

Flavour Physics

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1. Elementary constituents & fundamental interactions
2. Flavour-changing phenomena
3. Meson mixing & CP violation

Flavour Structure of the Standard Model

$$\begin{pmatrix} u & \nu_e \\ d & e^- \end{pmatrix}, \begin{pmatrix} c & \nu_\mu \\ s & \mu^- \end{pmatrix}, \begin{pmatrix} t & \nu_\tau \\ b & \tau^- \end{pmatrix}$$



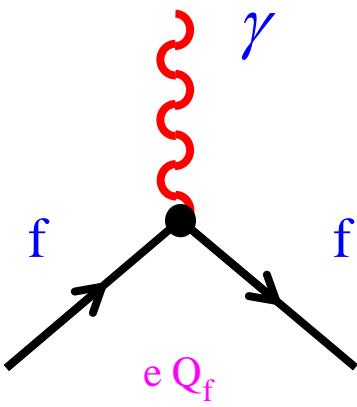
- Pattern of masses
- Flavour Mixing
- $c\bar{p}$



Related to SSB
Scalar Sector (Higgs)

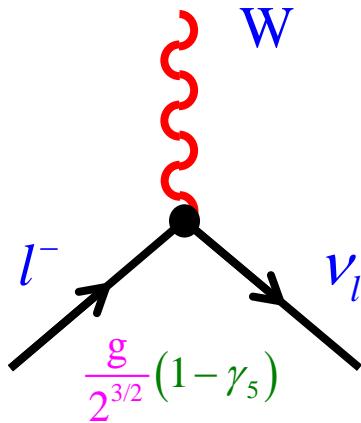
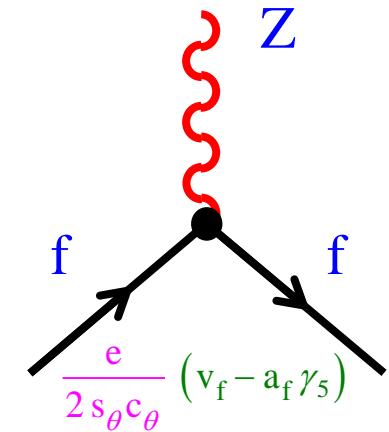
- | | |
|------------------------------|--|
| • Kaon Factories : u , d , s | • LHC : t , b , c |
| • $\tau c F$: c , τ | • LC, FCC : t , b , c , τ |
| • BF: b , c , τ | • νF : ν_e , ν_μ , ν_τ |

Universality: Family–Independent Couplings



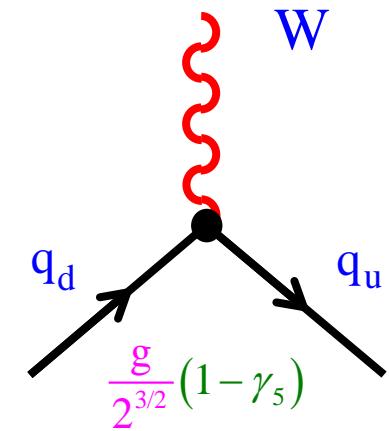
NEUTRAL
CURRENTS

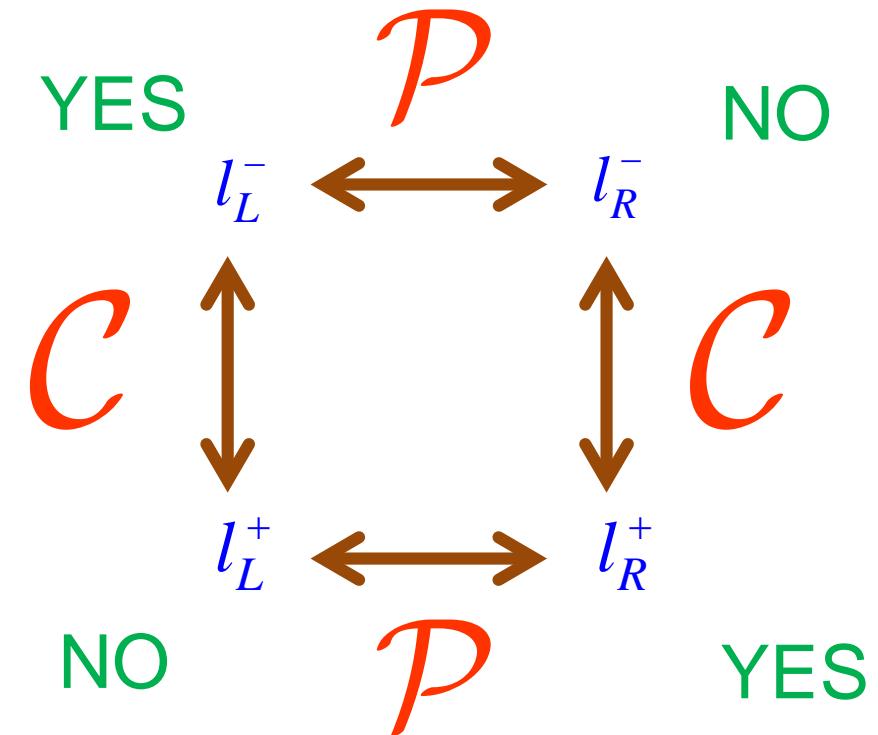
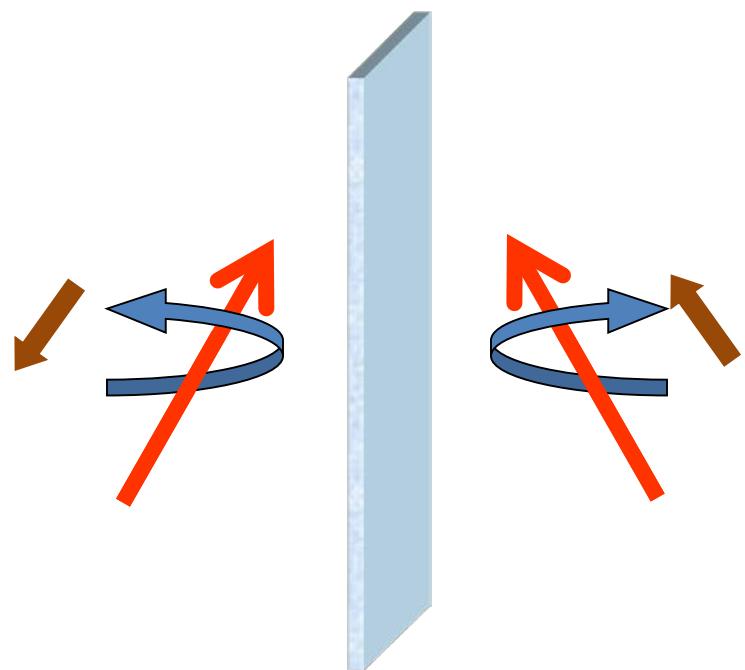
Flavour Conserving



CHARGED
CURRENTS

Flavour Changing
Left Handed





\mathcal{P} and \mathcal{C} in Weak Interactions
 CP still a good symmetry (1 family)

FERMION MASSES

Scalar – Fermion Couplings allowed by Gauge Symmetry

$$\mathcal{L}_Y = - (\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{v}_l, \bar{l})_L c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

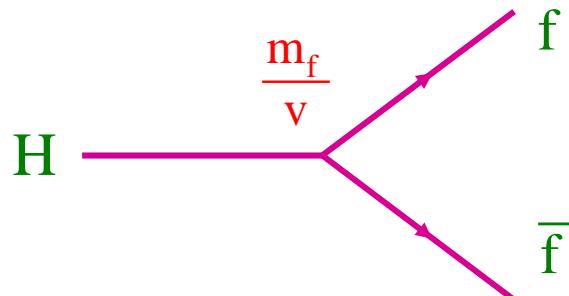
$$\phi^{(0)} \rightarrow \frac{1}{\sqrt{2}} (v + H) \quad , \quad \phi^{(+)} \rightarrow 0$$

↓ SSB

$$\mathcal{L}_Y = - \left(1 + \frac{H}{v} \right) \left\{ m_{q_d} \bar{q}_d q_d + m_{q_u} \bar{q}_u q_u + m_l \bar{l} l \right\}$$

Fermion Masses are
New Free Parameters

$$[m_{q_d}, m_{q_u}, m_l] = [c^{(d)}, c^{(u)}, c^{(l)}] \frac{v}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{v}$

FERMION GENERATIONS

$N_G = 3$ Identical Copies

Masses are the only difference

$$Q = 0$$

$$Q = -1$$

$$\begin{pmatrix} v'_j & u'_j \\ l'_j & d'_j \end{pmatrix}$$

$$Q = +2/3$$

$$Q = -1/3$$

$$(j = 1, \dots, N_G)$$

WHY ?

$$\mathcal{L}_Y = - \sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j \right)_L \left[c_{jk}^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c_{jk}^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\bar{v}'_j, \bar{l}'_j \right)_L c_{jk}^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

↓ SSB

$$\mathcal{L}_Y = - \left(1 + \frac{H}{v} \right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$[\mathbf{M}'_d, \mathbf{M}'_u, \mathbf{M}'_l]_{jk} = [c_{jk}^{(d)}, c_{jk}^{(u)}, c_{jk}^{(l)}] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$

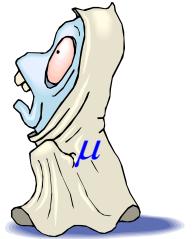
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathbf{S}_u \cdot \mathbf{U}_u$$

$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_f = \mathbf{H}_f^\dagger$$

$$\mathbf{U}_f \cdot \mathbf{U}_f^\dagger = \mathbf{U}_f^\dagger \cdot \mathbf{U}_f = 1$$

$$\mathbf{S}_f \cdot \mathbf{S}_f^\dagger = \mathbf{S}_f^\dagger \cdot \mathbf{S}_f = 1$$



$$\mathcal{L}_Y = - \left(1 + \frac{H}{v} \right) \left\{ \bar{d} \cdot \mathcal{M}_d \cdot d + \bar{u} \cdot \mathcal{M}_u \cdot u + \bar{l} \cdot \mathcal{M}_l \cdot l \right\}$$

$$\mathcal{M}_u = \text{diag}(m_u, m_c, m_t) ; \quad \mathcal{M}_d = \text{diag}(m_d, m_s, m_b) ; \quad \mathcal{M}_l = \text{diag}(m_e, m_\mu, m_\tau)$$

$$d_L \equiv \mathbf{S}_d \cdot d'_L \quad ; \quad u_L \equiv \mathbf{S}_u \cdot u'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$

$$d_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot d'_R \quad ; \quad u_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot u'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates
 \neq
 Weak Eigenstates

$$\bar{f}'_L f'_L = \bar{f}_L f_L \quad ; \quad \bar{f}'_R f'_R = \bar{f}_R f_R \quad \longrightarrow$$

$$\mathcal{L}'_{NC} = \mathcal{L}_{NC}$$

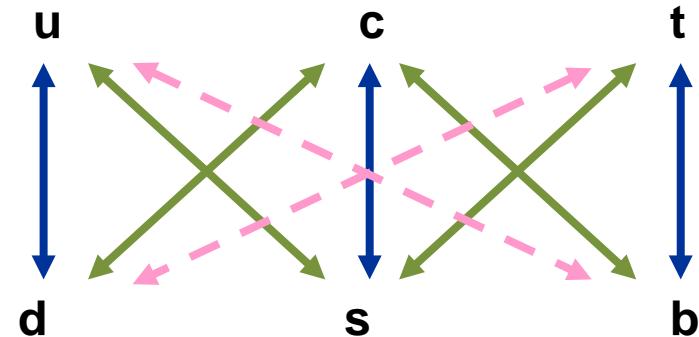
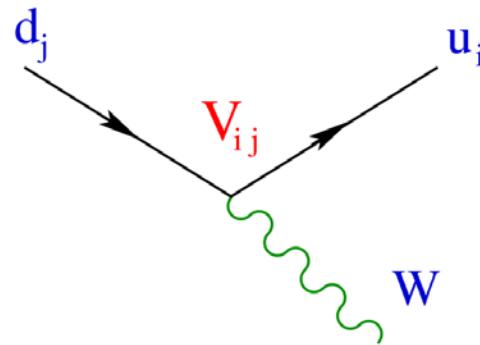
$$\bar{u}'_L d'_L = \bar{u}_L \cdot \mathbf{V} \cdot d_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \longrightarrow$$

$$\mathcal{L}'_{CC} \neq \mathcal{L}_{CC}$$

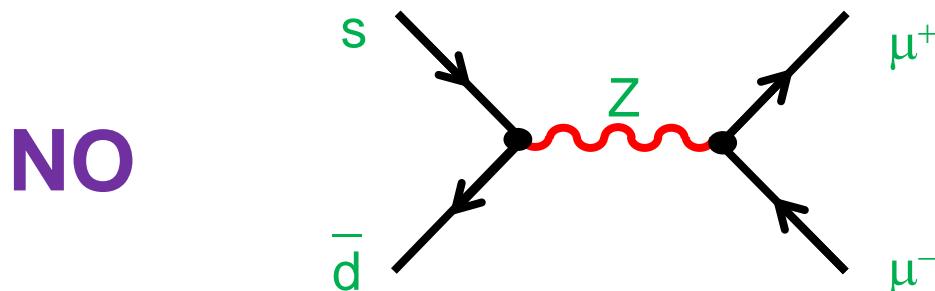
QUARK MIXING

Flavour Changing Charged Currents

$$\mathcal{L}_{\text{CC}} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) V_{ij} d_j + \sum_l \bar{v}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$

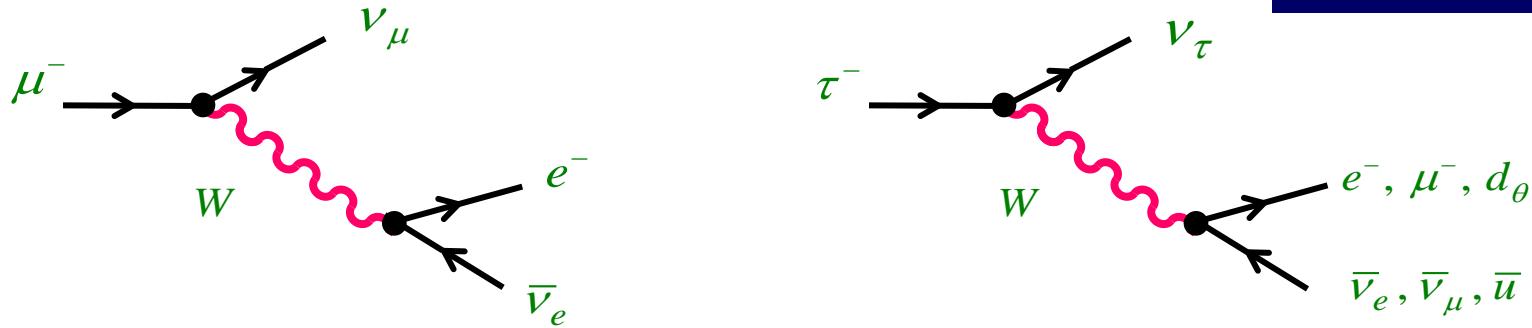


Flavour Conserving Neutral Currents (GIM)



LHCb, 2001.10354
 $\text{Br}(K_S \rightarrow \mu^+ \mu^-) < 2.1 \times 10^{-10}$
 (90% CL)

Weak Decays

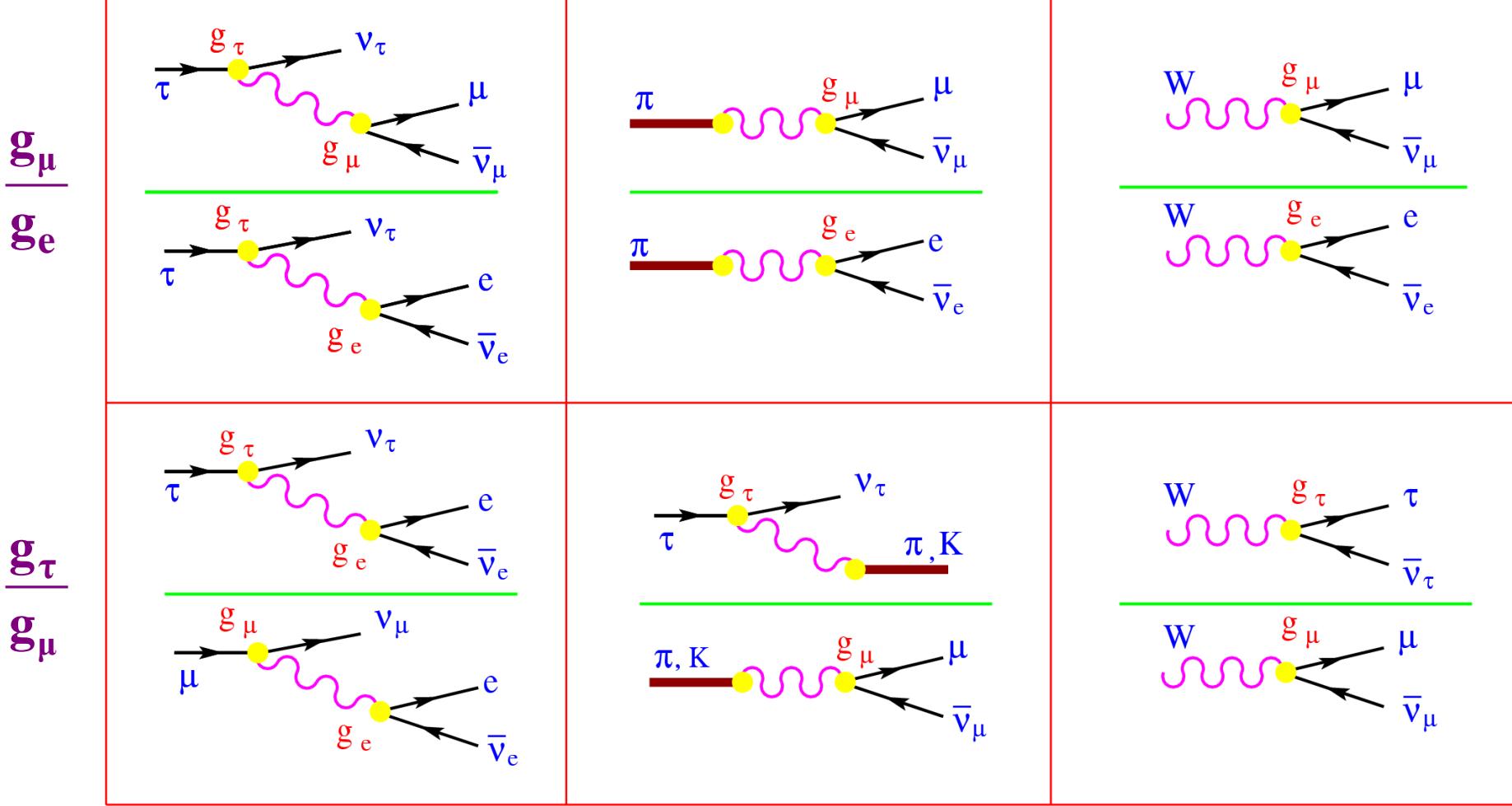


$$T(l \rightarrow \nu_l l' \bar{\nu}_{l'}) \sim \frac{g^2}{M_W^2 - q^2} \quad \xrightarrow{q^2 \ll M_W^2} \quad \frac{g^2}{M_W^2} = 4\sqrt{2} G_F$$

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192 \pi^3} f(m_e^2/m_\mu^2) r_{EW} \quad \longrightarrow \quad G_F = (1.166\,378\,7 \pm 0.000\,000\,6) \times 10^{-5} \text{ GeV}^{-2}$$

$$r_{EW} = \left[1 + \frac{\alpha(m_\mu)}{2\pi} \left(\frac{25}{4} - \pi^2 \right) + C_2 \frac{\alpha(m_\mu)^2}{\pi^2} \right] = 0.9958 \quad ; \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

LEPTON UNIVERSALITY



CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:2012.07099

$$|g_\mu / g_e|$$

$B_{\tau \rightarrow \mu} / B_{\tau \rightarrow e}$	1.0017 ± 0.0016
$B_{\pi \rightarrow \mu} / B_{\pi \rightarrow e}$	1.0010 ± 0.0009
$B_{K \rightarrow \mu} / B_{K \rightarrow e}$	0.9978 ± 0.0018
$B_{K \rightarrow \pi \mu} / B_{K \rightarrow \pi e}$	1.0010 ± 0.0025
$B_{W \rightarrow \mu} / B_{W \rightarrow e}$	0.998 ± 0.004

$$|g_\tau / g_\mu|$$

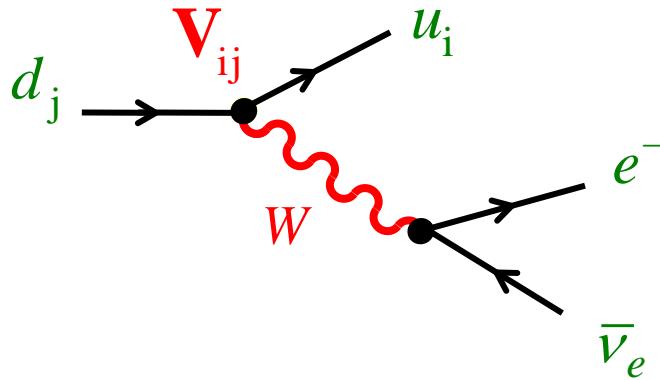
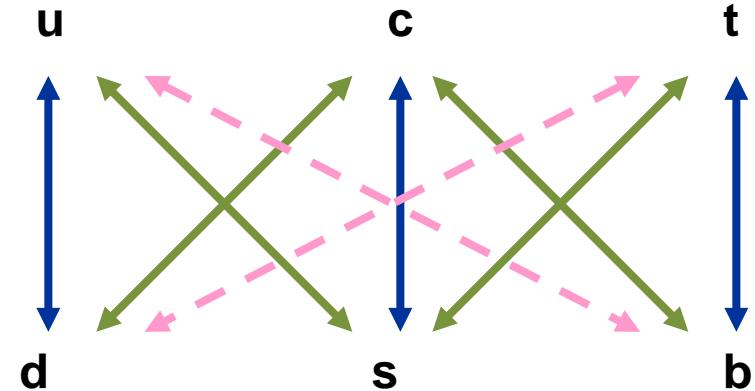
$B_{\tau \rightarrow e} \tau_\mu / \tau_\tau$	1.0011 ± 0.0014
$\Gamma_{\tau \rightarrow \pi} / \Gamma_{\pi \rightarrow \mu}$	0.9965 ± 0.0026
$\Gamma_{\tau \rightarrow K} / \Gamma_{K \rightarrow \mu}$	0.986 ± 0.007
$B_{W \rightarrow \tau} / B_{W \rightarrow \mu}$	1.004 ± 0.016

$$|g_\tau / g_e|$$

$B_{\tau \rightarrow \mu} \tau_\mu / \tau_\tau$	1.0028 ± 0.0015
$B_{W \rightarrow \tau} / B_{W \rightarrow e}$	1.022 ± 0.012

0.997 ± 0.011 (CMS preliminary)

Flavour Changing Charged Currents



$$\Gamma(d_j \rightarrow u_i e^- \bar{\nu}_e) \propto |V_{ij}|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} Determinations

PDG 2020

CKM entry	Value	Source
$ V_{ud} $	0.97370 ± 0.00014	Nuclear β decay
$ V_{us} $	0.2245 ± 0.0008	$K \rightarrow (\pi) \ell \nu$
$ V_{cd} $	0.221 ± 0.004	$D \rightarrow (\pi) \ell \nu, \bar{v} d \rightarrow c X$
$ V_{cs} $	0.987 ± 0.011	$D \rightarrow K \ell \nu, D_s \rightarrow \ell \nu$
$ V_{cb} $	0.0410 ± 0.0014	$b \rightarrow c \ell \nu, B \rightarrow D^{(*)} \ell \nu$
$ V_{ub} $	0.00382 ± 0.00024	$b \rightarrow u \ell \nu, B \rightarrow \pi \ell \nu$
$ V_{tb} $	1.013 ± 0.030	$p\bar{p}, pp \rightarrow tb + X$
$ V_{tb} / \sqrt{\sum_q V_{tq} ^2}$	$> 0.975 \text{ (95% CL)}$	$t \rightarrow bW / t \rightarrow qW$
$ V_{td} $	0.0080 ± 0.0003	$B_d^0 - \bar{B}_d^0$ mixing
$ V_{ts} $	0.0388 ± 0.0011	$B_s^0 - \bar{B}_s^0$ mixing

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1.028 \pm 0.061$$

$$\sum_j (|V_{uj}|^2 + |V_{cj}|^2) = 2.002 \pm 0.027 \quad (\text{LEP})$$

PDG 2018

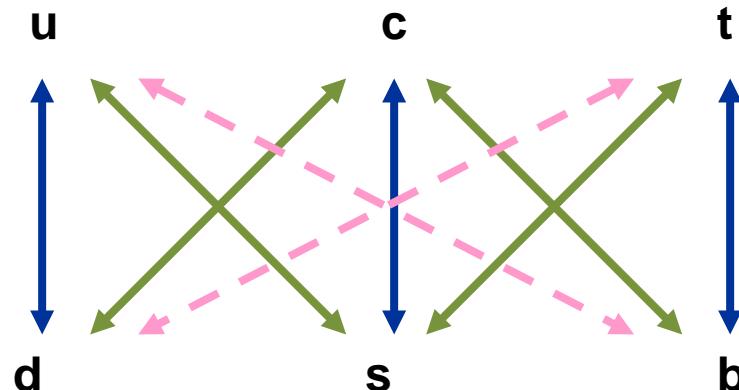
0.97420 (21)

0.2243 (5)

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \approx \sin \theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$$



QUARK MIXING MATRIX

- Unitary $N_G \times N_G$ Matrix: N_G^2 parameters

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

$\frac{1}{2} N_G (N_G - 1)$ moduli, $\frac{1}{2} N_G (N_G + 1)$ phases

- $2 N_G - 1$ arbitrary phases: $\bar{u}_i \mathbf{V}_{ij} d_j$

$$u_i \rightarrow e^{i\phi_i} u_i ; d_j \rightarrow e^{i\theta_j} d_j \quad \longrightarrow \quad \mathbf{V}_{ij} \rightarrow e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$



\mathbf{V}_{ij} Physical Parameters:

$$\frac{1}{2} N_G (N_G - 1) \text{ moduli} ; \quad \frac{1}{2} (N_G - 1) (N_G - 2) \text{ phases}$$

- $N_f = 2$: 1 angle, 0 phases (Cabibbo)

$$\mathbf{V} = \begin{bmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{bmatrix} \quad \xrightarrow{\text{Red Arrow}} \quad \text{No } \mathcal{CP}$$

- $N_f = 3$: 3 angles, 1 phase (CKM) $c_{ij} \equiv \cos \theta_{ij}$; $s_{ij} \equiv \sin \theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

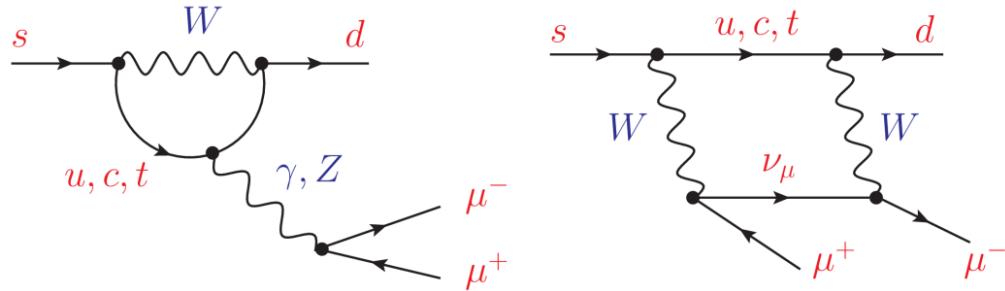
$$\lambda \approx \sin \theta_C \approx 0.225 ; A \approx 0.81 ; \sqrt{\rho^2 + \eta^2} \approx 0.4$$

$$\delta_{13} \neq 0 \quad (\eta \neq 0)$$



\mathcal{CP}

GIM Mechanism



$$\mathcal{M} \propto \sum_{i=u,c,t} V_{is} V_{id}^* F(m_i^2/M_W^2)$$

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0 \quad \rightarrow \quad \mathcal{M} = 0 \quad \text{if} \quad m_u = m_c = m_t$$

$$\tilde{F}(x) \equiv F(x) - F(0)$$



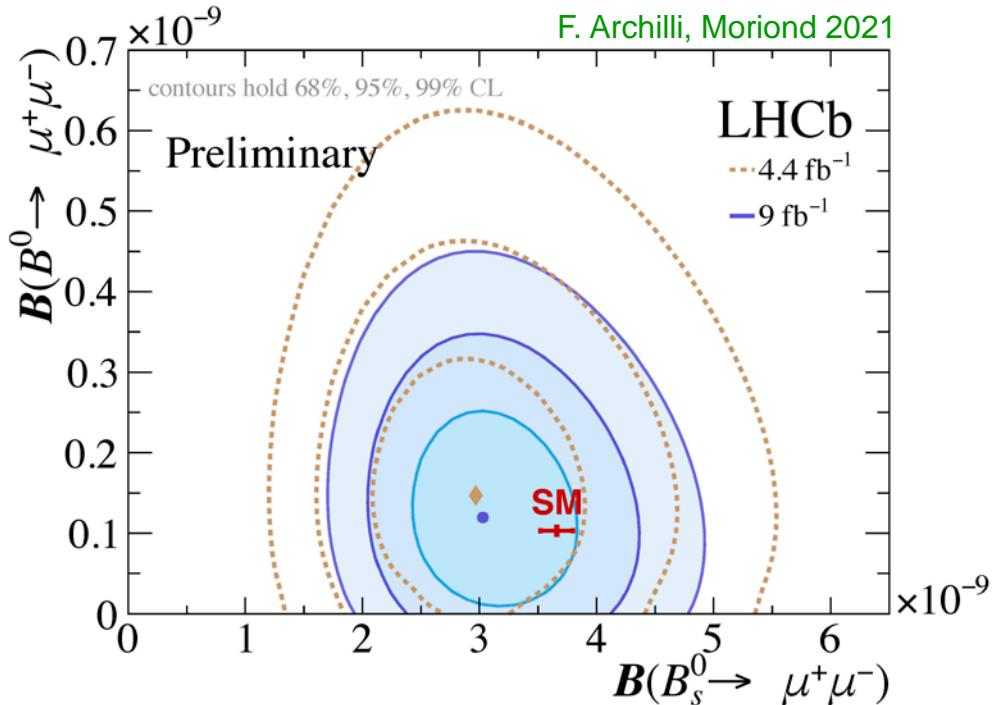
$$\begin{aligned} \mathcal{M} &\propto V_{cs} V_{cd}^* \tilde{F}(m_c^2/M_W^2) + V_{ts} V_{td}^* \tilde{F}(m_t^2/M_W^2) \\ &\approx -\lambda \tilde{F}(m_c^2/M_W^2) - \lambda^5 A^2 (1 - \rho + i\eta) \tilde{F}(m_t^2/M_W^2) \end{aligned}$$

- **Top contribution dominates. Strong suppression:** $\mathcal{M} \propto \frac{g^4}{16\pi^2} \left[\lambda^5 A^2 \frac{m_t^2}{M_W^2}, \lambda \frac{m_c^2}{M_W^2} \right]$
- **~~CP~~ effects fully governed by top contribution** $\left[\text{Im}(V_{cs} V_{cd}^*) = -\text{Im}(V_{ts} V_{td}^*) \right]$

Rare Decays

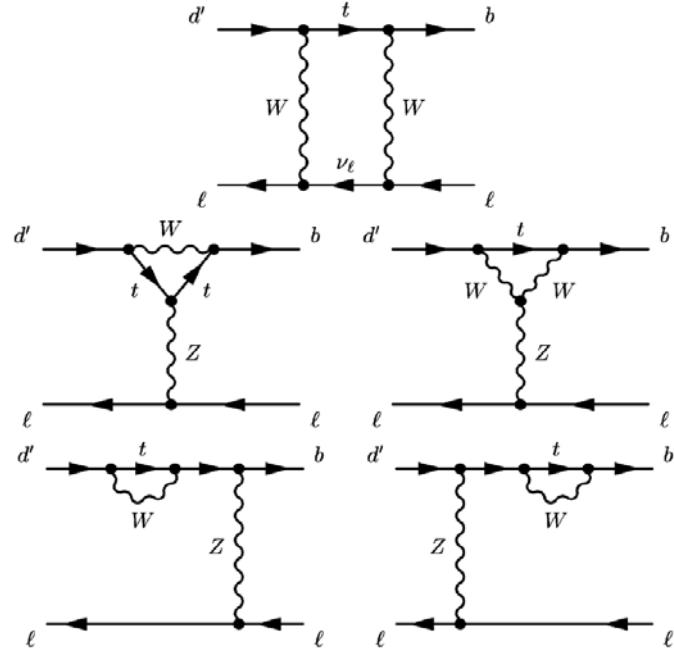
Loop & CKM suppression Sensitivity to New Physics

$B_{s,d} \rightarrow \mu^+ \mu^-$



$$\bar{B}(B_s^0 \rightarrow \mu^+ \mu^-)_{\text{exp}} = (2.9 \pm 0.4) \cdot 10^{-9}$$

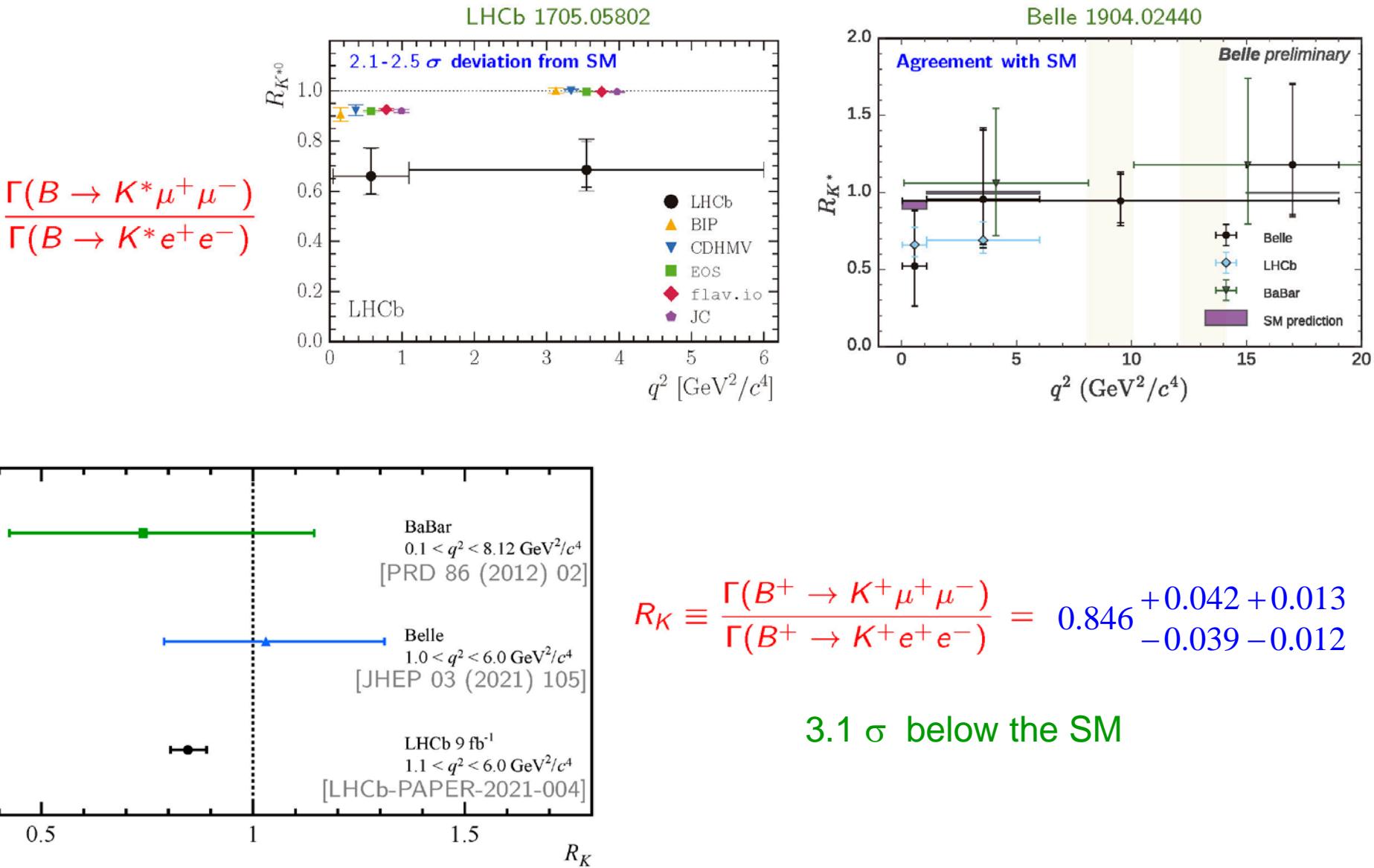
$$\bar{B}(B_d^0 \rightarrow \mu^+ \mu^-)_{\text{exp}} = (0.5^{+1.7}_{-1.5}) \cdot 10^{-10}$$



$$W^\pm \leftrightarrow H^\pm, \quad Z \leftrightarrow H^0, A^0$$

Sensitive to (pseudo)
scalar contributions

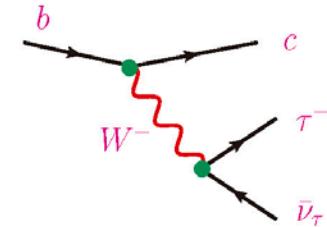
Violations of Lepton Flavour Universality



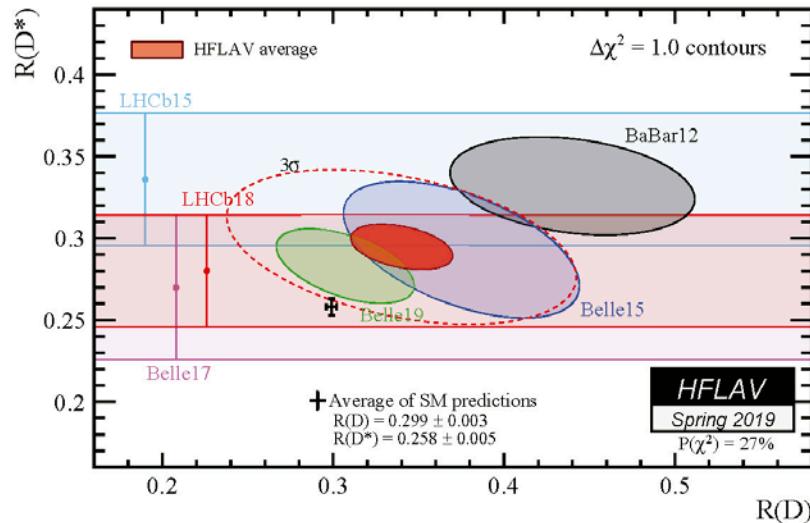
Flavour Anomaly (charged currents)

$$\mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu}_\ell)}$$

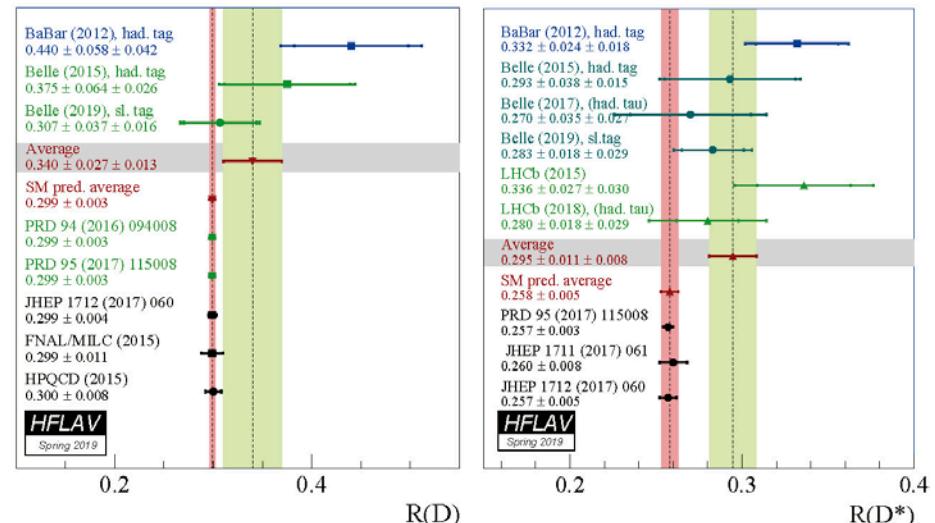
Tree-level process



3.08σ discrepancy



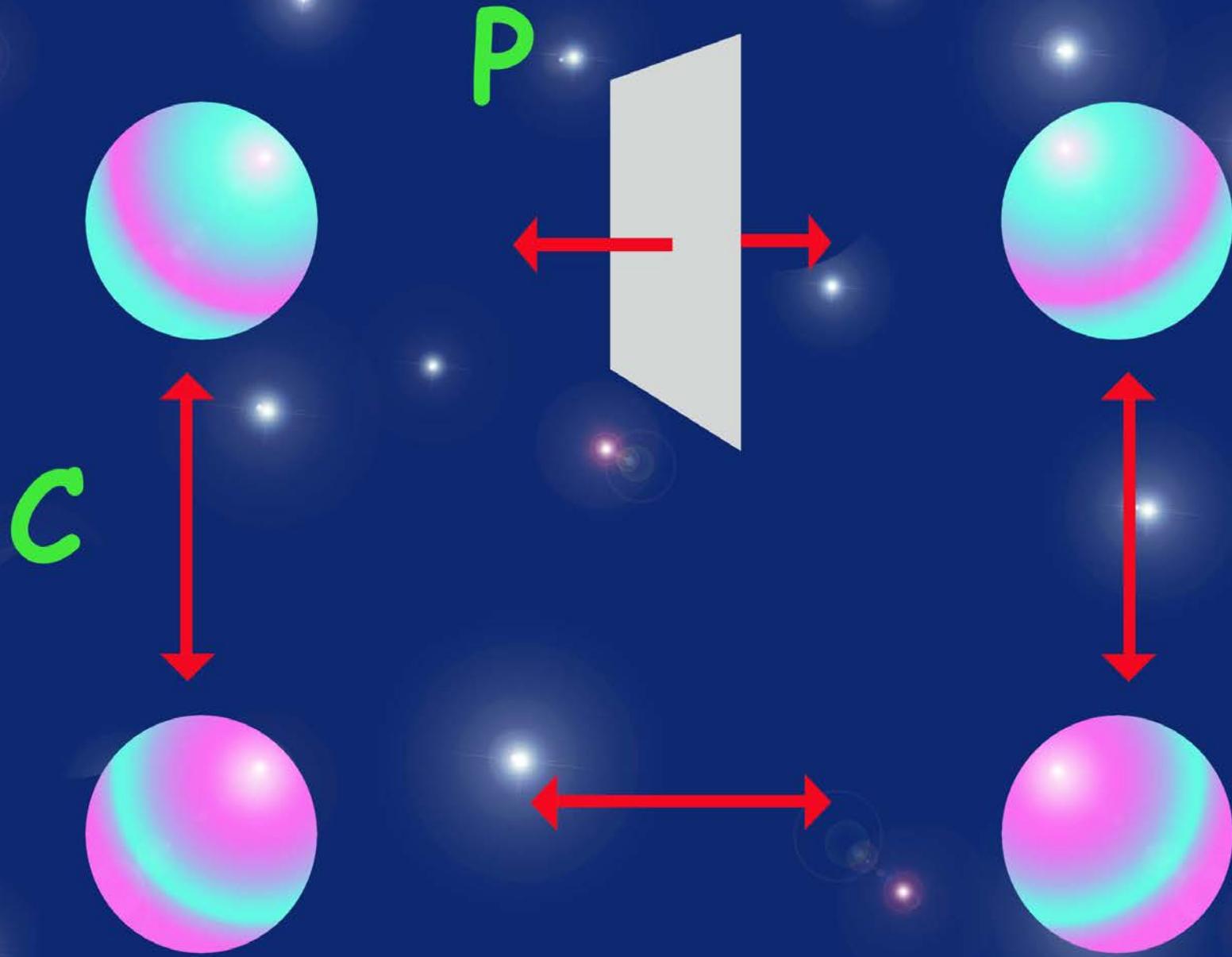
(3.2σ with more recent predictions)



LHCb, 1711.05623: $\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \rightarrow J/\psi \tau^- \bar{\nu}_\tau)}{\mathcal{B}(B_c \rightarrow J/\psi \mu^- \bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18 \quad (1.7\sigma) \quad \mathcal{R}_{J/\psi}^{\text{SM}} \approx 0.26 - 0.28$

Belle, 1903.03102: $F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \quad (1.6\sigma) \quad F_{L,\text{SM}}^{D^*} = 0.455 \pm 0.003$

Belle, 1612.00529: $\mathcal{P}_\tau^{D^*} = -0.38 \pm 0.51^{+0.21}_{-0.16} \quad \mathcal{P}_{\tau,\text{SM}}^{D^*} = -0.499 \pm 0.003$



- \mathcal{C}, \mathcal{P} : Violated maximally in weak interactions
- \mathcal{CP} : Symmetry of nearly all observed phenomena
- Slight ($\sim 0.2\%$) $\cancel{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\cancel{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter–Antimatter Asymmetry in our Universe
 \longrightarrow Baryogenesis
- Small $\cancel{\mathcal{CP}}$ in D^0 decays (LHCb, 2019)

\mathcal{CPT} Theorem: $\cancel{\mathcal{CP}} \leftrightarrow \cancel{T}$

Thus, $\cancel{\mathcal{CP}}$ requires:

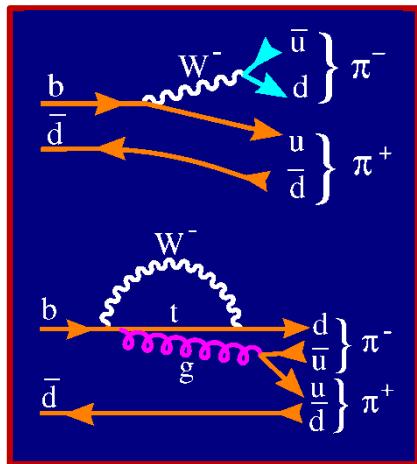
- Complex Phases
- Interferences

DIRECT

\mathcal{CP}

$$|\mathbf{T}(P \rightarrow f)| \neq |\mathbf{T}(\bar{P} \rightarrow \bar{f})|$$

\bar{B}_d^0



$$\mathbf{T}(P \rightarrow f) = T_1 e^{i\phi_1} e^{i\delta_1} + T_2 e^{i\phi_2} e^{i\delta_2}$$

\mathcal{CP}

$$\mathbf{T}(\bar{P} \rightarrow \bar{f}) = T_1 e^{-i\phi_1} e^{i\delta_1} + T_2 e^{-i\phi_2} e^{i\delta_2}$$

$$A_{P \rightarrow f}^{\text{CP}} \equiv \frac{\Gamma(P \rightarrow f) - \Gamma(\bar{P} \rightarrow \bar{f})}{\Gamma(P \rightarrow f) + \Gamma(\bar{P} \rightarrow \bar{f})} = \frac{-2 T_1 T_2 \sin(\phi_2 - \phi_1) \sin(\delta_2 - \delta_1)}{T_1^2 + T_2^2 + 2 T_1 T_2 \cos(\phi_2 - \phi_1) \cos(\delta_2 - \delta_1)}$$

One needs:

- **2 Interfering Amplitudes**
- **2 Different Weak Phases**
- **2 Different FSI Phases**

$$[\sin(\phi_2 - \phi_1) \neq 0]$$

$$[\sin(\delta_2 - \delta_1) \neq 0]$$

$$A_{CP}(B \rightarrow f) \equiv \frac{\text{Br}(\bar{B} \rightarrow \bar{f}) - \text{Br}(B \rightarrow f)}{\text{Br}(\bar{B} \rightarrow \bar{f}) + \text{Br}(B \rightarrow f)}$$

$$A_{CP}(B_d^0 \rightarrow \pi^- K^+) = -0.084 \pm 0.004 \quad (21 \sigma)$$

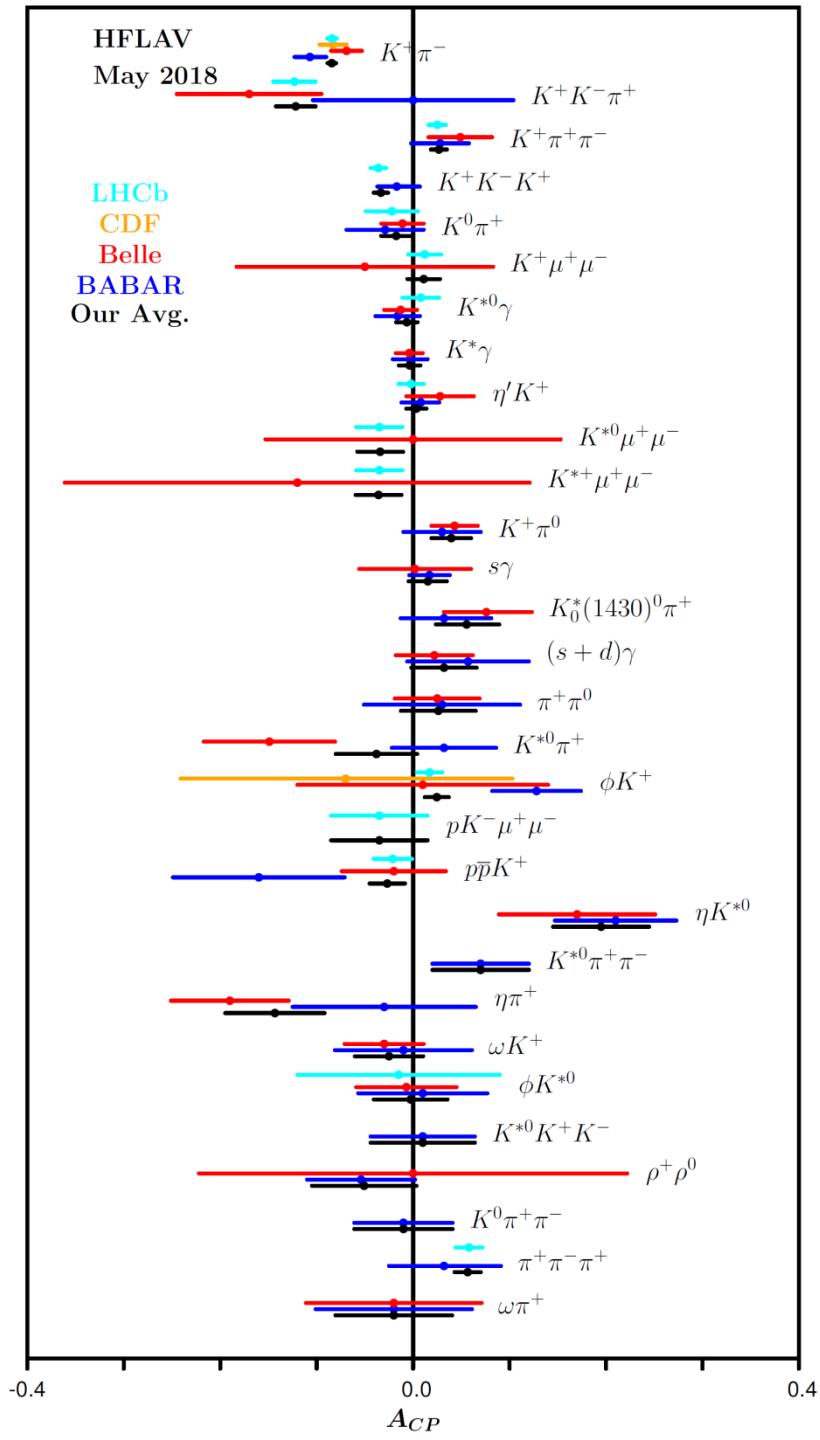
$$A(B_s^0 \rightarrow \pi^- K^+) = -0.213 \pm 0.017 \quad (12.5 \sigma)$$

$$A_{CP}(B^+ \rightarrow K^+ K^- \pi^+) = -0.122 \pm 0.021 \quad (5.8 \sigma)$$

Large & Interesting Signals

Big challenge: Get reliable SM predictions

Severe hadronic uncertainties

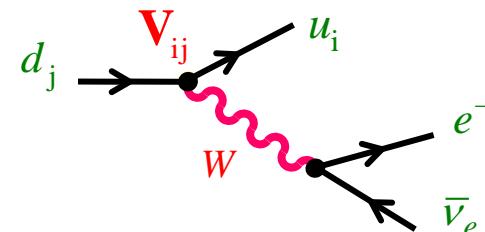


Backup

V_{ij} Determination

$(0^- \rightarrow 0^-)$

$K \rightarrow \pi \ell \nu, D \rightarrow K \ell \nu \dots$



$$\langle P'(k') | \bar{u}_i \gamma^\mu d_j | P(k) \rangle = C_{PP'} \left\{ (k+k')^\mu f_+(q^2) + (k-k')^\mu f_-(q^2) \right\}$$

$$\Gamma(P \rightarrow P' l \nu) = \frac{G_F^2 M_P^5}{192 \pi^3} |V_{ij}|^2 C_{PP'}^2 |f_+(0)|^2 I (1 + \delta_{RC})$$

$$I \approx \int_0^{(M_P - M_{P'})^2} \frac{dq^2}{M_P^8} \lambda^{3/2}(q^2, M_P^2, M_{P'}^2) \left| \frac{f_+(q^2)}{f_+(0)} \right|^2$$

$f_-(q^2)$ suppressed

$(k-k')^\mu \bar{l} \gamma_\mu (1 - \gamma_5) \nu_l \sim m_l$

- Measure the q^2 distribution $\rightarrow I$
- Measure Γ $\rightarrow f_+(0) |V_{ij}|$
- Get a theoretical prediction for $f_+(0)$ $\rightarrow |V_{ij}|$

Theory is always needed: Symmetries

PDG parametrization of the CKM matrix

$$\mathbf{V} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein: $s_{12} \equiv \lambda$, $s_{23} \equiv A\lambda^2$, $s_{13} e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$



$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

Standard Model \cancel{CP} : 3 fermion families needed

 \cancel{CP} 

$$\mathbf{H}(M_u^2) \cdot \mathbf{H}(M_d^2) \cdot \mathbf{J} \neq 0$$

$$\mathbf{H}(M_u^2) \equiv (m_t^2 - m_c^2) (m_c^2 - m_u^2) (m_t^2 - m_u^2)$$

$$\mathbf{H}(M_d^2) \equiv (m_b^2 - m_s^2) (m_s^2 - m_d^2) (m_b^2 - m_d^2)$$

$$\mathbf{J} = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \delta_{13} = |A^2 \lambda^6 \eta| < 10^{-4}$$

- Low-Energy Phenomena
- Small Effects $\sim \mathbf{J}$
- Big Asymmetries \longleftrightarrow Suppressed Decays
- B Decays are an optimal place for \cancel{CP} signals