

Flavour Physics

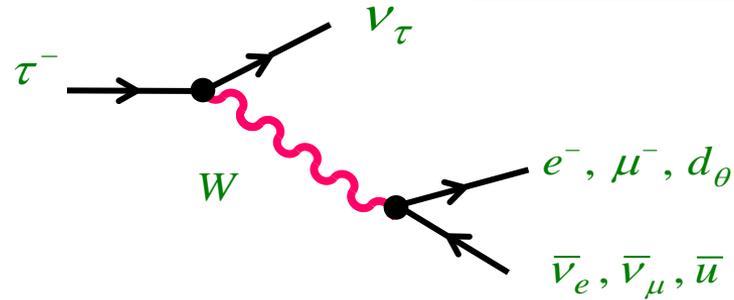
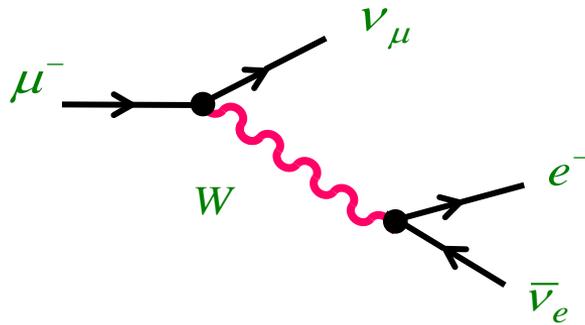
A. Pich

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1. Elementary constituents & fundamental interactions
2. Flavour-changing phenomena
3. Meson mixing & CP violation

Weak Decays



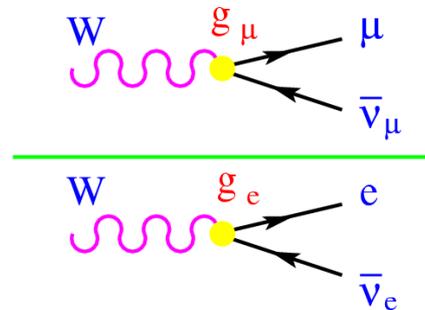
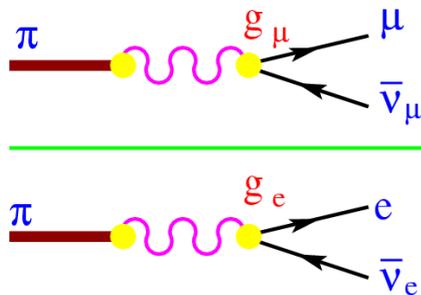
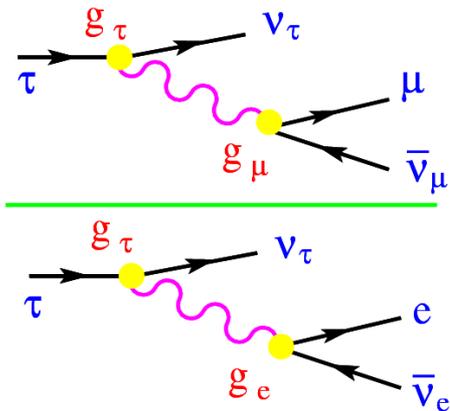
$$T(l \rightarrow \nu_l l' \bar{\nu}_{l'}) \sim \frac{g^2}{M_W^2 - q^2} \xrightarrow{q^2 \ll M_W^2} \frac{g^2}{M_W^2} = 4\sqrt{2} G_F$$

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192 \pi^3} f(m_e^2/m_\mu^2) r_{EW} \quad \longrightarrow \quad G_F = (1.166\,378\,7 \pm 0.000\,000\,6) \times 10^{-5} \text{ GeV}^{-2}$$

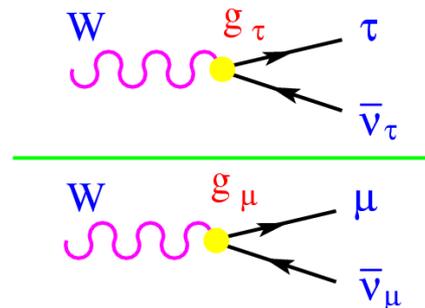
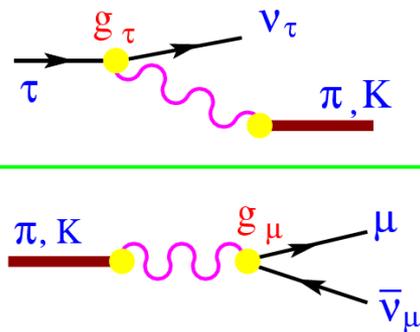
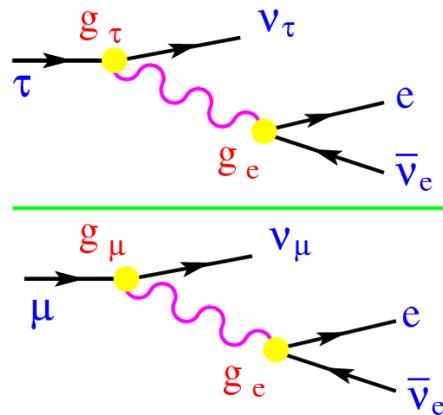
$$r_{EW} = \left[1 + \frac{\alpha(m_\mu)}{2\pi} \left(\frac{25}{4} - \pi^2 \right) + C_2 \frac{\alpha(m_\mu)^2}{\pi^2} \right] = 0.9958 \quad ; \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

LEPTON UNIVERSALITY

$\frac{\sigma_\mu}{\sigma_e}$



$\frac{\sigma_\tau}{\sigma_\mu}$



CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:2012.07099

$$|g_\mu / g_e|$$

$B_{\tau \rightarrow \mu} / B_{\tau \rightarrow e}$	1.0017 ± 0.0016
$B_{\pi \rightarrow \mu} / B_{\pi \rightarrow e}$	1.0010 ± 0.0009
$B_{K \rightarrow \mu} / B_{K \rightarrow e}$	0.9978 ± 0.0018
$B_{K \rightarrow \pi\mu} / B_{K \rightarrow \pi e}$	1.0010 ± 0.0025
$B_{W \rightarrow \mu} / B_{W \rightarrow e}$	0.998 ± 0.004

$$|g_\tau / g_\mu|$$

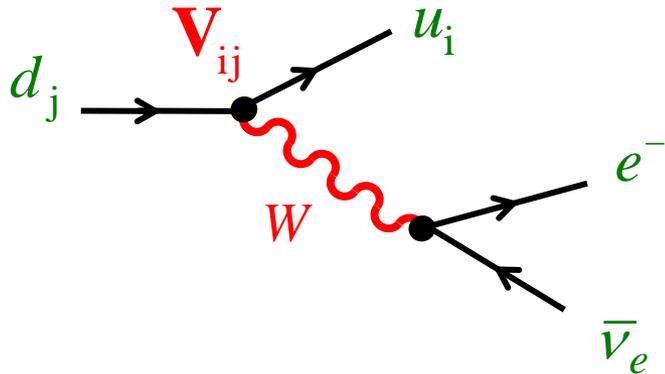
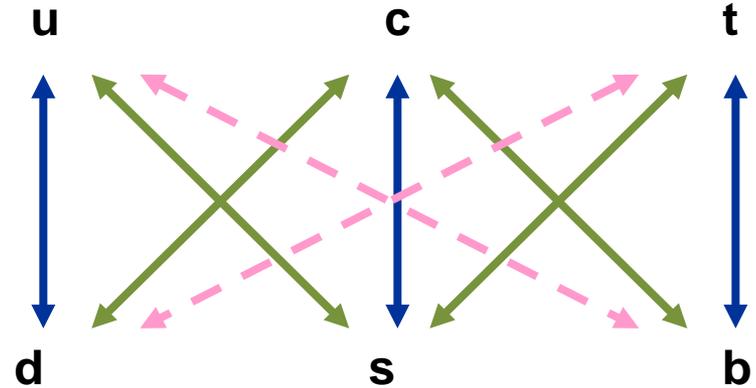
$B_{\tau \rightarrow e} \tau_\mu / \tau_\tau$	1.0011 ± 0.0014
$\Gamma_{\tau \rightarrow \pi} / \Gamma_{\pi \rightarrow \mu}$	0.9965 ± 0.0026
$\Gamma_{\tau \rightarrow K} / \Gamma_{K \rightarrow \mu}$	0.986 ± 0.007
$B_{W \rightarrow \tau} / B_{W \rightarrow \mu}$	1.004 ± 0.016

$$|g_\tau / g_e|$$

$B_{\tau \rightarrow \mu} \tau_\mu / \tau_\tau$	1.0028 ± 0.0015
$B_{W \rightarrow \tau} / B_{W \rightarrow e}$	1.022 ± 0.012

0.997 ± 0.011 (CMS preliminary)

Flavour Changing Charged Currents



$$\Gamma(d_j \rightarrow u_i e^- \bar{\nu}_e) \propto |\mathbf{V}_{ij}|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} Determinations

PDG 2020

CKM entry	Value	Source
$ V_{ud} $	0.97370 ± 0.00014	Nuclear β decay
$ V_{us} $	0.2245 ± 0.0008	$K \rightarrow (\pi) \ell \nu$
$ V_{cd} $	0.221 ± 0.004	$D \rightarrow (\pi) \ell \nu, \nu d \rightarrow c X$
$ V_{cs} $	0.987 ± 0.011	$D \rightarrow K \ell \nu, D_s \rightarrow \ell \nu$
$ V_{cb} $	0.0410 ± 0.0014	$b \rightarrow c \ell \nu, B \rightarrow D^{(*)} \ell \nu$
$ V_{ub} $	0.00382 ± 0.00024	$b \rightarrow u \ell \nu, B \rightarrow \pi \ell \nu$
$ V_{tb} $	1.013 ± 0.030	$p \bar{p}, pp \rightarrow tb + X$
$ V_{tb} / \sqrt{\sum_q V_{tq} ^2}$	> 0.975 (95% CL)	$t \rightarrow bW / t \rightarrow qW$
$ V_{td} $	0.0080 ± 0.0003	$B_d^0 - \bar{B}_d^0$ mixing
$ V_{ts} $	0.0388 ± 0.0011	$B_s^0 - \bar{B}_s^0$ mixing

PDG 2018

0.97420 (21)

0.2243 (5)

0.9994 ± 0.0005 (PDG 2018)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022$$

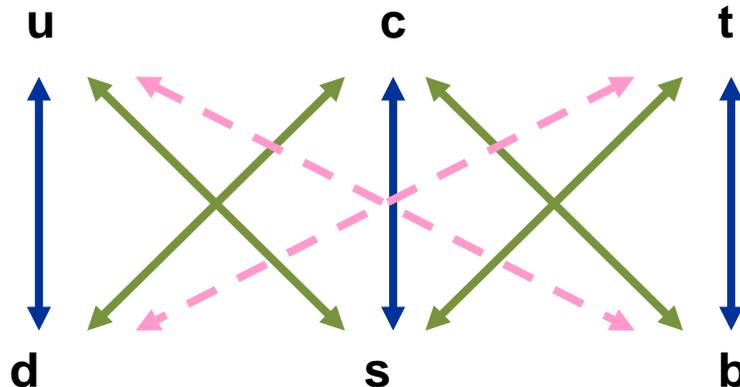
$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1.028 \pm 0.061$$

$$\sum_j (|V_{uj}|^2 + |V_{cj}|^2) = 2.002 \pm 0.027 \quad (\text{LEP})$$

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \approx \sin \theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$$



QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1} \quad \frac{1}{2} N_G (N_G - 1) \text{ moduli, } \frac{1}{2} N_G (N_G + 1) \text{ phases}$$

- $2 N_G - 1$ **arbitrary phases:** $\bar{u}_i \mathbf{V}_{ij} d_j$

$$u_i \rightarrow e^{i\phi_i} u_i \quad ; \quad d_j \rightarrow e^{i\theta_j} d_j \quad \longrightarrow \quad \mathbf{V}_{ij} \rightarrow e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$



\mathbf{V}_{ij} **Physical Parameters:**

$$\frac{1}{2} N_G (N_G - 1) \text{ moduli} \quad ; \quad \frac{1}{2} (N_G - 1) (N_G - 2) \text{ phases}$$

- $N_f = 2$: 1 angle, 0 phases (Cabibbo)

$$\mathbf{V} = \begin{bmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{bmatrix} \quad \longrightarrow \quad \text{No } \cancel{CP}$$

- $N_f = 3$: 3 angles, 1 phase (CKM) $c_{ij} \equiv \cos \theta_{ij}$; $s_{ij} \equiv \sin \theta_{ij}$

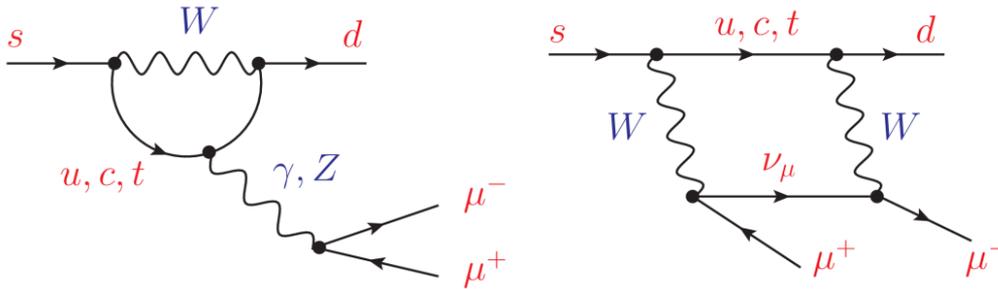
$$\mathbf{V} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda \approx \sin \theta_C \approx 0.225 \quad ; \quad A \approx 0.81 \quad ; \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$$

$$\delta_{13} \neq 0 \quad (\eta \neq 0) \quad \longrightarrow \quad \cancel{CP}$$

GIM Mechanism



$$\mathcal{M} \propto \sum_{i=u,c,t} V_{is} V_{id}^* F(m_i^2/M_W^2)$$

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0 \quad \longrightarrow \quad \mathcal{M} = 0 \quad \text{if} \quad m_u = m_c = m_t$$

$$\tilde{F}(x) \equiv F(x) - F(0)$$



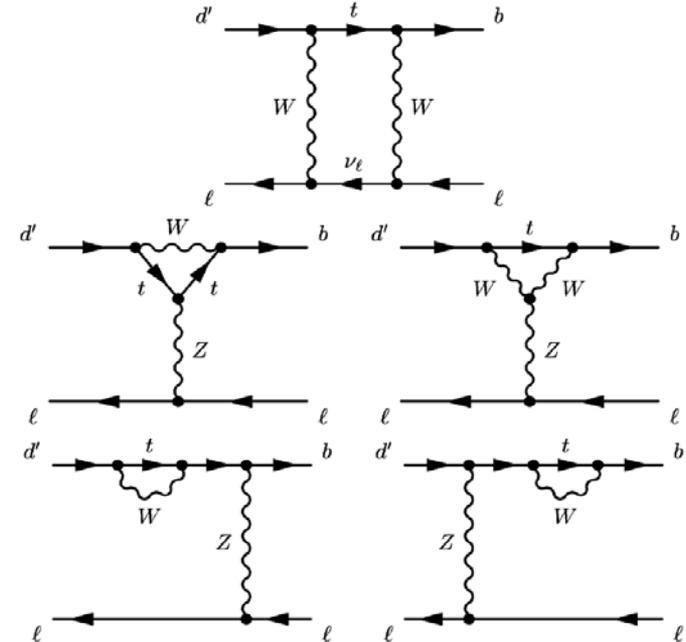
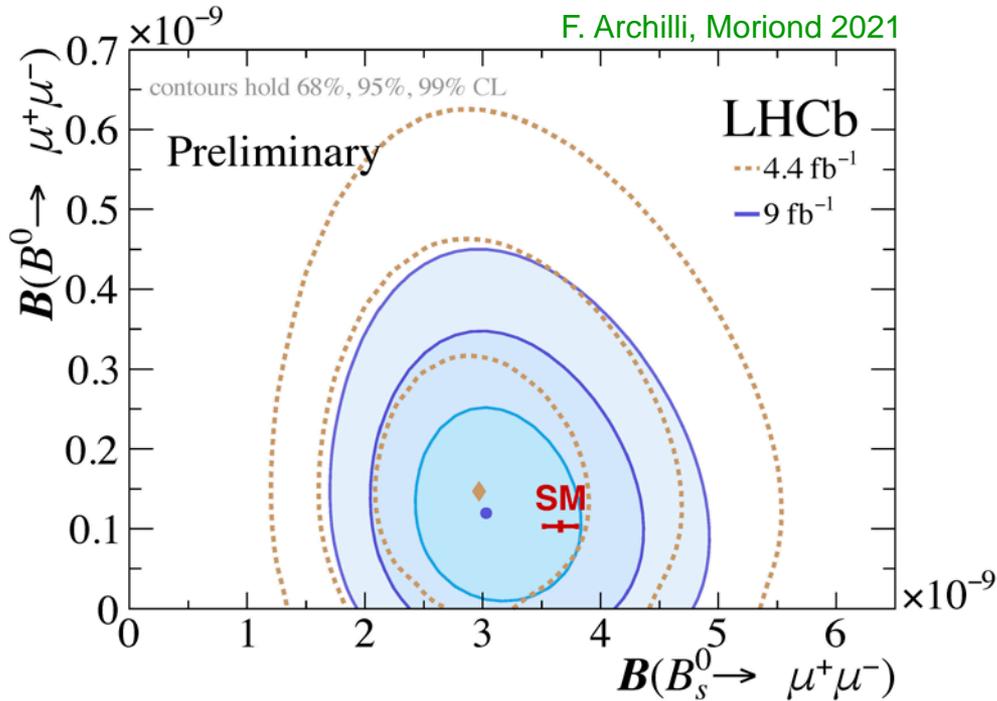
$$\begin{aligned} \mathcal{M} &\propto V_{cs} V_{cd}^* \tilde{F}(m_c^2/M_W^2) + V_{ts} V_{td}^* \tilde{F}(m_t^2/M_W^2) \\ &\approx -\lambda \tilde{F}(m_c^2/M_W^2) - \lambda^5 A^2 (1 - \rho + i\eta) \tilde{F}(m_t^2/M_W^2) \end{aligned}$$

- **Top contribution dominates. Strong suppression:** $\mathcal{M} \propto \frac{g^4}{16\pi^2} \left[\lambda^5 A^2 \frac{m_t^2}{M_W^2}, \lambda \frac{m_c^2}{M_W^2} \right]$
- **\mathcal{CP} effects fully governed by top contribution** $\left[\text{Im}(V_{cs} V_{cd}^*) = -\text{Im}(V_{ts} V_{td}^*) \right]$

Rare Decays

Loop & CKM suppression Sensitivity to New Physics

$$B_{s,d} \rightarrow \mu^+ \mu^-$$



$$W^\pm \leftrightarrow H^\pm, \quad Z \leftrightarrow H^0, A^0$$

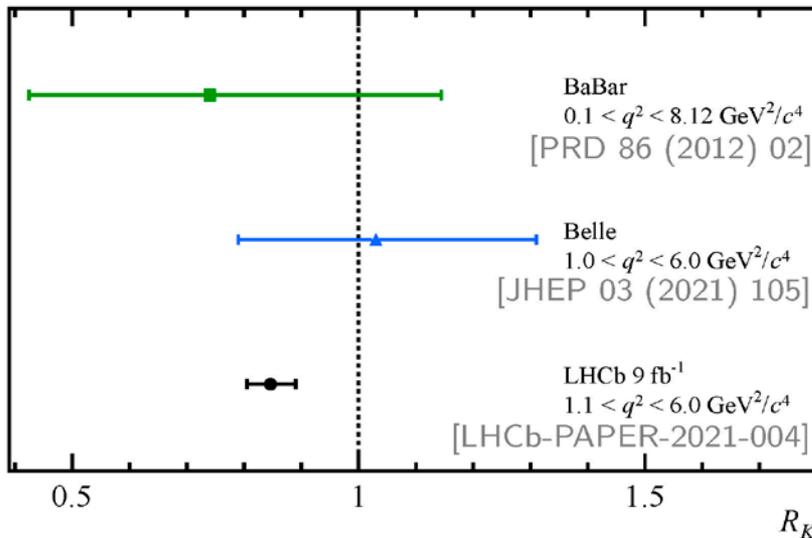
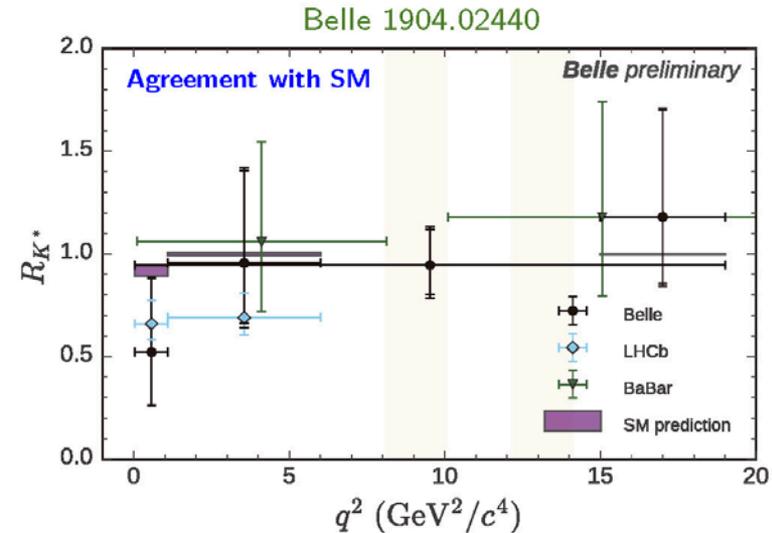
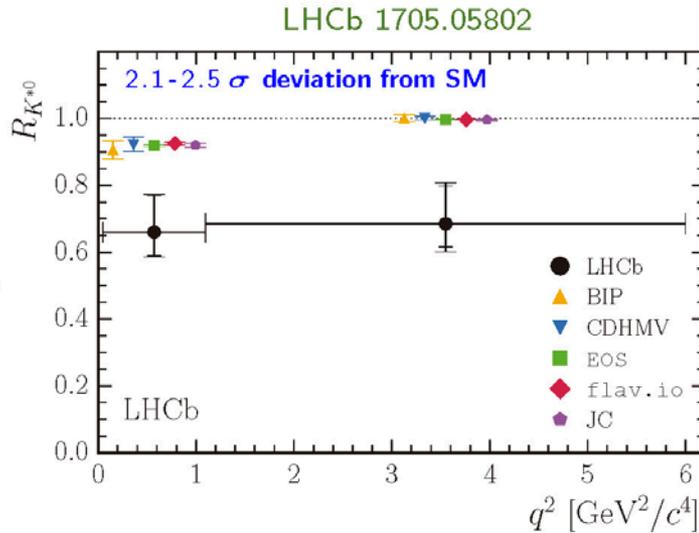
$$\bar{B}(B_s^0 \rightarrow \mu^+ \mu^-)_{\text{exp}} = (2.9 \pm 0.4) \cdot 10^{-9}$$

$$\bar{B}(B_d^0 \rightarrow \mu^+ \mu^-)_{\text{exp}} = \left(0.5^{+1.7}_{-1.5} \right) \cdot 10^{-10}$$

Sensitive to (pseudo) scalar contributions

Violations of Lepton Flavour Universality

$$\frac{\Gamma(B \rightarrow K^* \mu^+ \mu^-)}{\Gamma(B \rightarrow K^* e^+ e^-)}$$



$$R_K \equiv \frac{\Gamma(B^+ \rightarrow K^+ \mu^+ \mu^-)}{\Gamma(B^+ \rightarrow K^+ e^+ e^-)} = 0.846^{+0.042 + 0.013}_{-0.039 - 0.012}$$

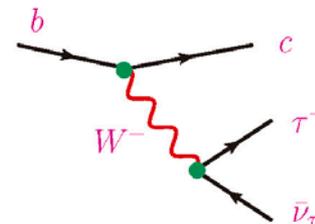
3.1 σ below the SM

Flavour Anomaly

(charged currents)

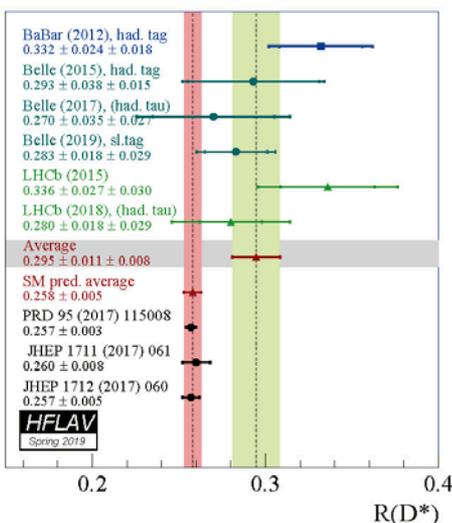
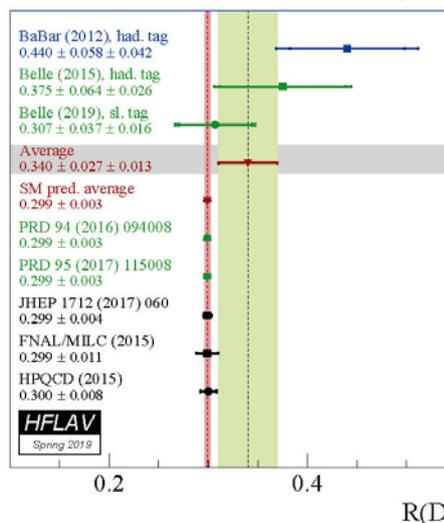
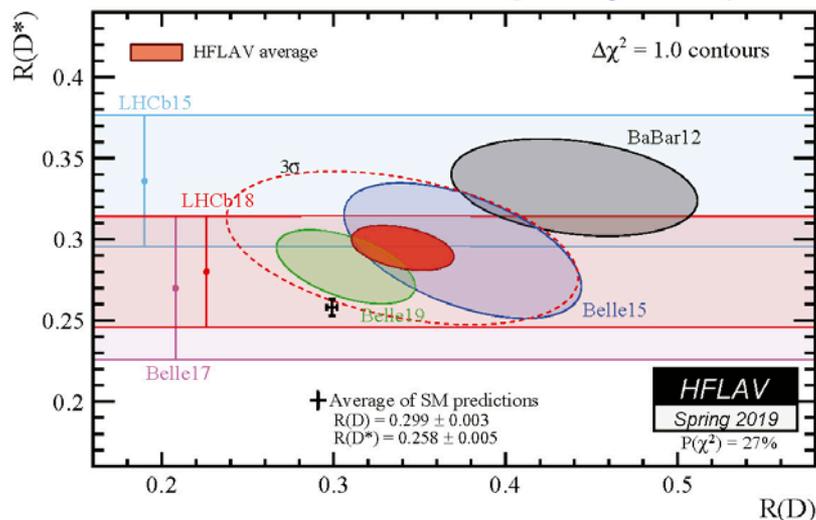
$$\mathcal{R}_{D^{(*)}} \equiv \frac{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell)}$$

Tree-level process



3.08 σ discrepancy

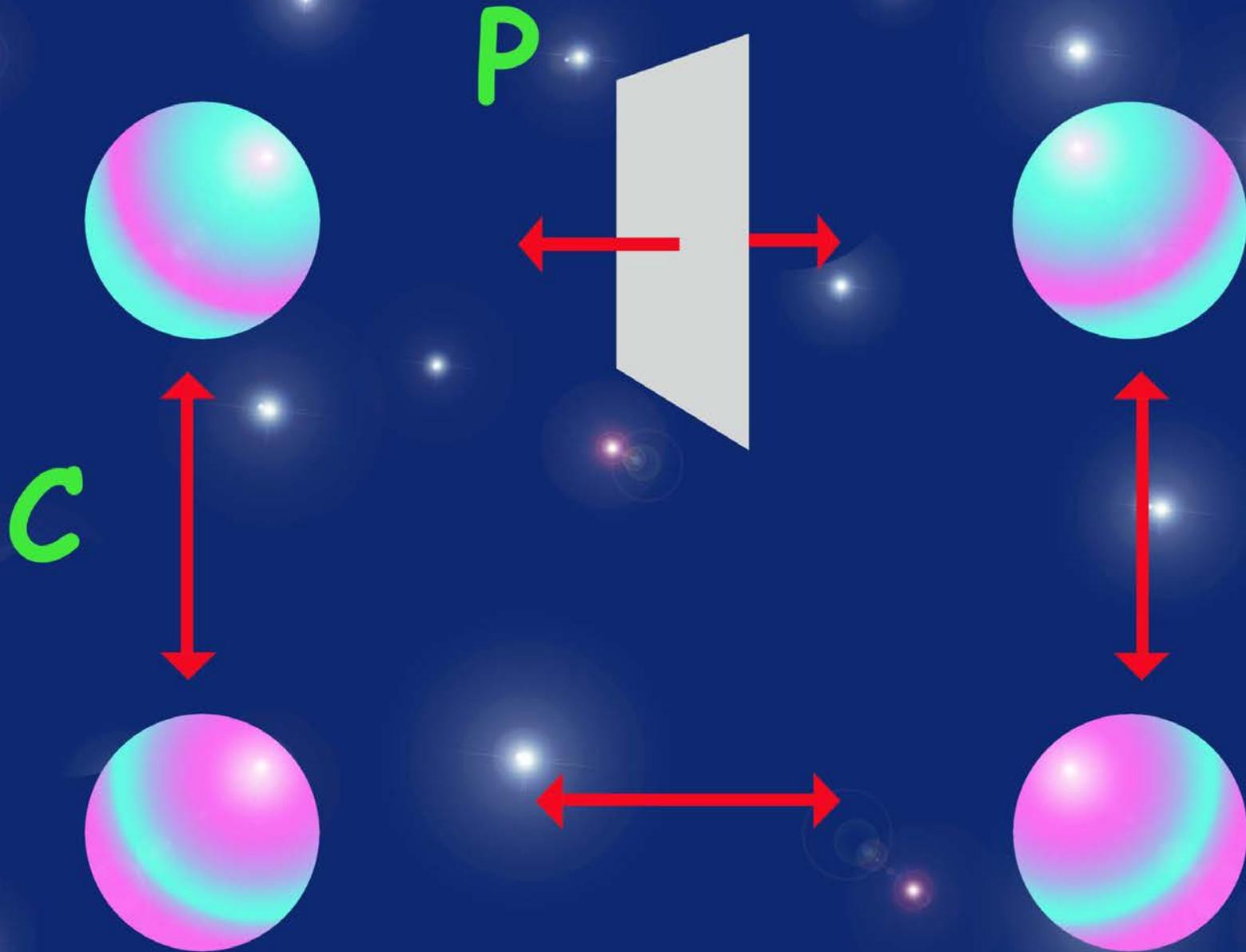
(3.2 σ with more recent predictions)



LHCb, 1711.05623: $\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \rightarrow J/\psi \tau \bar{\nu}_\tau)}{\mathcal{B}(B_c \rightarrow J/\psi \mu \bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18 \quad (1.7 \sigma) \quad \mathcal{R}_{J/\psi}^{\text{SM}} \approx 0.26 - 0.28$

Belle, 1903.03102: $F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \quad (1.6 \sigma) \quad F_{L,\text{SM}}^{D^*} = 0.455 \pm 0.003$

Belle, 1612.00529: $\mathcal{P}_\tau^{D^*} = -0.38 \pm 0.51^{+0.21}_{-0.16} \quad \mathcal{P}_{\tau,\text{SM}}^{D^*} = -0.499 \pm 0.003$



- \mathcal{C}, \mathcal{P} : Violated maximally in weak interactions
- \mathcal{CP} : Symmetry of nearly all observed phenomena
- Slight ($\sim 0.2\%$) $\cancel{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\cancel{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter–Antimatter Asymmetry in our Universe
 Baryogenesis
- Small $\cancel{\mathcal{CP}}$ in D^0 decays (LHCb, 2019)

\mathcal{CPT} Theorem: $\cancel{\mathcal{CP}} \longleftrightarrow \cancel{\mathcal{T}}$

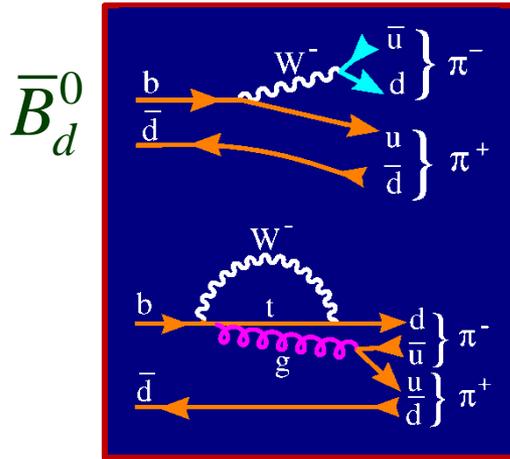
Thus, $\cancel{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

DIRECT

C/P

$$|\mathbf{T}(P \rightarrow f)| \neq |\mathbf{T}(\bar{P} \rightarrow \bar{f})|$$



$$\mathbf{T}(P \rightarrow f) = T_1 e^{i\phi_1} e^{i\delta_1} + T_2 e^{i\phi_2} e^{i\delta_2}$$

\downarrow $C\mathcal{P}$

$$\mathbf{T}(\bar{P} \rightarrow \bar{f}) = T_1 e^{-i\phi_1} e^{i\delta_1} + T_2 e^{-i\phi_2} e^{i\delta_2}$$

$$A_{P \rightarrow f}^{\text{CP}} \equiv \frac{\Gamma(P \rightarrow f) - \Gamma(\bar{P} \rightarrow \bar{f})}{\Gamma(P \rightarrow f) + \Gamma(\bar{P} \rightarrow \bar{f})} = \frac{-2 T_1 T_2 \sin(\phi_2 - \phi_1) \sin(\delta_2 - \delta_1)}{T_1^2 + T_2^2 + 2 T_1 T_2 \cos(\phi_2 - \phi_1) \cos(\delta_2 - \delta_1)}$$

One needs:

- 2 Interfering Amplitudes
- 2 Different Weak Phases $[\sin(\phi_2 - \phi_1) \neq 0]$
- 2 Different FSI Phases $[\sin(\delta_2 - \delta_1) \neq 0]$

DIRECT CP

$$A_{CP}(B \rightarrow f) \equiv \frac{\text{Br}(\bar{B} \rightarrow \bar{f}) - \text{Br}(B \rightarrow f)}{\text{Br}(\bar{B} \rightarrow \bar{f}) + \text{Br}(B \rightarrow f)}$$

$$A_{CP}(B_d^0 \rightarrow \pi^- K^+) = -0.084 \pm 0.004 \quad (21 \sigma)$$

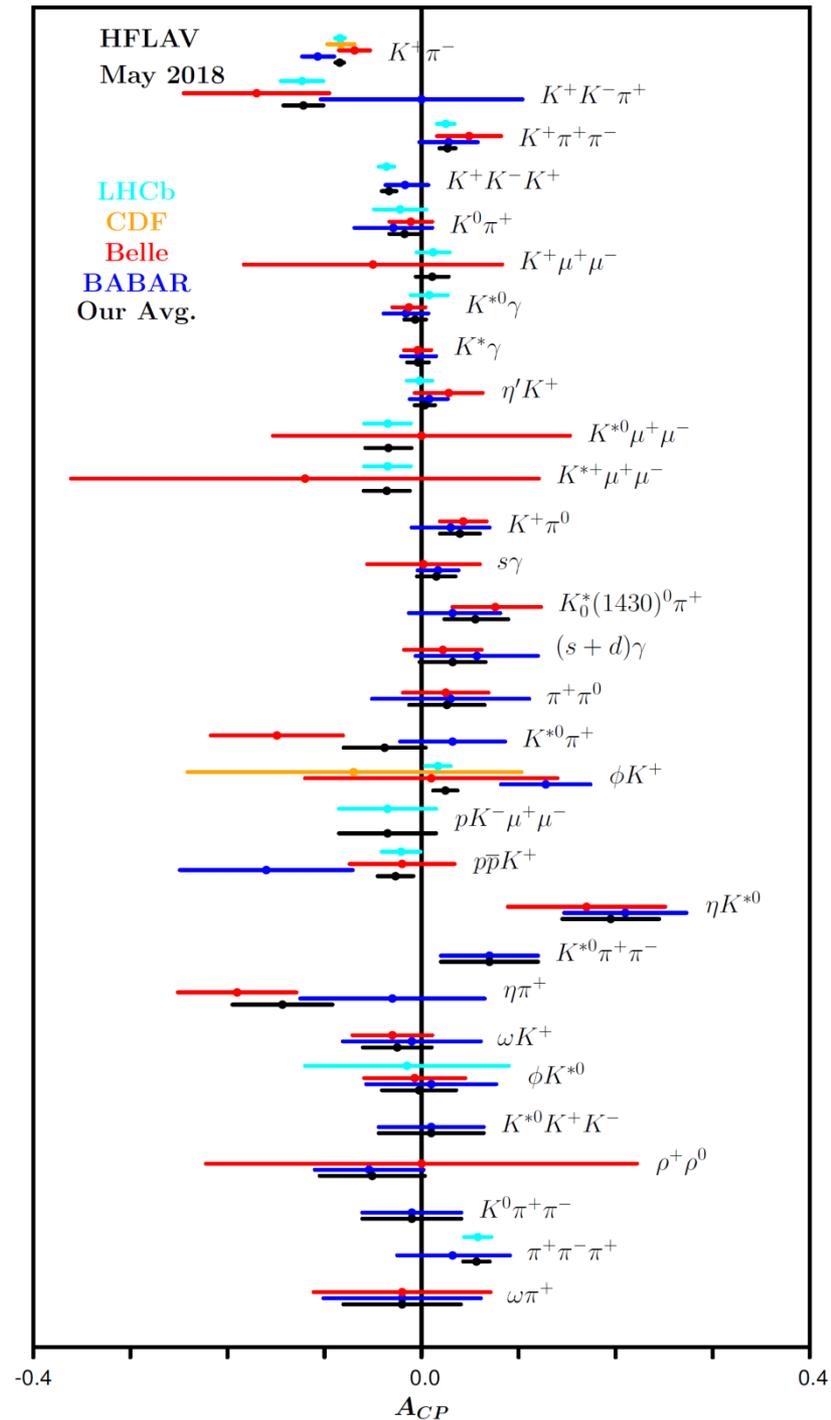
$$A(B_s^0 \rightarrow \pi^- K^+) = -0.213 \pm 0.017 \quad (12.5 \sigma)$$

$$A_{CP}(B^+ \rightarrow K^+ K^- \pi^+) = -0.122 \pm 0.021 \quad (5.8 \sigma)$$

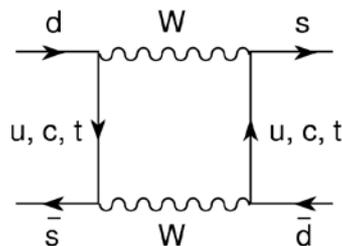
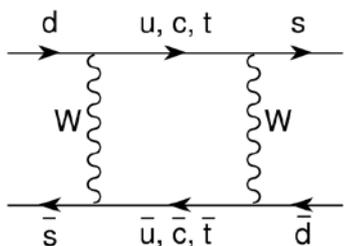
Large & Interesting Signals

Big challenge: Get reliable SM predictions

Severe hadronic uncertainties



INDIRECT \mathcal{CP} : $K^0 - \bar{K}^0$ MIXING



$$|K_{S,L}^0\rangle \sim p |K^0\rangle \mp q |\bar{K}^0\rangle$$

$$q/p \equiv (1 - \bar{\epsilon}_K)/(1 + \bar{\epsilon}_K)$$

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j S(r_i, r_j) \eta_{ij} \langle O_{\Delta S=2} \rangle$$

$$\langle O_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2 \right) \hat{B}_K$$

$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

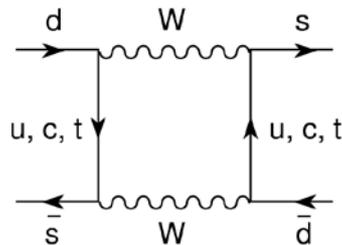
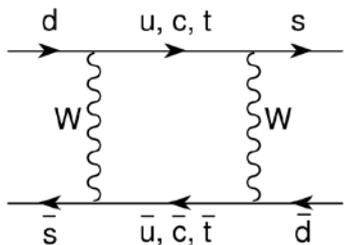
- **GIM Mechanism:** $\lambda_u + \lambda_c + \lambda_t = 0$

$$(M_{K_L} - M_{K_S}) / M_{K^0} = (7.00 \pm 0.01) \cdot 10^{-15}$$

- \mathcal{CP} : $\text{Im} \lambda_t = -\text{Im} \lambda_c \simeq \eta \lambda^5 A^2$

- **Hard GIM Breaking:** $S(r_i, r_i) \sim r_i \rightarrow$ **t quark**

INDIRECT \mathcal{CP} : $K^0 - \bar{K}^0$ MIXING



$$|K_{S,L}^0\rangle \sim p |K^0\rangle \mp q |\bar{K}^0\rangle$$

$$q/p \equiv (1 - \bar{\varepsilon}_K)/(1 + \bar{\varepsilon}_K)$$

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j S(r_i, r_j) \eta_{ij} \langle \mathcal{O}_{\Delta S=2} \rangle$$

$$\langle \mathcal{O}_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2 \right) \hat{B}_K$$

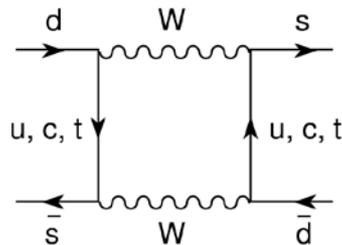
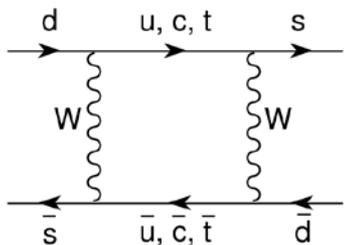
$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

$$\mathcal{C} |K^0\rangle = |\bar{K}^0\rangle \quad , \quad \mathcal{P} |K^0\rangle = -|K^0\rangle \quad , \quad \mathcal{CP} |K^0\rangle = -|\bar{K}^0\rangle$$

$$|K_{1,2}^0\rangle = \frac{1}{\sqrt{2}} \left(|K^0\rangle \mp |\bar{K}^0\rangle \right) \quad , \quad \mathcal{CP} |K_{1,2}^0\rangle = \pm |K_{1,2}^0\rangle$$

$$|K_S^0\rangle \simeq |K_1^0\rangle + \bar{\varepsilon}_K |K_2^0\rangle \quad , \quad |K_L^0\rangle \simeq |K_2^0\rangle + \bar{\varepsilon}_K |K_1^0\rangle$$

INDIRECT CP : $K^0 - \bar{K}^0$ MIXING



$$|K_{S,L}^0\rangle \sim p |K^0\rangle \mp q |\bar{K}^0\rangle$$

$$q/p \equiv (1 - \bar{\varepsilon}_K)/(1 + \bar{\varepsilon}_K)$$

$$K^0 \rightarrow \pi^- l^+ \nu_l \quad (\bar{s} \rightarrow \bar{u}) \quad ; \quad \bar{K}^0 \rightarrow \pi^+ l^- \bar{\nu}_l \quad (s \rightarrow u)$$

$$\frac{\Gamma(K_L^0 \rightarrow \pi^- l^+ \nu_l) - \Gamma(K_L^0 \rightarrow \pi^+ l^- \bar{\nu}_l)}{\Gamma(K_L^0 \rightarrow \pi^- l^+ \nu_l) + \Gamma(K_L^0 \rightarrow \pi^+ l^- \bar{\nu}_l)} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{2 \operatorname{Re}(\bar{\varepsilon}_K)}{1 + |\bar{\varepsilon}_K|^2} = (0.332 \pm 0.006)\%$$



$$\operatorname{Re}(\bar{\varepsilon}_K) = (1.66 \pm 0.03) \cdot 10^{-3}$$

$$\eta_{+-} \equiv \frac{T(K_L \rightarrow \pi^+ \pi^-)}{T(K_S \rightarrow \pi^+ \pi^-)} \approx \varepsilon_K$$

$$\eta_{00} \equiv \frac{T(K_L \rightarrow \pi^0 \pi^0)}{T(K_S \rightarrow \pi^0 \pi^0)} \approx \varepsilon_K$$

$$\varepsilon_K = (2.228 \pm 0.011) \cdot 10^{-3} e^{i\phi_\varepsilon}$$

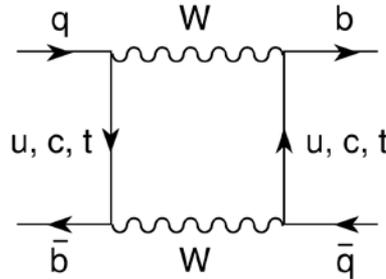
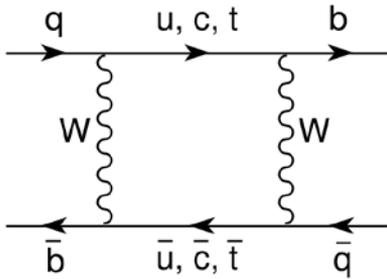
$$\phi_\varepsilon = (43.52 \pm 0.05)^\circ$$



Buras et al

$$\eta \left[(1 - \rho) A^2 + 0.22 \right] A^2 \hat{B}_K = 0.143$$

B⁰ – B⁰ MIXING



$$V_{ud} V_{ub}^* \sim V_{cd} V_{cb}^* \sim V_{td} V_{tb}^* \sim A\lambda^3$$

$$\langle \bar{B}^0 | \mathbf{H} | B^0 \rangle \sim |V_{td}|^2 S(r_t, r_t) \left(\frac{4}{3} M_B^2 f_B^2 \right) \hat{B}_B$$

$$\Delta M_{B_d^0} = (0.5065 \pm 0.0019) \text{ ps}^{-1}$$



$$|V_{td}|$$

- $\Delta M_{B_d^0} / \Gamma_{B_d^0} = 0.769 \pm 0.004$

- $\Delta M_{B_s^0} = (17.766 \pm 0.006) \text{ ps}^{-1}$

- $\Delta \Gamma_{B^0} / \Delta M_{B^0} \sim m_b^2 / m_t^2 \ll 1$

- $\text{Re}(\bar{\varepsilon}_{B_d^0}) = -0.0005 \pm 0.0004$

$$\Delta M_{B_s^0} / \Gamma_{B_s^0} = 26.89 \pm 0.11$$

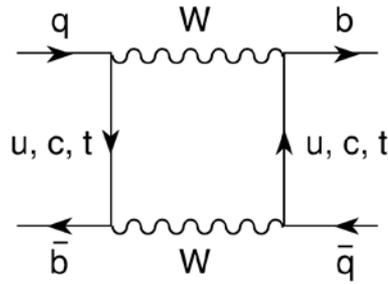
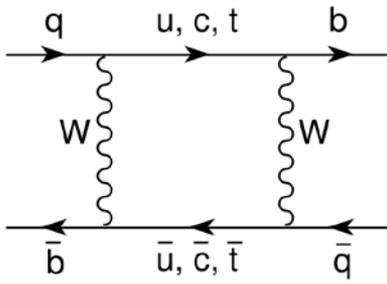
$$|V_{ts}|^2 \gg |V_{td}|^2$$

$$\Delta \Gamma_{B_s^0} / \Gamma_{B_s^0} = -0.124 \pm 0.008$$

$$\text{Re}(\bar{\varepsilon}_{B_s^0}) = -0.00015 \pm 0.00070$$

~~C/P~~ very small

$$|q/p| - 1 \sim m_c^2 / m_t^2$$



$$\mathbf{M} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

$$|B_{\mp}^0\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} \left(p |B^0\rangle \mp q |\bar{B}^0\rangle \right)$$

$$\frac{q}{p} \equiv \frac{1 - \bar{\epsilon}_B}{1 + \bar{\epsilon}_B} = \left(\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \right)^{1/2}$$

$$(\Delta M)^2 - \frac{1}{4} (\Delta \Gamma)^2 = 4 |M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta M \Delta \Gamma = 4 \operatorname{Re}(M_{12} \Gamma_{12}^*)$$

$$\Delta \Gamma / \Delta M \approx \Gamma_{12} / M_{12} \sim m_b^2 / m_t^2 \ll 1$$



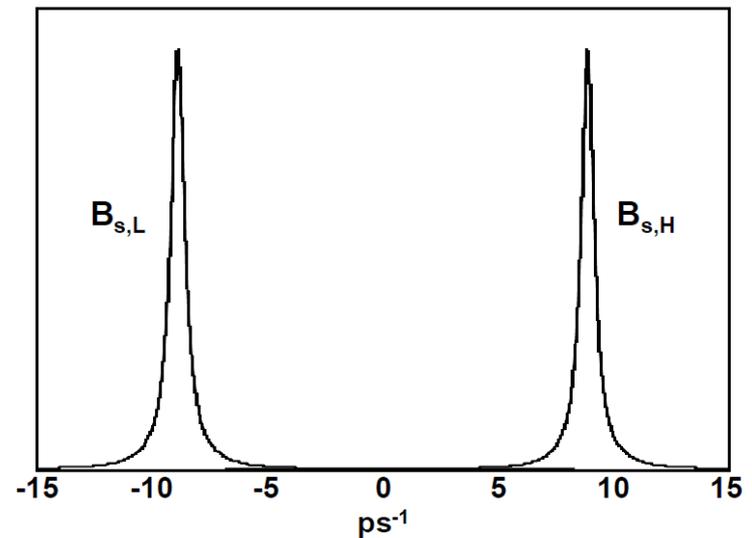
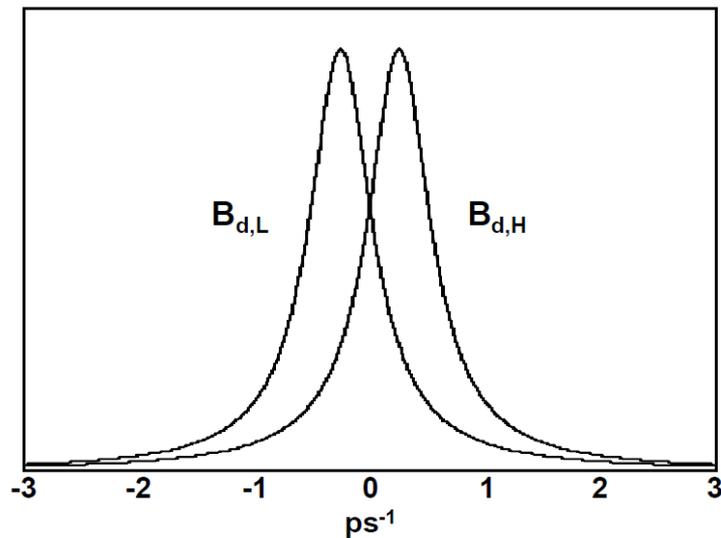
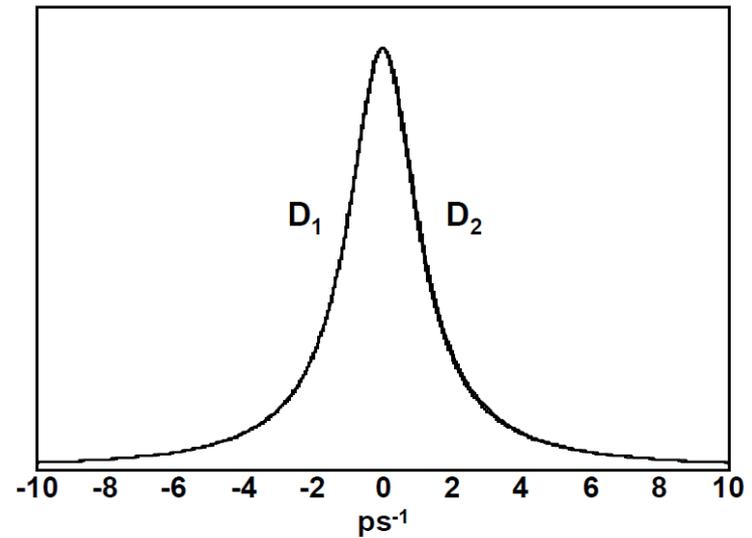
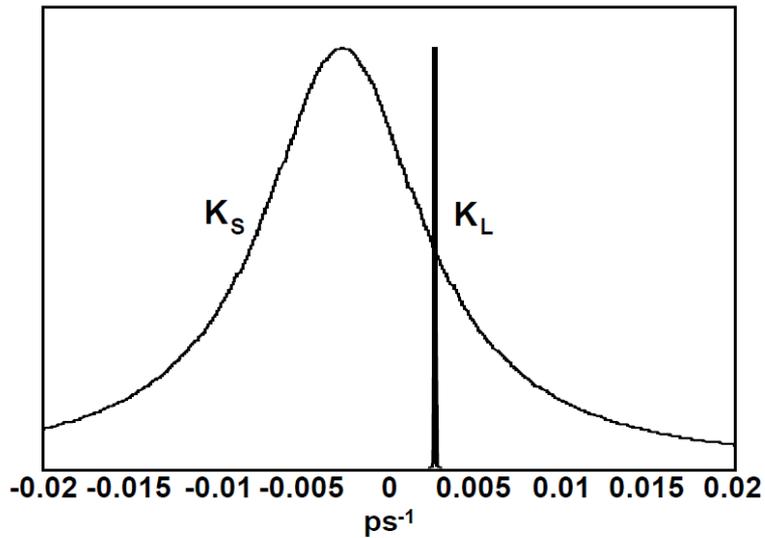
$$\left| \frac{q}{p} \right| \approx 1 + \frac{1}{2} \left| \frac{\Gamma_{12}}{M_{12}} \right| \sin \phi_{\Delta B=2}, \quad \phi_{\Delta B=2} \equiv \arg(M_{12} / \Gamma_{12})$$

$$\Delta M \equiv M_{B_+} - M_{B_-}$$

$$\Delta \Gamma \equiv \Gamma_{B_+} - \Gamma_{B_-}$$

$$\begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} g_1(t) & \frac{q}{p} g_2(t) \\ \frac{p}{q} g_2(t) & g_1(t) \end{pmatrix} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix}, \quad \begin{pmatrix} g_1(t) \\ g_2(t) \end{pmatrix} = e^{-iMt} e^{-\Gamma t/2} \begin{pmatrix} \cos \left[\left(\Delta M - \frac{i}{2} \Delta \Gamma \right) \frac{t}{2} \right] \\ -i \sin \left[\left(\Delta M - \frac{i}{2} \Delta \Gamma \right) \frac{t}{2} \right] \end{pmatrix}$$

Widths & Mass Differences

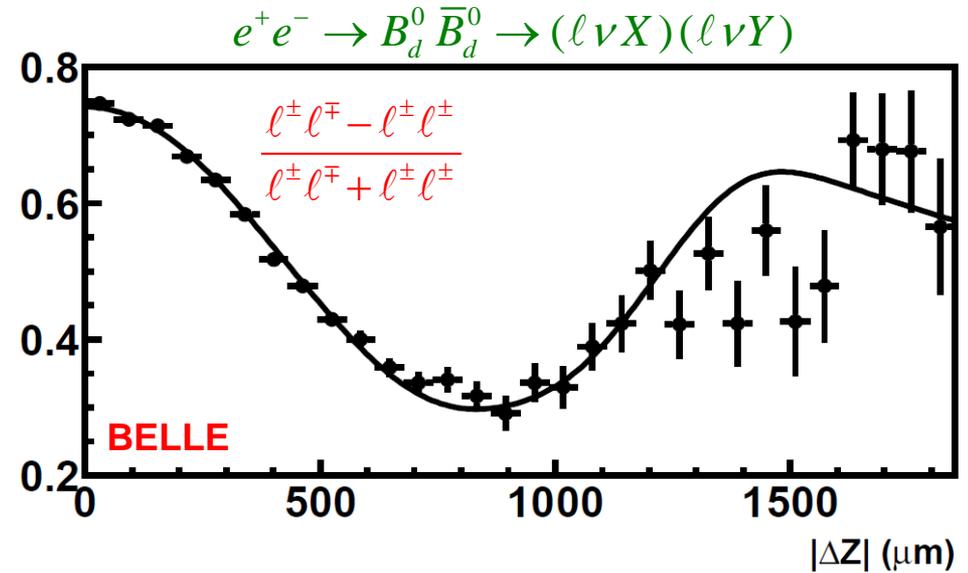


M. Gersabeck

Time Scales:

$$x \equiv \frac{\Delta M}{\Gamma} \quad , \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}$$

$$\text{Oscillation} \sim \sin \left[(x - iy) \Gamma t / 2 \right]$$



- \mathbf{K}^0 : $x \sim y \sim 1$
- \mathbf{D}^0 : $x \sim y \sim 0.01$
- \mathbf{B}_d : $x \sim 1$, $y \sim 0.01$
- \mathbf{B}_s : $x \sim 25$, $y \sim 0.05$

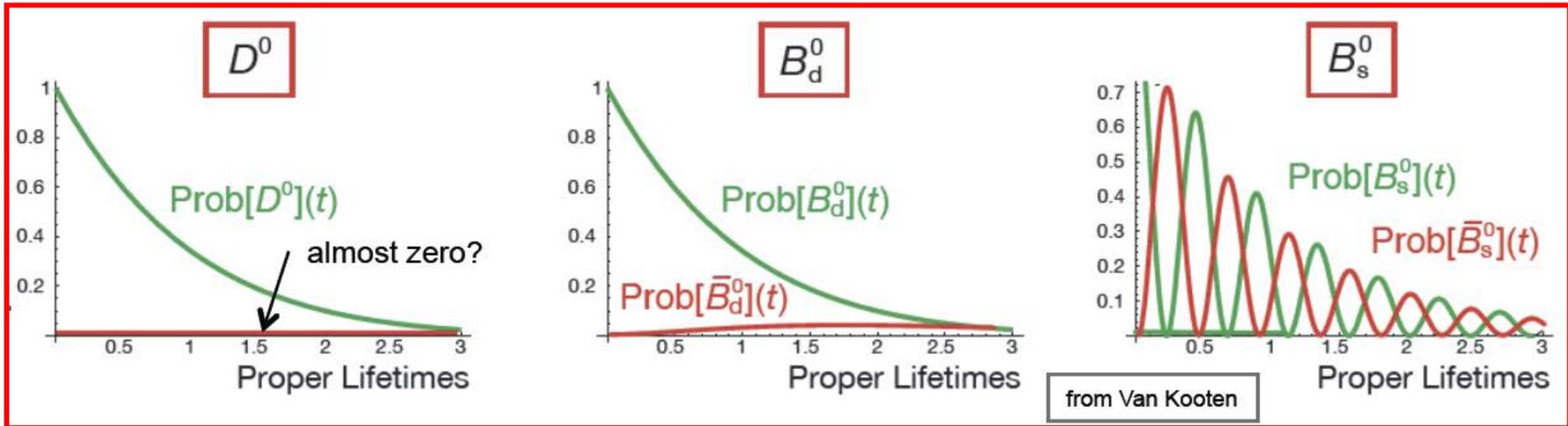
Slow oscillation (decays faster)

Fast oscillation (averages out to 0)

Time Scales:

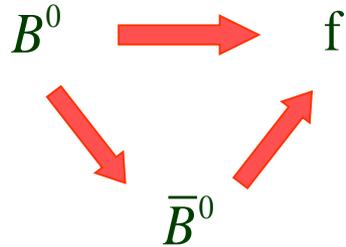
$$\text{Oscillation} \sim \sin \left[(x - iy) \Gamma t / 2 \right]$$

$$x \equiv \Delta M / \Gamma \quad , \quad y \equiv \Delta \Gamma / 2\Gamma$$



- \mathbf{K}^0 : $x \sim y \sim 1$
- \mathbf{D}^0 : $x \sim y \sim 0.01$ **Slow oscillation** (decays faster)
- \mathbf{B}_d : $x \sim 1$, $y \sim 0.01$
- \mathbf{B}_s : $x \sim 25$, $y \sim 0.05$ **Fast oscillation** (averages out to 0)

$B^0 - \bar{B}^0$ MIXING AND DIRECT CP



$$\begin{aligned} T_f &\equiv T[B^0 \rightarrow f] \quad ; \quad \bar{T}_f \equiv -T[\bar{B}^0 \rightarrow f] \quad ; \quad \bar{\rho}_f \equiv \bar{T}_f / T_f \\ T_{\bar{f}} &\equiv T[B^0 \rightarrow \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \equiv -T[\bar{B}^0 \rightarrow \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}} \end{aligned}$$

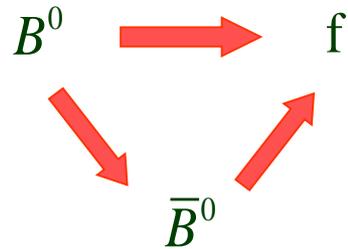
$$CP \ B^0 = -\bar{B}^0 \quad ; \quad CP \ f = \bar{f}$$

$$\begin{aligned} \Gamma[B^0(t) \rightarrow f] &\sim \frac{1}{2} e^{-\Gamma t} \left(|T_f|^2 + |\bar{T}_f|^2 \right) \left\{ 1 + C_f \cos(\Delta M t) - S_f \sin(\Delta M t) \right\} \\ \Gamma[\bar{B}^0(t) \rightarrow \bar{f}] &\sim \frac{1}{2} e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2 \right) \left\{ 1 - C_{\bar{f}} \cos(\Delta M t) + S_{\bar{f}} \sin(\Delta M t) \right\} \end{aligned}$$

$$C_f \equiv \frac{1 - |\bar{\rho}_f|^2}{1 + |\bar{\rho}_f|^2} \quad ; \quad S_f \equiv \frac{2 \operatorname{Im}\left(\frac{q}{p} \bar{\rho}_f\right)}{1 + |\bar{\rho}_f|^2} \quad ; \quad C_{\bar{f}} \equiv -\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad S_{\bar{f}} \equiv \frac{-2 \operatorname{Im}\left(\frac{p}{q} \rho_{\bar{f}}\right)}{1 + |\rho_{\bar{f}}|^2}$$

$$\Delta\Gamma \ll \Delta M \quad \longrightarrow \quad \frac{q}{p} \approx \sqrt{\frac{M_{12}^*}{M_{12}}} \approx \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} = e^{-2i\phi_M} \quad ; \quad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT CP



$$\begin{aligned} T_f &\equiv T[B^0 \rightarrow f] \quad ; \quad \bar{T}_f \equiv -T[\bar{B}^0 \rightarrow f] \quad ; \quad \bar{\rho}_f \equiv \bar{T}_f / T_f \\ T_{\bar{f}} &\equiv T[B^0 \rightarrow \bar{f}] \quad ; \quad \bar{T}_{\bar{f}} \equiv -T[\bar{B}^0 \rightarrow \bar{f}] \quad ; \quad \rho_{\bar{f}} \equiv T_{\bar{f}} / \bar{T}_{\bar{f}} \end{aligned}$$

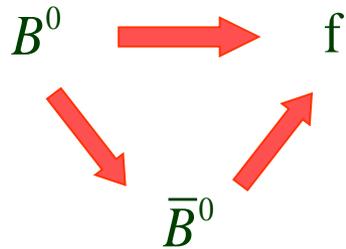
$$CP \ B^0 = -\bar{B}^0 \quad ; \quad CP \ f = \bar{f}$$

$$\begin{aligned} \Gamma[B^0(t) \rightarrow f] &\sim \frac{1}{2} e^{-\Gamma t} \left(|T_f|^2 + |\bar{T}_f|^2 \right) \left\{ 1 + C_f \cos(\Delta M t) - S_f \sin(\Delta M t) \right\} \\ \Gamma[\bar{B}^0(t) \rightarrow \bar{f}] &\sim \frac{1}{2} e^{-\Gamma t} \left(|\bar{T}_{\bar{f}}|^2 + |T_{\bar{f}}|^2 \right) \left\{ 1 - C_{\bar{f}} \cos(\Delta M t) + S_{\bar{f}} \sin(\Delta M t) \right\} \end{aligned}$$

$$C_f \equiv \frac{1 - |\bar{\rho}_f|^2}{1 + |\bar{\rho}_f|^2} \quad ; \quad S_f \equiv \frac{2 \operatorname{Im}\left(\frac{q}{p} \bar{\rho}_f\right)}{1 + |\bar{\rho}_f|^2} \quad ; \quad C_{\bar{f}} \equiv -\frac{1 - |\rho_{\bar{f}}|^2}{1 + |\rho_{\bar{f}}|^2} \quad ; \quad S_{\bar{f}} \equiv \frac{-2 \operatorname{Im}\left(\frac{p}{q} \rho_{\bar{f}}\right)}{1 + |\rho_{\bar{f}}|^2}$$

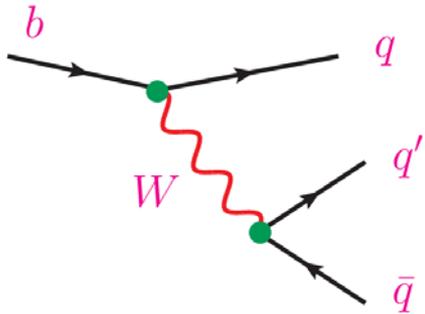
$$CP \text{ self-conjugate: } \bar{f} = \eta_f f \quad \longrightarrow \quad \begin{aligned} T_{\bar{f}} &= \eta_f T_f \quad ; \quad \bar{T}_{\bar{f}} = \eta_f \bar{T}_f \quad ; \quad \rho_{\bar{f}} \equiv 1/\bar{\rho}_f \\ C_{\bar{f}} &= C_f \quad ; \quad S_{\bar{f}} = S_f \end{aligned}$$

$B^0 - \bar{B}^0$ MIXING AND DIRECT CP



CP self-conjugate: $\bar{f} = \eta_f f$

$$\frac{q}{p} \approx \frac{V_{tb}^* V_{tq}}{V_{tb} V_{tq}^*} = e^{-2i\phi_M} \quad ; \quad \phi_M \approx \begin{cases} \beta & (B_d^0) \\ -\beta_s \approx -\lambda^2 \eta & (B_s^0) \end{cases}$$



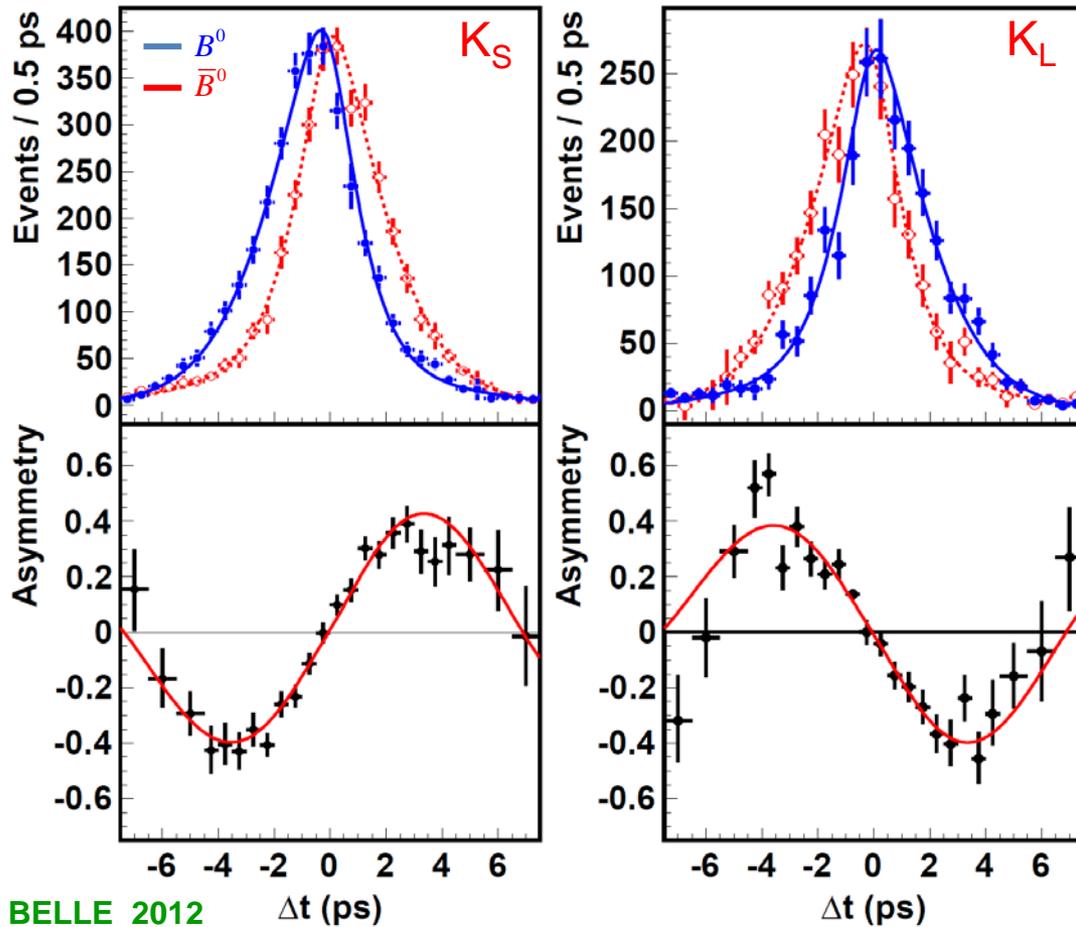
Assumption: **Only 1 decay amplitude**

$$\frac{A_{b \rightarrow q\bar{q}q'}}{A_{\bar{b} \rightarrow \bar{q}qq'}} = \frac{V_{qb} V_{qq'}^*}{V_{qb}^* V_{qq'}} = e^{-2i\phi_D} \quad \longrightarrow \quad \begin{aligned} \rho_{\bar{f}} &= \bar{\rho}_f^* = \eta_f e^{2i\phi_D} \\ C_f &= 0 \end{aligned}$$

$$\longrightarrow \frac{\Gamma(\bar{B}^0 \rightarrow \bar{f}) - \Gamma(B^0 \rightarrow f)}{\Gamma(\bar{B}^0 \rightarrow \bar{f}) + \Gamma(B^0 \rightarrow f)} = -\eta_f \sin(2\phi) \sin(\Delta M t) \quad ; \quad \phi = \phi_M + \phi_D$$

Direct information on the CKM matrix

$$\frac{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S) - \Gamma(B^0 \rightarrow J/\psi K_S)}{\Gamma(\bar{B}^0 \rightarrow J/\psi K_S) + \Gamma(B^0 \rightarrow J/\psi K_S)} = -\eta_f \sin(2\beta) \sin(\Delta M t)$$



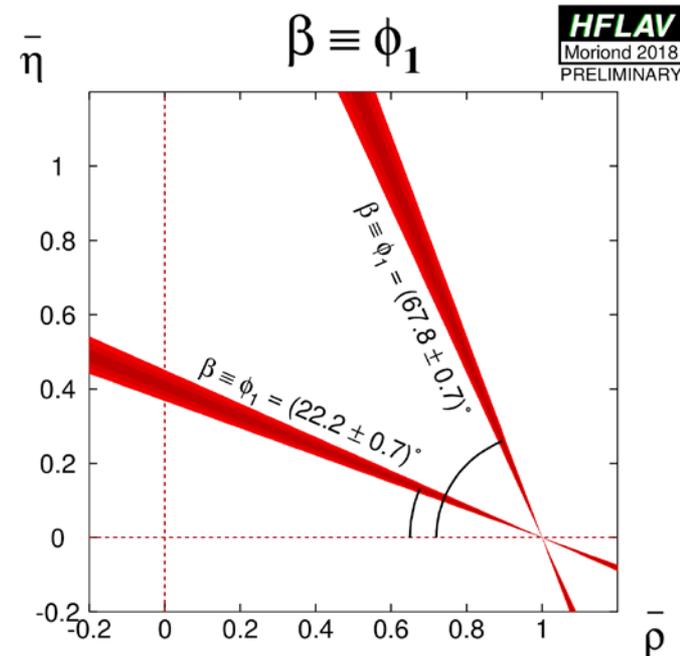
BELLE 2012

~~CP~~ Signal

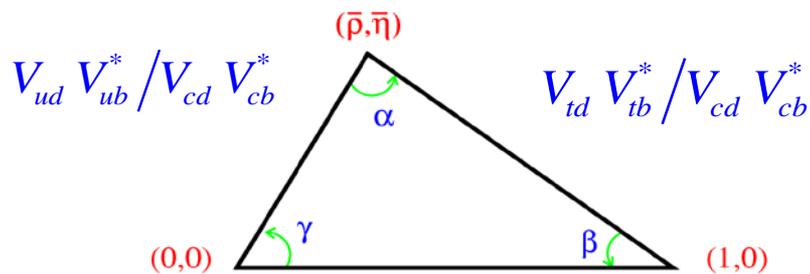
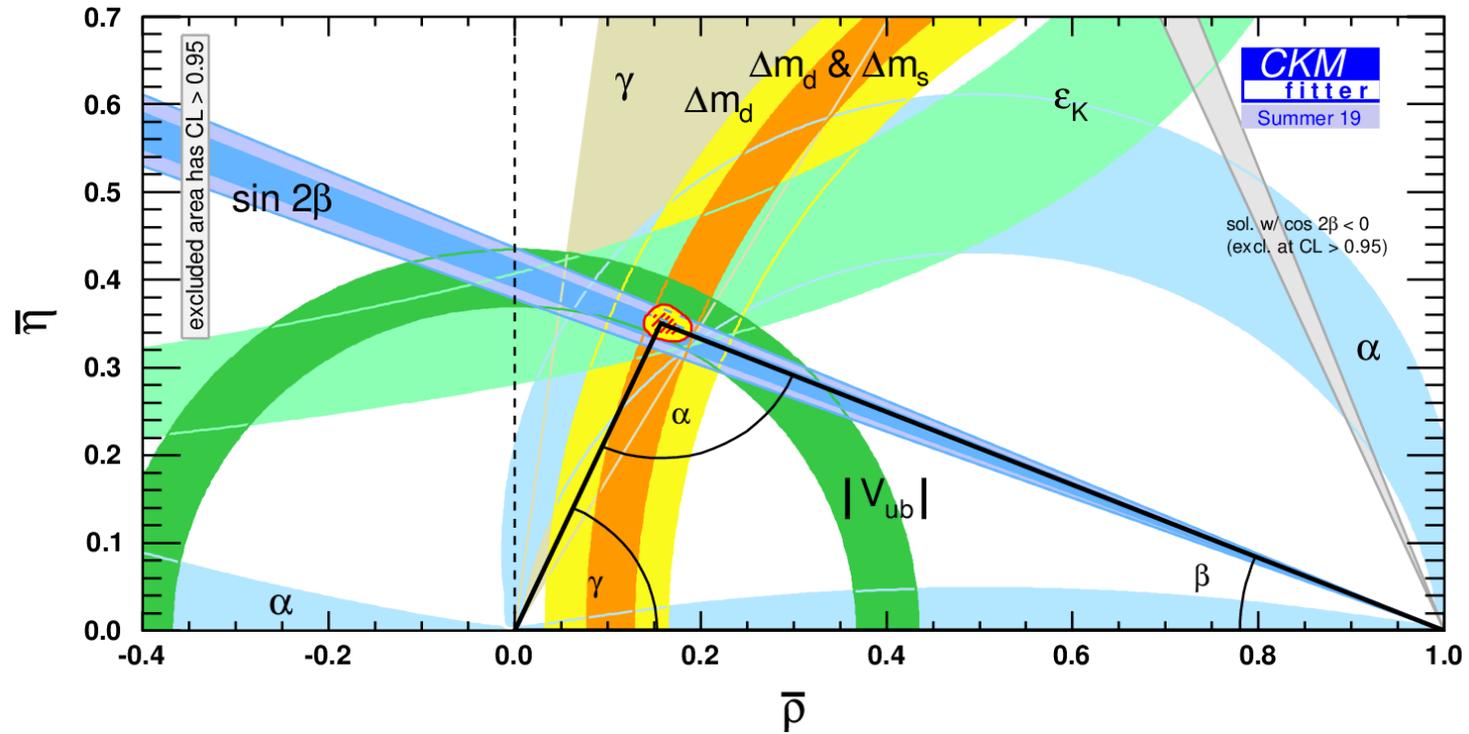
PDG 2020:

$$\sin(2\beta) = 0.701 \pm 0.017$$

$B^0 \rightarrow J/\psi K_{S,L}, \psi(2S) K_S, \chi_c K_S, \eta_c K_S$



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$



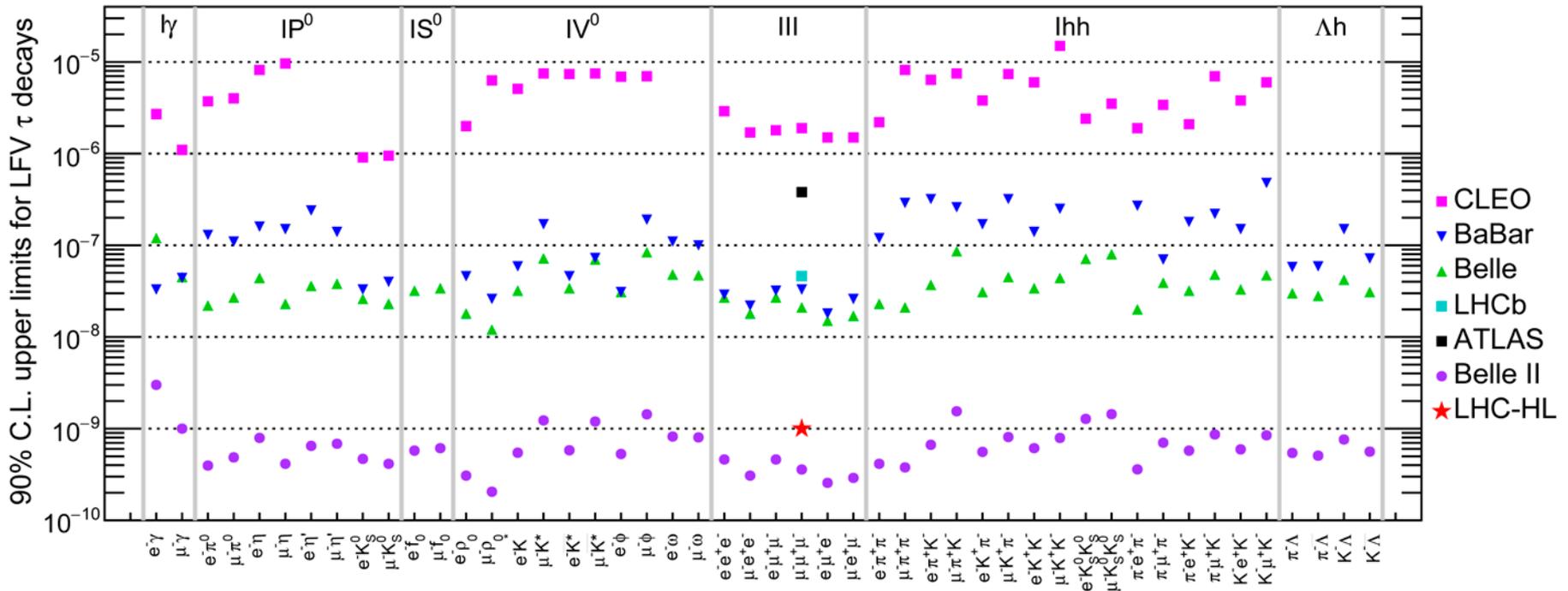
$$\text{UT}_{fit} \quad \bar{\eta} \equiv \eta \left(1 - \frac{1}{2} \lambda^2\right) = 0.348 \pm 0.010$$

$$\bar{\rho} \equiv \rho \left(1 - \frac{1}{2} \lambda^2\right) = 0.148 \pm 0.013$$

$$\alpha = 90.1 \pm 2.2^\circ ; \quad \beta = 23.8 \pm 1.3^\circ ; \quad \gamma = 65.8 \pm 2.2^\circ$$

Bounds on Lepton Flavour Violation

τ Decays (90% CL)



$$\text{Br}(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13} \quad (\text{MEG, 90\% CL})$$

$$\text{Br}(K_L \rightarrow \mu e) < 4.7 \times 10^{-12} \quad (\text{BNL-E871, 90\% CL})$$

$$\text{Br}(B^0 \rightarrow e \mu) < 1.0 \times 10^{-9} \quad (\text{LHCb, 90\% CL})$$

$$\text{Br}(\mu \rightarrow 3e) < 1.0 \times 10^{-12} \quad (\text{SINDRUM, 90\% CL})$$

$$\text{Br}(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11} \quad (\text{BNL-E865, 90\% CL})$$

$$\text{Br}(D^0 \rightarrow e \mu) < 1.3 \times 10^{-8} \quad (\text{LHCb, 90\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \mu) < 7.5 \times 10^{-7} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow e \tau) < 5.0 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(Z^0 \rightarrow \mu \tau) < 6.5 \times 10^{-6} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(H \rightarrow e \mu) < 6.1 \times 10^{-5} \quad (\text{ATLAS, 95\% CL})$$

$$\text{Br}(H \rightarrow e \tau) < 4.7 \times 10^{-3} \quad (\text{CMS, 95\% CL})$$

$$\text{Br}(H \rightarrow \mu \tau) < 2.5 \times 10^{-3} \quad (\text{ATLAS, 95\% CL})$$

SUMMARY

- **Flavour Structure and CP** are major pending questions
- **Related to SSB**  **Scalar Sector (Higgs)**
- Important **cosmological implications (Baryogenesis)**
- Sensitive to **New Physics: Flavour Anomalies!**

Intriguing signals (most anomalies related to 3rd family)

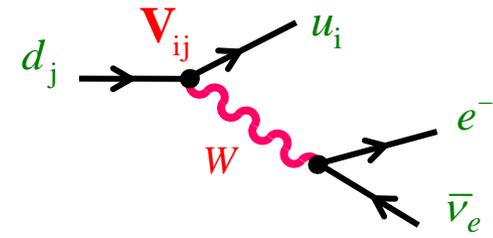
Many questions. Higher statistics & better systematics (QCD) needed

Eagerly awaiting new experimental results

Backup

V_{ij} Determination $(0^- \rightarrow 0^-)$

$K \rightarrow \pi \ell \nu, D \rightarrow K \ell \nu \dots$



$$\langle P'(k') | \bar{u}_i \gamma^\mu d_j | P(k) \rangle = C_{PP'} \left\{ (k+k')^\mu f_+(q^2) + (k-k')^\mu f_-(q^2) \right\}$$

$$\Gamma(P \rightarrow P' l \nu) = \frac{G_F^2 M_P^5}{192 \pi^3} |V_{ij}|^2 C_{PP'}^2 |f_+(0)|^2 \mathbf{I} (1 + \delta_{RC})$$

$f_-(q^2)$ suppressed

$$\mathbf{I} \approx \int_0^{(M_P - M_{P'})^2} \frac{dq^2}{M_P^8} \lambda^{3/2}(q^2, M_P^2, M_{P'}^2) \left| \frac{f_+(q^2)}{f_+(0)} \right|^2$$

$(k-k')^\mu \bar{l} \gamma_\mu (1-\gamma_5) \nu_l \sim m_l$

- Measure the q^2 distribution $\longrightarrow \mathbf{I}$
- Measure Γ $\longrightarrow f_+(0) |V_{ij}|$
- Get a theoretical prediction for $f_+(0)$ $\longrightarrow |V_{ij}|$

Theory is always needed: Symmetries

PDG parametrization of the CKM matrix

$$\mathbf{V} = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{13}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{13}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{13}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{13}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{13}} & c_{23} c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein:

$$s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13} e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$$



$$\mathbf{V} \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

Standard Model \cancel{CP} : 3 fermion families needed

$$\cancel{CP} \iff \mathbf{H}(M_u^2) \cdot \mathbf{H}(M_d^2) \cdot \mathbf{J} \neq 0$$

$$\mathbf{H}(M_u^2) \equiv (m_t^2 - m_c^2) (m_c^2 - m_u^2) (m_t^2 - m_u^2)$$

$$\mathbf{H}(M_d^2) \equiv (m_b^2 - m_s^2) (m_s^2 - m_d^2) (m_b^2 - m_d^2)$$

$$\mathbf{J} = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \delta_{13} = |A^2 \lambda^6 \eta| < 10^{-4}$$

- Low-Energy Phenomena

- Small Effects $\sim \mathbf{J}$

- Big Asymmetries \iff Suppressed Decays

- B Decays are an optimal place for \cancel{CP} signals

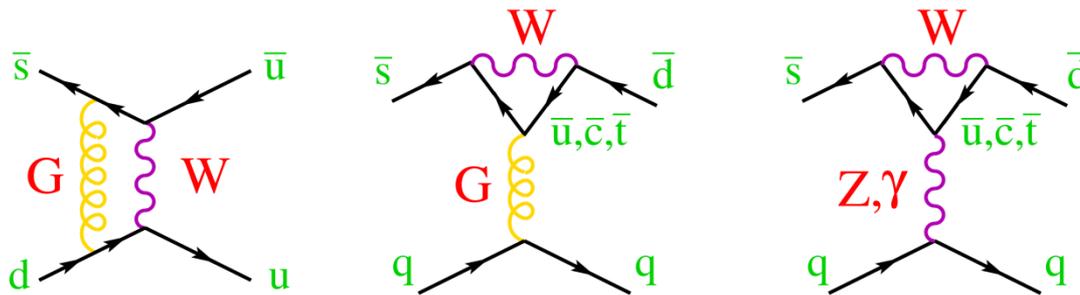
DIRECT C/P in $K \rightarrow \pi \pi$

$$\eta_{+-} \equiv \frac{T(K_L \rightarrow \pi^+ \pi^-)}{T(K_S \rightarrow \pi^+ \pi^-)} \approx \varepsilon_K + \varepsilon'_K$$

$$\eta_{00} \equiv \frac{T(K_L \rightarrow \pi^0 \pi^0)}{T(K_S \rightarrow \pi^0 \pi^0)} \approx \varepsilon_K - 2\varepsilon'_K$$

$$\text{Re}(\varepsilon'_K / \varepsilon_K) \approx \frac{1}{6} \left\{ 1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right\} = (16.6 \pm 2.3) \cdot 10^{-4}$$

NA48, NA31
KTeV, E731



$$\text{Re}(\varepsilon'_K / \varepsilon_K)_{\text{Th}} = (14 \pm 5) \cdot 10^{-4}$$

- Short-distance OPE

Ciuchini et al, Buras et al

- Long-distance χ PT

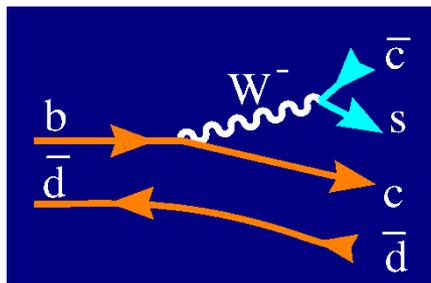
Pallante-Pich-Scimemi

Cirigliano-Ecker-Neufeld-Pich

Cirigliano-Gisbert-Pich-Rodríguez

$$\bar{B}_d^0 \rightarrow J/\Psi K_S^0$$

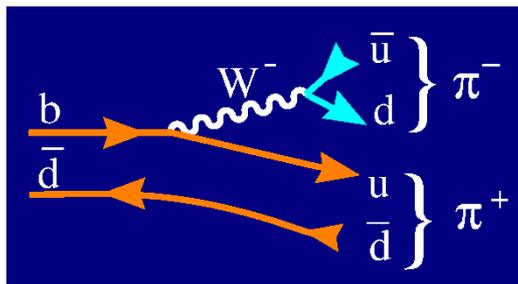
$$\phi \simeq \beta$$



$$V_{cb} V_{cs}^* \sim A\lambda^2$$

$$\bar{B}_d^0 \rightarrow \pi^+ \pi^-$$

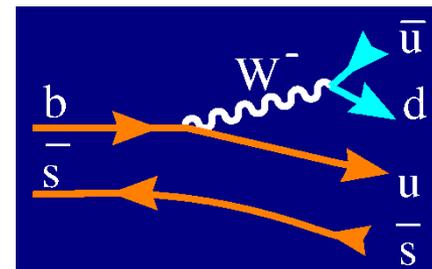
$$\phi \simeq \beta + \gamma = \pi - \alpha$$



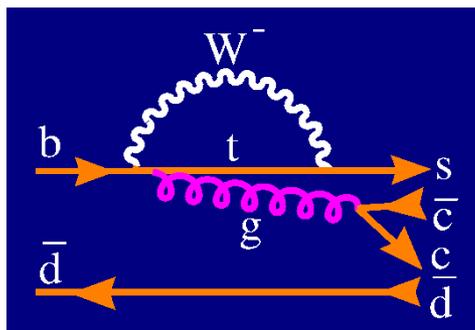
$$V_{ub} V_{ud}^* \sim A\lambda^3(\rho - i\eta)$$

$$\bar{B}_s^0 \rightarrow \rho^0 K_S^0$$

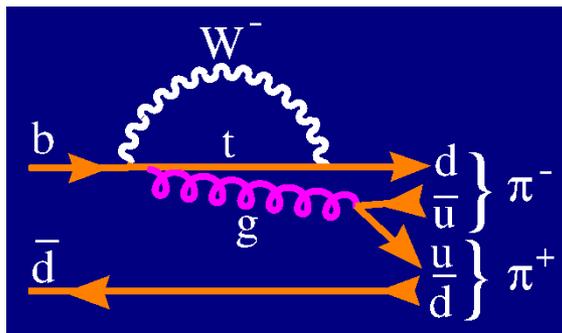
$$\phi \neq \gamma$$



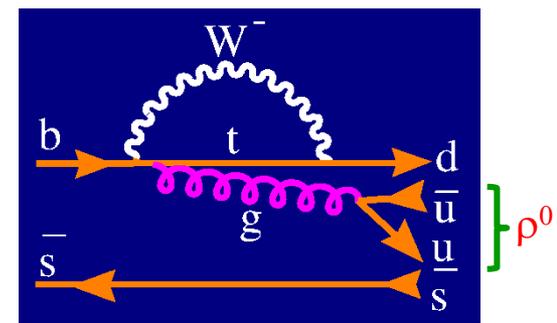
$$V_{ub} V_{ud}^* \sim A\lambda^3(\rho - i\eta)$$



$$V_{tb} V_{ts}^* \sim -A\lambda^2$$



$$V_{tb} V_{td}^* \sim A\lambda^3(1 - \rho + i\eta)$$



$$V_{tb} V_{td}^* \sim A\lambda^3(1 - \rho + i\eta)$$

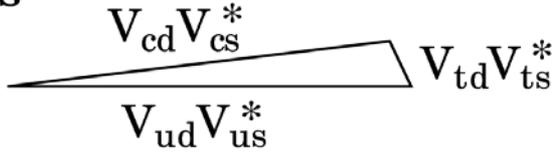
**

BAD

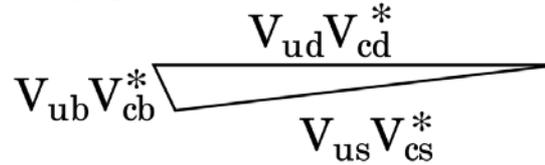
UNITARITY TRIANGLES

$$V_{ui} V_{uj}^* + V_{ci} V_{cj}^* + V_{ti} V_{tj}^* = 0 \quad (i \neq j)$$

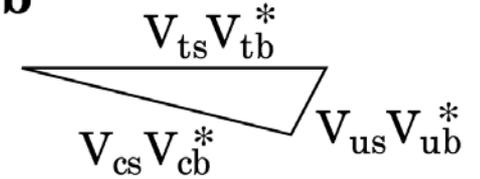
ds



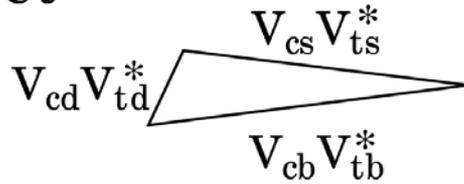
uc



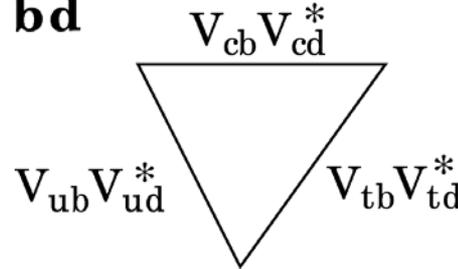
sb



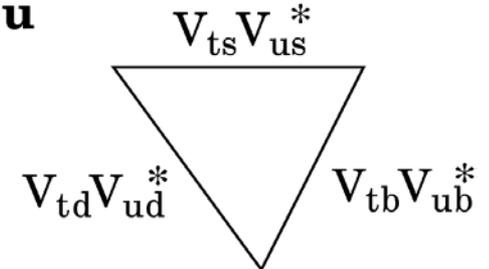
ct



bd



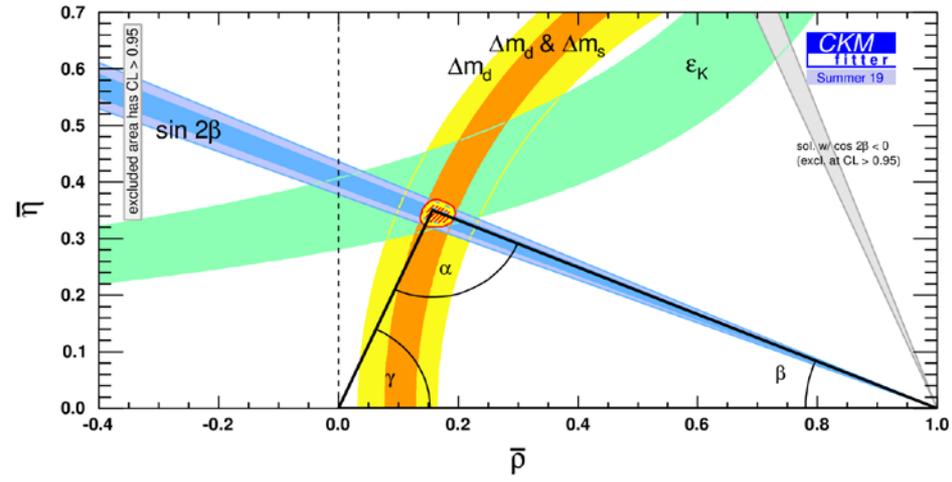
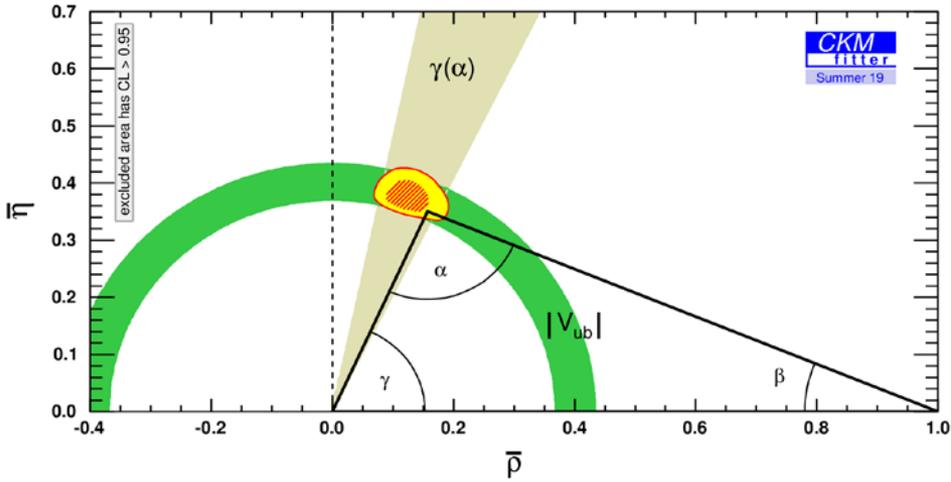
tu



$$V \approx \begin{bmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}(\lambda^4)$$

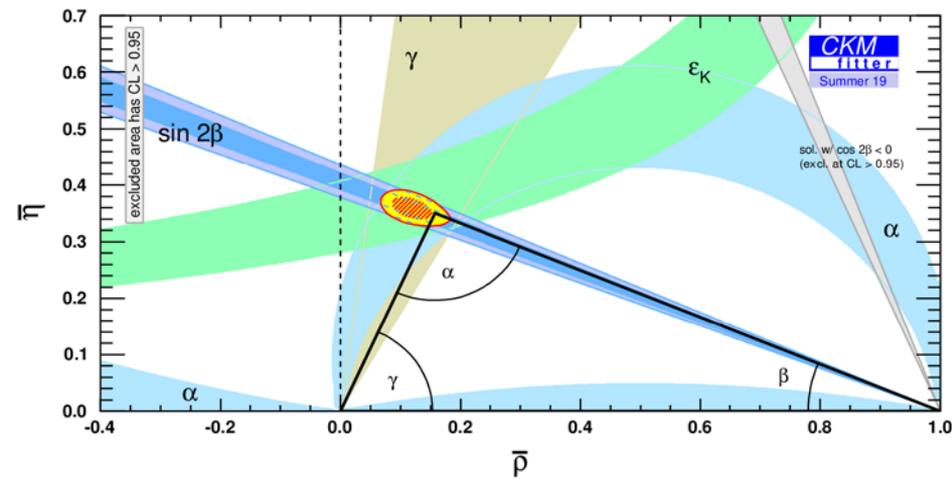
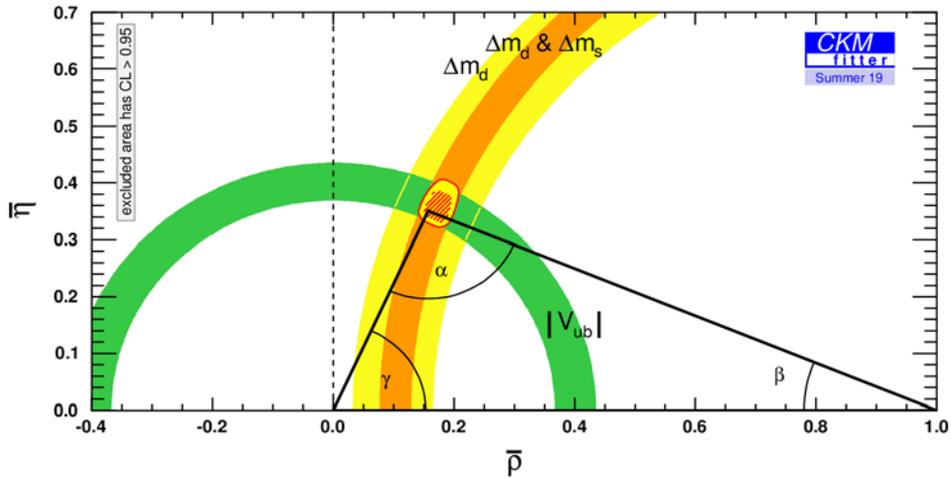
Tree-level determinations

Loop processes

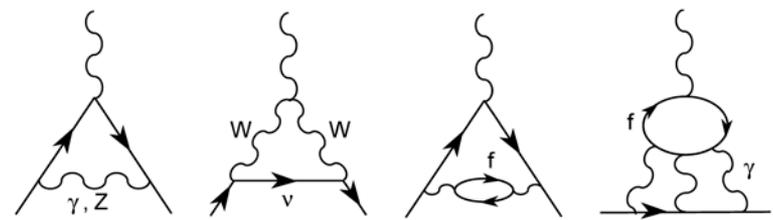


CP Conserving

CP Violating



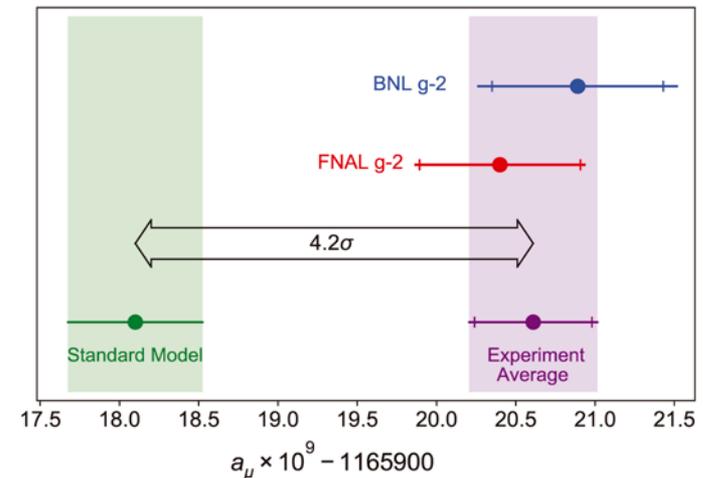
μ Anomalous Magnetic Moment



White Paper (2020)

G. Colangelo, Moriond EW 2021

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	-98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, $udsc$)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, uds)	79(35)
HLbL (phenomenology + lattice)	90(17)
<hr/>	
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^- , LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
<hr/>	
Total SM Value	116 591 810(43)
Experiment (E821)	116 592 089(63)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)



$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \cdot 10^{-11} \quad (4.2\sigma)$$

Much smaller discrepancy with the SM prediction obtained from BMW (lattice) or τ data