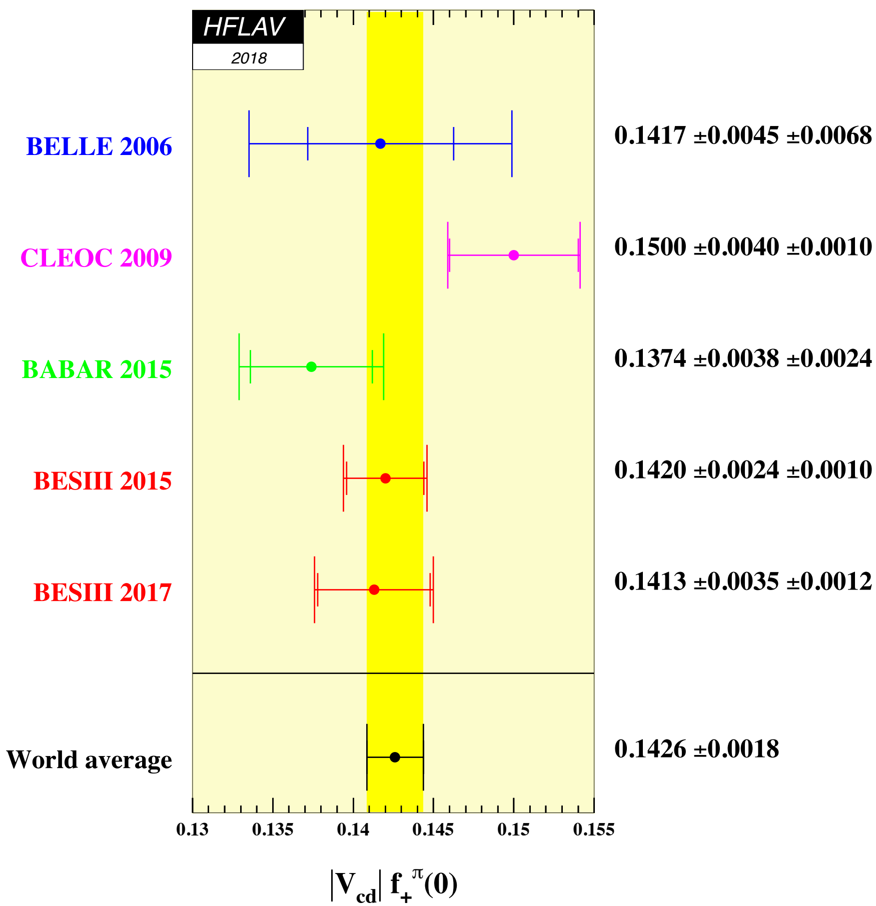
$$a_1/a_0 \equiv r_1 \qquad a_2/a_0 \equiv r_2$$

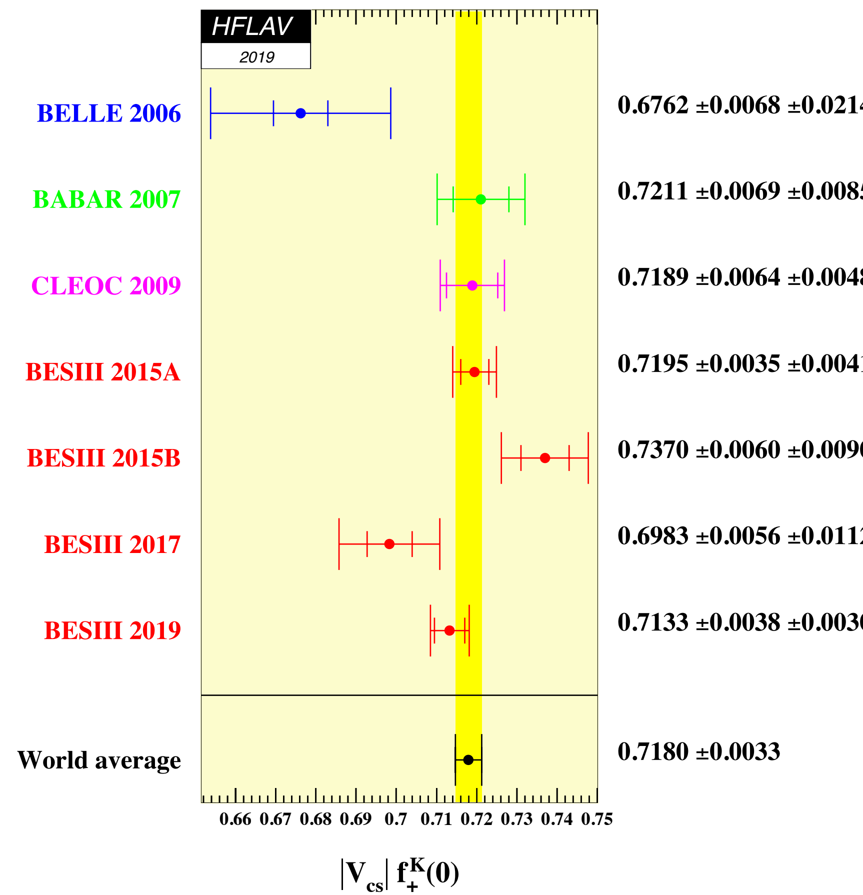


Semileptonic Decays

$D \to (K,\pi)\,\ell^+\nu$:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\, p_h^3}{24\pi^3}\,|V_{cs,cd}|^2\,|f_+(q^2)|^2$$





Using recent LQCD results:
ETM, PRD 96, 054514 (2017)
HPQCD, PRD 82, 114506; 84 114505 (2011)
FNAL/MILC arXiv:1901.08989

$f^K_+(0) = 0.760 \pm 0.011$

$f^\pi_+(0) = 0.634 \pm 0.015$

gives:

$|V_{cs}| = 0.943 \pm 0.004\ (exp) \pm 0.014\ (LQCD)$

$|V_{cd}| = 0.2249 \pm 0.0028\ (exp) \pm 0.0055\ (LQCD)$