Connections between hadron and lepton CPV

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Plan of my talk

1 Introduction

2 Quark CP violating phase in quark mass matrices

3 Linking leptonic CP violation to CKM CP phase

4 Predictions of leptonic CP violation

5 Summary

1 Introduction

Flavor mixing angles are much different between quarks and leptons !

PDG2018

$$|V_{\rm CKM}| = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

NuFIT 4.1 (2019)

$$|U|^{\text{with SK-atm}}_{3\sigma} = \begin{pmatrix} 0.797 \to 0.842 & 0.518 \to 0.585 & 0.143 \to 0.156 \\ 0.243 \to 0.490 & 0.473 \to 0.674 & 0.651 \to 0.772 \\ 0.295 \to 0.525 & 0.493 \to 0.688 & 0.618 \to 0.744 \end{pmatrix}$$

Theorists could not predict two large mixing angles !

Why?

Because we had not a reliable flavor theory.

Is flavor mixing of quarks and leptons correlated or not ?

Phenomenological suggestion

$$\theta_{13}^{\mathrm{PMNS}} \simeq \theta_{\mathrm{Cabibbo}}/\sqrt{2} = 0.16$$

Reactor Experiment: ~ 0.15

Cabibbo Hase

Cabibbo sized effect of lepton mixing

Antusch, Gross, Murer, Sluk, arXiv:1107.3728
Marzocca, Petcov, Romanino, Spinrath, arXiv:1108.0614

If this relation is not accidental,
we may be able to predict J_{CP} (PMNS) from J_{CP} (CKM)
(δ_{PMNS} from δ_{CKM}) .

Jarlskog 1985, Krastev-Petcov 1988

Search for clear correlations of quark/lepton CP violation through "Cabibbo Haze in lepton mixing"

A. Datta, L. Everett, P.Ramond, Phys.Lett.B620 (2005) 42

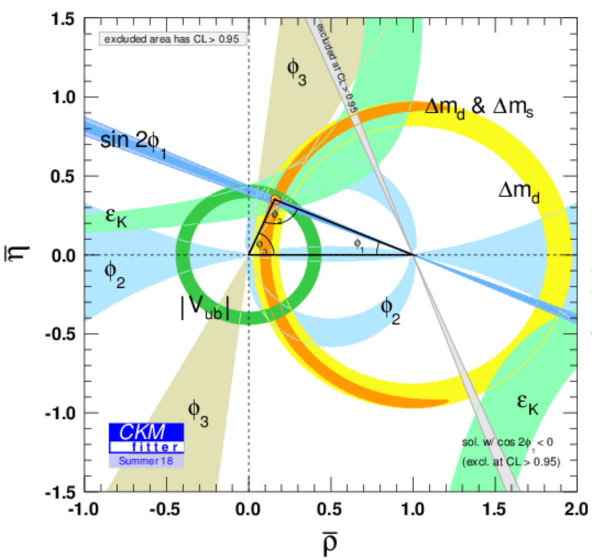
How large are CP violations of quarks and leptons ?

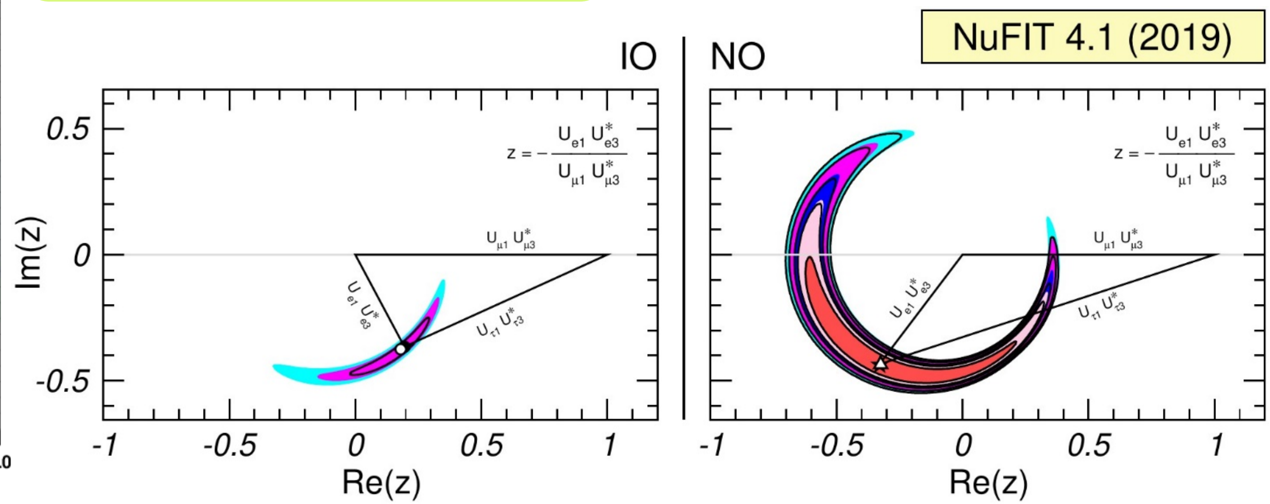
CP violating measures J_{CP}

PDG2018 $J_{CP}^{q} = (3.18 \pm 0.15) \times 10^{-5}$ $\delta_{CP} = (+73.5^{+4.2}_{-5.1})°$ for quarks

Best fit of NuFIT 4.1 $J_{CP}^{l} \simeq -2 \times 10^{-2}$ $\delta_{CP} = -138°$ for leptons

Unitarity Triangle





$m_3 < m_1 < m_2$ $m_1 < m_2 < m_3$

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CKM}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CKM}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CKM}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CKM}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CKM}}} & c_{23}c_{13} \end{pmatrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta^{\ell}_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta^{\ell}_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta^{\ell}_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta^{\ell}_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta^{\ell}_{\text{CP}}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{e3}U_{e1}^* + U_{\mu 3}U_{\mu 1}^* + U_{\tau 3}U_{\tau 1}^* = 0$$

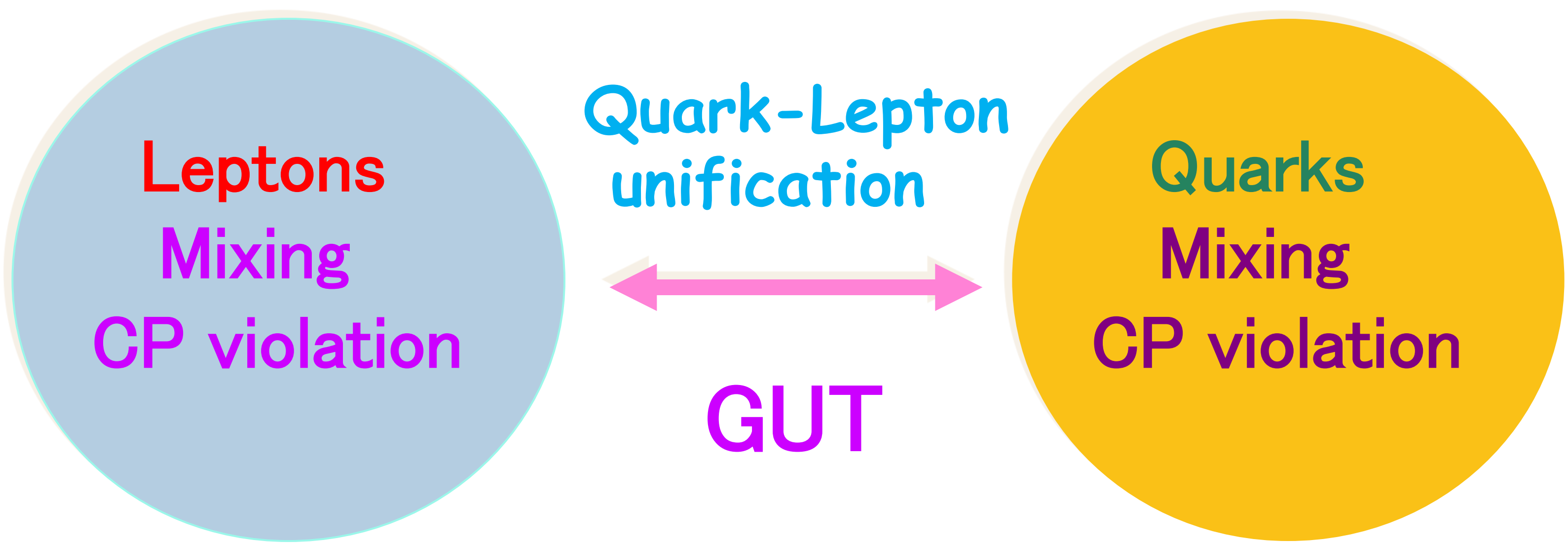
Majorana Neutrinos

Can we predict the lepton CP violation ?

We try to connect to lepton CP violation and quark CP violation as well as flavor mixing angles.

We need Two steps

- **Reproduce CKM matrix by using quark mass matrices**
- **Quark-lepton unification: SU(5) GUT or Pati-Salam GUT**



2 Quark CP violating phase in quark mass matrices

CKM matrix is obtained by quark mass matrices.

$$\mathcal{L}_Y = -\bar{u^i_L}(M_u)_{ij}u^j_R - \bar{d^i_L}(M_d)_{ij}d^j_R$$

$$V_{uL}^\dagger M_u V_{uR} = \mathrm{diag}(m_u, m_c, m_t)\ , \qquad V_{dL}^\dagger M_d V_{dR} = \mathrm{diag}(m_d, m_s, m_b)$$

$$V_{\mathrm{CKM}} \equiv V_{uL} V_{dL}^\dagger$$

We need mass matrices of up-type and down-type quarks.

Ex: Weinberg's Anzatz 1977 Vanishing (1,1) element of down mass matrix

$$M_{\mathrm{d}} = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \quad \Rightarrow \quad \theta_{\mathrm{Cabibbo}} \simeq \sqrt{\frac{m_d}{m_s}} \simeq 0.2$$

Let us discuss Down-type quark mass matrix M_d in the basis of diagonal Up-type quark mass matrix

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

One can take the diagonal basis in general.
Then, how can we take M_d ?

In general, down quark mass matrix has 3x3=9 complex parameters. Among them, 5 phases are removed by rephasing of quark fields because of diagonal M_u . **2 x 9 - 5 =13 parameters** **observables 7**

"Entities should not be multiplied unnecessarily." Occam's Razor

Remove unnecessary parameters in M_d in order to reproduce CKM mixing angles and CP violation.

M.Tanimoto, T.T.Yanagida, PTEP (2016) 043B03, arXiv:1601.04459
K. Harigaya, M. Ibe, T.T. Yanagida, PRD86(2012)013002, arXiv:1205.2198

Put 3 zeros in entries of M_d

There remains 6 complex parameters in M_d. (12 real parameters)

Among them, 5 phases are removed by re-definitions of down-type left- and right-quark fields.

Finally, there remain 6 real parameter and 1 phase.
It is easy to control CP violation due to one phase.

There are 20 patterns of mass matrices with 3 zeros.

One example:

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}$$

which is completely consistent with 4 CKM parameters and 3 down-type quark masses.
However, nontrivial problem in general even if # of parameters is 7.

Eigenvalues

$$m_d^2 + m_s^2 + m_b^2 = a^2 + a'^2 + b^2 + c^2 + c'^2 + d^2 ,$$
$$m_d^2 m_s^2 + m_s^2 m_b^2 + m_b^2 m_d^2 = a^2 a'^2 + a^2(c^2 + d^2) + a'^2(c'^2 + d^2) + c^2 c'^2 + b^2 d^2 - 2bcc'd\cos\phi$$
$$m_d^2 m_s^2 m_b^2 = a^2 a'^2 d^2 .$$

Eigenvectors

$$|V_{us}| \simeq \frac{ab}{m_s^2}\left|\sin\frac{\phi}{2}\right| , \quad |V_{cb}| \simeq \sqrt{2}\frac{c}{m_b}\left|\cos\frac{\phi}{2}\right| , \quad |V_{ub}| \simeq \frac{ac'}{m_b^2} , \quad \delta_{CP} \simeq \frac{1}{2}(\pi - \phi)$$

CP violating measure Jarlskog invariant

$$J_{CP}^q = \frac{1}{(m_b^2 - ms^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2)} a^2 bcc'd\sin\phi$$

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}$$

Constructiong CP violating measure

$$H_U \equiv M_U M_U^\dagger \ , \qquad H_D \equiv M_D M_D^\dagger$$

CP invariance leads to

$$\mathrm{Tr}\left([H_U, H_D]^3\right) = 0$$

$$\mathrm{Tr}\left([H_U, H_D]^3\right) = 6i \sum_{\alpha,\beta=u,c,t} \sum_{i,j=d,s,b} m_\alpha^4 m_\beta^2 m_i^4 m_j^2 \mathrm{Im}[V_{\alpha i} V_{\beta j} V_{\beta i}^* V_{\alpha j}^*]$$

$$\mathrm{Im}[V_{ij} V_{kl} V_{il}^* V_{kj}^*] = J_{\mathrm{CP}}^q \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln}$$

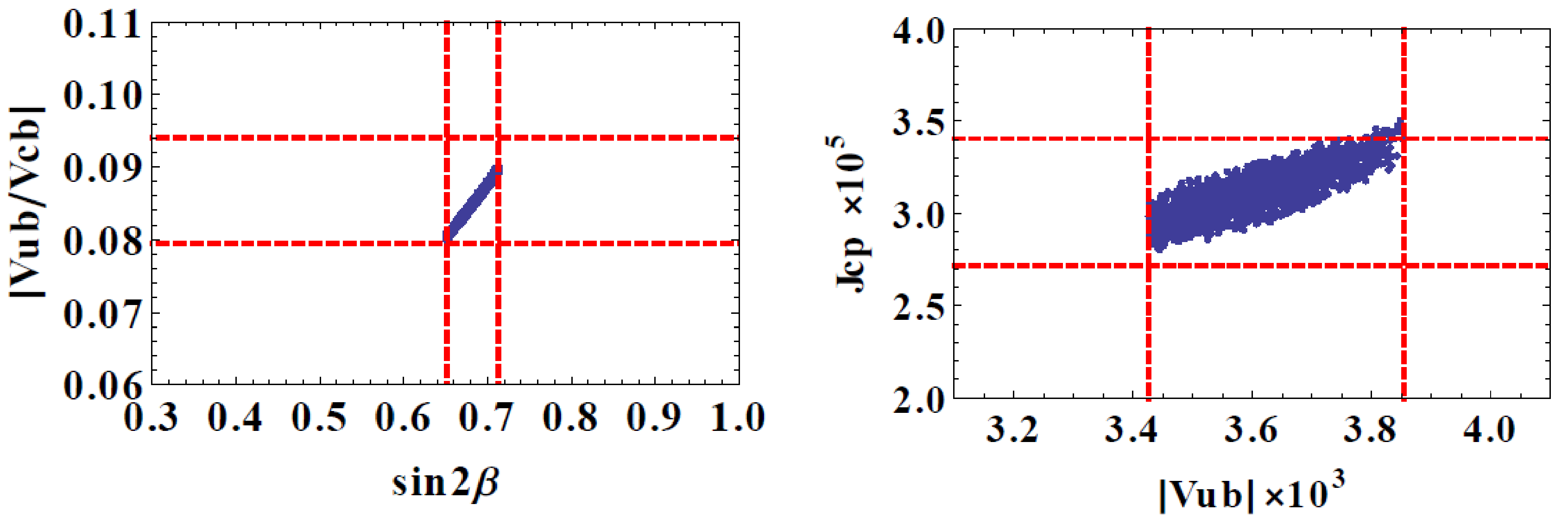
$$\mathrm{Tr}\left([H_U, H_D]^3\right) = 6i \Delta_u \Delta_d J_{\mathrm{CP}}^q$$

Jarlskog invariant

$$\Delta_u \equiv (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_u^2 - m_t^2)$$
$$\Delta_d \equiv (m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_d^2 - m_b^2)$$

$$|V_{us}| \simeq \frac{ab}{m_s^2}\left|\sin\frac{\phi}{2}\right| \; , \quad |V_{cb}| \simeq \sqrt{2}\frac{c}{m_b}\left|\cos\frac{\phi}{2}\right| \; , \quad |V_{ub}| \simeq \frac{ac'}{m_b^2} \; , \quad \delta_{CP} \simeq \frac{1}{2}(\pi - \phi)$$

Consistency check



	a_D [MeV]	a'_D [MeV]	b_D [MeV]	c_D [MeV]	c'_D [GeV]	d_D [GeV]	ϕ [$\circ$]
$M_D^{(1)}$	15-17.5	10-15	92-104	78-95	1.65-2.0	2.0-2.3	37-48

13 textures with 3 zeros consistent with observed CKM

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR} , \quad M_D^{(2)} = \begin{pmatrix} a'_D & a_D & 0 \\ 0 & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR} , \quad M_D^{(3)} = \begin{pmatrix} 0 & a_D & 0 \\ 0 & b_D\, e^{-i\phi} & c_D \\ a'_D & c'_D & d_D \end{pmatrix}_{LR}$$

$$M_D^{(4)} = \begin{pmatrix} 0 & a_D & c'_D \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & 0 & d_D \end{pmatrix}_{LR} , \quad M_D^{(5)} = \begin{pmatrix} a'_D & a_D & c'_D \\ 0 & b_D\, e^{-i\phi} & c_D \\ 0 & 0 & d_D \end{pmatrix}_{LR} , \quad M_D^{(6)} = \begin{pmatrix} 0 & a_D & c'_D \\ 0 & b_D\, e^{-i\phi} & c_D \\ a'_D & 0 & d_D \end{pmatrix}_{LR}$$

$$M_D^{(11)} = \begin{pmatrix} a'_D & a_D\, e^{-i\phi} & b_D \\ 0 & 0 & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR} , \quad M_D^{(12)} = \begin{pmatrix} 0 & a_D\, e^{-i\phi} & b_D \\ a'_D & 0 & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR} , \quad M_D^{(13)} = \begin{pmatrix} 0 & a_D\, e^{-i\phi} & b_D \\ 0 & 0 & c_D \\ a'_D & c'_D & d_D \end{pmatrix}_{LR}$$

$$M_D^{(14)} = \begin{pmatrix} a_D\, e^{i\phi} & a'_D & c'_D \\ b_D & 0 & c_D \\ 0 & 0 & d_D \end{pmatrix}_{LR} , \quad M_D^{(15)} = \begin{pmatrix} a_D\, e^{-i\phi} & a'_D & b_D \\ 0 & 0 & c_D \\ c'_D & 0 & d_D \end{pmatrix}_{LR} ,$$

$$M_D^{(16)} = \begin{pmatrix} 0 & a_D & b_D \\ a'_D & 0 & c_D\, e^{-i\phi} \\ c'_D & 0 & d_D \end{pmatrix}_{LR} , \quad M_D^{(17)} = \begin{pmatrix} a_D & a'_D & 0 \\ b_D & 0 & c_D\, e^{i\phi} \\ c'_D & 0 & d_D \end{pmatrix}_{LR}$$

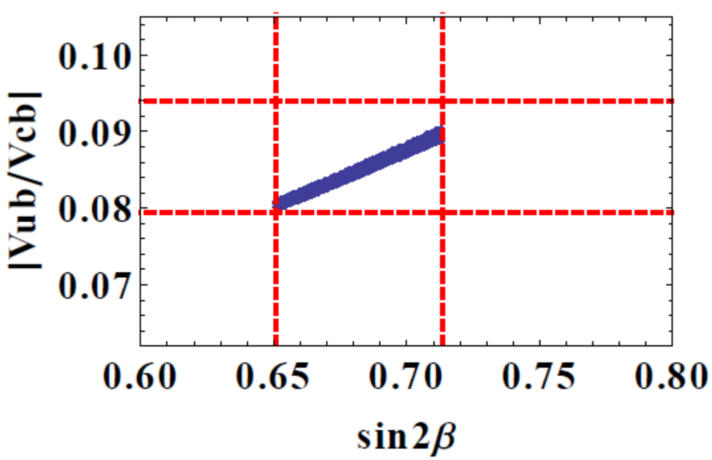
There are redundancies in our textures due to unitary transformation of the right-handed quarks.

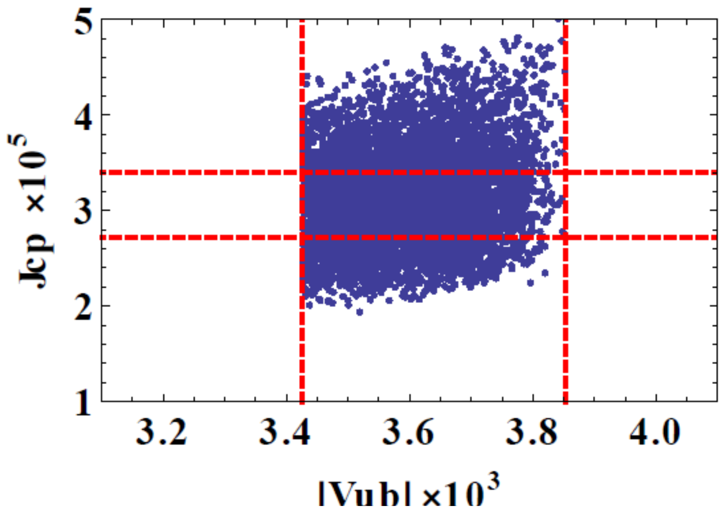
$$M_D^{(2)} \equiv M_D^{(16)} \equiv M_D^{(17)} , \qquad M_D^{(5)} \equiv M_D^{(14)} , \qquad M_D^{(11)} \equiv M_D^{(15)}$$

	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$	δ_{CP}	j_{CP}
$M_d^{(1)},\ M_d^{(2)},\ M_d^{(3)},\ M_d^{(16)},\ M_d^{(17)}$	$\frac{ab}{m_s^2}\left\lvert\sin\frac{\phi}{2}\right\rvert$	$\frac{\sqrt{2}c}{m_b}\left\lvert\cos\frac{\phi}{2}\right\rvert$	$\frac{ac'}{m_b^2}$	$\frac{1}{2}(\pi-\phi)$	$a^2bcc'd\sin\phi$
$M_d^{(4)},\ M_d^{(5)},\ M_d^{(6)},\ M_d^{(14)}$	$\frac{ab}{m_s^2}$	$\frac{c}{m_b}$	$\frac{c'}{m_b}$	ϕ	$abcc'd^2\sin\phi$
$M_d^{(11)},M_d^{(12)},\ M_d^{(13)},\ M_d^{(15)}$	$\frac{ac}{m_s^2}\frac{c'}{m_b}$	$\frac{c}{m_b}$	$\frac{b}{m_b}$	$\pi-\phi$	$abc^2c'd\sin\phi$

$$M_d^{(5)} = \begin{pmatrix} a' & a & c' \\ 0 & b\,e^{-i\phi} & c \\ 0 & 0 & d \end{pmatrix}_{LR}$$

$$J_{CP} = \frac{-1}{(m_b^2-m_s^2)(m_s^2-m_d^2)(m_d^2-m_b^2)}\, j_{CP}$$

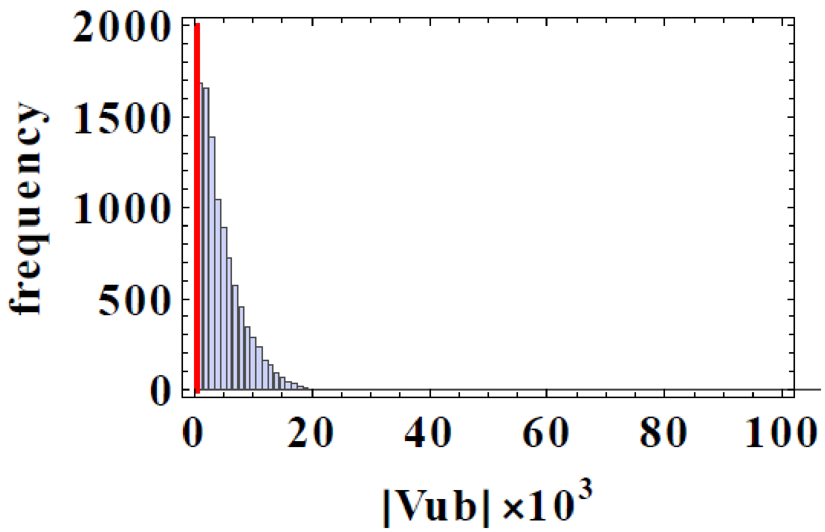




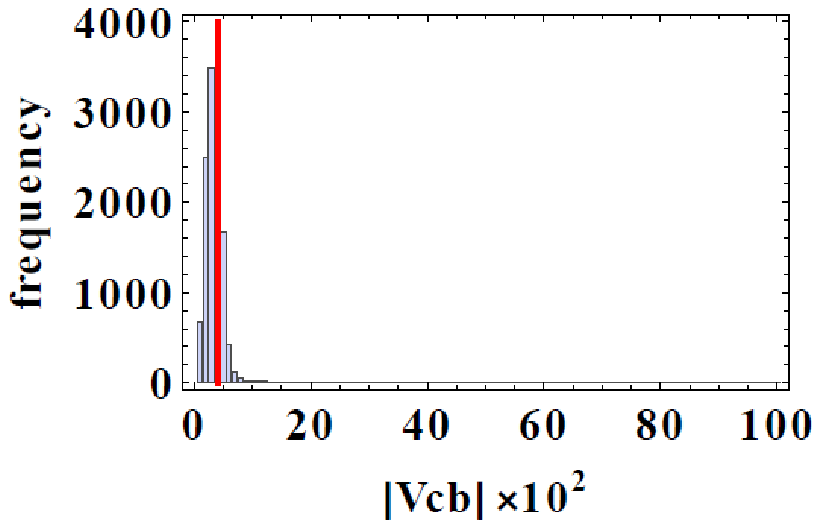
Benefit of Occam's Razor

The hierarchical mixing are naturally obtained.

Two examples



Only down quark masses are input.



Down quark masses and sin 2β are input.

3 Linking leptonic CP violation to CKM CP phase

Derivation of J_{cp} of leptons

$$\mathcal{L}_M = -\overline{u_L} M_U u_R - \overline{d_L} M_D d_R - \frac{1}{2}\overline{\nu_L} M_\nu (\nu_L)^c - \overline{e_L} M_E e_R + \text{h.c.}$$

$$H_i = M_i M_i^\dagger \ (i = U, D, \nu, E)$$

M^*_ν in Branco et al arXiv:1111.5332

$$\mathrm{Tr}([H_U, H_D]^3) = 6i J^q_{CP} \Delta_u \Delta_d$$

$$\Delta_u \equiv (m_u^2 - m_t^2)(m_u^2 - m_c^2)(m_c^2 - m_t^2) < 0, \qquad \Delta_d \equiv (m_d^2 - m_b^2)(m_d^2 - m_s^2)(m_s^2 - m_b^2) < 0.$$

$$\mathrm{Tr}([H_\nu, H_E]^3) = -6i J^l_{CP} \Delta_\nu \Delta_e$$

$$\Delta_\nu \equiv (m_1^2 - m_3^2)(m_1^2 - m_2^2)(m_2^2 - m_3^2) < 0, \qquad \Delta_e \equiv (m_e^2 - m_\tau^2)(m_e^2 - m_\mu^2)(m_\mu^2 - m_\tau^2) < 0$$

$$\mathrm{Im}[U_{k\alpha} U_{l\beta} U^*_{k\beta} U^*_{l\alpha}] = J^l_{\mathrm{CP}} \sum_{m,n} \varepsilon_{klm} \varepsilon_{\alpha\beta n} \qquad \mathrm{Im}[V_{ij} V_{kl} V^*_{il} V^*_{kj}] = J^q_{\mathrm{CP}} \sum_{m,n} \varepsilon_{ikm} \varepsilon_{jln}$$

$$U_{\mathrm{PMNS}} = U_E U_\nu^\dagger \qquad V_{\mathrm{CKM}} = V_u V_d^\dagger$$

Simple Exercise

Suppose $M_D = M_E$ (unrealistic) in the diagonal basis of M_U and M_ν

$$\mathrm{Tr}([H_U, H_D]^3) = 6i J^q_{CP} \Delta_u \Delta_d$$

$$V_{\mathrm{CKM}} = V_u V_d^\dagger \qquad U_{\mathrm{PMNS}} = U_E U_\nu^\dagger$$

$$\mathrm{Tr}([H_\nu, H_E]^3) = -6i J^l_{CP} \, \Delta_\nu \, \Delta_e$$

↓

$$6i J^q_{CP} \Delta_d = -6i J^l_{CP} \Delta_e$$

$$J^l_{CP} = -J^q_{CP} \quad \rightarrow \quad \delta^\ell_{CP} = -\delta^q_{CP}$$

Negative sign is preferred,
however, the relative magnitude is unrealistic !

Let us consider the realistic case:

Quark mass matrices by Georgi-Jarlskog in SU(5)
PLB 86B (1979) 297

$$M_{D,E} = \begin{pmatrix} 0 & 5_H & 0 \\ 5_H & \overline{45}_H & 0 \\ 0 & 0 & 5_H \end{pmatrix} \Rightarrow M_D = \begin{pmatrix} 0 & A & 0 \\ A' & B & 0 \\ 0 & 0 & C \end{pmatrix} \quad M_E = \begin{pmatrix} 0 & A' & 0 \\ A & -3B & 0 \\ 0 & 0 & C \end{pmatrix}$$

$$m_b = m_\tau \ , \quad 3m_s = m_\mu \ , \quad m_d = 3m_e$$

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}_L$$

$$10 \otimes 10 = \bar{5} \oplus \overline{45} \oplus \overline{50}$$

Higgs: 5_H, $\overline{45}_H$

$$\bar{5} \otimes 10 = 5 \oplus 45 \qquad 10 \otimes \bar{5} \otimes 5_H \ , \quad 10 \otimes \bar{5} \otimes \overline{45}_H$$

Quark mass matrices by using Occam's Razor:

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR} \qquad M_U = \mathrm{diag}\,\{m_u, m_c, m_t\}$$

M_E is derived in Pati-Salam symmetry or SU(5) GUT.

Pati-Salam symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$

$$F^{i\alpha a} = (\mathbf{4},\mathbf{2},\mathbf{1})^i = \begin{pmatrix} u_L^R & u_L^B & u_L^G & \nu_L \\ d_L^R & d_L^B & d_L^G & e_L^- \end{pmatrix}^i \qquad H^{\alpha b} = (\mathbf{4},\mathbf{1},\mathbf{2})$$

$$\bar{F}^i_{\alpha x} = (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})^i = \begin{pmatrix} \bar{d}_R^R & \bar{d}_R^B & \bar{d}_R^G & e_R^+ \\ \bar{u}_R^R & \bar{u}_R^B & \bar{u}_R^G & \bar{\nu}_R \end{pmatrix}^i \qquad \bar{H}_{\alpha x} = (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$$

Quark and Lepton Unification by SU(5) GUT

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{pmatrix}_L$$

Higgs: 5_H, 24_H, $\overline{45}_H$, 75_H

4 Predictions of leptonic CP violation

Y.Shimizu, K.Takagi, S.Takahashi, M.Tanimoto, arXiv:1901.06146

Consider diagonal M_ν

No extra Dirac CP phase except for Majorana phases

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\, e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}$$

Pati-Salam symmetry

$$M_E^{(1)} = \begin{pmatrix} 0 & a_E & 0 \\ a'_E & b_E\, e^{-i\phi} & c_E \\ 0 & c'_E & d_E \end{pmatrix}_{LR}$$

SU(5) GUT

$$M_E^{(1)} = \begin{pmatrix} 0 & a'_E & 0 \\ a_E & b_E\, e^{-i\phi} & c'_E \\ 0 & c_E & d_E \end{pmatrix}_{LR}$$

$$a_E = C_a a_D, \quad a'_E = C_{a'} a'_D, \quad b_E = C_b b_D, \quad c_E = C_c c_D, \quad c'_E = C_{c'} c'_D, \quad d_E = C_d d_D$$

$$a_E = C_a a_D, \quad a'_E = C_{a'} a'_D, \quad b_E = C_b b_D, \quad c_E = C_c c_D, \quad c'_E = C_{c'} c'_D, \quad d_E = C_d d_D$$

Suppose that the single mass operator dominates in each entry of the mass matrix like as Georgi-Jarlskog.
Then, CG coefficients are determined by group theory.

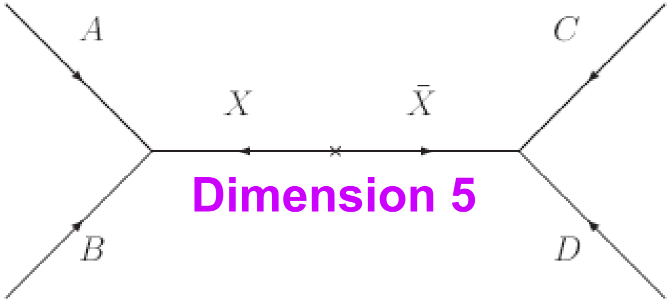
Pati-Salam

$$\text{dimension } 4 : (1, -3)\ , \qquad \text{dimension } 5 : (1, -3, 9)\ , \qquad \text{dimension } 6 : (0, \frac{3}{4}, 1, 2, -3)$$

SU(5)

$$\text{dimension } 4 : (1, -3)\ , \qquad \text{dimension } 5 : (-\frac{1}{2}, 1, \pm\frac{3}{2}, -3, \frac{9}{2}, 6, 9, -18)$$

5_H, $\overline{45}_H$



Dimension 5

S.Antusch, M.Spinrath, Phys.Rev.D79 (2009) 095004,arXiv:0902.4644

Pati-Salam symmetry

$$U_{\text{PMNS}} \simeq \begin{pmatrix} X_e & \frac{a_E}{2b_E \sin\frac{\phi}{2}} X_e & \frac{a_E}{2d_E \sin\frac{\phi}{2}} e^{i\phi/2} X_e \\ -\frac{a_E b_E}{m_\mu^2} \sin\frac{\phi}{2} Y_\mu & Y_\mu & \frac{c_E}{d_E} \cos\frac{\phi}{2} Y_\mu \\ \frac{a_E c'_E}{m_\tau^2} e^{i\pi/2} Z_\tau & -\frac{b_E d_E}{m_\tau^2} Z_\tau & Z_\tau \end{pmatrix}$$

negative

$$J_{CP}^l = \frac{1}{\Delta_e} a_E^2 b_E c_E c'_E d_E \sin\phi$$

$$J_{CP}^l / J_{CP}^q = -C_a^2\ C_b\ C_c\ C_{c'}\ C_d\ \Delta_d/\Delta_e$$

SU(5) GUT

$$U_{\text{PMNS}} \simeq \begin{pmatrix} X_e & \frac{a'_E}{2b_E \sin\frac{\phi}{2}} X_e & -\frac{a'_E c'_E (b_E d_E e^{i\phi} - c_E c'_E)}{\left|b_E d_E - c_E c'_E e^{i\phi}\right|^2} e^{i(\pi-\phi)/2} X_e \\ -\frac{2a'_E b_E}{m_\mu^2} \sin\frac{\phi}{2} Y_\mu & Y_\mu & \frac{c'_E}{d_E} Y_\mu \\ \frac{a'_E b_E}{c'^2_E - m_\tau^2} e^{-i(\pi+\phi)/2} Z_\tau & \frac{c'_E d_E}{c'^2_E - m_\tau^2} Z_\tau & Z_\tau \end{pmatrix}$$

$$J_{CP}^l = \frac{1}{\Delta_e} a'^2_E b_E c_E c'_E d_E \sin\phi$$

$$J_{CP}^l / J_{CP}^q = -C_{a'}^2\ C_b\ C_c\ C_{c'}\ C_d\ \Delta_d/\Delta_e$$

Relative sign of J_{CP} is determined by CG coefficients.

CG coefficients are constrained by observed mass eigenvalues as:

GUT Scale

$$C_a^2\, C_{a'}^2\, C_d^2 = \frac{m_e^2\, m_\mu^2\, m_\tau^2}{m_d^2\, m_s^2\, m_b^2} = 1.7 - 7.3 \qquad \frac{m_\tau^2}{m_b^2} \simeq \frac{C_d^2 d_D^2 + C_{c'}^2 c_D'^2}{d_D^2 + c_D'^2} = 0.99 - 1.1$$

$$\frac{m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2}{m_d^2 m_s^2 + m_s^2 m_b^2 + m_d^2 m_b^2} \simeq \frac{c_E^2 c_E'^2 + b_E^2 d_E^2 - 2 b_E c_E c_E' d_E \cos\phi}{c_D^2 c_D'^2 + b_D^2 d_D^2 - 2 b_D c_D c_D' d_D \cos\phi} = 15\text{-}26$$

Best choice of CG's to reproduce mass ratios

Pati-Salam $\quad C_a = 2, \quad C_{a'} = 1, \quad C_b = -3, \quad C_c = -3, \quad C_{c'} = 1, \quad C_d = 1$

$$J_{CP}^l = -8.1 \times 10^{-5}\,, \qquad \frac{m_e^2\, m_\mu^2\, m_\tau^2}{m_d^2\, m_s^2\, m_b^2} = 4\,, \qquad \frac{m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2}{m_d^2 m_s^2 + m_s^2 m_b^2 + m_d^2 m_b^2} = 8.5$$

$$\sin^2\theta_{12}^{\rm PMNS} \simeq 0.021, \qquad \sin^2\theta_{23}^{\rm PMNS} \simeq 0.012, \qquad \sin\theta_{13}^{\rm PMNS} \simeq 0.015, \qquad \delta_{CP}^l \simeq -20.4^\circ$$

SU(5) $\quad C_a = 1, \quad C_{a'} = \frac{3}{2}, \quad C_b = \frac{9}{2}, \quad C_c = 6, \quad C_{c'} = 1, \quad C_d = 1\,,$

$$J_{CP}^l = -2.5 \times 10^{-5}, \qquad \frac{m_e^2\, m_\mu^2\, m_\tau^2}{m_d^2\, m_s^2\, m_b^2} = 2.3, \qquad \frac{m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2}{m_d^2 m_s^2 + m_s^2 m_b^2 + m_d^2 m_b^2} = 20.6$$

$$\sin^2\theta_{12}^{\rm PMNS} \simeq 0.0022, \qquad \sin^2\theta_{23}^{\rm PMNS} \simeq 0.39, \qquad \sin\theta_{13}^{\rm PMNS} \simeq 0.038, \qquad \delta_{CP}^l \simeq -1.7^\circ$$

Best fit (NuFIT 4.0) $\quad J_{CP}^l \simeq -2 \times 10^{-2}$

Consider the case: M_ν is non-diagonal

Tri-bimaximal mixing pattern for neutrinos

Harrison, Perkins, Scott (2002) proposed

$$V_{\rm TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Charged lepton sector

Neutrino sector

$$U_{\rm PMNS} = \begin{pmatrix} \cos\phi & -e^{-i\sigma}\sin\phi & 0 \\ e^{i\sigma}\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{\rm TBM}$$

$$|U_{e3}| = \left| \frac{e^{-i\sigma}\sin\phi}{\sqrt{2}} \right|$$

If $\sin\Phi = \lambda$ (Cabibbo angle),

$$\theta_{13}^{\rm PMNS} \simeq \theta_{\rm Cabibbo}/\sqrt{2} = 0.16$$

Cabibbo Hase
Cabibbo sized effect of lepton mixing

Suppose

θ_{13} (from neutrino mass matrix) is supposed to be negligible small

Then, no extra Dirac CP phase except for Majorana phases

Majorana phases

$$U_\nu = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\cos\theta_{23}\sin\theta_{12} & \cos\theta_{12}\cos\theta_{23} & -\sin\theta_{23} \\ -\sin\theta_{12}\sin\theta_{23} & \cos\theta_{12}\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} P$$

Pati-Salam

$$J^l_{CP} \simeq -\frac{1}{2\Delta_e} a_E b_E d_E^2 (c_E'^2 + d_E^2) \sin(2\theta_{12}) \cos\theta_{23} \sin^2\theta_{23} \sin\phi$$

negative

SU(5)

$$J^l_{CP} \simeq \frac{1}{2\Delta_e} a'_E b_E d_E \sin(2\theta_{12})(c'_E \sin\theta_{23} - d_E \cos\theta_{23})(c'_E \cos\theta_{23} + d_E \sin\theta_{23})^2 \sin\phi$$

In the case of Tri-bimaximal mixing $\sin\theta_{12} = 1/\sqrt{3}$ and $\sin\theta_{23} = 1/\sqrt{2}$.

Pati-Salam $C_a = 2, \quad C_{a'} = 1, \quad C_b = -3, \quad C_c = \frac{3}{4}, \quad C_{c'} = 1, \quad C_d = 1$

Obs: -10^{-2}

$$J^l_{CP} \simeq -0.76 \times 10^{-2}, \qquad \frac{m_e^2\, m_\mu^2\, m_\tau^2}{m_d^2\, m_s^2\, m_b^2} = 4 \text{ (obs :}1.7 - 7.3),$$

$$\frac{m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2}{m_d^2 m_s^2 + m_s^2 m_b^2 + m_d^2 m_b^2} = 24 \text{ (obs :}15 - 26), \quad \frac{m_e^2 + m_\mu^2 + m_\tau^2}{m_d^2 + m_s^2 + m_b^2} = 1.0 \text{ (obs :}0.99 - 1.1)$$

$$\sin^2\theta_{12}^{\rm PMNS} \simeq 0.38, \quad \sin^2\theta_{23}^{\rm PMNS} \simeq 0.47, \quad \sin\theta_{13}^{\rm PMNS} \simeq 0.06, \quad \delta^l_{CP} \simeq -30^\circ$$

SU(5) $C_a = 1, \quad C_{a'} = \frac{9}{2}, \quad C_b = \frac{9}{2}, \quad C_c = \frac{9}{2}, \quad C_{c'} = -\frac{3}{2}, \quad C_d = -\frac{1}{2}$

$$J^l_{CP} \simeq -1.13 \times 10^{-2}, \qquad \frac{m_e^2\, m_\mu^2\, m_\tau^2}{m_d^2\, m_s^2\, m_b^2} = 5.06 \text{ (obs :}1.7 - 7.3),$$

$$\frac{m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_e^2 m_\tau^2}{m_d^2 m_s^2 + m_s^2 m_b^2 + m_d^2 m_b^2} = 26 \text{ (obs :}15 - 26), \quad \frac{m_e^2 + m_\mu^2 + m_\tau^2}{m_d^2 + m_s^2 + m_b^2} = 1.07 \text{ (obs :}0.99 - 1.1)$$

$$\sin^2\theta_{12}^{\rm PMNS} \simeq 0.28, \quad \sin^2\theta_{23}^{\rm PMNS} \simeq 0.85, \quad \sin\theta_{13}^{\rm PMNS} \simeq 0.153, \quad \delta^l_{CP} \simeq -113^\circ$$

M_ν is diagonal

M_D	$\mathrm{Tr}\left([H_\nu, H_E]^3\right)_{\mathrm{Pati-Salam}}$	$\mathrm{Tr}\left([H_\nu, H_E]^3\right)_{\mathrm{SU(5)}}$
$M_D^{(1)}$	$-6ia_E^2 b_E c_E c'_E d_E \Delta_\nu \sin\phi$	$-6ia'^2_E b_E c_E c'_E d_E \Delta_\nu \sin\phi$
$M_D^{(2)}$	$-6ia_E^2 b_E c_E c'_E d_E \Delta_\nu \sin\phi$	0
$M_D^{(3)}$	$-6ia_E^2 b_E c_E c'_E d_E \Delta_\nu \sin\phi$	$6ia'^2_E b_E c_E c'_E d_E \Delta_\nu \sin\phi$
$M_D^{(4)}$	$-6ia_E b_E c_E c'_E d_E^2 \Delta_\nu \sin\phi$	$-6ia_E a'^2_E b_E c_E c'_E \Delta_\nu \sin\phi$
$M_D^{(5)}$	$-6ia_E b_E c_E c'_E d_E^2 \Delta_\nu \sin\phi$	$6ia_E a'^2_E b_E c_E c'_E \Delta_\nu \sin\phi$
$M_D^{(6)}$	$-6ia_E b_E c_E c'_E d_E^2 \Delta_\nu \sin\phi$	0
$M_D^{(11)}$	$-6ia_E b_E c_E^2 c'_E d_E \Delta_\nu \sin\phi$	$-6ia_E a'^2_E b_E c'_E d_E \Delta_\nu \sin\phi$
$M_D^{(12)}$	$-6ia_E b_E c_E^2 c'_E d_E \Delta_\nu \sin\phi$	0
$M_D^{(13)}$	$-6ia_E b_E c_E^2 c'_E d_E \Delta_\nu \sin\phi$	$6ia_E a'^2_E b_E c'_E d_E \Delta_\nu \sin\phi$
$M_D^{(14)}$	$-6ia_E b_E c_E c'_E d_E^2 \Delta_\nu \sin\phi$	$-6ia_E a'^2_E b_E c_E c'_E \Delta_\nu \sin\phi$
$M_D^{(15)}$	$-6ia_E b_E c_E^2 c'_E d_E \Delta_\nu \sin\phi$	$6ia_E a'^2_E b_E c'_E d_E \Delta_\nu \sin\phi$
$M_D^{(16)}$	$-6ia'_E b_E^2 c_E c'_E d_E \Delta_\nu \sin\phi$	0
$M_D^{(17)}$	$-6ia_E^2 b_E c_E c'_E d_E \Delta_\nu \sin\phi$	0

$$\mathrm{Tr}\left([H_\nu, H_E]^3\right) = -6i\Delta_\nu\Delta_e J^l_{\mathrm{CP}}$$

M_ν is non-diagonal: Tri-bimaximal

M_D	$\mathrm{Tr}\left([H_\nu, H_E]^3\right)_{\text{Pati–Salam}}$	$\mathrm{Tr}\left([H_\nu, H_E]^3\right)_{\text{SU(5)}}$
$M_D^{(1)}$	$ia_E b_E (c_E'^2 + d_E^2) d_E^2 \Delta_\nu \sin\phi$	$ia_E' b_E (c_E' + d_E) d_E (d_E^2 - c_E'^2) \Delta_\nu \sin\phi$
$M_D^{(2)}$	$ia_E b_E (c_E'^2 + d_E^2) d_E^2 \Delta_\nu \sin\phi$	$ia_E a_E' b_E c_E (c_E' + d_E)^2 \Delta_\nu \sin\phi$
$M_D^{(3)}$	$ia_E b_E (c_E'^2 + d_E^2) d_E^2 \Delta_\nu \sin\phi$	$-ia_E' b_E c_E (d_E^2 - c_E'^2)(c_E + d_E) \Delta_\nu \sin\phi$
$M_D^{(4)}$	$ia_E b_E d_E^4 \Delta_\nu \sin\phi$	$ia_E' b_E d_E^4 \Delta_\nu \sin\phi$
$M_D^{(5)}$	$ia_E b_E d_E^4 \Delta_\nu \sin\phi$	$ia_E' b_E c_E (a_E - c_E') d_E^2 \Delta_\nu \sin\phi$
$M_D^{(6)}$	$ia_E b_E d_E^4 \Delta_\nu \sin\phi$	$-ia_E' b_E c_E d_E^3 \Delta_\nu \sin\phi$
$M_D^{(11)}$	$ia_E c_E c_E' d_E (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$ia_E a_E' d_E (c_E' d_E^2 + d_E^3 - c_E'^3 - c_E'^2 d_E) \Delta_\nu \sin\phi$
$M_D^{(12)}$	$ia_E c_E c_E' d_E (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$-ia_E a_E' b_E c_E (c_E' + d_E)^2 \Delta_\nu \sin\phi$
$M_D^{(13)}$	$ia_E c_E c_E' d_E (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$-ia_E a_E' b_E (d_E^2 - c_E'^2)(c_E' + d_E) \Delta_\nu \sin\phi$
$M_D^{(14)}$	$ia_E b_E d_E^4 \Delta_\nu \sin\phi$	$ia_E a_E' d_E^4 \Delta_\nu \sin\phi$
$M_D^{(15)}$	$ia_E c_E c_E' d_E (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$ia_E a_E' d_E^2 (c_E' d_E + 2c_E'^2 - d_E^2) \Delta_\nu \sin\phi$
$M_D^{(16)}$	$ib_E c_E c_E'^2 (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$ia_E' c_E \left\lvert a_E' d_E - c_E c_E' e^{i\phi} \right\rvert^2 \Delta_\nu \sin\phi$
$M_D^{(17)}$	$ia_E c_E c_E' d_E (c_E'^2 + d_E^2) \Delta_\nu \sin\phi$	$-ib_E c_E \left\lvert c_E c_E' - b_E d_E e^{i\phi} \right\rvert^2 \Delta_\nu \sin\phi$

$$\mathrm{Tr}\left([H_\nu, H_E]^3\right) = -6i\Delta_\nu \Delta_e J_{\mathrm{CP}}^l$$

5 Summary

We have connected to lepton CP violation and quark CP violation.

Quark-Lepton unification

$$M_D^{(1)} = \begin{pmatrix} 0 & a_D & 0 \\ a'_D & b_D\ e^{-i\phi} & c_D \\ 0 & c'_D & d_D \end{pmatrix}_{LR}$$

Occam's Razor

Diagonal up-quark mass matrix

Pati-Salam →

Related by CG's

SU(5) GUT →

$$M_E^{(1)} = \begin{pmatrix} 0 & a_E & 0 \\ a'_E & b_E\ e^{-i\phi} & c_E \\ 0 & c'_E & d_E \end{pmatrix}_{LR}$$

$$M_E^{(1)} = \begin{pmatrix} 0 & a'_E & 0 \\ a_E & b_E\ e^{-i\phi} & c'_E \\ 0 & c_E & d_E \end{pmatrix}_{LR}$$

Neutrino mass matrix
is Tri-bimaximal mixing

J_{CP} for quarks is positive
Sign of J_{CP} for leptons depends on CG's

Choosing relevant CG's, J_{CP} (lepton) $= -\,0.02$ is obtained !

One CP violating phase controls both CP violation of quarks/leptons.