An alternative paradigm of time-dependent fits at Belle II

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In the traditional approach for time-dependent fits (used at Belle and BaBar) the maximum of the unbinned likelihood is used

$$\max \ L = \prod_{i} P_{ev}(\Delta t) \quad \text{with} \quad P_{ev}(\Delta t) = \int_{-\infty}^{+\infty} d(\Delta t_{\text{true}}) \ P_{\text{theory}}(\Delta t_{\text{true}}) \ R(\Delta t - \Delta t_{\text{true}})$$

assuming that theory and detector effects are independent and the $\Delta t$ resolution function $R$ does not depend on true $\Delta t_{\text{true}}$

- this “independence” is not granted at Belle II (see slides below)
- a new paradigm of time-dependent fits is proposed which could deal with correlations between theory and detector effects:

VC, “The MPI Concept of Time-Dependent Fits at Belle II”, BELLE2-NOTE-PH-2019-023
VC, “The MPI Concept of Time-Dependent Fits at Belle II”, xFitter Workshop, Minsk, March 2019
https://indico.desy.de/indico/event/22011/session/7/contribution/24/material/slides/0.pdf
VC, “First look at the time-dependent CP violation using early Belle II data“, Lomonosov conference, Moscow, August 2019
https://yadi.sk/i/aOVi-ybK0pwmKA
+ presentations of VC at the TDCPV WG meetings
Time-dependent CP violation analyses at the asymmetric B-factories

The $B^0\bar{B}^0$ pairs from $\Upsilon(4S)$ are produced in a \textit{coherent, entangled quantum mechanical state}. When $B^0(\bar{B}^0)$ decays, the flavor wavefunction of other $\bar{B}^0(B^0)$ collapses and it propagates alone. One needs to measure decay times of both $B^0$s to observe CP violation.

\begin{align*}
\Delta t &= t_{B_{\text{CP}}} - t_{B_{\text{tag}}} \approx \Delta l / \gamma \beta c \\
\Delta M &= t_{B_{\text{CP}}} - t_{B_{\text{tag}}} \approx \Delta O / PQ_c
\end{align*}

$B^0$: CP violation, mixing and lifetime are coming together and can be found in the fit to $\Delta t$ distribution in data

\begin{align*}
P_{\text{sig}}(\Delta t) &= \frac{\exp \left(-\frac{|\Delta t|}{\tau} \right)}{4\tau} \left( 1 + q_{\text{tag}} \left( A_{C_{\text{CP}}} \cos(\Delta m \Delta t) \right) + S_{C_{\text{CP}}} \sin(\Delta m \Delta t) \right) \\
q_{\text{tag}} &= -\xi_{C_{\text{CP}}} \text{ for } B^0_{\text{tag}} \\
q_{\text{tag}} &= \xi_{C_{\text{CP}}} \text{ for } \bar{B}^0_{\text{tag}}
\end{align*}

$A_{C_{\text{CP}}}$: direct CPV

$S_{C_{\text{CP}}} = \sin(2\varphi_1)$ mixing-induced CPV
A new feature at Belle II – the tiny size of the beam spot

$e^+e^- \rightarrow Y(4S) \rightarrow B_\text{tag}^0 + B_\text{sig}^0 (J/\psi K_S^0)$

- the beam spot at Belle II (~400 μm in z) is comparable with the $B^0$ lifetime
  $\Rightarrow$ the $B^0$ decay position in $z$ is far away from the beam spot in the tails of the $\Delta t$ distribution

$\Delta t = t_{B\text{sig}} - t_{B\text{tag}}$ (ps)

$S_{CP} = 0.70$
New challenge for the time-dependent analysis

In the traditional approach for time-dependent fits (used at Belle and BaBar) the maximum of the unbinned likelihood is used

$$\max \ L = \prod_{i} P_{i, ev}(\Delta t) \quad \text{with} \quad P_{i, ev}(\Delta t) = \int_{-\infty}^{+\infty} d(\Delta t_{\text{true}}) \ P_{\text{theory}}(\Delta t_{\text{true}}) \ R(\Delta t - \Delta t_{\text{true}})$$

assuming that theory and detector effects are independent and the $\Delta t$ resolution function $R$ does not depend on true $\Delta t_{\text{true}}$

→ this is not granted at Belle II because of the tiny size of the beam spot, excellent precision of PXD and a need to make use the beam spot information for improvement of the $B_{\text{tag}}^0$ vertex position on the tag side:

the beam information helps to select tracks directly from $B_{\text{tag}}^0$ decay and remove displaced tracks from decays of charmed particles (Ds) or $K_S^0$

Several developments at Belle II to deal with the new challenge

1. Optimisation of the $B_{\text{tag}}^0$ vertexing on the tag side, e.g. “Btube constraint” (D. Sourav) applied to tag vertexing (T. Humair)
2. Further development of the traditional approach
3. A new paradigm of time-dependent fits

which is robust and could deal with correlations between theory and detector effects
An alternative paradigm of time-dependent fits at Belle II

- $P^i_{ev} (\Delta t) \rightarrow$ calculated numerically using weighted MC events (i.e. use convolution of theory and detector from simulation)

- variation of input physics parameters ($\tau, \Delta m, S_{CP}$ and $A_{CP}$) $\rightarrow$ by weighting of an auxiliary, “assistive” MC sample

- differences in the detector response between data and simulation $\rightarrow$ by downgrading (smearing) of the detector response in an auxiliary, “assistive” MC sample, using weighting of the simulated event

- physics parameters and the detector smearing $\rightarrow$ determined simultaneously in the TD CPV fit of the signal and control channels
New input physics parameters – analytic expression for weighting of MC

$P_{\text{theory}}(t^{B^0_{\text{first}}}, \Delta t) = \frac{\exp\left(-\frac{|t^{B^0_{\text{first}}}|}{4\tau}\right)}{4\tau} \frac{\exp\left(-\frac{|\Delta t|}{4\tau}\right)}{4\tau} [1 + q(A\cos(\delta m \Delta t) + S \sin(\delta m \Delta t))]$

Input from generator to simulation:

If values of $t^{B^0_{\text{first}}}$ and $\Delta t$ are defined (and frozen):
- simulation of the detector effects does not depend on $\tau$, $\delta m$, $A$, $S$
- only probability of event with given $t^{B^0_{\text{first}}}$ and $\Delta t$ depends on $\tau$, $\delta m$, $A$, $S$

→ Thus, MC sample generated with $\tau_0$, $\delta m_0$, $A_0$, $S_0$ can be used to get MC sample equivalent to simulation with $\tau$, $\delta m$, $A$, $S$ by the weighting of MC events:

$w = P_{\text{theory}}(t^{B^0_{\text{first}}}, \Delta t; \tau, \delta m, A, S) / P_{\text{theory}}(t^{B^0_{\text{first}}}, \Delta t; \tau_0, \delta m_0, A_0, S_0)$

---

simulation without CPV ($S=A=0$) (blue)

$B^0_{\text{tag}}$

simulation with CPV ($S=0.70,A=0$) (blue)

$B^0_{\text{tag}}$

MC without CPV reweighted to $S=0.70$ (red)

$B^0_{\text{tag}}$

→ MC with CPV and re-weighted MC without CPV are identical !!!
Treatment of discrepancies between data and MC in the detector response

prior the TD fit (once) → smearing of reconstructed quantities ($\Delta t$) in the MC sample
- very flexible and can have any level of complexity if there is a model for downgrading of the detector resolution
- the simplest and also very efficient smearing model: $\Delta t' = \Delta t + G(\alpha_{\text{smear}} \cdot \delta(\Delta t))$ ($\delta(\Delta t)$ – uncertainty of $\Delta t$)

during the TD fit (many times) → approximation of the “simplest smearing model” by the weighting of the MC sample
- could be directly included in the TD fit with the smearing factor and the physics parameters determined simultaneously

First, determine a simplified $\Delta t$ resolution function in a two gaussian fit of the simulated MC sample:

$$P_{\text{res.func}}(\Delta t - \Delta t_{\text{true}}) = f G_1(\mu_1, \sigma_1) + (1 - f) G_2(\mu_2, \sigma_2)$$

where $\mu_i = \mu_i^{\text{pull}} \cdot \delta(\Delta t)$ and $\sigma_i = \sigma_i^{\text{pull}} \cdot \delta(\Delta t)$
determined separately for positive and negative $\Delta t_{\text{true}}$

Then, analytic expression for weighting of MC events

$$w = \frac{P_{\text{res.func}}^{\text{new}}}{P_{\text{res.func}}}, \quad \text{where}$$

$$P_{\text{res.func}}^{\text{new}} = f G_1(\mu_1, \sigma_1^{\text{new}}) + (1 - f) G_2(\mu_2, \sigma_2^{\text{new}})$$

with $\sigma_i^{\text{new}} = \sqrt{\sigma_i^2 + (\alpha_{\text{smear}} \delta(\Delta t))^2}$
An example of alternative time-dependent CP violation fit of signal $B^0 \rightarrow J/\psi K_S^0$ and control $B^\pm \rightarrow J/\psi K^\pm$ channels for Belle II MC (500 fb$^{-1}$)

<table>
<thead>
<tr>
<th></th>
<th>$\tau(B^\pm)$ ps</th>
<th>$\tau(B^0)$ ps</th>
<th>$S$</th>
<th>$\delta m$ (ps$^{-1}$)</th>
<th>$\alpha_{smear}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ass. MC(BGx0)$</td>
<td>1.637</td>
<td>1.525</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$J/\psi(\mu\mu)K_S^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ass. MC(BGx0)$</td>
<td>1.637</td>
<td>1.525</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$J/\psi(ee)K_S^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ass. MC(BGx1)$</td>
<td>1.637</td>
<td>1.525</td>
<td>0.695</td>
<td>0.502</td>
<td></td>
</tr>
<tr>
<td>$J/\psi(\mu\mu)K^\pm$</td>
<td></td>
<td></td>
<td></td>
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</table>

<table>
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<th>$\tau(B^0)$ ps</th>
<th>$S$</th>
<th>$\delta m$ (ps$^{-1}$)</th>
<th>$\alpha_{smear}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$MC12b(BGx1)$ $J/\psi(\mu\mu, ee)K_S^0$</td>
<td>1.596 $\pm$ 0.036</td>
<td>1.554 $\pm$ 0.037</td>
<td>0.700 $\pm$ 0.059</td>
<td>0.536 $\pm$ 0.048</td>
<td>0 $\pm$ 0.63</td>
</tr>
<tr>
<td>$MC12b(BGx1)$ $J/\psi(\mu\mu)K^\pm$</td>
<td>1.596 $\pm$ 0.036</td>
<td>1.554 $\pm$ 0.037</td>
<td>0.701 $\pm$ 0.059</td>
<td>0.536 $\pm$ 0.048</td>
<td>0 $\pm$ 0.44</td>
</tr>
<tr>
<td>$MC12b(BGx1)$ combined</td>
<td></td>
<td></td>
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</tbody>
</table>

→ all results of the alternative TD CPV fits are consistent with the expectations within one sigma
Conclusions

- Large room for improvements of precision for the TD CPV measurements, by far not limited by systematics
  - a long term Belle II project aiming for 50 \( ab^{-1} \)

- Ultimate precision will require best methods for time-dependent analyses

- New challenges and new developments related to TD CPV analyses at Belle II
  - precision measurement
  - possible correlations of physics parameters and detector effects, e.g. due to the tiny size of the beam spot
    - optimization of vertexing on tag side
    - traditional & alternative approaches for TD fits
Time-dependent CP violation and the CKM unitarity matrix

- $B^0$ system has the largest CP violation effects in the SM described by the CKM unitarity matrix $V_{CKM}$

- Time-dependent CPV effects are related to interference between mixing and decay amplitudes (mixing-induced CPV)

$$A(t) = \frac{\Gamma(B^0 \to f_{CP}) - \Gamma(B^0 \to \bar{f}_{CP})}{\Gamma(B^0 \to f_{CP}) + \Gamma(B^0 \to \bar{f}_{CP})} = -\xi_f \sin(2\varphi_1) \sin(\Delta m t), \quad S_{CP} = \sin(2\varphi_1)$$

- $\varphi_1$ and $\varphi_2$ can be measured in TD CPV analyses of for $B^0 \to c\bar{c}s$, $q\bar{q}s$ and $B^0 \to u\bar{u}d$, e.g. for $B^0 \to J/\psi K_S^0$

$\rightarrow$ room for improvement of the CPV measurements at Belle II projections to 5 $ab^{-1}$ and 50 $ab^{-1}$ (arXiv:1808.10567)

<table>
<thead>
<tr>
<th>Channel</th>
<th>WA (2017)</th>
<th>5 $ab^{-1}$</th>
<th>50 $ab^{-1}$</th>
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<tbody>
<tr>
<td></td>
<td>$\sigma(S)$</td>
<td>$\sigma(A)$</td>
<td>$\sigma(S)$</td>
</tr>
<tr>
<td>$J/\psi K^0$</td>
<td>0.022</td>
<td>0.021</td>
<td>0.012</td>
</tr>
<tr>
<td>$\phi K^0$</td>
<td>0.12</td>
<td>0.14</td>
<td>0.048</td>
</tr>
<tr>
<td>$\eta/K^0$</td>
<td>0.06</td>
<td>0.04</td>
<td>0.032</td>
</tr>
<tr>
<td>$\omega K_S^0$</td>
<td>0.21</td>
<td>0.14</td>
<td>0.08</td>
</tr>
<tr>
<td>$K_S^0\pi^0\gamma$</td>
<td>0.20</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>$K_S^0\pi^0$</td>
<td>0.17</td>
<td>0.10</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Linearity checks - variations of $\tau(B^0)$, $\delta m$ and $S$

\begin{table}[h!]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & $S$ & $\Delta S$ & $\tau(\text{ps})$ & $\Delta \tau(\text{ps})$ & $\delta m(\text{ps}^{-1})$ & $\Delta(\delta m)$ \\
\hline
assistive MC & 0 & & 1.525 & & 0 & \\
\hline
CPV & 0.703 & & 1.525 & & 0.507 & \\
 & 0.7024 \pm 0.0029 & -0.0006 & 1.5233 \pm 0.0030 & -0.0017 & 0.5089 \pm 0.0023 & +0.0019 \\
\hline
CPV Down & 0.65 & & 1.500 & & 0.485 & \\
 & 0.6484 \pm 0.0030 & -0.0006 & 1.4944 \pm 0.0030 & -0.0056 & 0.4861 \pm 0.0026 & +0.0011 \\
\hline
CPV Up & 0.75 & & 1.550 & & 0.530 & \\
 & 0.7480 \pm 0.0028 & -0.0020 & 1.5478 \pm 0.0030 & -0.0022 & 0.5325 \pm 0.0021 & +0.0025 \\
\hline
\end{tabular}
\caption{The MPI TD CPV fit results with the MC samples “CPV”, “CPV Down” or “CPV Up” serving as data and with “no CPV” - as an assistive MC sample. The expected values of physics parameters and differences between fitted and expected values are shown as well.}
\end{table}

"perfect flavor tagging": $B^0$ or $\bar{B}^0$ from MC

$\rightarrow$ excellent agreement in a wide range of $\tau(B^0)$, $\delta m$ and $S$
Treatment of differences in detector response between data and MC

\[ B^0 \rightarrow J/\psi(\mu\mu) K_S^0 \]

"MC sample" no CPV

Phase 3

no CPV

\[ S = 0.695, \tau = 1.525, \text{dm} = 0.503 \]

Phase 3

"perfect flavor tagging": \( B^0 \) or \( \bar{B}^0 \) from MC

<table>
<thead>
<tr>
<th>MC as data</th>
<th>0.695</th>
<th>1.525</th>
<th>0.503</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph3 bk0</td>
<td>0.6948 \pm 0.0029</td>
<td>-0.0002</td>
<td>1.5211 \pm 0.0030</td>
</tr>
<tr>
<td>Ph3 bk1</td>
<td>0.6876 \pm 0.0031</td>
<td>-0.0074</td>
<td>1.5350 \pm 0.0032</td>
</tr>
<tr>
<td>Ph3 bk2</td>
<td>0.6816 \pm 0.0034</td>
<td>-0.0134</td>
<td>1.5463 \pm 0.0036</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ass. MC</th>
<th>0</th>
<th>1.525</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph3 bk2</td>
<td>0.695</td>
<td>-0.0005</td>
<td>1.525</td>
</tr>
<tr>
<td>fit 4 par.</td>
<td>0.6945 \pm 0.0046</td>
<td>-0.0007</td>
<td>1.5186 \pm 0.0074</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \Delta S )</th>
<th>( \Delta \tau(p) )</th>
<th>( \Delta m(\text{ps}^{-1}) )</th>
<th>( \Delta(\delta m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ph3 bk2</td>
<td>0.695</td>
<td>-0.0064</td>
<td>0.5055 \pm 0.0030</td>
</tr>
<tr>
<td>fit 4 par.</td>
<td>0.695</td>
<td>-0.0007</td>
<td>1.5186 \pm 0.0074</td>
</tr>
</tbody>
</table>

\( \alpha_{\text{smear}} \)

Multiplicative method: excellent (< \( \sigma \))

MC with additional smearing: very good (<2\( \sigma \))

TABLE XII: The MPI TD CPV fit results for the "Ph3 bk2" sample with an extra free parameter, \( \alpha_{\text{smear}} \), and with smearing of the assistive MC sample, "no CPV", prior the fit. The expected values of physics parameters and differences between fitted and expected values are shown as well.