Advanced Methods to Measure ϕ_2

2nd OPEN Belle II Physics Week

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USC UNIVERSIDADE DE SANTIAGO DE COMPOSTELA

Motivation

ϕ_2 is the least known parameter constraining the Unitarity Triangle

- Greatest potential for experimental impact in New Physics searches
- But ϕ_2 analyses arguably the most difficult
  - $b \to u$ transitions $\Rightarrow$ small signal, high background
  - π^0 almost always involved
  - Analysis of multiple channels normally required
  - High solution degeneracy, can only resolve with amplitude analysis
  - Isospin breaking $I = 1$ amplitudes distort the measurement
- 4 main systems to measure ϕ_2 (there are others)
  - $B \to \pi\pi$
  - $B^0 \to (\rho\pi)^0$
  - $B \to \rho\rho$
  - $B^0 \to a_1^{\pm}\pi^{\mp}$
- Cracking the 1° precision barrier will require innovation and cooperation

Outline

I. Developments

1. $B^0 \to (\rho\pi)^0$
   - — ρ^0-ω mixing
   - — The S-wave
2. $B \to \rho\rho$
   - — Resolving the final ambiguity
   - — Relativistic dynamics
   - — Finite ρ width
3. $B^0 \to a_1^{\pm}\pi^{\mp}$
   - — Ejecting 7 solutions
   - — Precision SU(3)
   - — Resonance pole parameters

II. Coordination

1. ϕ_2 combination
   - — Correlated systematics
   - — Interplay with LHCb

$B^0 \to (\rho\pi)^0$

Time-dependent, flavour-tagged, amplitude analysis of $B^0 \to \pi^+\pi^-\pi^0$

Dalitz Plot contains enough degrees of freedom to model strong penguin

Measure ϕ_2 without ambiguity in a single analysis

- A.E. Snyder and H.R. Quinn, "Measuring CP asymmetry in $B \to \rho\pi$ decays without ambiguities", Phys. Rev. **D 48** (1993) 2139 [SLAC-PUB-6056]

SU(2) isospin is a symmetry related to the exchange of u and d quarks

Broken by their mass difference and the electromagnetic interaction

In this analysis, isospin symmetry only relates the penguin amplitudes

- Triangular SU(2) approach relates tree amplitudes *eg.* $B \to \pi\pi$
- Isospin breaking effects expected to be smaller in $B^0 \to (\rho\pi)^0$

Isospin breaking from electroweak penguins, π^0-η-η' and ρ^0-ω mixing

Handle electroweak penguins and π^0-η-η' mixing in the ϕ_2 constraint

- M. Gronau and J. Zupan, "Isospin-breaking effects on α extracted in $B \to \pi\pi$, $\rho\rho$, $\rho\pi$", Phys. Rev. **D 71** (2005) 074017 [INSPIRE]

ρ^0-ω Mixing

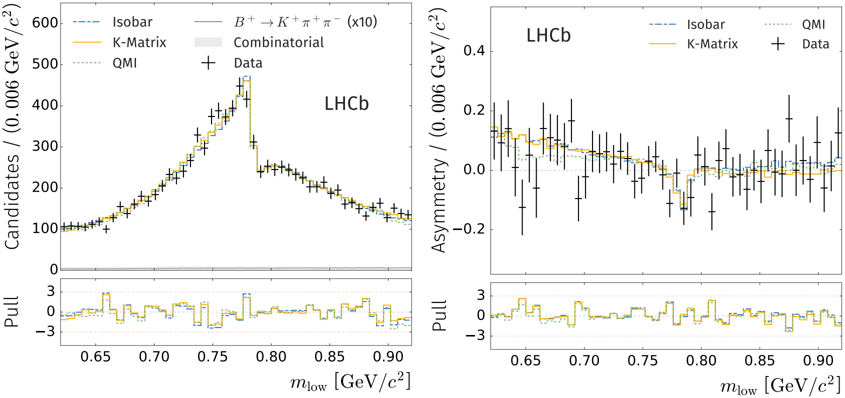
- ρ^0 and ω not exact eigenstates of isospin
- Physical ρ^0 contains small contribution from $I = 0$, *ie.* $\omega \to \pi^+\pi^-$
- Manifests as ρ^0-ω mixing
- To first order, simply add the ω resonance to the amplitude model
- But we can still do a little better
- Also account for electromagnetic mixing
  - P.E. Rensing, "Single electron detection for SLD CRID and multi-pion spectroscopy in K^-p interactions at $11\ \mathrm{GeV}/c$", SLAC-R-0421.

$$T_{\rho^0\text{-}\omega}(m) = c_{\rho^0} T_{\rho^0}(m) \left[\frac{1 + c_{\omega/\rho^0} \Delta T_\omega(m)}{1 - \Delta^2 T_{\rho^0}(m) T_\omega(m)} \right]$$

- $T_{\rho^0}(m)$: Gounaris-Sakurai for ρ^0
- $T_\omega(m)$: Breit-Wigner for ω
- c_i: Flavour-dependent complex free parameters of the model
- $\Delta \equiv \delta(m_{\rho^0} + m_\omega)$: where δ governs strength of electromagnetic mixing
  - $\delta = 0.00215 \pm 0.00035\ \mathrm{GeV}$
- m_i: Pole masses

ρ^0-ω Mixing in Action

Already being used in amplitude analysis of $B^+ \to \pi^+\pi^+\pi^-$

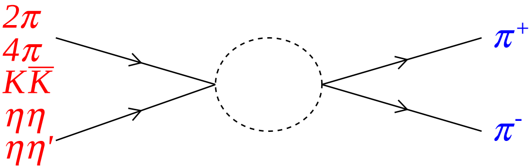


LHCb Collab., "Observation of several sources of CP violation in $B^+ \to \pi^+\pi^+\pi^-$ decays", LHCb-PAPER-2019-018.

LHCb Collab., "Amplitude analysis of the $B^+ \to \pi^+\pi^+\pi^-$ decay", LHCb-PAPER-2019-017.

The S-wave

- $L = 0$ component of $\pi^+\pi^-$ large compared to ρ^0
- Strong phase motion very difficult to describe
- Contains many broad, overlapping resonances: σ, $f_0(980)$, $f_0(1370)$, ...
  - Coherent sum of Breit-Wigners (Isobar) violates unitarity
  - With statistics available today, fits are now visibly bad
- Mass crosses many decay thresholds



- Long-distance elastic ($\pi^+\pi^- \to \pi^+\pi^-$) and inelastic rescattering present
- Reconstructed $B^0 \to \pi^+\pi^-\pi^0$ may not have been produced that way
  - Rescattering generates a phase
  - Further complicates dynamics

K-matrix

Experimentally-driven approaches to handle these effects

K-matrix

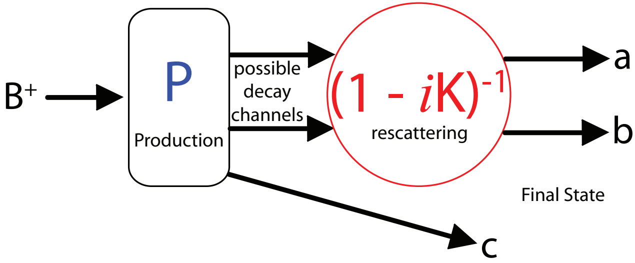
- Built on the premise of conserving 2-body unitarity

Originally developed for scattering experiments

K-matrix parameters obtained (fixed) from global fit to scattering data

Resonance poles understood within the context of particle rescattering

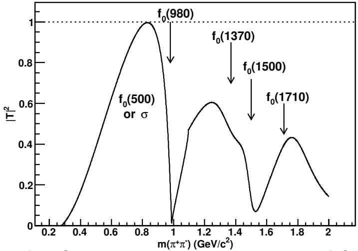
Modified for production environments



Replace initial state K-matrix with production vector of poles (free)

K-matrix

Describes S-wave as a whole



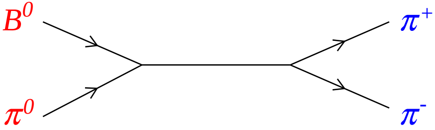
Issues

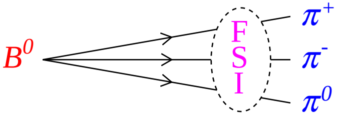
Interpretation of individual resonances not possible

No information above charm threshold

- $D\bar{D} \to \pi^+\pi^-$ rescattering possible

Scattering and production environments are not the same





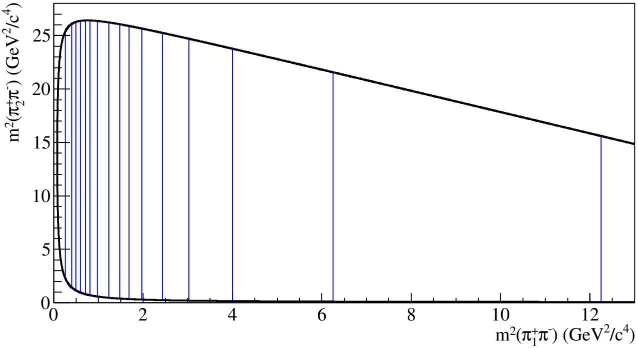
Final state interactions (FSI) in production environments

Quasi-model-independent Approach

Experimentally-driven approaches to handle these effects

Quasi-model-independent (QMI)

- Bin phase space and free magnitude and phase



FSI dynamics can vary along vertical helicity direction

At least absorbs average of FSI effects

Quantify FSI through comparison with scattering amplitudes

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Quasi-model-independent Approach

Issues

Choose wisely between constants in each bin and spline interpolation

- Scattering theory requires a cusp in the amplitude at each channel opening
- Sharp changes on scales less than a bin width possible, constants better
- Splines good for replacing single component *eg*. confirming resonances

The QMI is not a stand-alone method

- Whether splines or constants in each bin, constructs have no physical origin
- Number of bins and binning scheme is ultimately chosen *ad hoc*
- Bins highly localised, no guarantee there is anything to interfere with

Need to assess whether an analytic S-wave can be reproduced

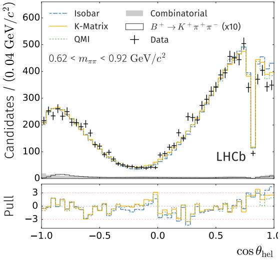
Generate pseudoexperiments with another approach (Isobar or K-matrix)

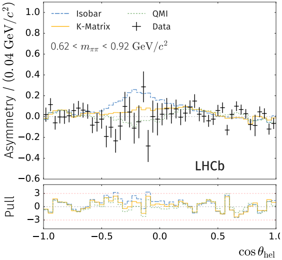
- Fit with QMI model
- Average deviation is the QMI inherent bias
- Should be a dominant systematic at amplitude level

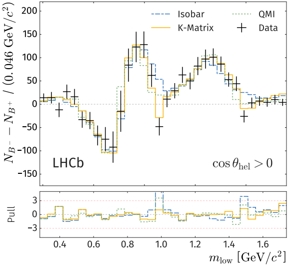
Similar to, but not quite the same as a fit bias (should also be estimated)

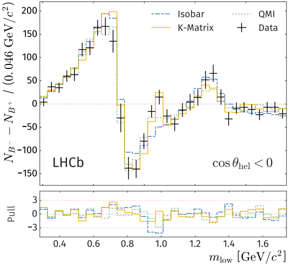
QMI Approach in Action

Amplitude analysis of $B^+ \to \pi^+\pi^+\pi^-$









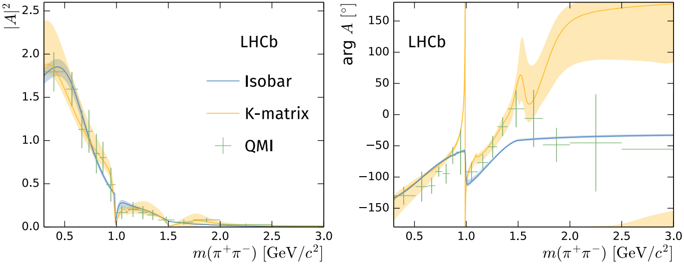
QMI picks up FSI

QMI binning too wide at $f_0(980)$

The S-wave

The QMI has no physical meaning and so cannot be the answer

But it points the way



Difference between the QMI and Isobar exposes gaps in our knowledge

Watch out for new dispersion relation approaches from theory

- Respects unitarity and analyticity
- New form factors to replace sum of Breit-Wigners in the Isobar model

Isobar approach has physical meaning

> The Isobar approach was the past, but it must also be our future.

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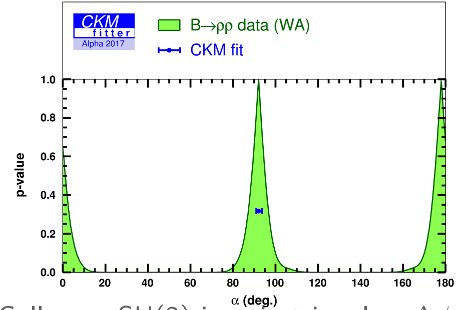
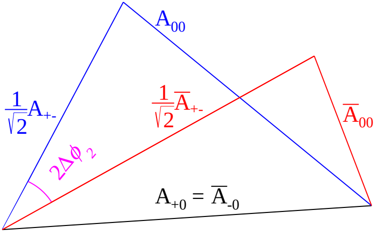
$B \to \rho\rho$

- $B \to \rho\rho$ currently gives the best constraint on ϕ_2
- Small penguin contribution leads to small $B^0 \to \rho^0\rho^0$ branching fraction





- Collapses SU(2) isospin triangles, $\Delta\phi_2 \sim 0$, leaving two solution for ϕ_2
- However, this poses an experimental challenge in $B^0 \to \rho^0\rho^0$
- Most dangerous physics background comes from $B^0 \to a_1^{\pm}\pi^{\mp}$
  - $\mathcal{B}(B^0 \to \rho^0\rho^0) \sim \mathcal{O}(10^{-6})$ interferes with $\mathcal{B}(B^0 \to a_1^{\pm}\pi^{\mp})\mathcal{B}(a_1^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) \sim \mathcal{O}(10^{-5})$

$B^0 \to \rho^0\rho^0$

$B^0 \to a_1^{\pm}\pi^{\mp}$ cannot be removed from the $B^0 \to \rho^0\rho^0$ analysis region

Time-dependent flavour-tagged amplitude analysis necessary to disentangle

$$\Gamma(\Delta t, q) \propto e^{-|\Delta t|/\tau_d}\Big[(|A|^2 + |\bar{A}|^2) - q(|A|^2 - |\bar{A}|^2)\cos\Delta m_d\Delta t + 2q\Im(\bar{A}A^*)\sin\Delta m_d\Delta t\Big]$$

A: phase space-dependent amplitude model for $B^0 \to \pi^+\pi^-\pi^+\pi^-$

Interfering contributions, except $B^0 \to a_1^{\pm}\pi^{\mp}$, already known to be small

All would be flavour-non-specific and sensitive to ϕ_2

Consider the limit of no penguin contribution

- Isobar approach: $A = \sum_i A_i(\Phi_4)$, $\bar{A} = \sum_i \lambda^i_{CP} A_i(\bar{\Phi}_4) = \sum_i \lambda^i_{CP} A_i(\Phi_4)$
- Φ_4 is 4-body phase space position
- CP-violation parameter would factorise, $\lambda^i_{CP} \to \lambda_{CP} = e^{i2\phi_2}$
- $\Im(\bar{A}A^*) = \Im(\lambda_{CP}AA^*) = \Im\lambda_{CP}|A|^2 = \sin 2\phi_2$

2 solutions remain, despite amplitude analysis

- Incidentally, this is why $B^0 \to K^0_S K^+ K^-$ doesn't work yet

$B^0 \to \pi^+\pi^-\pi^+\pi^-$

But we know $B^0 \to a_1^{\pm}\pi^{\mp}$ has a penguin contribution

3σ evidence for CP violation found at Belle

Belle Collab., "Measurement of Branching Fraction and First Evidence of CP Violation in $B^0 \to a_1^{\pm}(1260)\pi^{\mp}$ Decays", Phys.Rev. **D 86** (2012) 092012, INSPIRE.

CP violation also expected from theory

H.-Y. Cheng and K.-C Yang, "Hadronic charmless B decays $B \to AP$, Phys. Rev. **D 76** (2007) 114020, INSPIRE.

Experiment and theory happen to be in excellent agreement

Widen analysis region to include decent $B^0 \to a_1^{\pm}\pi^{\mp}$ contribution

Ensure $a_1^{\pm}$ hadronic form factor can be sufficiently understood

CP violation parameter can no longer factorise out of the Isobar sum

Effective ϕ_2^{00} could be determined without ambiguity

Extended $B \to \rho\rho$ Isospin Analysis

Implications for the SU(2) isospin triangle analysis

$$A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00}, \qquad \bar{A}^{+0} = \frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00}$$

Parameterise with free parameters as usual

J. Charles, O. Deschamps, S. Descotes-Genon and V. Niess, "Isospin analysis of charmless B-meson decays", Eur. Phys. J. **C 77** (2017) 574, INSPIRE.

Build physics observables from amplitudes

$$\frac{1}{\tau_B^{i+j}} \mathcal{B}^{ij} = \frac{|\bar{A}^{ij}|^2 + |A^{ij}|^2}{2}, \; \mathcal{A}_{CP}^{ij} = \frac{|\bar{A}^{ij}|^2 - |A^{ij}|^2}{|\bar{A}^{ij}|^2 + |A^{ij}|^2}, \; \mathcal{S}_{CP}^{ij} = \frac{2\Im(\bar{A}^{ij} A^{ij*})}{|\bar{A}^{ij}|^2 + |A^{ij}|^2}$$

Replace $B^0 \to \rho^0 \rho^0$ parameters

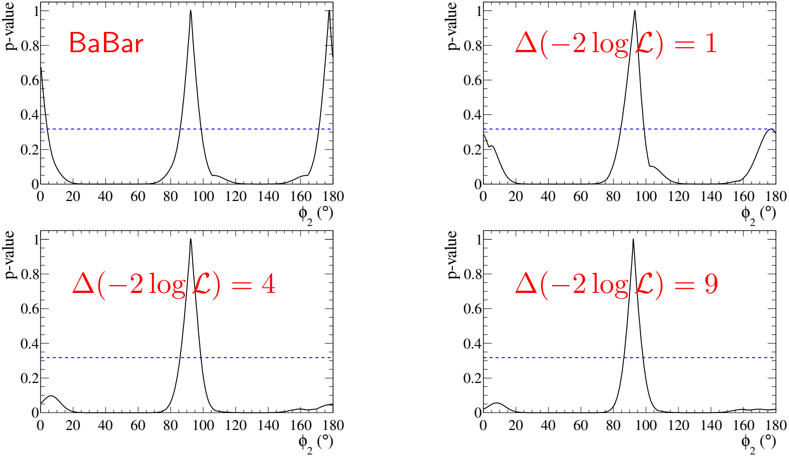
$$\mathcal{A}_{CP}^{00} \to |\lambda_{CP}^{00}| = \left| \frac{\bar{A}^{00}}{A^{00}} \right|, \qquad \mathcal{S}_{CP}^{00} \to \phi_2^{00} = \frac{\arg(\bar{A}^{00} A^{00*})}{2}$$

Extended $B \to \rho\rho$ Isospin Analysis Performance

Begin with BaBar input, Phys. Rev. Lett. **102** (2009) 141802, INSPIRE.

Assume 2 solutions for ϕ_2^{00} resolved with increasing significance

Insert ϕ_2^{00} likelihood profile into fit χ^2



Penguin amplitudes: The cause of, and solution to, ϕ_2's problems

Prospects for Resolving ϕ_2^{00} Solutions

Method relies on penguin from $B^0 \to a_1^{\pm}\pi^{\mp}$, not the dominant tree

Assume analysis region captures all $B^0 \to a_1^{\pm}\pi^{\mp}$

Estimate amount of data needed for penguin to play significant role

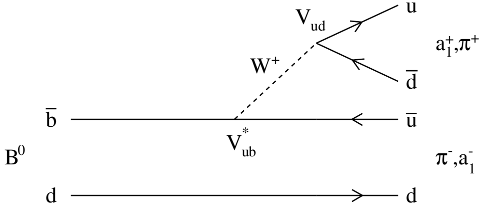
Generate pseudo-experiments based on current experimental results

Critical variable is $\Delta(-2\log\mathcal{L})$ between ϕ_2^{00} solutions

Observation of channel required for consideration in the ensemble test

- 5 contributions

$B^0 \to a_1^{\pm}\pi^{\mp}$

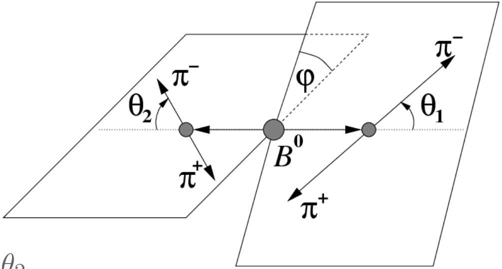


- In colour-favoured tree, produce a_1^+ from W^+, a_1^- from spectator
- Orbital S-wave between $\rho^0\pi^{\pm}$ from $a_1^{\pm}$

Prospects for Resolving ϕ_2^{00} Solutions

$B^0 \to \rho^0 \rho^0$

- 3 polarisations
- Transversity basis used up to this point



$A_0 \propto \cos\theta_1 \cos\theta_2$

$A_\perp \propto \sin\theta_1 \sin\theta_2 \sin\phi$

$A_\parallel \propto \sin\theta_1 \sin\theta_2 \cos\phi$

Rotationally invariant, eigenstates of CP

But there is a problem

Assumes vector mesons $\approx$ at rest with zero width, *ie.* not covariant

Relativistic Dynamics

- Need the benefits of transversity formalism while enforcing relativity
  - Covariant spin tensor formalism
- Based on Rarita-Schwinger conditions
  - Polarisation tensor orthogonal to momentum, symmetric and traceless
- Integral spin projections represented by tensors of the same rank
- Rarita-Schwinger conditions reduce independent elements to $2S+1$
- Spin-1, boost to arbitrary frame for covariance

$$\epsilon^{\mu}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow \epsilon^{\mu}(p,0) = \frac{1}{M}\begin{pmatrix} p_z \\ p_z p_x/(E+M) \\ p_z p_y/(E+M) \\ M + p_z^2/(E+M) \end{pmatrix}$$

$$\epsilon^{\mu}(\pm 1) = \frac{\mp 1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} \rightarrow \epsilon^{\mu}(p,\pm 1) = \frac{\mp 1}{M\sqrt{2}}\begin{pmatrix} p_x \pm i p_y \\ M + p_x(p_x \pm i p_y)/(E+M) \\ \pm iM + p_y(p_x \pm i p_y)/(E+M) \\ p_z(p_x \pm i p_y)/(E+M) \end{pmatrix}$$

- Couple with Clebsch-Gordon to generate higher spin tensors
  - Rarita-Schwinger conditions automatically satisfied

Relativistic Dynamics

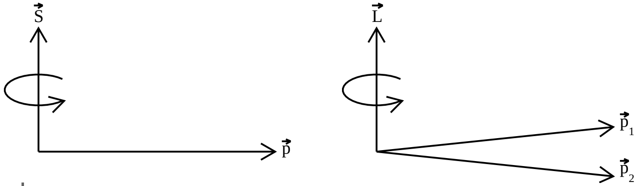
Sum over unobservable polarisation indices, spin-1 projection operator

$$P_1^{\mu\nu}(p) = \sum_{s_z} \epsilon^{\mu}(p, s_z)\epsilon^{*\nu}(p, s_z) = -g^{\mu\nu} + \frac{p^{\mu}p^{\nu}}{M^2}$$

Projects any 4-vector onto spin subspace spanned by polarisation tensors

Consider decay process $R \to m_1 m_2$

- Total momentum $p_R \equiv p_1 + p_2$, Relative momentum $q_R \equiv p_1 - p_2$



Orbital angular momentum tensor

$$L^{\mu_1 \dots \mu_L}(p_R, q_R) = (-1)^L P_L^{\mu_1 \dots \mu_L \, \nu_1 \dots \nu_L}(p_R) q_{R\nu_1} \dots q_{R\nu_L}$$

Spin and orbital angular momentum described by the projection operator

Relativistic Dynamics

Can describe $B^0 \to \rho^0 \rho^0$ spin in terms of orbital angular momentum

$A_S \propto L_a(q_1) L^a(q_2)$
$A_P \propto \epsilon_{abcd} p_B^d L^c(q_B) L^b(q_1) L^a(q_2)$
$A_D \propto L_{ab}(q_B) L^b(q_1) L^a(q_2)$

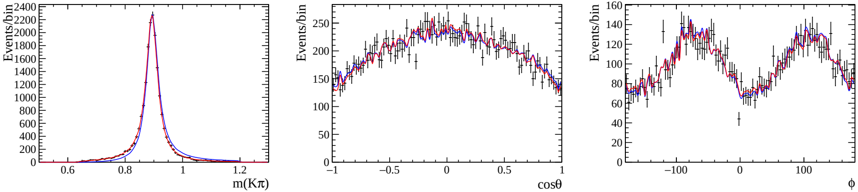
Solves another problem with transversity basis

Production Blatt-Weisskopf barrier factor now has meaning

The effects of ignoring relativistic invariance are quite large

eg. Generate $B_s^0 \to K^{*0} \bar{K}^{*0}$, transversity fit (red), covariant fit (blue)

LHCb Collab., JHEP **03** (2018) 140, INSPIRE.

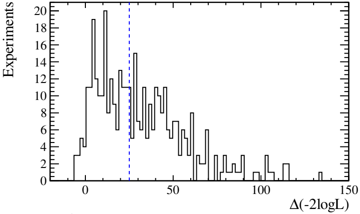


The angular distribution of transversity basis is not wrong

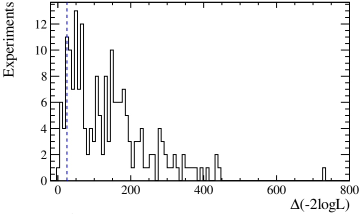
Relativistic invariance induces additional dynamics

Prospects for Resolving ϕ_2^{00} Solutions

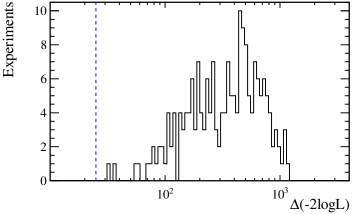
$2.5\ \mathrm{ab}^{-1}$: Effective Yield ~ 1500



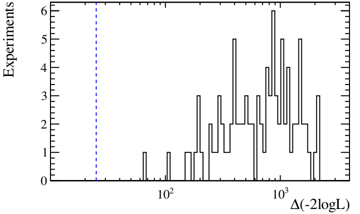
$10\ \mathrm{ab}^{-1}$: Effective Yield ~ 6000



$25\ \mathrm{ab}^{-1}$: Effective Yield ~ 15000



$50\ \mathrm{ab}^{-1}$: Effective Yield ~ 30000



- Spread includes hadronic systematic uncertainty
- Method guaranteed to work at Belle II, while $B^0 \to (\rho\pi)^0$ might not
  - J. Dalseno, "Resolving the ϕ_2 (α) ambiguity in $B \to \rho\rho$", JHEP **11** (2018) 193, INSPIRE

Finite ρ Width Effects

- Triangular SU(2) isospin symmetry relates tree amplitudes
- Isospin-breaking amplitudes larger than in $B^0 \to (\rho\pi)^0$
- Not affected by π^0-η-η' mixing effects
- Electroweak penguin shift calculable
- ρ^0-ω mixing can be handled in amplitude analysis
- Additional isospin-breaking effect, unique to $B \to \rho\rho$
  - Isospin analysis assumes equal ρ invariant masses
  - Amplitude can have antisymmetric component under exchange of masses
  - Still does not affect $B^0 \to \rho^0\rho^0$ as $I = 1$ always forbidden
  - A.F. Falk, Z. Ligeti, Y. Nir, H. Quinn, "Comment on extracting α from $B \to \rho\rho$, Phys.Rev. **D 69** (2004) 011502, INSPIRE
- Structure known, add to amplitude model, $c_{1,2}$ complex free parameters

$$|A_{I=1}|^2 \sim |c_1 \frac{\Delta m}{m_\rho} A_{\rho\rho}|^2, \qquad |A_{I=2}|^2 \sim |c_2 \left[\frac{\Delta m}{m_\rho}\right]^2 A_{\rho\rho}|^2$$

- Alternative suggestion
  - As narrow a ρ analysis region as the statistical error will allow
  - M. Gronau and J.L. Rosner, "Controlling ρ width effects for a precise value of α in $B \to \rho\rho$", Phys. Lett. **B 766** (2017) 345, INSPIRE

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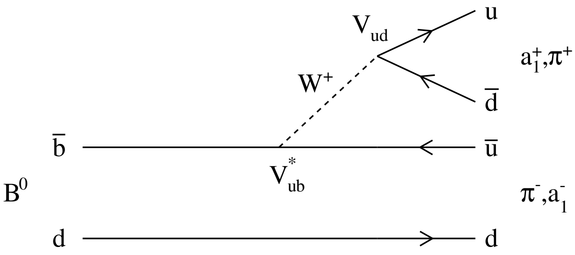
$B^0 \to a_1^\pm \pi^\mp$

8-fold degeneracy in current time-dependent flavour-tagged analysis



Sensitive to algebraic average of effective ϕ_2^+ and ϕ_2^- (4 solutions)

SU(2) solutions not practical, amplitude analysis of $B^0 \to (a_1\pi)^0$

Measure $B^{+0} \to K_{1A}^{0+}\pi^{+-}$ and $B^{+0} \to K^{0+}a_1^{+-}$ branching fractions

- K_{1A} is the ${}^3P_1$ partner of the a_1

$|\Delta\phi_2|$ from SU(3) analysis ($\times 2$ solutions)

- M. Gronau and J. Zupan, "Weak phase α from $B^0 \to a_1^\pm(1260)\pi^\mp$", Phys. Rev. **D 73** (2006) 057502, INSPIRE

$B^0 \to a_1^\pm \pi^\mp$

$B^0 \to a_1^\pm \pi^\mp$ was included in the $B^0 \to \rho^0 \rho^0$ analysis

$B^0 \to a_1^+ \pi^-$ and $a_1^- \pi^+$ distinguished in the amplitude analysis

Search for degeneracy by switching all λ_{CP}^i to second solution

Effective ϕ_2^+ and ϕ_2^- separately resolved with same significance as ϕ_2^{00}

Check impact on SU(3) analysis with pure penguin (B^+) modes

$$a_1^+\pi^- : A_d^+ = T^+ e^{+i\phi_3} + P^+,\ \ \bar{A}_d^+ = T^+ e^{-i\phi_3} + P^+,\ \ \lambda_{CP}^+ = \frac{\bar{A}_d^+}{A_d^+} e^{i(2\pi - 2\phi_1)}$$

$$a_1^-\pi^+ : A_d^+ = T^- e^{+i\phi_3} + P^-,\ \ \bar{A}_d^- = T^- e^{-i\phi_3} + P^-,\ \ \lambda_{CP}^- = \frac{\bar{A}_d^-}{A_d^-} e^{i(2\pi - 2\phi_1)}$$

Parameterise $\phi_3 = \pi - \phi_1 - \phi_2$

$$B^+ \to K_{1A}^0 \pi^+ : A_s^+ = -\frac{1}{\bar{\lambda}} \frac{f_{K_1}}{f_{a_1}} P^+$$

$$B^+ \to K^0 a_1^+ : A_s^- = -\frac{1}{\bar{\lambda}} \frac{f_K}{f_\pi} P^-$$

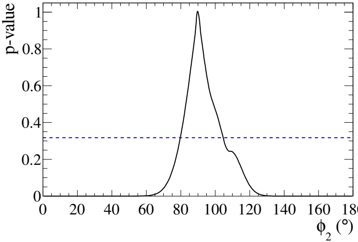
Factorisable SU(3) breaking, $\bar{\lambda} = |V_{us}/V_{ud}|$, f_i: decay constants

8 free parameters $T^\pm$ (tree), $P^\pm$ (penguin) and ϕ_2, fix $\arg(T^+) = 0$

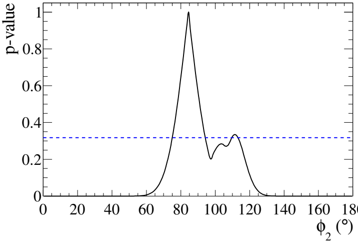
$B^0 \to a_1^{\pm}\pi^{\mp}$

- 9 physical observables
  - 4 branching fractions, $2\mathcal{B}_i/\tau_B = |\bar{A}_i|^2 + |A_i|^2$
  - 4 CP-violating parameters, $\lambda_{CP}^{\pm}$
  - 1 strong phase difference, $\arg(A_d^-/A_d^+)$
- Consider if Belle could resolve $\phi_2^{\pm}$
  - Set to theoretical values with scaled Belle uncertainties
- Take BaBar branching fractions for SU(3)-related B^+ channels
  - $B \to K_{1A}\pi$: BaBar Collab. Phys. Rev. **D 81** (2010) 052009, INSPIRE
  - $B \to a_1 K$: BaBar Collab. Phys. Rev. Lett. **100** (2008) 051803, INSPIRE

Most probable $\mathcal{B}(B^+ \to K_{1A}^0\pi^+)$



Mean $\mathcal{B}(B^+ \to K_{1A}^0\pi^+)$



Resolving ϕ_2 in $B^0 \to a_1^{\pm}\pi^{\mp}$

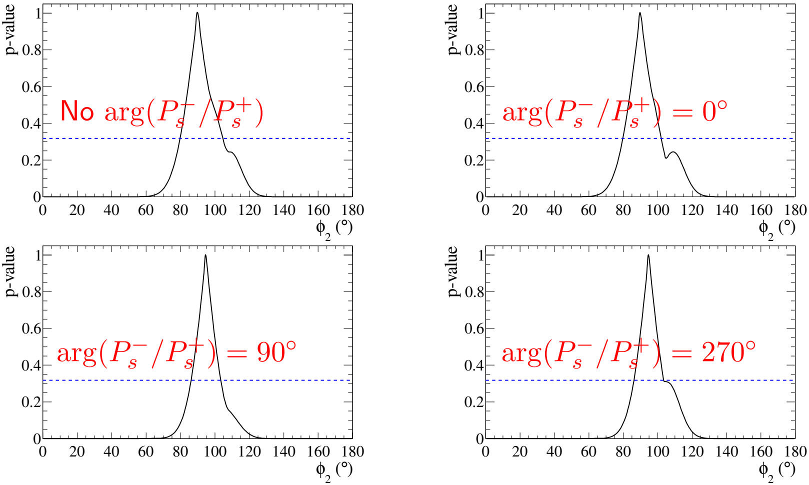
- Huge improvement over 8 distinct solutions with 1st generation errors
- Still some hint of multiple solutions
- $B^0 \to a_1^{\pm}\pi^{\mp}$ observables came from amplitude analysis
  - $A_d^{\pm}$, $\bar{A}_d^{\pm}$ amplitudes fully constrained
- Take another look at the $b \to s$ penguin system

$$K_{1A}^0\pi^+ : A_s^+ = -\frac{1}{\bar{\lambda}}\frac{f_{K_1}}{f_{a_1}}P^+, \qquad K^0 a_1^+ : A_s^- = -\frac{1}{\bar{\lambda}}\frac{f_K}{f_\pi}P^-$$

- Branching fractions essentially give the magnitude of $P^{\pm}$
- $B^+ \to K_{1A}^0\pi^+$ and $B^+ \to K^0 a_1^+$ share the same final state
- Amplitude analysis of $B^+ \to K_S^0\pi^+\pi^-\pi^+$ gives the missing information
  - Strong phase difference between $B^+ \to K_{1A}^0\pi^+$ and $K^0 a_1^+$ gives $\arg(A_s^-/A_s^+)$
- 8 free parameters for 10 physical observables
  - Overconstrained

Resolving ϕ_2 in $B^0 \to a_1^{\pm}\pi^{\mp}$

Check impact of ϕ_2 constraint with most probable $\mathcal{B}(B^+ \to K_{1A}^0\pi^+)$



Method can work even with current uncertainties

SU(3) breaking in $B^0 \to a_1^\pm \pi^\mp$

Factorisable SU(3)-breaking parameters already accounted for

Non-factorisable SU(3) breaking an additional source of uncertainty

- Other diagrams, theoretical uncertainties, other unknown effects

Additional real factors, $F^\pm_{\mathrm{SU(3)}}$

$$K^0_{1A}\pi^+ : A_s^+ = -\frac{F^+_{\mathrm{SU(3)}}}{\bar\lambda}\frac{f_{K_1}}{f_{a_1}}P^+, \qquad K^0 a_1^+ : A_s^- = -\frac{F^-_{\mathrm{SU(3)}}}{\bar\lambda}\frac{f_K}{f_\pi}P^-$$

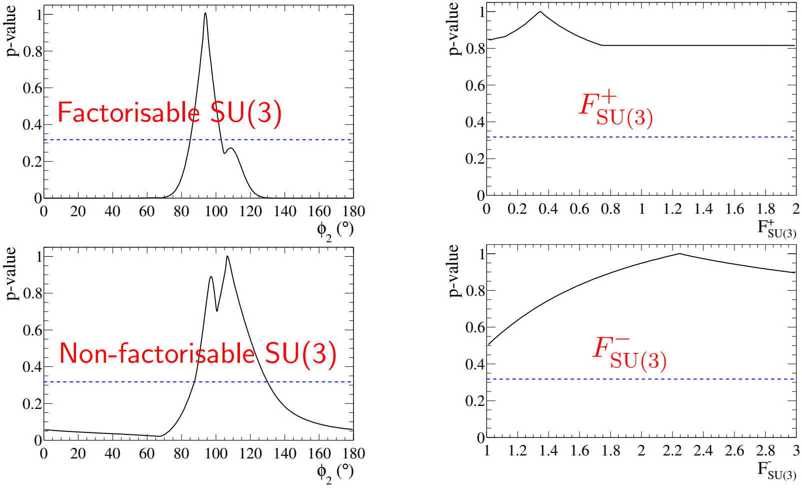
Unity in the limit of no non-factorisable SU(3)-breaking

$8 \to 10$ free parameters for 10 physics observables

- Might be able to get some sensitivity to SU(3)-breaking parameters
- If so, irreducible systematic uncertainty absorbed into statistical error

SU(3) breaking in $B^0 \to a_1^{\pm}\pi^{\mp}$

Best case scenario at $\arg(P_s^-/P_s^+) = 45°$



Emerging sensitivity to SU(3)-breaking, costs ϕ_2 precision

Prospects for Measuring $\arg(P_s^-/P_s^+)$

- Requires combination of 2 analyses into a single amplitude analysis
  - $B^+ \to K_{1A}^0 \pi^+$ and $K^0 a_1^+$
- In $B^0 \to \pi^+\pi^-\pi^+\pi^-$, a_1^+ and a_1^- do not overlap
- Similarly, neither would K_{1A}^0 and a_1^+ in $B^+ \to K_S^0 \pi^+\pi^-\pi^+$
- Need to involve another intermediate state
- In $B^0 \to \pi^+\pi^-\pi^+\pi^-$, this is achieved through $B^0 \to \rho^0\rho^0$
- So in $B^+ \to K_S^0 \pi^+\pi^-\pi^+$, this has to be $B^+ \to K^{*+}\rho^0$
  - Bundle in the f_L measurement, now combination of 3 analyses
- Assess feasibility of measuring $\arg(P_s^-/P_s^+)$
- Ensemble test based on current experimental results
- Determine width of $\arg(P_s^-/P_s^+)$ distribution
  - Expected uncertainty including amplitude model systematic error
- Not an asymmetry measurement where systematics can cancel
  - Controlling hadronic uncertainties critical to the method

Energy-dependent Widths

Most general Breit-Wigner propagator

$$T(s) = \frac{1}{M^2(s) - s - im_0\Gamma(s)},$$

Energy-dependent mass, $M^2(s)$

- Approximated by the pole mass
- Also a problem, can solve with dispersive analysis
- Solved for the ρ, Gounaris-Sakurai
- Very active field of research

Energy-dependent width, $\Gamma(s)$

Typically approximated by 2-body breakup momentum

- Generally fine for dominant vector resonances *eg.* K^*, ϕ
- Problematic elsewhere, *eg.* $f_2(1270) \to \pi\pi$, and especially for axial vectors

Need the best possible lineshape to study hadronic parameters

Energy-dependent Widths

$\Gamma(s)$ should represent the total width of the decay

ie. Sum over partial widths of all possible decays

Can further breakdown into couplings and decay phase space volumes

$$\Gamma(s) = \sum_i \Gamma_i(s) = \Gamma_0 \sum_i g_i \rho_i(s),$$

Γ_0: resonance width at the pole, $\Gamma(m_0^2)$

$\rho_i(s)$: energy-dependent phase space volume for decay channel i

For 2-body decay, this is the familiar breakup momentum

For n-body decays, this is the integral of its Dalitz Plot

eg. Spin-1 decays

$$\rho_n(s) = \frac{1}{2\sqrt{s}} \int \sum_{\lambda=0,\pm1} |A_\lambda(\Phi_n)|^2 d\Phi_n(s),$$

Take incoherent sum over unobservable polarisation indices

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Energy-dependent Widths

Reverse engineer partial width couplings from PDG and/or own analysis

Minimise a χ^2 based on the branching fractions

$$\chi^2 = \sum_i \left[\frac{\mathcal{B}_i^{\text{exp}} - \mathcal{B}_i^{\text{pred}}(g_i)}{\Delta \mathcal{B}_i^{\text{exp}}} \right]^2,$$

Branching fraction of partial width calulated on the fly

$$\mathcal{B}_i^{\text{pred}} \propto \int_{s_{\min}}^{\infty} \frac{m_0 \Gamma_i(s)}{|m_0^2 - s - i m_0 \Gamma(s)|^2} ds = \int_{s_{\min}}^{\infty} \frac{m_0 g_i \rho_i(s)}{|m_0^2 - s - i m_0 \Gamma(s)|^2} ds.$$

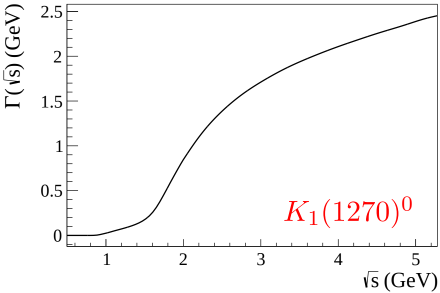
Constraints can be placed

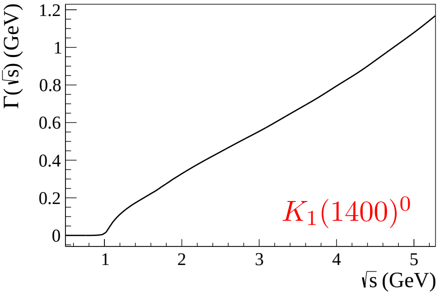
- $\sum_i \mathcal{B}_i$ scaled to 1
- Convention $\Gamma_0 = \Gamma(m_0^2) \Rightarrow \sum_i g_i = 1$ reduces free parameters by 1

Energy-dependent Widths

Decay Channel	$K_1(1270)^0$ Couplings	$K_1(1400)^0$ Couplings
$K\rho$	0.473 (constrained)	0.029 (constrained)
$K\omega$	0.124 ± 0.022	0.010 ± 0.010
$K^*(892)\pi$	0.184 ± 0.047	0.956 ± 0.028
$Kf_0(1370)$	0.016 ± 0.010	0.005 ± 0.005
$K_0^*(1430)\pi$	0.203 ± 0.030	—
χ^2	1.4×10^{-4}	1.2×10^{-4}

Total energy-dependent width, $\Gamma(s)$





Treatment of $K_1(1270)$-$K_1(1400)$ Mixing

Need to measure phase difference between $B^+ \to K^0_{1A}\pi^+$ and $K^0 a_1^+$

However, K_{1A} is not a mass eigenstate like the a_1

Mixed with K_{1B} due to strange and non-strange quark mass difference

$$A(B^+ \to K^0_{1A}\pi^+) = \sin\theta_{K_1} A(B^+ \to K_1(1270)^0\pi^+) + \cos\theta_{K_1} A(B^+ \to K_1(1400)^0\pi^+)$$

$$A(B^+ \to K^0_{1B}\pi^+) = \cos\theta_{K_1} A(B^+ \to K_1(1270)^0\pi^+) - \sin\theta_{K_1} A(B^+ \to K_1(1400)^0\pi^+)$$

Parametrise production amplitude in terms of mass eigenstates

Not sensitive to the mixing angle θ_{K_1}

- Poorly known at this time
- Thought to be around 34°
- Can be measured with high-statistics $B \to J/\psi K_1$ analysis
- $\mathcal{B}(B^+ \to K^0 a_1^+)$ and $\arg(P_s^-/P_s^+)$ insensitive to θ_{K_1}
- $\mathcal{B}(B^+ \to K^0_{1A}\pi^+)$ highly sensitive

No θ_{K_1} uncertainty profile, difficult to assign systematic

- Ignored in previous analysis
- BaBar Collab., "Measurement of branching fractions of B decays to $K_1(1270)\pi$ and $K_1(1400)\pi$ and determination of the CKM angle α from $B^0 \to a_1(1260)^\pm\pi^\mp$", Phys. Rev. **D 81** (2010) 052009 INSPIRE.

Prospects for Measuring P_s^-/P_s^+

Estimate total $B^+ \to K_S^0\pi^+\pi^-\pi^+$ signal yield at Belle II

Measuring relative to $B^+ \to K_{1A}^0\pi^+$

Estimate uncertainties from width of their distribution in ensemble test

Projected Yield	$\delta\|a_{B^+\to K^0 a_1(1260)^+}\|$ (%)	$\delta \arg(a_{B^+\to K^0 a_1(1260)^+})$ (°)
5000 ($5\ \mathrm{ab}^{-1}$)	17.7 (7.2) [12.3]	17.3 (13.3) [13.1]
10000 ($10\ \mathrm{ab}^{-1}$)	15.5 (4.6) [8.6]	14.4 (10.5) [9.4]
50000 ($50\ \mathrm{ab}^{-1}$)	14.1 (2.7) [4.2]	12.0 (7.6) [7.3]

Blue: hadronic parameters fixed in the fit

Red: statistical component of the total error

Total error does not scale well with increased data sample sizes

Due to limited knowledge of K_1 pole parameters

- *eg*. 20% uncertainty on $K_1(1270)$ width
- Must be determined in the fit

Grey: Total error with free axial vector pole parameters

- More sustainable analysis
- J. Dalseno. "Resolving the ϕ_2 (α) ambiguity in $B^0 \to a_1^\pm\pi^\mp$", JHEP **10** (2019) 191, INSPIRE

Outline

I. Developments

1. $B^0 \to (\rho\pi)^0$
   — ρ^0-ω mixing
   — The S-wave
2. $B \to \rho\rho$
   — Resolving the final ambiguity
   — Relativistic dynamics
   — Finite ρ width
3. $B^0 \to a_1^{\pm}\pi^{\mp}$
   — Ejecting 7 solutions
   — Precision SU(3)
   — Resonance pole parameters

II. Coordination

1. ϕ_2 combination
   — Correlated systematics
   — Interplay with LHCb

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ϕ_2 Average

- ϕ_2 doesn't come from a single analysis except in $B^0 \to (\rho\pi)^0$
- Combination of measurements generally needed to give ϕ_2
- Reliable ϕ_2 constraint should consider correlations between input
- Statistical correlations are straightforward
  - Comes directly from the fit
- Systematic correlations will require some degree of coordination
- Some are easy
  - *eg*. Number of $B\bar{B}$ events in branching fraction calculation
  - All branching fractions will have a correlation of $+1$ in this category
- Some might be not so trivial
  - *eg*. Particle identification (PID) efficiency correction
  - Depends on momentum spectrum
  - Upward fluctuation in the $B^+ \to \pi^+\pi^0$ branching fraction not necessarily the same as in $B^+ \to (\pi^+\pi^0)(\pi^+\pi^-)$
  - Tag-side interference sounds like a nightmare
- Some can be done externally
  - *eg*. Amplitude model
  - Fluctuation of ρ pole parameters propagated to each parameter

Input from LHCb

- May also want to consider cooperation with LHCb
- LHCb-only measurments of ϕ_2 not expected with Run 3
- Partial input only unless $B^0 \to a_1^{\pm}\pi^{\mp}$ method turns out to be viable
- Estimate their capabilities based on penalty terms relative to Belle II
  - π^0, flavour-tagging an order of magnitude less efficient
- 1 penalty term $\Rightarrow$ competitive in Run 3
- 2 penalty terms $\Rightarrow$ competitive in Run 6

Channel	Run 3 (2025)	Run 6 (2038)
$B^+ \to \pi^+\pi^0$	$\mathcal{B}$	
$B^0 \to \pi^+\pi^-$	$\mathcal{B}$, $\mathcal{A}_{CP}$, $\mathcal{S}_{CP}$	
$B^0 \to \pi^0\pi^0$	-	-
$B^0 \to (\rho\pi)^0$	$\|A\|^2 + \|\bar{A}\|^2$	ϕ_2
$B^+ \to \rho^+\rho^0$	$\mathcal{B}$, $\|A\|^2 + \|\bar{A}\|^2$	
$B^0 \to \rho^+\rho^-$	-	$\mathcal{B}$, $\|A\|^2 + \|\bar{A}\|^2$?
$B^0 \to \rho^0\rho^0$	$\mathcal{B}$, $\|A\|^2 + \|\bar{A}\|^2$, λ_{CP}^{00}	
$B^0 \to a_1^{\pm}\pi^{\mp}$	ϕ_2	

Summary

- Not a comprehensive list of things to consider/improve
  - *eg*. Didn't touch on π^0-η-η' mixing
- Amplitude analysis is very powerful
  - Degrees of freedom to take care of isospin-breaking effects
  - ρ^0-ω mixing and finite ρ width in $B \to \rho\rho$
- Active field of research, need to stay up to date with latest developments
  - Unitarity, relativity, analyticity, crossing symmetry
  - Model uncertainty can be controlled
- Enhanced methods to measure ϕ_2 without ambiguity identified
  - In $B \to \rho\rho$ by allowing $B^0 \to a_1^{\pm}\pi^{\mp}$ in $B^0 \to \pi^+\pi^-\pi^+\pi^-$ within SU(2)
  - In $B^0 \to a_1^{\pm}\pi^{\mp}$ by involving $B^+ \to K_S^0\pi^+\pi^-\pi^+$ within SU(3)
  - Non-factorisable SU(3) breaking parameters may also be constrained
- Don't worry too much about optimising analysis for the statistical error
  - Plan analyses back-to-front in Belle II instead
  - Spend time on new methods to reduce/eliminate dominant systematics
- Consider analysis within the wider context of the ϕ_2 average
  - Account for correlations between systematics for reliable constraint
  - Collaboration with LHCb may also be mutually beneficial