Introduction to

τ Physics

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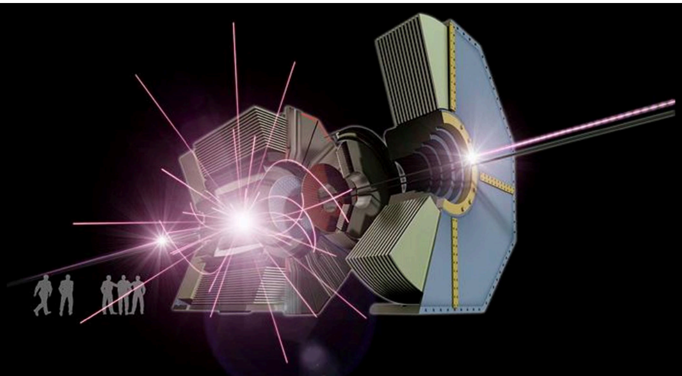


2021 Belle II Physics Week

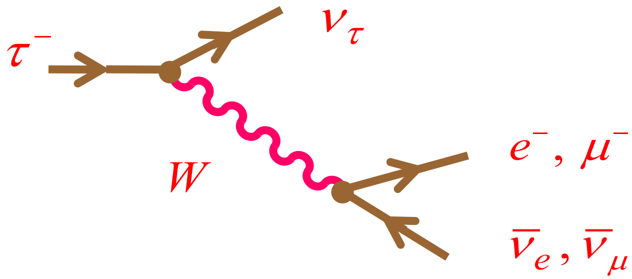
November 29 – December 3

Belle II

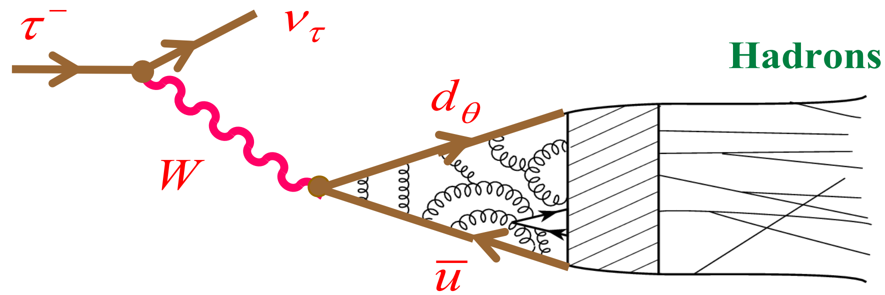
A.P., Prog. Part. Nucl. Phys. 75 (2014) 41

τ Physics

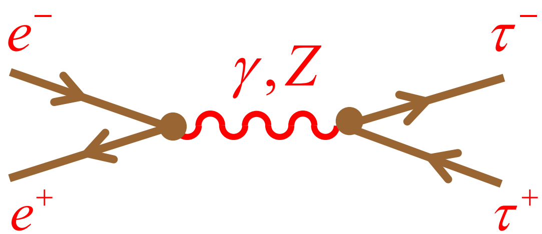
Decay



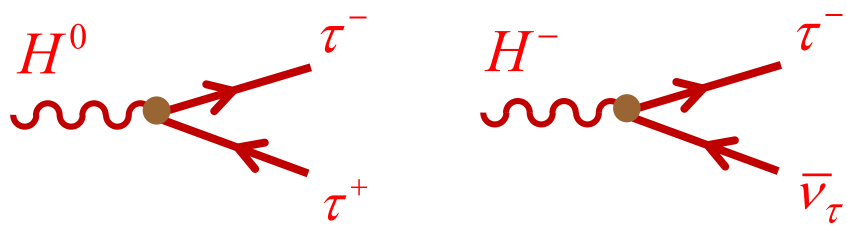
QCD



Production

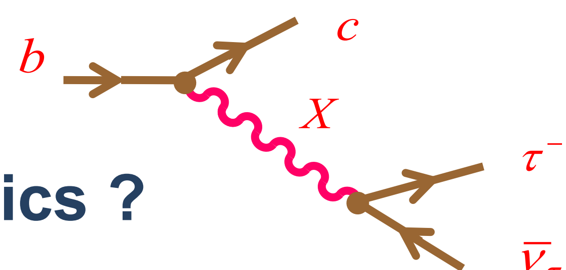


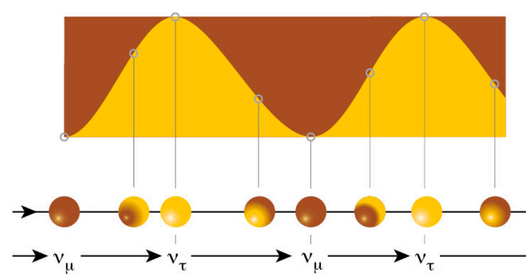
Higgs Interactions



?

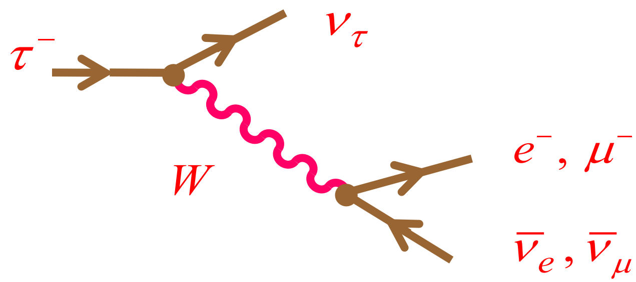
New Physics ?





Neutrinos

LEPTONIC DECAYS

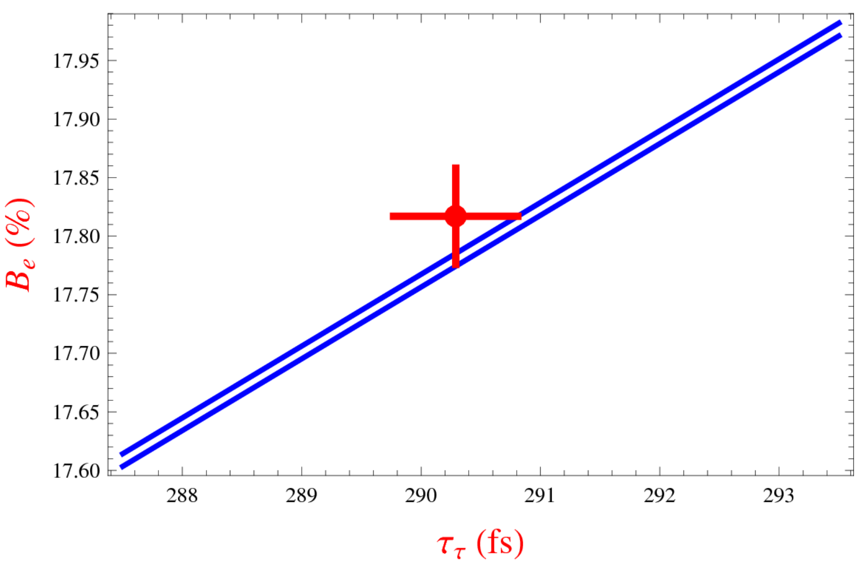


$$\Gamma(\tau \to \nu_\tau\, l\, \bar{\nu}_l) = \frac{G_F^2\, m_\tau^5}{192\,\pi^3}\, f(m_l^2/m_\tau^2)\, \left(1+\delta_{\rm RC}\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$B_e = \frac{B_\mu}{0.972561 \pm 0.000004} = \frac{\tau_\tau}{(1632.7 \pm 0.5)\times 10^{-15}\,\mathrm{s}}$$

τ_τ **(Belle),** m_τ **(BesIII)**



$$\left(B_\mu/B_e\right)_{\rm exp} = 0.9762 \pm 0.0028$$

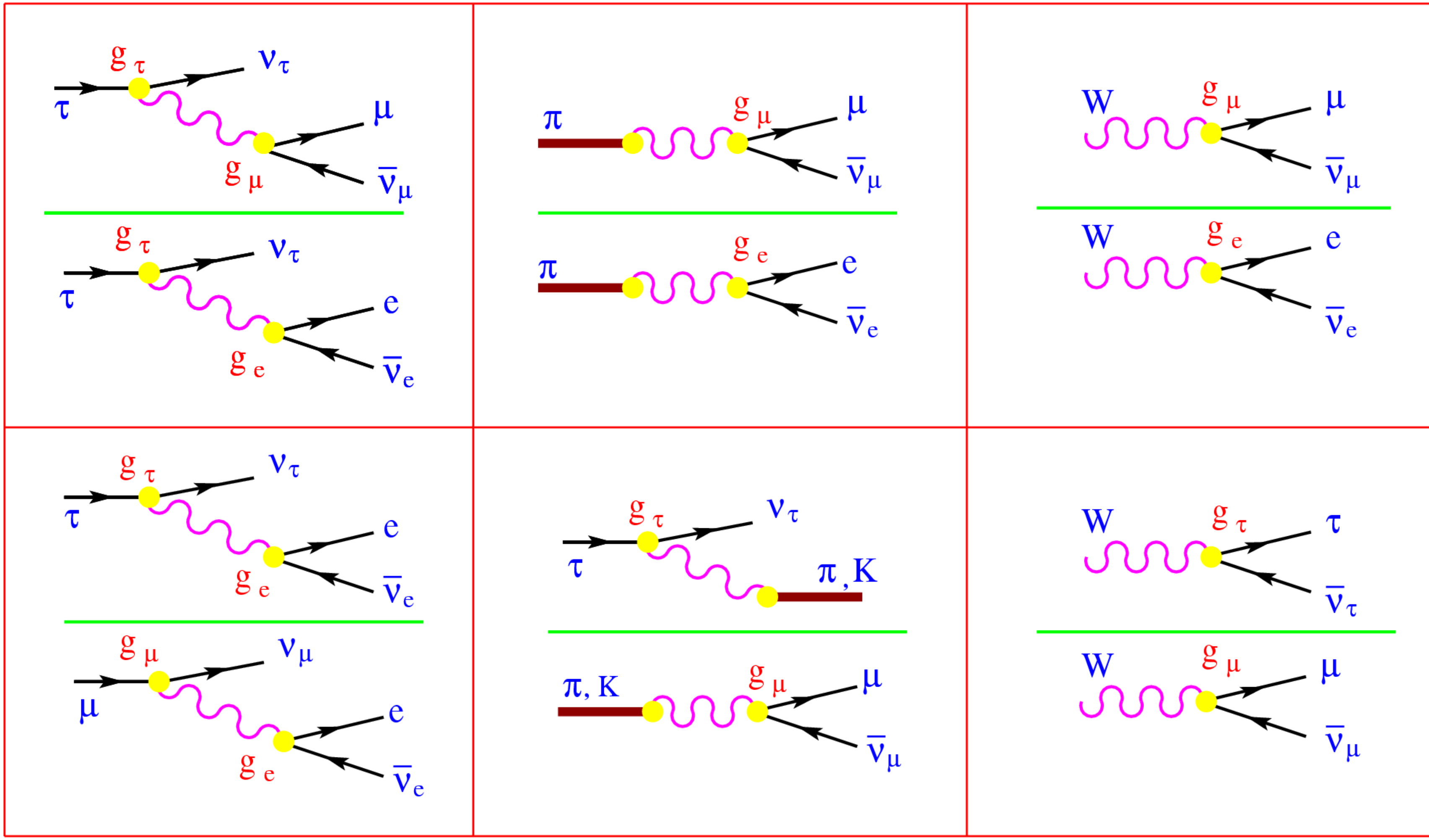
Non-BF:	0.9725 ± 0.0039
BaBar '10:	0.9796 ± 0.0039

$$B_e^{\rm univ} = \left(17.815 \pm 0.023\right)\%$$

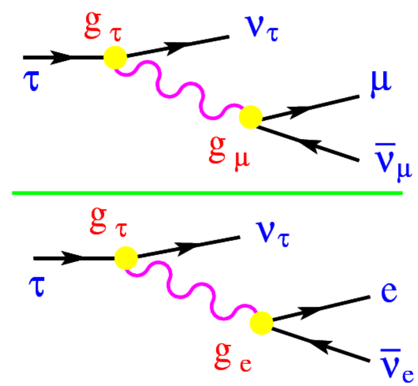
Lepton Universality

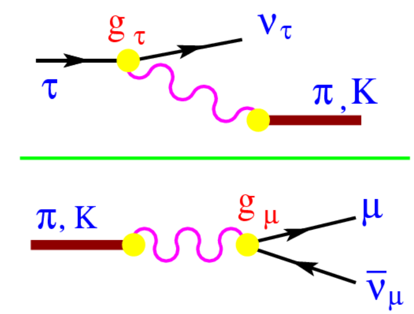
$\frac{g_\mu}{g_e}$

$\frac{g_\tau}{g_\mu}$



Lepton Universality





$|g_\mu / g_e|$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0017 ± 0.0016
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0010 ± 0.0009
$B_{K\to\mu}/B_{K\to e}$	0.9978 ± 0.0018
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.0010 ± 0.0025
$B_{W\to\mu}/B_{W\to e}$	0.998 ± 0.004

$|g_\tau / g_e|$

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0028 ± 0.0015
$B_{W\to\tau}/B_{W\to e}$	1.021 ± 0.012

$|g_\tau / g_\mu|$

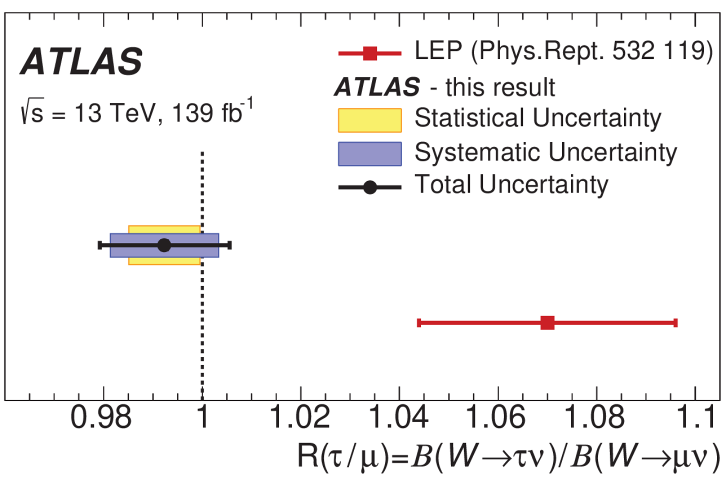
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0011 ± 0.0014
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9964 ± 0.0038
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.9857 ± 0.0078
$B_{W\to\tau}/B_{W\to\mu}$	1.004 ± 0.016

Lepton Universality in W decays

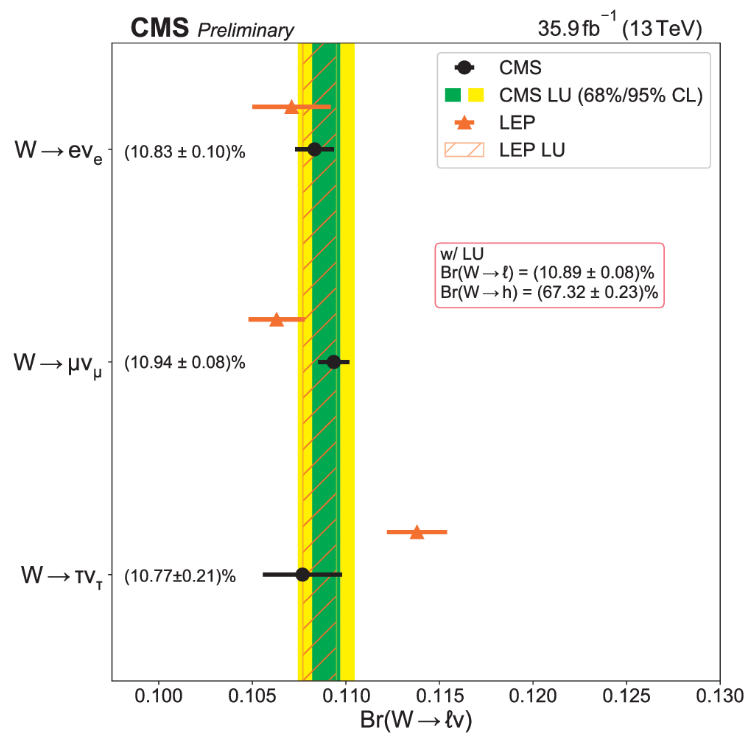
2109.06065

	LEP [389]	LHCb [396]	ATLAS [388, 397]	CMS [391] (prelim.)	Average (prelim.)
$\lvert g_\mu/g_e \rvert$	0.996 (10)	0.990 (9)	1.002 (5)	1.005 (6)	1.000 (3)
$\lvert g_\tau/g_\mu \rvert$	1.034 (13)		0.996 (7)	0.992 (10)	1.001 (10)
$\lvert g_\tau/g_e \rvert$	1.031 (13)			0.997 (11)	1.008 (12)

2007.14040



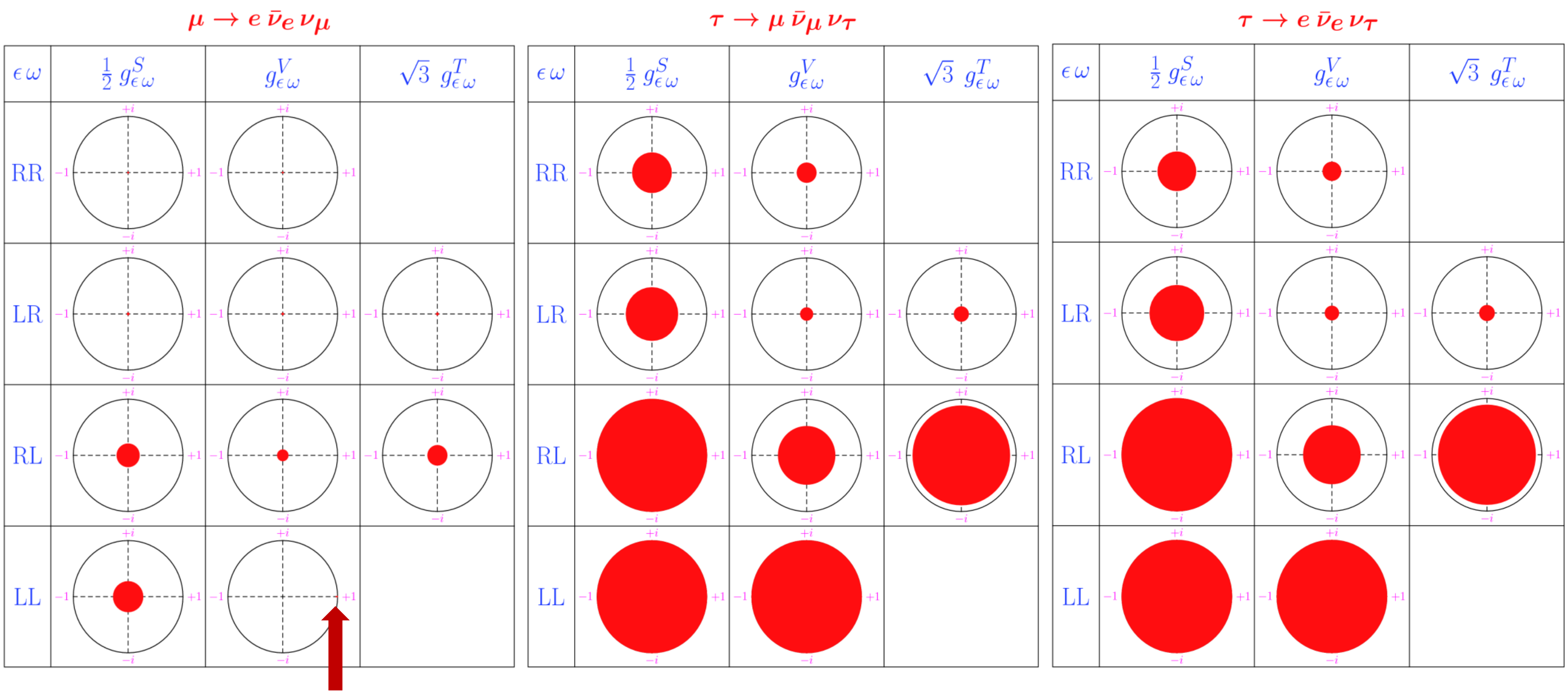
CMS PAS SMP-18-011 (2021)



Lorentz Structure: $\ell^- \to \ell'^- \bar{\nu}_{\ell'} \nu_\ell$

Effective Hamiltonian: $$\mathcal{H} = 4\,\frac{G_{\ell'\ell}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega}\, \left[\overline{\ell'_\epsilon}\,\Gamma^n (\nu_{\ell'})_\sigma\right] \left[\overline{(\nu_\ell)_\lambda}\,\Gamma_n\, \ell_\omega\right]$$

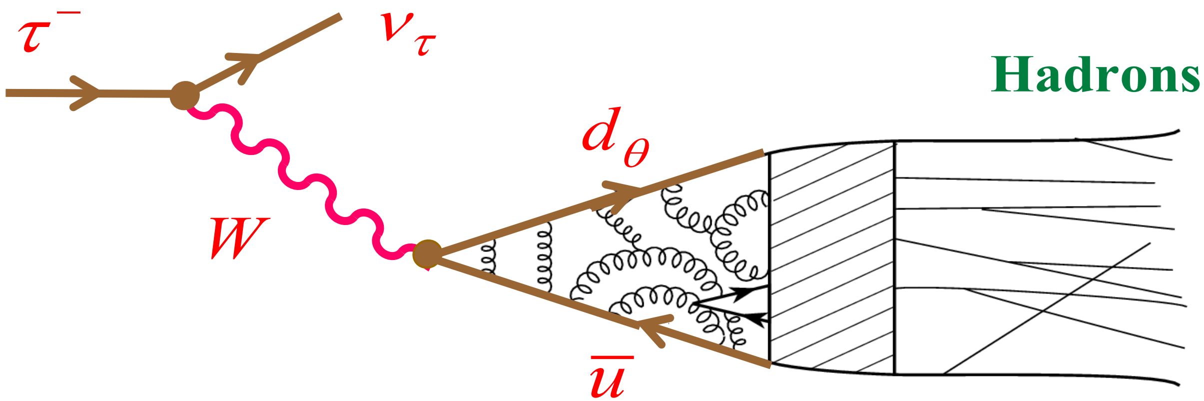
Normalization: $$\Gamma \propto \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) + 3\,\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2\right) \equiv 1$$



$|g^V_{LL}| > 0.960$ (90% CL)

High-precision τ data needed!

HADRONIC TAU DECAY



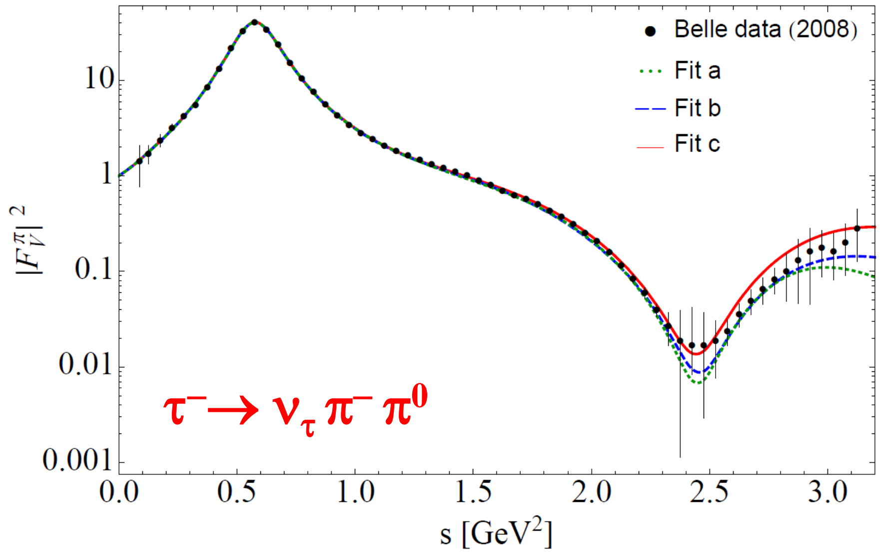
$$d_\theta = V_{ud}\, d + V_{us}\, s$$

Only lepton massive enough to decay into hadrons

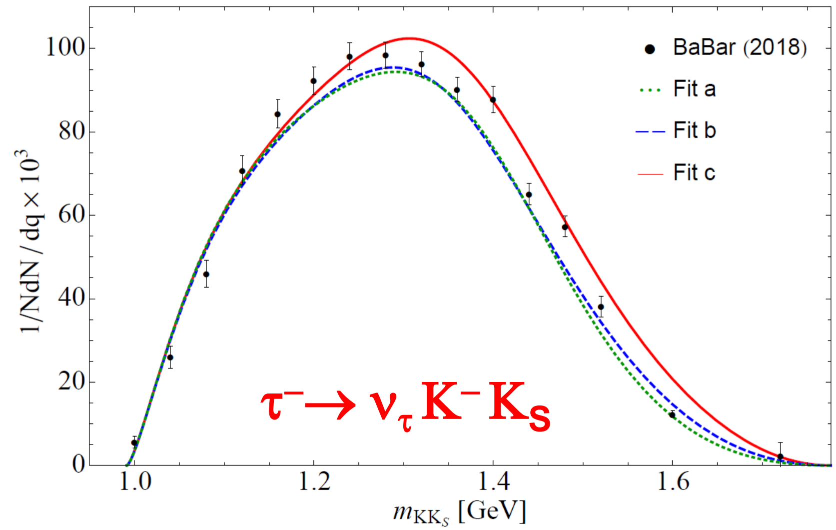
$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{Hadrons})}{\Gamma(\tau^- \to \nu_\tau\, e^-\, \overline{\nu}_e)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.637 \pm 0.011$$

$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.97256 = 3.6407 \pm 0.0072 \qquad ; \qquad R_\tau = \frac{\text{Br}(\tau^- \to \nu_\tau + \text{Hadrons})}{B_e^{\text{univ}}} = 3.6349 \pm 0.0082$$

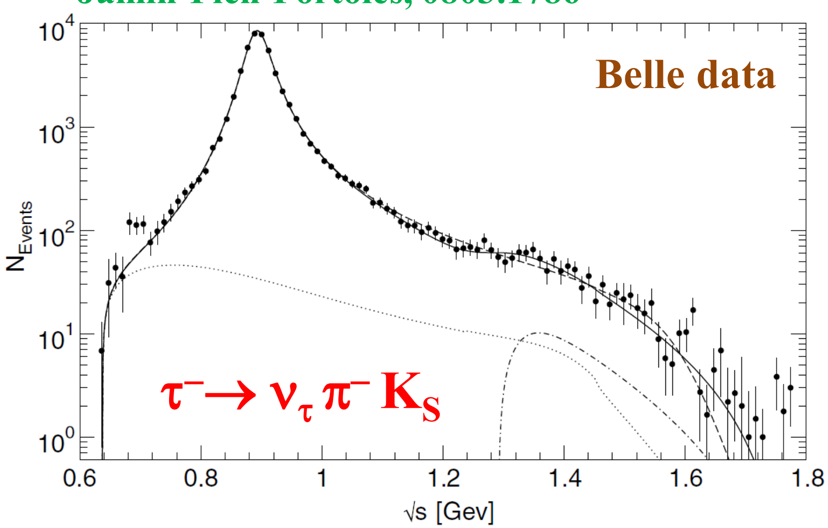
Invariant Mass Spectra



Gónzàlez-Solís – Roig, 1902.02273



Jamin-Pich-Portolés, 0803.1786

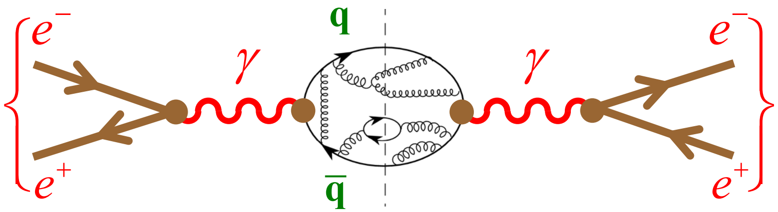


Useful tests of QCD Dynamics

Form Factors

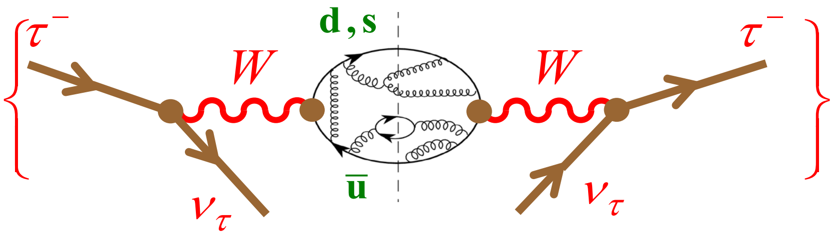
Non-perturbative parameters

Resonance Chiral Theory (RχT)

$\sigma \sim \mathrm{Im}$ 

$$\frac{\sigma(e^+e^-\to\mathrm{had})}{\sigma(e^+e^-\to\mu^+\mu^-)} = 12\pi\ \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i\int d^4x\ e^{iqx}\,\langle 0|T[J^\mu_{\mathrm{em}}(x)J^\nu_{\mathrm{em}}(0)]|0\rangle = \left(-g^{\mu\nu}q^2+q^\mu q^\nu\right)\Pi_{\mathrm{em}}(q^2)$$

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$ 

$$R_\tau \equiv \frac{\Gamma(\tau^-\to\nu_\tau+\mathrm{had})}{\Gamma(\tau^-\to\nu_\tau e^-\bar\nu_e)} = 12\pi\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left[\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2\left[\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right]$$

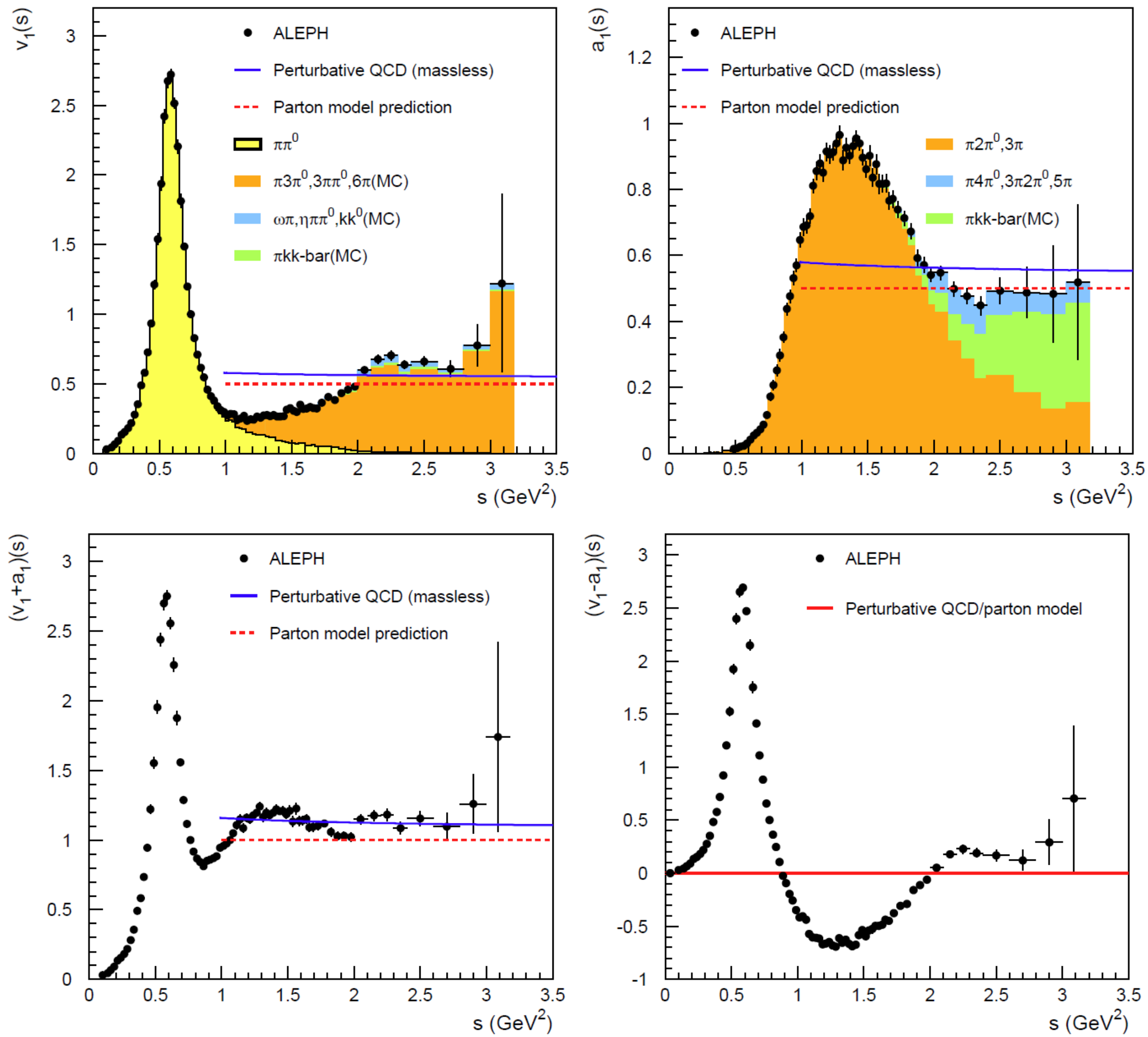
$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i\int d^4x\ e^{iqx}\,\langle 0|T[J^\mu_{ij}(x)J^\nu_{ij}(0)^\dagger]|0\rangle = \left(-g^{\mu\nu}q^2+q^\mu q^\nu\right)\Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu\ \Pi^{(0)}_{ij,J}(q^2)$$

SPECTRAL FUNCTIONS

Davier et al, 1312.1501

$$v_1(s) = 2\pi \,\mathrm{Im}\,\Pi^{(0+1)}_{ud,V}(s)$$

$$a_1(s) = 2\pi \,\mathrm{Im}\,\Pi^{(0+1)}_{ud,A}(s)$$



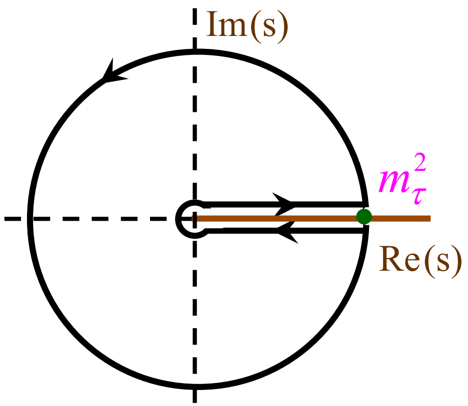
Better data needed

QCD Prediction of R_τ

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$

$x \equiv s/m_\tau^2$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C\, S_{\text{EW}}\,(1+\delta_{\text{P}}+\delta_{\text{NP}}) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\text{EW}} = 1.0201\ (3)$; $\delta_{\text{NP}} = -0.0064 \pm 0.0013$

Marciano-Sirlin, Braaten-Li, Erler — **Fitted from data (Davier et al)**

$\delta_{\text{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Baikov-Chetyrkin-Kühn

Perturbative $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$$K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101 \quad , \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08

$$\Rightarrow \quad \delta_P = \sum_{n=1} K_n\; A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{NP} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

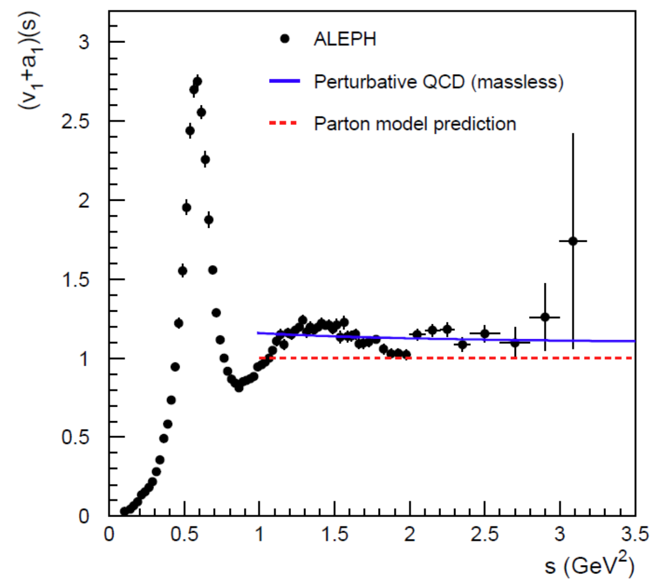
Suppressed by m_τ^6 **[additional chiral suppression in** $C_6\langle O_6\rangle^{V+A}$ **]**

Spectral Function Distribution

Moments:

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

Sensitivity to power corrections (k,l)



The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons

Detailed analyses by ALEPH, CLEO and OPAL

$$\delta_{NP} = -0.0064 \pm 0.0013$$

$$\alpha_s(m_\tau^2) = 0.332 \pm 0.005_{exp} \pm 0.011_{th}$$

Davier et al., 1312.1501
(ALEPH data)

Exhaustive Analysis of ALEPH Data

Rodríguez-Sánchez, Pich, 1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments [1]	$0.339\,^{+0.019}_{-0.017}$	$0.319\,^{+0.017}_{-0.015}$	$0.329\,^{+0.020}_{-0.018}$
Mod. ALEPH moments [2]	$0.338\,^{+0.014}_{-0.012}$	$0.319\,^{+0.013}_{-0.010}$	$0.329\,^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments [3]	$0.336\,^{+0.018}_{-0.016}$	$0.317\,^{+0.015}_{-0.013}$	$0.326\,^{+0.018}_{-0.016}$
s_0 dependence [4]	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform [5]	$0.328\,^{+0.014}_{-0.013}$	$0.318\,^{+0.015}_{-0.012}$	$0.323\,^{+0.015}_{-0.013}$
Combined value	$\mathbf{0.335 \pm 0.013}$	$\mathbf{0.320 \pm 0.012}$	$\mathbf{0.328 \pm 0.013}$

→ $$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

1) $\omega_{kl}(x) = (1+2x)\,(1-x)^{2+k}\,x^l$ $\qquad (k,l) = (0,0),\ (1,0),\ (1,1),\ (1,2),\ (1,3)$

2) $\tilde{\omega}_{kl}(x) = (1-x)^{2+k}\,x^l$ $\qquad (k,l) = (0,0),\ (1,0),\ (1,1),\ (1,2),\ (1,3)$

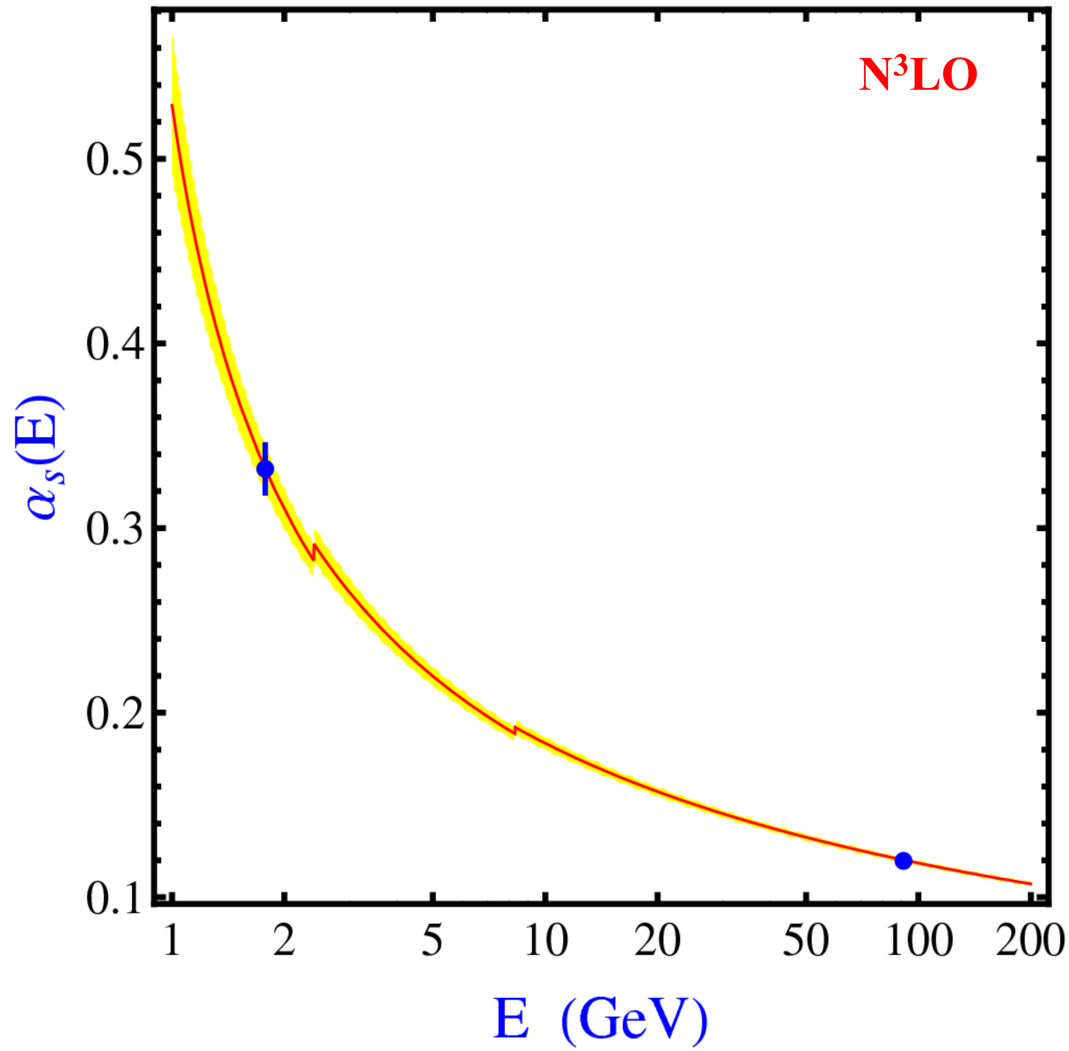
3) $\omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^{m} (k+1)\,x^k = 1 - (m+2)\,x^{m+1} + (m+1)x^{m+2}$, $\qquad 1 \le m \le 5$

4) $\omega^{(2,m)}(x)$ $\qquad 0 \le m \le 2$, 1 single moment in each fit

5) $\omega_a^{(1,m)}(x) = (1 - x^{m+1})\,\mathrm{e}^{-ax}$ $\qquad 0 \le m \le 6$

α_s at N^{3}LO from τ and Z

Rodríguez-Sánchez, Pich, 1605.06830



$$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$$

$$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1199 \pm 0.0029$$

Very precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0002 \pm 0.0015_\tau \pm 0.0029_Z$$

V_{us} Determination

Gámiz-Jamin-Pich-Prades-Schwab

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

$$\delta R^{kl}_{\tau} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)$$

$$\delta R^{00}_{\tau,\mathrm{th}} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.086\,(32)}_{m_s(2\text{ GeV}) = 94\,(6)\ \text{MeV}} = 0.240\,(32)$$

$$R^{00}_{\tau,S} = 0.1645\ (23)$$
$$R^{00}_{\tau,V+A} = 3.4709\ (79)$$
$$|V_{ud}| = 0.97370\ (14)$$

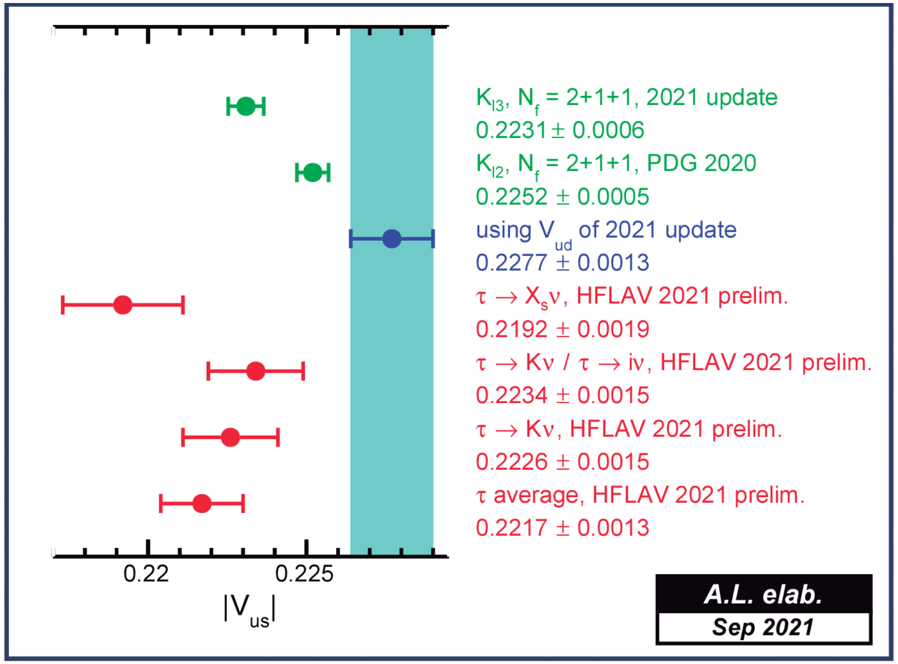
$$\Rightarrow \quad |V_{us}| = 0.2194 \pm 0.0016_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}$$

K_{l3}: $|V_{us}| = 0.2232 \pm 0.0006$ $\quad [f_+(0) = 0.9698 \pm 0.0017]$ $\quad$ FLAG 2021

Sizeable discrepancy. Improvements needed

V_{us} & V_{ud} Cabibbo Anomaly

A. Lusiani, TAU 2021



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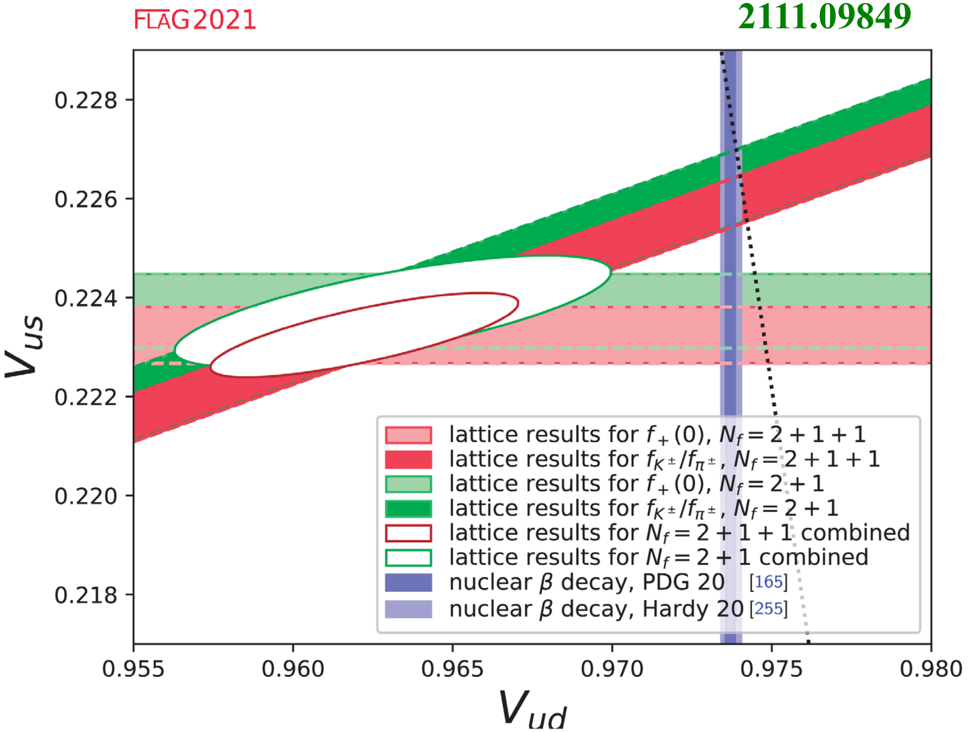
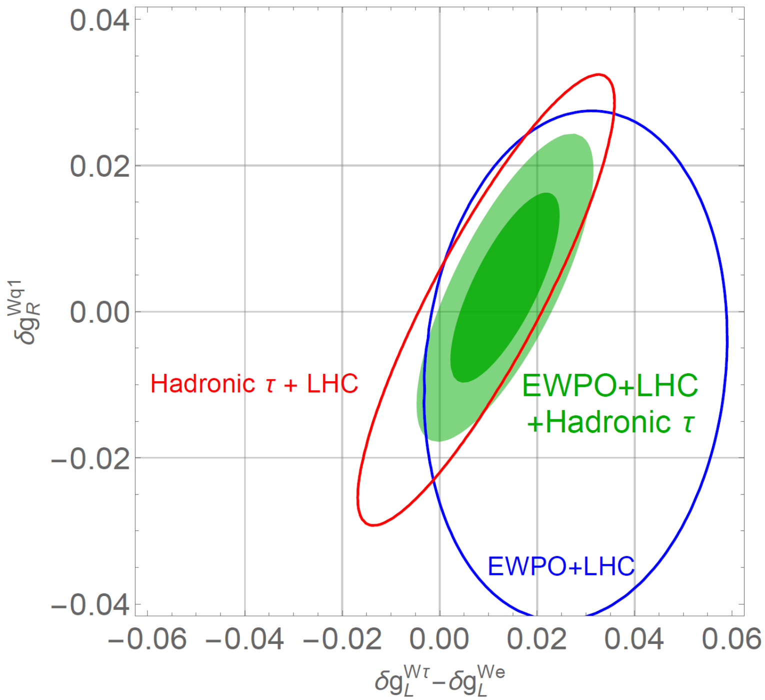


Sizeable violation of CKM unitarity

Hadronic τ Decay & New Physics

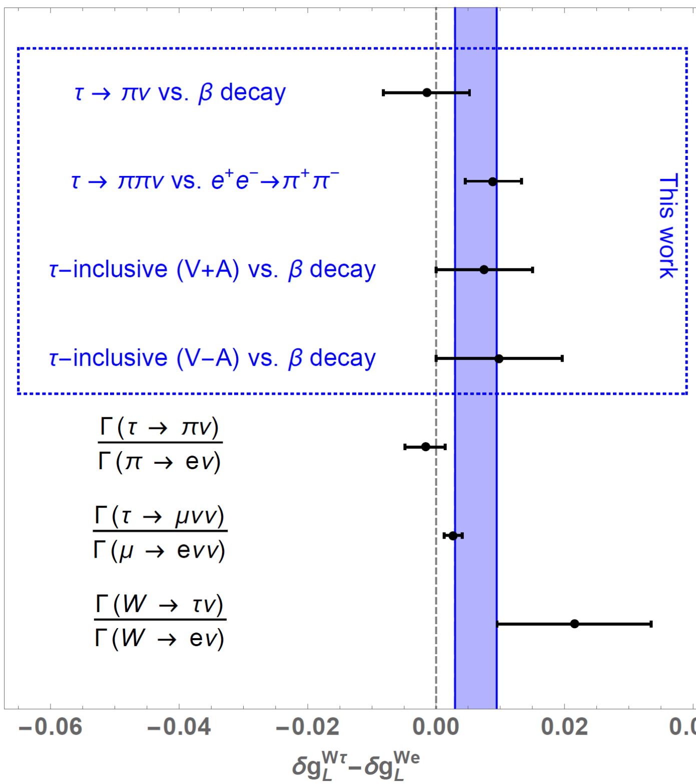
$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{ud}}{\sqrt{2}} \Big[\left(1+\epsilon_L^\tau\right) \bar\tau\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar u\gamma^\mu(1-\gamma_5)d \; + \; \epsilon_R^\tau \; \bar\tau\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar u\gamma^\mu(1+\gamma_5)d \\ + \; \bar\tau(1-\gamma_5)\nu_\tau \cdot \bar u \left[\epsilon_S^\tau - \epsilon_P^\tau\gamma_5\right] d \; + \; \epsilon_T^\tau \, \bar\tau\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar u\sigma^{\mu\nu}(1-\gamma_5)d \Big] + \text{h.c.}$$



Cirigliano, Falkowski, González-Alonso, Rodríguez-Sánchez, 1809.01161

$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee11} \qquad \epsilon_{S,P}^\tau = -\frac{1}{2}[c_{lequ} \pm c_{ledq}]^*_{\tau\tau 11}$$

$$\epsilon_R^\tau = \delta g_R^{Wq_1}, \qquad \epsilon_T^\tau = -\frac{1}{2}[c_{lequ}^{(3)}]^*_{\tau\tau 11} \, ,$$

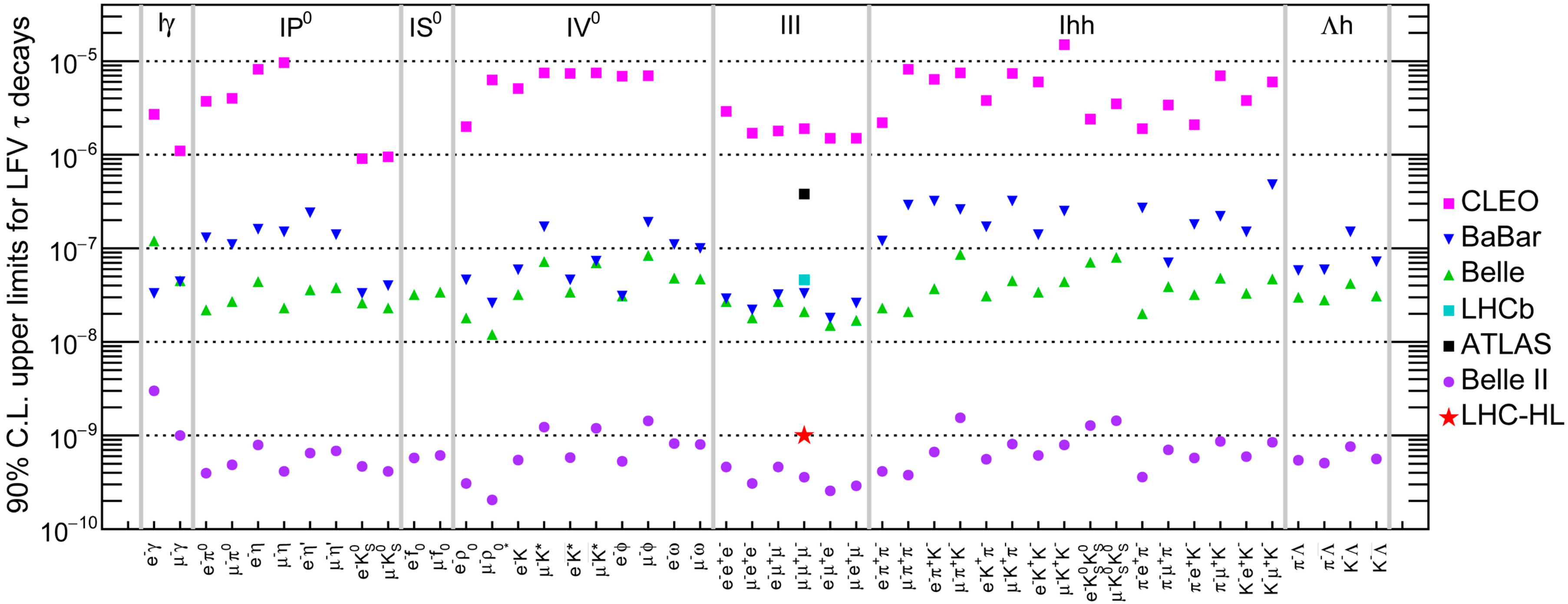


95% CL $\qquad$ 10^{-3} units

Coefficient	ATLAS $\tau\nu$	τ decays	τ and π decays
$[c_{\ell q}^{(3)}]_{\tau\tau 11}$	$[0.0, 1.6]$	$[-12.6, 0.2]$	$[-7.6, 2.1]$
$[c_{\ell equ}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-8.4, 4.1]$	$[-5.6, 2.3]$
$[c_{\ell edq}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-3.5, 9.0]$	$[-2.1, 5.8]$
$[c_{\ell equ}^{(3)}]_{\tau\tau 11}$	$[-3.3, 3.3]$	$[-10.4, -0.2]$	$[-8.6, 0.7]$

Bounds on Lepton Flavour Violation

τ Decays (90% CL)



$\mathrm{Br}(\mu\to e\gamma) < 4.2\times 10^{-13}$ (MEG, 90% CL) , $\mathrm{Br}(\mu\to 3e) < 1.0\times 10^{-12}$ (SINDRUM, 90% CL)

$\mathrm{Br}(K_L\to \mu e) < 4.7\times 10^{-12}$ (BNL-E871, 90% CL) , $\mathrm{Br}(K^+\to \pi^+\mu^+e^-) < 1.3\times 10^{-11}$ (BNL-E865, 90% CL)

$\mathrm{Br}(B^0\to e\mu) < 1.0\times 10^{-9}$ (LHCb, 90% CL) , $\mathrm{Br}(D^0\to e\mu) < 1.3\times 10^{-8}$ (LHCb, 90% CL)

$\mathrm{Br}(Z^0\to e\mu) < 7.5\times 10^{-7}$ (ATLAS, 95% CL) , $\mathrm{Br}(H\to e\mu) < 6.1\times 10^{-5}$ (ATLAS, 95% CL)

$\mathrm{Br}(Z^0\to e\tau) < 5.0\times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H\to e\tau) < 2.2\times 10^{-3}$ (CMS, 95% CL)

$\mathrm{Br}(Z^0\to \mu\tau) < 6.5\times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H\to \mu\tau) < 1.5\times 10^{-3}$ (CMS, 95% CL)

CP Asymmetry

$$A_\tau \equiv \frac{\Gamma\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S \nu_\tau\right)} = \left(-3.6 \pm 2.3 \pm 1.1\right) \cdot 10^{-3}$$

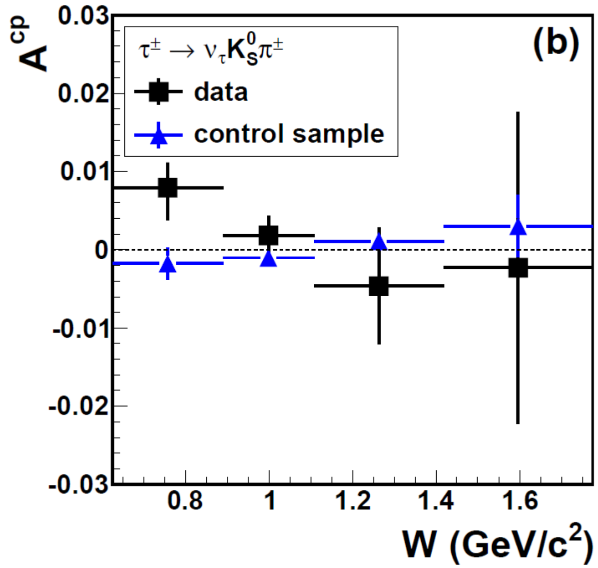
BaBar'11 $\left(\geq 0\, \pi^0\right)$

$$A_\tau^{\mathrm{SM}}\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) = \left(3.6 \pm 0.1\right) \cdot 10^{-3}$$

Bigi-Sanda, Grossman-Nir

2.8 σ discrepancy

Belle does not see any asymmetry at the 10^{-2} level



$$A_i^{\mathrm{CP}} \simeq \langle \cos\beta \cos\psi \rangle_i^{\tau^-} - \langle \cos\beta \cos\psi \rangle_i^{\tau^+}$$

bins (i) of $W = \sqrt{Q^2}$

$\beta = K_S$ direction in hadronic rest frame

$\psi = \tau$ direction

BaBar signal incompatible (with EFT) with other sets of flavour data

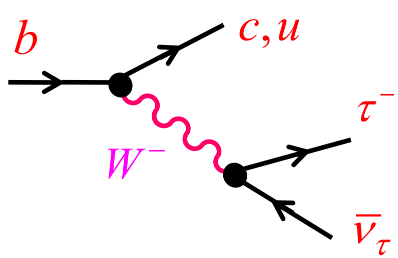
Cirigliano-Crivellin-Hoferichter, 1712.06595

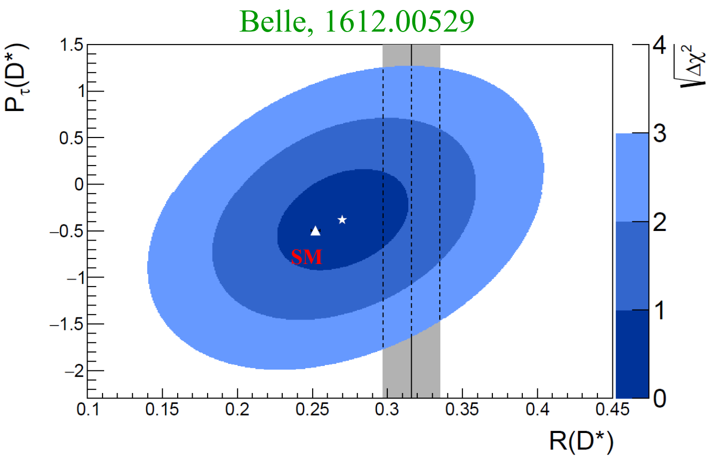
Rendón-Roig-Toledo, 1902.08143

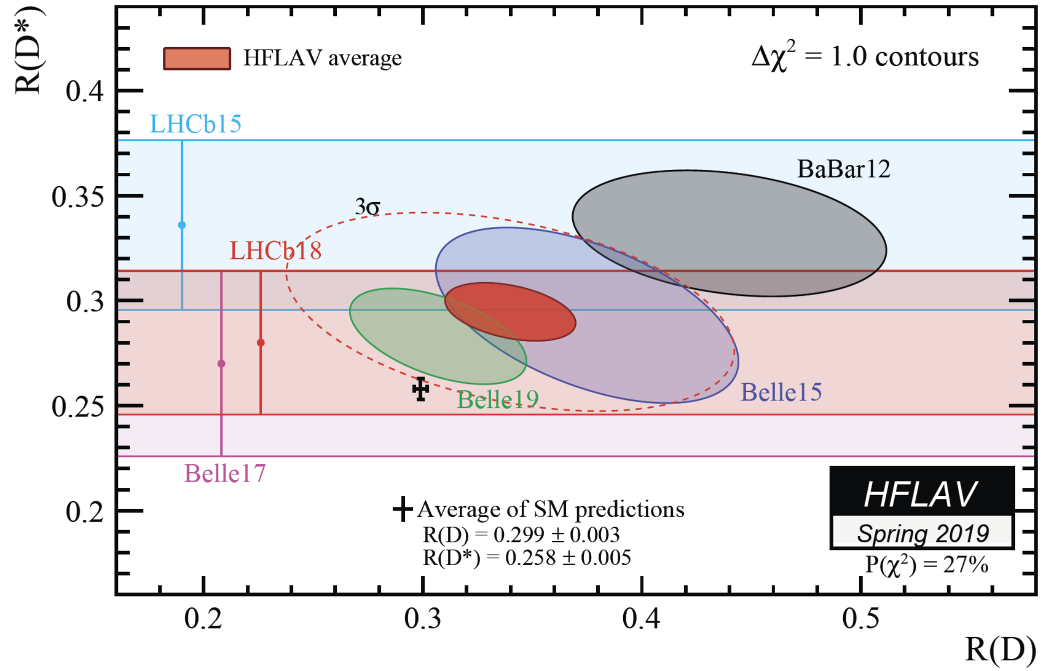
Flavour Anomaly

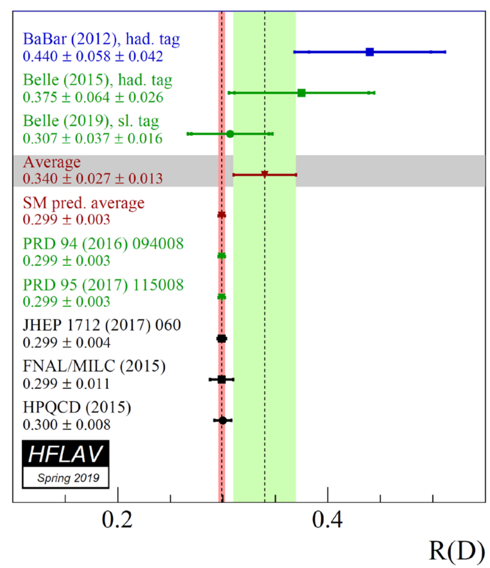
3.08 σ discrepancy

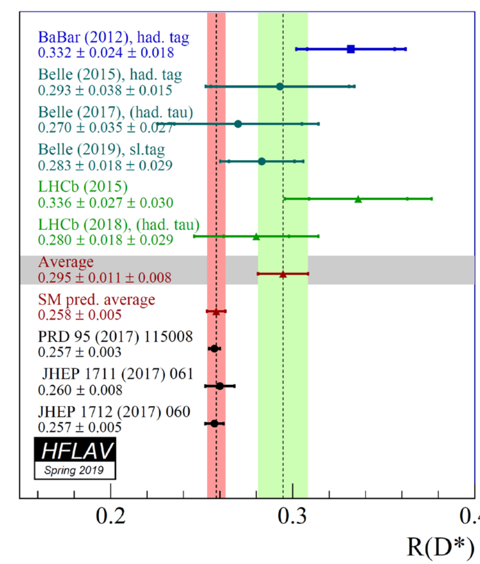
$$R(D^{(*)}) \equiv \frac{\mathrm{Br}(\bar{B} \to D^{(*)}\tau^- \bar{\nu}_\tau)}{\mathrm{Br}(\bar{B} \to D^{(*)}\ell^- \bar{\nu}_\ell)}$$











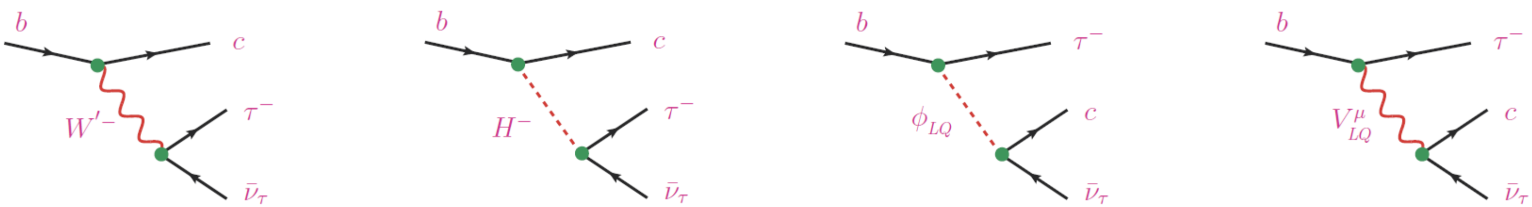
$$\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18 \qquad (1.7\,\sigma) \qquad \mathcal{R}^{\rm SM}_{J/\psi} \approx 0.26 - 0.28$$

$$F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \qquad (1.6\,\sigma) \qquad F_{L,\rm SM}^{D^*} = 0.455 \pm 0.003$$

Effective Field Theory Analysis

$$\mathcal{H}_{\text{eff}}^{b\to c\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[(1 + C_{V_L})\, \mathcal{O}_{V_L} + C_{V_R} \mathcal{O}_{V_R} + C_{S_R} \mathcal{O}_{S_R} + C_{S_L} \mathcal{O}_{S_L} + C_T \mathcal{O}_T \right] + \text{h.c.}$$

$$\mathcal{O}_{V_{L,R}} = (\bar{c}\,\gamma^\mu b_{L,R})\,(\bar{\ell}_L \gamma_\mu \nu_{\ell L})\,, \qquad \mathcal{O}_{S_{L,R}} = (\bar{c}\, b_{L,R})\,(\bar{\ell}_R \nu_{\ell L})\,, \qquad \mathcal{O}_T = (\bar{c}\,\sigma^{\mu\nu} b_L)\,(\bar{\ell}_R \sigma_{\mu\nu} \nu_{\ell L})$$



C_{V_L} $\qquad$ $C_{S_L}\,,\ C_{S_R}\quad (C_T)$ $\qquad$ $C_{V_L}\,,\ C_{S_L}\,,\ C_T$ $\qquad$ $C_{V_L}\,,\ C_{S_R}$

Many analyses (usually with single operator/mediator and partial data information)

Freytsis et al, Bardhan et al, Cai et al, Hu et al, Celis et al, Datta et al, Bhattacharya et al, Alonso et al, ...

Global fit to all data

(q^2 distributions included)

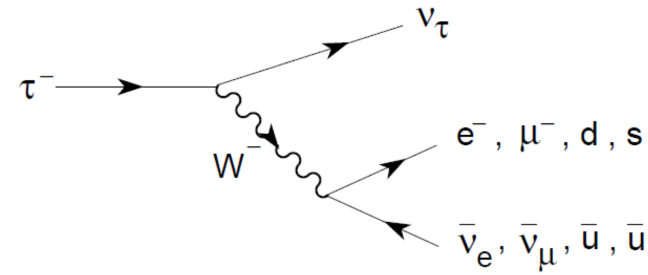
Murgui-Peñuelas-Jung-Pich, 1904.09311

$F_L^{D^*}$, $\mathcal{B}_{10}$	Min 1	Min 2
χ^2/d.o.f.	37.4/54	40.4/54
C_{LL}^V	$0.09\,^{+0.13}_{-0.12}$	$0.34\,^{+0.05}_{-0.07}$
C_{RL}^S	$0.09\,^{+0.12}_{-0.61}$	$-1.10\,^{+0.48}_{-0.07}$
C_{LL}^S	$-0.14\,^{+0.52}_{-0.07}$	$-0.30\,^{+0.11}_{-0.50}$
C_{LL}^T	$0.008\,^{+0.046}_{-0.044}$	$0.093\,^{+0.029}_{-0.030}$

$\mathcal{B}(B_c \to \tau\bar{\nu}) < 10\%$

$F_L^{D^*}$ included

SUMMARY



Many interesting τ topics

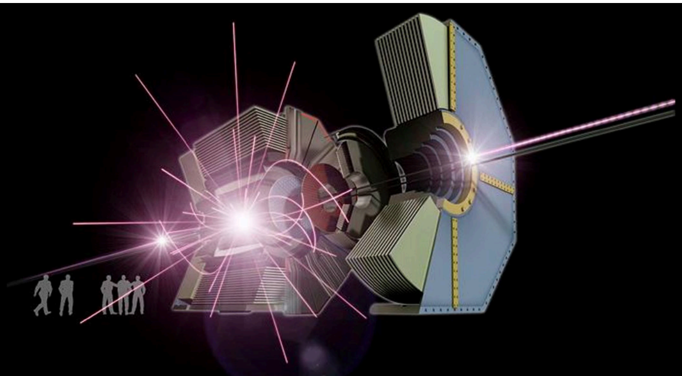
- Tests of QCD and the Electroweak Theory
- Looking for Signals of New Phenomena
- Superb Tool for New Physics Searches

Current anomalies: **Better data samples needed**

Lots of data will be produced @ Belle-II & LHC

Improving systematics brings a great reward

Backup



LORENTZ STRUCTURE

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{l'_\epsilon} \Gamma^n (\nu_{l'})_\sigma\right] \left[\overline{(\nu_l)_\lambda} \Gamma_n l_\omega\right]$$

90% CL — Fetscher-Gerber, PDG2020

$\mu \to e \overline{\nu}_e \nu_\mu$

$\lvert g^S_{RR}\rvert < 0.035$	$\lvert g^V_{RR}\rvert < 0.017$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.050$	$\lvert g^V_{LR}\rvert < 0.023$	$\lvert g^T_{LR}\rvert < 0.015$
$\lvert g^S_{RL}\rvert < 0.420$	$\lvert g^V_{RL}\rvert < 0.105$	$\lvert g^T_{RL}\rvert < 0.105$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2g^T_{RL}\rvert < 0.418$	
$Q_{RR} + Q_{LR} < 8.2 \times 10^{-4}$		

95% CL — Stahl, PDG2020

$\tau \to e \overline{\nu}_e \nu_\tau$

$\lvert g^S_{RR}\rvert < 0.70$	$\lvert g^V_{RR}\rvert < 0.17$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.99$	$\lvert g^V_{LR}\rvert < 0.13$	$\lvert g^T_{LR}\rvert < 0.082$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$

$\tau \to \mu \overline{\nu}_\mu \nu_\tau$

$\lvert g^S_{RR}\rvert < 0.72$	$\lvert g^V_{RR}\rvert < 0.18$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.95$	$\lvert g^V_{LR}\rvert < 0.12$	$\lvert g^T_{LR}\rvert < 0.079$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$

$\tau \to \pi \nu_\tau$

$\lvert g^V_R\rvert < 0.15$ $\quad$ $\lvert g^V_L\rvert > 0.992$

$\tau \to \rho \nu_\tau$

$\lvert g^V_R\rvert < 0.10$ $\quad$ $\lvert g^V_L\rvert > 0.995$

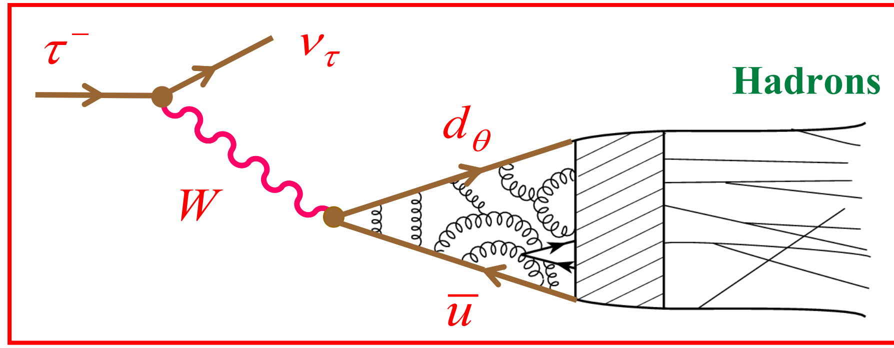
$\tau \to a_1 \nu_\tau$

$\lvert g^V_R\rvert < 0.16$ $\quad$ $\lvert g^V_L\rvert > 0.987$

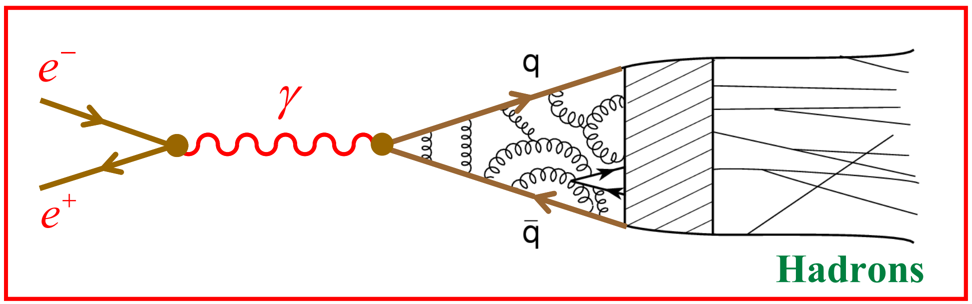
Only Lepton Massive Enough to Decay into Hadrons

$\tau^- \to \nu_\tau H^-$ probes the hadronic V-A current

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1-\gamma_5) u | 0 \rangle$$



$e^+e^- \to H^0$ probes the hadronic electromagnetic current



$$\langle H^0 | \sum_q Q_q \, \bar{q} \gamma^\mu q | 0 \rangle$$

Isospin: $$\frac{\Gamma(\tau^- \to \nu_\tau V^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{3\cos^2\theta_C}{2\pi\alpha^2} S_{\mathrm{EW}} \int_0^1 dx\, (1-x)^2 (1+2x)\, x\; \sigma^{I=1}_{e^+e^- \to V^0}(x m_\tau^2)$$

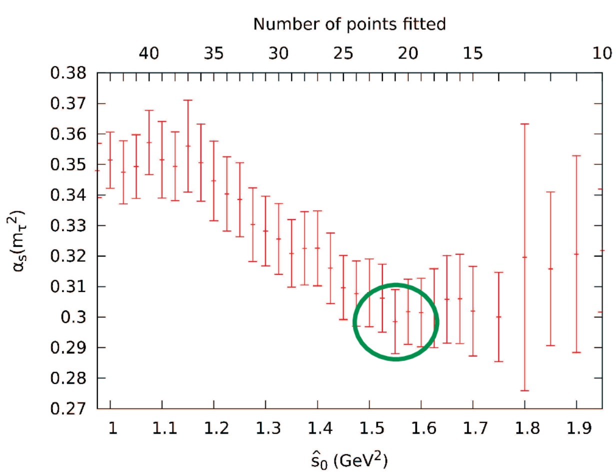
Models of Duality Violation

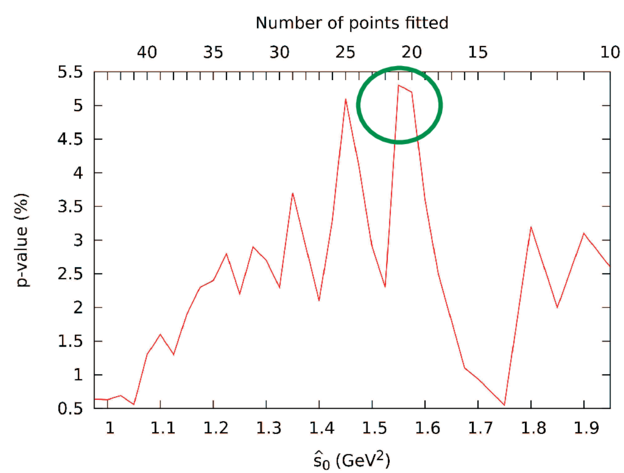
$$\Delta A^{\omega}_{V/A}(s_0) = \frac{i}{2}\oint_{|s|=s_0}\frac{ds}{s_0}\,\omega(s)\,\left\{\Pi_{V/A}(s)-\Pi^{\rm OPE}_{V/A}(s)\right\} = -\pi\int_{s_0}^{\infty}\frac{ds}{s_0}\,\omega(s)\,\Delta\rho^{\rm DV}_{V/A}(s)$$

Ansatz: $\Delta\rho^{\rm DV}_{V/A}(s) = s^{\lambda_{V/A}}\, e^{-(\delta_{V/A}+\gamma_{V/A}\, s)}\,\sin(\alpha_{V/A}+\beta_{V/A}\, s) \quad , \quad s > \hat{s}_0$

1) Boito et al.: $\lambda_{V/A} = 0 \quad , \quad \hat{s}_0 \sim 1.55\ {\rm GeV}^2 \quad , \quad \omega(x) = 1$

- Fit s_0 dependence: ➔ $\{A^{(00)}(s_0), \rho(s_0+\Delta s_0), \cdots, \rho(s_0+(n-1)\Delta s_0)\}$
- Direct fit of the spectral function. **OPE not valid**





Rodríguez-Sánchez, A.P.

FOPT , V

(too large errors in A)

Boito et al. value

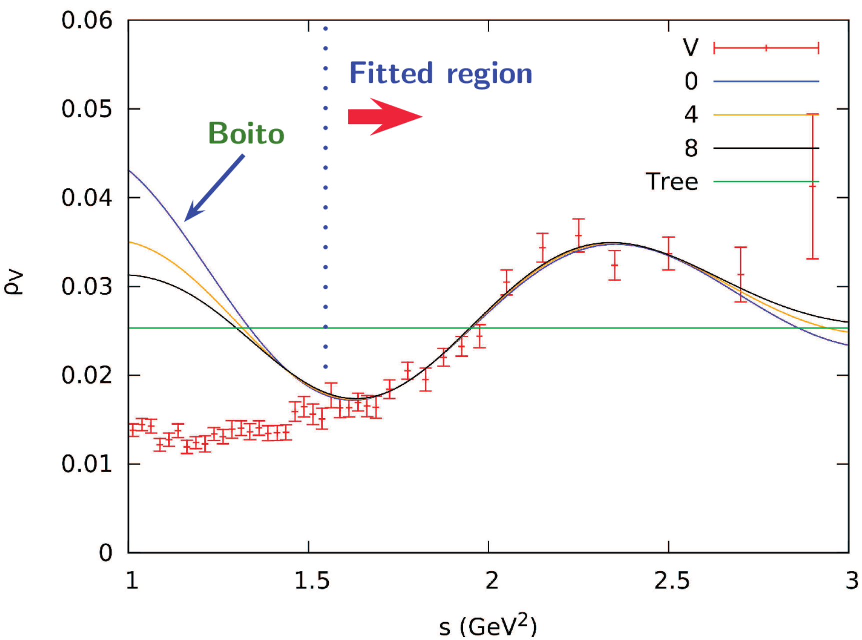
Bad quality fit **(Model dependence. Instabilities. Very low p-value)**

Ansatz: $\Delta\rho^{\rm DV}_{V/A}(s) \; = \; s^{\lambda_{V/A}}\, e^{-(\delta_{V/A}+\gamma_{V/A}\, s)}\, \sin\left(\alpha_{V/A} + \beta_{V/A}\, s\right) \quad , \quad s > \hat{s}_0$

2) $\lambda_V \geq 0$: $\quad \hat{s}_0 \sim 1.55\ \mathrm{GeV}^2 \quad , \quad \omega(x) = 1$

Rodríguez-Sánchez, A.P.

	λ_V	$\alpha_s(m_\tau^2)^{\rm FOPT}$	δ_V	γ_V	α_V	β_V	p-value
Boito	0	0.298 ± 0.010	3.6 ± 0.5	0.6 ± 0.3	-2.3 ± 0.9	4.3 ± 0.5	5.3 %
	1	0.300 ± 0.012	3.3 ± 0.5	1.1 ± 0.3	-2.2 ± 1.0	4.2 ± 0.5	5.7 %
	2	0.302 ± 0.011	2.9 ± 0.5	1.6 ± 0.3	-2.2 ± 0.9	4.2 ± 0.5	6.0 %
	4	0.306 ± 0.013	2.3 ± 0.5	2.6 ± 0.3	-1.9 ± 0.9	4.1 ± 0.5	6.6 %
	8	0.314 ± 0.015	1.0 ± 0.5	4.6 ± 0.3	-1.5 ± 1.1	3.9 ± 0.6	7.7 %

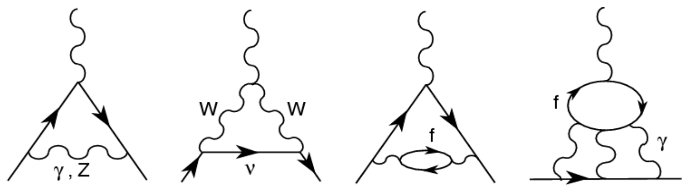


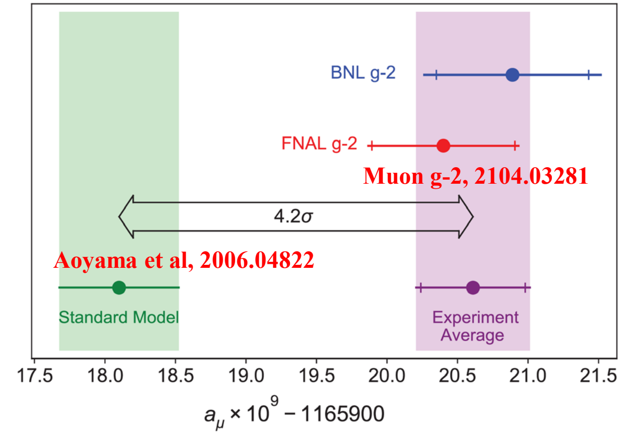
- **Fitted α_s is model dependent**
- **$\lambda_V = 0$ (Boito) gives the worse fit**
- **Fit quality & α_s increase with λ_V**

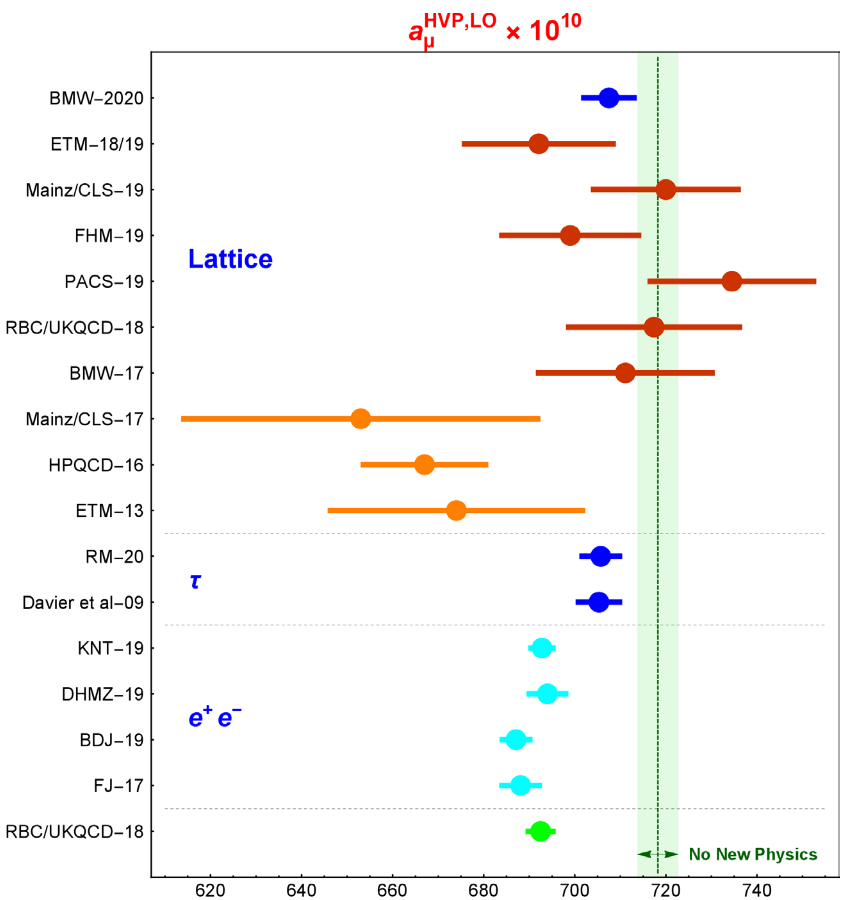
  → **closer to data at $s < \hat{s}_0$**
- **$\Delta\hat{s}_0$ → 3 times larger errors**

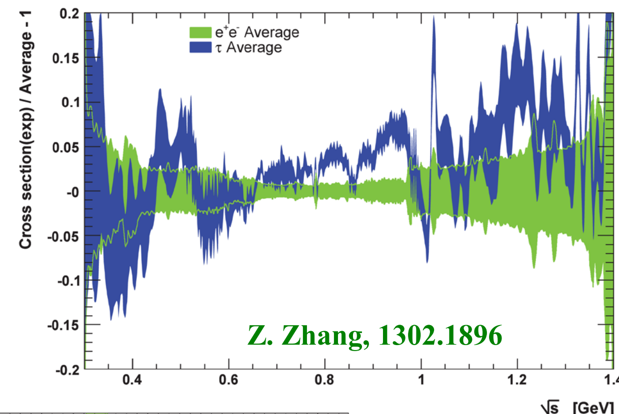
Not competitive & unreliable

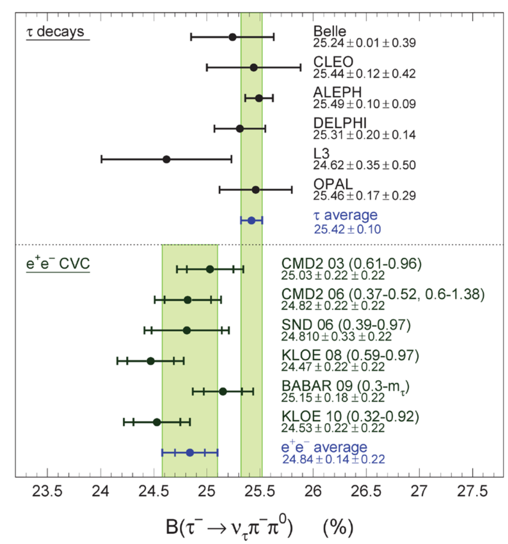
μ Anomalous Magnetic Moment





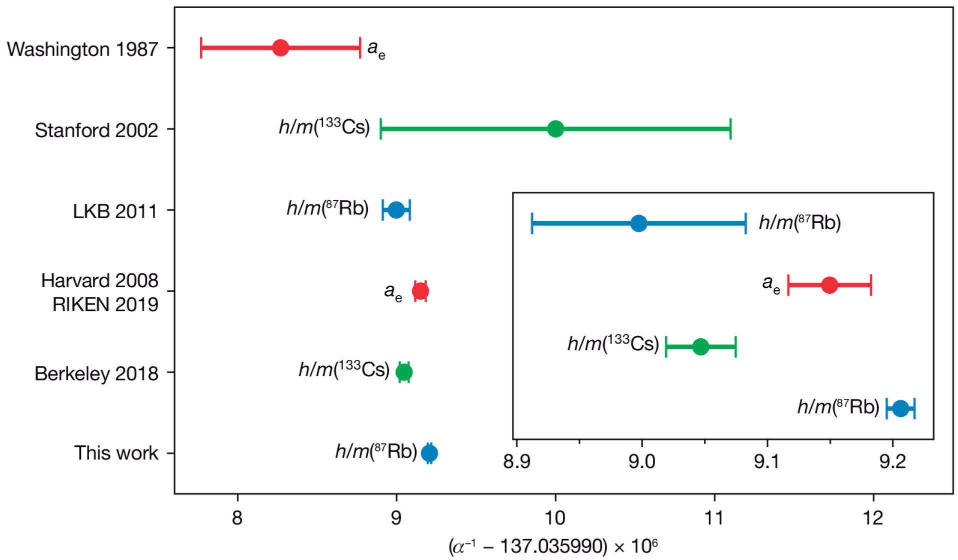






M. Davier
Tau 10

Electron Anomalous Magnetic Moment



Morel et al, Nature 588 (2020) 61

New measurement of α

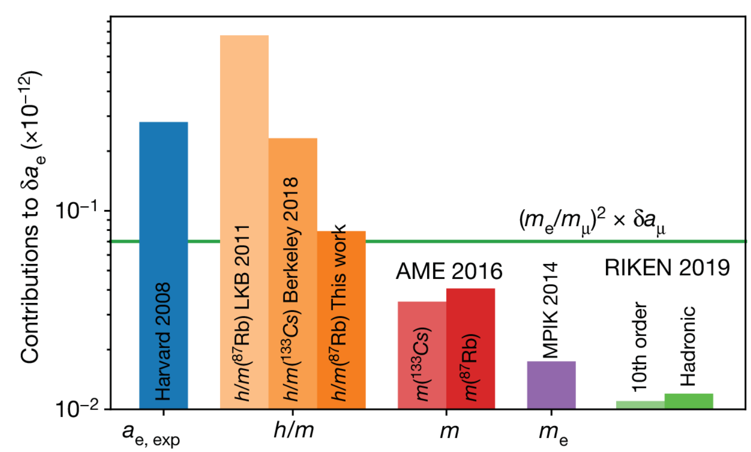
$$\alpha^{-1}(\mathrm{Rb}) = 137.035\,999\,206\,(11)$$

8.1×10^{-11} accuracy

$5.8\,\sigma$ discrepancy with Cs experiment

$$\Delta a_e \equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}}$$

$$= \begin{cases} (-8.8 \pm 3.6) \cdot 10^{-13} & (\mathrm{Cs},\ -2.4\sigma) \\ (+4.8 \pm 3.0) \cdot 10^{-13} & (\mathrm{Rb},\ +1.6\sigma) \end{cases}$$



τ Anomalous Magnetic Moment

Difficult to measure! $a_\tau^{\mathrm{exp}} = (-0.018 \pm 0.017)$ **DELPHI**

$-0.007 < a_\tau^{\mathrm{New\,Phys}} < 0.005$ **González-Springer, Santamaria, Vidal '00** **(LEP/SLD data)**

Eidelman, Passera

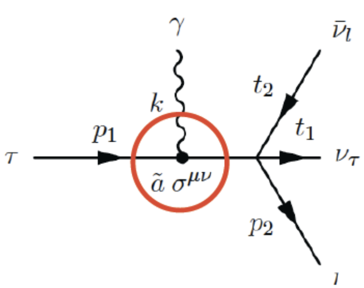
$10^8 \cdot a_\tau^{\mathrm{th}} =$	117 324	$\pm$ 2	QED
	+ 47.4	$\pm$ 0.5	EW
	+ 337.5	$\pm$ 3.7	hvp
	+ 7.6	$\pm$ 0.2	hvp NLO
	+ 5	$\pm$ 3	light-by-light
	= 117 721	**$\pm$ 5**	

Enhanced sensitivity to new physics: $(m_\tau/m_\mu)^2 = 283$

	Electron	Muon	Tau
a^{EW}/a^{HAD}	1/56	1/45	1/7
$a^{EW}/\delta a^{HAD}$	1.6	3	10

Essentially unknown

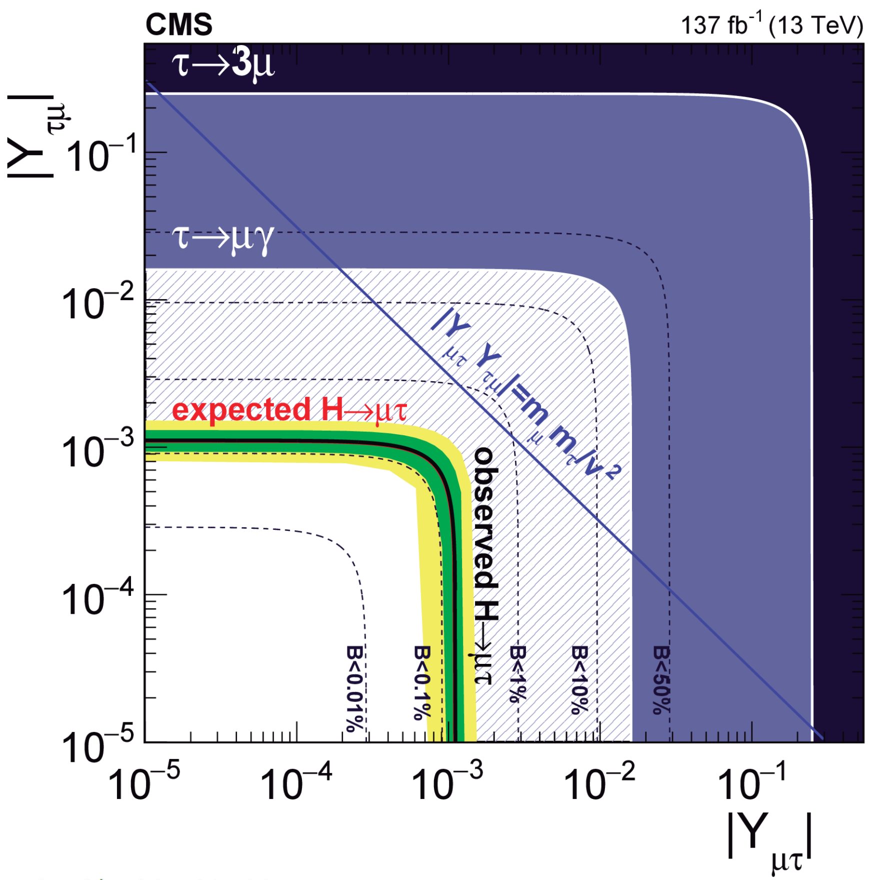
May be accessible at BFs through radiative leptonic decays **(Fael et al)** or with a polarized e⁻ beam **(Crivellin et al)**



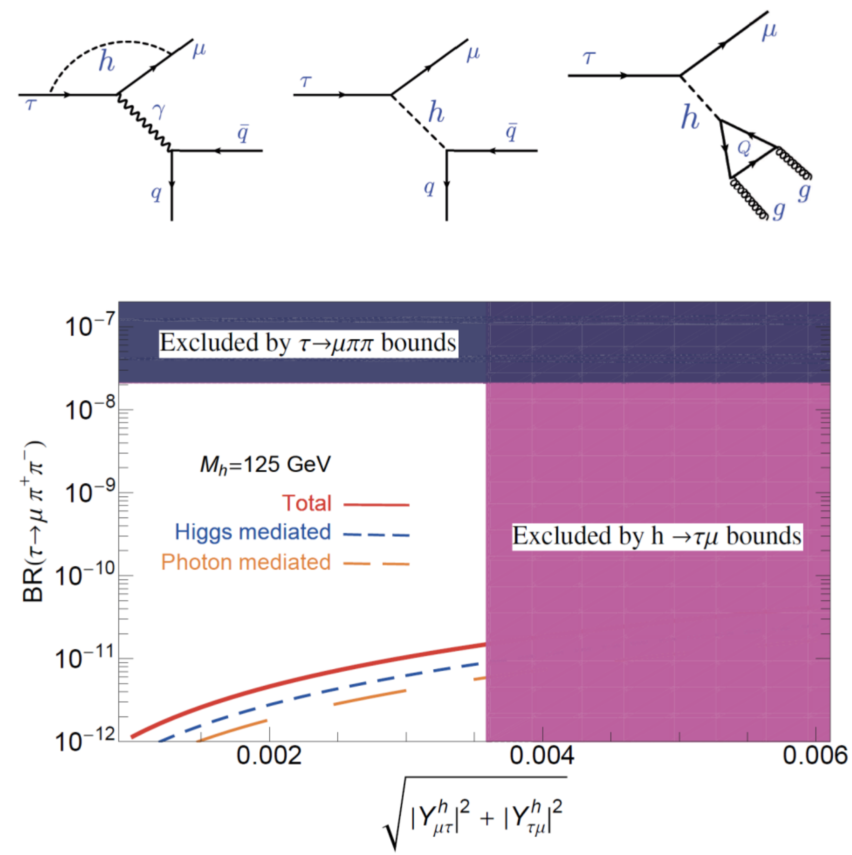
Flavour-Violating Higgs Couplings

$$\mathcal{L} = -H \,\{ Y_{e\mu}\, \bar{e}_L \mu_R + Y_{e\tau}\, \bar{e}_L \tau_R + Y_{\mu\tau}\, \bar{\mu}_L \tau_R + \cdots \}$$

$\mathrm{Br}\left(H \to \mu\tau\right) < 0.15\%$ (95% CL)



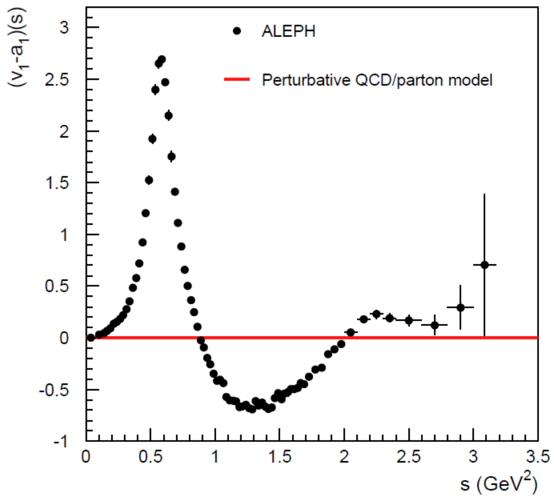
$\tau \to \mu\, \pi^+ \pi^-$ Celis et al., 1409.4439



τ Data Samples

ALEPH:	$3.3 \cdot 10^5$ reconstructed τ decays	
BaBar / Belle:	$1.4 \cdot 10^9$ $\tau^+\tau^-$ pairs	
Belle-II:	$4.6 \cdot 10^{10}$ $\tau^+\tau^-$ pairs	
sτcF:	$2.1 \cdot 10^{10}$ $\tau^+\tau^-$ pairs	(10^8 near threshold)

Luminosity is important. Systematics also!



Chiral Sum Rules

$$\Pi(s) \equiv \Pi_{VV}(s) - \Pi_{AA}(s)$$

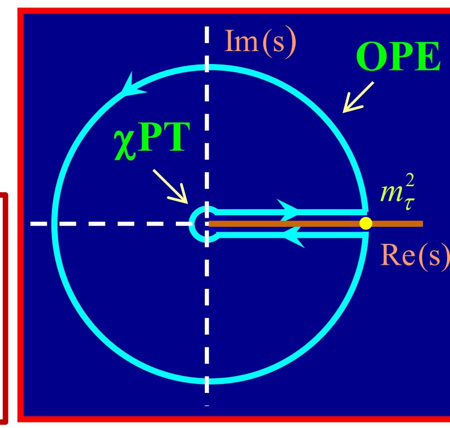
Pure non-perturbative quantity

$$\lim_{s\to\infty} s^2\,\Pi(s) = 0 \quad\rightarrow\quad \Pi^{\rm OPE}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \cdots$$

χPT (s→0):

$$\Pi(s) = \frac{2F^2}{s} - 8\,L_{10}^r(\mu^2) + \frac{1}{16\pi^2}\left(\frac{5}{3} - \ln\frac{-s}{\mu^2}\right) + 16\,C_{87}^r(\mu^2)\,\frac{s}{F^2} + \cdots$$

$$\int_{s_{\rm th}}^{s_0} ds\,\omega(s)\,\frac{1}{\pi}\,{\rm Im}\,\Pi(s) + \frac{1}{2\pi i}\oint_{|s|=s_0} ds\,\omega(s)\,\Pi(s) \;=\; 2\,f_\pi^2\,\omega(m_\pi^2) + {\rm Res}[\omega(s)\Pi(s),\, s=0]$$



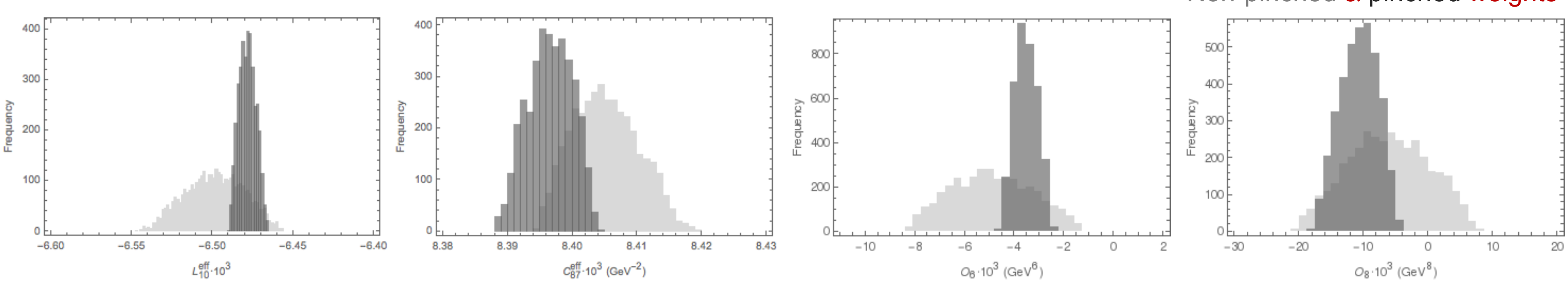
Statistical analysis:

González-Pich-Rodríguez, 1602.06112

$$C_{87}^{\rm eff} = (8.40 \pm 0.18)\cdot 10^{-3}\ {\rm GeV}^{-2} \qquad O_6 = (-3.6 \pm 0.7)\cdot 10^{-3}\ {\rm GeV}^6$$

$$L_{10}^{\rm eff} = (-6.48 \pm 0.05)\cdot 10^{-3}\ . \qquad O_8 = (-1.0 \pm 0.4)\cdot 10^{-2}\ {\rm GeV}^8$$

Non-pinched & pinched weights



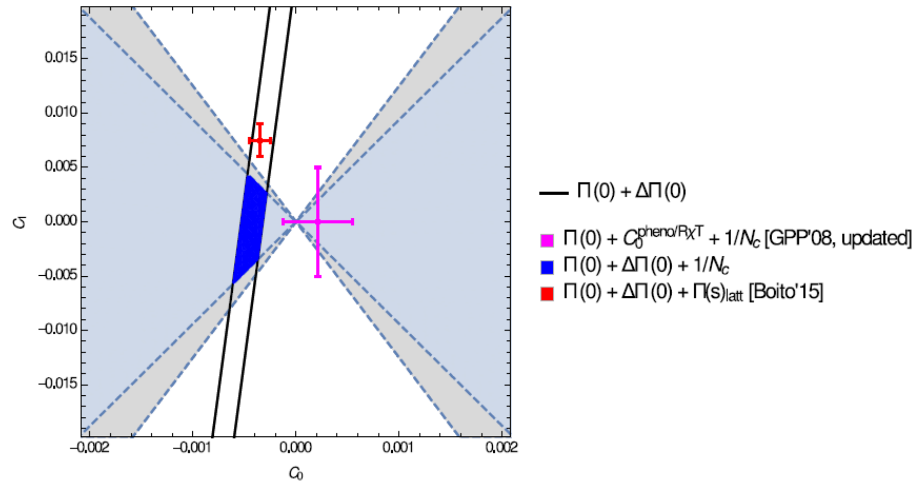
• χPT Parameters:

González-Alonso, Pich, Rodríguez-Sánchez, 1602.06112

$$L_{10}^{\rm eff} = L_{10}^r - 0.00126 + \mathcal{O}(p^6)$$

$$L_{10}^{\rm eff} = 1.53\, L_{10}^r + 0.263\, L_9^r - 0.00179 - \frac{1}{8}\left(\mathcal{C}_0^r + \mathcal{C}_1^r\right) + \mathcal{O}(p^8)$$

$$C_{87}^{\rm eff} = C_{87}^r + 0.296\, L_9^r + 0.00155 + \mathcal{O}(p^8)$$



- $\mathcal{O}(p^4)$ analysis: $L_{10}^r(M_\rho) = -(5.22 \pm 0.05)\cdot 10^{-3}$
- $\mathcal{O}(p^6)$ analysis: $L_{10}^r(M_\rho) = -(4.1 \pm 0.4)\cdot 10^{-3}$

  $C_{87}^r(M_\rho) = (5.10 \pm 0.22)\cdot 10^{-3}\ \mathrm{GeV}^{-2}$

• $\varepsilon'_K/\varepsilon_K$: $\mathcal{O}_6$ → $\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle$ → e.m. penguin contribution

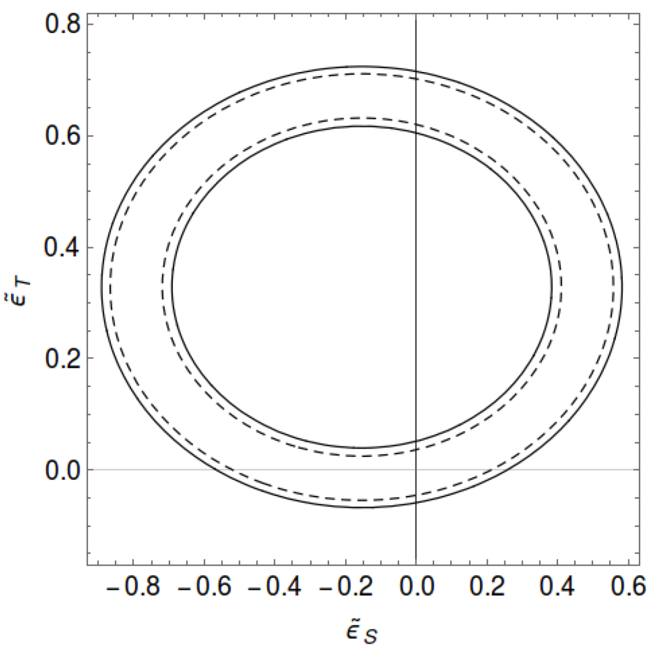
$$\left(\varepsilon'_K/\varepsilon_K\right)_{\rm EWP}^{I=2} = (-4.5 \pm 1.8)\cdot 10^{-4}$$

Pich-Rodríguez, 2102.09308

EFT analysis of $\tau \to \nu_\tau K\pi$

Rendón-Roig-Toledo, 1902.08143

$$\mathcal{L}_{cc} = -\frac{G_F V_{us}}{\sqrt{2}}(1+\epsilon_L+\epsilon_R)\Big[\bar\tau\gamma_\mu(1-\gamma_5)\nu_\ell\cdot\bar u[\gamma^\mu-(1-2\hat\epsilon_R)\gamma^\mu\gamma_5]s + \bar\tau(1-\gamma_5)\nu_\ell\cdot\bar u[\hat\epsilon_s-\hat\epsilon_p\gamma_5]s + 2\hat\epsilon_T\bar\tau\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell\cdot\bar u\sigma^{\mu\nu}s\Big] + \text{h.c.}$$



Best fit values	$\hat\epsilon_S$	$\hat\epsilon_T$	χ^2	χ^2 in the SM
Excluding $i = 5,6,7$ bins	$(1.3 \pm 0.9)\times 10^{-2}$	$(0.7 \pm 1.0)\times 10^{-2}$	$[72, 73]$	$[74, 77]$
Including $i = 5,6,7$ bins	$(0.9 \pm 1.0)\times 10^{-2}$	$(1.7 \pm 1.7)\times 10^{-2}$	$[83, 86]$	$[91, 95]$

⇒ $\Lambda_{NP} \geq 2-5$ TeV

Complementary to kaon and hyperon data analyses

τ's @ LHC

- **Excellent signature to probe New Physics**
  Difficult to identify light objects ($Z, W^{\pm}$) with only Jets
  QCD Jets orders of magnitude larger
  Must rely on leptons

- **LHC produces high-momenta τ's**
  Tightly collimated decay products (mini-jet like)
  Momentum reconstruction possible

- **Low multiplicity.** Good tagging efficiency

- **Heaviest lepton coupling to the Higgs** (4^{th} H Br)

- **Polarization information**