Introduction to accelerator physics at SuperKEKB

Demin Zhou
dmzhou@post.kek.jp

Accelerator theory group, Accelerator laboratory, KEK

With materials taken from

Y. Funakoshi, N. Iida, M. Masuzawa, K. Hosoyama, Y. Ohnishi, K. Ohmi

and special thanks to Diego Tonelli for coordination of my talk

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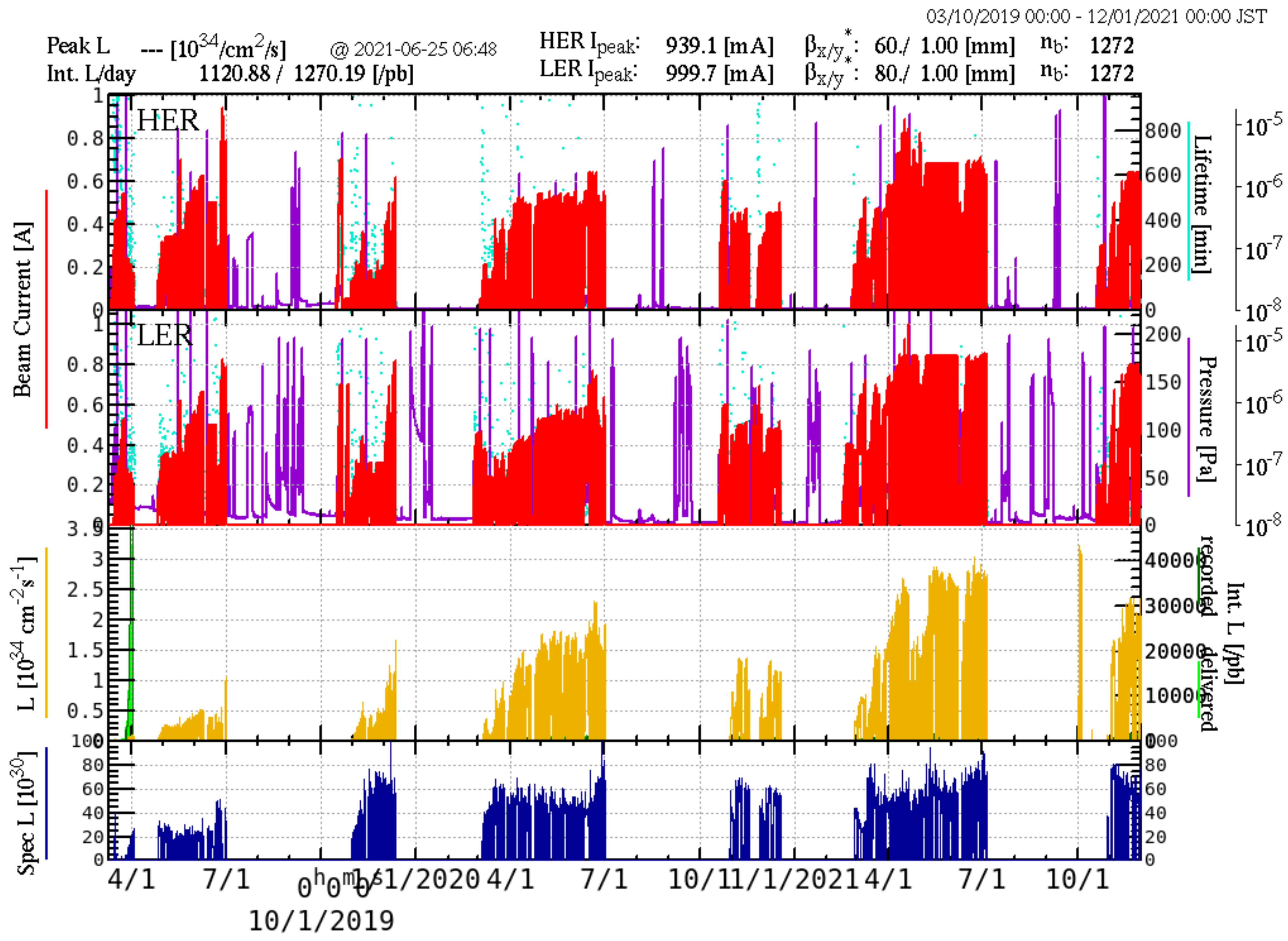
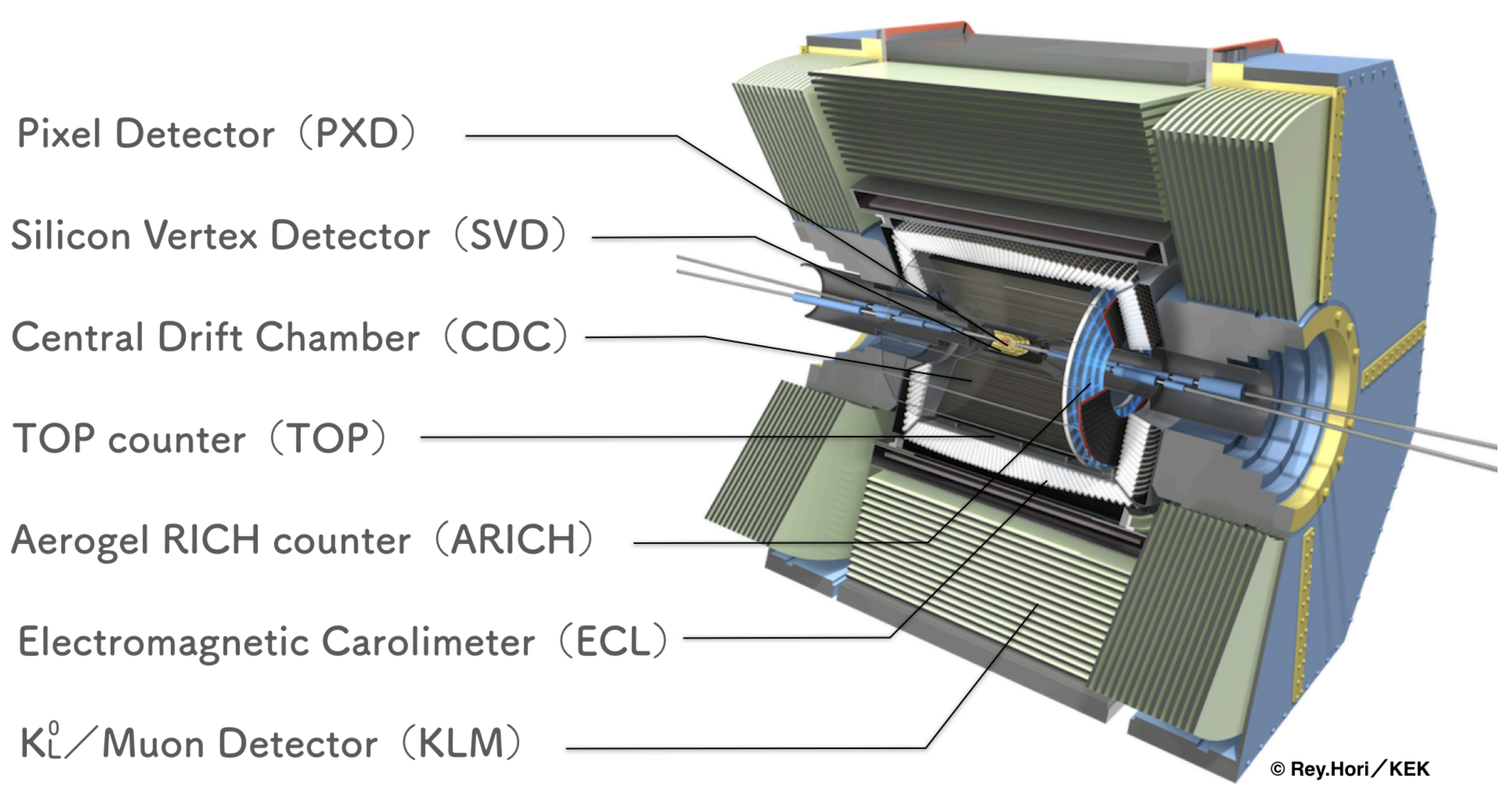
Outline

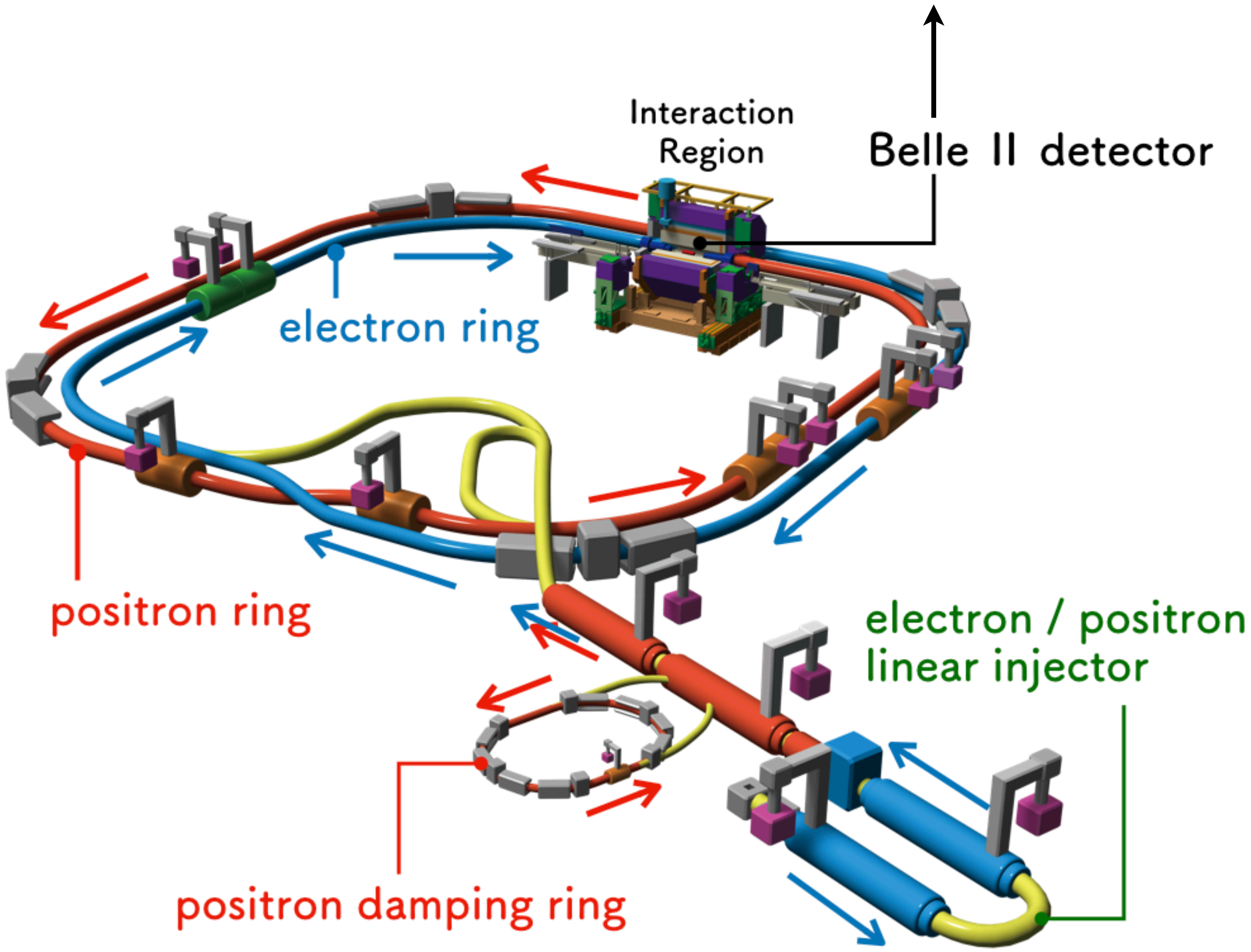
- Machine overview
- Luminosity
- Single-particle linear dynamics
- Design strategy for SuperKEKB
- Single-particle nonlinear dynamics
- Collective effects
  - Wake fields
  - Beam-beam interaction
- Machine tunings
- Challenges in accelerator physics at SuperKEKB
- Summary

Machine overview

- Phase-1: From February, 2016 to June, 2016 w/o QCS magnets and Belle II.
- Phase-2: From February, 2018 to July, 2018 w/ QCS and Belle II, w/o Vertex detector.
- Phase-3: Started from March, 2019 w/ Full Belle II.







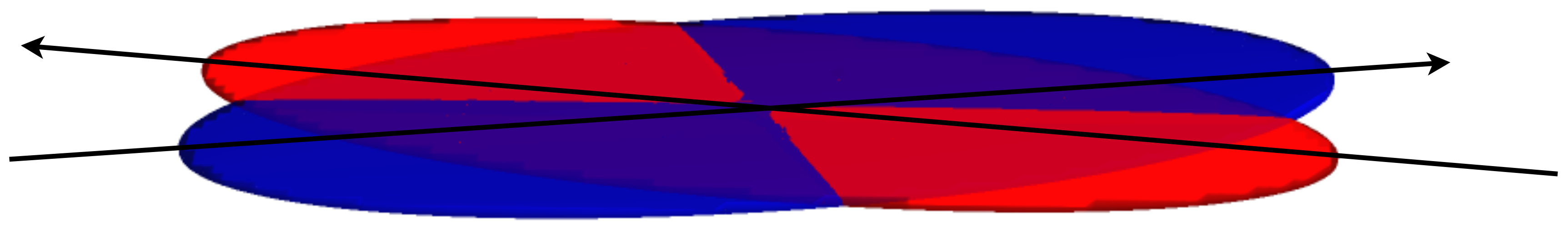
Machine overview

- Collision scheme (KEKB → SuperKEKB)
  - Beam energy E (LER/HER): 3.5/8 ⇒ 4/7 GeV.
  - Vertical beam-beam parameter ξ_y: 0.09 ⇒ 0.09.
  - Crab waist: Optional.
  - Luminosity L: 2.1 ⇒ 80 $\times 10^{34}$ cm^{-2}s^{-1}.

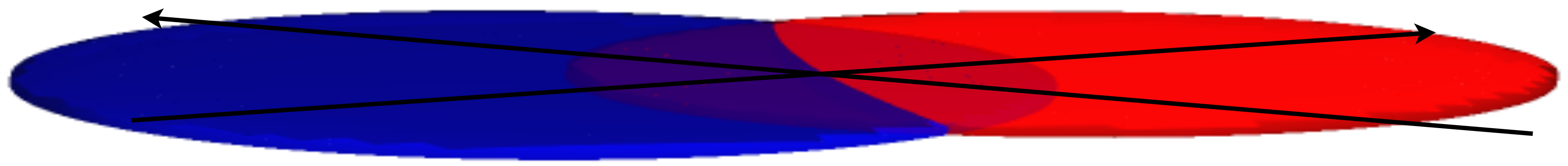
	KEKB (2009.06.17)		SKEKB (2021c)		SKEKB (Final design)	
	HER	LER	HER	LER	HER	LER
I_{bunch} (mA)	1.2	1.0	0.64	0.8	2.6	3.6
# bunch	1585		1272		2500	
ε_x (nm)	24	18	4.6	4.0	4.6	3.2
ε_y (pm)	150	150	40	40	12.9	8.64
β_x (mm)	1200	1200	60	80	25	32
β_y (mm)	5.9	5.9	1	1	0.3	0.27
σ_z (mm)	6	6	5	6	5	6
ν_x	44.511	45.506	45.533	44.525	45.53	44.53
ν_y	41.585	43.561	43.581	46.595	43.57	46.57
ν_s	0.0209	0.0246	0.0272	0.0233	0.028	0.0245
Crab waist	-		40%	80%		
Crossing angle (mrad)	0 (22)		83		83	

Schematic view of collision schemes

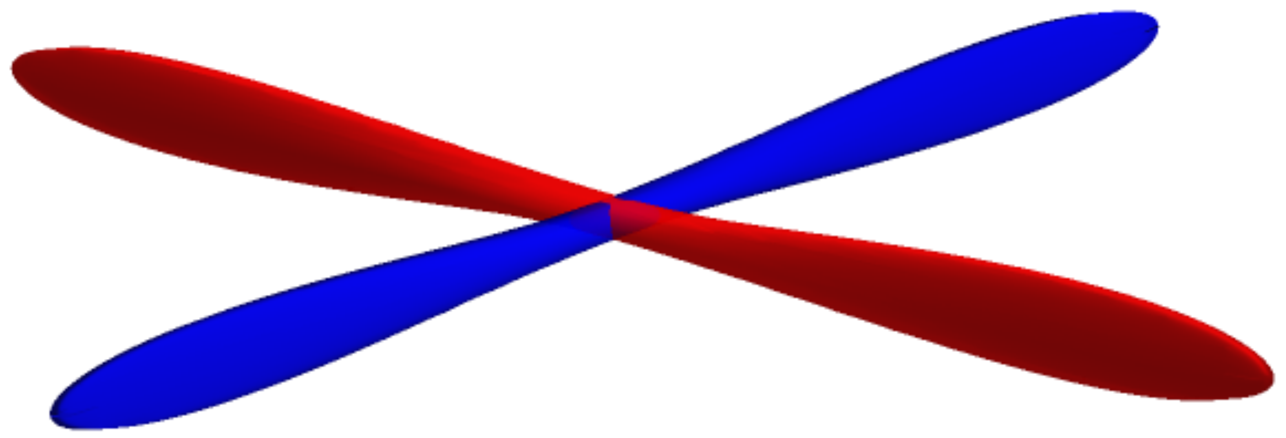
KEKB (Crossing angle)



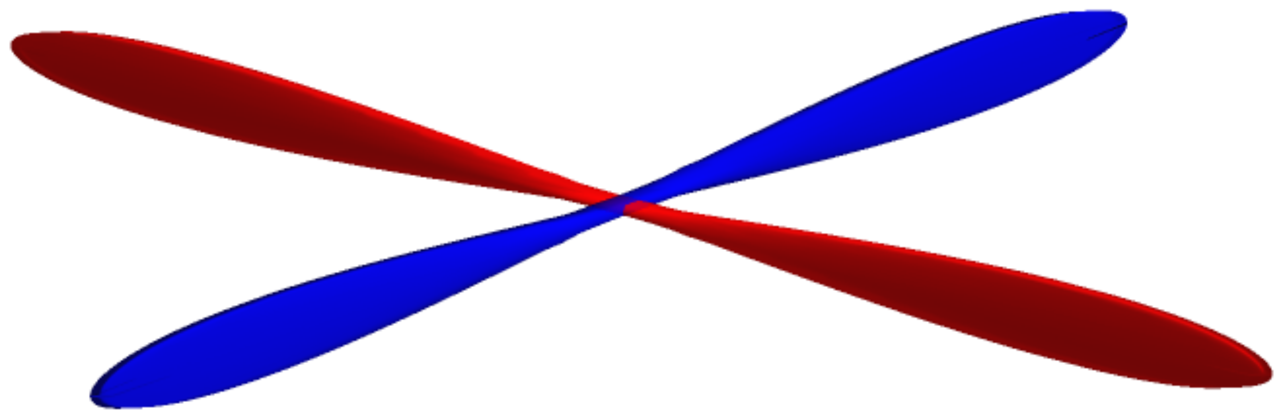
KEKB (Crab cavity)



SuperKEKB (2021c)



SuperKEKB (Final design)



Luminosity

- Luminosity is one of the most important performance parameter for a collider [1]:

$$L = \frac{N_+ N_-}{s_b} K \iiiint_{-\infty}^{\infty} \rho_+(x, y, s, -s_0)\rho_-(x, y, s, s_0) dx dy ds ds_0$$

- $N_\pm$ is the bunch population, s_b is the bunch spacing, K is defined by:

$$K = \sqrt{(\vec{v}_+ - \vec{v}_-)^2 - (\vec{v}_+ \times \vec{v}_-)^2/c^2}$$

- Usually 3D Gaussian distribution is a good approximation for the beam distribution:

$$\rho_+(x, y, s, s_0) = \frac{1}{(2\pi)^{3/2}\sigma_{x+}(s)\sigma_{y+}(s)\sigma_{z+}} e^{-\frac{x^2}{2\sigma_{x+}^2(s)} - \frac{y^2}{2\sigma_{y+}^2(s)} - \frac{(s-s_0)^2}{2\sigma_{z+}^2}}$$

- Note that the transverse beam sizes depend on the longitudinal position. This is called "hour-glass" effect:

$$\sigma_u(s) = \sigma_u^* \sqrt{1 + \frac{s^2}{\beta_u^{*2}}} \quad \textbf{with } u = x, y$$

- The beam sizes at interaction point (IP) can be simply written as:

$$\sigma_u^* = \sqrt{\beta_u^* \epsilon_u} \quad \textbf{with } u = x, y$$

- $\beta_{x,y}^*$ are the beta functions at IP and $\epsilon_{x,y}$ are the emittances in x and y directions. They are important concepts in accelerator physics and will be discussed later.

[1] W. Herr and B. Muratori, "Concept of luminosity", http://cds.cern.ch/record/941318

Luminosity

- After tedious calculations with approximations, the explicit form of luminosity formula with inclusion of "hour-glass" effect can be obtained:

$$L = L_0 R_\theta$$

$$L_0 = \frac{N_b N_+ N_- f}{2\pi\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}}}$$

$$R_\theta \approx \sqrt{\frac{2}{\pi}}a \cdot e^b K_0(b)$$

$$a = \frac{t_y}{\sqrt{2}} = \sqrt{\frac{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}{\left(\sigma_{z+}^2 + \sigma_{z-}^2\right)\left(\frac{\sigma_{y+}^{*2}}{\beta_{y+}^{*2}} + \frac{\sigma_{y-}^{*2}}{\beta_{y-}^{*2}}\right)}}$$

$$b = a^2\left(1 + \frac{\sigma_{z+}^2 + \sigma_{z-}^2}{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}}\tan^2\frac{\theta_c}{2}\right)$$

- The equation of R_θ is valid with the condition of $\beta_x^* \gg \beta_y^*$ which is obvious for SuperKEKB.
- With $b/a^2 \gg 1$ and $a \approx \beta_y^*/\sigma_z$ not too small (it means "hour-glass" effect is negligible), R_θ can be approximated by:

$$R_\theta \approx \frac{a}{\sqrt{b}} = \frac{1}{\sqrt{1 + \frac{\sigma_{z+}^2 + \sigma_{z-}^2}{\sigma_{x+}^{*2} + \sigma_{x-}^{*2}}\tan^2\frac{\theta_c}{2}}}$$

- The last approximation is to assume the so-called large Piwinski-angle:

$$\frac{\sigma_z}{\sigma_x}\tan\frac{\theta_c}{2} \gg 1$$

- With the above condition, we obtain a simple formula for luminosity:

$$L \approx \frac{N_b N_+ N_- f}{2\pi\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2}\tan\frac{\theta_c}{2}}$$

Luminosity

- Accelerator physics behind the luminosity at SuperKEKB

$$L \approx \frac{N_b N_+ N_- f}{2\pi\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2}\tan\frac{\theta_c}{2}}$$

N_b:
* Tolerance of hardwares
* Injection
* …

$N_+ N_-$:
* Impedance effects (TMCI, PWD, HOM, etc.)
* Beam-beam blowup
* …

$\sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}$:
* TMCI (Y-Z instability)
* Beam-beam blowup
* β_y^*, ϵ_y
* Optics correction
* Tunes $\nu_{x,y}$
* Machine imperfections
* …

$\sqrt{\sigma_{z+}^2 + \sigma_{z-}^2}$:
* Impedance effects
* Beam-beam blowup
* …

$\frac{\theta_c}{2}$:
* Coherent X-Z instability
* Beam-beam resonances (X-Y coupling)
* β_x^*
* Crab waist
* …

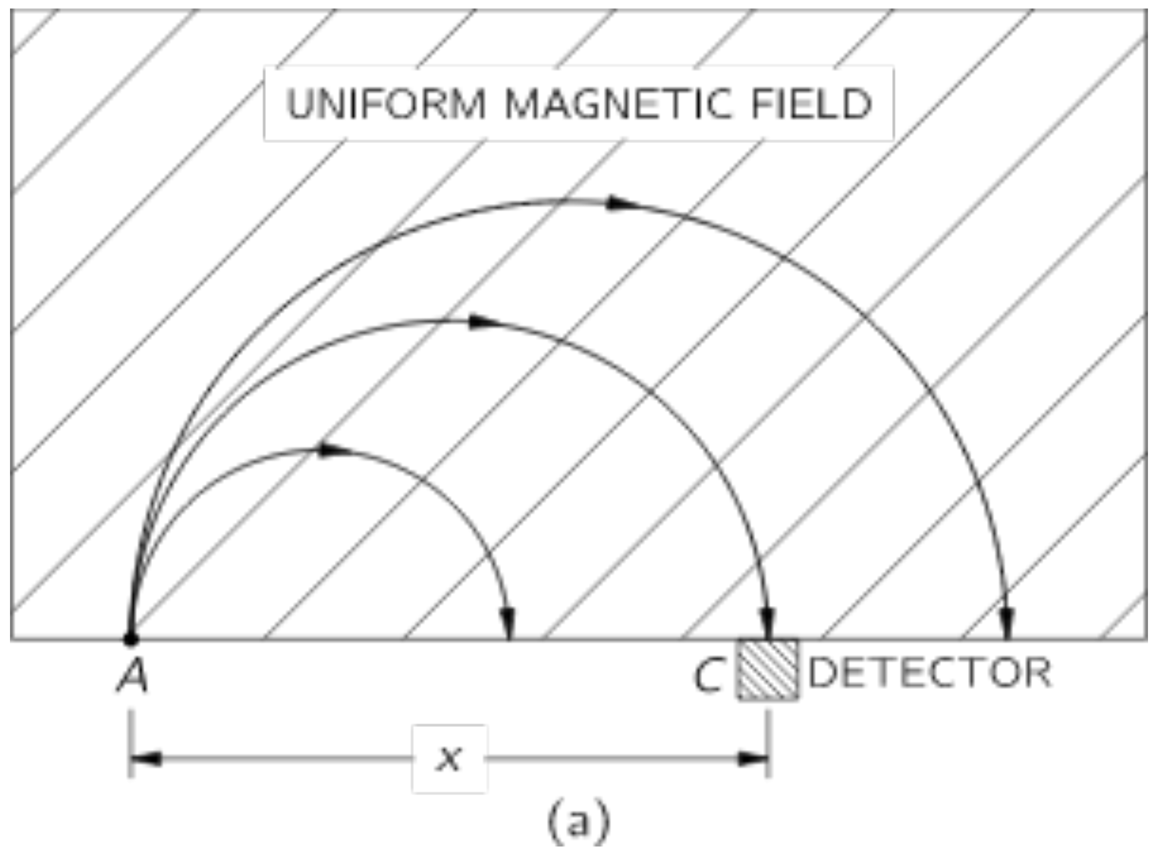
Note:
* $\sigma_{x\pm}^*$ do not appear in this luminosity formulae. But they play a role of "invisible hand" and have very important impact on beam dynamics, eventually affecting the luminosity.

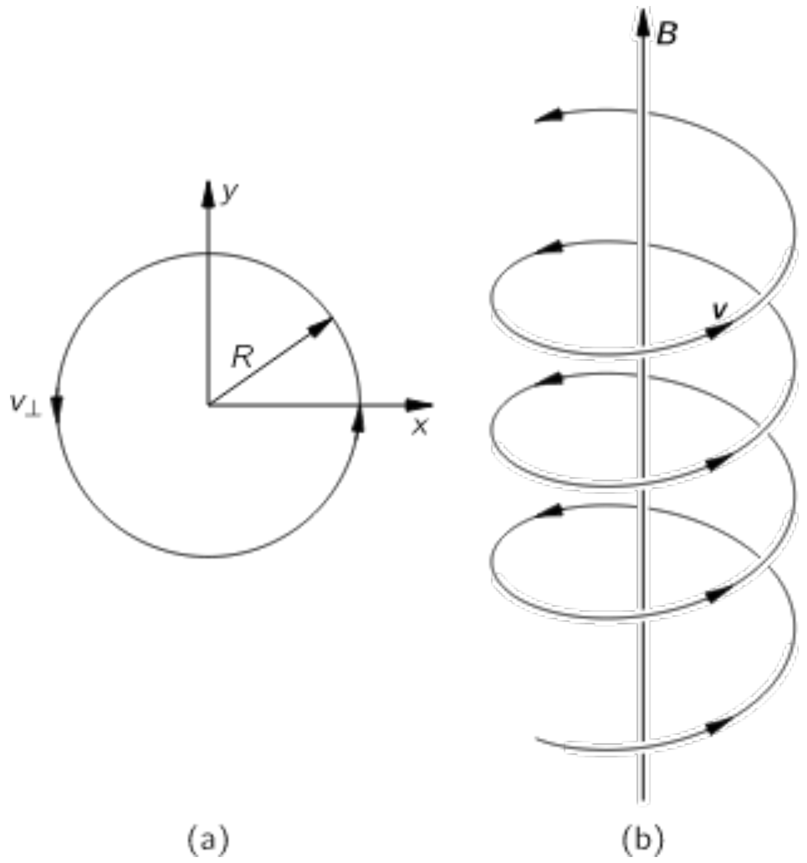
Single-particle linear dynamics

- Charge-particles' motion in electromagnetic field is governed by Lorentz force law:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

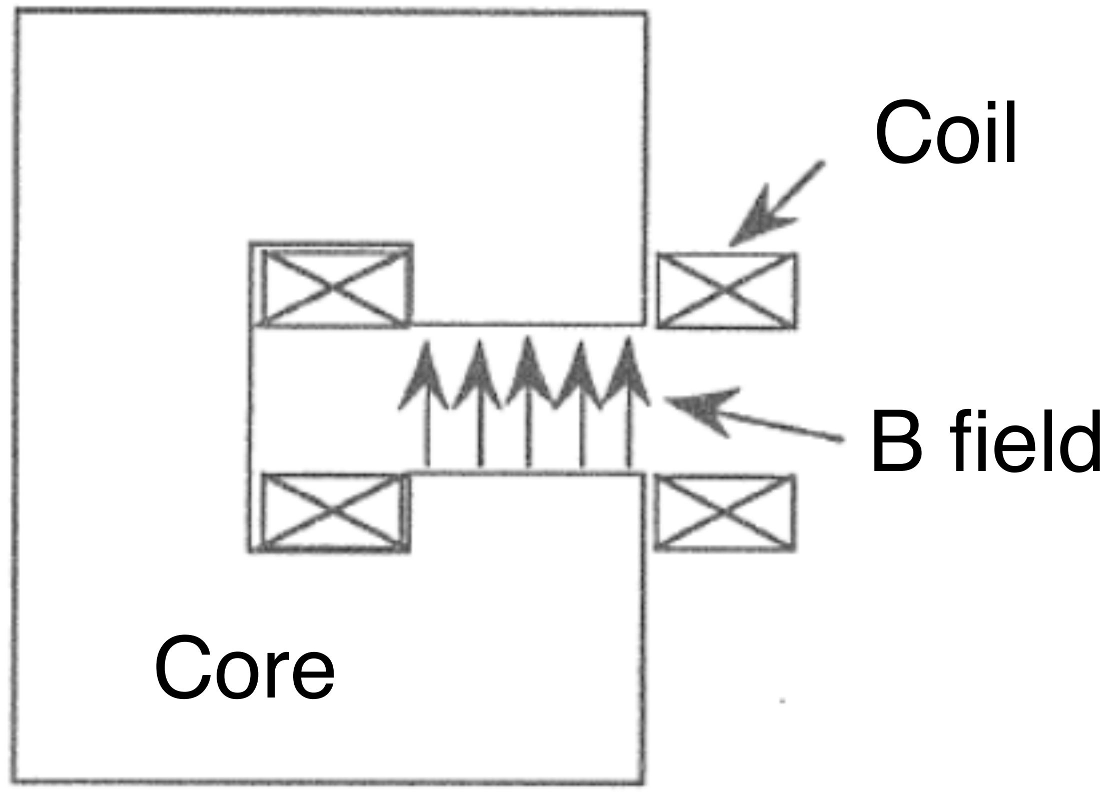
- In a uniform magnetic field a charged-particle takes circular motion [2].

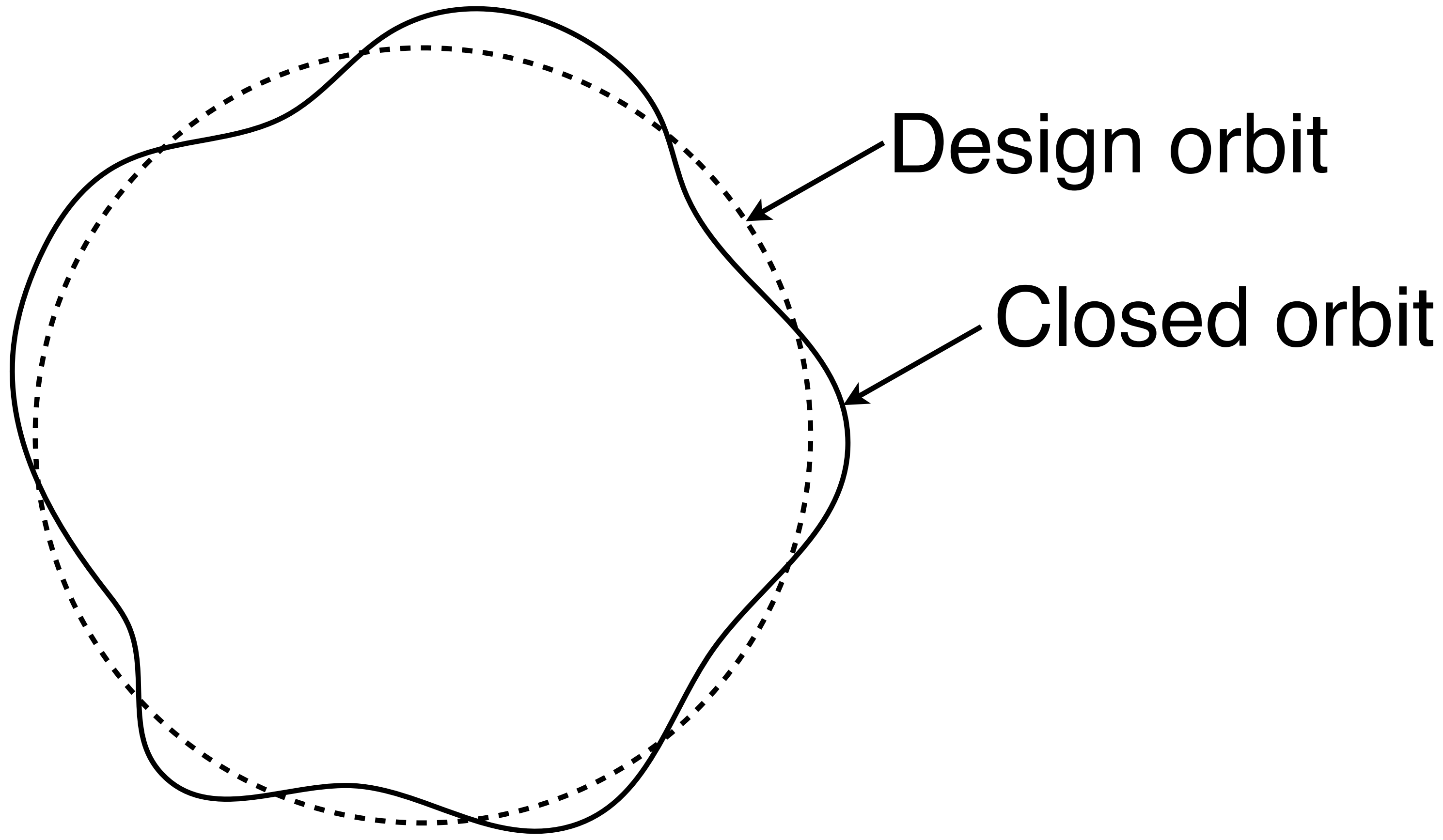




Photos from Ref.[2]

- Dipole magnets are used to create a circular particle accelerator and also to confine the beam along a closed orbit ("Fixed point"): The particle trajectory that closes on itself after one turn.

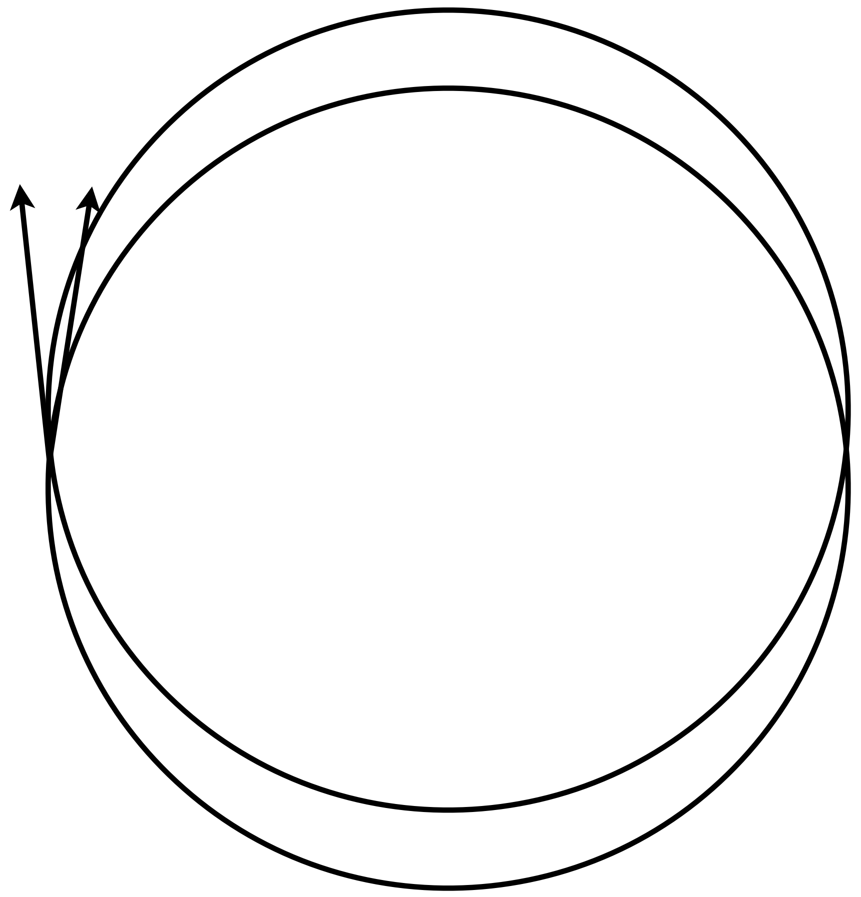




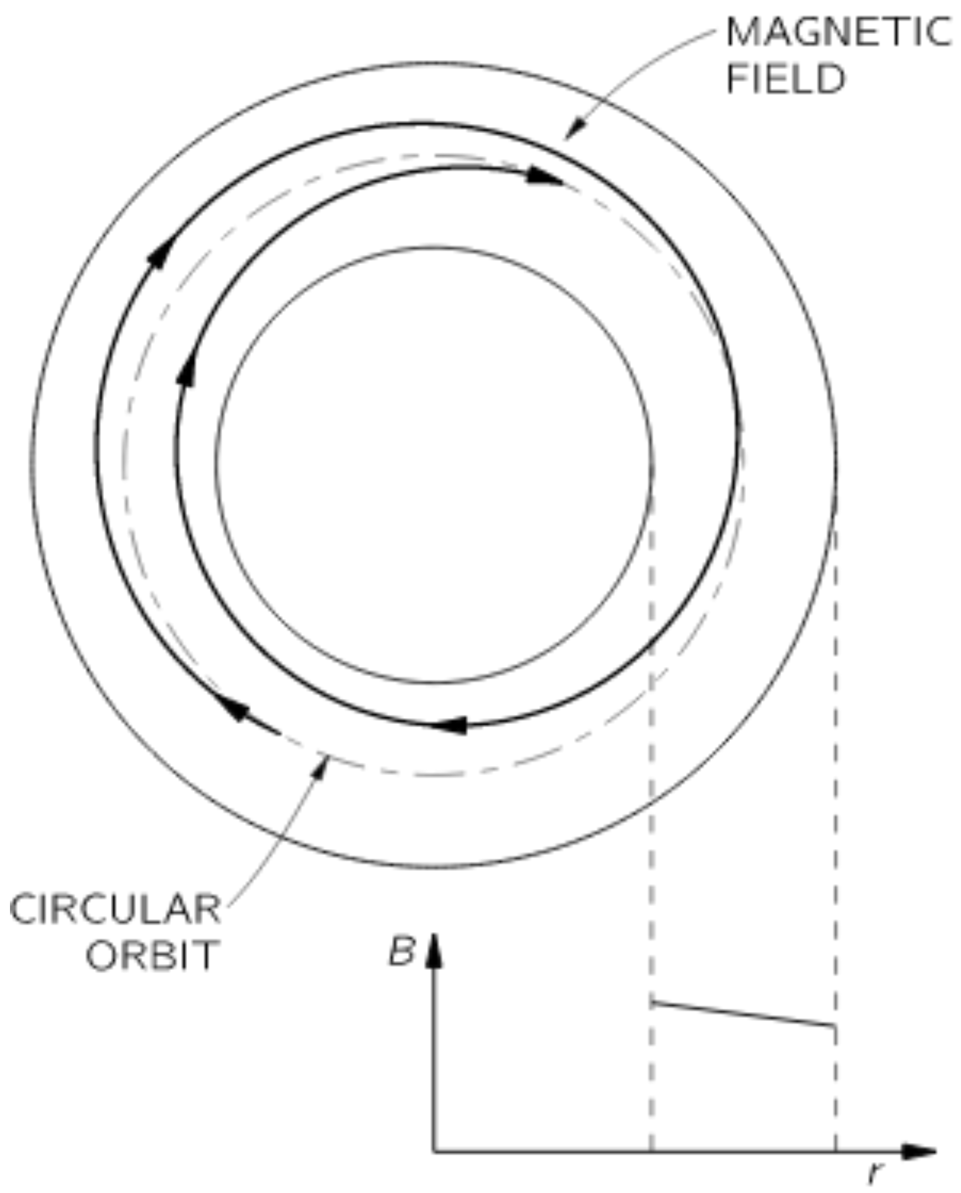
[2] R. Feynman, "The Feynman Lectures on Physics", https://www.feynmanlectures.caltech.edu/

Single-particle linear dynamics

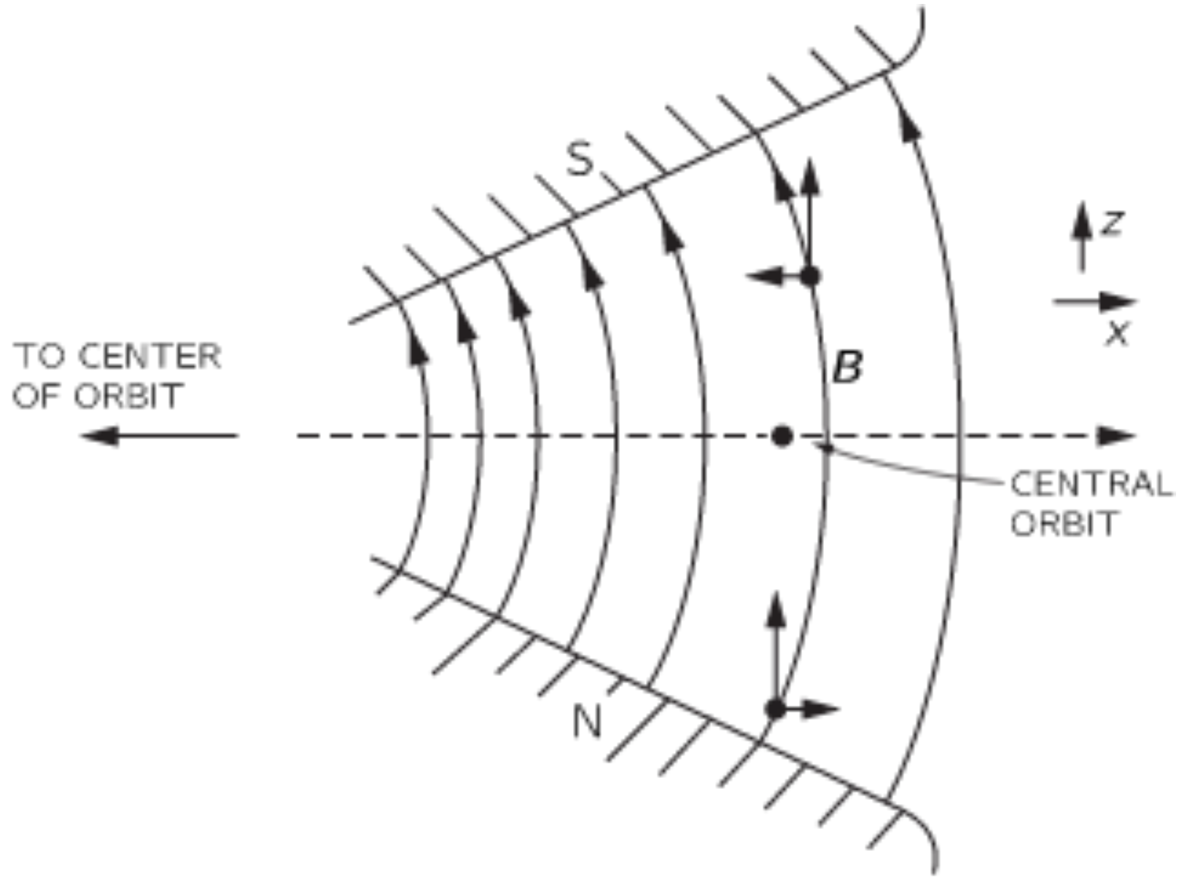
- Following the Lorentz force law, particles with different momentum and initial velocity will not follow the same trajectory. Therefore, external focusing force is necessary to confine a group of particle to moving around the closed orbit.



- “Weak” focusing was introduced by making a slope to the field gradient. With $-1 < n < 0$, the fields provide radial and vertical focusing simultaneously, and therefore the beam is stable: Particles are moving along a stable orbit.



$$-1 < n = \frac{dB/B}{dr/r} < 0$$

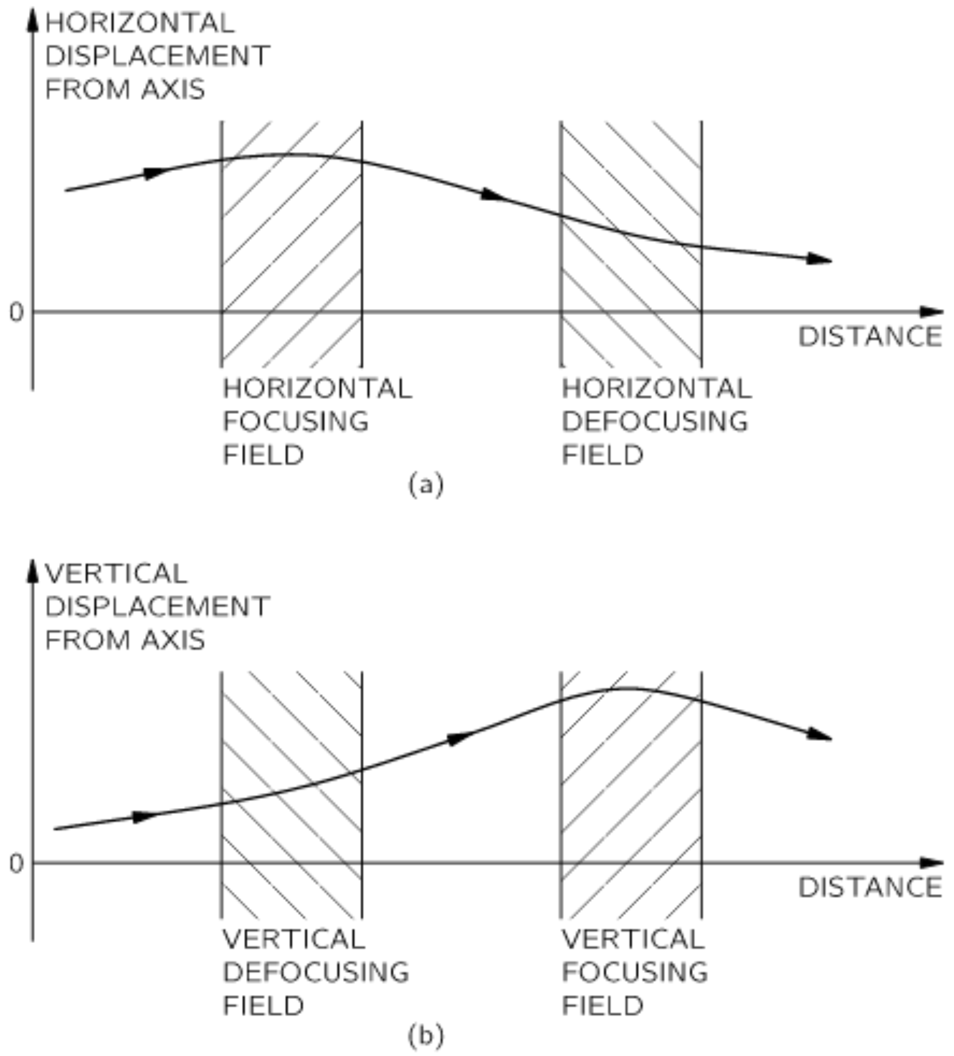


Photos from Ref.[2]

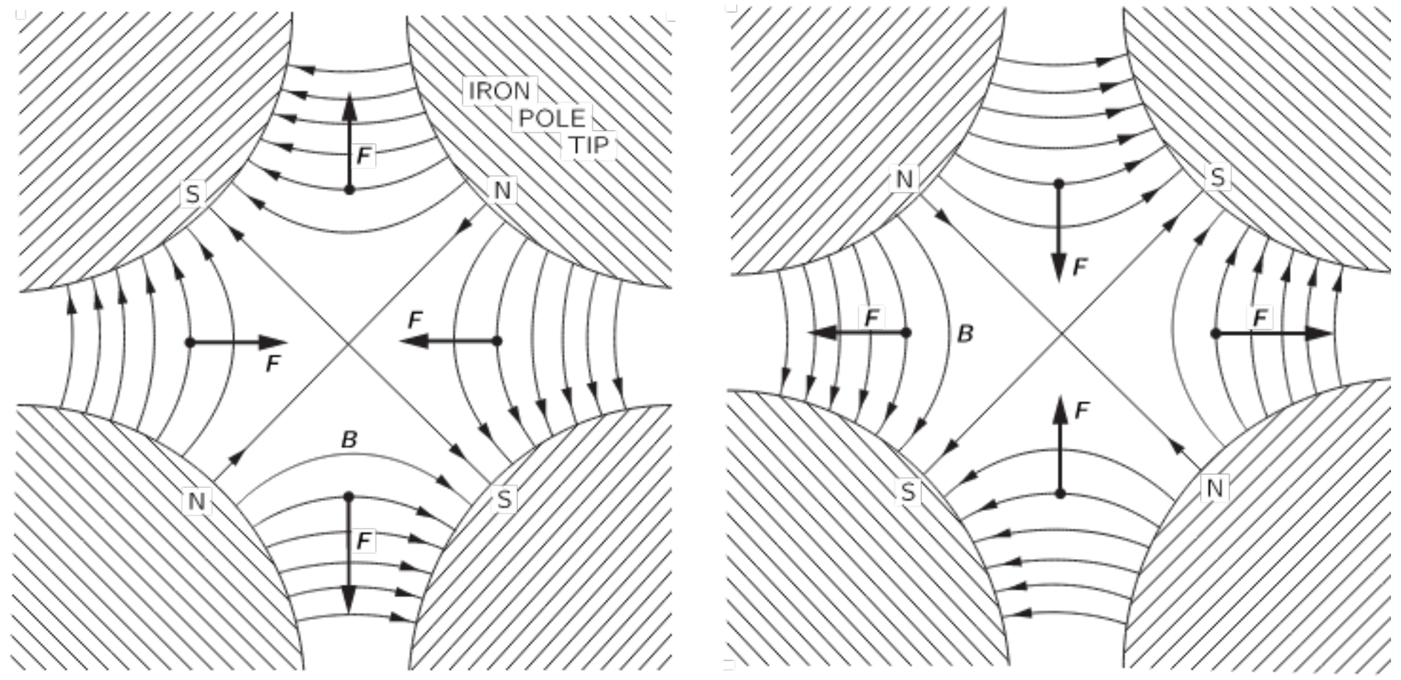
[2] R. Feynman, “The Feynman Lectures on Physics”, https://www.feynmanlectures.caltech.edu/

Single-particle linear dynamics

- Dipole magnets with field slope serve as combined-function magnets. The “strong” focusing or alternating-focusing was invented to provide more effective control of the charge beam. This principle introduces quadrupoles with alternating gradients to a beam line, and became the basis of most modern high-energy particle accelerators.



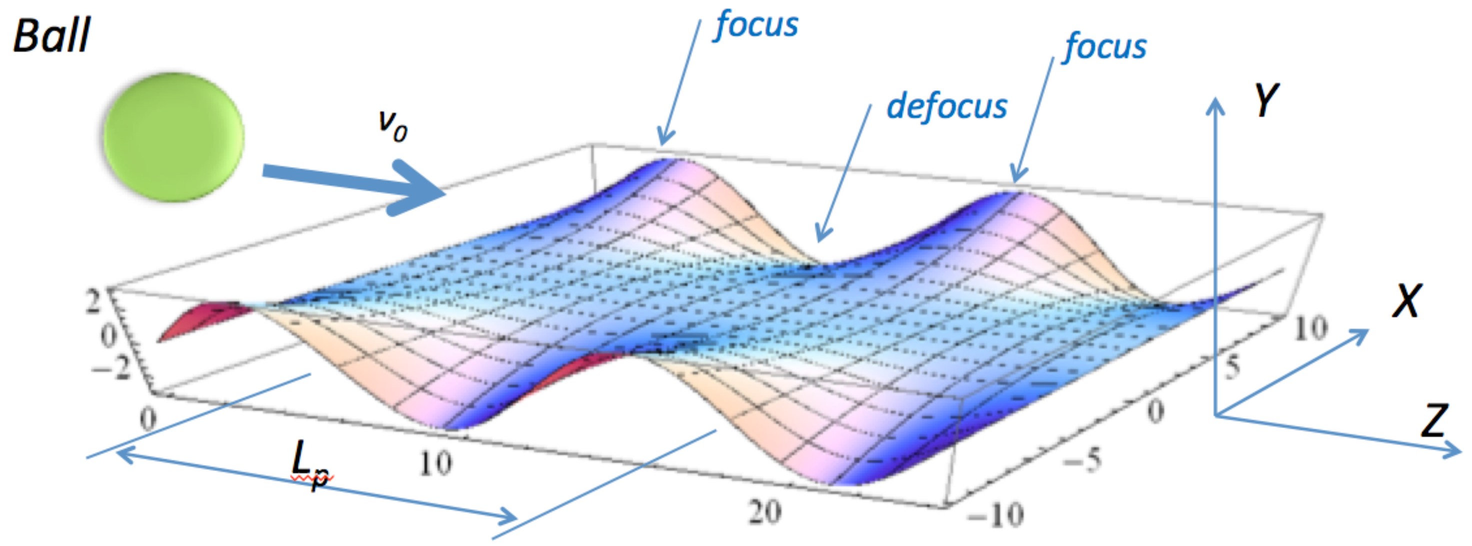
$$n = \frac{dB/B}{dr/r} \gg 1 \quad \text{or} \quad n = \frac{dB/B}{dr/r} \ll -1$$

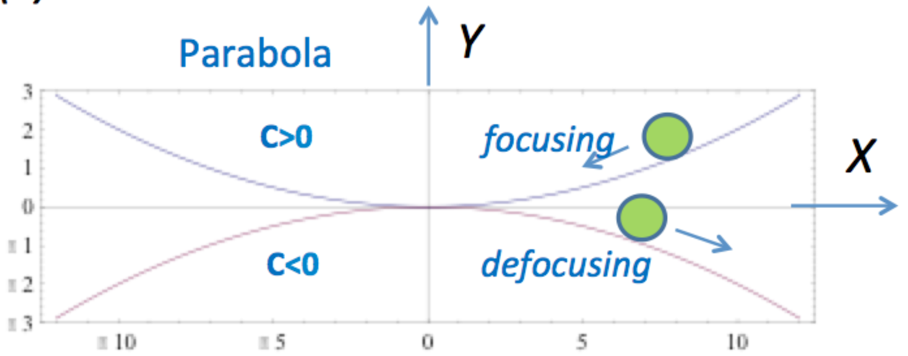


Photos from Ref.[2]

- The “strong” focusing shares the principle of Headstand pendulum, Segway, Balancing stick, Wavy surface coaster, …

Pictures from http://people.kth.se/~crro/segway_challenge/model.html





$$X'' = -D(z)X$$
$$Y'' = -E(z)Y$$

[2] R. Feynman, “The Feynman Lectures on Physics”, https://www.feynmanlectures.caltech.edu/

Single-particle linear dynamics

- With the basic elements dipole and quadrupole magnets defined, we can discuss the transverse linear dynamics of a particle moving around a closed orbit. The equation of motion is so-called Hill's equation:

$$x'' + K_1(s)x = 0$$

- $K_1(s)$ is related to the field gradient. For a quadrupole, there is:

$$K_1 \approx \frac{q}{P_0}\frac{\partial B_y}{\partial x}$$

- A beautiful solution of Hill's equation is given in terms of Courant-Snyder parameters:

$$x(s) = \sqrt{2\beta_x(s)J_x}\cos\phi_x(s)$$

- The constant J_x and s-dependent functions $\beta_x(s)$ and $\phi_x(s)$ have clear physical meanings as will be shown later. Applying $x(s)$ to Hill's equation gives:

$$\phi_x'(s) = \frac{1}{\beta_x(s)}$$

$$\beta_x''(s) - \frac{4 + \beta_x'(s)}{2\beta_x} + 2K_1(s)\beta_x(s) = 0$$

- Also we define derived function $\alpha_x(s)$, $\gamma_x(s)$ and momentum variable p_x:

$$\alpha_x(s) = -\frac{1}{2}\beta_x'(s) \qquad \gamma_x(s) = \frac{1+\alpha_x^2(s)}{\beta_x(s)}$$

$$p_x = \frac{v_x}{\beta_0 c}$$

Single-particle linear dynamics

- The phase-space coordinates of a particle can be expressed as:

$$x(s) = \sqrt{2\beta_x(s)J_x}\cos\phi_x(s)$$

$$p_x(s) = -\sqrt{\frac{2J_x}{\beta_x(s)}}\left(\sin\phi_x(s) + \alpha_x(s)\cos\phi_x(s)\right)$$

- Calculating the second-order moments gives beam size $\sigma_x(s)$, momentum spread $\sigma_{p_x}(s)$, and emittance ϵ_x:

$$\sigma_x(s) = \sqrt{\langle x^2\rangle} = \sqrt{\beta_x(s)\epsilon_x}$$

$$\sigma_{p_x}(s) = \sqrt{\langle p_x^2\rangle} = \sqrt{\gamma_x(s)\epsilon_x}$$

$$\langle xp_x\rangle = -\alpha_x(s)\epsilon_x$$

$$\epsilon_x = \langle J_x\rangle = \sqrt{\langle x^2\rangle\langle p_x^2\rangle - \langle xp_x\rangle^2}$$

- The linear transverse motion around the closed orbit is also called betatron motion. The key concepts for describing betatron motion are beta functions $\beta_{x,y}(s)$, emittances $\epsilon_{x,y}$, and phase advances $\phi_{x,y}(s)$ (An integral function of $\beta_{x,y}(s)$):

$$\phi_{x,y}(s) = \int^s \frac{1}{\beta_{x,y}(s')}ds'$$

- For storage rings like SuperKEKB, the one-turn phase advances are constants, defining betatron tunes:

$$\nu_{x,y} = \frac{1}{2\pi}\phi_{x,y}(C) = \frac{1}{2\pi}\int^C \frac{1}{\beta_{x,y}(s')}ds'$$

Single-particle linear dynamics

- A side effect of using dipole magnets to make a beam move along a circular orbit (it can be taken as transverse acceleration) is the so-called synchrotron radiation (SR). The radiation is especially important for electron/positron storage rings. The energy loss per turn is:

$$U_0 = \frac{C_\gamma}{2\pi}\beta_0^3 E_0^4 I_2$$

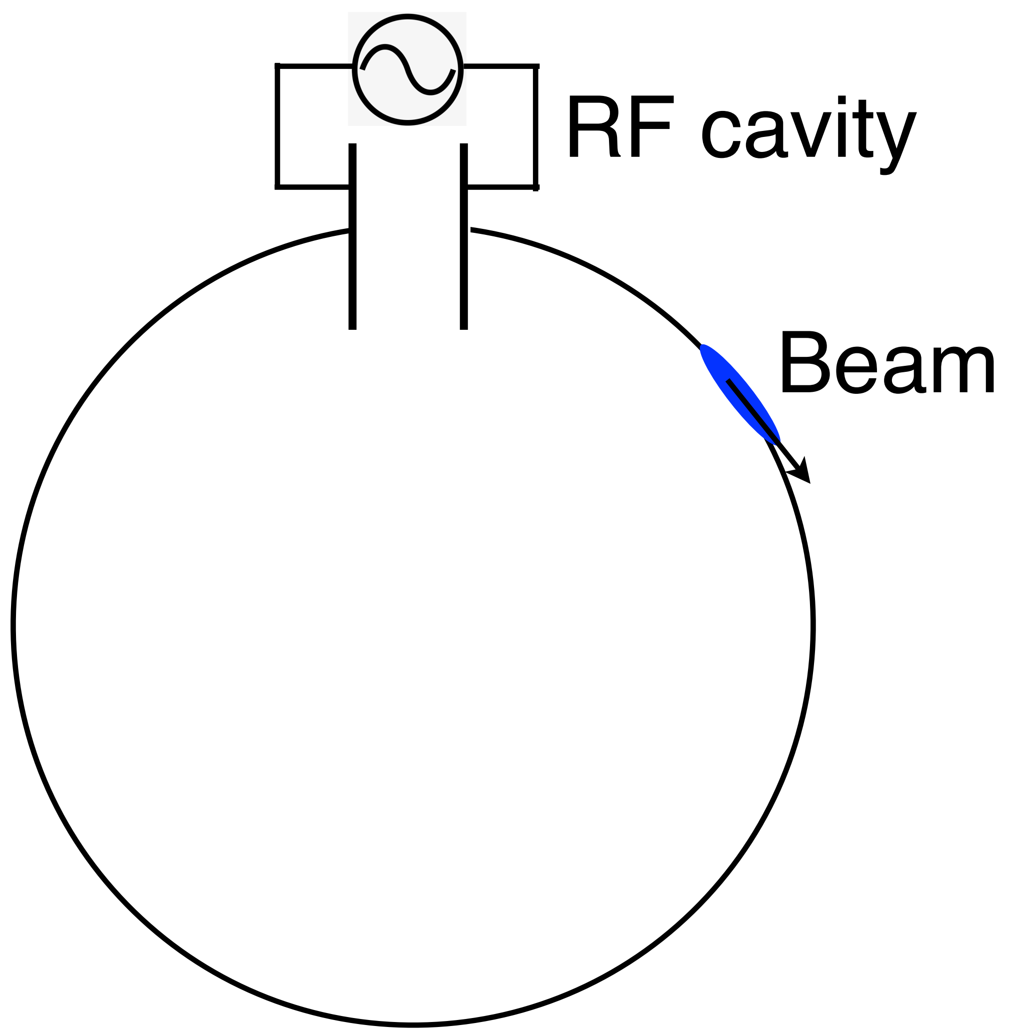
$$C_\gamma = \frac{q^2}{3\epsilon_0 (mc^2)^4} \qquad I_2 = \int^C \frac{1}{\rho^2(s)} ds$$

- For SuperKEKB, $U_0 \approx 1.76$ MeV and $U_0 \approx 2.43$ MeV for the positron and electron beams, respectively. The energy loss due to SR needs to be compensated by acceleration using radio frequency (RF) cavities.

- The compensation of SR loss is simple:

$$U_0 = qV_{\mathbf{rf}} \sin\phi_s$$

- Here V_{rf} is the total RF acceleration voltage, and ϕ_s is the synchronous RF phase.



Single-particle linear dynamics

- The effects of SR on beams in electron/ positron storage rings include:
  - Radiation damping: Average energy loss into SR
  - Quantum excitation: Random photon emission.
  - Equilibrium (Gaussian) distribution in x, y and z directions: Determining transverse emittances $\epsilon_{x,y}$ and energy spread σ_δ.
- The particles of a bunch take synchrotron oscillation around the synchronous phase.
- Similar to betatron tunes, there is a synchrotron tune ν_s which defines the frequency of synchrotron motion.
- The betatron (x and y) and synchrotron (z) tunes of a storage ring usually satisfy:

$$\nu_s \ll 1 \ll \nu_{x,y}$$

- The emittances of electron/positron storage rings usually satisfy:

$$\epsilon_y \ll \epsilon_x \ll \epsilon_z$$

- Another important parameter is the natural bunch length σ_{z0}:

$$\sigma_{z0} = \frac{C\alpha_p}{2\pi\nu_s}\sigma_\delta$$

- So far, most of the important concepts of accelerator physics at SuperKEKB are addressed. The next step is to discuss the practical design of SuperKEKB and strategies to achieve extremely high luminosity.

Design strategy for SuperKEKB

- The strategy of achieving high luminosity is reflected from the parameter table:
  - The beam sizes and crossing angle determine the luminosity.
  - The tunes and crossing angle strongly affect the beam instability (to be discussed).

	KEKB (2009.06.17)		SKEKB (2021c)		SKEKB (Final design)	
	HER	LER	HER	LER	HER	LER
I_{bunch} (mA)	1.2	1.0	0.64	0.8	2.6	3.6
# bunch	1585		1272		2500	
ε_x (nm)	24	18	4.6	4.0	4.6	3.2
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σ_z (mm)	6	6	5	6	5	6
ν_x	44.511	45.506	45.533	44.525	45.53	44.53
ν_y	41.585	43.561	43.581	46.595	43.57	46.57
ν_s	0.0209	0.0246	0.0272	0.0233	0.028	0.0245
Crab waist	-		40%	80%		
Crossing angle (mrad)	0 (22)		83		83	

$$\sigma^*_{x,y} = \sqrt{\beta^*_{x,y}\epsilon_{x,y}}$$

$$L \approx \frac{N_b N_+ N\_f}{2\pi\sqrt{\sigma^{*2}_{y+} + \sigma^{*2}_{y-}}\sqrt{\sigma^2_{z+} + \sigma^2_{z-}}\tan\frac{\theta_c}{2}}$$

$$\nu_s \ll 1 \ll \nu_{x,y}$$

$$\epsilon_y \ll \epsilon_x \ll \epsilon_z$$

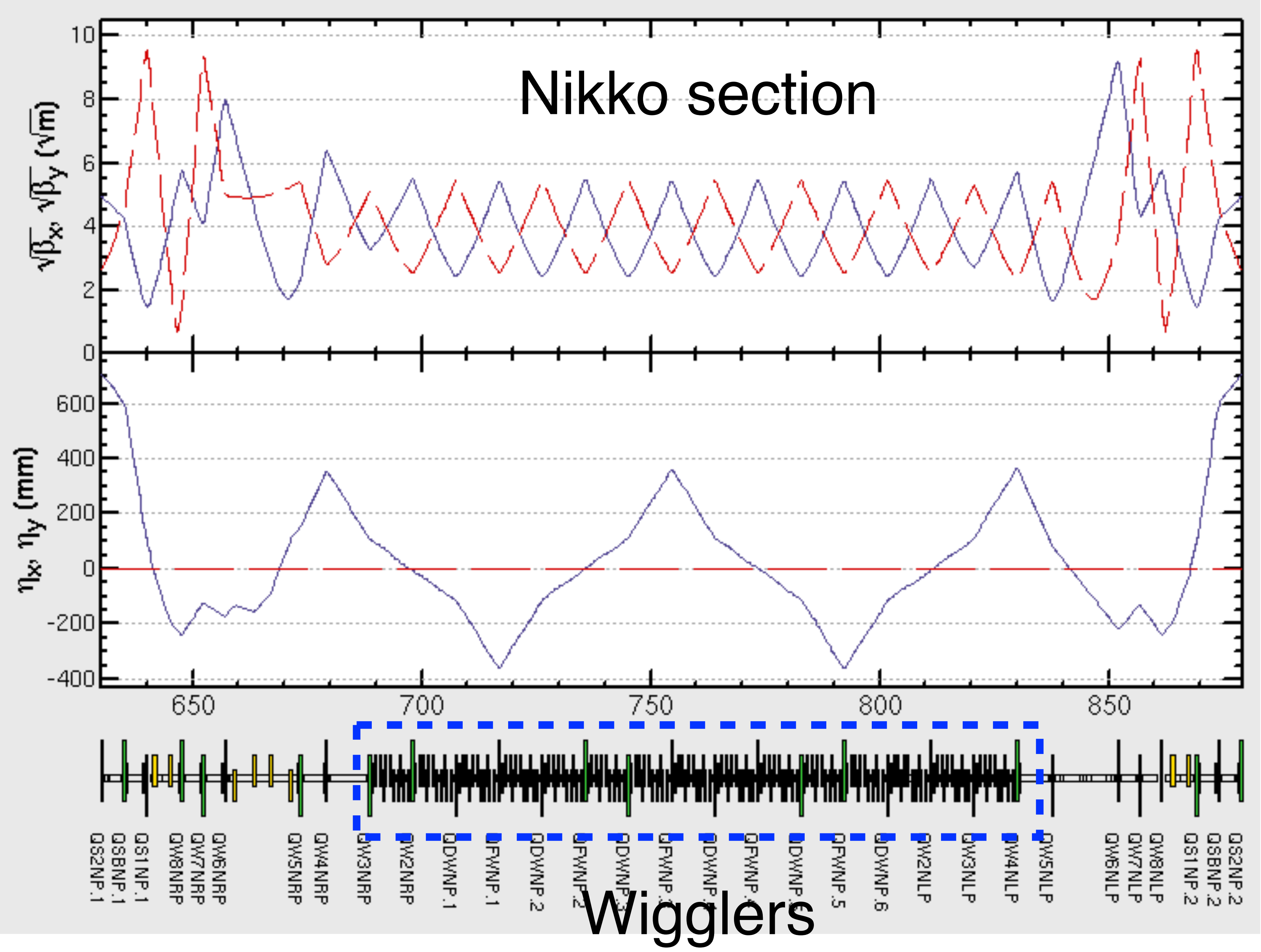
$$\sigma_y \ll \sigma_x \ll \sigma_z$$

Design strategy for SuperKEKB

- Optics design to achieve low emittance is the first task.

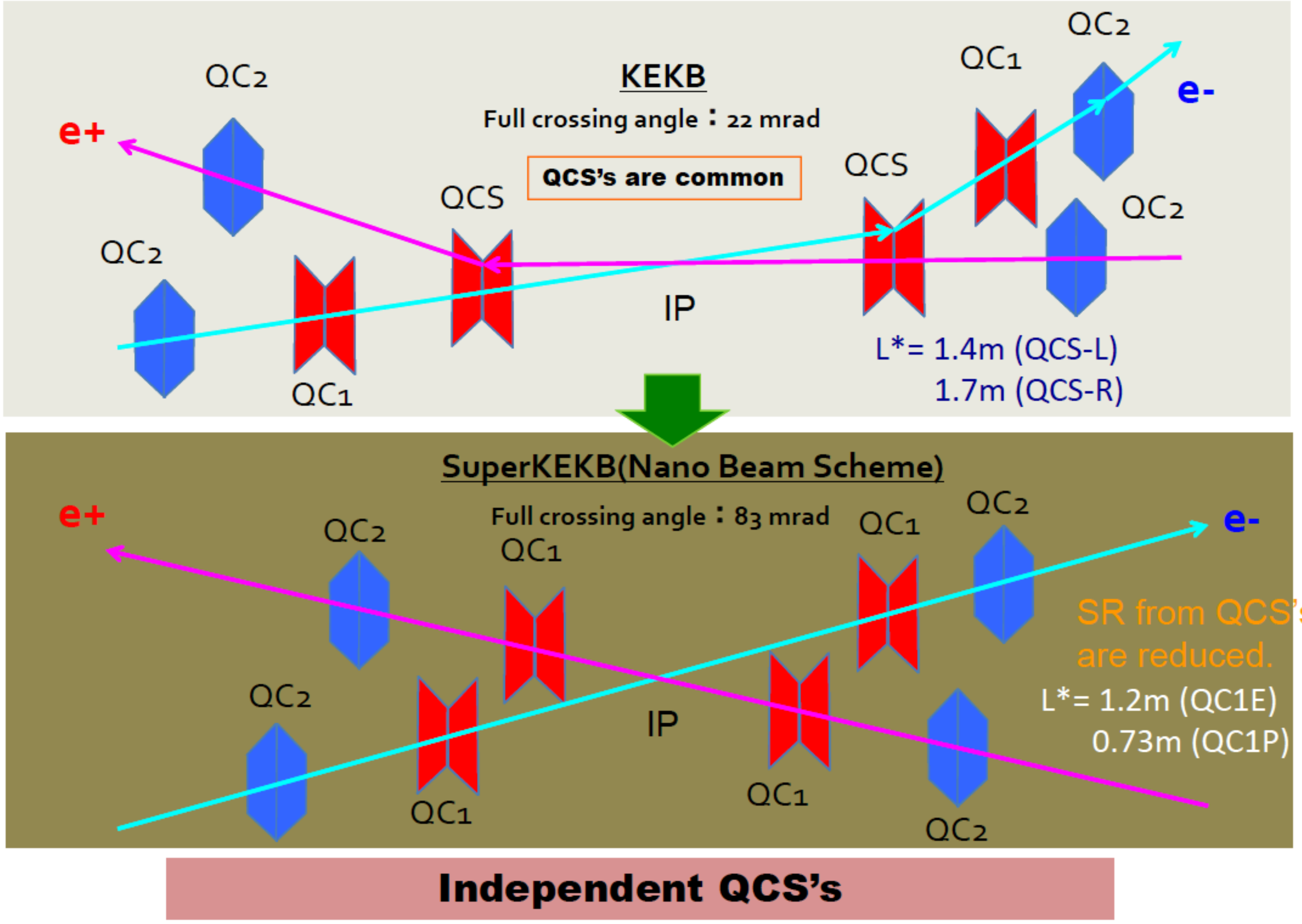
$$\epsilon_x = \frac{C_\gamma \gamma^2}{J_x} \frac{\int^{\text{Arc}} \frac{H}{|\rho|^3} ds + \int^{\text{Wiggler}} \frac{H}{|\rho|^3} ds}{\int^{\text{Arc}} \frac{1}{\rho^2} ds + \int^{\text{Wiggler}} \frac{1}{\rho^2} ds}$$

$$H = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$



- The next task is to design a final focus system to achieve very small beta functions at the IP. This is the most important but also most challenging part of SuperKEKB.

Final Focus Quadrupoles (QCS' s)

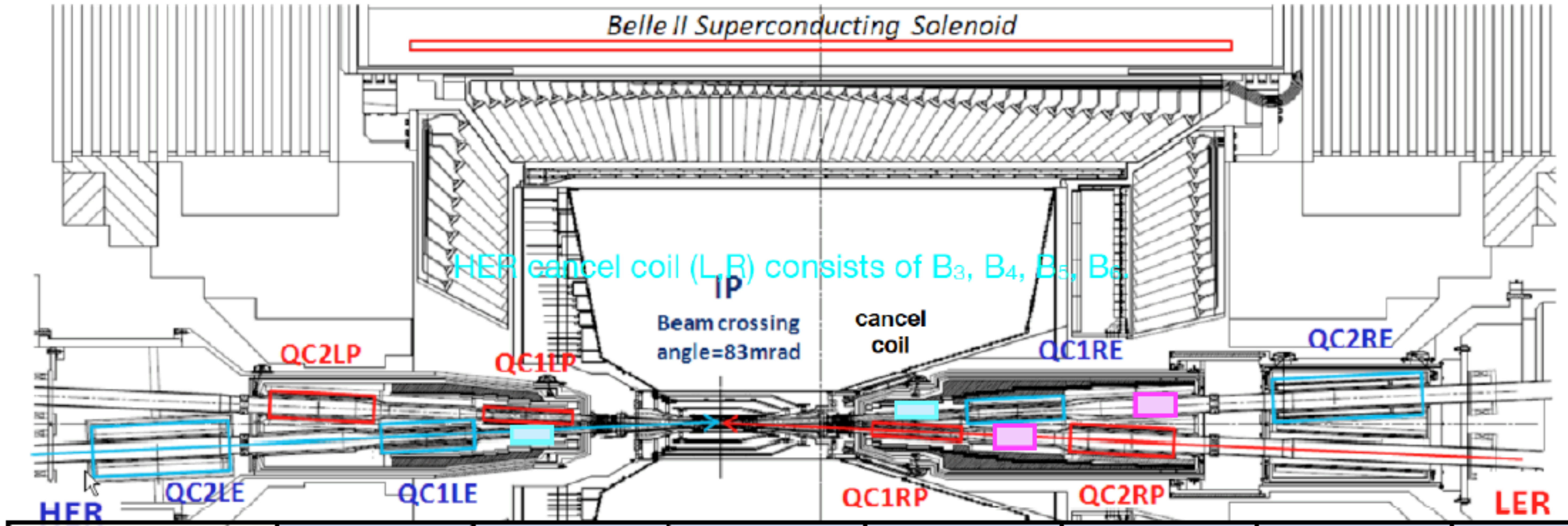


Design strategy for SuperKEKB

- The interaction region (IR) is very complicated with many correctors integrated.

Final Focus System: QCS

N. Ohuchi, Y. Arimoto

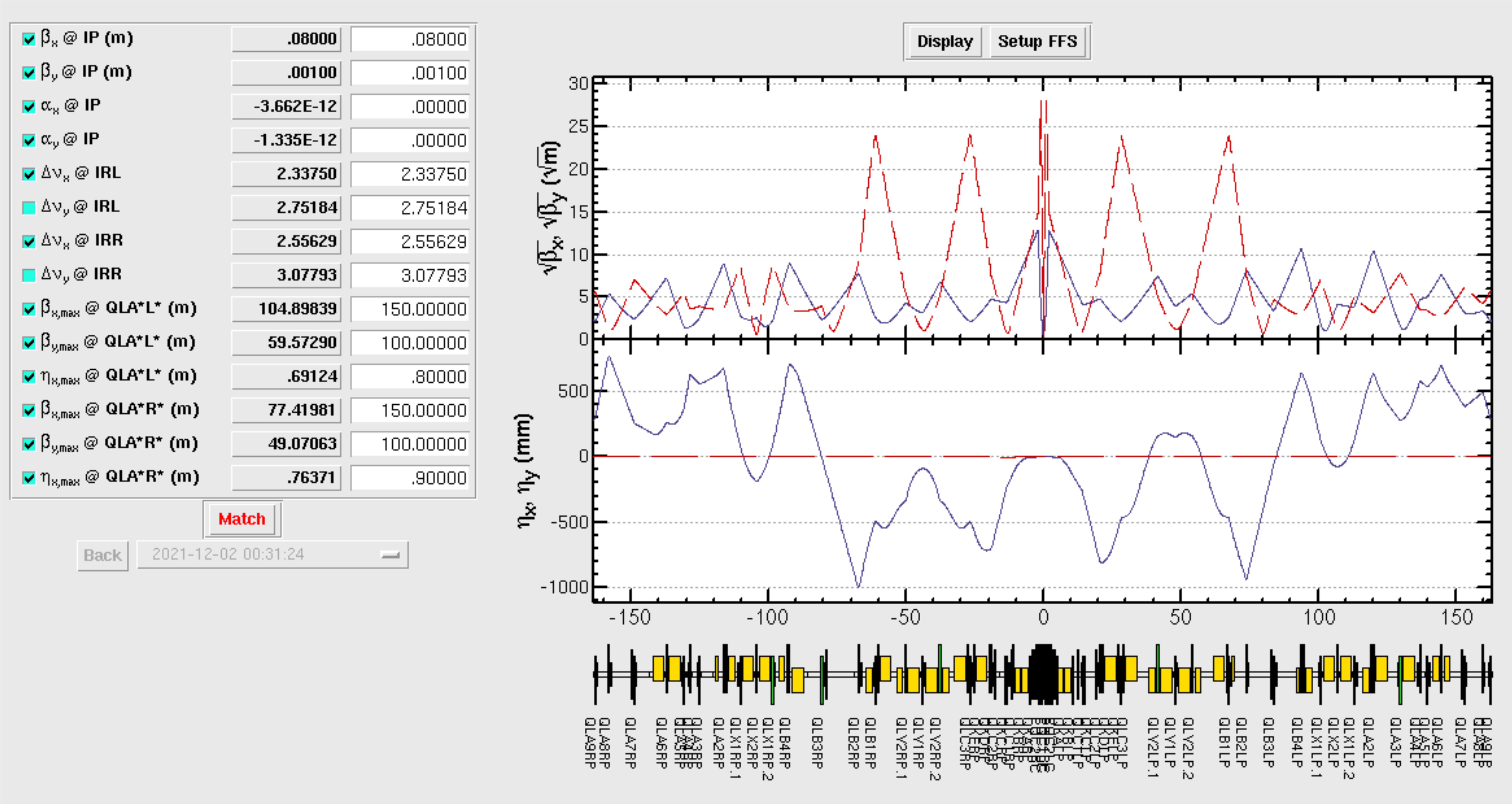


Design param.	Dipole	Skew dipole	Quad	Skew quad	Sextupole	Skew sext	Octupole
	B_1L (Tm)	A_1L (Tm)	B_2L/r_0 (T)	A_2L/r_0 (T)	B_3L/r_0^2 (T/m)	A_3L/r_0^2 (T/m)	B_4L/r_0^3 (T/m^2)
QC1LP	0.004	-0.002	-22.96	-9.50E-05			-27.0
QC2LP	-0.0217	0.022	11.48	0.0095			48.2
QC1RP	0.0050	-0.0086	-22.96	1.92E-05		0.0	-26.7
QC2RP	-0.0023	0.0214	11.54	-6.30E-06		0.0	
QC1RP-QC2RP					0.0		
QC1LE	0.030	0.0092	-26.94	-0.0729			8.9
QC2LE	0.000	-0.0016	15.27	0.0271			23.6
QC1RE	-0.0305	0.0053	-25.39	0.0653		0.0	
QC2RE	0.000	-0.0022	13.04	0.0559		0.0	
QC1RE-QC2RE					0.0		

Y. Ohnishi

Design strategy for SuperKEKB

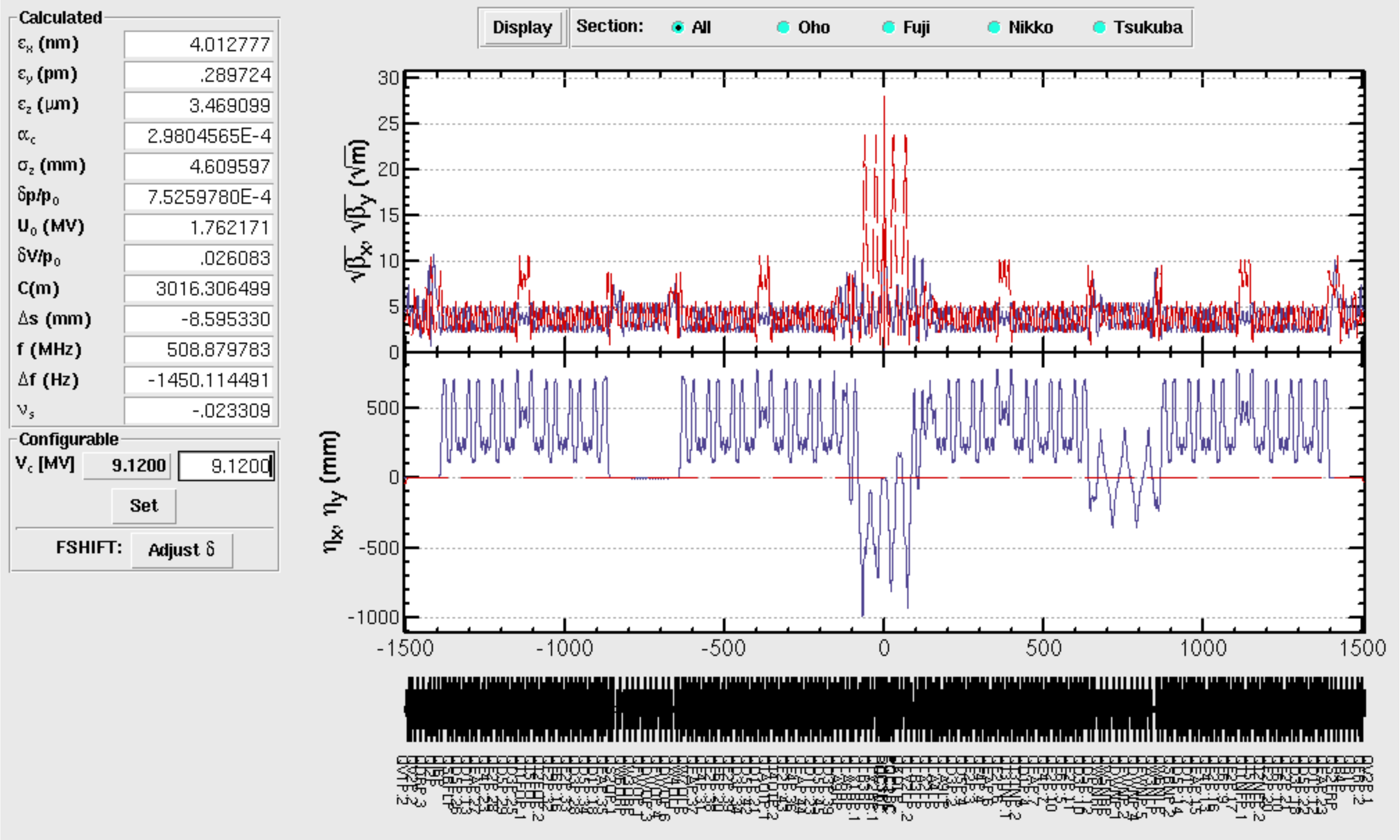
- IR optics of LER with $\beta_y^* = 1$ mm. Note that the beam sizes $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y} + \eta_{x,y}^2\sigma_\delta^2}$.



β_x @ IP (m)	.08000	.08000
β_y @ IP (m)	.00100	.00100
α_x @ IP	-3.662E-12	.00000
α_y @ IP	-1.335E-12	.00000
$\Delta\nu_x$ @ IRL	2.33750	2.33750
$\Delta\nu_y$ @ IRL	2.75184	2.75184
$\Delta\nu_x$ @ IRR	2.55629	2.55629
$\Delta\nu_y$ @ IRR	3.07793	3.07793
$\beta_{x,max}$ @ QLA*L* (m)	104.89839	150.00000
$\beta_{y,max}$ @ QLA*L* (m)	59.57290	100.00000
$\eta_{x,max}$ @ QLA*L* (m)	.69124	.80000
$\beta_{x,max}$ @ QLA*R* (m)	77.41981	150.00000
$\beta_{y,max}$ @ QLA*R* (m)	49.07063	100.00000
$\eta_{x,max}$ @ QLA*R* (m)	.76371	.90000

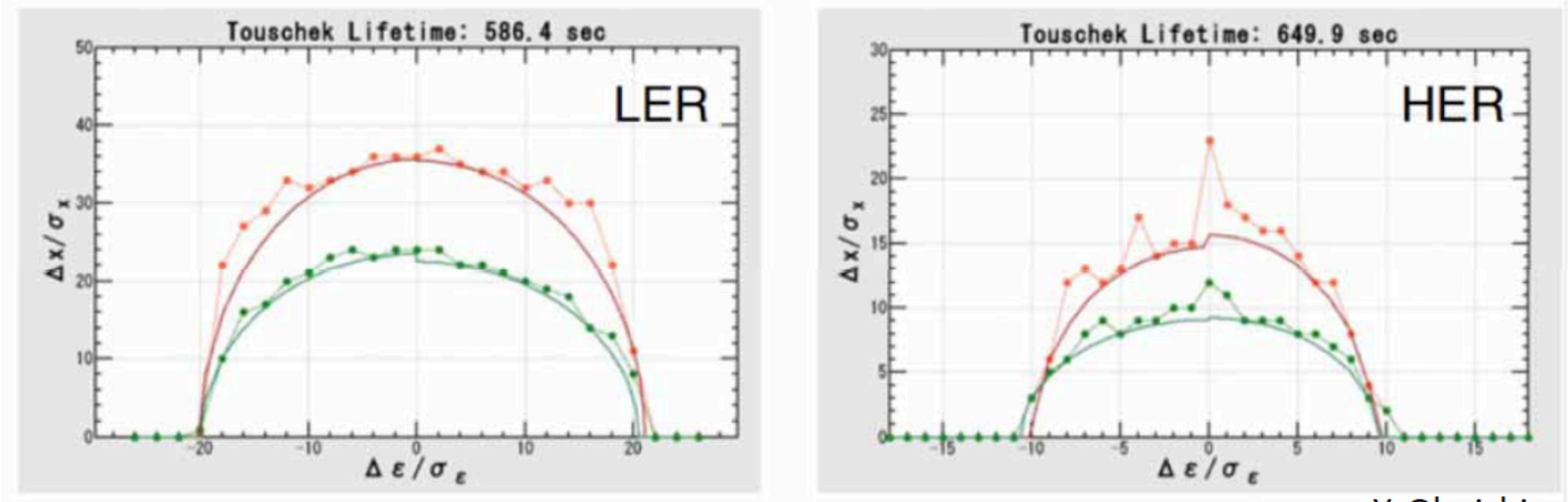
Design strategy for SuperKEKB

- Optics of LER with $\beta_y^* = 1$ mm. Note that the beam sizes $\sigma_{x,y} = \sqrt{\beta_{x,y}\epsilon_{x,y} + \eta_{x,y}^2\sigma_\delta^2}$.



Design strategy for SuperKEKB

- Squeezing $\beta^*_{x,y}$ results in short Touschek lifetime, which depends on:
  - Dynamic aperture: The region in $x - y$ space inside which particles can survive in certain number of turns.
  - Momentum aperture: The maximum momentum deviation δ_{max} below which particle can survive in certain number of turns.
- Short lifetime requires a very powerful and reliable injector.



w/o machine errors

Beam lifetime

	KEKB (design)		KEKB (operation)		SuperKEKB	
	LER	HER	LER	HER	LER	HER
Radiative Bhabha	21.3h	9.0h	6.6h	4.5h	28min.	20min.
Beam-gas	45h[a)]	45h[a)]			24.5min.[b)]	46min.[b)]
Touschek	10h	-			10min.	10min.
Total	5.9h	7.4h	~133min.	~200min.	6min.	6min.
Beam current	2.6A	1.1A	1.6A	1.1A	3.6A	2.6A
Loss Rate	0.12mA/s	0.04mA/s	0.23mA/s	0.11mA/s	10mA/s	7.2mA/s
					4nC@25Hz	2.9nC@25Hz

a) Bremsstrahlung
b) Coulomb scattering, sensitive to collimator setting

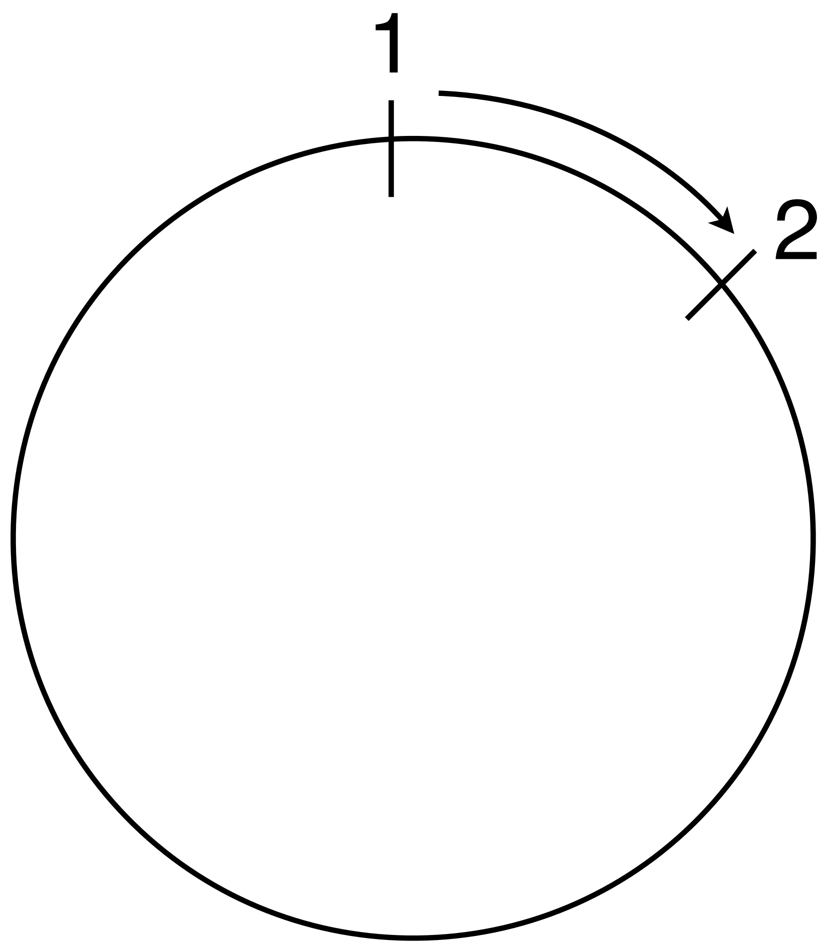
Single-particle nonlinear dynamics

- A full description of particle motion along a beam line requires powerful mathematical techniques.
- Suppose the particle's coordinates $(x, p_x, y, p_y, z, \delta)$, the linear transfer map from position 1 to position 2 can be described by transfer matrix:

$$\begin{pmatrix} x \\ px \\ y \\ p_y \\ z \\ \delta \end{pmatrix}_2 = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} & M_{36} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} & M_{46} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} & M_{56} \\ M_{61} & M_{62} & M_{63} & M_{64} & M_{65} & M_{66} \end{pmatrix} \begin{pmatrix} x \\ px \\ y \\ p_y \\ z \\ \delta \end{pmatrix}_1$$

- In a realistic accelerator, higher-order nonlinear correctors (such as sextupole magnets, octupole magnets, etc.) are often intentionally introduced to control the particle motion. But, more often unwanted nonlinear fields (or nonlinear kicks) appear in most of the elements of a beam line.
- The analysis of nonlinear dynamics relies on tools such as Hamilton's equations and Lie algebra methods. The transfer matrix for linear motion is then extended to transfer map for nonlinear motion:

$$\overrightarrow{X}_2 = \mathscr{M} \circ \overrightarrow{X}_1$$



Single-particle nonlinear dynamics

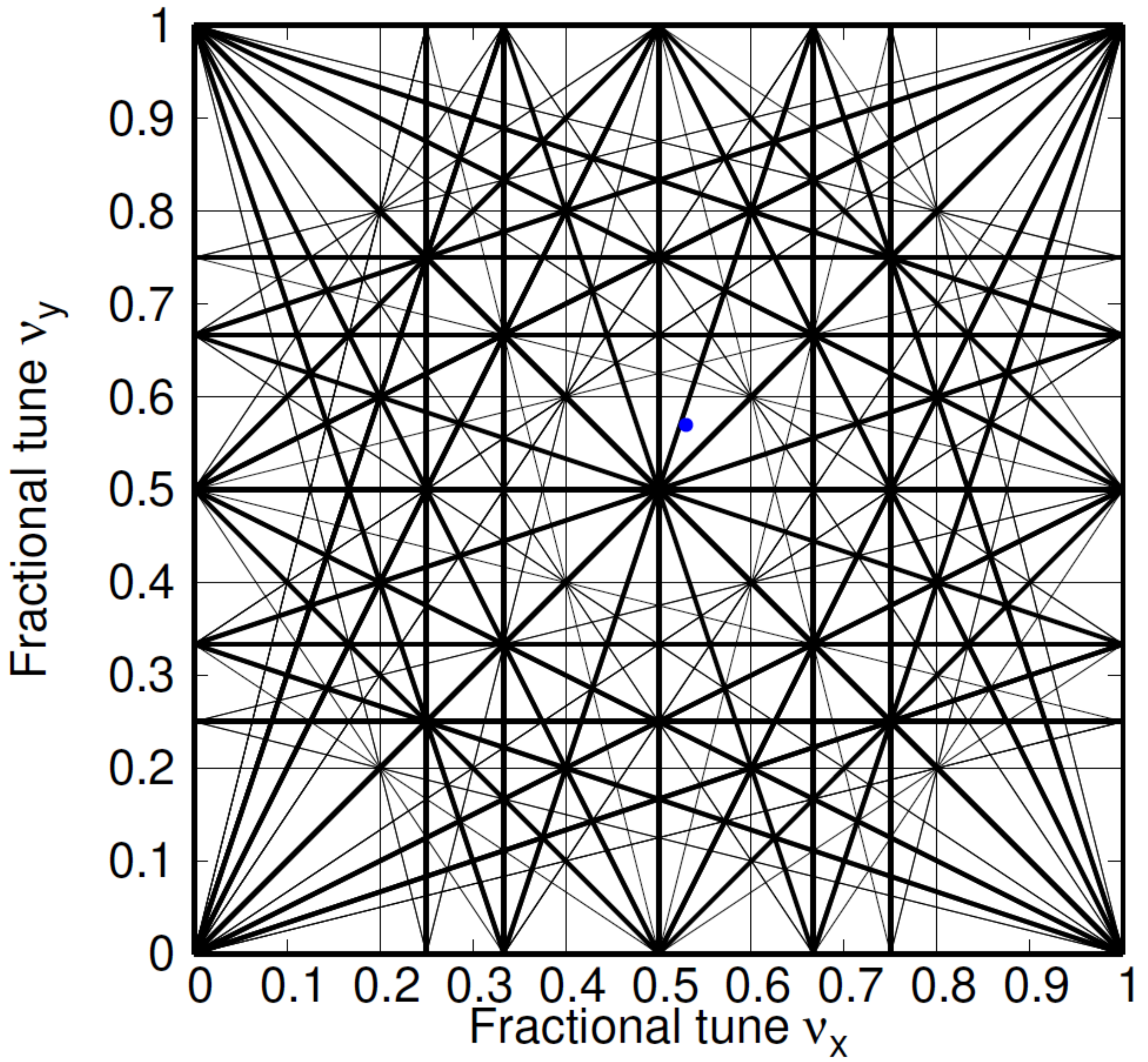
- For a storage ring, the particles take periodic motion because of periodic lattice. The nonlinear analysis of transfer maps usually results in strong correlation of dynamics with betatron resonances (X-Y coupling) and synchro-betatron resonances (X-Y-Z coupling):

$$m_x\nu_x + m_y\nu_y = \textbf{Integer}$$

$$m_x\nu_x + m_y\nu_y + m_s\nu_s = \textbf{Integer}$$

- When a storage ring is operating on a resonance, the kicks felt by particles will accumulate from turn to turn, leading to a large amplitude of betatron motion.
- Resonances are generally harmful to the beam quality (characterized by emittances, beam sizes, lifetime, detector background, etc.).
- Taking the fact of $\epsilon_y \ll \epsilon_x \ll \epsilon_z$ (~$1:10^3:10^6$) at SuperKEKB, any coupling from X- and/or Z-directions would make a significant change to ϵ_y, and consequently reduce luminosity.

- Usually higher-order resonances (= larger number of $|m_x|+|m_y|+|m_s|$) are less harmful than lower-order resonances. The working point $(\nu_{x0}, \nu_{y0}, \nu_{s0})$ should be away from dangerous resonances.
- Sometimes the resonances are correlated with single-particle dynamics, but more often they are correlated with collective effects. Collective effects depend on bunch/beam current.

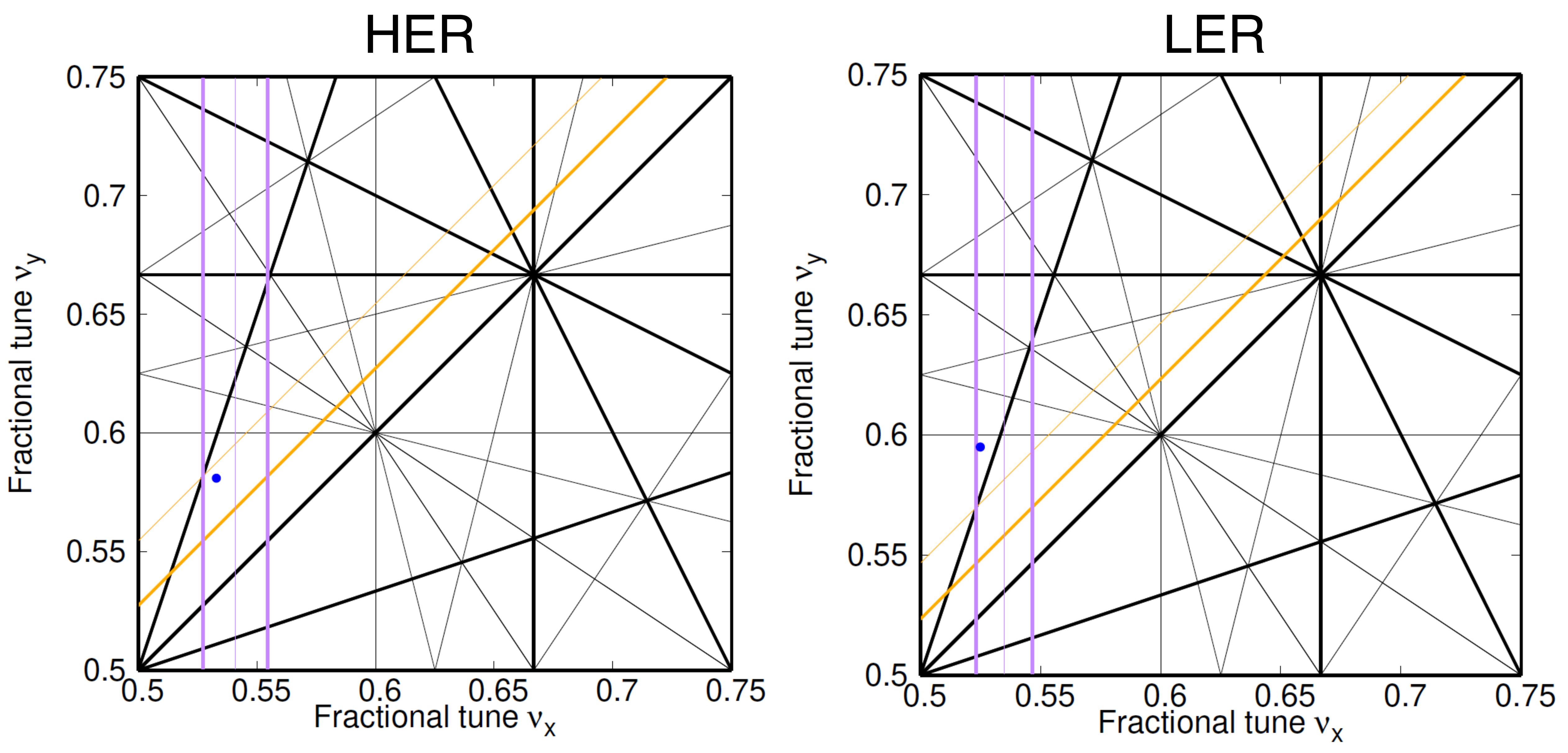


Resonance diagram with $|m_x|+|m_y| \le 5$. The blue dot shows the design working point of SuperKEKB.

Single-particle nonlinear dynamics

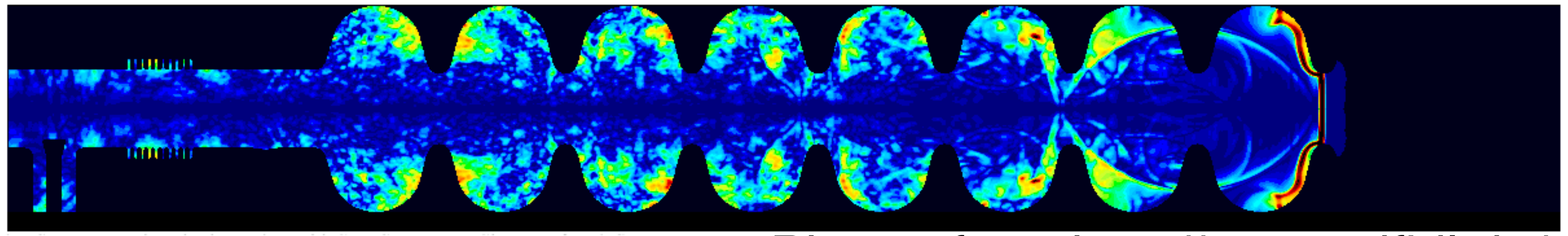
- At SuperKEKB, a list of dangerous resonances can be tentatively given:
  - Geometric lattice resonances with $|m_x| + |m_y| \leq 4$ mainly related to sextuples:
    $$m_x\nu_x + m_y\nu_y = \textbf{Integer}$$
  - Chromatic coupling resonances with $m_s = 1$ and 2 mainly related to nonlinear IR:
    $$\nu_x - \nu_y + m_s\nu_s = \textbf{Integer}$$
  - X-Z synchro-betatron resonances with $m_s = 1$ to 4 mainly related to dispersive sections (IR and Arcs) and beam-beam interaction:
    $$2\nu_x - m_s\nu_s = \textbf{Integer}$$
  - Y-Z synchro-betatron resonances mainly related to vertical impedances from small-gap collimators:
    $$2\nu_y - m_s\nu_s = \textbf{Integer}$$

- The most important beam-beam resonances with $m_y = 2$ and 4 due to large crossing angle and "hour-glass" effect:
  $$\nu_x - m_y\nu_y + \alpha = \textbf{Integer}$$
- Here α is a parameter related to incoherent beam-beam tune shift and synchrotron tune.
- The resonance diagram with synchro-betatron resonances can be plotted:



Collective effects

- The particles within a beam interact with each other in many ways.
- The "incoherent" collective effects include space charge, Touschek scattering, intrabeam scattering, …
- The "coherent" collective effects usually come from wake fields: The electromagnetic interaction of charged beam with its surroundings.

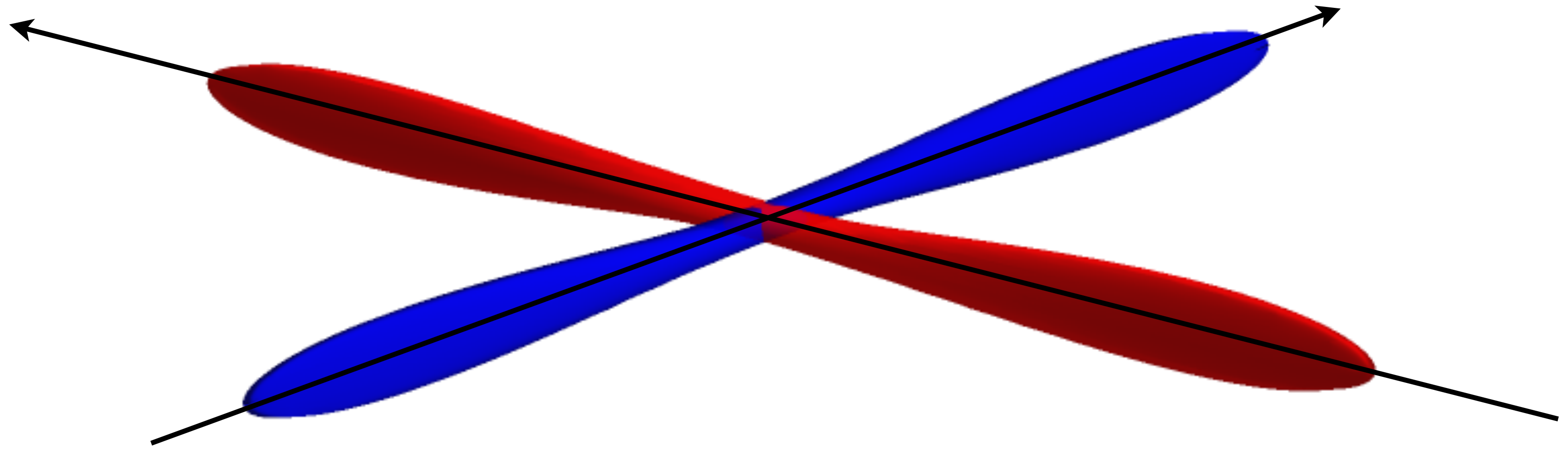


Photos from http://www.gdfidl.de/

Wake field is similar to turbulence by airplane

Photos from Wikipedia

- For a collider, the interaction of the two colliding beams, so-called beam-beam interaction, fundamentally sets a limit on luminosity performance.
- The beam-beam effect is a mixture of "incoherent" effects (similar to space-charge) and "coherent" effects (similar to wake fields).



SuperKEKB (2021c)

Collective effects

- The leading-order effect of longitudinal wake fields is bunch lengthening and incoherent synchrotron tune shift caused by so-called potential-well distortion.

$$\sigma_{z+} \approx \sigma_{z0+} + A_+ I_{b+}$$

$$\sigma_{z-} \approx \sigma_{z0-} + A_- I_{b-}$$

$$\nu_s \sigma_z = \frac{C\alpha_p}{2\pi}\sigma_\delta$$

- The momentum compaction α_p and momentum spread σ_δ can be taken as constants (determined by lattice design). Therefore, bunch lengthening cause decrease of synchrotron tune.
- Bunch lengthening will cause direct loss of luminosity at SuperKEKB (see luminosity formula), and also change the strength of beam-beam force.

- The linear part of beam-beam force act on the beam like a focusing quadrupole. It's direct effect is causing incoherent betatron tune shift. A simple estimate can be obtained [3]:

$$\xi^i_{x+} \approx \frac{r_e}{2\pi\gamma_+} \frac{N_- \beta^*_{x+}}{\sigma^2_{z-}\tan^2\frac{\theta_c}{2} + \sigma^{*2}_{x-}}$$

$$\xi^i_{y+} \approx \frac{r_e}{2\pi\gamma_+} \frac{N_- \beta^*_{y+}}{\sigma^*_{y-}\sqrt{\sigma^2_{z-}\tan^2\frac{\theta_c}{2} + \sigma^{*2}_{x-}}}$$

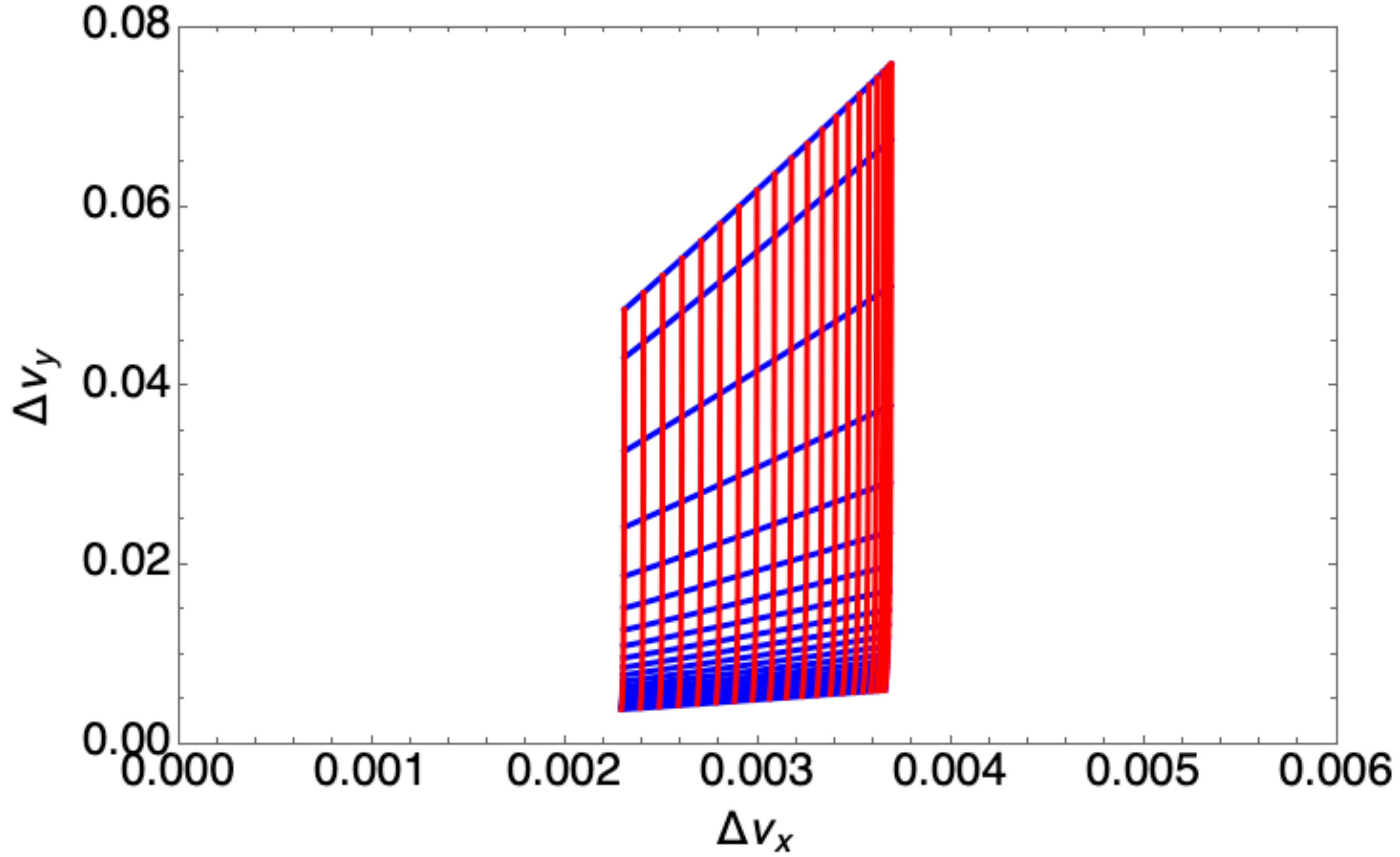
- ξ^i_x is the order of 0.003, and ξ^i_y is the order of 0.08 according to SuperKEKB design.

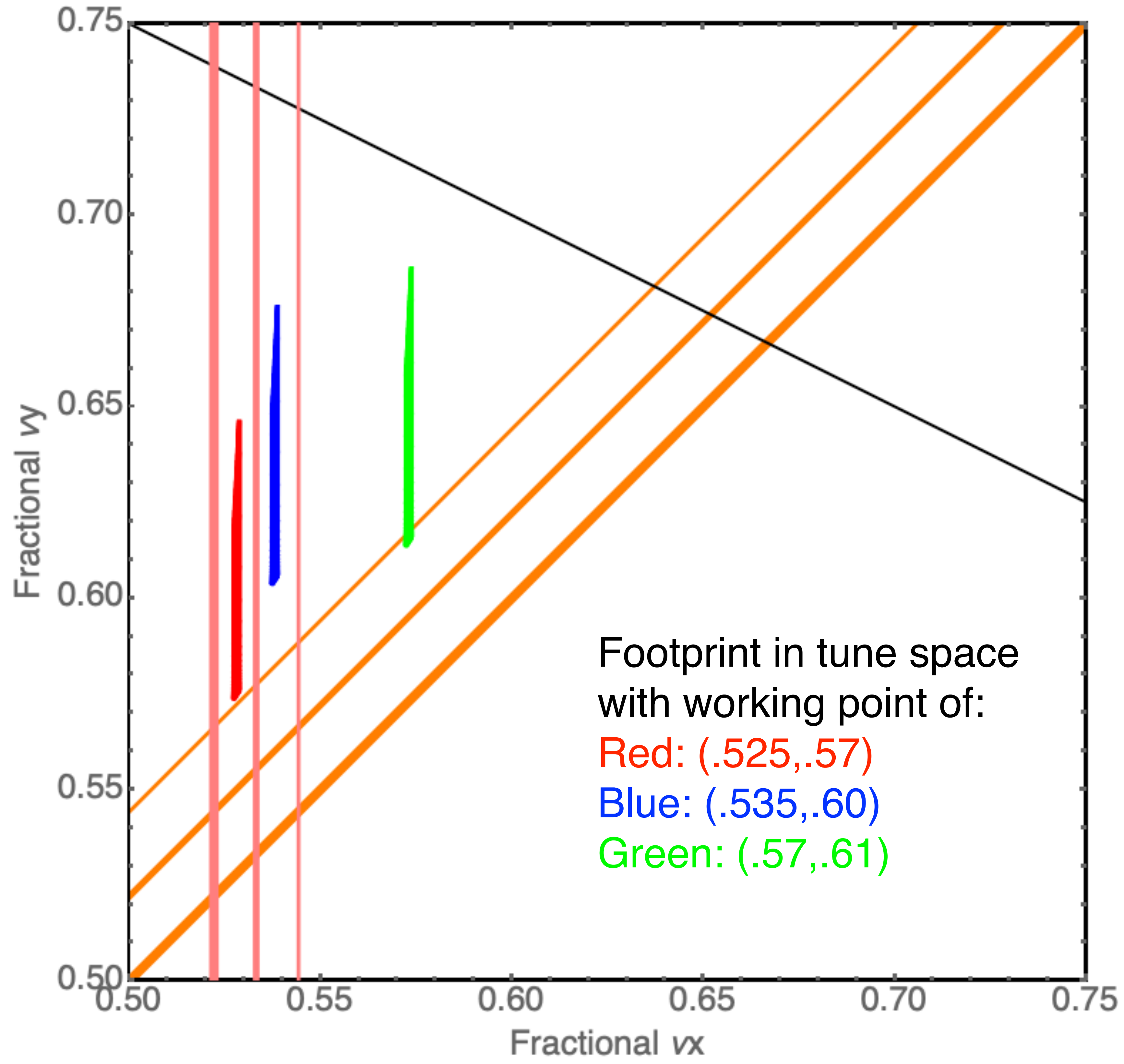
[3] P. Raimondi and M. Zobov, "Tune shift in beam-beam collisions with a crossing angle", DAFNE Tech. Note G-58 (2003).

Collective effects

- Due to the nonlinear nature of beam-beam force, the particles of a bunch have a footprint in the tune space.
- The incoherent beam-beam tune shift is useful to estimate the interplay of beam-beam and lattice resonances. In principle, the footprint of the beam in tune space should not overlap with any strong lattice resonances. An example of SuperKEKB LER is given for illustration.

	2021.07.01		Comments
	HER	LER	
I_{bunch} (mA)	0.80	1.0	
# bunch	1174		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	23	23	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.84	Natural bunch length (w/o MWI)
ν_x	45.532	44.525	Measured tune of pilot bunch
ν_y	43.582	46.593	Measured tune of pilot bunch
ν_s	0.0272	0.0221	Calculated from lattice
Crab waist	40%	80%	Lattice design





Collective effects

- Usually, beam-beam parameters ξ_{y+} and ξ_{y-} are calculated from luminosity:

$$L = \frac{1}{2er_e}\frac{\gamma_+ I_+}{\beta^*_{y+}}\xi_{y+} = \frac{1}{2er_e}\frac{\gamma_- I_-}{\beta^*_{y-}}\xi_{y-}$$

- From experiences of the colliders operated in the past decades, there is an upper limit (so-called beam-beam limit) for ξ_y. KEKB achieved $\xi_y \approx 0.09$. So we respect this fact and designed SuperKEKB with an assumption of $\xi_y \leq 0.09$.
- Given a beam-beam limit, high luminosity can be achieved by decreasing β^*_y or by increasing beam current I.
- What is the achievable ξ_y at SuperKEKB remains to be an open question.

- Since formulae for the luminosity of 3D Gaussian beam distribution are available, we can obtain the explicit forms of ξ_{y+} and ξ_{y-} in terms of beam parameters:

$$\xi_{y+} \approx \frac{r_e}{\pi \tan\frac{\theta_c}{2}}\frac{N_-\beta^*_{y+}}{\gamma_+\sqrt{\sigma^{*2}_{y+}+\sigma^{*2}_{y-}}\sqrt{\sigma^2_{z+}+\sigma^2_{z-}}}$$

$$\xi_{y-} \approx \frac{r_e}{\pi \tan\frac{\theta_c}{2}}\frac{N_+\beta^*_{y-}}{\gamma_-\sqrt{\sigma^{*2}_{y+}+\sigma^{*2}_{y-}}\sqrt{\sigma^2_{z+}+\sigma^2_{z-}}}$$

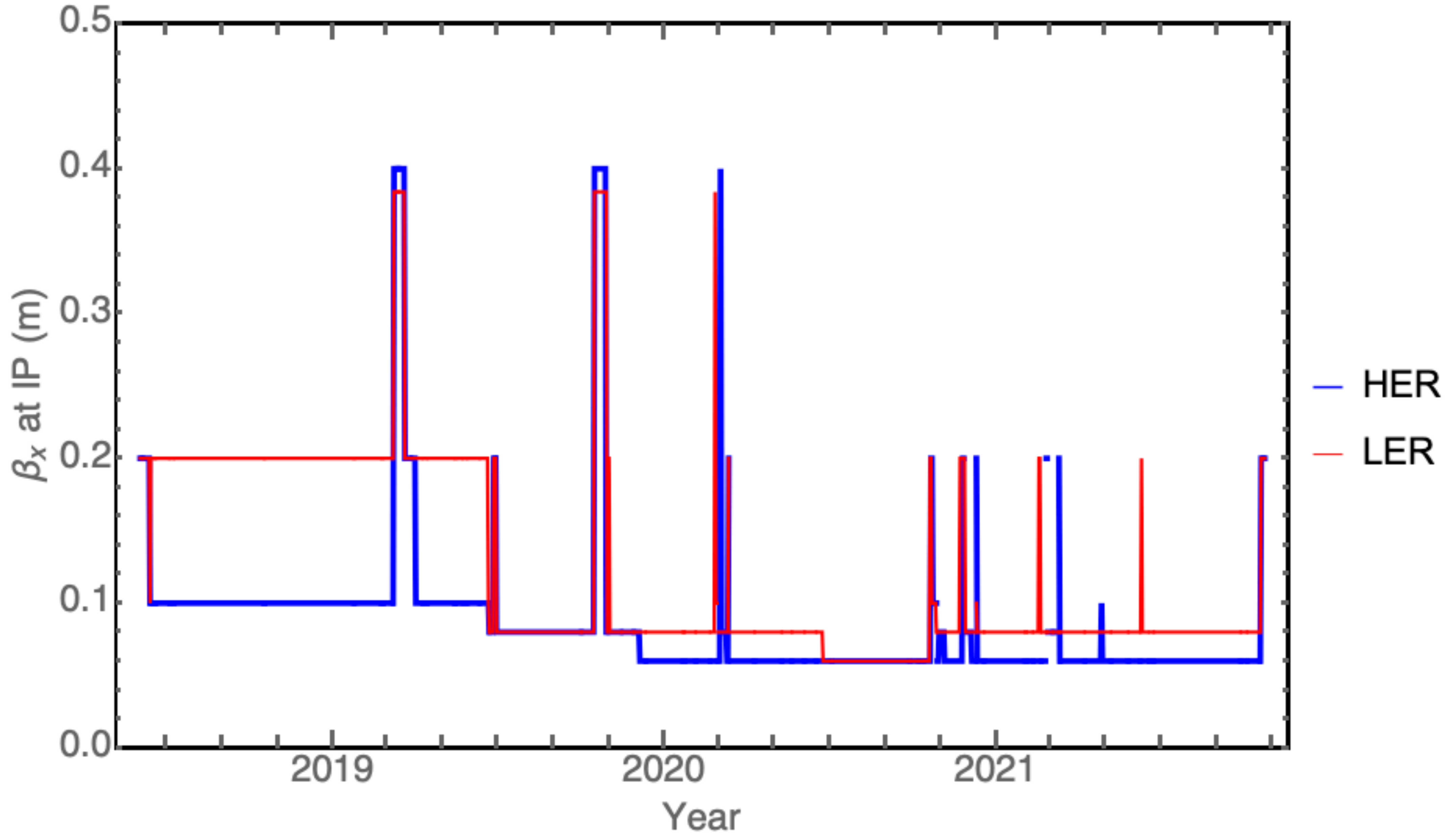
- If conditions of $\sigma_{z+} = \sigma_{z-}$, $\sigma_{y+} = \sigma_{y-}$, (balanced collision) $\sigma_x \gg \sigma_y$ (flat beams), and $\sigma_z \tan\frac{\theta_c}{2} \gg \sigma_x$ (large Piwinski angle) are satisfied, the beam-beam parameters $\xi_{y\pm}$ are equal to incoherent beam-beam tune shift $\xi^i_{y\pm}$.

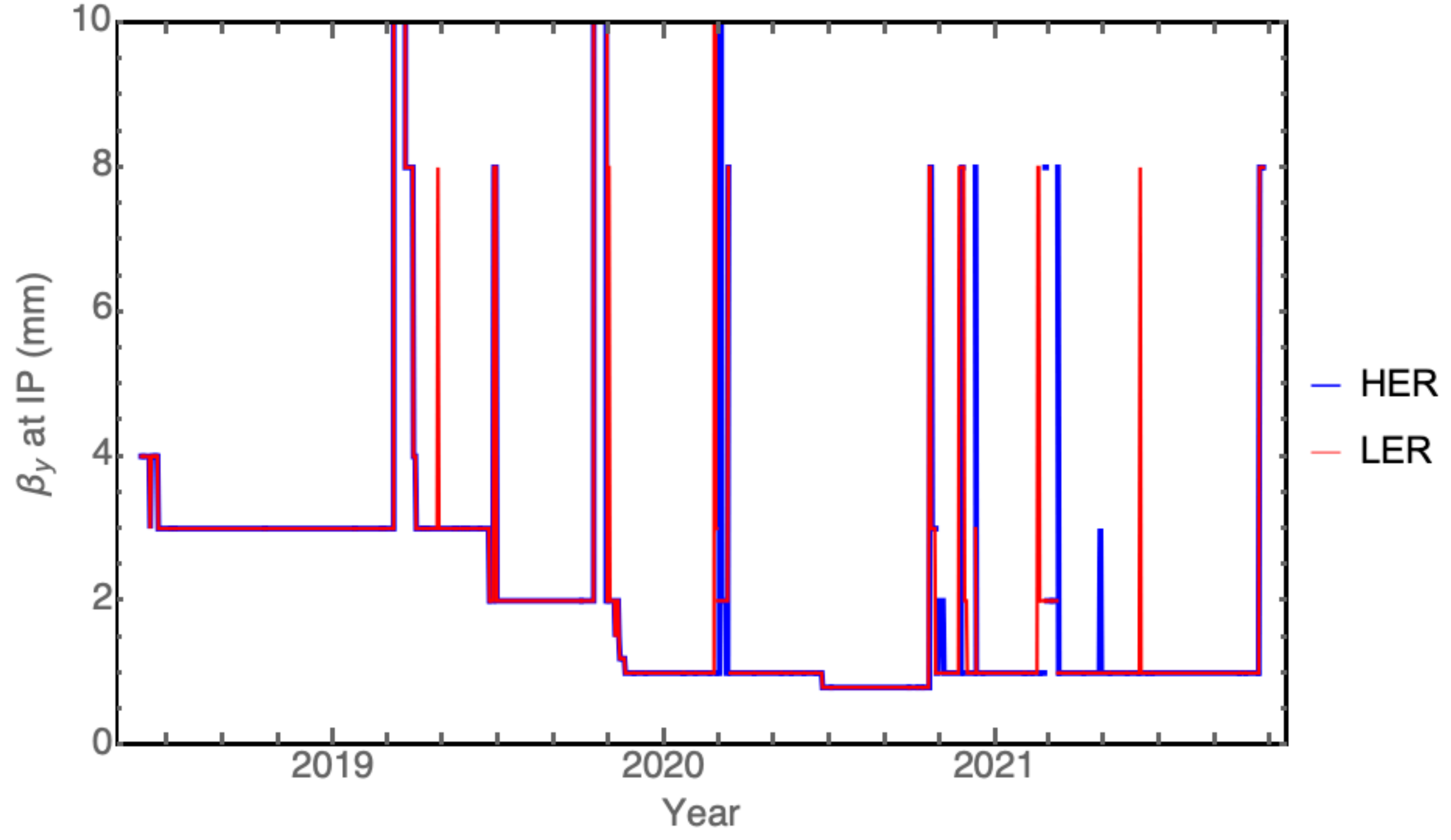
Collective effects

- With balanced collision conditions of $\sigma_{z+} = \sigma_{z-}$, $\sigma_{y+} = \sigma_{y-}$, and $\beta^*_{y+} = \beta^*_{y-}$, we can find a simple scaling law of ϵ_y vs β^*_y under a beam-beam limit:

$$\epsilon_y \propto \beta_y / \xi_y^2$$

- This scaling law implies that, the vertical emittance (single-beam emittance without collision) has to be small enough during the process of squeezing β^*_y.
- The right plots show history of $\beta^*_{x,y}$ since 2018.
- Currently, we are operating SuperKEKB with $\beta^*_y = 1$ mm. With crab waist implemented, squeezing β^*_y further meets challenges in:
  - Poor beam lifetime (side effect of crab waist)
  - Poor injection efficiency
  - High Belle II background
  - Low gain of luminosity





Collective effects

- Due to large crossing angle, the beam-beam resonances set a strong limit to the luminosity performance of SuperKEKB. Numerical simulations of beam-beam effects can be used for illustration.
- It is clear that crab waist strongly suppressed the beam-beam resonances and consequently creates a large area in tune space for good luminosity.
- Crab waist was not adopted in the initial design of SuperKEKB, but was partially implemented in 2020 as a remedy of beam-beam resonances.
- Using crab waist, a price we have to pay is loss of dynamic aperture and beam lifetime. Squeezing $\beta^*_{x,y}$ requires another remedy for the side effects of crab waist.

	2021.07.01		Comments
	HER	LER	
I_{bunch}	0.80	1.0	
# bunch	1174		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	23	23	Estimated from XRM data
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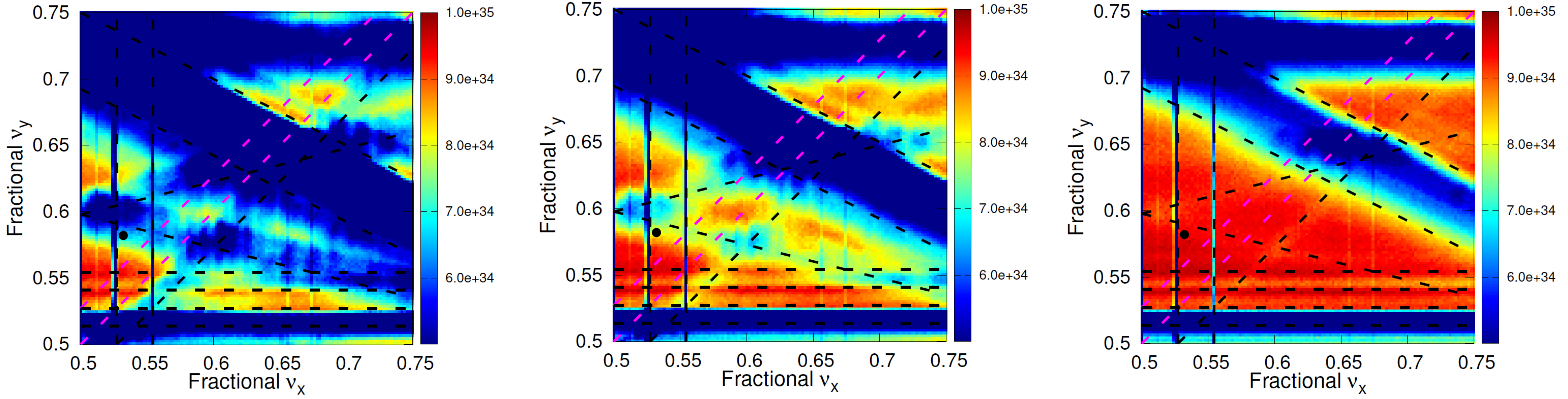
σ_{z0} (mm)	5.05	4.84	Natural bunch length (w/o
ν_x	45.532	44.525	Measured tune of pilot
ν_y	43.582	46.593	Measured tune of pilot
ν_s	0.0272	0.0221	Calculated from lattice
Crab	40%	80%	Lattice design

No crab waist | 40% crab waist | 80% crab waist

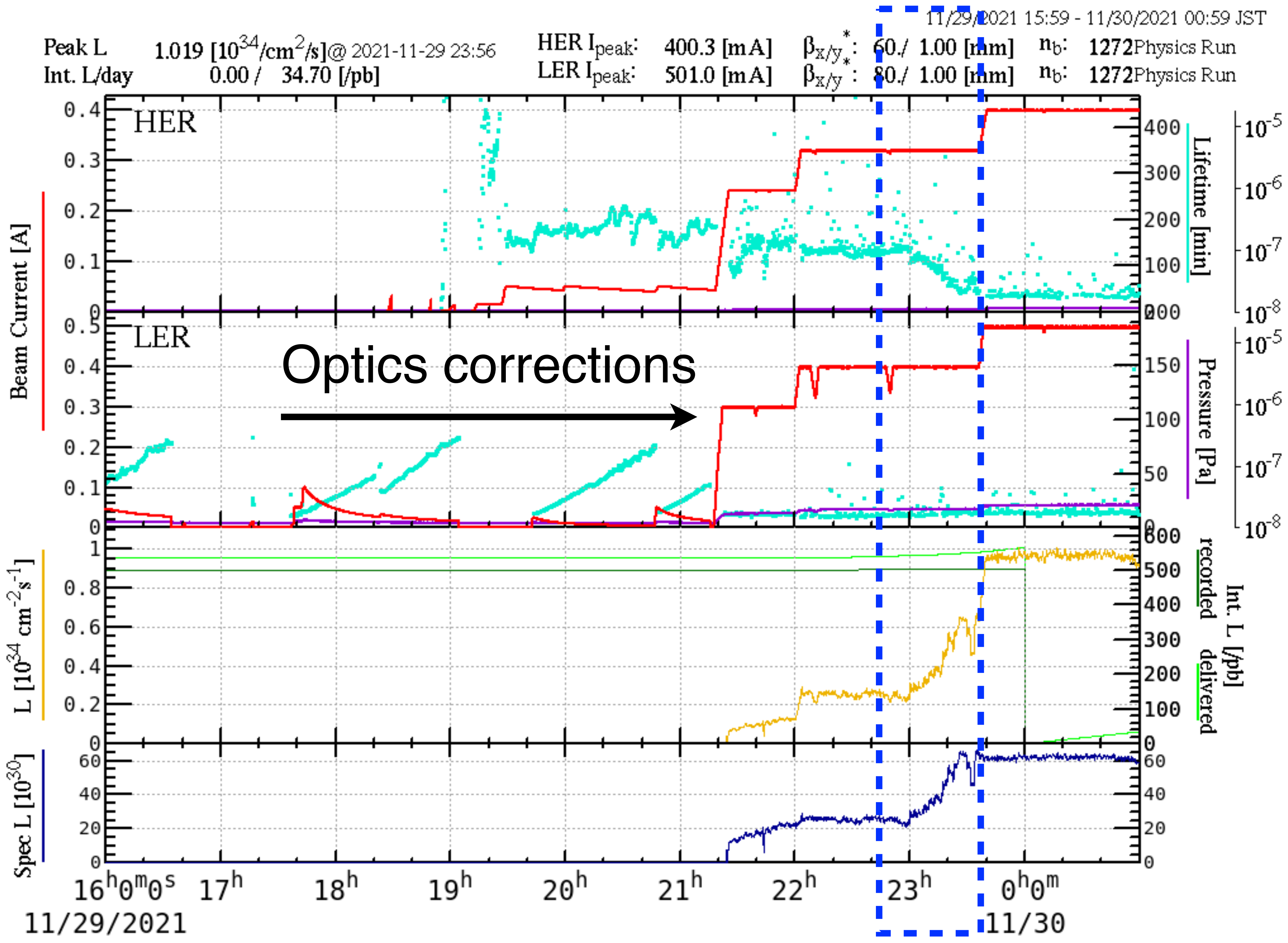


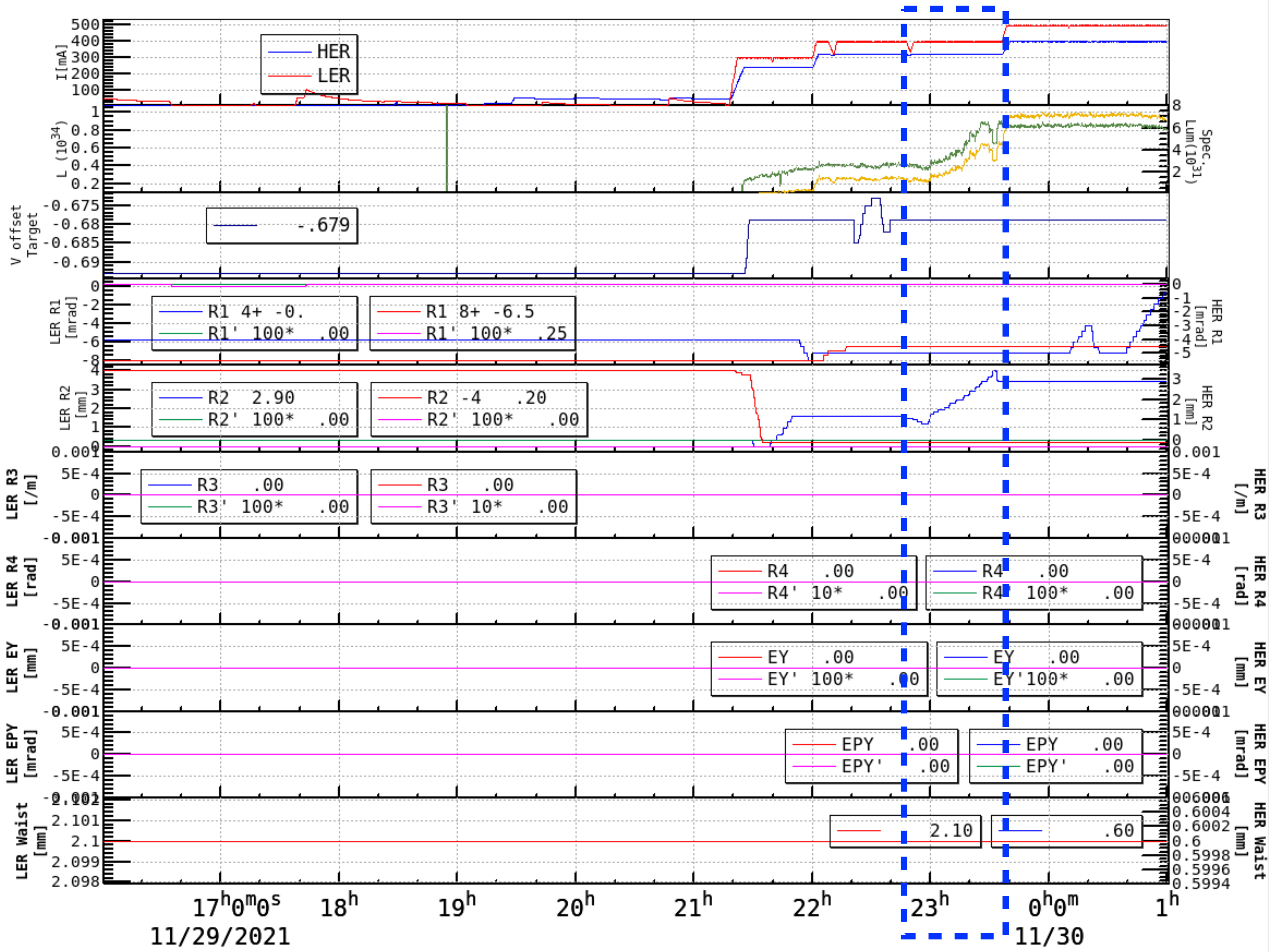
Machine tunings

- SuperKEKB is operated with may feedback (FB) systems working to stabilize the machine conditions. A far from full list gives the important FB systems:
  - Transverse and longitudinal FB systems (to suppress transverse and longitudinal coupled-bunch instabilities)
  - Continuous closed orbit correction (CCC) (to stabilize closed orbit of the beams)
  - Tune feedback (to compensate and stabilize the beam current dependent tunes)
  - So-called fast/slow iBump FB system (to stabilize the collision)
  - Injection FB system (to maintain the matching between linac and rings)
  - … …
- In addition to the automatic feedback system, the KCG/BCG/MSC (KEKB Commissioning Group/Belle II commissioning Group/Mitsubishi Company) shifters continuously observe the machine status (luminosity, beam sizes, lifetime, injection efficiency, detector background, etc.) and frequent manual tunings are done shift by shift.
- More than 100k of EPICS PVs (Process Variables) are constantly logged for post analysis to help understand the machine status.

Machine tunings

- The so-called luminosity optimization with IP knobs are frequently done by KCG shifters.
- The IP knobs are usually successful after fresh global optics correction (beta functions, coupling, dispersion).
- The global optics corrections do not control the optics parameters at IP well. So IP knobs serve as a next-step fine tuning.



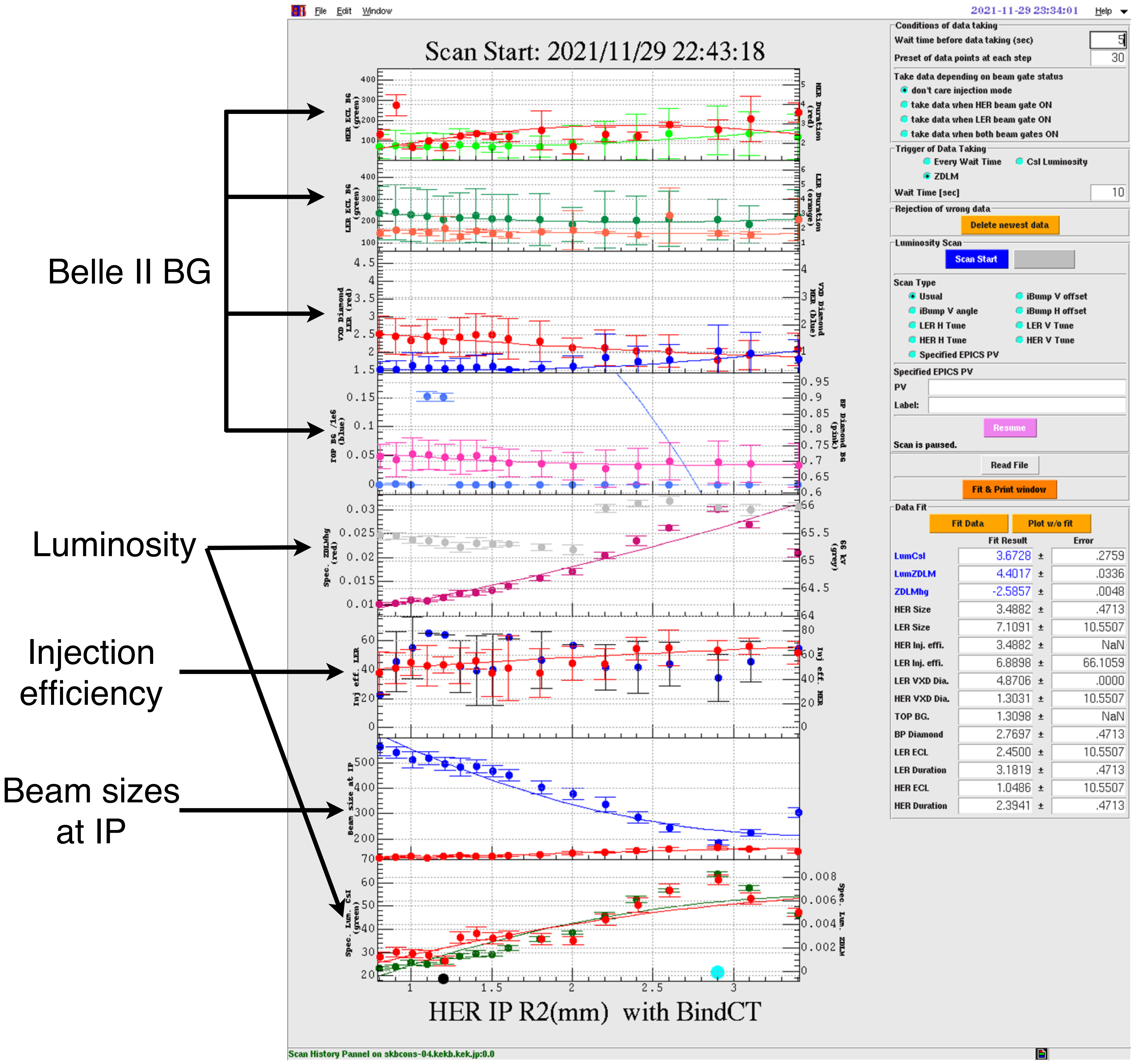


Machine tunings

- An example of successful IP knobs is shown.
- The principle of IP knobs is empirical:

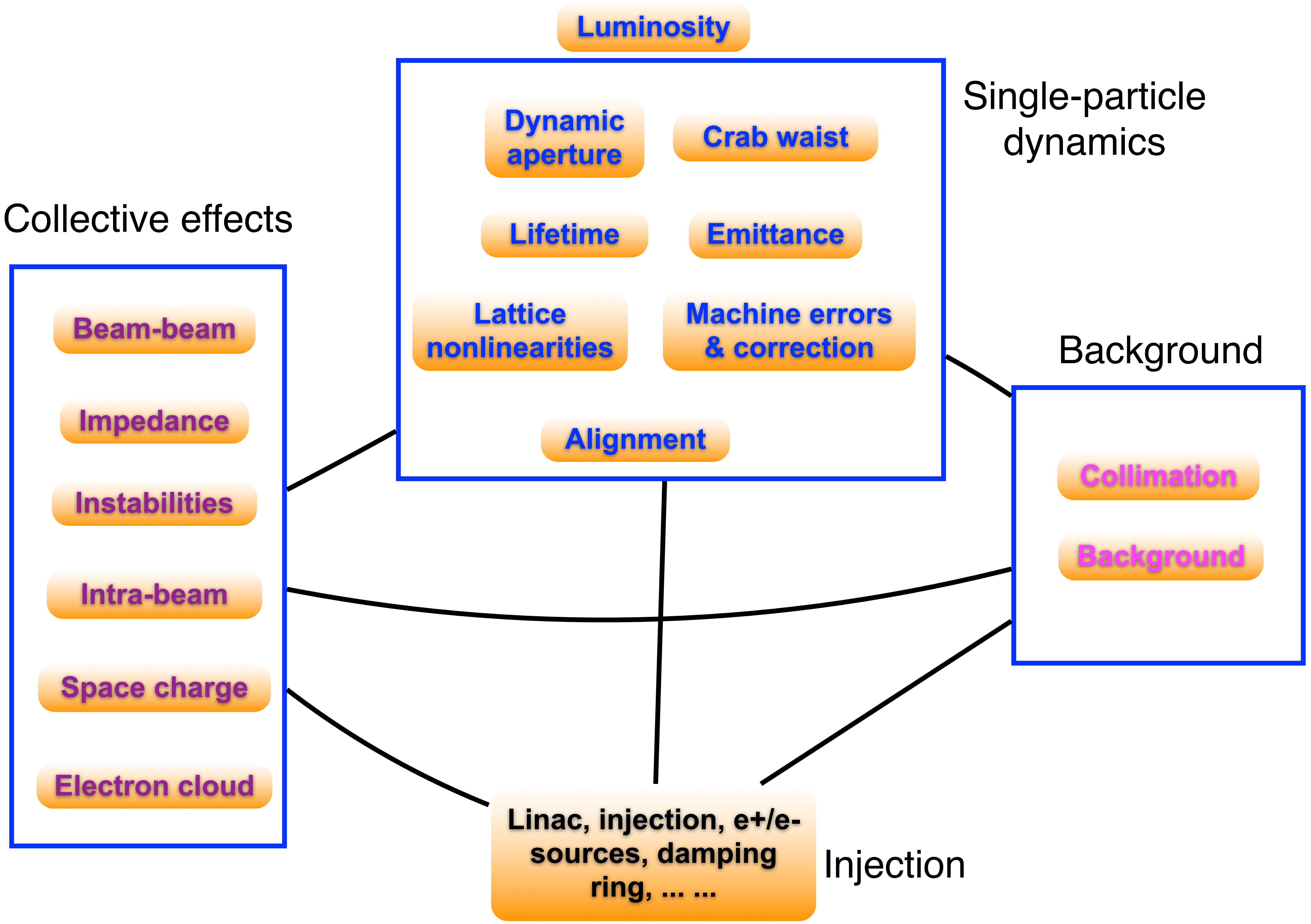
$$\frac{\partial L(\vec{R})}{\partial R_i} = 0 \quad \Rightarrow \quad R_i = 0$$

- The above criteria is not guaranteed since there are many parameters under control.
- Luminosity optimization is a challenging task.



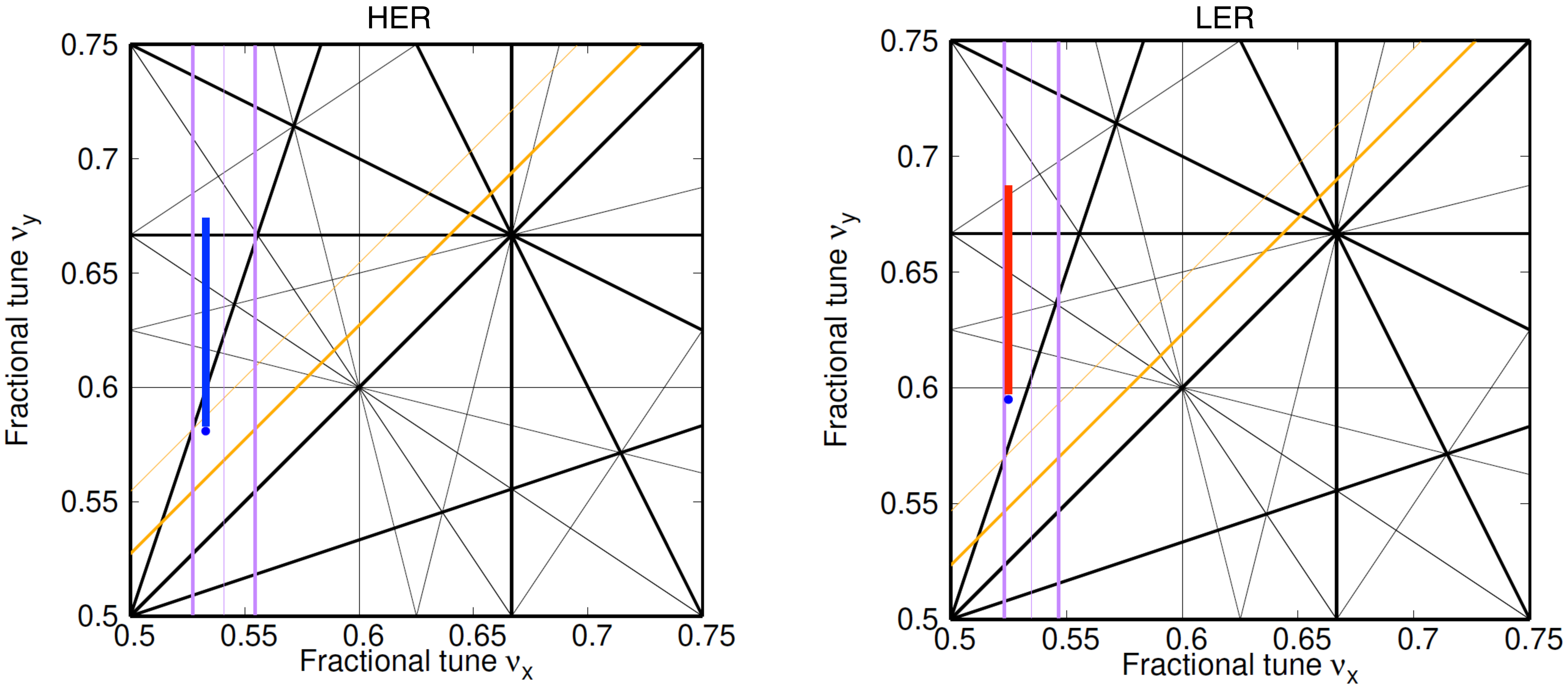
Challenges in accelerator physics at SuperKEKB

- The challenges in accelerator physics arise from the strong interplay of multiple beam dynamics.



Challenges in accelerator physics at SuperKEKB

- Using the resonance diagram for illustration: Beam-beam interaction and wake fields dynamic change the strengths, width and position of synchro-betatron resonance lines $m_x\nu_x + m_y\nu_y + m_s\nu_s =$ Integer. The footprint of the beam in tune space also depend on current and dynamically move around. It is difficult to locate the footprint in a region free from dangerous resonances.
- Crab waist is a key remedy, but achieving perfect crab waist is a new challenge.



Summary

- Fundamental concepts of accelerator physics at SuperKEKB
    - Beta functions, tunes, beam sizes, closed orbit, …
    - Luminosity, crab waist, …
- Linear and nonlinear beam dynamics
    - Coupling, chromatic coupling, resonances, …
- Collective effects
    - Wake fields, beam-beam, …
- Machine design, machine tuning
- Challenges in accelerator physics
    - Interplay of multiple beam dynamics

- References

    - Andrzej Wolski, “Beam dynamics in high energy particle accelerators”, Imperial College Press, 2014
    - Alexander Wu Chao, “Lectures on accelerator physics”, World Scientific, 2020
    - A. Chao et al., “Handbook of Accelerator Physics and Engineering”, 2013.