Describing Charm time dependent CPV in the Precision Era

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Plan

- Introduction
- Absorptive and dispersive CPV in $D^0 - \overline{D}^0$ mixing
- Time-dependent CPV phenomenology
- Approximate Universality, and implementation
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  - CF/DCS decays $D^0 \to K^\pm X$
  - CF/DCS decays $D^0 \to K^0 X \, , \overline{K}^0 X$
- Current Status
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Introduction

- In the SM, CP violation (CPV) in $D^0 - \overline{D}^0$ mixing and D decays enters at $O(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$, due to weak phase γ, yielding all 3 types of CPV:
  - direct CPV (dCPV)
  - CPV in pure mixing (CPVMIX): due to interference between dispersive and absorptive mixing amps
  - CPV in the interference of decays with and without mixing (CPVINT)
- Our interest here is in CPVMIX and CPVINT, both of which result from mixing, and which we refer to as "indirect CPV"

- Questions:
  - How large are the indirect CP asymmetries in the SM?
  - What is the appropriate minimal parametrization of indirect CPV?
  - How large is the current window for new physics (NP)?
  - Can this window be closed in the Belle-II / LHCb Precision Era ?
- Answers:
  - obtained via description of CPVINT in terms of pairs of dispersive and absorptive CPV phases ϕ_f^M and ϕ_f^Γ, for CP conjugate final states f, $\bar{f}$
  - they parametrize CPVINT contributions from interference of D^0 decays with and without dispersive mixing, and with and without absorptive mixing
  - they are separately measurable
  - SM estimates of ϕ_f^M, ϕ_f^Γ, and final state dependence (approximate universality) obtained from comparison to two "theoretical phases" ${\phi_2}^M$, ${\phi_2}^\Gamma$

Absorptive and Dispersive CPV

- time-evolution of linear combination $a|D^0\rangle + b|\overline{D}^0\rangle$ follows from Schrodinger equation,

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H \begin{pmatrix} a \\ b \end{pmatrix} \equiv (M - \tfrac{i}{2}\,\Gamma) \begin{pmatrix} a \\ b \end{pmatrix} .$$

  - transition amplitudes

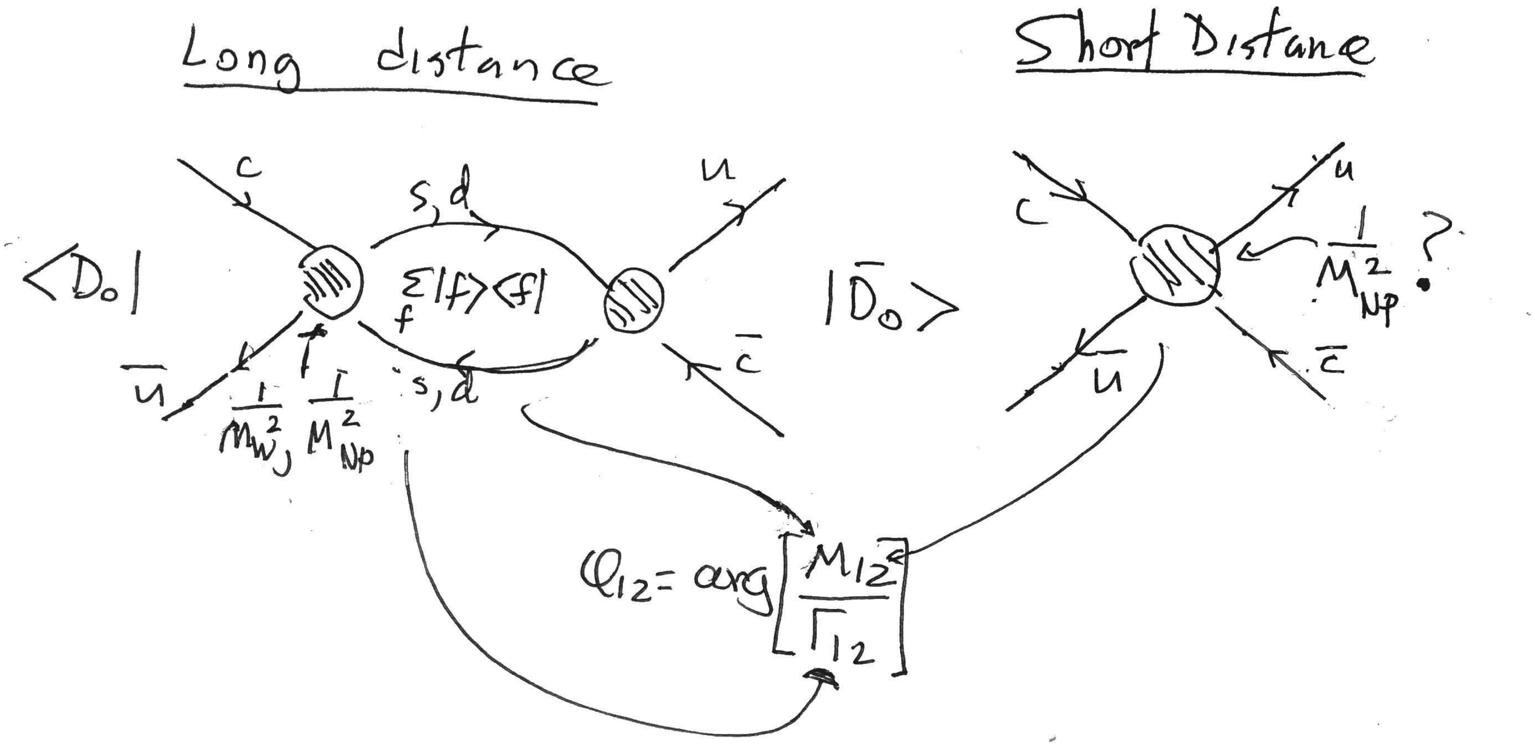
$$\langle D^0|H|\overline{D^0}\rangle = M_{12} - \frac{i}{2}\Gamma_{12}\,, \quad \langle \overline{D^0}|H|D^0\rangle = M_{12}^* - \frac{i}{2}\Gamma_{12}^*$$

    - M_{12} is the dispersive mixing amplitude
    - Γ_{12} is the absorptive mixing amplitude

- Mass eigenstates $|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$:

  - mass and width differences expressed in terms of parameters x, y

$$x = \frac{m_2 - m_1}{\Gamma_D}, \qquad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma_D}$$



- M_{12} is dispersive mixing: due to long-distance exchange of off-shell intermediate states; and short-distance effects
  - long distance dominates in SM
  - significant short distance would be new physics (NP)
- Γ_{12} is absorptive mixing: due to long distance exchange of on-shell intermediate states

- introduce three “theoretical" physical mixing parameters

$$x_{12} \equiv 2|M_{12}|/\Gamma_D, \quad y_{12} \equiv |\Gamma_{12}|/\Gamma_D, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$$

  - ϕ_{12} is the CPV phase responsible for CPVMIX, e.g. $A_{\rm SL}$
  - CP conserving observables: $|x| = x_{12} + O({\rm CPV}^2),\ |y| = y_{12} + O({\rm CPV}^2)$

- Time-evolved meson solutions, for $t \lesssim \tau_D$:

For $D^0(0) = D^0$, the mixed component at time t,

$$\langle \overline{D}^0 | D^0(t) \rangle = e^{-i\left(M_D - i\frac{\Gamma_D}{2}\right)t} \left(e^{-i\pi/2} M_{12}^* - \tfrac{1}{2}\Gamma_{12}^*\right) t, \ldots$$

  - the phase $\pi/2$ is a CP-even “dispersive strong phase", originating from the time derivative. It contributes to strong phase differences required for non-vanishing time dependent CPV

- The CPVMIX “wrong sign" semileptonic CP asymmetry:

$$a_{\rm SL} \equiv \frac{\Gamma(D^0(t) \to \ell^- X) - \Gamma(\overline{D^0}(t) \to \ell^+ X)}{\Gamma(D^0(t) \to \ell^- X) + \Gamma(\overline{D^0}(t) \to \ell^+ X)} ,$$
$$= \frac{|\langle \overline{D}^0 | D^0(t)\rangle|^2 - |\langle \overline{D}^0 | D^0(t)\rangle|^2}{|\langle \overline{D}^0 | D^0(t)\rangle|^2 + |\langle \overline{D}^0 | D^0(t)\rangle|^2} .$$

The semileptonic decay amplitude factors are cancelled in second relation, given negligible direct CPV $|\bar{A}_{\ell^- X}| = |A_{\ell^+ X}|$.

- Solutions for mixed components $\langle \overline{D}^0 | D^0(t)\rangle$, $\langle D^0 | \overline{D}^0(t)\rangle$ $\Rightarrow$

$$a_{\rm SL} = \frac{2 x_{12}\, y_{12}}{x_{12}^2 + y_{12}^2} \, \sin\phi_{12} .$$

- The CP-even phase difference between the interfering dispersive and absorptive mixing amplitudes, required to obtain CPVMIX, provided by the dispersive mixing phase $\pi/2$

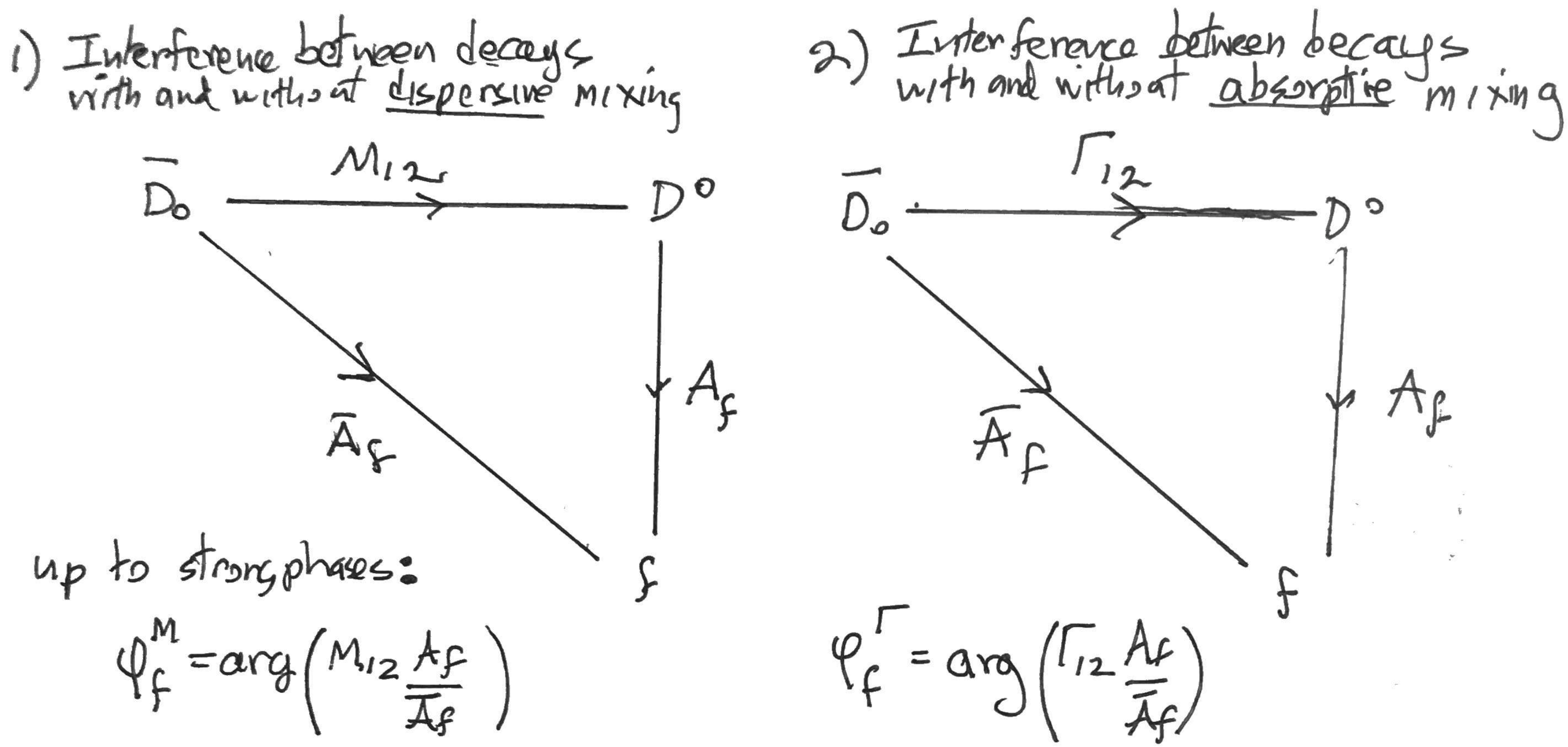
The dispersive and absorptive CPV phases ϕ_f^M, ϕ_f^Γ in hadronic decays

- Hadronic $D^0(t)$, $\overline{D}^0(t)$ decay amplitudes sum over contributions with/without mixing:

$$A(D^0(t) \to f) = \bar{A}_f \, \langle \overline{D}^0 | D^0(t) \rangle + A_f \, \langle D^0 | D^0(t) \rangle,$$
$$A(\overline{D}^0(t) \to f) = A_f \, \langle D^0 | \overline{D}^0(t) \rangle + \bar{A}_f \, \langle \overline{D}^0 | \overline{D}^0(t) \rangle \,.$$

where $A_f \equiv \langle f | \mathcal{H} | D^0 \rangle \,, \;\; \bar{A}_f \equiv \langle f | \mathcal{H} | \bar{D}^0 \rangle$ are the decay ampltiudes

- ϕ_f^M and ϕ_f^Γ are the CPV phase differences between the two interfering amplitudes



Relation to “phenomenological” CPVINT parameters

- The more familiar “phenomenological" CPV observables are

$$\text{CPVMIX}: \quad \left|\frac{q}{p}\right| - 1$$

$$\text{CPVINT}: \ \phi_{\lambda_f} = \arg\left(\frac{q}{p}\frac{\overline{A}_f}{A_f}\right), \quad \text{up to strong phase difference, for } f \neq \bar{f}$$

- Relation to absorptive and dispersive CPVINT phases

$$\left|\frac{q}{p}\right| - 1 = \frac{x_{12}\,y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2} + O(\text{CPV}^3), \qquad \text{where} \quad \phi_{12} = \phi_f^M - \phi_f^\Gamma$$

$$\sin 2\phi_{\lambda_f} = -\left(\frac{x_{12}^2\sin 2\phi_f^M + y_{12}^2 \sin 2\phi_f^\Gamma}{x_{12}^2 + y_{12}^2}\right) + O(\text{CPV}^3)$$

  - ϕ_{λ_f} is a sum over ϕ_f^M and ϕ_f^Γ, weighted by the the relative dispersive and absorptive contributions to the CP averaged mixing probability, $x_{12}^2/(x_{12}^2 + y_{12}^2)$ and $y_{12}^2/(x_{12}^2 + y_{12}^2)$
  - $\phi_{12} = \phi_f^M - \phi_f^\Gamma \Rightarrow$ same number of CPV quantities in each description

Time dependent CPV phenomenology

I. Phenomenology of SCS decays to CP eigenstates

The phases ϕ_f^M, ϕ_f^Γ enter the decay widths via the dimensionless observables λ_f^M, λ_f^Γ:

- for SCS decays to CP-eigenstate final states:

  $\bar{f} = \eta_f^{CP}\, f$, where $\eta_f^{CP} = +(-)$ for f a CP-even (odd) final state

$$\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|}\,\frac{A_f}{\overline{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\overline{A}_f}\right| e^{i\,\phi_f^M}\,, \qquad \lambda_f^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\,\frac{A_f}{\overline{A}_f} = \eta_f^{CP}\left|\frac{A_f}{\overline{A}_f}\right| e^{i\,\phi_f^\Gamma}\,.$$

- recall CP asymmetries require both a CPV phase difference (ϕ), and a CP-even phase difference (δ), between interfering amplitudes $\Rightarrow$ $A_{CP} \propto \sin\phi \sin\delta$

- Trivial strong phase difference between A_f, $\overline{A}_f$ $\Rightarrow$ the only CP-even phase available for generation of CP asymmetries is the dispersive phase $\pi/2$

- Therefore, for CP-eigenstate final states, in general, CPVINT is purely dispersive and $\propto x_{12} \sin\phi_f^M$

- General expressions for time-dependent decay widths in terms of λ_f^M, λ_f^Γ follow from $|A(\overline{D}^0(t) \to f)|^2$, etc. (modified expressions for decays to $K^0 X$, $\overline{K}^0 X$)

$$\Gamma(\overline{D}^0(t) \to f) = e^{-\tau} |\bar{A}_f|^2 \Big\{ 1 + \tau \, \mathrm{Re}\big[e^{-i\delta_M} \lambda_f^M x_{12} - \lambda_f^\Gamma y_{12}\big]$$

$$+ \frac{\tau^2}{4} \left(|\lambda_f^M|^2 x_{12}^2 + |\lambda_f^\Gamma|^2 y_{12}^2 + 2 x_{12} \, y_{12} \, \mathrm{Im}\left[\lambda_f^{M\,*} \lambda_f^\Gamma\right] \right) \Big\},$$

with similar expressions for $\Gamma(D^0(t) \to f)$, $\Gamma(\overline{D}^0(t) \to \bar{f})$, $\Gamma(D^0(t) \to \bar{f})$

- time-dependent decay widths for SCS decays to CP eigenstates ($\tau \equiv \Gamma_D t$), e.g. $f = K^+K^-,\ \pi^+\pi^-,\ \rho^0\pi^0,\ K^{*+}K^{*-},\ \rho^+\rho^-$

$$\Gamma(D^0(t) \to f) = e^{-\tau}|A_f|^2 \left(1 + c_f^+\,\tau + c_f'^+\,\tau^2\right)\,,$$
$$\Gamma(\overline{D}^0(t) \to f) = e^{-\tau}|\bar{A}_f|^2 \left(1 + c_f^-\,\tau + c_f'^-\,\tau^2\right)\,,$$

where the coefficients $c_f^\pm$, $c_f'^\pm$ satisfy

$$c_f^\pm = \eta_{CP}^f \left[\mp x_{12}\sin\phi_f^M - y_{12}\cos\phi_f^\Gamma\,(1 \mp a_f^d)\right],$$
$$c_f'^\pm = \tfrac{1}{4}(x_{12}^2 + y_{12}^2)\left(1 \pm a_{\rm SL} \mp 2a_f^d\right)\,,$$

and the direct CP asymmetry $a_f^d \equiv 1 - \left|\bar{A}_f/A_f\right| = -2r_f\sin\phi_f\sin\delta_f$

- traditional to express SCS widths as exponentials, neglecting $O(\tau^2)$ dependence:

$$\Gamma(D^0(t) \to f) = |A_f|^2 \exp[-\hat{\Gamma}_{D^0\to f}\,\tau],\quad \Gamma(\overline{D^0}(t) \to f) = |\bar{A}_f|^2 \exp[-\hat{\Gamma}_{\overline{D^0}\to f}\,\tau],$$

where $\hat{\Gamma}_{D^0/\overline{D}^0\to f} = 1 - c^\pm$

(should revisited in the precision era, for the CP conserving part)

- The time-dependent CPVINT asymmetry:

$$\Delta Y_f = -A_\Gamma \equiv \frac{(c_f^+ - c_f^-)}{2} = \frac{\hat{\Gamma}_{\overline{D}^0 \to f} - \hat{\Gamma}_{D^0 \to f}}{2}$$

  - In terms of the CPVINT parameters,

$$\Delta Y_f = \eta_{CP}^f \, (-x_{12} \sin \phi_f^M + a_f^d \, y_{12})$$

  - confirmation that CPVINT is purely dispersive (up to dCPV effects)
  - can only probe ϕ_f^Γ with non-CP eigenstate final states
  - the dCPV contribution is disentangled via time-integrated measurements
  - Compare to phenomenological parametrization:

$$\Delta Y_f = \frac{y}{2} \cos \phi_{\lambda_f} \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) - \frac{x}{2} \sin \phi_{\lambda_f} \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) + a_f^d \, |y|$$

    - the physical interpretation is obscured

- The CP conserving observable y_{CP}^{f} (for CP-eigenstate final states),

$$y_{CP}^{f} \equiv -\frac{(c_f^+ + c_f^-)}{2} = \frac{\hat{\Gamma}_{D^0 \to f_{CP}} + \hat{\Gamma}_{\overline{D^0} \to f_{CP}}}{2} - 1$$

  - In terms of the CPVINT parameters

$$y_{CP}^{f} = \eta_f^{CP} y_{12} \cos \phi_f^{\Gamma}$$

    - exp. avg. over $f = K^+K^-, \pi^+\pi^-$ $\Rightarrow$ $y_{CP}^{f}/\eta_f^{CP} > 0$
    - combining with global fit result $\phi_{12} = \phi_f^M - \phi_f^{\Gamma} \approx 0$ (rather than π), we learn that

$$\phi_f^M \approx 0, \qquad \phi_f^{\Gamma} \approx 0$$

      (rather than $\approx \pi$)

II. Phenomenology of CF/DCS Decays to $K^\pm X$

- For CF/DCS decays to $K^\pm X$, e.g. $K^+\pi^-$, $K^+\pi^-\pi^0$ (and SCS decays to non-CP eigenstates, e.g. $KK\pi\pi$, $\pi\pi\pi\pi$), have two pairs of observables: one for f, one for $\bar{f}$:

$$\lambda_f^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_f}{\overline{A}_f} = -\left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^M - \Delta_f)}, \quad \lambda_f^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_f}{\overline{A}_f} = -\left|\frac{A_f}{\overline{A}_f}\right| e^{i(\phi_f^\Gamma - \Delta_f)},$$

$$\lambda_{\bar{f}}^M \equiv \frac{M_{12}}{|M_{12}|}\frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} = -\left|\frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}}\right| e^{i(\phi_f^M + \Delta_f)}, \quad \lambda_{\bar{f}}^\Gamma \equiv \frac{\Gamma_{12}}{|\Gamma_{12}|}\frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}} = -\left|\frac{A_{\bar{f}}}{\overline{A}_{\bar{f}}}\right| e^{i(\phi_f^\Gamma + \Delta_f)}.$$

  - Δ_f = strong phase difference between $\overline{A}_f$ (DCS) and A_f (CF), and between $A_{\bar{f}}$ (DCS) and $\overline{A}_{\bar{f}}$ (CF)

  - the total CP-even phase difference between decays with and without mixing is $\Delta_f - \pi/2$ (dispersive) and Δ_f (absorptive) $\Rightarrow$

    - the time dependent CPVINT asymmetries are

$$\propto x_{12}\, \sin\phi_f^M\, \cos\Delta_f \quad \text{(dispersive mixing)}$$

$$\propto y_{12}\, \sin\phi_f^\Gamma\, \sin\Delta_f \quad \text{(absorptive mixing)}$$

- in the SM, and NP models with negligible dCPV in CF/DCS decays, the time-dependent decay widths for the “wrong sign” decays $D^0 \to \bar{f}$ and $\overline{D}^0 \to f$, e.g. $\bar{f} = K^+\pi^-$, are:

$$\Gamma(D^0(t) \to \bar{f}) = e^{-\tau}|A_f|^2 \left(R_f + \sqrt{R_f}c_f^+\,\tau + c_f'^+\,\tau^2\right) ,$$
$$\Gamma(\overline{D}^0(t) \to f) = e^{-\tau}|A_f|^2 \left(R_f + \sqrt{R_f}c_f^-\,\tau + c_f'^-\,\tau^2\right) ,$$

  - $R_f = |A_{\bar{f}}/A_f|^2 = O(\lambda^2)$ (ratio of DCS to CF decay widths), and the coefficients satisfy,

$$\begin{aligned} c_f^\pm &= -\left[x_{12}\sin\Delta_f + y_{12}\cos\Delta_f\right] \mp x_{12}\sin\phi_f^M\cos\Delta_f \pm y_{12}\sin\phi_f^\Gamma\sin\Delta_f, \\ c_f'^\pm &= \tfrac{1}{4}(x_{12}^2 + y_{12}^2)\;[1 \pm a_{\rm SL}]\,. \end{aligned}$$

- the wrong sign CP asymmetry at linear order in τ:

$$\delta c_f \equiv \frac{1}{2}(c_f^+ - c_f^-) = -x_{12} \sin \phi_f^M \cos \Delta_f + y_{12} \sin \phi_f^\Gamma \sin \Delta_f$$

  - confirms expected Δ_f dependence for dispersive and absorptive CPV
  - as expected, non-CP eigenstate final states (non-trivial Δ_f) yield sensitivity to ϕ_f^Γ
  - the expression for the CP asymmetry in the ($|q/p|$, ϕ_{λ_f}) parametrization is a mess, again obscuring the physics

$$-2\,\delta c_f = \left[x \cos \phi_{\lambda_f} \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + y \sin \phi_{\lambda_f} \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right] \sin \Delta_f$$
$$+ \left[y \cos \phi_{\lambda_f} \left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x \sin \phi_{\lambda_f} \left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right] \cos \Delta_f$$

III. Phenomenology of CF/DCS decays to $K^0 X$, $\overline{K}^0 X$, e.g. $K_S\,\pi^+\pi^-$

- two-step transitions $D^0 \to [K_{S,L} \to \pi^+\pi^-] + X$, to CP conjugate final states $f = [\pi^+\pi^-]X,\ \bar{f} = \overline{[\pi^+\pi^-]X}$
  - for example, for $X = \pi^+\pi^-$, $\bar{f}$ and f related by interchanging the Dalitz plot variables: $(p_K + p_{\pi^+})^2 \leftrightarrow (p_K + p_{\pi^-})^2$
- Extra care is required for these decays: must account for CPV in $K^0 - \overline{K}^0$ mixing, i.e. ϵ_K, in order to achieve sensitivity to charm CPVINT in the SM
- the neutral K mass eigenstates are given by

$$|K_S\rangle = p_K|K^0\rangle + q_K|\overline{K}^0\rangle, \qquad |K_L\rangle = p_K|K^0\rangle - q_K|\overline{K}^0\rangle\,.$$

  - To excellent approximation,

$$\frac{q_K}{p_K} = \frac{A_0}{\overline{A}_0}\,(1 - 2\,\epsilon_K)\,, \qquad \left|\frac{q_K}{p_K}\right| = 1 - 2\,\mathrm{Re}[\epsilon_K]$$

    - $\epsilon_K \cong (1.62 + i\,1.53) \times 10^{-3}$, and A_0 denotes the $K^0 \to (\pi\pi)_{I=0}$ amplitude

- CF/DCS decays to $K^0 X$, $\overline{K}^0 X$, in general, have four pairs of CPVINT observables,

$$\lambda^M_{K_a X},\ \lambda^\Gamma_{K_a X}\ \text{ and }\ \lambda^M_{\overline{K_a X}},\ \lambda^\Gamma_{\overline{K_a X}}, \qquad a = S\,, L$$

- In the SM, and for negligible new weak phases in CF/DCS decays, the DCS amplitudes can be neglected to very good approximation (will quantify later)

- in this limit, the CPVINT observables reduce to two pairs,

$$\lambda_f^{M\,(\Gamma)} \equiv \lambda^{M,(\Gamma)}_{K_S X} = -\lambda^{M,(\Gamma)}_{K_L X} = \left| \frac{p_K\, A_{\overline{K}^0 X}}{q_K\, \overline{A}_{K^0 X}} \right| e^{i(\phi_f^{M\,(\Gamma)} - \Delta_f)},$$

$$\lambda_{\bar f}^{M\,(\Gamma)} \equiv \lambda^{M,(\Gamma)}_{\overline{K_S X}} = -\lambda^{M,(\Gamma)}_{\overline{K_L X}} = \left| \frac{p_K\, \overline{A}_{K^0 X}}{q_K\, A_{\overline{K}^0 X}} \right| e^{i(\phi_f^{M\,(\Gamma)} + \Delta_f)},$$

for CP conjugate final states $f = [\pi^+\pi^-]X$, $\quad \bar f = \overline{[\pi^+\pi^-]X}$

  - Note that $\mathrm{Im}[\epsilon_K]$ feeds into ϕ_f^M, ϕ_f^Γ via $\arg(p_K/q_K)$

- the time dependent decay widths depend on two elapsed time intervals: t and t', at which the D and K decay, following their respective production
  - Kaon time evolution conveniently described in the mass basis:

$$|K_S(t)\rangle = e^{-iM_S t} e^{-\Gamma_S t/2} |K_S\rangle\,, \quad |K_L(t)\rangle = e^{-iM_L t} e^{-\Gamma_L t/2} |K_L\rangle$$

  - the time dependent decay amplitudes, e.g. for

$$D^0(t) \to [K_{S,L}(t') \to \pi^+\pi^-] + X\,,$$

    are obtained by summing over the intermediate $K_S X$ and $K_L X$ states, evolved over time interval t'

- the absolute value squared of the decay amplitudes, e.g.

$$A_f(t,t') = \sum_{a=S,L} A(K_a \to \pi^+\pi^-) \times e^{-(iM_a + \frac{1}{2}\Gamma_a)t'} (A_{K_a X} \langle D^0 | D^0(t)\rangle + \overline{A}_{K_a X} \langle \overline{D}^0 | D^0(t)\rangle)\,,$$

  expressed in terms of $\lambda^{M,\Gamma}_{K_{S,L}X}$, $\lambda^{M,\Gamma}_{\overline{K_{S,L}X}}$, yields decay widths in terms of $\phi_f^{M,\Gamma}$ and ϵ_K

- for example, the resulting decay width for $D^0(t) \to f = [\pi^+\pi^-]X \quad (\tau \equiv \Gamma_D t)$:

$$\Gamma_f(t,t') = e^{-\tau}|\overline{A}_{+-}|^2|A_{\overline{K}^0 X}|^2 \Big\{ e^{-\Gamma_S t'}\left[c^+ + \sqrt{R_f}\, c_f^+\, \tau + R_f\, c'^+\, \tau^2\right] +$$
$$e^{-\Gamma_K t'}\Big[(b^+ + \sqrt{R_f}\, b_f^+ \tau + R_f\, b'^+ \tau^2)\cos(\Delta M_K t')$$
$$+ (d^+ + \sqrt{R_f}\, d_f^+ \tau + R_f\, d'^+ \tau^2)\sin(\Delta M_K t') \Big] \Big\},$$

  - $\overline{A}_{+-} \equiv \langle \pi^+\pi^-|H|\overline{K}^0\rangle$, $R_f \equiv \left|\overline{A}_{K^0 X}/A_{\overline{K}^0 X}\right|^2$ is the ratio of CF decay rates, $\Delta M_K \equiv M_L - M_S$, $\Gamma_K \equiv (\Gamma_L + \Gamma_S)/2$,

$$c^+ = 1 + 2\,\mathrm{Re}[\epsilon_K], \quad b, d = O(\epsilon_K)$$
$$c_f^+ = (x_{12} - y_{12}\sin\phi_f^\Gamma)\,\sin\Delta_f - (y_{12} + x_{12}\sin\phi_f^M)\cos\Delta_f$$

  - the pure K_S contribution is $\propto e^{-\Gamma_S t'}$,
  - the $K_L - K_S$ interference contribution is $\propto e^{-\Gamma_K t'}$, and is $O(\epsilon_K)$,
  - the pure K_L contribution is $O(\epsilon_K^2)$ and negligible

- CP asymmetries will be discussed in the context of approximate universality, also taking into account the ϵ_K dependence of $\phi_f^{M,\Gamma}$

Approximate Universality

- we have parameterized indirect CPV in terms of final state dependent pairs of dispersive and absorptive phases ϕ_f^M , ϕ_f^Γ.

- to arrive at a minimal parametrization of indirect CPV effects in the precision era, we need to understand the final state dependence

- accomplished via a U-spin flavor symmetry decomposition of the SM mixing amplitudes - this also yields estimates of ϕ_f^M , ϕ_f^Γ in the SM

- can write the SM $D^0 - \overline{D}^0$ mixing amplitudes as, (CKM factors $\lambda_i \equiv V_{ci} V_{ui}^*$)

$$\Gamma_{12}^{\rm SM} = -\sum_{i,j=d,s} \lambda_i \lambda_j \Gamma_{ij}\,, \qquad M_{12}^{\rm SM} = -\sum_{i,j=d,s,b} \lambda_i \lambda_j M_{ij}$$

  - at quark level, Γ_{ij}, M_{ij} identified with $(\bar{u}c)^2$ box diagrams, containing internal i and j quarks

  - they have internal quark flavor structures

$$\Gamma_{ss}, M_{ss} \sim (\bar{s}s)^2\,, \quad \Gamma_{dd}, M_{dd} \sim (\bar{d}d)^2\,, \quad \Gamma_{sd}, M_{sd} \sim (\bar{s}s)(\bar{d}d)\,,$$

U-spin decomposition ($SU(2)$ for $d-s$ rotations)

- using CKM unitarity ($\lambda_d + \lambda_s + \lambda_b = 0$), obtain

$$\Gamma_{12}^{\rm SM} = \frac{(\lambda_s - \lambda_d)^2}{4}\,\Gamma_2 + \frac{(\lambda_s - \lambda_d)\lambda_b}{2}\,\Gamma_1 + \frac{\lambda_b^2}{4}\,\Gamma_0$$

  - $\Gamma_{2,1,0}$ are the $\Delta U_3 = 0$ elements of ΔU= 2, 1, 0 multiplets, respectively
  - can be seen from their flavor structures

$$\Gamma_2 = \Gamma_{ss} + \Gamma_{dd} - 2\Gamma_{sd} \;\sim\; (\bar{s}s - \bar{d}d)^2 = O(\epsilon^2)\,,$$
$$\Gamma_1 = \Gamma_{ss} - \Gamma_{dd} \;\sim\; (\bar{s}s - \bar{d}d)(\bar{s}s + \bar{d}d) = O(\epsilon)\,,$$
$$\Gamma_0 = \Gamma_{ss} + \Gamma_{dd} + 2\Gamma_{sd} \;\sim\; (\bar{s}s + \bar{d}d)^2 = O(1)\,.$$

  - the orders in the U-spin breaking parameter ϵ are shown, corresponding to the power of the U-spin breaking "spurion" $\sim \epsilon\,(\bar{s}s - \bar{d}d)$ required to construct each Γ_i
  - decomposition of $M_{12}^{\rm SM}$ is analogous (with exception of contributions to M_1, M_0 containing internal b quarks)

CPV in mixing

- small $|\lambda_b/\lambda_s| \sim 0.7 \times 10^{-3} \Rightarrow$ mass and width differences (x_{12} , y_{12}) are due to M_2 and Γ_2, even though $O(\epsilon^2)$
  - U-spin breaking is large:
    - inclusive OPE approach yields $\Gamma_{ij} \sim \Gamma_D \Rightarrow \epsilon^2 = O(20\%)$ in Γ_2
    - exclusive approach: consensus that $y_{12} \sim 1\%$ requires high multiplicity final states, due to large U-spin breaking near threshold

- CPV in mixing arises at $O(\epsilon)$, due to Γ_1 and M_1 ($\lambda_b \propto e^{i\,\gamma}$)
  - neglect the $O(\lambda_b^2)$ effects of Γ_0, M_0

- introduce the “theoretical” phases

$$\phi_2^\Gamma \equiv \arg\left[\frac{\Gamma_{12}}{(\lambda_s-\lambda_d)^2\,\Gamma_2}\right],\quad \phi_2^M \equiv \arg\left[\frac{M_{12}}{(\lambda_s-\lambda_d)^2\,M_2}\right],$$
$$\phi_2 \equiv \arg\left[\frac{q}{p}\,(\lambda_s-\lambda_d)^2\,\Gamma_2\right]$$

  - ϕ_2^Γ, ϕ_2^M, ϕ_2 are the theoretical analogs of ϕ_f^M, ϕ_f^Γ, ϕ_{λ_f}, respectively
  - they are defined w.r.t the direction of the dominant $\Delta U = 2$ mixing amplitudes in the complex plane $\propto (\lambda_s-\lambda_d)^2$, rather than $\overline{A}_f/A_f$
  - they sum over the contributions of all intermediate states relative to this direction
  - can ultimately be measured on the lattice for SM

- the phases are related as

$$\sin 2\phi_2 = -\frac{x_{12}^2\sin 2\phi_2^M + y_{12}^2\sin 2\phi_2^\Gamma}{x_{12}^2+y_{12}^2} + O(\mathrm{CPV}^3)$$

- rough SM estimates of ϕ_2^Γ, and similarly for ϕ_2^M:

$$\phi_2^\Gamma \approx \mathrm{Im}\left(\frac{2\lambda_b}{\lambda_s - \lambda_d}\frac{\Gamma_1}{\Gamma_2}\right) \sim \left|\frac{\lambda_b}{\theta_c}\right| \sin\gamma \times \frac{1}{\epsilon},$$

  - used $\Gamma_1/\Gamma_2\,,\ M_1/M_2 = O(1/\epsilon)$
  - CKM fits yield

$$\phi_2^\Gamma \sim \phi_2^M \sim (2.2\times 10^{-3}) \times \left[\frac{0.3}{\epsilon}\right],$$

    and ϕ_2, ϕ_{12} of same order, barring large cancelations

- alternative SM estimate of ϕ_2^Γ, via the relation $|\Gamma_2| \cong |y|\Gamma_D/\lambda_s^2$

$$|\phi_2^\Gamma| = \left|\frac{\lambda_b\,\lambda_s\,\sin\gamma}{y}\right| \frac{|\Gamma_1|}{\Gamma_D} \approx 0.005\,\frac{|\Gamma_1|}{\Gamma_D} \sim 0.005\,\epsilon\,,$$

  - in last relation used $\Gamma_1 \sim \epsilon\Gamma_D$ (recall $\Gamma_{ij} \sim \Gamma_D$ in inclusive approach)
  - the two estimates for ϕ_2^Γ are consistent (they coincide for $\epsilon \approx 0.4$)

Approximate Universality in the SM

- the misalignments $\delta\phi_f$ between the measured phases ϕ_f^M, ϕ_f^Γ, ϕ_{λ_f}, and their theoretical counterparts are equal in magnitude,

$$\delta\phi_f = \phi_f^\Gamma - \phi_2^\Gamma = \phi_f^M - \phi_2^M = \phi_2 - \phi_{\lambda_f},$$

  - in general, up to strong phases, $\delta\phi_f = \arg\left[\frac{A_f}{\overline{A}_f}(\lambda_s - \lambda_d)^2\right]$

- what are the misalignments in the various classes of decays? or, what is the uncontrolled theoretical error on measurements of ϕ_2^M, ϕ_2^Γ?

- CF/DCS decays to $K^\pm X$, e.g. $K^+\pi^-$, $K^+\pi^-\pi^0$:

$$\delta\phi_f = \arg\left[-\frac{V_{cs}^* V_{ud}}{V_{cd} V_{us}^*}(\lambda_s - \lambda_d)^2\right] = O\left(\frac{\lambda_b^2}{\lambda_s^2}\right) \sim 4 \times 10^{-5}$$

  - the misalignment is negligible, i.e. $\delta\phi_f \sim 10^{-2}\,\phi_2^{M,\Gamma}$

- CF/DCS decays to $K^0 X$, $\overline{K}^0 X$, e.g. $K_S \pi^+ \pi^-$: including the effects of kaon CPV,

$$\delta\phi_f = 2\,\mathrm{Im}[\epsilon_K] + \left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma = 3.7 \times 10^{-3},$$

is precisely known, up to two corrections of $O(0.1\ \phi_2^{M,\Gamma})$ which can be neglected:

  - an $O(\lambda^2)$ multiplicative final state dependent DCS amplitude correction, $\sim\ 2\,\lambda^2 \mathrm{Im}[\epsilon_K] \sim 1.5 \times 10^{-3}$

  - a contribution of $O(10^{-4})$ related to ϵ'/ϵ

- SCS decays, e.g. K^+K^-, $\pi^+\pi^-$: for CP eigenstate final states

$$\delta\phi_f = -2r_f \cos\delta_f \sin\gamma = -a_f^d \cot\delta_f \sim a_f^d$$

and the CP asymmetry is corrected as,

$$\Delta Y_f/\eta_{CP}^f = -x_{12}(\sin\phi_f^M + 2r_f \cos\delta_f \sin\gamma) - 2\, y_{12}\, r_f \sin\delta_f \sin\gamma$$

  - $r_f = |P/T|$ is the relative magnitude of the subleading QCD penguin amplitude, ϕ_f and δ_f are the weak and strong phase differences

  - formally, $\delta\phi_f/\phi_2^{M,\Gamma} = O(\epsilon)$, but U-spin $\Rightarrow$ $\delta\phi_{K^+K^-} \sim -\delta\phi_{\pi^+\pi^-}$, or

$$\frac{1}{2}(\phi^{M,\Gamma}_{K^+K^-} + \phi^{M,\Gamma}_{\pi^+\pi^-}) = \phi_2^{M,\Gamma}[1 + O(\epsilon^2)]$$

  - while ϵ could be large, e,.g. ~ 0.4, an $O(\epsilon^2)$ suppression of QCD penguin pollution in the average is beneficial

- Approximate universality generalizes beyond the SM under conservative assumptions regarding subleading decay amplitudes containing new weak (CPV) phases:
  - they can be neglected in CF/DCS decays: exotic flavor structure would be required to evade ϵ_K constraint
  - in SCS decays, they are of similar magnitude to, or smaller than SM QCD penguins, as hinted at by ΔA_{CP}
  - these assumptions can ultimately be tested via dCPV measurements
- NP is most likely to appear in ϕ_2^M via dispersive short distance mixing amplitudes
- Exotic invisible or missing energy D^0 decays, e.g. to axions, would contribute to both ϕ_2^M and ϕ_2^Γ

CPVINT in $D^0 \to K_S \pi^+\pi^-$

- we can now incorporate ϵ_K into the $K_S\pi^+\pi^-$ time dependent CP asymmetries. For example, [asymmetries entering at $O(\tau^2)$ are negligible]

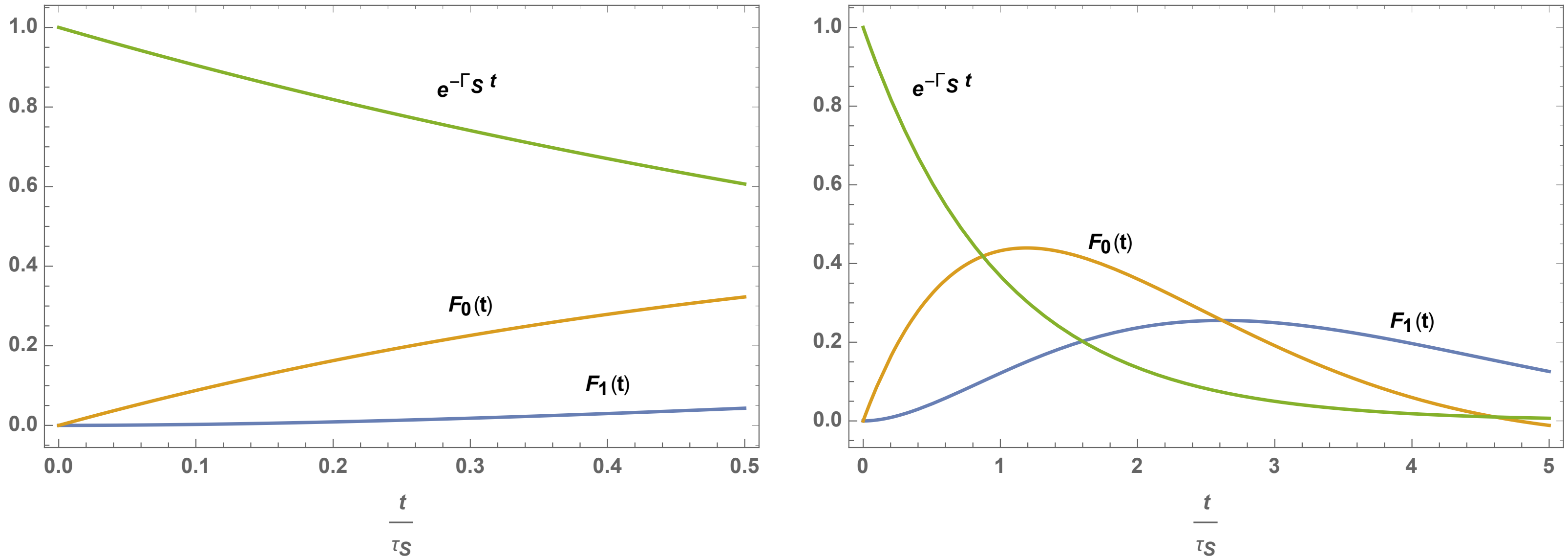
$$\Gamma_f - \overline{\Gamma}_{\bar f} = -2\, e^{-\tau} |\overline{A}_{+-}|^2 |A_{\overline{K}^0 X}|^2 \Big\{ \epsilon_R\, F_0(t') + \sqrt{R_f}\,\tau \Big[(x_{12}\cos\Delta_f + y_{12}\sin\Delta_f)\,\epsilon_I\, F_1(t') \\ + \Big(x_{12}\cos\Delta_f \sin\Big(\phi_2^M + \Big|\frac{\lambda_b}{\lambda_s}\Big| \sin\gamma\Big) + y_{12}\sin\Delta_f \sin\Big(\phi_2^\Gamma + \Big|\frac{\lambda_b}{\lambda_s}\Big|\sin\gamma\Big)\Big)\, e^{-\Gamma_S t'} \Big] \Big\},$$

where $\epsilon_R \equiv \mathrm{Re}[\epsilon_K]$, $\epsilon_I \equiv \mathrm{Im}[\epsilon_K]$, and

$$F_0(t) = -e^{-\Gamma_S t} + e^{-\Gamma_K t}\Big(\cos\Delta m_K t + \frac{\epsilon_I}{\epsilon_R}\sin\Delta m_K t\Big),$$

$$F_1(t) = e^{-\Gamma_S t} - e^{-\Gamma_K t}\Big(\cos\Delta m_K t - \frac{\epsilon_R}{\epsilon_I}\sin\Delta m_K t\Big)$$

- F_0 is associated with dCPV, agrees with Grossman, Nir 2012
- F_1 and $e^{-\Gamma_S t'}$ are associated with the contributions of ϵ_K and $\phi_2^{M,\Gamma}$
- $\epsilon_R/\epsilon_I = 1$ up to a $\approx 5\%$ correction



Shown are $F_0(t)$, $F_1(t)$, and $\exp[-\Gamma_S t]$, plotted over a short time interval of relevance to LHCb (left), and a longer time interval of relevance to Belle-II (right)

- over the time scale for observed K^0's at LHCb, e.g. $t' \lesssim 0.5\tau_S$, F_1 is suppressed down to the few percent level, while $e^{-\Gamma_S t'} = O(1)$
  - ϵ_K effects in the CPVINT asymmetries can be neglected at LHCb
- over the Belle-II time scale, e.g. $t' \lesssim 10\tau_S$, the cancelation in F_1 subsides, and ϵ_K ultimately dominates the SM CPVINT asymmetries.
  - at Belle-II, ϵ_K is an important, but its a precisely known systematic effect

Current Status

- Superweak Approximation: in the past, sensitivity to ϕ_{12} of $O(100)$ mrad probed short-distance NP
  - was appropriate to neglect the effects of weak phases in subleading decay ampltiudes in indirect CPV observables. In this limit,

$$\phi_f^M = \phi_2^M = \phi_{12}, \quad \phi_f^\Gamma = 0\,, \quad \phi_{\lambda_f} = \phi_2$$

    e.g. $\Delta Y_F = -\eta_{CP}\, x_{12} \sin\phi_2^M$

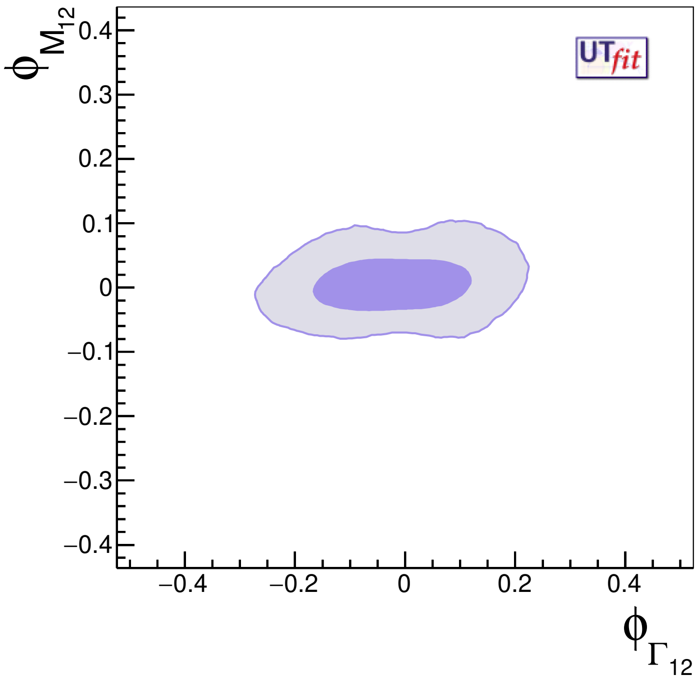
  - the superweak global fit is highly constrained, since there is only one CPV phase controlling all indirect CPV phenomena
  - currently, HFLAV obtains

$$\phi_2^M = -0.004 \pm 0.016\ (1\sigma)$$

  - comparison with the SM estimate, $\phi_2^M = O(0.2\%)$, implies that there is an order of magnitude window for NP at 95% CL

Approximate Universality global fit

- the approximate universality global fit is less constrained, given there are now two CPVINT phases, ϕ_2^M and ϕ_2^Γ



Preliminary: ϕ_2^Γ vs. ϕ_2^M at 68% CL, 95% CL

- error on $\phi_2^M \approx \pm 0.027$ [rad] is approximately a factor of three smaller than on ϕ_2^Γ
- largely due to the observable $\Delta Y_f = -A_\Gamma$, which only depends on ϕ_2^M

Conclusion

- the description of indirect CPV in terms of the absorptive and dispersive phases ϕ_f^M, ϕ_2^Γ is simpler, and far more physically transparent than ϕ_{λ_f}, $|q/p| - 1$
- ultimately, the goal is to measure the two corresponding theory phases ϕ_2^M, ϕ_2^Γ
- approximate universality: fortunately, there is minimal uncontrolled pollution from the decay amplitudes
  - CF/DCS decays: to excellent approximation, it is negligible in the CF/DCS decays in the SM, and in models with negligible new weak phases in these decays
  - SCS decays: there is uncontrolled final state dependent pollution, formally of $O(\epsilon)$ for individual modes, and of $O(\epsilon^2)$ for the sum $\phi_{K^+K^-}^{M,\Gamma} + \phi_{\pi^+\pi^-}^{M,\Gamma}$
    - in the future, at SM sensitivity, it will be instructive to compare the SCS and CF/DCS measurements
- ϕ_2^M and ϕ_2^Γ can, in principle, be measured on the lattice - this will become crucial for a precision test of the SM