## Global fits from $b \rightarrow$ sll decays

## The $b \rightarrow$ s $\ell \ell$ anomalies

## $b \rightarrow s \mu^{+} \mu^{-}$anomaly

Several LHCb measurements deviate from Standard model (SM) predictions by $2-3 \sigma$ :

- Angular observables in $B \rightarrow K^{*} \mu^{+} \mu^{-}$.

LHCb, arXiv:2003.04831, arXiv:2012.13241

- Branching ratios of $B \rightarrow K \mu^{+} \mu^{-}, B \rightarrow K^{*} \mu^{+} \mu^{-}$, and $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$.

LHCb, arXiv:1403.8044, arXiv:1506.08777, arXiv:1606.04731, arXiv:2105.14007


## Hints for LFU violation in $b \rightarrow s \ell^{+} \ell^{-}$decays

Measurements of lepton flavor universality (LFU) ratios $R_{K^{*}}^{[0.045,1.1]}, R_{K^{*}}^{[1.1,6]}, R_{K}^{[1,6]}$ show deviations from SM by 2.3, 2.5, and 3.1 $\sigma$.

LHCb, arXiv:1705.05802, arXiv:2103.11769
Belle, arXiv:1904.02440, arXiv:1908.01848

$$
R_{K^{(*)}}=\frac{B R\left(B \rightarrow K^{(*)} \mu^{+} \mu^{-}\right)}{B R\left(B \rightarrow K^{(*)} e^{+} e^{-}\right)}
$$



## Combination of $B_{s, d} \rightarrow \mu^{+} \mu^{-}$measurements

Measurements of $\operatorname{BR}\left(B_{s, d} \rightarrow \mu^{+} \mu^{-}\right)$by LHCb, CMS, and ATLAS show combined deviation from SM by about $2 \sigma$.


## Theoretical Framework

## $b \rightarrow s \ell \ell$ in the weak effective theory

- Effective Hamiltonian at scale $m_{b}: \quad \mathcal{H}_{\text {eff }}^{\text {bset }}=\mathcal{H}_{\text {eff, }}^{\text {bsl }}+\mathcal{H}_{\text {eff, }}^{\text {bse had }}$
- Semileptonic operators: $\left(\mathcal{N}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \frac{e^{2}}{16 \pi^{2}}\right)$

$$
\begin{aligned}
& \mathcal{H}_{\mathrm{eff}, \mathrm{sl}}^{\text {bsel }}=-\mathcal{N}\left(C_{7}^{b s} O_{7}^{b s}+C_{7}^{b s} O_{7}^{b s}+\sum_{\ell} \sum_{i=9,10, s, P}\left(C_{i}^{b s e l} O_{i}^{\text {bsel }}+C_{i}^{\prime b s e \ell} O_{i}^{\text {bsel }}\right)\right)+\text { h.c. } \\
& O_{9}^{\text {bsel }}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \quad O_{9}^{\text {bsel }}=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \\
& O_{10}^{\text {bsel }}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), \quad O_{10}^{\text {bssel }}=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), \\
& O_{7}^{b s}=\frac{m_{b}}{e}\left(\bar{s} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu}, \quad O_{7}^{\prime b s}=\frac{m_{b}}{e}\left(\bar{s} \sigma_{\mu \nu} P_{L} b\right) F^{\mu \nu}, \\
& O_{S}^{b s e \ell}=m_{b}\left(\bar{s} P_{R} b\right)(\bar{\ell} \ell), \quad O_{S}^{\text {bsel }}=m_{b}\left(\bar{s} P_{L} b\right)(\bar{\ell} \ell), \\
& O_{P}^{b s \ell \ell}=m_{b}\left(\bar{s} P_{R} b\right)\left(\bar{\ell} \gamma_{5} \ell\right), \quad O_{P}^{\text {bssel }}=m_{b}\left(\bar{s} P_{L} b\right)\left(\bar{\ell} \gamma_{5} \ell\right) .
\end{aligned}
$$

## $b \rightarrow s \ell \ell$ in the weak effective theory

- Effective Hamiltonian at scale $m_{b}: \quad \mathcal{H}_{\text {eff }}^{\text {bse }}=\mathcal{H}_{\text {eff, sl }}^{\text {bsel }}+\mathcal{H}_{\text {eff, had }}^{\text {bse }}$
- Semileptonic operators: $\left(\mathcal{N}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \frac{e^{2}}{16 \pi^{2}}\right)$

$$
\begin{aligned}
& \mathcal{H}_{\mathrm{eff}, \mathrm{sl}}^{b s \ell \ell}=-\mathcal{N}\left(C_{7}^{b s} O_{7}^{b s}+C_{7}^{\prime b s} O_{7}^{\prime b s}+\sum_{\ell} \sum_{i=9,10, S, P}\left(C_{i}^{b s \ell \ell} O_{i}^{b s \ell \ell}+C_{i}^{\prime b s \ell \ell} O_{i}^{\prime b s \ell \ell}\right)\right)+\text { h.c. } \\
& O_{9}^{b s \ell \ell}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \quad O_{9}^{\prime b s \ell \ell}=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right) \text {, } \\
& O_{10}^{\text {bsel }}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), \quad O_{10}^{\text {bs }} \text { bl }=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), \\
& O_{7}^{b s}=\frac{m_{b}}{e}\left(\bar{s} \sigma_{\mu \nu} P_{R} b\right) F^{\mu \nu}, \quad O_{7}^{\prime b s}=\frac{m_{b}}{e}\left(\bar{s} \sigma_{\mu \nu} P_{L} b\right) F^{\mu \nu}, \\
& O_{S}^{b s e \ell}=m_{b}\left(\bar{s} P_{R} b\right)(\bar{\ell} \ell), \quad O_{S}^{\text {bsel }}=m_{b}\left(\bar{s} P_{L} b\right)(\bar{\ell} \ell), \\
& O_{P}^{\text {bs } \ell \ell}=m_{b}\left(\bar{s} P_{R} b\right)\left(\bar{\ell} \gamma_{5} \ell\right), \quad O_{P}^{\text {bs } \ell \ell}=m_{b}\left(\bar{s} P_{L} b\right)\left(\bar{\ell} \gamma_{5} \ell\right) .
\end{aligned}
$$

- Hadronic operators:

$$
\begin{aligned}
& \mathcal{H}_{\text {eff, had }}^{\text {bsel }}=-\mathcal{N} \frac{16 \pi^{2}}{e^{2}}\left(C_{8}^{b s} O_{8}^{b s}+C_{8}^{\text {bs }} O_{8}^{\text {bs }}+\sum_{i=1 . .6} C_{i}^{\text {bsel }} O_{i}^{b s}\right)+\text { h.c. } \\
& \text { e.g. } \quad O_{1}^{b s}=\left(\bar{s} \gamma_{\mu} P_{L} T^{a} c\right)\left(\bar{c} \gamma^{\mu} P_{L} T^{a} b\right), \quad O_{2}^{b s}=\left(\bar{s} \gamma_{\mu} P_{L} c\right)\left(\bar{c} \gamma^{\mu} P_{L} b\right) .
\end{aligned}
$$

Theory of $B \rightarrow$ M $\ell$ decays $\left(M=K, K^{*}, \phi\right)$

$$
\begin{aligned}
\mathcal{M}(B \rightarrow M \ell \ell) & =\langle M \ell \ell| \mathcal{H}_{\mathrm{eff}}^{\text {bsel }}|B\rangle \\
& =\mathcal{N}\left[\left(\mathcal{A}_{V}^{\mu}+\mathcal{H}^{\mu}\right) \bar{u}_{\ell} \gamma_{\mu} v_{\ell}+\mathcal{A}_{A}^{\mu} \bar{u}_{\ell} \gamma_{\mu} \gamma_{5} v_{\ell}+\mathcal{A}_{S} \bar{u}_{\ell} v_{\ell}+\mathcal{A}_{p} \bar{u}_{\ell} \gamma_{5} v_{\ell}\right]
\end{aligned}
$$



$$
\begin{aligned}
\mathcal{A}_{V}^{\mu}= & -\frac{2 i m_{b}}{q^{2}} C_{7}\langle M| \bar{s} \sigma^{\mu \nu} q_{\nu} P_{R} b|B\rangle+C_{9}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle \\
& +\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{A}^{\mu}= & C_{10}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{S}= & C_{S}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{P}= & C_{P}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right)
\end{aligned}
$$

Theory of $B \rightarrow$ M $\ell$ decays $\left(M=K, K^{*}, \phi\right)$

$$
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\mathcal{M}(B \rightarrow M \ell \ell) & =\langle M \ell \ell| \mathcal{H}_{\mathrm{eff}}^{\text {bsel }}|B\rangle \\
& =\mathcal{N}\left[\left(\mathcal{A}_{V}^{\mu}+\mathcal{H}^{\mu}\right) \bar{u}_{\ell} \gamma_{\mu} v_{\ell}+\mathcal{A}_{A}^{\mu} \bar{u}_{\ell} \gamma_{\mu} \gamma_{5} v_{\ell}+\mathcal{A}_{S} \bar{u}_{\ell} v_{\ell}+\mathcal{A}_{p} \bar{u}_{\ell} \gamma_{5} v_{\ell}\right]
\end{aligned}
$$

Local:


$$
\begin{aligned}
\mathcal{A}_{V}^{\mu}= & -\frac{2 i m_{b}}{q^{2}} C_{7}\langle M| \bar{s} \sigma^{\mu \nu} q_{\nu} P_{R} b|B\rangle+C_{9}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle \\
& +\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{A}^{\mu}= & C_{10}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{S}= & C_{S}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
\mathcal{A}_{P}= & C_{P}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right)
\end{aligned}
$$



$$
\begin{array}{r}
\mathcal{H}^{\mu}=\frac{-16 i \pi^{2}}{q^{2}} \sum_{i=1 . .6,8} C_{i} \int d x^{4} e^{i q \cdot x}\langle M| T\left\{j_{\mathrm{em}}^{\mu}(x), O_{i}(0)\right\}|B\rangle \\
j_{\mathrm{em}}^{\mu}=\sum_{q} Q_{q} \bar{q} \gamma^{\mu} q
\end{array}
$$

## Form factors



$$
\begin{aligned}
\mathcal{A}_{V}^{\mu}= & -\frac{2 i m_{b}}{q^{2}} C_{7}\langle M| \bar{s} \sigma^{\mu \nu} q_{\nu} P_{R} b|B\rangle+C_{9}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle \\
& +\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right) \\
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\mathcal{A}_{P}= & C_{P}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right)
\end{aligned}
$$

- Wilson coefficients: short-distance UV physics, perturbative
- Form Factors: hadronic physic, non-perturbative, a main source of uncertainty
- Not all $\langle M| \bar{s} \Gamma_{i} b|B\rangle$ matrix elements independent:
- 3 form factors for spin zero final states $M=\boldsymbol{K}$
- 7 form factors for spin one final states $M=K^{*}, \phi$


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$$
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\mathcal{A}_{V}^{\mu}= & -\frac{2 i m_{b}}{q^{2}} C_{7}\langle M| \bar{s} \sigma^{\mu \nu} q_{\nu} P_{R} b|B\rangle+C_{9}\langle M| \bar{s} \gamma^{\mu} P_{L} b|B\rangle \\
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\mathcal{A}_{P}= & C_{P}\langle M| \bar{s} P_{R} b|B\rangle+\left(P_{L} \leftrightarrow P_{R}, C_{i} \rightarrow C_{i}^{\prime}\right)
\end{aligned}
$$

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- Form Factors: hadronic physic, non-perturbative, a main source of uncertainty
- Not all $\langle M| \bar{s} \Gamma_{i} b|B\rangle$ matrix elements independent:
- 3 form factors for spin zero final states $M=\boldsymbol{K}$
- 7 form factors for spin one final states $M=K^{*}, \phi$
- Determination of form factors
- high $q^{2}$ : Lattice QCD
- low $q^{2}$ : Light-Cone Sum Rules (LCSR)

Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945
Gubernari, Kokulu, van Dyk, arXiv:1811.00983
Ball, Zwicky, arXiv:hep-ph/0406232

- low + high $q^{2}$ : Combined fit to LCSR + lattice

Bharucha, Straub, Zwicky, arXiv:1503.05534
Gubernari, Kokulu, van Dyk, arXiv:1811.00983 Altmannshofer, Straub, arXiv:1411.3161

## Non-local matrix elements



$$
\begin{array}{r}
\mathcal{H}^{\mu}=\frac{-16 i \pi^{2}}{q^{2}} \sum_{i=1 . .6,8} C_{i} \int d x^{4} e^{i q \cdot x}\langle M| T\left\{j_{\mathrm{em}}^{\mu}(x), O_{i}(0)\right\}|B\rangle \\
j_{\mathrm{em}}^{\mu}=\sum_{q} Q_{q} \bar{q} \gamma^{\mu} q
\end{array}
$$

- Leading terms for $q^{2}<6 \mathrm{GeV}^{2}$ from QCD factorization (QCDF)

Beneke, Feldmann, Seidel, arXiv:hep-ph/0106067

- Subleading terms not calculable in QCDF, a main source of uncertainty
- Large subleading terms could mimic new physics in $C_{9}$
e.g. Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, arXiv:1512.07157


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$$

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e.g. Ciuchini, Fedele, Franco, Mishima, Paul, Silvestrini, Valli, arXiv:1512.07157
- Several compatible approaches to estimate subleading terms at low $q^{2}$
- LCSR estimates

Khodjamirian, Mannel, Pivovarov, Wang, arXiv:1006.4945 Gubernari, van Dyk, Virto, arXiv:2011.09813

- order of magnitude estimate parameterized as polynomial in $q^{2}$

Descotes-Genon, Hofer, Matias, Virto, arXiv:1407.8526, arXiv:1510.04239
Arbey, Hurth, Mahmoudi, Neshatpour, arXiv:1806.02791
Altmannshofer, Straub, arXiv:1411.3161

- fit of sum of resonances to data

Blake, Egede, Owen, Pomery, Petridis, arXiv:1709.03921

- analyticity + experimental data on $b \rightarrow s c \bar{c} \quad$ Bobeth, Chrzaszcz, van Dyk, Virto, arXiv:1707.07305


## Uncertainties of observables

$-\boldsymbol{B} \rightarrow \boldsymbol{K} \mu \mu, \boldsymbol{B} \rightarrow \boldsymbol{K}^{*} \mu \mu$, and $\boldsymbol{B}_{\mathrm{s}} \rightarrow \phi \mu \mu$ branching fractions: fully affected by uncertainties from form factors and non-local matrix elements

- Optimized angular observables:
reduced impact of form factor uncertainties
- $B_{s} \rightarrow \mu \mu$ branching fraction

Small uncertainties (no hadron in final state, $B_{s}$ decay constant from lattice)

- LFU observables

Tiny hadronic uncertainties in SM (but can be larger in the presence of new physics)

## New physics interpretation

## New physics in $b \rightarrow$ s $\ell \ell$ in the weak effective theory

- Effective Hamiltonian at scale $m_{b}: \quad \mathcal{H}_{\text {eff }}^{\text {bsel }}=\mathcal{H}_{\mathrm{eff}, \mathrm{SM}}^{\text {bsel }}+\mathcal{H}_{\mathrm{eff}, \mathrm{NP}}^{\text {bse }}$

$$
\mathcal{H}_{\mathrm{eff}, \mathrm{NP}}^{\text {bsel }}=-\mathcal{N} \sum_{\ell=e, \mu} \sum_{i=9,10, S, P}\left(C_{i}^{b s \ell \ell} O_{i}^{\text {bsel }}+C_{i}^{\prime b s \ell \ell} O_{i}^{\prime b s \ell \ell}\right)+\text { h.c. }
$$

- Operators considered here $(\ell=e, \mu)$

$$
\begin{array}{ll}
O_{9}^{b s l \ell}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), & O_{9}^{\prime b s \ell \ell}=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \ell\right), \\
O_{10}^{b s \ell \ell}=\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), & O_{10}^{\prime \text { bsel }}=\left(\bar{s} \gamma_{\mu} P_{R} b\right)\left(\bar{\ell} \gamma^{\mu} \gamma_{5} \ell\right), \\
O_{S}^{b s \ell \ell}=m_{b}\left(\bar{s} P_{R} b\right)(\bar{\ell} \ell), & O_{S}^{\prime \text { bsel }}=m_{b}\left(\bar{s} P_{L} b\right)(\bar{\ell} \ell) \\
O_{P}^{b s \ell \ell}=m_{b}\left(\bar{s} P_{R} b\right)\left(\bar{\ell} \gamma_{5} \ell\right), & O_{P}^{\text {bs } \ell \ell}=m_{b}\left(\bar{s} P_{L} b\right)\left(\bar{\ell} \gamma_{5} \ell\right) .
\end{array}
$$

- Not considered here
- Dipole operators: strongly constrained by radiative decays.
e.g. [arXiv:1608.02556]
- Four quark operators: dominant effect from RG running above $m_{B}$.

Jäger, Leslie, Kirk, Lenz [arXiv:1701.09183]

## Setup

- Quantify agreement between theory and experiment by $\chi^{2}$ function

$$
\chi^{2}(\vec{C})=\left(\vec{O}_{\text {exp }}-\vec{o}_{\mathrm{th}}(\vec{C})\right)^{\top}\left(C_{\text {exp }}+C_{\mathrm{th}}\right)^{-1}\left(\vec{O}_{\text {exp }}-\vec{O}_{\mathrm{th}}(\vec{C})\right) .
$$

- theory errors and correlations in covariance matrix $C_{\text {th }}$
- experimental errors and available correlations in covariance matrix $C_{\text {exp }}$
- Theory errors depend on new physics Wilson coefficients $\boldsymbol{C}_{\mathrm{th}}(\overrightarrow{\boldsymbol{C}})$
- $\Delta \chi^{2}$ and pull

$$
\begin{gathered}
\text { pull }_{1 \mathrm{D}}=1 \sigma \cdot \sqrt{\Delta \chi^{2}}, \quad \text { where } \Delta \chi^{2}=\chi^{2}(\overrightarrow{0})-\chi^{2}\left(\vec{C}_{\text {best fit }}\right) . \\
\text { pull }_{2 \mathrm{D}}=1 \sigma, 2 \sigma, 3 \sigma, \ldots \text { for } \quad \Delta \chi^{2} \approx 2.3,6.2,11.8, \ldots
\end{gathered}
$$

- New physics scenarios Weak Effective Theory (WET) at scale 4.8 GeV


## Setup

- Quantify agreement between theory and experiment by $\chi^{2}$ function

$$
\chi^{2}(\vec{C})=\left(\vec{O}_{\exp }-\vec{O}_{\mathrm{th}}(\vec{C})\right)^{\top}\left(C_{\exp }+C_{\mathrm{th}}(\vec{C})\right)^{-1}\left(\vec{O}_{\exp }-\vec{O}_{\mathrm{th}}(\vec{C})\right) .
$$

- theory errors and correlations in covariance matrix $C_{\text {th }}$
- experimental errors and available correlations in covariance matrix $C_{\text {exp }}$
- Theory errors depend on new physics Wilson coefficients $C_{\text {th }}(\vec{C}) \quad$ *NEW*
- $\Delta \chi^{2}$ and pull

$$
\begin{gathered}
\text { pull }_{1 D}=1 \sigma \cdot \sqrt{\Delta \chi^{2}}, \quad \text { where } \Delta \chi^{2}=\chi^{2}(\overrightarrow{0})-\chi^{2}\left(\vec{C}_{\text {best fit }}\right) . \\
\text { pull }_{2 D}=1 \sigma, 2 \sigma, 3 \sigma, \ldots \text { for } \quad \Delta \chi^{2} \approx 2.3,6.2,11.8, \ldots
\end{gathered}
$$

- New physics scenarios Weak Effective Theory (WET) at scale 4.8 GeV


## Scenarios with a single Wilson coefficients

| Wilson coefficient | $\begin{aligned} & \qquad \rightarrow \text { s } \mu \mu \\ & \text { best fit } \end{aligned}$ |  | $\begin{aligned} & \text { LFU, } B_{s} \rightarrow \mu \mu \\ & \text { best fit } \quad \text { pull } \end{aligned}$ |  | all rare $B$ decays best fit pull |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $C_{9}^{\text {bs } \mu \mu}$ | $-0.87_{-0.18}^{+0.19}$ | $4.3 \mathrm{\sigma}$ | $-0.74_{-0.21}^{+0.20}$ | 4.1 $\sigma$ | $-0.80{ }_{-0.14}^{+0.14}$ | $5.7 \sigma$ |
| $C_{10}^{\text {bs } \mu \mu}$ | +0.49 ${ }_{-0.25}^{+0.24}$ | $1.9 \sigma$ | +0.60 ${ }_{-0.14}^{+0.14}$ | $4.7 \sigma$ | $+0.55_{-0.12}^{+0.12}$ | $4.8 \sigma$ |
| $C_{9}^{\text {bs } \mu \mu}$ | +0.39 ${ }_{-0.26}^{+0.27}$ | 1.5 $\sigma$ | $-0.32_{-0.17}^{+0.16}$ | $2.0 \sigma$ | $-0.14_{-0.13}^{+0.13}$ | $1.0 \sigma$ |
| $C_{10}^{\text {bs }}{ }^{\text {m }}$ | -0.10-0.16 | $0.6 \sigma$ | +0.06 ${ }_{-0.12}^{+0.12}$ | 0.5\% | +0.04-0.10 | $0.4 \sigma$ |
| $C_{9}^{\text {bs } \mu \mu}=C_{10}^{\text {bs } \mu \mu}$ | -0.34-0.16 | $2.1 \sigma$ | +0.43 ${ }_{-0.18}^{+0.18}$ | $2.4 \sigma$ | $-0.01_{-0.12}^{+0.12}$ | 0.1 $\sigma$ |
| $C_{9}^{\text {bs } \mu \mu}=-C_{10}^{\text {bs } \mu \mu}$ | $-0.60_{-0.12}^{+0.13}$ | $4.3 \sigma$ | $-0.35_{-0.08}^{+0.08}$ | $4.6 \sigma$ | $-0.41_{-0.07}^{+0.07}$ | $5.9 \sigma$ |

Only small pull for

- Coefficients with $\ell=e$ (cannot explain $b \rightarrow s \mu \mu$ anomaly and $B_{s} \rightarrow \mu \mu$ )
- Scalar coefficients (can only reduce tension in $B_{s} \rightarrow \mu \mu$ )


## Scenarios with a single Wilson coefficients

| Wilson coefficient |  | $b \rightarrow s \mu \mu$ | pull | LFU, $B_{s} \rightarrow \mu \mu$ |  | all rare $B$ decays |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\square}{2}$ | $C_{9}^{\text {bs } \mu \mu}$ | $-0.87{ }_{-0.18}^{+0.19}$ | $4.3 \sigma$ | $-0.74_{-0.21}^{+0.20}$ | $4.1 \sigma$ | $-0.80_{-0.14}^{+0.14}$ | $5.7 \sigma$ |
| $\bigcirc$ | $C_{10}^{\text {bs } \mu \mu}$ | $+0.49_{-0.25}^{+0.24}$ | $1.9 \sigma$ | $+0.60_{-0.14}^{+0.14}$ | $4.7 \sigma$ | $+0.55_{-0.12}^{+0.12}$ | $4.8 \sigma$ |
| z | $C_{9}^{b s \mu \mu}=-C_{10}^{\text {bs } \mu \mu}$ | $-0.60{ }_{-0.12}^{+0.13}$ | $4.3 \sigma$ | $-0.35_{-0.08}^{+0.08}$ | $4.6 \sigma$ | $-0.41_{-0.07}^{+0.07}$ | $5.9 \sigma$ |
|  | $C_{9}^{\text {bs }}$ \% | $-0.96{ }_{-0.18}^{+0.19}$ | $4.6 \sigma$ | $-0.74{ }_{-0.21}^{+0.20}$ | $4.1 \sigma$ | $-0.83_{-0.14}^{+0.14}$ | $5.9 \sigma$ |
| $\sum^{ \pm}$ | $C_{10}^{\text {bs } \mu \mu}$ | $+0.51_{-0.22}^{+0.22}$ | $2.3 \sigma$ | $+0.60_{-0.14}^{+0.14}$ | $4.7 \sigma$ | $+0.56_{-0.12}^{+0.12}$ | $4.9 \sigma$ |
| $\omega$ | $C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}$ | $-0.64{ }_{-0.17}^{+0.16}$ | $4.3 \sigma$ | $-0.35_{-0.08}^{+0.08}$ | $4.6 \sigma$ | $-0.41_{-0.07}^{+0.07}$ | $5.9 \sigma$ |

Visible effect of theory errors depending on new physics

## Scenarios with two Wilson coefficients



- Before Moriond 2021

WET at 4.8 GeV

## Scenarios with two Wilson coefficients



- After Moriond 2021:
- $\boldsymbol{R}_{\boldsymbol{K}}$ : smaller uncertainty
- $\boldsymbol{B}_{s} \rightarrow \mu \mu$ : smaller uncertainty, better agreement with $b \rightarrow s \mu \mu$

[^0]
## Scenarios with two Wilson coefficients



Combination of $B_{s} \rightarrow \mu^{+} \mu^{-}$and NC LFU observables ( $R_{K}, R_{K^{*}}, D_{P_{4^{\prime}, 5^{\prime}}}$ )

- NCLFU obs. \& $B_{s} \rightarrow \mu \mu$ : very clean theory prediction, insensitive to universal $C_{9}^{\text {univ. }}$
- $b \rightarrow$ s $\mu \mu$ sensitive to univ. coeff. possibly afflicted by underestimated hadr. uncert.
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[^1]
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- After Moriond 2021:
- LFU obs. \& $B_{s} \rightarrow \mu \mu$ : smaller uncertainty, better agreement with $b \rightarrow s \mu \mu$

[^2]
## Scenarios with two Wilson coefficients



- Global fit in $C_{9}^{b s \mu \mu}{ }_{-} C_{10}^{b s \mu \mu}$ plane prefers negative $C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}$
- Tension between fits to $b \rightarrow s \mu \mu$ observables and $R_{K} \& R_{K^{*}}$ could be reduced by LFU contribution to $\boldsymbol{C}_{9}$

[^3]
## Scenarios with two Wilson coefficients

- Before Moriond 2021


WET at 4.8 GeV

- Perform two-parameter fit in space of $\boldsymbol{C}_{9}^{\text {univ. }}$ and $\Delta C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}$ :

$$
\begin{aligned}
C_{9}^{b s e e} & =C_{9}^{b s \tau \tau}=C_{9}^{\text {univ. }} \\
C_{9}^{b s \mu \mu} & =C_{9}^{\text {univ. }}+\Delta C_{9}^{b s \mu \mu} \\
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\end{aligned}
$$

scenario first considered in
Algueró et al., arXiv:1809.08447

- Preference for non-zero $C_{9}^{\text {univ. }}$
- could be mimicked by hadronic effects
- can arise from RG effects:


Bobeth, Haisch, arXiv:1109.1826
Crivellin, Greub, Müller, Saturnino, arXiv:1807.02068

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WET at 4.8 GeV

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## RG effect in SMEFT

RG effects require scale separation

- Consider SMEFT


Possible operators:
$-\left[0_{l q}^{(3)}\right]_{3323}=\left(\bar{I}_{3} \gamma_{\mu} \tau^{a} l_{3}\right)\left(\bar{q}_{2} \gamma^{\mu} \tau^{a} q_{3}\right):$ Might also explain $R_{D^{(*)}}$ anomalies!
$-\left[O_{l q}^{(1)}\right]_{3323}=\left(\bar{I}_{3} \gamma_{\mu} \boldsymbol{I}_{3}\right)\left(\overline{\boldsymbol{q}}_{2} \gamma^{\mu} \boldsymbol{q}_{3}\right):$


Strong constraints from $B \rightarrow K \nu \nu$ require $\left[\mathbf{C}_{l q}^{(1)}\right]_{3323} \approx\left[C_{l q}^{(3)}\right]_{3323}$

- $\mathbf{U}_{1}$ vector leptoquark $(\mathbf{3}, \mathbf{1})_{2 / 3}$ couples LH fermions

$$
\mathcal{L}_{U_{1}} \supset g_{l q}^{j i}\left(\bar{q}^{i} \gamma^{\mu} \dot{p}\right) U_{\mu}+\text { h.c. }
$$

- Generates semi-leptonic operators at tree-level

$$
\left[C_{l q}^{(1)}\right]_{i j k l}=\left[C_{l q}^{(3)}\right]_{i j k l}=-\frac{g_{l q}^{j k} g_{l q}^{i / *}}{2 M_{U}^{2}}
$$



## Scenarios with two Wilson coefficients

Constraint on scalar coefficients


- Before Moriond 2021


## WET at 4.8 GeV

## Scenarios with two Wilson coefficients

Constraint on scalar coefficients


- After Moriond 2021:
- Region corresponding to mass eigenstate rate asymmetry $A_{\Delta \Gamma}=-1$ excluded at $1 \sigma$

[^4]
## Scenarios with two Wilson coefficients

Constraint on scalar coefficients


- After Moriond 2021:
- Region corresponding to mass eigenstate rate asymmetry $A_{\Delta \Gamma}=-1$ excluded at $1 \sigma$
- Clear effect of new, more precise measurement of effective $B_{S} \rightarrow \mu \mu$ lifetime $\tau_{\text {eff }}$

[^5]
## Summary and Outlook

## Summary

- Updated measurements of $R_{K}$ and $B_{s} \rightarrow \mu \mu$ (new $B_{s} \rightarrow \phi \mu \mu$ data not included yet)
- New physics in the single muonic Wilson coefficients $C_{9}^{b s \mu \mu}, C_{10}^{b s \mu \mu}$, and $C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}$ gives clearly better fit to data than SM (pull ${ }_{1 D} \gtrsim 5 \sigma$ ).
- Slight tension between $R_{K^{(*)}}$ and $b \rightarrow s \mu \mu$ in $C_{9}^{b s \mu \mu} C_{10}^{b s \mu \mu}$ scenario can be reduced by lepton flavor universal $C_{9}^{\text {univ. }}$.


## Outlook

Some directions for improving theory predictions and global fits

- Updated $B \rightarrow$ Kौ€ LCSR form factors including $\mathcal{O}\left(\alpha_{s}\right)$ corrections

Gubernari, Kokulu, van Dyk, arXiv:1811.00983 Ball, Zwicky, arXiv:hep-ph/0406232

- Implement recent results on non-local matrix elements into global fits

Bobeth, Chrzaszcz, van Dyk, Virto, arXiv:1707.07305 Gubernari, van Dyk, Virto, arXiv:2011.09813

- $B \rightarrow K^{*}$ beyond narrow width on the lattice: study $B \rightarrow K \pi$ transition amplitude

Briceño, Hansen, Walker-Loud, arXiv:1406.5965

- $B \rightarrow K^{*}$ beyond narrow width in LCSR: implement into global fits

Descotes-Genon, Khodjamirian, Virto, arXiv:1908.02267

- Non-local $B \rightarrow K \ell \ell$ matrix elements on the lattice


## Backup slides

## The global picture in the SMEFT

## The global picture in the SMEFT



$$
\begin{aligned}
& {\left[C_{l q}^{(1)}\right]_{3323}=\left[C_{l q}^{(3)}\right]_{3323} \quad \Rightarrow \quad C_{9}^{\text {univ. }} \quad \text { (RG effect) }} \\
& {\left[C_{l q}^{(1)}\right]_{2223}=\left[C_{l q}^{(3)}\right]_{2223} \quad \Rightarrow \quad \Delta C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}}
\end{aligned}
$$

- Clear preference for non-zero $\left[C_{l q}^{(1)}\right]_{3323}=\left[C_{l q}^{(3)}\right]_{3323}$


## The global picture in the SMEFT


$\left[C_{l q}^{(1)}\right]_{3323}=\left[C_{l q}^{(3)}\right]_{3323} \quad \Rightarrow \quad C_{9}^{\text {univ. }} \quad$ (RG effect)
$\left[C_{l q}^{(1)}\right]_{2223}=\left[C_{l q}^{(3)}\right]_{2223} \Rightarrow \Delta C_{9}^{b s \mu \mu}=-C_{10}^{b s \mu \mu}$

- Clear preference for non-zero $\left[C_{l q}^{(1)}\right]_{3323}=\left[C_{l q}^{(3)}\right]_{3323}$
- $R_{D^{(*)}}$ explanation:

Very good agreement between $R_{D(*)}$, $R_{K^{(*)}}$ and $b \rightarrow s \mu \mu$ explanations

## The global picture in the SMEFT


$\left[C_{l q}^{(1)}\right]_{3323}=\left[C_{l q}^{(3)}\right]_{3323} \quad \Rightarrow \quad C_{9}^{\text {univ. }} \quad$ (RG effect)
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- $R_{D^{(*)}}$ explanation:

Very good agreement between $R_{D(*)}$, $R_{K^{(*)}}$ and $b \rightarrow s \mu \mu$ explanations

- Only a simple SMEFT scenario $\Rightarrow$ Consider explicit models that yield this coefficients
$\Rightarrow$ Good candidate: $\mathbf{U}_{1}$ Leptoquark


## Correlation effects in the global likelihood

## Slightly different results by different groups

Descotes-Genon, PS, Talk at Beyond the Flavour Anomalies
https://conference.ippp.dur.ac.uk/event/876/

|  | All |  |  | LFUV |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1D Hyp. | $1 \sigma$ | Pull ${ }_{\text {SM }}$ | p -value | e $1 \sigma$ | Pullsm | p -value |
| $\mathcal{C}_{9 \mu}^{\mathrm{NP}} \quad[-1$ | [-1.19, -0.88] | 6.3 | 37.5\% | \% [-1.25, -0.61] | 3.3 | 60.7\% |
| $\mathcal{C}_{9 \mu}^{\mathrm{NP}}=-\mathcal{C}_{10 \mu}^{\mathrm{NP}} \quad[-0$ | [-0.59, -0.41] | 5.8 | 25.3\% | \% [-0.50, -0.28] | 3.7 | 75.3\% |
| $\mathcal{C}_{9 \mu}^{\mathrm{NP}}=-\mathcal{C}_{9}{ }^{\prime} \mu \quad[-1$ | [-1.17, -0.87] | 6.2 | 34.0\% | \% [-2.15, -1.05] | 3.1 | 53.1\% |
| Coefficient | type | best fit |  | $1 \sigma$ | pull $_{10}=$ | $\sqrt{\Delta \chi^{2}}$ |
| $C_{9}^{\text {bs }}$ / ${ }^{\text {a }}$ | $L \otimes V$ | -0.93 |  | [-1.07, -0.79] | 6.2 |  |
| $\mathrm{C}_{0}^{\text {bs } \mu \mu}$ | $R \otimes V$ | +0.14 |  | $[-0.02,+0.31]$ | 0.9 |  |
| $\mathrm{C}_{10}^{\text {bs }}{ }^{\text {en }}$ | $L \otimes A$ | +0.71 |  | [ $+0.58,+0.84]$ | 5.7 |  |
| $C_{10}^{\text {bs } \mu \mu}$ | $R \otimes A$ | -0.20 |  | [-0.29, -0.08] | 1.7 |  |
| $C_{9}^{\text {bs } \mu \mu}=C_{10}^{\text {bs } \mu \mu}$ | $\mu \quad L \otimes R$ | +0.15 |  | +0.02, +0.29] | 1.2 |  |
| $C_{9}^{\text {bs } \mu \mu}=-C_{10}^{\text {bs } \mu \mu}$ | $\mu \mu \quad L \otimes L$ | -0.53 |  | [-0.61, -0.46] | 6.9 |  |

## $C_{9}$ vs. $C_{9}=-C_{10}$ with global likelihood

Likelihood contours for different sets of observables taken into account


- Most groups doing fits of $b \rightarrow$ slौ observables do not include $\Delta F=2$ obs.: They do not depend on $b \rightarrow$ sll Wilson coefficients
- In global likelihood, $\Delta F=2$ obs. naturally included (globa!!)
- Choice whether to include them or not: clear difference in $\mathrm{C}_{10}^{b s \mu \mu}$ direction (red contour vs. blue contour)
- This explained the differences between the different groups!

Why does the inclusion of $\Delta F=2$ observables has such an impact on the fit in the $C_{10}^{b s \mu \mu}$ direction if $\Delta F=2$ observables do not depend on $C_{10}^{b s \mu \mu}$ ?

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Theory correlations...

## Correlations in a toy example

- Correlations for observables $\mathrm{O}_{1}, \mathrm{O}_{2}$ (uncertainties $\sigma_{1,2}$, correlation coeff. $\rho$ ):

$$
-2 \ln \mathcal{L}\left(O_{1}, O_{2}\right)=\frac{1}{1-\rho^{2}}\left(\frac{D_{1}^{2}}{\sigma_{1}^{2}}+\frac{D_{2}^{2}}{\sigma_{2}^{2}}-2 \rho \frac{D_{1} D_{2}}{\sigma_{1} \sigma_{2}}\right), \quad D_{1,2}=\left(O_{1,2}-\hat{O}_{1,2}\right)
$$

- If $D_{1}\left(C_{10}\right)$ depends on $C_{10}$ and $D_{2}$ is constant in $C_{10}$, then $\Delta \ln \mathcal{L}$ between $C_{10}=0$ and $C_{10}=\tilde{C}_{10}$ yields

$$
\Delta \ln \mathcal{L} \propto \frac{D_{1}^{2}(0)-D_{1}^{2}\left(\tilde{C}_{10}\right)}{\sigma_{1}^{2}}-2 \rho D_{2} \frac{D_{1}(0)-D_{1}\left(\tilde{C}_{10}\right)}{\sigma_{1} \sigma_{2}}
$$

- First term is present whether we include $\mathrm{O}_{2}$ or not (up to $\frac{1}{1-\rho^{2}}$ prefactor)
- Second term makes a difference
- if $\rho \neq 0$, i.e. $\boldsymbol{O}_{\mathbf{1}}$ and $\boldsymbol{O}_{\mathbf{2}}$ are correlated
- if $D_{2} \neq 0$, i.e. experimental estimate $\hat{\boldsymbol{O}}_{2}$ shows deviation from SM prediction $\mathrm{O}_{2}$


## Correlations in the global likelihood

The same is true for $\Delta F=2$ observables, in particular $\epsilon_{K}$ :

- theory predictions of $\epsilon_{K}$ and $B R\left(B_{s} \rightarrow \mu \mu\right)$ are correlated, $B R\left(B_{s} \rightarrow \mu \mu\right)$ depends on $C_{10}$
- experimental estimate of $\epsilon_{K}$ shows deviation from SM prediction

Should we include $\boldsymbol{\Delta F}=\mathbf{2}$ observables in $b \rightarrow s \ell \ell$ fit or not?
Two different assumptions:

- Including them and only varying $C_{10}$ means we assume all other Wilson Coefficients $C_{i}=0$, i.e. we fix the SM point in these directions
- Excluding them is (nearly) equivalent to setting certain $C_{i} \neq 0$ such that theory prediction and experimental estimate of $\Delta F=2$ observables agree
Bayesian approach: marginalise over "nuisance coefficients" $C_{i}$
- Including them and only varying $C_{10}$ corresponds to prior on $C_{i}$ strongly peaked around SM value $C_{i}=0$
- Excluding them is equivalent to flat prior that allows the posterior for $C_{i}$ to be peaked around $C_{i} \neq 0$


## What can we learn from this?

- There are different assumptions we can make by including or excluding certain observables
- It is not obvious if there is a "correct" one, but we should be aware of the differences
- The $\Delta \chi^{2}$ values between best-fit point and SM point can be different and one has to think about what "SM point" actually means if one does not fix $C_{i}=0$


[^0]:    WET at 4.8 GeV

[^1]:    WET at 4.8 GeV

[^2]:    WET at 4.8 GeV

[^3]:    WET at 4.8 GeV

[^4]:    WET at 4.8 GeV

[^5]:    WET at 4.8 GeV

