Time-Dependent Analysis Dissection

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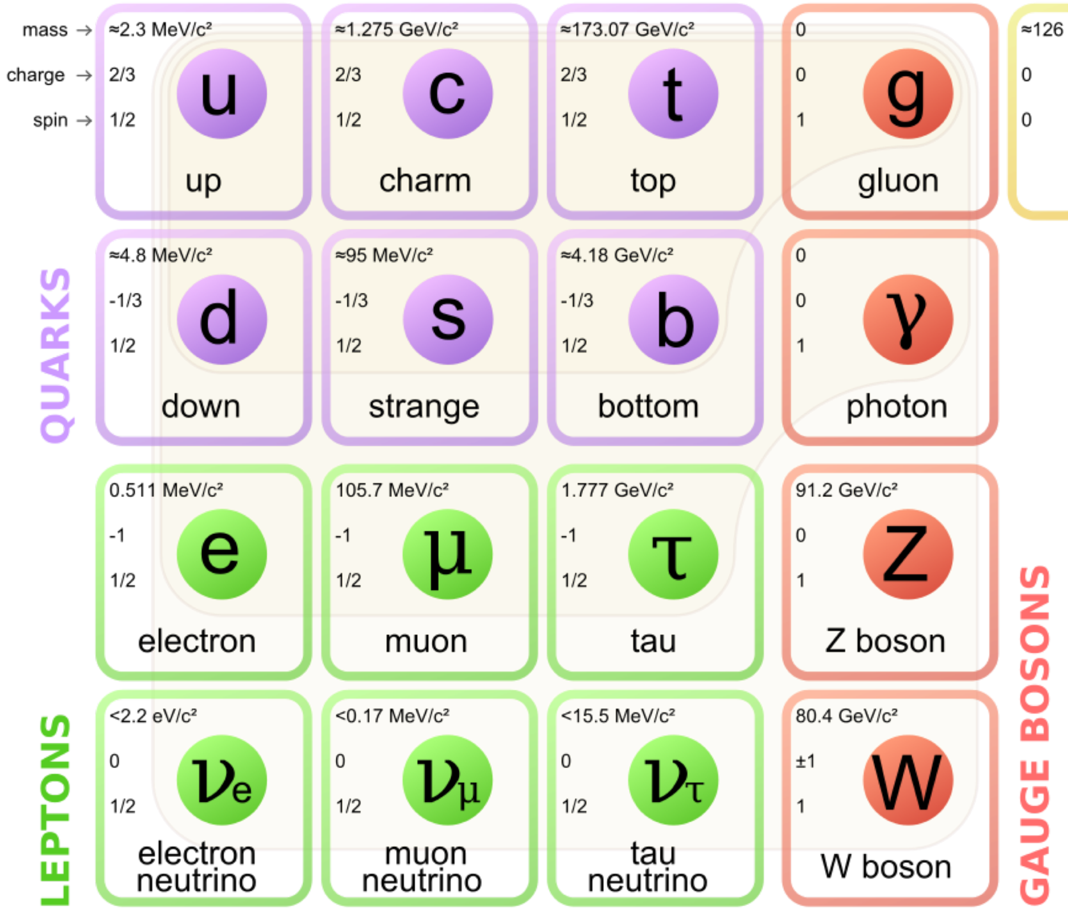
Particle List in the SM Framework

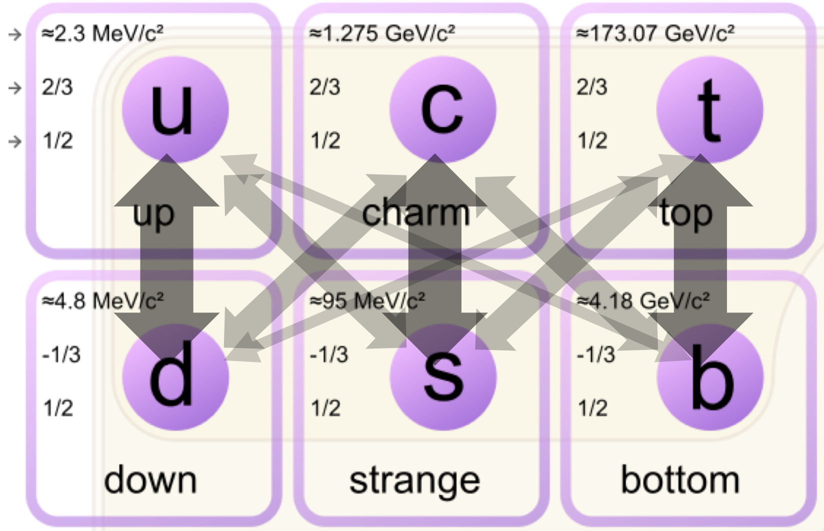


×6 quarks
The particle class for the constituents of *p*, *n*, ...

Kobayashi Maskawa Theory

- In 1973, when only 3 kinds of quarks (u, d, s) were known, M. Kobayashi and T. Maskawa proposed a new theory that gives an answer to the CP violation puzzle discovered by J. W. Cronin *et al*. in 1964.

- They needed a "complex" coupling-strength constant between two quarks to answer the puzzle. They realized that the complex constant is obtained by increasing the number of quark variations from 3 to 6.



- A matrix of coupling strengths between 3 up-type quarks and 3 down-type quarks is called Cabibbo-Kobayashi-Maskawa matrix. According to the KM theory, the elements V_{ub} and V_{td} are complex.

$$\begin{pmatrix} V_{ud} & V_{us} & \boldsymbol{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ \boldsymbol{V_{td}} & V_{ts} & V_{tb} \end{pmatrix} \equiv V_{CKM}$$

- By 1995, all the predicted kinds of quarks were discovered.
- **Question: is V_{td} really complex as M. Kobayashi and T. Maskawa predicted?**

CKM Matrix

- The CKM matrix is a unitary matrix: $\begin{pmatrix} V_{ud} & V_{us} & \boldsymbol{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ \boldsymbol{V_{td}} & V_{ts} & V_{tb} \end{pmatrix}^{\dagger} \begin{pmatrix} V_{ud} & V_{us} & \boldsymbol{V_{ub}} \\ V_{cd} & V_{cs} & V_{cb} \\ \boldsymbol{V_{td}} & V_{ts} & V_{tb} \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

  From the unitarity condition, 6 equations are derived.

  (a) $V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$

  (b) $V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$

  (c) $V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$

  (d) $V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$

  (e) $V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$

  (f) $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

- From physics discussion, the Wolfenstein parameterization is obtained:

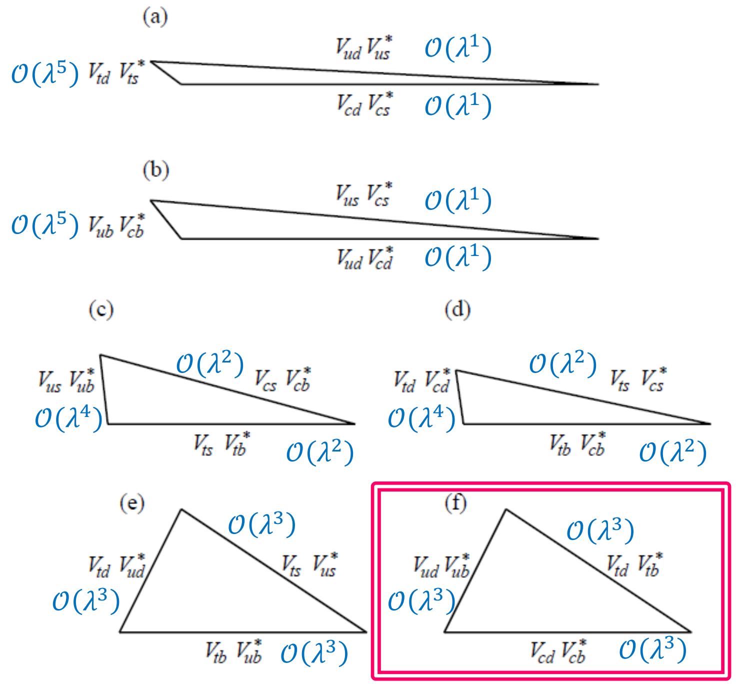
$$V_{\text{CKM}} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - \boldsymbol{i\eta}) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-\boldsymbol{i\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

  - You need to remember that V_{td} and V_{ub} are complex.
  - You need to remember $\lambda \approx 0.2$ plus the order of λ for each element.
  - You need to remember $A \approx 0.8$.

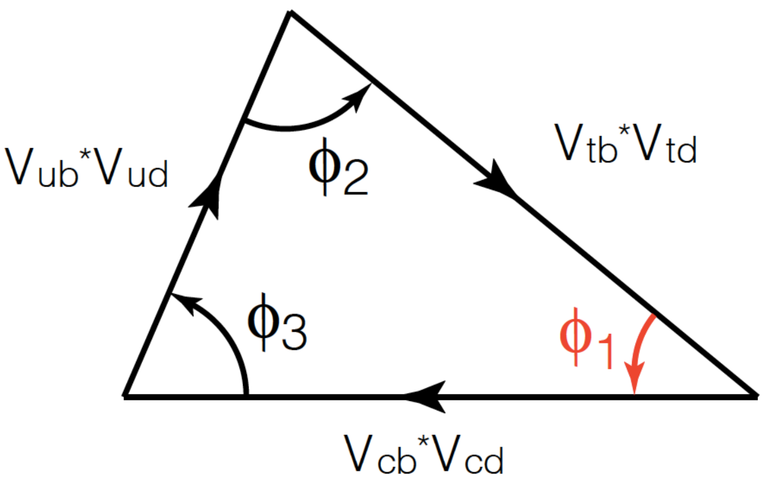
CKM Triangle

- Each of the equation forms a triangle on the complex plane.

- The bottom right triangle, which is associated to the equation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ is moderately large.



- By assuming $V_{ud}V_{ub}^*$, $V_{cd}V_{cb}^*$, and $V_{td}V_{tb}^*$ are vectors, we can draw a triangle associated to the equation on the complex plane, which is called "CKM triangle".



Interior angle definition

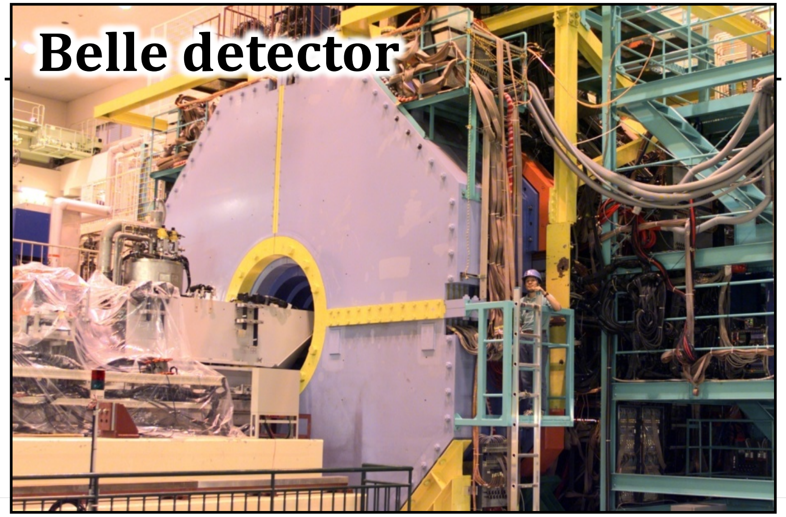
$$\phi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{\boldsymbol{V_{td}}V_{tb}^*}\right) = \pi - \arg(\boldsymbol{V_{td}})$$

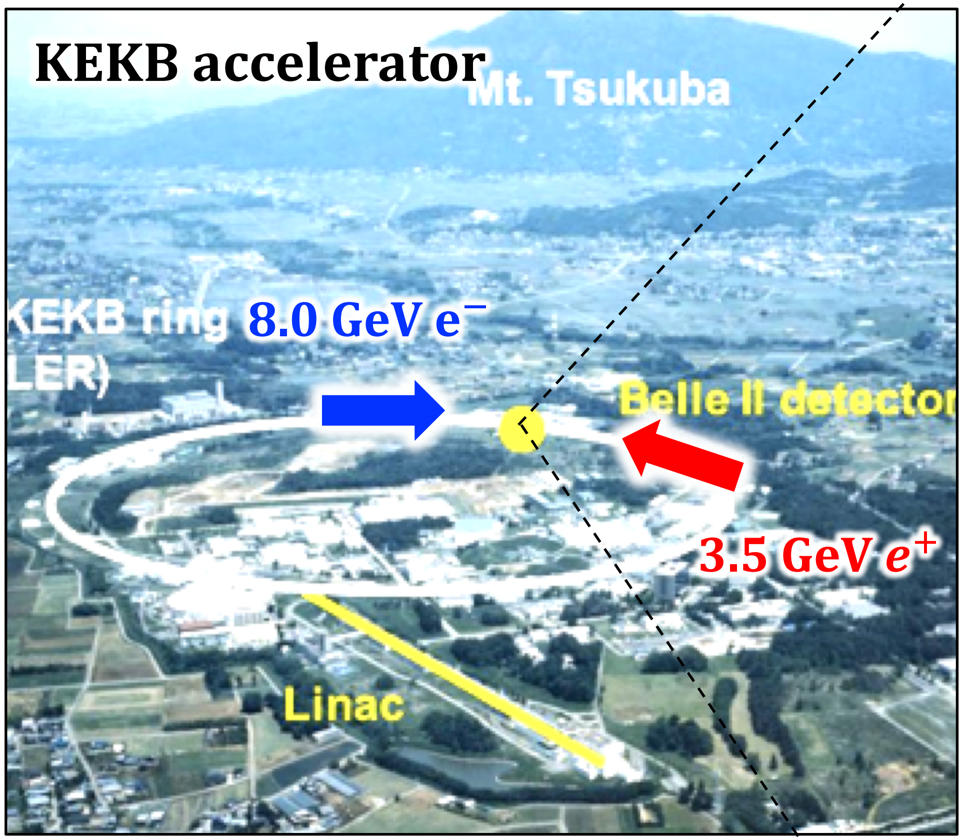
$$\phi_2 \equiv \arg\left(-\frac{\boldsymbol{V_{td}}V_{tb}^*}{V_{ud}\boldsymbol{V_{ub}^*}}\right)$$

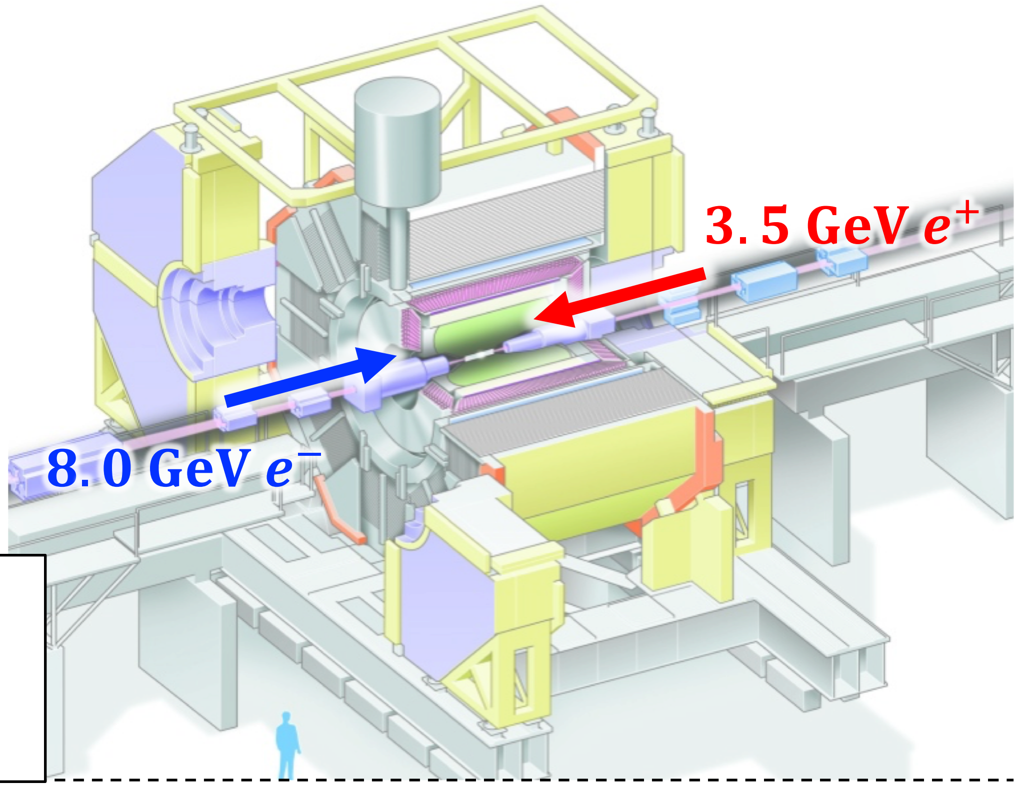
$$\phi_3 \equiv \arg\left(-\frac{V_{ud}\boldsymbol{V_{ub}^*}}{V_{cd}V_{cb}^*}\right)$$

If the KM theory is correct, $\phi_1 \neq 0, \pi$.

Belle ~~II~~ Experiment







- The Belle experiment verified the KM theory in 2001 by measuring the *CP* violation in the *B* meson system.

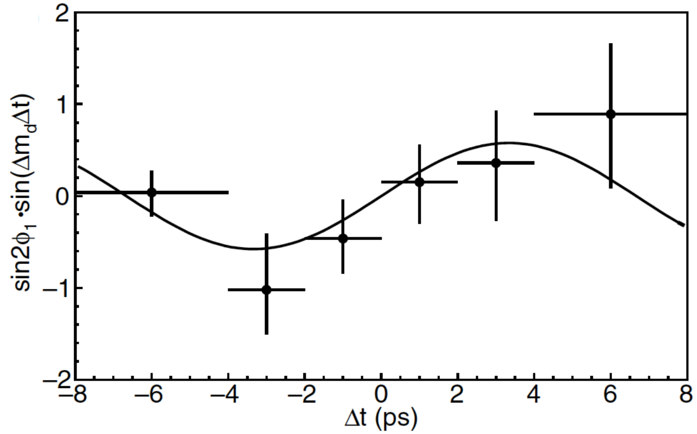
Belle ~~II~~ Experiment

Belle Collab., Phys. Rev. Lett. **86**, 2509 (2001).

2001/02/09 ($\sim 10\times 10^6$ $B\bar{B}$ pairs)

$$\sin 2\phi_1 = +0.58^{+0.32}_{-0.34}{}^{+0.09}_{-0.10}$$

First report

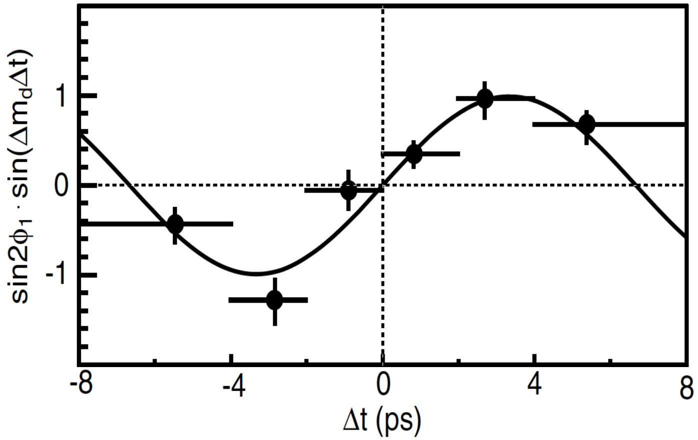


Belle Collab., Phys. Rev. Lett. **87**, 091802 (2001).

2001/07/18 ($\sim 29\times 10^6$ $B\bar{B}$ pairs)

$$\boldsymbol{\sin 2\phi_1 = +0.99 \pm 0.14 \pm 0.06}$$

Confirmation of $\boldsymbol{\phi_1 \neq 0}$

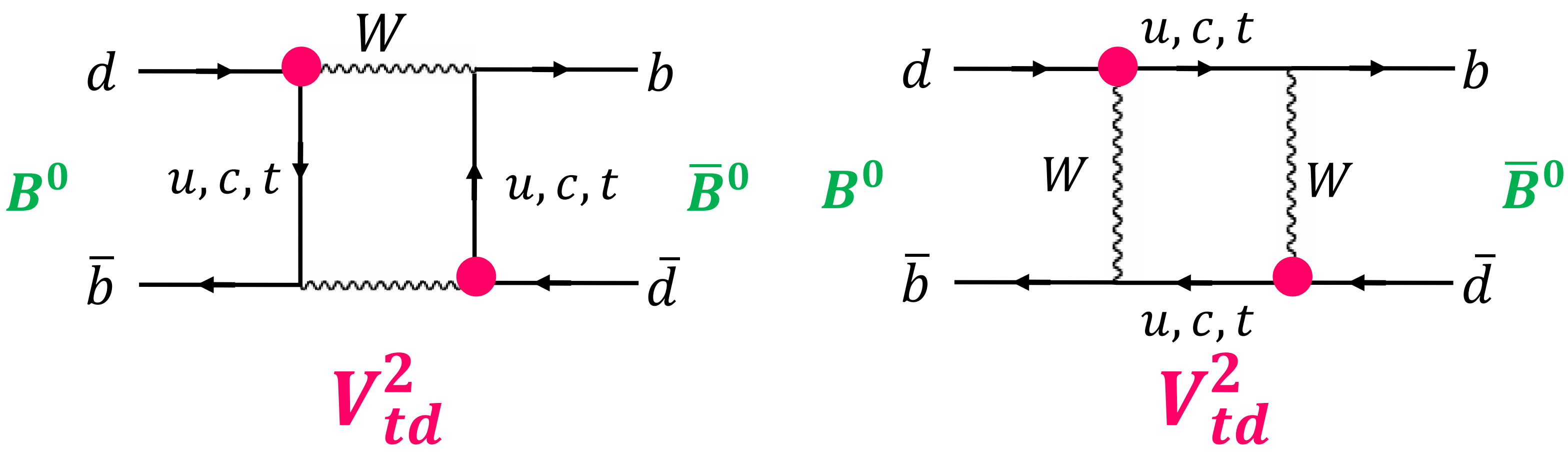


By demonstrating $\sin 2\phi_1 \neq 0$, Belle verified the KM theory in 2001. They used the "**time-dependent analysis**" method to obtain $\sin 2\phi_1$. The same method will be used in NP search in Belle II. Dissect the time-dependent analysis method.

Where is V_{td}?

Neutral-meson mixing

- A neutral meson produced by any means "mixes" with its *CP* partner until it decays. A particle initially produced in the $|X^0\rangle$ state stochastically stays in the $|X^0\rangle$ state or moves to the $|\bar{X}^0\rangle$ state. **No relevance with the pair creation.**

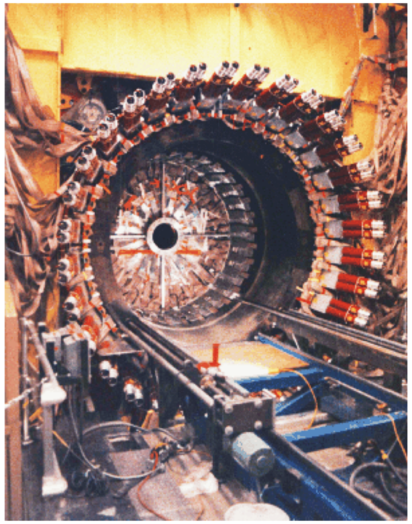


V_{td} in the B^0-$\bar{B}^0$ mixing

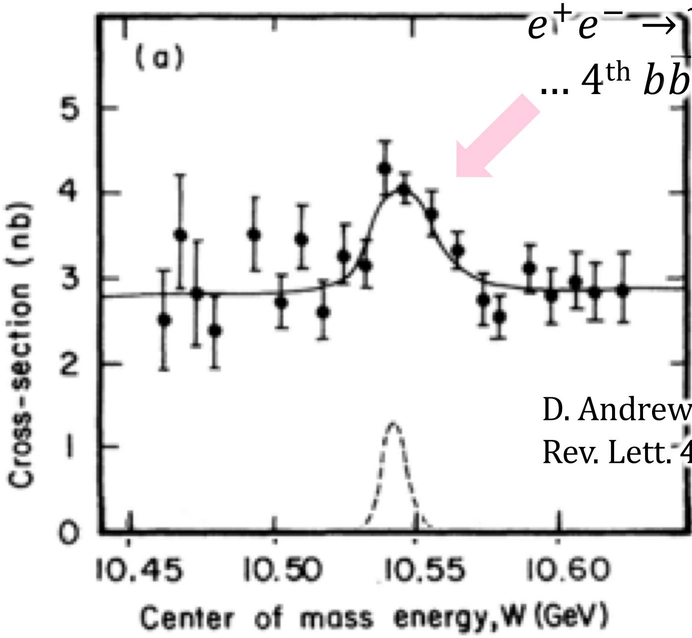
- The amplitude associated to the B^0-$\bar{B}^0$ mixing includes V_{td}^2. The B^0-$\bar{B}^0$ mixing would be a good probe to measure $\sin 2\phi_1$, where $\phi_1 \equiv \pi - \arg(V_{td})$.

Digression: B^0 Discovery (1980)

https://www.classe.cornell.edu/

Cornell Electron Storage Ring (left)
and CLEO-II detector (right)

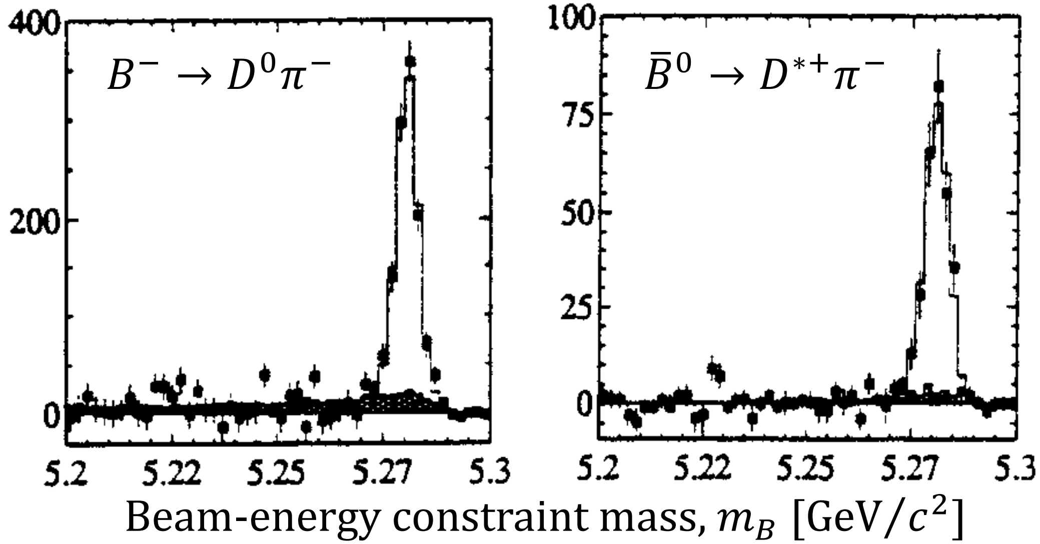


$e^+e^- \to \Upsilon(4S)$
… 4^{th} $b\bar{b}$ resonance

D. Andrews *et al.*, Phys. Rev. Lett. **45**, 219 (1980).

$\Upsilon(4S) \to B^0\bar{B}^0$ (50%)
$\to B^+B^-$ (50%)

$$m_B^2 = E_{\text{beam}}^2 - (\vec{p}_D + \vec{p}_\pi)^2$$



AIP Conf. Proc. **424**, 75 (1998).

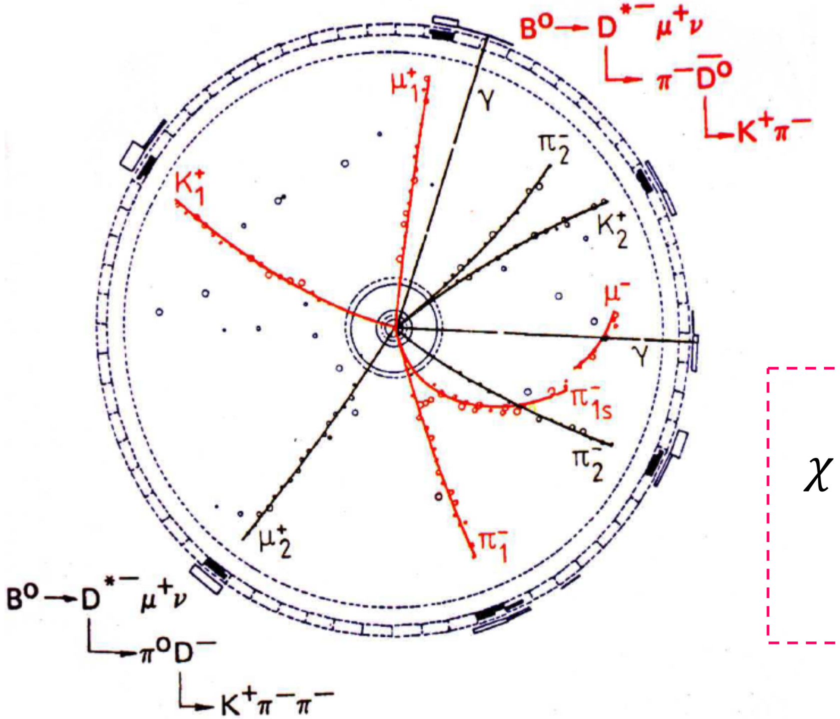
Digression: B^0-$\bar{B}^0$ Mixing (1986, 1987)

- B^0- $\bar{B}^0$ mixing: $\chi = \dfrac{N(B^0 \to \bar{B}^0)}{N(B^0 \to B^0) + N(B^0 \to \bar{B}^0)}$. The $\chi = 0$ if no mixing.

$N(B^0 \to B^0)$ ... unmixed $N(B^0 \to \bar{B}^0)$ ... mixed

Evidence for the B^0- $\bar{B}^0$ mixing

- A $\bar{B}^0$ pairly-produced with a B^0 from the e^+-e^- collision by the DORIS II accelerator had changed to a B^0.



https://argus-fest.desy.de/e301/e305/wsp_arg_new.pdf

ARGUS Collaboration, Phys. Lett.**B192**, 245 (1987).

$$\chi = \frac{N(B^0 \to \bar{B}^0)}{N(B^0 \to B^0) + N(B^0 \to \bar{B}^0)} = 0.17 \pm 0.05$$

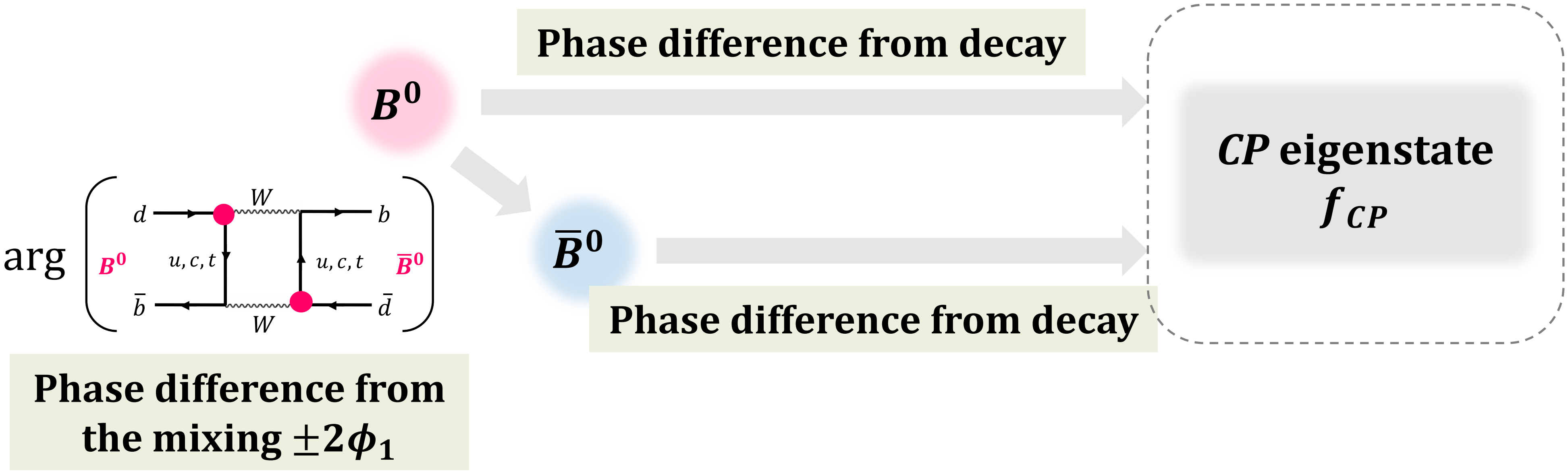
$\chi \equiv x^2/2(1+x^2), x \equiv \Delta m_{B^0}/\Gamma_{B^0}$.

$$\boldsymbol{\chi_{B^0}^{\mathrm{PDG2020}} = 0.1858 \pm 0.0011}$$

Mixing-Induced *CP* Violation

- *CP* eigenstate f_{CP}: a state to which both B^0 and $\bar{B}^0$ can decay.
  $J/\psi\, K_S^0, \phi K_S^0, J/\psi\, \pi^0, \pi^+\pi^-$, ...
- Flavor specific final state:
  a state to which only either of B^0 and $\bar{B}^0$ can decay.
  $B^0 \to D^{*-}\ell^+\nu_\ell \Leftrightarrow \bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$, ...

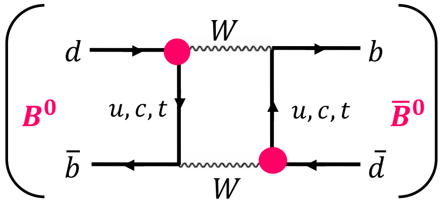
- There are two possible paths from B^0 to f_{CP}: $B^0 \to f_{CP}$ and $B^0 \to \bar{B}^0 \to f_{CP}$.
  They may have different weak phases as below.



Mixing-Induced *CP* Violation

Phase difference from decay

$\boldsymbol{B^0}$

arg 

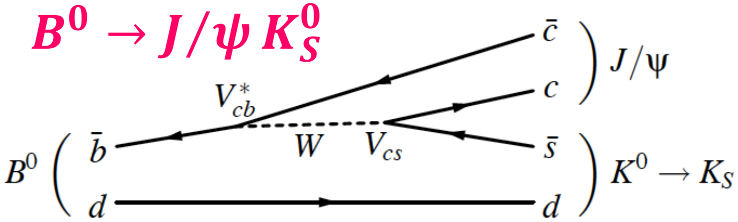
$\boldsymbol{\bar{B}^0}$

Phase difference from decay

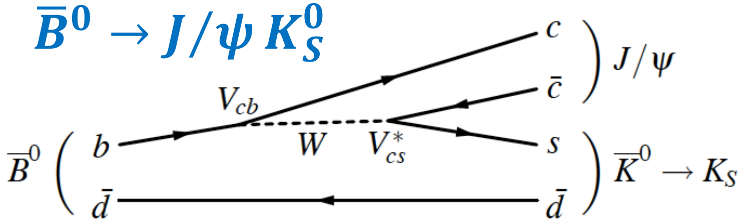
***CP* eigenstate**
$\boldsymbol{f_{CP}}$

Phase difference from the mixing $\pm 2\phi_1$

$\boldsymbol{B^0 \to J/\psi\, K^0_S}$



$\boldsymbol{\bar{B}^0 \to J/\psi\, K^0_S}$



$$\arg\left(B^0 \to J/\psi\, K^0_S\right) = \arg(V^*_{cb} V_{cs}) = 0$$

$$\arg\left(\bar{B}^0 \to J/\psi\, K^0_S\right) = \arg(V_{cb} V_{cs}) = 0$$

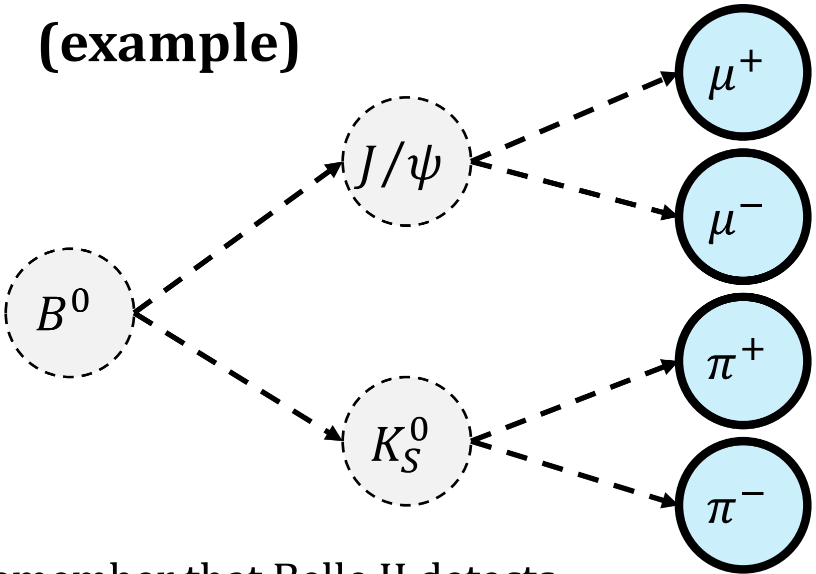
Remember only $\arg(V_{td})$ and $\arg(V_{ub})$ are non zero.

We can extract $\boldsymbol{\phi_1}$ by analyzing the $\boldsymbol{B \to J/\psi\, K^0}$ and other $\boldsymbol{(c\bar{c})K^0}$ modes.

Event Reconstruction

- We detect the $B \to J/\psi\, K_S^0$ decay by trying to reconstruct the B-meson candidate mass from particles recorded by the detector.

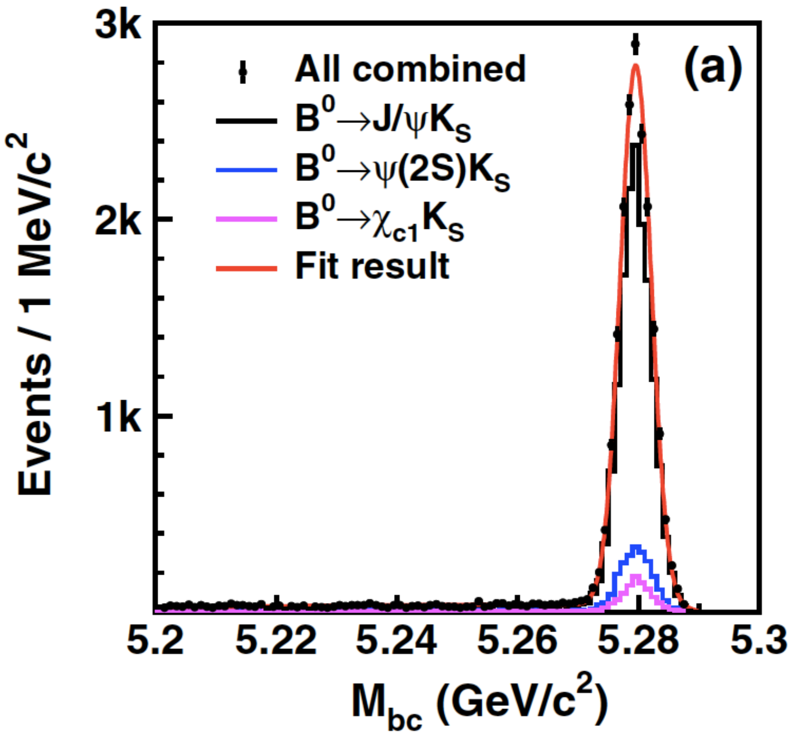
Event reconstruction (example)



Remember that Belle II detects only $\pi^\pm,\ K^\pm,\ e^\pm,\ \mu^\pm,\ p(\bar{p})$, and γ.

$$\left(E_{B^0}^{\text{cand}}, \vec{p}_{B^0}^{\text{cand}}\right) = \sum_i^{\mu^+,\mu^-,\pi^+,\pi^-} (E_i, \vec{p}_i)$$

Belle Collaboration, Phys. Rev. Lett. **108**, 171802 (2012).



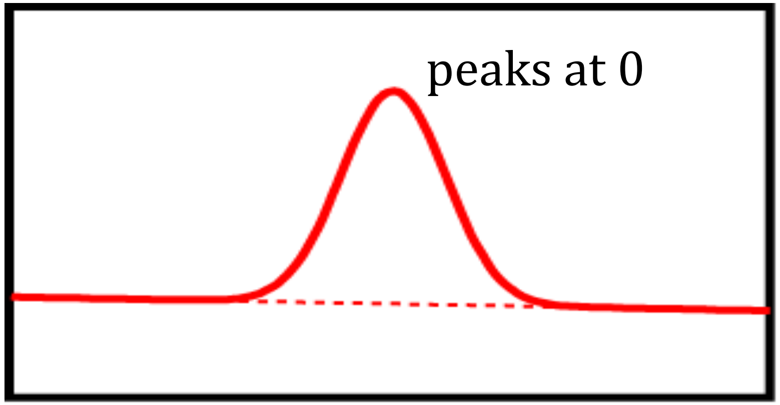
Luckily, the TD analysis is less affected to by the event reconstruction procedure.

Event Reconstruction

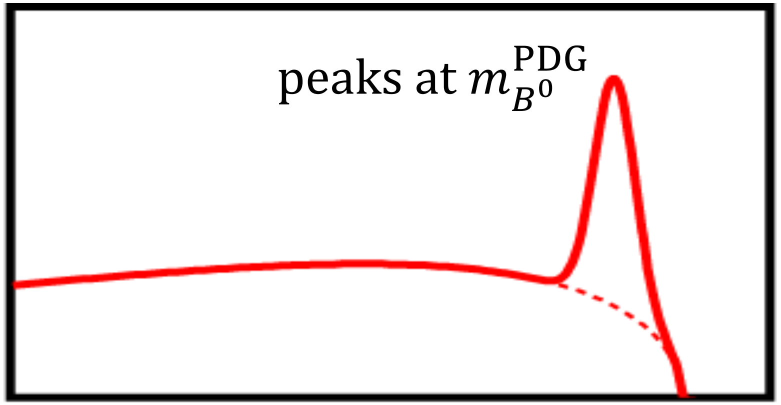
- Typical distributions of ΔE and m_{BC} of the reconstructed B candidates are …

$$\Delta E \equiv E_{B^0}^{\text{cand}} - E_{\text{beam}}^{\text{CMS}}$$

$$m_{BC} \equiv \sqrt{E_{\text{beam}}^2 - \left(\vec{p}_{B^0}^{\,\text{cand}}\right)^2}$$



Typical ΔE dist. model
- Signal: Gaussian(s)
- BG: 1st order poly



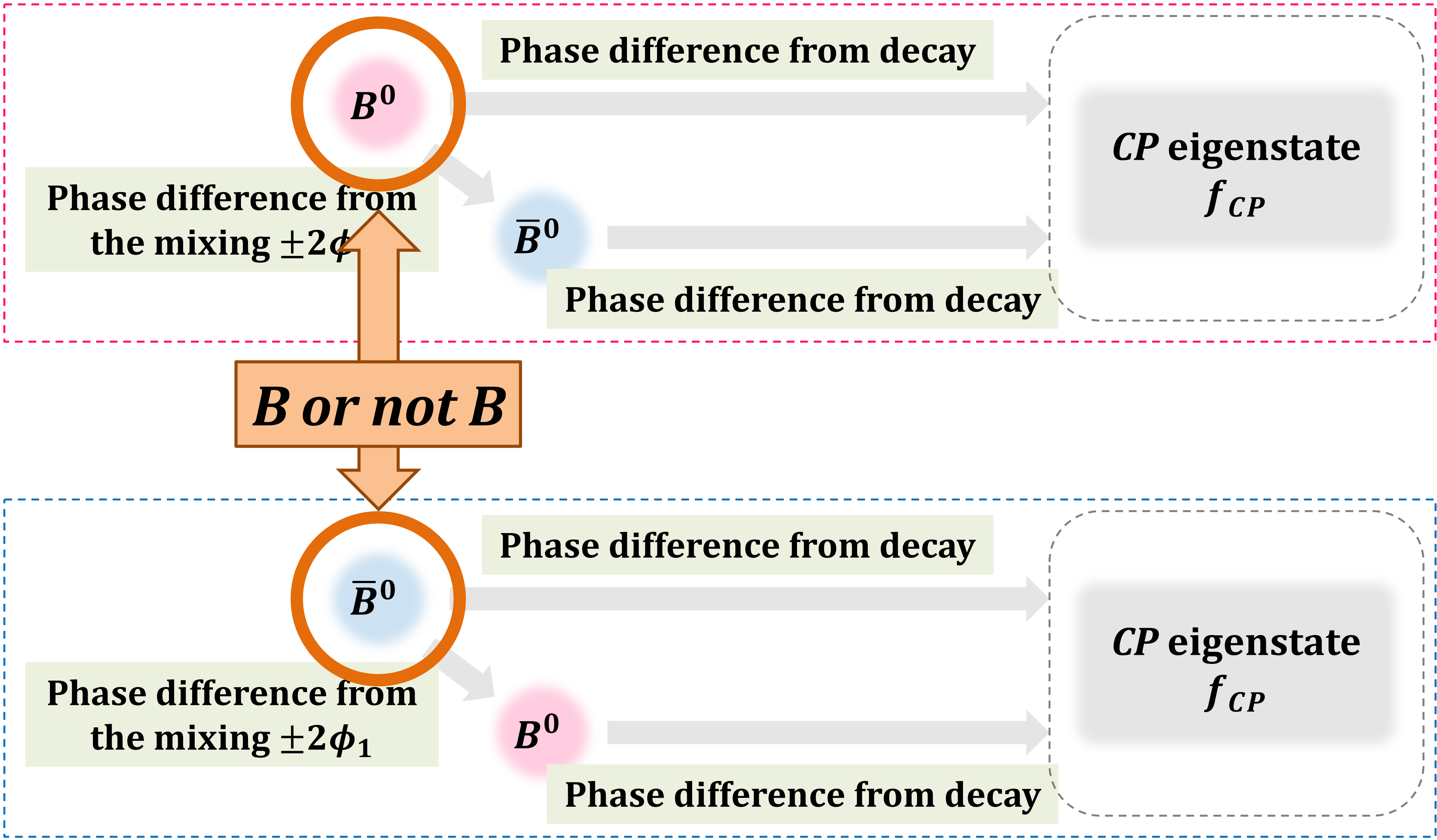
Typical m_{BC} dist. model
- Signal: Gaussian(s)
- BG: ARGUS function

$$ARGUS(x) \equiv E_{\text{beam}}^{\text{CMS}} x \sqrt{1 - x^2} \exp[d(1 - x)^2]$$
$$x \equiv m_{BC} / E_{\text{beam}}^{\text{CMS}}$$

- The overall shape is be determined by a 2D, 3D, or … ML fit method (but sometimes ΔE only (1D)).

- The event-by-event signal probability calculated from the determined signal and background shapes is used later when determining the ϕ_1.

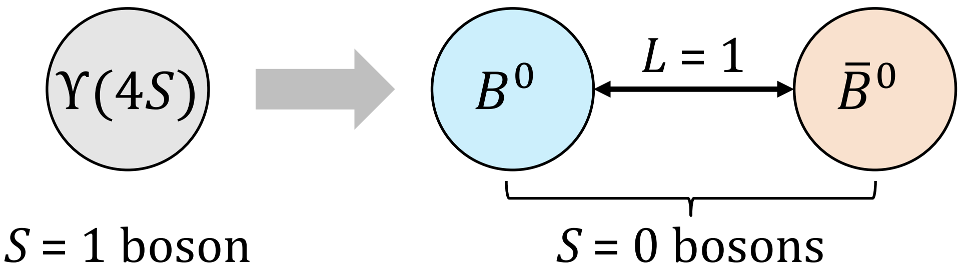
Flavor Tagging



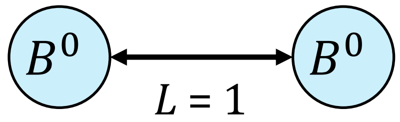
B^0-$\bar{B}^0$ Coherence

- Just *forget* about the B^0-$\bar{B}^0$ mixing.
- When B^0 and $\bar{B}^0$ are pair-produced from the $\Upsilon(4S) \to B^0\bar{B}^0$ decay, the two B-mesons take neither (B^0, B^0) nor $(\bar{B}^0, \bar{B}^0)$ state but only the $(B^0, \bar{B}^0)$ state.

[proof]



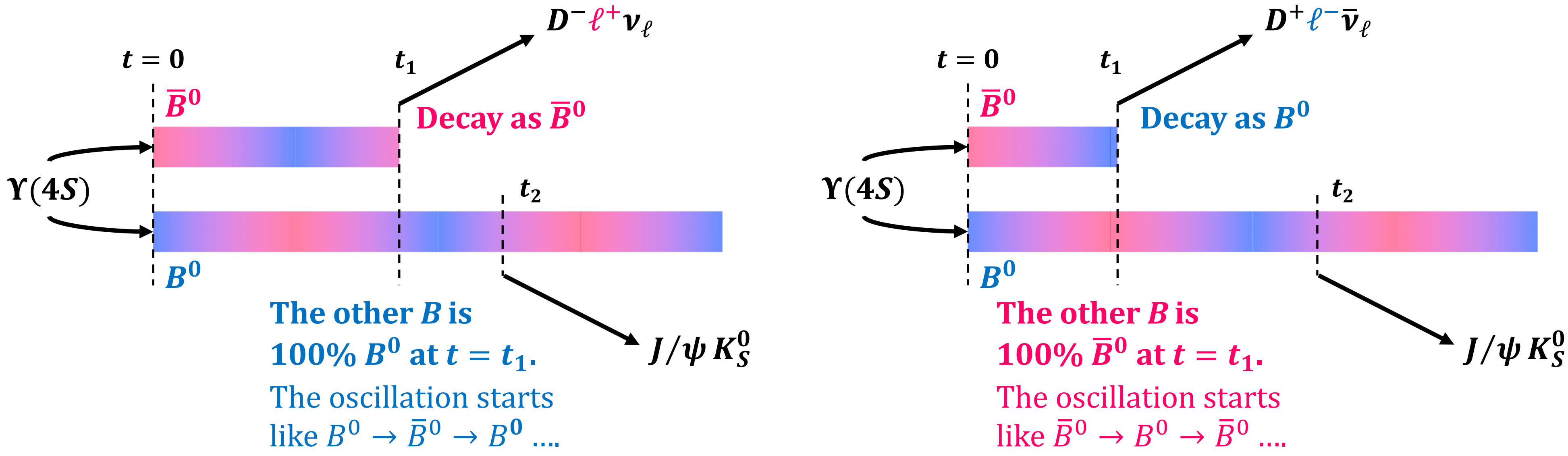
$\Upsilon(4S)$ is a $S = 1$ boson, and B^0, $\bar{B}^0$ are $S = 0$ boson. Because of the angular momentum conservation, the orbital angular momentum L between the two B mesons is $L = 1$.



If both of the two B mesons take the same particle state B^0, the wavefunction of the system is exchange-symmetric of the two B mesons because of the Bose-Einstein statistics. However, the wavefunction must be exchange-antisymmetric of two same particles with $L = 1$. These two statements are inconsistent. The same inconsistency is true for the $\bar{B}^0$- $\bar{B}^0$ system.

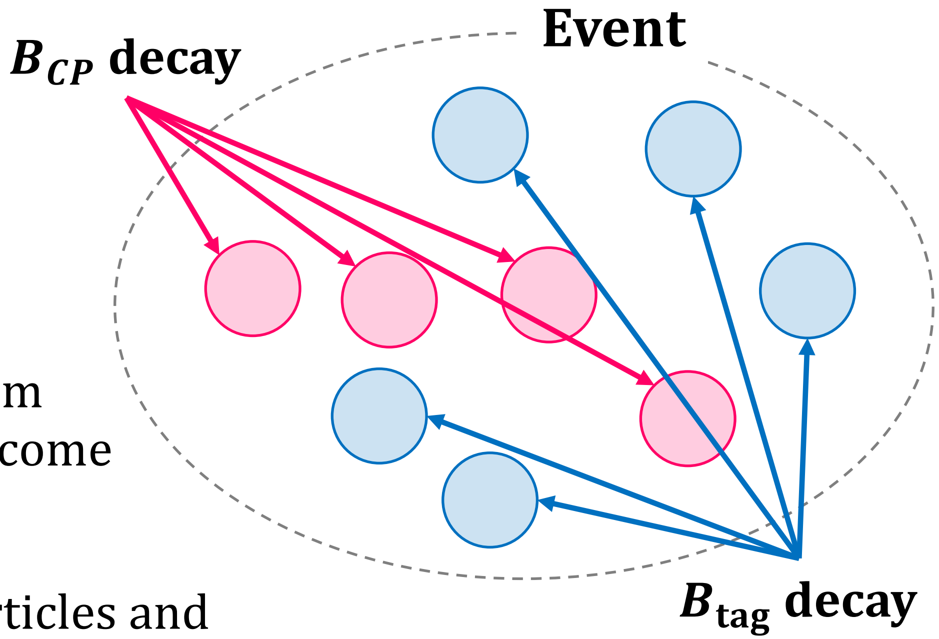
Thence, **only the $(B^0, \bar{B}^0)$ state is allowed**. □

B^0-$\bar{B}^0$ Mixing And B^0-$\bar{B}^0$ Coherence



- The flavor of the two B mesons can be known only stochastically until they decay to some other particles. … **B^0-$\bar{B}^0$ mixing**
- One of the two B mesons B_1 decays to a flavor specific state. The B_1 flavor can be known from the decay products.
- The flavor of the other B meson B_2 at the time of the B_1 decay t=t_1 is opposite to the B_1 flavor. … **B^0-$\bar{B}^0$ coherence**
- The B_2 flavor can be known only statistically after t_1 until it decays to some other particles. … **B^0-$\bar{B}^0$ mixing**

Flavor Tagging



Prescription

- Reconstruct $B_{CP} \to J/\psi\, K_S^0$.
- Remove all daughter particles of B_{CP} from the event. Assume the rest of the event come from the other B meson, B_{tag}.
- Examine the assumed B_{tag} daughter particles and determine B_{tag} flavor, B^0 or $\bar{B}^0$, from the daughter-particle properties.
- The opposite flavor to the B_{tag} flavor is the B_{CP} flavor (this step can be skipped).

Example

- The $B_{\text{tag}} \to D\ell\nu$ decay (a flavor specific decay) tends to produce a high momentum ℓ. The lepton charge corresponds to the B_{tag} flavor by one-to-one: $B^0 \to D^{*-}\ell^+\nu_\ell$ and $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$.
- The B_{tag} flavor of an event with a high momentum $\ell^+(\ell^-)$ in ROE is estimated to be $B^0(\bar{B}^0)$.

Flavor Tagging

Implementations

Characteristic B decays

$$\overline{B}^0 \to D^{*+}\ \overline{\nu}_\ell\ \ell^-$$
$$\hookrightarrow D^0\ \pi^+$$
$$\hookrightarrow X\ K^-$$

$$\overline{B}^0 \to D^+\ \pi^-\ (K^-)$$
$$\hookrightarrow K^0\ \nu_\ell\ \ell^+$$

$$\overline{B}^0 \to \Lambda_c^+\ X^-$$
$$\hookrightarrow \Lambda\ \pi^+$$
$$\hookrightarrow p\ \pi^-$$

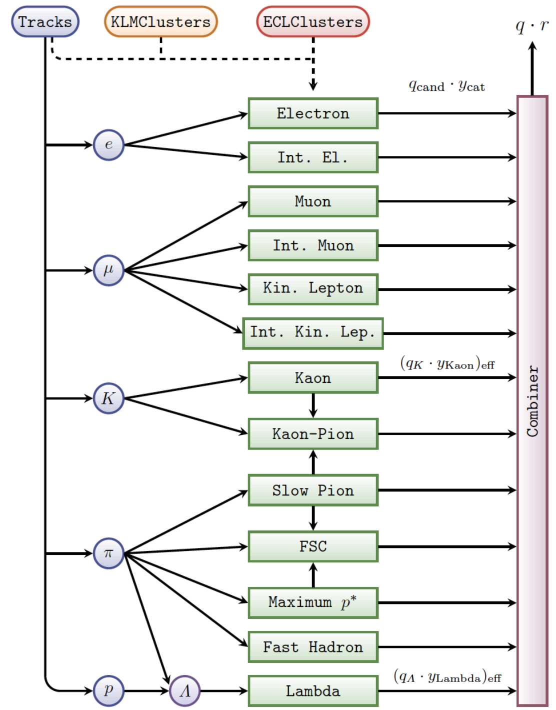
Parameter classification

Calculate B^0-likelihood and $\bar{B}^0$-likelihood for each category.

Categories	Targets for $\bar{B}^0$
Electron	e^-
Intermediate Electron	e^+
Muon	μ^-
Intermediate Muon	μ^+
Kinetic Lepton	ℓ^-
Intermediate Kinetic Lepton	ℓ^+
Kaon	K^-
Kaon-Pion	K^-, π^+
Slow Pion	π^+
Maximum p^*	ℓ^-, π^-
Fast-Slow-Correlated (FSC)	ℓ^-, π^+
Fast Hadron	π^-, K^-
Lambda	Λ

Final likelihood

Combine all the likelihoods.



- The Belle II flavor tagger employs boosted decision tree technology relying on the MC perfectness. The flavor tagger must be calibrated with the real data.

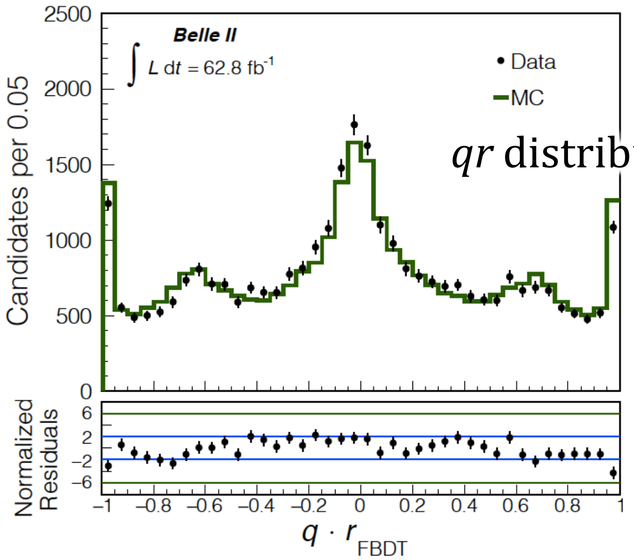
Flavor Tagging

Output of the flavor tagger

q: $q = +1$ for B^0 tag
$q = -1$ for $\bar{B}^0$ tag

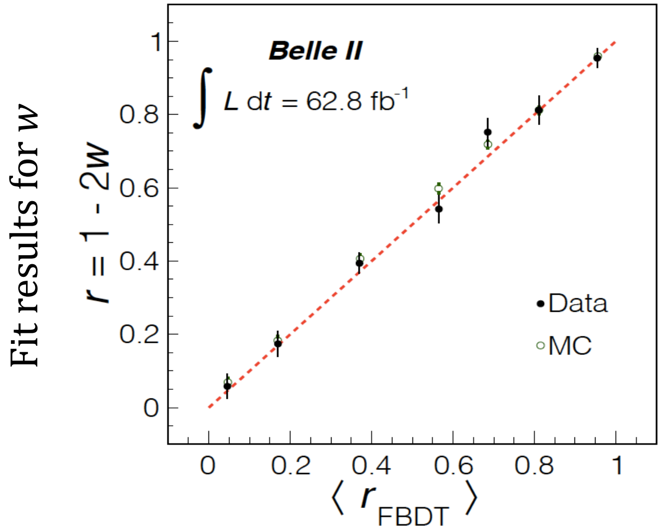
r: $0 \leq r \leq 1$
$r = 0$ for no flavor information
$r = 1$ for unambiguous flavor information

Wrong tagging probability $w = (1-r)/2$



qr distribution

Calibration



Averaged r, output from the tagger, in the bin

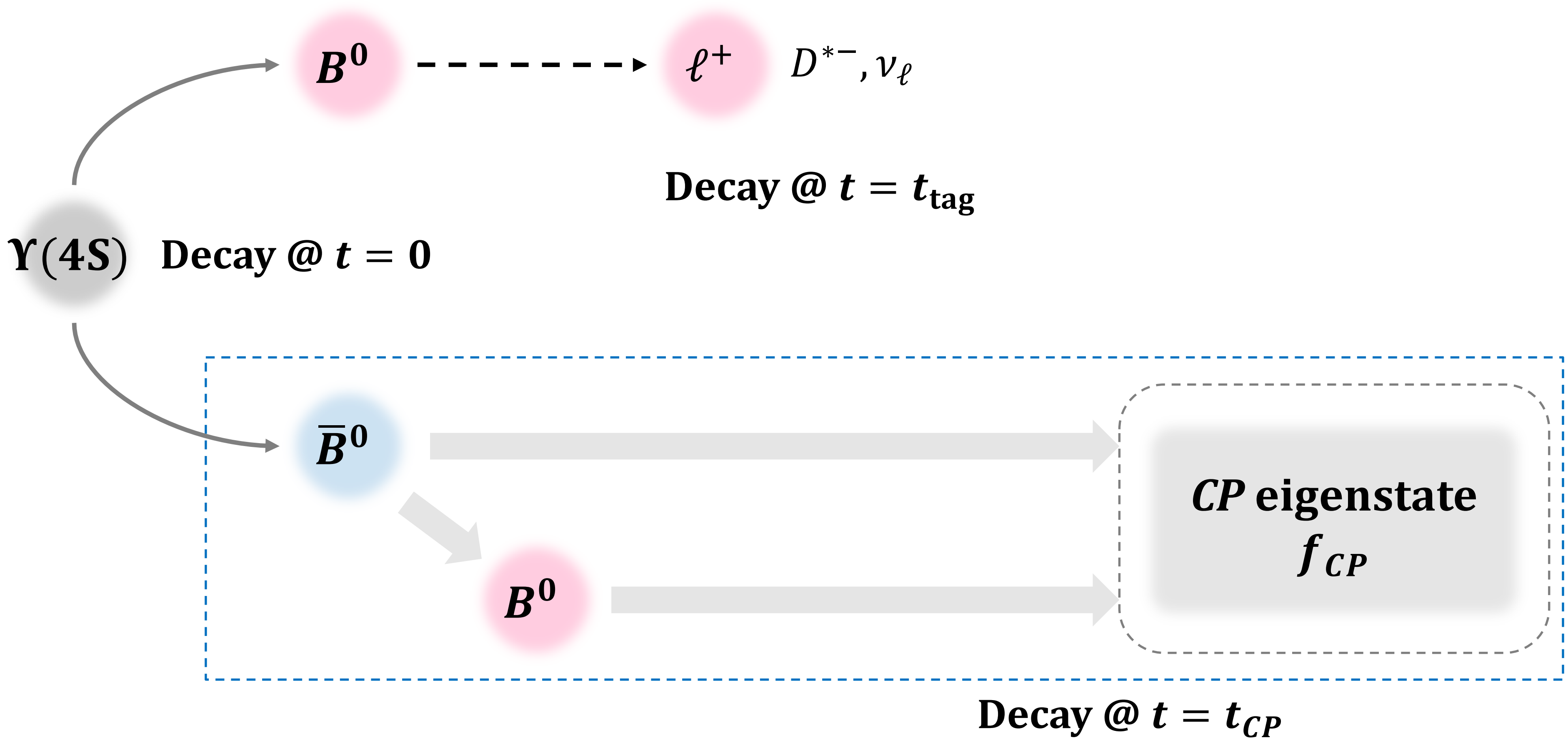
Effective tagging efficiency: ϵ_{eff}

$$\epsilon_{\mathrm{eff}} = \sum_{bin} \epsilon_i (1 - 2w_i)^2 = (33.8 \pm 3.6 \pm 1.6)\%$$

F. Abudinén *et al.* (Belle II), arXiv:2008.02707.

Effective efficiency ↑↑ when efficiency ↑
and wrong tagging probability ↓

B-Meson Decay Time



- It is known that, for the $B_{CP} \to J/\psi\, K_S^0$ case, $\Delta t \equiv t_{CP} - t_{\text{tag}}$ distributes in

$$\boxed{P(\Delta t) = \frac{1}{2} \exp\left[-\frac{|\Delta t|}{\tau_{B^0}}\right](1 \pm \sin 2\phi_1 \sin \Delta m_d \Delta t)}$$

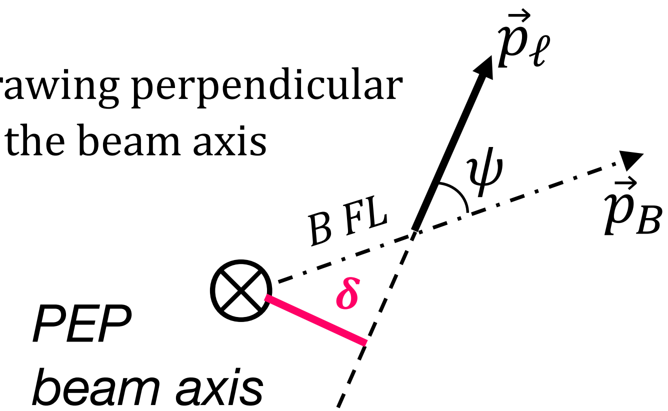
the sign is the same as the lepton charge form the B_{tag} decay.

Digression: *B* Lifetime (1983)

E. Fernandez *et al.* (MAC), Phys. Rev. Lett. **51**, 1022 (1983);
N. S. Lockyer *et al.* (Mark II), Phys. Rev. Lett. **51**, 1316 (1983).

- The *B* mesons are unstable particle → they decay with a finite lifetime.

Drawing perpendicular to the beam axis



Closest approach $\boldsymbol{\delta}$: the distance from the lepton track ℓ to the beam axis, called an "impact parameter"

$$\tau_B = \frac{\langle \delta \rangle}{\langle \beta\gamma \sin\psi \sin\theta \rangle c}$$

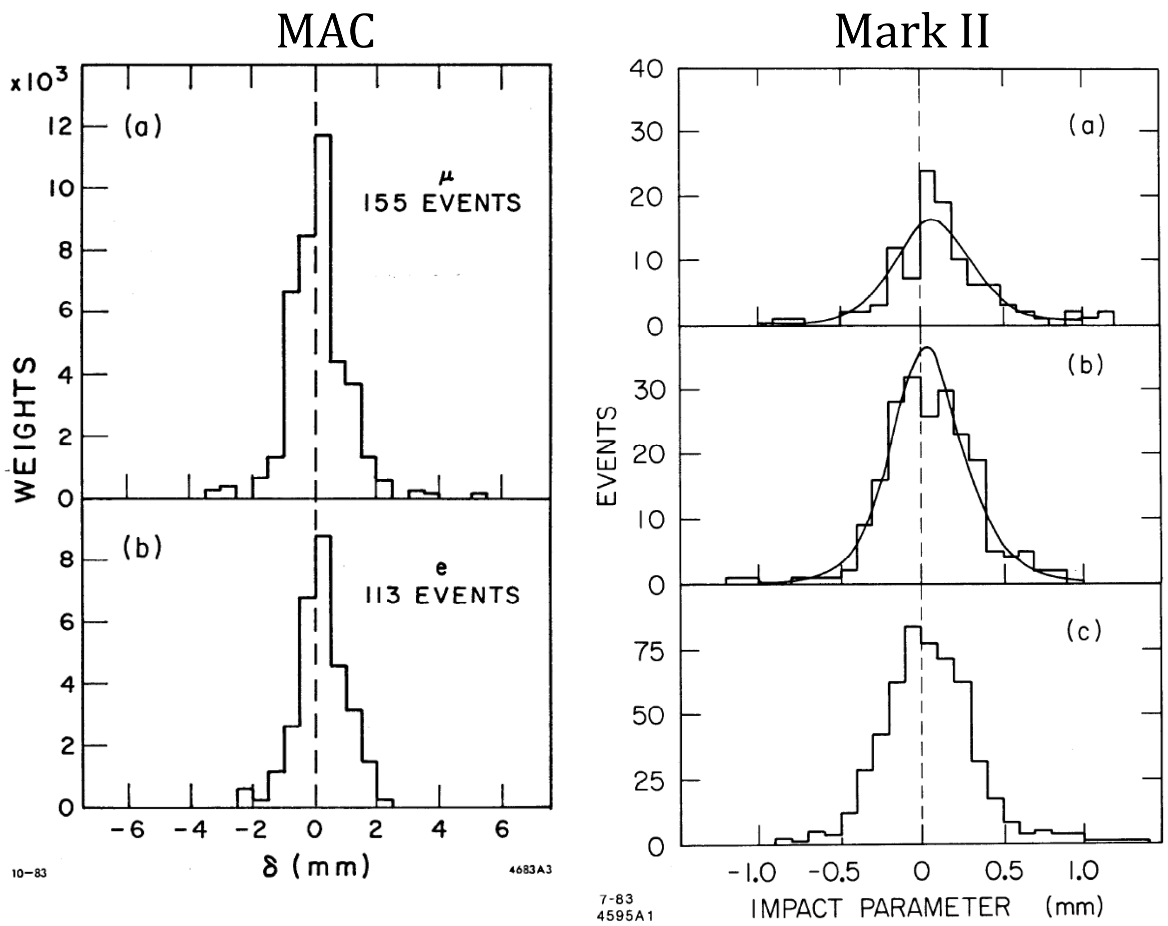
θ ... polar angle of the track not shown in the figure

$$\tau_B^{\text{MAC}} = (1.8 \pm 0.6 \pm 0.4)\ \text{ps}$$

$$\tau_B^{\text{Mark II}} = \left(1.2^{+0.45}_{-0.36} \pm 0.3\right)\ \text{ps}$$

$$\boldsymbol{\tau_{B^0}^{\text{PDG2020}} = 1.514 \pm 0.004\ \text{ps}}$$

PEP experiments @ SLAC

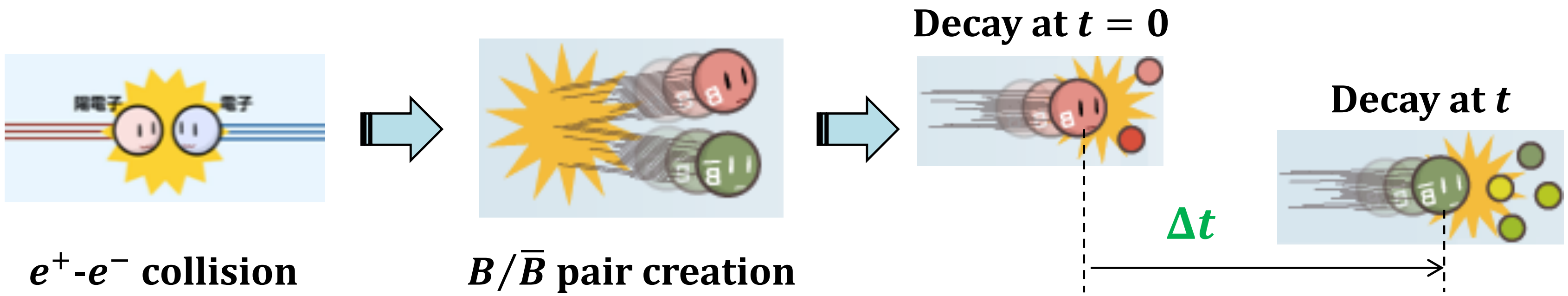


Decay Time Measurement

- $\langle \Delta t \rangle = \tau_B \approx 1.5$ ps. Measurement of a time of $\mathcal{O}(1 \text{ ps})$ is not easy.

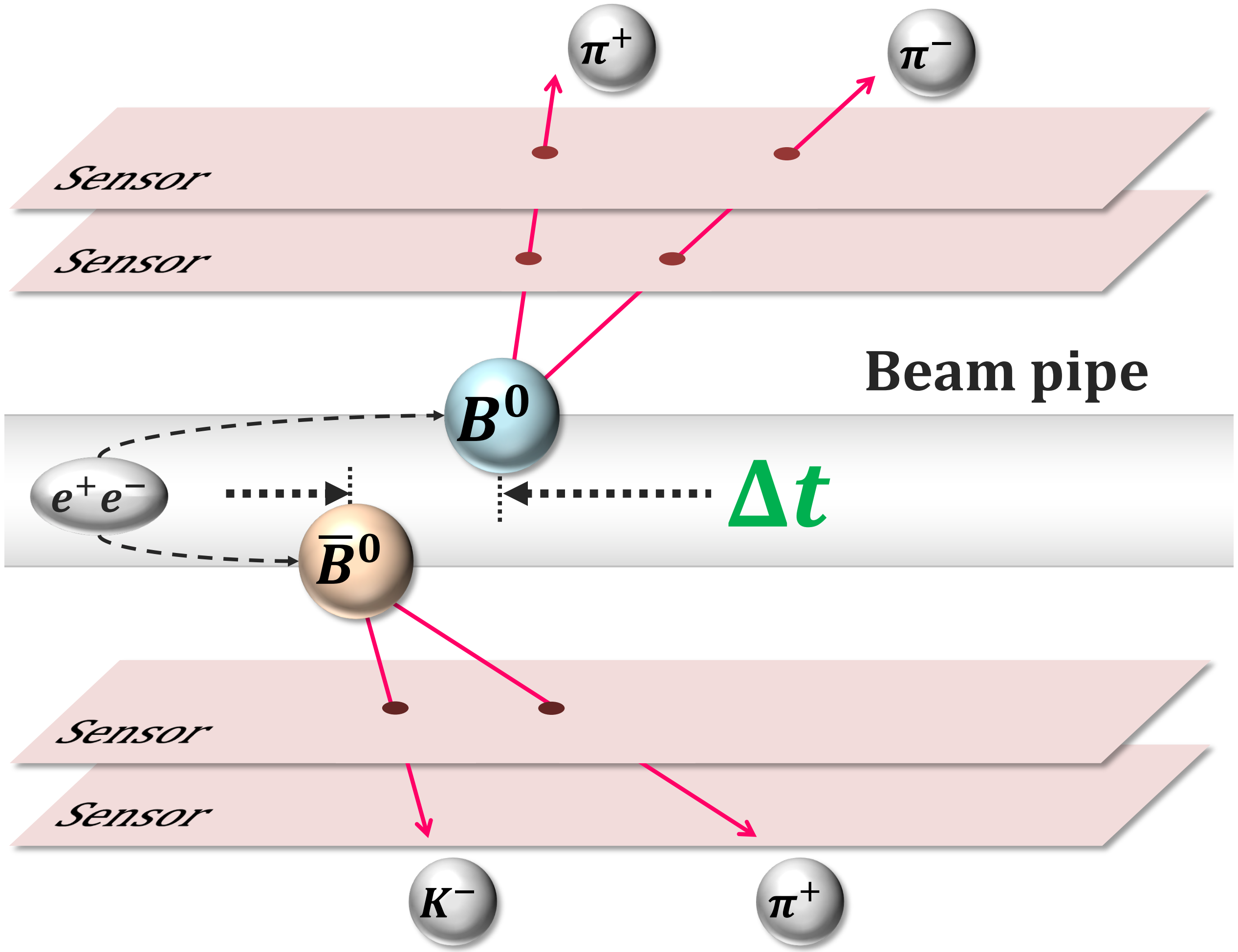
  Remember the average of an exponential distribution is its decay constant.

- The issue is solved by producing two mesons with a fixed momentum.



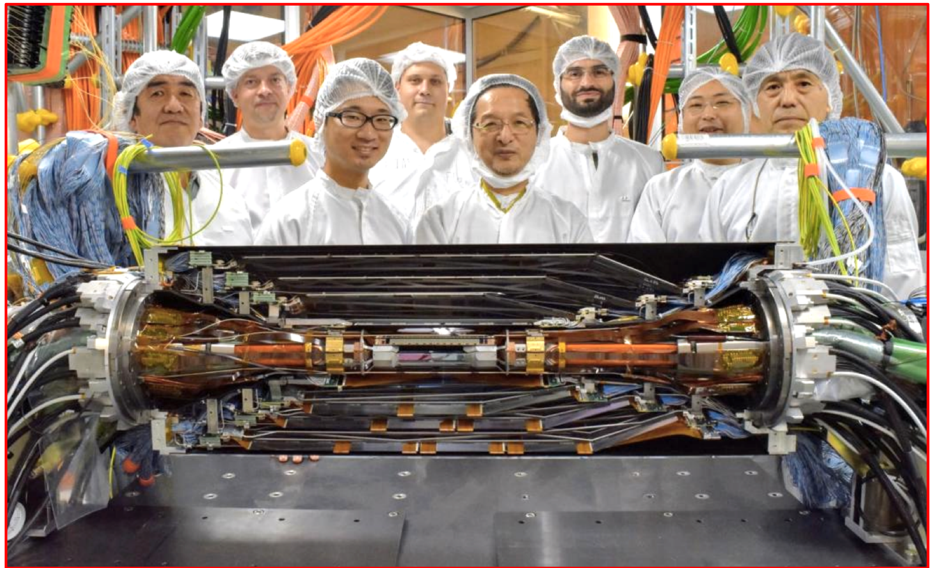
- At Belle II, B-mesons pairs are produced from a $\Upsilon(4S)$ decay created by a collision of 7.0 GeV e^- to 4.0 GeV e^+ provided by SuperKEKB.
- Luckily because $m_{\Upsilon(4S)} \approx m_B + m_{\bar{B}}$, the speed of the produced B meson is approximately the same as $\Upsilon(4S)$ that means $(\beta\gamma)_B \approx (\beta\gamma)_{\Upsilon(4S)}$.
- $\beta_{\Upsilon(4S)} = p_{\Upsilon(4S)}/E_{\Upsilon(4S)} \approx 3/11 = 0.27 \rightarrow (\beta\gamma)_{\Upsilon(4S)} = 0.28$.
- The typical distance of the two B-decay positions: $\langle \Delta z \rangle \approx (\beta\gamma)_B c \langle t \rangle \approx 130\mu\text{m}$. Measurement of a length of $\mathcal{O}(100\ \mu\text{m})$ is feasible.

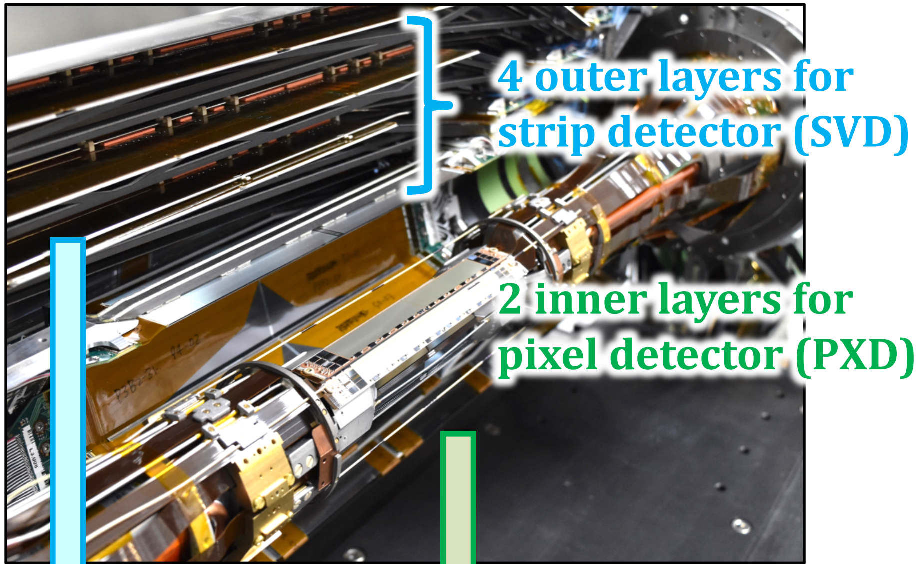
Decay Time Measurement

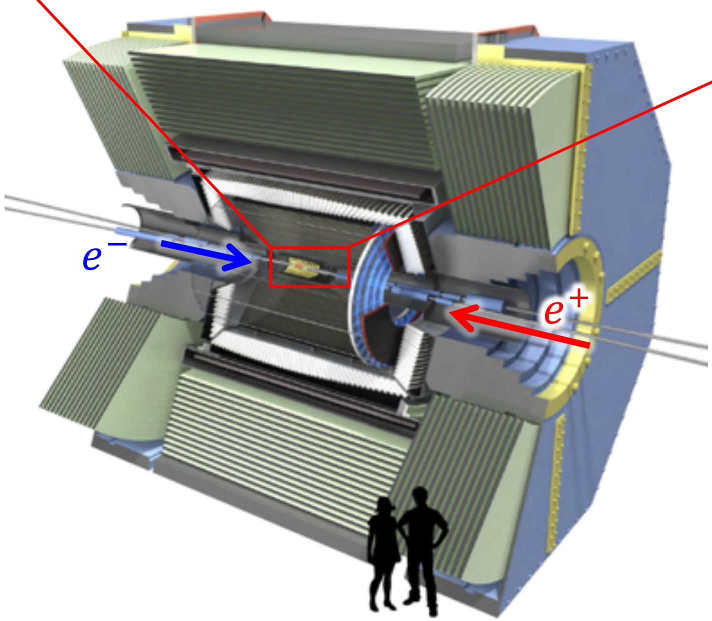


$$\Delta t \equiv t_{CP} - t_{\text{tag}} \approx (z_{CP} - z_{\text{tag}})/(\beta\gamma)_B c$$

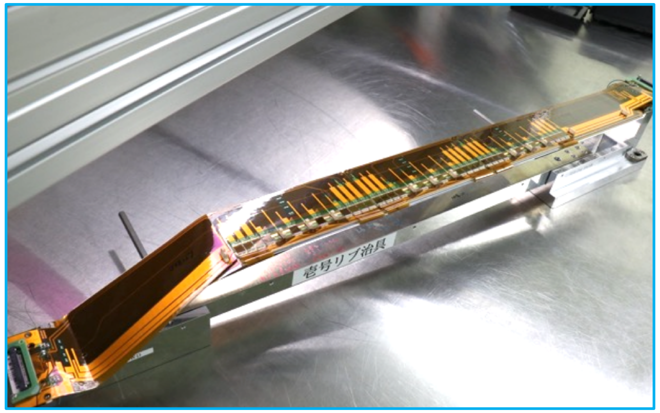
Belle II Vertex Detectors







45 cylindrically arranged SVD ladders



10 cylindrically arranged PXD ladders

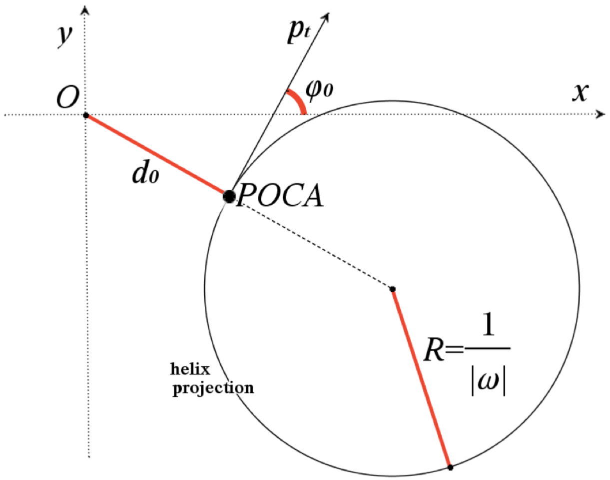
Determination of the *B*-Decay Position

- Charged particle trajectory in a magnetic field = **helix**

helix parameter $\equiv (d_0, \phi_0, \omega, z_0, \tan\lambda)$

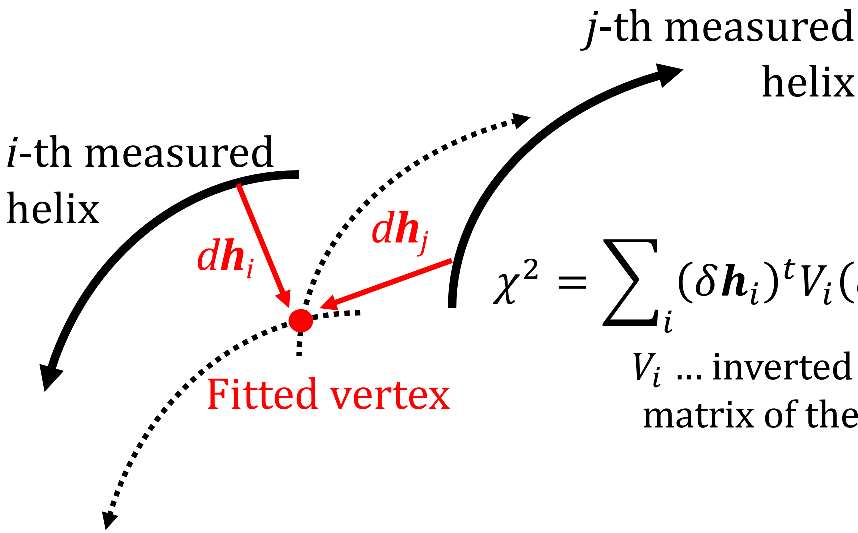
Belle II (BELLE2-NOTE-TE-2018-003)

$(x^P, y^P, z^P, p_x^P, p_y^P, p_z^P)$ at
POCA = Point of Closest Approach



$$
\begin{aligned}
x^P &= d_0 \sin\phi_0 & \quad p_x^P &= \cos\phi_0/\alpha\omega \\
y^P &= -d_0 \cos\phi_0 & \quad p_y^P &= \sin\phi_0/\alpha\omega \\
z^P &= z_0 & \quad p_z^P &= \tan\lambda/\alpha\omega
\end{aligned}
$$

- The decay position (called vertex) is determined wit the χ^2-minimizing method.



$$\chi^2 = \sum_i (\delta\boldsymbol{h}_i)^t V_i (\delta\boldsymbol{h}_i)$$

V_i ... inverted error matrix of the helix

The vertex that gives the minimum χ^2 is taken as the fitted vertex (KFit).
When the “IP constraint” is applied to KFit, $\chi^2 + \chi^2_{\mathrm{IP}}$ is minimized where χ^2_{IP} accounts for the IP spread.

Typical vertex resolution: $\delta z \approx$ **50 μm**

Determination of sin2ϕ_1

- The unbinned maximum-likelihood (ML) fit method us used.
- Suppose N events are recorded as signal candidates and $(\Delta t, q)$ is measured for each. The probability to obtain *the set* of the N events is

$$\mathcal{L}(\sin 2\phi_1) = \mathcal{P}(\Delta t_1, q_1; \sin 2\phi_1)\times\mathcal{P}(\Delta t_2, q_2; \sin 2\phi_1)\times \cdots\times\mathcal{P}(\Delta t_N, q_N; \sin 2\phi_1)$$

$$= \prod_i^N \mathcal{P}(\Delta t_i, q_i; \sin 2\phi_1)$$

Extended ML fit is out of the scope of the lecture.

- Because $\mathcal{L}(\sin 2\phi_1)$ tends to be very small for computers, the log likelihood is commonly used.

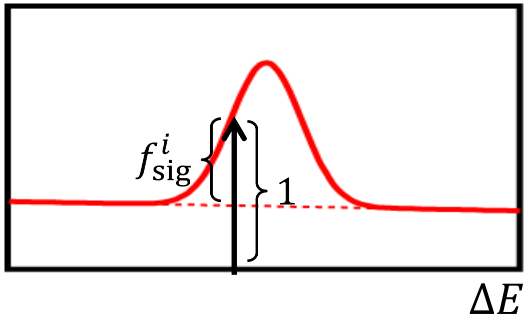
$$\ln \mathcal{L}(\sin 2\phi_1) = \sum_i^N \ln \mathcal{P}(\Delta t_i, q_i; \sin 2\phi_1)$$

- Take the $\sin 2\phi_1$ value that maximizes $\mathcal{L}(\sin 2\phi_1)$ as the the estimated $\sin 2\phi_1$ value.

Determination of $\sin 2\phi_1$

$$P(\Delta t) = \frac{1}{2}\exp\left[-\frac{|\Delta t|}{\tau_{B^0}}\right](1 \pm \sin 2\phi_1 \sin \Delta m_d \Delta t)$$

$$\boldsymbol{\mathcal{P}(\Delta t_i, q_i; \sin 2\phi_1)} = + \begin{cases} f_{\mathrm{sig}}^i \times \mathcal{P}_{\mathrm{sig}}(\Delta t_i, q_i; \sin 2\phi_1) = P(\Delta t_i, q_i; \sin 2\phi_1) \otimes R(\Delta t) \\ (1 - f_{\mathrm{sig}}^i) \times \mathcal{P}_{\mathrm{sig}}(\Delta t_i, q_i; \sin 2\phi_1) = \cdots \end{cases}$$



Empirical distribution accounting for apparatus limitations

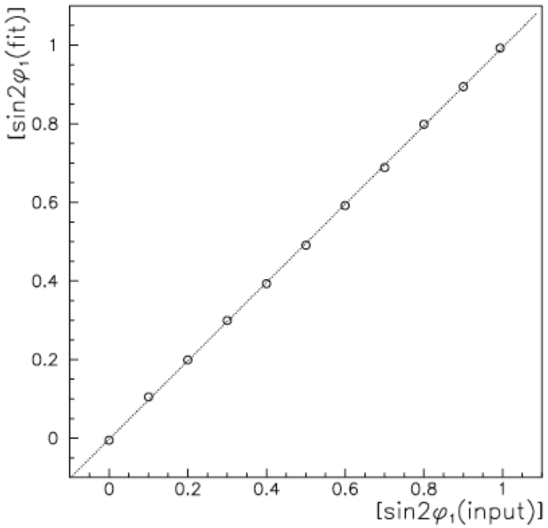
- Detector resolution
- $b \to c$ effect to B_{tag} decay vertex reconstruction
- $\Delta t \approx \Delta z / \beta\gamma c$ approximation

Empirical distributions that model the background Δt distribution well

- Typically determined from MC samples or
- Sideband events ($\Delta E, m_{BC}$, ...)

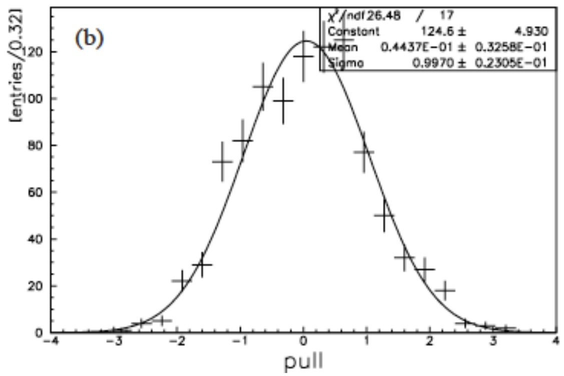
Determination of $\sin 2\phi_1$

Linearity test



Pull distribution test

- Generate a number of "experiments" and perform the $\sin 2\phi_1$ fit for each experiment.



- Check the $p \equiv (s_{\text{fit}} - s_{\text{gen}})/\sigma_s$ distribution.
- If the analysis is healthy, it distributes in the standard normal Gaussian.

Null asymmetry test

- Perform the $\sin 2\phi_1$ fit to flavor specific B-decay samples (real data).
- If the analysis is healthy, the fitted $\sin 2\phi_1$ value must be consistent with zero.

Systematic error estimation

- The major sources to the $\sin 2\phi_1$ systematic error are:
  - Vertex reconstruction procedure
  - $R(\Delta t)$ modelling and parameters
  - Wrong tagging probability
  - Tag side interference

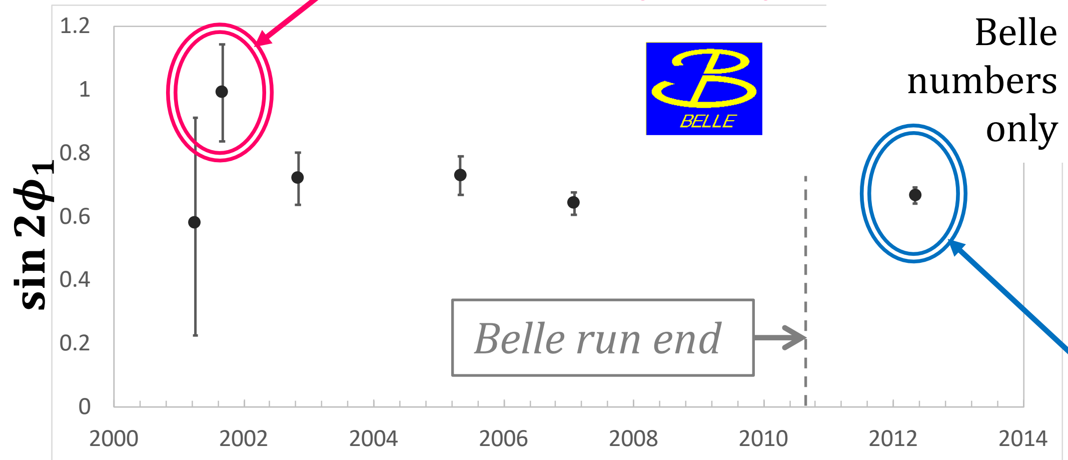
Time Propagation of the $\sin 2\phi_1$ Value

Belle Collab., Phys. Rev. Lett. **87**, 091802 (2001).

Belle 29 $MB\bar{B}$)
$\sin 2\phi_1 = +0.99 \pm 0.14 \pm 0.06$

Belle Collab., Phys. Rev. Lett. **108**, 171802 (2012).

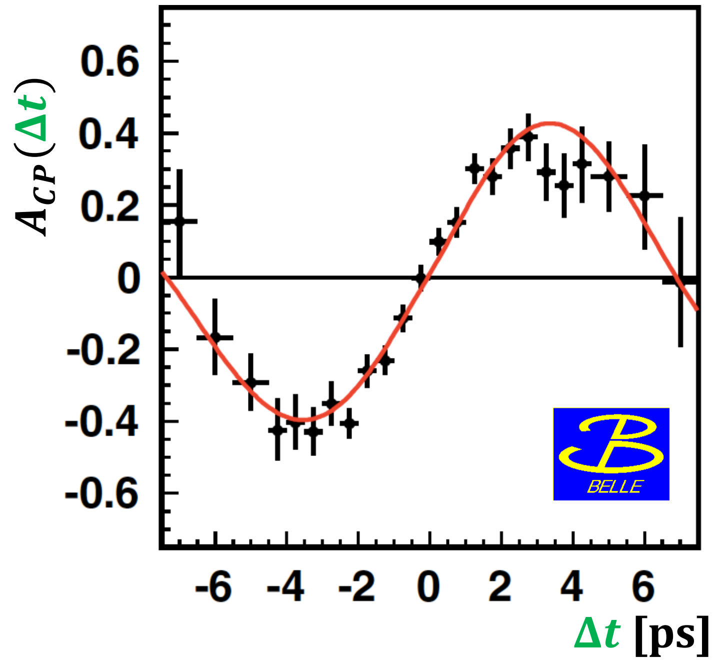
We were very lucky.



Belle numbers only

→ year

Belle final (772 $MB\bar{B}$)

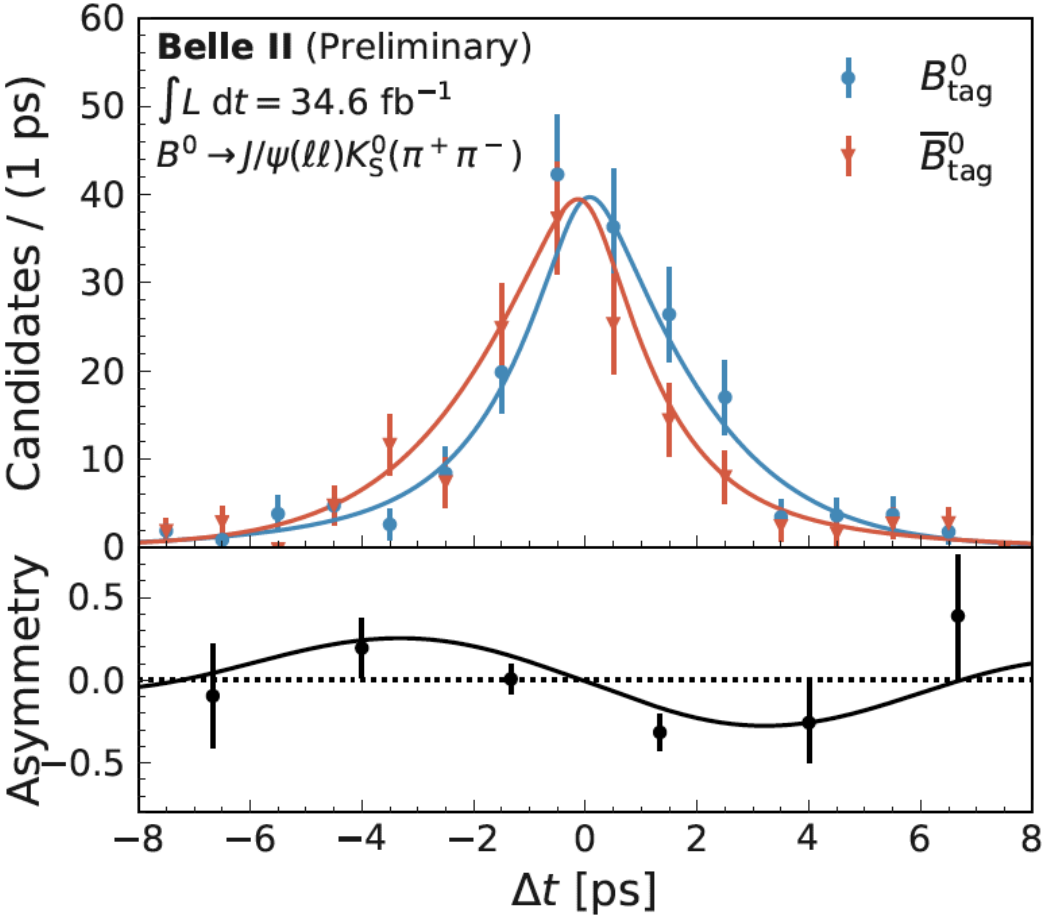


$\sin 2\phi_1 = +0.667 \pm 0.023 \pm 0.012$

$(\sin 2\phi_1)_{\mathrm{WA}} = +0.699 \pm 0.017$

$\phi_1 = (22.2 \pm 0.7)^\circ$ (HFLAV 2021)

Measurement of sin2ϕ_1 at Belle II



Prompt measurements of time-dependent *CP*-violation and mixing (BELLE2-NOTE-PL-2020-011)

(Preliminary)

$$\mathbf{\sin 2\phi_1 = 0.55 \pm 0.21 \pm 0.04}$$

Consistent with the HFLAV value
$(\sin 2\phi_1)_{\mathrm{WA}} = +0.699 \pm 0.017$

Yesterday's discovery is today's calibration

R. Feynman

... and tomorrow's background!

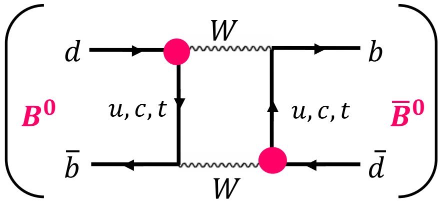
V. Telegdi

The full analysis is ongoing.

NP Search in the Mixing-Induced *CP* Violation

Phase difference from decay

$\boldsymbol{B^0}$

arg 

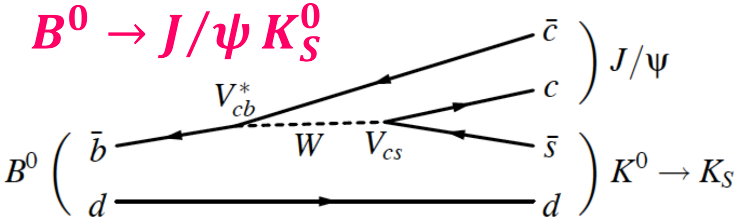
$\boldsymbol{\bar{B}^0}$

Phase difference from decay

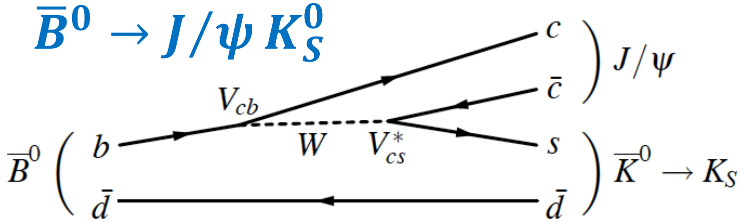
***CP* eigenstate**
$\boldsymbol{f_{CP}}$

Phase difference from the mixing $\pm 2\phi_1$

$\boldsymbol{B^0 \to J/\psi\, K_S^0}$



$\boldsymbol{\bar{B}^0 \to J/\psi\, K_S^0}$



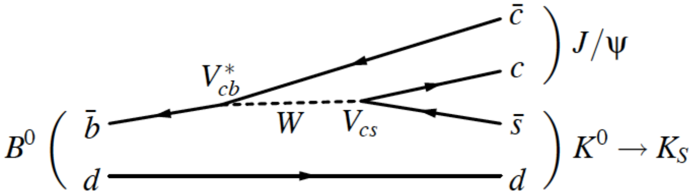
$$\arg\left(B^0 \to J/\psi\, K_S^0\right) = \arg(V_{cb}^* V_{cs}) = 0$$

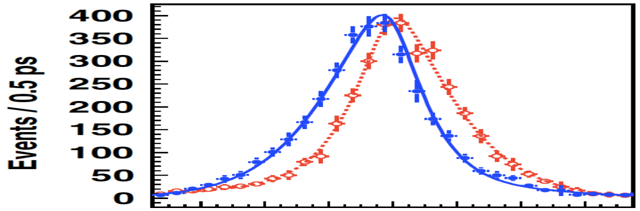
$$\arg\left(\bar{B}^0 \to J/\psi\, K_S^0\right) = \arg(V_{cb} V_{cs}) = 0$$

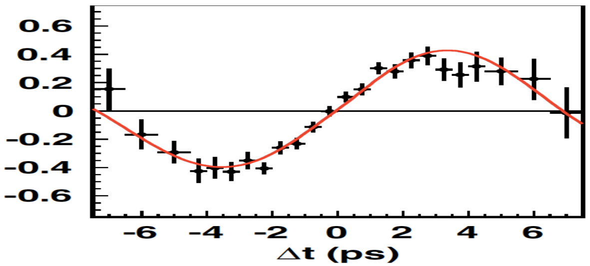
Variety of $\sin 2\phi_1^{\mathrm{eff}}$ measurement becomes available by replacing the $b \to c\bar{c}s$ with other transitions

CPV in the $b \to c\bar{c}q$ Family

$b \to c\bar{c}s\ (B \to J/\psi\, K^0)$ … reference

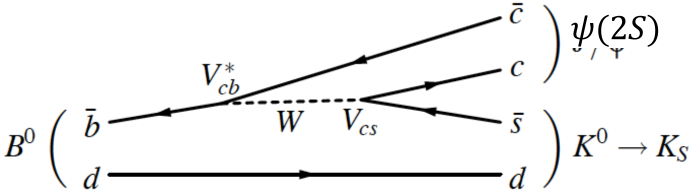






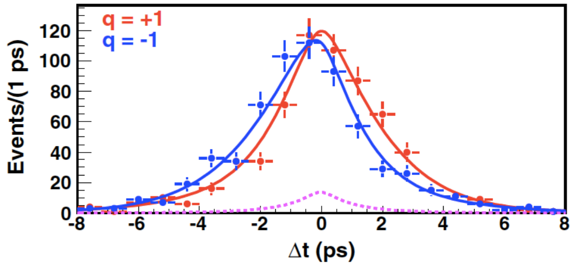
$S_{J/\psi K_S^0} = \sin 2\phi_1 = 0.699 \pm 0.017$ … reference

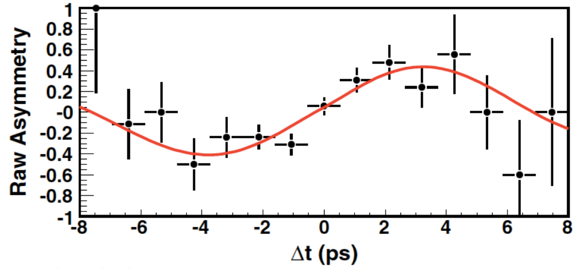
$b \to c\bar{c}s\ (B \to \psi(2S)K^0)$



Same diagram as $B \to J/\psi\, K^0$
$\Rightarrow$ same $\sin 2\phi_1$?

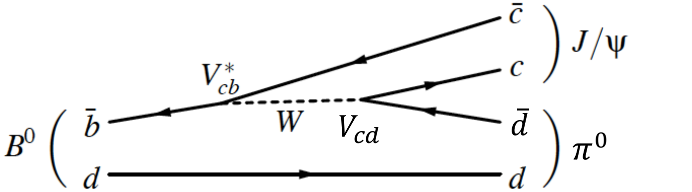
Phys. Rev **D77**, 091103(R) (2008).





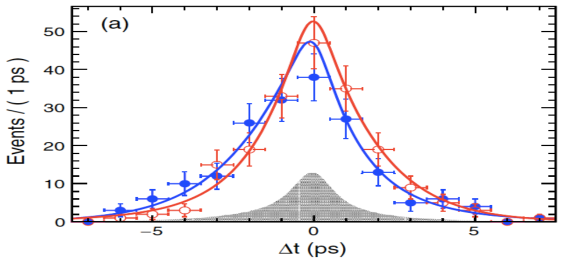
$S_{\psi(2S)K_S^0} = 0.72 \pm 0.09 \pm 0.03$ … same, no surprise

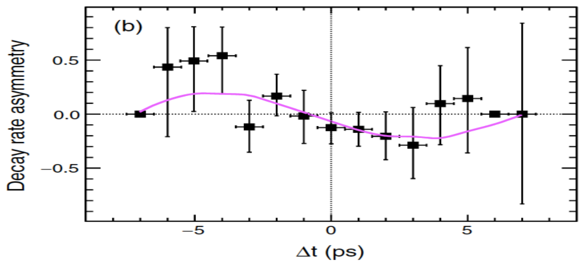
$b \to c\bar{c}d\ (B \to J/\psi\, \pi^0)$



$(b \to c\bar{c}s)\times(V_{cd}/V_{cs})$, $\arg(V_{cd}/V_{cs}) = 0$
$\Rightarrow$ same $\sin 2\phi_1$?

Phys. Rev **D98**, 112008 (2018).

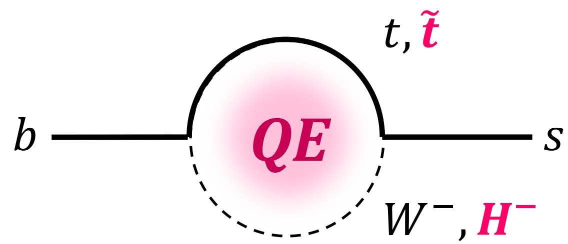




$S_{J/\psi\pi^0} = -0.59 \pm 0.19 \pm 0.03$ … same, no surprise

E. Kou, P. Urquijo *et al.*,
Prog. Theor. Exp. Phys.
2019, 123C01 (2019).

CPV in the $b \to sq\bar{q}$ Family



- New heavy particles ($\tilde{t}, H^{\pm}$, ...), which are expected to be heavier than $m_{NP} \gtrsim \mathcal{O}(1)$ TeV$/c^2$ can appear in the $b \to s$ loop thanks to the quantum effect.
- $\mathcal{S}_{sq\bar{q}} = \sin 2\phi_1^{sq\bar{q}} \approx \mathcal{S}_{c\bar{c}s} = \sin 2\phi_1$ in the SM because the diagram includes neither V_{td} nor V_{ub}. Observation of $\Delta\mathcal{S} \equiv \mathcal{S}_{sq\bar{q}} - \mathcal{S}_{c\bar{c}s} \neq 0 \to$ discovery of the NP.
- **Golden mode of NP search =** $B^0 \to \eta' K_S^0$ for its theoretically accurate prediction of $\Delta\mathcal{S}$: $\Delta\mathcal{S}^{\text{theo}} \approx [0.00, 0.03]$. arXiv:hep-ph/0505075

Belle, JHEP **1410**, 165 (2014).

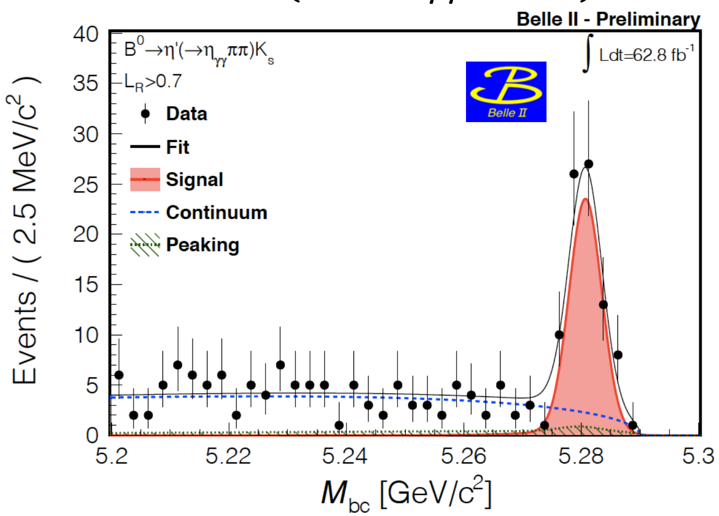
$$\boxed{\textbf{Belle: } \mathcal{S}_{\eta' K^0} = 0.68 \pm 0.07 \pm 0.03}$$

- The Belle II analysis is ongoing. We expect $\sigma(\Delta\mathcal{S}^{\text{exp}}) \approx 0.02$ when the full Belle II dataset is used. This corresponds to a sensitivity of $m_{\text{NP}} \approx \mathcal{O}(100)$ TeV$/c^2$ particles.

Preliminary; BELLE2-CONF-PH-2021-005.

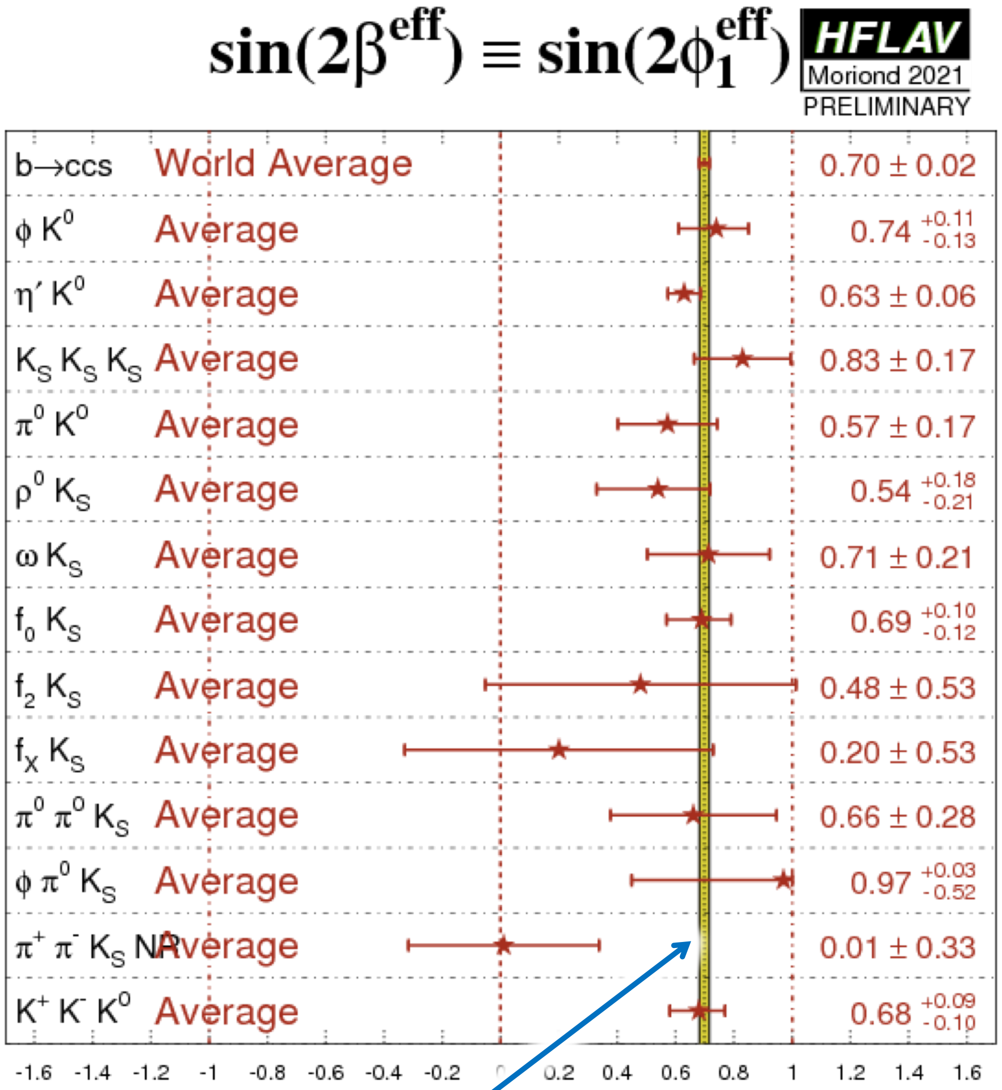
$$\boxed{\mathcal{B}(B^0 \to \eta' K^0) = [59.9^{+5.8}_{-5.5} \pm 2.7] \times 10^{-6}}$$

$\eta' K_S^0$ $(\eta' \to \eta_{\gamma\gamma}\pi^+\pi^-)$



CPV in the $b \to sq\bar{q}$ Family

- More $b \to sq\bar{q}$ modes. They are waiting for you.

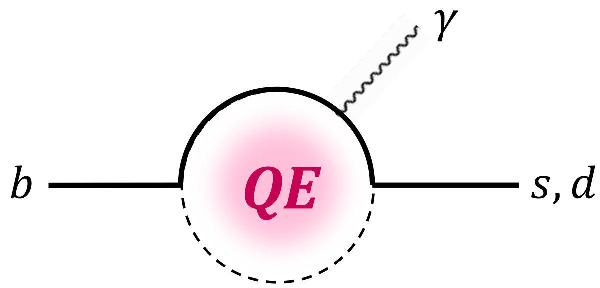


$$(\sin 2\phi_1)_{c\bar{c}s} = 0.699 \pm 0.017$$

CPV in the $b \to sq\bar{q}$ Family

- $K^{*0}\gamma$ is not a complete *CP* eigenstate because the photons from $B^0(\bar{B}^0) \to K^{*0}\gamma$ are predominantly RH'ed (LH'ed).
- The *CP*-violating parameter $\mathcal{S}_{K^{*0}\gamma}$ is suppressed by $\mathcal{S}_{K^{*0}\gamma} \approx -(2m_s/m_b)\times \sin 2\phi_1 = -(2.3 \pm 1.6)\%$.

P. Ball, G. W. Jones, and R. Zwicky Phys.Rev.D**75**, 054004 (2007).



- The $\mathcal{S}_{K^{*0}\gamma}$ value may deviate from the SM prediction if the new particle couples with a RH'ed fermion.

Belle, Phys. Rev. D **74**, 111104(R) (2006).

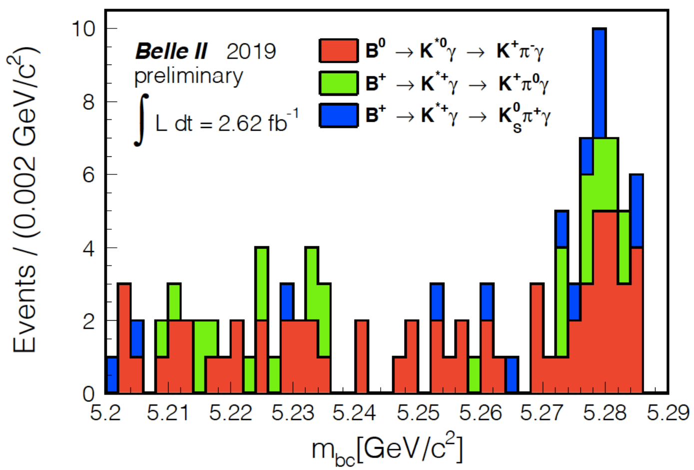
Belle: $\boldsymbol{\mathcal{S}_{K^{*0}\gamma} = -0.10 \pm 0.31 \pm 0.07}$

$\boldsymbol{\mathcal{A}_{K^{*0}\gamma} = -0.20 \pm 0.20 \pm 0.06}$

- The Belle II analysis is ongoing. We expect $\sigma(\mathcal{S}_{K^{*0}\gamma}) \approx 0.031$ and $\sigma(\mathcal{A}_{K^{*0}\gamma})$ when the full Belle II dataset is used.

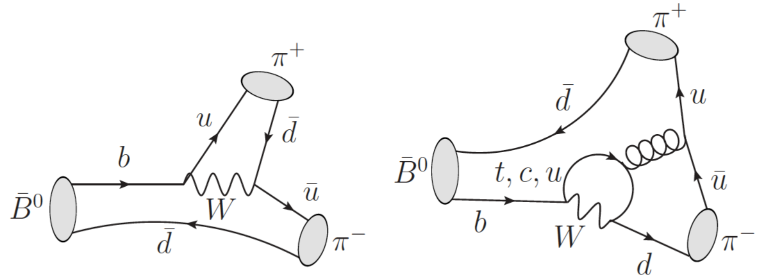
E. Kou, P. Urquijo *et al.*, Prog. Theor. Exp. Phys. 2019, 123C01 (2019).

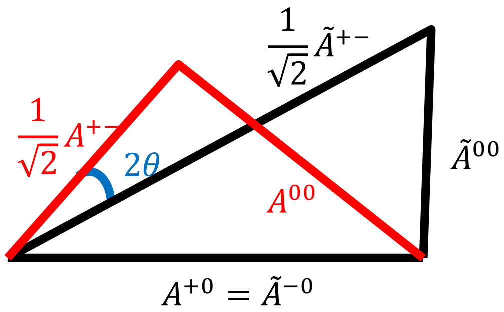
BELLE2-NOTE-PL-2019-021



Measurement of ϕ_2

- ϕ_2 is measured with the $b \to u\bar{u}d$ transition.
- In the ϕ_2 case, the penguin (right) contribution is not negligible compared to the tree (left); we determine ϕ_2 by combining 6 $b \to u\bar{u}d$ B decays.





$A^{+-} \equiv \mathrm{Amp}(B \to \pi^+\pi^-); \tilde{A}^{+-} \equiv \mathrm{Amp}(\bar{B} \to \pi^+\pi^-);$
$A^{+0} \equiv \mathrm{Amp}(B \to \pi^+\pi^0);\ \tilde{A}^{-0} \equiv \mathrm{Amp}(\bar{B} \to \pi^-\pi^0);$
$A^{00} \equiv \mathrm{Amp}(B \to \pi^0\pi^0);\ \ \tilde{A}^{00} \equiv \mathrm{Amp}(\bar{B} \to \pi^0\pi^0).$

$$\mathcal{S}_{\pi\pi} = \sqrt{1 - \mathcal{A}_{\pi\pi}^2}\,\sin(2\boldsymbol{\phi_2} + \boldsymbol{2\theta})$$

$\mathcal{S}_{\pi\pi}$ and $\mathcal{A}_{\pi\pi}$ are obtained with the TD method.

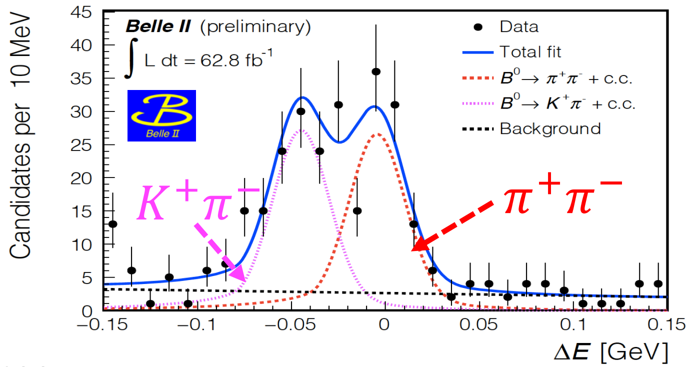
Belle, Phys. Rev. D **88**, 092003 (2013).

$$\textbf{Belle:}\ \boldsymbol{\mathcal{S}_{\pi^+\pi^-} = -0.64 \pm 0.08 \pm 0.03}\ etc$$
$$\boldsymbol{\mathcal{A}_{\pi^+\pi^-} = +0.33 \pm 0.06 \pm 0.03}\ etc$$

- The Belle II analysis is ongoing. We expect $\sigma(\phi_2) \approx 0.6°$ when the full Belle II dataset is used.

E. Kou, P. Urquijo *et al.*, Prog. Theor. Exp. Phys. 2019, 123C01 (2019).

$\boldsymbol{B^0 \to \pi^+\pi^-}$



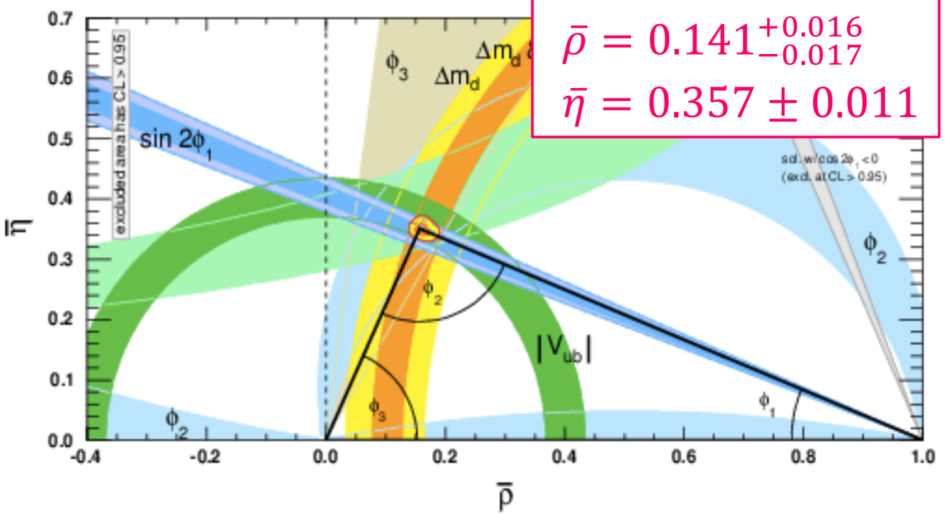
Preliminary; arXiv:2106.03766.

Preliminary

$$\boldsymbol{\mathcal{B}(B^+ \to \pi^+\pi^0) = [5.5^{+1.0}_{-0.9} \pm 0.7]\times 10^{-6}}$$
$$\boldsymbol{\mathcal{B}(B^0 \to \pi^+\pi^-) = [5.8 \pm 0.7 \pm 0.3]\times 10^{-6}}$$

CKM Triangle – Current Status

CKMfitter 2019, PDG2020



Apex position

$\bar{\rho} = 0.141^{+0.016}_{-0.017}$

$\bar{\eta} = 0.357 \pm 0.011$

$$\phi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = (22.56^{+0.47}_{-0.40})^\circ$$

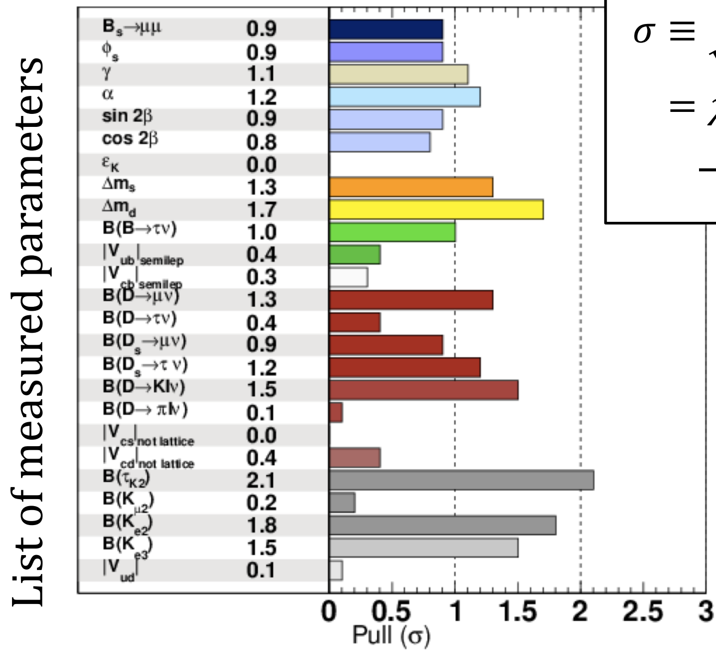
$$\phi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = (91.7^{+1.7}_{-1.1})^\circ$$

$$\phi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = (65.8^{+0.94}_{-1.29})^\circ$$

$$|V_{cb}| = 0.04162^{+0.00026}_{-0.00080}$$

$$|V_{ub}| = 0.003683^{+0.000075}_{-0.000061}$$

PDG 2020



$$\sigma \equiv \sqrt{\chi^2_{x;\mathrm{min}}} = \chi^2_{\text{with data on } X} - \chi^2_{\text{without data on } X}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985 \pm 0.0005$$

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.025 \pm 0.020$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{tb}|^2 = 0.9970 \pm 0.0018$$

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1.026 \pm 0.022$$

The unitarity of the CKM matrix holds surprisingly well (except the first relation).

Summary

- **The time-dependent analysis has been introduced.**
- **Key ingredients of the time-dependent analysis**:
  event reconstruction, flavor tagging, vertex fitting, and
  maximum likelihood fitting.
- **Time-dependent analysis an essential method**:
  in Belle, when we verified the KM theory,
  in Belle II, when we search for new physics.

An extra lecture (mainly on mathematics) is planned tomorrow morning (CET).