

Introduction to CP-violation in beauty and charm: beauty physics

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- How can CP-violation be observed with b-quarks?
 - can be observed by comparing CP-conjugated decay rates in various ways, both with and w/out time dependence

$$a_{CP}(f) = \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})}$$

- can manifest itself in charm $\Delta B=1$ transitions (direct CP-violation)

$$\Gamma(B \rightarrow f) \neq \Gamma(CP[B] \rightarrow CP[f]) \quad \text{dCPV}$$

- or in $\Delta B=2$ transitions (indirect CP-violation): mixing $|B_{1,2}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$

$$R_m^2 = |q/p|^2 = \left| \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta m - (i/2)\Delta\Gamma} \right|^2 = 1 + A_m \neq 1 \quad \text{CPVmix}$$

- or in the interference b/w decays ($\Delta B=1$) and mixing ($\Delta B=2$)

$$\lambda_f = \frac{q}{p} \frac{\overline{A}_f}{A_f} = R_m e^{i(\phi+\delta)} \left| \frac{\overline{A}_f}{A_f} \right| \quad \text{CPVint}$$

Recall from yesterday: CP-violation in the SM

★ CP-violation in the SM is related to a single phase of the CKM matrix

- there are MULTIPLE ways to parameterize CKM matrix

- Wolfenstein parameterization (parameters: $\lambda \sim 0.22$, $A \sim 0.83$, $\rho \sim 0.15$, $\eta \sim 0.35$)

$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

- Buras-Wolfenstein parameterization (with $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$)

$$V = \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix} \quad (\text{note } \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*})$$

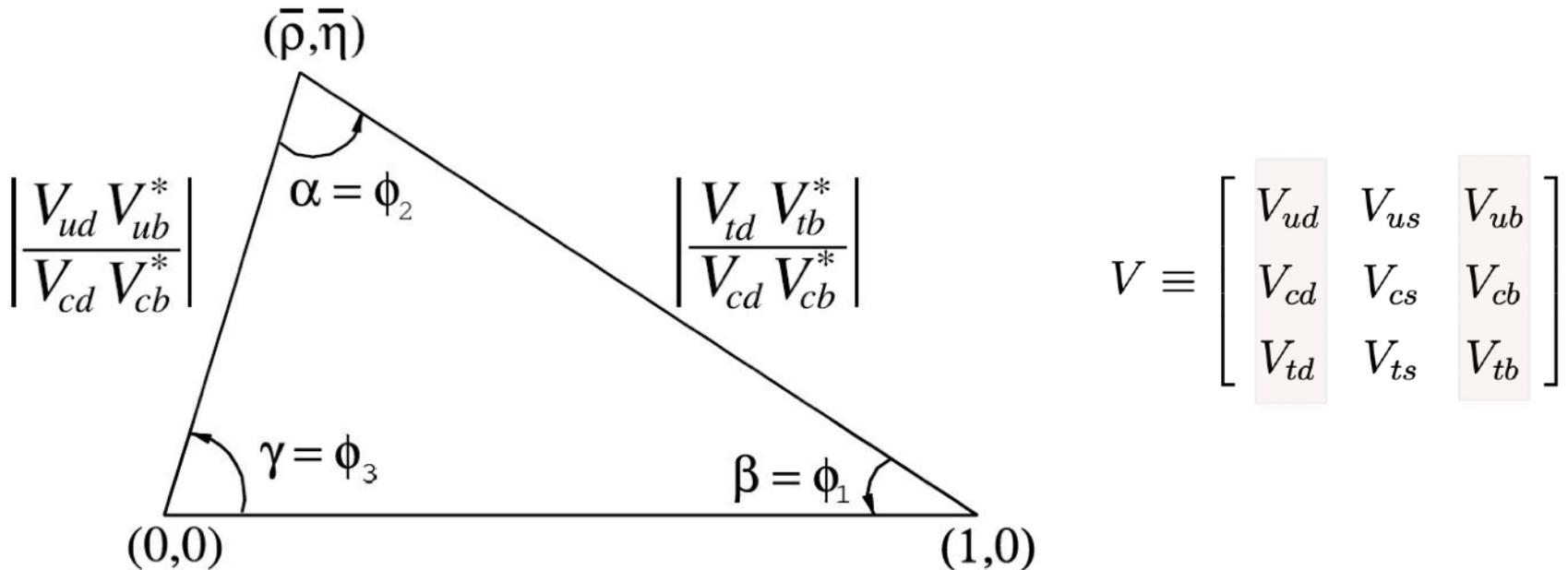
- off-diagonal terms in relations $VV^\dagger=1$ look like triangles in a complex plane

★ CP-violation in flavor transition s can be learned by studying the CKM matrix

Recall from yesterday: CP-violation in the SM

★ There is a single phase of the CKM matrix for 3-generation SM

- off-diagonal terms in unitarity relations $VV^\dagger=1$ look like triangles in a complex plane (ρ,η) , e.g. $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ Each term is $\mathcal{O}(\lambda^3)$



- angles are
 - $\phi_1(\beta) = \arg [-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$ phase of V_{td} in Wolfenstein param
 - $\phi_2(\alpha) = \arg [-V_{td}V_{tb}^*/V_{ud}V_{ub}^*]$
 - $\phi_3(\gamma) = \arg [-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*]$ phase of V_{ub} in Wolfenstein param

2. Time-independent (direct) CP-violation

★ Direct CP-violating asymmetries probe CP-violation in $\Delta B=1$ amplitudes

- CP-asymmetries compare partial rates of CP-conjugated decays

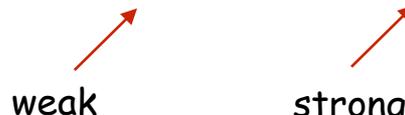
$$a_{CP}(f) = \frac{\Gamma(B \rightarrow f) - \Gamma(\bar{B} \rightarrow \bar{f})}{\Gamma(B \rightarrow f) + \Gamma(\bar{B} \rightarrow \bar{f})} \quad (\text{both charged and neutral B's})$$

- a non-vanishing decay asymmetry requires that a decay amplitude
 - contain several components each of which has its own strong and weak phases
 - strong phases: do not change under CP transformation of the decay amplitude
 - weak phases: flip sign under CP transformation of the decay amplitude

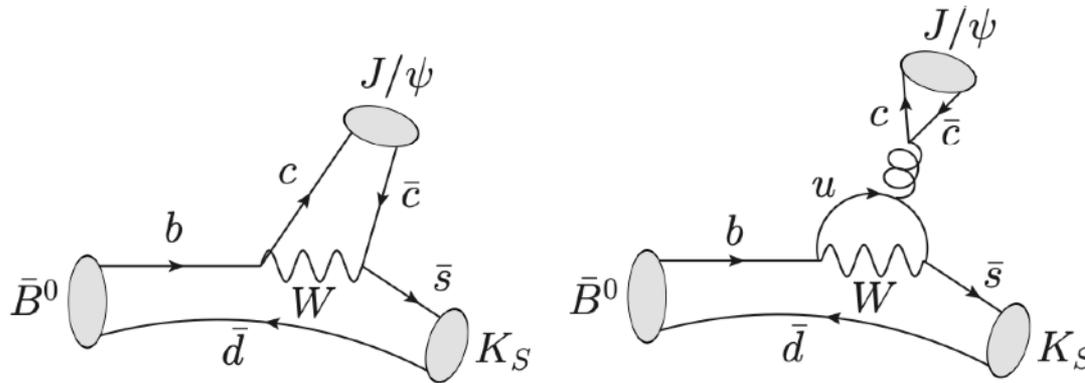
$$A(B \rightarrow f) \equiv A_f = |A_{f1}|e^{i\delta_1}e^{i\theta_1} + |A_{f2}|e^{i\delta_2}e^{i\theta_2}$$

- Now we can form the CP-asymmetry

$$a_{CP}(f) = 2r_f \sin(\theta_1 - \theta_2) \sin(\delta_1 - \delta_2) \quad \text{with} \quad r_f = \left| \frac{A_{f2}}{A_{f1}} \right|$$


weak strong

- How can one compute the amplitudes (especially the strong phase difference)
 - QCD factorization with Bander-Silverman-Soni mechanism



- experimental fits to flavor flow/flavor SU(3) amplitude basis

Loopless studies of CP-violation: CKM angle ϕ_3

★ There are ways to study CP-violation without penguin loops



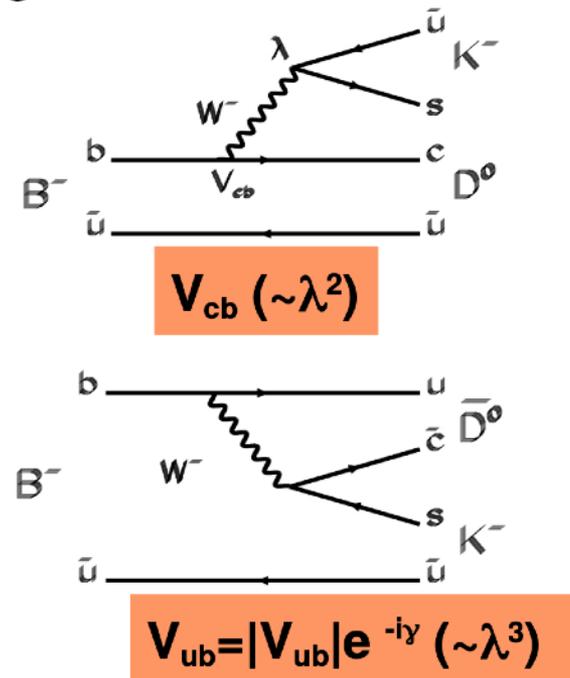
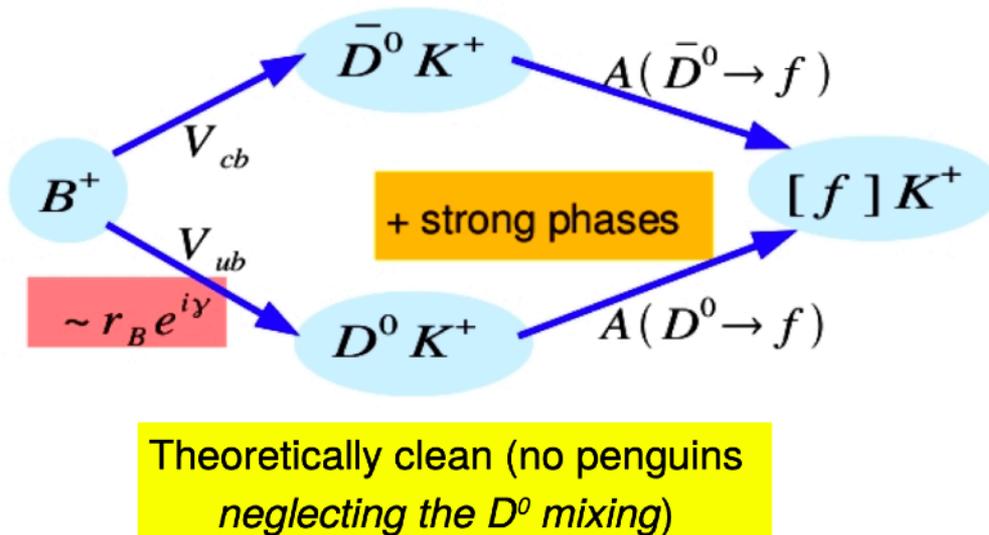
- cleanest signals involve interference of $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$

- **via** $B^\pm \rightarrow D \left(\left[D^0, \bar{D}^0, D_{CP} \right] \rightarrow f \right) K^\pm$ GWS (Gronau, Wyler, London)
- **via** $B^\pm \rightarrow D \left(\left[D^0, \bar{D}^0 \right] \rightarrow K\pi \right) K^\pm$ ADS(Atwood, Dunietz, Soni)
- **via** $B^\pm \rightarrow D \left(D \rightarrow KK^* \right) K^\pm$ GLS (Grossman, Ligeti, Soffer)
- **via** $B^\pm \rightarrow D \left(D \rightarrow \text{multibody} \right) K^\pm$ (Giri, Grossman, Soffer, Zupan, Atwood, Soni)

| Process | Observable | Theory | Sys. dom. (Discovery) [ab ⁻¹] | vs LHCb | vs Belle | Anomaly | NP |
|------------------|-------------------|--------|---|---------|----------|---------|-----|
| ● GGSZ | ϕ_3 | *** | >50 | ** | *** | * | ** |
| ● GLW | ϕ_3 | *** | >50 | ** | *** | * | ** |
| ● ADS | ϕ_3 | ** | >50 | ** | *** | * | *** |
| ● Time-dependent | $\phi_3 - \phi_2$ | ** | - | ** | ** | * | * |

CKM angle ϕ_3 : final state triangles

- ⊙ $D^{(*)}K^{(*)}$ decays: from BRs and BR ratios, no time-dependent analysis, just rates
- ⊙ the phase γ is measured exploiting interferences: two amplitudes leading to the same final states
- ⊙ some rates can be really small: $\sim 10^{-7}$



M. Bona

CKM angle ϕ_3 : final state triangles

GLW(*Gronau, London, Wyler*) method:

more sensitive to r_B

uses the CP eigenstates $D^{(*)0}_{CP}$ with final states:

K^+K^- , $\pi^+\pi^-$ (CP-even), $K_S\pi^0$ (ω, ϕ) (CP-odd)

$$R_{CP\pm} = 1 + r_B^2 \pm 2r_B \cos \gamma \cos \delta_B \quad A_{CP\pm} = \frac{\pm 2r_B \sin \gamma \sin \delta_B}{1 + r_B^2 \pm 2r_B \cos \gamma \cos \delta_B}$$

ADS(*Atwood, Dunietz, Soni*) method: B^0 and \bar{B}^0 in the same final state with $D^0 \rightarrow K^+\pi^-$ (suppr.) and $\bar{D}^0 \rightarrow K^+\pi^-$ (fav.)

$$R_{ADS} = r_B^2 + r_{DCS}^2 + 2r_B r_{DCS} \cos \gamma \cos(\delta_B + \delta_D)$$

the most sensitive way to γ

D^0 Dalitz plot with the decays $B^- \rightarrow D^{(*)0}[K_S\pi^+\pi^-] K^-$

three free parameters to extract: γ , r_B and δ_B

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CKM angle ϕ_3 : initial state triangles

★ One can also use a fact that initial state at Belle II is quantum coherent

- which means that initial state can be CP-tagged
- can be done for both B_d (at $\Upsilon(4S)$) or B_s (at $\Upsilon(5S)$). For B_s

$$A_{\text{CP}} = A(B_s^{\text{CP}} \rightarrow D_s^- K^+) = (A_1 + A_2)/\sqrt{2},$$

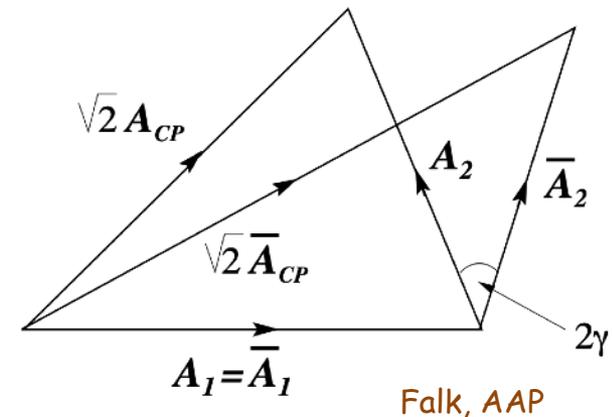
$$\bar{A}_{\text{CP}} = A(B_s^{\text{CP}} \rightarrow D_s^+ K^-) = (\bar{A}_1 + \bar{A}_2)/\sqrt{2}.$$

- measuring all amplitudes,

$$\alpha = \frac{2|A_{\text{CP}}|^2 - |A_1|^2 - |A_2|^2}{2|A_1||A_2|},$$

$$\bar{\alpha} = \frac{2|\bar{A}_{\text{CP}}|^2 - |\bar{A}_1|^2 - |\bar{A}_2|^2}{2|\bar{A}_1||\bar{A}_2|},$$

$$\sin 2\gamma = \pm \left(\alpha \sqrt{1 - \bar{\alpha}^2} - \bar{\alpha} \sqrt{1 - \alpha^2} \right)$$



- analysis is similar for $B_d \rightarrow D\pi$ is similar, but coefficients are e time-dependent

3. Time-dependent CP-asymmetries

★ Time-dependent CP-asymmetries probe CP-violation in $\Delta B=2$ amplitudes

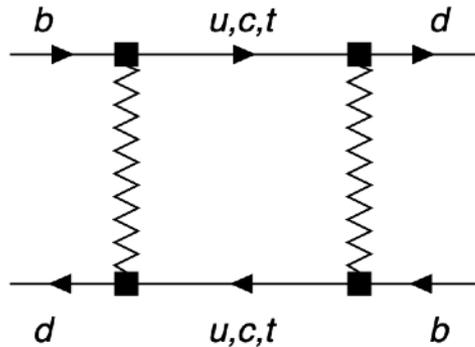
- SM: CP-violation in $\Delta B=2$ and $\Delta B=1$ transitions have the same origin, this fact does not have to be true in general NP model
- it most conveniently can be probed in transitions that involve mixing
 - use time-dependent CP asymmetries due to the interference between B-mixing and B decay amplitudes
 - interference between the two neutral B meson evolution eigenstates generates the time-dependent CP asymmetry

$$a_{CP}(f, t) = \frac{\Gamma(B(t) \rightarrow f) - \Gamma(\bar{B}(t) \rightarrow \bar{f})}{\Gamma(B(t) \rightarrow f) + \Gamma(\bar{B}(t) \rightarrow \bar{f})}$$

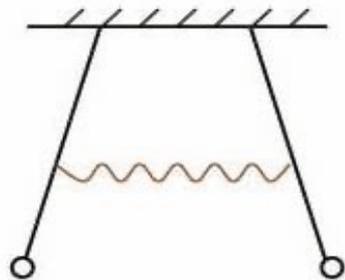
- Need to develop a formalism for time-dependent decays

Time dependent decay amplitudes

★ In the SM, neutral B-mesons can mix via weak interaction diagrams



- only at one loop in the Standard Model, so can be sensitive to possible quantum effects due to new physics particles
- $\Delta B = 2$ interactions couple dynamics of B^0 and \bar{B}^0
- We need to study simultaneous time evolution,



Coupled oscillators

$$|B(t)\rangle = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} = a(t)|B^0\rangle + b(t)|\bar{B}^0(t)\rangle$$

- This is very similar to the case of coupled pendula in classical mechanics

Time dependent decay amplitudes

- Time dependence: coupled Schrodinger equations

- note that CPT-invariance requires that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$

$$i \frac{d}{dt} |B(t)\rangle = \left[M - i \frac{\Gamma}{2} \right] |B(t)\rangle \equiv \begin{bmatrix} A & p^2 \\ q^2 & A \end{bmatrix} |B(t)\rangle$$

Q: this Hamiltonian is clearly non-hermitian! What is goin on?

- Non-diagonal Hamiltonian: need to diagonalize the mass matrix

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

("switch from flavor to mass eigenstates")

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

- In the mass basis the mass matrix is diagonal, i.e.

$$Q^{-1} \left[M - i \frac{\Gamma}{2} \right] Q = \begin{pmatrix} M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2 \end{pmatrix}$$

- ... with mass and lifetime differences: $\Delta M = M_H - M_L$ & $\Delta\Gamma = \Gamma_L - \Gamma_H$

$$\text{Note that } m = \frac{M_H + M_L}{2} = M_{11} = M_{22} \quad \& \quad \Gamma = \frac{\Gamma_L + \Gamma_H}{2} = \Gamma_{11} = \Gamma_{22}$$

Time dependent decay amplitudes

- The transformation matrices that diagonalize the Hamiltonian are

$$Q = \begin{pmatrix} p & p \\ q & -q \end{pmatrix} \quad \text{and} \quad Q^{-1} = \frac{1}{2pq} \begin{pmatrix} q & p \\ q & -p \end{pmatrix}$$

- To find the time development of the flavor eigenstates one needs to transform the evolution equation back to the flavor basis

$$\begin{bmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{bmatrix} = Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} \begin{bmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{bmatrix}$$

- ... which gives for the time evolution matrix in the flavor basis

$$Q \begin{pmatrix} e^{-iM_L t - \Gamma_L t/2} & 0 \\ 0 & e^{-iM_H t - \Gamma_H t/2} \end{pmatrix} Q^{-1} = \begin{pmatrix} g_+(t) & \frac{q}{p} g_-(t) \\ \frac{p}{q} g_-(t) & g_+(t) \end{pmatrix} \quad \text{Nierste}$$

$$\text{with} \quad \begin{aligned} g_+(t) &= e^{-imt} e^{-\Gamma t/2} \left[\cosh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta M t}{2} - i \sinh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta M t}{2} \right], \\ g_-(t) &= e^{-imt} e^{-\Gamma t/2} \left[-\sinh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta M t}{2} + i \cosh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta M t}{2} \right]. \end{aligned}$$

Time dependent decay amplitudes

- This procedure provides a picture of how B-states evolve due to flavor oscillations,

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle$$

$$|\bar{B}^0(t)\rangle = \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle$$

with

$$g_+(t) = e^{-imt} e^{-\Gamma t/2} \left[\cosh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta M t}{2} - i \sinh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta M t}{2} \right],$$

$$g_-(t) = e^{-imt} e^{-\Gamma t/2} \left[-\sinh \frac{\Delta\Gamma t}{4} \cos \frac{\Delta M t}{2} + i \cosh \frac{\Delta\Gamma t}{4} \sin \frac{\Delta M t}{2} \right].$$

- The only thing left is to relate q/p , ΔM and $\Delta\Gamma$ to original parameters of H

secular equation: $(\Delta M + i\frac{\Delta\Gamma}{2})^2 = 4 \left(M_{12} - i\frac{\Gamma_{12}}{2} \right) \left(M_{12}^* - i\frac{\Gamma_{12}^*}{2} \right)$

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2 \qquad \Delta M \Delta\Gamma = -4 \operatorname{Re}(M_{12}\Gamma_{12}^*)$$

- Finally, the ratio $\frac{q}{p} = -\frac{\Delta M + i\Delta\Gamma/2}{2M_{12} - i\Gamma_{12}} = -\frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta M + i\Delta\Gamma/2}$

Phases and amplitudes

- The B-meson states can have an arbitrary phase, so only relative phase is physical, which implies that there are three quantities that define B-mixing

$$|M_{12}|, \quad |\Gamma_{12}|, \quad \text{and} \quad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

- ... which gives for the mixing parameters

$$\Delta M \simeq 2|M_{12}| \quad \text{and} \quad \Delta\Gamma \simeq 2|\Gamma_{12}|\cos\phi$$

- ... and, up to a good approximation, to the phase of the box diagram,

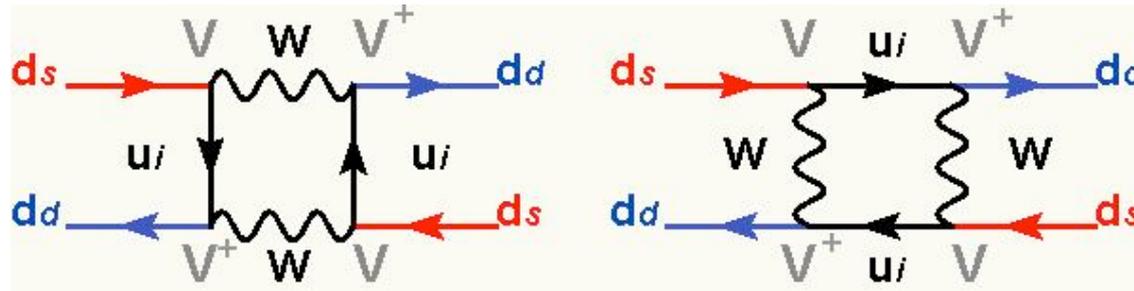
$$\frac{q}{p} = -\frac{M_{12}^*}{M_{12}} = \frac{V_{tb}^*V_{tq}}{V_{tb}V_{tq}^*} \quad \text{and} \quad \left|\frac{q}{p}\right|^2 = 1 - a = 1 - \text{Im}\frac{\Gamma_{12}}{M_{12}}$$

We can calculate B-mixing parameters in the SM: any sign of New Physics?

FCNC in the SM: GIM-mechanism

Glashow-Iliopoulos-Maiani (GIM) mechanism

- There are no $\Delta Q=2$ interactions in the Standard Model...
- ... but we can make them via a “two-step process” (loop diagram):

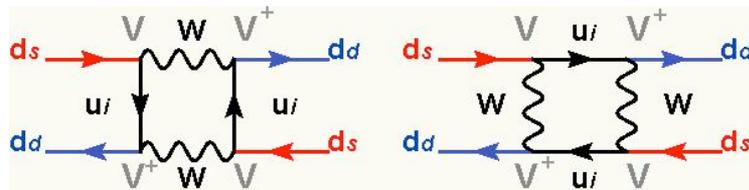


- Let's calculate them! For each internal quark type we get

$$\sim g^4 \left(V_{is} V_{id}^\dagger V_{js} V_{jd}^\dagger \right) \int \frac{d^4 k}{(4\pi)^4} \frac{(\text{some gamma matrices}) (k^2)}{(k - m_i)(k - m_j)(k^2 - m_W^2)^2}$$

Divergent: not good...

- However, CKM matrix is unitary:
- contribution of different internal flavors comes with different signs!



top: $(V_{tb}V_{td}^\dagger V_{tb}V_{td}^\dagger) \sim (1 \times A\lambda^3)(1 \times A\lambda^3)$
 top-charm: $(V_{tb}V_{td}^\dagger V_{cb}V_{cd}^\dagger) \sim (1 \times A\lambda^3)(A\lambda^2 \times (-\lambda))$

- Thus, in the limit where $k \gg m_i, m_j, M_W$:

top:
$$g^4 (A\lambda^3)^2 \int \frac{d^4k}{(4\pi)^4} \frac{(\text{some gamma matrices})(k^2)}{(\not{k})(\not{k})(k^2)^2}$$

top-charm:
$$-g^4 (A\lambda^3)^2 \int \frac{d^4k}{(4\pi)^4} \frac{(\text{some gamma matrices})(k^2)}{(\not{k})(\not{k})(k^2)^2}$$

... and similarly for other quarks

Cancellation of divergences!

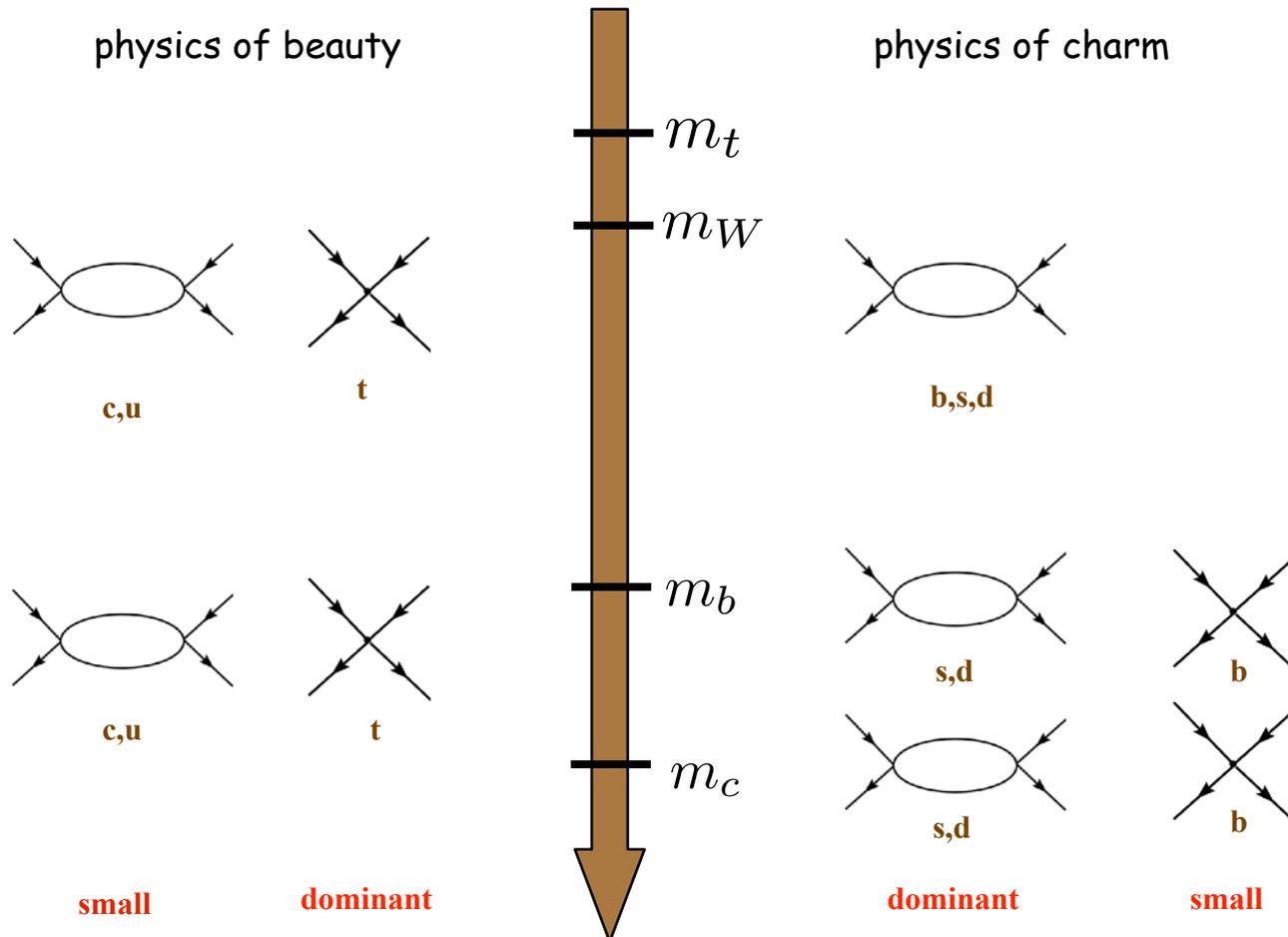
$$A \propto \sum_i m_i^2 (V_{is} V_{ib}^*)^2 g_k (m_i^2)$$

Glashow-Iliopoulos-Maiani

Introduction: energy scales

★ Modern approach to flavor physics calculations: effective field theories

★ It is important to understand relevant energy scales for the problem at hand

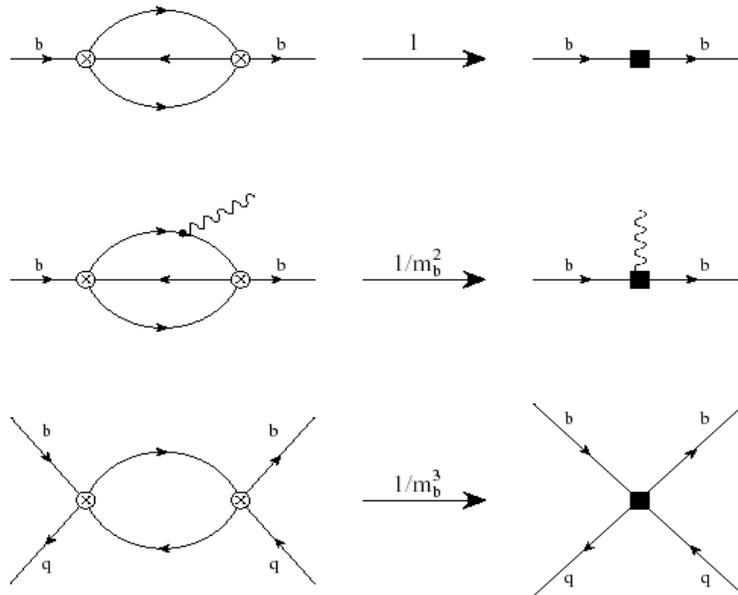


Theoretical expectations

- Assume quark-hadron duality: relate width to forward matrix element

$$\Gamma(H_b) = \frac{1}{2M_b} \langle H_b | T | H_b \rangle = \frac{1}{2M_b} \langle H_b | \text{Im} i \int d^4x T \{ H_{eff}^{\Delta B=1}(x) H_{eff}^{\Delta B=1}(0) \} | H_b \rangle$$

- This correlator can be expanded using OPE



I. Bigi, M. Shifman, A. Vainshtein, M. Voloshin, N. Uraltsev, A. Falk, A. Manohar, M. Wise, M. Neubert, C. Sachrajda, P. Colangelo, F. de Fazio,

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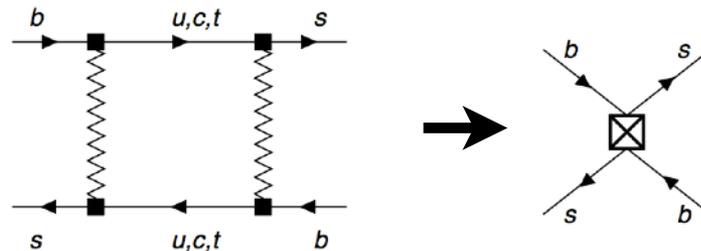
$$\Gamma(H_b) = \sum_k \frac{C_k(\mu)}{m_b^k} \langle H_b | O_k^{\Delta B=0}(\mu) | H_b \rangle$$

What are the results?

Standard Model contributions

Both ΔM_{B_s} and $\Delta\Gamma_{B_s}$ can be computed in the limit $m_b \rightarrow \infty$:

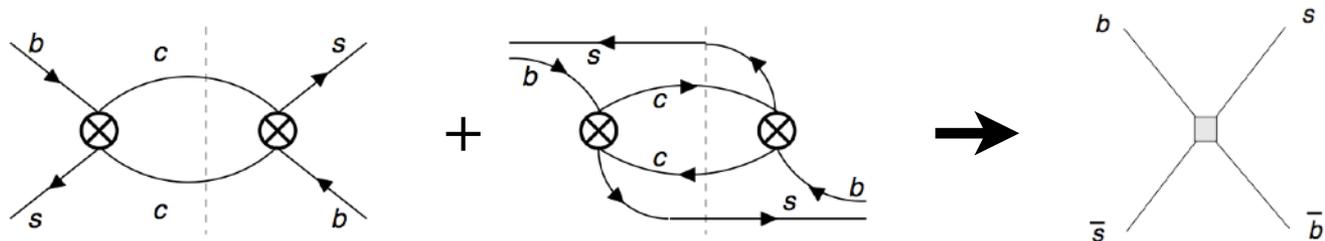
ΔM_{B_s} :



A. Buras, M. Jamin, P. Weisz

$$M_{12}(B_s) = \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 (V_{tb} V_{ts}^*)^2 \hat{\eta}_B S_0(x_t) f_{B_s}^2 B$$

$\Delta\Gamma_{B_s}$:



A. Lenz, U. Nierste

$$\Gamma_{21}(B_s) = \sum_i \frac{C_k(\mu)}{m_h^k} \langle B_s | \mathcal{O}_k^{\Delta B=2}(\mu) | \bar{B}_s \rangle.$$

$$\frac{\Delta\Gamma_s}{\Gamma_s} \approx 0.137 \pm 0.027$$

Not so easy: SM contributions to $\Delta\Gamma_{B_s}$

$\Delta\Gamma_{B_s}$: a calculation yields:

$$\Gamma_{21}(B_s) = - \frac{G_F^2 m_b^2}{12\pi(2M_{B_s})} (V_{cb}^* V_{cs})^2 [[F(z) + P(z)] \langle Q \rangle + [F_S(z) + P_S(z)] \langle Q_S \rangle + \delta_{1/m} + \delta_{1/m^2}]$$

★ ... with operators

WC (incl. pQCD corr): Beneke et al, Ciuchini et al

$$\left. \begin{aligned} Q &= (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A}, & Q_S &= (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S-P} \\ \tilde{Q} &= (\bar{b}_i s_j)_{V-A} (\bar{b}_j s_i)_{V-A}, & \tilde{Q}_S &= (\bar{b}_i s_j)_{S-P} (\bar{b}_j s_i)_{S-P} \end{aligned} \right\} \begin{aligned} \langle Q \rangle &= 2 \frac{1+N_c}{N_c} f_{B_s}^2 M_{B_s}^2 B \\ \langle Q_S \rangle &= \frac{1-2N_c}{N_c} \frac{M_{B_s}^4}{(m_b+m_s)^2} f_{B_s}^2 B_S \end{aligned}$$

★ ... so the result (up to $1/m_b^2$) is:

$$\begin{aligned} \Delta\Gamma_{B_s} &= \left[0.0005B + 0.1732B_s + 0.0024B_1 - 0.0237B_2 - 0.0024B_3 - 0.0436B_4 \right. \\ &+ 2 \times 10^{-5}\alpha_1 + 4 \times 10^{-5}\alpha_2 + 4 \times 10^{-5}\alpha_3 + 0.0009\alpha_4 - 0.0007\alpha_5 \\ &+ 0.0002\beta_1 - 0.0002\beta_2 + 6 \times 10^{-5}\beta_3 - 6 \times 10^{-5}\beta_4 - 1 \times 10^{-5}\beta_5 \\ &\left. - 1 \times 10^{-5}\beta_6 + 1 \times 10^{-5}\beta_7 + 1 \times 10^{-5}\beta_8 \right] \quad (\text{ps}^{-1}). \end{aligned}$$

A.Badin, F. Gabbiani, A.A.P.
Phys. Lett. B653, 230 (2007)

Time-dependent CP-asymmetries

- ★ Time-dependent CP-asymmetries probe CP-violation in $\Delta B=2$ amplitudes
 - Now we know how to deal with time-dependent rates

$$\Gamma(M(t) \rightarrow f) = \mathcal{N}_f |\langle f|S|M(t)\rangle|^2$$

$$\Gamma(\bar{M}(t) \rightarrow f) = \mathcal{N}_f |\langle f|S|\bar{M}(t)\rangle|^2$$

- ... which can be calculated using the developed formalism, $\lambda_f = \frac{q}{p} \frac{\bar{A}_f}{A_f}$

$$\Gamma(M(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma t}{2} + \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \operatorname{Re} \lambda_f \sinh \frac{\Delta\Gamma t}{2} - \operatorname{Im} \lambda_f \sin(\Delta M t) \right\},$$

$$\Gamma(\bar{M}(t) \rightarrow f) = \mathcal{N}_f |A_f|^2 \frac{1}{1 - a} e^{-\Gamma t} \left\{ \frac{1 + |\lambda_f|^2}{2} \cosh \frac{\Delta\Gamma t}{2} - \frac{1 - |\lambda_f|^2}{2} \cos(\Delta M t) - \operatorname{Re} \lambda_f \sinh \frac{\Delta\Gamma t}{2} + \operatorname{Im} \lambda_f \sin(\Delta M t) \right\}.$$

Time-dependent CP-asymmetries

★ Various time-dependent CP-asymmetries can now be formed

- The flavor-specific CP-asymmetry (aka semileptonic CP asymmetry)

$$a_{\text{fs}} \equiv \frac{\Gamma(\bar{M}(t) \rightarrow f) - \Gamma(M(t) \rightarrow \bar{f})}{\Gamma(\bar{M}(t) \rightarrow f) + \Gamma(M(t) \rightarrow \bar{f})} = \frac{1 - (1 - a)^2}{1 + (1 - a)^2} = a + \mathcal{O}(a^2).$$

- CP-asymmetry for decays to CP-eigenstates (such as $f_{\text{CP}} = J/\psi K_S$, etc.)

$$\begin{aligned} a_{f_{\text{CP}}}(t) &= \frac{\Gamma(\bar{M}(t) \rightarrow f_{\text{CP}}) - \Gamma(M(t) \rightarrow f_{\text{CP}})}{\Gamma(\bar{M}(t) \rightarrow f_{\text{CP}}) + \Gamma(M(t) \rightarrow f_{\text{CP}})} \\ &= -\frac{A_{\text{CP}}^{\text{dir}} \cos(\Delta M t) + A_{\text{CP}}^{\text{mix}} \sin(\Delta M t)}{\cosh(\Delta\Gamma t/2) + A_{\Delta\Gamma} \sinh(\Delta\Gamma t/2)} + \mathcal{O}(a) \end{aligned}$$

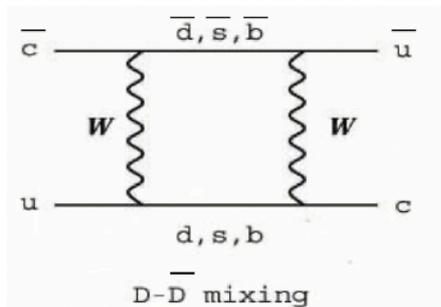
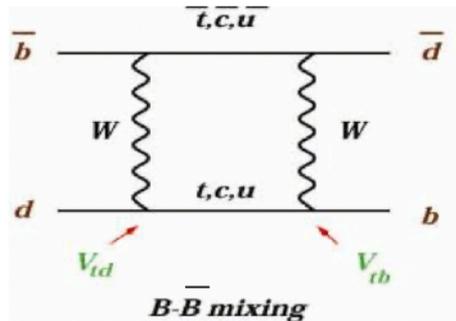
$$\text{where } A_{\text{CP}}^{\text{dir}} = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad A_{\text{CP}}^{\text{mix}} = -\frac{2 \text{Im } \lambda_f}{1 + |\lambda_f|^2} \quad \text{and} \quad A_{\Delta\Gamma} = -\frac{2 \text{Re } \lambda_f}{1 + |\lambda_f|^2}$$

Belle II studies for time-dependent CPV

| Process | Observable | Theory | Sys. dom. (Discovery) [ab ⁻¹] | vs LHCb | vs Belle | Anomaly | NP |
|-----------------------------------|------------|--------|---|---------|----------|---------|-----|
| ● $B \rightarrow J/\psi K_S^0$ | ϕ_1 | *** | 5-10 | ** | ** | * | * |
| ● $B \rightarrow \phi K_S^0$ | ϕ_1 | ** | >50 | ** | *** | * | *** |
| ● $B \rightarrow \eta' K_S^0$ | ϕ_1 | ** | >50 | ** | *** | * | *** |
| ● $B \rightarrow \rho^\pm \rho^0$ | ϕ_2 | *** | >50 | * | *** | * | * |
| ● $B \rightarrow J/\psi \pi^0$ | ϕ_1 | *** | >50 | * | *** | - | - |
| ● $B \rightarrow \pi^0 \pi^0$ | ϕ_2 | ** | >50 | *** | *** | ** | ** |
| ● $B \rightarrow \pi^0 K_S^0$ | S_{CP} | ** | >50 | *** | *** | ** | ** |

Things to take home

- We discuss how CP-violation can be studied with B-mesons
 - would D-mixing be different?
 - would CP-violation studies in charm be different?



| $\bar{D}^0 - D^0$ mixing | $\bar{B}^0 - B^0$ mixing |
|---|---|
| <ul style="list-style-type: none"> • intermediate down-type quarks • SM: b-quark contribution is negligible due to $V_{cd}V_{ub}^*$ • $rate \propto f(m_s) - f(m_d)$ (zero in the SU(3) limit) <p>Falk, Grossman, Ligeti, and A.A.P. Phys.Rev. D65, 054034, 2002 2nd order effect!!!</p> | <ul style="list-style-type: none"> • intermediate up-type quarks • SM: t-quark contribution is dominant • $rate \propto m_t^2$ (expected to be large) |
| <ol style="list-style-type: none"> 1. Sensitive to long distance QCD 2. Small in the SM: New Physics! (must know SM x and y) | <ol style="list-style-type: none"> 1. Computable in QCD (*) 2. Large in the SM: CKM! |

(*) up to matrix elements of 4-quark operators

- We shall see tomorrow how to deal with that

