# Inclusive $B \to X_{s,d}\ell^+\ell^-$: getting ready for 50 $\mathrm{ab}^{-1}$

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KEK, Belle II physics week
October 28, 2019

T. Huber, EL, M. Misiak, D. Wyler — hep-ph/0512066 (NPB)
T. Huber, T. Hurth, EL — 1503.04849 (JHEP)
T. Huber, T. Hurth, J. Jenkins, EL, Q. Qin, K. Vos — 1908.07507 (JHEP)
T. Huber, T. Hurth, J. Jenkins, EL, Q. Qin, K. Vos — in preparation

# Outline

- Introductory remarks
  short distance physics, NP contributions, typical spectrum
- Anomalies in exclusive modes
  global fits
  open theoretical issues (form factors, power corrections, resonances)
- Theory of inclusive decays
  OPE and its breakdown
  Krüger-Sehgal description of $u\bar{u}$ and $c\bar{c}$ resonances in the singlet channel
  Resonant octet contributions *[work in progress]*
  Non-local power corrections
  Cascades
  $m_X$ cuts
  QED radiation, Monte Carlo study and experiment/theory interplay
- Phenomenology
  SM predictions, New Physics reach, comparison with exclusive

# Introduction: operators

SM operator basis:

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[ \sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$

- Magnetic & chromo-magnetic

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$
$$Q_8 = \frac{g}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

- Semileptonic

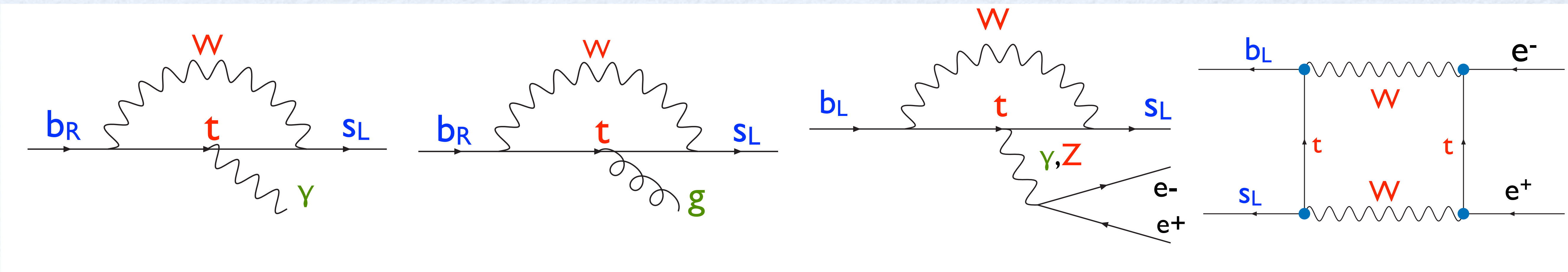
$$Q_9 = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \ell)$$
$$Q_{10} = \frac{\alpha_{\rm em}}{4\pi} (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Everything is known very well ($V_{ub} V_{uq}$ contribution is small for $b \to s\ell\ell$ but important for $b \to d\ell\ell$)

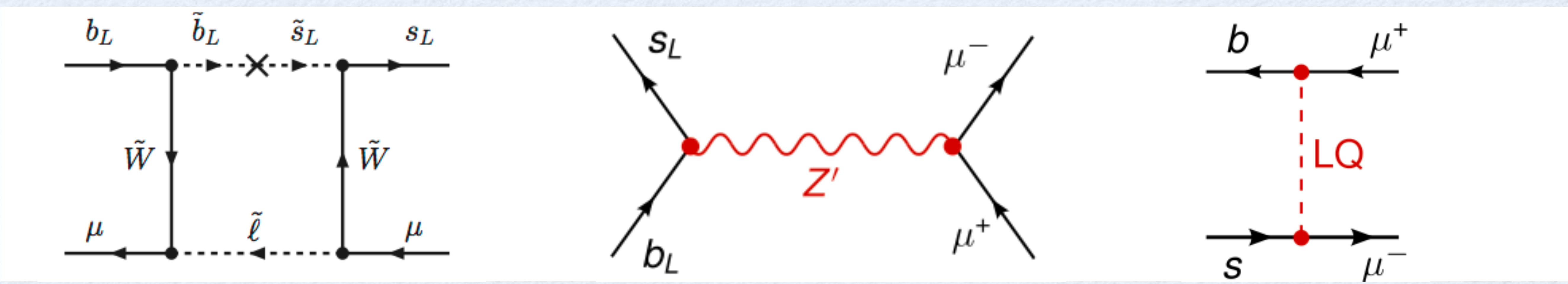
- To address $b \to s\mu\mu$ anomalies, the lepton universality breaking operators $Q_{9,10}^{ee}$ and $Q_{9,10}^{\mu\mu}$ have been considered as well

# Introduction: SM vs New Physics

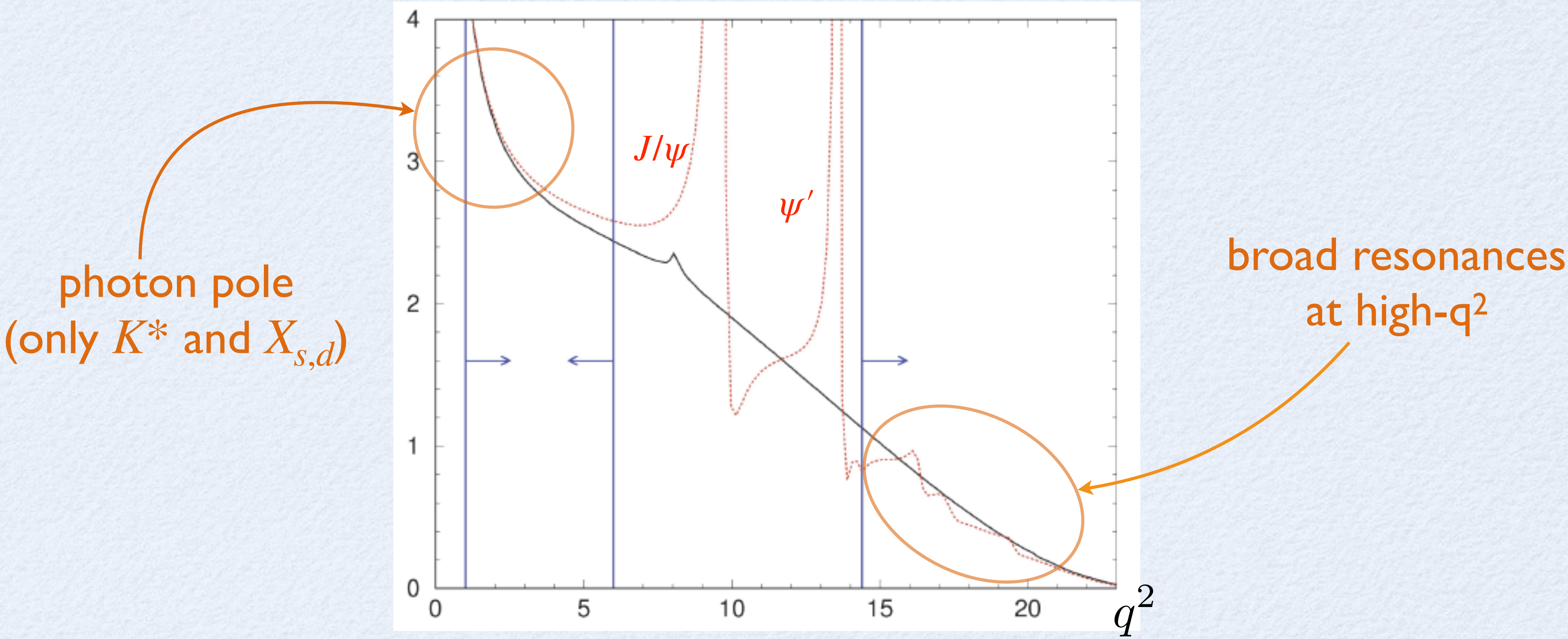
SM contributions:




NP contributions:



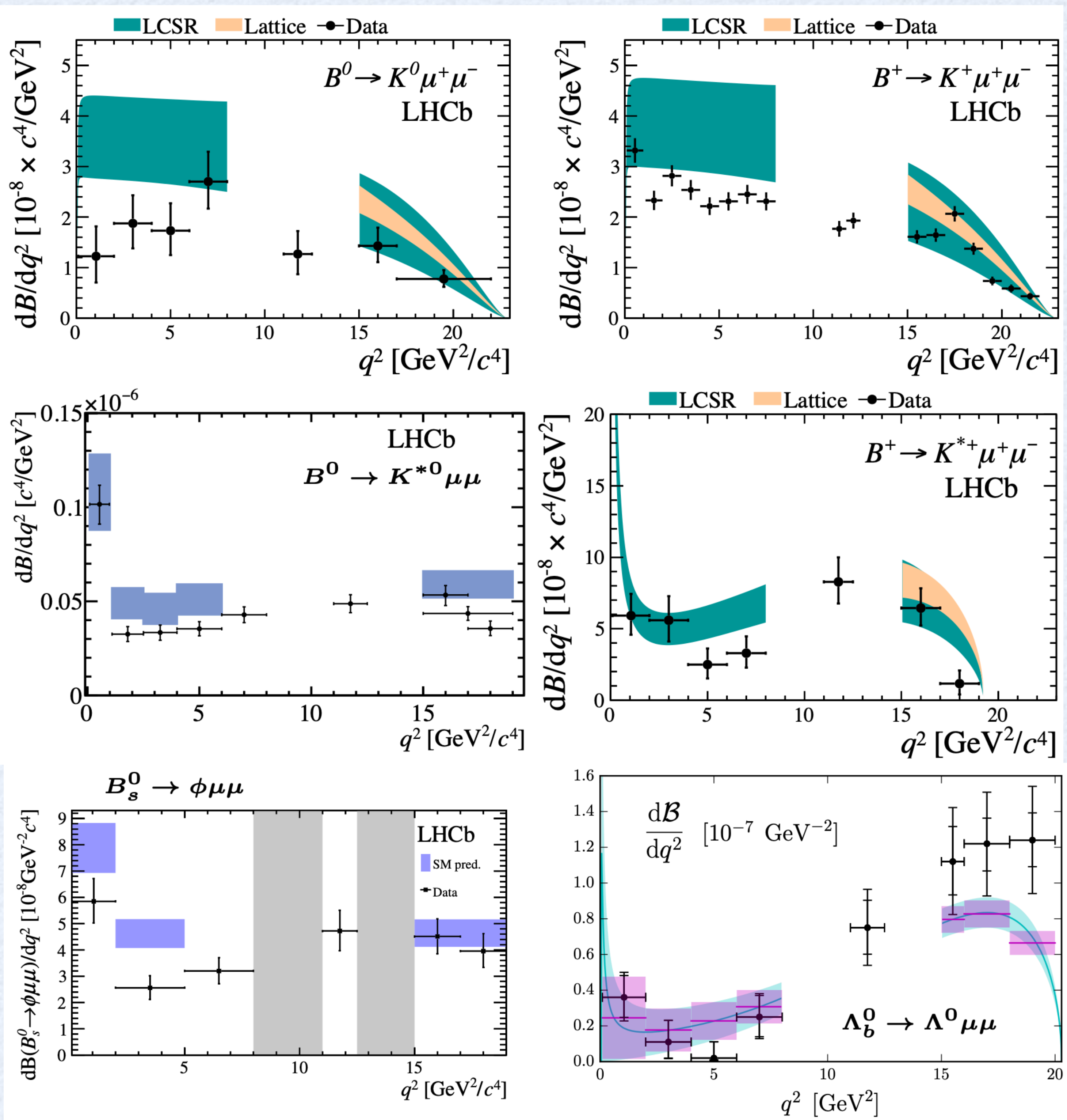
# Introduction: typical spectrum



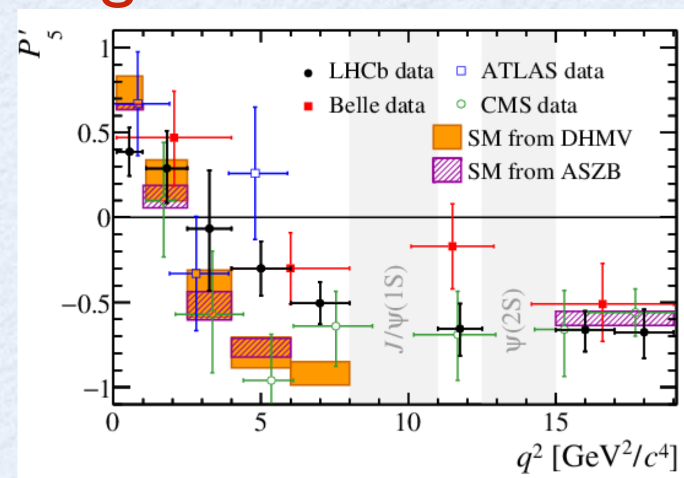

- Intermediate charmonium resonances contribute via:
  $B \to (K, K^*, X_s)\, \psi_{\bar{c}c} \to (K, K^*, X_s)\, \ell^+\ell^-$
- Contributions of  and  have to be dropped
- Theory at low-$q^2$ and high-$q^2$ presents different challenges

# Exclusive modes: anomalies

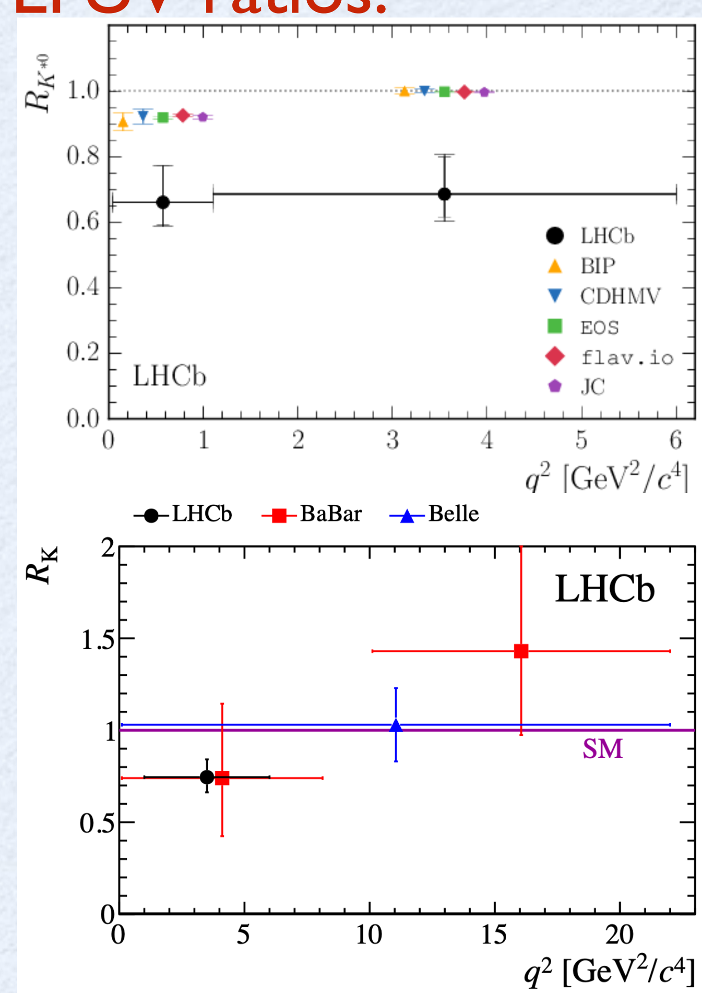
## Branching Ratios:




## Angular observables:

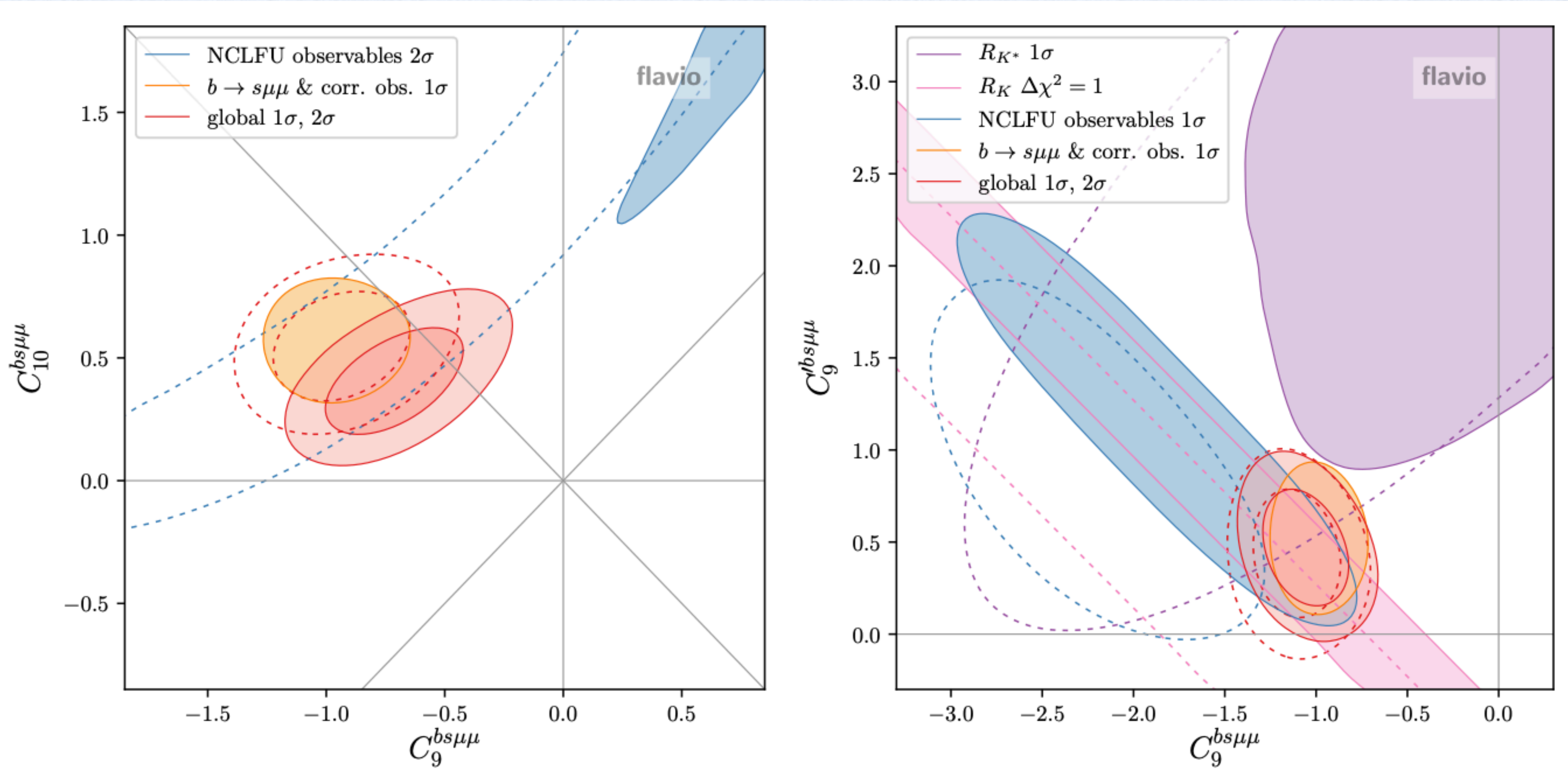



## LFUV ratios:



# Exclusive modes: global fits

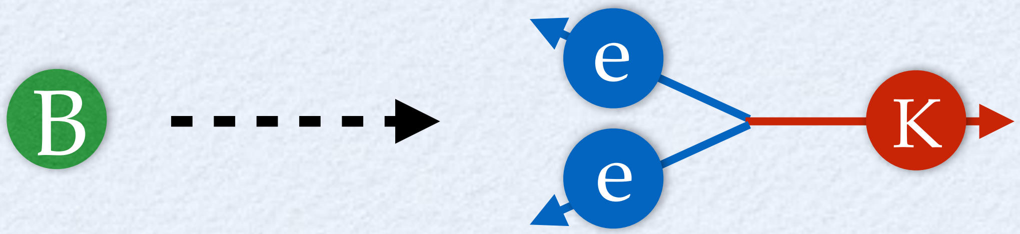
- $[C_9^{bs\mu\mu}, C_{10}^{bs\mu\mu}] = [-0.73, 0.40]$
  pull = $6.3\sigma$
- $[C_9^{bs\mu\mu}, C_9'^{bs\mu\mu}] = [-1.06, 0.47]$
  pull = $6.0\sigma$




[Aebischer et al, 1903.10434]

# Exclusive modes: theoretical frameworks

- The central problem is the calculation of matrix elements:
  $\langle K^{(*)}\ell\ell|O(y)|B\rangle \approx \langle K^{(*)}|T\,J_\mu^{\rm em}(x)\,O(y)|B\rangle$
- At low-q² the K(*) has large energy (large recoil):

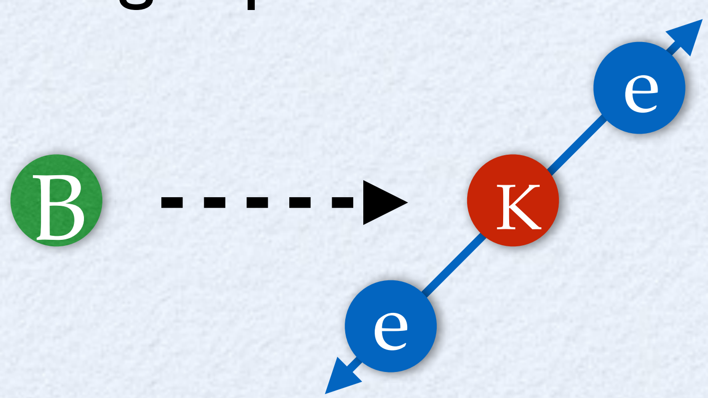



⇨ Soft-Collinear Effective Theory

The large energy of the K(*) introduces three scales: $m_b^2$, $\Lambda m_b$ and $\Lambda^2$:

$$\langle K^{(*)}|TJ_\mu^{\rm em}(x)O(y)|B\rangle \sim C \times [\,\text{Form Factor} + \phi_B \star J \star \phi_{K^{(*)}}] + O\left(\Lambda/m_b\right)$$

- At high-q² the K(*) does not recoil:




⇨ $(x-y)^2 \sim \frac{1}{q^2} \sim \frac{1}{m_b^2}$ ⇨ local OPE

$$\langle K^{(*)}|TJ_\mu^{\rm em}(x)O(y)|B\rangle \sim C \times [\,\text{Form Factor}] + O\left(\Lambda/m_b\right)$$

# Exclusive modes: issues

- Form factors
  - lattice QCD (high-q²): $B \to K$ complete, $B \to K^*$ and $B_s \to \phi$ ongoing
  - LCSR (low-q²): some uncertainties have to be ball-parked (power corrections, …) but get access to all form factors (including baryons)

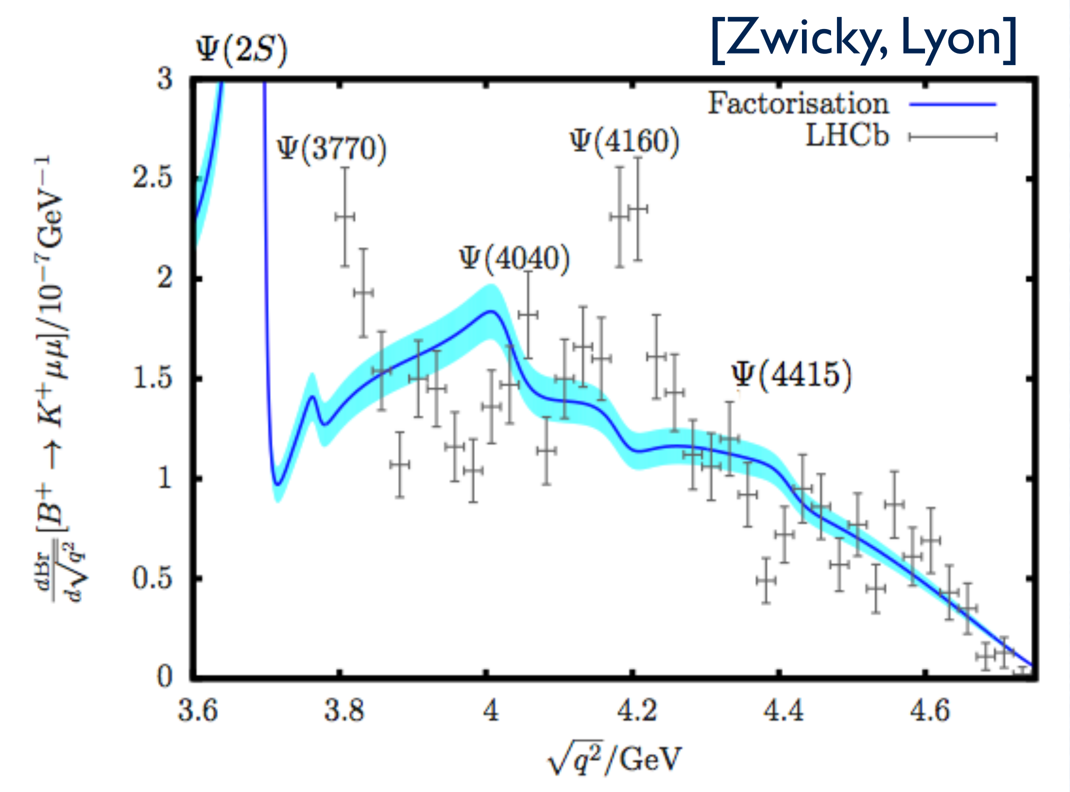
- Power corrections
  - Presently incalculable
  - In global fits they are taken into account via nuisance parameters
  - If no form factors relations are used, their impact is not sizable because they are essentially confined to the the matrix element $\langle K^{(*)} | T J_\mu^{\rm em} O_2 | B \rangle$

[Fermilab/MILC and EL, 1903.10434]

  - If form factors relations are used ⇒ construct "clean observables" (e.g. $P_5'$)

# Exclusive modes: issues

- Resonances at high-$q^2$
  - Unsurprisingly naive factorization fails to reproduce the resonant pattern observed in $B \to K\mu\mu$ at high-$q^2$
  - The OPE and quark-hadron duality lead to a reliable prediction for the integrated high-$q^2$ branching ratio [Beylich, Buchalla]
  - Within naive factorization the contribution of the "wiggles" is non-negligible
  - This has led to some uneasiness about our ability to use the high-$q^2$ region effectively



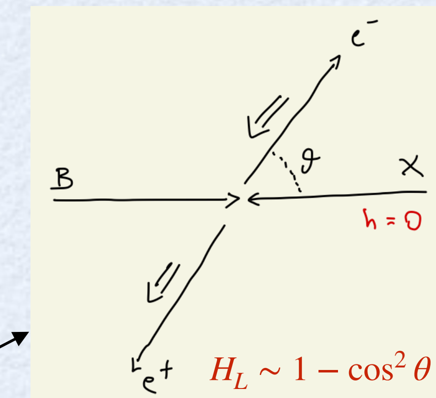

- All these anomalies need confirmation at Belle II
  - different systematics
  - access to more observables (inclusive modes)

# Inclusive theory: observables

- $B \to K\ell\ell$

$$\frac{d^2\Gamma^K}{dq^2\ d\cos\theta_\ell} = a + b\ \cos\theta_\ell + c\ \cos\theta_\ell^2$$

  - In the SM $b$ is suppressed by the lepton mass



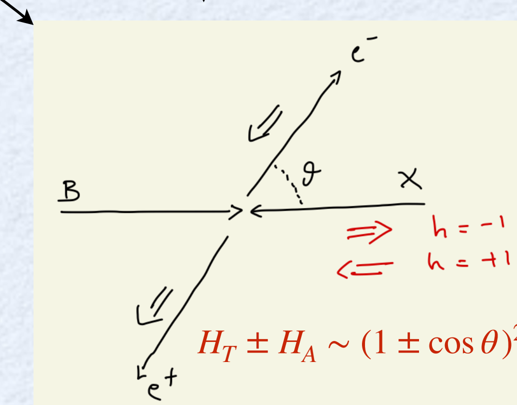

- $B \to X_s\ell\ell$

$$\frac{d^2\Gamma^{X_s}}{dq^2\ d\cos\theta_\ell} = \frac{3}{8}\left[(1+\cos^2\theta_\ell)\ H_T + 2(1-\cos^2\theta_\ell)\ H_L + 2\cos\theta_\ell\ H_A\right]$$

$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right]$$

$$H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

$$H_A \sim -4\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[C_{10}(C_9 + 2\frac{m_b^2}{q^2}C_7)\right]$$

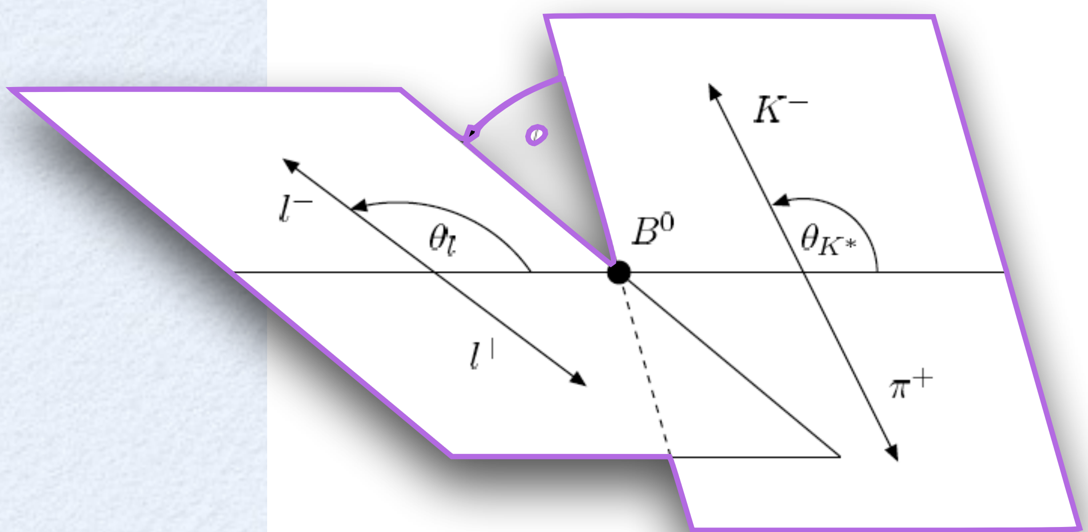



  - In the SM $H_A$ is not suppressed by the lepton mass
  - There are similar contributions from non-SM operators but there is no interference between V+A and V-A structures
  - At low $q^2$ ($\hat{s} < 0.3$) $H_T$ is suppressed ($C_7 < 0$)

# Inclusive theory: observables

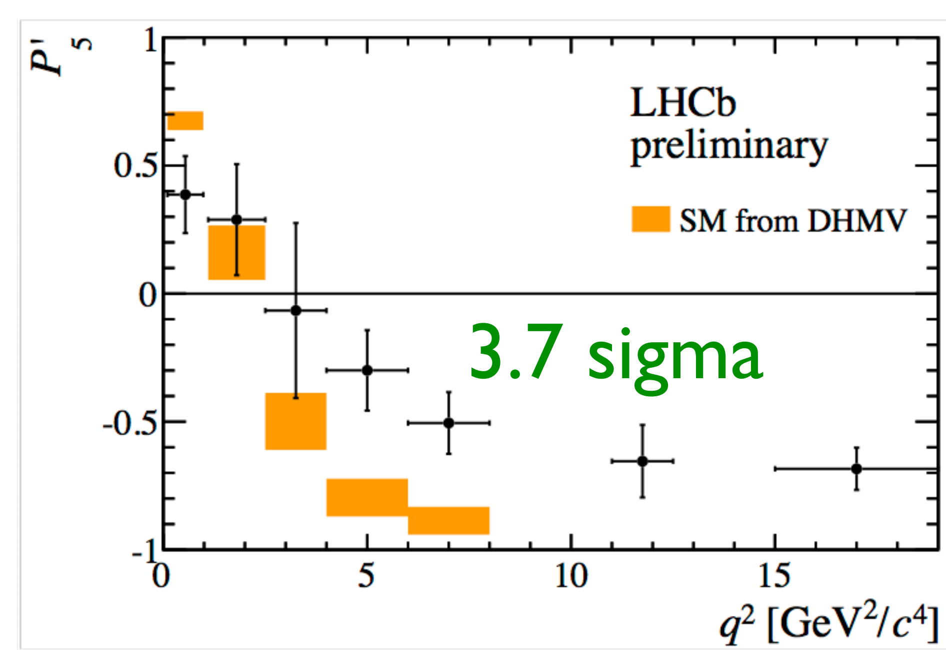
- $B \to K^*\ell\ell \to K\pi\ell\ell$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}}\bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_{\mathrm{L}})\sin^2\theta_K + F_{\mathrm{L}}\cos^2\theta_K$$
$$+\tfrac{1}{4}(1-F_{\mathrm{L}})\sin^2\theta_K\cos 2\theta_l$$
$$-F_{\mathrm{L}}\cos^2\theta_K\cos 2\theta_l + S_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi$$
$$+S_4\sin 2\theta_K\sin 2\theta_l\cos\phi + S_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+\tfrac{4}{3}A_{\mathrm{FB}}\sin^2\theta_K\cos\theta_l + S_7\sin 2\theta_K\sin\theta_l\sin\phi$$
$$+S_8\sin 2\theta_K\sin 2\theta_l\sin\phi + S_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



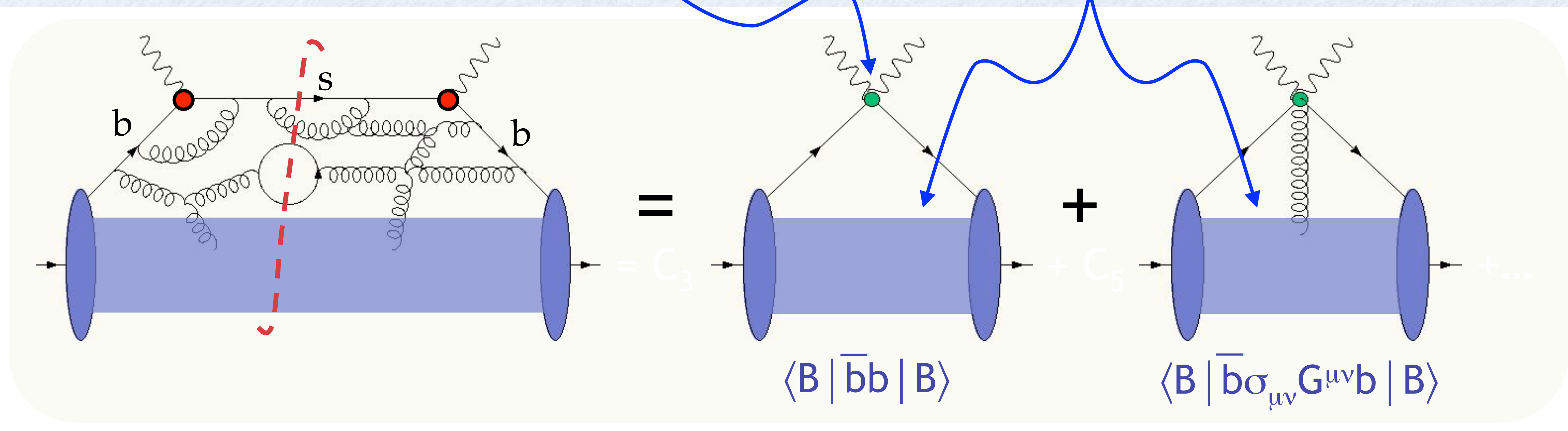

"clean ratios"

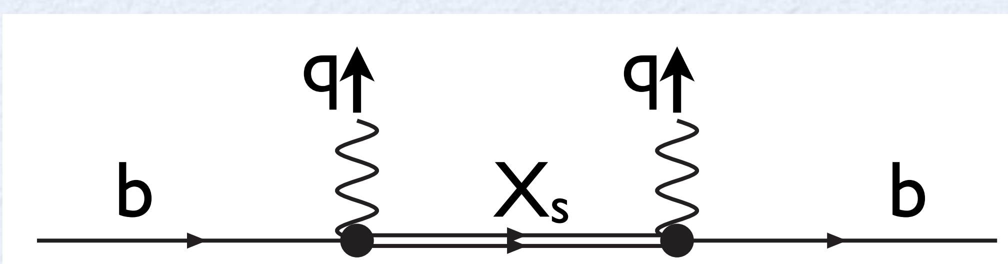
$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_{\mathrm{L}}(1-F_{\mathrm{L}})}}$$



# Inclusive theory: OPE

$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right] + O\left(\frac{\Lambda_{QCD}^2}{m_b^2}, \frac{\Lambda_{QCD}^3}{m_b^3}, \frac{\Lambda_{QCD}^2}{m_c^2}, \ldots\right)$$




$$\begin{aligned} p_{X_s}^2 &= (p_b - q)^2 = m_b^2 + q^2 - 2 m_b q_0 \\ &< m_b^2 + q^2 - 2 m_b \sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2 \end{aligned}$$

OPE is an expansion in $\Lambda_{\text{QCD}}/(m_b - \sqrt{q^2})$ and breaks down at $q^2 \sim m_b^2$

# Inclusive theory: OPE

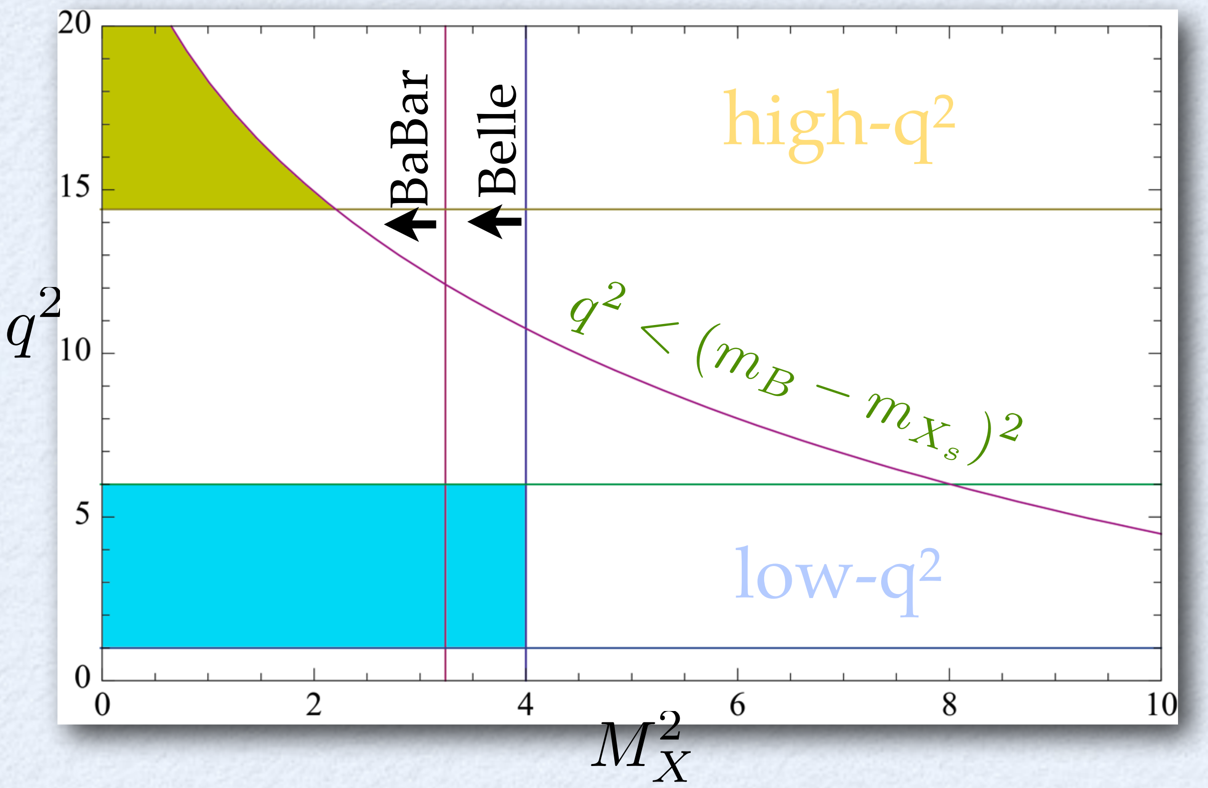
- The breakdown of the OPE at high-$q^2$ results in large power corrections:
  - power corrections account for the almost totality of the high-$q^2$ integrated branching ratio
  - the poorly known matrix elements required to evaluate $1/m_b^3$ power corrections are responsible for the large uncertainty

- Power corrections proportional to $C_{9,10}^2$ are identical to the power corrections which appear in $\bar{B}^0 \to X_u \ell \nu$
  - Introduce a new observable obtained by normalizing the rate to the semileptonic rate with the same $q^2$ cut [Ligeti et al]:

$$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^{1} d\hat{s} \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{d\hat{s}}}{\int_{\hat{s}_0}^{1} d\hat{s} \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell \nu)}{d\hat{s}}} \qquad [\hat{s} = s/m_b^2 = q^2/m_b^2]$$

# Inclusive theory: $m_X$ cuts

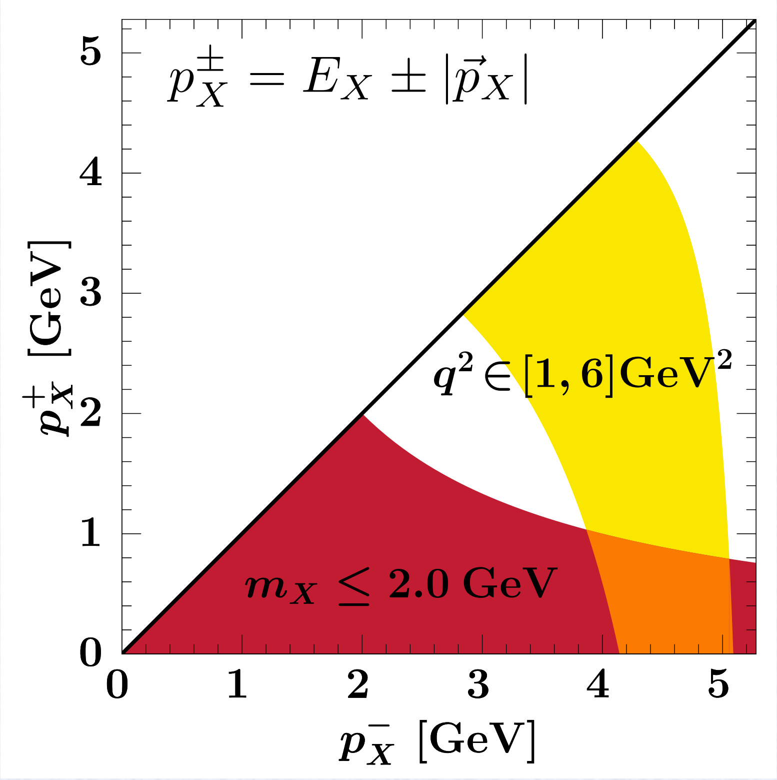
- $m_X$ cuts are required to suppress background from double semileptonic decays (both same side and opposite side):
  - $B \to (X_c \to X_s \ell^+ \nu)\ell^- \bar{\nu} = X_s \ell \ell$ + missing energy
  - $ee \to (B \to (X_c \to X_s)\ell^- \bar{\nu})(\bar{B} \to (X_c \to X_s)\ell^+ \nu) = X_s \ell \ell$ + missing energy
- These cuts introduce sensitivity to a hard collinear scale (of order 2 GeV) and the rate becomes dependent on the B meson shape function




- The high-q² region is unaffected
- Current BaBar and Belle analyses correct using a Fermi motion model
- Better modeling can be achieved within SCET and by using $B \to X_s \gamma$ and $B \to X_u \ell \nu$ data to extract the shape function

# Inclusive theory: $m_X$ cuts

- Kinematics:

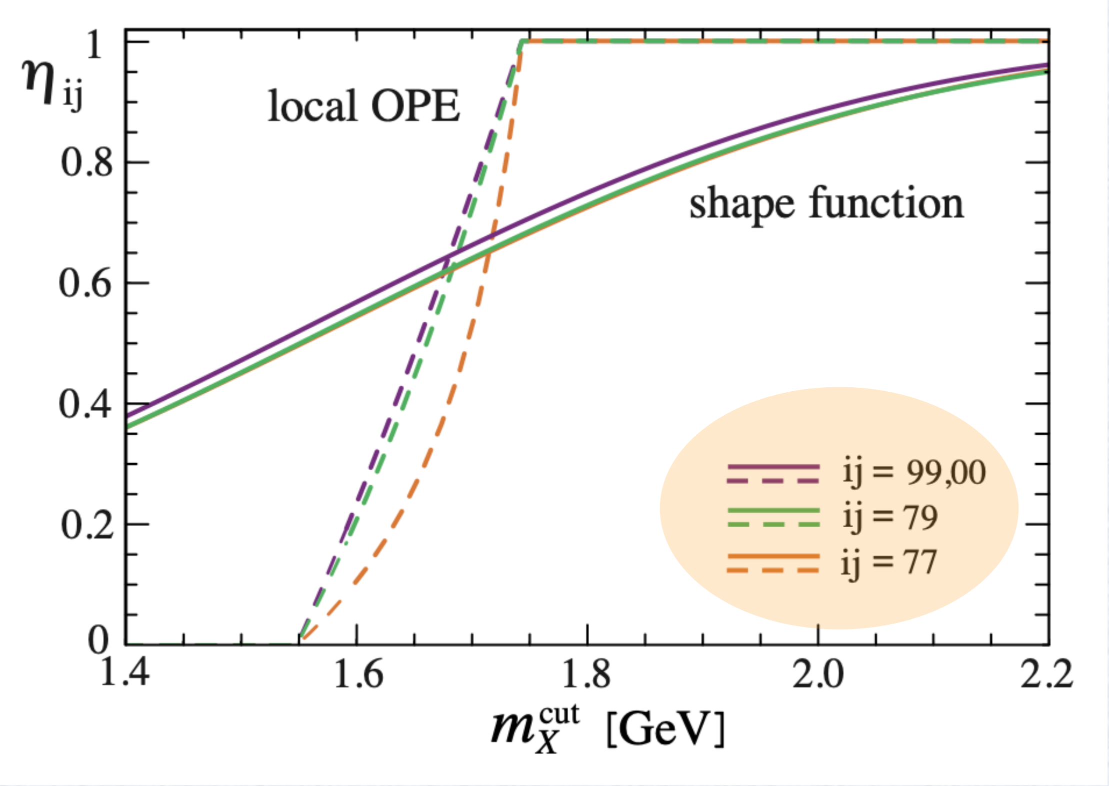



$$p_X^+ \ll p_X^- \Longrightarrow m_X^2 \ll E_X^2$$

X is hard-collinear:

$$\Lambda^2 \ll m_X^2 \sim \Lambda m_b \ll m_b^2$$

- The impact of the cuts is universal ($\eta = \Gamma_{\rm cut}/\Gamma$): [Lee, Ligeti, Stewart, Tackmann]



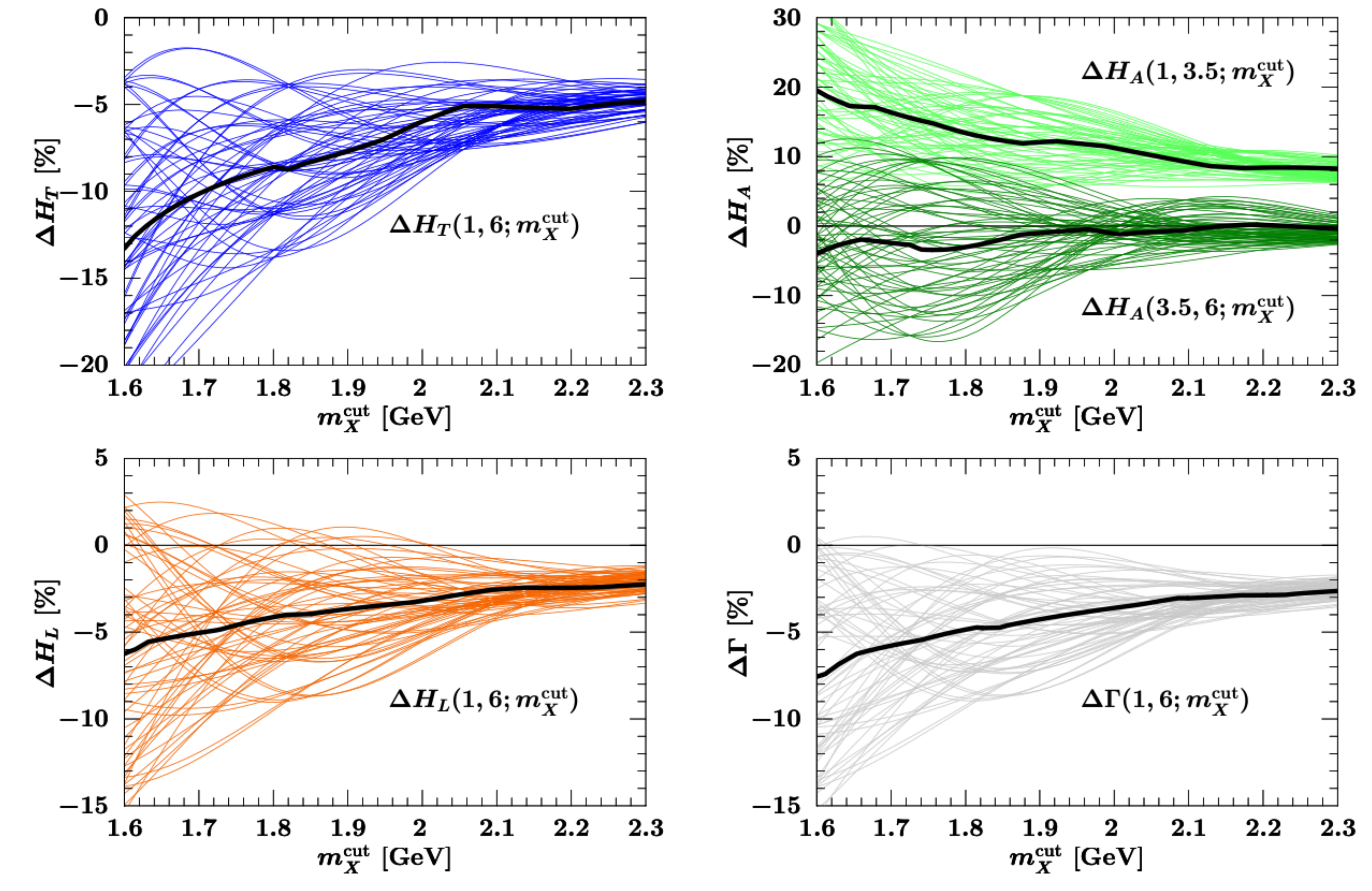

- Since the universality of the cuts extends to $B \to X_u \ell \nu$, the following ratio is minimally sensitive to the shape function modeling:

$$\frac{\Gamma(B \to X_s \ell \ell)_{\rm cut}}{\Gamma(B \to X_u \ell \nu)_{\rm cut}}$$

*[same $m_X$ cut]*

# Inclusive theory: $m_X$ cuts

- Current status of shape function modeling:
  [Lee, Ligeti, Stewart, Tackmann; Bell, Beneke, Huber, Li]




The same-color curves correspond to a sampling of potential shape functions

# Inclusive theory: resonances

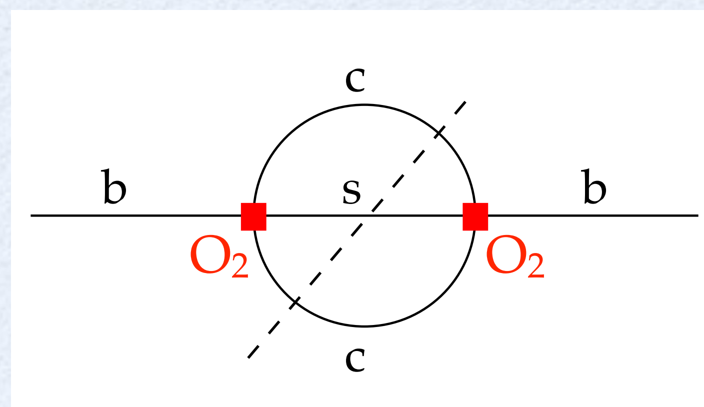
- Optical theorem: [Beneke, Buchalla, Neubert, Sachrajda]

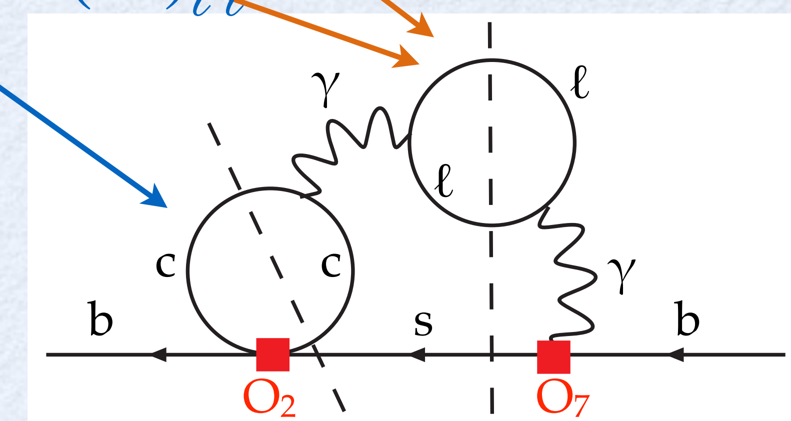
$$\mathrm{Im}\left[\sum_{ij}\langle\bar{B}|T\,Q_i(0)\,Q_j(x)|\bar{B}\rangle\right]\sim\Gamma(\bar{B}\to X_s)\neq\Gamma(\bar{B}\to X_s\ell^+\ell^-)$$

$b\to s(c\bar{c})_{\rm had}$

$b\to s(c\bar{c})_{\ell\ell}$

$b\to s\ell^+\ell^-$




$$\mathrm{BR}(B\to X_s)\sim 10^{-2}$$

$$\mathrm{BR}(B\to X_s(J/\psi,\psi')\to X_s\ell\ell)\sim 10^{-4}$$ → Experimental cuts

$$\mathrm{BR}(B\to X_s\ell\ell)\sim 10^{-6}$$ → Need to control charmonium contamination away from $\psi(1s,2s)$

# Inclusive theory: resonances

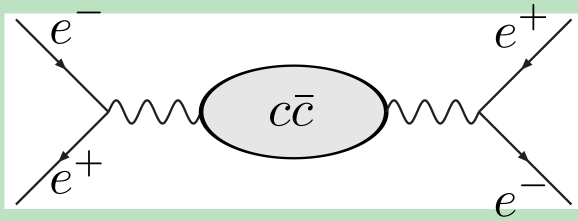
- The charmonium in $B \to X_s(\psi_{cc} \to \ell\ell)$ can be produced by an underlying color singlet and color octet quark transition:
  - the color singlet contribution is modeled exactly over the whole $q^2$ spectrum using $R_{had}$ data for both on- and off-shell charmonium (Krüger-Sehgal mechanism)
  - off-shell color octet effects at high-$q^2$ are taken into account by $1/m_c^2$ corrections [Voloshin; Buchalla, Isidori, Rey]
  - off-shell color octet effects at low-$q^2$ can be described within SCET and yield so-called resolved contributions which at present can only be estimated [Voloshin; Buchalla, Isidori, Rey]
  - on-shell color octet effects at high-$q^2$ are under study (at low-$q^2$ there is no on-shell charmonium)

- Cascade decays $B \to X_s(\psi_{cc} \to X_s'\ell\ell)$:
  - on-shell effects do not interfere and can be measured and subtracted from the experimental measurement or added to the theory prediction (luckily they turn out to have negligible impact)

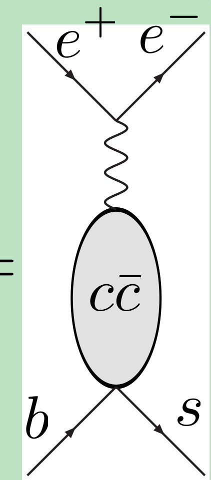
# Inclusive theory: resonant color singlet production

- Kruger-Sehgal mechanism:

$$R_{\rm had}^{c\bar c} = \frac{\sigma(e^+e^- \to c\bar c \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} =$$




$$\langle O_2 \rangle =$$

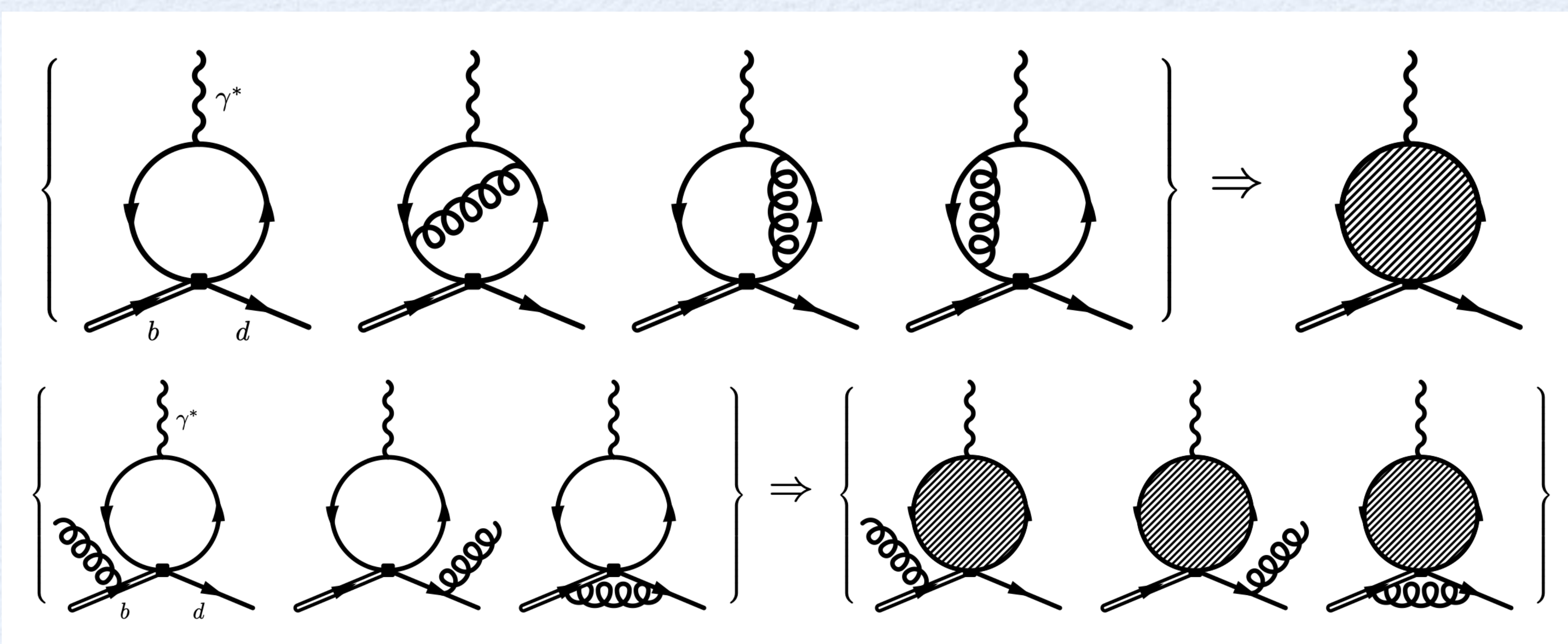



$$\mathrm{Im}[h_c] = \frac{\pi}{3} R_{\rm had}$$

$$\mathrm{Re}[h_c] = \mathrm{Re}[h_c(s_0)] + \frac{s-s_0}{\pi}\int_0^\infty \frac{\mathrm{Im}[h_q(t)]}{(t-s)(t-s_0)}dt$$

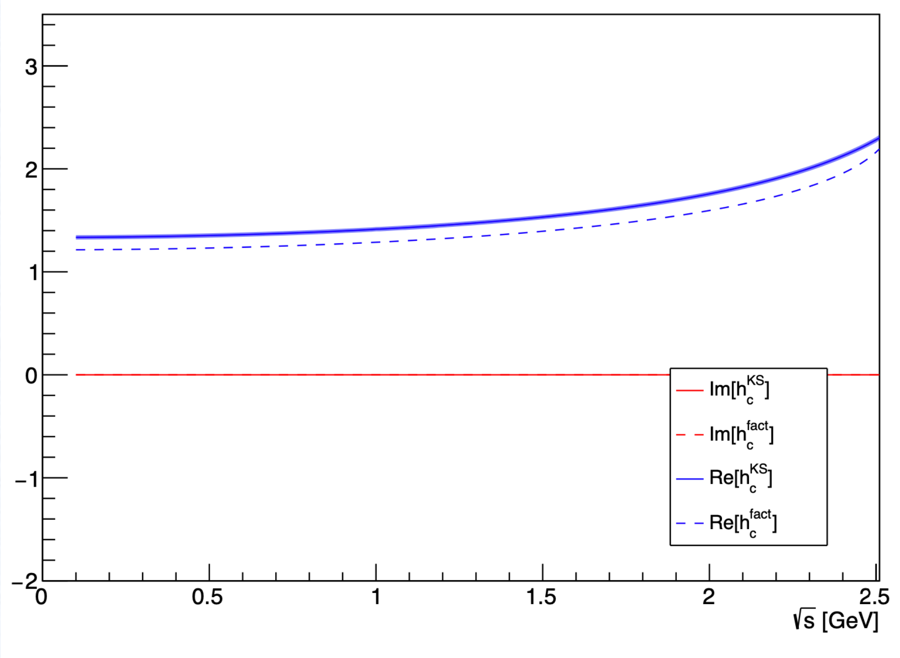
perturbative for $s_0 \sim -\mu_b^2$

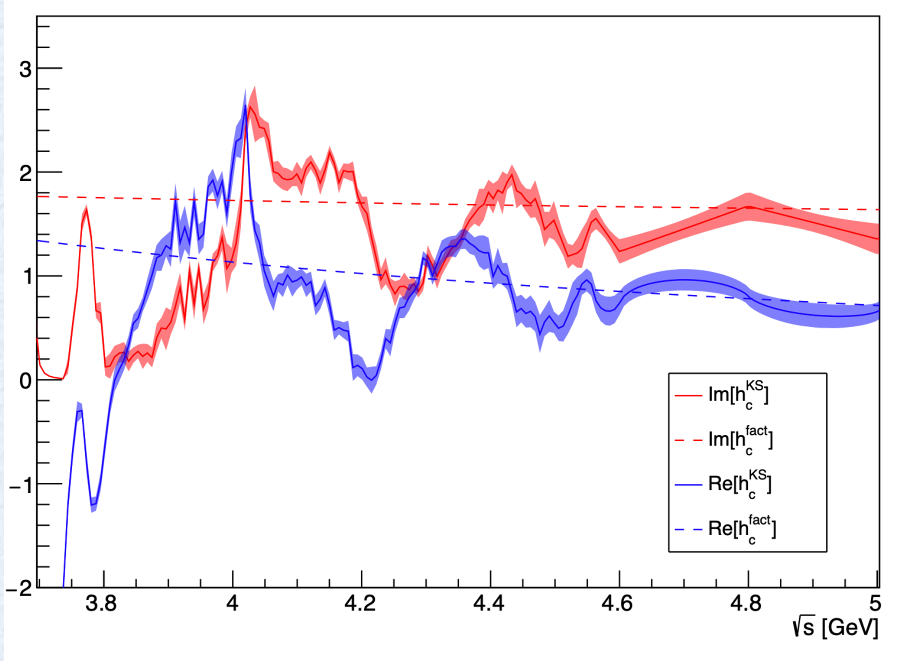
- We can include NLO effects [separation of two-loop perturbative functions provided by de Boer]



# Inclusive theory: resonant color singlet production

- Using updated $R_{had}$ data [BESII, BaBar, ALEPH; Keshavarzi, Nomura, Teubner] and perturbation theory (program `rhad`) for asymptotically large s [Harlander, Steinhauser]

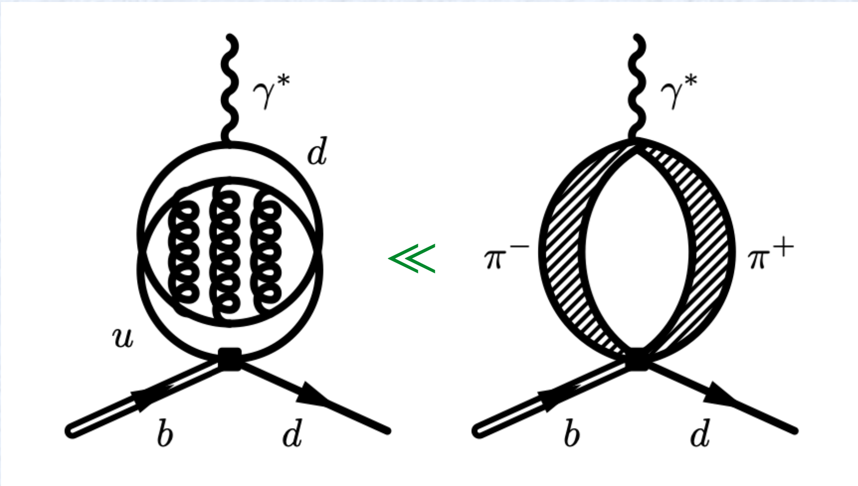


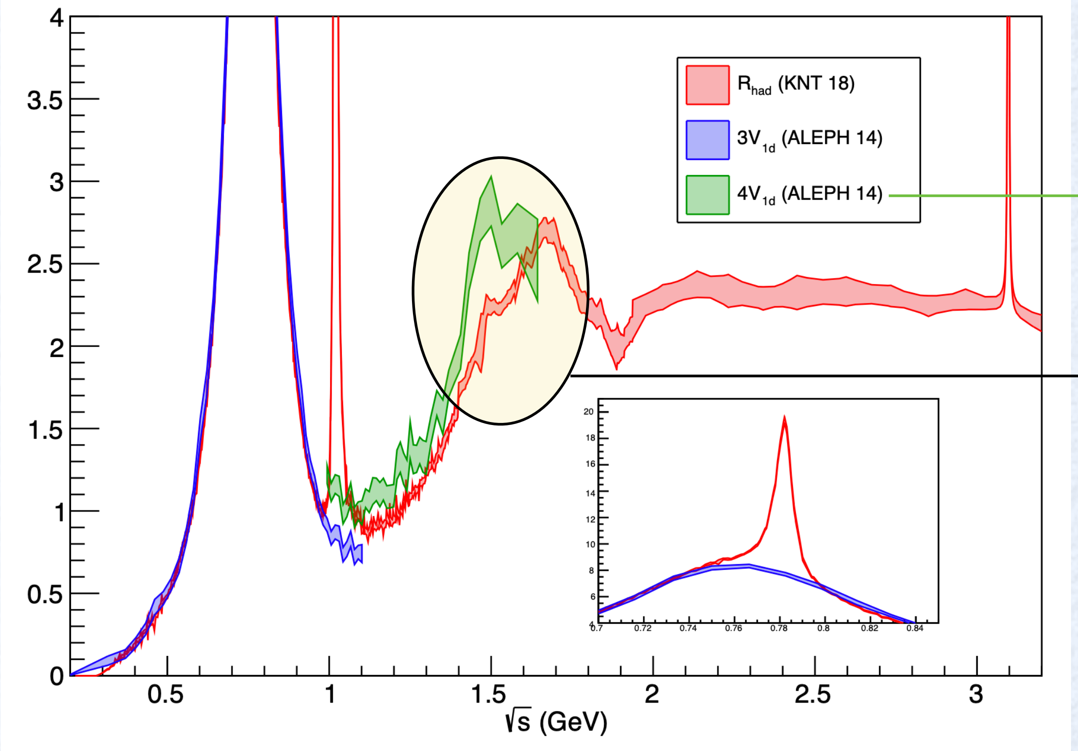

- Impact at low-$q^2$ is small (about 2%): perturbation theory and dispersive approaches agree because below threshold we are mostly sensitive to the total integral over $R_{had}$ which is well described in perturbation theory
- Impact at high-$q^2$ region is large (about -10%): details of the resonant structure matters

# Inclusive theory: resonant color singlet production

- For $B \to X_d \ell\ell$ we need to include $u\bar{u}$ resonant effects
- Considerable complications arise because we need to estimate $\langle J_q J_{q'} \rangle$ correlators with $q, q' = u, d, s$ whose relative size at low-q² is not described by perturbation theory at all




- Using both Isospin SU(2) and SU(3) we were able to express the $u\bar{u}, d\bar{d}$ and $s\bar{s}$ KS functions in terms of $R_{had}$ and $\tau$ decay data only

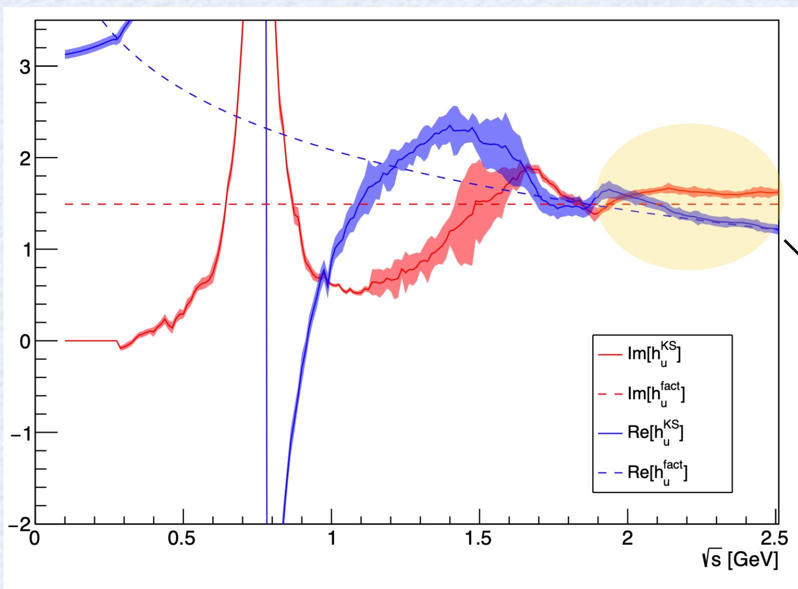


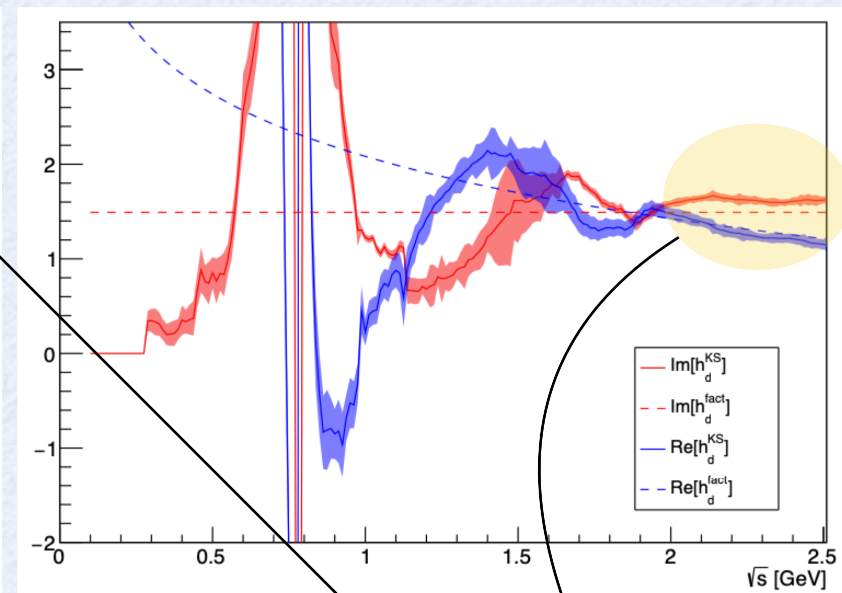

Rhad predicted from $\tau$ data using Isospin SU(3)

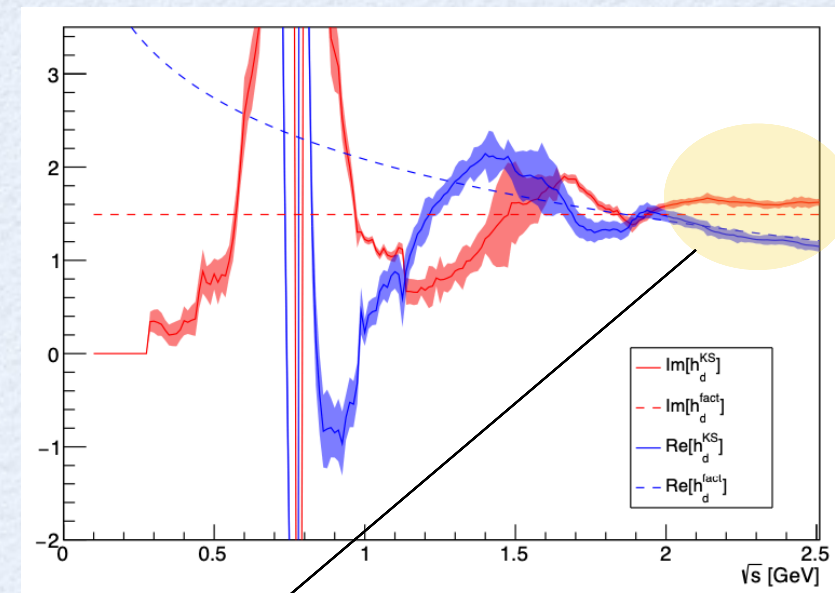
We use its deviation from the actual Rhad measurement (in red) as an estimate of SU(3) breaking effects

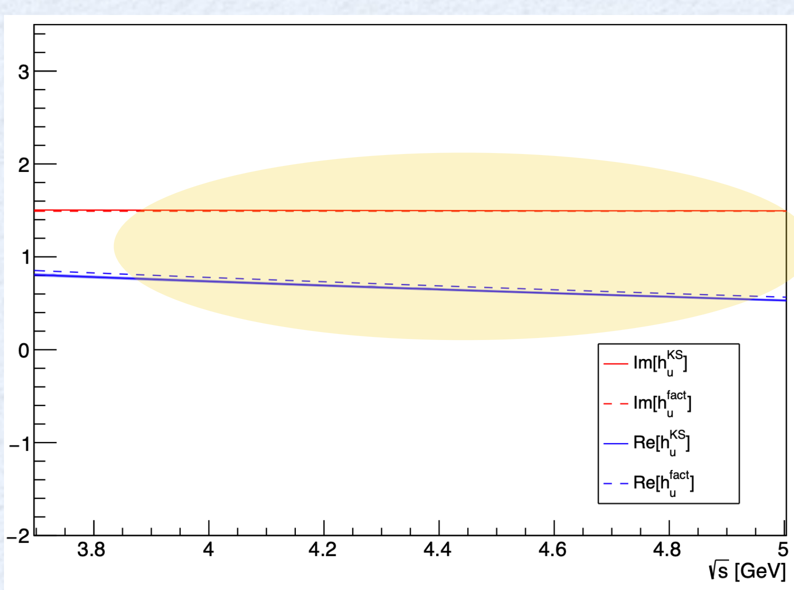
# Inclusive theory: resonant color singlet production

- For $B \to X_d \ell \ell$ we need to include $u\bar{u}$ resonant effects



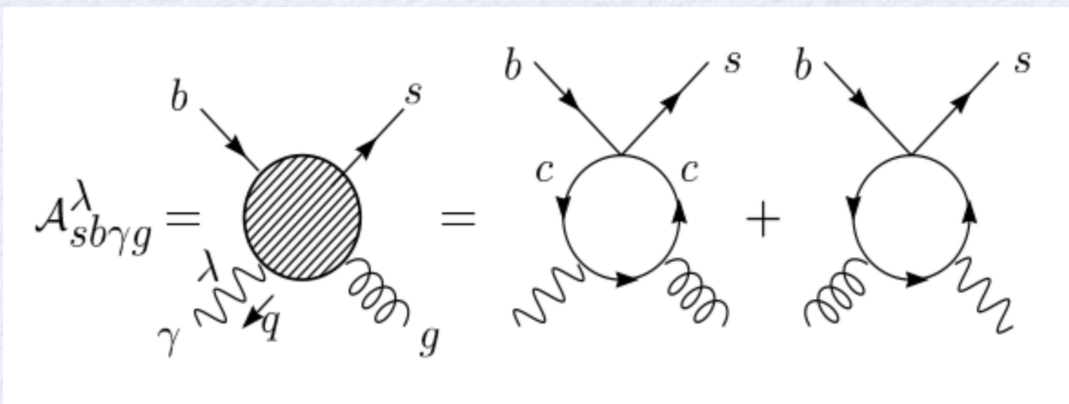




Very good asymptotic agreement
with perturbation theory

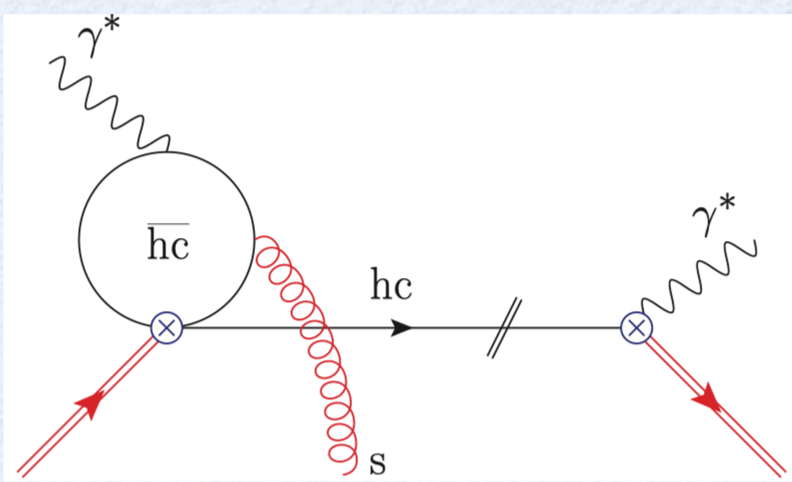
# Inclusive theory: non-resonant color octet

- Non-resonant color octet effects at high-q² can be calculated in perturbation theory and it scales as $\Lambda^2_{\rm QCD}/q^2$ [Buchalla, Isidori, Rey]:




$$\Longrightarrow \frac{\langle B|\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

- At low-q² and with a cut on $m_X$ the charm loop is hard-collinear and needs to be treated using SCET [Hurth, Benzke, Fickinger, Turczyk]:



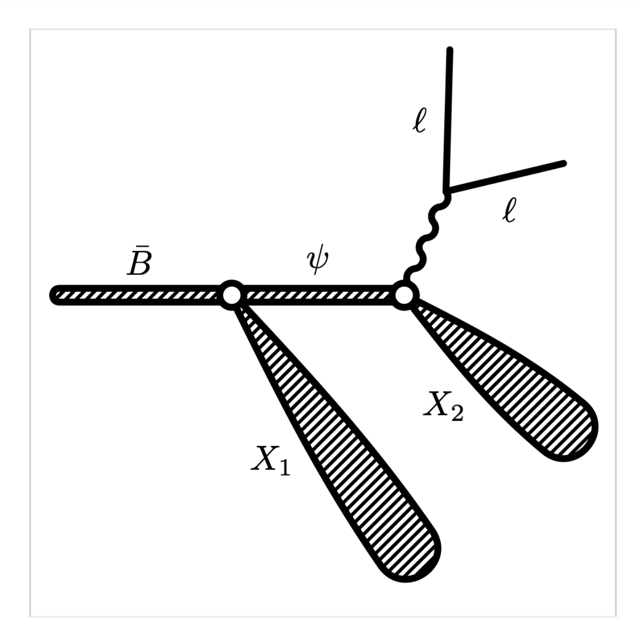

- Power corrections stay non-local after $m_X$ cut is released ⇒ so-called resolved contributions
- Depend on mostly unknown subleading B shape functions
- Work in progress on explicit estimate [Benzke, Hurth, Turczyk]
- For the time being, we use rough estimates to asses an irreducible uncertainty of about 5%

# Inclusive theory: cascades

- Cascade decays $B \to X_1(\psi \to X_2 \ell \ell)$ constitute another long distance effect
  [Buchalla, Isidori, Rey; Beneke, Buchalla, Neubert, Sachrajda]

- Effects are potentially very large:

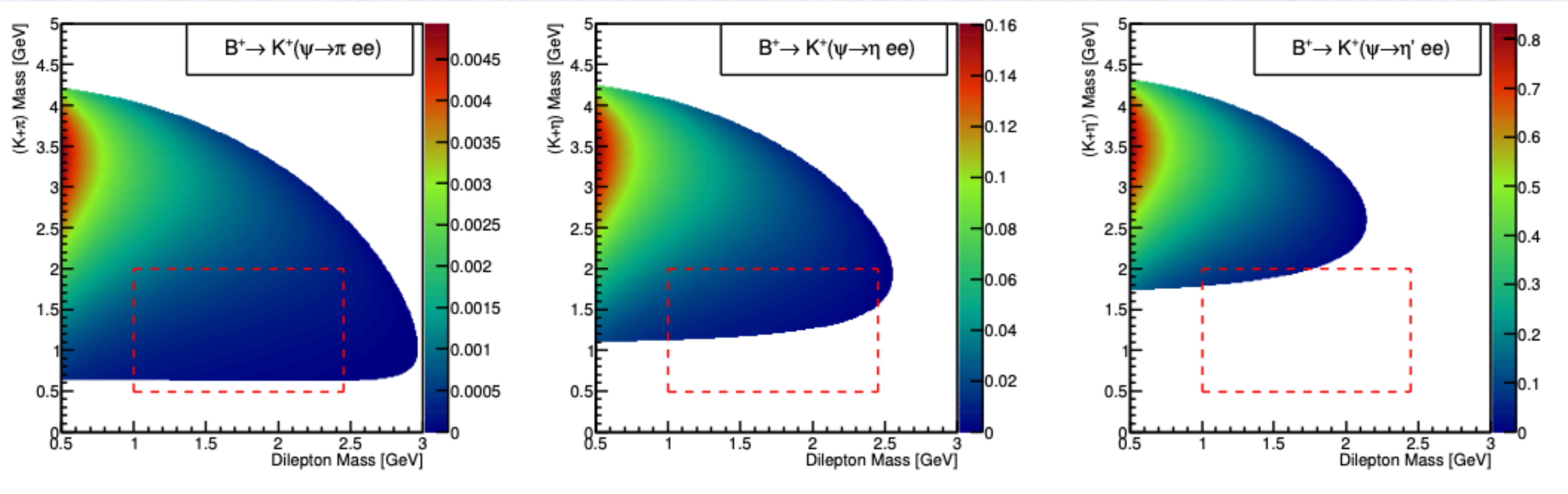
| | $\mathcal{B} \times 10^3$ | | $\mathcal{B} \times 10^5$ |
|---|---|---|---|
| $\bar{B} \to X_s \psi$ | $7.8 \pm 0.4$ | $\psi \to \eta \ell^+ \ell^-$ | $1.43 \pm 0.07$ |
| $\bar{B} \to X_s \psi'$ | $3.07 \pm 0.21$ | $\psi \to \eta' \ell^+ \ell^-$ | $6.59 \pm 0.18$ |
| $\bar{B} \to X_s \chi_{c1}$ | $3.09 \pm 0.22$ | $\psi \to \pi^0 \ell^+ \ell^-$ | $0.076 \pm 0.014$ |
| $\bar{B} \to X_s \chi_{c2}$ | $0.75 \pm 0.11$ | $\psi' \to \eta' \ell^+ \ell^-$ | $0.196 \pm 0.026$ |
| $\bar{B} \to X_s \eta_c$ | $4.88 \pm 0.97$ [111] | | |
| $\bar{B} \to X_s \chi_{c0}$ | $3.0 \pm 1.0$ [112] | | |
| $\bar{B} \to X_s h_c$ | $2.4 \pm 1.0^\dagger$ [53] | | |
| $\bar{B} \to X_s \eta'_c$ | $0.12 \pm 0.22^\dagger$ [113] | | |




  - For instance, the $\eta'$ contribution alone yields a contribution which is of the same order as the short distance $b \to s \ell \ell$:
    $\mathrm{BR}(B \to X_s J/\psi)\mathrm{BR}(J/\psi \to \eta' \ell \ell) = 5.1 \times 10^{-7}$

# Inclusive theory: cascades

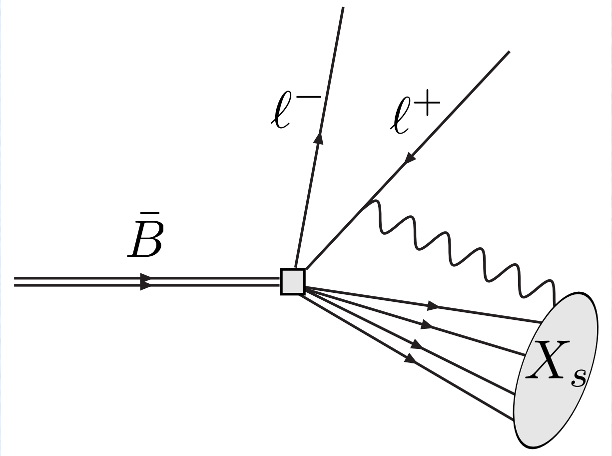
- Even though the inclusive process $J/\psi \to X\ell\ell$ has not be studied yet, we can study cascade effects as sum over exclusive
- This background is concentrated at low-$q^2$:

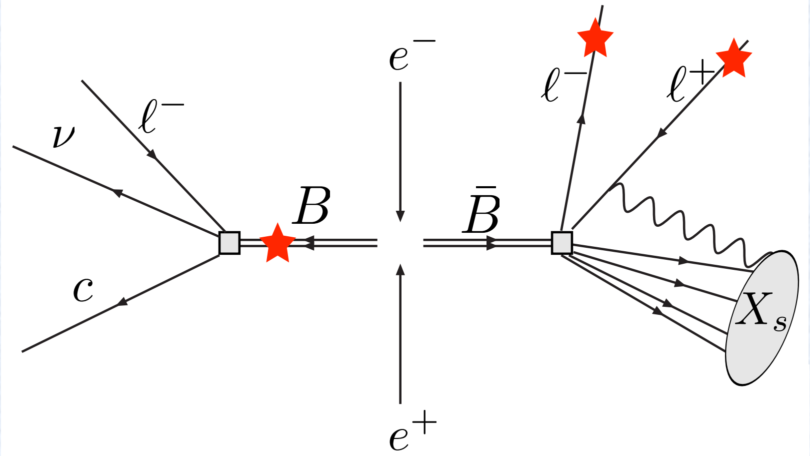



- After imposing $m_X < 2$ GeV this background becomes $\ll 1\,\%$ !

# QED radiation: theory vs experiment

- Photons emitted by the final state leptons (especially electrons) should be technically included in the Xs system:



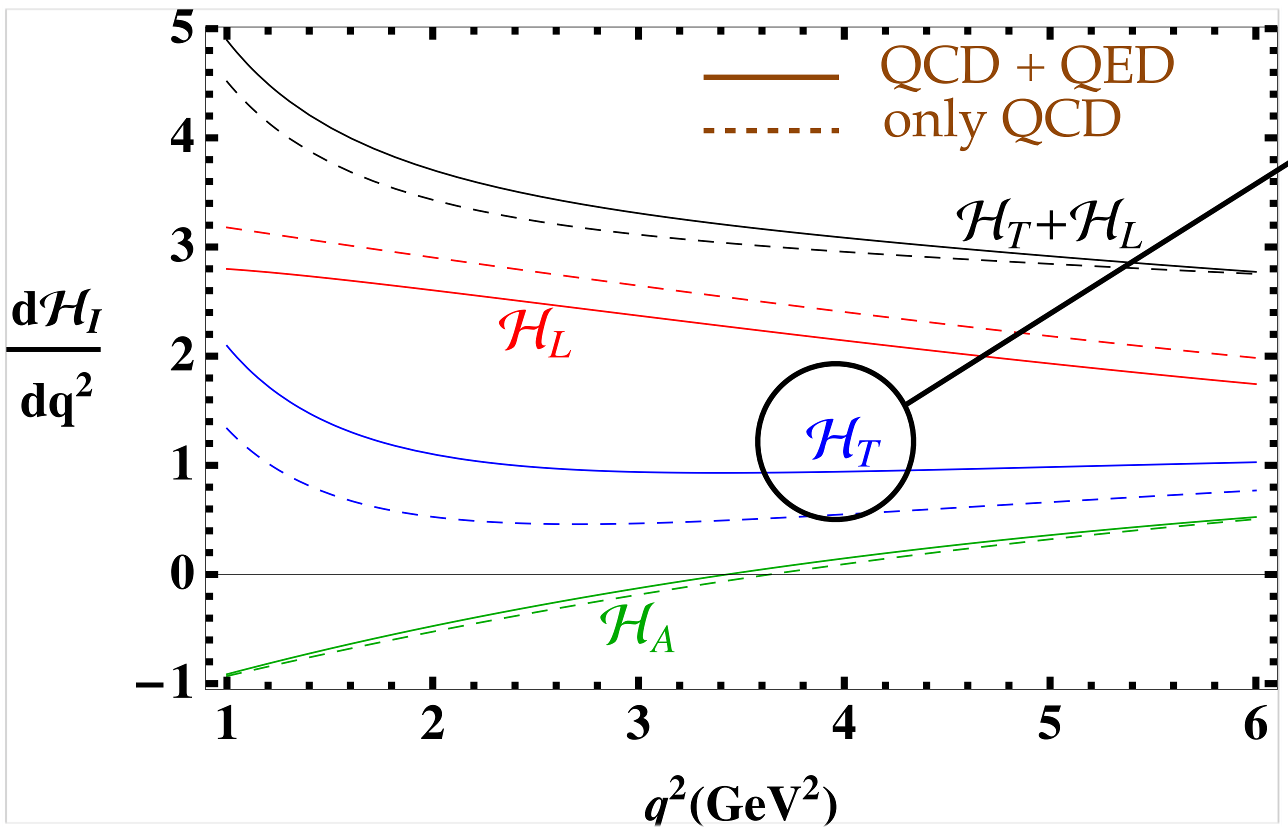

- This implies large $\alpha_{em}\log(m_e/m_b)$ at low and high-$q^2$
- The logs cancel in the total rate that is however inaccessible (resonances)
- At BaBar and Belle most but not all of these photons are included in the Xs system
- Need Monte Carlo studies (EVTGEN+PHOTOS) to find the correction factor:

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 1.65\%$$

$$\frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}} - 1 = 6.8\%$$

# QED radiation: size of the effect

- Impact of collinear photon radiation is huge on some observables
- Cross check with Monte Carlo study (EVTGEN + PHOTOS)




Shift on $H_T$ is $\sim 70\,\%$ !

$H_T$ is smaller than $H_L$ ($\hat{s} < 0.3$ and $C_7 < 0$):

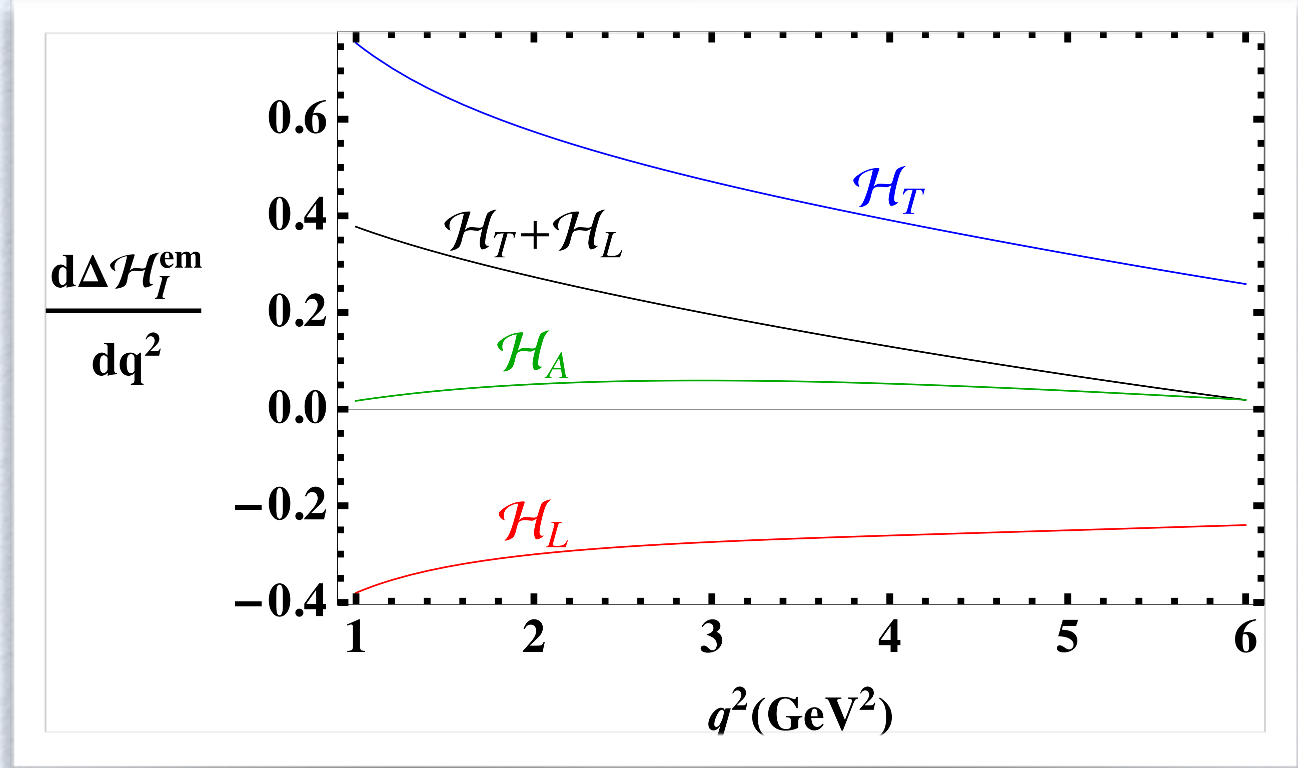
$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right]$$

$$H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$

| | $q^2 \in [1,6]$ GeV$^2$ | | | $q^2 \in [1,3.5]$ GeV$^2$ | | | $q^2 \in [3.5,6]$ GeV$^2$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$ | $\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$ | $\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$ |
| $\mathcal{B}$ | 100 | 5.1 | 5.1 | 54.6 | 3.7 | 6.8 | 45.4 | 1.4 | 3.1 |
| $\mathcal{H}_T$ | 19.5 | 14.1 | 72.5 | 9.5 | 8.8 | 92.1 | 10.0 | 5.4 | 53.6 |
| $\mathcal{H}_L$ | 80.0 | -8.7 | -10.9 | 44.7 | -4.7 | -10.6 | 35.3 | -4.0 | -11.3 |
| $\mathcal{H}_A$ | -3.3 | 1.4 | -43.6 | -7.2 | 0.8 | -10.7 | 4.0 | 0.6 | 16.2 |

# QED radiation: size of the effect

- We calculated the effect of collinear photon radiation and found large effects on some observables

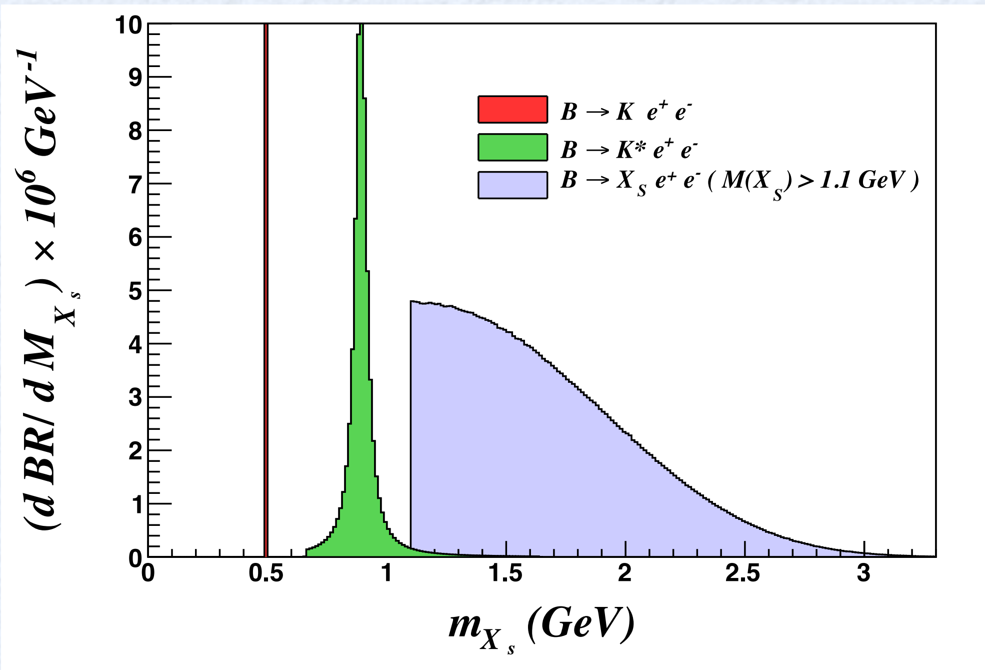


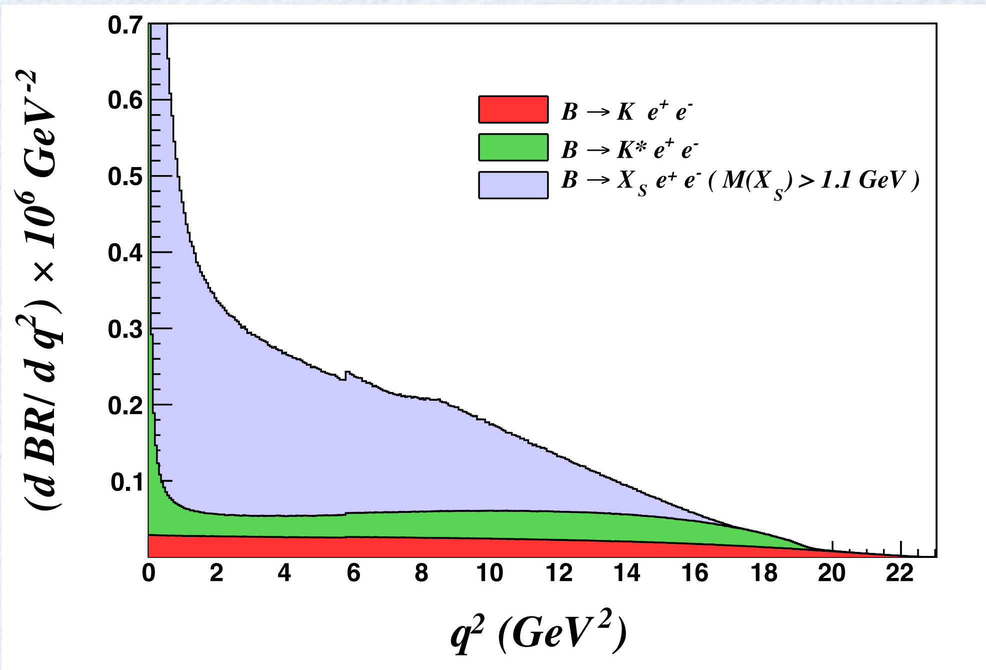

Size of QED contributions to the $H_T$ and $H_L$ is similar

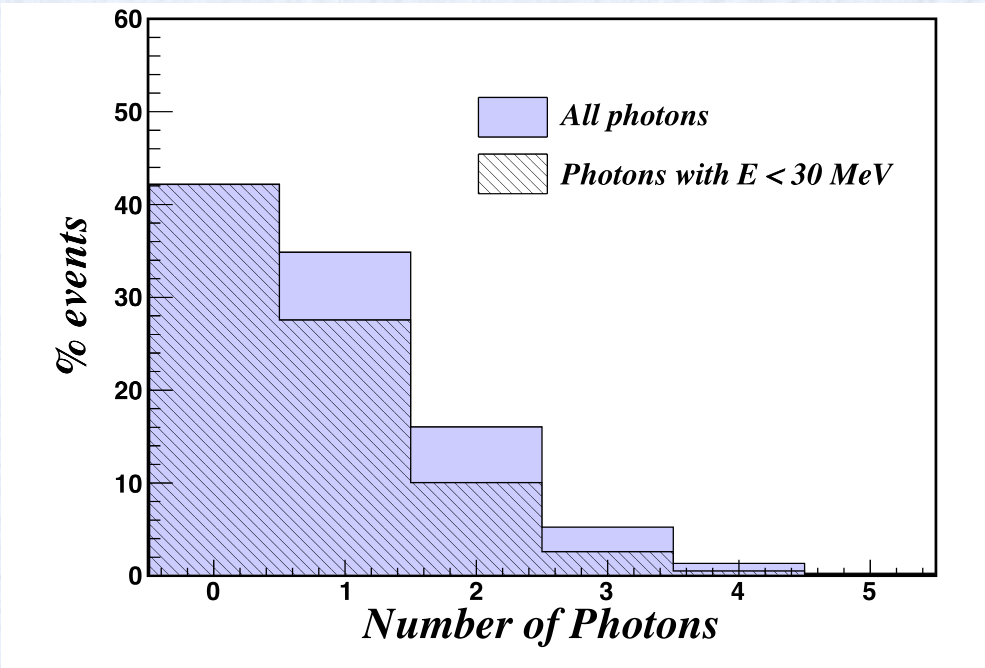
| | $q^2 \in [1,6]$ GeV$^2$ | | | $q^2 \in [1,3.5]$ GeV$^2$ | | | $q^2 \in [3.5,6]$ GeV$^2$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$ | $\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$ | $\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$ |
| $\mathcal{B}$ | 100 | 5.1 | 5.1 | 54.6 | 3.7 | 6.8 | 45.4 | 1.4 | 3.1 |
| $\mathcal{H}_T$ | 19.5 | 14.1 | 72.5 | 9.5 | 8.8 | 92.1 | 10.0 | 5.4 | 53.6 |
| $\mathcal{H}_L$ | 80.0 | -8.7 | -10.9 | 44.7 | -4.7 | -10.6 | 35.3 | -4.0 | -11.3 |
| $\mathcal{H}_A$ | -3.3 | 1.4 | -43.6 | -7.2 | 0.8 | -10.7 | 4.0 | 0.6 | 16.2 |

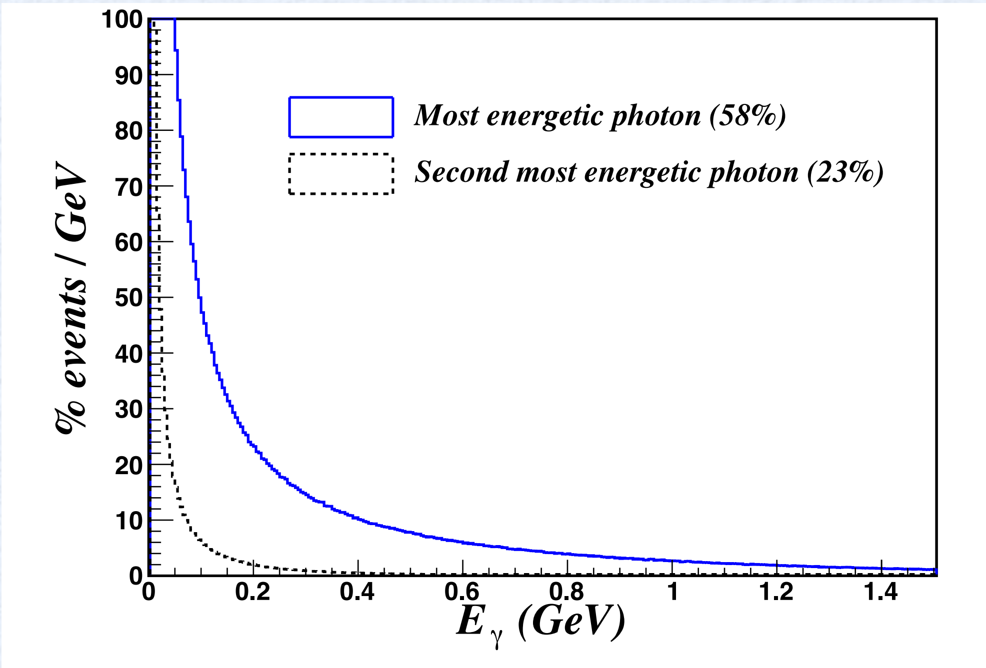
# QED radiation: Monte Carlo check

- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)
  [Many thanks to K. Flood, O. Long and C. Schilling]



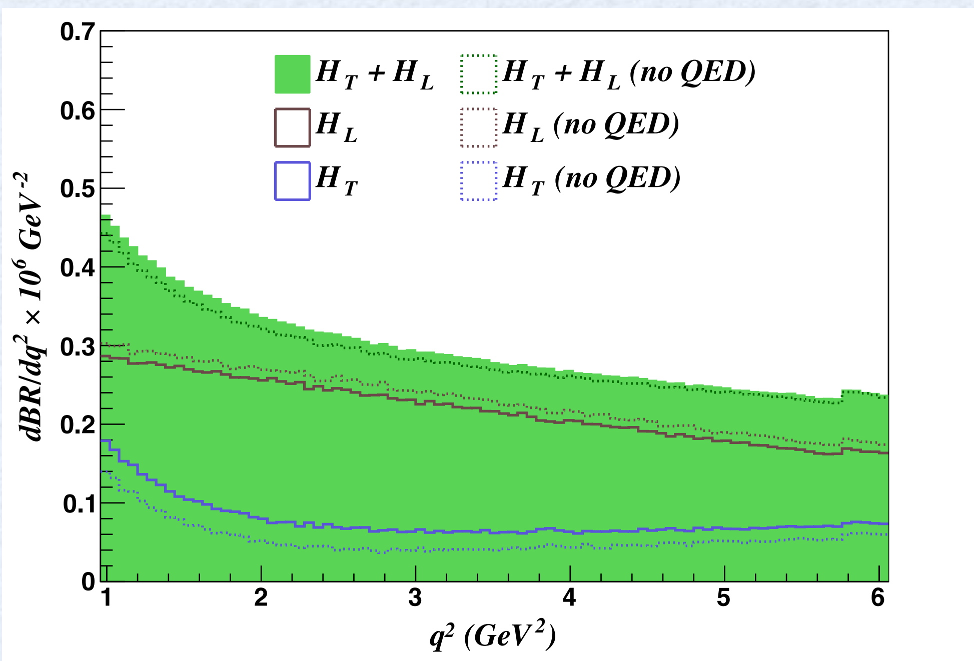



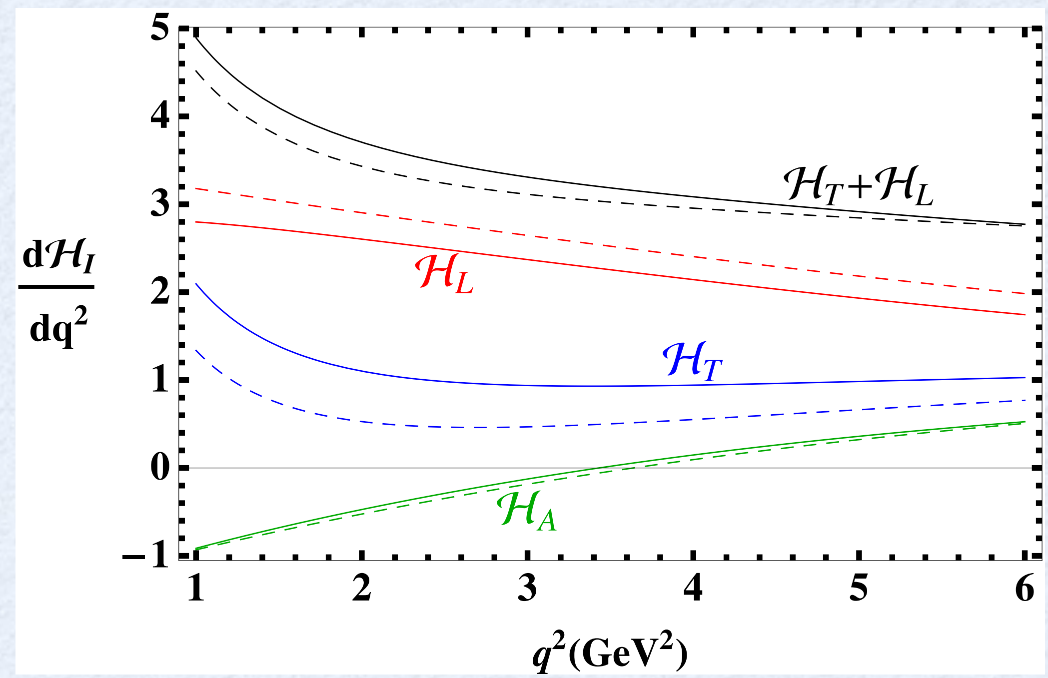



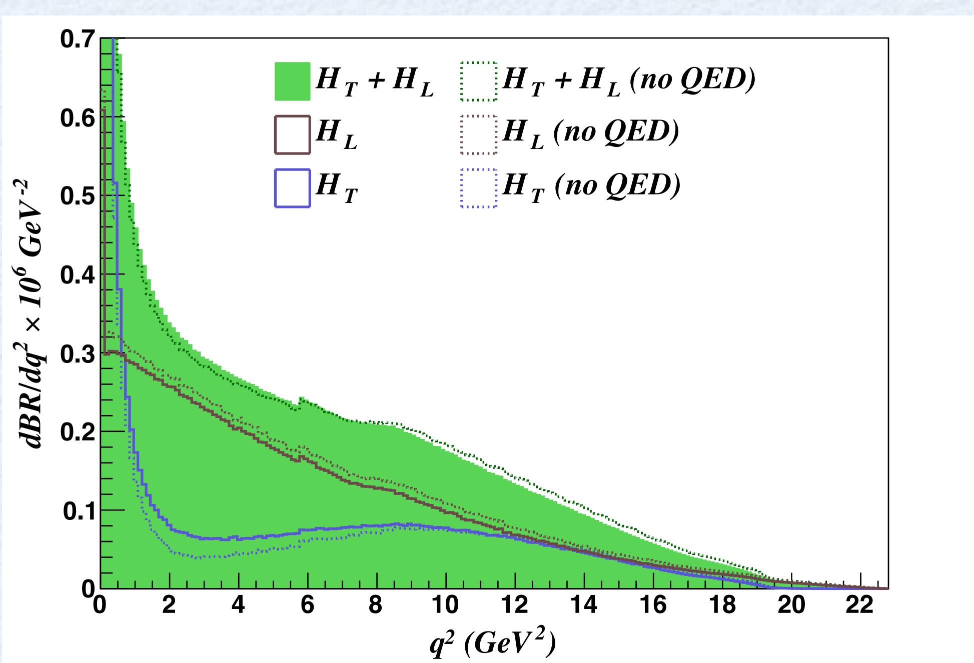



# QED radiation: Monte Carlo check

- The Monte Carlo study reproduces the main features of the analytical results




Monte Carlo:

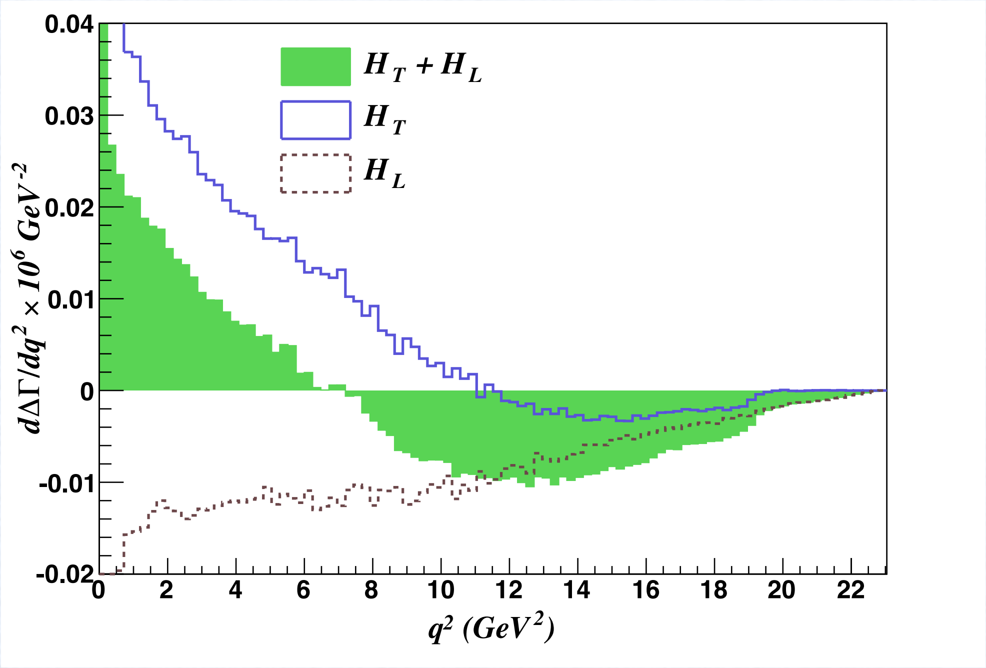
| | $q^2 \in [1,6]$ GeV$^2$ | | |
|---|---|---|---|
| | $\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$ |
| $\mathcal{B}$ | 100 | 3.5 | 3.5 |
| $\mathcal{H}_T$ | 19.0 | 8.0 | 43.0 |
| $\mathcal{H}_L$ | 81.0 | -4.5 | -5.5 |

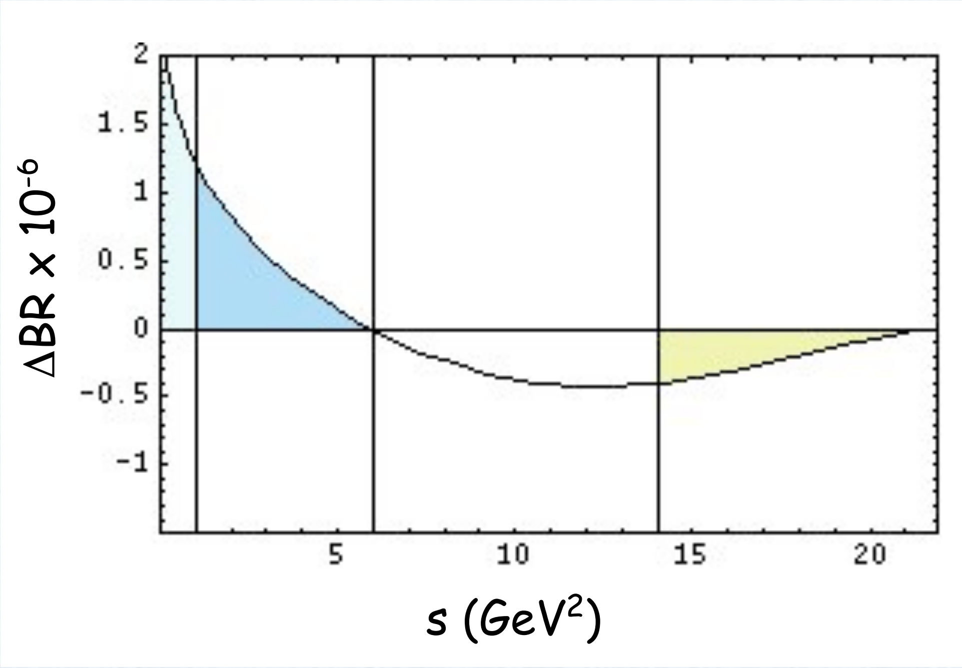
Analytical:

| | $q^2 \in [1,6]$ GeV$^2$ | | |
|---|---|---|---|
| | $\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$ | $\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$ |
| $\mathcal{B}$ | 100 | 5.1 | 5.1 |
| $\mathcal{H}_T$ | 19.5 | 14.1 | 72.5 |
| $\mathcal{H}_L$ | 80.0 | -8.7 | -10.9 |

# QED radiation: Monte Carlo check

- The Monte Carlo study reproduces the main features of the analytical results:




- Take home points on QED radiation and treatment of photons:
  - Large impact (up to $70\,\%$ for $H_T$)
  - Strong dependence on the observable (e.g. $H_T$) and on the shape of the spectrum (as shown by the comparison between theory and EVTGEN+PHOTOS)
- Experimental strategies:
  - be as inclusive as possible (i.e. include photons in $X_s$ system)
  - "remove" collinear photons effects with PHOTOS (be wary of dependence on the shape of the EVTGEN generated spectrum)

# Inputs

| | |
|---|---|
| $\alpha_s(M_z) = 0.1181(11)$ | $m_e = 0.51099895$ MeV |
| $\alpha_e(M_z) = 1/127.955$ | $m_\mu = 105.65837$ MeV |
| $s_W^2 \equiv \sin^2\theta_W = 0.2312$ | $m_\tau = 1.77686$ GeV |
| $\|V_{ts}^* V_{tb}/V_{cb}\|^2 = 0.96403(87)$ [118] | $\overline{m}_c(\overline{m}_c) = 1.275(25)$ GeV |
| $\|V_{ts}^* V_{tb}/V_{ub}\|^2 = 123.5(5.3)$ [118] | $m_b^{1S} = 4.691(37)$ GeV [119, 120] |
| $\|V_{td}^* V_{tb}/V_{cb}\|^2 = 0.04195(78)$ [118] | $\|V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\| = 0.02022(44)$ [118] |
| $\|V_{td}^* V_{tb}/V_{ub}\|^2 = 5.38(26)$ [118] | $\arg\left[V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\right] = 115.3(1.3)^\circ$ [118] |
| $\mathcal{B}(B \to X_c e\bar{\nu})_{\rm exp} = 0.1065(16)$ [121] | $\|V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\| = 0.420(10)$ |
| $m_B = 5.2794$ GeV | $\arg\left[V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\right] = -88.3(1.4)^\circ$ |
| $M_Z = 91.1876$ GeV | $m_{t,\rm pole} = 173.1(0.9)$ GeV |
| $M_W = 80.379$ GeV | $C = 0.568(7)(10)$ [122] |
| $\mu_b = 5^{+5}_{-2.5}$ GeV | $\mu_0 = 120^{+120}_{-60}$ GeV |
| $f_{\rm NV} = (0.02 \pm 0.16)\ {\rm GeV}^3$ | $\lambda_2^{\rm eff} = 0.130(21)\ {\rm GeV}^2$ [48] |
| $f_{\rm V} - f_{\rm NV} = (0.041 \pm 0.052)\ {\rm GeV}^3$ | $\lambda_1 = -0.267(90)\ {\rm GeV}^2$ [48] |
| $[\delta f]_{SU(3)} = (0 \pm 0.04)\ {\rm GeV}^3$ | $\rho_1 = 0.038(70)\ {\rm GeV}^3$ [48] |
| $[\delta f]_{SU(2)} = (0 \pm 0.004)\ {\rm GeV}^3$ | |

Dominant uncertainties at high-$q^2$

# Inputs: HQET matrix elements

- Power corrections affects mainly high-q² where the OPE breaks down:

$$\lambda_1 \equiv \frac{1}{2m_B}\langle B|\bar{h}_v(iD)^2 h_v|B\rangle$$

$$\lambda_2 \equiv \frac{1}{12m_B}\langle B|\bar{h}_v(-i\sigma_{\mu\nu})G^{mu\nu} h_v|B\rangle$$

$$\rho_1 \equiv \frac{1}{2m_B}\langle B|\bar{h}_v iD_\mu(iv\cdot D)iD^\mu h_v|B\rangle$$

$$\rho_2 \equiv \frac{1}{6m_B}\langle B|\bar{h}_v iD^\mu(iv\cdot D)iD^\nu h_v(-i\sigma_{\mu\nu})|B\rangle$$

  - Extracted in the kinetic scheme from moments of the $B \to X_c \ell \nu$ spectrum [Gambino, Healey, Turczyk]
  - Converted to the pole scheme
  - In $b \to s\ell\ell$ $\lambda_2$ and $\rho_2$ appear in the combination $\lambda_2^{\rm eff} \equiv \lambda_2 - \frac{\rho_2}{m_b}$

$$f_q^{0,\pm} \equiv \frac{1}{2m_B}\langle B^{0,\pm}|Q_1^q - Q_2^q|B^{0,\pm}\rangle$$

$$Q_1^q = \bar{h}_v\gamma_\mu(1-\gamma_5)q\;\bar{q}\gamma^\mu(1-\gamma_5)h_v\,,$$

$$Q_2^q = \bar{h}_v(1-\gamma_5)q\;\bar{q}(1+\gamma_5)h_v\,.$$

  - Weak annihilation contributions ($q = u, d, s$ is the flavor of the spectator quark)

# Inputs: Weak Annihilation

- In the isospin SU(3) limit there are only two WA matrix elements:

$$f_{\rm V} \equiv f_u^{\pm} \overset{SU(2)}{=} f_d^0$$

$$f_{\rm NV} \equiv f_u^0 \overset{SU(2)}{=} f_d^{\pm} \overset{SU(3)}{=} f_s^0 \overset{SU(2)}{=} f_s^{\pm}$$

- Numerically we adopt upper limits extracted from $D^{0,\pm}$ and $D_s$ decays rescaled by a factor $m_B f_B^2/(m_D f_D^2)$ [following the analysis of Gambino, Kamenik]
- We found that $f_{\rm NV}$ and $f_{\rm NV} - f_{\rm V}$ are mostly uncorrelated
- We estimate SU(2) and SU(3) breaking effects following [Ligeti, Tackmann]
- Taking into account the adopted normalizations, we need:

$$\mathcal{B}(B \to X_s \ell^+ \ell^-) \Longrightarrow \begin{cases} f_s = f_{\rm NV} \\ f_u = (f_{\rm V} + f_{\rm NV})/2 \end{cases}$$

$$\mathcal{R}(s_0, B \to X_s \ell^+ \ell^-) \Longrightarrow \begin{cases} (f_s + f_u^0)/2 = f_{\rm NV} \\ f_s - f_u^0 = [\delta f]_{SU(3)} \end{cases}$$

$$\mathcal{B}(B \to X_d \ell^+ \ell^-) \text{ and } \mathcal{R}(s_0, B \to X_d \ell^+ \ell^-) \Longrightarrow \begin{cases} (f_d + f_u)/2 = (f_{\rm V} + f_{\rm NV})/2 \\ f_d - f_u = [\delta f]_{SU(2)} \end{cases}$$

# $B \to X_s \ell\ell$: experimental status and SM predictions

- Branching ratios
  - World averages from BaBar (424 fb$^{-1}$) and Belle (140 fb$^{-1}$):

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{\exp}_{low} = \left(1.58 \pm 0.30\right) \times 10^{-6} \qquad \delta_{\exp} = 23\,\% \qquad q^2 \in [1,6]\ \mathrm{GeV}^2$$

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{\exp}_{high} = (4.8 \pm 1.0) \times 10^{-7} \qquad \delta_{\exp} = 21\,\% \qquad q^2 > 14.4\ \mathrm{GeV}^2$$

  - SM predictions [preliminary]:

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{\mathrm{SM}}_{low} = \left(1.75 \pm 0.13\right) \times 10^{-6} \qquad \delta_{\exp} = 7.4\,\% \qquad q^2 \in [1,6]\ \mathrm{GeV}^2$$

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{\mathrm{SM}}_{high} = \left(2.21 \pm 0.68\right) \times 10^{-7} \qquad \delta_{\exp} = 31\,\% \qquad q^2 > 14.4\ \mathrm{GeV}^2$$

- Forward-backward asymmetry (non-optimal binning)
  - Belle: $\bar{A}^{\exp}_{\mathrm{FB}} = \begin{cases} 0.34 \pm 0.24 \pm 0.02 & q^2 \in [0.2,4.3]\mathrm{GeV}^2 \\ 0.04 \pm 0.31 \pm 0.05 & q^2 \in [4.3,7.3(8.1)]\mathrm{GeV}^2 \end{cases}$
  - SM: $\bar{A}^{\mathrm{SM}}_{\mathrm{FB}} = \begin{cases} -0.077 \pm 0.006 & q^2 \in [0.2,4.3]\mathrm{GeV}^2 \\ 0.05 \pm 0.02 & q^2 \in [4.3,7.3(8.1)]\mathrm{GeV}^2 \end{cases}$

[not updated with new inputs]

# $B \to X_s \ell\ell$: complete SM predictions

- Branching ratios [preliminary]

$$\mathcal{B}[1,6]_{ee} = (1.78 \pm 0.08_{\text{scale}} \pm 0.02_{m_t} \pm 0.04_{C,m_c} \pm 0.02_{m_b} \pm 0.01_{\alpha_s} \pm 0.002_{\text{CKM}} \pm 0.03_{\text{BR}_{\text{sl}}} \pm 0.01_{\lambda_2} \pm 0.09_{\text{resolved}}) \cdot 10^{-6}$$

$$= 1.78\,(1 \pm 7.5\%) \cdot 10^{-6}$$

$$\mathcal{B}[1,6]_{\mu\mu} = 1.73\,(1 \pm 7.4\%) \cdot 10^{-6}$$

$$\mathcal{B}[>14.4]_{ee} = (2.04 \pm 0.28_{\text{scale}} \pm 0.02_{m_t} \pm 0.03_{C,m_c} \pm 0.19_{m_b} \pm 0.002_{\text{CKM}} \pm 0.03_{\text{BR}_{\text{sl}}} \pm 0.01_{\alpha_s} \pm 0.13_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \cdot 10^{-7}$$

$$= 2.04\,(1 \pm 46\%) \cdot 10^{-7}$$

$$\mathcal{B}[>14.4]_{\mu\mu} = 2.38\,(1 \pm 36\%) \cdot 10^{-7}$$

- Scale uncertainties and resolved contributions dominate at low-$q^2$
- Scale uncertainties and power corrections dominate at high-$q^2$

# $B \to X_s \ell\ell$: complete SM predictions

- $H_T$ and $H_L$ ($\mathrm{BR} = H_T + H_L$) and ($[1,3.5], [3.5,6]$) $\mathrm{GeV}^2$ breakdown

$$\mathcal{B}[1,3.5]_{ee} = 0.982\,(1 \pm 6.8\%) \cdot 10^{-6} \qquad \mathcal{B}[1,3.5]_{\mu\mu} = 0.944\,(1 \pm 6.7\%) \cdot 10^{-6}$$
$$\mathcal{B}[3.5,6]_{ee} = 0.798\,(1 \pm 8.4\%) \cdot 10^{-6} \qquad \mathcal{B}[3.5,6]_{\mu\mu} = 0.785\,(1 \pm 8.4\%) \cdot 10^{-6}$$
$$\mathcal{B}[1,6]_{ee} = 1.78\,(1 \pm 7.5\%) \cdot 10^{-6} \qquad \mathcal{B}[1,6]_{\mu\mu} = 1.73\,(1 \pm 7.4\%) \cdot 10^{-6}$$
$$H_T[1,3.5]_{ee} = 2.91\,(1 \pm 6.5\%) \cdot 10^{-7} \qquad H_T[1,3.5]_{\mu\mu} = 2.09\,(1 \pm 5.7\%) \cdot 10^{-7}$$
$$H_T[3.5,6]_{ee} = 2.43\,(1 \pm 8.2\%) \cdot 10^{-7} \qquad H_T[3.5,6]_{\mu\mu} = 1.94\,(1 \pm 8.2\%) \cdot 10^{-7}$$
$$H_T[1,6]_{ee} = 5.34\,(1 \pm 7.1\%) \cdot 10^{-7} \qquad H_T[1,6]_{\mu\mu} = 4.03\,(1 \pm 6.9\%) \cdot 10^{-7}$$
$$H_L[1,3.5]_{ee} = 6.35\,(1 \pm 5.5\%) \cdot 10^{-7} \qquad H_L[1,3.5]_{\mu\mu} = 6.79\,(1 \pm 5.3\%) \cdot 10^{-7}$$
$$H_L[3.5,6]_{ee} = 4.97\,(1 \pm 5.8\%) \cdot 10^{-7} \qquad H_L[3.5,6]_{\mu\mu} = 5.34\,(1 \pm 5.9\%) \cdot 10^{-7}$$
$$H_L[1,6]_{ee} = 1.13\,(1 \pm 5.3\%) \cdot 10^{-6} \qquad H_L[1,6]_{\mu\mu} = 1.21\,(1 \pm 5.8\%) \cdot 10^{-6}$$

[not updated with new inputs]

- Error breakdown is similar to the branching ratio one

# $B \to X_s \ell\ell$: complete SM predictions

- $H_A$ and zero-crossing $(q_0^2)$ $[\bar{A}_{\rm FB} = \frac{3}{4}\frac{H_A}{H_T + H_L}]$

$$
\begin{aligned}
H_A[1,3.5]_{ee} &= (-1.03 \pm 0.04_{\rm scale} \pm 0.01_{m_t} \pm 0.02_{C,m_c} \pm 0.02_{m_b} \\
&\quad \pm 0.01_{\alpha_s} \pm 0.003_{\rm CKM} \pm 0.01_{{\rm BR}_{\rm sl}}) \cdot 10^{-7} \\
&= -1.03\ (1 \pm 4.9\%) \cdot 10^{-7} \\
H_A[3.5,6]_{ee} &= (+0.73 \pm 0.11_{\rm scale} \pm 0.01_{m_t} \pm 0.04_{C,m_c} \pm 0.05_{m_b} \\
&\quad \pm 0.02_{\alpha_s} \pm 0.002_{\rm CKM} \pm 0.01_{{\rm BR}_{\rm sl}}) \cdot 10^{-7} \\
&= 0.73\ (1 \pm 16\%) \cdot 10^{-7} \\
H_A[1,3.5]_{\mu\mu} &= -1.10\ (1 \pm 11\%) \cdot 10^{-7} \\
H_A[3.5,6]_{\mu\mu} &= 0.67\ (1 \pm 18\%) \cdot 10^{-7}
\end{aligned}
$$

$$
\begin{aligned}
(q_0^2)_{ee} &= (3.46 \pm 0.10_{\rm scale} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.06_{m_b} \pm 0.02_{\alpha_s})\ {\rm GeV}^2 \\
&= 3.46\ (1 \pm 3.2\%)\ {\rm GeV}^2 \\
(q_0^2)_{\mu\mu} &= 3.58\ (1 \pm 3.4\%)\ {\rm GeV}^2
\end{aligned}
$$

[not updated with new inputs]

- Error breakdown is similar to the branching ratio one

# $B \to X_s \ell\ell$: SM predictions

[not updated with new inputs]

- $\mathcal{R}(s_0) = \Gamma_{s>s_0}(\bar{B} \to X_s \ell\ell)/\Gamma_{s>s_0}(\bar{B}^0 \to X_u \ell\nu)$:

$$\begin{aligned}
\mathcal{R}(14.4)_{ee} &= (2.25 \pm 0.12_{\text{scale}} \pm 0.03_{m_t} \pm 0.02_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.20_{\text{CKM}} \\
&\quad \pm 0.02_{\lambda_2} \pm 0.14_{\rho_1} \pm 0.08_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\
&= 2.25\ (1 \pm 14\%) \cdot 10^{-3} \\
\mathcal{R}(14.4)_{\mu\mu} &= (2.62 \pm 0.09_{\text{scale}} \pm 0.03_{m_t} \pm 0.01_{C,m_c} \pm 0.01_{m_b} \pm 0.01_{\alpha_s} \pm 0.23_{\text{CKM}} \\
&\quad \pm 0.0002_{\lambda_2} \pm 0.09_{\rho_1} \pm 0.04_{f_u^0+f_s} \pm 0.12_{f_u^0-f_s}) \cdot 10^{-3} \\
&= 2.62\ (1 \pm 11\%) \cdot 10^{-3}
\end{aligned}$$

- Impact of $m_b^{-2,-3}$ power corrections $(\lambda_2, \rho_1)$ and weak annihilation $(f_q^a)$ is reduced:

$\delta_{m_b^{-2},m_b^{-3}} : 29\,\% \to 6\,\%$

$\delta_{\text{WA}} : 27\,\% \to 6\,\%$

- The largest source of uncertainty is $V_{ub}$

# $B \to X_s \ell \ell$: new observables

- At leading order in QED and at all orders in QCD, the double differential width is a quadratic polynomial: $\Gamma \sim a \cos^2\theta + b\, cos\theta + c$
- $\Gamma$ receives non polynomial log-enhanced QED corrections
- We can build new observables by projecting out with Legendre polynomials:

$$H_I(q^2) = \int_{-1}^{1} \frac{d^2\Gamma}{dq^2 dz} W_I(z) dz$$

$$W_T = \frac{2}{3} P_0(z) + \frac{10}{3} P_2(z)$$

$$W_L = \frac{1}{3} P_0(z) - \frac{10}{3} P_2(z)$$

$$W_A = \frac{4}{3} \text{sign}(z)$$

$$W_3 = P_3(z)$$

$$W_4 = P_4(z)$$

→ new observables

# $B \to X_s \ell \ell$: lepton flavor universality violation

| | | $R_X^{\mu/e}$ |
|---|---|---|
| $\mathscr{B}[1,6]_{ee} = 1.78\,(1 \pm 0.075) \cdot 10^{-6}$ | $\mathscr{B}[1,6]_{\mu\mu} = 1.73\,(1 \pm 0.074) \cdot 10^{-6}$ | 0.97 |
| $\mathscr{B}[>14.4]_{ee} = 2.04\,(1 \pm 0.46) \cdot 10^{-7}$ | $\mathscr{B}[>14.4]_{\mu\mu} = 2.38\,(1 \pm 0.36) \cdot 10^{-7}$ | 1.17 |
| $H_T[1,6]_{ee} = 5.34(1 \pm 0.07) \times 10^{-7}$ | $H_T[1,6]_{\mu\mu} = 4.03(1 \pm 0.07) \times 10^{-7}$ | 0.75 |
| $H_L[1,6]_{ee} = 1.13(1 \pm 0.05) \times 10^{-6}$ | $H_L[1,6]_{\mu\mu} = 1.21(1 \pm 0.06) \times 10^{-6}$ | 1.07 |
| $H_A[1,3.5]_{ee} = -1.03(1 \pm 0.05) \times 10^{-7}$ | $H_A[1,3.5]_{\mu\mu} = -1.10(1 \pm 0.05) \times 10^{-7}$ | 1.07 |
| $H_A[3.6,6]_{ee} = 0.73(1 \pm 0.16) \times 10^{-7}$ | $H_A[3.6,6]_{\mu\mu} = 0.67(1 \pm 0.18) \times 10^{-7}$ | 0.92 |
| $H_3[1,6]_{ee} = 8.92(1 \pm 0.13) \times 10^{-9}$ | $H_3[1,6]_{\mu\mu} = 3.71(1 \pm 0.13) \times 10^{-9}$ | 0.42 |
| $H_4[1,6]_{ee} = 8.41(1 \pm 0.09) \times 10^{-9}$ | $H_4[1,6]_{\mu\mu} = 3.50(1 \pm 0.09) \times 10^{-9}$ | 0.42 |

- Scale uncertainties dominate at low-$q^2$
- Power corrections and scale uncertainties dominate at high-$q^2$
- Log-enhanced QED corrections at low and high-$q^2$ are correlated

# $B \to X_d \ell\ell$: SM predictions

- Branching ratios

$$\mathscr{B}[1,6]_{ee} = (7.81 \pm 0.37_{\rm scale} \pm 0.08_{m_t} \pm 0.17_{C,m_c} \pm 0.08_{m_b} \pm 0.04_{\alpha_s} \pm 0.15_{\rm CKM}$$
$$\pm 0.12_{\rm BR_{sl}} \pm 0.05_{\lambda_2} \pm 0.39_{\rm resolved}) \cdot 10^{-8}$$
$$= 7.81\ (1 \pm 7.8\%) \cdot 10^{-8}$$
$$\mathscr{B}[1,6]_{\mu\mu} = 7.59\ (1 \pm 7.8\%) \cdot 10^{-8}$$

$$\mathscr{B}[>14.4]_{ee} = (0.86 \pm 0.12_{\rm scale} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.08_{m_b} \pm 0.02_{\rm CKM} \pm 0.02_{\rm BR_{sl}}$$
$$\pm 0.06_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \cdot 10^{-8}$$
$$= 0.86\ (1 \pm 45\%) \cdot 10^{-8}$$
$$\mathscr{B}[>14.4]_{\mu\mu} = 1.00\ (1 \pm 39\%) \cdot 10^{-8}$$

- Scale and resolved uncertainties dominate at low-$q^2$ (hard to improve)
- Power corrections and scale uncertainties dominate at high-$q^2$

# $B \to X_d \ell\ell$: SM predictions

- Ratio $\mathcal{R}(s_0)$

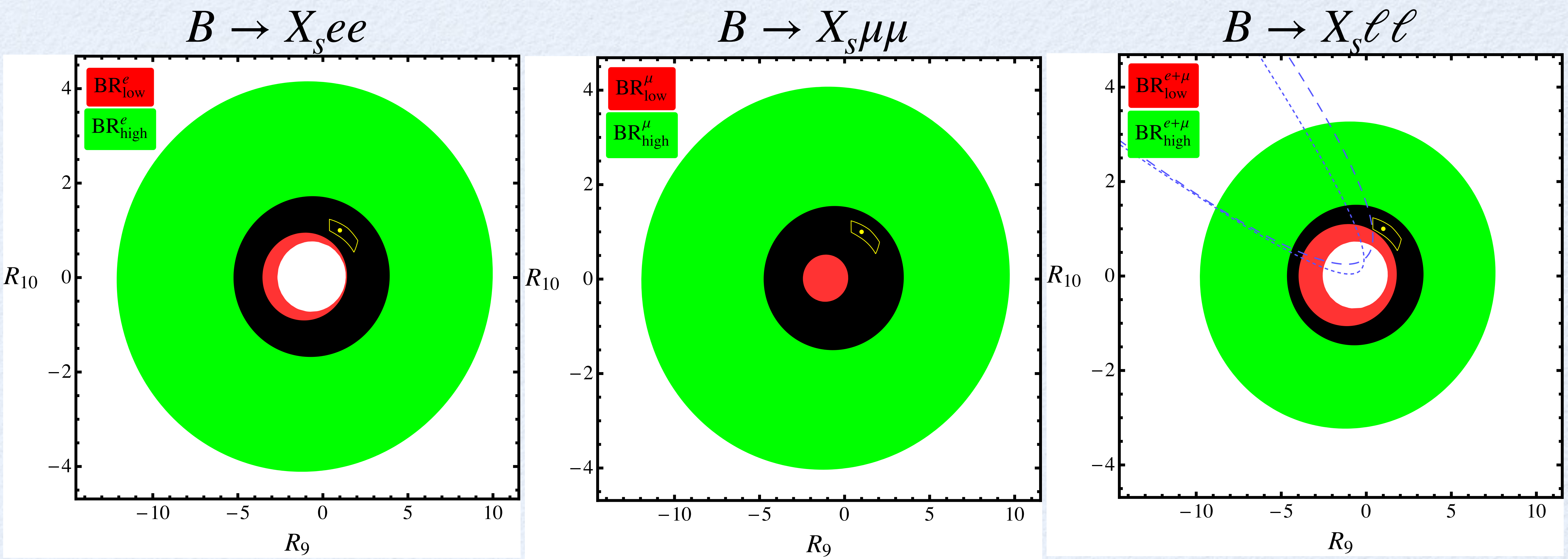
$$\mathcal{R}(14.4)_{ee} = (0.93 \pm 0.02_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{\text{CKM}}$$
$$\pm 0.004_{\lambda_2} \pm 0.06_{\rho_1} \pm 0.05_{f_{u,d}}) \times 10^{-4}$$
$$= 0.93\ (1 \pm 9.7\%) \times 10^{-4}$$
$$\mathcal{R}(14.4)_{\mu\mu} = 1.10\ (1 \pm 6.4\%) \times 10^{-4}$$

- Forward-backward asymmetry and zero-crossing

$$H_A[1,3.5]_{ee} = -0.41\ (1 \pm 9.8\%) \cdot 10^{-8}$$
$$H_A[3.5,6]_{ee} = 0.40\ (1 \pm 18\%) \cdot 10^{-8}$$
$$H_A[1,3.5]_{\mu\mu} = -0.44\ (1 \pm 9.1\%) \cdot 10^{-8}$$
$$H_A[3.5,6]_{\mu\mu} = 0.37\ (1 \pm 19\%) \cdot 10^{-8}$$
$$(q_0^2)_{ee} = 3.28 \pm 0.11_{\text{scale}} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b}$$
$$\pm 0.03_{\alpha_s} \pm 0.004_{\text{CKM}} \pm 0.001_{\lambda_2} \pm 0.06_{\text{resolved}} = 3.28 \pm 0.14$$
$$(q_0^2)_{\mu\mu} = 3.39 \pm 0.14$$

# Wilson coefficients fits

- 95 % CL constraints in the $[R_9, R_{10}]$ plane ($R_i = C_i(\mu_0)/C_i^{SM}(\mu_0)$):




- Note that $C_9^{SM}(\mu_0) = 1.6$ and $C_9^{SM}(\mu_0) = -4.3$
- Best fits from the exclusive anomaly translate in $R_9 \sim -0.45$ and $R_{10} \sim -0.09$

# Belle II reach

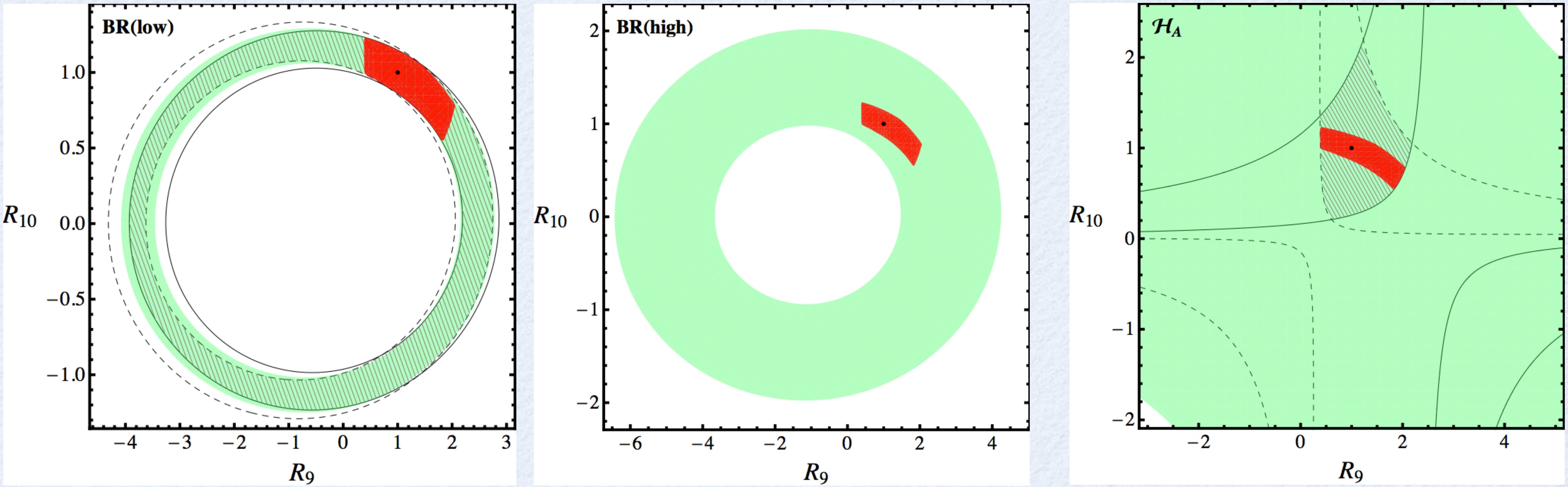
- Projected reach with $50\ \mathrm{ab}^{-1}$ of integrated luminosity

$$\mathcal{O}_{\rm exp} = \int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\hat{s}, z]\, d\hat{s}\, dz\ ,$$

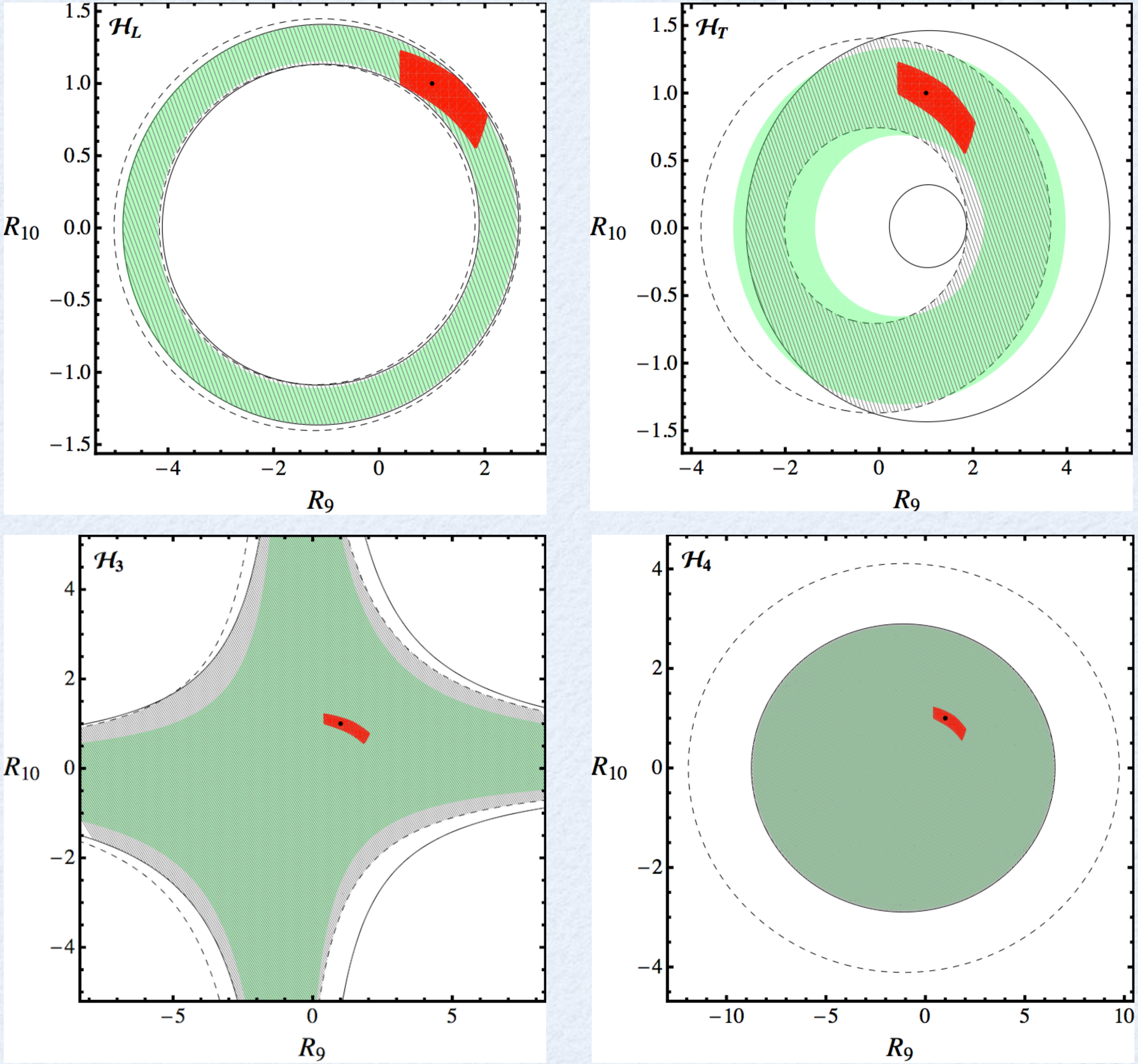
$$\delta\mathcal{O}_{\rm exp} = \left[\int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\hat{s}, z]^2\, d\hat{s}\, dz\right]^{\frac{1}{2}}$$

weight (Legendre polynomial)

| | $[1, 3.5]$ | $[3.5, 6]$ | $[1, 6]$ | $> 14.4$ |
|---|---|---|---|---|
| $\mathcal{B}$ | 3.7 % | 4.0 % | 3.0 % | 4.1% |
| $\mathcal{H}_T$ | 24 % | 21 % | 16 % | - |
| $\mathcal{H}_L$ | 5.8 % | 6.8 % | 4.6 % | - |
| $\mathcal{H}_A$ | 37 % | 44 % | 200 % | - |
| $\mathcal{H}_3$ | 240 % | 180 % | 150 % | - |
| $\mathcal{H}_4$ | 140 % | 360 % | 140 % | - |

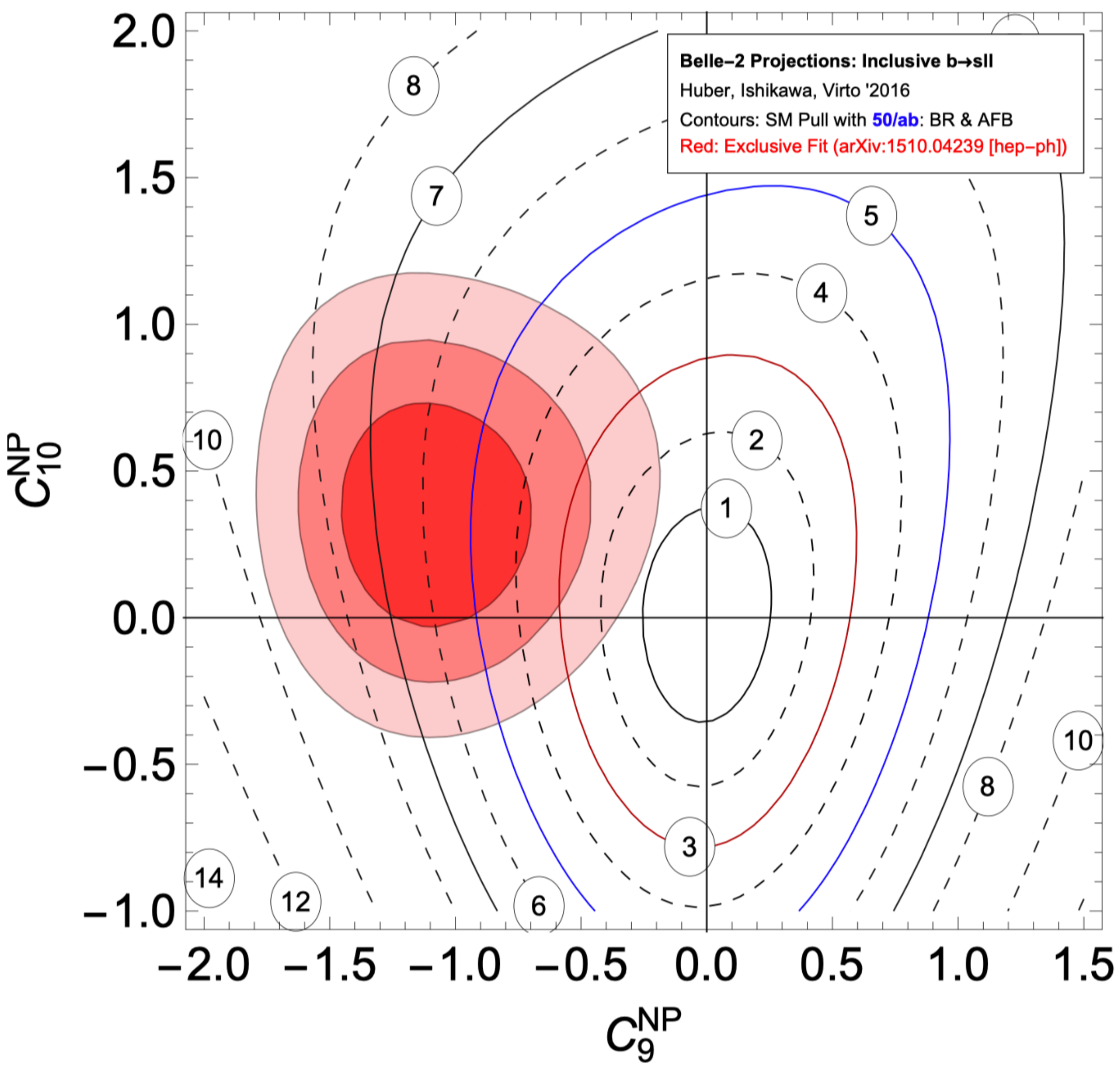


# Belle II reach



# Inclusive/exclusive interplay

[Ishikawa, Virto, Huber, Belle II physics book, 1808.1056, sec. 9.4.5]




[See also Hurth,Mahmoudi'13; Hurth,Mahmoudi,Neshatpour'14]